Firm Entry and Market Interdependence*

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Abstract

Using Chinese Customs data, we study how unilateral anti-dumping (AD) tariffs on Chinese products affect exports to various destination countries. We first document that investigated products experience a decline in total sales in both the countries that impose the tariffs (focal countries) and those that do not (third countries). A decomposition exercise further suggests that the simultaneous decline in exports across countries is primarily driven by a reduction in the number of entrants. In contrast, individual exporters' sales decrease in the focal countries but increase in the third countries. To explain these findings, we develop an industry equilibrium framework based on firms' interdependent entry decisions across multiple markets. We show that the entry margin alone can generate cross-market policy spillovers and heterogeneous tariff responses that align with the data. Finally, we calibrate our model to quantify the global impact of a unilateral tariff on trade and welfare.

Keywords. Market interdependence, Firm entry, HDIA and HSA Preferences

JEL Classification. D12; F12; F13; F14

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1 Introduction

The use of country-specific tariffs, such as anti-dumping (AD) and countervailing duties, has surged in recent years. Among these, the U.S.–China trade war has drawn significant attention in both policy circles and academic debates over the past decade. Although these tariffs and trade disputes are typically country-specific or country-pair-specific, their effects often spill over to "bystander" countries—nations that neither impose the tariffs nor are directly targeted by them. This is due to the intrinsic interconnectedness of global export markets. However, existing literature tends to examine third-country effects either at the aggregate level or at the firm level. This segmented approach often yields conflicting empirical results and divergent policy conclusions, highlighting the need for a more unified analytical framework.[1]

In this paper, we present a unified analysis of the impact of country-specific tariffs by integrating both firm-level and aggregate-level evidence. Our study makes empirical and theoretical contributions and offers policy insights based on a rich quantitative model. Empirically, we examine how AD tariffs affect Chinese exports to both imposing (focal) countries and third countries, analyzing these effects at both the aggregate and firm levels. A key finding is that the responses in third markets diverge: aggregate exports decline, while surviving firms tend to increase their sales. Theoretically, we rationalize these patterns using an industry equilibrium model that incorporates firms' interdependent entry decisions across multiple markets and variable markups. The model yields closed-form expressions for tariff responses at both the aggregate and firm levels across all markets. Quantitatively, we calibrate the model using a flexible demand system and entry technology to match the observed export patterns. We then use the model to evaluate the broader implications of alternative trade policies.

Our study focuses on the case of AD tariffs imposed on China, as these tariffs represent the most common form of trade barrier, and China is their most frequent target. Between 2000 and 2011, 11% of Chinese exports to the United States were subject to AD investigations, with tariffs ultimately applied in most cases. These tariffs are typically both prohibitive—most

[1]Regarding AD and countervailing duties, Bown and Crowley (2007) find that U.S. tariffs on Japanese exports led to trade deflection, with Japanese exports shifting to third countries. In contrast, Bown and Crowley (2010) and Crowley et al. (2018) document a trade destruction effect, with Chinese exports to third markets declining following the imposition of temporary tariffs. Similarly, Albornoz-Crespo et al. (2021) find that Argentinian exports to third markets fell after the U.S. removed preferential tariffs on Argentine imports. In the context of the U.S.–China trade war, Fajgelbaum et al. (2024) find positive co-movements in aggregate exports from bystander countries to both the U.S. and the rest of the world—evidence that challenges the standard trade diversion narrative.

exceeding 50%—and long-lasting, often remaining in effect for over eight years. Moreover, the availability of detailed export transaction data enables analysis of tariff responses at the firm–product–country level. Taken together, these features make China an ideal context for studying the multifaceted effects of AD tariffs.

We begin our analysis by identifying four stylized facts about the impact of AD tariffs on Chinese exports. First, the initiation of AD investigations leads to a decline in both the total export value and quantity of the investigated products in focal and third markets. Second, these reductions in total exports are primarily driven by changes in the number of exporters, particularly in third markets. Third, the decline in the number of exporting firms is largely attributable to reduced firm entry, rather than firm exit. Fourth, at the firm level, individual exporters experience a decrease in export value to focal markets but an increase in export value to third markets-accompanied by price increases for the investigated products in both types of markets.

