The Effect of Intangible Capital on Offshoring *

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Job Market Paper

This version: January 3, 2026

Abstract

This paper analyzes the impact of intangible capital on firms' offshoring decisions, aggregate productivity, and external competitiveness. I develop a two-country offshoring model with endogenous intangible investment that captures its unique scalability in a framework featuring heterogeneous firm entry and exit. We find that the introduction of intangible capital successfully accounts for the post-GFC U.S. data, characterized by productivity improvement and real exchange rate appreciation. This occurs as productivity shocks induce domestic firm entry, which lowers average productivity and raises domestic prices, thereby intensifying the Harrod–Balassa–Samuelson effect. Incorporating intangible capital temporarily reduces aggregate productivity gains in the short run, but these gains are amplified over longer horizons. This research contributes by endogenously modeling the scalability of intangible capital, a previously understudied factor in offshoring models.

JEL Codes: F12, F23, F41, O33

*I am deeply grateful to my advisors, Fabio Ghironi, Theo Eicher, and Andrei Zlate, for their support and guidance. I also thank Gianluca Benigno, Paul Bergin, and Francesco Zanetti for their invaluable advice, as well as the participants at the University of Washington Macroeconomics Seminar, for their useful comments. All errors are my own.

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1 Introduction

Since the Global Financial Crisis (GFC), the US economy has experienced a slight increase in aggregate productivity accompanied by a weakening of external competitiveness due to the appreciation of the real exchange rate (RER) (Figures 1 & 2). Changes in productivity have been regarded as a key determinant of long-run real exchange rates (Corsetti et al., 2008a; Rabanal and Rubio-Ramirez, 2015). The Harrod–Balassa–Samuelson (HBS) effect is the representative mechanism explaining this, wherein faster productivity growth in the tradable sector relative to the non-tradable sector leads to rising prices and RER appreciation. Considering recent data, the observed dynamics suggest a need for closer attention to structural changes within the US tradable sector.

Central to these shifts is the sharp post-GFC increase in intangible investment. Previous studies suggest that higher intangible capital investment by firms participating in Global Value Chains (GVCs) leads to distinct productivity improvements (Karpowicz and Suphaphiphat, 2020). Specifically, the expansion of intangible investments, such as R&D, can foster endogenous productivity improvements in US firms, inducing a persistent appreciation of the RER (Gornemann, Guerrón Quintana, and Saffie, 2025). Thus, the expansion of intangible investment is a key to explaining both US endogenous growth and recent RER dynamics.

This rise in intangible investment is not confined to domestic borders; it is also expanding internationally, facilitating GVC participation and enhancing value appropriation (Jona-Lasinio, Manzocchi, and Valentina Meliciani, 2019). Since 2000, as US multinationals have deepened their GVC integration, intangible investment has increasingly been sourced globally, alongside a rise in R&D offshoring (Baldwin and Evenett, 2015). To analyze how intangible offshoring affects corporate decisions, it is necessary to grasp the unique scalability of intangible capital. Since intangible capital is non-physical, offshored intangible capital can be replicated across borders at near-zero marginal cost and deployed across multiple production sites (Haskel and Westlake, 2017; Hsieh and Rossi-Hansberg, 2023). This scalability, unlike tangible capital which is fixed in location, provides a strong incentive for firms to combine intangible capital with foreign production sites to maximize profits.

However, the cross-border relocation of intangible capital implies a tradeoff for US productivity. On one hand, offshoring intangible capital can enhance aggregate productivity through efficient

resource reallocation. By deepening global integration, offshoring drives the least efficient firms out of the market and shifts resources toward more efficient ones (Melitz, 2003). On the other hand, relocating intangibles to foreign affiliates may contract domestic production or shift profits overseas, risking a statistical decline in US aggregate productivity (Guvenen, Mataloni, Rassier, and Ruhl, 2022).

Consequently, the international allocation of intangible capital is a crucial determinant of not only firm-level production and profits but also aggregate productivity. Yet, few studies have provided an integrated analysis of how intangible investment by offshoring firms acts as an endogenous driver of productivity changes and subsequently impacts the RER. Against this background, this paper develops a theoretical model endogenizing the scalability of intangible capital to analyze: (1) the role intangible capital plays in firms' offshoring decisions, and (2) the general equilibrium effects of intangible capital on aggregate productivity and external competitiveness. Using this framework, we aim to elucidate how the surge in intangible investment after the GFC can simultaneously explain the improvement in aggregate productivity and the appreciation of the real exchange rate.

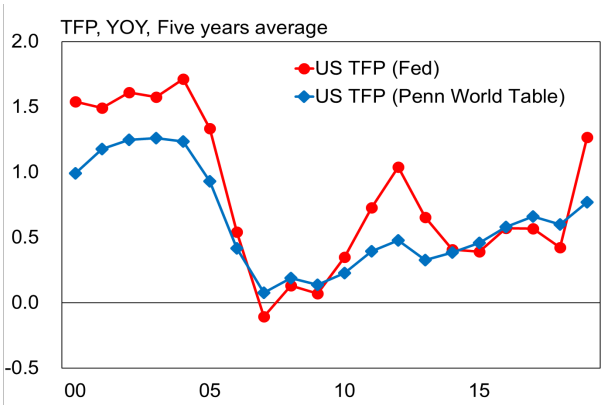


Figure 1: Total Factor Productivity in the U.S.
Source: San Francisco Fed, Penn World Table

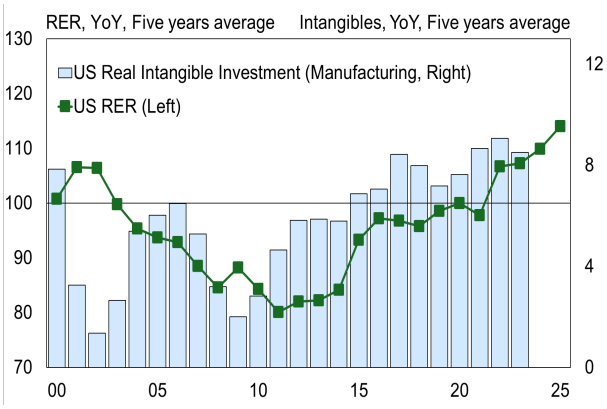


Figure 2: RER & Intangible Investment in the U.S.
Source: BIS, BEA

This paper develops an offshoring model that incorporates intangible capital investment as an endogenous decision for heterogeneous firms across two economies (Home and Foreign). Firms endogenously choose whether to invest in intangible capital or offshore production based on their productivity. Similar to Ghironi and Melitz (2005, GM05), the model features endogenous firm entry, heterogeneous labor productivity, and endogenous export decisions. Following Zlate (2016, Zlate16), the model also includes endogenous offshoring decisions for Home firms, assuming an

asymmetric offshoring framework where only Home firms offshore due to wage differentials. The model reflects a vertical FDI setup, with Home firms establishing offshore plants in Foreign to exploit lower costs and reshipping output back to serve the domestic market.

To incorporate heterogeneous firms' intangible investment, I adopt De Ridder's (2024, De Ridder24) assumption: firms incur fixed costs to invest in intangible capital, which reduces marginal costs. Intangible capital is modeled as a fixed cost to reflect its scalability. Unlike tangible capital, intangibles have low marginal replication costs (Haskel and Westlake, 2017; Hsieh and Rossi-Hansberg, 2023). This scalability shifts firms' cost structures, minimizing incremental costs of scaling up production while increasing fixed costs (De Ridder24). For example, intangible capital like R&D or software involves recurring fixed expenses (e.g., software licenses, R&D wages) but negligible marginal replication costs.

After paying sunk entry costs, Home firms draw idiosyncratic productivity levels and decide whether to invest in intangible capital or offshore production. As these decisions are made post-entry, firms choose on a period-by-period basis. Firms then optimize price and quantity based on realized productivity.

Home firms operate under three production modes: offshoring with intangible investment, domestic production with intangible investment, or domestic production without it. Unlike Zlate16, offshoring firms in this paper must invest in intangibles—incurring additional fixed costs while benefiting from lower marginal costs.[1] Domestic firms, previously undifferentiated in Zlate16, are now categorized by intangible investment. Home firms can serve the domestic market using domestic or foreign labor. Offshoring firms combine their productivity with foreign labor, incurring offshoring fixed costs, intangible fixed costs, and iceberg trade costs each period. Only firms productive enough to cover these costs offshore. Domestic firms investing in intangible capital combine firm-specific productivity with domestic labor, incurring intangible fixed costs. Only firms with productivity above the intangible investment cutoff invest in intangibles. A subset of firms exports, but only those with productivity above the export cutoff engage in exporting.

The benchmark model incorporating intangible capital yields several key distinctions compared

[1]This is consistent with empirical evidence showing that offshoring firms exhibit greater R&D intensity and technology investment than non-offshoring firms (Bøler, Moxnes, and Ulltveit-Moe, 2015; Dachs, Ebersberger, Kinkel, and Som, 2015; Fritsch and Gorg, 2015), and that multinationals show higher innovation rates and offshore more R&D (Gumpert, Manova, Rujan, and Schnitzer, 2024).

to the model without intangibles (Zlate16). First, the introduction of intangible capital predicts a stronger RER appreciation in response to positive productivity shocks, successfully reproducing the post-GFC RER dynamics when calibrated with U.S. empirical TFP data. The mechanism driving this appreciation is as follows: A positive productivity shock boosts firm profits, triggering firm entry in the Home economy. This expansion increases Home household income and demand for Foreign goods, thereby inducing excess demand for Foreign labor and driving up Foreign wages. Consequently, the relative cost of Home labor falls (a short-run depreciation of the terms of labor), which further stimulates domestic entry. Crucially, this heightened entry allows lower-productivity firms to enter, leading to a decline in average domestic productivity, a rise in domestic prices, and ultimately, an amplification of RER appreciation.

In this context, the scalable nature of intangible capital—which lowers marginal costs—amplifies the increase in profits compared to the model without intangibles. This enhanced profitability further expands Home demand for foreign goods, exacerbating the rise in foreign wages and the depreciation of relative labor costs. As a result, domestic entry is significantly higher than in the model without intangibles, leading to a steeper rise in domestic prices and a stronger Harrod–Balassa–Samuelson (HBS) effect. This mechanism theoretically rationalizes the empirical pattern of RER appreciation accompanying productivity growth observed in the U.S. post-GFC (Figures 1 & 2), highlighting the importance of accounting for intangibles in understanding recent RER dynamics.

Second, using the Dynamic Olley-Pakes decomposition (Melitz and Polanec, 2015), I analyze aggregate productivity dynamics following a permanent positive productivity shock. I find that while incorporating intangibles temporarily dampens aggregate productivity gains in the short run, it amplifies them in the long run. Initially, the shock increases profits for intangible-investing firms (both domestic and offshoring), increasing the number of active firms and lowering average productivity due to the entry of less efficient firms. However, over time, as the market stabilizes, the number of inefficient entrants declines, revealing substantial productivity gains.

Comparative analysis shows that the benchmark model generates larger and faster aggregate productivity gains than the model without intangibles. This is because intangible investment increases demand for foreign-produced goods, thereby raising foreign wages and relatively slowing the entry of offshore firms, which results in a comparatively quicker and larger average

offshore productivity gain. This suggests that when firms combine intangible capital with offshoring, the reallocation of resources toward more efficient firms occurs more rapidly, and, consequently, aggregate productivity gains are larger compared to offshoring without intangible capital investment. Thus, the benchmark model reaffirms Melitz's (2003) argument—the productivity-enhancing role of trade-within the context of intangible offshoring.

Third, the benchmark model better captures the expansion of exports following a productivity shock compared to the model without intangibles. This is because the introduction of intangible capital reduces marginal costs, thereby improving the price competitiveness of Home exporters. This feature allows the model to better match the procyclicality of U.S. export data observed since the GFC, indicating a theoretical role for intangibles in explaining export cyclicality.

Fourth, incorporating intangibles reduces the procyclicality of offshore value-added (VA), achieving a closer fit to post-GFC data compared to the model without intangibles. In the benchmark model, the rise in domestic income drives up foreign labor demand and wages, which acts as a cost-push factor for offshore firms employing foreign labor. Consequently, even as Home income rises, the increased cost of foreign labor dampens offshore VA. This result aligns precisely with the data showing a reduction in the procyclicality of U.S. offshore VA following the GFC, confirming that incorporating intangibles better explains this stylized fact than Zlate16.

This research advances the offshoring literature by incorporating the scalable nature of intangible capital into firms' endogenous production decisions—a cost structure (shifting from marginal to fixed costs) that remains underexplored in offshoring models despite its relevance for productive firms (Haskel et al., 2017; Hsieh et al., 2023; De Ridder, 2024). Building on models of endogenous offshoring such as Zlate16, which did not fully account for this unique cost structure, this study endogenizes scalability within heterogeneous firms' offshoring choices. While the causal link from foreign outsourcing to increased R&D has been explored (Bernard et al., 2020; Bøler et al., 2015), the simultaneous determination of offshoring and intangible investment remains relatively under-researched. Unlike recent dynamic partial-equilibrium frameworks (Gumpert et al., 2025; Liu, 2024), this paper employs a general equilibrium model to comprehensively examine the economy-wide impact of intangible investment's scalability within the offshoring model.

To the best of my knowledge, this is the first paper to theoretically elucidate the empirical link between rising intangible investment, productivity growth, and RER appreciation within a

framework of endogenous producer entry and exit. While Ghironi and Melitz (2005) established that endogenous producers' entry and exit decisions motivate RER appreciation—validating the HBS effect—and Zlate16 subsequently found that offshoring dampens RER appreciation by lowering consumer prices, this paper identifies a novel theoretical channel. I show that endogenously determined intangible capital stimulates domestic firm entry, which drives up domestic prices and amplifies RER appreciation. Thus, this study provides a theoretical mechanism that rationalizes the simultaneous increase in intangible investment, productivity, and RER appreciation, thereby explaining the strengthening of the HBS effect observed in the U.S. since the GFC.

Outline The remainder of the paper proceeds as follows: Section 2 presents the benchmark two-country heterogeneous firm model with the inclusion of intangible capital under an offshoring setup. Section 3 details the calibration. Section 4 discusses the model dynamics and evaluates the model performance through empirical TFP calibration. Finally, Section 5 concludes. The Appendix includes analytical derivations and supplementary figures and tables. Furthermore, it outlines empirical results that back up the assumptions of the benchmark model and presents the findings of the event-study framework.

2 The Model

The model is composed of two economies, Home and Foreign, each with a representative household and a continuum of monopolistically competitive firms with heterogeneous labor productivity producing differentiated varieties. Home firms can produce domestically or offshore for their domestic market, while Foreign firms produce only domestically due to Home's higher labor costs. Additionally, a subset of firms from both countries produce domestically to export.

This section details the representative household's and firms' problems in the Home economy under financial autarky. For simplicity, labor is the sole production input. Appendix B provides full model summaries.

2.1 Household Problem

Households maximize their expected intertemporal utility, which depends on their consumption

level (C_t). The expected intertemporal utility function is:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}}{1-\gamma} \tag{1}$$

The parameter $\beta \in (0,1)$ is a discount factor, and $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution. The consumption basket (C_t) is defined over a continuum of goods Ω, where the elasticity of substitution across goods is constant, $\theta > 1$.

The household purchases the consumption basket C_t, including varieties produced domestically by Home firms without intangible investment ($\omega \in \Omega_t^{NI}$), with intangible investment ($\omega \in \Omega_t^{I}$), varieties produced offshore ($\omega \in \Omega_t^{V}$), and varieties produced by Foreign firms and imported by Home ($\omega \in \Omega_t^{F}$), all with the symmetric elasticity of substitution $\theta > 1$:

$$C_t = \left[\underbrace{\int_{z_{min}}^{z_{D,t}^I} y_{D,t}^{NI}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{NI}} + \underbrace{\int_{z_{D,t}^I}^{z_{V,t}} y_{D,t}^{I}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{I}} + \underbrace{\int_{z_{V,t}}^{\infty} y_{V,t}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{V}} + \underbrace{\int_{z_{X,t}^*}^{\infty} y_{X,t}^{*}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{F}} \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

$[z_{min}, \infty)$ is the support interval for the idiosyncratic productivity of Home firms. Firms exceeding the endogenous threshold $z_{D,t}^I$ invest in intangible capital for the domestic market, while those above $z_{V,t}$ offshore production. The composition of the consumption basket changes over time since the number of firms is time-variant and firms re-optimize their intangible investment, offshoring and exporting strategies every period. $p_t(\omega)$ denotes the home currency price of a good $\omega \in \Omega_t$, and the consumption-based price index for the home economy is $P_t = (\int_{\omega \in \Omega} p_t(\omega)^{1-\theta} d\omega)^{1/(1-\theta)}$. The household's demand for each individual good ω is $c_t(\omega) = (p_t(\omega)/P_t)^{-\theta} C_t = \rho_t(\omega)^{-\theta} C_t$, where $\rho_t(\omega)$ is the real price of each variety and $\omega \in \Omega_t^{NI} \cup \Omega_t^{I} \cup \Omega_t^{V} \cup \Omega_t^{F}$.

2.2 Firm entry

In each period, an unbounded mass of potential entrants exists in both Home and Foreign. Following GM05, firm entry occurs each period in both countries and requires paying a sunk cost $f_{E,t}$, measured in effective labor units of the respective country. Upon entry, each firm draws its productivity level z from a common distribution $G(z)$ supported over the interval $[z_{min}, \infty)$,

which remains fixed thereafter.

New entrants are forward-looking and calculate the expected stream of future profits $\tilde{d}_t$ in every period. Entrants at time t only start producing at time $t+1$, introducing a one-period time-to-build lag in the model. The exogenous exit shock occurs at the end of the time period with the probability δ. Prospective home entrants in period t compute their expected post-entry value $\tilde{v}_t$ given by the present discounted value of their expected stream of profit $\{\tilde{d}_s\}_{s=t+1}^{\infty}$:

$$\tilde{v}_t(z) = E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_s}{C_t}\right)^{-\gamma} \tilde{d}_s(z) \tag{3}$$

Thus, $N_{E,t}$ new firms are created every period t and $(1-\delta)N_{E,t}$ firms start producing at t+1. The law of motion for the number of active firms is: $N_{t+1} = (1-\delta)(N_t + N_{E,t})$. The equation (4) also represents the average value of incumbent firms after production has occurred. Entry occurs until the average firm value is equalized with the entry cost, leading to the free entry condition $\tilde{v}_t = \frac{w_t}{Z_t} f_{E,t}$.

2.3 Firms' choice of markets and production strategies

Each period, the N_t active firms endogenously choose their production locations and whether to invest in intangible capital. The setup is as follows: (1) To serve the home market, domestic firms can choose three types of production methods: Producing domestically with intangible investment, producing domestically without intangible investment or producing in an offshore plant with intangible investment. Intangible investment offers the advantage of reducing marginal costs of production but incurs intangible fixed costs each period. Similarly, offshoring provides the advantage of lower wage costs but entails offshoring fixed and trade costs per period. It is essential to note that all firms, regardless of their choices regarding intangible investment or offshore production, are exclusively focused on serving their domestic market and do not engage in foreign market activities. (2) A subset of firms in each economy exports to serve foreign markets. This study focuses on intangible-intensive manufacturing firms. Given the empirical prevalence of vertical foreign direct investment (FDI) in manufacturing, this paper abstracts from outsourcing driven by horizontal FDI aimed at capturing foreign market share. Consequently, I assume that

firms serving foreign markets produce domestically and export, subject to a fixed cost, as in GM05.

2.3.1 Intangible investment decision

This paper models intangible capital investment as recurring fixed costs that reduce marginal production costs, reflecting the scalable nature of intangibles. Scalability refers to the minimal marginal costs of reproducing intangible assets, which alters firms' cost structures by shifting costs from marginal to fixed components (De Ridder24). For instance, incorporating intangible capital such as R&D or software, with negligible replication costs into production, raises fixed costs, such as recurring expenses for software licensing fees or R&D wages, while keeping marginal costs low. As a result, firms' production decisions increasingly hinge on fixed costs. Moreover, the impact of this investment depends on firms' heterogeneous productivity: more productive firms achieve larger marginal cost reductions from the same fixed investment (Veugelers et al., 2020)[2].

