Capital Structure and ESG Integration: A Security-Design Approach

Dongkyu Chang Keeyoung Rhee Aaron Yoon*

January 13, 2026

Abstract

We analyze how borrowers' capital structure affects their incentives to integrate ESG. Borrowers may undertake socially valuable but financially underperforming projects when doing so lowers expected repayments to outside investors. This repayment saving is larger under more repayment-sensitive securities – contracts whose payoffs respond more strongly to project profitability. However, investor competition endogenously reduces contract sensitivity, undermining ESG incentives. Financially motivated borrowers therefore prefer debt, a possible explanation for the growing trend of debt in green financing. Under asymmetric information, ESG-oriented borrowers can credibly signal their type by issuing equity when some investors are *non-consequentialists*, who willingly incur monetary losses for direct green financing.

Keywords: capital structure, ESG, greenwashing, moral hazard, security design

JEL Codes: D82, D86, G32, G38

*Chang: City University of Hong Kong (email: donchang@cityu.edu.hk); Rhee: Sungkyunkwan University (SKKU) (email: ky.rhee829@gmail.com); Yoon: Kellogg School of Management at Northwestern University (email: aaron.yoon@kellogg.northwestern.edu). The authors are grateful to Pierre Chaigneau, Briana Chang, Yeon-Koo Che, Dong Beom Choi, Jaewon Choi, Lin William Cong, Ronald A. Dye, Jeffrey Ely, Pingyang Gao, Jungsuk Han, Sunwoo Hwang, Jeong Ho (John) Kim, Kyungmin (Teddy) Kim, Soohun Kim, Robert Korajczyk, Jangwoo Lee, Jongsub Lee, Alessandro Lizzeri, Andrey Malenko, Nadya Malenko, Julien Manili, Ji Yeol Jimmy Oh, Joel Shapiro, Kevin Smith, Swaminathan Sridharan, Jan Starmans, and Shixiang Xia as well as participants at the 2024 Wellington Finance Summit, 2024 Econometric Society Australasian Meeting, the 10th NTHU-UNSW-SMU Symposium on Sustainable Finance and Economics, the 14th Accounting Research Workshop, the 2nd Annual Conference of the Accounting and Economics Society, 2025 Junior Accounting Theory Conference, 2025 World Congress of the Econometric Society, the 2nd Asian Finance Theory Group Conference, and seminars at Academia Sinica, Chinese University of Hong Kong, National Taiwan University, National Tsing Hua University, and University of Hong Kong for their helpful comments and suggestions. This paper was supported by SKKU Academic Research Support Program, Sungkyunkwan University, 2024.

1 Introduction

Over the past decade, environmental, social, and governance (ESG) investing has seen substantial growth in interest and capital inflows. However, it has recently faced increasing skepticism and political pushback, particularly in the United States (Bloomberg, 2024). A major concern fueling this backlash is the prevalence of greenwashing by both corporate issuers and investors (Reuters, 2020; SEC, 2022). In particular, it remains difficult to verify whether firms that receive ESG-designated capital subsequently integrate ESG principles into their management and investment decisions as promised. This difficulty is compounded by several structural factors: ESG is an amorphous concept (i.e., not clearly defined); ESG metrics tend to be complex and opaque; and objective and verifiable information about firms' ESG performance remains scarce (Berg, Koelbel and Rigobon, 2022; Serafeim and Yoon, 2023; Gipper, Ross and Shi, 2022).

Amid growing skepticism about the effectiveness of ESG investing, one segment of the market has continued to expand rapidly: ESG-linked debt. The premise behind using debt for sustainable finance is the market belief that ESG factors directly affect default risk and borrower performance (Chatterji, Levine and Toffel, 2021; OECD, 2021; S&P Global, 2023). Furthermore, the nature of debt contracts – narrowing lenders' focus to borrowers' ability to meet fixed obligations – makes ESG-linked debt particularly appealing to investors seeking measurable outcomes (Preqin, 2025). This perception has fueled the growth of green bonds, sustainability-linked loans, and ESG-tied private credit facilities. Market data show that ESG debt issuance has exceeded $1 trillion annually since 2021 and is projected to surpass $7 trillion in cumulative volume by 2025 (BloombergNEF, 2025), establishing ESG debt as a major pillar of sustainable finance. However, it is not clear whether a borrower's ESG performance is systematically linked to the repayment capacity or credit fundamentals. Moreover, the limited availability of accurate and verifiable ESG metrics makes it difficult to design credible performance-based incentives – such as interest rate discounts or penalties in sustainability-linked loans – without inviting further greenwashing.

These challenges motivate several important research questions. When ESG follow-through is difficult to monitor and cannot be cleanly contracted on, can standard debt-based financing discipline firms to honor ESG commitments? If not, which security structures can credibly align incentives, and what kinds of market environments foster (or hinder) the adoption of such instruments? To address these questions, this paper develops a security-design framework that isolates a key contract feature – how expected repayments vary with the

firm's ESG choice (i.e., the contract's sensitivity) – through which capital structure shapes incentives for ESG integration and follow-through. Because ESG follow-through is hard to contract on, the relevant margin is how the contract's cash-flow mapping endogenously shapes the borrower's *ex-post* choice.

In the model, a borrowing firm raises external capital to finance a new investment project, while outside investors compete by offering securities that specify repayment schedules contingent on realized cash flows. We distinguish between two investor types: green investors, who value the social return generated by the firm's ESG follow-through, and brown investors, who care only about financial payoffs. The firm chooses between a *green* project that generates positive social value but is financially less attractive, and a *brown* project that is financially more attractive but generates no social value. The key friction is that the firm's project choice is non-contractible, capturing the practical difficulty investors face in verifying whether firms genuinely implement their ESG commitments (Liang, Sun and Teo, 2022; Kim and Yoon, 2023).

The firm's incentive to pursue green investment depends on how the financing instrument maps realized cash flows into repayments. Even though the green project is financially underperforming, "doing ESG" can be a privately attractive choice because the green project also lowers expected repayment to investors. Accordingly, the firm is more likely to select the green project when the investors' claim is sufficiently *sensitive* to the project choice – that is, when the repayment reduction from green investment is large. For instance, equity-like contracts are more sensitive because their payoffs move closely with cash flows, whereas debt-like contracts are relatively insensitive because repayments are largely fixed at face value. Moreover, when investors hold a larger claim on cash flows, the contract becomes more repayment-sensitive, strengthening the firm's repayment-savings motive and making the green project a more attractive choice. Finally, the sensitivity threshold required to induce green investment declines with the firm's intrinsic valuation of social returns, so more ESG-oriented firms follow through even under contracts with relatively modest repayment sensitivity.

Building on this mechanism, we show that investor competition generates a non-monotonic relation between a borrower's intrinsic social preference and ESG adoption. When the firm is anticipated to choose the financially attractive brown project under its weak social preference, brown investors can win the competition by offering the contracts under which they retain only a small repayment claim while remaining financially viable under the brown project. Such concessionary terms compress repayment sensitivity, weaken the financial consequences of choosing the green project, and reinforce incentives for brown investment. At the

other extreme, when the firm is anticipated to choose the financially unattractive green project due to its strong intrinsic social preference, brown investors must demand a large repayment claim on cash flows to compensate for the project's lower profitability. Brown investors' demand for a large claim on cash flows pushes equilibrium contracts toward greater repayment sensitivity, which in turn further reinforces ESG follow-through. Once green investment is financed by brown investors, green investors can free-ride on the social return as a positive externality without directly funding the firm.

When the firm's social preference lies in an intermediate range, it would choose the brown project absent additional financial support, so inducing green investment requires green investors to subsidize the firm. Crucially, because project choice is non-contractible, such a subsidy is effective only if it is embedded in a contract that is sufficiently sensitive so that green investment materially lowers the firm's expected repayment. However, only the green investors are willing to subsidize the firm because they internalize the social return generated by green investment. Green investors can implement the subsidy by offering concessionary repayment terms under which they retain only a small repayment claim, which allows them to outbid brown investors who are unwilling to make loss-making offers. At the same time, green investors must ensure that the winning contract remains highly sensitive – through equity-like contracts – so that the firm indeed follows through on the green project *ex post*.

A central implication is that competition among investors with heterogeneous social preferences can undermine ESG integration by endogenously compressing contract sensitivity. As investors compete more aggressively to win funding, green investors must offer increasingly favorable terms – retaining a smaller repayment claim on cash flow – which reduces repayment sensitivity. As sensitivity substantially decreases, the firm may become indifferent and partially revert toward financial motives, choosing the brown project with positive probability. This contrasts with Oehmke and Opp (2025), where stronger competition strengthens socially valuable investment when project choice is contractually enforceable.

We further analyze public capital markets, where firms rely on standardized securities – debt or equity – that differ sharply in sensitivity. Because equity is structurally more sensitive, it more effectively supports ESG follow-through in our framework. However, when the firm's intrinsic preference for social value is weak, it may strategically choose debt *ex ante* to weaken ESG incentives: by making its payoff less sensitive to ESG follow-through, debt issuance effectively self-incentivizes the firm to pursue brown investment *ex post*. This perspective helps rationalize why debt-based instruments remain prevalent in retail green capital markets even amid growing concerns about follow-through and greenwashing.

Lastly, we examine whether firms can credibly signal their ESG commitment through their security issuance choices when their intrinsic preference for social value is privately known. We show that such signaling is feasible and incentive-compatible when ESG-oriented firms issue equity-like sensitive securities rather than debt. However, this requires some green investors to be non-consequentialists – who derive private utility from direct green financing and thus can internalize part of the subsidy rather than free-ride on others' green financing. However, if this private utility is excessively large, financially motivated (brown) firms may mimic ESG-oriented firms by issuing equity, eroding the credibility of equity as a signaling device of ESG commitment. The analysis thus suggests a "reverse pecking order" in ESG financing: ESG-oriented firms credibly signal their commitment and obtain cheaper financing by issuing equity-like securities, provided that non-consequentialist pro-ESG investors places a moderate value on direct engagement in green investment.

Our analysis directly suggests the following implications. First, promoting ESG investment requires attention to the sensitivity of financing instruments: policies that indiscriminately favor low-sensitivity securities – such as standardized green bonds – may unintentionally weaken firms' incentives for genuine ESG integration. Second, the model's "reverse pecking order" highlights that ESG-oriented firms can use equity issuance as a signaling device of their commitment, whereas financially driven firms favor debt. Regulatory frameworks that recognize this heterogeneity, rather than prescribing a uniform debt-based approach to sustainable finance, can better align private financing incentives with social objectives.

Our analysis also provides empirically testable predictions, a foundation for evaluating sustainable-finance practices in real markets. First, the model predicts that ESG outcomes are stronger among equity-financed firms, weaker when capital-market competition reduces security sensitivity, and that increased reliance on debt-based ESG instruments is associated with higher risk of greenwashing. Moreover, the model predicts that in markets dominated by overly enthusiastic non-consequentialist investors that are pro-ESG, the reverse pecking order breaks down as financially motivated firms mimic ESG-oriented issuers, eroding the credibility of equity as a signaling device.

This paper proceeds as follows. Section 2 reviews the related literature. Section 3 presents the model, and Section 4 characterizes the incentive conditions governing the firm's investment decision. Section 5 provides the full equilibrium analysis and discusses policy implications. Section 6 extends the main framework to analyze ESG financing in public capital markets and to establish a pecking-order theory for ESG investing under asymmetric information. Finally, Section 7 concludes. All proofs are relegated to the appendix.

2 Related Literature

Recent empirical research documents a pronounced gap between firms' ESG commitments and actual behavior. Liang, Sun and Teo (2022) show that U.S. hedge funds endorsing the UN PRI attract capital inflows but do not improve ESG performance or returns. On the corporate side, Raghunandan and Rajgopal (2022*b*) find little evidence that Business Roundtable firms meaningfully engage in stakeholder-oriented practices following public pledges.

A central reason for this disconnect is the difficulty of evaluating and enforcing ESG activities. ESG auditing suffers from weak standards, limited assurance expertise, and substantial disagreement among rating agencies (Azarmsa and Shapiro, 2022; Berg, Koelbel and Rigobon, 2022; Gipper, Ross and Shi, 2022). While prior work documents benefits from environmental enforcement and mandatory disclosure (Choy et al., 2021; Christensen et al., 2017; Grewal, Riedl and Serafeim, 2019; Krueger et al., 2021; Fiechter, Hitz and Lehmann, 2022; Aghamolla and An, 2023; Xue, 2023; Manili, 2024), monitoring ESG follow-through remains challenging even in regulated environments, as ESG actions are often difficult to verify ex post. Our first contribution is to show that firms' capital structure plays a key role in shaping ESG incentives precisely under such frictions.

A growing theory literature studies how financial contracting can induce ESG integration. Chowdhry, Davies and Waters (2019) argue that high borrowing costs tilt firms toward social objectives by compressing financial payoffs. Oehmke and Opp (2025) show that ESG-oriented lenders can promote ESG investment through enforceable covenants, while Feldhütter and Pedersen (2022) demonstrate that issuing green bonds need not lower firms' overall cost of capital.

Our approach differs in several important respects. We adopt a security-design framework that endogenizes payoff sensitivity, generalizing Chowdhry, Davies and Waters (2019). Unlike Oehmke and Opp (2025), we allow for greenwashing by making ESG actions non-contractible. We show that ESG incentives are governed primarily by the sensitivity of repayments to investment profitability rather than borrowing costs *per se*, and that intense competition can weaken ESG incentives by compressing this sensitivity. When firms are privately informed about their ESG orientation, our model delivers a reverse pecking order: genuinely ESG-oriented firms may optimally issue equity to credibly signal commitment and attract subsidized capital from ESG-motivated investors. This signaling channel contrasts with the cost-of-capital irrelevance result in Feldhütter and Pedersen (2022).

We also relate to theory work on ESG disclosure and greenwashing under asymmetric information (Friedman, Heinle and Luneva, 2021; Bonham and Riggs-Cragun, 2022; Edmans, Levit and Schneemeier, 2022; Cartellier, Tankov and Zerbib, 2023; Chaigneau and Sahuguet, 2025). Furthermore, Barbalau and Zeni (2022) and Allen, Barbalau and Zeni (2023) study ESG-linked securities whose payoffs depend on measured ESG outcomes. These studies typically assume the availability of (noisy) ESG metrics. By contrast, we consider an environment in which ESG actions are unverifiable and non-contractible, and show that ESG incentives can instead be shaped indirectly through the repayment structure – and, in particular, the sensitivity – of financial contracts.

Our analysis also builds on classic financial contracting insights. High leverage can distort investment incentives and discourage long-term value creation (Jensen, 1986; Maksimovic and Titman, 1991), while debt's payoff structure can generate risk shifting and adverse selection (Inderst and Mueller, 2006; Dang, Gorton and Holmström, 2013). Equity, by contrast, can be a robust contract under uncertainty (Admati and Pfleiderer, 1994; Ravid and Spiegel, 1997; Carroll, 2015; Malenko and Tsoy, 2023). We extend this literature by showing how equity financing can strengthen incentives for long-run social value creation and identifying market conditions under which equity issuance and ESG integration arise endogenously.

Finally, our paper speaks to the rise of green bonds and skepticism about their effectiveness. While green bond issuance is often interpreted as a signal of ESG commitment (Flammer, 2021; Tang and Zhang, 2020), both practitioner accounts and recent evidence suggest limited real effects (Aswani and Rajgopal, 2022; Bhagat and Yoon, 2023). Our theory provides an economic explanation for this pattern and suggests that equity financing may be more effective than debt in inducing meaningful ESG integration, with implications for investors and regulators.

3 Model

In this section, we first present the details of the model analyzed in this paper. We then introduce preliminary mathematical properties that play a critical role in the equilibrium analysis. Finally, we discuss the relevance and limitation of the model's key assumptions.

3.1 Setup

Consider a static economy with a firm undertaking a financial project and investors competing to fund the firm's project. When the game begins, each investor competitively makes a financial contract offer to fund the firm's project. Among the offers, the firm borrows from an investor who provides the most favorable offer. The firm then chooses a certain type of financial project that maximizes its total payoff. At the end of the game, the firm's project generates cash flow, which is distributed to the firm and the funding investor according to the payment terms they agreed to. We assume every player in the economy is risk-neutral.

The firm chooses one of two financial projects, green (G) or brown (B), each of which requires the firm to pay a unit of initiation cost. Project G refers to ESG investment that generates a higher social return but low financial profit, while project B represents traditional investment that generates a lower social return but high financial profit. Specifically, we assume that project G yields a stochastic cash flow $x \in [0, \infty)$, a continuous random variable drawn from a density function $f_G(\cdot)$ (and the corresponding distribution function $F_G(\cdot)$). This project also generates a constant social return $\phi > 0$ in the form of an externality. That is, every investor can enjoy ϕ once the firm chooses project G, even though she does not directly finance the firm. Note that the main results of our paper remains unchanged even if the social return ϕ from project G is a random variable; since every player is risk-neutral, each player will consider the expected value of ϕ when that player values the social return.

Like project G, project B also yields the stochastic cash flow x, a continuous random variable drawn from a density function $f_B(\cdot)$ (and the corresponding distribution function $F_B(\cdot)$). However, in contrast to project G, project B generates no social return. We further assume the monotone likelihood ratio property (MLRP) between project B and G such that $f_B(x)/f_G(x)$ increases with x. In other words, project B (G) is more likely to generate a high (low) cash flow than project G (B).[1] For the exposition, we define

$$\mathbb{E}_i[h(x)] := \int_0^\infty h(x) f_i(x) dx$$

for any integrable function $h(\cdot)$ and $i \in \{G, B\}$.

The firm is assumed to be cashless, so it has to finance its project externally from

[1] In the security design literature, the MLRP assumption is often introduced for ranking different investment projects based on their stochastic cash flow (Inderst and Mueller, 2006; Axelson, 2007). Furthermore, Milgrom (1981) finds that many standard continuous distributions satisfy the MLRP.

an outside investor. In the capital market, many investors, each endowed with unit of cash, compete to invest in the firm's project in Bertrand fashion. Specifically, each investor, if she wants to fund the firm's project, bids with a *security* defined by a continuous function $s : [0, \infty) \to [0, \infty)$ that specifies financial repayment, conditional on every realized cash flow x. As standard in the literature of security design, we restrict investors to offer securities $s(x)$ satisfying the following conditions for every $x \in [0, \infty)$: *(i)* both $s(x)$ and $x - s(x)$ weakly increase with x; *(ii)* $0 \leq s(x) \leq x$ (Nachman and Noe, 1994; Hart and Moore, 1995; DeMarzo, Kremer and Skrzypacz, 2005).

Of all available offers by bidding investors, the firm finances from an investor who makes the most favorable bid by issuing the security to the investor corresponding to her bid. If multiple investors make the most favorable offer, the firm randomly chooses one of the investors with equal likelihood. Formally, if the firm decides to accept a security $s(\cdot)$ and there are $n \in \mathbb{N}$ investors who bid with that same security $s(\cdot)$, the firm ends up contracting with each of them with probability $1/n$. We interpret this randomization as a move of a chance node rather than the firm's voluntary randomization.

We make several assumptions to incorporate key characteristics with regard to ESG in the capital markets into our model. The first assumption is that the firm and investors have heterogeneous preferences for the social return generated by the firm's project G. First, there is a "green" investor whose utility is a linear sum of the expected financial payoff and the social return with equal weight. If the green investor wins the lending game with her bid $s(\cdot)$, her utility is

$$(\mathbb{E}_i[s(x)] - 1) + \phi \mathbb{I}_G(i) \text{ for each } i \in \{G, B\}, \tag{1}$$

where $\mathbb{I}_G : \{G, B\} \to \{0, 1\}$ is an indicator function such that $\mathbb{I}_G(G) = 1$ and $\mathbb{I}_G(B) = 0$, respectively. Since the social return from "green investment" in project G is materialized as externality, the green investor's payoff will be still ϕ upon the firm's selection of project G, even though the green investor does not fund the firm directly:

$$\phi \mathbb{I}_G(i) \text{ for each } i \in \{G, B\}. \tag{2}$$

This green investor represents a group of market participants recently more salient in the capital market, that is, those who heavily consider the social impacts of their investments. The assumption of a single green investor is made for analytical convenience: the main results

of our paper still stand even when we allow multiple green investors.

There are also "brown" investors, who are akin to traditional financial investors in the standard corporate finance literature, primarily pursuing monetary income. Each brown investor's utility is equal to the expected financial payoff if he funds the firm directly with his security bid $s(x)$:

$$\mathbb{E}_i[s(x)] - 1 \text{ for each } i \in \{G, B\}\,. \tag{3}$$

However, any brown investor will get zero utility if he loses the funding competition.

For simplicity, brown investors' social preferences are normalized to zero. The main analytical results presented below remain unchanged as long as brown investors place a smaller weight on corporate social returns than a green investor does.

Lastly, the firm also considers the non-financial consequence of its project choice, but not as heavily as the green investor. Specifically, the firm's utility is represented by a linear sum of its residual income with a unit weight and social return with a different weight $\lambda \in (0, 1)$:

$$\mathbb{E}_i[x - s(x)] + \lambda\phi\mathbb{I}_G(i) \text{ for each } i \in \{G, B\}\,. \tag{4}$$

Indeed, the parameter λ captures the firm's inherent preference for social value. In this reduced-form formulation, a higher λ means that the firm places a greater private value on implementing the ESG-oriented (green) project, holding fixed its financial payoffs. Throughout the paper, λ serves as a parsimonious index of the firm's ESG orientation, while our main focus is on how the financing contract's sensitivity interacts with λ to discipline (or fail to discipline) ESG follow-through. Section 3.3 provides the economic interpretation of λ and clarifies what the model captures – and abstracts from – regarding ESG integration, thereby helping to interpret the results that follow.

The second assumption is perhaps more important: the firm's project choice after funding from an investor cannot be contractible. This assumption is crucial for capturing corporate greenwashing and the limited availability of ESG assurance, which are salient features of real-world ESG finance. That is, the firm makes an investment decision to maximize its own payoff, even though this decision leads to lower production of the social return, possibly against the preferences of the green investor.

Lastly, we assume the following regularity conditions that incorporate potential incen-

tive misalignment between the green investor and the firm into our model.

Assumption 1. *(i)* $\mathbb{E}_G[x] + \phi > \mathbb{E}_B[x] > \mathbb{E}_G[x] + \lambda\phi$*; (ii)* $\mathbb{E}_B[x] > \mathbb{E}_G[x] > 1$*; (iii)* $\phi \leq 1$.

Assumption 1-(i) means that project G generates higher total surplus than project B from the green investor' perspective ($\mathbb{E}_G[x] + \phi > \mathbb{E}_B[x]$), while the firm will invest in project B if it can self-finance ($\mathbb{E}_B[x] > \mathbb{E}_G[x] + \lambda\phi$). Assumption 1-(ii) means that both project G and B are financially profitable, while project B is more lucrative. This assumption rules out a trivial equilibrium in which project G is never chosen due to a negative net present value.[2] Lastly, Assumption 1-(iii) means that the green investor is unwilling to sacrifice all of the financial payoffs, even if the firm invests in project G. Analytically, Assumption 1-(iii) is made to rule out a trivial equilibrium in which a (green) investor charges zero interest for funding the firm's green project.

Before turning to equilibrium analysis, we introduce a contract characteristic – repayment sensitivity – that summarizes how repayment terms respond to the firm's project choice and will serve as the key sufficient statistic in what follows.

3.2 Technical Preliminaries: Sensitivity of Securities

In this section, we discuss some mathematical properties of securities that facilitate the understanding of our main analysis. First, we formalize some standard types of securities commonly issued and traded in the capital market.

Definition 1.

(i) A debt contract with face value D is defined as $d(x, D) := \min\{x, D\}$*;*

(ii) A (levered) equity contract with a strike price S is defined as $c(x, S) := \max\{x - S, 0\}$.