The divergence between firm-level and aggregate-level responses to tariffs in third markets points to key considerations for modeling their effects. On one hand, aggregation is crucial: models that focus solely on individual firm behavior fail to capture the broader implications of tariffs. On the other hand, firm-level patterns—particularly the role of entry decisions—highlight that firms' entry decisions provide a micro-foundation for the observed co-movement in total exports across markets.

We then propose an industry equilibrium model based on firms' simultaneous entry decisions into multiple markets. Our model extends the framework of Melitz (2003) along two key dimensions. First, we introduce a two-tier nested demand structure for exported goods. Foreign consumers demand country-specific imported composites (the outer nest), each of which aggregates all varieties sourced from the same country (the inner nest). For the inner nest, we adopt the homothetic demand with directly implicit additivity (HDIA) preferences developed by Matsuyama and Ushchev (2017). Compared to the standard CES demand system, HDIA preferences offer greater flexibility—allowing the model not only to capture price responses to tariffs but also to analyze the entry margin under variable markups. For the outer nest, we assume that demand for country-specific imported composites decreases with the ideal price index of the underlying varieties.

Second, we generalize the entry technology in Melitz (2003), where firms pay a single, common entry cost to access all export markets. In our framework, firms instead make market-specific entry investments: after incurring an initial entry cost, firms choose—prior to the realization of their productivity—their ex ante entry intensity (rate) for each export

market, subject to a constraint.[2] Our entry technology assumes a constant elasticity of substitution across market entries, governed by a parameter β, which captures the degree of interdependence between market entry decisions. When $\beta = 1$, market entries are perfect complements—entering one market grants access to all others, as in the original Melitz model. In contrast, when $\beta = 0$, market entries are perfect substitutes, and decisions are entirely independent across markets. We do not impose restrictions on β, instead estimating it empirically from the data.

To illustrate the model's core mechanism, we consider a special case with homogeneous firms. This simplification allows us to approximate, to a first order, changes in key variables—whether at the firm or market level—as functions of changes in the number of exporters in each market (N_i) and market-specific tariff shocks. We then focus on the dynamics of N_i and show that changes in the number of exporters are weighted averages of changes in total export entrants and the corresponding market-specific tariff changes. These expressions reveal that even in markets without tariff changes, the number of exporters can be indirectly affected through adjustments at the entry margin—that is, through changes in the total number of firms entering export markets.

The intuition behind the entry margin is straightforward. Individual firms base their export decisions on the expected profits from serving *all* markets. Under free entry, these expected profits must equal the constant entry cost. When a unilateral tariff reduces profitability in the focal market, firms must offset the loss by earning higher profits elsewhere—specifically, in third markets—in order to break even. This mechanism explains why individual exporters tend to experience lower sales in the focal market but higher sales in third markets.

With constant consumer demand, the increase in individual exporters' profits in third markets arises solely from reduced competition—implying a decline in the number of exporters in those markets. A reduction in exporters in one market reflects fewer total export entrants, which in turn leads to a contraction in the number of exporters across all markets. In this way, entry decisions serve as the transmission channel through which tariff shocks in the focal market spill over into third markets, ultimately causing a decline in total export sales globally.

Importantly, the entry margin operates independently of industry-level increasing returns to scale (IRS). While simultaneous declines in sales to both focal and third markets might

[2]This entry investment decision can be interpreted as a managerial allocation problem, where managers distribute limited attention across multiple tasks. Such problems are commonly modeled in span-of-control frameworks, e.g., Lucas (1978), Rosen (1982), Antràs et al. (2006), Garicano and Rossi-Hansberg (2006), Caliendo and Rossi-Hansberg (2012), Chen (2017), Gumpert (2018), and Chen and Suen (2019).

suggest the presence of IRS, we show that the entry margin alone can generate the observed tariff responses—even when the industry-level production function exhibits constant returns to scale (i.e., Hopenhayn (1992)).[3] The mechanism is straightforward: a reduction in third-market sales requires higher equilibrium prices in those markets. Although IRS is one way to link a smaller number of firms to higher prices, under constant returns to scale, similar price increases can arise through reduced supply or increased markups driven by less competition.