The following analysis will explore the cost dynamics of firms with and without intangible investments. If a firm with heterogeneous productivity z does not undertake intangible capital investment, its marginal cost is equivalent to wages measured in units of effective labor.

$$mc_t(z) = \frac{w_t}{Z_t z} \tag{4}$$

If a firm with heterogeneous productivity z invests in intangible capital, its marginal costs will decrease by a factor of ψ_t^{ξ}. ψ_t denotes the fraction by which marginal costs are reduced, and ξ represents the cost elasticity of intangibles.

$$mc_t(z) = \psi_t^{\xi} \frac{w_t}{Z_t z} \text{ where } \psi_t < 1 \tag{5}$$

However, firms must incur a fixed cost, measured in units of consumption, in order to engage in intangible capital investment.

$$\phi_t \frac{w_t}{Z_t} (\psi_t^{-\xi} - 1) \tag{6}$$

Here, ϕ_t represents the share of intangible fixed costs in total fixed costs. Countries with lower ϕ_t have more efficient firms, reflecting higher intangible capital efficiency, as they achieve greater

[2]The scalability—where marginal costs decline with output—can also be confirmed using Compustat Segment Historical data. Related analysis is provided in Appendix A, Figure 10.

marginal cost reductions for the same fixed costs.

Firms invest in intangible capital only if the resulting profit $d^I_{D,t}(z)$ meets or exceeds the profit without it, $d^{NI}_{D,t}(z)$ (i.e., $d^I_{D,t}(z) \geq d^{NI}_{D,t}(z)$). Since intangible investment entails fixed costs, only firms with sufficiently high productivity can justify the investment.

2.3.2 The timing of production and intangible investment decisions

After entry, firms draw their idiosyncratic productivity z from the common distribution $G(z)$. They then choose their production location and whether to invest in intangible capital based on their productivity. Without intangible investment, marginal costs are $\frac{w_t}{Z_t z}$; with investment, firms pay a fixed cost to reduce marginal costs to $\psi_t^{\xi} \frac{w_t}{Z_t z}$. Highly productive firms may both invest in intangibles and offshore production. Firms make these decisions simultaneously and then set prices and quantities to maximize profits. Figure 9 illustrates the timing of these decisions.

Since the decision to undertake intangible capital investment is made after the realization of productivity, firms cannot determine whether to make such investments prior to entering the market. Therefore, intangible capital investment decisions are made on a period-by-period basis according to the idiosyncratic productivity realized after entry. This setup can be considered a more realistic representation of the recurrent costs that firms must incur to maintain R&D, software, and information technology investments.

Since firms observe productivity only after entry, they cannot decide on intangible investment beforehand. Instead, they choose investment each period based on realized idiosyncratic productivity. This framework better captures the recurring costs firms incur to sustain R&D, software, and IT investments.

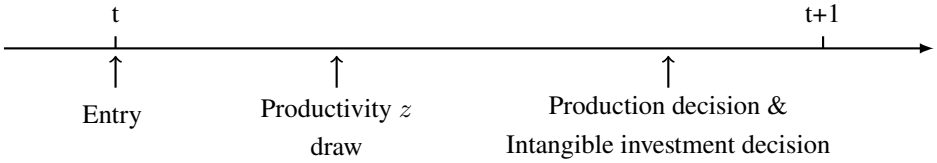


Figure 9: The timing of production and intangible investment decisions

2.3.3 Production decision

A home firm with idiosyncratic productivity z chooses between two production strategies each period to serve the home market: (a) Produce domestically, with output $y_{D,t}(z) = Z_t z l_t(z)$, as a

function of aggregate productivity Z_t, the firm-specific labor productivity z, and domestic labor $l_t(z)$; or (b) Produce offshore, with output $y_{V,t}(z) = Z_t^* z l_t^*$, using foreign labor $l_t^*(z)$, subject to aggregate foreign productivity Z^*, while retaining its idiosyncratic productivity z abroad.

Within a monopolistically competitive framework, a firm with idiosyncratic productivity z optimally chooses between three production modes, factoring in the effect of intangible investment on profits: domestic production with intangible investment, domestic production without intangible investment, and offshore production with intangible investment. Equations (7)–(9) present the profit functions for these modes.

$$\max_{\{\rho_{D,t}^I(z)\}} d_D^I(z) = \left(\rho_{D,t}^I(z) - \psi_{D,t}^{\xi} \frac{w_t}{Z_t z}\right) y_{D,t}(z) - \phi_{D,t} \frac{w_t}{Z_t} (\psi_{D,t}^{-\xi} - 1) \tag{7}$$

$$\max_{\{\rho_{D,t}^{NI}(z)\}} d_D^{NI}(z) = \left(\rho_{D,t}^{NI}(z) - \frac{w_t}{Z_t z}\right) y_{D,t}(z) \tag{8}$$

$$\max_{\{\rho_{V,t}(z)\}} d_V(z) = \left(\rho_{V,t}(z) - \tau Q_t \psi_{V,t}^{\xi} \frac{w_t^*}{Z_t^* z}\right) y_{V,t}(z) - f_V Q_t \frac{w_t^*}{Z_t^*} - Q_t \phi_{V,t} \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \tag{9}$$

where $\rho_{D,t}^{NI}(z)$, $\rho_{D,t}^I(z)$, and $\rho_{V,t}(z)$ represent the relative prices for domestic production without intangible investment, domestic production with intangible investment, and offshore production with intangible investment, respectively. w_t and w_t^* denote real wages in the home and foreign markets, and Q_t represents the real exchange rate. Thus, the cost of producing a unit of output depends on whether production is domestic or offshore and whether intangible investment is undertaken. In other words, this decision is influenced not only by effective labor costs $\frac{w_t}{Z_t}$ and $\frac{w_t^* Q_t}{Z_t^*}$ across countries, but also by firm-specific labor productivity z and the marginal cost reduction fraction ψ_t.

Home firms investing in intangible capital incur fixed costs while gaining the ability to reduce marginal costs. For domestic producers, intangible investment lowers marginal costs by a factor of $\psi_{D,t}^{\xi}$; for offshore producers, by $\psi_{V,t}^{\xi}$. Following Gumpert, Manova, Rujan, and Schnitzer (2025), I assume $\psi_{D,t} > \psi_{V,t}$, reflecting higher fixed costs for intangible investment in an offshore plant. A lower $\psi_{V,t}$ indicates both higher fixed costs and greater marginal cost reduction efficiency for offshore firms. Fixed costs are independent of firm-specific productivity z. However, the fixed costs associated with intangible investment mean that only firms with relatively high idiosyncratic productivity z engage in such investment.

Moreover, home firms that offshore production incur a fixed cost of f_V units of foreign effective labor, covering the establishment and maintenance of the offshore facility. They also face an iceberg trade cost $\tau > 1$ when transporting goods produced overseas back to the home country.

The demand for the variety of firm z, produced domestically or offshore considering intangible investment, is $y^I_{D,t}(z) = \rho^I_{D,t}(z)^{-\theta} C_t$, $y^{NI}_{D,t}(z) = \rho^{NI}_{D,t}(z)^{-\theta} C_t$ or $y_{V,t}(z) = \rho_{V,t}(z)^{-\theta}\ C_t$ respectively, where C_t is aggregate Home consumption. Profit maximization yields equilibrium prices: $\rho^I_{D,t}(z) = \frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t z}$, $\rho^{NI}_{D,t}(z) = \frac{\theta}{\theta-1}\frac{w_t}{Z_t z}$ and $\rho_{V,t}(z) = \frac{\theta}{\theta-1}\tau Q_t \psi^{\xi}_{V,t}\ \frac{w^*_t}{Z^*_t z}$ for the three production methods. These prices show that firms with intangible capital investment price differently from non-intangible investing firms by a factor of ψ^{ξ}, allowing them to reduce prices proportionally to marginal cost reductions. The corresponding profits for each production cases are as follows: $d^I_{D,t}(z) = \frac{1}{\theta}\rho^I_{D,t}(z)^{1-\theta} C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1), d^{NI}_{D,t}(z) = \frac{1}{\theta}\rho^{NI}_{D,t}(z)^{1-\theta} C_t$ and $d_{V,t}(z) = \frac{1}{\theta}\rho_{V,t}(z)^{1-\theta} C_t - f_V Q_t \frac{w^*_t}{Z^*_t} - Q_t \phi_{V,t}\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t} - 1)$.

Each period, domestic firms decide whether to invest in intangible capital. A firm with productivity z compares domestic profit with intangible investment $d^I_{D,t}(z)$ and without such investment $d^{NI}_{D,t}(z)$. The cutoff productivity $z^I_{D,t}$ on the support interval $[z_{min}, \infty)$ is where the two profits are equal; at this threshold, the firm is indifferent between investing and not investing.

$$z^I_{D,t} = \{z : d^I_{D,t}(z^I_{D,t}) = d^{NI}_{D,t}(z^I_{D,t})\} \tag{10}$$

Similarly, a firm with productivity z determines its production location each period by comparing profits from domestic production with intangible investment, $d^I_{D,t}(z)$, and from offshoring, $d_{V,t}(z)$. The productivity cutoff $z_{V,t}$ on the support interval $[z_{min}, \infty)$ denotes the level at which the firm is indifferent, earning the same profit from either location.

$$z_{V,t} = \{z : d^I_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})\} \tag{11}$$

The model implies that only relatively more productive domestic firms can invest in intangible capital, and even more productive firms among them can pursue offshore production with intangible investment. Since intangible investment entails fixed costs, only firms with productivity above the investment threshold ($z > z^I_{D,t}$) gain enough from intangible capital to offset these costs.[3]

[3]Intangible investing firms are more productive, a fact supported by the Compustat Historical Segment data showing high average productivity for firms in the highest intangible quantile. See empirical fact (iii) and Figure 12 in Appendix A for details. Based on empirical fact (ii), intangible investing firms exhibit higher net sales.

Similarly, offshore production involves additional fixed and iceberg trade costs, such that only firms with idiosyncratic productivity above the offshoring cutoff ($z > z_{V,t}$) can generate sufficient gains to offset these costs. This implication is consistent with empirical evidences: multinational firms are more likely to invest in R&D (Castellani et al., 2015); and multinational firms are both more productive and earn higher R&D returns than domestic firms (Belderbos et al., 2014; Rahko, 2021). The Compustat Segment Historical data also backs up the finding that offshoring firms are more productive than non-offshoring firms, which is presented as stylized fact (iii) in Appendix A. Furthermore, the model's setup, where offshoring firms always undertake intangible investment and incur fixed costs, is motivated by stylized fact (iv) identified in Appendix A using Compustat Historical Segment data: offshoring firms are more likely to undertake intangible investments compared to domestic firms.

Additionally, the productivity cutoffs $z^I_{D,t}$ and $z_{V,t}$ respond to changes in relative effective labor costs across countries, thereby influencing the extensive margin over the business cycle. A decline in the relative cost of Home effective labor lowers domestic prices and raises Home firms' revenues, leading more Home firms to invest in intangibles at equilibrium. Similarly, a decline in the relative cost of Foreign effective labor increases the profitability of offshoring, resulting in a higher share of firms choosing to offshore.

Existence of the equilibrium productivity cutoff

The intangible investment cutoff ($z^I_{D,t}$) In equilibrium, the existence of intangible investment productivity cutoff $z^I_{D,t}$ requires lower price for domestically producing firms with intangible investment compared to the domestic firms without intangible investment. To clarify this, I rewrite the per-period profits from domestically producing firms with intangible investment and that of without intangible investment: $d^I_D(z^I_{D,t}) = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t z^I_{D,t}}\right)^{1-\theta} C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)$ and $d^{NI}_D(z^I_{D,t}) = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t z^I_{D,t}}\right)^{1-\theta} C_t$.

Figure 4 plots the three profit functions against the idiosyncratic productivity parameter $z^{\theta-1}$ over the support interval $[z_{min}, \infty)$. Domestic production without intangible investment incurs no fixed costs, resulting in a zero vertical intercept. Domestic production with intangible investment incurs intangible fixed costs, yielding a vertical intercept of $-\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)$. Offshoring incurs both intangible and offshoring fixed costs, resulting in a vertical intercept of

$-f_V \frac{w_t^* Q_t}{Z_t^*} - \phi_{V,t} \frac{w_t^* Q_t}{Z_t^*} (\psi_{V,t}^{-\xi} - 1)$.

The model predicts that the productivity cutoff $z_{D,t}^I$ exists in equilibrium only if the slope of the profit function for domestic production with intangible investment exceeds that of production without it: $slope\{d_{D,t}^I(z)\} > slope\{d_{D,t}^{NI}(z)\}$. When this condition holds, domestic production with intangible investment yields higher profits for firms with productivity above $z_{D,t}^I$ compared to domestic production without intangible investments. The slope inequality is equivalent to the condition $\psi_{D,t}^{\xi(1-\theta)} > 1$. Since $\psi_{D,t} < 1, \xi > 0$, and $\theta > 1$, this condition always holds. It implies that domestic firms investing in intangibles can charge lower prices than those without, ensuring a unique intangible investment cutoff. To avoid a corner solution, profits for domestic firms without intangible investment must exceed those with intangible investment at z_{min} : $d_{D,t}^{NI}(z_{min}) > d_{D,t}^I(z_{min})$.

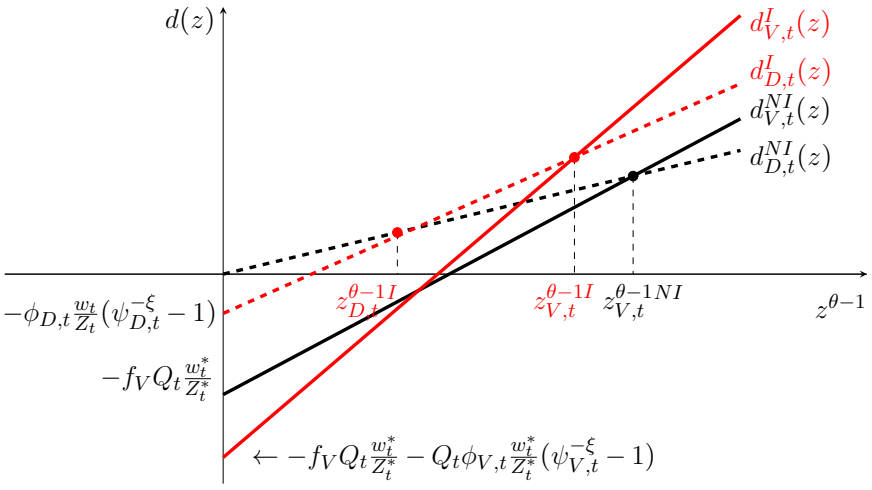


Figure 3: The productivity cutoff

The offshoring cutoff $(z_{V,t})$ In equilibrium, the existence of productivity cutoff $z_{V,t}$ requires a cross-country disparity in effective labor costs, ensuring that a subset of domestic firms find offshore production profitable. To illustrate this, I express per-period profits as: $d_{D,t}^I(z) = M_t \left(\psi_{D,t}^{\xi} \frac{w_t}{Z_t}\right)^{1-\theta} z^{\theta-1} - \phi_{D,t} \frac{w_t}{Z_t} (\psi_{D,t}^{-\xi} - 1)$ and $d_{V,t}(z) = M_t \left(\tau \psi_{V,t}^{\xi} \frac{w_t^* Q_t}{Z_t^*}\right)^{1-\theta} z^{\theta-1}$ $-f_V \frac{w_t^* Q_t}{Z_t^*} - \phi_{V,t} \frac{w_t^* Q_t}{Z_t^*} (\psi_{V,t}^{-\xi} - 1)$, where $M_t = \frac{1}{\theta} \left(\frac{\theta}{\theta-1}\right)^{1-\theta} C_t$ measures the size of Home market.

The model predicts that a productivity cutoff $z_{V,t}$ exists in equilibrium only if the slope of the offshoring profit function exceeds that of domestic production with intangible investment: $slope\{d_{V,t}(z)\} > slope\{d_{D,t}^I(z)\}$. This condition implies that offshoring is more profitable for

firms with productivity above $z_{V,t}$.

The slope inequality is equivalent to the condition $\tau(\psi_{V,t}/\psi_{D,t})^{\xi}TOL_t < 1$, where $TOL_t = \frac{Q_t w^*/Z_t^*}{w_t/Z_t}$ denotes the relative cost of effective labor between the foreign and home country in the same currency. This condition requires the effective foreign wage, adjusted for relative marginal cost reductions, to be sufficiently low to offset fixed and iceberg trade costs ($\tau > 1$), incentivizing some domestic intangible investing firms to offshore. To avoid a corner solution, profits for domestic producers with intangible investment must exceed those of offshoring firms at z_{min} ($d^I_{D,t}(z_{min}) > d_{V,t}(z_{min})$). The model's calibration and the scale of macroeconomic shocks guarantee that this condition holds true in all periods.

2.3.4 Exporting

Firms in each economy can serve both domestic and foreign markets through exports, as in GM05. A Home firm with productivity z exports to the foreign market by using domestic labor $l_{X,t}$ to produce $y_{X,t}(z) = Z_t z l_{X,t}(z)$. Foreign firms similarly export to Home. Profit maximization yields the equilibrium export price and profit function for a Home exporter with productivity z: $\rho_{X,t}(z) = \frac{\theta}{\theta-1}\tau^* Q_t^{-1}\frac{w_t}{Z_t z}$ and $d_{X,t}(z) = \frac{1}{\theta}\rho_{X,t}(z)^{1-\theta}C_t^* Q_t - f_X\frac{w_t}{Z_t}$, where C_t^* is foreign aggregate consumption. Exporting generates additional profits but requires fixed costs (f_X) in units of Home effective labor and iceberg trade costs τ^*. Only Home firms with $z \geq z_{X,t}$ can export profitably, where $z_{X,t} = inf\{z \mid d_{X,t}(z) > 0\}$.

2.4 Aggregation over heterogeneous firms

This section reformulates the offshoring model with four representative Home firms: (1) domestic production without intangible investment, (2) domestic production with intangible investment, (3) offshore production with intangible investment, and (4) domestic production for export. Offshoring occurs only from Home to Foreign. Accordingly, Foreign has three representative firms: domestic production without intangible investment, domestic production with intangible investment, and exporters to Home.

2.4.1 Average productivity, prices, and profits

Figure 5 shows average productivity for three representative Home firms: those producing domestically without intangible investment, those producing domestically with intangible investment, and those producing offshore. Specifically, Figure 4 plots the density of firm-specific labor productivity z over the support interval $[z_{min}, \infty)$. $N_{D,t}^{NI}$ firms with productivity $z < z_D^{NI}$ produce domestically without intangible investment, and their average productivity is $\tilde{z}_{D,t}^{NI}$. Additionally, $N_{D,t}^{I}$ firms with productivity $z_{D,t}^{I} < z < z_{V,t}$ have an average productivity of $\tilde{z}_{D,t}^{I}$. Finally, $N_{V,t}$ firms with productivity $(z > z_{V,t})$ have an average productivity of $\tilde{z}_{V,t}$. Given that firm-specific labor productivity levels z are randomly drawn from a common distribution $G(z)$ with density $g(z)$, the three average productivity levels are:

$$\tilde{z}_{D,t}^{NI} = \left[\frac{1}{G(z_{D,t}^{I})}\int_{z_{min}}^{z_{D,t}^{I}} z^{\theta-1} g(z)dz\right]^{\frac{1}{\theta-1}}, \quad \tilde{z}_{D,t}^{I} = \left[\frac{1}{G(z_{V,t}) - G(z_{D,t}^{I})}\int_{z_{D,t}^{I}}^{z_{V,t}} z^{\theta-1} g(z)dz\right]^{\frac{1}{\theta-1}},$$

$$\tilde{z}_{V,t} = \left[\frac{1}{1 - G(z_{V,t})}\int_{z_{V,t}}^{\infty} z^{\theta-1} g(z)dz\right]^{\frac{1}{\theta-1}} \tag{12}$$

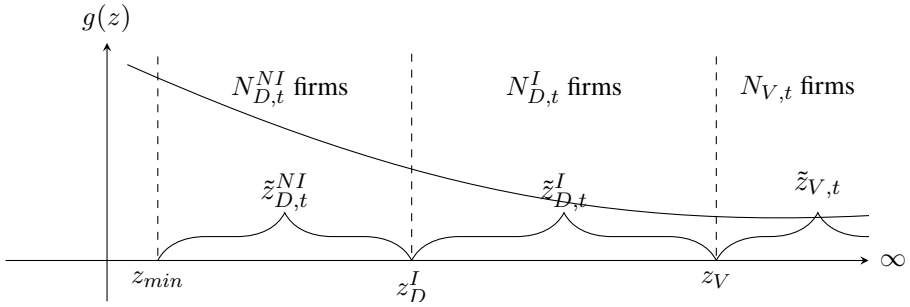


Figure 4: Average productivity of firms producing at Home

Assuming firm-specific labor productivity z follows a Pareto distribution with density $g(z) = kz_{min}^{k}/z^{k+1}$ and cumulative distribution $G(z) = 1 - (z_{min}/z)^{k}$ over $[z_{min}, \infty)$, the average productivity levels can be expressed in terms of the cutoffs $z_{D,t}^{I}$ and $z_{V,t}$:

$$\tilde{z}_{D,t}^{NI} = \nu z_{min} z_{D,t}^{I}\left[\frac{z_{D,t}^{I\,k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_{D,t}^{I\,k} - z_{min}^{k}}\right]^{\frac{1}{\theta-1}}, \quad \tilde{z}_{D,t}^{I} = \nu z_{D,t}^{I} z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{I\,k-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{I\,k}}\right]^{\frac{1}{\theta-1}},$$

$$\tilde{z}_{V,t} = \nu z_{V,t} \tag{13}$$

where the cutoff is $z_{V,t} = z_{min}(N_t/N_{V,t})^{1/k}$, with parameters $\nu \equiv \left[\frac{k}{k-(\theta-1)}\right]^{\frac{1}{\theta-1}}$ and $k > \theta - 1$.

