Heterogeneous Downward Nominal Wage Rigidity: Foundations of a Nonlinear Phillips Curve*

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This draft: October 13, 2025

Abstract

We propose a model with heterogeneous downward nominal wage rigidity across individual labor varieties. The model delivers a smooth, convex wage Phillips curve linking current wage inflation to current unemployment that is relatively steep at high levels of inflation and relatively flat at low levels of inflation. Using U.S. regional data, we estimate a smooth nonlinear wage Phillips curve that nests the linear case and find strong evidence of nonlinearity, consistent with the model's predictions. These results suggest that the cost of lowering demand-driven high inflation is low in terms of unemployment, and that the cost of reducing demand-driven high unemployment is low in terms of inflation. Although the model features occasionally binding constraints for individual labor varieties, there are no such constraints in the aggregate, making the model amenable to perturbation analysis.

Keywords: Downward nominal wage rigidity, nonlinear wage Phillips curve, unemployment, inflation, NAIRU.
JEL Classification: E24, E31, E32.

*An earlier version of this paper circulated under the title "Heterogeneous Downward Nominal Wage Rigidity: Foundations of a Static Wage Phillips Curve," NBER Working Paper No. 30774, December 2022. We thank Francisco Ciocchini, Pablo Cuba Borda, Giulia Gitti, Ina Hajdini, Juan Herreño, Bart Hobijn, Cristina Manea, and Brenda Samaniego de la Parra, for helpful comments and suggestions and Giovanni Bonfanti, Patricio Goldstein, and Lucas Rosso for superb research assistance. We also thank for comments seminar participants at the Cowles Foundation, the San Francisco Fed, the St Louis Fed, the Minneapolis Fed, the Dallas Fed, the Cleveland Fed, the Board of Governors, the central bank of Chile, PUC Santiago de Chile, Brown, Columbia, Notre Dame, Penn, the 2024 Midwest Macroeconomics Meeting, Rutgers, the National Bank of Poland, CUNY, ESSEC (Paris), the 2024 Women in Macro Conference, CEBRA, and the 2024 NBER SI.

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1 Introduction

The seminal empirical work of Phillips (1958) documented a negative relationship between unemployment and wage inflation and suggested the possibility that this relationship could be nonlinear, exhibiting a relatively steep slope at high levels of inflation and a relatively flat slope at low levels. The question of nonlinearity in the wage Phillips curve remained mostly dormant until the second decade of the 21st century, when two significant macroeconomic events rekindled interest: one involving a large reduction in unemployment without much of an increase in inflation (the exit from the global financial crisis), and another involving a reduction in high inflation without much of an increase in unemployment (the tightening following the inflation spike after the Covid-19 pandemic). More broadly, understanding the nature of nonlinearity in the Phillips curve is important because it can shed light on the possibility that both the cost of stabilizing high inflation in terms of unemployment and the cost of reducing high unemployment in terms of inflation may be relatively low.

This paper proposes a model of a nonlinear wage Phillips curve due to heterogeneous downward nominal wage rigidity. Specifically, wage rigidity is assumed to vary in intensity across a continuum of labor varieties. The nominal wage of each labor variety is bounded below by the average wage prevailing in the previous period times a variety-specific scalar. In all respects other than the heterogeneity of downward nominal wage rigidity, the model economy is standard; households and firms operate in competitive markets and are rational and forward looking.

In equilibrium the model delivers a nonlinear wage Phillips curve. An increase in wage inflation raises the fraction of labor varieties that are not constrained by the wage lower bound. As a result, the fraction of the labor force suffering involuntary unemployment falls. These effects imply a negative relationship between current wage inflation and current unemployment. Importantly, the sensitivity of the implied relationship between unemployment and wage inflation changes at different levels of inflation. For low levels of inflation, a large measure of workers is stuck at their wage lower bound. As a result, an increase in inflation, by lowering the real value of the wage lower bound, raises employment for a large measure of workers. Thus, equilibrium unemployment is relatively sensitive to changes in inflation. By contrast, for high levels of inflation, the mass of workers with a binding wage constraint is small, so an increase in inflation stimulates employment, but only for a small group of workers, rendering unemployment relatively insensitive to changes in inflation.

A conceptual contribution of this paper is to show that market power and price setting are not prerequisites for introducing nominal rigidity. The model does not make the standard assumption that workers possess market power and instead treats households and firms

as price takers in both product and factor markets. The implied Phillips curve is a contemporaneous relationship between unemployment and wage inflation, in line with Phillips' empirical formulation. In this respect, the proposed framework departs from the New Keynesian model, which features a forward-looking Phillips curve linking unemployment not only to current wage inflation but also to future expected wage inflation. The reason why the new-Keynesian model generates an expectations-augmented wage Phillips curve is that it assumes that workers have market power. This assumption together with the assumption of nominal wage rigidity implies that the wage setting decision is forward looking, as today's nominal wage choice impacts the entire expected future path of the worker's real wage. The assumption that workers have market power can be justified in economies with a strong presence of labor unions, but is less tenable in economies, like the United States, in which secularly a small fraction of the labor force is unionized.

We identify the parameters of the model using moments stemming from both macro and micro data that include no information about the curvature of the wage Phillips curve. The calibrated model predicts a wage Phillips curve that is smooth and nonlinear. We stress the smoothness of the implied nonlinear Phillips curve in light of econometric evidence suggesting the lack of a kink in the relationship between wage growth and unemployment (Bernanke and Blanchard, 2023). The implied degree of nonlinearity of the wage Phillips curve is quantitatively significant. For example, reducing inflation from 6 to 5 percent, raises unemployment by 0.3 percentage points, while reducing inflation from 2 to 1 percent raises unemployment by 3 percentage points.

We find that the predicted Phillips curve captures relatively well the nonlinear relationship between unemployment and nominal wage growth observed in the U.S. economy over the past four decades. In particular, it provides theoretical support for the relatively low cost in terms of employment resulting from the stabilization efforts in the aftermath of the Covid-19 inflation spike as well as for the missing inflation observed during the recovery from the 2008 financial crisis. For the pandemic era, the model predicts that in 2020 and 2021, the U.S. economy was hit by adverse supply shocks, but that the inflation spike of 2022 was primarily due to demand shocks.

We explore the smooth nonlinearity of the Phillips curve further by estimating a parametric nonlinear wage Phillips curve that nests a linear version as a special case using regional U.S. data from combined statistical areas (CSAs). We find statistically significant evidence of a nonlinear component. A version of the model calibrated to match the location and slope, but not the curvature, of the regional Phillips curve generates a degree of nonlinearity consistent with the estimated empirical relationship.

In spite of the aforementioned differences with the new-Keynesian framework, for regular

fluctuations of inflation around the intended target, under plausible calibrations, the proposed model delivers equilibrium dynamics that are quantitatively similar to those associated with the standard new-Keynesian model with wage rigidity. An implication of this result is that the assumption that workers have market power does not appear to play a crucial role, at least for standard calibrations of the model considered in the related literature. In sum, the proposed model globally delivers a nonlinear Phillips curve—which is important for understanding unusual events like the pandemic inflation and the great contraction—and locally preserves the dynamic properties of the new-Keynesian model—which is important for understanding normal fluctuations like those observed during the great moderation (roughly 1985 to 2007).

Finally, the paper makes a methodological contribution. One impediment that has limited a more widespread adoption of models with downward nominal wage rigidity in monetary analysis in spite of their empirical appeal, is the difficulty to approximate their equilibrium conditions due to the presence of occasionally binding constraints. This is most relevant for medium scale models used for policy analysis. This paper contributes to overcoming this impediment. Unlike standard models with homogeneous downward nominal wage rigidity, the proposed model is amenable to perturbation analysis, which is the standard method used to approximate and estimate equilibrium dynamics. Although in the present formulation there are occasionally binding constraints at the level of individual labor varieties, in the aggregate the equilibrium conditions do not feature such restrictions, thereby allowing for the differentiation of the aggregate equilibrium conditions around the deterministic steady-state.

Related Literature

The empirical relevance of downward nominal wage rigidity has been extensively documented by, among others, Card and Hyslop (1997), Kahn (1997), Altonji and Devereux (2000), Gottschalk (2005), Barattieri, Basu, and Gottschalk (2014), Daly and Hobijn (2014), Schmitt-Grohé and Uribe (2016), Jo (2025), and Grigsby, Hurst, and Yildirmaz (2021).

Cross-sectional heterogeneity in the degree of downward nominal wage rigidity, the friction that gives rise to a convex wage Phillips curve in the model proposed in this paper, has been documented using administrative, survey, and experimental data. Using data on nominal wage changes from 1991 to 1997 from a Swiss representative labor force survey and a representative administrative database, Fehr and Goette (2005) estimate significant dispersion in the cost of cutting nominal wages across individual workers. This finding is consistent with experimental results by Fehr and Falk (1999) and Fehr and Gächter (2000) showing that respondents vary with respect to their nominal fairness standards. In turn, Bewley (1999) finds that fairness considerations are the main reason why firms are reluctant to cut wages during recessions. Specifically, Bewley conducts a survey of more than 300 firm

managers in the northeastern United States during the recession of the early 1990s and finds that the most common answer of why firms were reluctant to cut wages was that wage cuts undermine morale, which disrupts productivity in the workplace.

There is also evidence of heterogeneous downward nominal wage rigidity at the layoff margin. Davis and Krolikowski (2024) conduct a survey of new unemployment insurance recipients in Illinois over the period September 2018 to July 2019, during which the labor market was relatively tight. They find that survey respondents varied significantly in the nominal wage cut that they would have been willing to accept to maintain their last job, even when controlling for individual characteristics including race, gender, education, and job tenure.

Heterogeneity in nominal wage rigidity stemming from the fact that the timing of wage changes varies across firms and sectors and is determined independently of the state of the business cycle has been documented using large representative administrative data sets (see Murray, 2021, for the United States; Faia and Pezone, 2023, and Fanfani, 2023, for Italy; and Adamopoulou, Díez-Catalán, and Villanueva, 2024, for Spain). All of these studies demonstrate that heterogeneity is substantial and has significant real effects. To the best of our knowledge, the present paper is the first one to introduce heterogeneous downward nominal wage rigidity into a general equilibrium setting.

This paper is also related to a literature on the role of nominal wage rigidity for macroeconomic adjustment. In the context of the new-Keynesian framework, sticky wages à la Calvo was introduced by Erceg, Henderson, and Levin (2000). The derivation of a linear wage Phillips curve associated with that model is presented in Casares (2010) and Galí (2011). Kim and Ruge-Murcia (2009) study a model with nominal wage rigidity à la Rotemberg (1982) with an asymmetric wage adjustment cost function. They estimate the parameters of this cost function and find that wage cuts are more costly than wage increases. They estimate that downward nominal wage rigidity implies a positive optimal inflation rate of about 35 basis points. Elsby (2009) studies downward nominal wage rigidity in the context of a model in which firms have monopsony power in the labor market and Benigno and Ricci (2011) in a model in which workers have monopoly power. There is also a literature combining labor search frictions and nominal rigidities including Faia (2008), Gertler, Sala, and Trigari (2008), and Dupraz, Nakamura, and Steinsson (2022). Unlike the present study, these papers are not concerned with the global nonlinearity of the Phillips curve.

There is a literature that has examined the effects of nominal wage rigidity more globally. The starting point is the empirical estimate by Phillips (1958) of a negative nonlinear relation between wage inflation and unemployment. Phillips hypothesized that the observed nonlinearity might be the consequence of downward nominal wage rigidity, but did not pro-

vide a theoretical model. Harding, Lindé, and Trabandt (2022, 2023) show, using nonlinear numerical approximation methods, that the new-Keynesian model of Smets and Wouters (2007), which features a Kimball aggregator for both intermediate goods and labor varieties, when reparameterized to allow for stronger real rigidities implies a convex relationship between the contemporaneous equilibrium values of the output gap and inflation. Specifically, in the Smets and Wouters model the strength of real rigidities is increasing in the curvature of the Kimball aggregator for intermediate goods. Smets and Wouters use a value of 10 for this curvature parameter, whereas Harding, Lindé, and Trabandt, based on a reestimation of the model, use a value of 64.5. A difference with the present study is that the Smets and Wouters model assumes bilateral wage rigidity, whereas the present study assumes that wages are downwardly rigid, which has been shown to be more empirically compelling. Elsewhere (Schmitt-Grohé and Uribe, 2012, 2013, 2016, 2017) we have investigated the implications of downward nominal wage rigidity for macroeconomic adjustment in dynamic general equilibrium models of open and closed economies. In contrast to the present formulation, these earlier studies maintain a homogeneous lower bound on nominal wages. This class of models yields a limiting case of nonlinearity, characterized by an L-shaped Phillips curve, horizontal at all positive levels of unemployment and vertical at zero unemployment. This type of Phillips curve does not fully align with the observed convex but relatively smooth relation between unemployment and wage inflation. A further distinction between the heterogeneous and homogeneous versions of the downward nominal wage rigidity model is that the latter is not amenable to perturbation analysis due to the occasionally binding constraint in its aggregated equilibrium conditions.

Daly and Hobijn (2014) introduce downward nominal wage rigidity in a sticky wage model à la Erceg, Henderson, and Levin (2000). Using a nonlinear approximation method that tracks the distribution of individual wages, they show that the model captures qualitatively the observed cross-sectional distribution of wage changes. Further, Daly and Hobijn show that their model delivers a bent wage Phillips curve. Two differences of the present model with this (and other formulations of wage stickiness) is that in the present model workers are assumed not to have market power (a feature that, as pointed out above, is relevant for modeling the U.S. economy, given its low degree of unionization), and that workers are not obliged to work more hours than desired at the going wage.

In contemporaneous work, Benigno and Eggertsson (2023) add downward nominal wage rigidity to a new-Keynesian model with labor search frictions. Wages are assumed to be flexible when the labor market is tight and downwardly rigid when it is not. This assumption introduces a kink in a piece-wise linearized Phillips curve relating price inflation to labor market tightness (and other variables). By contrast, the model proposed here predicts a

nonlinear, convex relationship between wage growth and unemployment but without a kink, which is more consistent with existing empirical estimates. For example, Bernanke and Blanchard (2023) find no evidence of kinks in the relationship between wage growth and market slack when unemployment is linked to wage growth in a smooth and convex fashion.[1] Also, the assumption of wage flexibility when the labor market is tight, maintained in Benigno and Eggertsson (2023), is not entirely consistent with the evidence in Davis and Krolikowski (2024) discussed above, indicating that even in tight labor markets nominal wages display a high degree of downward rigidity. The model with heterogeneous downward nominal wage rigidity proposed here is consistent with this evidence, because it predicts that even when the labor market is tight, unemployment is the consequence of downward nominal wage rigidity. A further difference with the work of Benigno and Eggertsson (2023) is that the Phillips curve predicted by their formulation with a single kink can capture a steepening of the Phillips curve but not a flattening, and therefore does not address the missing inflation observed in the recovery from the great recession. By contrast, the wage Phillips curve predicted by the present model becomes increasingly flatter as unemployment rises thereby capturing the missing inflation phenomenon.

Our paper is also related to a body of work on the empirical estimation of Phillips curves. Galí (2011) and Galí and Gambetti (2019) estimate linear wage Phillips curves on aggregate U.S. data. We use results from the latter paper to calibrate the baseline version of the model. Our estimates of nonlinear wage Phillips curves using regional data builds on a number of empirical contributions. Fitzgerald and Nicolini (2014), McLeay and Tenreyro (2019), and Hazell et al. (2022) use regional data to estimate linear price Phillips curves. The low inflation exit from the great recession of 2008 and the labor-market resilience during the post-Covid-19 disinflation spurred empirical work on non-linearities in the Phillips curve. Leduc and Wilson (2017) relate the missing inflation post great recession to a flattening of the Phillips curve, and Crust, Lansing, and Petrosky-Nadeau (2023) interpret the missing unemployment post Covid-19 as a steepening of the Phillips curve. Using state-level data and piecewise non-linear specifications, Leduc, Marti, and Wilson (2019) and Hooper, Mishkin, and Sufi (2020) find little support for nonlinearities. By contrast, Kumar and Orrenius (2016), using pre-Covid state-level data and piecewise linear specifications of the wage Phillips curve, find strong evidence of nonlinearities. Using more recent data from U.S. metropolitan statistical areas, Cerrato and Gitti (2022) find that post Covid-19 the slope of regional price Phillips curves was three times larger than pre Covid-19. Using the same data, Gitti (2024) estimates kinked Phillips curves linking price inflation and labor market tightness and finds support

[1] See in particular Bernanke and Blanchard (2023) Section IV, and especially the regression results in Table A10 and the discussion in footnote 27.

for this type of nonlinearity. Our estimates of regional wage Phillips curves contribute to this literature along three dimensions: First, consistent with the proposed theoretical model in which workers lack market power, our specification excludes expected inflation. This is important because it simplifies the choice of valid controls. Second, we estimate a smooth, continuous nonlinear wage Phillips curve rather than a piecewise linear one. Third, we proxy wage inflation using an employment cost index based measure, which mitigates composition bias.

The remainder of the paper is organized as follows. Section 2 develops the heterogeneous downward nominal wage rigidity model. Section 3 derives the model implied wage Phillips curve, discusses its shifters, and gauges its ability to explain observed patterns of wage inflation and unemployment. Section 4 estimates a nonlinear empirical wage Phillips curve on regional data and compares it with its theoretical counterpart. Section 5 characterizes the long-run wage Phillips curve implied by the model and extends the model to allow for endogenous labor supply and idiosyncratic productivity shocks. Section 6 shows that locally the model behaves like the standard new-Keynesian model of wage rigidity. Section 7 concludes.

2 The Model

The model features firms that use a variety of differentiated labor inputs. Nominal wages are downwardly rigid and the degree of rigidity varies across labor varieties. Households and firms are price takers in product and factor markets. Like existing models with nominal rigidity, the adjustment is non-Walrasian. A difference with existing models in the new-Keynesian tradition is that nominal rigidity is introduced without requiring market power and price setting.

2.1 Firms

Firms use labor to produce a final good. Profits are given by

$$P_t a_t F(h_t) - W_t h_t,$$

where P_t denotes the product price level, h_t denotes labor, W_t denotes the nominal wage rate, a_t is an exogenous productivity shock, and $F(\cdot)$ is an increasing and concave production function. The optimality condition determining the demand for labor is

$$a_t F'(h_t) = \frac{W_t}{P_t}, \tag{1}$$

which equates the marginal product of labor to the real wage.

The labor input h_t is assumed to be a composite of a continuum of labor varieties h_{jt} for $j \in [0,1]$. The aggregation technology is of the form

$$h_t = \left[\int_0^1 h_{jt}^{1-\frac{1}{\eta}} dj\right]^{\frac{1}{1-\frac{1}{\eta}}}, \tag{2}$$

where $\eta > 0$ is the elasticity of substitution across labor varieties.[2] The firm chooses the quantity of each labor variety h_{jt} to minimize its total labor cost, $\int_0^1 W_{jt} h_{jt} dj$, subject to the aggregation technology (2), given its desired amount of the labor composite h_t and taking as given the wage of each variety of labor, denoted W_{jt}. This cost minimization problem yields the demand for labor of type j

$$h_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\eta} h_t, \tag{3}$$

where

$$W_t = \left[\int_0^1 W_{jt}^{1-\eta} dj\right]^{\frac{1}{1-\eta}} \tag{4}$$

is the cost-minimizing price of one unit of aggregate labor, that is, when h_{jt} is chosen optimally for all j, the aggregate wage rate W_t satisfies $W_t h_t = \int_0^1 W_{jt} h_{jt} dj$.

2.2 Households

The representative household has preferences over streams of consumption, denoted c_t, described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{5}$$

where $\beta \in (0,1)$ is a subjective discount factor, and $U(\cdot)$ is an increasing and concave period utility function.

The household supplies inelastically $\bar{h}$ units of labor of each variety $j \in [0,1]$.[3] The economy faces an exogenous natural rate of unemployment denoted u_t^n. The natural rate of unemployment reflects frictions in the labor market unrelated to nominal rigidity (Friedman, 1968). More broadly, we interpret variations in u_t^n as aggregate supply shocks. The effective supply of labor of each variety is then given by $\bar{h}(1-u_t^n)$. Sometimes the household will not be able to sell all the units of labor it supplies. In these circumstances, employment is demand-determined and the household suffers involuntary unemployment above the natural

[2] Section 5.3 introduces heterogeneity in labor productivity across workers.

