Behavioral Expectations*

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(Most Recent Version Here)

Abstract

I study a model where firms form inflation expectations using dual-process reasoning. When the state of the economy is familiar, firms employ System 1 - a fast, heuristic mode of thinking that is prone to systematic errors. When the state of the economy is unfamiliar, firms trigger System 2 - a deliberative mode of thinking that is more accurate, but also more costly. The model features a Phillips curve with a state- and history-dependent slope and, due to strategic complementarities in learning, can exhibit multiple equilibria. The optimal policy response to cost-push shocks involves front-loading the cognitive costs associated with System 2 and, for shocks that are not too large, greater price stability than under rationality.

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1 Introduction

In order to make decisions, individuals have to forecast equilibrium variables. This task can be challenging, as equilibrium variables depend on fundamentals and those same forecasts. The dominant approach in macroeconomics is to assume that forecasts are correct: the data-generating process used by the individuals in the model is the same as the data-generating process of the model. While economists have long questioned this rational expectations hypothesis, they have struggled to provide an incontrovertible alternative.

In this paper, I turn to the consensus of the cognitive psychology literature and develop a model in which individuals use dual-process reasoning to form inflation expectations. I then study the implications for the behavior of the Phillips curve and for optimal monetary policy.

Dual-process reasoning is a framework that provides a taxonomy of the different ways in which people make decisions (Evans (2008)). System 1 processes are fast, intuitive, but prone to systematic errors. System 2 processes are slow and deliberative, but cognitively more costly. Due to these cognitive costs, System 2 tends to be triggered only in unfamiliar situations. Recent empirical evidence suggests that dual-process reasoning might be a useful device for modeling expectation formation. Baley and Turén (2023) and Baley and Turén (2025) document that professional forecasters revise their inflation expectations infrequently, but that the frequency and size of revisions increase in periods of high volatility. Pfäuti (2023) estimates that attention to inflation doubles when inflation exceeds 4%.

The model employs some of the techniques developed by Ilut and Valchev (2023). In the model, firms understand that inflation is a linear function of the average inflation expectation and marginal costs, but are uncertain about the loadings on those variables. I model this uncertainty by assuming that firms extract signals on equilibrium inflation, subject to cognitive costs. The optimal information acquisition decision is to rely on a prior forecasting function when the state of the economy is familiar - System 1 - and to update the forecasting function if the state is unfamiliar - System 2. The law of motion for forecasts has the flavor of adaptive expectations, except that (1) the learning coefficient is state-dependent, and (2), as the state of the economy becomes more unfamiliar, expectations become closer to rational expectations.

The model can exhibit multiple equilibria. When deciding the optimal information acquisition strategy, each firm takes the average expectation as given. The expectation formation process induces a fixed point in expectations: the average expectation must be such that it is optimal for individual firms to have expectations that aggregate to that same average expectation. If the degree of strategic complementarity in price setting is sufficiently high, then there might be both System-1 equilibria, in which individuals do not adjust their expectations be-

cause the average expectation varies little, and System-2 equilibria, in which individuals adjust their expectations because the average expectation varies substantially.

The model can also generate a Phillips curve with a state- and history-dependent slope. When output varies little, the Phillips curve is linear, but flatter than under rationality. When output varies above a threshold, the Phillips curve is nonlinear, with a slope that converges to the rational expectations analogue as output deviations grow. Moreover, a large output deviation today widens the System 1 region in the future and, because it induces learning, also makes the Phillips curve steeper.

The central bank wishes to avoid triggering System 2 if possible. In the case of a persistent, one-time shock, it is optimal to trigger System 2 initially and have firms rely on System 1 forever after. For medium shocks, this policy is associated with price stability. Higher inflation, in absolute terms, decreases current utility due to cognitive costs and due to belief dispersion. For large shocks, it might be optimal to increase inflation by more than under rationality in order to widen the System 1 region in the future.

Section 2 describes the setup and the expectation formation process. Section 3 develops the properties of the equilibrium, and Section 4 characterizes the optimal policy. Section 5 concludes.

2 The Setup

The model is based on (Sargent, 1999). Time is discrete, infinite, and indexed by $t = 0, 1, \ldots$. There is a continuum of measure 1 of private agents indexed by $i \in [0, 1]$. I start by describing the structural equations of the model given arbitrary inflation expectations.

2.1 Temporary Equilibrium and Payoffs

Let $\mathbb{E}_{i,t}[\pi_t]$ be the beginning-of-period inflation expectation of individual i. Actual inflation, π_t, is determined by the Phillips curve

$$\pi_t = \omega \overline{\mathbb{E}}_t \pi_t + (1 - \omega)(\kappa y_t + u_t), \quad \omega \in (0, 1), \quad \kappa > 0 \tag{1}$$

where $\overline{\mathbb{E}}_t[\pi_t] \equiv \int_0^1 \mathbb{E}_{i,t}[\pi_t]\, di$ is the average inflation expectation, y_t is the output gap, and u_t is a AR(1) cost-push shock,

$$u_t = \rho u_{t-1} + \sigma_\nu \nu_t, \quad \nu_t \overset{\text{iid}}{\sim} \mathcal{N}(0, 1), \quad |\rho| < 1.$$

The utility of the central bank is

$$V_0 = \mathbb{E}_0^* \left[\sum_{t=0}^{\infty} \beta^t U_t \right], \ \beta \in [0,1)$$

where

$$U_t \equiv -\frac{\lambda}{2} y_t^2 - \frac{\gamma}{2} \left\{ \left(\frac{1-\omega}{\omega} \right) \pi_t^2 + \omega \text{Var}_i \left[e_{i,t} \right] \right\} - \int_0^1 \mathcal{I}_{i,t} di, \ \gamma > 0, \tag{2}$$

$$e_{i,t} = \mathbb{E}_{i,t} \left[\pi_t \right] - \pi_t,$$

$$\text{Var}_i \left[e_{i,t} \right] \equiv \int_0^1 \left(e_{i,t} - \bar{e}_t \right)^2 di,$$

$$\bar{e}_t \equiv \int_0^1 e_{i,t} di,$$

and $\mathcal{I}_{i,t}$ is the cognitive cost of individual i. $\mathbb{E}_t^*$ is the model-consistent expectation, and

$$\lambda = \left(\frac{1-\omega}{\omega} \right) \kappa.$$

I assume that the end-of-period utility of individual i in period t is

$$v_{i,t} = -\frac{\gamma\omega}{2} \left(\mathbb{E}_{i,t} \pi_t - \pi_t \right)^2 - \mathcal{I}_{i,t}, \ \gamma > 0,$$

Appendix E describes the New Keynesian microfoundations of this model. Firms are subject to a Calvo friction, but they reset their prices at the beginning of the period before observing the prices set by other firms.[1][2] In the context of these microfoundations, ω is the fraction of firms that reset their prices, λ is the elasticity of real marginal costs to output, and γ is related to the elasticity of demand.

Given an expectation formation process, the problem of the central bank is to solve

$$\max_{\{y_t, \pi_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0^* U_t \ \text{ s.t. } \ \pi_t = \omega \overline{\mathbb{E}}_t \pi_t + (1-\omega) \left(\kappa y_t + u_t \right).$$

This formulation assumes that the central bank can implement any $\{y_t, \pi_t\}_{t=0}^{\infty}$ that satisfies the Phillips curve and that the policy instrument does not directly affect expectation formation. In Appendix C, I study a version of the model in which that is not the case.

[1] For simplicity, I also assume firms are myopic. This assumption implies that equation (1) does not include a forward-looking term.

[2] The model thus differs from (Sargent, 1999), in that a pricing friction generates a long-run trade-off between inflation and output.

2.2 Expectation Formation

Under rationality, $\mathbb{E}_{i,t}\pi_t = \overline{\mathbb{E}}_t\pi_t = \pi_t$, and equation (1) becomes

$$\pi_t = \kappa y_t + u_t. \tag{3}$$

The parameter ω, which relates to the degree of strategic complementarity, does not affect the rational expectations equilibrium other than through κ. This result is due to a general equilibrium effect on expectations that perfectly offsets a partial equilibrium effect on inflation.[3]

I depart from rational expectations by assuming that agents have a limited understanding of general equilibrium. While agents are aware that the temporary equilibrium inflation mapping (1) is a linear function of $\overline{\mathbb{E}}_t\pi_t$, y_t, and u_t, they face uncertainty about the coefficients of that mapping. This modeling technique is a specialization of Ilut and Valchev (2023) to the case of linear functions.

Let

$$\mathbf{x}_t \equiv \begin{bmatrix} \overline{\mathbb{E}}_t\pi_t \\ y_t \\ u_t \end{bmatrix}, \quad \boldsymbol{\theta}^* \equiv \begin{bmatrix} \omega \\ (1-\omega)\,\kappa \\ 1-\omega \end{bmatrix}.$$

Equilibrium inflation is determined by

$$\pi_t = f^*(\mathbf{x}_t) = \mathbf{x}_t^\top \boldsymbol{\theta}^*.$$

Private agents believe that

$$\pi_t = f(\mathbf{x}_t) = \mathbf{x}_t^\top \boldsymbol{\theta},$$

where $\boldsymbol{\theta}$ is random. Individual i enters period t with a prior distribution for $\boldsymbol{\theta}$,

$$\boldsymbol{\theta} \mid \mathscr{I}_{i,t-1} \sim \mathcal{N}\left(\boldsymbol{\mu}_{i,t-1}, \boldsymbol{\Sigma}_{i,t-1}\right),$$

where $\boldsymbol{\mu}_{i,-1}$ and $\boldsymbol{\Sigma}_{i,-1}$ is given. I assume throughout that $\boldsymbol{\Sigma}_{i,-1} = \boldsymbol{\Sigma}_{-1}$ for all i. The prior distribution on $\boldsymbol{\theta}$ induces a prior distribution on $f(\mathbf{x})$. That is, for any $\mathbf{x}$, $\tilde{\mathbf{x}}$,

$$\mathbb{E}_{i,t-1}\left[f(\mathbf{x})\right] = \mathbf{x}^\top \boldsymbol{\mu}_{i,t-1} \equiv m_{i,t-1}(\mathbf{x})$$

$$\mathrm{Var}_{i,t-1}\left[f(\mathbf{x})\right] = \mathbf{x}^\top \boldsymbol{\Sigma}_{i,t-1}\mathbf{x} \equiv \sigma^2_{i,t-1}(\mathbf{x})$$