Foreign firms exclusively serve their domestic market through domestic production, with average productivity given by:

$$\tilde{z}_{D,t}^{NI*} = \nu z_{min}^{*} z_{D,t}^{I*} \left[\frac{z_{D,t}^{I*\,k-(\theta-1)} - z^{*k-(\theta-1)}_{min}}{z_{D,t}^{I*\,k} - z^{*k}_{min}}\right]^{\frac{1}{\theta-1}}, \quad \tilde{z}_{D,t}^{I*} = \nu z_{D,t}^{I*} \tag{14}$$

The average productivity levels of exporting firms are similar to GM05:

$$\tilde{z}_{X,t} = \nu z_{min}\left(\frac{N_t}{N_{X,t}}\right)^{1/k} \text{ and } \tilde{z}_{X,t}^{*} = \nu z_{min}^{*}\left(\frac{N_{D,t}^{*}}{N_{X,t}^{*}}\right)^{1/k} \tag{15}$$

Based on the average productivity of firms engaged in domestic production (with and without intangible investment), offshoring, and exporting, Table 1 presents the average prices and profits for each production type. Therefore, the aggregate price indexes for Home and Foreign are: $1 = N_{D,t}^{NI}(\tilde{\rho}_{D,t}^{NI}(z))^{1-\theta} + N_{D,t}^{I}(\tilde{\rho}_{D,t}^{I}(z))^{1-\theta} + N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta} + N_{X,t}^{*}(\tilde{\rho}_{X,t}^{*}(z^{*}))^{1-\theta}$ and $1 = N_{D,t}^{*NI}(\tilde{\rho}_{D,t}^{*NI}(z^{*}))^{1-\theta} + N_{D,t}^{*I}(\tilde{\rho}_{D,t}^{*I}(z^{*}))^{1-\theta} + N_{X,t}(\tilde{\rho}_{X,t}(z))^{1-\theta}$. Similarly, the total profits of firms from Home and Foreign are $N_t\tilde{d}_t(z) = N_{D,t}^{NI}\tilde{d}_{D,t}^{NI}(z) + N_{D,t}^{I}\tilde{d}_{D,t}^{I}(z) + N_{V,t}\tilde{d}_{V,t}(z) + N_{X,t}\tilde{d}_{X,t}(z)$ and $N_t^{*}\tilde{d}_t^{*}(z^{*}) = N_{D,t}^{*NI}\tilde{d}_{D,t}^{*NI}(z^{*}) + N_{D,t}^{*I}\tilde{d}_{D,t}^{*I}(z^{*}) + N_{X,t}^{*}\tilde{d}_{X,t}^{*}(z^{*})$.

Origin	Production	Market	Intangible	Average Prices	Average Profits
Home	Home	Home	Y	$\tilde{\rho}_{D,t}^{I} = \frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t\tilde{z}_{D,t}^{I}}$	$\tilde{d}_{D,t}^{I} = \frac{1}{\theta}\tilde{\rho}_{D,t}^{I\,1-\theta}C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi}-1)$
Home	Home	Home	N	$\tilde{\rho}_{D,t}^{NI} = \frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}_{D,t}^{NI}}$	$\tilde{d}_{D,t}^{NI} = \frac{1}{\theta}\tilde{\rho}_{D,t}^{NI\,1-\theta}C_t$
Home	Foreign	Home	Y	$\tilde{\rho}_{V,t} = \frac{\theta}{\theta-1}\tau Q_t\psi_{V,t}^{\xi}\frac{w_t^{*}}{Z_t^{*}\tilde{z}_{V,t}}$	$\tilde{d}_{V,t} = \frac{1}{\theta}\tilde{\rho}_{V,t}^{1-\theta}C_t - f_V Q_t\frac{w_t^{*}}{Z_t^{*}} - Q_t\phi_{V,t}\frac{w_t^{*}}{Z_t^{*}}(\psi_{V,t}^{-\xi}-1)$
Home	Home	Foreign	N	$\tilde{\rho}_{X,t} = \frac{\theta}{\theta-1}\tau^{*}Q_t^{-1}\frac{w_t}{Z_t\tilde{z}_{X,t}}$	$\tilde{d}_{X,t} = \frac{1}{\theta}\tilde{\rho}_{X,t}^{1-\theta}C_t^{*}Q_t - f_X\frac{w_t}{Z_t}$
Foreign	Foreign	Foreign	Y	$\tilde{\rho}_{D,t}^{*I} = \frac{\theta}{\theta-1}\psi_{D,t}^{*\xi*}\frac{w_t^{*}}{Z_t^{*}\tilde{z}_{D,t}^{*I}}$	$\tilde{d}_{D,t}^{*I} = \frac{1}{\theta}\tilde{\rho}_{D,t}^{*I\,1-\theta}C_t^{*} - \phi_{D,t}^{*}\frac{w_t^{*}}{Z_t^{*}}(\psi_{D,t}^{*\,-\xi}-1)$
Foreign	Foreign	Foreign	N	$\tilde{\rho}_{D,t}^{*NI} = \frac{\theta}{\theta-1}\frac{w_t^{*}}{Z_t^{*}\tilde{z}_{D,t}^{*NI}}$	$\tilde{d}_{D,t}^{*NI} = \frac{1}{\theta}\tilde{\rho}_{D,t}^{*NI\,1-\theta}C_t^{*}$
Foreign	Foreign	Home	N	$\tilde{\rho}_{X,t}^{*} = \frac{\theta}{\theta-1}\tau Q_t\frac{w_t^{*}}{Z_t^{*}\tilde{z}_{X,t}^{*}}$	$\tilde{d}_{X,t}^{*} = \frac{1}{\theta}\tilde{\rho}_{X,t}^{*\,1-\theta}C_tQ_t^{-1} - f_X^{*}\frac{w_t^{*}}{Z_t^{*}}$

Table 1: Average prices and profits

2.4.2 Indifference conditions for offshoring and exporting

At the productivity cutoff, the two production strategies yield equivalent average productivity levels, establishing a linkage between the average profits of firms at distinct productivity cutoffs. First, at the intangible investment cutoff $z_{D,t}^{I}$, a connection emerges between the average profits of domestic firms with and without intangible investment. Second, at the offshoring cutoff $z_{V,t}$, a link

appears between the average productivity of offshore firms and those producing domestically with intangible investment.

$$\begin{aligned}\tilde{d}^{I}_{D,t} =& \frac{k}{k-(\theta-1)}\left(\frac{z^{I}_{D,t}z_{V,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1}\left[\frac{z^{k-\theta+1}_{V,t}-z^{I\,k-\theta+1}_{D,t}}{z^{k}_{V,t}-z^{I\,k}_{D,t}}\right]\tilde{d}^{NI}_{D,t} \\ &+\left\{\frac{k}{k-(\theta-1)}z^{\theta-1}_{V,t}\left[\frac{z^{k-\theta+1}_{V,t}-z^{I\,k-\theta+1}_{D,t}}{z^{k}_{V,t}-z^{I\,k}_{D,t}}\right]-1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)\end{aligned} \tag{16}$$

$$\begin{aligned}\tilde{d}_{V,t} =& \frac{k}{k-(\theta-1)}\left(\frac{z_{V,t}}{\tilde{z}^{I}_{D,t}}\right)^{\theta-1}\left\{\tilde{d}^{I}_{D,t}+\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)\right\} \\ &+\frac{\theta-1}{k-(\theta-1)}\left\{f_V Q_t\frac{w^*_t}{Z^*_t}+\phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V}-1)\right\}\end{aligned} \tag{17}$$

Additionally, since firms at the productivity cutoff $z_{X,t}$ earn zero profits from exporting, the average export profits are derived as shown in GM05:

$$\tilde{d}_{X,t} = \frac{\theta-1}{k-(\theta-1)}f_X\frac{w_t}{Z_t},\;\; \tilde{d}^{*}_{X,t} = \frac{\theta-1}{k-(\theta-1)}f^{*}_X\frac{w^*_t}{Z^*_t} \tag{18}$$

The derivations of equations (16)-(18) are provided in Appendix D.

2.5 Household Budget Constraint and Intertemporal Choices

The representative Home household entering period t holds two types of assets: domestic risk-free bond holdings B_t, and mutual fund share holdings x_t. Each period, the household begins with share holdings x_t in a mutual fund of N_t firms with average market value $\tilde{v}_t$. It also holds Home risk-free real bonds, B_t, denominated in units of the Home consumption basket. The household receives dividends equal to the average firm profit $\tilde{d}_t$ in proportion with the stock of firms N_t, the real wage w_t for $L \equiv 1$ supplied inelastically, and real rates of return r_t from the Home bonds. The household allocates these resources between purchases of bonds and shares to be carried into next period and consumption. Each period, the household purchases two types of assets, risk-free bonds B_{t+1} and mutual fund shares x_{t+1}. The mutual fund comprises N_t incumbent Home firms (producing domestically or offshore) and $N_{E,t}$ new entrants in period t. The average market value of each firm is $\tilde{v}_t$, representing the net present value of expected future profits.

The period budget constraint (in units of consumption) is:

$$(1+r_t)B_t + (\tilde{v}_t + \tilde{d}_t)N_t x_t + w_t L \geq B_{t+1} + \tilde{v}_t(N_t + N_{E,t})x_{t+1} + C_t \tag{19}$$

The household maximizes its expected intertemporal utility subject to equation (19). The Euler equations for bonds is:

$$C_t^{-\gamma} = \beta(1+r_{t+1})E_t(C_{t+1})^{-\gamma} \tag{20}$$

The Euler equations for stocks is as follows, with the rate of firm exit δ described in Section 2.2:

$$\tilde{v}_t = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{v}_{t+1} + \tilde{d}_{t+1})\right] \tag{21}$$

With the consumption basket C_t set as the numeraire, the Home price index is normalized such that $1 = \left(\int_{\Omega_t} \rho_t(\omega)^{1-\theta} d\omega\right)^{\frac{1}{1-\theta}}$.

2.6 Aggregation over heterogeneous firms

In a financial autarky, household income from labor, stocks, and bonds is used for consumption and investment in new firms within each country:

$$C_t + N_{E,t}\tilde{v}_t + B_{t+1} = w_t L + N_t \tilde{d}_t + (1+r_t)B_t \tag{22}$$

$$C_t^* + N_{E,t}^*\tilde{v}_t^* + B_{t+1}^* = w_t^* L^* + N_{D,t}^*\tilde{d}_t^* + (1+r_t^*)B_t^* \tag{23}$$

The current account in the Home is given by:

$$\begin{aligned} CA_t = \underbrace{N_{X,t}(\tilde{\rho}_{X,t}(z))^{1-\theta}C_t^* Q_t}_{\text{(a) Exports}} + \underbrace{N_{V,t}\tilde{d}_{V,t}(z)}_{\text{(b) Repatriated offshore profits}} \\ - \underbrace{N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta}C_t^* Q_t}_{\text{(c) Value added offshore in Foreign}} - \underbrace{N_{X,t}^*(\tilde{\rho}_{X,t}^*(z^*))^{1-\theta}C_t}_{\text{(d) Imports from Foreign}} \end{aligned} \tag{24}$$

Under financial autarky, the balanced current account condition ($CA_t = 0$) requires that the sum of (a) export value added and (b) repatriated offshore profits equals the sum of (c) the value added by offshoring affiliates imported into Home and (d) imports from Foreign firms.

As a result, the benchmark model of financial autarky for the Home economy consists of 19 equations and 19 endogenous variables: $N_t, N_{D,t}^I, N_{D,t}^{NI}, N_{V,t}, N_{X,t}, N_{E,t}, \widetilde{d}_t, \widetilde{d}_{D,t}^I, \widetilde{d}_{D,t}^{NI}, \widetilde{d}_{V,t}, \widetilde{d}_{X,t}, \widetilde{z}_{D,t}^I, \widetilde{z}_{D,t}^{NI}, \widetilde{z}_{V,t}, \widetilde{z}_{X,t}, \widetilde{v}_t, r_t, w_t, C_t$.

Since Foreign firms do not produce in Home, the Foreign economy reduces to 16 equations and variables, with no counterparts for $N_{V,t}, \tilde{d}_{V,t}, \tilde{z}_{V,t}$. The domestic productivity of a representative Foreign firm $\tilde{z}^*_{D,t}$ is constant over time.

3 Calibration

The model calibrates $\beta = 0.99$ to match a 4% annual interest rate, and the relative risk aversion coefficient is $\gamma = 2$. Following GM05, the intra-temporal elasticity of substitution is $\theta = 3.8$, and the probability of firm exit is $\delta = 0.025$.

Following Zlate16, the Pareto distribution parameter k is set to 4.2, and the iceberg trade cost to $\tau = 1.2$. The fixed costs of offshoring (f_V) and exporting (f_X and f^*_X) are set to 0.095, 0.040, and 0.032, respectively. These parameter values ensure that the steady-state model replicates the observed significance of offshoring in the Foreign economy, as well as the role of exports in both the Foreign and Home economies. The lower bound of productivity support is normalized to $z_{min} = z^*_{min} = 1$.

According to Zlate16, the sunk entry cost is set to be higher in Foreign than in Home ($f^*_E = 4f_E$ and $f_E = 1$) to reflect the greater regulatory burden of starting a business abroad, thereby introducing an asymmetry in the effective labor costs between the two countries in steady state. Consequently, the steady-state output, the number of firms, the labor demand, and the effective wage are relatively lower in Foreign. This calibration is consistent with the 1.5 times higher cost of business start-up procedures in China compared to the United States in purchasing power parity terms in 2015 (World Bank Database, 2015).

The calibrated values for k, τ, f_V, f_X, f^*_X, along with asymmetric sunk entry costs, result in a steady-state value for the terms of labor that is less than one ($TOL = \frac{Qw^*/Z^*}{w/Z} = 0.75$). This indicates that the steady-state cost of effective labor in Foreign is 75% of that in Home, providing an incentive for some Home firms to offshore production.

Following De Ridder24's estimates of intangible cost share (ϕ), I set $\phi = 0.215$ (Home) and $\phi^* = 0.279$ (Foreign), reflecting higher US intangible investment efficiency. I assume US R&D efficiency (ϕ) is lower than trading partners (ϕ^*), using France's (ϕ^*) as a proxy for Foreign. The intangible cost elasticities, ξ and ξ^*, are set to 2, following De Ridder24. Setting these to 0 nests the model in Zlate16's framework without intangible investment.

The reduction fraction of marginal costs for Home firms with intangible investment is set at 0.94. This indicates that these firms can reduce their marginal costs by $1 - \psi_{D,t}^{\xi} = 1 - 0.94^2 = 0.12$, or 12%. The reduction fraction for Home firms engaged in offshoring is set at 0.92, implying a marginal cost reduction of $1 - \psi_{V,t}^{\xi} = 1 - 0.92^2 = 0.15$, or 15%. Following Gumpert et al. (2025), this paper assumes $\psi_{D,t} > \psi_{V,t}$; therefore offshore intangible investment requires higher fixed costs than domestic intangible investment. Similarly, the reduction fraction for Foreign firms with intangible investment is set at 0.96, resulting in a marginal cost reduction of $1 - \psi_{D,t}^{*\xi^*} = 1 - 0.96^2 = 0.08$, or 8%. Therefore, the most productive offshoring firms can achieve an average marginal cost reduction of 15% through intangible investment, while domestic Home firms can reduce costs by 12%, and Foreign domestic firms by 8%. All parameterizations are summarized in Table 2.

Parameter	Description	Value	Source
β	Discount rate	0.99	GM05
γ	The coefficient of relative risk aversion	2	
θ	The intra-temporal elasticity of substitution	3.8	
δ	The probability of firm exit	0.025	
k	The Pareto distribution parameter	4.2	Zlate16
τ	The iceberg trade cost	1.2	
f_E	The sunk entry cost, Home	1	
f_E^*	The sunk entry cost, Foreign	4	
f_V	The fixed costs of offshoring	0.095	
f_X	The exporting fixed cost, Home	0.040	
f_X^*	The exporting fixed cost, Foreign	0.032	
ξ, ξ^*	Intangibles cost elasticity	2	De Ridder24
ϕ	Share of intangible costs in fixed costs, Home	0.215	
ϕ^*	Share of intangible costs in fixed costs, Foreign	0.279	
ψ_D	The reduction fraction of marginal costs, Home	0.94	
ψ_V	The reduction fraction of marginal costs for offshoring, Home	0.92	
ψ_D^*	The reduction fraction of marginal costs, Foreign	0.96	

Table 2: Calibration parameters and steady-state targets

4 Results

4.1 Impulse responses

To illustrate the mechanisms and aggregate effects of intangible investment and offshoring, I log-linearize the model around the steady state and compute impulse responses to a permanent

1% productivity shock in Home. Aggregate productivity follows $logZ_{t+1} = \rho\ logZ_t + u_t$, with persistence $\rho = 0.99$.

4.1.1 Benchmark model

Figures 5 show impulse responses, with solid lines representing the benchmark model and dashed lines the model without intangible investment (Zlate16). The shock makes Home more attractive to firms, increasing firm entry and boosting Home income. Higher income allows Home consumers to increase demand for both Home and Foreign goods. The rise in demand for Foreign varieties raises Foreign labor demand and wages, causing a short-run appreciation of Foreign wages and a depreciation of the terms of labor ($TOL_t = \frac{Qw_t^*/Z_t^*}{w_t/Z_t}$). TOL measures the cost of effective labor in the Foreign country relative to that in the Home country. In the long run, however, TOL appreciates. As aggregate productivity remains above its steady-state level for several periods after the shock, firm entry gradually rises, increasing the total number of firms (N). The resulting increase in labor demand pushes up the cost of effective labor in Home relative to Foreign, driving a gradual appreciation (decline) of TOL over time.

The benchmark model generates a greater increase in the average productivity of offshoring firms ($\tilde{z}_V$) following a positive productivity shock than the model without intangible investment. The short-run TOL depreciation raises offshoring costs, limiting firm entry into offshore production ($N_V \downarrow$). As a result, only more productive firms choose to offshore, increasing the average productivity of offshoring firms.

The introduction of intangible capital leads to a larger increase in Home firms' profits, which further raises Home income. This prolongs the excess demand for Foreign labor, extending the period of TOL depreciation. Thus, the subsequent appreciation of TOL is also dampened over time compared to Zlate16. As a result, the prolonged depreciation restrains the expansion of offshoring firms, leading to a more pronounced increase in average offshore productivity.

Intangible capital amplifies RER appreciation under a positive productivity shock, strengthening the HBS effect for two reasons. First, intangible capital induces short-run TOL depreciation, increasing Home firm entry, which lowers average productivity and raises domestic prices, further amplifying RER appreciation. Second, the greater increase in domestic entry leads to higher consumption of Home-produced varieties, shifting expenditure from Foreign-produced varieties to

Home-produced varieties, further strengthening RER appreciation. (A detailed explanation of how expenditure switching induces RER appreciation can be found in section 4.2.) This mechanism helps explain the empirical pattern in Figure 1 & 2: the post-GFC increase in U.S. intangible investment, combined with productivity growth, may have driven the observed RER appreciation.

The introduction of intangible capital boosts Home exports by reducing firms' marginal costs, enhancing price competitiveness during a Home productivity shock. This allows Home firms to offer lower prices than Foreign firms, significantly increasing Home exports.