Suppose the firm finances its project with a debt contract $d(x, D) = \min\{x, D\}$. When the firm's project generates high cash flow (i.e., $x \geq D$), a funding investor will always get D

[2] In practice, firms often construct carbon abatement cost curves, sometimes with the help of consultants such as McKinsey ("Beating the abatement cost curve for growth," McKinsey, April 13, 2021, https://www.mckinsey.com/capabilities/operations/our-insights/net-zero-or-bust-beating-the-abatement-cost-curve-for-growth), to identify and choose green projects that lead to cost savings and positive NPV. See https://www.climateworkscentre.org/resource/how-to-read-a-marginal-abatement-cost-curve/.

whether x is close to but above D or substantially higher than x. Furthermore, the investor's senior claim is fully secured by the project in the event $x < D$, i.e., the firm defaults on servicing its debt. Next, suppose the firm finances from an investor with a levered equity contract $c(x, S) = \max\{x - S, 0\}$ (hereafter referred to as "equity"). Under such financing terms, the investor effectively becomes a residual claimant of cash flow of the underlying project; it is as if the investor owes a fixed amount of S to have a claim on the remaining cash flow $x - S$. At the same time, the investor is protected by limited liability if the firm's project yields a low cash flow $x < S$.

It is worth noting that we deliberately abstract away from any additional contractual rights that are often associated with specific securities. For instance, debt holders may obtain control rights over borrowers' assets or collateral in the event of default, and equity holders can influence corporate decisions through voting rights. We omit these features to isolate the cash-flow channel: our focus is on how the allocation of future cash flows specified by the security issued to outside investors shapes the firm's incentives for ESG integration.[3]

We next introduce a key characteristic of securities coined "sensitivity," which is crucial to our main analysis. Formally, the sensitivity of a security $s(\cdot)$ is defined as $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)]$, i.e., the difference of the expected payment to a funding investor between two projects. Note first that, since MLRP implies the first order stochastic dominance (FOSD), we have $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] \geq 0$ for any weakly increasing security $s(\cdot)$, where the inequality is strict whenever $s(x)$ is strictly increasing for a positive measure of x.

The sensitivity of a security $s(\cdot)$ is determined by how steeply $s(x)$ changes with x, particularly for high values of x. To provide intuition, consider an example of a binary random variable $x \in \{x_l, x_h\}$ with $x_l < x_h$ and $Pr(x = x_h) = p \in (0, 1)$. If $s(x_h) - s(x_l)$ is small, the expected payoff of $s(x)$ barely changes with p. An extreme case is $s(x) = x_l$ for every x: the expected payoff of $s(x)$ stays at x_l for every $p \in (0, 1)$. However, if $s(x_h) - s(x_l)$ is large, then the expected payoff of $s(x)$ significantly increases with p. For example, suppose $s(x) = x$ for every $x \in \{x_l, x_h\}$. Then the expected value of $s(x)$ is $p(x_h - x_l)$, whose derivative with respect to p is $x_h - x_l$, which is the highest of all other securities.

One question of interest is whether and how securities are ordered by their sensitivity. Since we impose few restrictions on the forms of the securities, infinitely many types of securities are feasible for bidding in our model, inhibiting us from ordering them completely by

[3] As shown in Section 5 and Section 6, the firm has stronger incentives to choose project G when it issues equity rather than debt, even absent contractual control rights (e.g., governance or voting rights).

sensitivity. However, as the following lemma states, we can find the least and most sensitive securities under the MLRP of the stochastic cash flow x between project B and G.

Lemma 1 (ranking securities by sensitivity)**.**

(i) $\mathbb{E}_B[d(x,D)] - \mathbb{E}_G[d(x,D)]$ *is increasing in* D*.*

(ii) $\mathbb{E}_B[c(x,S)] - \mathbb{E}_G[c(x,S)]$ *is decreasing in* S*.*

(iii) Fix any security $s(\cdot)$ *that is not a debt contract. For each* $i = G$ *and* B*, there exists* $D_i \geq 0$ *such that* $\mathbb{E}_i[s(x)] = \mathbb{E}_i[d(x,D_i)]$ *and* $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] > \mathbb{E}_B[d(x,D_i)] - \mathbb{E}_G[d(x,D_i)]$*.*

(iv) Fix any security $s(\cdot)$ *that is not an equity contract. For each* $i = G$ *and* B*, there exists* $S_i \geq 0$ *such that* $\mathbb{E}_i[s(x)] = \mathbb{E}_i[c(x,S_i)]$ *and* $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] < \mathbb{E}_B[c(x,S_i)] - \mathbb{E}_G[c(x,S_i)]$*.*

(v) $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] < \phi$ *for any security* $s(\cdot)$*.*

We first analyze how the sensitivity of standard securities, such as debt and equity, is determined. First, the slope of a debt contract $d(x,D) = \min\{x, D\}$ is positive and steep (indeed equal to one) if $x \leq D$ but is completely flat (i.e., equal to zero) if $x > D$. Thus, the range of x with which $d(x,D)$ increases expands as face value D increases, making the debt contract more sensitive to the firm's project selection (Lemma 1-(i)). By contrast, the slope of an equity contract $c(x,S) = \max\{x - S, 0\}$ is completely flat if $x \leq S$ but is the steepest if $x > S$. Hence, the subset of x with which $c(x,S)$ increases shrinks as the strike price S increases, making the equity contract less sensitive (Lemma 1-(ii)). Note that an increase in D also increases both $\mathbb{E}_G[d(x,D)]$ and $\mathbb{E}_B[d(x,D)]$. Similarly, decreasing S increases both $\mathbb{E}_G[c(x,S)]$ and $\mathbb{E}_B[c(x,S)]$. In short, standard securities become more sensitive as the firm gives an investor a larger share of cash flow.

Lemma 1-(iii) and (iv) are the replication of DeMarzo, Kremer and Skrzypacz (2005). Indeed, the MLRP is useful for ranking securities based on their sensitivity. Figure 1 depicts the repayment schedules of several securities, such as debt contracts, equity contracts, and an arbitrary security, which facilitates comparing the sensitivity of the securities. First, the debt contract is the least sensitive of all securities with the same expected payment under a specific project type. The intuition is as follows: Compared to the other securities, the slope of the debt contract is the highest (equal to one) for relatively low values of x (i.e., $x \leq D$) but is the lowest (equal to zero) for relatively high values of x (i.e., $x > D$). Thus, for any arbitrary

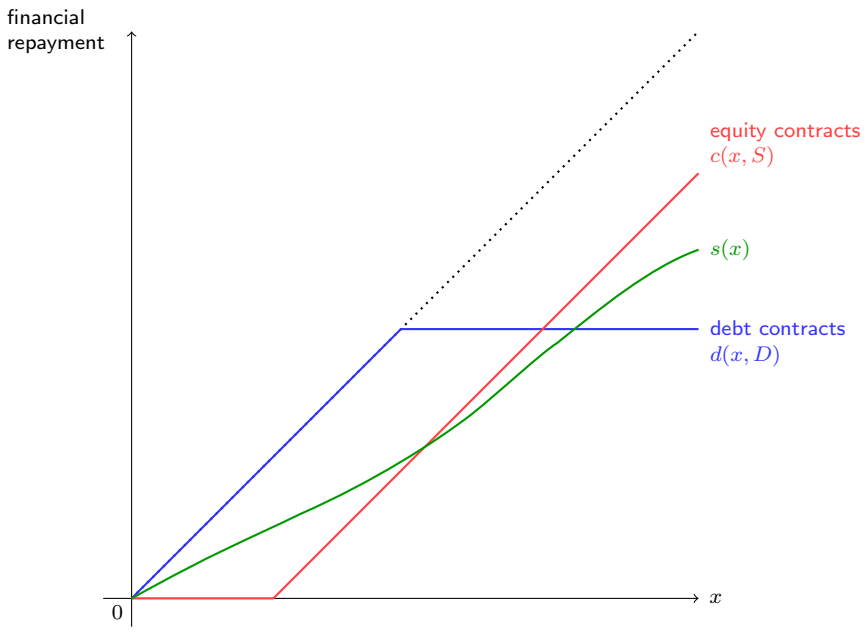


Figure 1 – Sensitivity of securities

security $s(\cdot)$, we have $s(x) < d(x, D)$ for relatively small values of x but have $s(x) > d(x, D)$ for relatively large values of x. The MLRP implies that low (high) values of x are more likely to be realized under project G (B). Hence, the debt contract $d(x, D)$ has a relatively high expected payoff compared to $s(x)$ under project G, while the opposite holds true under project B, implying that debt contracts are less sensitive than any other securities.

In contrast, equity contracts are the most sensitive security with respect to project selection. Specifically, compared to other securities, the slope of an equity contract $c(x, S)$ is the lowest (equal to zero) for relatively low realizations of x (i.e., $x < S$), and the highest (equal to one) for relatively high values (i.e., $x > S$). As a result, for any other security $s(\cdot)$, we have $s(x) > c(x, S)$ for small x and $s(x) < c(x, S)$ for large x. By the MLRP, this implies that $c(x, S)$ yields a relatively higher expected payoff than $s(x)$ under project B, and a relatively lower one under project G, indicating that equity contracts are strictly more sensitive than other security types.

Among these, $s(x) = x$ – i.e., a pure equity contract with zero strike price – is the most sensitive equity contract. However, by Assumption 1-(i), its sensitivity is bounded above by ϕ. Consequently, every security admissible for investor bidding in out model must have sensitivity no greater than ϕ (Lemma 1-(v)). Taken together, these preliminaries show that standard instruments such as debt and equity represent extreme points in the sensitivity ranking. The

next subsection clarifies how our modeling choices map to ESG settings in practice, and what economic forces we intentionally abstract from – to keep the focus on this sensitivity channel.

3.3 Interpretation of the Model

Having laid out the contracting environment and sensitivity ranking of securities, we now discuss how to interpret the model's key assumptions in the context of ESG integration.

To highlight the key analytical findings presented below, we abstract from certain features of real-world capital markets and corporate finance for expositional clarity. First, following the existing ESG literature (Edmans, Levit and Schneemeier, 2022; Oehmke and Opp, 2025), we assume that borrowers' ESG investments generate positive externalities. In this setting, "green" investors derive social surplus from firms' ESG follow-through even without directly financing green investment. In this sense, the green investors in our model are *consequentialists* who care solely about final ESG outcomes. As discussed in footnote 5 of Section 4, our main results remain qualitatively robust when we allow for *non-consequentialist* green investors who value direct engagement and may be willing to offer concessionary terms to promote project G. However, as shown in Lemma 2, such concessionary lending cannot be sustained in equilibrium because it dilutes incentives and ultimately encourages borrowers to prioritize financial returns over ESG objectives.

We further introduce the parameter λ to capture the borrowing firm's inherent preference for social value in a reduced-form manner. This utility representation is consistent with the approaches taken by the theoretical literature on ESG integration in financial markets (Chowdhry, Davies and Waters, 2019; Chaigneau and Sahuguet, 2025; Dangl et al., 2025; Oehmke and Opp, 2025). Moreover, Chang, Rhee and Yoon (2024, p. 6) survey empirical work suggesting that socially responsible investments can indirectly enhance long-term firm value through multiple channels, including preferential regulatory treatment following philanthropic activities (Waddock and Graves, 1997), increased labor productivity via improved worker morale (Turban and Greening, 1997), and stronger consumer awareness through reputation building (Fombrun and Shanley, 1990).

As a reduced-form index, λ allows us to focus on the contracting implications of heterogeneous ESG orientations without committing to a specific microfoundation for how ESG engagement maps into long-run firm value. Meanwhile, such a reduced-form treatment abstracts from dynamic feedback channels through which ESG engagement may endogenously

affect future cash flows, investor bases, or regulatory outcomes. More generally, the framework applies to any non-contractible mission choice that creates non-monetary spillovers, with ESG serving as a leading case. Lastly, in Section 6.2, we relax the complete-information assumption by allowing the firm to be privately informed about λ.

Next, we assume that the borrowing firm undertakes its new project at a fixed scale. This simplification enables us to emphasize the role of security design in incentivizing the firm to meet its ESG obligations. Our model can be interpreted in two distinct ways. First, it captures the firm's decision on how to allocate proceeds based on the surplus generated per unit of capital: the firm will allocate more of its external capital to the green project if it delivers a higher total payoff per dollar spent than the alternative. Second, the model is well-suited to the cases where firms incur fixed costs to undertake a green transition. For instance, achieving zero carbon emissions may require manufacturers to fully redesign their production processes, including the complete reconstruction of existing facilities. In addition, firms may need to invest substantial capital in research and development (R&D) to achieve technological breakthroughs critical to greening their operations.

Furthermore, we simplify the firm's ESG integration decision to a binary choice between two financial projects. However, our main results remain robust to richer ex-post strategy spaces. For instance, in Chowdhry, Davies and Waters (2019), the firm can continuously allocate its attention between two projects – assigning a share $\gamma \in [0, 1]$ to project G and $1 - \gamma$ to project B – with each project more likely to succeed as it receives greater attention. Despite this flexibility, the combination of linear payoffs and the technology of attention allocation yields corner solutions, where the firm ultimately concentrates its attention entirely on one project. It is important, however, not to overly emphasize the literal implications of this stylization. An equilibrium in which the firm selects project G should be viewed as reflecting a stronger commitment to ESG integration, rather than a full-scale transition to ESG activities.

In our setting, the central agency problem is not shirking in costly effort but ex-post ESG follow-through – whether the borrower implements the promised project once financing is secured. More broadly, while we do not explicitly model costly managerial effort, the core contracting force emphasized in the paper – how a security's payoff profile shapes the borrower's incentives for ESG integration – continues to operate when effort affects project outcomes. In such extensions, repayment sensitivity still governs the strength of incentives, while the quantitative cutoff values possibly shift.

We also emphasize that our research focuses on how the sensitivity of securities influences the firm's incentive to pursue green investment, rather than on the specific roles of different security types in green financing. In this context, the assumption of MLRP serves primarily to facilitate the ranking of securities by their sensitivity. Specifically, Lemma 1 demonstrates that, under the MLRP, debt and equity – standard security types issued through public offerings – represent the least and most sensitive securities, respectively. Our main results remain qualitatively unchanged even in the absence of the assumption, provided that the securities eligible for bidding can still be completely ordered by sensitivity.

Similarly, to preserve generality in our analysis, we do not restrict the types of securities that investors may offer to fund the firm's project. This flexibility reflects real-world private capital markets for ESG, where venture capitalists often tailor repayment terms to their advantage when funding startup social enterprises that typically lack strong credit histories or sufficient collateral and therefore face limited access to public retail markets (Yang and Zeng, 2019). Another example is project finance, where borrowing firms isolate the risks of capital-intensive ESG projects to attract institutional investors such as pension funds. Our results are also robust to the scenarios in which borrowing firms collateralize existing assets, as long as the resulting financial contracts can be completely ranked by sensitivity. Furthermore, as shown in Section 6.1, our analysis can be extended to situations where the firm raises capital by issuing standard securities, such as debt or equity, as is typical in public capital markets.

Lastly, we restrict attention to an economically relevant class of securities that are monotone in cash flows: both the repayment schedule $s(x)$ and the borrower's residual payoff $x - s(x)$ are assumed to be non-decreasing. This restriction is standard in security design and rules out contracts that would perversely reward lower cash-flow realizations, thereby creating incentives for value destruction, earnings manipulation, or renegotiation. Allowing non-monotone repayment schedules could mechanically implement outcomes via state-contingent "penalties" and "rewards," but such provisions are unlikely to be enforceable or robust when cash-flow reporting and renegotiation are salient concerns (Townsend, 1979). In the same spirit, as noted in Section 4, we abstract from contractual control and voting rights and focus on the cash-flow implications of financing terms. Our goal is to isolate how repayment sensitivity within this contract class shapes borrowers' incentives for ESG follow-through.

4 Ex-Post Incentive Conditions for Green Investment

We now proceed to the main analysis. We first analyze how the sensitivity of investors' security bids influences the incentive structure of the firm's project selection, which will eventually feed back into the investors' bidding strategies and thus shape the equilibrium outcome.

Fix a security $s^*(\cdot)$ that the firm accepts from a winning investor in the lending competition. Then the firm will receive the expected payoff $\mathbb{E}_G[x - s^*(x)] + \lambda\phi$ from project G and $\mathbb{E}_B[x - s^*(x)]$ from project B, respectively. Hence, the firm's optimal project selection is type G if and only if $\mathbb{E}_G[x - s^*(x)] + \lambda\phi \geq \mathbb{E}_B[x - s^*(x)]$. Then doing simple algebra yields the following result:

Lemma 2. *Let $s^*(\cdot)$ denote an equilibrium security offer the firm accepts. Then, the firm chooses project G if*

$$\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi, \tag{5}$$

and the firm chooses project B if

$$\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi. \tag{6}$$

Lemma 2 reveals an important necessary condition for the green investment: the security the firm issues to raise capital must be sufficiently sensitive to incentivize the firm to choose project G. How can the security's sensitivity influence the firm's incentive for project selection? The answer lies in the expected payoff of the security held by a funding investor. By choosing project G instead of B, the firm can reduce the expected payment to the funding investor by $\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)]$. This effect amplifies as the repayment schedule specified by the security $s^*(\cdot)$ becomes more sensitive.[4] Furthermore, project G partially compensates the firm for its financial under-profitability by generating the non-financial social return ϕ, which the firm adds to its payoff with a discounted weight λ. In sum, the total benefit of choosing project G can outweigh its cost when the security $s^*(\cdot)$ is strongly sensitive and the firm's preference for social value is sufficiently strong (i.e., λ is sufficiently high).

Furthermore, the minimum security sensitivity level supporting the green investment

[4]Nevertheless, the positive impact of lowering the expected payment to the funding investor itself cannot overcome the lower profitability of project G than B. Namely, the firm suffers a net financial loss if it switches from project B to G, due to the decrease in the expected return of the project; $\mathbb{E}_G[x - s^*(x)] < \mathbb{E}_B[x - s^*(x)]$ since $x - s^*(x)$ is an increasing function of x and F_B is strict FOSD over F_G.

decreases with λ, a proxy of the firm's preference for social value. Suppose the firm has a weak preference for social return. In that case, the green investor must offer strongly sensitive securities enough for the firm to pursue financial gains from reducing the expected payment to the funding investor by choosing project G. In this context, we can conjecture that equity financing may facilitate ESG integration compared to debt financing, particularly for a relatively low λ; equity contracts are the most sensitive securities, while debt contracts are the least (Lemma 1-(iii) and (iv)). This conjecture will be analytically confirmed in Section 6.1.

However, we should not hastily judge that an investor's bid with a strongly sensitive security always guarantees the green investment in equilibrium. Indeed, competitive pressure in the capital market may result in an insensitive security as an equilibrium funding term the firm accepts. Certainly, the borrowing cost is positively correlated with the sensitivity of the security held by the funding investor. For example, consider a green investor bidding with an equity contract $c(x, S) = \max\{x - S, 0\}$. Given a low λ, the green investor may have to increase the sensitivity of her equity contract offer by decreasing S substantively (Lemma 1-(ii)). However, lowering S increases the total repayment the firm owes, no matter what project it chooses afterward. Hence, the firm may want to borrow from another investor who bids to fund the project with an insensitive security, even though such a security bid may lead to the investment in project B. In other words, a security bid must be strongly sensitive to induce the green investment *ex post*, while the firm will reject such an offer *ex ante* due to the associated high borrowing cost.[5] This dilemma indeed hampers the firm's choice of project G, which will be shown in Section 5 and Section 6.1.

From the discussion above, one can find that a key parameter that pins down the equilibrium outcome will be λ, representing the firm's preference for the social return. If λ is high, every security bid that can yield the green investment does not need to be strongly sensitive, making such bid a cheap financing option. However, if λ is low, the firm may find a security bid incentivizing the green investment too costly. In what follows, we will analyze how these countervailing effects of the security sensitivity determine the equilibrium outcome for each value of λ.

[5]For a similar reason, our main results are not qualitatively changed even if the green investor is a *non-consequentialist*; i.e., she receives an extra utility from funding project G directly (Dangl et al., 2025). Although the green investor can offer cheaper financing to the firm than her competitors by internalizing her extra utility from the direct green investment, she will not exercise her competitive advantage in the funding game. Indeed, securities claiming low expected repayments tend to be insensitive (Lemma 1), which adversely incentivizes the firm to choose project B rather than G and thus removes the green investor's additional utility, a source of subsidized financing. See Section 3 of Chang, Rhee and Yoon (2024) for more details.

5 Equilibrium Analysis

In this section, we analyze how the firm's equilibrium project selection changes with its preference for social return. We continue to analyze how the firm's capital structure supports or hampers green investments.

For exposition, we first introduce new notations.

Definition 2.

(i) $D_y^i := \min\{D \geq 0 : \mathbb{E}_i[d(x, D)] \geq y\}$ *for each* $i \in \{G, B\}$;

(ii) $S_y^i := \min\{S \geq 0 : \mathbb{E}_i[c(x, S)] \geq y\}$ *for each* $i \in \{G, B\}$.

D_y^i is the face value of a debt contract that yields the expected value y under project $i \in \{G, B\}$. Similarly, S_y^i is the strike price of an equity contract that yields the expected value y under project $i \in \{G, B\}$. Note from Definition 1 that D_y^i increases with y, while S_y^i decreases with y. Furthermore, the MLRP dominance of $f_B(\cdot)$ over $f_G(\cdot)$ implies that $D_y^G > D_y^B$ and $S_y^G < S_y^B$ for every $y > 0$.

In what follows, we establish a subgame perfect equilibrium, which is referred to as the equilibrium going forward, where all investors employ pure strategies. We hereafter refer to $s^*(\cdot)$ as the equilibrium security bid the firm issues to the funding investor.[6] In equilibrium, the firm accepts an equilibrium security $s^*(\cdot)$ only if it maximizes its expected payoff. Furthermore, after accepting the issuance of $s^*(\cdot)$, the firm chooses project G with positive probability only if $s^*(\cdot)$ satisfies (5) and chooses project B with positive probability only if $s^*(\cdot)$ satisfies (6). Moreover, investors cannot strictly benefit from deviating from their equilibrium bids. As discussed earlier, a key parameter determining the equilibrium outcome is λ, representing the firm's preference for the social return ϕ. Figure 2 depicts how the equilibria presented in Theorem 1 - 4 below are characterized by λ.

5.1 Equilibrium with Market-Rate Investment

We first characterize an equilibrium with *market-rate* investment in competitive capital markets, where the firm's payments to a funding investor, specified by the equilibrium security,

[6]The notation $s^*(\cdot)$ implicitly assumes that the firm chooses $s^*(\cdot)$ with probability one among all feasible securities. This is indeed the case in all equilibria discussed in this section. Restricting attention to such equilibria does not result in any loss of generality. Indeed, we allow the firm to randomize over multiple securities in all proofs in the appendix and confirm that all results remain valid.

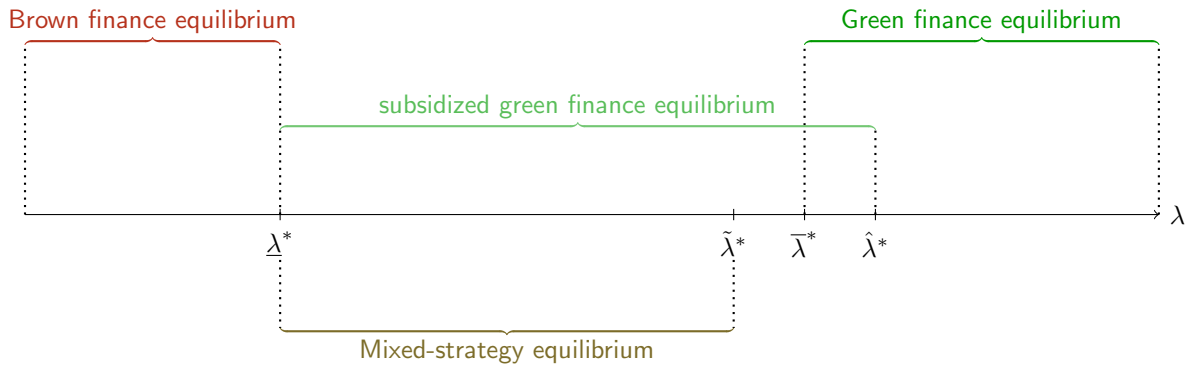


Figure 2 – Equilibrium characterization

break even in expectation – i.e., $\mathbb{E}_B[s^*(x)] = 1$ ($\mathbb{E}_G[s^*(x)] = 1$, *resp.*) when the firm chooses project B (G, *resp.*). Intuitively, such an equilibrium will arise when brown investors dominate the green investor in competition in the capital market; these investors will fund the firm only when the expected payments specified by their securities guarantee a nonnegative financial payoff. In what follows, we show that this dominance of brown investors is likely to hold when the firm's inherent social preference is either sufficiently strong or sufficiently weak, so that its ex-post project choice is less affected by the sensitivity of the security issued to the funding investor.