[3]This property is a more general feature of rational expectations equilibria. See the discussion in Angeletos et al. (2025).

$$\text{Cov}_{i,t-1}\left[f\left(\mathbf{x}\right),f\left(\tilde{\mathbf{x}}\right)\right]=\mathbf{x}^{\top}\boldsymbol{\Sigma}_{i,t-1}\tilde{\mathbf{x}}\equiv\sigma_{i,t-1}\left(\mathbf{x},\tilde{\mathbf{x}}\right).$$

Since $f\left(\mathbf{x}\right)$ and $f\left(\tilde{\mathbf{x}}\right)$ are jointly normal, $f\left(\mathbf{x}\right)$ follows a Gaussian process,

$$f\left(\mathbf{x}\right)\mid\mathscr{I}_{i,t-1}\sim\mathcal{GP}\left[m_{i,t-1}\left(\mathbf{x}\right),\sigma_{i,t-1}\left(\mathbf{x},\tilde{\mathbf{x}}\right)\right].$$

Each individual can extract a costly normal signal with perceived distribution

$$s_{i,t}=f\left(\mathbf{x}_{t}\right)+\sigma_{\epsilon,i,t}\epsilon_{i,t},\ \ \epsilon_{i,t}\sim\mathcal{N}\left(0,1\right).$$

The intra-period timing of information acquisition is:

1. $\mathbf{x}_t$ is realized, and each individual i chooses $\sigma_{\epsilon,i,t}$;
2. $s_{i,t}^{*}=f^{*}\left(\mathbf{x}_{t}\right)+\sigma_{\epsilon,i,t}\epsilon_{i,t}$ is realized, and the prior moments are updated according to Bayes' law;
3. Each individual i sets her expectation $\mathbb{E}_{i,t}\pi_t$;
4. Inflation π_t is realized, and individual i obtains payoff

$$v_{i,t}=-\frac{\gamma\omega}{2}\left(\mathbb{E}_{i,t}\pi_{t}-\pi_{t}\right)^{2}-\mathcal{I}_{i,t},$$

where $\mathcal{I}_{i,t}$ is proportional to Shannon mutual information,

$$\mathcal{I}_{i,t}=\frac{\iota}{2}\ln\left[\frac{\sigma_{i,t-1}^{2}\left(\mathbf{x}_{t}\right)}{\sigma_{i,t}^{2}\left(\mathbf{x}_{t}\right)}\right],\ \ \iota>0.$$

The problem of individual i is

$$\max_{\sigma_{\epsilon,i,t}\geq0}-\frac{\gamma\omega}{2}\min_{\mathbb{E}_{i,t}\pi_{t}}\mathbb{E}\left[\left(\mathbb{E}_{i,t}\pi_{t}-f\left(\mathbf{x}_{t}\right)\right)^{2}\mid\mathscr{I}_{i,t}\right]-\frac{\iota}{2}\ln\left[\frac{\sigma_{i,t-1}^{2}\left(\mathbf{x}_{t}\right)}{\sigma_{i,t}^{2}\left(\mathbf{x}_{t}\right)}\right].$$

Given any information set $\mathscr{I}_{i,t}$, the optimal expectation is equal to the posterior mean,

$$\mathbb{E}_{i,t}\left[\pi_{t}\right]=m_{i,t}\left(\mathbf{x}_{t}\right). \tag{4}$$

Substituting in the objective, the problem of choosing the signal variance is

$$\max_{\sigma_{i,t}^{2}(\mathbf{x}_{t})}-\frac{\gamma\omega}{2}\sigma_{i,t}^{2}\left(\mathbf{x}_{t}\right)-\frac{\iota}{2}\ln\left[\frac{\sigma_{i,t-1}^{2}\left(\mathbf{x}_{t}\right)}{\sigma_{i,t}^{2}\left(\mathbf{x}_{t}\right)}\right]\ \ \text{s.t.}\ \ \sigma_{i,t}^{2}\left(\mathbf{x}_{t}\right)\leq\sigma_{i,t-1}^{2}\left(\mathbf{x}_{t}\right),$$

where the constraint ensures that the solution is consistent with Bayes' rule.

The solution is

$$\sigma_{i,t}^2(\mathbf{x}_t) = \min\left\{\sigma_{i,t-1}^2(\mathbf{x}_t), \overline{\sigma}^2\right\}, \tag{5}$$

where $\overline{\sigma}^2 \equiv \iota/(\gamma\omega)$. Equation (5) states that when the prior uncertainty at the current state is low, $\sigma_{i,t-1}^2(\mathbf{x}_t) \leq \overline{\sigma}^2$, individual i is confident in her prior and relies on System 1. When the prior uncertainty at the current state is high, $\sigma_{i,t-1}^2(\mathbf{x}_t) > \overline{\sigma}^2$, individual i finds herself in an unfamiliar situation, triggers System 2, and draws a signal.

The optimal solution for $\sigma_{i,t}^2(\mathbf{x}_t)$ does not depend on the signal realization at t. Together with the assumption that the initial prior variance is common across agents, this fact implies that $\sigma_{i,t}(\mathbf{x}, \tilde{\mathbf{x}}) = \sigma_t(\mathbf{x}, \tilde{\mathbf{x}})$ for all i and t. The laws of motion for $m_{i,t}(\mathbf{x})$ and $\sigma_t(\mathbf{x}, \tilde{\mathbf{x}})$ are

$$m_{i,t}(\mathbf{x}) = m_{i,t-1}(\mathbf{x}) + \alpha_t(\mathbf{x})\left[\mathbf{x}_t^\top \boldsymbol{\theta}^* + \sigma_{\epsilon,t}\epsilon_{i,t} - m_{i,t-1}(\mathbf{x}_t)\right], \tag{6}$$

and

$$\sigma_t(\mathbf{x}, \tilde{\mathbf{x}}) = \sigma_{t-1}(\mathbf{x}, \tilde{\mathbf{x}}) - \alpha_t(\mathbf{x})\,\sigma_{t-1}(\mathbf{x}_t, \tilde{\mathbf{x}}), \tag{7}$$

where

$$\alpha_t(\mathbf{x}) = \alpha_t(\mathbf{x}_t) \times \frac{\sigma_{t-1}(\mathbf{x}, \mathbf{x}_t)}{\sigma_{t-1}^2(\mathbf{x}_t)},$$

$$\sigma_{\epsilon,t} = \sqrt{\frac{1-\alpha_t(\mathbf{x}_t)}{\alpha_t(\mathbf{x}_t)}\sigma_{t-1}^2(\mathbf{x}_t)},$$

and

$$\alpha_t(\mathbf{x}_t) = \max\left\{0, 1 - \frac{\overline{\sigma}^2}{\sigma_{t-1}^2(\mathbf{x}_t)}\right\}. \tag{8}$$

These laws of motion follow from the standard formulas for the conditional distribution of normal variables.

The moment recursions (6)-(7) have two additional useful representations. First, due to linearity, posterior moments can be written as

$$m_{i,t}(\mathbf{x}) = \mathbf{x}^\top \boldsymbol{\mu}_{i,t}$$

and

$$\sigma_t(\mathbf{x}, \tilde{\mathbf{x}}) = \mathbf{x}^\top \boldsymbol{\Sigma}_t \tilde{\mathbf{x}},$$

where

$$\boldsymbol{\mu}_{i,t} = \boldsymbol{\mu}_{i,t-1} + \mathbf{K}_t\left[\mathbf{x}_t^\top \boldsymbol{\theta}^* + \sigma_{\epsilon,t}\epsilon_{i,t} - m_{i,t-1}(\mathbf{x}_t)\right], \tag{9}$$

$$\boldsymbol{\Sigma}_t = \left(\mathbf{I} - \mathbf{K}_t \mathbf{x}_t^\top\right) \boldsymbol{\Sigma}_{t-1}, \tag{10}$$

and

$$\mathbf{K}_t \equiv \frac{\alpha_t(\mathbf{x}_t)}{\sigma_{t-1}^2(\mathbf{x}_t)} \boldsymbol{\Sigma}_{t-1} \mathbf{x}_t.$$

Second, (6)-(7) also induce a metric on the state space. Let

$$\|\mathbf{x}\|_t \equiv \left\| \sqrt{\boldsymbol{\Sigma}_t / \overline{\sigma}^2} \mathbf{x} \right\|$$

be an uncertainty-weighted norm, and

$$\cos_t(\mathbf{x}, \tilde{\mathbf{x}}) \equiv \cos\left(\sqrt{\boldsymbol{\Sigma}_t / \overline{\sigma}^2} \mathbf{x}, \sqrt{\boldsymbol{\Sigma}_t / \overline{\sigma}^2} \tilde{\mathbf{x}} \right)$$

an uncertainty-weighted cosine. Then

$$\|\mathbf{x}\|_t = \sqrt{\sigma_t^2(\mathbf{x}) / \overline{\sigma}^2}, \;\; \cos_t(\mathbf{x}, \tilde{\mathbf{x}}) = \frac{\sigma_t(\mathbf{x}, \tilde{\mathbf{x}})}{\sqrt{\sigma_t^2(\mathbf{x})} \sqrt{\sigma_t^2(\tilde{\mathbf{x}})}}.$$

Using equations (6) and (7), the laws of motion for $\|\cdot\|_t$ and $\cos_t(\cdot, \cdot)$ are

$$\|\mathbf{x}\|_t = \|\mathbf{x}\|_{t-1} \sqrt{1 - \alpha_t(\mathbf{x}_t) \cos_{t-1}^2(\mathbf{x}, \mathbf{x}_t)} \tag{11}$$

and

$$\cos_t(\mathbf{x}, \tilde{\mathbf{x}}) = \frac{\cos_{t-1}(\mathbf{x}, \tilde{\mathbf{x}}) - \alpha_t(\mathbf{x}_t) \cos_{t-1}(\mathbf{x}, \mathbf{x}_t) \cos_{t-1}(\mathbf{x}_t, \tilde{\mathbf{x}})}{\sqrt{1 - \alpha_t(\mathbf{x}_t) \cos_{t-1}^2(\mathbf{x}, \mathbf{x}_t)} \sqrt{1 - \alpha_t(\mathbf{x}_t) \cos_{t-1}^2(\tilde{\mathbf{x}}, \mathbf{x}_t)}}. \tag{12}$$

If individuals rely on System 1 in period t, the metric is not updated. If individuals trigger System 2 in period t, the vector norm decreases the most for vectors that are collinear with $\mathbf{x}_t$, and stays unchanged for vectors that are orthogonal to $\mathbf{x}_t$. Equation (11) thus gives a precise meaning to similarity between states.[4]

The definition of $\|\cdot\|_t$ and the optimality condition (5) implies that System 2 is triggered at t if $\|\mathbf{x}_t\|_{t-1}^2 > 1$. Let $\tau < t$ denote the last period in which System 2 was triggered. Then $\alpha_\tau(\mathbf{x}_\tau) > 0$. Iterating on (11) until τ, the threshold rule for System 2 at t is

$$\|\mathbf{x}_t\|_{\tau-1}^2 \left[1 - \alpha_\tau(\mathbf{x}_\tau) \cos_{\tau-1}^2(\mathbf{x}_t, \mathbf{x}_\tau)\right] > 1. \tag{13}$$

The following Lemma establishes several properties of the learning decision.