In the quantitative section of the paper, we calibrate the model by matching the responses of aggregate trade variables to the tariff shock. We then conduct a counterfactual analysis to highlight the role of entry interdependence in transmitting tariff shocks across countries. Typically, calibrating the structural parameters of an industry-equilibrium model requires solving the model in full. However, by leveraging theoretical results derived from log-linearization and sufficient statistics, we can infer these structural parameters without fully solving the model. The objective of the calibration is to demonstrate how key parameters can be cleanly identified from empirical moments.

To ensure transparency in parameter identification, we analyze a simplified case with homogeneous firms operating in symmetric markets. Additionally, we assume that the fixed cost of exporting is zero. These assumptions reduce the number of parameters that require calibration and make the mapping from model parameters to empirical moments highly transparent. Finally, we adopt a specific functional form for the demand system: the Klenow-Willis version of the Kimball demand function (i.e., Klenow and Willis (2016)).

The calibrated parameters of the model are sensible and underscore the importance of the two key innovations introduced in our framework. First, the calibrated value of the entry-dependence parameter, β, lies between zero and one—neither too close to either extreme—highlighting the need to allow for a general form of entry interdependence. Second, the implied super-elasticity of the Klenow-Willis version of the Kimball demand is positive, emphasizing the importance of incorporating endogenous markups into the model.[4]

We then examine how the parameter β influences the transmission of a tariff shock across markets by considering two alternative cases: $\beta = 0$ (no interdependence) and $\beta = 1$ (as in Melitz (2003)). To isolate the role of β in the transmission mechanism, we compare the responses to a market-specific tariff shock across three environments: $\beta = 0$, $\beta = 1$, and the calibrated value. For comparability, we adjust the entry efficiency parameters, h_i, so that all three scenarios share the same initial equilibrium allocation prior to the shock. We find

[3] Results are available upon request.

[4] The super-elasticity refers to the elasticity of the elasticity. It governs how the price elasticity of demand varies with the relative quantity sold by a firm.

that the model with $\beta = 1$ substantially overpredicts the spillover effect to the third market and underpredicts the impact on the focal market. This discrepancy is primarily driven by differences in the number of exporting firms across scenarios.

Literature Review

Our paper contributes to the following four strands of literature. First, we contribute to the extensive literature on market interdependence in the context of international trade. One strand of this literature examines market interdependence through the lens of firms' decision-making processes (e.g., Antràs et al. (2017), Morales et al. (2019), Albornoz-Crespo et al. (2021), Almunia et al. (2021), Alfaro-Ureña et al. (2024)). These studies emphasize how market interdependence shapes firms' export behavior and how production decisions at the intensive margin drive policy spillovers in partial equilibrium settings. In contrast, our paper focuses on how entry decisions at the extensive margin generate market interdependence within an industry equilibrium framework. We further show that the resulting general equilibrium effects—specifically changes in the number of exporters—endogenously influence firms' intensive margin responses.

Another strand of research on market interdependence examines the heterogeneous effects of unilateral trade policies across countries (e.g., Breinlich et al. (2022), Fajgelbaum et al. (2024)). This work connects to the broader literature on trade models with increasing returns to scale (IRS), pioneered by Krugman (1980).[5] Our contribution to this literature lies in identifying an alternative channel through which tariffs generate aggregate effects—namely, the entry margin—supported by firm-level evidence and a robust theoretical framework.