We first analyze equilibrium where the firm chooses project B. The following statement reveals that such an equilibrium arises from relatively low values of λ.

Theorem 1 (brown finance equilibrium)**.** *Define* $\underline{\lambda}^* > 0$ *as*

$$\underline{\lambda}^* := \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)]\right\}\right]. \tag{7}$$

Then there exists an equilibrium in which the firm chooses project B *after accepting a security offer* $s^*(\cdot)$ *such that* $\mathbb{E}_B[s^*(x)] = 1$ *if and only if* $\lambda \leq \underline{\lambda}^*$.

The equilibrium in Theorem 1 is called the "brown finance equilibrium" from the firm's choice of project B. From (7), we have

$$\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \text{ for every } \lambda < \underline{\lambda}^*.$$

This inequality means that the firm's inherent preference for social value is too weak to invest in socially valuable project even if it issues equity, the most sensitive security that



Figure 3 – Brown finance equilibrium: the firm with $\lambda \leq \underline{\lambda}^*$, depicted in brown, finances project B.

maximizes the benefit of reducing the expected repayment to a funding investor (Lemma 2-(iv)). Hence, investors, particularly those with brown type, rationally infer that the firm will choose project B, no matter what type of security it issues to finance its project. Therefore, the brown investors competitively bid with an equilibrium security $s^*(\cdot)$ which breaks even for the selected funding investor, conditional on project B (i.e., $\mathbb{E}_B[s^*(x)] = 1$). Figure 3 graphically illustrates the range of λ (i.e., $\lambda \leq \underline{\lambda}^*$) that supports the brown finance equilibrium.

Investors' equilibrium security offer strategy is indeterminate. Nonetheless, one equilibrium outcome is that a selected brown investor wins the lending competition and holds an equity contract $c(\cdot, S_1^B)$, which breaks even contingent on the firm's choice of project B (i.e., $\mathbb{E}_G[c(x, S_1^B)] = 1$). Then it is impossible for the green investor to make any security offer that provides a low borrowing cost enough to win the funding competition and has strong sensitivity enough to induce the firm's choice of project G. Note that the green investor is willing to sacrifice its financial profit up to ϕ once the firm certainly switches to project G. To make the deviating security bid more attractive than the equilibrium security $c(\cdot, S_1^B)$, the green investor must take a financial loss by claiming a smaller share of cash flow on average than the equilibrium security offer $c(\cdot, S_1^B)$ to the extent that $\mathbb{E}_G[s''(x)] \leq \mathbb{E}_G[c(x, S_1^B)]$ – if not, the firm, even conditional on its choice of project G, will get a strictly higher payoff from accepting $c(\cdot, S_1^B)$ rather than $s''(\cdot)$. However, a lower borrowing cost means insensitivity of $s''(\cdot)$. In addition, $s''(\cdot)$ is essentially less sensitive than the equity contract $c(x, S_1^B)$ inducing the firm's choice of project B. Therefore, the green investor's deviation offer must be strictly less sensitive than the brown investor's equilibrium security offer, failing to induce the firm's selection of project G but only incurring a financial loss to the green investor.

The infeasibility of securities supporting green investment highlights the green investor's dilemma in the funding competition with brown investors, as we briefly discussed at the end of Section 4. It is necessary for any security bid from the green investor to be sufficiently sensitive to incentivize the firm to reduce the expected payment to the funding investor by choosing project G. An increase in sensitivity, however, places a large repay-

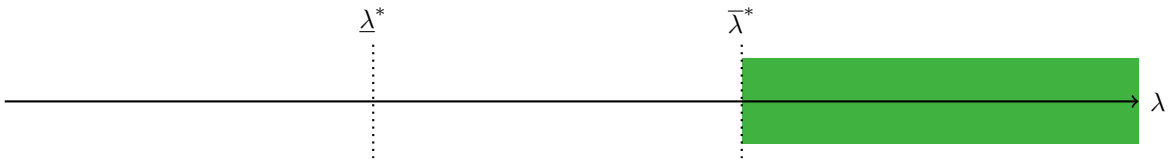


Figure 4 – Green finance equilibrium: the firm with $\lambda \geq \overline{\lambda}^*$, depicted in green, finances project G.

ment burden on the firm. In particular, any security inducing the green investment must be strongly sensitive to project selection when the firm has a weak preference for social return. Since the firm prefers lowering the repayment burden, it will borrow from an investor who offers a security that is insensitive to project selection.

We next analyze the equilibrium where the firm chooses project G. In contrast to the brown finance equilibrium, such equilibrium will arise from sufficiently high values of λ.

Theorem 2 (green finance equilibrium)**.** *Define* $\overline{\lambda}^* > \underline{\lambda}^*$ *as*

$$\overline{\lambda}^* := \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)]\right\}\right]. \tag{8}$$

Then there exists an equilibrium in which the firm chooses project G *after accepting a security offer* $s^*(\cdot)$ *such that* $\mathbb{E}_G[s^*(x)] = 1$ *if and only if* $\lambda \geq \overline{\lambda}^*$.

The equilibrium in Theorem 2 is called the "green finance equilibrium" simply because the firm chooses project G. Condition (8) is rewritten as

$$\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] > \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \text{ for every } \lambda > \overline{\lambda}^*.$$

Following from Lemma 2, the above inequality means that the firm's inherent preference for social value is too high to choose project B even if it issues debt, the least sensitive security. Therefore, brown investors rationally infer that the firm's ex-post choice must be project G and thus competitively offer a security $s^*(\cdot)$ that breaks even contingent on the firm's choice of project G – i.e., $\mathbb{E}_G[s^*(x)] = 1$. Figure 4 graphically illustrates the set of λ (i.e., $\lambda \geq \overline{\lambda}^*$) that supports the green finance equilibrium.

Like in the brown finance equilibrium, the investors' bidding strategies in the green finance equilibrium are indeterminate. One example is that a selected brown investor holds the debt contract $s^*(\cdot) = d(\cdot, D_1^G)$ which breaks even under the firm's choice of project G

(that is, $\mathbb{E}_G[d(x, D_1^G)] = 1$). Indeed, brown investors find it impossible to deviate profitably by offering a security that charges a smaller share of cash flow than $d(\cdot, D_1^G)$ and induces the firm's choice of project B: even the debt contract $d(\cdot, D_1^B)$ – the least sensitive security that breaks even under project B – fails to incentivize the firm to choose project B. This suggests that even the least sensitive debt facilitates the firm's ESG integration when it has a sufficiently strong inherent preference for social value.

Interestingly, the green investor has no incentive to outbid the brown investor's security $d(\cdot, D_1^G)$ to win the lending competition. First, even if the green investor does not directly fund the firm, she enjoys the free-riding benefit of the social return ϕ as positive externality from the brown investor's financing of project G. Furthermore, to win the lending competition against the brown investors, the green investor's (deviation) security bid, say $s'(\cdot)$, must claim a smaller share of cash flow to an extent that the firm can enjoy a lower borrowing cost on average than issuing the equilibrium security $d(\cdot, D_1^G)$. That is, the green investor must offer an insensitive security compared to the equilibrium security. However, such a deviation strategy will only harm the green investor's welfare: she may suffer a financial loss $\mathbb{E}_G[s'(x)] < \mathbb{E}_G[d(x, D_1^G)] = 1$ without gaining an extra social return; even worse, the green investor's deviation strategy may adversely incentivize the firm to choose project B, eliminating the social return she could obtain if she does not bid at all.

5.2 Equilibrium with Subsidized Financing

We now characterize the equilibrium when the firm has a moderate preference for social value, that is, for intermediate values of $\lambda \in [\underline{\lambda}^*, \overline{\lambda}^*]$. The key distinction from the equilibria discussed in Section 5.1 is that the firm's ex-post project choice now depends on its funding structure, as captured by the sensitivity of the securities it issues.

One immediate observation is that no equilibrium as those in Theorem 1 and 2 can exist in this region. Suppose, for contradiction, that a green finance equilibrium exists in which the winning investor funds project G and receives zero expected financial payoff for some $\lambda < \overline{\lambda}^*$. From condition (8), we have

$$\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi.$$

Hence, a brown investor can profitably deviate: she can offer an insensitive debt contract $d(\cdot, D)$ with D slightly larger than D_1^B that *(i)* induces brown investment – i.e., $\mathbb{E}_B[d(x, D)] -$

$\mathbb{E}_G[d(x,D)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$; *(ii)* lowers the firm's borrowing cost – i.e., $\mathbb{E}_G[d(x,D)] < 1$; and *(iii)* yields a strictly positive financial payoff – i.e., $\mathbb{E}_B[d(x,D)] > 1$.

Conversely, suppose there exists a brown finance equilibrium in which the winning investor funds project B and earns zero expected financial payoff for some $\lambda > \underline{\lambda}^*$. From condition (7), we have

$$\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] > \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi.$$

Hence, the green investor can profitably deviate by offering a sensitive equity contract $c(\cdot, S)$ with S slightly higher than S_1^B that *(i)* induces green investment – i.e., $\mathbb{E}_B[c(x,S)] - \mathbb{E}_G[c(x,S)] > \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$; *(ii)* reduces the firm's borrowing cost – i.e., $\mathbb{E}_B[c(x,S)] < 1$; and *(iii)* provides a strictly positive payoff – i.e., $\mathbb{E}_G[c(x,S)] - 1 + \phi > 0$.[7] That is, the green investor can win the lending competition against brown investors by sacrificing her financial gains to achieve socially valuable green investment.

Thus, in equilibrium, the winning investor ends up providing *subsidized* financing – incurring a financial loss from directly funding the firm. Because brown investors care only about financial returns, the green investor – who internalizes social returns – must be the winner of the lending competition. Moreover, the equilibrium security must be sufficiently sensitive to induce green investment, ensuring that the green investor's financial loss is offset by the non-financial return. We refer to this outcome as the "subsidized green finance equilibrium": the green investor funds the firm with a security that delivers an expected financial payoff below the investment cost, yet successfully incentivizes the firm to pursue project G.

Theorem 3 (subsidized green finance equilibrium)**.** *Define* $\hat{\lambda}^* > \overline{\lambda}^*$ *as*

$$\hat{\lambda}^* := \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_G[d(x, D^G_{1-\phi})]\right\}\right]. \tag{9}$$

Then there exists an equilibrium in which the firm chooses project G after accepting a security offer $s^*(\cdot)$ such that $\mathbb{E}_G[s^*(x)] \in [1-\phi, 1)$ if and only if $\lambda \in [\underline{\lambda}^*, \hat{\lambda}^*]$.

Figure 5 graphically illustrates the range of λ (i.e., $\lambda \in [\underline{\lambda}^*, \hat{\lambda}^*]$) that supports the subsidized green finance equilibrium. In equilibrium, the green investor must offer a security

[7] Since $\mathbb{E}_B[c(x, S_1^B)] = 1$, we must have $\mathbb{E}_G[c(x, S_1^B)] > 1 - \phi$, which follows from the fact that every security must have sensitivity strictly less than ϕ. This guarantees the existence of an equity contract $c(\cdot, S)$ with $S = S_1^B + \varepsilon$ for a small $\varepsilon > 0$ such that $\mathbb{E}_B[c(x,S)] < 1$ and $\mathbb{E}_G[c(x,S)] > 1 - \phi$.



Figure 5 – Subsidized green finance equilibrium: the firm with $\lambda \in [\underline{\lambda}^*, \overline{\lambda}^*]$, depicted in light green, finances project G at a subsidized borrowing rate (i.e., $\mathbb{E}_G[s^*(x)] < 1$).

offer $s^*(\cdot)$ which incurs a financial loss lower than ϕ and induces green investment. Recall that condition (7) implies

$$\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$$

for every $\lambda \geq \underline{\lambda}^*$, where $c(\cdot, S_1^B)$ is the equity contract that breaks even under the firm's choice of financially outperforming project B. This condition guarantees the existence of an equilibrium security $s^*(\cdot)$ that *(i)* gives a financial loss to the green investor (i.e., $\mathbb{E}_B[s^*(x)] \leq 1$); *(ii)* ensures the green investor to earn zero payoff for funding project G at least (i.e., $\mathbb{E}_G[s^*(x)] - 1 + \phi \geq 0$); and *(iii)* is sufficiently sensitive to induce the firm's choice of project G (i.e., $\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] > \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$.). One example of the equilibrium security offer is the equity contract $c(\cdot, S^*)$, where the strike price S^* is slightly higher than S_1^B.[8]

Indeed, the green investor's security bid necessarily claims a small share of cash flow to the firm to the extent that brown investors cannot outbid the green investor with any security offer that yields at least zero financial payoff. For example, suppose a brown investor deviates and offers the least sensitive debt contract $d(\cdot, D_1^B)$ that breaks even under the firm's choice of project B. However, the green investor's equilibrium security bid $s^*(\cdot)$ is designed to provide a lower cost of borrowing to the firm even under brown investment (i.e., $\mathbb{E}_B[s^*(x)] \leq 1 = \mathbb{E}_B[d(x, D_1^B)]$). Hence, no brown investor will not attempt to beat the green investor's equilibrium security bid, which will only incur a financial loss.

However, without active bidding by brown investors, the green investor as a single winning bidder would eventually hold market power, empowering her to charge a larger share of cash flow to the firm than the equilibrium security $s^*(\cdot)$ specifies. Indeed, the green investor, where possible, can strictly increase her payoff by raising her share of cash flow, which will definitely make up her financial loss and even increase the sensitivity of the security in her

[8] Indeed, there exists a $S^* > S_1^B$ such that the security offer $c(\cdot, S^*)$ yields $\mathbb{E}_G[c(x, S^*)] \in (1 - \phi, 1)$ (Lemma 1-(ii)), $\mathbb{E}_B[c(x, S^*)] < 1$ since $\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)] < \phi$ (Lemma 1-(v)), and $\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)] > \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$ for every $\lambda > \underline{\lambda}^*$ (condition (7)).

holding, strengthening the firm's incentive to make green investment. Therefore, to make the green investor's equilibrium bidding strategy incentive compatible, at least one brown investor must place a "losing" bid with a security $\hat{s}(\cdot)$ such that *(i)* the firm will never accept the losing bid unless the green investor claims more cash flow as repayment than her equilibrium security offer specifies; but *(ii)* the green investor will severely lose her payoff once the firm accepts the brown investor's losing bid. The first condition is necessary to ensure that a selected brown investor placing a losing bid will never suffer a financial loss on the equilibrium path, making his bidding strategy incentive compatible.

One example of such a losing security bid is a debt contract $d(\cdot, \hat{D})$ such that *(i)* it induces brown investment if the firm accepts (i.e., $\mathbb{E}_B[d(x,\hat{D})] - \mathbb{E}_G[d(x,\hat{D})] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$) and *(ii)* the firm (weakly) prefers borrowing from the green investor (i.e., $\mathbb{E}_B[x - d(x,\hat{D})] = \mathbb{E}_G[x - c(x,S^*)] + \lambda\phi$). Suppose the green investor deviates and claims a larger share of cash flow as repayment by bidding with a security $s'(\cdot)$ such that $\mathbb{E}_G[s'(x)] > \mathbb{E}_G[c(x,S^*)]$. Due to the increased cost of green investment, the firm will borrow from the brown investor placing the losing bid and invest in project B instead, which will worsen off the deviating green investor.

The upper threshold $\hat{\lambda}^*$ of λ in (9) indeed concerns the feasibility of both winning and losing security bids for the establishment of the subsidized green finance equilibrium. Indeed, condition (9) is rewritten as

$$\mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_G[d(x, D^G_{1-\phi})] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \text{ for all } \lambda \leq \hat{\lambda}^*,$$

enabling to construct a losing security bid $d(\cdot, \hat{D})$ corresponding to a winning security bid $c(\cdot, S^*)$ that characterize the subsidized green finance equilibrium for every $\lambda \in [\underline{\lambda}^*, \hat{\lambda}^*]$. Conversely, if λ exceeds the upper threshold $\hat{\lambda}^*$, the firm will prefer the social return of project G too strongly to choose project B under any losing bid designed to prevent the green investor from raising her bid.[9]

We illustrate an example of the subsidized green finance equilibrium for the case $\lambda = \hat{\lambda}^*$

[9]For example, suppose the green investor offers $s^*(\cdot) = d(\cdot, D^G_{1-\phi})$. Since the firm is indifferent between project G with the winning bid and project B with the losing bid, the losing bid $\hat{s}(\cdot)$ must satisfy $\mathbb{E}_B[x - \hat{s}(x)] = \mathbb{E}_G[x - d(x, D^G_{1-\phi})] + \lambda\phi$ or equivalently $\mathbb{E}_B[\hat{s}(x)] - \mathbb{E}_G[d(x, D^G_{1-\phi})] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. This implies $\mathbb{E}_B[\hat{s}(x)] < \mathbb{E}_B[d(x, D^G_{1-\phi})]$. However, even if $\hat{s}(\cdot)$ is a debt contract $d(\cdot, \hat{D})$, we only have $\hat{D} < D^G_{1-\phi}$. Hence, we must have $\mathbb{E}_B[\hat{s}(x)] - \mathbb{E}_G[\hat{s}(x)] > \mathbb{E}_B[\hat{s}(x)] - \mathbb{E}_G[d(x, D^G_{1-\phi})] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, leading the firm to choose project G, even with the losing bid. Consequently, there is no feasible losing bid to support the subsidized green finance equilibrium.

established by an equity contract $c(\cdot, S^*)$ such that $\mathbb{E}_G[c(x, S^*)] = \mathbb{E}_G[d(x, D^G_{1-\phi})]$ as a winning bid and a debt contract $d(\cdot, D^G_{1-\phi})$ as a losing bid. First, from (9), the winning bid induces green investment:

$$\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)] > \mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_G[d(x, D^G_{1-\phi})] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \hat{\lambda}^*\phi.$$

On the contrary, the losing bid, if accepted by the firm, will induce brown investment:

$$\mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_G[d(x, D^G_{1-\phi})] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \hat{\lambda}^*\phi.$$

Meanwhile, the green investor's winning bid $c(\cdot, S^*)$ satisfies the participation constraint:

$$\mathbb{E}_G[c(x, S^*)] - 1 + \phi = \mathbb{E}_G[d(x, D^G_{1-\phi})] - 1 + \phi \geq 0.$$

Furthermore, the firm weakly prefers to accept the winning bid:

$$\mathbb{E}_G[x - c(x, S^*)] + \hat{\lambda}^*\phi = \mathbb{E}_G[x - d(x, D^G_{1-\phi})] = \mathbb{E}_B[x - d(x, D^G_{1-\phi})].$$

Lastly, the green investor's winning security bid is incentive compatible. Suppose the green investor deviates by offering $s'(\cdot)$ that charges a larger share of cash flow than the winning bid, i.e., $\mathbb{E}_G[s'(x)] > \mathbb{E}_G[c(x, S^*)]$. Even though such an offer is sufficiently sensitive to yield green investment, it will be rejected outright by the firm. Instead, the firm will accept the losing bid and make brown investment, giving zero payoff to the green investor:

$$\mathbb{E}_G[x - s'(x)] + \hat{\lambda}^*\phi < \mathbb{E}_G[x - c(x, S^*)] + \hat{\lambda}^*\phi = \mathbb{E}_B[x - d(x, D^G_{1-\phi})].$$

It is worth noting that both the green finance equilibrium in Theorem 2 and the subsidized green finance equilibrium in Theorem 3 subsume key insights from prior studies on socially responsible investment. For example, Chowdhry, Davies and Waters (2019) show that corporate borrowers are incentivized to pursue non-pecuniary social returns over monetary profit when lenders demand a substantially large share of project cash flow. Under such financing conditions, borrowers retain only a small portion of financial returns – even from profitable but socially harmful projects – thereby making socially responsible investments relatively more attractive. Importantly, these social returns are not appropriated by investors but instead are commonly shared, regardless of the financing terms.

The economic mechanism highlighted in Chowdhry, Davies and Waters (2019) is closely

related to our green finance equilibrium, in which borrowers accept relatively high funding costs to ensure lenders break even financially. However, our analysis goes further: in the subsidized green finance equilibrium, the funding investor suffers a strictly negative financial payoff. This allows us to isolate a key force driving green investment – namely, the borrower's incentive to minimize the expected repayment to the investor. To achieve this, the equilibrium contract must be sufficiently sensitive to the firm's profitability (Lemma 2-(i)). In our framework, the high borrowing cost is not the direct cause of green investment; rather, it is an outcome of using strongly sensitive securities to implement the desired incentive structure.

In contrast to Chowdhry, Davies and Waters (2019), Oehmke and Opp (2025) argue that green investors must accept financial losses to induce green investment, but do so by incorporating binding covenants into financial contracts. Their framework mirrors our subsidized green finance equilibrium in that the green investor willingly takes a loss to support green investment and to deter counter-bidding from brown investors, which would otherwise steer firms toward brown projects. Both analyses highlight a common feature: the green investor's acceptance of financial loss as a strategic barrier against brown investor entry.

Nevertheless, the underlying mechanisms differ significantly. In Oehmke and Opp (2025), the green investor can contractually enforce the firm's project choice via covenants. This allows her to offer a discounted funding rate conditional on the firm committing to green investment. Competitive pressure from brown investors further leads the green investor to offer even deeper discounts, encouraging the firm to size up its green investment.

In our framework, by contrast, the green investor cannot contractually enforce the firm's project choice. She must not only accept a financial loss to outbid brown investors but also design a sufficiently sensitive security to ensure the firm voluntarily selects project G. Crucially, cheap financing alone does not induce green investment unless paired with strong sensitivity. Moreover, the need for subsidized financing arises only in the presence of brown investors, whose bids typically feature low sensitivity – thereby weakening borrowers' incentives to integrate ESG concerns. Our analysis thus suggests that the entry of brown investors into the ESG capital market may hinder ESG integration, especially when borrower commitments are non-binding and subject to ex-post deviation.

Another notable feature of the subsidized green finance equilibrium is that each investor's belief about the degree of competition in the capital market is endogenously determined, naturally giving rise to a multiple equilibria outcome. As discussed above, the green investor's security bid $s^*(\cdot)$ – and the associated financial loss $\mathbb{E}_G[s^*(x)] - 1$ – must be pinned

down by the brown investor's losing bid $\mathbb{E}_B[\hat{s}(x)]$, ensuring that the firm weakly prefers the winning offer in equilibrium; that is, $\mathbb{E}_B[x - \hat{s}(x)] = \mathbb{E}_G[x - s^*(x)] + \lambda\phi$. If the brown investor is believed to demand only a small share of the project's cash flow in their losing bid, the green investor must also lower her share of cash flow accordingly, and vice versa.

This endogenously strong competitiveness not only increases the green investor's financial loss but also reduces the sensitivity of the winning security bid. In the extreme, the winning bid may become too insensitive to guarantee green investment – an implication formalized in the following theorem.