[3] Section 5.2 endogenizes the supply of labor.

rate. Formally, households supply labor of each variety j subject to the constraint

$$h_{jt} \leq \bar{h}(1 - u_t^n). \tag{6}$$

Each period $t \geq 0$, households can trade a nominally risk free discount bond denoted B_t that pays the interest rate i_t when held between periods t and $t+1$. In addition, each period the household pays real lump-sum taxes in the amount τ_t and receives profits from the ownership of firms in the amount ϕ_t. Its sequential budget constraint is then given by

$$c_t + \frac{B_t/P_t}{1+i_t} + \tau_t = \int_0^1 \frac{W_{jt}}{P_t} h_{jt} dj + \frac{B_{t-1}/P_{t-1}}{1+\pi_t} + \phi_t,$$

where

$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 \tag{7}$$

denotes the inflation rate. The household chooses contingent plans for bond holdings and consumption to maximize its lifetime utility subject to its sequential budget constraint and some no-Ponzi game borrowing limit, taking as given ϕ_t, i_t, P_t, and W_{jt}, for $j \in [0,1]$. The optimality conditions associated with consumption and bond holdings give rise to the Euler equation

$$U'(c_t) = \beta(1+i_t) E_t \frac{U'(c_{t+1})}{1+\pi_{t+1}}. \tag{8}$$

2.3 Heterogeneous Downward Nominal Wage Rigidity

Each period $t \geq 0$, the nominal wage of every variety $j \in [0,1]$ is assumed to be subject to a lower bound constraint of the form

$$W_{jt} \geq \gamma(j) W_{t-1}, \tag{9}$$

where $\gamma(j)$ is a positive and increasing function governing the degree of downward nominal wage rigidity of labor variety j. The wage lower bound is assumed to depend on the past average wage rate, W_{t-1}, instead of on the past variety-specific wage rate, W_{jt-1}, to facilitate aggregation.

The rationale for introducing heterogeneity in downward nominal wage rigidity is rooted in empirical and experimental studies documenting two facts: (i) fairness standards play a significant role in shaping downward nominal wage rigidity, and (ii) there is substantial heterogeneity in these standards across workers and sectors. A detailed discussion of this evidence is presented in the literature review in the introduction. Incorporating fairness stan-

dards explicitly into the model would be desirable, but lies beyond the scope of this paper. An additional motivation for introducing heterogeneity in downward nominal wage rigidity is empirical evidence indicating that labor productivity varies across firms and workers, and that firms find it easier to cut wages when worker productivity is low (Fehr and Goette, 2005; Bewley, 1999; Campbell and Kamlani, 1997). We develop a model that incorporates this form of heterogeneity in section 5.3.

The function $\gamma(\cdot)$ need not be interpreted as representing a fixed ordering of labor varieties. For example, welders could be represented by $j = 0.45$ in period t and by $j = 0.73$ in period $t + 1$. This could occur, for example, because employment in welding became more regulated. The present formulation of downward nominal wage rigidity nests the homogeneous case studied in Schmitt-Grohé and Uribe (2016), which obtains when the function $\gamma(j)$ is independent of j.

The labor market closes with a slackness condition at the level of each labor variety,

$$[\bar{h}(1 - u_t^n) - h_{jt}][W_{jt} - \gamma(j)W_{t-1}] = 0. \tag{10}$$

This condition implies that when an occupation exhibits unemployment above the natural rate, the wage must be at its lower bound. Conversely, if the wage is strictly above the lower bound, the occupation must be at full employment, defined as an unemployment rate equal to the natural rate. Condition (10) has the same logical status as a market clearing condition in a Walrasian setting. Specifically, in a Walrasian environment, the wage W_{jt} would adjust to ensure that the market for each labor variety clears at the desired quantities, $[\bar{h}(1 - u_t^n) - h_{jt}] = 0$, for all j. In the present non-Walrasian environment, however, because of nominal rigidity, W_{jt} might not be able to adjust to ensure market clearing, giving rise to involuntary unemployment.

2.4 The Government

The central bank sets the nominal interest rate according to a Taylor rule of the form

$$1 + i_t = \frac{1 + \pi^*}{\beta} \left(\frac{1 + \pi_t}{1 + \pi^*} \right)^{\alpha_\pi} \left(\frac{y_t}{y} \right)^{\alpha_y} \mu_t, \tag{11}$$

where π^* denotes the central bank's inflation target, y_t denotes aggregate output, y denotes the steady-state value of y_t, α_π and α_y are parameters, and μ_t is an exogenous and stochastic monetary shock.

We assume that fiscal policy is passive in the sense that government solvency is satisfied independently of the path of the price level.

2.5 Equilibrium

In equilibrium, aggregate output is given by

$$y_t = a_t F(h_t). \tag{12}$$

Market clearing in the goods market requires that consumption equal output,

$$c_t = y_t. \tag{13}$$

We are now ready to define a competitive equilibrium.

Definition 1 (Competitive Equilibrium)**.** *A competitive equilibrium is a set of processes* c_t*,* y_t*,* h_t*,* h_{jt}*,* W_t*,* W_{jt}*,* P_t*,* π_t*, and* i_t *satisfying* (1)*,* (3)*,* (4)*, and* (6)–(13) *for all* $j \in [0,1]$ *and* $t \geq 0$*, given the initial wage* W_{-1} *and the exogenous disturbances* a_t*,* μ_t*, and* u_t^n*.*

Next, we show that the equilibrium conditions can be written in terms of a single labor variety.

2.6 Equilibrium in j^* Form

We consider an equilibrium in which for every $t \geq 0$ there exists a cutoff labor variety denoted $j_t^* \in (0,1)$ that operates at full employment, $h_{j_t^* t} = \bar{h}(1-u_t^n)$, and for which the wage lower bound holds with equality, $W_{j_t^* t} = \gamma(j_t^*)W_{t-1}$. Evaluating the labor demand (3) at $j = j_t^*$ yields the condition

$$\bar{h}(1-u_t^n) = \left(\frac{\gamma(j_t^*)}{1+\pi_t^W}\right)^{-\eta} h_t, \tag{14}$$

where

$$\pi_t^W \equiv \frac{W_t}{W_{t-1}} - 1 \tag{15}$$

denotes wage inflation in period t.

All varieties $j < j_t^*$ also pay the wage $\gamma(j_t^*)W_{t-1}$ and operate at full employment. To see this, suppose first, contrary to the claim, that $W_{jt} < W_{j_t^* t}$ for some $j < j_t^*$. Then, by (3) we have that $h_{jt} = (W_{jt}/W_t)^{-\eta} h_t > (W_{j_t^* t}/W_t)^{-\eta} h_t = \bar{h}(1-u_t^n)$, which violates the time constraint (6). Intuitively, since at $W_{j_t^* t}$ there is full employment, a wage lower than $W_{j_t^* t}$ would induce a demand for labor in excess of full employment, which is impossible. Suppose now that, contrary to the claim, $W_{jt} > W_{j_t^* t}$ for some $j < j_t^*$. Then by the same logic $h_{jt} < \bar{h}(1-u_t^n)$. Further, $W_{jt} > W_{j_t^* t} = \gamma(j_t^*)W_{t-1} > \gamma(j)W_{t-1}$. So we have that in this case $\bar{h}(1-u_t^n) - h_{jt} > 0$ and $W_{jt} - \gamma(j)W_{t-1} > 0$, which violates the slackness condition (10).

It also follows that all labor varieties $j > j_t^*$ are stuck at their wage lower bound and suffer involuntary unemployment. To see this, use (3) and (9) to write, for any $j > j_t^*$, $h_{jt} = (W_{jt}/W_t)^{-\eta} h_t \leq (\gamma(j)W_{t-1}/W_t)^{-\eta} h_t < (\gamma(j_t^*)W_{t-1}/W_t)^{-\eta} h_t = \bar{h}(1-u_t^n)$. This shows that all labor varieties $j > j_t^*$ suffer involuntary unemployment. It then follows from the slackness condition (10) that $W_{jt} = \gamma(j)W_{t-1}$, that is, wages of all labor varieties $j > j_t^*$ are stuck at their lower bounds.

Summing up, in the equilibrium we are considering, we have that[4]

$$\begin{cases} h_{jt} = \bar{h}(1-u_t^n) \text{ and } W_{jt} = \gamma(j_t^*)W_{t-1} & \text{for } j \leq j_t^* \\ h_{jt} < \bar{h}(1-u_t^n) \text{ and } W_{jt} = \gamma(j)W_{t-1} & \text{for } j > j_t^* \end{cases} . \tag{16}$$

The cutoff labor variety j_t^* is an important object in this model because it governs the extensive margin of unemployment, that is, how many occupations will operate below potential. Put differently, j_t^* is a measure of tightness in the labor market: The larger j_t^* is, the smaller the number of labor varieties suffering Keynesian unemployment will be.

The facts that varieties below j_t^* are fully employed, $h_{jt} = \bar{h}(1-u_t^n)$ for $j < j_t^*$, and that varieties above j_t^* suffer involuntary unemployment, $h_{jt} < \bar{h}(1-u_t^n)$ for $j > j_t^*$, means that for varieties $j < j_t^*$ employment is supply-determined and that for varieties $j > j_t^*$ employment is demand-determined. This marks a difference with the new-Keynesian model with Calvo-type nominal wage rigidity in which employment is demand-determined for all labor varieties at all times. This difference arises because, unlike in the Calvo-type wage stickiness model, in the present model workers are not forced to work more hours than they wish at the prevailing wage, which is arguably a more realistic assumption.

Next, we show how the labor-market tightness indicator j_t^* is linked to wage inflation and unemployment, providing the building blocks for the wage Phillips curve implied by the model. We begin by writing the wage aggregation equation (4) as

$$\begin{aligned} W_t^{1-\eta} &= \int_0^1 W_{jt}^{1-\eta} dj \\ &= \int_0^{j_t^*} [\gamma(j_t^*)W_{t-1}]^{1-\eta} dj + \int_{j_t^*}^1 [\gamma(j)W_{t-1}]^{1-\eta} dj \\ &= W_{t-1}^{1-\eta} \left[j_t^* \gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta} dj \right]. \end{aligned}$$

The second equality follows from (16). Using the definition of wage inflation given in (15)

[4]Appendix A presents a graphical derivation of condition (16).

to eliminate W_t/W_{t-1} yields

$$(1+\pi_t^W)^{1-\eta} = j_t^*\gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta}dj. \tag{17}$$

According to this expression, wage inflation is increasing in the cutoff labor variety j_t^*. To understand why, suppose that the cutoff variety increases from $j_t^{*'}$ to $j_t^{*''} > j_t^{*'}$. Then, all varieties from 0 to $j_t^{*'}$ are unconstrained before and after the increase in j_t^*. As a result, their wages increase from $\gamma(j_t^{*'})W_{t-1}$ to $\gamma(j_t^{*''})W_{t-1}$. Varieties j between $j_t^{*'}$ and $j_t^{*''}$ were constrained before the change and become unconstrained after. For these workers, the wage rate increases from $\gamma(j)W_{t-1} < \gamma(j_t^{*''})W_{t-1}$ to $\gamma(j_t^{*''})W_{t-1}$. Finally labor varieties $j > j_t^{*''}$ are constrained before and after the change in j_t^*, so their wages remain unchanged. Since for every variety j the nominal wage either increases or stays the same, it follows that the aggregate wage, W_t, and hence wage inflation, π_t^W, increase. Because j_t^* reflects labor-market tightness, equation (17) says that wage inflation is increasing in labor-market tightness.[5]

We next establish how unemployment is linked to labor-market tightness. The aggregate unemployment rate, denoted u_t, is defined as the integral of the unemployment rates across all labor varieties. Formally,

$$\begin{aligned}
u_t &= \int_0^1 \left(\frac{\bar{h}-h_{jt}}{\bar{h}}\right) dj \\
&= u_t^n j_t^* + \int_{j_t^*}^1 \left(\frac{\bar{h}-h_{jt}}{\bar{h}}\right) dj \\
&= u_t^n j_t^* + (1-j_t^*) - \frac{h_t}{\bar{h}} \int_{j_t^*}^1 \left(\frac{W_{jt}}{W_t}\right)^{-\eta} dj \\
&= u_t^n j_t^* + (1-j_t^*) - \left(\frac{W_{t-1}}{W_t}\right)^{-\eta} \frac{h_t}{\bar{h}} \int_{j_t^*}^1 \gamma(j)^{-\eta} dj.
\end{aligned}$$

The second and fourth equalities follow from (16) and the third from (3). Using equilibrium condition (14) and the definition of wage inflation given in (15) to eliminate $(W_{t-1}/W_t)^{-\eta}\, h_t$ and rearranging, we can write

$$u_t = u_t^n + (1-u_t^n)\left[(1-j_t^*) - \int_{j_t^*}^1 \left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{-\eta} dj\right]. \tag{18}$$

The right-hand side of equation (18) is decreasing in j_t^*. It follows that as the labor market tightness indicator j_t^* increases, the unemployment rate falls. This is intuitive because all

[5] For equation (17) to hold with equality at all times it must be the case that in equilibrium wage inflation be neither too high nor too low so as to rule out the corner solutions $j_t^* = 0$ and $j_t^* = 1$.

activities below the cutoff threshold j_t^* operate at full employment, so the higher the cutoff threshold is, the smaller the set of activities displaying unemployment above the natural rate will be.

Wage inflation and price inflation are linked by the identify

$$1 + \pi_t^W = \frac{w_t}{w_{t-1}}(1 + \pi_t), \tag{19}$$

where $w_t \equiv W_t/P_t$ denotes the real wage rate.

We can now define the competitive equilibrium in j^* form as follows:

Definition 2 (Competitive Equilibrium in j^* Form)**.** *A competitive equilibrium is a set of processes* j_t^**,* u_t*,* y_t*,* h_t*,* c_t*,* $w_t \equiv W_t/P_t$*,* i_t*,* π_t*, and* π_t^W*, satisfying* (1)*,* (8)*,* (11)–(14)*, and* (17)–(19)*, given the initial condition* w_{-1} *and the exogenous stochastic processes* a_t*,* μ_t*, and* u_t^n*.*

The Keynesian features of the model appear in (14), (17), and (18). All other equilibrium conditions are standard components of optimizing monetary models with or without nominal rigidity. In Appendix B, we show that monetary shocks have real consequences in this environment.

3 The Wage Phillips Curve

Given the natural rate of unemployment, u_t^n, equations (17) and (18) parametrically represent a contemporaneous relationship involving only unemployment and wage inflation (u_t and π_t^W). Further, u_t and π_t^W are negatively related. To see this, recall that equation (17) implies that π_t^W is increasing in j_t^* and that equation (18) implies that u_t is decreasing in j_t^*. Thus, the model's implied relationship between unemployment and wage inflation represents a downward sloping wage Phillips curve.

We note that the model implies a contemporaneous wage Phillips curve. In particular, it does not feature future expected inflation. In this sense, the present model departs from the new-Keynesian framework in which the wage Phillips curve is forward looking (Erceg, Henderson, and Levin, 2000; Galí, 2011). In both models, households and firms are rational, optimizing, and forward looking. The reason why the new-Keynesian model produces a forward-looking Phillips curve is its assumption that workers have monopoly power. By contrast, in the heterogeneous downward nominal wage rigidity model proposed here, households and firms are assumed to be price takers in the labor market. In this way, the present model provides microfoundations to Phillips's original formulation of a contemporaneous wage Phillips curve (Phillips, 1958). The following proposition summarizes this result.

Proposition 1 (The Wage Phillips Curve)**.** *The model with heterogeneous downward nominal wage rigidity implies a contemporaneous negative relationship between wage inflation,* π_t^W*, and the unemployment rate,* u_t*. This relationship is parametrically defined by equations* (17) *and* (18) *and depends on the exogenous supply shock* u_t^n*.*

The supply shock u_t^n is therefore a shifter of the wage Phillips curve, whereas all other disturbances, such as monetary shocks, productivity shocks, and preference shocks, cause movements along a given Phillips curve.

3.1 Curvature of the Wage Phillips Curve

To illustrate the properties of the wage Phillips curve implied by the model, we consider a linear functional form for $\gamma(j)$ and calibrate the parameters of the model. Specifically, assume that

$$\gamma(j) = (1 + \pi^*)^\delta(\Gamma_0 + \Gamma_1 j). \tag{20}$$

Here, the parameter $\delta \in [0, 1]$ captures the degree of wage indexation to long-run inflation, and the parameters $\Gamma_0, \Gamma_1 > 0$ govern the degree of downward nominal wage rigidity. This formulation implies that the degree of downward nominal wage rigidity distributes uniformly across varieties, with lower bound $(1 + \pi^*)^\delta\Gamma_0$ and upper bound $(1 + \pi^*)^\delta(\Gamma_0 + \Gamma_1)$. In Appendix E, we consider a case in which the degree of downward nominal wage rigidity has a Pareto distribution.

In general, one can obtain an exact numerical solution for the Phillips curve. This is the approach we take here.[6] To find the exact numerical solution for the Phillips curve, we calibrate the model as follows. The time unit is a quarter. The calibration period is 1986 to 2007. This period is of interest because it excludes the missing inflation episode in the recovery from the financial crisis of 2008 and the missing unemployment episode in the post Covid-19 disinflation, which are two periods often associated, respectively, with an unusually flat and an unusually steep portion of the Phillips curve.

We calibrate Γ_0 and Γ_1 to meet two restrictions. First, we impose that the Phillips curve contains the point π_t^W equal to 3 percent per year and u_t equal to 6 percent. These values correspond to the medians of wage inflation and unemployment observed over the calibration period. Second, we require that at that point the slope of the wage Phillips curve be -0.74, that is, a one-percentage point increase in the unemployment rate is associated with a 74-basis-point decline in the annual wage inflation rate. This value for the slope of the wage

[6]Depending on the assumed distribution of the wage rigidity parameter $\gamma(j)$, one can even obtain the Phillips curve in closed form. As shown in Appendix E this is, for example, the case when $\gamma(j)$ has a Pareto distribution.

Table 1: Parameter Values

Parameter	Value	Description
Γ_0	0.978	Parameter of the $\gamma(j)$ function
Γ_1	0.031	Parameter of the $\gamma(j)$ function
δ	1	Wage indexation parameter of the $\gamma(j)$ function
π^*	$1.03^{1/4} - 1$	Steady state inflation rate
u^n	0.04	Natural rate of unemployment
η	11	Elasticity of substitution across labor varieties
β	0.99	Subjective discount factor
σ	1	Inverse of intertemporal elasticity of substitution
θ	5	Inverse of Frisch elasticity of labor supply
α	0.75	Labor elasticity of output
α_π	1.5	Inflation coefficient of Taylor rule
α_y	0.125	Output coefficient of Taylor rule
ρ_μ	0.5	Persistence of monetary shock
ρ_a	0.9	Persistence of technology shock

Note. The time unit is a quarter.

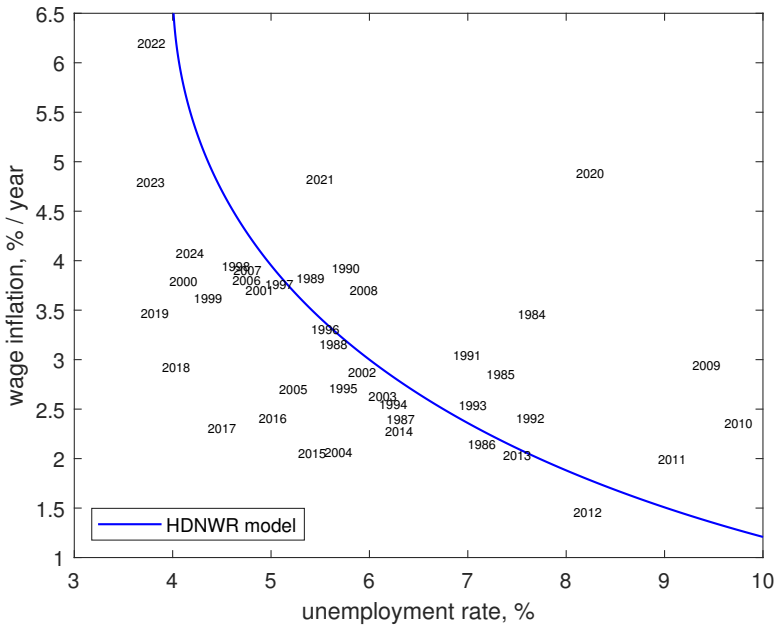
Phillips curve is taken from Galí and Gambetti (2019), who estimate a linear wage Phillips curve using U.S. data from 1986 to 2007.