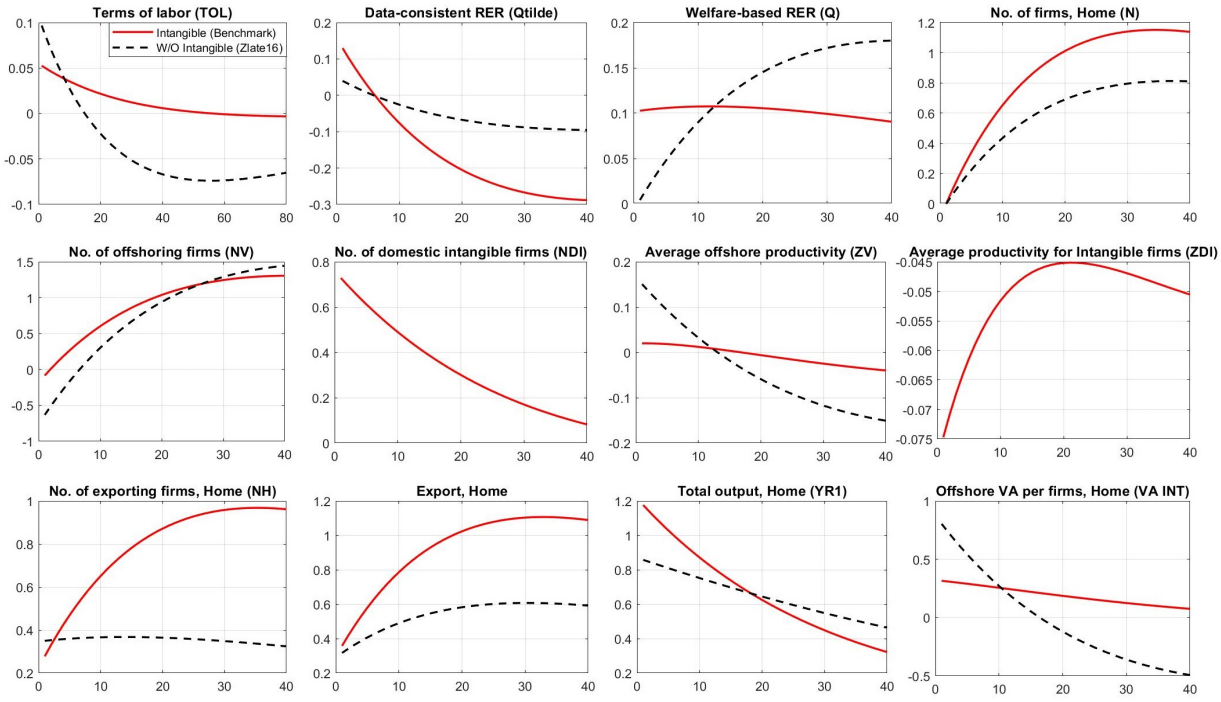


Figure 5: Comparing Impulse Responses: Benchmark Model and Model without Intangibles

Compared to the steady state in the model without intangible investment (Zlate16), the steady state of our model offers a foundation for assessing how intangible investment influences firms' long-term offshoring decisions. As shown in Figure 3, incorporating intangible capital lowers the offshoring productivity cutoff ($z_V \downarrow$), which in turn increases the number of offshore firms. This outcome stems from the mechanism in which intangible investment raises fixed costs but reduces marginal costs, thereby boosting offshore profits and encouraging more firms to choose offshoring. As a result, the productivity cutoff for offshoring falls relative to the model without intangible capital—consistent with our calibration results and the findings in Figure 3.

Furthermore, the introduction of intangible capital leads to a greater depreciation of the welfare-consistent RER. Welfare-consistent RER measures differences in a consumer's welfare derived from spending a given normal amount in each market ($Q_t \equiv \epsilon P_t^* / P_t$), capturing pure variety effects. A value above 1 indicates depreciation, meaning consumers in Home enjoy greater welfare due to access to more varieties. In Zlate16, the steady-state RER was 1.272—above 1—due to asymmetries in wages and sunk entry costs between Home and Foreign. With the introduction of intangible capital, lower marginal costs and prices encourage more firms to enter the Home market. This widens the variety gap and amplifies price differences between countries, leading to a further depreciation of the welfare-consistent RER, which rises to 1.485.

In addition, incorporating intangible investment dampens the appreciation of the data-consistent RER in the steady state. The data-consistent RER differs from the welfare-consistent RER in that it captures real price data and purges out pure variety effects. As shown in Table 3, the RER rises from 0.760 to 0.914 with intangible investment. Although the RER remains below 1—indicating continued appreciation—the extent of appreciation is clearly dampened. This effect arises for several reasons. First, relative to the Zlate16 model, intangible investment enables offshoring firms to transfer domestic wage pressure more effectively from Home to Foreign, thereby dampening TOL appreciation in the long run. Second, by enabling more efficient offshore production, intangibles weaken Foreign firms' export competitiveness. This leads to expenditure switching from less competitive imported varieties to cheaper offshore-produced goods, further dampening RER appreciation. Finally, as intangibles lower the prices of both offshoring and domestic firms, domestic consumers benefit from lower prices, reinforcing the moderation of RER appreciation.

	Benchmark	Zlate16	Difference
Offshoring cutoff, Home (z_V)	2.685	2.978	-0.293
Welfare-consistent RER (Q)	1.485	1.272	0.213
Data-consistent RER ($\widetilde{Q}$)	0.914	0.760	0.154
Share of the number of firms ($\frac{N}{N^*}$)	8.129	6.314	1.815

Table 3: Steady State Results: Benchmark vs. Zlate16

4.2 Real Exchange Rate Dynamics

The welfare consistent RER ($Q_t \equiv \epsilon P_t^* / P_t$) measures differences in a consumer's welfare derived

from spending a given normal amount in each market, capturing pure variety effects. When the number of Home varieties increases more than that of Foreign, the welfare consistent RER (Q_t) depreciates and exceeds 1. In contrast, the data-consistent RER captures real price data, but it purges out the pure variety effect. As in GM05, the data-consistent RER ($\widetilde{Q}_t$) is related to the welfare-consistent RER (Q_t) by the equation:

$$\widetilde{Q}_t = \left(\frac{N_t^*}{N_t}\right)^{\frac{1}{\theta-1}} Q_t \tag{25}$$

where $N_t = N_{D,t}^{NI} + N_{D,t}^{I} + N_V + N_X^*$ denotes the total number of varieties available to Home consumers, and $N_t^* = N_{D,t}^{NI*} + N_{D,t}^{I*} + N_X$ represents the total number of products available to Foreign consumers.

To derive the data-consistent RER, I decompose welfare-based price indexes P_t and P_t^* into components representing (i) product variety and (ii) average prices as $P_t = (N_{D,t}^{I} + N_{D,t}^{NI} + N_{V,t} + N_{X,t}^{*})^{\frac{1}{\theta-1}} \widetilde{P}_t$ for the Home country and $P_t^* = (N_{D,t}^{*I} + N_{D,t}^{*NI} + N_{X,t})^{\frac{1}{\theta-1}} \widetilde{P}_t^*$ for the Foreign country. The resulting expression for the data-consistent RER is:

$$\widetilde{Q}_{CPI,t}^{1-\theta} = \left(\frac{N_{D,t}^{I} + N_{D,t}^{NI} + N_{V,t} + N_{X,t}^{*}}{N_{D,t}^{*I} + N_{D,t}^{*NI} + N_{X,t}}\right) \frac{N_{D,t}^{*NI}\left(\frac{TOL_t}{\widetilde{z}_{D,t}^{*NI}}\right)^{1-\theta} + N_{D,t}^{*I}\left(\psi_{D,t}^{*\xi*}\frac{TOL_t}{\widetilde{z}_{D,t}^{*I}}\right)^{1-\theta} + N_{X,t}\left(\frac{\tau_t^*}{\widetilde{z}_{X,t}^{*}}\right)^{1-\theta}}{N_{D,t}^{NI}\left(\frac{1}{\widetilde{z}_{D,t}^{NI}}\right)^{1-\theta} + N_{D,t}^{I}\left(\frac{\psi_{D,t}^{\xi}}{\widetilde{z}_{D,t}^{I}}\right)^{1-\theta} + N_{V,t}\left(\frac{\psi_{V,t}^{\xi}\tau TOL_t}{\widetilde{z}_{D,t}^{I}}\right)^{1-\theta} + N_{X,t}^{*}\left(\frac{\tau TOL_t}{\widetilde{z}_{X,t}^{*}}\right)^{1-\theta}} \tag{26}$$

where the terms of labor $TOL_t = \frac{Qw_t^*/Z_t^*}{w_t/Z_t}$ measures the cost of effective labor in the Foreign country relative to that in the Home country; the iceberg trade costs τ_t and τ_t^* affect the imports of Home and Foreign, respectively. I log-linearize the data-consistent RER in equation (27) to derive economic implications. The derivation process is provided in the appendix F, and the final equation is as follows.

Analytical results The log-linearized version of (26) is:

$$\widetilde{\mathsf{Q}}_t = (s_D - s_V + s_D^* - 1)\mathsf{TOL}_t \tag{C1}$$

$$+ s_V(\tilde{\mathsf{z}}_{V,t} - \mathsf{t}_t) + (s_D - s_D^I - s_V)\tilde{\mathsf{z}}_{D,t}^{NI} + s_D^I\tilde{\mathsf{z}}_{D,t}^{I} \tag{C2,C3,C4}$$

$$+ (1 - s_D)(\tilde{\mathsf{z}}_{X,t}^{*} - \mathsf{t}_t) - (1 - s_D^*)(\tilde{\mathsf{z}}_{X,t} - \mathsf{t}_t^*) - (s_D^* - s_D^{*I})\tilde{\mathsf{z}}_{D,t}^{*NI} - s_D^{*I}\tilde{\mathsf{z}}_{D,t}^{*I} \tag{C5}$$

$$- \ s_V\xi\Psi_{V,t} - s_D^I\xi\Psi_{D,t} + s_D^{*I}\xi^*\Psi_{D,t}^* \tag{C6}$$

$$+\frac{1}{\theta-1}\left(s_V-\frac{N_V}{N_D+N_V+N_X^*}\right)\left(\mathsf{N}_{V,t}-\mathsf{N}_{X,t}^*\right) \tag{C7}$$

$$+\frac{1}{\theta-1}\left[\begin{array}{l}\left(\frac{N_D^{*NI}}{N_D^{*NI}+N_D^{*I}+N_X}-(s_D^*-s_D^{*I})\right)\left(\mathsf{N}_{D,t}^{*NI}-\mathsf{N}_{X,t}\right)\\ -\left(\frac{N_D^{NI}}{N_D^{NI}+N_D^I+N_V+N_X^*}-(s_D-s_D^I-s_V)\right)\left(\mathsf{N}_{D,t}^{NI}-\mathsf{N}_{X,t}^*\right)\end{array}\right] \tag{C8}$$

$$+\frac{1}{\theta-1}\left[\begin{array}{l}\left(\frac{N_D^{*I}}{N_D^{*NI}+N_D^{*I}+N_X}-s_D^{*I}\right)\left(\mathsf{N}_{D,t}^{*I}-\mathsf{N}_{X,t}\right)\\ -\left(\frac{N_D^I}{N_D^{NI}+N_D^I+N_V+N_X^*}-s_D^I\right)\left(\mathsf{N}_{D,t}^I-\mathsf{N}_{X,t}^*\right)\end{array}\right] \tag{C9}$$

Variables denoted in sans-serif font represent percent deviations from their respective steady states. Parameter s_D represents the steady-state share of Home's spending on varieties produced by Home firms, both domestically and offshore. Within s_D, s_D^I is the share spent on varieties produced by the domestic intangible firms, and s_V on varieties produced by offshoring firms. The remainder, s_D^{NI}, is denoted $s_D - s_D^I - s_V$, representing spending on varieties produced by domestic firms without intangible investment. Similarly, s_D^* is the steady-state share of Foreign's spending on varieties produced by Foreign firms domestically, with s_D^{*I} and $(s_D^* - s_D^{*I})$ denoting the shares for intangible and non-intangible firms, respectively. The average productivities of Foreign intangible and non-intangible firms, $\tilde{z}_D^{*I}$ and $\tilde{z}_D^{*NI}$, remain constant over time.

The calibration ensures that: (a) $(s_D - s_V + s_D^*) > 1$, as domestically produced varieties represent more than half of the consumption expenditure in each country; (b) $\left(\frac{N_D^I}{N_D^{NI}+N_D^I+N_V+N_X^*} - s_D^I\right) > 0$ and $\left(\frac{N_D^{*I}}{N_D^{*NI}+N_D^{*I}+N_X} - s_D^{*I}\right) > 0$, indicating that less productive intangible-investing firms have smaller market shares than their share in the total number of varieties in both countries; (c) $\left(\frac{N_D^{NI}}{N_D^{NI}+N_D^I+N_V+N_X^*} - (s_D - s_D^I - s_V)\right) > 0$ and $\left(\frac{N_D^{*NI}}{N_D^{*NI}+N_D^{*I}+N_X} - (s_D^* - s_D^{*I})\right) > 0$, meaning that less productive non-intangible firms also have smaller market shares than their share in the total number of varieties in both countries; and (d) $\left(s_V - \frac{N_V}{N_D+N_V+N_X^*}\right) > 0$, showing that more productive offshoring firms hold larger market shares than their share in total varieties. As in Zlate16, the model captures that more productive offshoring firms dominate market shares compared to less productive domestic firms, consistent with the empirical findings of Kurz (2006).

The log-linearized equation (26) shows nine channels (C1-C9) through intangible capital that affect the data-consistent RER: (1) price of non-traded varieties (via TOL_t); (2) offshore prices

(via the average offshore productivity $\tilde{\mathsf{z}}_{V,t}$, and the magnitude of trade costs t_t); (3) price of domestic non-intangible varieties (through average non-intangible firm productivity $\tilde{\mathsf{z}}^{NI}_{D,t}$); (4) price of domestic intangible varieties (via the average productivity of intangible firms $\tilde{\mathsf{z}}^{I}_{D,t}$); (5) relative import prices (via the average productivity of Foreign exporters $(\tilde{\mathsf{z}}^{*}_{X,t} - \mathsf{t}_t)$ and that of Home exporters $(\tilde{\mathsf{z}}_{X,t} - \mathsf{t}^{*}_t)$; (6) marginal cost reductions $(-s_V\xi\Psi_{V,t} - s^{I}_{D}\xi\Psi_{D,t} + s^{*I}_{D}\xi^{*}\Psi^{*}_{D,t})$; (7) relative difference in varieties between offshore varieties and imported varieties $\left(\mathsf{N}_{V,t} - \mathsf{N}^{*}_{X,t}\right)$; (8) relative difference in varieties between domestic non-intangible varieties and imported varieties $(\mathsf{N}^{NI}_{D,t} - \mathsf{N}^{*}_{X,t})$; and (9) relative difference in varieties between domestic intangible varieties and imported varieties $(\mathsf{N}^{I}_{D,t} - \mathsf{N}^{*}_{X,t})$.

Impulse responses Intangible investment amplifies data-consistent RER appreciation following a positive Home aggregate productivity shock. This is because it boosts domestic firm entry, lowering average productivity and raising domestic prices, leading to RER appreciation. The greater variety of Home goods further shifts consumer spending from Foreign to domestic, reinforcing this appreciation. Thus, intangibles strengthen the HBS effect, explaining the post-GFC RER appreciation alongside increased US intangible investment. This operates through channels C3, C4, C8, and C9. Figure 5 illustrates this pronounced RER appreciation, and we explore the contributing channels, specifically the additional impact of intangible investment compared to Zlate (2010).

(C1) Price changes of non-traded varieties Without intangibles, a Home aggregate productivity increase encourages firm entry and lead to an appreciation of the terms of labor in the long run (i.e., TOL_t decreases). This results in a relative increase in the average price of non-traded varieties in Home, causing an appreciation of the data-consistent RER (i.e., $\widetilde{Q}^{CPI}_t$ decreases). Intangible investment can dampen this RER appreciation through two channels: (a) It leads to a dampened appreciation of TOL compared to Zlate (2010) (i.e., causes TOL_t to decrease by a smaller amount) because offshoring firms' intangible investments more significantly transfer wage pressures abroad. (b) Simultaneously, intangibles increase the impact of TOL on RER by lowering the spending share on offshore varieties and raising it on domestic varieties. This is seen in the coefficient of TOL_t in channel C1 $(s_D - s_V + s^{*}_D - 1)$, which increases as the offshore share (s_V) falls and the domestic share (s_D) rises. Without intangible capital, TOL appreciation decreases the domestic spending share (s_D). However, intangible capital dampens the TOL

appreciation in the long run, preventing a sufficient decrease in the relative cost of hiring foreign labor for offshoring firms and mitigating upward pressure on domestic labor costs. This limits the expansion of offshoring firms, reducing the offshore spending share (s_V), and conversely, increases the domestic spending share (s_D). Consequently, intangible capital increases the coefficient ($s_D - s_V + s_D^* - 1$), further contributing to the dampened TOL appreciation.

(C2) Price changes of offshore varieties This channel mirrors Zlate (2010). Rising Foreign wages reduce the number of offshoring firms but increase their average productivity. Intangible investment amplifies this average productivity increase ($\tilde{z}_{V,t}$), leading to a larger price drop in offshore varieties and dampening RER appreciation. However, long-run TOL appreciation increases offshoring profitability, growing the number of offshoring firms and decreasing their average productivity $\tilde{z}_{V,t}$, raising their average price over time and contributing to RER appreciation.

(C3 and C4) Price changes of domestically produced goods (without and with intangible capital) Intangible capital incorporation increases Home firms' profits and income, depreciating TOL. This initially raises the number of both non-intangible (N_D^{NI}) and intangible (N_D^{I}) firms, decreases their respective average productivities ($\tilde{z}_{D,t}^{NI}$ and $\tilde{z}_{D,t}^{I}$), and increases their average prices, leading to RER appreciation. However, as TOL appreciates over time, the increase in the number of both types of domestic firms slows, causing their average productivities to rise and average prices to fall, thus dampening the RER appreciation effect.

(C5) Relative import price changes Without intangible investment, offshoring increases Foreign labor demand, putting upward pressure on Foreign wages and weakening the export competitiveness of Foreign firms. As a result, the average productivity of surviving Foreign exporters ($\tilde{z}_{X,t}^*$) rises relative to that of their Home counterparts ($\tilde{z}_{X,t}$), lowering the average price of imports in Home relative to Foreign and dampening RER appreciation. With intangible investment, this effect is amplified. Offshoring firms that invest in intangibles reduce marginal costs, further undermining the price competitiveness of Foreign firms. As a result, the average productivity of surviving Foreign exporters ($\tilde{z}_{X,t}^*$) rises further relative to their Home counterparts ($\tilde{z}_{X,t}$), lowering the average price of imports in Home and further dampening RER appreciation.

(C6) Changes in the fraction of marginal cost reduction The incorporation of intangible investment reduces marginal costs, leading to lower production prices. When the price of Home's offshore-produced varieties falls by $\Psi_{V,t}$, it causes RER depreciation. Similarly, a price decline

in domestically produced varieties by $\Psi_{D,t}$ due to intangible investment also contributes to RER depreciation. In contrast, a decline in the price of Foreign's domestically produced varieties by $\Psi^*_{D,t}$ leads to RER appreciation.

As discussed in (C1), intangible investment raises the spending share of domestically produced varieties with intangible investment in total spending (s^I_D) and reduces the share on offshore varieties (s_V). As a result, the depreciation effect on the RER caused by offshore varieties is diminished, while that from domestic intangible varieties becomes more pronounced.

(C7) Expenditure switching from imports towards offshore varieties Similar to the channel discussed in Zlate (2010), offshoring reduces Foreign export competitiveness. Since offshoring requires intangible investment, Foreign export competitiveness decreases even further. Consequently, Home consumers shift their expenditure from less competitive Foreign varieties to relatively cheaper offshore varieties, more significantly dampening RER appreciation.

(C8 and C9) Expenditure switching from imports towards domestic varieties (without and with intangible capital) When there is a positive aggregate productivity shock, the incorporation of intangibles generates higher profits for Home firms, resulting in a rise in Home income. Consequently, Home consumers spend more on Foreign-produced varieties, which increases Foreign labor demand and Foreign wages. This short-run TOL depreciation causes more domestic firms—both those investing in intangibles and those not—to enter the Home market. This firm entry in the Home country increases the number of domestic varieties ($\mathsf{N}^I_{D,t}$ and $\mathsf{N}^{NI}_{D,t}$) compared to the number of imported foreign varieties ($\mathsf{N}^*_{X,t}$). Consequently, Home consumers shift their expenditure away from imported varieties toward domestically produced varieties, despite their relatively higher prices. This shift leads to an RER appreciation. However, as TOL appreciates in the long run, the number of domestically producing firms decreases. This reduction leads to a rise in average productivity and a decrease in average prices, easing the appreciation pressure on RER.

4.3 Model Performance: Empirical TFP Calibration

To assess the benchmark model's ability to explain actual U.S. data, I evaluate its performance using quarterly U.S. TFP data from the San Francisco Fed. The Foreign TFP series was constructed by linearly interpolating annual China TFP data from the Penn World Table into

quarterly observations, extending the series post-2020 using labor productivity growth rates from CEIC.

Simulations using real TFP data demonstrate that the model effectively captures post-GFC TFP improvements and RER appreciation in the U.S. Figure 6 illustrates the productivity shock dynamics generated by the model when injected with real TFP shocks. The model accurately replicates the historical trend of U.S. TFP declining after 2000 and subsequently increasing following the GFC.