Theorem 4 (mixed-strategy equilibrium)**.** *Define* $\tilde{\lambda}^* \in [\underline{\lambda}^*, \hat{\lambda}^*)$ *as*

$$\tilde{\lambda}^* := \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[c(x, S^G_{1-\phi})] - \mathbb{E}_G[c(x, S^G_{1-\phi})]\right\}\right]. \tag{10}$$

Then there exist mixed-strategy equilibria if and only if $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$*. In each mixed-strategy equilibrium for a given* λ*, the firm randomizes its project selection by choosing project* B *with some probability* $\beta^* \in (0, \tilde{\beta}^*]$*, where* $\tilde{\beta}^* < 1$ *for any* $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$ *and* $\tilde{\beta}^*$ *converges to zero as* λ *approaches* $\tilde{\lambda}^*$*.*

In the "mixed-strategy" equilibrium, the firm randomizes its project selection by choosing project B with probability $\beta^* \in (0, 1)$, i.e., the firm partially pursues financial profit. Figure 6 graphically illustrates the mixed-strategy equilibrium outcome. The strategy profile in the mixed-strategy equilibrium resembles that in the subsidized green finance equilibrium. First, the green investor places a winning but loss-making security bid $s^*(\cdot)$. Furthermore, another (brown) investor must place a losing bid with an insensitive security $\tilde{s}(\cdot)$ such that the green investor will lose the lending competition and suffer the loss of social return should she deviate by slightly increasing her share of cash flow to reduce the financial loss.

A key feature of the mixed-strategy equilibrium is that the firm is indifferent between two projects to randomize in its project selection:

$$\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi. \tag{11}$$

Particularly, to make sure the green investor cannot deviate, she must bid with an equity contract, i.e., $s^*(\cdot) = c(\cdot, \tilde{S}^*)$ for some S^* to satisfy (11). Conditions (9) and (10) guarantee the existence of such S^* such that the green investor gets a positive total payoff under project G (i.e., $\mathbb{E}_G[c(x, \tilde{S}^*)] - 1 + \phi > 0$) for every $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$. Namely, unless the winning bid

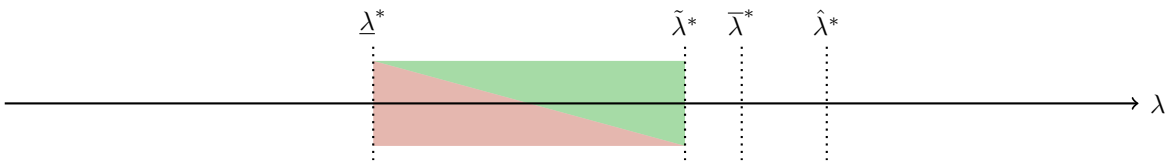


Figure 6 – Mixed-strategy equilibrium: the firm with $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*]$ randomizes in financing project G (depicted in light green) and B (depicted in light brown) at a subsidized borrowing rate (i.e., $\mathbb{E}_G[s^*(x)] < \mathbb{E}_B[s^*(x)] < 1$).

$s^*(\cdot)$ is an equity contract, the green investor can alternatively offer an equity contract $c(\cdot, \tilde{S}')$ that *(i)* is slightly but strictly more sensitive than $s^*(\cdot)$ (i.e., $\mathbb{E}_B[c(x, \tilde{S}')] - \mathbb{E}_G[c(x, \tilde{S}')] > \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$); and *(ii)* provides a lower borrowing cost to the firm but a positive payoff to the deviating green investor (i.e., $1 - \phi < \mathbb{E}_G[c(x, \tilde{S}')] < \mathbb{E}_G[s^*(x)]$). With this deviation bidding strategy, the green investor can discretely increase her total payoff by inducing the firm to choose project G with probability one.

In equilibrium, the firm chooses project B with probability $\beta^* \in (0, 1)$ and G with $1 - \beta^*$, respectively. Although this equilibrium mixed strategy is indeterminate, it must give at least non-negative total payoff to the green investor with the winning bid:

$$\beta^*[\mathbb{E}_B[c(x, \tilde{S}^*)] - 1] + (1 - \beta^*)[\mathbb{E}_G[c(x, \tilde{S}^*)] - 1 + \phi] \geq 0.$$

Like in the subsidized green finance equilibrium, the green investor will suffer disutility from the firm's choice of project B. On the other hand, the green investor enjoys a positive total payoff from the social return if the firm chooses project G. Therefore, the firm's mixed strategy should not choose project B with a higher probability than the upper bound $\tilde{\beta}^*$.

Notice that the equilibrium contract $c(\cdot, \tilde{S}^*)$ generates the largest financial loss to the green investor among all other securities satisfying the firm's indifference condition (11). This follows from Lemma 1-(iv): among all securities that have the same level of sensitivity, equity contracts charge the lowest expected repayment under project G. Thus, any other securities that also make the firm indifferent between the two projects necessarily generate a lower financial loss to the green investor than $c(\cdot, \tilde{S}^*)$ does.

This observation has an interesting implication: increased competitiveness in the ESG capital market may hamper ESG integration. An intensified competition would push the brown investors to offer more generous financing conditions to the firm; in other words, their losing bid $\tilde{s}(\cdot)$ would claim a smaller share of cash flow on average as repayment. If this is

what the green investor believes about the brown investors' bidding strategy, she will respond by charging the firm an even smaller share of cash flow, which in turn lowers the sensitivity of her equilibrium security bid (Lemma 1-(i) and (ii)). With this relatively insensitive security, the firm's incentive to make brown investment is partially recovered compared to the subsidized green finance equilibrium, where the firm surely chooses project G with probability one.[10] In short, endogenous competitiveness in the ESG capital market creates uncertainty, possibly leading to the *market failure* in ESG integration: borrowers may partially keep prioritizing pursuing financial profit to environmental or social values when investors are intensely competitive for funding borrowers' future investments.

This result starkly contrasts with that of Oehmke and Opp (2025), who argue that the green investor's offer of subsidized financing promotes green investments. Further, assuming the borrowing firm can scale up its investment, the green investment is made even on a larger scale when the green investor intensely competes with brown investors. Their claim critically relies on the assumption that the firm's project selection is contractually enforceable, so that the green investor can explicitly tie green investment to subsidized financing. However, with the lack of covenants dictating the firm's ex-post behavior, our analysis reveals that an overly competitive ESG capital market can adversely preserve the firm's incentive to pursue financial profit by choosing project B rather than reduce the payment to the funding investor by choosing project G. Our result also resonates with Chang, Rhee and Yoon (2024), pointing out that intensified competition between investors to provide capital to ESG projects generally deters rather than facilitates ESG integration.

The mechanism underlying Theorem 4 also provides a cautionary implication for policy design of ESG investing: fostering competition among financiers is not unambiguously beneficial for ESG integration. When investors compete by making funding terms increasingly favorable to borrowers, the winning contract may become too insensitive to the firm's ex-post project choice: aggressive concessional bidding reduces the investor's repayment claim and repayment sensitivity, thereby weakening the borrower's private gain from choosing the green project and, in equilibrium, permitting partial reversion toward brown investment. From a policy perspective, this implies that expanding the supply of ESG capital or intensifying competition may backfire by weakening the incentive mechanisms that lead to meaningful ESG follow-through.

[10] As indicated in Figure 2, multiple equilibria coexist for $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$. The validity of the observation here also depends on which equilibrium is selected among them.

6 Debt versus Equity in Public ESG Capital Markets

How can green investors eliminate the possibility of brown investments when the firm's preference for social return is neither strong nor weak? Recall that the mixed-strategy equilibrium in Theorem 4 can exist only when investors are allowed to bid with varying types of securities with different sensitivities. One question of interest then arises: can we eliminate the uncertainty of capital market equilibrium outcomes and induce green investment if every investor can bid with only a single type standard security, such as a debt or equity contract? Indeed, this restriction on the investors' bidding strategies is more suited to public capital markets, where corporate borrowers selectively issue either debt or equity to finance their future investments.

To address this question, we extend our model by assuming that the firm chooses, prior to entering the capital market, whether to issue a debt or equity contract to finance its project. We restrict the firm's choice to debt and equity contracts, rather than allowing the full range of general security types. Moreover, once the firm selects a security type, it must issue the same type to all investors. Once the security type is selected, outside investors compete by submitting contract offers, and the firm accepts the most favorable bid. For instance, if the firm opts for a debt contract $d(\cdot, D)$, the investor offering the lowest face value D wins the lending competition. If the firm chooses an equity contract $c(\cdot, S)$, the investor offering the highest strike price S is selected. We then analyze the firm's optimal security issuance strategy across different informational environments. First, we consider a benchmark case in which the firm's ESG preference parameter λ is publicly known. Next, we analyze equilibrium with asymmetric information, where λ is privately known to the firm.

6.1 Equilibrium with Complete Information on ESG Preferences

We first analyze how the firm's security issuance strategy is determined when λ, representing the firm's inherent ESG preference, is public information. The following observation shows that the firm's ex-post investment decision is indeed pinned down by its ex-ante security choice, while its ex-ante security choice eventually depends on its intrinsic preference for social value over financial profit.

Theorem 5.

(i) Suppose the firm issues debt contracts to finance its project. Then there exists only a

pure-strategy equilibrium where the firm chooses project G if $\lambda \geq \overline{\lambda}^*$ and chooses project B otherwise.

(ii) Suppose the firm issues equity contracts to finance its project. Then there exists only a pure-strategy equilibrium where the firm chooses project G if $\lambda \geq \underline{\lambda}^*$ and chooses project B otherwise.

(iii) When Assumption 1-(i) holds, the firm maximizes its ex-ante expected payoff by issuing debts.

To extract meaningful economic insights from Theorem 5, we focus on cases where the firm has an intermediate preference for social value, that is, $\lambda \in [\underline{\lambda}^*, \overline{\lambda}^*]$. When $\lambda < \underline{\lambda}^*$ (respectively, $\lambda > \overline{\lambda}^*$), the firm strictly prefers project B (respectively, project G), regardless of the type of security used for external financing.

Parts (i) and (ii) of the theorem show that equity financing facilitates green investment, while debt financing tends to discourage it. The intuition is straightforward. Consider first the case where the firm raises capital through debts. In this case, the investor offering the lowest face value wins the lending competition and finances the firm's project. For the green investor to support green investment, she must submit a bid with a relatively high face value such that the firm finds it optimal to choose project G under debt financing, i.e.,

$$\mathbb{E}_B[d(x, D_G)] - \mathbb{E}_G[d(x, D_G)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi.$$

However, from condition (8), we know that

$$\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \text{ for all } \lambda < \overline{\lambda}^*.$$

Therefore, to ensure the firm's choice of project G, the green investor must bid a face value D_G substantially larger than D_1^B, ensuring the debt contract is sufficiently sensitive to induce green investment (Lemma 1-(i)). Yet this creates an opening for profitable deviation by a brown investor. Specifically, condition (8) implies the existence of a face value $D_B \in (D_1^B, D_G)$ such that

$$\mathbb{E}_B[d(x, D_B)] - \mathbb{E}_G[d(x, D_B)] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi.$$

Since $D_B < D_G$, the firm strictly prefers the brown investor's offer due to its lower borrowing cost. Moreover, the firm will choose project B, yielding the brown investor a strictly positive financial payoff – i.e., $\mathbb{E}_B[d(x, D_B)] > \mathbb{E}_B[d(x, D_1^B)] = 1$.

Consider the next case where the firm issues equity. Then condition (7) implies

$$\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_B[c(x, S_1^B)] > \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \text{ for all } \lambda > \underline{\lambda}^*.$$

That is, even the equity contract that breaks even an investor under the selection of project B induces the firm's choice of project G. Therefore, the brown investors rationally infer that the firm's ex-post selection must be project G and thus offer $c(\cdot, S_1^G)$ to ensure they will not incur any financial loss from funding green investment. Like we showed in the green finance equilibrium as in Theorem 2, the green investor now has no incentive to outbid the brown investors' competitive equity bid but enjoys the free-riding benefit ϕ.

In sum, the structural insensitivity of debt contracts fails to align the firm's incentives with green investment objectives, instead leading to brown investment even when the firm's preference for social value is relatively strong (i.e., $\lambda \geq \underline{\lambda}^*$). By contrast, the inherently high sensitivity of equity contracts restores the firm's incentive to reduce expected repayment to outside investors, thereby making financially under-performing green investment even when the firm's preference for social value is relatively weak (i.e., $\lambda \leq \overline{\lambda}^*$).

The desirability of equity financing for inducing green investment sharply contrasts with the established literature advocating the superiority of debt financing in the presence of asymmetric information. Numerous studies, including Myers and Majluf (1984), Nachman and Noe (1994), DeMarzo and Duffie (1999), and Dang, Gorton and Holmström (2013), argue that the informational insensitivity of debt minimizes the severity of adverse selection between borrowers and lenders. Because the expected payoff from debt is relatively unresponsive to borrower quality, financially unproductive firms extract only limited informational rents by mimicking productive ones. In addition, DeMarzo, Kremer and Skrzypacz (2005) show that debt issuance can serve as a credible signal of firm quality. Similarly, Townsend (1979) and Innes (1990) demonstrate the optimality of debt in addressing moral hazard: debt contracts reward effort by granting large residual claims on high cash flow, thereby incentivizing managers to increase project returns. In both adverse selection and moral hazard contexts, the appeal of debt financing stems from its financial payoff structure.

A key ingredient that differentiates our analysis from the literature is the consideration of non-pecuniary social benefits in corporate decision-making. As discussed by Lemma 2, the additional social return attached to project G is necessary for the firm to have an incentive to reduce the expected payment to the funding investors, despite the resulting net financial loss. Furthermore, the firm's incentive to lower its average payment strengthens as the securities

issued by the firm to finance its investment are sensitive to the investment decision. Consequently, firms with more equity can better incorporate ESG. Further, investors can consider financing the firm through equity to promote socially desirable outcomes. This could maximize corporate borrowers' incentive to integrate ESG by increasing the sensitivity of their capital structure.

Nevertheless, part (iii) reveals that a competitive capital market facilitates debt financing. Under Bertrand competition, the winning investor always earns zero expected financial payoff from funding the firm's project, regardless of the security type chosen *ex ante*. As a result, since there is no additional gains in expected repayment under either debt or equity financing, the firm simply selects the security type that maximizes its own total surplus. By Assumption 1-(i), the firm would prefer project B if it were able to self-finance. Accordingly, in equilibrium, it chooses to finance its project with debt.

Our findings suggest that imposing a uniform restriction on security bids – such as limiting them to a standard type – can in fact *exacerbate* the incentive misalignment in green investment. Regardless of the security type, borrowers rationally anticipate facing the lowest possible borrowing cost in a competitive ESG capital market. As a result, their choice of financing instrument depends entirely on their subjective assessment of total surplus – that is, the combined financial and non-financial (social) returns – associated with each project. Notably, when borrowers have a relatively weak preference for social value and would opt for brown investment under self-financing, they will rationally select debt contracts, given their structural insensitivity and compatibility with brown investment incentives.

We make two final remarks before concluding this section. First, testable predictions for empirical research could be drawn from our analysis. Specifically, a researcher could examine whether companies with a higher proportion of equity financing demonstrate a greater commitment to ESG initiatives compared to those primarily financed through debt. Recent related papers explore a similar theme but from a different angle. For example, Kräussl, Rauh and Stefanova (2024) empirically examine the effects of ESG ratings on firms' security issuance and find that higher environmental scores are associated with increases in equity issuance and decreases in debt issuance.

Second, considering equity as a means to deter greenwashing is highly relevant amid growing scrutiny of ESG claims and green finance. Recently, there have been concerns about greenwashing among bond issuers. For instance, Aswani and Rajgopal (2022) find that firms issuing green bonds often exhibit little ESG follow-through, and the market fails to see through

this. Although greenwashing in equity investing has also been documented (Kim and Yoon, 2023; Raghunandan and Rajgopal, 2022*a*), more recent data reveal a decline in greenwashing in the space (Pástor, n.d.; Pástor, Stambaugh and Taylor, 2023). Moreover, equity markets are generally more transparent and subject to stricter regulation, facilitating monitoring and verification of a company's ESG commitments. Our theory not only provides a foundation for empirically testable hypotheses but also contributes to the broader discourse on enhancing the credibility and effectiveness of ESG integration in corporate finance.

6.2 The Pecking Order Theory: Revisited for ESG Integration

In this section, we extend our analysis to a setting where the firm's true preference for social value is privately known. In standard capital structure theory, the seminal work of Myers and Majluf (1984) shows that highly productive firms can signal their quality by issuing debt before equity, thereby reducing the cost of external financing. Building on this insight, we examine whether corporate borrowers can similarly benefit by signaling their strong preference for social value to pro-ESG investors, and whether such signaling is more effectively achieved through debt or equity issuance.

We further extend our model in Section 6.1 by assuming that the firm privately knows its preference for social value before it chooses whether to issue debt or equity. Specifically, we assume that λ takes one of two values, $\lambda_B \in [0, \underline{\lambda}^*)$ and $\lambda_G \in (\underline{\lambda}^*, \overline{\lambda}^*)$ (thus, $\lambda_B < \lambda_G$). The firm with $\lambda = \lambda_B$ (respectively, the firm with $\lambda = \lambda_G$) is referred to as "brown" (respectively, "green"), based on its project choice in the case where λ is publicly revealed to the market. The firm with $\lambda = \lambda_B$ will always choose project B, regardless of the sensitivity of the security it issues for external financing (Theorem 1), while the green firm with $\lambda = \lambda_G$ may choose project G if the security held by the funding investor is sufficiently sensitive (Theorem 5-(i) and (ii)). Finally, we also focus on the case in which both $\lambda = \lambda_B$ and $\lambda = \lambda_G$ satisfy Assumption 1, i.e., neither green nor brown firm will prefer investing in project G should they self-finance their projects.

Since we study whether the firm can use its choice of security issuance as a signaling instrument, we throughout focus on a separating equilibrium in which green and brown firms issue different types of securities and choose different projects. First, the following lemma characterizes the green firm's signaling strategy.

Lemma 3. *In any separating equilibrium where two firms issue different types of security*

and choose different projects, the green firm must finance its project with an equity contract, whereas the brown firm must finance its project with a debt contract.

Due to the structural insensitivity of debt, the green firm is rationally anticipated to choose project B *ex post* if it issues debt (Theorem 5-(i)). Hence, the only possible way the green firm can signal its relatively strong preference for social value is to issue equity, leading to its ex-post choice of project G (Theorem 5-(ii)). However, the green firm will not choose to issue equity unless doing so yields a benefit from revealing its true social preference. For example, suppose the green firm is funded by a brown investor through an equity contract. Since any brown investor selected to fund the firm must be guaranteed at least a zero financial payoff, the brown investor must offer the break-even equity contract $c(\cdot, S_1^G)$, yielding the green firm's expected payoff as follows:

$$\mathbb{E}_G[x - c(x, S_1^G)] + \lambda_G \phi = \mathbb{E}_G[x] - 1 + \lambda_G \phi.$$

Meanwhile, the brown firm must issue debt to finance its brown investment, prompting brown investors to offer a breakeven debt contract under project B, i.e., $d(\cdot, D_1^B)$ such that $\mathbb{E}_B[d(x, D_1^B)] = 1$. The green firm could then profitably deviate by mimicking the brown firm and issuing the same debt contract, yielding a strictly higher payoff:

$$\mathbb{E}_B[x - d(x, D_1^B)] = \mathbb{E}_B[x] - 1 > \mathbb{E}_G[x] - 1 + \lambda_G \phi = \mathbb{E}_G[x - c(x, S_1^G)] + \lambda_G \phi.$$

This result aligns with Theorem 5-(iii), which shows that even the green firm may prefer brown investment if the expected borrowing cost does not vary with its choice of security issuance.

Therefore, the green firm must secure more favorable financing terms through equity issuance, which serves as a credible commitment to green investment. The key advantage of such signaling lies in the potential for subsidized financing from a green investor, who is willing to forgo up to ϕ in financial return to support green investment. Namely, for equity financing to be incentive compatible, the expected repayment the green firm makes to a funding (green) investor under an equilibrium equity contract $c(\cdot, S^*)$ must be sufficiently low:

$$\mathbb{E}_B[x - d(x, D_1^B)] \leq \mathbb{E}_G[x - c(x, S^*)] + \lambda_G \phi \implies \mathbb{E}_G[c(x, S^*)] \leq 1 - (\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda_G \phi).$$

This raises the next question: under what circumstances will a green investor offer such a discount lending to the firm by internalizing her social gains from funding project G? To address

this, we first generalize market structure by allowing multiple competitive green investors. We also introduce a new type of green investor defined as follows.

Definition 3 (Dangl et al. (2025)). *A green investor is a non-consequentialist if she gets an extra utility $\rho \in (0,1)$ from funding the firm's project G directly.*

Throughout our analysis so far, we have focused on a *consequentialist* green investor – one who values the social return equally, no matter who finances the socially valuable project. By contrast, a *non-consequentialist* green investor derives *additional* utility ρ from directly contributing to the green investment and thus values her own role in the funding decision.[11] Formally, if a non-consequentialist green investor funds the firm using a security $s(\cdot)$, her payoff is given by

$$\mathbb{E}_i[s(x)] - 1 + (\phi + \rho)\mathbb{I}_G(i) \text{ for each } i \in \{G, B\},$$

where $\mathbb{I}_G(i)$ is the indicator function whose value is one (respectively, zero) if and only if $i = G$ (respectively, $i = B$). However, if the non-consequentialist green investor does not directly fund the firm, her payoff is simply equal to

$$\phi\mathbb{I}_G(i).$$

As was seen above in Theorem 2, a consequentialist green investor passively secures the free-riding benefit whenever another investor is expected to fund green investment. Unlike a consequentialist, a non-consequentialist green investor may not stay passive to free-ride on the brown investors' financing of green investment. While a non-consequentialist investor still receives the social return ϕ without directly funding project G, she obtains a total payoff of $\mathbb{E}_G[c(x,S)] - 1 + \rho + \phi$ if she wins the lending competition with an equity contract $c(x,S)$ sensitive enough to induce green investment. Thus, non-consequentialist green investors have an incentive to actively bid as long as doing so does not entail an excessive financial loss, i.e., $\mathbb{E}_G[c(x,S)] \geq 1 - \rho$. This implies a non-consequentialist green investor can hold an incentive-compatible equity contract to finance the green firm if ρ, the additional utility of direct green financing, is sufficiently high. However, if a non-consequentialist green investor offers an excessively large discount in lending for securing the high utility of direct green financing, the brown firm may have an ex-ante incentive to mimic the green firm by issuing equity instead of debt.

[11] A number of theoretical and empirical studies – such as Goldstein et al. (2022), Gollier and Pouget (2014), and Green and Roth (2025) – discusses the extra non-pecuniary utility from direct engagement with green firms ("warm glow") and divestment in brown firms ("cold prickle").

Consequently, the existence of a separating equilibrium critically depends on the structure of competition in the ESG capital market, which may shape the incentive of non-consequentialist green investor(s) – if one exists – to actively engage in the lending game.

Theorem 6.

(i) The separating equilibrium cannot exist if there is at most one non-consequentialist green investor;

(ii) When there are multiple non-consequentialist green investors, there exist two thresholds $0 \leq \underline{\rho}^* \leq \overline{\rho}^*$ *such that the separating equilibrium exists if and only if* $\rho \in [\underline{\rho}^*, \overline{\rho}^*]$. *In equilibrium, the green firm issues equity and a selected non-consequentialist green investor offers a contract* $c(\cdot, S^*)$ *which incurs a financial loss, i.e.,* $\mathbb{E}_G[c(x, S^*)] < 1$.

If the green firm reveals its type by issuing equity, brown investors will competitively bid using an equity contract $c(\cdot, S_1^G)$ that breaks even under the firm's ex-post choice of project G, i.e., $\mathbb{E}_G[c(x, S_1^G)] = 1$. However, consequentialist green investors have no incentive to incur a financial loss to outbid brown investors; instead, they remain passive in the lending competition and free-ride on the social return ϕ. Even if a single non-consequentialist green investor is present in the market, she is unlikely to offer a substantial lending discount. Rather, she can outbid brown investors and secure the additional utility from directly financing the green project by marginally increasing the strike price, thereby minimizing her financial loss. In such market environments, the green firm cannot reduce its borrowing cost through equity issuance and will therefore prefer to pursue financial profit by issuing debt and investing in the brown project.