In addition to these restrictions, to find Γ_0 and Γ_1 we need to assign values to u_t^n, η, δ, and π^*. We fix the natural rate of unemployment u_t^n at its steady-state value, denoted u^n. We set u^n equal to 0.04 (or 4 percent) to match the minimum unemployment rate observed over the calibration period 1986 to 2007. This value is in line with the 2024 Long-Range Consensus Forecast of the Blue Chip Survey (2024) of 4.1 percent for the period 2031-2035 and with the average of the U.S. full employment unemployment rate of 4.1 percent estimated by Michaillat and Saez (2024) over the period 1930 to 2024.

We set the elasticity of substitution across labor varieties to 11 ($\eta = 11$). This number is an average of the values used in Erceg, Henderson, and Levin (2000), Christiano, Eichenbaum, and Evans (2005), and Galí (2015). We assume full indexation of wages ($\delta = 1$), as in much of the related literature (Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007). We set π^* to 3 percent per year, which corresponds to the median value of wage inflation over the calibration period.

The resulting values for Γ_0 and Γ_1 are $\Gamma_0 = 0.978$ and $\Gamma_1 = 0.031$. We note that the calibration of the Phillips curve uses no information about curvature or about the period starting with the great contraction of 2008. The top panel of Table 1 summarizes the parameter values used in the computation of the Phillips curve. (The bottom panel of this table is discussed in section 6.)

Figure 1: The U.S. Wage Phillips Curve



Notes. The figure shows with a solid line the wage Phillips curve implied by the calibrated heterogeneous downward nominal wage rigidity (HDNWR) model for $u_t^n = u^n = 0.04$. The figure also shows the (u_t, π_t^W) pairs observed in annual U.S. data over the period 1984 to 2024.

Figure 1 shows with a solid line the exact wage Phillips curve predicted by the calibrated heterogeneous downward nominal wage rigidity model in the space (u_t, π_t^W). The supply shock u_t^n is set at its steady-state value of 4 percent. By construction, when the unemployment rate is 6 percent, the annual wage inflation rate is 3 percent. Also by construction, at that point, the slope of the Phillips curve is equal to -0.74.

However, the curvature of the relationship between unemployment and wage inflation is endogenously determined. The predicted Phillips curve is nonlinear, relatively steep at high levels of inflation and relatively flat at low levels of inflation implying that the costs in terms of unemployment of reducing high inflation and that the costs in terms of inflation of reducing high unemployment are both relatively small. For example, holding the shifter u_t^n constant, lowering inflation from 6 to 5 percent would increase the unemployment rate by only 0.3 percentage points, whereas lowering inflation from 2 to 1 percent would increase the unemployment rate by 3 percentage points.

The nonlinearity of the predicted wage Phillips curve provides a unified explanation for the apparent flattening of the Phillips curve (or missing inflation) in the recovery from the 2008 great contraction and for the resilience of the labor market during the tightening cycle that curbed the post Covid-19 inflation. This feature of the data is apparent in Figure 1, which, along with the predicted wage Phillips curve, displays annual observations of U.S.

unemployment and wage inflation for the period 1984 to 2024.[7]

At the same time, the model also predicts that the Phillips curve displayed significant shifts during the sample period. For example, as we will see in section 3.3 below, the model predicts that during the pandemic (2020 and 2021) the economy was buffeted by negative supply shocks (increases in u_t^n), which shifted the position of the wage Phillips curve to the right.

3.2 Shifters of the Wage Phillips Curve

Each data point in Figure 1 should be interpreted, up to possible measurement error, as being associated with a Phillips curve drawn for a different value of the supply shock u_t^n and possibly other factors determining its position, such as the long-run inflation rate π^*. Figure 2 shows how these shifters affect the position of the wage Phillips curve.

A negative aggregate supply shock in the form of an increase in the natural rate of unemployment, u_t^n, shifts the Phillips curve up and to the right. Intuitively, an exogenous increase in the number of unemployed workers requires more grease in the labor market (i.e., an increase in wage inflation) to maintain a given rate of overall unemployment.

Similarly, an increase in the long-run inflation rate (an increase in π^*) or an increase in the degree of downward nominal wage rigidity (an increase in Γ_0 or Γ_1) shifts the Phillips curve up and to the right. The intuition behind these effects is as follows. Wage inflation acts as a lubricant of the labor market because the higher wage inflation is, the larger the number of activities that are not constrained by the wage lower bound will be. An increase in π^*, Γ_0, or Γ_1 raises the wage lower bound. Thus, the economy needs more lubricant to maintain the same level of unemployment.

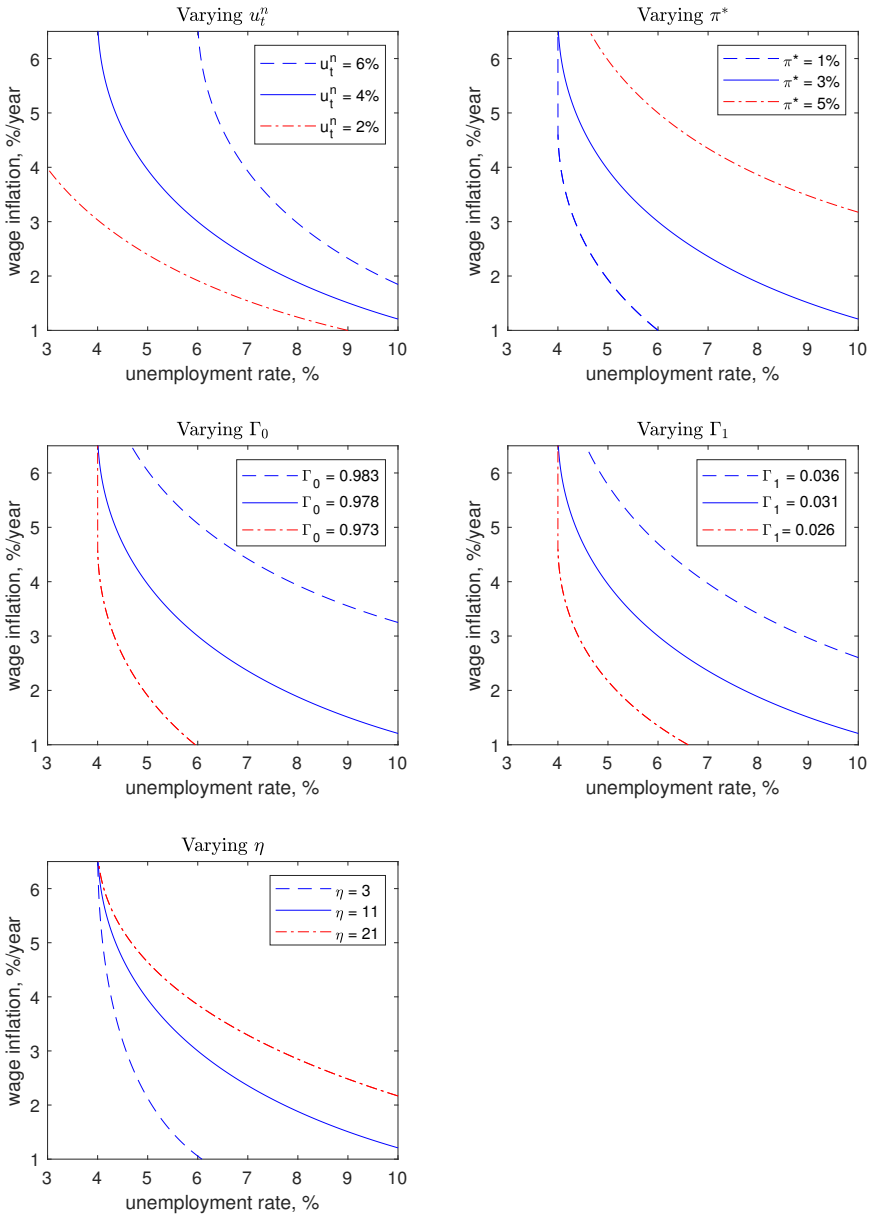
An increase in the elasticity of substitution across labor varieties, η, flattens the wage Phillips curve. Intuitively, the larger is η, the more sensitive labor demand becomes to changes in the relative wage rate. Thus, an increase in inflation, by reducing the relative wage of constrained labor varieties, causes a larger increase in employment the greater η is.

3.3 Demand and Supply Shocks in the Pandemic Era

According to conventional wisdom, the Covid-19 pandemic caused negative supply shocks due to lockdowns and other restrictions. These types of shocks are captured by u_t^n in the

[7]The unemployment rate is the arithmetic mean of monthly unemployment rates, FRED series UNRATE. Annual wage inflation is computed as the average of year-over-year monthly wage inflation. The measure of monthly nominal wages is Average Hourly Earnings of Production and Nonsupervisory Employees, FRED series AHETPI. This choice of this measure of wage inflation follows Galí and Gambetti (2019). In section 3.3, we consider a measure of wage inflation based on the employment cost index (ECI).

Figure 2: Shifters of the Wage Phillips Curve



Notes. Solid lines correspond to the baseline calibration. The variable u_t^n is the shock to the natural rate of unemployment. The parameter π^* is the inflation target. The parameters Γ_0 and Γ_1 pertain to the wage lower bound function $\gamma(j) = (1+\pi^*)^{\delta}(\Gamma_0 + \Gamma_1 j)$ (equation 20). The degree of indexation, δ, is set at 1. The parameter η represents the elasticity of substitution across labor varieties.

Table 2: Aggregate Supply Shocks During the Pandemic

		AHE		ECI	
	Actual Unemployment	Actual Wage Inflation	Predicted Supply Shocks	Actual Wage Inflation	Predicted Supply Shocks
Year	u_t	π_t^W	$u_t^n - u^n$	π_t^W	$u_t^n - u^n$
2020	8.10	4.88	3.70	2.92	2.05
2021	5.36	4.83	0.92	4.03	0.42
2022	3.64	6.20	-0.40	5.25	-0.62
2023	3.63	4.84	-0.81	4.62	-0.91

Note. Wage inflation is expressed in percent per year and the unemployment rate and the supply shock in percent.

model, which, as explained earlier, is a shifter of the Phillips curve, meant to capture cost-push shocks such as labor lockdowns, global supply chain disruptions, and movements in the price of commodities that serve as inputs of production. At the same time, both monetary and fiscal policy reacted aggressively during and after the pandemic, suggesting that demand forces—movements along a given Phillips curve—were also at work. An open question is what was the timing and impact of demand and supply shocks in the pandemic era.

The Phillips curve plotted in Figure 1 is calibrated with data prior to the global financial crisis and hence prior to the pandemic. Thus, there is nothing in the Phillips curve of Figure 1 that targets the pandemic years. In that figure, the supply shock u_t^n is set at its steady-state value ($u_t^n = u^n = 4$ percent). That is, the plot displays the relationship between wage inflation and unemployment predicted by the model in the absence of supply shocks. This is how a Phillips curve is typically depicted in the related literature. But this does not mean that the model predicts no supply shocks shifting the Phillips curve during the pandemic—or in other periods for that matter.

Assuming that the pandemic did not affect the long-run level of inflation, π^*—arguably the case of greatest empirical relevance (Brent and Smith, 2023; Lebow and Peneva, 2024; Schmitt-Grohé and Uribe, 2024)—and assuming no measurement errors in the observed values of unemployment and wage inflation (u_t and π_t^W), the supply shocks hitting the economy can be backed out by evaluating equilibrium conditions (17) and (18) at the actually observed levels of u_t and π_t^W. This yields a system of two equations in two unknowns, u_t^n (the object of interest) and the equilibrium cutoff variety j_t^* for each period t.

Table 2 displays the results of this exercise. The second and third columns display the actually observed values of unemployment and wage inflation, u_t and π_t^W, and the fourth column displays the supply shocks, $u_t^n - u^n$, that according to the model hit the economy in the pandemic years. The model predicts that the largest negative supply shock occurred

in 2020. This is consistent with the fact that 2020 was characterized by severe lockdowns in the United States. The second largest negative supply shock occurs the following year, 2021, when lockdowns were still in place but had been relaxed in many parts of the country. According to the model, by 2022 the negative supply shocks had ceased, which is in line with the lifting of restrictions and the recovery of aggregate activity observed at the time. This analysis suggests that the predicted curvature of the Phillips curve is not at odds with the prediction that the economy was buffeted by significant supply shocks during the worst of the pandemic.

The fact that according to the model by 2022 supply conditions had largely returned to normal implies that the model interprets the 2022 inflation surge as driven by demand shocks. This finding is in line with the empirical analyses of Bergholt et al. (2024) and Giannone and Primiceri (2024) identifying demand shocks as the key drivers of the post-Covid inflation surge.[8] The wage dynamics post Covid-19 predicted by the HDNWR model are also consistent with the observed wage compression documented in Autor, Dube, and McGrew (2023). Specifically, the HDNWR model predicts that when the economy moves up the wage Phillips curve (wage inflation is high and unemployment is low), a larger number of workers are unconstrained (j_t^* is relatively high). The equilibrium implication of this effect is that a larger fraction of wages are clustered around this cut-off point, that is, $W_{jt} = W_{j_t^*}$ for a larger fraction of workers, reducing wage inequality.

A number of authors have noted that the measurement of wage inflation can differ substantially when using average hourly earnings (AHE), as in the construction of the third column of Table 2, and when using the employment cost index (ECI), particularly during sharp contractions in employment (Solon, Barsky, and Parker, 1994, among others). A common concern is that if low-wage workers are disproportionately affected during downturns, AHE may suffer from an upward composition bias, overstating true wage inflation. The ECI, by contrast, is a Laspeyres index of compensation and thus mitigates this bias.

To assess whether the finding that the HDNWR model attributes the post-Covid inflation surge primarily to demand rather than supply shocks, depends on the wage measure used, we redo the exercise using the ECI to measure wage inflation.[9] The result appears in the last two columns of Table 2. Consistent with the composition-bias hypothesis, in 2020 (the height of the pandemic shutdown), ECI-based wage inflation is noticeably lower than AHE-

[8] Another notable inflation-stabilization episode in the postwar era is the Volcker disinflation. However, the present framework, in its current form, is not applicable for explaining the economic effects of that episode—and in particular the high sacrifice ratio it entailed—because it assumes anchored inflation expectations, which, as is widely believed, was not the case during the Volcker disinflation (see Mishkin, 2007).

[9] ECI wage inflation is constructed from the BLS series CIU2020000000000A, which reports wages and salaries for all private industry workers.

based wage inflation, 2.92 versus 4.88 percent. The implied contractionary supply shock in that year is correspondingly smaller (2.05 versus 3.70), though still substantial. More importantly, the conclusion that the post-Covid inflation spike was not driven by adverse supply conditions remains unchanged. Under both wage-inflation measures, inflation peaks in 2022 (5.25 and 6.20 percent under the ECI- and AHE-based measures, respectively), and in both cases the model infers that by 2022 the supply shock had dissipated, with u_t^n below its steady-state value u^n. Hence, the model continues to predict that the 2022 inflation surge was unlikely to be driven by supply shocks.

4 Nonlinearity: Evidence from U.S. Regional Data

Here, we explore whether the wage Phillips curve features nonlinearities using a panel of data from U.S. Combined Statistical Areas (CSAs). The advantage of employing regional data is that it provides more variation in wage inflation and unemployment than using national data. The use of regional data in the estimation of linear Phillips curves has been adopted by Fitzgerald and Nicolini (2014), McLeay and Tenreyro (2019), and Hazell et al. (2022), among others. Nonlinear regional Phillips curves have been estimated by Kumar and Orrenius (2016), Leduc and Wilson (2017), Leduc, Marti, and Wilson (2019), Hooper, Mishkin, and Sufi (2020), Cerrato and Gitti (2022), and Gitti (2024), among others. Our approach differs from these latter studies in three respects: First, in line with our theoretical model in which workers are assumed not to have market power, our Phillips curve does not include expected inflation. This distinction is relevant for the set of controls that are valid in estimation. Second, also in line with the predictions of the HDNWR model, we estimate a smooth nonlinear wage Phillips curve, as opposed to a piecewise linear specification. And third, our measure of inflation is inflation of hourly wages as measured by the employment cost index (ECI), instead of price inflation, annual wage and salary income per worker, or average hourly earnings. We use an employment cost index based measure of wage inflation, as it mitigates concerns about composition bias.

We estimate four empirical specifications of the regional wage Phillips curve. The dependent variable is the wage inflation rate in region i in quarter t, denoted π_{it}^W, expressed in percent per year. All specifications include as dependent variables a constant, the unemployment rate in region i in quarter t, u_{it}, expressed in percent, the reciprocal of the unemployment rate, $1/u_{it}$, which captures nonlinearity, and an endogenous control variable, denoted c_{it}, meant to capture regional supply disturbances. The need to control for supply disturbances arises from the fact that the wage Phillips curve predicted by the HDNWR model contains the supply shock u_{it}^n. Since this supply shock is a shifter of the wage Phillips

curve—rather than a source of movements along it—failing to control for it would result in the regression error being correlated with unemployment, which would create endogeneity bias in the estimated coefficients. We do not control for expected inflation, as is done in related empirical studies, because the HDNWR model implies a contemporaneous relationship between wage inflation and unemployment.

The first specification we consider includes the right-hand side variables discussed above and a time trend, denoted t,

$$\pi_{it}^{W} = \alpha_0 + \alpha_1 u_{it} + \alpha_2 u_{it}^{-1} + \alpha_3 c_{it} + \alpha_4 t + \epsilon_{it}, \qquad \text{(Model 1)}$$

where ϵ_{it} is the regression error. The second specification adds regional fixed effects, denoted γ_i,

$$\pi_{it}^{W} = \alpha_0 + \alpha_1 u_{it} + \alpha_2 u_{it}^{-1} + \alpha_3 c_{it} + \alpha_4 t + \gamma_i + \epsilon_{it}. \qquad \text{(Model 2)}$$

The third specification replaces the time trend t with time fixed effects, denoted γ_t,

$$\pi_{it}^{W} = \alpha_0 + \alpha_1 u_{it} + \alpha_2 u_{it}^{-1} + \alpha_3 c_{it} + \gamma_t + \gamma_i + \epsilon_{it}. \qquad \text{(Model 3)}$$

The time fixed effect, γ_t, controls for nationwide shocks, which could be shifters of the regional Phillips curve (nationwide supply shocks) or movements along a given regional Phillips curve (nationwide demand shocks). It follows that adding time fixed effects alleviates the endogeneity bias though at the cost of removing some potentially useful information for identifying α_1 and α_2. Following Leduc and Wilson (2017), McLeay and Tenreyro (2019), and Hazell et al. (2022), the fourth, and last, specification replaces the unemployment rate with the unemployment gap. We proxy the unemployment gap by $u_{it} - \bar{u}$, where $\bar{u}$ reflects the steady-state value of the natural rate of unemployment. Consistent with the calibration strategy of the HDNWR model, we proxy $\bar{u}$ by the floor of the minimum unemployment rate in the regional panel, which equals 2 percent ($\bar{u} = 2$),

$$\pi_{it}^{W} = \alpha_0 + \alpha_1 (u_{it} - \bar{u}) + \alpha_2 (u_{it} - \bar{u})^{-1} + \alpha_3 c_{it} + \gamma_t + \gamma_i + \epsilon_{it}. \qquad \text{(Model 4)}$$

All specifications nest a linear Phillips curve when $\alpha_2 = 0$.