[4]The law of motion for the cosine follows from a standard Gram-Schmidt decomposition.

Lemma 1. *Consider the decision of whether to draw a signal in period t. Then:*

1. *If $\|\mathbf{x}_t\|_{\tau-1}^2 \leq 1$, individuals rely on System 1;*
2. *Given $\cos_{\tau-1}^2(\mathbf{x}_t, \mathbf{x}_\tau) \equiv \chi$, individuals trigger System 2 if the $(\tau-1)$-norm is large enough,*

$$\|\mathbf{x}_t\|_{\tau-1} > \sqrt{\frac{1}{1-\alpha_\tau(\mathbf{x}_\tau)\chi}};$$

3. *Given $\|\mathbf{x}_t\|_{\tau-1} \equiv \delta > 1$, individuals trigger System 2 if the $(\tau-1)$-cosine with $\mathbf{x}_\tau$ is sufficiently close to zero,*

$$-\min\left\{1; \sqrt{\left(1-\frac{1}{\delta^2}\right)\frac{1}{\alpha_\tau(\mathbf{x}_\tau)}}\right\} < \cos_{\tau-1}(\mathbf{x}_t, \mathbf{x}_\tau) < \min\left\{1; \sqrt{\left(1-\frac{1}{\delta^2}\right)\frac{1}{\alpha_\tau(\mathbf{x}_\tau)}}\right\}.$$

Individuals are more likely to trigger System 2 if the current state is orthogonal to past states according to the uncertainty-weighted metric.

2.3 Equilibrium

I now characterize the implementable set of actions (π_t, y_t) in period t, as well as the utility function U_t. Equation (1) implies that the implementable set depends on the average expectation $\overline{\mathbb{E}}_t[\pi_t]$. Equation (2) shows that U_t also depends on the variances and on the cross-sectional forecast error dispersion.

I start by characterizing $\overline{\mathbb{E}}_t[\pi_t]$. Equation (4) implies that the average inflation expectation is equal to the average posterior evaluated at $\mathbf{x}_t$. Let $\overline{m}_t(\mathbf{x}) \equiv \int_0^1 m_{i,t}(\mathbf{x})\, di$. Then

$$\begin{aligned}\overline{m}_t(\mathbf{x}) &\equiv \int_0^1 m_{i,t}(\mathbf{x})\, di \\ &= \overline{m}_{t-1}(\mathbf{x}) + \alpha_t(\mathbf{x})\left[\mathbf{x}_t^\top \boldsymbol{\theta}^* - \overline{m}_{t-1}(\mathbf{x}_t)\right],\end{aligned}$$

and, with $\overline{\boldsymbol{\mu}}_t \equiv \int_0^1 \boldsymbol{\mu}_{i,t} di$,

$$\overline{\boldsymbol{\mu}}_t = \overline{\boldsymbol{\mu}}_{t-1} + \mathbf{K}_t \mathbf{x}_t^\top \left(\boldsymbol{\theta}^* - \overline{\boldsymbol{\mu}}_{t-1}\right).$$

The average inflation expectation in period t is

$$\overline{\mathbb{E}}_t[\pi_t] = \overline{m}_t(\mathbf{x}_t). \tag{14}$$

$\overline{\mathbb{E}}_t[\pi_t]$ shows up in both sides of equation (14). The fixed point of (14) corresponds to the

average expectation that is consistent with the individually optimal forecasts chosen by private agents. In general, it does not correspond to the rational expectation.

I now turn to the the cross-sectional belief dispersion. Let

$$\Delta_{e,t}(\mathbf{x}) \equiv \int_0^1 \left[m_{i,t}(\mathbf{x}) - \overline{m}_t(\mathbf{x})\right]^2 di$$

be the dispersion of the forecasting function evaluated at a state $\mathbf{x}$. Then $\text{Var}_i\left[e_{i,t}\right] = \Delta_{e,t}(\mathbf{x}_t)$. The laws of motion for the prior moments imply the following Lemma.

Lemma 2. *The belief dispersion function can be written as*

$$\Delta_{e,t}(\mathbf{x}) = \mathbf{x}^\top \mathbf{\Omega}_t \mathbf{x},$$

where

$$\mathbf{\Omega}_t = \left(\mathbf{I} - \mathbf{K}_t \mathbf{x}_t^\top\right) \mathbf{\Omega}_{t-1} \left(\mathbf{I} - \mathbf{K}_t \mathbf{x}_t^\top\right)^\top + \sigma_{e,t}^2 \mathbf{K}_t \mathbf{K}_t^\top.$$

Evaluated at $\mathbf{x}_t$,

$$\Delta_{e,t}(\mathbf{x}_t) = \left[1 - \alpha_t(\mathbf{x}_t)\right]^2 \Delta_{e,t-1}(\mathbf{x}_t) + \left[1 - \alpha_t(\mathbf{x}_t)\right] \alpha_t(\mathbf{x}_t) \sigma_{t-1}^2(\mathbf{x}_t).$$

Proof. In Appendix A. □

The previous Lemma shows that $\Delta_{e,t}(\mathbf{x}_t)$ is the sum of two terms. The first term relates to System-1 belief dispersion, $\Delta_{e,t-1}(\mathbf{x}_t)$. The second term relates to belief dispersion induced by private signals.

The effect of System-1 dispersion is unambiguous. Given a Kalman weight at t, $\alpha_t(\mathbf{x}_t)$, higher $\Delta_{e,t-1}(\mathbf{x}_t)$ translates to higher disagreement at t. This effect is small if the weight on private signals, $\alpha_t(\mathbf{x}_t)$, is high.

The effect of private signals is ambiguous. Even though the average expectation converges to the rational expectation as $\alpha_t(\mathbf{x}_t)$ increases, for $\alpha_t(\mathbf{x}_t) \in [0, 0.5]$, private signals increase dispersion because the signal precision is low. For $\alpha_t(\mathbf{x}_t) \in [0.5, 1]$ the signals become more precise, and so dispersion goes down.

Let Γ_t be the implementable set of actions in period t. Then Γ_t consists of the pairs (π_t, y_t) characterized by following equations, which I repeat here for convenience:

$$\pi_t = \omega \overline{\mathbb{E}}_t\left[\pi_t\right] + (1 - \omega)(\kappa y_t + u_t),$$

$$\overline{\mathbb{E}}_t\left[\pi_t\right] = \mathbf{x}_t^\top \overline{\boldsymbol{\mu}}_t, \tag{15}$$

$$\overline{\boldsymbol{\mu}}_t = \overline{\boldsymbol{\mu}}_{t-1} + \mathbf{K}_t \mathbf{x}_t^\top \left(\boldsymbol{\theta}^* - \overline{\boldsymbol{\mu}}_{t-1}\right), \tag{16}$$

$$\boldsymbol{\Sigma}_t = \left(\mathbf{I} - \mathbf{K}_t \mathbf{x}_t^\top\right) \boldsymbol{\Sigma}_{t-1},$$

$$\mathbf{K}_t = \frac{\alpha_t\left(\mathbf{x}_t\right)}{\mathbf{x}_t^\top \boldsymbol{\Sigma}_{t-1} \mathbf{x}_t} \boldsymbol{\Sigma}_{t-1} \mathbf{x}_t,$$

and

$$\alpha_t\left(\mathbf{x}_t\right) = \max\left\{0, 1 - \frac{\overline{\sigma}^2}{\mathbf{x}_t^\top \boldsymbol{\Sigma}_{t-1} \mathbf{x}_t}\right\}.$$

It follows that the implementable set can be written as $\Gamma_t = \Gamma\left(u_t, \overline{\boldsymbol{\mu}}_{t-1}, \boldsymbol{\Sigma}_{t-1}\right)$. Therefore the Bellman equation of the central bank is

$$V(\boldsymbol{\zeta}) = \max_{(\pi, y) \in \Gamma(u, \overline{\boldsymbol{\mu}}, \boldsymbol{\Sigma})} \left\{\mathcal{U}(y, \pi, \boldsymbol{\Sigma}, \boldsymbol{\Omega}) + \beta \mathbb{E}^*\left[V\left(\boldsymbol{\zeta}'\right) \mid \boldsymbol{\zeta}, \pi, y\right]\right\},$$

where $\boldsymbol{\zeta} \equiv (u, \overline{\boldsymbol{\mu}}, \boldsymbol{\Sigma}, \boldsymbol{\Omega})$, and $\overline{\boldsymbol{\mu}}_{-1}$, $\boldsymbol{\Sigma}_{-1}$, $\boldsymbol{\Omega}_{-1}$ are given. I assume throughout that $\boldsymbol{\Omega}_{-1} = \mathbf{0}$.