A key obstacle to subsidized green financing arises when there is only a single non-consequentialist green investor, who thus holds strong market power. In this case, she has little incentive to internalize her utility from direct financing of project G, as her competitive advantage in security bidding is preserved without offering more favorable terms. By contrast, when multiple non-consequentialist green investors are present in the capital market, competition among them to secure ρ – the extra utility from direct green financing – drives down the green firm's cost of borrowing through equity issuance. Specifically, these non-consequentialist green investors will compete for lending by fully internalizing their utility from financing project G directly, lowering the expected payoff of the equilibrium equity contract $c(\cdot, S^*)$. For the non-consequentialist green investors to give a large lending discount,

ρ – the utility from direct green financing – must be sufficiently high (i.e., $\rho \geq \underline{\rho}^*$) to ensure that the green firm's equilibrium strategy of equity issuance is incentive compatible.

However, an overly competitive equity market may distort the brown firm's financing incentives, inducing its deviation to equity financing. Specifically, when ρ becomes excessively large, the lending discount offered by non-consequentialist green investors to any equity issuer perceived as green type increases substantially. Anticipating this large subsidized cost of borrowing, the brown firm finds its prescribed equilibrium strategy of issuing debt strictly dominated and mimics the green type by issuing equity. Such mimicking cannot support the lending discount offered by non-consequentialist green investors because they cannot fully internalize the utility of funding green investment due to the concern of possible financing of the brown firm, thereby collapsing the separating equilibrium. Hence, to sustain the incentive compatibility of debt financing for the brown firm, ρ must be bounded above – that is, $\rho \leq \overline{\rho}^*$.

Key takeaways from our analysis are as follows. First, there is a *reverse* pecking order in capital structure to facilitate ESG integration: corporate borrowers with a strong intrinsic preference for ESG can secure cheap financing by signaling their commitment through *equity* issuance. This contrasts with the classical pecking order theory of Myers and Majluf (1984), arguing that companies can signal their productivity by prioritizing debt to equity for external financing. A key economic insight is that companies with higher financial profitability are more likely to meet the repayment obligations and thus secure a larger residual income under debt financing, whereas under-profitable companies prefer to share downside risks with outside investors under an equity contract. In contrast, our analysis shows that ESG-oriented borrowing firms may secure cheap financing by revealing their strong ESG preferences through equity issuance to pro-ESG investors, who are willing – if necessary – to tolerate financial losses in order to subsidize the borrowing costs of socially valuable but financially underperforming investments.

However, subsidized green equity financing can only materialize when pro-ESG investors have a clear incentive to internalize the future social value generated by their investments. Our analysis suggests that this incentive arises when multiple *non-consequentialist* pro-ESG investors actively compete to directly finance ESG-oriented firms in capital markets. In contrast, *consequentialist* pro-ESG investors – who care only about the final outcome of ESG investing – have no incentive to offer discounted equity financing and instead free-ride on the contributions of others to green investment. Moreover, when there is only a single non-consequentialist investor, her strong competitive advantage allows her to secure direct funding opportunities without offering a meaningful discount, thereby failing to induce ESG-oriented

firms to signal their commitment through equity issuance. These findings imply that competition among non-consequentialist ESG-friendly investors facilitates establishing a reverse pecking order in ESG investing – one that prioritizes equity over debt.

Nevertheless, our analysis highlights a seemingly counterintuitive but important implication for ESG investing: overly enthusiastic non-consequentialist investors may inadvertently hinder, rather than promote, green financing. When such investors offer heavily subsidized funding to any equity issuer perceived as ESG-oriented, non-ESG firms may strategically mimic green firms by issuing equity to access cheaper capital. This adverse selection problem undermines the ability of genuinely ESG-oriented firms to credibly signal their commitment through equity issuance. As a result, non-consequentialist investors lose the incentive to compete in financing truly green projects, ultimately eroding the effectiveness of subsidized financing for ESG integration in equity markets. This finding suggests that pro-ESG investors' moderate willingness, rather than unconditional willingness, to subsidize ESG investment may better support credible signaling and foster more effective ESG integration in practice.

Our analysis also provides an alternative interpretation of ESG investing through the lens of *additionality* – that is, the extent to which investors' capital and contract choices induce green investment that would not arise under the counterfactual market allocation. In our framework, non-consequentialist green investors play a pivotal role in generating such additionality because they are willing to provide concessional financing to directly promote ESG follow-through. However, the adverse-selection mechanism highlighted above reveals that exceptionally generous subsidization by non-consequentialists can undermine additionality by encouraging non-ESG firms to mimic ESG-oriented issuers and thereby eroding the informativeness of equity issuance. The key concern, therefore, is whether market conditions prevent indiscriminate, aggressive concessional bidding that collapses this signaling channel. Practically, our results suggest that ESG capital provided by non-consequentialist investors is most effective when market conditions temper "over-enthusiastic" subsidization and preserve a meaningful wedge between genuinely ESG-oriented issuers and imitators, rather than when subsidies are offered unconditionally to any issuer labeled as ESG-oriented.

7 Conclusion

This paper theoretically dissects the economic interaction between corporate issuers' capital structure and their incentive to pursue ESG investments. The crucial economic friction on

ESG integration is that lenders cannot adopt any incentive scheme directly conditional on borrowers' ESG performance, which is hard to verify without credible and transparent ESG metrics that are audited and assured. Given that we do not currently have such a widely accepted and developed assurance system, it is important to understand when corporate borrowers follow through on their ESG commitments. This paper highlights the significant role of companies' capital structure in their incentive for ESG integration.

The key takeaways are as follows. First, the sensitivity of securities issued to finance companies' new projects critically shapes their incentive to integrate ESG. Companies can reduce the expected payments to their funding investors by pursuing ESG projects, which tend to be financially less profitable. The financial gains from repayment reduction, combined with the social return that ESG projects generate, can outweigh the loss in the value of their assets – the expected cash flow of the companies' projects, particularly when the expected payoffs of securities held by the funding investors substantially change with the profitability of the companies' projects specified as the underlying assets.

Second, market-based mechanisms alone may not be sufficient to ensure ESG integration by corporate issuers. In private capital markets where outside investors have broad flexibility in designing securities, intensified competition among investors can lead to reduced sensitivity in security bids. In public capital markets, where firms choose among standardized securities such as debt or equity, debt financing tends to be preferred by firms placing less weight on social returns. Taken together, market dynamics may ultimately undermine firms' incentives to prioritize social value over financial returns, hampering rather than facilitating ESG integration. Nevertheless, in the presence of asymmetric information about corporate borrowers' true willingness to engage in ESG investing, a moderate level of investor competition can foster a favorable market environment for ESG integration. Specifically, pro-ESG firms can access lower-cost capital by signaling their commitment to ESG through equity issuance to non-consequentialist ESG-friendly investors – those willing to accept financial losses in exchange for the utility from directly supporting ESG investment.

Our analysis provides a theoretical explanation for the recent rise in debt issuance within green finance. It also reveals that the role of security sensitivity in shaping moral hazard in ESG integration diverges fundamentally from its counterpart in standard corporate finance. Finally, we propose a novel pecking order theory of capital structure for ESG integration, highlighting corporate borrowers' potential advantages of equity issuance in financing socially valuable projects. We hope this research will inform future policy discussions and inspire empirical work on greenwashing and ESG-related incentives in corporate finance.

References

Admati, Anat, and Paul Pfleiderer. 1994. "Robust Financial Contracting and the Role of Venture Capitalists." *Journal of Finance*, 49(2): 371–402.

Aghamolla, Cyrus, and Byeong-Je An. 2023. "Mandatory vs. Voluntary ESG Disclosure, Efficiency, and Real Effects." *Nanyang Business School Research Paper No. 22-41.*

Allen, Franklin, Adelina Barbalau, and Federica Zeni. 2023. "Reducing carbon using regulatory and financial market tools." *http://dx.doi.org/10.2139/ssrn.4357160*.

Aswani, Jitendra, and Shivaram Rajgopal. 2022. "Rethinking the Value and Emission Implications of Green Bonds." *https://ssrn.com/abstract=4215882*.

Axelson, Ulf. 2007. "Security Design with Investor Private Information." *Journal of Finance*, 62(6): 2587–2632.

Azarmsa, Ehsan, and Joel Shapiro. 2022. "The Market for ESG Ratings." Unpublished Manuscript.

Barbalau, Adelina, and Federica Zeni. 2022. "The Optimal Design of Green Securities." *http://dx.doi.org/10.2139/ssrn.4155721*.

Berg, Florian, Julian F Koelbel, and Roberto Rigobon. 2022. "Aggregate Confusion: The Divergence of ESG Ratings." *Review of Finance*, 26(6): 1315–1344.

Bhagat, Sanjai, and Aaron Yoon. 2023. "Corporate Green Bonds: Market Response and Corporate Response." *https://ssrn.com/abstract=4492883*.

Bloomberg. 2024. "Backlash Against ESG Seen in Sharp Decline of Fund Launches." *https://www.bloomberg.com/news/articles/2024-07-18/backlash-against-esg-seen-in-sharp-decline-of-fund-launches*.

BloombergNEF. 2025. "Sustainable Finance Market Outlook 2025." Accessed 2025-05-18.

Bonham, Jonathan, and Amoray Riggs-Cragun. 2022. "Motivating ESG Activities through Contracts, Taxes and Disclosure Regulation." *Chicago Booth Research Paper 22-05.*

Carroll, Gabriel. 2015. "Robustness and Linear Contracts." *American Economic Review*, 105(2): 536–563.

Cartellier, Fanny, Peter Tankov, and Olivier David Zerbib. 2023. "Can Investors Curb Greenwashing?" *https://ssrn.com/abstract=4644741*.

Chaigneau, Pierre, and Nicolas Sahuguet. 2025. "Executive Compensation with Social and Environmental Performance." *Review of Finance*, 29(3): 779–818.

Chang, Dongkyu, Keeyoung Rhee, and Aaron Yoon. 2024. "Financial Market Structure for ESG Integration." *https://ssrn.com/abstract=4325281*.

Chatterji, Aaron, David Levine, and Michael Toffel. 2021. "How Well Do Socially Responsible Investing Strategies Perform?" Harvard Business Review.

Chowdhry, Bhagwan, Shaun William Davies, and Brian Waters. 2019. "Investing for Impact." *Review of Financial Studies*, 32(3): 864–904.

Choy, Stacey, Shushu Jiang, Scott Liao, and Emma Wang. 2021. "Public Environmental Enforcement and Private Lender Monitoring: Evidence from Environmental Covenants." *http://dx.doi.org/10.2139/ssrn.3860178*.

Christensen, Hans B., Eric Floyd, Lisa Liu, and Mark Maffett. 2017. "The Real Effects of Mandated Information on Social Responsibility in Financial Reports: Evidence from Mine-Safety Records." *Journal of Accounting and Economics*, 64(2): 284–304.

Dangl, Thomas, Michael Halling, Jin Yu, and Josef Zechner. 2025. "Social Preferences and Corporate Investment." *Journal of Financial Economics*, forthcoming.

Dang, Tri Vi, Gary Gorton, and Bengt Holmström. 2013. "The Information Sensitivity of a Security." *Unpublished Working Paper, Yale University.*

DeMarzo, Peter, and Darrell Duffie. 1999. "A Liquidity-Based Model of Security Design." *Econometrica*, 67(1): 65–99.

DeMarzo, Peter M, Ilan Kremer, and Andrzej Skrzypacz. 2005. "Bidding with Securities: Auctions and Security Design." *American Economic Review*, 95(4): 936–959.

Edmans, Alex, Doron Levit, and Jan Schneemeier. 2022. "Socially Responsible Divestment." *http://dx.doi.org/10.2139/ssrn.4093518*.

Feldhütter, Peter, and Lasse Heje Pedersen. 2022. "Is Capital Structure Irrelevant with ESG Investors?" *https://ssrn.com/abstract=4227547*.

Fiechter, Peter, Jörg-Markus Hitz, and Nico Lehmann. 2022. "Real Effects of a Widespread CSR Reporting Mandate: Evidence from the European Union's CSR Directive." *Journal of Accounting Research*, 60(4): 1499–1549.

Flammer, Caroline. 2021. "Corporate Green Bonds." *Journal of Financial Economics*, 142(2): 499–516.

Fombrun, Charles, and Mark Shanley. 1990. "What's in a Name? Reputation Building and Corporate Strategy." *Academy of Management Journal*, 33(2): 233–258.

Friedman, Henry L, Mirko Stanislav Heinle, and Irina Luneva. 2021. "A Theoretical Framework for ESG Reporting to Investors." *https://ssrn.com/abstract=3932689*.

Gipper, Brandon, Samantha Ross, and Shawn Shi. 2022. "ESG Assurance in the United States." *http://dx.doi.org/10.2139/ssrn.4263085*.

Goldstein, Itay, Alexandr Kopytov, Lin Shen, and Haotian Xiang. 2022. "On ESG Investing: Heterogeneous Preferences, Information, and Asset Prices." *http://www.nber.org/papers/w29839*.

Gollier, Christian, and Sébastien Pouget. 2014. "The "Washing Machine": Investment Strategies and Corporate Behavior with Socially Responsible Investors." TSE Working Paper.

Green, Daniel, and Benjamin Roth. 2025. "The Allocation of Socially Responsible Capital." *Journal of Finance*, 80(2): 755–781.

Grewal, Jody, Edward J Riedl, and George Serafeim. 2019. "Market Reaction to Mandatory Nonfinancial Disclosure." *Management Science*, 65(7): 3061–3084.

Hart, Oliver, and John Moore. 1995. "Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management." *American Economic Review*, 85(3): 567–585.

Inderst, Roman, and Holger M Mueller. 2006. "Informed Lending and Security Design." *Journal of Finance*, 61(5): 2137–2162.

Innes, Robert D. 1990. "Limited Liability and Incentive Contracting with Ex-ante Action Choices." *Journal of Economic Theory*, 52(1): 45–67.

Jensen, Michael C. 1986. "Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers." *American Economic Review*, 76(2): 323–329.

Kim, Soohun, and Aaron Yoon. 2023. "Analyzing Active Fund Managers' Commitment to ESG: Evidence from the United Nations Principles for Responsible Investment." *Management science*, 69(2): 741–758.

Kräussl, Roman, Joshua Rauh, and Denitsa Stefanova. 2024. "ESG Ratings and Firms' Financing Decisions." *Unpublished Manuscript*.

Krueger, Philipp, Zacharias Sautner, Dragon Yongjun Tang, and Rui Zhong. 2021. "The Effects of Mandatory ESG Disclosure Around the World." *European Corporate Governance Institute – Finance Working Paper No. 754/2021, Swiss Finance Institute Research Paper No. 21-44*.

Liang, Hao, Lin Sun, and Melvyn Teo. 2022. "Responsible Hedge Funds." *Review of Finance*, 26(6): 1585–1633.

Maksimovic, Vojislav, and Sheridan Titman. 1991. "Financial Policy and Reputation for Product Quality." *Review of Financial Studies*, 4(1): 175–200.

Malenko, Andrey, and Anton Tsoy. 2023. "Asymmetric Information and Security Design under Knightian Uncertainty." *Journal of Finance*, forthcoming.

Manili, Julien. 2024. "Optimism and Voluntary Disclosure: A Theory of Corporate Greenwashing." *https://ssrn.com/abstract=4704323*.

Milgrom, Paul R. 1981. "Rational Expectations, Information Acquisition, And Competitive Bidding." *Econometrica*, 49(4): 921–943.

Myers, Stewart C, and Nicholas S Majluf. 1984. "Corporate Financing and Investment Decisions When Firms Have Information that Investors Do Not Have." *Journal of Financial Economics*, 13(2): 187–221.

Nachman, David C, and Thomas H Noe. 1994. "Optimal Design of Securities under Asymmetric Information." *Review of Financial Studies*, 7(1): 1–44.

OECD. 2021. "ESG Investing and Climate Risk Integration in Financial Markets." Accessed 2025-05-18.

Oehmke, Martin, and Marcus Opp. 2025. "A Theory of Socially Responsible Investment." *Review of Economic Studies*, 92(2): 1193–1225.

Pástor, Lubos. n.d.. "Sustainable Investing in Equilibrium."

Pástor, L'uboš, Robert F Stambaugh, and Lucian A Taylor. 2023. "Green Tilts." *http://www.nber.org/papers/w29839*.

Preqin. 2025. "Global Private Debt Report 2025." Accessed 2025-05-18.

Raghunandan, Aneesh, and Shiva Rajgopal. 2022*a*. "Do ESG Funds Make Stakeholder-Friendly Investments?" *Review of Accounting Studies*, 27(3): 822–863.

Raghunandan, Aneesh, and Shivaram Rajgopal. 2022*b*. "Do Socially Responsible Firms Walk the Talk?" *http://dx.doi.org/10.2139/ssrn.3609056*.

Ravid, S Abraham, and Matthew Spiegel. 1997. "Optimal Financial Contracts for a Start-up with Unlimited Operating Discretion." *Journal of Financial and Quantitative Analysis*, 32(3): 269–286.

Reuters. 2020. "Volkswagen Says Diesel Scandal Has Cost It 31.3 Billion Euros." *https://www.reuters.com/article/business/volkswagen-says-diesel-scandal-has-cost-it-313-billion-euros-idUSKBN2141JA/*.

SEC. 2022. "SEC Proposes to Enhance Disclosures by Certain Investment Advisers and Investment Companies About ESG Investment Practices." [Press Release], May 25, 2022, https://www.sec.gov/newsroom/press-releases/2022-92.

Serafeim, George, and Aaron Yoon. 2023. "Stock Price Reactions to ESG News: The Role of ESG Ratings and Disagreement." *Review of Accounting Studies*, 28: 1500–1530.

S&P Global. 2023. "The Rising Importance of Sustainability in Credit Risk." Accessed 2025-05-18.

Tang, Dragon Yongjun, and Yupu Zhang. 2020. "Do Shareholders Benefit from Green Bonds?" *Journal of Corporate Finance*, 61: 101427.

Townsend, Robert M. 1979. "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory*, 21(2): 265–293.

Turban, Daniel B, and Daniel W Greening. 1997. "Corporate Social Performance and Organizational Attractiveness to Prospective Employees." *Academy of Management Journal*, 40(3): 658–672.

Waddock, Sandra A, and Samuel B Graves. 1997. "The Corporate Social Performance – Financial Performance Link." *Strategic Management Journal*, 18(4): 303–319.

Xue, Hao. 2023. "ESG Disclosure, Market Forces, and Investment Efficiency." *https://ssrn.com/abstract=4344253*.

Yang, Ming, and Yao Zeng. 2019. "Financing Entrepreneurial Production: Security Design with Flexible Information Acquisition." *Review of Financial Studies*, 32(3): 819–863.

A Appendix: Proofs

A.1 Proof of Lemma 1

The proof of parts (i) – (iv) suffices to establish the following lemma.

Lemma A.1. *Fix any two securities $s(\cdot)$ and $t(\cdot) : [0, \infty) \to [0, \infty)$ such that either $\mathbb{E}_G[s(x)] = \mathbb{E}_G[t(x)]$ or $\mathbb{E}_B[s(x)] = \mathbb{E}_B[t(x)]$ holds. Then, $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] \geq \mathbb{E}_B[t(x)] - \mathbb{E}_G[t(x)]$ if there exists a cutoff $x^* \in [0, \infty)$ such that*

$$\begin{aligned} s(x) &\geq t(x) \quad \text{for any } x \geq x^*, \\ s(x) &\leq t(x) \quad \text{for any } x \leq x^*. \end{aligned} \tag{A.1}$$

Furthermore, $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] > \mathbb{E}_B[t(x)] - \mathbb{E}_G[t(x)]$ if the inequalities in (A.1) hold for a positive measure of $x \in [0, \infty)$.

Proof. In this proof, we focus on the case that $\mathbb{E}_G[s(x)] = \mathbb{E}_G[t(x)]$. The proof for the other case is essentially identical hence omitted here. Indeed, we have the following inequalities:

$$\begin{aligned} \mathbb{E}_B[s(x) - t(x)] &= \int_{x \leq x^*} [s(x) - t(x)] \frac{f_B(x)}{f_G(x)} f_G(x) dx + \int_{x > x^*} [s(x) - t(x)] \frac{f_B(x)}{f_G(x)} f_G(x) dx \\ &\geq \frac{f_B(x^*)}{f_G(x^*)} \int_{x \leq x^*} [s(x) - t(x)] f_G(x) dx + \frac{f_B(x^*)}{f_G(x^*)} \int_{x > x^*} [s(x) - t(x)] f_G(x) dx \end{aligned}$$

where the inequality follows from MLRP and (A.1). Thus,

$$\mathbb{E}_B[s(x) - t(x)] \geq \frac{f_B(x^*)}{f_G(x^*)} \int_0^\infty [s(x) - t(x)] f_G(x) dx = \frac{f_B(x^*)}{f_G(x^*)} \mathbb{E}_G[s(x) - t(x)] = 0,$$

which implies $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] \geq \mathbb{E}_B[t(x)] - \mathbb{E}_G[t(x)]$. The above inequalities hold strictly whenever the inequalities (A.1) hold for a positive measure of $x \in [0, \infty)$. *Q.E.D.*

To prove part (v), fix any security $s(\cdot)$ that satisfies the properties stated in the lemma. Let $\varepsilon > 0$ be defined such that $\mathbb{E}_G[x - \varepsilon] = \mathbb{E}_G[s(x)]$. Note that $\mathbb{E}_B[x - s(x)] \geq \mathbb{E}_G[x - s(x)] = \varepsilon$ by the FOSD property, and thus, we also have $\mathbb{E}_B[x - \varepsilon] \geq \mathbb{E}_B[s(x)]$. Then, $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] \leq \mathbb{E}_B[x - \varepsilon] - \mathbb{E}_G[x - \varepsilon] = \mathbb{E}_B[x] - \mathbb{E}_G[x] < \phi$, where the last strict inequality follows from Assumption 1-(i).

A.2 Proof of Theorem 1

Note that $\lambda \leq \underline{\lambda}^*$ if and only if

$$\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \tag{A.2}$$

where $\mathbb{E}_G[c(x, S_1^B)] < \mathbb{E}_B[c(x, S_1^B)] = 1$ by MLRP and the construction of S_1^B. We first prove the sufficiency of (A.2) and then the necessity of (A.2).

A.2.1 Proof of Sufficiency

Let $s^*(x) := c(x, S_1^B)$ and suppose that all brown investors offer the same contract s^*. The green investor abstains from offering any security acceptable to the firm (e.g., the green investor offers $s(x) \equiv x$). Because the sensitivity of s^* is weakly less than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, it is (weakly) optimal for the firm to accept s^* and then choose project B. Thus, it suffices to show that all investors also find it weakly optimal to offer s^*, given that all other investors also offer the same contract. Note that all investors get zero expected payoff from offering s^*.

First, we prove that it is optimal for each brown investor to offer s^*. Suppose that a brown investor deviates and offers s' such that, if accepted, the firm would choose project B. For s' to be a profitable deviation, we must have $\mathbb{E}_B[s'(x)] > 1$. However, such s' will be rejected outright since $\mathbb{E}_B[s'(x)] > 1 = \mathbb{E}_B[s^*(x)]$. Next, suppose that a brown investor deviates and offers s'' such that, if accepted, the firm would choose project G. For s'' to be profitable, we must have $\mathbb{E}_G[s''(x)] > 1$. However, such an offer will be rejected outright since $\mathbb{E}_G[s''(x)] > 1 > \mathbb{E}_G[s^*(x)] = \mathbb{E}_G[c(x, S_1^B)]$.