In addition, our paper contributes to the growing literature on variable markups and their implications for trade (e.g., Matsuyama and Ushchev (2017), Mrázová and Neary (2017), Arkolakis et al. (2019), Dhingra and Morrow (2019), Baqaee et al. (2023)). In particular, we build on the framework developed by Baqaee et al. (2023), extending their single-market model to one with multiple markets and interdependent market entry. Moreover, we focus on the distributional effects of demand shocks—such as tariffs—across markets.

Lastly, our paper relates to the extensive literature on temporary trade barriers, particularly anti-dumping (AD) tariffs (e.g., Prusa (2001), Bown and Crowley (2007), Blonigen and Prusa (2008), Bown and Crowley (2010), Lu et al. (2013), Blonigen and Prusa (2016),

[5] Notably, Venables (1987) and Ossa (2011) study the firm-relocation effects of import tariffs in Krugman-style models, highlighting how tariffs reduce the number of foreign exporters and the resulting welfare consequences.

Crowley et al. (2018), Felbermayr and Sandkamp (2020), Sandkamp (2020), Steinbach and Khederlarian (2022)). This body of work typically relies on reduced-form firm-level evidence to argue that firms' intensive margin decisions drive the observed effects of tariffs. In contrast, we identify and explain a "missing intercept" problem in the context of AD tariffs: even without invoking complex firm-level adjustments on the intensive margin, changes in the extensive margin alone can generate firm-level responses consistent with empirical findings. This insight calls for caution in interpreting existing reduced-form evidence. More broadly, our findings underscore the need for unified frameworks that jointly consider extensive and intensive margins when evaluating the effects of trade policy.

2 The Impact of Anti-dumping Investigations

In this section, we estimate the impact of anti-dumping investigations on Chinese exports. We begin by introducing our data, followed by a description of the empirical specifications and presentation of the results. We examine the impact on exports to countries that initiate investigations into a particular product ("focal" and "focal controls"), as well as to countries that do not investigate the product ("third" and "third controls"). We decompose changes in aggregate exports into quantity and price effects, as well as into the extensive margin (the number of firms) and the intensive margin (average exports per firm). Finally, we conduct a firm-level analysis and compare firm-level responses to those observed at the market level.

2.1 Data

Our analysis utilizes several key data sources. First, we use Chinese customs data from 2000 to 2011, which cover the universe of Chinese exporters' transactions. The original data is at the firm-HS8 (Harmonized System)-country level, and we aggregate the data to the HS6-country or firm-HS6 levels. This dataset has been widely used in the literature to study exporter behaviors (Manova and Zhang, 2012). In our online appendix, we show that the aggregate exports in the Chinese customs data are almost identical to the data reported by UN Comtrade, and the market-level responses to AD are also similar if we use the Comtrade data instead.

Second, we use the Temporary Trade Barriers Database (TTBD) from Bown et al. (2020), which provides detailed information on anti-dumping (AD) cases, including the initiation country and date, the targeting country, and the affected products. Bown et al. (2020) classify products using the finest level of the HS codes available for each country, and we use

the first six digits (HS6) because only the classification of the first six digits are consistent across countries. We focus on AD cases initiated between 2000 and 2011 against China.

Table 1: Top 15 investigating countries against China, 2000-2011

		Cases initiated between 2000 and 2011			
Country	% of Chinese exports	# cases	# HS6	% of HS6	% of exports to destination
India	1.97	107	181	3.75	15.99
United States	18.95	92	266	4.84	11.20
European Union	19.32	75	172	3.05	8.19
Turkey	0.70	61	124	2.83	7.41
Argentina	0.30	56	125	3.20	17.93
Brazil	1.12	42	72	1.65	5.72
Canada	1.45	27	85	1.73	4.73
Australia	1.59	26	36	0.73	4.39
Mexico	0.99	25	41	0.96	3.14
Colombia	0.20	25	71	1.86	5.41
South Africa	0.60	22	31	0.68	3.00
Peru	0.16	13	54	1.43	3.69
Korea	4.60	13	27	0.49	0.59
Indonesia	1.22	11	34	0.69	1.70
Thailand	1.11	10	35	0.68	2.01