Furthermore, RER dynamics generated using actual U.S. TFP shocks closely match the observed U.S. RER trajectory (Figure 7). The benchmark model, which incorporates intangible capital, proves highly effective at capturing the pattern of RER depreciation in the 2000s followed by appreciation after the GFC. In particular, the benchmark model outperforms the Zlate16 model (which excludes intangibles) in tracking these depreciation and appreciation trends. As shown in Figure 7, the model without intangibles (Zlate16) quantitatively underestimates the changes in RER dynamics compared to the benchmark. Although the trends are qualitatively similar, the model without intangibles overestimates currency depreciation during the 2000s and subsequently overstates the appreciation following the GFC relative to the benchmark.

These results suggest that the rise in firm profits and the consequent increase in domestic firm entry driven by intangible investment are pivotal in explaining the post-GFC RER appreciation and the Harrod–Balassa–Samuelson (HBS) effect. Thus, incorporating intangibles into an offshoring framework is essential for explaining productivity enhancements in the tradable sector and the associated real exchange rate appreciation.

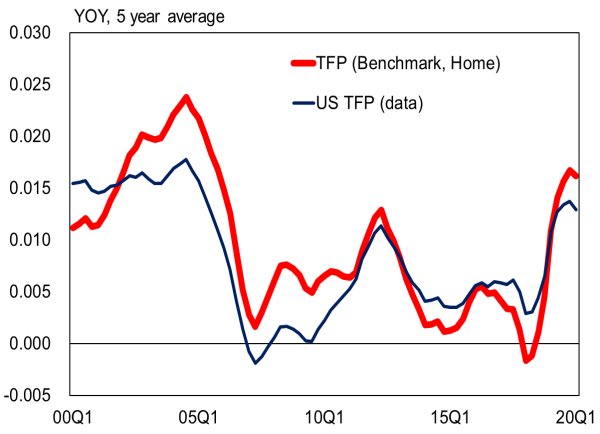


Figure 6: Productivity Growth in the U.S.
Source: Federal Reserve Bank of San Francisco

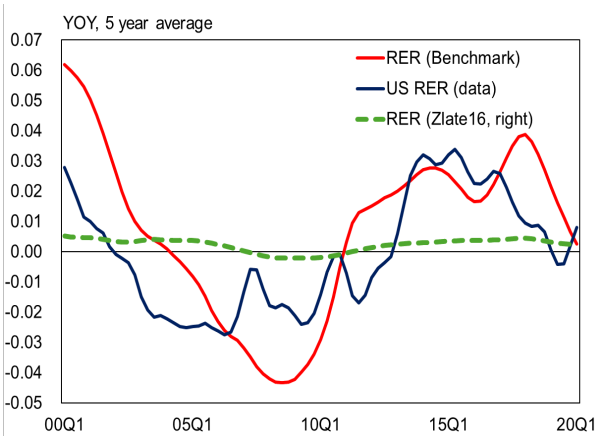


Figure 7: Real Exchange Rate Changes in the U.S.
Source: BIS

4.4 The Business Cycle Properties

The aggregate productivies Z_t and Z_t^* from the steady state follow the bivariate process:

$$\begin{bmatrix} logZ_t \\ logZ_t^* \end{bmatrix} = \begin{bmatrix} \rho_Z & \rho_{ZZ^*} \\ \rho_{Z^*Z} & \rho_{Z^*} \end{bmatrix} \begin{bmatrix} logZ_{t-1} \\ logZ_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix}$$

with the persistence parameters ρ_Z and $\rho_Z^* < 1$, spillovers ρ_{ZZ^*} and $\rho_{Z^*Z} \geq 1$, and normally-distributed, zero-mean technology shocks ε_t and ε_t^*.

To compare the effects of introducing intangibles, I derived the moments generated by the benchmark model using the parameters of Zlate16. Home exhibits a more persistent productivity process than Foreign ($\rho_Z = 0.996 > \rho_{Z^*} = 0.951$). Spillovers from Foreign to Home are negligible ($\rho_{ZZ^*} = 0.003$), whereas Home-to-Foreign spillovers remain positive ($\rho_{Z^*Z} = 0.049$). To ensure comparability, the shock variance is set equal for both countries using the average standard deviation across the two economies (0.0095^2), and the covariance is fixed at 0.2422×10^{-4}.

The model generates volatilities for output, consumption, and firm entry that are close to the US data, although the volatilities of offshore VA and exports are generally less than in the data. The introduction of intangibles to Zlate16 increased the standard deviation of output, offshore VA, and exports, allowing for volatility closer to real data.

The correlation between U.S. real GDP and offshore value-added, exports, consumption, and firm entry was found to be procyclical. The introduction of intangibles reduces the procyclicality of offshore value-added while increasing that of exports, yielding a closer fit to post-GFC data than Zlate16. The inclusion of intangibles raises domestic income, which boosts demand for foreign-produced varieties and, in turn, increases foreign labor demand and wages, raising offshore costs. This mechanism dampens the rise in offshore value-added relative to the model without intangibles, consistent with the observed post-GFC decline in business cycle procyclicality. Conversely, by enhancing the price competitiveness of exporters, intangibles expand exports and amplify their procyclicality, aligning with U.S. export patterns after the GFC.

Examining cross-country correlations, foreign exports are procyclical with home output, but their cyclicality weakens with the introduction of intangibles—consistent with the post-GFC decline ($corr(Y, EX^*) \downarrow$). The introduction of intangibles reduces the cross-country correlation of output while increasing the cross-country correlation of consumption. Therefore, compared to

Zlate16, intangibles exacerbate the consumption–output anomaly (BKK puzzle, $corr(Y, Y^*) \ll corr(C, C^*)$). Intangibles lower output–extensive margin correlation, aligning more closely with real data ($corr(Y, N_V) \downarrow$). While intangibles slightly reduce the relative consumption–RER correlation ($corr(C/C^*, \tilde{Q}) \downarrow$), the correlation remains strongly positive, failing to resolve the consumption–RER anomaly (Backus–Smith puzzle).

		Data		Model		
		2000–23	2010–23	Benchmark	Zlate16	GM05
% Standard deviation						
Output [1]		1.41	1.39	1.18	1.14	1.08
Standard deviation relative to GDP						
OffshoreVA [2]	BEA Census	4.20 4.13	5.06 4.30	1.08	0.99	–
Export		3.27	3.09	1.07	1.02	0.95
Consumption		1.06	1.26	0.76	0.82	0.81
New entrants [3]	Census	2.28	2.01	2.83	2.62	2.91
Correlation with GDP						
OffshoreVA [4]	BEA Census	0.82 0.58	0.58 0.59	0.73	0.84	–
Export		0.87	0.84	0.84	0.82	0.83
Consumption		0.92	0.95	1.00	0.99	0.99
New entrants [5]	Census	0.71	0.29	0.99	1.02	0.95
Cross-country correlation						
(Y, EX^*) [6]	US, G20 US, G10	0.55 0.66	0.50 0.65	0.82	0.84	0.87
(Y, Y^*)	US, G20 [7] US, G10	0.56 0.77	0.67 0.81	0.32	0.35	0.22
(C, C^*)	US, G20 US, G10	0.37 0.58	0.54 0.78	0.84	0.40	0.80
(Y, N_V)	No. of Foreign Affiliates [8]	0.48	0.36	0.59	0.97	–
$(C/C^*, \tilde{Q})$	US, G20 US, G10	0.01 -0.12	-0.01 -0.15	0.51	0.63	0.37

Table 4: International Business Cycles with Intangible Investment

Source: BEA, BIS, BLS, Census Bureau, IMF

Note: (1) The real GDP is used. (2) BEA data: Manufacturing imports from affiliates, 2000–2022; Census data: Imports from related trade in manufacturing industries, 2005–2023. All offshore VA data are deflated using the BLS import price index excluding petroleum. (3) Center for Economic Studies, Business Dynamics Statistics: Firm age 0 –5 Years. (4) BEA data: Manufacturing imports from affiliates, 2000–2022; Census data: Imports from related trade in manufacturing industries, 2005–2023. All offshore VA data are deflated using the BLS import price index excluding petroleum. (5) Center for Economic Studies, Business Dynamics Statistics: Firm age 0 –5 Years. (6) Average of correlations between individual countries' HP-filtered cycles and the U.S. cycle. Deviations from the HP-filtered series of U.S. imports by country are averaged. Nominal imports are used due to the lack of real import data by country. (7) Average of correlations between individual countries' HP-filtered cycles and the U.S. cycle. (8) Number of affiliates with assets, sales, or net income (+/-) greater than £25 million, AMNE data, year: 2000–2020.

4.5 A Dynamic Decomposition of Aggregate Productivity

I apply the dynamic Olley-Pakes decomposition (Melitz and Polanec, 2015) to analyze how intangible investment affects aggregate productivity dynamics. Aggregate productivity change from period t to t+1 is decomposed into: (1) changes in average productivity among survivors, (2) changes in the covariance between market share and productivity, (3) the market share-weighted productivity contribution of entrants relative to survivors, and (4) that of exiters. In this paper's model, components (1) and (2) are further disaggregated by non-intangible investing firms, intangible investing firms, and offshoring firms. The decomposition is formalized in equation (27).

Figure 8 shows the aggregate productivity decomposition of the benchmark model with intangibles in response to a permanent aggregate productivity shock. Since intangibles increase profits, they substantially raise the number of firms investing in intangibles in the short run, thereby dampening the initial productivity gains. In particular, the short-term decline in productivity gain is most pronounced due to the increased numbers of offshore firms and domestic intangible firms. Over time, however, the growth in the numbers of offshore and domestic intangible firms slows, allowing these firms to contribute positively to productivity gains. Figure 9 presents the results for Zlate16, which excludes intangibles. When a productivity shock occurs, the number of offshore firms rises sharply, contributing the most to the short-term reduction in productivity gains. Similar to the model with intangibles, as the expansion of offshore firms slows over time, they begin to contribute positively to productivity gains.

As shown in Figures 8 and 9, the introduction of intangibles not only amplifies the magnitude of aggregate productivity gains but also shortens the time required to transition to positive gains. This is because intangible investment increases demand for foreign-produced goods, thereby raising foreign wages and relatively slowing the entry of offshore firms, which results in a comparatively quicker and larger average offshore productivity gain. This suggests that when firms combine intangible capital with offshoring, the reallocation of resources toward more efficient firms occurs more rapidly, and, consequently, aggregate productivity gains are larger compared to offshoring without intangible capital investment. Thus, the benchmark model reaffirms Melitz's (2003) argument—the productivity-enhancing role of trade-within the context of intangible offshoring.

$$
\begin{aligned}
\triangle z_{t+1} &= \tilde{z}_{S,t+1} - \tilde{z}_{S,t} + \triangle cov_{S,t+1} + s_{E,t+1}(\Phi_{E,t+1} - \Phi_{S,t+1}) - s_{EX,t}(\Phi_{EX,t} - \Phi_{S,t}) \\
&= \underbrace{\triangle \tilde{z}^{NI}_{D,t+1} + \triangle \tilde{z}^{I}_{D,t+1} + \triangle \tilde{z}_{V,t+1}}_{\substack{\text{Average Productivity Changes} \\ \text{(Non-intangible / Intangible / Offshore firms)}}} \\
&+ \underbrace{\triangle cov_{NI,t+1} + \triangle cov_{I,t+1} + \triangle cov_{V,t+1}}_{\substack{\text{Changes in the Covariance Between Market Share and Productivity} \\ \text{(Non-intangible / Intangible / Offshore firms)}}} \\
&+ \underbrace{\frac{s_{E,t+1}\left\{ s_{E,t+1}\tilde{z}_{E,t+1} - \left(s^{NI}_{D,t+1}\tilde{z}^{NI}_{D,t+1} + s^{I}_{D,t+1}\tilde{z}^{I}_{D,t+1} + s_{V,t+1}\tilde{z}_{V,t+1} \right) \right\}}{(1-\delta)(s_{E,t} + s^{NI}_{D,t} + s^{I}_{D,t} + s_{V,t})}}_{\text{Market Share–Weighted Productivity Contribution of Entrants Relative to Survivors}} \\
&- \underbrace{\frac{s_{EX,t}\left\{ s_{EX,t}\tilde{z}_{EX,t} - \left(s^{NI}_{D,t}\tilde{z}^{NI}_{D,t} + s^{I}_{D,t}\tilde{z}^{I}_{D,t} + s_{V,t}\tilde{z}_{V,t} \right) \right\}}{(1-\delta)(s_{E,t-1} + s^{NI}_{D,t-1} + s^{I}_{D,t-1} + s_{V,t-1})}}_{\text{Market Share–Weighted Productivity Contribution of Exiters Relative to Survivors}}
\end{aligned}
\tag{27}
$$

where S: surviving firms (non-intangible/ intangible/ offshore firms), E: entrant firms, EX: exit firms, $s_{E,t}$: aggregate market share of entrant firms, $s_{EX,t}$: aggregate market share of exit firms, $\Phi_{E,t}$: aggregate productivity of entrant firms, $\Phi_{S,t}$: aggregate productivity of surviving firms

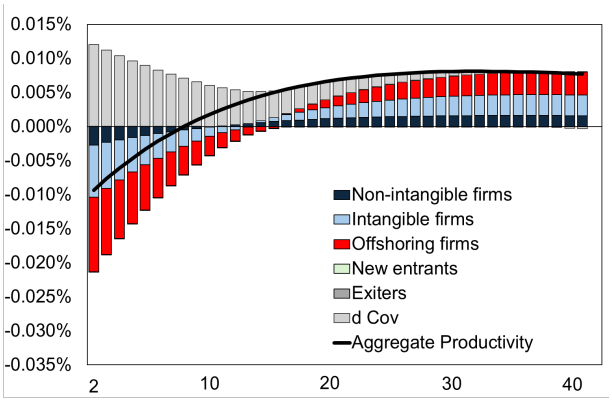


Figure 8: Accumulated Aggregate Productivity Change

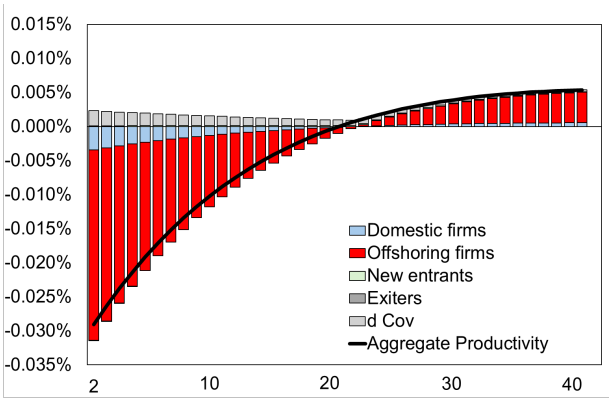


Figure 9: Accumulated Aggregate Productivity Change (Zlate16)

5 Conclusion

This paper develops a general equilibrium offshoring model where firms endogenously choose both offshoring and intangible investment. By integrating the scalable nature of intangible capital—modeled as a fixed cost that lowers marginal costs—we examine how intangibles influence firms' offshoring decisions, aggregate productivity, and external competitiveness.

The model successfully reproduces the U.S. RER appreciation following the GFC and provides a theoretical mechanism for the intensified Harrod–Balassa–Samuelson (HBS) effect. We show that following a positive productivity shock, intangible capital boosts profits, inducing the entry of lower-productivity firms. This compositional shift lowers average domestic productivity and raises domestic prices, thereby amplifying RER appreciation.

Using the Dynamic Olley-Pakes decomposition, I find that intangibles not only amplify aggregate productivity gains but also accelerate their realization. Intangible-driven increase in Foreign wages slows offshore entry and thereby generates quicker and larger average offshore productivity improvements than in the model without intangibles. Consequently, explicitly incorporating intangible investment demonstrates that trade-induced productivity improvements are substantially larger and materialize faster than previously thought, reinforcing the resource allocation channel highlighted by Melitz (2003).

Furthermore, the benchmark model captures the expansion of exports following a productivity shock more effectively than a model without intangibles, closely matching the increased export procyclicality observed in U.S. data. By reducing marginal costs, intangibles enhance the price competitiveness of exporters, leading to export growth. This consistency with empirical data suggests that intangibles are a crucial factor in explaining the procyclicality of exports.

On the other hand, the introduction of intangibles raise the foreign wages associated with production in Foreign plants, thereby reducing the value added (VA) and the extensive margin of offshore firms. Since intangible investment leads to a simultaneous increase in domestic production and a decrease in offshore VA, a pattern that parallels the declining procyclicality of offshore VA in U.S. data. These results underscore the necessity of incorporating intangibles to accurately evaluate the decisions of offshore firms.

In conclusion, this paper highlights the central role of intangibles in determining aggregate productivity and the RER. By bridging the gap between endogenous intangible investment and open economy macroeconomics, the findings offer significant implications for policy. As the global economy increasingly shifts toward intangible assets, our framework suggests that traditional trade models may underestimate the speed and magnitude of productivity gains from trade, while misdiagnosing the drivers of exchange rate fluctuations.

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Appendix A. Empirical Facts and Event Study

This study utilizes Compustat Segment data to establish stylized facts regarding the impact of intangible capital investment on offshoring decisions and aggregate productivity. These empirical findings provide the rationale for the assumptions adopted in the benchmark model presented in the main text.

The Compustat Segment data covers 28,900 North American firms, representing 67% of the total database. This dataset is instrumental for analyzing the operating structures and geographical segmentation of firms expanding into foreign markets. It is uniquely suited for this study as it not only facilitates the identification of offshore plant locations but also contains the micro-level data—specifically R&D and Selling, General, & Administrative (SG&A) expenses—necessary to construct aggregate measures of intangible investment.

Firms are identified using the "GEOTP" variable within the operating segment type classification. Because this variable details the geographic locations of operations, it allows for a clear distinction between the domestic plants and foreign affiliates of multinational corporations. Following Corrado et al. (2006) and Eisfeldt and Papanikolaou (2013), intangible investment is defined as the sum of R&D expenses and organizational capital investment, the latter calculated as 30% of SG&A expenses. For the purpose of this analysis, total factor productivity (TFP) is approximated as sales per employee, while marginal cost is proxied by the ratio of cost of goods sold (COGS) to sales, assuming COGS represents variable costs. Manufacturing industries are defined by NAICS codes 311–339. Based on this data, I document the following stylized facts:

(i) The scalability of intangible investment (Figure 10): Manufacturing firms demonstrate scalability, characterized by a negative correlation between intangible investment and marginal costs. This finding validates the assumption in the Section 3 benchmark model that intangible investment drives reductions in marginal cost.

(ii) Revenue growth with intangible investment (Figure 11): There is a positive relationship between intangible investment and average sales. As shown in Figure 11, average sales rise distinctively as manufacturing firms progress to higher R&D quantiles. This corroborates the model's implication that intangible capital accumulation contributes to increased firm profitability.

(iii) Productivity sorting into intangible investment and offshoring (Figure 12): More productive firms make higher intangible investments and engage more in offshoring. The benchmark model

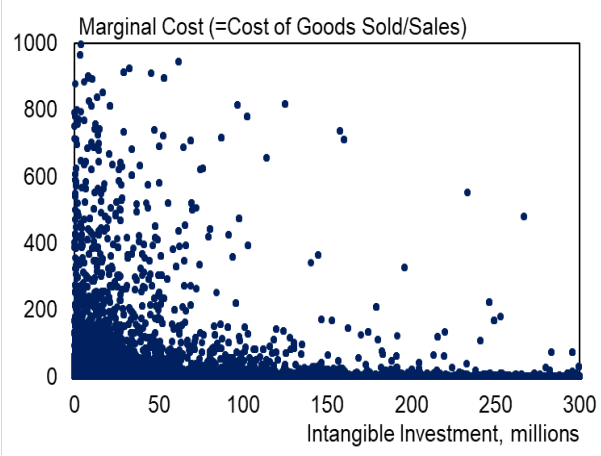


Figure 10: Scalability of Intangible Investment
Source: Compustat Segment

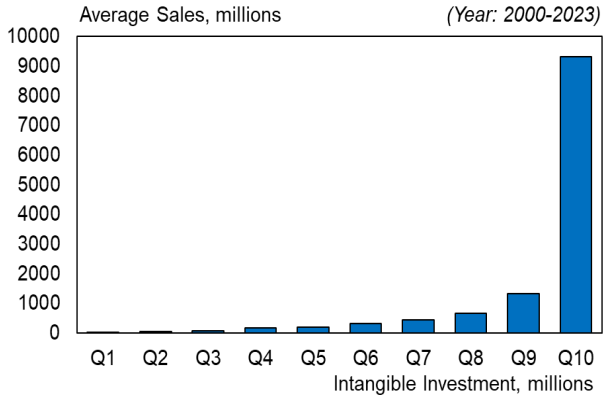


Figure 11: Quantiles of Intangible Investment and Average Sales (Manufacturing)
Source: Compustat Segment

relies on the assumption that only productive firms can overcome the fixed costs of intangible investment and offshoring. Figure 11 validates this assumption empirically: average productivity clearly increases alongside intangible investment quantiles. Additionally, offshore firms display higher average productivity than their domestic counterparts. These findings provide empirical evidence that high-productivity firms are significantly more likely to undertake both offshoring and intangible investment.