Next, we prove that the green investor also has no incentive to deviate. Similar to brown investors, the green investor finds it suboptimal to offer s' such that, if accepted, the firm would choose project B. Suppose the green investor deviates and offers $s''(\cdot) \neq s^*(\cdot)$ such that, if accepted, the firm would choose project G. The firm will find it strictly optimal to choose project G only if

$$\mathbb{E}_B[s''(x)] - \mathbb{E}_G[s''(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)].$$

Moreover, we must have $\mathbb{E}_G[s''(x)] > \mathbb{E}_G[s^*(x)]$. First, if $s''(\cdot) = c(\cdot, S'')$ for some $S'' > 0$, we must have $S'' < S_1^B$ since $s''(\cdot) \neq s^*(\cdot)$, implying $\mathbb{E}_G[s''(x)] > \mathbb{E}_G[s^*(x)]$. Next, to show $\mathbb{E}_G[s''(x)] > \mathbb{E}_G[s^*(x)]$ when $s''(\cdot)$ is not an equity contract, suppose to the con-

trary $\mathbb{E}_G[s''(x)] \leq \mathbb{E}_G[s^*(x)]$. Then there must exist an equity contract $c(\cdot, S^G_{\mathbb{E}_G[s''(x)]})$ such that $\mathbb{E}_B[c(x, S^G_{\mathbb{E}_G[s''(x)]})] - \mathbb{E}_G[c(x, S^G_{\mathbb{E}_G[s''(x)]})] \geq \mathbb{E}_B[s''(x)] - \mathbb{E}_G[s''(x)]$. Furthermore, since $\mathbb{E}_G[c(x, S^G_{\mathbb{E}_G[s''(x)]})] = \mathbb{E}_G[s''(x)] \leq \mathbb{E}_G[s^*(x)] = \mathbb{E}_G[c(x, S^B_1)]$, we must have $S^G_{\mathbb{E}_G[s''(x)]} \geq S^B_1$. Since $s''(\cdot)$ is not an equity contract, $S^G_{\mathbb{E}_G[s''(x)]} \geq S^B_1$ implies that $s''(\cdot)$ is strictly less sensitive than $s^*(\cdot)$ by Lemma 1-(iv), a contradiction. However, $\mathbb{E}_G[s''(x)] > \mathbb{E}_G[s^*(x)]$ implies that the firm will reject s'' since $\mathbb{E}_G[x - s''(x)] + \lambda\phi < \mathbb{E}_G[x - s^*(x)] + \lambda\phi \leq \mathbb{E}_B[x - s^*(x)]$. Therefore, there is no feasible deviation offer that induces the firm to choose project G.

A.2.2 Proof of Necessity

In this proof, let $\mathbf{s}^*$ generically denote the set of securities that the firm accepts with positive probability in equilibrium. Suppose to the contrary that

$$\mathbb{E}_B[c(x, S^B_1)] - \mathbb{E}_B[c(x, S^B_1)] > \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$$

but there exists an equilibrium in which all securities in $\mathbf{s}^*$ induce project B, and furthermore, $\mathbb{E}_B[s(x)] = 1$ for all $s(\cdot) \in \mathbf{s}^*$. Note that in such an equilibrium, every investor earns zero expected payoff. We show that the green investor could strictly benefit by unilaterally deviating and offering $c(x, S^B_1)$, which should not be the case in equilibrium. By supposition, the sensitivity of $c(\cdot, S^B_1)$ exceeds $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Thus, the firm strictly prefers project G to project B once it accepts $c(\cdot, S^B_1)$. Furthermore, we have

$$\mathbb{E}_G[x - c(x, S^B_1)] + \lambda\phi > \mathbb{E}_B[x - c(x, S^B_1)] = \mathbb{E}_B[x - 1] = \mathbb{E}_B[x - s(x)] \quad \forall s \in \mathbf{s}^*, \tag{A.3}$$

and thus, the firm will indeed accept $c(x, S^B_1)$ if offered by the green investor. On the other hand, the green investor will obtain

$$\mathbb{E}_G[c(x, S^B_1)] - 1 + \phi = \mathbb{E}_G[c(x, S^B_1)] - \mathbb{E}_B[c(x, S^B_1)] + \phi > 0,$$

where the last inequality holds because $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] < \phi$ for any security $s(\cdot)$, as stated in Lemma 1-(v). Thus, the deviation indeed strictly benefits the green investor, a contradiction.

A.3 Proof of Theorem 2

Note that $\lambda \geq \overline{\lambda}^*$ if and only if

$$\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \tag{A.4}$$

where $\mathbb{E}_B[d(x, D_1^B)] = 1 > \mathbb{E}_G[d(x, D_1^B)]$ by MLRP and the construction of D_1^B. By Lemma 1, (A.4) also implies

$$\mathbb{E}_B[c(x, S_1^G)] - \mathbb{E}_G[c(x, S_1^G)] \geq \mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi. \tag{A.5}$$

We first prove the sufficiency of (A.4) and then the necessity of (A.4).

A.3.1 Proof of Sufficiency

Let $s^*(\cdot) := c(\cdot, S_1^G)$ such that $\mathbb{E}_B[c(x, S_1^G)] > \mathbb{E}_G[c(x, S_1^G)] = 1$ and suppose that all brown investors offer the same security $s^*(\cdot)$. The green investor withdraws from bidding for a project by offering a security that the firm never finds it profitable to accept. From (A.4), the sensitivity of $s^*(\cdot)$ is weakly larger than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, and thus, it is (weakly) optimal for the firm to accept $s^*(\cdot)$ and then choose project G. Thus, it suffices to show that all investors find it suboptimal to deviate from the bidding strategies described above. Note that brown investors and the green investor respectively obtain 0 and ϕ as their expected payoffs from the above bidding strategies.

Suppose that an investor deviates and offers s' such that, if accepted, the firm would choose project G. For s' to be profitable, we must have $\mathbb{E}_G[s'(x)] > 1$. However, such s' will be rejected outright since $\mathbb{E}_G[s'(x)] > 1 = \mathbb{E}_G[s^*(x)] = \mathbb{E}_G[c(x, S_1^G)]$.

Next, suppose that an investor deviates and offers s'' such that, if accepted, the firm would choose project B. For s'' to be profitable, we must have $\mathbb{E}_B[s''(x)] > 1$. However, by Lemma 1-(iii), there exists a debt contract $d(\cdot, D'')$ such that $\mathbb{E}_B[s''(x)] = \mathbb{E}_B[d(x, D'')]$ and thus $D'' > D_1^B$. By Lemma 1-(i), we must have $\mathbb{E}_B[s''(x)] - \mathbb{E}_G[s''(x)] \geq \mathbb{E}_B[d(x, D'')] - \mathbb{E}_G[d(x, D'')] > \mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)]$, which implies

$$\mathbb{E}_B[s''(x)] - \mathbb{E}_G[s''(x)] > \mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi.$$

Thus, the firm would choose project G once it accepts s'', which contradicts the supposition

that s'' would induce project B.

A.3.2 Proof of Necessity

In this proof, let $\mathbf{s}^*$ generically denote the set of securities that the firm accepts with positive probability in equilibrium. Suppose to the contrary that

$$\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_B[d(x, D_1^B)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$$

but there is an equilibrium in which all securities in $\mathbf{s}^*$ induce project G, and furthermore, $\mathbb{E}_G[s(x)] = 1$ for all $s(\cdot) \in \mathbf{s}^*$. Note that all brown investors obtain zero payoff, whereas the green investor gets the payoff ϕ in such an equilibrium.

Now, we prove that such an equilibrium never exists by showing that a brown investor could strictly benefit by unilaterally deviating and offering $d(x, D_{1+\varepsilon}^B)$, where $\varepsilon > 0$ is a small positive number such that

$$\mathbb{E}_B[d(x, D_{1+\varepsilon}^B)] - \mathbb{E}_B[d(x, D_{1+\varepsilon}^B)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi, \tag{A.6}$$

$$\mathbb{E}_B[x] - 1 - \varepsilon > \mathbb{E}_G[x] - 1 + \lambda\phi, \tag{A.7}$$

where (A.6) and (A.7) hold true for any sufficiently small $\varepsilon > 0$ by Lemma 1 and Assumption 1-(i), respectively. The sensitivity of $d(x, D_{1+\varepsilon}^B)$ is strictly less than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Thus, the firm will strictly prefer project B to project G once it accepts $d(x, D_{1+\varepsilon}^B)$. Furthermore,

$$\mathbb{E}_B[x - d(x, D_{1+\varepsilon}^B)] = \mathbb{E}_B[x] - 1 - \varepsilon > \mathbb{E}_G[x] - 1 + \lambda\phi = \mathbb{E}_G[x - s(x)] + \lambda\phi \quad \forall s \in \mathbf{s}^*,$$

where the last equality follows from $\mathbb{E}_G[s(x)] = 1$ for all $s \in \mathbf{s}^*$. Hence, the firm will accept $d(x, D_{1+\varepsilon}^B)$ if offered by a brown investor. The deviating brown investor will then obtain

$$\mathbb{E}_B[d(x, D_{1+\varepsilon}^B)] - 1 = 1 + \varepsilon - 1 = \varepsilon > 0.$$

Thus, the deviation indeed strictly benefits the deviating brown investor.

A.4 Proof of Theorem 3

In this proof, let $\mathbf{s}^*$ generically denote the set of securities that the firm accepts with positive probability in equilibrium. Note that $\lambda \in [\underline{\lambda}^*, \hat{\lambda}^*]$ if and only if

$$\begin{aligned}\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] &\geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \\ &\geq \mathbb{E}_B[d(x, D_{1-\phi}^G)] - \mathbb{E}_G[d(x, D_{1-\phi}^G)], \end{aligned} \tag{A.8}$$

where $\mathbb{E}_B[c(x, S_1^B)] = 1 > \mathbb{E}_G[c(x, S_1^B)] > 1-\phi$ and $\mathbb{E}_G[d(x, D_{1-\phi}^G)] = 1-\phi < \mathbb{E}_B[d(x, D_{1-\phi}^G)] < 1$ (Lemma 1-(v)). Hence, we have $S_{1-\phi}^G > S_1^B$ and $D_{1-\phi}^G < D_1^B$, and therefore, at least one of the two inequalities in (A.8) must hold strictly. We first prove the sufficiency of the condition (A.8) and then prove its necessity.

A.4.1 Preliminary Observations

Observation A.1. *Suppose* (A.8) *holds. Then, there are securities* s^* *and* $\hat{s}$ *such that* $s^* \neq \hat{s}$, *and*

(i) $1 - \phi \leq \mathbb{E}_G[s^*(x)] < 1$ *and* $\mathbb{E}_B[\hat{s}(x)] \leq 1$;

(ii) $\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$;

(iii) $\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \mathbb{E}_B[x - \hat{s}(x)] \geq \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi$.

Proof. Note first that $S_{1-\phi}^G > S_1^B$, which is immediate from Lemma 1-(v). There are three subcases. First, suppose $\mathbb{E}_B[c(x, S_{1-\phi}^G)] - \mathbb{E}_G[c(x, S_{1-\phi}^G)] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. In this case,

$$\begin{aligned}E_B[c(x, S_{1-\phi}^G)] - \mathbb{E}_G[c(x, S_{1-\phi}^G)] &\leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \\ &\leq \mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)],\end{aligned}$$

where only one inequality can weakly hold since $S_{1-\phi}^G > S_1^B$. Also, there is $S^* \in [S_1^B, S_{1-\phi}^G]$ such that $\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$ and $1-\phi \leq \mathbb{E}_G[c(x, S^*)] < \mathbb{E}_B[c(x, S^*)] \leq 1$. Now, let $s^*(\cdot) = c(\cdot, S^*)$ and $\hat{s}(\cdot) = d(\cdot, D^B_{\mathbb{E}_B[c(x,S^*)]})$. Then $s^*(\cdot)$ and $\hat{s}(\cdot)$ clearly satisfy (i) and (ii). To show (iii) also holds, note that the sensitivity of $d(\cdot, D^B_{\mathbb{E}_B[c(x,S^*)]})$ is strictly less than the sensitivity of $c(\cdot, S^*)$, while the sensitivity of $c(\cdot, S^*)$ equals $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Thus,

$$\mathbb{E}_G[x - c(x, S^*)] + \lambda\phi = \mathbb{E}_B[x - d(x, D^B_{\mathbb{E}_B[c(x,S^*)]})] \geq \mathbb{E}_G[x - d(x, D^B_{\mathbb{E}_B[c(x,S^*)]})] + \lambda\phi,$$

and hence (iii) holds true.

Second, suppose that

$$\begin{aligned}\mathbb{E}_B[c(x, S^G_{1-\phi})] - \mathbb{E}_G[c(x, S^G_{1-\phi})] &> \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \\ &= \mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_B[d(x, D^G_{1-\phi})].\end{aligned}$$

Define $s^*(\cdot) := \alpha c(\cdot, S^G_{1-\phi}) + (1-\alpha)d(\cdot, D^G_{1-\phi})$, where we choose the weight $\alpha \in (0,1)$ so that the sensitivity of $s^*(\cdot)$ is larger than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Then,

$$\begin{aligned}&\mathbb{E}_G[s^*(x)] = \alpha\mathbb{E}_G[c(x, S^G_{1-\phi})] + (1-\alpha)\mathbb{E}_G[d(x, D^G_{1-\phi})] = 1-\phi, \\ &\mathbb{E}_B[s^*(x)] < \mathbb{E}_G[s^*(x)] + \phi = 1 - \phi + \phi = 1,\end{aligned}$$

where the strict inequality follows from Lemma 1-(v). Thus, $s^*(\cdot)$ satisfies both properties (i) and (ii) above. In addition, let $\hat{s}(\cdot) := d(\cdot, D^G_{1-\phi})$. Note that $s^*(\cdot)$ is not a debt contract (because $\alpha \in (0,1)$), implying $s^*(\cdot) \neq \hat{s}(\cdot)$. Furthermore, $\mathbb{E}_G[\hat{s}(x)] = 1-\phi$ and $\mathbb{E}_B[\hat{s}(x)] < \mathbb{E}_G[\hat{s}(x)] + \phi \leq 1$ by Lemma 1-(v), implying $\hat{s}(\cdot)$ satisfies (i). Lastly,

$$\begin{aligned}&\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \mathbb{E}_G[x] - 1 + \phi + \lambda\phi, \\ &\mathbb{E}_B[x - \hat{s}(x)] = \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi = \mathbb{E}_G[x] - 1 + \phi + \lambda\phi,\end{aligned}$$

where the equality $\mathbb{E}_B[x - \hat{s}(x)] = \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi$ holds because the sensitivity of $\hat{s}(\cdot)$ equals $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$ by construction. Hence, $s^*(\cdot)$ and $\hat{s}(\cdot)$ also satisfy (iii).

Finally, suppose that

$$\begin{aligned}\mathbb{E}_B[c(x, S^G_{1-\phi})] - \mathbb{E}_G[c(x, S^G_{1-\phi})] &> \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \\ &> \mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_B[d(x, D^G_{1-\phi})].\end{aligned}$$

Define $s^*(\cdot) := \alpha c(\cdot, S^G_{1-\phi}) + (1-\alpha)d(\cdot, D^G_{1-\phi})$, where we choose $\alpha \in (0,1)$ such that the sensitivity of $s^*(\cdot)$ is exactly equal to $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Then,

$$\begin{aligned}&\mathbb{E}_G[s^*(x)] = \alpha\mathbb{E}_G[c(x, S^G_{1-\phi})] + (1-\alpha)\mathbb{E}_G[d(x, D^G_{1-\phi})] = 1-\phi, \\ &\mathbb{E}_B[s^*(x)] < \mathbb{E}_G[s^*(x)] + \phi = 1 - \phi + \phi = 1,\end{aligned}$$

where the strict inequality in the last line follows from Lemma 1-(v). Thus, $s^*(\cdot)$ satisfies both properties (i) and (ii) above. In addition, let $\hat{s}(\cdot) = d(\cdot, D^B_{\mathbb{E}_B[s^*(x)]})$. Note that $s^*(\cdot)$ is

not a debt contract (because $\alpha \in (0,1)$), implying $s^*(\cdot) \neq \hat{s}(\cdot)$. Furthermore, $\mathbb{E}_B[\hat{s}(x)] = \mathbb{E}_B[s^*(x)] < \mathbb{E}_G[s^*(x)] + \phi = 1$ (Lemma 1-(v)), hence $\hat{s}(\cdot)$ satisfies (i). Finally,

$$\mathbb{E}_B[x - s^*(x)] = \mathbb{E}_G[x - s^*(x)] + \lambda\phi = \mathbb{E}_G[x] - 1 + \phi + \lambda\phi,$$
$$\mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi \leq \mathbb{E}_B[x - \hat{s}(x)] = \mathbb{E}_B[x - s^*(x)],$$

where the inequality in the last line holds because $\hat{s}(\cdot)$ is less sensitive than $s^*(\cdot)$ and the sensitivity of $s^*(\cdot)$ equals $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$ by construction and the equality in the last line holds because $\mathbb{E}_B[\hat{s}(x)] = \mathbb{E}_B[s^*(x)]$. Hence, $s^*(\cdot)$ and $\hat{s}(\cdot)$ also satisfy (iii). *Q.E.D.*

Observation A.2. *In any equilibrium such that the firm chooses project G with probability 1 and $\mathbb{E}_G[s(x)] \in [1-\phi, 1)$ for any $s \in \mathbf{s}^*$:*

(i) *$\mathbf{s}^*$ is a singleton (i.e., $\mathbf{s}^* = \{s^*\}$), and only the green investor offers s^*, where $\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$.*

(ii) *A brown investor must offer a security $\hat{s} \neq s^*$ (which is not accepted by the firm) that satisfies Observation A.1-(iii). That is,*

$$\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \max\{\mathbb{E}_B[x - \hat{s}(x)], \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi\} = \mathbb{E}_B[x - \hat{s}(x)].$$

Proof. To show (i), note that brown investors have no preference for the social value of project G, and thus, they will never offer $s \in \mathbf{s}^*$ whose expected return is less than 1. Thus, $\mathbf{s}^* = \{s^*\}$ must be a singleton, and it must be the green investor who offers s^*. In addition, to ensure that s^* induces the firm to choose project G, the sensitivity of s^* must be weakly larger than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Note that (i) also implies

$$\mathbb{E}_G[x - s^*(x)] + \lambda\phi \geq \max\{\mathbb{E}_B[x - s(x)], \mathbb{E}_G[x - s(x)] + \lambda\phi\}$$

for any security $s(\cdot)$ offered by a brown investor in equilibrium because, otherwise, the green investor's offer s^* is never accepted by the firm, which contradicts (i).

To show (ii), it suffices to show that a brown investor offers $\hat{s}(\cdot)$ such that

$$\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \max\{\mathbb{E}_B[x - \hat{s}(x)], \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi\} = \mathbb{E}_B[x - \hat{s}(x)]. \tag{A.9}$$

First, suppose the contrary to the first equality in (A.9) that

$$\mathbb{E}_G[x - s^*(x)] + \lambda\phi > \max\{\mathbb{E}_B[x - \hat{s}(x)], \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi\}$$

for any security $\hat{s}(\cdot)$ a brown investor offers in equilibrium. Suppose that the green investor deviates by offering $s'(\cdot) := c(\cdot, S^G_{\mathbb{E}_G[s^*(x)]+\varepsilon})$, where $\varepsilon > 0$ is a small positive number. Then, by construction, $s'(\cdot)$ is more sensitive than s^* hence also induces the firm to choose project G if accepted (Lemma 1-(iv)). Furthermore, provided that $\varepsilon > 0$ is small, we have

$$\mathbb{E}_G[x - s'(x)] + \lambda\phi = \mathbb{E}_G[x - s^*(x)] + \lambda\phi - \varepsilon > \max\{\mathbb{E}_B[x - \hat{s}(x)], \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi\}$$

for all $\hat{s}$ offered by brown investors in equilibrium. Hence, the firm will accept $s'(\cdot)$ with probability 1. As a result, the green investor strictly benefits from such a deviation, which should not be possible in equilibrium.

Finally, suppose the contrary to the second equality in (A.9) that

$$\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi > \mathbb{E}_B[x - \hat{s}(x)] \tag{A.10}$$

for any brown investor' offer $\hat{s}(\cdot)$ such that $\mathbb{E}_G[x-s^*(x)]+\lambda\phi = \max\{\mathbb{E}_G[x-s^*(x)]+\lambda\phi, \mathbb{E}_B[x-\hat{s}(x)]\}$. In equilibrium, the green investor earns $\mathbb{E}_G[s^*(x)]-1+\phi < \phi$. But if the green investor withdraws her offer $s^*(\cdot)$, the firm will accept one of the brown investors' offers that satisfy (A.10) and then choose project G. Thus, the green investor can get ϕ from withdrawing her offer, a strictly higher payoff than that from offering $s^*(\cdot)$, which should not be possible in equilibrium. *Q.E.D.*

A.4.2 Proof of Sufficiency

Suppose that the green investor offers $s^*(\cdot)$, while all other investors offer $\hat{s}(\cdot)$, where $s^*(\cdot)$ and $\hat{s}(\cdot)$ are the securities as constructed in Observation A.1. The firm employs the following decision rule: the firm chooses $s^*(\cdot)$ (both on and off the path) if both $s^*(\cdot)$ and $\hat{s}(\cdot)$ are available; the firm will choose project G if it accepts $s^*(\cdot)$; the firm will choose project B if it accepts $\hat{s}(\cdot)$. By Observation A.1-(ii) and (iii), this decision rule is optimal for the firm.

Now we show that, given the other investors' offer strategies and the firm's decision rule as stipulated above, no investor can strictly benefit from any deviation. First, consider a brown investor. In order to win the competition to fund the firm's project and to induce the

firm to choose project B, a brown investor must offer a security $s'(\cdot)$ such that $\mathbb{E}_B[s'(x)] \leq \mathbb{E}_B[\hat{s}(x)] \leq 1$. However, such a deviation will never result in a strictly higher payoff for the brown investor and thus is strictly suboptimal. Similarly, in order to win the competition to fund the firm's project and to induce the firm to choose project G, a brown investor must offer a security $s''(\cdot)$ such that $\mathbb{E}_G[s''(x)] \leq \mathbb{E}_G[s^*(x)] < 1$, which would generate a negative payoff to the deviating brown investor. In summary, brown investors have no incentive to deviate from $\hat{s}(\cdot)$, because such a deviation could never generate a strictly positive financial payoff.

Next, consider the green investor. The green investor obtains the payoff $\mathbb{E}_G[s^*(x)] - 1 + \phi \geq 0$ in equilibrium. Similar to the brown investors, the green investor cannot obtain a positive payoff from deviating to any contract that would induce project B. Therefore, we focus on the case that the green investor deviates to $s''(\cdot)$ such that *(a)* the firm finds it optimal to accept $s''(\cdot)$ over $\hat{s}(\cdot)$ and *(b)* $s''(\cdot)$ would induce the firm to choose project G. In order for $s''(\cdot)$ to yield the green investor a strictly higher payoff than $s^*(\cdot)$, we must have $\mathbb{E}_G[s''(x)] > \mathbb{E}_G[s^*(x)]$. However, the firm would find it more profitable to accept $\hat{s}$ over $s''(\cdot)$ because

$$\mathbb{E}_G[x - s''(x)] + \lambda\phi < \mathbb{E}_G[x - s^*(x)] + \lambda\phi = \mathbb{E}_B[x - \hat{s}(x)],$$

where the last equality follows from Observation A.1-(iii).

A.4.3 Proof of Necessity

Suppose to the contrary that, despite (A.8) fails, there is an equilibrium such that *(i)* the firm chooses project G with probability 1 and *(ii)* the firm's expected payment lies in $[1-\phi, 1)$. By Observation A.2, we may assume without loss that $\mathbf{s}^* = \{s^*\}$ is a singleton where $s^*(\cdot)$ is offered by the green investor, $\mathbb{E}_G[s^*(x)] \in [1-\phi, 1)$, and $\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Note that all brown investors obtain zero payoff in any such an equilibrium.