Our empirical findings in this section use only the Chinese customs data and TTBD. Combining these two datasets, we identify HS6 products that are subject to AD investigations from 2000 and 2011 and the relative importance of these products among Chinese exports. In Table 1, we list the top 15 countries that initiated AD cases against China between 2000 and 2011. By the number of cases (products) investigated, India, the United States and the European Union (EU) are the three largest initiators of AD cases against China. The United States and EU are also the two largest destination markets for Chinese exports, each making up around 20% of total export values between 2000 and 2011. The products subject to AD investigations account for a significant share of Chinese exports to these countries. For example, the products subject to U.S. investigations account for 11% of Chinese exports to the U.S.

2.2 Empirical specifications

We want to study the impact of AD investigations on Chinese exports in different markets. For each product subject to AD investigations, we first define focal and third markets, distinguishing between treated and control products. In our analysis, a market is a country-HS6 product pair. When studying the impact of AD on Chinese exports in the country

that initiated the investigation, we define the "focal treated market" as the product being treated and the "focal control market" as the products with the same first four digit code (HS4) but different HS6 in the same country. For the same treated products, we define the "third treated market" as the same HS6 product in countries that do not initiate such an investigation. The corresponding control group includes products with the same HS4 code but different HS6 codes sold in the same set of countries. We then compare the changes in exports to the focal and third markets with those in their corresponding control groups (after the initiation of the AD investigation).

In Table 2, we illustrate our definitions of treated and control groups using an actual example. In Jan./2004, the U.S. initiated an AD case on "certain frozen and canned warm water shrimp and prawns" (HS6 code 160520). The case received preliminary and final determinations in July 2004 and Feb 2005, respectively. We define the focal treated market as frozen and canned shrimps and prawns (160520) sold in the U.S. Within the HS4 code of 1605, there are two other six-digit products, preserved crabs (160510) and lobsters (160530). We define the focal control market as those two products sold in the U.S. Other countries that do not initiate AD investigations against Chinese preserved shrimps and prawns belong to "third markets", such as Japan. A typical third treated market is frozen and canned shrimps and prawns (160520) sold in Japan, while a typical third control markets are preserved crabs (160510) and lobsters (160530) sold in Japan.

Table 2: Defining focal (third) markets and control groups: an example

Market	Country, i	HS6, h	HS6 Description
Focal treated	United States	160520	shrimps and prawns
Focal control	United States	160510	crab
		160530	lobster
Third treated	Japan	160520	shrimps and prawns
Third control	Japan	160510	crab
		160530	lobster

Notes: focal/third treated and control markets in the example of the U.S. AD case on shrimps and prawns.

We first employ an event-study dynamic specification to analyze the impact of AD investigations on Chinese exports. For either focal or third markets, the specification is given by

the following equation:

$$y_{iht} = \sum_{k=-K}^{K} \beta_k \text{Treated}_{ih} \mathbb{1}\{t - E_{ih} = k\} + \delta_{ih} + \delta_{is(h)t} + \varepsilon_{iht}. \tag{1}$$

In this equation, E_{ih} represents the year of the investigation. We set $K = 10$ as the largest time-to-treat, grouping $k \geq K$ and $k \leq -K$ together. Here, i denotes the destination country, h is an HS6 product, and $s(h)$ is the HS4 category of h. The δ terms are fixed effects, and standard errors are clustered at the HS6-product (h) level.

Our main dependent variable is the export value, denoted as $\ln R$. We further decompose $\ln R$ into the number of firms ($\ln N$) and average revenue per firm ($\ln \bar{r}$). Additionally, we decompose $\ln R$ into total quantity ($\ln \tilde{Q}$) and unit value ($\ln p^m$).