(iv) Offshore firms tend to have higher intangible investment and higher sales compared to non-offshore firms (Figure 13): Figure 13 presents a scatter plot of intangible investment against sales, distinguishing between offshore and domestic entities. The data reveals a significant positive correlation between sales and intangible investment for offshore firms relative to non-offshore firms. This pattern is consistent with the benchmark model, in which offshore firms achieve higher profits through intangible investment and therefore face stronger incentives to invest in intangibles. Based on this empirical fact, I assume—purely for simplicity in the benchmark—that offshore firms necessarily undertake intangible investment.

(v) Positive sales–intangible correlation in high-tech sectors (Figure 14): In manufacturing, intangible investments exhibit a stronger positive correlation with net sales growth within high-tech industries.[4] This observation confirms that among firms with significant intangible assets, technology-intensive firms achieve superior sales growth rates. Consequently, this suggests

[4]High-tech manufacturing includes industries classified under NAICS codes 325 (Chemicals), 334 (Computer and Electronic Products), 336 (Transportation Equipment), 333 (Machinery), 335 (Electrical Equipment), and 339 (Miscellaneous Manufacturing).

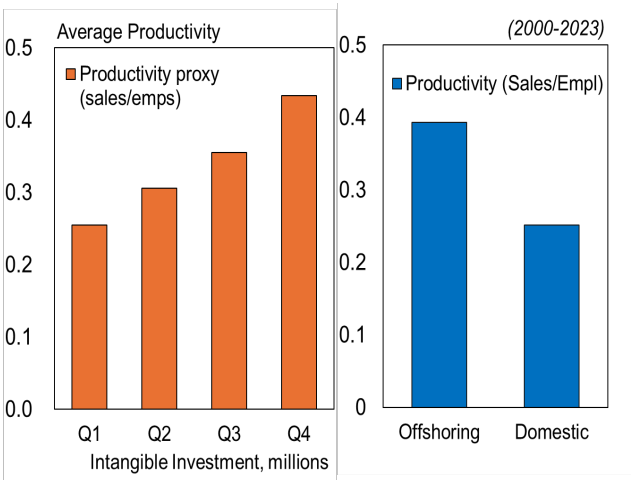


Figure 12: Intangibles Quantile & Productivity, Offshore vs. Domestic Productivity
Source: Compustat Segment

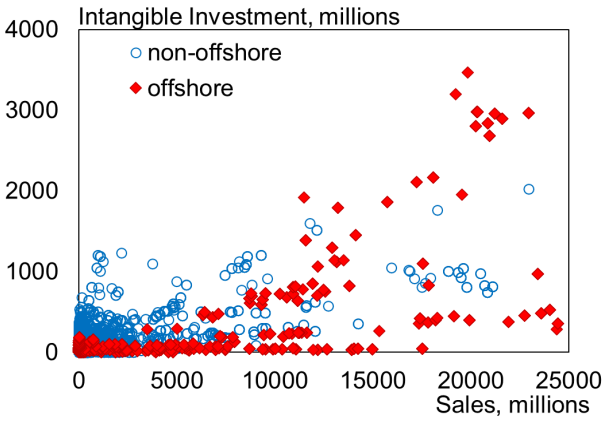


Figure 13: Offshore vs. Non-offshore Intangibles
Source: Compustat Segment
Note: Exclude data for non-offshore firms whose segments are classified as market, division, or product and service.

that the benchmark model is particularly effective for analyzing the dynamics of high-technology manufacturing firms.

(vi) Positive correlation between exports and intangible investment (Figure 15): For manufacturing firms, a rise in intangible investment is associated with increased export activity. In the benchmark model, both exporting and intangible investment entail fixed costs, meaning only firms capable of covering these costs can engage in them. In other words, firms with higher productivity are more likely to export and invest in intangibles. Figure 15 demonstrates this fact, showing that highly productive firms tend to increase both their exports and intangible investments.

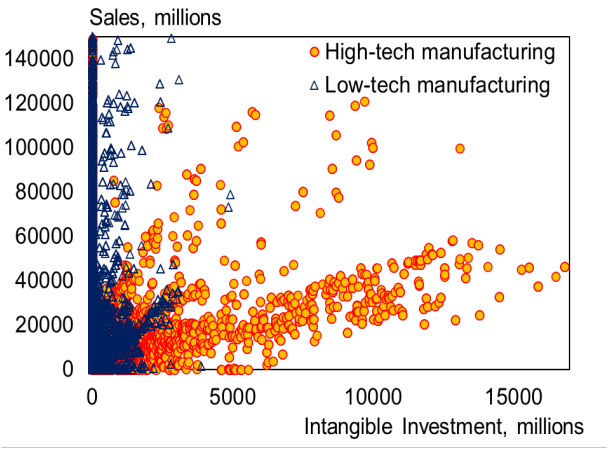


Figure 14: High- and Low-Tech Sales & Intangibles
Source: Compustat Segment

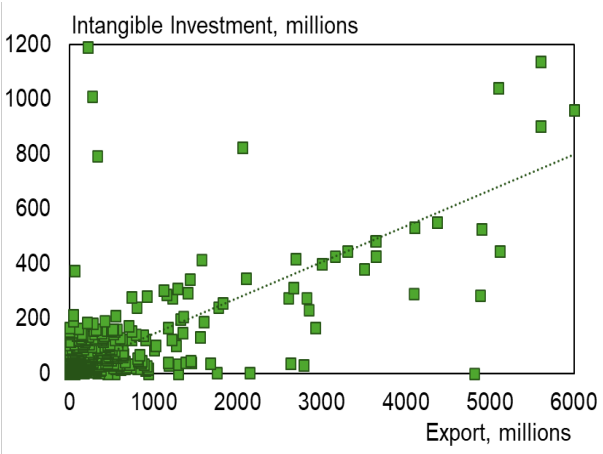


Figure 15: Export and Intangible Investment
Source: Compustat Segment

I executed an Event study using Equation (28) and Compustat Segment data to investigate the impact of intangible capital on sales, exports, offshore value-added, and productivity. To investigate how the introduction of intangible investment affects the decision-making of offshoring firms, I defined the event as a firm's R&D-to-sales ratio exceeding 10% compared to the previous year ($D_{i,t+j} = 1$ if firm i's R&D-to-sales ratio $\geq 10\%$ at relative year τ). To account for highly productive firms that already conduct intangible investment, I chose not to define the event as a transition from zero R&D investment to positive R&D. Relative year $j = 0$ corresponds to the event year when the R&D-to-sales ratio first exceeds 10%, and $j = -1$ is used as the omitted reference period. The unit of the event window is years.

$$y_{i,t} = \sum_{j=-5}^{7} \gamma_j D_{i,t+j} + \delta_i + \delta_t + \beta X_{i,t} + \epsilon_{i,t}, \tag{28}$$

$y_{i,t}$ represents sales, offshore value-added (VA), marginal cost, exports, and productivity, and γ_j captures the effect at relative year j. δ_i and δ_t capture firm and year fixed effects, respectively, and $X_{i,t}$ represents control variables.

The event study was conducted on manufacturing firms and, specifically, on offshore firms within that group. First, the Intangible investment of manufacturing firms was found to significantly increase net sales, export, and productivity at the 95% confidence level, relative to the control group, which experienced a comparatively smaller increase in R&D investment. The investment also significantly decreased marginal cost, thereby demonstrating a scalable nature. Specifically, as shown in Figure 16, the increase in intangible investment led to a significant rise in manufacturing firms' net sales starting from one year post-event (Figure 16 (a)), and a significant immediate decrease in marginal cost for two years following the event (Figure 16 (b)). Furthermore, export significantly increased for up to two years post-event following the rise in intangible investment (Figure 16 (c)), and productivity significantly increased starting from the fourth year post-event (Figure 16 (d)).

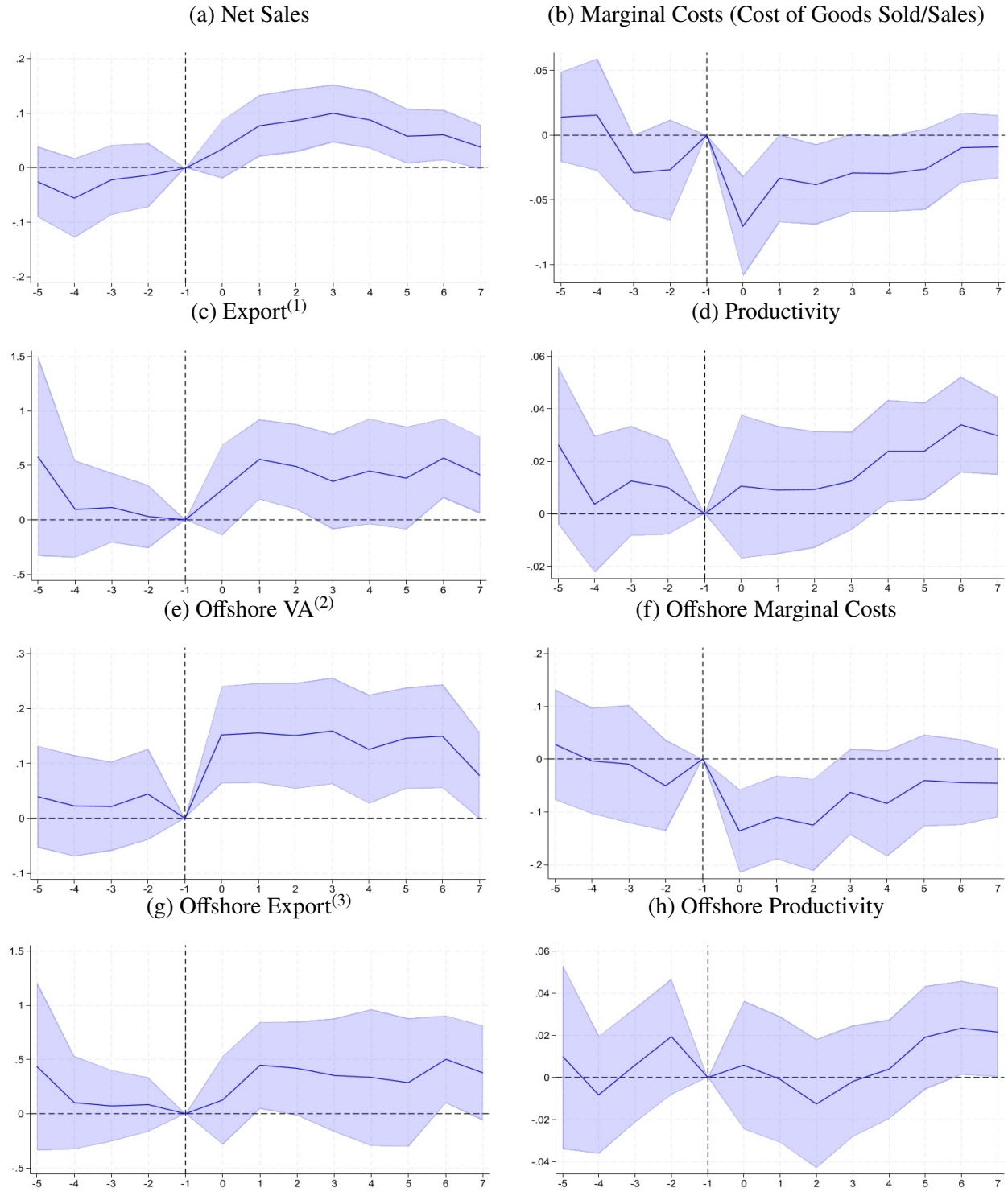
For offshore firms, the increase in intangible investment significantly increased offshore VA (Value Added) and also significantly reduced marginal cost. Offshore VA showed a significant increase over seven years post-event (Figure 16 (e)), and marginal cost significantly decreased for two years post-event, demonstrating scalability (Figure 16 (f)). Conversely, the degree of significant increase in offshore firms' exports and productivity was weaker compared to the overall

manufacturing firms. Nevertheless, export significantly increased in the first year post-event following the increase in offshore firms' intangible investment (Figure 16 (g)), and offshore productivity also showed a significant increase starting from the sixth year (Figure 16 (h)).

The detailed table of the Event study estimation results can be found in Appendix A.

Based on these results, we confirm, using micro-level evidence, that an increase in intangible investment significantly boosts the sales, export, and productivity of both manufacturing and offshore firms, while significantly lowering their marginal cost. This finding implies that the assumptions in the benchmark model—that firms with intangible investment experience increased profit and lower marginal cost—are supported by empirical evidence. Furthermore, the model's foundation, which posits that firms with intangible investment expand their exports and have higher productivity, is also corroborated by this event study.

Figure 16: Manufacturing Firms & Offshore Firms



Source: Compustat Segment
Note: Sample period is 2000–2023. For the dependent variables—net sales, offshore VA, export, marginal costs, and productivity measures—the inverse hyperbolic sine (IHS) transformation (asinh$(x) = \ln(x + \sqrt{x^2 + 1})$) was applied. (1) Missing values for exports are removed. (2) Offshoring VA is measured using sales of vertical offshore plants. (3) Missing values for exports are removed.

Table 5: Event Study Results

	Manufacturing Firms							
Dependent Variables	Log(Sales)	Log(Marg. Costs)	Log(Export)	Log(Productivity)	Log(Offsh. VA)	Log(Offsh. MC)	Log(Offsh. Export)	Log(Offsh. Prod.)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
4 years before	-0.025	0.014	0.583	0.026	0.040	0.028	0.439	0.010
	(0.033)	(0.018)	(0.464)	(0.015)	(0.047)	(0.053)	(0.394)	(0.022)
3 years before	-0.055	0.016	0.099	0.004	0.023	-0.003	0.104	-0.008
	(0.037)	(0.022)	(0.227)	(0.013)	(0.047)	(0.051)	(0.218)	(0.014)
2 years before	-0.022	-0.029	0.112	0.013	0.022	-0.009	0.074	0.006
	(0.033)	(0.015)	(0.162)	(0.011)	(0.041)	(0.057)	(0.168)	(0.014)
1 year before	-0.014	-0.027	0.029	0.010	0.044	-0.050	0.086	0.019
	(0.030)	(0.020)	(0.146)	(0.009)	(0.042)	(0.044)	(0.127)	(0.014)
Event	0.034	-0.070***	0.276	0.010	0.152***	-0.136***	0.124	0.006
	(0.027)	(0.020)	(0.212)	(0.014)	(0.045)	(0.040)	(0.208)	(0.016)
1 year after	0.077***	-0.033*	0.556***	0.009	0.155***	-0.111***	0.447**	-0.001
	(0.029)	(0.017)	(0.187)	(0.012)	(0.046)	(0.040)	(0.203)	(0.015)
2 years after	0.087***	-0.038**	0.489**	0.009	0.150***	-0.125***	0.417*	-0.012
	(0.029)	(0.016)	(0.199)	(0.011)	(0.049)	(0.045)	(0.220)	(0.016)
3 years after	0.100***	-0.029*	0.353	0.013	0.159***	-0.062	0.356	-0.002
	(0.027)	(0.015)	(0.223)	(0.010)	(0.049)	(0.041)	(0.266)	(0.014)
4 years after	0.088***	-0.030**	0.446*	0.024**	0.126**	-0.084	0.334	0.004
	(0.027)	(0.015)	(0.247)	(0.010)	(0.051)	(0.051)	(0.320)	(0.012)
5 years after	0.058**	-0.026*	0.385	0.024**	0.146***	-0.041	0.291	0.019
	(0.026)	(0.016)	(0.239)	(0.009)	(0.047)	(0.044)	(0.301)	(0.013)
6 years after	0.060**	-0.010	0.566***	0.034***	0.150***	-0.044	0.503**	0.024**
	(0.023)	(0.014)	(0.186)	(0.009)	(0.048)	(0.041)	(0.206)	(0.011)
7 years after	0.038*	-0.009	0.411**	0.030***	0.078*	-0.045	0.376*	0.022**
	(0.021)	(0.012)	(0.179)	(0.008)	(0.040)	(0.033)	(0.222)	(0.011)
Pre-trend	No	Yes	Yes	Yes	No	No	Yes	No
Firm lin. time trends	Yes	No	No	Yes	No	No	No	Yes
FE (year)	No	No	Yes	No	Yes	Yes	Yes	No
FE (firm)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Mean dep. Var.	4.6886	0.8716	3.0913	0.3459	5.0892	0.9215	3.0651	0.3766
# Cluster (firm)	4,307	4,421	413	4,525	1,404	1,403	360	1,573
N	95,996	100,763	5,172	127,255	21,823	21,617	4,449	30,247
Prob > F	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
R square	0.9696	0.8508	0.9457	0.6242	0.9599	0.7361	0.9499	0.8301

Notes: Robust standard errors in parentheses, clustered at the firm level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Dependent variables (export, offshore VA, productivity) are inverse hyperbolic sine transformed. Event study coefficients are relative to year -1 (normalized to 0). All regressions include firm fixed effects.

Appendix B. Model Summary (Financial Autarky)

Average profit (Home)	$\tilde{d}^I_{D,t}(z) = \frac{1}{\theta}\tilde{\rho}^I_{D,t}(z)^{1-\theta}C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)$ $\tilde{d}^{NI}_{D,t}(z) = \frac{1}{\theta}\tilde{\rho}^{NI}_{D,t}(z)^{1-\theta}C_t$ $\tilde{d}_{V,t}(z) = \frac{1}{\theta}\tilde{\rho}_{V,t}(z)^{1-\theta}C_t - f_V Q_t \frac{w^*_t}{Z^*_t} - Q_t\phi_{V,t}\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t} - 1)$ $\tilde{d}_{X,t}(z) = \frac{1}{\theta}\tilde{\rho}_{X,t}(z)^{1-\theta}C^*_t Q_t - f_X\frac{w_t}{Z_t}$
Average profit (Foreign)	$\tilde{d}^{I*}_{D,t}(z^*) = \frac{1}{\theta}\tilde{\rho}^{*I}_{D,t}(z^*)^{1-\theta}C^*_t - \phi^*_{D,t}\frac{w^*_t}{Z^*_t}(\psi^{*-\xi^*}_{D,t} - 1)$ $\tilde{d}^{NI*}_{D,t}(z^*) = \frac{1}{\theta}\tilde{\rho}^{*NI}_{D,t}(z^*)^{1-\theta}C^*_t$ $\tilde{d}^*_{X,t}(z^*) = \frac{1}{\theta}\tilde{\rho}^*_{X,t}(z^*)^{1-\theta}C_t Q_t^{-1} - f^*_X\frac{w^*_t}{Z^*_t}$
Average price (Home)	$\tilde{\rho}^I_{D,t}(z) = \frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t\tilde{z}^I_{D,t}}$ $\tilde{\rho}^{NI}_{D,t}(z) = \frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}^{NI}_{D,t}}$ $\tilde{\rho}_{V,t}(z) = \frac{\theta}{\theta-1}\tau Q_t\psi^{\xi}_{V,t}\frac{w^*_t}{Z^*_t\tilde{z}_{V,t}}$ $\tilde{\rho}_{X,t}(z) = \frac{\theta}{\theta-1}\tau^* Q_t^{-1}\frac{w_t}{Z_t\tilde{z}_{X,t}}$
Average price (Foreign)	$\tilde{\rho}^{I*}_{D,t}(z^*) = \frac{\theta}{\theta-1}\psi^{*\xi^*}_{D,t}\frac{w^*_t}{Z^*_t\tilde{z}^{I*}_{D,t}}$ $\tilde{\rho}^{NI*}_{D,t}(z^*) = \frac{\theta}{\theta-1}\frac{w^*_t}{Z^*_t\tilde{z}^{NI*}_{D,t}}$ $\tilde{\rho}^*_{X,t}(z^*) = \frac{\theta}{\theta-1}\tau Q_t\frac{w^*_t}{Z^*_t\tilde{z}^*_{X,t}}$
Free entry	$\tilde{v}_t = \frac{w_t}{Z_t}f_{E,t}$ $\tilde{v}^*_t = \frac{w^*_t}{Z^*_t}f^*_{E,t}$
Total number of firms	$N_t = N^I_{D,t} + N^{NI}_{D,t} + N_{V,t}$ $N^*_t = N^{I*}_{D,t} + N^{NI*}_{D,t}$
Law of motion, firms	$N_{t+1} = (1-\delta)(N_t + N_{E,t})$ $N^*_{t+1} = (1-\delta)(N^*_t + N^*_{E,t})$
Aggregate accounting	$C_t + N_{E,t}\tilde{v}_t(z) = w_t + N_t\tilde{d}_t(z)$ $C^*_t + N^*_{E,t}\tilde{v}^*_t(z^*) = w^*_t + N^*_{D,t}\tilde{d}^*_t(z^*)$
Consumption price index	$1 = N^I_{D,t}(\tilde{\rho}^I_{D,t}(z))^{1-\theta} + N^{NI}_{D,t}(\tilde{\rho}^{NI}_{D,t}(z))^{1-\theta} + N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta} + N^*_{X,t}(\tilde{\rho}^*_{X,t}(z))^{1-\theta}$ $1 = N^{I*}_{D,t}(\tilde{\rho}^{I*}_{D,t}(z^*))^{1-\theta} + N^{NI*}_{D,t}(\tilde{\rho}^{NI*}_{D,t}(z^*))^{1-\theta} + N_{X,t}(\tilde{\rho}_{X,t}(z^*))^{1-\theta}$
Total profits	$N_t\tilde{d}_t(z) = N^I_{D,t}\tilde{d}^I_{D,t}(z) + N^{NI}_{D,t}\tilde{d}^{NI}_{D,t}(z) + N_{V,t}\tilde{d}_{V,t}(z) + N_{X,t}\tilde{d}_{X,t}(z)$ $N^*_t\tilde{d}_t(z^*) = N^{I*}_{D,t}\tilde{d}^{I*}_{D,t}(z^*) + N^{NI*}_{D,t}\tilde{d}^{NI*}_{D,t}(z^*) + N^*_{X,t}\tilde{d}^*_{X,t}(z^*)$
Euler equations (Bonds)	$C_t^{-\gamma} = \beta(1+r_{t+1})E_t(C_{t+1})^{-\gamma}$ $C_t^{*-\gamma} = \beta(1+r^*_{t+1})E_t(C^*_{t+1})^{-\gamma}$