Step I: We first show that the first inequality in (A.8) could not fail. In other words, if any inequality fails in (A.8), it must be the second inequality. Suppose for contradiction

$$\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi > \mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)].$$

Consider the equity contract $c(\cdot, S^G_{\mathbb{E}_G[s^*(x)]})$. By Lemma 1-(iv), $c(\cdot, S^G_{\mathbb{E}_G[s^*(x)]})$ is more sensitive than $s^*(\cdot)$ and thus also induces the firm to choose project G. Furthermore, the firm's expected

payment to an investor is identical under $c(\cdot, S^G_{\mathbb{E}_G[s^*(x)]})$ and $s^*(\cdot)$. Hence, the firm must be indifferent between $s^*(\cdot)$ and $c(\cdot, S^G_{\mathbb{E}_G[s^*(x)]})$.

Now, note that there exists $S^* \in (S^G_{\mathbb{E}_G[s^*(x)]}, S^B_1)$ such that $\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, and hence, $c(\cdot, S^*)$ would induce the firm to choose project B if accepted by the firm. Suppose that a brown investor deviates by offering $s'(\cdot) = c(\cdot, S^*)$. The striking price S^* is strictly larger than $S^G_{\mathbb{E}_G[s^*(x)]}$, and hence, the firm will find $s'(\cdot)$ strictly more profitable than $s^*(\cdot)$. Hence, the firm would accept $s'(\cdot)$ and then choose project B, which yields the deviating brown investor

$$\mathbb{E}_B[c(x, S^*)] - 1 > \mathbb{E}_B[c(x, S^B_1)] - 1 \geq 0,$$

contradicting the earlier observation that all brown investors earn zero payoff in equilibrium.

Step II: To complete our proof for the necessity of (A.8), we show that the second inequality in (A.8) could not fail either. Suppose for contradiction

$$\begin{aligned}\mathbb{E}_B[c(x, S^B_1)] - \mathbb{E}_G[c(x, S^B_1)] &> \mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_G[d(x, D^G_{1-\phi})] \\ &> \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi.\end{aligned}$$

Consider $\hat{s}(\cdot)$ that is offered by a brown investor and also satisfies Observation A.2-(ii). The sensitivity of $\hat{s}(\cdot)$ must be strictly less than $d(\cdot, D^G_{1-\phi})$, and thus, $\mathbb{E}_G[\hat{s}(x)] < \mathbb{E}_G[d(x, D^G_{1-\phi})] = 1 - \phi$. If $\mathbb{E}_G[\hat{s}(x)] \geq \mathbb{E}_G[d(x, D^G_{1-\phi})]$, there exists a debt contract $d(\cdot, D^G_{\mathbb{E}_G[\hat{s}(x)]})$ such that $D^G_{\mathbb{E}_G[\hat{s}(x)]} \geq D^G_{1-\phi}$, implying $\hat{s}(\cdot)$ is weakly more sensitive than $d(\cdot, D^G_{1-\phi})$. On the other hand, Observation A.2-(ii) implies $\mathbb{E}_G[s^*(x)] \leq \mathbb{E}_G[\hat{s}(x)] < 1 - \phi$. But then, the green investor's expected payoff from offering $s^*(\cdot)$, $\mathbb{E}_G[s^*(x)] - 1 + \phi$, is strictly less than zero, which is impossible in equilibrium.

A.5 Proof of Theorem 4

Note that $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$ if and only if

$$\begin{aligned}\mathbb{E}_B[c(x, S^B_1)] - \mathbb{E}_G[c(x, S^B_1)] &\geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \\ &> \mathbb{E}_B[c(x, S^G_{1-\phi})] - \mathbb{E}_G[c(x, S^G_{1-\phi})]. \qquad \text{(A.11)}\end{aligned}$$

In this section, we will first prove the necessity of (A.11) for the existence of a mixed-strategy equilibrium, and then we will discuss how $\tilde{\beta}^*$ varies as λ changes between $\underline{\lambda}^*$ and $\tilde{\lambda}^*$. Finally, we will prove the sufficiency of (A.11).

In this proof, let $\mathbf{s}^*$ generically denote the set of securities that the firm accepts with positive probability in equilibrium. For each $k \in \{B, G\}$, $\mathbf{s}_k^* \subset \mathbf{s}^*$ denotes the collection of securities in $\mathbf{s}^*$ that induce the selection of project k with positive probability if accepted by the firm. Note that the firm is indifferent between all securities in $\mathbf{s}^* = \mathbf{s}_B^* \cup \mathbf{s}_G^*$. Thus, whenever both $\mathbf{s}_B^*$ and $\mathbf{s}_G^*$ are non-empty,

$$\mathbb{E}_B[x - s_B(x)] = \mathbb{E}_G[x - s_G(x)] + \lambda\phi \quad \forall (s_B, s_G) \in \mathbf{s}_B^* \times \mathbf{s}_G^*. \tag{A.12}$$

Furthermore, for any $k \in \{B, G\}$, $\mathbb{E}_k[s'(x)] = \mathbb{E}_k[s''(x)]$ for any $s', s'' \in \mathbf{s}_k^*$ because the firm must be indifferent between any two securities in $\mathbf{s}_k^*$.

A.5.1 Preliminary Observation

We first make two preliminary observations, Observation A.3–A.4, which hold regardless of whether (A.11) holds or fails.

Observation A.3. *Consider any equilibrium such that $\mathbf{s}_B^* \neq \emptyset$ and $\mathbf{s}_G^* \neq \emptyset$. For any $s_G \in \mathbf{s}_G^*$ and $s_B \in \mathbf{s}_B^*$, we have the following properties:*

(i) $\mathbb{E}_G[s_G(x)] < \mathbb{E}_B[s_B(x)] \leq 1$.

(ii) $\mathbb{E}_G[s_G(x)] \leq \mathbb{E}_G[s_B(x)] < \mathbb{E}_B[s_B(x)] \leq \mathbb{E}_B[s_G(x)]$.

(iii) $\mathbb{E}_G[s_G(x)] > \mathbb{E}_B[s_G(x)] - \phi \geq \mathbb{E}_B[s_B(x)] - \phi$.

Proof. The first inequality $\mathbb{E}_G[s_G(x)] < \mathbb{E}_B[s_B(x)]$ in (i) directly follows (A.12) and Assumption 1-(i). Note that the first inequality in (i) also implies that the equilibrium payoff of any brown investor is strictly bounded from above by $r_B - 1$, where $r_B > 0$ is a non-negative number such that $\mathbb{E}_B[s_B(x)] = r_B$ for all $s_B \in \mathbf{s}_B^*$; brown investors' equilibrium payoffs never exactly attain $r_B - 1$ due to the firm's randomization in equilibrium.

To prove the second inequality in (i), suppose to the contrary that $r_B = \mathbb{E}_B[s_B(x)] > 1$ for any $s_B(\cdot) \in \mathbf{s}_B^*$. From the discussion in the previous paragraph, all brown investors earn strictly less than $r_B - 1$ in equilibrium. Now, suppose that a brown investor deviates by

offering $\hat{s}(\cdot) = d(\cdot, S^B_{r_B-\varepsilon})$, where $r_B - \varepsilon > 1$. The firm prefers $\hat{s}$ to any $s_B(\cdot) \in \mathbf{s}^*_B$ because $r_B > \mathbb{E}_B[\hat{s}(x)]$. Furthermore, by Lemma 1-(iii), $\hat{s}(\cdot)$ is less sensitive than any security in $\mathbf{s}^*_B$. Hence, the firm will accept $\hat{s}$ and then choose project B under $\hat{s}(\cdot)$. Thus, the deviating brown investor would obtain $r_B - \varepsilon - 1$. This payoff is strictly larger than her equilibrium payoff if we choose $\varepsilon > 0$ sufficiently small, which should not be possible in equilibrium.

To prove (ii), note first that

$$\mathbb{E}_B[x - s_B(x)] = \mathbb{E}_G[x - s_G(x)] + \lambda\phi \geq \mathbb{E}_B[x - s_G(x)] \implies \mathbb{E}_B[s_G(x)] \geq \mathbb{E}_B[s_B(x)],$$
$$\mathbb{E}_G[x - s_B(x)] + \lambda\phi \leq \mathbb{E}_B[x - s_B(x)] = \mathbb{E}_G[x - s_G(x)] + \lambda\phi \implies \mathbb{E}_G[s_G(x)] \leq \mathbb{E}_G[s_B(x)]$$

by construction of $\mathbf{s}^*_B$ and $\mathbf{s}^*_G$. In addition, $\mathbb{E}_G[s_B(x)] < \mathbb{E}_B[s_B(x)]$ by MLRP, which completes the proof of (ii). Finally, the two inequalities in (iii) immediately follow Lemma 1-(v) and $\mathbb{E}_B[s_G(x)] \leq \mathbb{E}_B[s_B(x)]$ in (ii), respectively. *Q.E.D.*

Observation A.4. *In any equilibrium such that $\mathbf{s}^*_B \neq \emptyset$ and $\mathbf{s}^*_G \neq \emptyset$, we have the following:*

(i) Only the green investor offers a security in $\mathbf{s}^*_G$. Thus, $\mathbf{s}^*_G$ is a singleton.

(ii) The security in $\mathbf{s}^*_G$ is the equity $c(\cdot, S^*)$ such that $\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$.

Proof. Fix any $s(\cdot) \in \mathbf{s}^*_G$. $\mathbb{E}_G[s(x)] < 1$ by Observation A.3-(i). There are two subcases. First, suppose $s(\cdot) \notin \mathbf{s}^*_B$. Then, the firm will choose project G with probability 1 under $s(\cdot)$, and thus, $s(\cdot)$ only generates a strictly negative financial payoff for a funding investor. Next, suppose $s(\cdot) \in \mathbf{s}^*_B$. By Observation A.3-(i), $\mathbb{E}_G[s(x)] < \mathbb{E}_B[s(x)] \leq 1$. Since the firm chooses both projects B and G with positive probabilities, $s(\cdot)$ also generates a strictly negative financial payoff for a funding investor. Hence, any brown investor would find it strictly suboptimal to offer any security in $\mathbf{s}^*_G$, which completes the proof of (i).

To show (ii), let s_G denote the unique security in $\mathbf{s}^*_G$. The green investor must offer s_G in equilibrium. Furthermore, s_G induces project G with positive probability, and hence, $\mathbb{E}_B[s_G(x)] - \mathbb{E}_G[s_B(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Finally, the green investor's expected payoff in equilibrium is

$$V_G := (1 - \beta^*)(\mathbb{E}_G[s_G(x)] - 1 + \phi) + \alpha^*(\mathbb{E}_B[s_G(x)] - 1) \tag{A.13}$$

where $1-\beta^* \in (0,1)$ is the probability that the firm accepts s_G and project G in equilibrium, and $\alpha^* \in [0, 1-\beta^*]$ is the probability that the firm accepts s_G and chooses project B. Note that $\alpha^* = 0$ if $s_G \notin \mathbf{s}_B^*$.

Now, suppose for contradiction with (ii) one of the following two fails:

$$\text{(a)}\ \ s_G(\cdot) \text{ is an equity;} \qquad \text{(b)}\ \ \mathbb{E}_B[s_G(x)] - \mathbb{E}_G[s_G(x)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi. \tag{A.14}$$

In either case, suppose that the green investor deviates by offering $s_\varepsilon(\cdot) := c(\cdot, S^G_{\mathbb{E}_G[s_G(x)]-\varepsilon})$, where $\varepsilon > 0$ is sufficiently small such that the sensitivity of $s_\varepsilon(\cdot)$ is strictly larger than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. By construction, we have $\mathbb{E}_G[s_\varepsilon(x)] < \mathbb{E}_G[s_G(x)]$, and therefore, the firm would strictly prefer $s_\varepsilon(\cdot)$ to $s_G(\cdot)$. Furthermore, the firm will choose project G under $s_\varepsilon(\cdot)$ because the sensitivity of $s_\varepsilon(\cdot)$ exceeds $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Thus, the green investor's expected payoff from the deviation will be

$$V_\varepsilon := \mathbb{E}_G[s_\varepsilon(x)] - 1 + \phi = \mathbb{E}_G[s_G(x)] - \varepsilon - 1 + \phi. \tag{A.15}$$

We will complete the proof by showing that V_ε in (A.15) is strictly larger than V_G in (A.13), provided that $\varepsilon > 0$ is sufficiently small. There are two subcases. First, suppose $\mathbb{E}_B[s_B(x)] = 1$ for any $s_B \in \mathbf{s}_B^*$. In this case,Observation A.3-(ii) also implies $\mathbb{E}_B[s_G(x)] \geq \mathbb{E}_B[s_B(x)] = 1$. If $\mathbb{E}_B[s_G(x)] > \mathbb{E}_B[s_B(x)] = 1$, the firm never accepts $s_G(\cdot)$ to choose project B, and thus, $\alpha^* = 0$. Hence, regardless of whether the inequality $\mathbb{E}_B[s_G(x)] \geq \mathbb{E}_B[s_B(x)]$ holds strictly or not, the second term in (A.13) must be zero. On the other hand, we have $\mathbb{E}_G[s_G(x)] > \mathbb{E}_B[s_B(x)] - \phi \geq 1 - \phi$ from Observation A.3-(iii). Hence,

$$V_G - V_\varepsilon = -\beta^*(\mathbb{E}_G[s_G(x)] - 1 + \phi) + \varepsilon < 0.$$

Next, consider the case $\mathbb{E}_B[s_B(x)] < 1$ for any $s_B \in \mathbf{s}_B^*$. In this case, any brown investor would suffer a negative financial payoff from offering a security in $\mathbf{s}_B^*$, and thus, we necessarily have $\mathbf{s}_G^* = \mathbf{s}_B^* = \{s_G\}$, implying $s_B(\cdot) = s_G(\cdot)$. Hence, $\mathbb{E}_B[s_G(x)] = \mathbb{E}_B[s_B(x)] < 1$, and thus the second term in (A.13) is negative. On the other hand, the green investor must obtain a non-negative payoff in equilibrium; thus, we also necessarily have $\mathbb{E}_G[s_G(x)] - 1 + \phi > 0$. In

summary,

$$\begin{aligned} V_G - V_\varepsilon &= (1-\beta^*)(\mathbb{E}_G[s_G(x)] - 1 + \phi) + \alpha^*(\mathbb{E}_B[s_G(x)] - 1) - (\mathbb{E}_G[s_G(x)] - \varepsilon - 1 + \phi) \\ &< -\beta^*(\mathbb{E}_G[s_G(x)] - 1 + \phi) + \varepsilon < 0 \end{aligned}$$

whenever $\varepsilon > 0$ is sufficiently small. *Q.E.D.*

In what follows, we will denote by s_G^* a unique security in $\mathbf{s}_G^*$.

Observation A.5. *In any equilibrium such that $\mathbf{s}_B^* \neq \emptyset$ and $\mathbf{s}_G^* = \{s_G^*\} \neq \emptyset$, there must be a brown investor who offers $\tilde{s}(\cdot) \neq s_G^*(\cdot)$ (which may not belong to $\mathbf{s}_B^* \cup \mathbf{s}_G^*$) such that*

$$\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[\tilde{s}(x)] - \mathbb{E}_G[\tilde{s}(x)] \quad \textit{and} \quad \mathbb{E}_B[\tilde{s}(x)] \leq \mathbb{E}_B[s_G^*(x)]. \tag{A.16}$$

Furthermore, if we additionally assume $\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, there must be a brown investor who offers $\tilde{s}(\cdot) \neq s_G^*(\cdot)$ (which may not belong to $\mathbf{s}_B^* \cup \mathbf{s}_G^*$) such that

$$\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[\tilde{s}(x)] - \mathbb{E}_G[\tilde{s}(x)] \ \textit{and} \ \mathbb{E}_B[\tilde{s}(x)] = \mathbb{E}_B[s_G^*(x)] \leq 1. \tag{A.17}$$

Proof.

Step I: Suppose for contradiction that (A.16) fails for any security $\tilde{s}(\cdot)$ offered by a brown investor in equilibrium. We claim that the firm strictly prefers $s_G^*(\cdot)$ to any securities offered by the brown investors. First, consider a brown investor's offer $\tilde{s}(\cdot)$ such that $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi < \mathbb{E}_B[\tilde{s}(x)] - \mathbb{E}_G[\tilde{s}(x)]$. The firm will choose project G under any such $\tilde{s}(\cdot)$. On the other hand, $s_G^*(\cdot)$ is the equity contract whose sensitivity is equal to $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, as identified in Observation A.4-(ii). Thus, by Lemma 1, we have $\mathbb{E}_G[\tilde{s}(x)] > \mathbb{E}_G[s_G^*(x)]$, and hence the firm strictly prefers $s_G^*(\cdot)$ to $\tilde{s}(\cdot)$.

Next, consider a brown investor's offer $\tilde{s}(\cdot)$ such that $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[\tilde{s}(x)] - \mathbb{E}_G[\tilde{s}(x)]$ and $\mathbb{E}_B[\tilde{s}(x)] > \mathbb{E}_B[s_G^*(x)]$. The firm obtains $\mathbb{E}_B[x - \tilde{s}(x)]$ from accepting any such $\tilde{s}(\cdot)$, while the firm obtains $\mathbb{E}_G[x - s_G^*(x)] + \lambda\phi$ from accepting $s_G^*(\cdot)$. Note that

$$\mathbb{E}_B[x - \tilde{s}(x)] < \mathbb{E}_B[x - s_G^*(x)] \leq \mathbb{E}_G[x - s_G^*(x)] + \lambda\phi$$

where the weak inequality follows from the assumption $s_G^* \in \mathbf{s}_G^*$. Hence, the firm strictly prefers $s_G^*(\cdot)$ to $\tilde{s}(\cdot)$.

Because the firm strictly prefers $s_G^*(\cdot)$ to any other securities offered by the brown investors, the firm would accept $s_G^*(\cdot)$ with probability 1 in equilibrium. The green investor's equilibrium payoff is

$$(1-\beta^*)(\mathbb{E}_G[s_G^*(x)]-1+\phi)+\beta^*(\mathbb{E}_B[s_G^*(x)]-1) \tag{A.18}$$

where $1-\beta^* \in (0,1)$ is the probability that the firm chooses project G under $s_G^*(\cdot)$. By Observation A.3-(iii), this equilibrium payoff is strictly less than $\mathbb{E}_G[s_G^*(x)]-1+\phi$.

Step II: Now, suppose that the green investor deviates by offering $s_\varepsilon(\cdot) := c(\cdot, S^G_{\mathbb{E}_G[s_G^*(x)]+\varepsilon})$, where $\varepsilon > 0$ is sufficiently small so that the firm still strictly prefers $s_\varepsilon(\cdot)$ to all other securities offered by brown investors. $s_\varepsilon(\cdot)$ is more sensitive than s_G^*, and thus, $s_\varepsilon(\cdot)$ would induce the firm to choose project G for sure. Thus, the green investor's payoff from the deviation is $\mathbb{E}_G[s_\varepsilon(x)]-1+\phi = \mathbb{E}_G[s_G^*(x)]+\varepsilon-1+\phi$, which is strictly larger than the green investor's equilibrium payoff in (A.18). Hence, if (A.16) fails for any security $\tilde{s}(\cdot)$ offered by a brown investor, the green investor can strictly benefit from offering $s_\varepsilon(\cdot)$, which should be impossible in equilibrium. This proves Observation A.5-(i).

Step III: Finally, to prove Observation A.5-(ii), assume that $\mathbb{E}_B[c(x,S_1^B)]-\mathbb{E}_G[c(x,S_1^B)]$ is weakly larger than $\mathbb{E}_B[x]-\mathbb{E}_G[x]-\lambda\phi$, which implies $\mathbb{E}_B[s_G^*(x)] \leq 1$. Suppose for contradiction that

$$\mathbb{E}_B[x]-\mathbb{E}_G[x]-\lambda\phi \geq \mathbb{E}_B[\tilde{s}(x)]-\mathbb{E}_G[\tilde{s}(x)] \quad \text{and} \quad \mathbb{E}_B[\tilde{s}(x)] < \mathbb{E}_B[s_G^*(x)] \leq 1 \tag{A.19}$$

for all $\tilde{s}(\cdot) \neq s_G^*(\cdot)$ offered by a brown investor in equilibrium. Because $\mathbb{E}_B[\tilde{s}(x)] < \mathbb{E}_B[s_G^*(x)]$, the firm strictly prefers $\tilde{s}(\cdot)$ to $s_G^*(\cdot)$ if the firm plans to choose project B. Hence, $s_G^*(\cdot) \notin \mathbf{s}_B^*$, and $\mathbf{s}_B^* \backslash \{s_G^*\}$ must include at least one $\tilde{s}(\cdot)$, offered by a brown investor, that satisfies (A.19). However, since $\mathbb{E}_B[\tilde{s}(x)] < \mathbb{E}_B[s_G^*(x)] \leq 1$ by the supposition, $\tilde{s}(\cdot)$ generates a negative financial payoff to any brown investor who offers $\tilde{s}(\cdot)$. Hence, brown investors will never offer $\tilde{s}(\cdot)$ in equilibrium, a contradiction. *Q.E.D.*

A.5.2 Proof of Necessity

Suppose that, although (A.11) fails, there is an equilibrium such that the firm chooses B with probability $\beta^* \in (0,1)$. Note that we always have $\mathbb{E}_B[c(x,S_1^B)]-\mathbb{E}_G[c(x,S_1^B)] > \mathbb{E}_B[c(x,S^G_{1-\phi})]-\mathbb{E}_G[c(x,S^G_{1-\phi})]$ because $\mathbb{E}_B[c(x,S^G_{1-\phi})] < \mathbb{E}_G[c(x,S^G_{1-\pi})]+\phi = 1 = \mathbb{E}_B[c(x,S_1^B)]$.

Step I: We first show that the first inequality in (A.11) always holds true. In other words, it must be the second inequality if any inequality in (A.11) fails. Suppose for contradiction that

$$\begin{aligned}\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi &> \mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] \\ &> \mathbb{E}_B[c(x, S_{1-\phi}^G)] - \mathbb{E}_G[c(x, S_{1-\phi}^G)].\end{aligned} \tag{A.20}$$

From Observation A.4-(i), we must have $\mathbf{s}_G^* = \{s_G^*\}$, where $s_G^*(\cdot) = c(\cdot, S^*)$ and $\mathbb{E}_B[c(x, S^*) - \mathbb{E}_G[c(x, S^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Additionally, from (A.20) and Lemma 1, $s_G^*(\cdot) = c(\cdot, S^*)$ is strictly more sensitive than $c(\cdot, S_1^B)$, implying $S^* < S_1^B$ and $\mathbb{E}_B[s_G^*(x)] > \mathbb{E}_B[c(x, S_1^B)] = 1$. Furthermore, because the sensitivity of $s_G^*(\cdot)$ is equal to $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, we have

$$\mathbb{E}_G[x - s_G^*(x)] + \lambda\phi = \mathbb{E}_B[x - s_G^*(x)] = \mathbb{E}_B[x - s_B(x)] \quad \Longrightarrow \mathbb{E}_B[s_B(x)] = \mathbb{E}_B[s_G^*(x)] > 1$$

for any $s_B \in \mathbf{s}_B^*$.

Due to the firm's randomization in equilibrium, the brown investors succeed in funding the firm's project with a probability strictly less than 1. Thus, the brown investors' equilibrium payoffs are strictly less than

$$\max_{s_B \in \mathbf{s}_B^*} \mathbb{E}_B[s_B(x)] - 1 = \mathbb{E}_B[s_G^*(x)] - 1 > 0.$$

Now, suppose that a brown investor deviates by offering $s'(\cdot) := d(\cdot, D^B_{\mathbb{E}_B[s_G^*(x)]-\varepsilon})$, where $\varepsilon > 0$ is sufficiently small. For all $s_B(\cdot) \in \mathbf{s}_B^*$, we have $\mathbb{E}_B[s'(x)] < \mathbb{E}_B[s_B(x)] = \mathbb{E}_B[s_G^*(x)]$, and thus, the firm would accept $s'(\cdot)$ for sure. Furthermore, by Lemma 1-(iii), the sensitivity of $s'(\cdot)$ is strictly less than the sensitivity of $s_G^*(\cdot)$, hence the firm will choose project B under $s'(\cdot)$. As a result, the deviating brown investor would earn a strictly positive financial payoff $\mathbb{E}_B[s_G^*(x)] - \varepsilon - 1 > 0$ from offering $s'(\cdot)$. Provided that $\varepsilon > 0$ is sufficiently small, this payoff is strictly larger than the deviating brown investor's payoff from the equilibrium strategy, which should be impossible in equilibrium.