The data are taken from the Bureau of Labor Statistics (BLS). The regions correspond to CSAs. The number of CSAs included in the sample is determined by the availability of quarterly wage and price inflation data, the latter being required for the construction of the endogenous control (c_{it}), as explained below. The BLS provides such data for the 15 largest CSAs. We dropped two CSAs for which no price inflation data were available prior to 2018, leaving 13 CSAs that together account for about 40 percent of total U.S. employment over

the sample period. The time dimension spans 2006:Q4 to 2025:Q2 for all CSAs except one, which begins in 2009:Q3.[10] The length of the panel is dictated by the availability of ECI data at the quarterly frequency. The resulting dataset includes two recent episodes that have been identified as pushing the economy into, respectively, the flat and steep portions of the wage Phillips curve: the missing inflation associated with the recovery from the global financial crisis and the missing unemployment associated with the tightening in response to the post-Covid-19 inflation spike.

The data source for wage inflation, π_{it}^W, is BLS (2025a). Wage inflation is measured by the twelve-month change in the ECI, as the three-month change, which would have been preferable, is not published at the CSA level. The data source for the unemployment rate, u_{it}, is BLS (2025b). In line with Gordon (2013), we use energy price inflation differentials to control for supply shocks. Specifically, the regional supply disturbance c_{it} is proxied by the difference between two inflation differentials, namely, the differential between regional energy price inflation, denoted π_{it}^E, and national energy price inflation, denoted π_t^E, and the differential between regional core inflation, denoted π_{it}^C, and national core inflation, denoted π_t^C, that is, $c_{it} \equiv (\pi_{it}^E - \pi_t^E) - (\pi_{it}^C - \pi_t^C)$. This control is meant to capture regional shifters of the wage Phillips curve. The data sources for energy and core inflation are the regional and national CPI data sets published by the BLS (2025c).[11] Appendix D provides more details about the data sources. All models are estimated using ordinary least squares.

The top panel of Table 3 presents the estimation results for the full sample, 2006:Q4 to 2025:Q2. For all four models, the coefficient on $(u_{it} - \bar{u})^{-1}$ is significantly positive at conventional levels of confidence. Hence, one can reject the null hypothesis that the Phillips curve is linear. Figure 3 displays with dashed lines the estimated empirical wage Phillips curves for Models 1 to 4 and with dots the data residualized with respect to the controls (c_{it}, γ_t, γ_i, and t, as applicable). All Phillips curves display a negative and noticeably convex shape. The estimated slopes at unemployment rates of 3, 5.1 (sample median), and 9 percent, respectively, are $(-0.67, -0.26, -0.11)$ for Model 1, $(-0.78, -0.30, -0.13)$ for Model 2, $(-0.45, -0.25, -0.17)$ for Model 3, and $(-0.48, -0.20, -0.18)$ for Model 4. These

[10]The 13 CSAs included are Atlanta-Athens-Clarke County-Sandy Springs, GA-AL CSA; Boston-Worcester-Providence, MA-RI-NH CSA; Chicago-Naperville, IL-IN-WI CSA; Dallas-Fort Worth, TX-OK CSA; Detroit-Warren-Ann Arbor, MI CSA; Houston-Pasadena, TX CSA; Los Angeles-Long Beach, CA CSA; Miami-Port St. Lucie-Fort Lauderdale, FL CSA; New York-Newark, NY-NJ-CT-PA CSA; Philadelphia-Reading-Camden, PA-NJ-DE-MD CSA; San Jose-San Francisco-Oakland, CA CSA; Seattle-Tacoma, WA CSA (the CSA with the later start in 2009:Q3); and Washington-Baltimore-Arlington, DC-MD-VA-WV-PA CSA. The two CSAs dropped from the sample are Phoenix-Mesa, AZ CSA, and Minneapolis-St. Paul, MN-WI CSA.

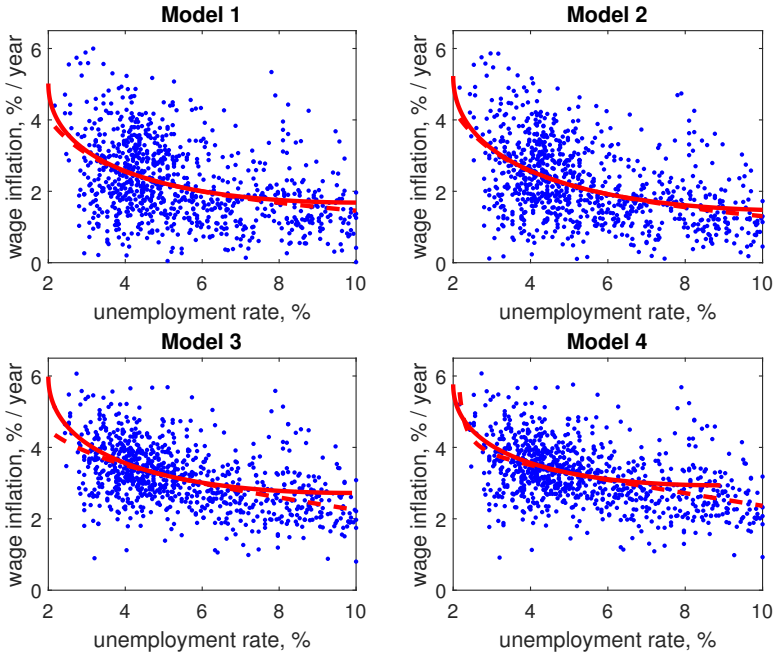
[11]The BLS publishes CPI data at the metropolitan statistical area (MSA) level, which is not always identical to the CSA level. In such cases, we use the MSA containing the largest city of the corresponding CSA.

Table 3: Estimation of the U.S. Wage Phillips Curve Using Regional Data

	$(u_{it} - \bar{u})$	$(u_{it} - \bar{u})^{-1}$	const.	Adj. R^2	Endog. Control	Time Trend	Region Fixed Effect	Time Fixed Effect
Full Sample (2006:Q4–2025:Q2)								
Model 1	-0.041	5.633	1.304	0.385	Yes	Yes	No	No
	(0.17)	(0.000)	(0.001)					
Model 2	-0.044	6.616	1.083	0.416	Yes	Yes	Yes	No
	(0.142)	(0.000)	(0.008)					
Model 3	-0.14	2.782	3.373	0.628	Yes	No	Yes	Yes
	(0.000)	(0.016)	(0.000)					
Model 4	-0.171	0.314	3.69	0.629	Yes	No	Yes	Yes
	(0.000)	(0.005)	(0.000)					
Pre-Covid-19 Sample (2006:Q4-2019:Q4)								
Model 1	-0.185	3.578	3.376	0.354	Yes	Yes	No	No
	(0.000)	(0.003)	(0.000)					
Model 2	-0.24	3.309	3.848	0.44	Yes	Yes	Yes	No
	(0.000)	(0.004)	(0.000)					
Model 3	-0.222	4.918	3.276	0.456	Yes	No	Yes	Yes
	(0.000)	(0.006)	(0.000)					
Model 4	-0.274	0.524	3.87	0.452	Yes	No	Yes	Yes
	(0.000)	(0.063)	(0.000)					

Notes. p-values are reported in parentheses. In Models 1–3 $\bar{u} = 0$, and in Model 4 $\bar{u} = 2$. Cross-sectional coverage: 13 large U.S. Combined Statistical Areas (40 percent of U.S. employment). The number of observations is 964 for the full sample and 678 for the pre-Covid-19 sample.

Figure 3: Theoretical and Estimated Wage Phillips Curves: Regional Data

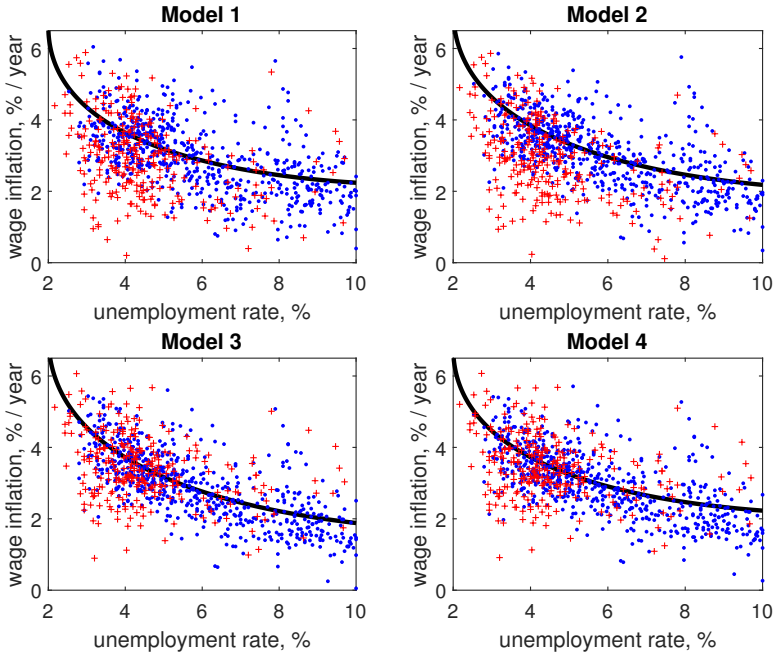


Notes. The figure shows with solid lines the wage Phillips curve predicted by the HDNWR model, with dashed lines the estimated empirical wage Phillips curve, and with dots the data after residualizing wage inflation with respect to the controls of the respective estimated empirical models. The horizontal axis is truncated at an unemployment rate of 10 percent.

estimates imply that, on average, an increase in the unemployment rate from 3 to 4 percent is associated with a decline in wage inflation of 0.60 percentage points, whereas an increase in the unemployment rate from 9 to 10 percent is associated with a 0.15 percentage point fall in wage inflation. This represents a significant flattening of the wage Phillips curve as unemployment increases from unusually low to unusually high levels in response to demand shocks.

Figure 3 also displays with solid lines the associated Phillips curves predicted by the HDNWR model. The HDNWR model is calibrated following the same strategy as in the calibration on aggregate data. Specifically, we match the position and the slope of the estimated Phillips curve at the median rate of unemployment in the panel, 5.1 percent, and its slope at that point, -0.26, -0.30, -0.25, and -0.20, for Models 1 to 4, respectively. We set the steady-state natural rate of unemployment to the floor of the minimum rate of unemployment observed in the panel, which equals 2 percent. These three moments pin down the theoretical structural parameters Γ_0, Γ_1, and u^n. We note that no information about the curvature of the estimated Phillips curves is used in the calibration of the HDNWR model. Overall, the HDNWR model captures the estimated significant degree of nonlinearity in

Figure 4: The Pre-Covid-19 Regional Wage Phillips Curve



Notes. For each empirical model, the figure shows with a solid line the wage Phillips curve predicted by the HDNWR model when calibrated to match the position and slope of the empirical wage Phillips curve estimated on pre-Covid data, 2006:Q4 to 2019:Q4, with dots the pre-Covid residualized data, and with plus signs the post-2019:Q4 residualized data.

the Phillips curve relatively well.[12] On average over the calibrations based on Models 1 to 4, the slope of the Phillips curve implied by the HDNWR model falls from -0.75 when the unemployment rate is 3 percent to -0.03 when the unemployment rate is 9 percent, compared with a decline from -0.60 to -0.15 in the estimated empirical Phillips curves.

A natural question is whether the estimated nonlinearity of the Phillips curve is driven by the post-Covid-19 inflation spike and, in particular, by the absence of labor-market slack during the tightening aimed at curbing it. The bottom panel of Table 3 reports results from a subsample that excludes the periods following 2019:Q4. The coefficient on $(u_{it} - \bar{u})^{-1}$ remains significant at conventional levels, indicating that the presence of nonlinearity can also be detected in the pre-Covid-19 sample.

We now ascertain whether, when calibrated to pre-Covid data, the wage Phillips curve predicted by the HDNWR model also displays a meaningful degree of nonlinearity. The calibration strategy is the same as the one based on the full sample, which, as noted, does not use information about the curvature of the empirical wage Phillips curve. Figure 4 displays the wage Phillips curve calibrated with pre-Covid information. The implied tradeoff between

[12]The theoretical counterpart of Model 4 of the empirical wage Phillips curve supports a maximum of 9 percent unemployment, at which point all workers are constrained, i.e., $j_t^* = 0$.

unemployment and wage inflation is significantly steeper at low unemployment rates than at high unemployment rates. The average slope of the Phillips curve implied by the HDNWR model over the 4 calibrations falls from -1.01 to -0.13 as the unemployment rate increases from 3 percent to 9 percent.

To gauge the out-of-sample performance of the model, Figure 4 also shows the residualized data for the out-of-sample period 2020:Q1 to 2025:Q2. The out-of-sample data is more dispersed, but for the empirical models that include time fixed effects (Models 3 and 4) its distribution above and below the wage Phillips curve predicted by the HDNWR model is similar to that of the in-sample data. This finding suggests that the HDNWR model does a relatively good job at capturing the relationship between unemployment and wage inflation over the post 2019:Q4 period.

5 Extensions

This section characterizes the implied long-run wage Phillips curve and extends the model to allow for endogenous labor supply and cross-sectional heterogeneity in labor productivity.

5.1 The Long-Run Wage Phillips Curve

In the long run wage inflation and price inflation are both equal to the long-run inflation rate π^*. Specifically, because consumption is constant in the steady state, the Euler equation (8) implies the long-run Fisher relationship

$$i = \frac{1+\pi}{\beta} - 1.$$

This expression and the Taylor rule (11) imply that in the steady state inflation must be at its target level,

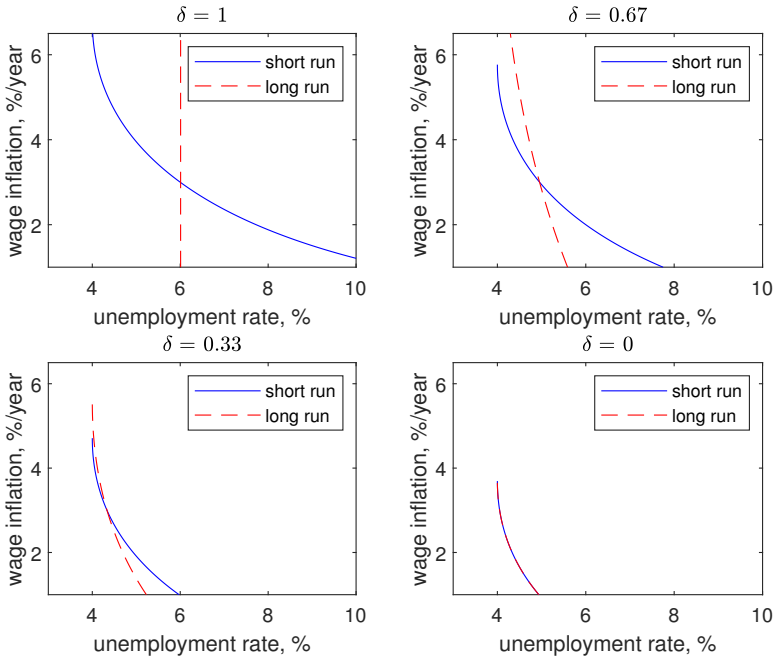
$$\pi = \pi^*.$$

Since in the steady state the real wage is constant, equilibrium condition (19) implies that wage inflation must equal product-price inflation,

$$\pi^W = \pi^*.$$

Equilibrium conditions (17) and (18) evaluated at $u_t = u$, $\pi_t^W = \pi^*$, and $u_t^n = u^n$ constitute a relationship between inflation and unemployment in the steady state, which we call the long-run wage Phillips curve. It follows immediately that in the absence of wage indexation

Figure 5: The Long-Run Wage Phillips Curve



Notes. The parameter δ pertains to the wage lower bound function $\gamma(j) = (1+\pi^*)^{\delta}(\Gamma_0 + \Gamma_1 j)$, given in equation (20). The figure shows that the long-run wage Phillips curve is in general downward sloping and steeper than the (short-run) Phillips curve. The long-run wage Phillips curve is vertical when $\delta = 1$ (baseline calibration) and identical to the (short-run) wage Phillips curve when $\delta = 0$.

($\delta = 0$), that is, when the function $\gamma(\cdot)$ is independent of π^*, the Phillips curve and the long-run Phillips curve coincide. But this ceases to be the case when wages are indexed to steady-state inflation. To see this, consider again the linear functional form for $\gamma(\cdot)$ given in equation (20). In this case, equilibrium conditions (17) and (18) evaluated at the steady state become

$$(1+\pi^W)^{(1-\eta)(1-\delta)} = j^* \widetilde{\gamma}(j^*)^{1-\eta} + \int_{j^*}^{1} \widetilde{\gamma}(j)^{1-\eta} dj, \tag{21}$$

$$u = u^n + (1-u^n)\left[(1-j^*) - \int_{j^*}^{1} \left(\frac{\widetilde{\gamma}(j)}{\widetilde{\gamma}(j^*)}\right)^{-\eta} dj\right], \tag{22}$$

where $\widetilde{\gamma}(j) \equiv \Gamma_0 + \Gamma_1 j$.

It is clear from (21) and (22) that under full wage indexation ($\delta = 1$, the baseline calibration) the long-run wage Phillips curve is perfectly vertical in the space (u, π^W). This is intuitive. Under full indexation, an increase in inflation fails to inject grease in the labor market in the long run, as indexation soaks it up one for one. By contrast, under imperfect indexation ($\delta < 1$), only a fraction δ of an increase in inflation is absorbed by indexation

and the rest is grease to the labor market.

To see more precisely what happens for intermediate degrees of wage indexation, Figure 5 displays the long-run wage Phillips curve for four different degrees of wage indexation, $\delta = 1, 2/3, 1/3$, and 0. For comparison, it also displays the corresponding (short-run) Phillips curves. The figure illustrates that absent full indexation the long-run wage Phillips curve is downward sloping and that as the degree of wage indexation goes down the slope of the long-run wage Phillips curve falls. The long-run wage Phillips curve rotates around the point $(u, \pi^W) = (0.06, 0)$ counterclockwise as δ declines. To see why this is so, recall that the calibration targets an unemployment rate of 6 percent and assumes full indexation. When $\delta = 1$, the left-hand side of (21) is equal to 1, regardless of the value of π^W. This uniquely pins down the steady-state value of j^* and by equation (22) also the steady state value of u. When $\delta < 1$ and inflation is zero ($\pi^W = 0$), then the left-hand side of equation (21) is also equal to 1, regardless of the value of δ. Thus, the long-run wage Phillips curve must contain the point $(u, \pi^W) = (0.06, 0)$ for any value of δ.

When $\delta = 1$, the unemployment rate associated with the rotation point $(u, \pi^W) = (0.06, 0)$ can be interpreted as the non-accelerating inflation rate of unemployment (NAIRU), because it is the rate of unemployment that can be sustained in the long run at the target rate of inflation.

Comparing the long-run and the short-run Phillips curves, the figure shows that for positive degrees of wage indexation $\delta \in (0, 1]$, the long-run Phillips curve is steeper than its short-run counterpart. The intuition why the wage Phillips curve is steeper in the long run is as follows. In the short run, movements in the inflation rate are not accompanied by movements in the long-run rate of inflation, so they grease the labor market one for one. By contrast, to the extent that δ is greater than zero, only a fraction $(1 - \delta)$ of an increase in inflation greases the labor market in the long run.

5.2 The HDNWR Model with Endogenous Labor Supply

Until now, we have considered an economy with an inelastic labor supply. This section shows that the main results are robust to assuming that the labor supply is endogenous.

Specifically, we replace the lifetime utility function (5) with the function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(c_t) - \int_0^1 V(h_{jt}^s) dj \right],$$

where h_{jt}^s denotes the amount of labor of type j supplied in period t, and $V(\cdot)$ is a convex

labor disutility function. To facilitate aggregation, we use the functional form

$$V(h) = \frac{h^{1+\theta}}{1+\theta}, \tag{23}$$

which is often used in the related literature (e.g., Galí, 2015). As before, there can be rationing in the labor market: for each labor type j, at the going wage W_{jt} households may not be able to sell all the units of labor they offer. The household sets its desired supply of labor of variety j to equate the marginal rate of substitution between labor and consumption to the variety-specific real wage. Formally, the supply of labor of type j is given by

$$\frac{V'(h^s_{jt})}{U'(c_t)} = w_{jt}, \tag{24}$$

where $w_{jt} \equiv W_{jt}/P_t$. We continue to assume that there is an exogenous amount of involuntary unemployment unrelated to wage stickiness, embodied in the variable u^n_t denoting the natural rate of unemployment. The restriction that employment, h_{jt}, is voluntary now takes the form

$$h_{jt} \leq h^s_{jt}(1-u^n_t).$$

This expression says that the household is not willing to have more members employed than the ones it voluntarily supplies to the market net of the ones that are naturally unemployed.