Euler equations (Shares)	$\tilde{v}_t = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{v}_{t+1}+\tilde{d}_{t+1})\right]$ $\tilde{v_t}^* = \beta^*(1-\delta^*)E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}(\tilde{v}^*_{t+1}+\tilde{d}^*_{t+1})\right]$
Average productivity	$\tilde{z}^I_{D,t} = \nu z_{min} z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)}-z_{min}^{k-(\theta-1)}}{z_{V,t}^k - z_{min}^k}\right]^{\frac{1}{\theta-1}}$ $\tilde{z}^{NI}_{D,t} = \nu z_{min} z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)}-z_{min}^{k-(\theta-1)}}{z_{V,t}^k - z_{min}^k}\right]^{\frac{1}{\theta-1}}$ $\tilde{z}_{V,t} = \nu z_{min}\left(\frac{N_t}{N_{V,t}}\right)^{1/k}$ $\tilde{z}_{X,t} = \nu z_{min}\left(\frac{N_t}{N_{X,t}}\right)^{1/k}$ $\tilde{z}^{I*}_{D,t} = \nu z^*_{min}\left(\frac{N^*_t}{N^{I*}_{D,t}}\right)^{1/k}$ $\tilde{z}^*_{X,t} = \nu z^*_{min}\left(\frac{N^*_t}{N^*_{X,t}}\right)^{1/k}$
Balanced trade	$N_{X,t}(\tilde{\rho}_{X,t}(z))^{1-\theta}C^*_t Q_t + N_{V,t}\tilde{d}_{V,t}(z) = N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta}C^*_t Q_t + N^*_{X,t}(\tilde{\rho}^*_{X,t}(z^*))^{1-\theta}C_t$

Appendix C. Average Productivity

C.1 The Average Productivity of Offshoring Firms

The average productivity of Home firms that produce offshore is obtained by integrating over the range of the support interval above the offshoring productivity cutoff:

$$\tilde{z}_{V,t} = \left[\frac{1}{1-G(z_{V,t})}\int_{z_{V,t}}^{\infty} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} = \left[\left(\frac{z_{V,t}}{z_{min}}\right)^{k}\int_{z_{V,t}}^{\infty} z^{\theta-1}\frac{kz_{min}^{k}}{z^{k+1}}dz\right]^{\frac{1}{\theta-1}}$$
$$= \left[\left(\frac{z_{V,t}}{z_{min}}\right)^{k}\frac{kz_{min}^{k}}{k-(\theta-1)}z_{V,t}^{\theta-1-k}\right]^{\frac{1}{\theta-1}} = \nu z_{V,t}, \text{ where } \nu \equiv \left[\frac{k}{k-(\theta-1)}\right]^{\frac{1}{\theta-1}}$$

C.2 The Average Productivity of Domestic Firms

The average productivity of Home firms that produce domestically with intangible investment is obtained by integrating over the range of the support interval above the intangible productivity cutoff and below the offshoring productivity cutoff:

$$\tilde{z}_{D,t}^{I} = \left[\frac{1}{G(z_{V,t})-G(z_{D,t}^{I})}\int_{z_{D,t}^{I}}^{z_{V,t}} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} = \left[\frac{(z_{D,t}^{I}z_{V,t})^{k}}{z_{min}^{k}(z_{V,t}^{k}-z_{D,t}^{I\,k})}\int_{z_{D,t}^{I}}^{z_{V,t}} z^{\theta-1}\frac{kz_{min}^{k}}{z^{k+1}}dz\right]^{\frac{1}{\theta-1}}$$
$$= \left[\frac{(z_{D,t}^{I}z_{V,t})^{k}}{z_{V,t}^{k}-z_{D,t}^{I\,k}}\frac{k}{k-(\theta-1)}(z_{D,t}^{I\,\theta-1-k}-z_{V,t}^{\theta-1-k})\right]^{\frac{1}{\theta-1}} = \nu\left[\frac{(z_{D,t}^{I}z_{V,t})^{k}}{z_{V,t}^{k}-z_{D,t}^{I\,k}}\left(\frac{1}{z_{D,t}^{I\,k-(\theta-1)}}-\frac{1}{z_{V,t}^{k-(\theta-1)}}\right)\right]^{\frac{1}{\theta-1}}$$
$$= \nu z_{D,t}^{I}z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)}-z_{D,t}^{I\,k-(\theta-1)}}{z_{V,t}^{k}-z_{D,t}^{I\,k}}\right]^{\frac{1}{\theta-1}}$$

The average productivity of Home firms that produce domestically without intangible investment is obtained by integrating over the range of the support interval above the minimum level of the productivity and below the intangible productivity cutoff:

$$\tilde{z}_{D,t}^{NI} = \left[\frac{1}{G(z_{D,t}^{I})}\int_{z_{min}}^{z_{D,t}^{I}} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} = \left[\frac{z_{D,t}^{Ik}}{z_{D,t}^{I\,k}-z_{min}^{k}}\int_{z_{min}}^{z_{D,t}^{I}} z^{\theta-1}\frac{kz_{min}^{k}}{z^{k+1}}dz\right]^{\frac{1}{\theta-1}}$$
$$= \left[\frac{z_{D,t}^{Ik}}{z_{D,t}^{I\,k}-z_{min}^{k}}\frac{kz_{min}^{k}}{k-(\theta-1)}(z_{min}^{\theta-1-k}-z_{D,t}^{I\theta-1-k})\right]^{\frac{1}{\theta-1}} = \nu\left[\frac{(z_{min}z_{D,t}^{I})^{k}}{z_{D,t}^{I\,k}-z_{min}^{k}}\left(\frac{1}{z_{min}^{k-(\theta-1)}}-\frac{1}{z_{D,t}^{I\,k-(\theta-1)}}\right)\right]^{\frac{1}{\theta-1}}$$
$$= \nu z_{min}z_{D,t}^{I}\left[\frac{z_{D,t}^{I\,k-(\theta-1)}-z_{min}^{k-(\theta-1)}}{z_{D,t}^{I\,k}-z_{min}^{k}}\right]^{\frac{1}{\theta-1}}$$

Appendix D. Average Profits from Domestic Production with and without Intangible Investment, and from Offshore Production

D.1 The relationship between the average profit of offshoring firms and the average profit of domestic intangible firms

The average profit of Home firms that produce domestically with intangible investment is:

$$\begin{aligned}
\tilde{d}^I_{D,t} = d^I_{D,t}(\tilde{z}^I_{D,t}) &= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t\tilde{z}^I_{D,t}}\right]^{1-\theta} C_t - \phi_{D.t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t}\right]^{1-\theta} C_t\tilde{z}^{I\theta-1}_{D,t} - \phi_{D.t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t}\right]^{1-\theta} C_t(\nu z^I_{D,t}z_{V,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t}-z^{Ik-(\theta-1)}_{D,t}}{z^k_{V,t}-z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_tz_{V,t}}\right]^{1-\theta} C_t(\nu z^I_{D,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t}-z^{Ik-(\theta-1)}_{D,t}}{z^k_{V,t}-z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= d^I_{D,t}(z_{V,t})(\nu z^I_{D,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t}-z^{Ik-(\theta-1)}_{D,t}}{z^k_{V,t}-z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)
\end{aligned}$$

The average profit of the Foreign firms producing offshore is:

$$\begin{aligned}
\tilde{d}_{V,t} = d_{V,t}(\tilde{z}_{V,t}) &= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{V,t}\tau Q_t\frac{w^*_t}{Z^*_t\tilde{z}_{V,t}}\right]^{1-\theta} C_t - f_VQ_t\frac{w^*_t}{Z^*_t} - \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{V,t}\tau Q_t\frac{w^*_t}{Z^*_t}\right]^{1-\theta} C_t\tilde{z}^{\theta-1}_{V,t} - f_VQ_t\frac{w^*_t}{Z^*_t} - \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1) \\
&= \left\{\frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{V,t}\tau Q_t\frac{w^*_t}{Z^*_tz_{V,t}}\right]^{1-\theta} C_t - f_VQ_t\frac{w^*_t}{Z^*_t} - \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)\right\}\nu^{\theta-1} \\
&\quad + (\nu^{\theta-1}-1)\left\{f_VQ_t\frac{w^*_t}{Z^*_t} + \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)\right\} \\
&= \frac{k}{k-(\theta-1)}d_{V,t}(z_{V,t}) + \frac{\theta-1}{k-(\theta-1)}\left\{f_VQ_t\frac{w^*_t}{Z^*_t} + \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)\right\}
\end{aligned}$$

The Home firm with productivity equal to the cutoff $z_{V,t}$ is indifferent between producing domestically or offshore. I use the equality of profits at the productivity cutoff, $d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})$:

$$\begin{aligned}
\tilde{d}_{V,t} &= \frac{k}{k-(\theta-1)} d_{V,t}(z_{V,t}) + \frac{\theta-1}{k-(\theta-1)} \left\{ f_V Q_t \frac{w_t^*}{Z_t^*} + \phi_{V,t} Q_t \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \right\} \\
&= {z_{D,t}^I}^{1-\theta} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^I}^{k-(\theta-1)}}{z_{V,t}^k - {z_{D,t}^I}^k} \right]^{-1} \left\{ \tilde{d}_{D,t}^I(z_{V,t}) + \phi_{D,t} \frac{w_t}{Z_t} (\psi_{D,t}^{-\xi} - 1) \right\} \\
&\quad + \frac{\theta-1}{k-(\theta-1)} \left\{ f_V Q_t \frac{w_t^*}{Z_t^*} + \phi_{V,t} Q_t \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \right\} \\
&= \frac{k}{k-(\theta-1)} \left(\frac{z_{V,t}}{\tilde{z}_{D,t}^I} \right)^{\theta-1} \tilde{d}_{D,t}^I(z_{V,t}) + \frac{\theta-1}{k-(\theta-1)} \left\{ f_V Q_t \frac{w_t^*}{Z_t^*} + \phi_{V,t} Q_t \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \right\}
\end{aligned}$$

The calculation process utilizes the domestic intangible firms' average productivity equation derived in Appendix C.

$$\tilde{z}_{D,t}^I = \nu z_{D,t}^I z_{V,t} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^I}^{k-(\theta-1)}}{z_{V,t}^k - {z_{D,t}^I}^k} \right]^{\frac{1}{\theta-1}}$$

$$\Rightarrow {z_{D,t}^I}^{1-\theta} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^I}^{k-(\theta-1)}}{z_{V,t}^k - {z_{D,t}^I}^k} \right]^{-1} = \nu^{\theta-1} \left(\frac{z_{V,t}}{\tilde{z}_{D,t}^I} \right)^{\theta-1} = \frac{k}{k-(\theta-1)} \left(\frac{z_{V,t}}{\tilde{z}_{D,t}^I} \right)^{\theta-1}$$

D.2 The relationship between the average profit of domestic intangible firms and the average profit of domestic non-intangible firms

The average profit of Home firms that produce domestically without intangible investment is:

$$\begin{aligned}
\tilde{d}_{D,t}^{NI} &= d_{D,t}^{NI}(\tilde{z}_{D,t}^{NI}) = \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}_{D,t}^{NI}}\right]^{1-\theta} C_t = \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right]^{1-\theta} C_t\tilde{z}_{D,t}^{NI\theta-1} \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right]^{1-\theta} C_t(\nu z_{min} z_{D,t}^{I})^{\theta-1}\left[\frac{z_{D,t}^{Ik-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_{D,t}^{Ik} - z_{min}^{k}}\right] \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t z_{D,t}^{I}}\right]^{1-\theta} C_t(\nu z_{min})^{\theta-1}\left[\frac{z_{D,t}^{Ik-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_{D,t}^{Ik} - z_{min}^{k}}\right] \\
&= d_{D,t}^{NI}(z_{D,t}^{I})(\nu z_{min})^{\theta-1}\left[\frac{z_{D,t}^{Ik-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_{D,t}^{Ik} - z_{min}^{k}}\right]
\end{aligned}$$

The average profit of Home firms that produce domestically with intangible investment is:

$$\begin{aligned}
\tilde{d}_{D,t}^{I} &= d_{D,t}^{I}(\tilde{z}_{D,t}^{I}) = \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t\tilde{z}_{D,t}^{I}}\right]^{1-\theta} C_t - \phi_{D.t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t}\right]^{1-\theta} C_t\tilde{z}_{D,t}^{I\theta-1} - \phi_{D.t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t}\right]^{1-\theta} C_t(\nu z_{D,t}^{I} z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t z_{D,t}^{I}}\right]^{1-\theta} C_t(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= (\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right]\left\{\frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t z_{D,t}^{I}}\right]^{1-\theta} C_t - \phi_{D.t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1)\right\} \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= (\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right] d_{D,t}^{I}(z_{D,t}^{I}) \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1)
\end{aligned}$$

The Home firm with productivity equal to the cutoff $z^I_{D,t}$ is indifferent between producing domestically with intangible investment or without it. I use the equality of profits at the productivity cutoff, $d^{NI}_{D,t}(z^I_{D,t}) = d^I_{D,t}(z^I_{D,t})$:

$$
\begin{aligned}
\tilde{d}^I_{D,t} &= (\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] d^I_{D,t}(z^I_{D,t}) \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= (\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right]\left\{(\nu z_{min})^{1-\theta}\left[\frac{z_{D,t}^{I\,k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\,k}_{D,t} - z^k_{min}}\right]^{-1}\tilde{d}^{NI}_{D,t}(z^I_{D,t})\right\} \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= (z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right](z_{min})^{1-\theta}\left[\frac{z_{D,t}^{I\,k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\,k}_{D,t} - z^k_{min}}\right]^{-1}\tilde{d}^{NI}_{D,t}(z^I_{D,t}) \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= \frac{k}{k-(\theta-1)}\left(\frac{z^I_{D,t}z_{V,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right]\tilde{d}^{NI}_{D,t}(z^I_{D,t}) \\
&\quad + \left\{\frac{k}{k-(\theta-1)}z_{V,t}^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1)
\end{aligned}
$$

The calculation process utilizes the domestic non-intangible firms' average productivity equation derived in Appendix C.

$$
\tilde{z}^{NI}_{D,t} = \nu z_{min} z^I_{D,t}\left[\frac{z_{D,t}^{I\,k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\,k}_{D,t} - z^k_{min}}\right]^{\frac{1}{\theta-1}}
$$

$$
\Rightarrow z_{min}^{1-\theta}\left[\frac{z_{D,t}^{I\,k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\,k}_{D,t} - z^k_{min}}\right]^{-1} = \nu^{\theta-1}\left(\frac{z^I_{D,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1} = \frac{k}{k-(\theta-1)}\left(\frac{z^I_{D,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1}
$$

Appendix E. Asymmetric Steady State

In this appendix, I derive the steady-state solution for the offshoring model incorporating intangible capital, accounting for cross-country asymmetry in the cost of effective labor (i.e., the terms of labor, $TOL < 1$).

To ascertain the unique steady-state solution, I obtain a numerical solution by resolving a system of 17 non-linear equations with 17 endogenous variables. The specific equations, numbered (29) through (45), are detailed herein. The variables in question represent the steady-state magnitudes of: z_V (the offshoring productivity cutoff in Home); z_H (the exporting productivity cutoff in Home); TOL (the terms of labor); $\frac{C}{C^*Q}$ (the real consumption ratio in units of the same consumption basket); Q (the real exchange rate); $\frac{d_D^I}{w}$, $\frac{d_D^{NI}}{w}$, $\frac{d_V}{w}$, $\frac{d_X}{w}$ (Home's real firm profits, specifically for domestic production with and without intangible investment, offshore production, and export market production, all divided by the real wage in Home); $\frac{d_D^{I*}}{w^*}$, $\frac{d_D^{NI*}}{w^*}$ (Foreign's real firm profits from domestic production, with and without intangible investment, divided by the real wage in Foreign); z_X^* (the exporting productivity cutoff in Foreign); ρ_X (the average exporting price in Home), ρ_X^*(the average exporting price in Foreign), $\frac{N}{N^*}$(the ratio of the number of firms between Home and Foreign). Once these 17 variables are numerically determined, I utilize them to derive the steady-state values of all remaining variables. For the purpose of calculating the steady-state solution, the following price and profit formulations are instrumental.

Using average profit and price in appendix B, this appendix computes the steady-state solution. Introducing $v = \frac{\beta(1-\delta)}{1-\beta(1-\delta)}d$, $N_E = \frac{\delta}{1-\delta}N$, and $v = f_E w$ into the Home firms' total profits, we can derive the first equation of the system.

$$\frac{1-\beta(1-\delta)}{\beta(1-\delta)}f_E = \frac{N_D^I}{N}\frac{\widetilde{d}_D^I}{w} + \frac{N_D^{NI}}{N}\frac{\widetilde{d}_D^{NI}}{w} + \frac{N_V}{N}\frac{\widetilde{d}_V}{w} + \frac{N_X}{N}\frac{\widetilde{d}_X}{w} \tag{29}$$

where $\frac{N_X}{N} = \left(\frac{1}{z_X}\right)^k$, $\frac{N_D^I}{N} = \left(\frac{1}{z_D^I}\right)^k - \left(\frac{1}{z_V}\right)^k$, $\frac{N_D^{NI}}{N} = 1 - \left(\frac{1}{z_D^I}\right)^k$, $\frac{N_V}{N} = \left(\frac{1}{z_V}\right)^k$.