3 Phillips Curve

In this section, I describe the properties of the Phillips curve. I start with a simple Lemma.

Lemma 3. *Suppose that $\overline{\boldsymbol{\mu}}_{-1} = \boldsymbol{\theta}^*$. Then $\overline{\boldsymbol{\mu}}_t = \boldsymbol{\theta}^*$ for all t, and $\overline{\mathbb{E}}_t[\pi_t] = \mathbb{E}^*[\pi_t] = \kappa y_t + u_t$.*

Proof. The proof follows immediately from iterating on equation (16). □

This Lemma shows that the average expectation differs from the rational expectation only insofar as individuals have an initial average bias. This Lemma does not show that under the assumption that $\overline{\boldsymbol{\mu}}_{-1} = \boldsymbol{\theta}^*$, optimal policy coincides with the solution under rationality. I develop this point in Section 4.

$\overline{\boldsymbol{\mu}}_{-1} = \mathbf{0}$ is a natural choice for an initial average bias. This assumption means that, on average, a firm initially believes that inflation does not respond to shocks. $\overline{\boldsymbol{\mu}}_{-1} = \mathbf{0}$ can thus be interpreted as a partial equilibrium prior.

I make two simplifying assumptions.

Assumption 1. *There is no cognitive uncertainty on the effect of the average inflation expectation on actual inflation. That is, $\overline{\mu}_{-1,1} = \omega$ and the first row and column of $\boldsymbol{\Sigma}_{-1}$ are the zero vector.*

Assumption 2. *Individuals are aware that pricing only depends on the expected price level $\mathbb{E}_{i,t}[p_t]$ and on real marginal costs, $mc_t = ((1-\omega)/\omega) w_t$, where $w_t = \kappa y_t + u_t$.*

The first assumption implies that equation (15), that defines equilibrium expectations, has a unique fixed point. In Appendix (D), I show that without this assumption, there are multiple equilibria for expectations.[5] For now, I assume that multiplicity away.[6]

Assumption 2 reduces the dimensionality of the problem. It is a natural assumption, since the best-response of a resetting firm can be written as

$$p_{i,t}^{*} - p_{t-1} = \overline{\mathbb{E}}_{i,t}\left[\pi_t\right] + \left(\frac{1-\omega}{\omega}\right) w_t.$$

I relax this assumption in Appendix B.

These two assumptions imply that individuals understand that the temporary equilibrium relation can be written as

$$\pi_t = \omega \overline{\mathbb{E}}_t\left[\pi_t\right] + \theta w_t,$$

where θ is a random coefficient. The fixed point of equation (15) takes the simple form

$$\overline{\mathbb{E}}_t\left[\pi_t\right] = \frac{\overline{\mu}_t}{1-\omega} w_t, \tag{17}$$

and the implementable set simplifies to[7]

$$\pi_t = \left[\omega \frac{\overline{\mu}_t}{1-\omega} + (1-\omega)\right] w_t, \tag{18}$$

$$\overline{\mu}_t = \overline{\mu}_{t-1} + \alpha_t\left[(1-\omega) - \overline{\mu}_{t-1}\right], \tag{19}$$

$$\sigma_t^2 = (1-\alpha_t)\,\sigma_{t-1}^2,$$

and

$$\alpha_t = \max\left\{0; 1 - \frac{\overline{\sigma}^2}{w_t^2 \sigma_{t-1}^2}\right\}.$$

The recursion for disagreement is

$$\Omega_t = (1-\alpha_t)^2\,\Omega_{t-1} + \alpha_t\,(1-\alpha_t)\,\sigma_{t-1}^2.$$

Let $w_{r,t-1}$ be a reference value of w brought from the past. It is easy to show that the recur-

[5] If ω, the degree of strategic complementarity, is high enough, then there might be both "no-learning" and "learning" equilibria.

[6] Assumption 1 is similar to assuming that individuals forecast the rational expectations equilibrium, rather than the equilibrium in an economy where agents do not have rational expectations. Assumption 1 has the advantage of preserving correctly specified beliefs.

[7] In one dimension, $\cos_{t-1}^2(\cdot,\cdot) = 1$ for nonzero scalars, so the cosine terms drop out.

sion for the variance can be written as

$$\sigma_t^2 = \begin{cases} \frac{\overline{\sigma}^2}{w_{r,t-1}^2}, & \text{if } w_t^2 \leq w_{r,t-1}^2 \\ \frac{\overline{\sigma}^2}{w_t^2}, & \text{if } w_t^2 > w_{r,t-1}^2 \end{cases}.$$

The Kalman gain parameter is

$$\alpha_t = \max\left\{0; 1 - \frac{w_{r,t-1}^2}{w_t^2}\right\}, \tag{20}$$

and the reference is updated according to

$$w_{r,t}^2 = \begin{cases} w_{r,t-1}^2, & w_t^2 \leq w_{r,t-1}^2 \\ w_t^2, & w_t^2 > w_{r,t-1}^2 \end{cases}.$$

In the scalar case, individuals trigger System 2 whenever marginal costs increase above a System 1 threshold. The threshold is updated whenever System 2 is triggered.

I now study the Phillips curve in the case where $\overline{\mu}_{-1} = 0$.

Lemma 4. *Suppose that $\overline{\mu}_{-1} = 0$, and let $\mathcal{PC}_t(w)$ denote the Phillips curve. Then*

1. *For any t, $\mathcal{E}_t'(w) \in [1-\omega, 1)$,*
2. *For $w^2 \leq w_{r,t-1}^2$, $\mathcal{PC}_t'(w) = \vartheta_t$, where ϑ_t is independent of w,*
3. *For $w^2 > w_{r,t-1}^2$, $\mathcal{PC}_t(w)/w = \vartheta_t\left(w^2\right)$,*
4. *As $w^2 \to \infty$, $\vartheta_t\left(w^2\right) \to 1$.*

Proof. The proof follows directly from equations (18)-(20). □

When $\overline{\mu}_{-1} = 0$, the slope of the Phillips curve is lower than under rationality at any t. For $w_t^2 \leq w_{r,t-1}^2$, the Phillips curve is linear in w_t. For $w_t > w_{r,t-1}$, the Phillips curve is nonlinear, and the slope converges to 1 as $w_t^2 \to \infty$. If $w_t > w_{r,t-1}$, individuals trigger System 2 and widen the System 1 region at $t+1$. These properties are illustrated in Figure 1.

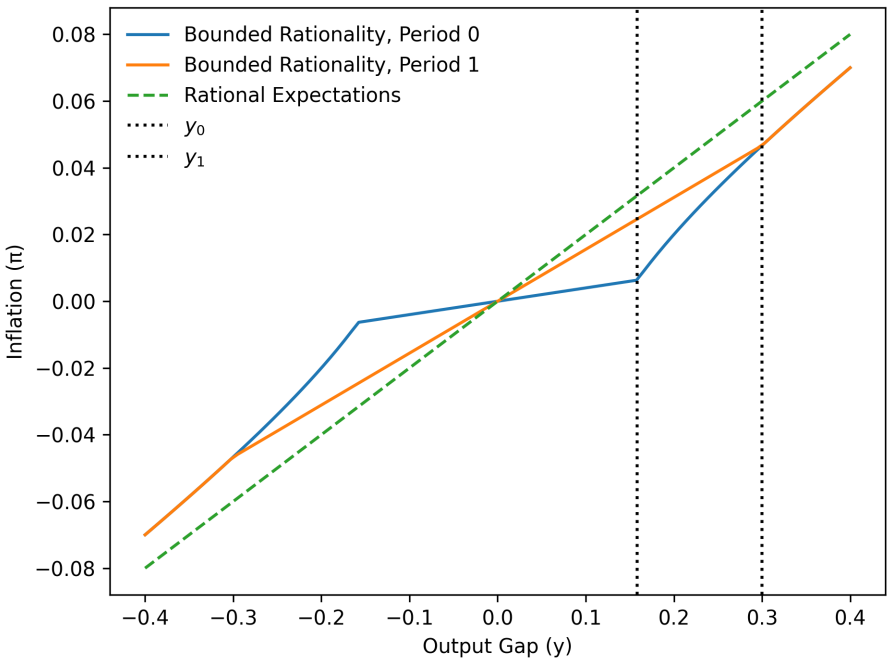


Figure 1: Phillips Curve

4 Optimal Policy

To write the problem of the central bank recursively, let $\overline{\mu}$ be the prior mean, Ω the beginning-of-period belief dispersion variable, and w_r the reference brought from the past. Let also

$$\mathcal{U}_{\text{aloc}}(\pi, y) = -\frac{1}{2}\left(\frac{1-\omega}{\omega}\right)\left(\kappa y^2 + \gamma\pi^2\right),$$

and

$$\mathcal{U}_{\text{info}}(z, z_r, \Omega) = -\left[z\Omega' - \overline{\sigma}^2 \ln(1-\alpha)\right],$$

where

$$\Omega' = (1-\alpha)^2\Omega + \alpha(1-\alpha)\frac{\overline{\sigma}^2}{z_r}, \tag{21}$$

$$\alpha = \max\left\{0, 1 - \frac{z_r}{z}\right\},$$

and $z \equiv w^2$, $z_r \equiv w_r^2$. Then the period utility of the central bank is

$$\mathcal{U} = \mathcal{U}_{\text{aloc}}(\pi, y) + \frac{\gamma\omega}{2}\mathcal{U}_{\text{info}}(z, z_r, \Omega).$$

The following Lemma establishes a useful property of belief dispersion in the scalar case.