The household's budget constraint and the optimality conditions associated with consumption and bond holdings are unchanged. The firm's demand for labor of variety $j \in [0,1]$, given by equation (3), is also unchanged.

As before, we consider an equilibrium in which each period $t \geq 0$ there is a cutoff labor variety, j^*_t, that operates at full employment,

$$h_{j^*_t t} = h^s_{j^*_t t}(1-u^n_t), \tag{25}$$

and for which the wage constraint holds with equality,

$$W_{j^*_t t} = \gamma(j^*_t)W_{t-1}. \tag{26}$$

Combining these two conditions with the labor demand (3) and the labor supply (24) yields

$$\frac{V'\left(\frac{h_t}{1-u^n_t}\left(\frac{\gamma(j^*_t)}{1+\pi^W_t}\right)^{-\eta}\right)}{U'(c_t)} = \frac{\gamma(j^*_t)w_{t-1}}{1+\pi_t}, \tag{27}$$

where $w_t \equiv W_t/P_t$ denotes the real wage in period t. It can be shown that, as in the case of an inelastic labor supply, all labor varieties $j < j_t^*$ operate at full employment and are paid the same wage as variety j_t^*. Also, all varieties $j > j_t^*$ are constrained by the wage lower bound and suffer unemployment above the natural rate.

With an endogenous labor supply, the unemployment rate is the ratio of unemployed labor to the total labor supply. Formally,

$$u_t = \frac{\int_0^1 (h_{jt}^s - h_{jt}) dj}{\int_0^1 h_{jt}^s dj}.$$

Using the functional form (23) for the disutility of labor and equations (3), (24), (25), and (26), we can rewrite the unemployment rate as

$$u_t = u_t^n + (1 - u_t^n) \frac{\int_{j_t^*}^1 \left[\left(\frac{\gamma(j)}{\gamma(j_t^*)} \right)^{\frac{1}{\theta}} - \left(\frac{\gamma(j)}{\gamma(j_t^*)} \right)^{-\eta} \right] dj}{j_t^* + \int_{j_t^*}^1 \left(\frac{\gamma(j)}{\gamma(j_t^*)} \right)^{\frac{1}{\theta}} dj}. \tag{28}$$

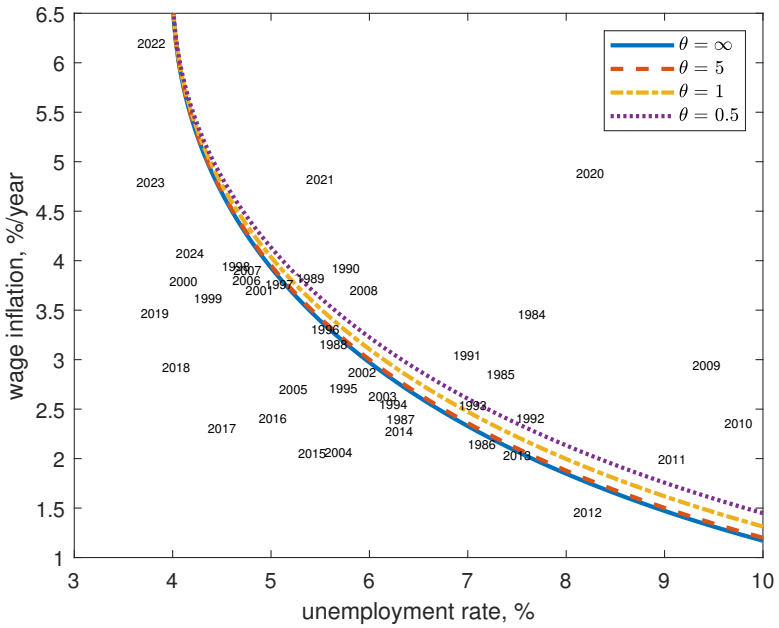
Note that as the elasticity of labor supply approaches zero ($\theta \to \infty$), equations (27) and (28) converge to equations (14) and (18) with $\bar{h}$ normalized to 1, and the model becomes the one with inelastic labor supply studied in sections 2 and 3.

Appendix C provides the definition of a competitive equilibrium in the economy with an endogenous labor supply.

As in the case of an inelastic labor supply, the model features a wage Phillips curve linking current unemployment, u_t, and current wage inflation, π_t^W. The equations implicitly defining the Phillips curve are (17) and (28).

The Phillips curve now features a new parameter, θ, representing the inverse of the wage elasticity of labor supply. To depict the implied short-run wage Phillips curve we assign a value of 5 to this parameter following Galí (2015). This value implies a labor supply elasticity of 0.2. The parameters Γ_0 and Γ_1 of the variety-specific wage lower bound function $\gamma(j) = (1 + \pi^*)(\Gamma_0 + \Gamma_1 j)$ were recalibrated using the same targets for the steady-state unemployment-inflation pair and for the slope of the Phillips curve at that point as in the economy with an inelastic labor supply. The resulting values are $\Gamma_0 = 0.9781$ and $\Gamma_1 = 0.0310$, which are the same as those associated with the HDNWR model with inelastic labor supply up to the third significant digit. Figure 6 displays with a solid line the predicted wage Phillips curve. It is virtually identical to the one predicted by the economy with an inelastic labor supply. This is not surprising given that the calibrated labor supply elasticity is relatively small. The figure also shows that as the labor supply elasticity increases, the

Figure 6: The Wage Phillips Curve in the Model with Endogenous Labor Supply



Note. See notes to Figure 1.

wage Phillips curve becomes flatter. However, quantitatively the differences are minor.

5.3 Heterogeneous Labor Productivity

Thus far we have taken the cross-sectional distribution of the wage lower bound, $\gamma(j)$, as given. In this section, we provide a microfoundation. A recent literature has emphasized the role of heterogeneity in labor productivity for macroeconomic adjustment (see, for example, Kaplan, Moll, and Violante, 2018; Bayer, Born, and Luetticke, 2024, and the references cited therein). Motivated by this literature, here we incorporate idiosyncratic productivity shocks into the model and show that this source of heterogeneity can give rise to a convex relationship between wage inflation and unemployment. Because workers suffering negative idiosyncratic productivity shocks require a fall in their real wage to remain fully employed, they are the most likely to be affected by downward nominal wage rigidity. The empirical literature has additionally documented that firms have an easier time cutting wages when worker productivity is low (Fehr and Goette, 2005; Bewley, 1999; and Campbell and Kamlani, 1997). This fact amounts to cross-sectional heterogeneity in the degree of downward nominal wage rigidity. To isolate this effect, we assume that given the level of labor productivity, all workers have the same wage lower bound, but that the wage lower bound is lower for less productive workers. All other aspects of the model are as in the baseline formulation.

The labor input, h_t, is assumed to be a composite of efficiency units of labor varieties with the aggregation technology $h_t = \left[\int_0^1 \left(z_{jt} h_t(z_{jt})\right)^{1-\frac{1}{\eta}} dj\right]^{\frac{1}{1-\frac{1}{\eta}}}$, where $z_{jt} > 0$ is a stochastic variety-specific level of labor productivity with density function $f(\cdot)$ and $h_t(z_{jt})$ is the level of employment of workers with productivity z_{jt}. It is convenient to conduct the analysis in the domain of labor productivities as opposed to in the domain of labor varieties. To this end, we perform a change of variable as follows. Assume without loss of generality that z_{jt} is increasing in j. It follows that $j = \mathcal{F}(z_{jt})$, where $\mathcal{F}(\cdot)$ is the cumulative density function associated with the density function $f(\cdot)$. Now introduce the change of variable $z = z_{jt}$. Then we have that $dj = f(z)dz$. Note also that $z \to 0$ as $j \to 0$ and that $z \to \infty$ as $j \to 1$. The aggregation technology can then be written as $h_t = \left[\int_0^\infty \left(z h_t(z)\right)^{1-\frac{1}{\eta}} f(z)dz\right]^{\frac{1}{1-\frac{1}{\eta}}}$. The firm chooses employment of each variety to maximize profits, $P_t a_t F(h_t) - \int_0^\infty W_t(z) h_t(z) f(z) dz$, taking P_t and $W_t(z)$ as given, where $W_t(z)$ is the nominal wage of labor variety z. The firm's cost-minimizing demand for labor with productivity z is

$$h_t(z) = z^{\eta-1} \left(\frac{W_t(z)}{W_t}\right)^{-\eta} h_t, \text{ with } W_t = \left[\int_z \left(\frac{W_t(z)}{z}\right)^{1-\eta} f(z)dz\right]^{\frac{1}{1-\eta}}. \tag{29}$$

When $h_t(z)$ is chosen optimally, the aggregate wage rate W_t satisfies $W_t h_t = \int_z W_t(z) h_t(z) f(z) dz$. The demand for aggregate units of labor, h_t, satisfies the optimality condition $P_t a_t F'(h_t) = W_t$.

The household supplies inelastically $\bar{h}$ units of labor of each variety z, subject to the constraint $h_t(z) \leq \bar{h}(1 - u_t^n)$. The nominal wage of every variety z is assumed to be subject to a lower bound constraint of the form

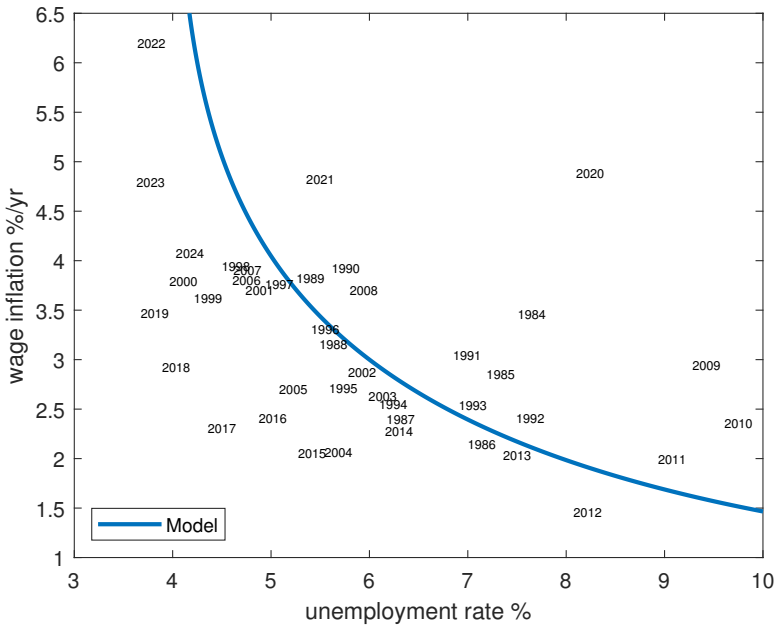
$$W_t(z) \geq z^\xi \gamma W_{t-1}. \tag{30}$$

The parameter $\gamma > 0$ measures the common degree of downward nominal wage rigidity across productivity levels. The parameter $\xi \geq 0$ measures the degree of productivity specific wage rigidity. It aims to capture the empirical regularity that workers experiencing lower productivity are more willing to tolerate wage cuts (Fehr and Goette, 2005). The higher ξ is, the more sensitive the degree of downward wage rigidity to changes in productivity will be. The labor market closes with a slackness condition imposed at the level of each labor variety z:

$$[\bar{h}(1 - u_t^n) - h_t(z)][W_t(z) - z^\xi \gamma W_{t-1}] = 0. \tag{31}$$

Appendix F derives the Phillips curve implied by this model and describes its calibration. In particular, it shows that there is a productivity cutoff z_t^* such that workers with this level

Figure 7: The Wage Phillips Curve of the Heterogeneous Labor Productivity Model



Note. See notes to Figure 1.

of productivity are fully employed and their wage constraint holds with equality. This cutoff is of interest because, as in the baseline model, it determines the entire cross-sectional distribution of wages and employment. For any $z > z_t^*$, there is full employment, $h_t(z) = \bar{h}(1-u_t^n)$, and the wage rate is unconstrained, $W_t(z) > z^{\xi}\gamma W_{t-1}$. And for any $z < z_t^*$, there is involuntary unemployment, $h_t(z) < \bar{h}(1-u_t^n)$, and the wage rate is stuck at its lower bound, $W_t(z) = z^{\xi}\gamma W_{t-1}$. In words, relatively productive workers (those with $z > z_t^*$) are unaffected by downward nominal wage rigidity and are fully employed, whereas relatively unproductive workers (those with $z < z_t^*$) are constrained by downward nominal wage rigidity, which prevents their wages from falling to a level that is consistent with full employment. This result is consistent with the empirical fact that employment of low-skill workers is more procyclical than that of high-skill workers (Solon, Barsky, and Parker, 1994).

Figure 7 displays the wage Phillips curve implied by the model. As in the baseline model the Phillips curve is convex, implying a relatively small cost in terms of inflation of reducing high unemployment as well as a relatively small cost in terms of unemployment of reducing high levels of inflation. The intuition behind this result is as follows. Take, for example, a situation in which the unemployment rate is high. In this case, a large number of workers is constrained by the wage lower bound. These workers have relatively low productivity, and their wages are too high for firms to have an incentive to fully employ them. A given increase in wage inflation reduces the relative wage of all constrained workers, that is, $W_t(z)/W_t$ goes

down for this type of worker, fostering firms' demand for their services. Because the mass of unemployed workers is large, the reduction in unemployment is also relatively large. If the initial situation is one in which unemployment is relatively low, the effect of an increase in wage inflation is qualitatively the same, that is, the relative wage of workers whose wages are constrained by the wage lower bound goes down, stimulating demand for this type of labor. However, the pool of workers constrained by the wage lower bound is small, so the fall in unemployment will also be relatively small.

5.4 Implications for Wage Change Distributions

There is an empirical literature studying the cross-sectional distribution of year-over-year changes in hourly wages of individual workers using micro data. (See, for example, Daly and Hobijn, 2014; and Jo, 2025.) This literature has documented that the mass of workers experiencing an exact zero change from one year to the next in their nominal hourly wage is cyclical, in particular, it increases with the unemployment rate. This stylized fact is often cited as evidence of downward nominal wage rigidity. So it is natural to be curious about whether the model proposed in the present paper can speak to this stylized fact.

In the heterogeneous downward nominal wage rigidity model, negative demand shocks lead to movements down a given wage Phillips curve. As the economy moves down the wage Phillips curve, j_t^* falls. Therefore, the mass of constrained workers, $1 - j_t^*$, increases with unemployment. And the larger the negative demand shock is, the larger the mass of constrained workers will be. All workers experiencing a binding wage constraint in consecutive periods, will have the same wage change. To see this, consider a labor variety j that is stuck at the lower bound, so that $W_{jt} = \gamma(j)W_{t-1}$. If the wage constraint is binding for this worker in two consecutive periods, then W_{jt}/W_{jt-1} equals W_{t-1}/W_{t-2}. Now, if the economy is in the relatively flat part of the Phillips curve, then the negative demand shock causes a relatively small change in aggregate wage inflation. This has implications for wage changes of individual workers. The model does qualitatively capture the notation that the mass of workers receiving the same wage change goes up in a recession and that this wage change is likely to be small. However, the model falls short of predicting that the mass of workers stuck at their wage lower bound constraint experience an exact zero nominal wage growth, as suggested by the survey-based microdata studies cited above.

6 Regular Dynamics

We have established that the heterogeneous downward nominal wage rigidity (HDNWR) model predicts that the unemployment costs of reducing inflation are much lower at high inflation rates than at low inflation rates. This result concerns the global properties of the model. A natural question is whether for regular fluctuations in a neighborhood around the intended inflation target, the HDNWR model produces equilibrium dynamics that are consistent with conventional intuition. To address this question we compare its predicted dynamics to those induced by the most widely used framework for nominal wage rigidities, namely, the new-Keynesian model with Calvo wage staggering.

We find that for calibrations and shock processes typically considered in the related literature, the predicted dynamics are fairly similar. This result is of interest because unlike the HDNWR model, the new-Keynesian model with Calvo staggering features a forward-looking wage Phillips curve. The claim in this section is not that the similarity in the dynamics predicted by the two models is necessarily valid for all possible calibrations and shock specifications. Rather the claim is that it is valid for conventional ones.

To derive this result the section characterizes the equilibrium dynamics of the HDNWR model and compares them to those implied by a new-Keynesian (NK) model with Calvo-type wage stickiness. The HDNWR model considered here is the one with endogenous labor supply developed in section 5.2.

We have shown that in spite of the fact that the HDNWR model with inelastic or elastic labor supply features occasionally binding constraints at the level of individual varieties of labor, its complete set of equilibrium conditions does not. This means that the model is amenable to a characterization of the equilibrium dynamics using perturbation methods. We summarize this result in the following proposition:

Proposition 2 (HDNWR and Perturbation)**.** *The equilibrium dynamics of the HDNWR model with inelastic or elastic labor supply described in Definitions 2 and C1 (Appendix C), respectively, can be approximated using perturbation techniques.*

Thus, to obtain the implied impulse responses of the HDNWR model to exogenous shocks we can follow the customary approach of linearizing the equilibrium conditions around the nonstochastic steady state. The quantitative analysis that follows adopts this approach.

The calibration of the model is summarized in Table 1. The parameters appearing in the top panel of the table were already discussed in section 3. We assume a period consumption subutility function of the form $U(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$ and a production function of the form $F(h) = h^{\alpha}$. Following Galí (2015), we set $\sigma = 1$, $\alpha = 0.75$, $\beta = 0.99$, $\theta = 5$, $\alpha_{\pi} = 1.5$, and $\alpha_{y} = 0.5/4$.

Here we study the dynamic response to a monetary shock. Appendix G considers productivity shocks. The monetary shock μ_t in the Taylor rule (11) is assumed to follow an autoregressive process of order one

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \epsilon_t^\mu, \tag{32}$$

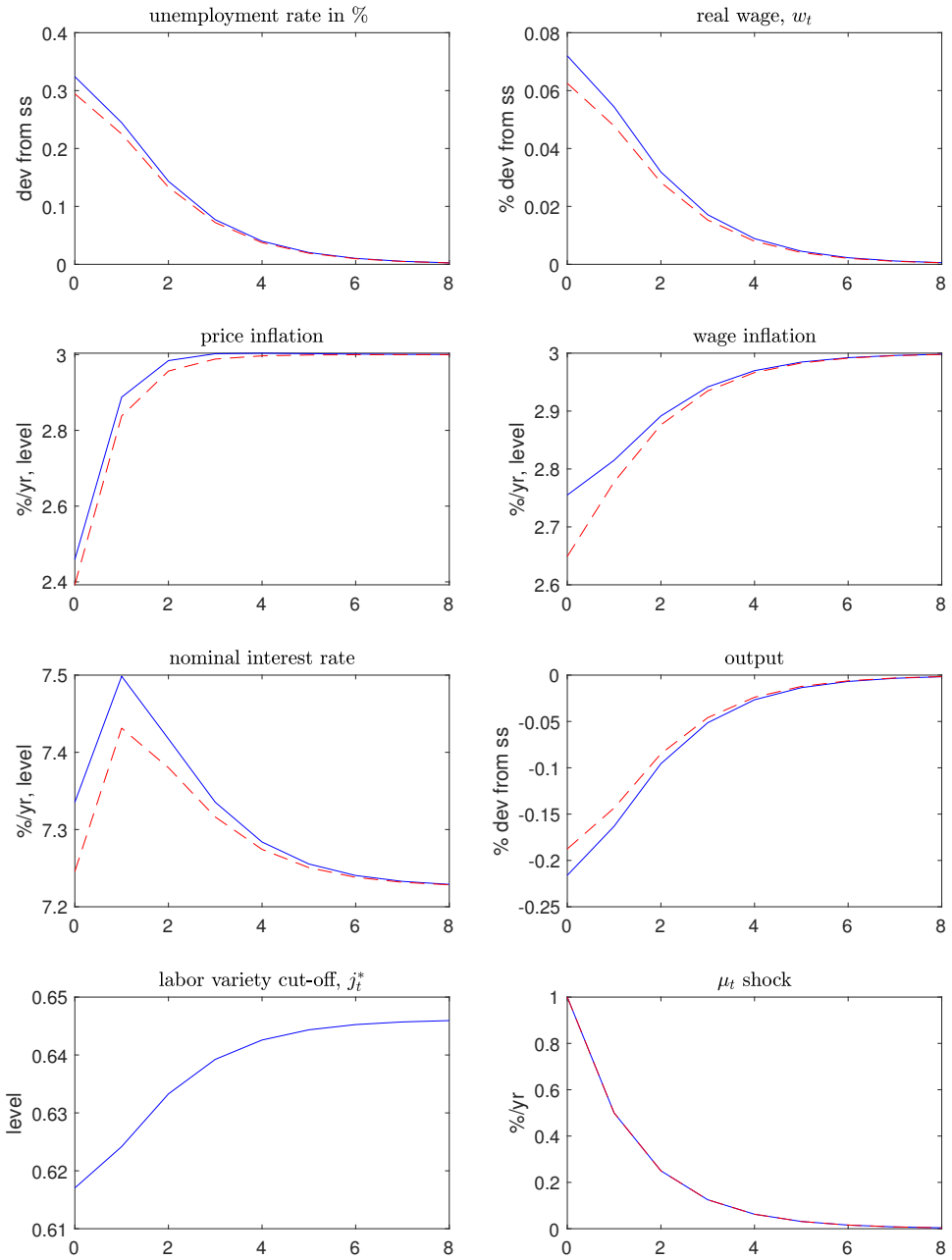
where ϵ_t^μ is a mean zero i.i.d. innovation, and $\rho_\mu \in [0, 1)$ is a parameter. Following Galí (2015), we set $\rho_\mu = 0.5$.