Then, the profit formulas in the Home country are represented as follows:

$$\frac{\widetilde{d}_D^I}{w} = \frac{k}{k-(\theta-1)}f_X\frac{C}{C^*Q}Q^{1-\theta}\tau^{*(\theta-1)}\psi_D^{\xi(1-\theta)}\left(\frac{z_D^I z_V}{z_X}\right)^{\theta-1}\left[\frac{z_V^{k-(\theta-1)} - z_D^{I\,k-(\theta-1)}}{z_V^k - z_D^{I\,k}}\right] - \phi_D(\psi_D^{-\xi}-1) \tag{30}$$

$$\frac{\widetilde{d}_D^{NI}}{w} = \frac{k}{k-(\theta-1)} f_X \frac{C}{C^*Q} Q^{1-\theta} \tau^{*(\theta-1)} \left(\frac{z_{min} z_D^I}{z_X} \right)^{\theta-1} \left[\frac{z_D^{I\,k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_D^{I\,k} - z_{min}^k} \right] \tag{31}$$

$$\frac{\widetilde{d}_V}{w} = \frac{k}{k-(\theta-1)} f_X \frac{C}{C^*Q} Q^{1-\theta} \left(\frac{\tau^*}{\tau} \right)^{(\theta-1)} \psi_V^{\xi(1-\theta)} TOL^{1-\theta} \left(\frac{z_V}{z_X} \right)^{\theta-1} - f_V TOL - \phi_V TOL(\psi_V^{-\xi} - 1) \tag{32}$$

$$\frac{\widetilde{d}_X}{w} = \frac{\theta-1}{k-(\theta-1)} f_X \tag{33}$$

$$\begin{aligned} \frac{\widetilde{d}_V}{w} =& z_D^{I\,1-\theta} \left[\frac{z_V^{k-(\theta-1)} - z_D^{I\,k-(\theta-1)}}{z_V^k - z_D^{I\,k}} \right]^{-1} \left\{ \frac{\widetilde{d}_D^I}{w} + \phi_D(\psi_D^{-\xi} - 1) \right\} \\ & - \frac{k}{k-(\theta-1)} \phi_D(\psi_D^{-\xi} - 1) + \frac{\theta-1}{k-(\theta-1)} TOL \left\{ f_V + \phi_V(\psi_V^{-\xi} - 1) \right\} \end{aligned} \tag{34}$$

$$\begin{aligned} \frac{\widetilde{d}_D^I}{w} =& \left(\frac{z_V}{z_{min}} \right)^{\theta-1} \left[\frac{z_V^{k-(\theta-1)} - z_D^{I\,k-(\theta-1)}}{z_V^k - z_D^{I\,k}} \right] \left[\frac{z_D^{I\,k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_D^{I\,k} - z_{min}^k} \right]^{-1} \frac{\widetilde{d}_D^{NI}}{w} \\ & + \left\{ \frac{k}{k-(\theta-1)} z_V^{\theta-1} \left(\frac{z_V^{k-(\theta-1)} - z_D^{I\,k-(\theta-1)}}{z_V^k - z_D^{I\,k}} \right) - 1 \right\} \phi_D(\psi_D^{-\xi} - 1) \end{aligned} \tag{35}$$

Then, the profit formulas in the Foreign country are represented as follows:

$$\frac{\widetilde{d}_D^{*\,I}}{w^*} = \frac{k}{k-(\theta-1)} f_X^* \frac{C^*Q}{C} (\tau^* Q)^{(\theta-1)} \psi_D^{*\xi^*(1-\theta)} \left(\frac{z_D^{I*}}{z_X^*} \right)^{\theta-1} - \phi_D^*(\psi_D^{*-\xi^*} - 1) \tag{36}$$

$$\frac{\widetilde{d}_D^{*\,NI}}{w^*} = \frac{k}{k-(\theta-1)} f_X^* \frac{C^*Q}{C} (\tau^* Q)^{(\theta-1)} \left(\frac{z_{min}^* z_D^{I*}}{z_X^*} \right)^{\theta-1} \left[\frac{z_D^{I*\,k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{z_D^{I*\,k} - z_{min}^{*k}} \right] \tag{37}$$

$$\frac{\widetilde{d}_D^{*\,I}}{w^*} = z_{min}^{*\,1-\theta} \left[\frac{z_D^{I*\,k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{z_D^{I*\,k} - z_{min}^{*k}} \right]^{-1} \frac{\widetilde{d}_D^{NI*}}{w^*} - \frac{\theta-1}{k-(\theta-1)} \phi_D^*(\psi_D^{*-\xi^*} - 1) \tag{38}$$

The total profit expression in the Foreign country is represented as follows:

$$\begin{aligned}\frac{1-\beta^*(1-\delta^*)}{\beta^*(1-\delta^*)}f_E^* =& \frac{k}{k-(\theta-1)}f_X^*\frac{C^*Q}{C}Q^{\theta-1}\tau^{\theta-1}\left(\frac{z_{min}^* z_D^{I*}}{z_X^*}\right)^{\theta-1}\left[\frac{z_D^{I*\,k-(\theta-1)}-z_{min}^{*k-(\theta-1)}}{z_D^{I*k}-z_{min}^{*k}}\right](1-z_D^{I*-k}) \\ &+\left\{\frac{k}{k-(\theta-1)}f_X^*\frac{C^*Q}{C}Q^{\theta-1}\tau^{\theta-1}\left(\frac{z_D^{I*}}{z_X^*}\right)^{\theta-1}\psi_D^{*\xi^*(1-\theta)}-\phi_D^*(\psi_D^{*\xi^*}-1)\right\}z_D^{I*-k} \\ &+\frac{\theta-1}{k-(\theta-1)}f_X^* z_X^{*-k}\end{aligned} \tag{39}$$

The consumption ratio in units of the Home consumption basket is:

$$\frac{C}{C^*Q}=\frac{f_X^*}{f_X}TOL^{\theta}Q^{\theta-1}\left(\frac{\tau^* z_X}{\tau z_X^*}\right)^{\theta-1} \tag{40}$$

Under the condition of a balanced current account, it follows that:

$$\begin{aligned}&z_V^{-k}TOL\left[\frac{k(\theta-1)}{k-(\theta-1)}\left(\frac{z_V}{z_X^*}\right)^{\theta-1}\psi_V^{\xi(1-\theta)}f_X^*-\left(f_V+\phi_V(\psi_V^{-\xi}-1)\right)\right] \\ &=\frac{k\theta}{k-(\theta-1)}\left\{\frac{1}{z_X^{*k}}\left(\frac{N}{N^*}\right)^{-1}f_X^*TOL-\frac{1}{z_X^k}f_X\right\}\end{aligned} \tag{41}$$

The real exchange rate in the steady state is as follows:

$$Q^{1-\theta}=\left(\frac{N}{N^*}\right)^{-1}\frac{\Lambda}{\Omega} \tag{42}$$

$$\begin{aligned}\Lambda =&(1-z_D^{I*-k})TOL^{1-\theta}(z_{min}^* z_D^{I*})^{\theta-1}\left[\frac{z_D^{I*\,k-(\theta-1)}-z_{min}^{*k-(\theta-1)}}{z_D^{I*\,k}-z_{min}^{*k}}\right]+z_D^{I*-k}\psi_D^{*\xi^*(1-\theta)}\left(\frac{TOL}{z_D^{I*}}\right)^{1-\theta} \\ &+z_X^{-k}\frac{N}{N^*}\left(\frac{\tau^*}{z_X}\right)^{1-\theta}\end{aligned}$$

$$\begin{aligned}\Omega =&(1-z_D^{I\,-k})(z_{min}z_D^I)^{\theta-1}\left[\frac{z_D^{I\,k-(\theta-1)}-z_{min}^{k-(\theta-1)}}{z_D^{I\,k}-z_{min}^k}\right]\\&+(z_D^{I\,-k}-z_V^{-k})\psi_D^{\xi(1-\theta)}(z_D^I z_V)^{\theta-1}\left[\frac{z_V^{k-(\theta-1)}-z_D^{I\,k-(\theta-1)}}{z_V^k-z_D^{I\,k}}\right]+z_V^{-k}\psi_V^{\xi(1-\theta)}\left(\frac{\tau TOL}{z_V}\right)^{1-\theta}\\&+z_X^{*-k}\left(\frac{N}{N^*}\right)^{-1}\left(\frac{\tau TOL}{z_X^*}\right)^{1-\theta}\end{aligned}$$

The equations for the export price are as follows:

$$\frac{\theta k}{k-(\theta-1)}f_X\frac{\widetilde{\rho}_X^{\theta-1}}{TOL}=1+\frac{1-\beta^*}{\beta^*(1-\delta^*)}f_E^*\frac{\widetilde{\rho}_X^{\theta-1}}{\Xi} \tag{43}$$

$$\frac{\theta k}{k-(\theta-1)}f_X^*TOL\widetilde{\rho}_X^{*\theta-1}=1+\frac{1-\beta}{\beta(1-\delta)}f_E\frac{\widetilde{\rho}_X^{*\theta-1}}{\Gamma} \tag{44}$$

$$\frac{N}{N^*}\left(\frac{\tilde{\rho}_X}{\tilde{\rho}_X^*}\right)^{\theta-1}=\frac{\Xi}{\Gamma} \tag{45}$$

where

$$\begin{aligned}\Xi =&(1-z_D^{*I\,-k})\left(\frac{\tau^*}{TOL}\right)^{\theta-1}\left(\frac{z_{min}^*z_D^{I*}}{z_X}\right)^{\theta-1}\left[\frac{z_D^{I*\,k-(\theta-1)}-z_{min}^{*k-(\theta-1)}}{z_D^{I*\,k}-z_{min}^{*k}}\right]\\&+z_D^{*I\,-k}\left(\frac{\tau^*}{TOL}\right)^{\theta-1}\psi_D^{*-\xi^*(\theta-1)}\left(\frac{z_D^{I*}}{z_X}\right)^{\theta-1}+z_X^{-k}\frac{N}{N^*}\end{aligned}$$

$$\begin{aligned}\Gamma =&(1-z_D^{I\,-k})(\tau TOL)^{\theta-1}\left(\frac{z_{min}z_D^I}{z_X^*}\right)^{\theta-1}\left[\frac{z_D^{I\,k-(\theta-1)}-z_{min}^{k-(\theta-1)}}{z_D^{I\,k}-z_{min}^k}\right]\\&+(z_D^{I\,-k}-z_V^{-k})(\tau TOL)^{\theta-1}\psi_D^{-\xi(\theta-1)}\left(\frac{z_D^I z_V}{z_X^*}\right)^{\theta-1}\left[\frac{z_V^{k-(\theta-1)}-z_D^{I\,k-(\theta-1)}}{z_V^k-z_D^{I\,k}}\right]\\&+z_V^{-k}\psi_V^{-\xi(\theta-1)}\left(\frac{z_V}{z_X^*}\right)^{\theta-1}+z_X^{*-k}\left(\frac{N}{N^*}\right)^{-1}\end{aligned}$$

Appendix F. The Real Exchange Rate Log-linearization

The data-consistent real exchange rate is $\widetilde{Q}_{CPI,t}^{1-\theta} = \frac{{N_t}^*}{N_t}\left(\frac{P_t^*\epsilon_t}{P_t}\right)^{1-\theta} = \frac{N_t^*}{N_t}Q_t^{1-\theta}$, where $N_t \equiv N_{D,t}^{NI} + N_{D,t}^{I} + N_{V,t} + N_{X,t}^*$ and ${N_t}^* \equiv N_{D,t}^{NI*} + N_{D,t}^{I*} + N_{X,t}$ represents the total number of available varieties for consumers in both Home and Foreign countries.

The real exchange rate is derived by using the equilibrium prices for each type of production in the respective countries.

$$\begin{aligned}
\widetilde{Q}_{CPI,t}^{1-\theta} &= \frac{N_t}{N_t^*}\frac{N_{D,t}^{NI*}(\tilde{\rho}_{D,t}^{NI*}P_t^*\epsilon_t)^{1-\theta} + N_{D,t}^{I*}(\tilde{\rho}_{D,t}^{I*}P_t^*\epsilon_t)^{1-\theta} + N_{X,t}(\tilde{\rho}_{X,t}P_t^*\epsilon_t)^{1-\theta}}{N_{D,t}^{NI}(\tilde{\rho}_{D,t}^{NI}P_t)^{1-\theta} + N_{D,t}^{I}(\tilde{\rho}_{D,t}^{I}P_t)^{1-\theta} + N_{V,t}(\tilde{\rho}_{V,t}P_t)^{1-\theta} + N_{X,t}^*(\tilde{\rho}_{X,t}^*P_t)^{1-\theta}} \\
&= \frac{N_t}{N_t^*}\frac{N_{D,t}^{NI*}\left(\frac{w_t^*P_t^*\epsilon_t}{Z_t^*\tilde{z}_{D,t}^{NI*}}\right)^{1-\theta} + N_{D,t}^{I*}\left(\frac{\psi_{D,t}^{*\xi^*}w_t^*P_t^*\epsilon_t}{Z_t^*\tilde{z}_{D,t}^{I*}}\right)^{1-\theta} + N_{X,t}\left(\frac{\tau_t^*Q^{-1}w_tP_t^*}{Z_t\tilde{z}_{X,t}}\right)^{1-\theta}}{N_{D,t}^{NI}\left(\frac{w_tP_t}{Z_t\tilde{z}_{D,t}^{NI}}\right)^{1-\theta} + N_{D,t}^{I}\left(\frac{\psi_{D,t}^{\xi}w_tP_t}{Z_t\tilde{z}_{D,t}^{I}}\right)^{1-\theta} + N_{V,t}\left(\frac{\tau_t\psi_{V,t}^{\xi}Q_tw_t^*P_t}{Z_t^*\tilde{z}_{V,t}}\right)^{1-\theta} + N_{X,t}^*\left(\frac{\tau_tw_t^*P_tQ_t}{Z_t^*\tilde{z}_{X,t}^*}\right)^{1-\theta}} \\
&= \frac{\frac{N_{D,t}^{NI*}}{N_t^*}\left(\frac{TOL_t}{\tilde{z}_{D,t}^{NI*}}\right)^{1-\theta} + \frac{N_{D,t}^{I*}}{N_t^*}\left(\frac{\psi_{D,t}^{*\xi^*}TOL_t}{\tilde{z}_{D,t}^{I*}}\right)^{1-\theta} + \frac{N_{X,t}}{N_t^*}\left(\frac{\tau_t^*}{\tilde{z}_{X,t}}\right)^{1-\theta}}{\frac{N_{D,t}^{NI}}{N_t}\left(\frac{1}{\tilde{z}_{D,t}^{NI}}\right)^{1-\theta} + \frac{N_{D,t}^{I}}{N_t}\left(\frac{\psi_{D,t}^{\xi}}{\tilde{z}_{D,t}^{I}}\right)^{1-\theta} + \frac{N_{V,t}}{N_t}\left(\frac{\psi_{V,t}^{\xi}\tau_tTOL_t}{\tilde{z}_{V,t}}\right)^{1-\theta} + \frac{N_{X,t}^*}{N_t}\left(\frac{\tau_tTOL_t}{\tilde{z}_{X,t}^*}\right)^{1-\theta}}
\end{aligned}$$

In the steady state, I assume $f_E = f_E^*, \tau = \tau^*$. I adhere to the notation used in Zlate (2010), $s_D \equiv N_D^{NI}\left(\frac{w}{Z\tilde{z}_D^{NI}}\right)^{1-\theta} + N_D^{I}\left(\psi_{D,t}^{\xi}\frac{w}{Z\tilde{z}_D^{I}}\right)^{1-\theta} + N_V\left(\tau TOL\psi_V^{\xi}\frac{w}{Z\tilde{z}_V}\right)^{1-\theta}$. Expression s_D represents the steady state share of Home spending on varieties produced by Home firms, both domestically and offshore. Expression $s_V \equiv N_V\left[\left(\frac{\tau TOL\psi_V^{\xi}}{\psi_D^{\xi}}\right)^{1-\alpha}\frac{w}{Zz_V}\right]^{1-\theta}$ denotes the steady-state share of Home spending on varieties produced by the offshoring Home firms only. (Note that $s_D - s_V > 0$) Expression s_D^* denotes the steady state share of Home spending on varieties produced by Home intangible firms. Expression $s_D^* \equiv N_D^*\left(Q\frac{\psi_D^{*\xi^*}}{\psi_D^{\xi}}\frac{w^*}{Z\tilde{z}_D}\right)^{1-\theta}$ represents the steady-state share of foreign spending on varieties produced by Foreign domestic firms. Expression $s_D^{I*} \equiv N_D^*\left(Q\frac{\psi_D^{*\xi^*}}{\psi_D^{\xi}}\frac{w^*}{Z\tilde{z}_D}\right)^{1-\theta}$ represents the steady-state share of foreign spending on varieties produced by Foreign intangible firms. The average productivity of foreign firms producing domestically, denoted by $\tilde{z}_D^*$, remains constant over time.

The log-linearized version of the data-consistent real exchange rate ($\widetilde{\mathsf{Q}}_t^{1-\theta}$) can be written as:

$$\widetilde{\mathsf{Q}}_t = \frac{\overline{NUM}dDEN_t - \overline{DEN}dNUM_t}{(\theta-1)\overline{NUMDEN}} = \frac{1}{\theta-1}\left[\frac{dDEN_t}{\overline{DEN}} - \frac{dNUM_t}{\overline{NUM}}\right]$$

Using above information, I derived the log-linearized version of the data-consistent real exchange rate ($\widetilde{\mathsf{Q}}_t^{1-\theta}$):

$$
\begin{aligned}
\widetilde{\mathsf{Q}}_t = \frac{1}{\theta-1}\Bigg[& (s_D - s_D^* - s_V)\Big\{\widetilde{\mathsf{N}}_{D,t}^{NI} - \widetilde{\mathsf{N}}_t - (1-\theta)\tilde{\mathsf{z}}_{D,t}^{NI}\Big\} \\
& + s_D^I\Big\{\widetilde{\mathsf{N}}_{D,t}^{I} - \widetilde{\mathsf{N}}_t + (1-\theta)(\xi\Psi_{D.t} - \tilde{\mathsf{z}}_{D,t}^{I})\Big\} \\
& + s_V\Big\{\widetilde{\mathsf{N}}_{V,t} - \widetilde{\mathsf{N}}_t + (1-\theta)(\mathsf{t}_t + \mathsf{TOL}_t - \tilde{\mathsf{z}}_{V,t} - \xi\Psi_{V,t})\Big\} \\
& + (1-s_D)\Big\{\widetilde{\mathsf{N}}_{X,t}^{*} - \widetilde{\mathsf{N}}_t + (1-\theta)(\mathsf{t}_t + \mathsf{TOL}_t - \tilde{\mathsf{z}}_{X,t}^{*})\Big\} \\
& - (s_D^* - s_D^{I*})\Big\{\widetilde{\mathsf{N}}_{D,t}^{NI*} - \widetilde{\mathsf{N}}_t^{*} + (1-\theta)(\mathsf{TOL}_t - \tilde{\mathsf{z}}_{D,t}^{NI*})\Big\} \\
& - s_D^{I*}\Big\{\widetilde{\mathsf{N}}_{D,t}^{I*} - \widetilde{\mathsf{N}}_t^{*} + (1-\theta)(\xi^*\Psi_{D.t}^{*} + \mathsf{TOL}_t - \tilde{\mathsf{z}}_{D,t}^{I*})\Big\} \\
& - (1-s_D^*)\Big\{\widetilde{\mathsf{N}}_{X,t} - \widetilde{\mathsf{N}}_t^{*} + (1-\theta)(\mathsf{t}_t^{*} - \tilde{\mathsf{z}}_{X,t})\Big\}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
= {} & (s_D - s_V + s_D^* - 1)\mathsf{TOL}_t && \text{(C1)} \\
& + s_V(\tilde{\mathsf{z}}_{V,t} - \mathsf{t}_t) + (s_D - s_D^I - s_V)\tilde{\mathsf{z}}_{D,t}^{NI} + s_D^I\tilde{\mathsf{z}}_{D,t}^{I} && \text{(C2–C4)} \\
& + (1-s_D)(\tilde{\mathsf{z}}_{X,t}^{*} - \mathsf{t}_t) - (1-s_D^*)(\tilde{\mathsf{z}}_{X,t} - \mathsf{t}_t^*) - (s_D^* - s_D^{*I})\tilde{\mathsf{z}}_{D,t}^{*NI} - s_D^{*I}\tilde{\mathsf{z}}_{D,t}^{*I} && \text{(C5)} \\
& - s_V\xi\Psi_{V,t} - s_D^I\xi\Psi_{D,t} + s_D^{*I}\xi^*\Psi_{D,t}^{*} && \text{(C6)} \\
& + \frac{1}{\theta-1}\left(s_V - \frac{N_V}{N_D + N_V + N_X^*}\right)(\mathsf{N}_{V,t} - \mathsf{N}_{X,t}^{*}) && \text{(C7)} \\
& + \frac{1}{\theta-1}\left[\begin{array}{l}\left(\dfrac{N_D^{*NI}}{N_D^{*NI} + N_D^{*I} + N_X} - (s_D^* - s_D^{*I})\right)(\mathsf{N}_{D,t}^{*NI} - \mathsf{N}_{X,t}) \\ \quad - \left(\dfrac{N_D^{NI}}{N_D^{NI} + N_D^{I} + N_V + N_X^*} - (s_D - s_D^I - s_V)\right)(\mathsf{N}_{D,t}^{NI} - \mathsf{N}_{X,t}^{*})\end{array}\right] && \text{(C8)} \\
& + \frac{1}{\theta-1}\left[\begin{array}{l}\left(\dfrac{N_D^{*I}}{N_D^{*NI} + N_D^{*I} + N_X} - s_D^{*I}\right)(\mathsf{N}_{D,t}^{*I} - \mathsf{N}_{X,t}) \\ \quad - \left(\dfrac{N_D^{I}}{N_D^{NI} + N_D^{I} + N_V + N_X^*} - s_D^{I}\right)(\mathsf{N}_{D,t}^{I} - \mathsf{N}_{X,t}^{*})\end{array}\right] && \text{(C9)}
\end{aligned}
$$