Step II: Next, we show that the second inequality in (A.11) could not fail. Suppose for contradiction

$$\begin{aligned}\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] &> \mathbb{E}_B[c(x, S_{1-\phi}^G)] - \mathbb{E}_G[c(x, S_{1-\phi}^G)] \\ &\geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi.\end{aligned} \tag{A.21}$$

By Observation A.4-(i), we have $\mathbf{s}_G^* = \{s_G^*\}$, where $s_G^*(\cdot) = c(\cdot, S^*)$ such that $\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. However, Lemma 1-(ii) implies that $S^* \geq S^G_{1-\phi}$ and $S^* > S^B_1$, which implies $\mathbb{E}_G[s_G^*(x)] \leq 1 - \phi$ and $\mathbb{E}_B[s_G^*(x)] < 1$. Hence, s_G^* should not be included in $\mathbf{s}_B^*$; otherwise, the green investor's expected payoff is strictly less than zero, which is not possible in equilibrium. Pick any $s_B \in \mathbf{s}_B^*$, which must be offered by a brown investor. However, because the firm is willing to accept $s_B(\cdot)$ only if $\mathbb{E}_B[s_B(x)] \leq \mathbb{E}_B[s_G^*(x)] < 1$, this brown investor would suffer a payoff strictly less than zero, which is also should be impossible in equilibrium.

A.5.3 Identification of $\tilde{\beta}^*$

Suppose (A.11) holds, and consider an equilibrium (if any) such that the firm chooses B with some probability $\beta^* \in (0, 1)$. Let $s_G^*(\cdot)$ denote the unique security in $\mathbf{s}_G^*$. Recall some preliminary observations: first, by Observation A.4-(i), $s_G^*(\cdot) = c(\cdot, S^*)$ and $\mathbb{E}_B[s_G^*(x)] - \mathbb{E}_G[s_G^*(x)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$; next, by (A.11) and Lemma 1, we also have $S^G_{1-\phi} > S^* \geq S^B_1$, which implies

$$1 \geq \mathbb{E}_B[c(x, S^*)] = \mathbb{E}_B[s_G^*(x)] > \mathbb{E}_G[s_G^*(x)] > 1 - \phi. \tag{A.22}$$

Next, we calculate the green investor's payoff in equilibrium. There are two subcases. First, if the firm only accepts $s_G^*(\cdot)$ in equilibrium, the green investor's equilibrium payoff is equal to

$$\beta^*(\mathbb{E}_G[c(x, S^*)] - 1 + \phi) + (1 - \beta^*)(\mathbb{E}_B[c(x, S^*)] - 1). \tag{A.23}$$

Second, consider the case where the firm also accepts an offer $s_B(\cdot) \in \mathbf{s}_B^*$ from a brown investor. Making such a contract $s_B(\cdot)$ is mutually profitable for both the firm and the brown investor only if $\mathbb{E}_B[c(x, S^*)] \geq \mathbb{E}_B[s_B(x)] \geq 1$. Combined with (A.22), this observation implies $\mathbb{E}_B[c(x, S^*)] = 1$, which makes the second term in (A.23) zero and hence irrelevant. Therefore, the green investor's equilibrium payoff is still given by (A.23) in this case, regardless of the probability that the firm accepts an offer from the brown investors in equilibrium.

In equilibrium, the green investor's payoff must be non-negative, which is the case if

and only if

$$0 < \beta^* \leq \tilde{\beta}^* := \frac{\mathbb{E}_G[c(x, S^*)] - 1 + \phi}{\phi - (\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)])} = \frac{\mathbb{E}_G[c(x, S^*)] - 1 + \phi}{\phi - (\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi)}. \tag{A.24}$$

As λ increases, the sensitivity of $c(\cdot, S^*)$ decreases, and thus, $\mathbb{E}_G[c(x, S^*)]$ decreases. As $\lambda \to \tilde{\lambda}^*$, $\mathbb{E}_G[c(x, S^*)]$ converges to $\mathbb{E}_G[c(x, S^G_{1-\phi})]$ so the upper bound $\tilde{\beta}^*$ converges to zero. On the other hand, $\tilde{\beta}^*$ converges to 1 as $\lambda \to \underline{\lambda}^*$.

A.5.4 Proof of Sufficiency

Suppose (A.11) holds. For any $\beta^* \in (0, \tilde{\beta}^*)$, where $\tilde{\beta}^*$ is the cutoff identified in (A.24), one can easily construct a mixed-strategy equilibrium as follows. (i) The green investor offers the equity $s^*(\cdot) = c(\cdot, S^*)$ constructed in Observation A.4. (ii) All brown investors offer $\tilde{s}(\cdot) \neq s^*(\cdot)$ that satisfies (A.17) in Observation A.5. (iii) If no investor deviates from the above offer strategies, the firm accepts $s^*(\cdot)$ and then chooses projects B and G with probabilities β^* and $1 - \beta^*$, respectively. (iv) If any investor unilaterally deviates, the firm accepts a security $s(\cdot)$ such that $\mathbb{E}_B[s(x)]$ is lowest among all available offers and then chooses project B.

First of all, the sensitivity of $s^*(\cdot) = c(\cdot, S^*)$ is exactly equal to $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, and thus, the firm indeed finds projects B and G indifferent under $s^*(\cdot)$. It is straightforward that the other parts of the strategy specified in (iii) and (iv) are also optimal for the firm. In particular, note that Lemma 1-(iii) guarantees that the firm finds project B more profitable under any security $s(\cdot) \neq s^*(\cdot)$ (including $\tilde{s}(\cdot)$) such that $\mathbb{E}_B[s(x)] \leq \mathbb{E}_B[s^*(x)]$.

Next, the green investor obtains non-negative payoff from offering $s^*(\cdot)$ as discussed in Section A.5.3. Furthermore, the green investor has no incentive to deviate from $s^*(\cdot)$ because any deviation would result in the firm selecting project B while generating a weakly negative financial payoff for the green investor. All brown investors also have no incentive to deviate from $\tilde{s}(\cdot)$ for the same reason.

A.6 Proof of Theorem 5

In this proof, let $\mathcal{S}$ denote the set of all securities that satisfy all the regularity conditions we imposed (i.e., $0 \leq s(x) \leq x$ for all $x \geq 0$, and both $s(x)$ and $x - s(x)$ are weakly increasing in x). Recall from Lemma 1:

(a) The debt contract $d(\cdot)$ is the least sensitive among all securities in $\mathcal{S}$ such that the expected payment under each project $k \in \{B, G\}$ coincides with $\mathbb{E}_k[d(x)]$. Furthermore, between any two debt contracts $d(\cdot)$ and $\tilde{d}(\cdot)$,

$$\begin{aligned} &\mathbb{E}_B[d(x)] - \mathbb{E}_G[d(x)] > \mathbb{E}_B[\tilde{d}(x)] - \mathbb{E}_G[\tilde{d}(x)] \\ \iff &\mathbb{E}_B[d(x)] > \mathbb{E}_B[\tilde{d}(x)] \\ \iff &\mathbb{E}_G[d(x)] > \mathbb{E}_G[\tilde{d}(x)]. \end{aligned}$$

(b) The equity contract $c(\cdot)$ is the most sensitive among all securities in $\mathcal{S}$ such that the expected payment under each project $k \in \{B, G\}$ coincides with $\mathbb{E}_k[c(x)]$. Furthermore, between any two equity contracts $c(\cdot)$ and $\tilde{c}(\cdot)$,

$$\begin{aligned} &\mathbb{E}_B[c(x)] - \mathbb{E}_G[c(x)] > \mathbb{E}_B[\tilde{c}(x)] - \mathbb{E}_G[\tilde{c}(x)] \\ \iff &\mathbb{E}_B[c(x)] > \mathbb{E}_B[\tilde{c}(x)] \\ \iff &\mathbb{E}_G[c(x)] > \mathbb{E}_G[\tilde{c}(x)]. \end{aligned}$$

Proof of (i): In what follows, we consider the case that all investors are restricted to offering debt contracts only, as assumed in Theorem 5-(i). The properties (a) and (b), as stated above, are the only properties we utilized in our proof of Theorem 1 - 4. Therefore, if only debt contracts are available, Theorem 1 and 2 still hold, with modifications to $\underline{\lambda}^*$ and $\overline{\lambda}^*$ as follows:

$$\begin{aligned} \underline{\lambda}^* &= \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)]\right\}\right], \\ \overline{\lambda}^* &= \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)]\right\}\right], \end{aligned}$$

which implies $\underline{\lambda}^* = \overline{\lambda}^*$.

To prove that no other equilibrium exists, first consider an equilibrium in which the firm chooses project G with probability one under the borrowing term $\mathbb{E}_G[s^*(x)] \in [1-\phi, 1)$. Note that Observation A.2 in Section A.4 still hold. Thus, if such an equilibrium exists, we

must have D^* and $\tilde{D} \neq D^*$ such that

$$\mathbb{E}_B[d(x, D^*)] - \mathbb{E}_G[d(x, D^*)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[d(x, \tilde{D})] - \mathbb{E}_G[d(x, \tilde{D})], \tag{A.25}$$

$$\mathbb{E}_G[x - d(x, D^*)] + \lambda\phi = \mathbb{E}_B[x - d(x, \tilde{D})]. \tag{A.26}$$

Note that (A.25) implies $D^* > \tilde{D}$. On the other hand, (A.26) is equivalent to

$$\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi = d(x, \tilde{D}) - d(x, D^*).$$

where $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi > 0$ by Assumption 1. Thus, (A.26) implies $\tilde{D} > D^*$. This contradicts the earlier observation that $\tilde{D} > D^*$. Hence, there is no equilibrium such that the firm chooses project G with probability one under the borrowing term $\mathbb{E}_G[s^*(x)] \in [1 - \phi, 1)$.

Next, consider a mixed-strategy equilibrium in which the firm randomizes its project selection by choosing project B with some probability $\beta^* \in (0, 1)$ and project G with probability $1 - \beta^*$. All the arguments in the proof of Theorem 4 remain valid once we replace the equity in each step of the proof with the debt contract with the same expected payment to the funding investor. Thus, an equilibrium of our interest here could exist only if $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$, where $\underline{\lambda}^*$ and $\tilde{\lambda}^*$ in the statement of Theorem 4 are modified as follows:

$$\underline{\lambda}^* = \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)]\right\}\right],$$

$$\tilde{\lambda}^* = \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D_{1-\phi}^G)] - \mathbb{E}_G[d(x, D_{1-\phi}^G)]\right\}\right].$$

Both Observation A.4 and A.5 in Section A.5 must also remain to hold. Thus, whenever such a mixed-strategy equilibrium exists, there are two different debt contracts $d(\cdot, D^*)$ and $d(\cdot, \tilde{D})$ (hence, $D \neq \tilde{D}$) such that (i) both $d(\cdot, D^*)$ and $d(\cdot, \tilde{D})$ are offered in equilibrium, (ii) the firm accepts $d(\cdot, D^*)$ with positive probability, and (iii) the following two conditions are satisfied simultaneously:

$$\mathbb{E}_B[d(x, \tilde{D})] \leq \mathbb{E}_B[d(x, D^*)],$$

$$\mathbb{E}_B[d(x, D^*)] - \mathbb{E}_G[d(x, D^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[d(x, \tilde{D})] - \mathbb{E}_G[d(x, \tilde{D})].$$

However, the condition $\mathbb{E}_B[d(x, \tilde{D})] \leq \mathbb{E}_B[d(x, D^*)]$ holds between two different debt contracts only if $D^* > \tilde{D}$ and $\mathbb{E}_B[d(x, \tilde{D})] < \mathbb{E}_B[d(x, D^*)]$. In that case, the firm strictly prefers $d(\cdot, \tilde{D})$ to $d(\cdot, D^*)$, which contradicts the supposition (ii) above. Therefore, there is no mixed-strategy

equilibrium such that the firm randomizes its project selection.

Proof of (ii): Next, we restrict all investors to offer equity only, as assumed in Theorem 5-(ii). Once again, the above properties (a) and (b) are the only properties we exploited in our proof of Theorem 1 - 4. Thus, if only equity contracts are available, Theorem 1 and 2 remain to hold, with modifications to $\underline{\lambda}^*$ and $\overline{\lambda}^*$ as follows:

$$\underline{\lambda}^* = \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)]\right\}\right],$$
$$\overline{\lambda}^* = \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)]\right\}\right].$$

Note that $\underline{\lambda}^* = \overline{\lambda}^*$. Finally, we can also prove that no other equilibrium exists in a way essentially identical to the proof for the case where only debt contracts are available. Therefore, we omit the details.

Proof of (iii): From parts (i) and (ii), the firm's expected repayment is one, whether it issues debt or equity, implying its payoff is $\mathbb{E}_B[x] - 1$ from debt financing and $\mathbb{E}_G[x] - 1 + \lambda\phi$ from equity financing, respectively. By Assumption 1-(i), we have

$$\mathbb{E}_B[x] - 1 > \mathbb{E}_G[x] - 1 + \lambda\phi.$$

Hence, the firm issues debt and chooses project B.

A.7 Proof of Lemma 3

Suppose to the contrary that the green firm issues debt. If the green firm is believed to choose project G, its funding term $d(\cdot, D^*)$ must satisfy

$$\mathbb{E}_B[d(x, D^*)] - \mathbb{E}_G[d(x, D^*)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda_G\phi.$$

However, since $\lambda_G < \overline{\lambda}^*$, we have $\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda_G\phi$, implying $D^* > D_1^B$. Then a brown investor will profitably deviate by offering $d(\cdot, D_1^B + \varepsilon)$ for some $0 < \varepsilon < D^* - D_1^B$, which is strictly more favorable to the green firm than $d(\cdot, D^*)$ and induces the green firm to choose project B, a contradiction.

A.8 Proof of Theorem 6

To characterize the separating equilibrium, define

$$\underline{\rho}^* := \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda_G \phi,$$
$$\overline{\rho}^* := \mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] = 1 - \mathbb{E}_G[c(x, S_1^B)].$$

We begin with two preliminary observations regarding $\underline{\rho}^*$ and $\overline{\rho}^*$.

Lemma A.2. *(i)* $0 < \underline{\rho}^* < \overline{\rho}^* < 1$*;* *(ii)* $S^G_{1-\overline{\rho}^*} = S_1^B$ *and* $S^G_{1-\rho} \leq S_1^B$ *if and only if* $\rho \leq \overline{\rho}^*$*.*

Proof. The statement (ii) immediately follows from the definition of $\overline{\rho}^*$ and $S^G_{1-\rho}$:

$$\begin{aligned} & \overline{\rho}^* = 1 - \mathbb{E}_G[c(x, S_1^B)] \quad \text{and} \quad \mathbb{E}_G[c(x, S^G_{1-\overline{\rho}^*})] = 1 - \overline{\rho}^* \\ \Longrightarrow \quad & \mathbb{E}_G[c(x, S_1^B)] = \mathbb{E}_G[c(x, S^G_{1-\overline{\rho}^*})] \quad \text{and} \quad S_1^B = S^G_{1-\overline{\rho}^*}. \end{aligned}$$

To prove (i), note first that since $0 < \mathbb{E}_G[c(x, S_1^B)] < \mathbb{E}_B[c(x, S_1^B)] = 1$, we must have $\overline{\rho}^* = 1 - \mathbb{E}_G[c(x, S_1^B)] < 1$. Furthermore,

$$\begin{aligned} \overline{\rho}^* = \mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] &> \underline{\rho}^* = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda_G \phi \\ &> \mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] > 0, \end{aligned}$$

where the first two inequalities follow from $\lambda_G \in (\underline{\lambda}^*, \overline{\lambda}^*)$, and the last inequality follows from the MLRP property. Thus, $(\underline{\rho}^*, \overline{\rho}^*)$ is non-empty and included in $(0, 1)$. *Q.E.D.*

To prove part (i), consider a fictitious equilibrium in which the green and brown firms choose projects B and G, respectively. Without loss of generality, suppose that the green firm issues equity while the brown firm issues debt in this equilibrium. In what follows, we focus on the subgame that begins after the green firm chooses to issue equity. Let S^* denote the equilibrium strike price (i.e., the highest strike price among all investors) offered in this subgame. Then, S^* must be weakly greater than S_1^G; otherwise, a brown investor would deviate by offering a slightly higher strike price $S' \in (S^*, S_1^G)$ to secure a positive financial profit, $\mathbb{E}_G[c(x, S')] - 1 > 0$. Hence, in this subgame, no investor earns a positive financial profit in equilibrium.

First, consider the case in which there is no non-consequentialist green investor; thus, there is only a consequentialist green investor. Then, S^* must be S_1^G by Theorem 5. With this

borrowing term, the green firm's expected payoff is $\mathbb{E}_G[x - c(x, S_1^G)] + \lambda_G \phi = \mathbb{E}_G[x] - 1 + \lambda_G \phi$. However, if the green firm deviates and issues debt to mimic the brown firm, it obtains $\mathbb{E}_B[x - d(x, D_1^B)] = \mathbb{E}_B[x] - 1 > \mathbb{E}_G[x] - 1 + \lambda_G \phi$. Therefore, issuing equity is not incentive compatible for the green firm.

Now, consider the case in which there is a single non-consequentialist green investor; thus, there are two green investors—one non-consequentialist and one consequentialist. There are two subcases.

First, consider the subcase in which a single investor offers S^*. This investor must be the non-consequentialist green investor, who obtains an extra payoff $\rho > 0$ from directly funding the firm's project G. However, the non-consequentialist green investor can profitably deviate by offering a strike price S' that is still higher than all other investors' strike prices (so the firm accepts S'), but lower than S^* (so the firm chooses project G after accepting S'). This deviation increases the investor's financial payoff without sacrificing her non-financial payoff from funding the green project, which contradicts equilibrium behavior.

Next, consider the subcase in which $n > 1$ investors offer S^*. Again, one of these investors must be the non-consequentialist green investor, whose expected payoff from offering S^* is given by:

$$\frac{1}{n} \left(\mathbb{E}_G[c(x, S^*)] - 1 + \rho \right) + \phi > 0.$$

There are two possibilities. If $\mathbb{E}_G[c(x, S^*)] - 1 + \rho \leq 0$, then the remaining $n - 1$ investors who offer S^* incur an expected financial loss and thus have an incentive to deviate to a lower strike price to avoid that loss. If $\mathbb{E}_G[c(x, S^*)] - 1 + \rho > 0$, then the non-consequentialist green investor has an incentive to deviate to a slightly higher strike price so that the firm accepts her offer with probability one. Either case contradicts equilibrium and therefore cannot occur in equilibrium.

To prove part (ii), suppose there are $n_{NC} \geq 2$ non-consequentialist green investors. Consider any separating equilibrium (if any) in which the green firm chooses project G after issuing equity, while the brown firm chooses project B after issuing debt. From **??**, the brown firm issues debt with face value D_1^B in this equilibrium. Let S^* denote the equilibrium strike price of the equity contract issued by the green firm.

All players must have no incentive to deviate from the their equilibrium strategies. Hence, the following conditions (S1)–(S4) constitute necessary and sufficient conditions for the separating equilibrium:

(S1) Non-consequentialist green investors must break even under $c(\cdot, S^*)$:

$$\frac{1}{n_{NC}}\left(\mathbb{E}_G[c(x,S^*)]-1+\rho\right)+\phi=\phi \quad \Longleftrightarrow \quad S^*=S^G_{1-\rho}.$$

(S2) $c(\cdot, S^*)$ must be sensitive enough to induce green investment:

$$\mathbb{E}_B[c(x,S^*)]-\mathbb{E}_G[c(x,S^*)]\geq \mathbb{E}_B[x]-\mathbb{E}_G[x]-\lambda_G\phi.$$

(S3) The green firm must prefer issuing equity:

$$\mathbb{E}_B[x-d(x,D_1^B)]=\mathbb{E}_B[x]-1\leq \mathbb{E}_G[x-c(x,S^*)]+\lambda_G\phi.$$

(S4) The brown firm must prefer issuing debt:

$$\mathbb{E}_B[x-d(x,D_1^B)]=\mathbb{E}_B[x]-1\geq \max\left\{\mathbb{E}_B[x-c(x,S^*)],\mathbb{E}_G[x-c(x,S^*)]+\lambda_B\phi\right\}.$$

To understand why (S1) must hold in equilibrium, note that Bertrand competition among investors would induce all non-consequentialist green investors (and no others) to offer S^* in the subgame after the firm chooses to issue equity. In equilibrium, each non-consequentialist green investor must be indifferent between offering S^* (in which case she directly funds the firm and obtains an extra utility ρ with probability $1/n_{NC}$) and withholding her offer (in which case she still enjoys the social value ϕ). The left- and right-hand sides of the equation in (S1) represent the expected payoffs in each case.

From now on, we impose $S^*=S^G_{1-\rho}$ in (S2)–(S4) without loss. In the remainder of the proof, we show that conditions (S2)–(S4) jointly hold if and only if $\rho\in[\underline{\rho}^*,\overline{\rho}^*]$. First, (S2) holds only if $\rho\leq\overline{\rho}^*$ and $\lambda_G\geq\underline{\lambda}^*$. Indeed, $S^*=S^G_{1-\rho}\leq S^B_1$ whenever $\rho\leq\overline{\rho}^*$ (by Lemma A.2), and thus,

$$\begin{aligned}\mathbb{E}_B[c(x,S^*)]-\mathbb{E}_G[c(x,S^*)] &\geq \mathbb{E}_B[c(x,S^B_1)]-\mathbb{E}_G[c(x,S^B_1)] && \because S^B_1\geq S^*\\ &\geq \mathbb{E}_B[x]-\mathbb{E}_G[x]-\lambda_G\phi. && \because \lambda_G\geq\underline{\lambda}^*\end{aligned}$$

Second, it is immediate from the definition of $\underline{\rho}^*$ that (S3) holds if and only if $\rho\geq\underline{\rho}^*$.

Finally, we prove that (S4) holds if and only if $\rho\leq\overline{\rho}^*$. To prove the "only if" direction,

note that the inequality in (S4) implies

$$\mathbb{E}_B[x] - 1 \geq \mathbb{E}_B[x - c(x, S^G_{1-\rho})],$$

which is equivalent to

$$\mathbb{E}_B[c(x, S^G_{1-\rho})] \geq 1 = \mathbb{E}_B[c(x, S^B_1)] \quad \Longleftrightarrow \quad S^G_{1-\rho} \leq S^B_1 \quad \Longleftrightarrow \quad \rho \leq \overline{\rho}^*.$$

Thus, (S4) holds only if $\rho \leq \overline{\rho}^*$. To prove the "if" direction, suppose $\rho \leq \overline{\rho}^*$. Then by Lemma A.2-(ii), we have $\mathbb{E}_B[x] - 1 \geq \mathbb{E}_B[x - c(x, S^G_{1-\rho})]$. Hence, (S4) holds if the following inequality also holds:

$$\mathbb{E}_B[x] - 1 \geq \mathbb{E}_G[x - c(x, S^G_{1-\rho})] + \lambda_B \phi.$$

Indeed,

$$\begin{aligned}
\mathbb{E}_B[x] - 1 &= \mathbb{E}_B[x - c(x, S^B_1)] \\
&\geq \mathbb{E}_G[x - c(x, S^B_1)] + \lambda_B \phi \\
&\geq \mathbb{E}_G[x - c(x, S^G_{1-\rho})] + \lambda_B \phi,
\end{aligned}$$

where the first inequality follows from $\lambda_B \leq \underline{\lambda}^*$ and the second from Lemma A.2-(ii).