Figure 8 displays with solid lines the impulse response to a one percent annualized increase in μ_t predicted by the HDNWR model. In equilibrium this monetary contraction results in a 0.11 percentage point increase in the policy interest rate (from its steady-state value of 7.23 percent to 7.34 percent). The increase in the interest rate is smaller than the increase in μ_t because of the contemporaneous adjustment of the endogenous variables that enter the Taylor rule, π_t and y_t. The HDNWR model predicts that the tightening in monetary conditions is deflationary. An efficient adjustment of the labor market would require a fall in nominal wages large enough to perfectly offset the fall in prices. However, due to the presence of downward nominal wage rigidity, the decline in nominal wages is insufficient. That is, a larger number of job varieties become constrained by the lower bound on nominal wages. This frictional adjustment is reflected in a decline in the labor variety cutoff $\jmath_t^*$. In turn, the fact that the real wage is inefficiently high for more labor varieties causes an increase in involuntary unemployment and hence a decline in output and consumption.

For comparison, we consider a canonical NK model with wage staggering. This model departs from the HDNWR model only in its wage setting module. Specifically, we assume that wages are set in a Calvo fashion as in Erceg, Henderson, and Levin (2000) and define the unemployment rate as in Galí (2011). A detailed derivation of the NK model we use here can be found in a technical appendix (Schmitt-Grohé and Uribe, 2022). All parameters of the NK model that are common to the HDNWR model are assigned the same values, namely, those given in Table 1. The common parameters are π^*, η, β, σ, θ, α, α_π, α_y, and ρ_μ. As in the HDNWR model, in the NK model we assume full indexation of wages to steady-state inflation.

It remains to explain how we calibrate the degree of nominal wage rigidity in the NK model. We cannot directly adopt the strategy used to calibrate the HDNWR model, namely, to match the slope of the implied wage Phillips curve to the one estimated by Galí and Gambetti (2019). The reason is that the empirical Phillips curve estimated by Galí and Gambetti is not forward looking and therefore does not have a natural theoretical counterpart in the NK model. Instead, we assume that the fraction of types of labor that cannot reoptimize

Figure 8: Impulse Responses to a Monetary Tightening in the HDNWR and NK Models



Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the monetary shock is 1 percent per annum. The horizontal axes measure quarters after the shock.

wages in any given period in the NK model is equal to the steady-state fraction of types of labor that are stuck at the wage lower bound in the HDNWR model. Formally, letting θ_w denote the fraction of wages that are not set optimally in any given period in the NK model, we impose

$$\theta_w = 1 - j^*,$$

where j^* is the deterministic steady-state value of j_t^*, the fraction of labor varieties that are not stuck at their wage lower bounds in the HDNWR model. The resulting value of θ_w is 0.35.[13]

Figure 8 displays with dashed lines the response of the NK model to a 1 percent per annum increase in the monetary shock μ_t. The figure shows that for most variables the responses predicted by the NK and HDNWR models are quite close.

To obtain some intuition for this result, consider the linear versions of the wage Phillips curves in the HDNWR and NK models, which can be written, respectively, as

$$\hat{\pi}_t^W = \kappa_1 \hat{u}_t$$

and

$$\hat{\pi}_t^W = \beta E_t \hat{\pi}_{t+1}^W + \kappa_2 \hat{u}_t,$$

where a $\hat{}$ superscript denotes deviations from the nonstochastic steady state. Iterating the NK Phillips curve forward, yields

$$\hat{\pi}_t^W = \sum_{t=0}^{\infty} \beta^j \kappa_2 E_t \hat{u}_{t+j}.$$

Assume for simplicity that in equilibrium unemployment follows an AR(1) law of motion of the form $E_t \hat{u}_{t+j} = \lambda(\rho, \chi)^j \hat{u}_t$, where the persistence parameter $\lambda(\rho, \chi)$ is an endogenous object that depends not only on the persistence of the exogenous shock in question, $\rho = \rho_\mu$, but also on the vector χ containing other structural parameters of the NK model that influence the endogenous persistence in unemployment.[14] Then, we have that

$$\hat{\pi}_t^W = \frac{\kappa_2}{1 - \lambda(\rho, \chi)\beta} \hat{u}_t.$$

The two models will deliver more similar dynamics the more similar are κ_1 and $\kappa_2/(1 -$

[13] This value is low relative to those typically used to calibrate NK models. For example, Galí (2015) sets θ_w to 0.75. To address this issue, in Appendix G we also consider a calibration in which $\theta_w = 1 - j^* = 0.75$.

[14] In the NK model with wage stickiness and a Taylor rule endogenous persistence arises from past real wages being a state variable.

$\lambda(\rho, \chi)\beta$). More importantly, the similarity of the two models will not be much affected by changes in the persistence of the exogenous shock, ρ, if $\lambda(\rho, \chi)$ is relatively insensitive to ρ. As it turns out, λ is not too sensitive to changes in ρ for values of $\rho = \rho_\mu$ between 0 and its respective calibrated value. For this range, the endogenous persistence in unemployment built in the NK model dominates the persistence induced by the exogenous shocks. As the exogenous shocks become highly serially correlated, their persistence might dominate the persistence of unemployment. In this range, the dynamics of the HDNWR and NK models can be quantitatively dissimilar.

7 Conclusion

This paper contributes to understanding nonlinearities in the trade off between wage inflation and unemployment. To this end, it proposes a model with heterogeneous downward nominal wage rigidity. This innovation results in a smooth, convex wage Phillips curve. At high inflation levels, the predicted Phillips curve is relatively steep, indicating that the costs of reducing high inflation can be low in terms of employment. Conversely, at high levels of unemployment, the curve is relatively flat, suggesting that the cost of reducing high unemployment can be low in terms of inflation.

An empirical contribution of the paper is the estimation, using U.S. regional data, of a wage Phillips curve with a smooth nonlinear component that nests as a special case a linear Phillips curve. The result of this estimation suggests that the U.S. wage Phillips curve is significantly nonlinear. The wage Phillips curve predicted by the proposed model captures reasonably well the tradeoff between wage inflation and unemployment implied by this estimate.

Methodologically, the paper contributes to macroeconomic modeling in three ways. First, it shows that market power is not a prerequisite for introducing nominal rigidity in dynamic general equilibrium models. In the proposed model, consumers, workers, and firms are price takers in product and factor markets, and yet nominal prices fail to adjust fast enough to guarantee market clearing at all times.

This result is relevant for modeling nominal rigidity in markets lacking a significant degree of market concentration. In the context of the present paper, it is of use to model nominal wage rigidity in an economy with low levels of labor unionization like the United States. Introducing nominal rigidity without market power implies that the Phillips curve is a contemporaneous relationship between a measure of inflation and a measure of aggregate slack, not mediated by short-run inflationary expectations, much in line with the Phillips curve estimated by Phillips himself.

Second, unlike in the standard new-Keynesian model of nominal wage rigidity, in the model proposed in the present study employment is not demand-determined. That is, workers are not forced to work more hours than they wish at the prevailing wage. We believe that this property makes the theoretical environment more realistic.

Finally, the paper provides a theoretical framework with downward nominal wage rigidity that is amenable to analysis using perturbation techniques. This property arises because, although the model features occasionally binding constraints at the micro level, no such constraints appear at the aggregate level. This is not the case in models with homogeneous downward nominal wage rigidity, which, we suspect, has impeded their adoption for the formulation, computation, and estimation of medium scale models for policy evaluation. In this regard, the proposed framework aims to lower the entry barrier for models with this empirically compelling type of nominal rigidity.

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Online Appendix

A Graphical Determination of Wages and Employment Across Labor Varieties Given h_t and W_t/W_{t-1}

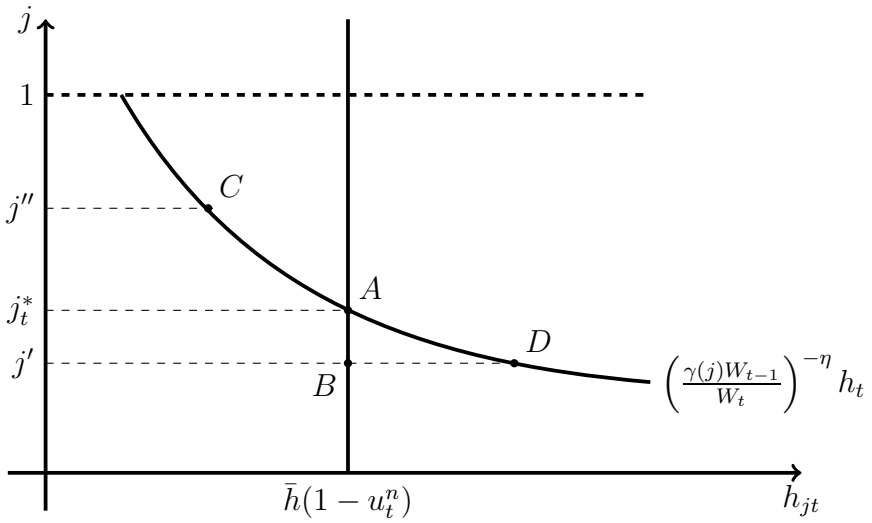
Figure A1 provides a graphical explanation of the determination of wages and employment across labor varieties, given the aggregate variables h_t and W_t/W_{t-1}. The downward sloping curve represents the demand for labor of each variety, h_{jt}, as a function of j when the variety-specific wage equals its lower bound, $W_{jt} = \gamma(j)W_{t-1}$. The vertical line represents the labor supply net of natural unemployment, $\bar{h}(1-u_t^n)$, as a function of j. The intersection of the two lines at point A determines the cutoff variety j_t^*. This is because at point A there is full employment and the wage constraint exactly binds, which are the two conditions defining j_t^*.

Points located to the right of the downward sloping line are infeasible because they imply that $W_{jt} < \gamma(j)W_{t-1}$, which violates the wage lower bound. Points located to the left of the downward sloping line imply that the wage lower bound is slack, $W_{jt} > \gamma(j)W_{t-1}$. Points located to the right of the vertical line are infeasible because they violate the resource constraint $h_{jt} \leq \bar{h}(1-u_t^n)$. Points located to the left of the vertical line imply involuntary unemployment, $h_{jt} < \bar{h}(1-u_t^n)$. Points to the left of both the downward sloping and the vertical lines are infeasible because they imply that the wage lower bound is slack, $W_{jt} > \gamma(j)W_{t-1}$, and that there is involuntary unemployment, $h_{jt} < \bar{h}(1-u_t^n)$, which violates the slackness condition $(W_{jt} - \gamma(j)W_{t-1})(h_{jt} - \bar{h}(1-u_t^n)) = 0$.

Since points located to the right of either curve or to the left of both are infeasible, it follows that in equilibrium pairs (j, h_{jt}) must lie on the vertical line if $j < j_t^*$ and on the downward sloping curve if $j > j_t^*$. For example, for variety $j' < j_t^*$ in the figure, the equilibrium is at point B, where there is full employment and the wage is unconstrained, i.e., workers receive a wage strictly above the lower bound $\gamma(j')W_{t-1}$. How much above? If the wage rate of variety j' were at its lower bound, $\gamma(j')W_{t-1}$, then there would be excess demand given by the distance between points B and D. Since wages are upwardly flexible, the wage rate has to increase until the excess demand disappears. It is clear from the figure that this occurs when variety j' earns the same wage as variety j_t^*, namely, $\gamma(j_t^*)W_{t-1}$. By contrast, for variety $j'' > j_t^*$, the equilibrium is at point C, where there is involuntary unemployment and the wage lower bound is binding, i.e., the wage is stuck at its lower bound.

The fact that the equilibrium is on the vertical line for $j < j_t^*$ and on the downward sloping curve for $j > j_t^*$ means that for varieties $j < j_t^*$ employment is supply determined and that for varieties $j > j_t^*$ employment is demand determined.

Figure A1: Determination of Wages and Employment Across Labor Varieties



Notes. The downward sloping line depicts the demand for labor when the wage constraint is binding in the space (h_{jt}, j), where $j \in [0,1]$ indexes labor varieties and h_{jt} denotes the quantity of labor of variety j demanded by firms. The vertical line depicts the supply of labor net of natural unemployment as a function of the labor variety j. In the figure, the aggregate variables h_t and W_t/W_{t-1} are taken as given.

The triangular area located above the downward sloping line, to the left of the vertical line, and below 1 represents the aggregate amount of unemployment above the natural rate.

B Real Effects of Monetary Disturbances

This appendix shows that in the model with heterogeneous downward nominal wage rigidity, monetary disturbances have real consequences. For convenience, it begins by reproducing the definition of the competitive equilibrium in j^* form.

Definition B1 (Competitive Equilibrium in j^* Form)**.** *A competitive equilibrium is a set of processes* j_t^*, u_t, y_t, h_t, w_t, i_t, π_t, *and* π_t^W, *satisfying*

$$y_t = a_t F(h_t), \tag{B1}$$

$$U'(y_t) = \beta(1+i_t)E_t\frac{U'(y_{t+1})}{1+\pi_{t+1}}, \tag{B2}$$

$$a_t F'(h_t) = w_t, \tag{B3}$$

$$1 + i_t = \frac{1+\pi^*}{\beta}\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\alpha_\pi}\left(\frac{y_t}{y}\right)^{\alpha_y}\mu_t, \tag{B4}$$

$$1 + \pi_t^W = \frac{w_t}{w_{t-1}}(1+\pi_t), \tag{B5}$$

$$\bar{h}(1-u_t^n) = \left(\frac{\gamma(j_t^*)}{1+\pi_t^W}\right)^{-\eta} h_t, \tag{B6}$$

$$(1+\pi_t^W)^{1-\eta} = j_t^*\gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta}dj,, \tag{B7}$$

and

$$u_t = u_t^n + (1-u_t^n)\left[(1-j_t^*) - \int_{j_t^*}^1 \left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{-\eta} dj\right]. \tag{B8}$$

given the initial condition w_{-1} and the exogenous stochastic processes a_t, μ_t, and u_t^n.

The last three equations give rise to real effects of monetary disturbances. Equation (B6) says that there is one labor variety, j_t^*, for which there is full employment and the wage constraint just binds. Equation (B7) says that wage inflation is a weighted average of the wage increase across varieties relative to the average wage prevailing the previous period. Equation (B8) says that unemployment is the sum of natural unemployment and Keynesian unemployment. In turn, Keynesian unemployment is decreasing in market tightness j_t^*.

To see that monetary disturbances have real effects, it suffices to consider, as an example, a situation in which the economy is initially in steady state and in period 0 experiences an unexpected purely transitory fall in the monetary disturbance μ_t. Suppose that after the shock there is perfect foresight. Suppose, contrary to the claim, that the fall in μ_t does not affect the real allocation (y_t or h_t for any $t \geq 0$). Then, by the Euler equation (B2) and the Taylor rule (B4), we have that the inflation rate π_t must change either at $t = 0$ or at $t = 1$ or both. Also, by the labor demand (B3), the real wage w_t must stay constant, otherwise h_t would move. Then, by (B5), wage inflation, π_t^W, must change either at $t = 0$ or at $t = 1$ or both. In turn, by (B6), j_t^* must change either at $t = 0$ or at $t = 1$ or both, but in such a way as to keep constant the ratio $\gamma(j_t^*)/(1+\pi_t^W)$, otherwise h_t would be affected. But, according to (B7), the ratio $\gamma(j_t^*)/(1+\pi_t^W)$ can stay constant only if $\gamma'(j) = 0$, which is a contradiction.

C Definition of Equilibrium in the Economy with Endogenous Labor Supply

The following definition provides the full set of conditions that constitute a competitive equilibrium in the economy with endogenous labor supply.