Lemma 5. *Let $\delta_t = z_{r,t}\Omega_t$. If $\delta_{-1} < \overline{\sigma}^2$, δ_t is non-decreasing and bounded above by $\overline{\sigma}^2$.*

Proof. First suppose that $z_t \leq z_{r,t}$. Then $z_{r,t} = z_{r,t-1}$, $\Omega_t = \Omega_{t-1}$, and $\delta_t = \delta_{t-1}$. Now suppose that $z_t > z_{r,t-1}$. Then $z_{r,t} = z_t$, and $\alpha_t = 1 - (z_{r,t-1}/z_{r,t})$. From equation (21),

$$\begin{aligned}\delta_t &= (1-\alpha_t)^2 z_{r,t}\Omega_{t-1} + \alpha_t (1-\alpha_t) z_{r,t}\frac{\overline{\sigma}^2}{z_{r,t-1}} \\ &= \left(\frac{z_{r,t-1}}{z_{r,t}}\right)^2 z_{r,t}\Omega_{t-1} + \alpha_t\overline{\sigma}^2 \\ &= (1-\alpha_t)\,\delta_{t-1} + \alpha_t\overline{\sigma}^2.\end{aligned}$$

Therefore

$$\delta_t = (1-\alpha_t)\,\delta_{t-1} + \alpha_t\overline{\sigma}^2.$$

A simple induction argument shows the result. □

This Lemma implies that whenever System 2 is triggered, belief dispersion increases. It follows that

$$z\Omega' = \begin{cases} \frac{z}{z_r}\delta, & \text{if } z \leq z_r \\ (1-\alpha)\,\delta + \alpha\overline{\sigma}^2, & \text{if } z > z_r \end{cases},$$

and, with some abuse of notation, $\mathcal{U}_{\text{info}}(z, z_r, \Omega) = \mathcal{U}_{\text{info}}(z, z_r, \delta)$.

Optimal policy under rationality Under rationality, $\mathbb{E}_{i,t}\pi_t = \overline{\mathbb{E}}_t\pi_t$ and equation (1) becomes

$$\pi_t = \kappa y_t + u_t = w_t.$$

The problem of the central bank is

$$V(u) = \max_{(y,\pi)} \left\{ -\frac{\lambda}{2}y^2 - \frac{\gamma}{2}\left(\frac{1-\omega}{\omega}\right)\pi^2 + \beta\mathbb{E}\left[V(u') \mid u\right] \text{ s.t. } \pi = \kappa y + u \right\}.$$

The first-order conditions imply that

$$y_t^* = -\frac{\gamma}{1+\gamma\kappa}u_t, \quad \pi_t^* = \frac{1}{1+\gamma\kappa}u_t.$$

The solution is standard. In response to a demand shock, the central bank faces no output-inflation trade-off and implements the first-best. In response to a positive cost-push shock, the central bank tolerates some inflation in order to partially stabilize output.

To isolate the role of cognitive uncertainty, I first study optimal policy under the assumption that $\overline{\mu}_{-1} = 1 - \omega$. Under this assumption, the central bank faces the same output-inflation trade-off as under rationality.

The following theorem partially characterizes the solution of the static problem of the central bank.

Theorem 1. *Suppose that $\beta = 0$, and let π^{BR} be the optimal inflation level under bounded rationality. If $\overline{\mu}_{-1} = 1 - \omega$, and $\delta_{-1} < \overline{\sigma}^2$, then $\left|\pi^{BR}\right| \leq |\pi^*|$, with strict inequality if $\delta > 0$.*

Proof. With abuse of notation,

$$\mathcal{U}_{\text{aloc}}(\pi, y) = \mathcal{U}_{\text{aloc}}(\pi, u) = -\frac{\lambda}{2\kappa^2}\left[(\pi - u)^2 + \gamma\kappa\pi^2\right].$$

Then

$$\frac{\partial \mathcal{U}_{\text{aloc}}(\pi, u)}{\partial \pi} = -\frac{\lambda}{\kappa^2}(1 + \gamma\kappa)(\pi - \pi^*).$$

Now consider

$$\mathcal{U}_{\text{info}}(z, z_r, \delta) = \begin{cases} -\frac{z}{z_r}\delta & \text{if } z \leq z_r \\ -\left\{[1 - \alpha(z)]\,\delta + \alpha(z)\,\overline{\sigma}^2 - \overline{\sigma}^2 \ln[1 - \alpha(z)]\right\}, & \text{if } z > z_r \end{cases}.$$

First note that

$$\alpha(z) = 1 - \frac{z_r}{z} \Longrightarrow \alpha'(z) = \frac{z_r}{z^2} > 0.$$

If $z \leq z_r$,

$$\frac{\partial \mathcal{U}_{\text{info}}(z, z_r, \delta)}{\partial z} = -\frac{\delta}{z_r} \leq 0.$$

If $z > z_r$,

$$\frac{\partial \mathcal{U}_{\text{info}}(z, z_r, \delta)}{\partial z} = -\alpha'(z)\left\{\left(\overline{\sigma}^2 - \delta\right) + \frac{z}{z_r}\overline{\sigma}^2\right\} < 0.$$

Since $w = \pi$ when $\overline{\mu}_{-1} = 1 - \omega$, it follows that any interior solution,

$$\begin{aligned} &\frac{\partial \mathcal{U}_{\text{aloc}}(\pi, u)}{\partial \pi} + \frac{\gamma\omega}{2}\frac{\partial \mathcal{U}_{\text{info}}(z, z_r, \delta)}{\partial z}\frac{dz}{d\pi} = 0 \\ \Longleftrightarrow\ &\pi = \frac{\pi^*}{1 - \frac{\gamma\omega\kappa^2}{\lambda(1+\gamma\kappa)}\frac{\partial \mathcal{U}_{\text{info}}(z, z_r, \delta)}{\partial z}}. \end{aligned}$$

Since $\partial \mathcal{U}_{\text{info}}/\partial z \leq 0$, it follows that $|\pi| \leq |\pi^*|$, with strict inequality if $\delta > 0$. □

I now solve the dynamic problem of the central bank.

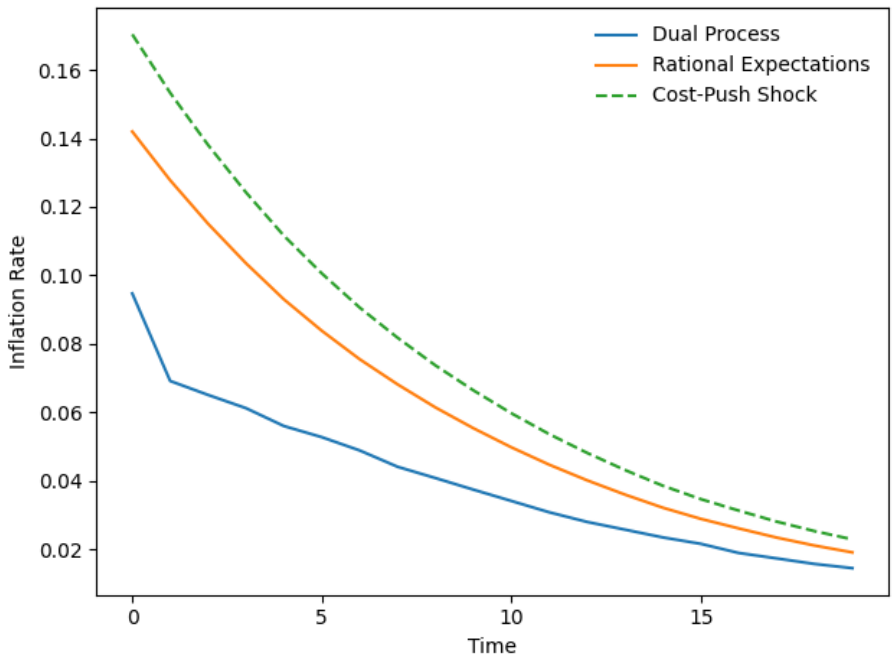


Figure 2: Impulse response function of inflation to a cost-push shock.

Conditional on System 1, the dynamic policy coincides with the static policy.

System 2 activation has a dynamic trade-off. On the one hand, triggering System 2 increases belief dispersion and makes inflation more costly in the future. This effect is permanent: once System 2 is triggered, belief dispersion never vanishes. On the other hand, it widens the System 1 region and mitigates the need to trigger System 2 in the future.

Figure 2 plots the (generalized) impulse response function of inflation to a cost-push shock, defined as

$$\text{IRF}_t = \mathbb{E}_0 \left[Y_t \mid \nu_0 = \overline{\nu} \right] - \mathbb{E}_0 \left[Y_t \mid \nu_0 = 0 \right],$$

where Y_t is the variable of interest. I compute these functions through Monte Carlo simulation.

In response to a cost-push shock, the central bank triggers System 2 initially. Belief dispersion increases and the (System 1) optimal policy features greater price stability than under rationality forever after. Figure 3 plots the share of sequences in which System 2 is activated.

Initial prior mean of zero When $\overline{\mu}_{-1} = 0$, the analogous of Theorem 1 does not hold. The reason is that when $\overline{\mu}_{-1} = 0$, the Phillips curve becomes flatter and more inflation is required to generate the same deviation in the output gap. However, the following Lemma suggests that if $\kappa\gamma$ is low, as in standard calibrations, the optimal policy still features more price stability than under rationality.

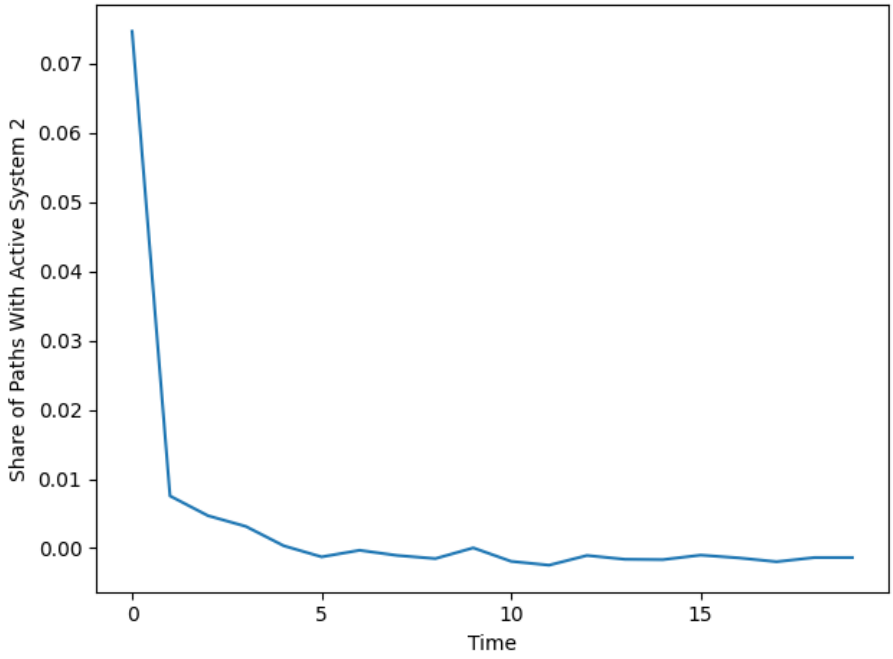


Figure 3: Impulse response function of System 2 activation.