As we show in the next section, we find that, relative to the control groups, Chinese exports of the treated products present very strong pre-trends, and many of these pre-trends are linear in time. To address this issue, we estimate a linear pre-trend and assume that it would have continued without the treatment, following the approach of Dobkin et al. (2018) and Khederlarian and Steinbach (2022). The modified specification is given by:

$$\begin{aligned} y_{iht} =& \beta_{-K} \text{Treated}_{ih} \mathbb{1}\{t - E_{ih} = -K\} + \sum_{k=0}^{K} \beta_k \text{Treated}_{ih} \mathbb{1}\{t - E_{ih} = k\} \\ &+ \gamma \text{Treated}_{ih} \times (t - E_{ih}) \times \mathbb{1}\{t < E_{ih}\} + \delta_{ih} + \delta_{is(h)t} + \varepsilon_{iht}. \end{aligned} \tag{2}$$

In this specification, $\gamma \text{Treated}_{ih} \times (t - E_{ih}) \times \mathbb{1}\{t < E_{ih}\}$ represents the linear pre-trend for the treated group. The treatment effect at time $k \geq 0$ equals $\beta_k - \gamma(k-1)$, consistent with our normalization that the treatment effect at time $t = E_{ih} - 1$ is zero.

In addition to the level regressions (1) and (2), we also consider the corresponding first-difference regressions. For example, when the dependent variable is the log number of exporters, N_{iht}, the first-difference regression related to (1) is given by:

$$\Delta \ln N_{iht} = \sum_{k=-K}^{K} \beta_k \text{Treated}_{ih} \Delta \mathbb{1}\{t - \text{EventTime}_{ih} = k\} + \delta_{is(h)t} + \varepsilon_{iht}. \tag{3}$$

This specification is useful when we further approximate $\Delta \ln N_{iht}$ with percentage growth rates and decompose the growth rates into different margins. For example, when the growth

rate of N_{iht} is small, we can approximate $\Delta \ln N_{iht}$ with the mid-point growth rate as

$$\Delta \ln N_{iht} \approx \frac{N_{iht} - N_{iht-1}}{(N_{iht} + N_{iht-1})/2}.$$

Because the change in the number of exporters must equal the difference between the number of entrants and exiting firms, we can further decompose the mid-point growth rates into the entry and exit margins as

$$\frac{N_{iht} - N_{iht-1}}{(N_{iht} + N_{iht-1})/2} = \frac{E_{iht}}{(N_{iht} + N_{iht-1})/2} + \frac{-X_{iht-1}}{(N_{iht} + N_{iht-1})/2},$$

where E_{iht} denotes the number of entrants (firms that do not export at time $t-1$ but at time t) and X_{iht-1} denotes the number of exiters (firms that export at time $t-1$ but not at time t). When components have trends, we can also apply the method of extrapolating the pretrends as in equation (2).

When analyzing the impact of AD at the firm level, we aggregate the firm-country-product-year level data at the firm-market-product-year level, where a market can be either focal or third. For either market, we estimate a similar dynamic specification as in equation (1):

$$y_{fht} = \sum_{k=-K}^{K} \beta_k \text{Treated}_h \mathbb{1}\{t - E_h = k\} + \delta_{fh} + \delta_{s(h)t} + \varepsilon_{fht}, \tag{4}$$

where f is the firm subscript, and the fixed effects are firm-HS6 and HS4-year. At the firm level, we focus on three outcome variables: export values, quantities and prices. Since the treatment is at the product-year level, we cluster standard errors at the HS6 product level.