Definition C1 (Competitive Equilibrium with Endogenous Labor Supply)**.** *A competitive equilibrium in the economy with endogenous labor supply is a set of processes* j_t^**,* y_t*,* h_t*,* u_t*,* w_t*,* i_t*,* π_t*, and* π_t^W*, satisfying* (B1)-(B5)*,* (B7)*,* (27)*, and* (28)*, given the initial condition* w_{-1} *and the stochastic processes* a_t*,* μ_t*, and* u_t^n*.*

Table D1: Data Sources and BLS Code

Variable	Geographical Area	BLS Code
Employment Cost Index	Atlanta-Athens-Clarke County-Sandy Springs, GA-AL CSA	CIU20200000000LAA
Employment Cost Index	Boston-Worcester-Providence, MA-RI-NH CSA	CIU20200000000LBA
Employment Cost Index	Chicago-Naperville, IL-IN-WI CSA	CIU20200000000LCA
Employment Cost Index	Dallas-Fort Worth, TX-OK CSA	CIU20200000000LDA
Employment Cost Index	Detroit-Warren-Ann Arbor, MI CSA	CIU20200000000LEA
Employment Cost Index	Houston-Pasadena, TX CSA	CIU20200000000LFA
Employment Cost Index	Los Angeles-Long Beach, CA CSA	CIU20200000000LGA
Employment Cost Index	New York-Newark, NY-NJ-CT-PA CSA	CIU20200000000LIA
Employment Cost Index	Philadelphia-Reading-Camden, PA-NJ-DE-MD CSA	CIU20200000000LJA
Employment Cost Index	San Jose-San Francisco-Oakland, CA CSA	CIU20200000000LKA
Employment Cost Index	Seattle-Tacoma, WA CSA	CIU20200000000LLA
Employment Cost Index	Washington-Baltimore-Arlington, DC-MD-VA-WV-PA CSA	CIU20200000000LMA
Employment Cost Index	Miami-Port St. Lucie-Fort Lauderdale, FL CSA	CIU20200000000LNA
Unemployment Rate	Atlanta-Athens-Clarke County-Sandy Springs, GA-AL CSA	LAUCA131220000000003
Unemployment Rate	New York-Newark, NY-NJ-CT-PA CSA	LAUCA364080000000003
Unemployment Rate	Boston-Worcester-Providence, MA-RI-NH CSA	LAUCA251480000000003
Unemployment Rate	Chicago-Naperville, IL-IN-WI CSA	LAUCA171760000000003
Unemployment Rate	Dallas-Fort Worth, TX-OK CSA	LAUCA482060000000003
Unemployment Rate	Detroit-Warren-Ann Arbor, MI CSA	LAUCA262200000000003
Unemployment Rate	Houston-Pasadena, TX CSA	LAUCA482880000000003
Unemployment Rate	Los Angeles-Long Beach, CA CSA	LAUCA063480000000003
Unemployment Rate	Miami-Port St. Lucie-Fort Lauderdale, FL CSA	LAUCA123700000000003
Unemployment Rate	Philadelphia-Reading-Camden, PA-NJ-DE-MD CSA	LAUCA424280000000003
Unemployment Rate	San Jose-San Francisco-Oakland, CA CSA	LAUCA064880000000003
Unemployment Rate	Seattle-Tacoma, WA CSA	LAUCA535000000000003
Unemployment Rate	Washington-Baltimore-Arlington, DC-MD-VA-WV-PA CSA	LAUCA115480000000003
Core CPI	National	CUUR0000SA0L1E
Core CPI	Boston-Cambridge-Newton, MA-NH	CUURS11ASA0L1E
Core CPI	New York-Newark-Jersey City, NY-NJ-PA	CUURS12ASA0L1E
Core CPI	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	CUURS12BSA0L1E
Core CPI	Chicago-Naperville-Elgin, IL-IN-WI	CUURS23ASA0L1E
Core CPI	Detroit-Warren-Dearborn, MI	CUURS23BSA0L1E
Core CPI	Washington-Arlington-Alexandria, DC-VA-MD-WV	CUURS35ASA0L1E
Core CPI	Miami-Fort Lauderdale-West Palm Beach, FL	CUURS35BSA0L1E
Core CPI	Atlanta-Sandy Springs-Roswell, GA	CUURS35CSA0L1E
Core CPI	Dallas-Fort Worth-Arlington, TX	CUURS37ASA0L1E
Core CPI	Houston-The Woodlands-Sugar Land, TX	CUURS37BSA0L1E
Core CPI	Los Angeles-Long Beach-Anaheim, CA	CUURS49ASA0L1E
Core CPI	San Francisco-Oakland-Hayward, CA	CUURS49BSA0L1E
Core CPI	Seattle-Tacoma-Bellevue, WA	CUURS49DSA0L1E
Energy CPI	National	CUUR0000SA0E
Energy CPI	Boston-Cambridge-Newton, MA-NH	CUURS11ASA0E
Energy CPI	New York-Newark-Jersey City, NY-NJ-PA	CUURS12ASA0E
Energy CPI	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	CUURS12BSA0E
Energy CPI	Chicago-Naperville-Elgin, IL-IN-WI	CUURS23ASA0E
Energy CPI	Detroit-Warren-Dearborn, MI	CUURS23BSA0E
Energy CPI	Washington-Arlington-Alexandria, DC-VA-MD-WV	CUURS35ASA0E
Energy CPI	Miami-Fort Lauderdale-West Palm Beach, FL	CUURS35BSA0E
Energy CPI	Atlanta-Sandy Springs-Roswell, GA	CUURS35CSA0E
Energy CPI	Dallas-Fort Worth-Arlington, TX	CUURS37ASA0E
Energy CPI	Houston-The Woodlands-Sugar Land, TX	CUURS37BSA0E
Energy CPI	Los Angeles-Long Beach-Anaheim, CA	CUURS49ASA0E
Energy CPI	San Francisco-Oakland-Hayward, CA	CUURS49BSA0E
Energy CPI	Seattle-Tacoma-Bellevue, WA	CUURS49DSA0E

D Series Codes for the Regional Data

Table D1 displays the BLS series codes for all the time series used in the estimation of regional wage Phillips curves presented in section 4.

E Pareto Distribution for γ

The baseline calibration assumes that the degree of variety-specific downward nominal rigidity is uniformly distributed over the interval $[(1+\pi^*)\Gamma_0, (1+\pi^*)(\Gamma_0+\Gamma_1)]$. Here, we consider a case in which the degree of downward nominal wage rigidity has a Pareto distribution, with a minimum value $\gamma_m > 0$ and tail index $\alpha > 2$. Specifically, indexing labor varieties by the degree of downward nominal wage rigidity, $\gamma \in (\gamma_m, \infty)$, we have that the lower bound on

nominal wages now is

$$W_t(\gamma) \geq (1+\pi^*)^{\delta}\gamma W_{t-1}.$$

The cumulative distribution of γ, denoted $\mathcal{F}(\gamma)$, takes the form

$$\mathcal{F}(\gamma) = 1 - \left(\frac{\gamma_m}{\gamma}\right)^{\alpha}.$$

Under this formulation, the three equilibrium conditions that distinguish the DNWRA model from the neo-classical model, namely, equations (14), (17), and (18), become

$$\bar{h}(1-u_t^n) = \left(\frac{(1+\pi^*)^{\delta}\gamma_t^*}{1+\pi_t^W}\right)^{-\eta} h_t, \tag{E1}$$

$$(1+\pi_t^W)^{1-\eta} = \left[(1+\pi^*)^{\delta}\gamma_t^*\right]^{1-\eta}\left[1-\left(\frac{\gamma_m}{\gamma_t^*}\right)^{\alpha}\left(\frac{\eta-1}{\eta+\alpha-1}\right)\right] \tag{E2}$$

and

$$u_t = u_t^n + (1-u_t^n)\left(\frac{\gamma_m}{\gamma_t^*}\right)^{\alpha}\frac{\eta}{\eta+\alpha}, \tag{E3}$$

where γ_t^* is the cutoff value of γ defined as the labor variety that is fully employed and for which the wage constraint is just binding. All labor varieties with $\gamma > \gamma_t^*$ suffer involuntary unemployment and are stuck at the wage lower bound, and all varieties $\gamma < \gamma_t^*$ display full employment and a slack nominal wage constraint.

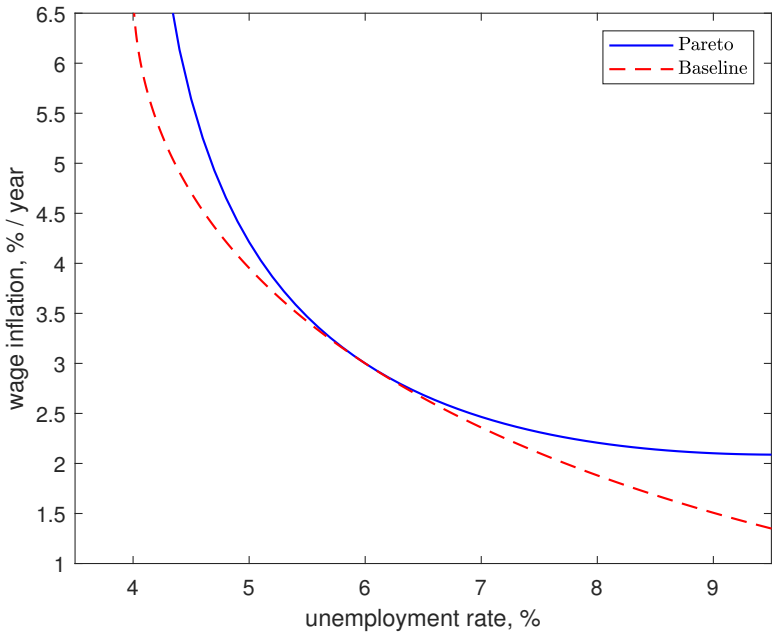
Combining (E2) and (E3) to eliminate γ_t^* yields the wage Phillips curve. As it turns out, when γ is distributed Pareto across labor varieties, the wage Phillips curve has a closed form solution:

$$\pi_t^W = (1+\pi^*)^{\delta}\left(\frac{1-u_t^n}{u_t-u_t^n}\frac{\eta}{\eta+\alpha}\right)^{\frac{1}{\alpha}}\gamma_m\left[1-\left(\frac{u_t-u_t^n}{1-u_t^n}\right)\left(\frac{\eta+\alpha}{\eta}\right)\left(\frac{\eta-1}{\eta+\alpha-1}\right)\right]^{\frac{1}{1-\eta}} - 1. \tag{E4}$$

It follows immediately that, holding the natural rate of unemployment constant, wage inflation is a decreasing function of unemployment.

We calibrate γ_m and α by imposing the same identification assumptions as in the baseline case. That is, we require that the Phillips curve crosses the point $(u_t, \pi_t^W) = (6\%, 3\%)$, and that at that point its slope is -0.74. This yields $\gamma_m = 0.9923$ and $\alpha = 178.1350$. Figure E1 displays with a solid line the wage Phillips curve implied by the model when γ has a Pareto distribution. In the figure, the natural rate of unemployment is held constant at its steady-state value of 4 percent. The wage Phillips curve continues to be nonlinear, steeper at high levels of inflation and low unemployment and flatter at high levels of unemployment

Figure E1: The Wage Phillips Curve: Pareto Distribution for $\boldsymbol{\gamma}$



Notes. The figure shows with a solid line the wage Phillips curve implied by the model with a Pareto distribution for γ holding u_t^n constant at its steady state value of 4 percent. The figure also shows with a dashed line the wage Phillips curve implied the baseline model, which features a uniform distribution for γ.

and low inflation. For comparison, the figure also displays, with a dashed line, the Phillips curve obtained under the baseline calibration, which assumes a uniform distribution for γ. By construction, the two curves coincide at $(u_t, \pi_t^W) = (6\%, 3\%)$, and have the same slope at that point. The Phillips curve associated with the Pareto distribution has more curvature. Intuitively, the Pareto distribution tends to concentrate a larger mass of γ values near the minimum value γ_m than does the uniform distribution. As a result, under the Pareto distribution the model behaves closer to the homogeneous downward nominal wage rigidity case than under a uniform distribution.

F The Phillips Curve with Heterogeneous Labor Productivity

In this appendix, we derive a parametric representation of the wage Phillips curve in the HDNWR model with heterogeneous labor productivity presented in section 5.3. The proofs of claims are collected in section F.2.

F.1 Parametric Representation of the Wage Phillips Curve

We make the following assumption about the parameter ξ:

Assumption F1. *The parameter ξ satisfies $\eta(1-\xi) > 1$.*

This assumption states that wage rigidity cannot increase too quickly as productivity rises.

We consider an equilibrium in which for every $t \geq 0$ there exists a cutoff labor productivity, denoted z_t^*, such that workers with this level of productivity are fully employed,

$$h_t(z_t^*) = \bar{h}(1 - u_t^n) \tag{F1}$$

and for which the wage lower bound holds with equality,

$$W_t(z_t^*) = z_t^{*\xi} \gamma W_{t-1}. \tag{F2}$$

This cutoff is of interest because, as in the baseline model, it determines the entire cross-sectional distribution of wages and employment. Claim F1 in section F.2 shows that for any $z > z_t^*$, there is full employment, $h_t(z) = \bar{h}(1 - u_t^n)$, and the wage rate is unconstrained, $W_t(z) > z^\xi \gamma W_{t-1}$. Claim F2 in section F.2 shows that for any $z < z_t^*$, there is involuntary

unemployment, $h_t(z) < \bar{h}(1-u_t^n)$, and the wage rate is stuck at its lower bound, $W_t(z) = z^{\xi}\gamma W_{t-1}$.

Next we analyze the relationship between the cutoff labor productivity level z_t^* and wage inflation. To this end, following steps similar to those given in the derivation of equation (17) of the baseline model, we write the wage inflation rate as

$$(1+\pi_t^W)^{1-\eta} = \int_{z<z_t^*} \left(\frac{\gamma}{z^{1-\xi}}\right)^{1-\eta} f(z)dz + \int_{z>z_t^*} \left(\frac{\gamma}{z_t^{*1-\xi}}\left(\frac{z}{z_t^*}\right)^{-\frac{1}{\eta}}\right)^{1-\eta} f(z)dz. \tag{F3}$$

This equation implies that wage inflation and the cutoff productivity level are negatively related. The formal proof is in Claim F3 in section F.2. This result is intuitive: higher inflation reduces the real value of the wage lower bound. In turn this allows the real wage of constrained workers to fall. As a consequence workers with productivity levels below but close to z_t^*, which were previously unemployed, now become fully employed. This means that z_t^* falls with wage inflation.

The aggregate unemployment rate is defined as

$$u_t = \int_z \frac{\bar{h} - h_t(z)}{\bar{h}} f(z)dz.$$

As shown above, highly productive workers, namely those with $z > z_t^*$, are fully employed, $h_t(z) = \bar{h}(1-u_t^n)$. Thus the unemployment rate for highly productive workers is u_t^n. Relatively unproductive workers, those with $z < z_t^*$, suffer involuntary unemployment. Evaluating the labor demand schedule, equation (29), at $z < z_t^*$ and at $z = z_t^*$ and taking ratios, we have

$$\frac{h_t(z)}{h_t(z_t^*)} = \left(\frac{z}{z_t^*}\right)^{\eta(1-\xi)-1},$$

which says that the lower productivity is, the lower the employment level will be (recall that by Assumption F1, $\eta(1-\xi) > 1$). This is so because, due to downward nominal wage rigidity, the marginal product of labor falls more quickly than the nominal wage. Unemployment can then be written as

$$u_t = u_t^n + (1-u_t^n)\int_{z<z_t^*} \left[1 - \left(\frac{z}{z_t^*}\right)^{\eta(1-\xi)-1}\right] f(z)dz. \tag{F4}$$

According to this expression, the unemployment rate is increasing in the productivity threshold z_t^*. The formal proof is in Claim F4 in section F.2. This is intuitive, because when z_t^* increases, some workers who were previously fully employed become unemployed.

Equations (F3) and (F4) parametrically represent a contemporaneous negative relationship between wage inflation and unemployment. Thus the model with heterogeneous labor productivity of workers and downward nominal wage rigidity implies that in equilibrium the economy moves along a downward sloping wage Phillips curve.

We assume that $\ln(z)$ follows a normal distribution with mean 0 and standard deviation σ_z. Then, equations (F3) and (F4) can be written as

$$\begin{aligned}(1+\pi_t^W)^{1-\eta} &= \gamma^{1-\eta} e^{\frac{(\eta-1)^2(1-\xi)^2\sigma_z^2}{2}} \Phi\left(\frac{\ln z_t^* - (\eta-1)(1-\xi)\sigma_z^2}{\sigma_z}\right) \\ &\quad + \gamma^{1-\eta} z_t^{*(\eta-1)(1-\xi)+\frac{1}{\eta}-1} e^{\frac{\left(1-\frac{1}{\eta}\right)^2\sigma_z^2}{2}} \left[1 - \Phi\left(\frac{\ln z_t^* - \left(1-\frac{1}{\eta}\right)\sigma_z^2}{\sigma_z}\right)\right]\end{aligned} \tag{F5}$$

and

$$u_t = u_t^n + (1-u_t^n)\left[\Phi\left(\frac{\ln z_t^*}{\sigma_z}\right) - z_t^{*1-(1-\xi)\eta} e^{\frac{(\eta(1-\xi)-1)^2\sigma_z^2}{2}} \Phi\left(\frac{\ln z_t^* - (\eta(1-\xi)-1)\sigma_z^2}{\sigma_z}\right)\right], \tag{F6}$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution.

Following McKay and Wolf (2023) we set the standard deviation of $\ln(z)$ equal to 0.2825.[1] As in the baseline model, to guarantee that the long-run wage Phillips curve is vertical, we assume that there is full indexation to long-run inflation, that is, we assume that $\gamma = (1+\pi^*)\tilde{\gamma}$, where $\tilde{\gamma}$ is a parameter. In the steady state, wage inflation is equal to the inflation target π^*. The parameters π^* and η, and the steady state value of u_t^n take the values given in Table 1. We set the steady-state unemployment rate equal to 6 percent as in the calibration of the baseline model. Following the same calibration strategy as in the baseline model, we set $\tilde{\gamma}$ and ξ to ensure that the pair $(u_t, \pi_t^W) = (u, \pi^*)$ lies on the wage Phillips curve and that at that point the slope of the wage Phillips curve is equal to -0.74. The implied values are $\tilde{\gamma} = 1$ and $\xi = 0.8835$.

F.2 Proofs of Claims

Claim F1 (Full employment for all labor varieties $z > z_t^*$)**.** *If Assumption F1 holds, then for all $z > z_t^*$ there is full employment, $h_t(z) = \bar{h}(1-u_t^n)$ and the wage rate is unconstrained, $W_t(z) > z^{\xi}\gamma\tilde{W}_{t-1}$.*

[1] Specifically, McKay and Wolf (2023, Appendix B1) postulate a continuum of regular workers indexed by $j \in [0,1]$ with productivity $z_{j,t}$ following the AR(1) process $\ln z_{j,t} = 0.9^{1/4} \ln z_{j,t-1} + \sigma_\epsilon \epsilon_{j,t}$, with $\epsilon_{j,t}$ following a standard normal distribution. They estimate σ_ϵ to be 0.064. The implied unconditional standard deviation of $\ln z_{j,t}$, which is the relevant object for the present analysis, is 0.2825. McKay, Alisdair, and Christian Wolf, "Optimal Policy Rules in HANK," mimeo, March 13, 2023.

Proof. Suppose that contrary to the claim, $h_t(z) < \bar{h}(1-u_t^n)$ for some $z > z_t^*$. Then by equilibrium condition (31), the wage lower bound must bind, $W_t(z) = z^{\xi}\gamma\tilde{W}_{t-1}$, or $\tilde{W}_t(z) = z^{\xi-1}\gamma\tilde{W}_{t-1}$, Using this expression to eliminate $\tilde{W}_t(z)$ from the labor demand function (3) yields

$$zh_t(z) = \left(\frac{z^{\xi-1}\gamma\tilde{W}_{t-1}}{\tilde{W}_t}\right)^{-\eta} h_t.$$

By definition, for the cutoff productivity z_t^*, the wage lower bound also binds, so the above expression also holds when evaluated at z_t^*,

$$z_t^* h_(z_t^*) = \left(\frac{z_t^{*\xi-1}\gamma\tilde{W}_{t-1}}{\tilde{W}_t}\right)^{-\eta} h_t.$$

Taking the ratio of these two expressions, we have that

$$\frac{h_t(z)}{h(z_t^*)} = \left(\frac{z}{z_t^*}\right)^{\eta(1-\xi)-1}.$$

By Assumption at the cutoff productivity z_t^* there is full employment, we can write this expression as

$$h_t(z) = \left(\frac{z}{z_t^*}\right)^{\eta(1-\xi)-1} \bar{h}(1-u_t^n).$$

Since, by assumption $z > z_t^*$ and $\eta(1-\xi)-1 > 0$, we have that

$$h_t(z) > \bar{h}(1-u^n),$$

which is a contradiction. We have thus shown that for $z > z_t^*$, $h_t(z) = \bar{h}(1-u_t^n)$. It remains to show that $W_t(z) > z^{\xi}\gamma\tilde{W}_{t-1}$. Evaluate the labor demand function (29) at z and at z_t^* and use the fact that $h_t(z) = h(z_t^*) = \bar{h}(1-u_t^n)$ and that $W(z_t^*) = z_t^{*\xi}\gamma\tilde{W}_{t-1}$. This yields

$$z\bar{h}(1-u_t^n) = \left(\frac{W_t(z)}{z\tilde{W}_t}\right)^{-\eta} h_t$$

and

$$z_t^*\bar{h}(1-u_t^n) = \left(\frac{z_t^{*\xi-1}\gamma\tilde{W}_{t-1}}{\tilde{W}_t}\right)^{-\eta} h_t.$$

Dividing the first expression by the second, and solving for $W_t(z)$ we obtain

$$W_t(z) = \left(\frac{z}{z_t^*}\right)^{\frac{\eta(1-\xi)-1}{\eta}} z^{\xi}\gamma\tilde{W}_{t-1} > z^{\xi}\gamma\tilde{W}_{t-1},$$

where the inequality follows from Assumption F1 and the fact that $z > z_t^*$. Thus we have shown that (30) holds with strict inequality for $z > z_t^*$, which is what we had wanted to show. □

Claim F2 (Unemployment for all labor varieties $z < z_t^*$)**.** *If Assumption F1 holds, then for all $z < z_t^*$, there is involuntary unemployment, $h_t(z) < \bar{h}(1 - u_t^n)$ and $W_t(z) = z^\xi \gamma \tilde{W}_{t-1}$.*