Theorem 2. *Suppose that* $\beta = 0$*, and let*

$$\overline{\vartheta} \equiv \omega \frac{\overline{\mu}}{1-\omega} + (1-\omega)\,.$$

If $\overline{\mu} \leq 1-\omega$ *and* $\kappa\gamma\overline{\vartheta} < 1$*, the interior solution under System 1 features greater price stability than under rationality.*

Proof. In Appendix A. □

5 Conclusion

In this paper, I developed a model in which firms have a limited understanding of general equilibrium and use dual-process reasoning to form inflation expectations. The model generates multiple equilibria, and is consistent with a nonlinear, as well as history-dependent Phillips curve. The optimal policy involves greater price stability than under rationality for shocks that are not extreme.

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A Proofs

A.1 Lemma 2

Let

$$\mathcal{D}_{e,t}(\mathbf{x},\tilde{\mathbf{x}}) \equiv \int_0^1 \left[m_{i,t}(\mathbf{x}) - \overline{m}_t(\mathbf{x})\right]\left[m_{i,t}(\tilde{\mathbf{x}}) - \overline{m}_t(\tilde{\mathbf{x}})\right] di,$$

and

$$\Delta_{e,t}(\mathbf{x}) \equiv \mathcal{D}_{e,t}(\mathbf{x},\mathbf{x}).$$

This function is the kernel of posterior means. Then

$$\begin{aligned}
\mathcal{D}_{e,t}(\mathbf{x},\tilde{\mathbf{x}}) &= \mathrm{Cov}\left[m_{i,t}(\mathbf{x}), m_{i,t}(\tilde{\mathbf{x}})\right] \\
&= \mathrm{Cov}\Big\{m_{i,t-1}(\mathbf{x}) + \alpha_t(\mathbf{x})\left[\mathbf{x}_t^\top\boldsymbol{\theta}^* + \sigma_{\epsilon,t}\epsilon_{i,t} - m_{i,t-1}(\mathbf{x}_t)\right], \\
&\quad m_{i,t-1}(\tilde{\mathbf{x}}) + \alpha_t(\tilde{\mathbf{x}})\left[\mathbf{x}_t^\top\boldsymbol{\theta}^* + \sigma_{\epsilon,t}\epsilon_{i,t} - m_{i,t-1}(\mathbf{x}_t)\right]\Big\} \\
&= \mathrm{Cov}\left[m_{i,t-1}(\mathbf{x}), m_{i,t-1}(\tilde{\mathbf{x}})\right] - \alpha_t(\tilde{\mathbf{x}})\,\mathrm{Cov}\left[m_{i,t-1}(\mathbf{x}), m_{i,t-1}(\mathbf{x}_t)\right] \\
&\quad + \alpha_t(\mathbf{x})\,\alpha_t(\tilde{\mathbf{x}})\,\sigma^2_{\epsilon,t} - \alpha_t(\mathbf{x})\,\mathrm{Cov}\left[m_{i,t-1}(\mathbf{x}_t), m_{i,t-1}(\tilde{\mathbf{x}})\right] \\
&\quad + \alpha_t(\mathbf{x})\,\alpha_t(\tilde{\mathbf{x}})\,\mathrm{Cov}\left[m_{i,t-1}(\mathbf{x}_t), m_{i,t-1}(\mathbf{x}_t)\right] \\
&= \mathcal{D}_{e,t-1}(\mathbf{x},\tilde{\mathbf{x}}) - \left[\alpha_t(\tilde{\mathbf{x}})\,\mathcal{D}_{e,t-1}(\mathbf{x},\mathbf{x}_t) + \alpha_t(\mathbf{x})\,\mathcal{D}_{e,t-1}(\tilde{\mathbf{x}},\mathbf{x}_t)\right] + \alpha_t(\mathbf{x})\,\alpha_t(\tilde{\mathbf{x}})\left[\Delta_{e,t-1}(\mathbf{x}_t) + \sigma^2_{\epsilon,t}\right].
\end{aligned}$$

Evaluated at $(\mathbf{x}_t, \mathbf{x}_t)$

$$\Delta_{e,t}(\mathbf{x}_t) = \left[1 - \alpha_t(\mathbf{x}_t)\right]^2 \Delta_{e,t-1}(\mathbf{x}_t) + \alpha_t(\mathbf{x}_t)\left[1 - \alpha_t(\mathbf{x}_t)\right]\sigma^2_{t-1}(\mathbf{x}_t)$$

But in fact we can write

$$\begin{aligned}
\Delta_{e,t}(\mathbf{x}) &= \int_0^1 \left[\mathbf{x}^\top\boldsymbol{\mu}_{i,t} - \mathbf{x}^\top\overline{\boldsymbol{\mu}}_t\right]^2 di \\
&= \mathbf{x}^\top\boldsymbol{\Omega}_t\mathbf{x},
\end{aligned}$$

where

$$\boldsymbol{\Omega}_t \equiv \int_0^1 \left(\boldsymbol{\mu}_{i,t} - \overline{\boldsymbol{\mu}}_t\right)\left(\boldsymbol{\mu}_{i,t} - \overline{\boldsymbol{\mu}}_t\right)^\top di.$$

Now note that

$$\boldsymbol{\mu}_{i,t} - \overline{\boldsymbol{\mu}}_t = \left(\mathbf{I} - \mathbf{K}_t\mathbf{x}_t^\top\right)\left(\boldsymbol{\mu}_{i,t-1} - \overline{\boldsymbol{\mu}}_{t-1}\right) + \sigma_{\epsilon,t}\epsilon_{i,t}\mathbf{K}_t,$$

where

$$\mathbf{K}_t \equiv \frac{\alpha_t(\mathbf{x}_t)}{\sigma^2_{t-1}(\mathbf{x}_t)}\boldsymbol{\Sigma}_{t-1}\mathbf{x}_t.$$

It follows that

$$\mathbf{\Omega}_t = \left(\mathbf{I} - \mathbf{K}_t \mathbf{x}_t^\top\right) \mathbf{\Omega}_{t-1} \left(\mathbf{I} - \mathbf{K}_t \mathbf{x}_t^\top\right)^\top + \sigma_{\epsilon,t}^2 \mathbf{K}_t \mathbf{K}_t^\top.$$

A.2 Theorem 2

In that case,

$$\mathcal{U}_{\text{info}}(z, z_r, \delta) = \begin{cases} -\frac{z}{z_r}\delta & \text{if } z \leq z_r \\ -\left\{[1-\alpha(z)]\,\delta + \alpha(z)\,\overline{\sigma}^2 - \overline{\sigma}^2 \ln[1-\alpha(z)]\right\}, & \text{if } z > z_r \end{cases}$$

and

$$\overline{\mu}'(z) = \overline{\mu} + \alpha(z)\,[(1-\omega) - \overline{\mu}].$$

and

$$\pi = \vartheta(z)\, w,$$

where

$$\vartheta(z) = \left[\omega \frac{\overline{\mu}'(z)}{1-\omega} + (1-\omega)\right]$$

Then

$$\mathcal{U} = -\frac{1}{2}\left(\frac{1-\omega}{\omega}\right)\left[\frac{1}{\kappa}(w-u)^2 + \gamma\vartheta^2(z)\, w^2\right] + \frac{\gamma\omega}{2}\mathcal{U}_{\text{info}}(z, z_r, \delta),$$

where

$$\mathcal{U}_{\text{info}}(z, z_r, \delta) = \begin{cases} -\frac{z}{z_r}\delta & \text{if } z \leq z_r \\ -\left\{[1-\alpha(z)]\,\delta + \alpha(z)\,\overline{\sigma}^2 - \overline{\sigma}^2 \ln[1-\alpha(z)]\right\}, & \text{if } z > z_r \end{cases}.$$

Now consider system 1. Then

$$\vartheta(z) = \left[\omega \frac{\overline{\mu}}{1-\omega} + (1-\omega)\right] \equiv \overline{\vartheta} \in [1-\omega, 1]$$

and $\mathcal{U}_{\text{info}}(z, z_r, \delta)$. It follows that

$$\begin{aligned} &-\left(\frac{1-\omega}{\omega}\right)\left[\frac{1}{\kappa}(w-u) + \gamma\overline{\vartheta}^2 w\right] - \gamma\omega\frac{\delta}{z_r}w = 0 \\ \iff w &= \frac{1}{1 + \kappa\gamma\overline{\vartheta}^2 + \kappa\gamma\omega\left(\frac{\omega}{1-\omega}\right)\frac{\delta}{z_r}}u \end{aligned}$$

and therefore

$$\pi = \frac{\overline{\vartheta}}{1 + \kappa\gamma\overline{\vartheta}^2 + \kappa\gamma\omega\left(\frac{\omega}{1-\omega}\right)\frac{\delta}{z_r}}u.$$

There is greater price stability than under rationality if

$$0 < \left(1 - \kappa\gamma\overline{\vartheta}\right)\left(1 - \overline{\vartheta}\right) + \kappa\gamma\omega\left(\frac{\omega}{1-\omega}\right)\frac{\delta}{z_r}.$$

B Relaxing Assumption 2

In this section, I relax assumption 2 by assuming instead that, at the beginning of any t, individuals only recall the latest drawn signal.