Proof. Suppose, contrary to the claim, that for $z < z_t^*$, $h_t(z) = \bar{h}(1 - u_t^n)$. Then evaluate equilibrium condition (29) at z to obtain

$$z\bar{h}(1 - u_t^n) = \left(\frac{\tilde{W}_t(z)}{\tilde{W}_t}\right)^{-\eta} h_t = \left(\frac{W_t(z)}{z\tilde{W}_t}\right)^{-\eta} h_t.$$

Evaluate (29) at z_t^* and use the fact that by definition $h(z_t^*) = \bar{h}(1 - u_t^n)$. This yields

$$z_t^*\bar{h}(1 - u_t^n) = \left(\frac{W_t(z_t^*)}{z_t^*\tilde{W}_t}\right)^{-\eta} h_t$$

Dividing the above equation by this expression we obtain

$$\frac{z}{z_t^*} = \left(\frac{z_t^* W_t(z)}{z W_t(z_t^*)}\right)^{-\eta}.$$

Solve for $W_t(z)$ and note that the maintained assumptions, namely, that $\eta > 0$ and $\eta(1-\xi) > 1$, imply that $0 < 1 - \frac{1}{\eta} < 1$. This yields

$$W_t(z) = \left(\frac{z}{z_t^*}\right)^{1-\frac{1}{\eta}} W_t(z_t^*) = \left(\frac{z}{z_t^*}\right)^{1-\frac{1}{\eta}} z_t^{*\xi}\gamma\tilde{W}_{t-1} = \left(\frac{z}{z_t^*}\right)^{\frac{\eta(1-\xi)-1}{\eta}} z^\xi\gamma\tilde{W}_{t-1} < z^\xi\gamma\tilde{W}_{t-1},$$

which violates (30). Thus for $z < z_t^*$, $h_t(z) = \bar{h}(1 - u_t^n)$ is impossible. The only equilibrium outcome therefore is $h_t(z) < \bar{h}(1 - u_t^n)$. It then follows immediately from equilibrium condition (31) that $W_t(z) = z^\xi\gamma\tilde{W}_{t-1}$. □

Claim F3 (Relation between π_t^W and z_t^*)**.** *If Assumption F1 holds, then in equilibrium π_t^W and z_t^* are negatively related.*

Proof. Let RHS denote the right-hand side of equilibrium condition (F3), that is,

$$RHS \equiv \int_{z<z^*} \left(\frac{\gamma}{z^{1-\xi}}\right)^{1-\eta} f(z)dz + \int_{z>z^*} \left(\frac{\gamma}{z_t^{*1-\xi}}\left(\frac{z}{z_t^*}\right)^{-\frac{1}{\eta}}\right)^{1-\eta} f(z)dz$$

and let LHS denote the left-hand side of equilibrium condition (F3), that is,

$$LHS = (1 + \pi_t^W)^{1-\eta}.$$

Totally differentiating (F3) yields

$$\frac{dLHS}{d\pi_t^W} d\pi_t^W = \frac{dRHS}{dz_t^*} dz_t^*.$$

Rearrange to obtain

$$\frac{d\pi_t^W}{dz_t^*} = \frac{\frac{dRHS}{dz_t^*}}{\frac{dLHS}{d\pi_t^W}}.$$

The derivative of RHS with respect to z_t^* is given by

$$\frac{dRHS}{dz_t^*} = (\eta - 1) \frac{\frac{\eta(1-\xi)-1}{\eta}}{z_t^*} \int_{z>z^*} \left(\frac{\gamma}{{z_t^*}^{1-\xi}} \left(\frac{z}{z_t^*} \right)^{-\frac{1}{\eta}} \right)^{1-\eta} f(z)dz > 0,$$

where the inequality follows from Assumption F1 and the fact that the assumption that $\eta > 0$ together with Assumption F1 imply that $\eta > 1$. The derivative of LHS with respect to π_t^W is given by

$$\frac{dLHS}{d\pi_t^W} = (1 - \eta)(1 + \pi_t^W)^{-\eta} < 0,$$

where the inequality follows from the fact that the assumption that $\eta > 0$ together with Assumption F1 imply that $\eta > 1$. Thus we have that

$$\frac{d\pi_t^W}{dz_t^*} = \frac{\frac{dRHS}{dz_t^*}}{\frac{dLHS}{d\pi_t^W}} < 0.$$

□

Claim F4 (Relation between u_t and z_t^*)**.** *If Assumption F1 holds, then in equilibrium u_t and z_t^* are positively related.*

Proof. Differentiating equilibrium condition (F4) with respect to z_t^* yields

$$\frac{d\, u_t}{d\, z_t^*} = (1 - u_t^n) \frac{\eta(1-\xi) - 1}{z_t^*} \int_{z<z_t^*} \left(\frac{z}{z_t^*} \right)^{\eta(1-\xi)-1} f(z)dz > 0,$$

where the inequality follows from Assumption F1. □

G Regular Dynamics: Additional Results

This appendix complements the results of section 6 in two ways. First, it considers a calibration with more wage rigidity, to make it comparable with existing calibrations of the NK model. Second, it compares the equilibrium responses of the HDNWR and NK models to productivity shocks.

G.1 Higher Wage Stickiness

The calibration of the NK model in section 6 delivers a value of θ_w, governing the degree of nominal wage rigidity, of 0.35, which is low relative to those typically used to calibrate NK models. For example, Galí (2015) sets θ_w to 0.75. To address this issue, here we also consider a calibration in which $\theta_w = 1 - j^* = 0.75$. In this case, we recalibrate the parameters Γ_0 and Γ_1 of the function $\gamma(j)$ in the HDNWR model. Specifically, we continue to impose that the steady-state unemployment-inflation pair match its observed median value but drop the requirement that the model match the slope of the Galí-Gambetti wage Phillips curve and instead target a value of 0.75 for $1 - j^*$. The resulting values of Γ_0 and Γ_1 are 0.9908 and 0.0175.

Figure G1 compares the impulse responses of the HDNWR and NK models when $\theta_w = 1 - j^* = 0.75$. Because now wages are more rigid than in the case in which $\theta_w = 1 - j^* = 0.35$ considered in the body of the paper (section 6), the HDNWR and NK models predict a more subdued response of wage inflation and a larger response of unemployment. The important point for the purpose of the present discussion, however, is that both models deliver quite similar dynamics.

G.2 Response to a Technology Shock

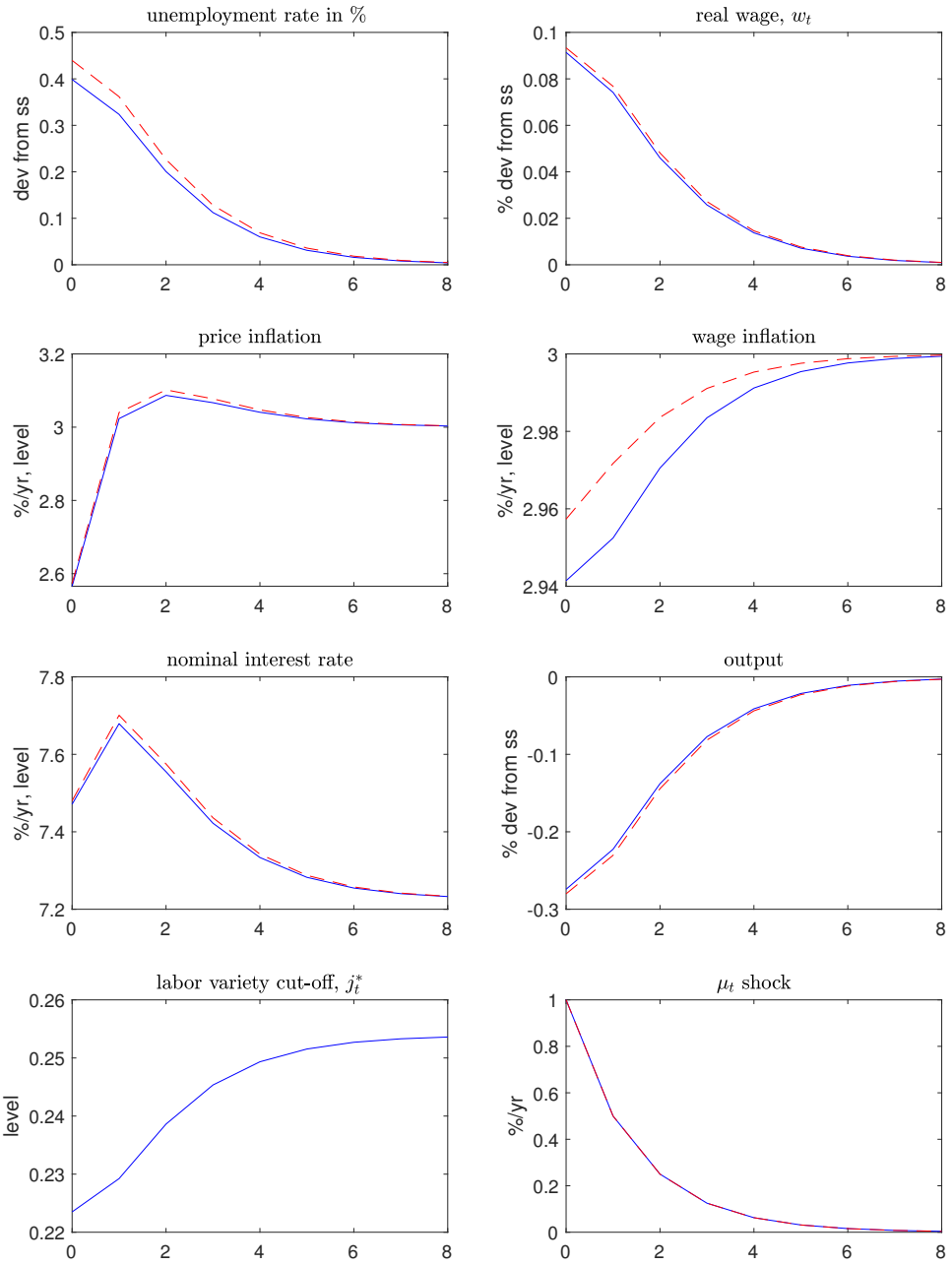
Figure G2 displays the response of the HDNWR model to a 1-percent positive productivity shock, that is, a 1 percent increase in the exogenous variable a_t buffeting the production function (12). The shock is assumed to follow a first-order autoregressive process of the form

$$\ln a_t = \rho_a \ln a_{t-1} + \epsilon_t^a,$$

where ϵ_t^a is a mean-zero i.i.d. disturbance and ρ_a is a parameter. Following Galí (2015), we set ρ_a equal to 0.9. All other parameters take their baseline values given in Table 1 in the body of the paper. This calibration implies that $1 - j^* = 0.35$.

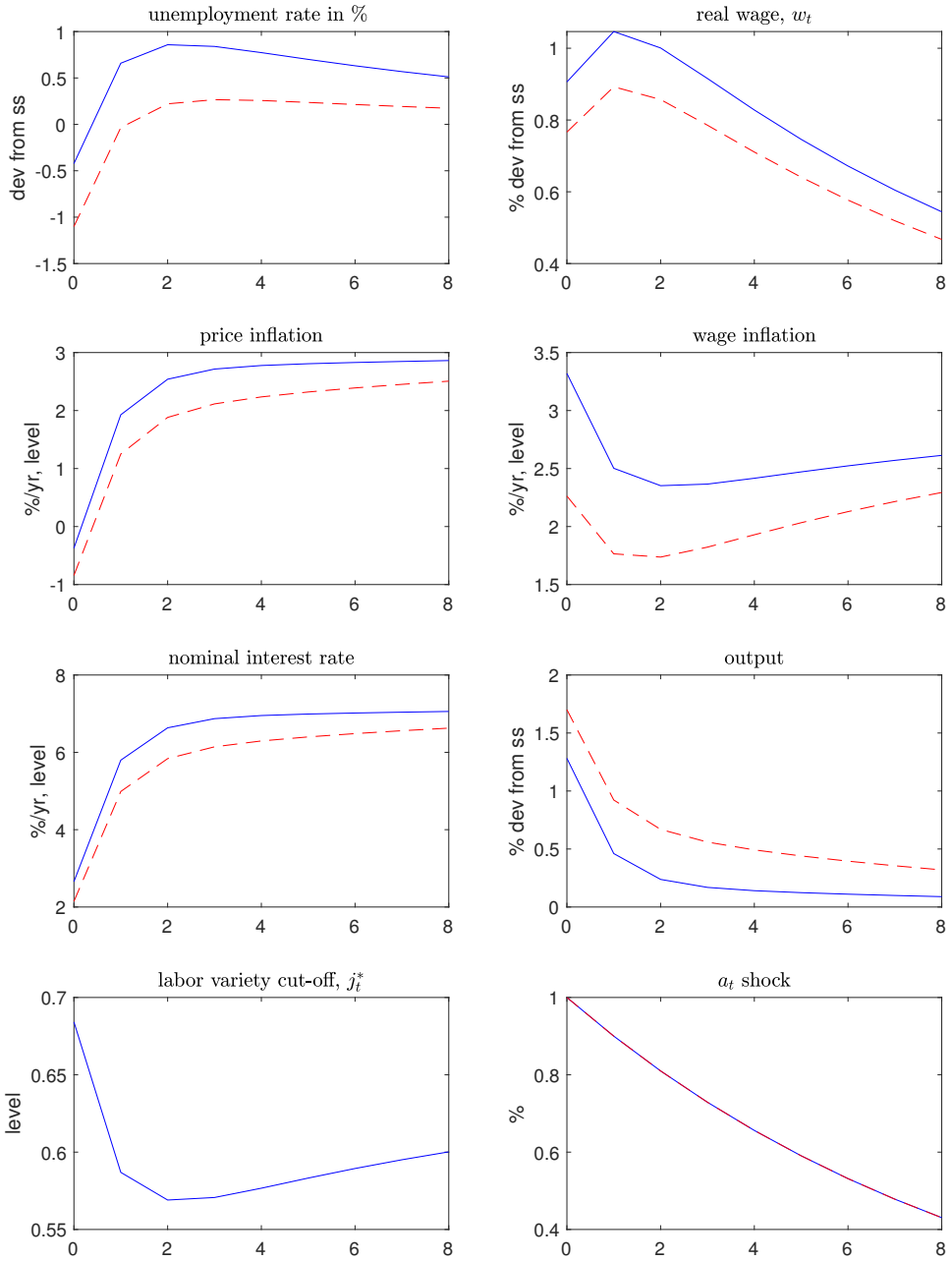
The increase in output following the positive technology shock puts downward pressure on product-price inflation. The increase in labor productivity following the technological

Figure G1: Impulse Responses to a Monetary Tightening in the HDNWR and NK Models with a Higher Degree of Wage Rigidity ($\theta_w = 1 - j^* = 0.75$)



Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the monetary shock is 1 percent per annum. The horizontal axes measure quarters after the shock.

Figure G2: Impulse Responses to a Productivity Shock in the HDNWR and NK Models for $1 - j^* = \theta_w = 0.35$



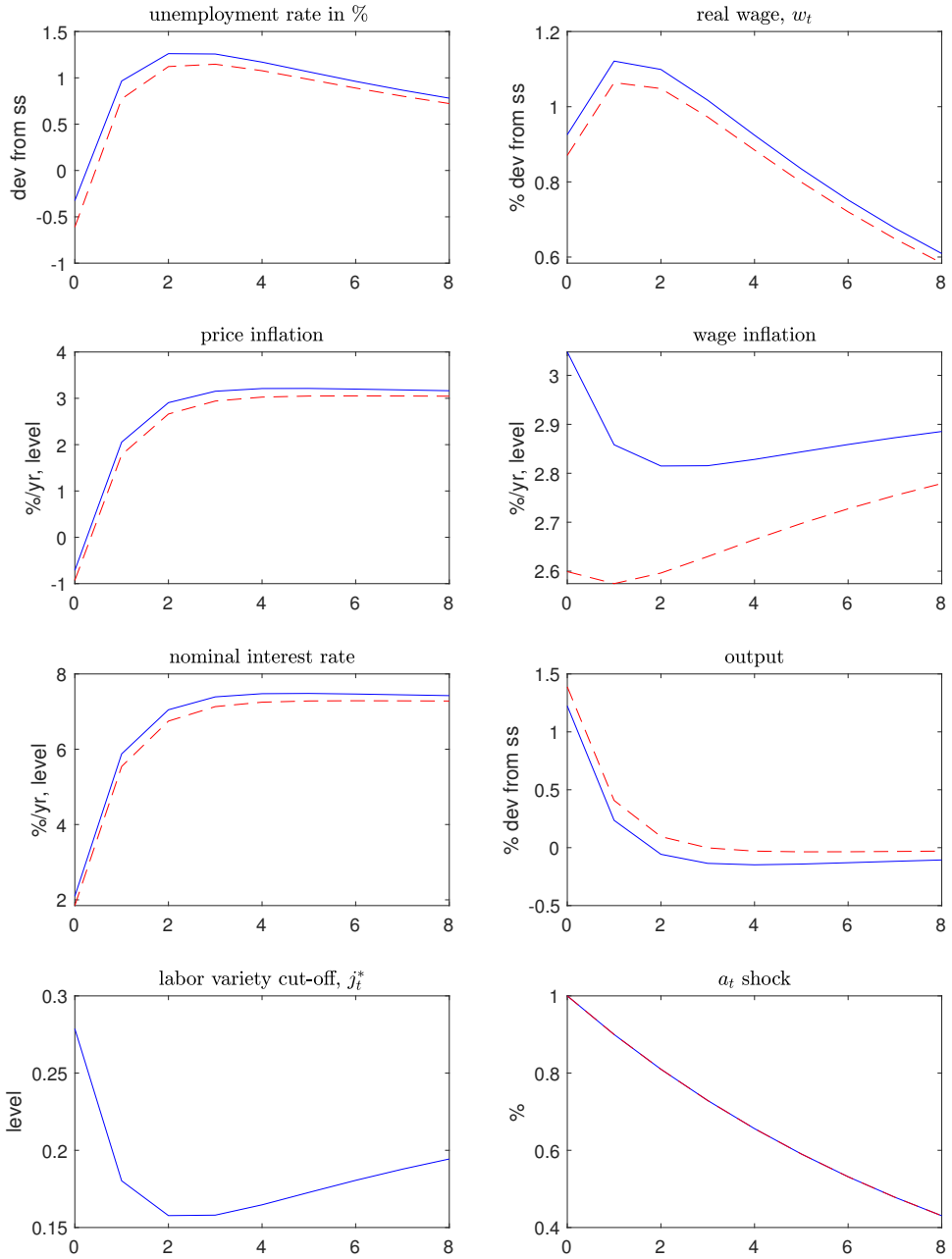
Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the shock is 1 percent. The horizontal axes measure quarters after the shock.

improvement pushes nominal wages up. This relaxes the wage constraint for some wage varieties (j_t^* goes up on impact), inducing a fall in unemployment in the initial period. As the technology shock begins to return to its stationary position, real wages fall. However, due to the presence of wage rigidity, they fall at a slower pace than the one consistent with full employment. As a result, unemployment rises and remains above steady state throughout the transition.

The response of the NK model to the positive productivity shock, shown with dashed lines in Figure G2, is similar. The main difference is that under the present calibration in the NK model unemployment experiences a larger decline on impact and a smaller subsequent increase. This difference in the response of unemployment is due to the relatively low value picked for the degree of wage rigidity ($\theta_w = 0.35$). When we set θ_w to the more conventional value of 0.75 and recalibrate the HDNWR model to target $1 - j^* = 0.75$, then the response of unemployment to the technological improvement is almost the same in both models. This result is shown in Figure G3. The two models produce virtually the same responses to the positive productivity shock not only for unemployment but also for most of the other endogenous variables displayed in the figure.

Taken together, the results presented in this appendix and in section 6 in the body of the paper suggest that for regular fluctuations in a neighborhood around the steady state, at least for the conventional calibrations considered here, the forward-looking nature of the wage Phillips curve in the NK framework, which up to first order is the key difference between the HDNWR model and the NK model, does not appear to play a crucial role for the predicted response to monetary and productivity disturbances.

Figure G3: Impulse Responses to a Productivity Shock in the HDNWR and NK Models with a Higher Degree of Wage Rigidity ($\theta_w = 1 - j^* = 0.75$)



Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the shock is 1 percent. The horizontal axes measure quarters after the shock.