Consider any $\overline{\boldsymbol{\mu}}_{-1}, \boldsymbol{\Sigma}_{-1}$. If individuals rely on System 1,

$$\overline{\boldsymbol{\mu}}_0 = \overline{\boldsymbol{\mu}}_{-1}, \ \ \boldsymbol{\Sigma}_0 = \boldsymbol{\Sigma}_{-1}.$$

If individuals draw a signal, they update the moments to

$$\overline{\boldsymbol{\mu}}_0 = \overline{\boldsymbol{\mu}}_{-1} + \mathbf{K}_0\mathbf{x}_0^\top\left(\boldsymbol{\theta}^* - \overline{\boldsymbol{\mu}}_{-1}\right),$$

$$\boldsymbol{\Sigma}_0 = \left(\mathbf{I} - \mathbf{K}_0\mathbf{x}_0^\top\right)\boldsymbol{\Sigma}_{-1},$$

and disagreement is

$$\Delta_{e,0} = \left[1 - \alpha_0\left(\mathbf{x}_0\right)\right]\alpha_0\left(\mathbf{x}_0\right)\mathbf{x}_0^\top\boldsymbol{\Sigma}_{-1}\mathbf{x}_0.$$

Now consider the following period. If there is no learning, the central bank moves to the same state. If there is learning, there is an intra-period updating, but in the following period individuals forget.

It follows that the problem is characterized by two value functions. In the initial period, the state is u. In the following periods, the state is $(u, \mathbf{x}_r)$, where $\mathbf{x}_r$ is the reference state. The value function is

$$V_0\left(u\right) = \max\left\{V_{0,NL}\left(u\right), V_{0,L}\left(u\right)\right\},$$

where

$$V_{0,\mathrm{NL}}\left(u\right) = \max_{(\pi,y)\in\Gamma_{0,\mathrm{NL}}}\left\{\mathcal{U}_{\mathrm{aloc}} + \beta\mathbb{E}\left[V_0\left(u'\right) \mid u\right]\right\},$$

$$V_{0,\mathrm{L}}\left(u\right) = \max_{(\pi,y)\in\Gamma_{0,\mathrm{L}}}\left\{\mathcal{U}_{\mathrm{aloc}} + \frac{\gamma\omega}{2}\mathcal{U}_{\mathrm{info}} + \beta\mathbb{E}\left[V_1\left(u', y_r, u_r\right)\right]\right\}.$$

In subsequent periods, the value function is

$$V_1(u, y_r, u_r) = \max\left\{V_{1,\text{NL}}(u, y_r, u_r), V_{1,\text{L}}(u, y_r, u_r)\right\}$$

where

$$V_{1,\text{NL}}(u, y_r, u_r) = \max_{(\pi,y)\in\Gamma_{1,\text{NL}}} \left\{\mathcal{U}_{\text{aloc}} + \frac{\gamma\omega}{2}\mathcal{U}_{\text{info}} + \beta\mathbb{E}\left[V_1\left(u', y_r, u_r\right)\right]\right\},$$

$$V_{1,\text{L}}(u, y_r, u_r) = \max_{(\pi,y)\in\Gamma_{1,\text{L}}} \left\{\mathcal{U}_{\text{aloc}} + \frac{\gamma\omega}{2}\mathcal{U}_{\text{info}} + \beta\mathbb{E}\left[V_1\left(u', y_r', u_r'\right)\right]\right\}.$$

C Behavioral Households

In the previous sections, I assumed that changes in the policy instrument did not have an effect on the expectation formation process. Implicitly, I assumed that households were fully rational.

In this section I study a static model in which households are behavioral. There is a continuum of measure one of households indexed by h. To simplify, I assume that firms are rational. Let i_0 be the nominal interest rate gap, σ the intertemporal elasticity of substitution, and $\tau_0 \equiv -(1/\sigma)\, i_0$. The temporary equilibrium conditions consist of a behavioral IS curve:

$$y_0 = (1-\beta)\,\overline{\mathbb{E}}_0[y_0] + \beta\tau_0, \quad \beta \in (0,1).$$

and a rational expectations Phillips curve:

$$\pi_0 = \kappa y_0 + u_0.$$

Utility of household h is

$$v_h = -\frac{\chi}{2}\left(\mathbb{E}_{h,0}[y_0] - y_0\right)^2 - \mathcal{I}_h,$$

where $\mathcal{I}_h$ is again a cognitive cost, and the utility of the central bank is

$$U_0 = -\frac{\lambda}{2}\left(y_0^2 + \tilde{\gamma}\pi_0^2\right) - \chi\left[\frac{1}{2}\text{Var}\left[\mathbb{E}_{h,0}[y_0]\right] + \frac{\overline{\sigma}^2}{2}\int_0^1 \mathcal{I}_h dh.\right]$$

These equations can be microfounded by a New Keynesian structure in which households have non-rational expectations on aggregate income.

Analogously to the baseline model, I assume that households understand that aggregate income satisfies

$$y_0 = (1-\beta)\,\overline{\mathbb{E}}_0[y_0] + \theta\tau_0,$$

where θ is random. In this section, I assume that $\overline{\mu}_{-1} = \beta$ throughout. It follows that $y_0 = \tau_0$, and, with a cognitive cost that is proportional to Shannon mutual information,

$$U_0 = \mathcal{W}_{\text{aloc}}(\tau_0) + \frac{\chi}{2}\mathcal{W}_{\text{info}}(\tau_0),$$

where

$$\mathcal{W}_{\text{aloc}}(\tau_0) \equiv -\frac{\lambda}{2}\left[\tau_0^2 + \tilde{\gamma}(\kappa\tau_0 + u_0)^2\right]$$

$$\mathcal{W}_{\text{info}}\left(\tau_0^2\right) = \begin{cases} 0, & \text{if } \tau_0^2 \leq \overline{\tau}_0^2 \\ -\overline{\sigma}^2\left\{\alpha\left(\tau_0^2\right) - \ln\left[1-\alpha\left(\tau_0^2\right)\right]\right\}, & \text{if } \tau_0^2 > \overline{\tau}_0^2 \end{cases},$$

and

$$\alpha(\tau_0) = 1 - \frac{\overline{\tau}_0^2}{\tau_0^2}.$$

where $\overline{\tau}_0$ is a reference value of the instrument. Here I have assumed that initial belief dispersion is equal to zero.

Conditional on System 1, the optimal policy for τ_0 coincides with rational expectations,

$$\tau_0 = -\frac{\tilde{\gamma}\kappa}{1+\tilde{\gamma}\kappa^2}u_0 = \tau_0^*.$$

This property is due to the fact that belief dispersion is initially zero.

Conditional on System 2, the first-order condition for an interior solution implies that

$$\tau_0 = \frac{1}{1 - \frac{\chi}{\lambda(1+\tilde{\gamma}\kappa^2)}\mathcal{W}'_{\text{info}}\left(\tau_0^2\right)}\tau_0^*,$$

where $\mathcal{W}'_{\text{info}}\left(\tau_0^2\right) < 0$. At an interior optimum, $|\tau_0| < |\tau_0^*|$. Note, moreover, that

$$\lim_{\tau_0^2 \to \overline{\tau}_0^{2+}} \mathcal{W}'_{\text{info}}\left(\tau_0^2\right) = -\frac{2\overline{\sigma}^2}{\overline{\tau}_0^2} \neq 0$$

while $\lim_{\tau_0^2 \to \overline{\tau}_0^{2-}} \mathcal{W}'_{\text{info}}\left(\tau_0^2\right) = 0$. These facts imply the following Lemma.

Lemma 6. *Under bounded rationality, the optimal policy for the instrument, τ_0^{BR}, satisfies*

$$\tau_0^{BR} = \begin{cases} \tau_0^*, & \text{if } u_0 \leq u_\ell \\ sign(u)\sqrt{\overline{\tau}_0^2}, & \text{if } u_\ell < u_0 \leq u_h \\ \tilde{\vartheta}\left[\left(\tau_0^{BR}\right)^2\right]\tau_0^*, & \text{if } u_0 > u_h \end{cases},$$

for some bounds u_ℓ, u_h, and where $\tilde{\vartheta}\left[\left(\tau_0^{BR}\right)^2\right] \in (0,1)$.

When the cost-push shock is small, $u_0 \leq u_\ell$ the central bank is able to implement the rational policy without triggering System 2. Since the implementability condition $\pi_0 = \kappa\tau_0 + u_0$ is the same as under rationality, it is optimal to do so. When the cost-push shock has medium size, $u_\ell < u_0 \leq u_h$, the central bank sets the interest rate at the boundary of the System 1 region. The optimal policy involves a lumpy interest rate. Finally, when the cost-push shock is large enough, the central bank changes the interest rate, but to a level that is smaller in absolute terms than under rationality.

Since the optimal interest rate gap changes by less than under rationality, so does the output gap. This fact implies that when households are behavioral, it is optimal for the central bank to tolerate higher inflation than under rationality for medium to large cost-push shocks.

D Multiplicity

Suppose that people believe

$$\pi_t = \theta_\pi \pi_{e,t} + (1-\omega)\, w_t.$$

In that case, we have

$$\alpha_t = \begin{cases} 0, & \text{if } \pi_{e,t}^2 \leq \pi_{r,e,t-1}^2 \\ 1 - \frac{\pi_{r,e,t-1}^2}{\pi_{e,t}^2}, & \text{if } \pi_{e,t} > \pi_{r,e,t-1}^2 \end{cases}.$$

Then

$$\overline{\mu}_t = \overline{\mu}_{t-1} + \alpha_t \left(\omega - \overline{\mu}_{t-1}\right).$$

First suppose that $\pi_{e,t}^2 \leq \pi_{r,e,t-1}^2$. Then

$$\begin{aligned} \pi_{e,t} &= \overline{\mu}_{t-1}\pi_{e,t} + (1-\omega)\, w_t \\ \iff \pi_{e,t} &= \left(\frac{1-\omega}{1-\overline{\mu}_{t-1}}\right) w_t. \end{aligned}$$

Then $\overline{\mu}_{t-1} \leq \omega \iff 1-\overline{\mu}_{t-1} \geq 1-\omega$, so $\pi_{e,t}/w_t < 1$. So this branch has a solution as long as

$$\left[\left(\frac{1-\omega}{1-\overline{\mu}_{t-1}}\right) w_t\right]^2 \leq \pi_{r,e,t-1}^2.$$

Now consider the other branch. Then

$$\begin{aligned}
\pi_{e,t} &= \left[\overline{\mu}_{t-1} + \left(1 - \frac{\pi^2_{r,e,t-1}}{\pi^2_{e,t}}\right)\left(\omega - \overline{\mu}_{t-1}\right)\right]\pi_{e,t} + (1-\omega)\, w_t \\
\iff &\left\{1 - \overline{\mu}_{t-1} - \left(1 - \frac{\pi^2_{r,e,t-1}}{\pi^2_{e,t}}\right)\left(\omega - \overline{\mu}_{t-1}\right)\right\}\pi_{e,t} = (1-\omega)\, w_t
\end{aligned}$$

First let's see if this is always strictly positive.

$$\begin{aligned}
&1 - \overline{\mu}_{t-1} - \left(1 - \frac{\pi^2_{r,e,t-1}}{\pi^2_{e,t}}\right)\left(\omega - \overline{\mu}_{t-1}\right) > 0 \\
\iff &1 - \overline{\mu}_{t-1} > \left(1 - \frac{\pi^2_{r,e,t-1}}{\pi^2_{e,t}}\right)\left(\omega - \overline{\mu}_{t-1}\right)
\end{aligned}$$

Note that $0 \leq \overline{\mu}_{t-1} < \omega$. Therefore $1 - \overline{\mu}_{t-1} > \omega - \overline{\mu}_{t-1} > 0$ because $\omega < 1$. So this condition always holds with strictly inequality. Now

$$\begin{aligned}
g\left(\pi_{e,t}\right) &= \left\{1 - \overline{\mu}_{t-1} - \left(1 - \frac{\pi^2_{r,e,t-1}}{\pi^2_{e,t}}\right)\left(\omega - \overline{\mu}_{t-1}\right)\right\}\pi_{e,t} \\
&= \left\{1 - \overline{\mu}_{t-1} - \left(\omega - \overline{\mu}_{t-1}\right) + \frac{\pi^2_{r,e,t-1}}{\pi^2_{e,t}}\left(\omega - \overline{\mu}_{t-1}\right)\right\}\pi_{e,t} \\
&= (1-\omega)\,\pi_{e,t} + \frac{\pi^2_{r,e,t-1}}{\pi_{e,t}}\left(\omega - \overline{\mu}_{t-1}\right).
\end{aligned}$$

Now

$$g'\left(\pi_{e,t}\right) = 1 - \omega - \frac{\pi^2_{r,e,t-1}}{\pi^2_{e,t}}\left(\omega - \overline{\mu}_{t-1}\right).$$

Note that

$$\begin{aligned}
&g'\left(\pi_{e,t}\right) > 0 \\
\iff &\pi^2_{e,t} > \left(\frac{\omega - \overline{\mu}_{t-1}}{1-\omega}\right)\pi^2_{r,e,t-1}.
\end{aligned}$$

This condition always holds in the relevant region, $\pi_{e,t}^2 > \pi_{r,e,t-1}^2$, if

$$\begin{aligned} &\frac{\omega - \overline{\mu}_{t-1}}{1-\omega} < 1 \\ \iff &\omega - \overline{\mu}_{t-1} < 1 - \omega \\ \iff &2\omega < 1 + \overline{\mu}_{t-1}. \end{aligned}$$

Therefore, if $\overline{\mu}_{t-1} \geq 2\omega - 1$, there are no multiple equilibria. This condition always holds when $\omega \leq 0.5$. If $\overline{\mu}_{t-1} < 2\omega - 1$, then $g'(\pi_{e,t})$ has the following properties:

$$g'(\pi_{e,t}) \begin{cases} > 0, & \text{if } \pi_{e,t} < -\sqrt{\left(\frac{\omega-\overline{\mu}_{t-1}}{1-\omega}\right)\pi_{r,e,t-1}^2} \\ = 0, & \text{if } \pi_{e,t} = -\sqrt{\left(\frac{\omega-\overline{\mu}_{t-1}}{1-\omega}\right)\pi_{r,e,t-1}^2} \\ < 0 & \text{if } -\sqrt{\left(\frac{\omega-\overline{\mu}_{t-1}}{1-\omega}\right)\pi_{r,e,t-1}^2} < \pi_{e,t} < -\sqrt{\pi_{r,e,t-1}^2} \\ > 0, & \text{if } -\sqrt{\pi_{r,e,t-1}^2} \leq \pi_{e,t} \leq \sqrt{\pi_{r,e,t-1}^2} \\ < 0, & \text{if } \sqrt{\pi_{r,e,t-1}^2} < \pi_{e,t} < \sqrt{\left(\frac{\omega-\overline{\mu}_{t-1}}{1-\omega}\right)\pi_{r,e,t-1}^2} \\ = 0, & \text{if } \pi_{e,t} = \sqrt{\left(\frac{\omega-\overline{\mu}_{t-1}}{1-\omega}\right)\pi_{r,e,t-1}^2} \\ > 0, & \text{if } \pi_{e,t} > \sqrt{\left(\frac{\omega-\overline{\mu}_{t-1}}{1-\omega}\right)\pi_{r,e,t-1}^2} \end{cases}.$$

Note that the extrema are $\pm\ell\left(\overline{\mu}_{t-1}, \pi_{r,e,t-1}^2\right)$, where:

$$\ell\left(\overline{\mu}_{t-1}, \pi_{r,e,t-1}^2\right) \equiv (1-\omega)\sqrt{\left(\frac{\omega - \overline{\mu}_{t-1}}{1-\omega}\right)\pi_{r,e,t-1}^2} + \frac{\pi_{r,e,t-1}^2}{\sqrt{\left(\frac{\omega-\overline{\mu}_{t-1}}{1-\omega}\right)\pi_{r,e,t-1}^2}}\left(\omega - \overline{\mu}_{t-1}\right).$$

Let

$$\overline{w}\left(\overline{\mu}_{t-1}, \pi_{r,e,t-1}^2\right) \equiv \frac{1-\overline{\mu}_{t-1}}{1-\omega}\sqrt{\pi_{r,e,t-1}^2}.$$

If

$$w_t \in \left(-\overline{w}\left(\overline{\mu}_{t-1}, \pi_{r,e,t-1}^2\right), -\frac{\ell\left(\overline{\mu}_{t-1}, \pi_{r,e,t-1}^2\right)}{1-\omega}\right) \cup \left(\frac{\ell\left(\overline{\mu}_{t-1}, \pi_{r,e,t-1}^2\right)}{1-\omega}, \overline{w}\left(\overline{\mu}_{t-1}, \pi_{r,e,t-1}^2\right), \right),$$

there are three equilibria for $\pi_{e,t}$.

These equilibria are Pareto-ranked. Suppose w_t is such that there are multiple equilibria.

All such equilibria have the same y_t, and therefore the same y_t^2. However, they have different levels of $\pi_{e,t}^2$. Since utility is strictly decreasing in $\pi_{e,t}^2$, and π_t is increasing in $\pi_{e,t}$, it follows that the equilibrium with smaller $\pi_{e,t}^2$ Pareto-dominates the remaining equilibria (if the central bank is myopic).

E Microfoundations for the Baseline Model

Consider a myopic version of Calvo pricing where, in log-deviations, the optimal reset price, p_t^*, is

$$p_t^* = p_t + \tilde{mc}_t, \tag{22}$$

$\tilde{mc}_t$ is the real marginal cost, and p_t is the price level. Each resetting firm sets its price at the beginning of the period, before observing the prices set by its competitors. Under this assumption, the price set by a resetting firm can be written as

$$p_{i,t} = e_{i,t} + p_t^*, \tag{23}$$

where $e_{i,t} \equiv \mathbb{E}_{i,t}[p_t] - p_t = \mathbb{E}_{i,t}[\pi_t] - \pi_t$ is the forecast error of inflation. In each period, a fraction ω of firms reset their prices, and a fraction $1-\omega$ keeps their prices constant,

$$p_{i,t} = p_{i,t-1}. \tag{24}$$

In log-deviations, the price level is

$$p_t = \int_0^1 p_{i,t} di. \tag{25}$$

Let $\bar{e}_t \equiv \overline{\mathbb{E}}_t[\pi_t] - \pi_t$ be the average forecast error. Equations (23)-(25) imply that

$$p_t^* - p_t = -\bar{e}_t + \left(\frac{1-\omega}{\omega}\right)\pi_t. \tag{26}$$

Let $\Delta_t \equiv \int_0^1 (p_{i,t} - p_t)^2 \, di$ be price dispersion in period t. Then

$$\begin{aligned}
\Delta_t &= \int_0^1 (p_{i,t} - p_t)^2 \, di \\
&= \omega \int_0^1 (e_{i,t} + p_t^* - p_t)^2 \, di + (1-\omega) \int_0^1 (p_{i,t-1} - p_t)^2 \, di \\
&= \omega \left[\int_0^1 e_{i,t}^2 di + 2\bar{e}_t (p_t^* - p_t) + (p_t^* - p_t)^2 \right] + (1-\omega) \left(\Delta_{t-1} + \pi_t^2 \right) \\
&= \omega \text{Var}_i [e_{i,t}] + \left(\frac{1-\omega}{\omega} \right) \pi_t^2 + (1-\omega) \Delta_{t-1},
\end{aligned}$$

where the last line uses equation (26). This equation is the standard recursion for the Calvo price dispersion found in (Woodford, 2003), plus a term proportional to the dispersion in forecast errors.

It follows that

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{1}{1 - \beta(1-\omega)} \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{1-\omega}{\omega} \right) \pi_t^2 + \omega \text{Var}_i (e_{i,t}) \right].$$

Let λ be the elasticity of real marginal costs to the output gap, and ξ_t an aggregate shock to marginal costs. Then

$$\tilde{mc}_t = \lambda y_t + \xi_t. \tag{27}$$

This shock can be modeled as a time-varying labor subsidy. Equations (22), (26), and (27) imply equation (1), with the transformation

$$u_t \equiv \frac{\omega}{1-\omega} \xi_t.$$

The objective function (2) can be derived as the standard second-order approximation to the utility of a representative household, where

$$\gamma \equiv \frac{\theta}{1 - \beta(1-\omega)},$$

and $\theta > 1$ is the demand elasticity implied by the standard Dixit-Stiglitz aggregator.