2.3 Market-level responses to AD investigations

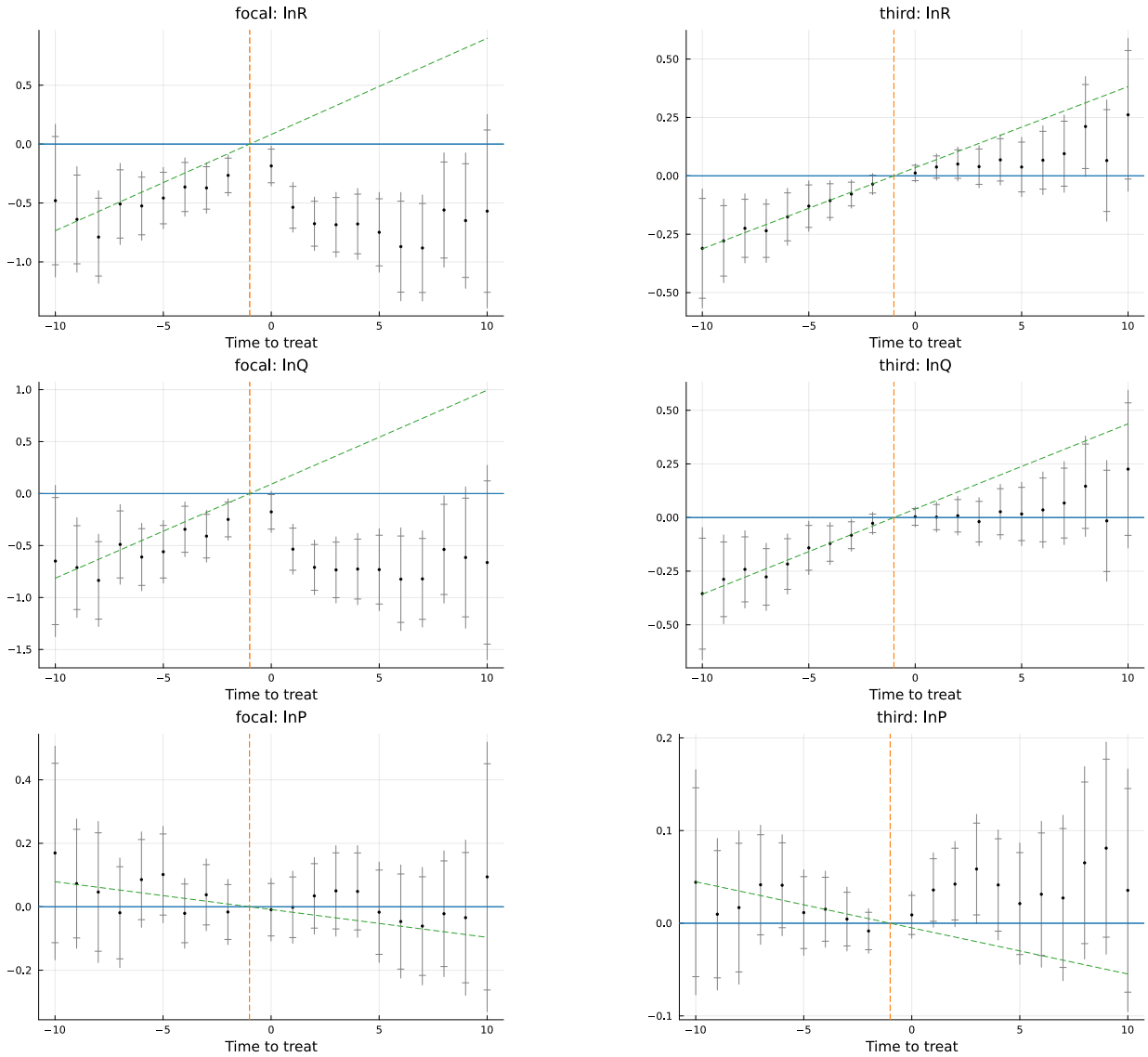
We document three key stylized facts in our market-level analysis. Several points are worth mentioning before proceeding. First, we use EU as a single "destination economy" which has a time-varying set of countries in all regressions, as all countries in EU share the same AD and trade policies. Second, we require the AD cases to satisfy the following three restrictions to exclude unusual cases: (1) lasting for at least 5 years; (2) being revoked later than 2011 (or not being revoked at all); (3) having a unique initiation year. We call these AD cases the restricted AD cases.

Fact 1. With the initiation of anti-dumping, export value and quantity drop in both the

focal and *third* markets.

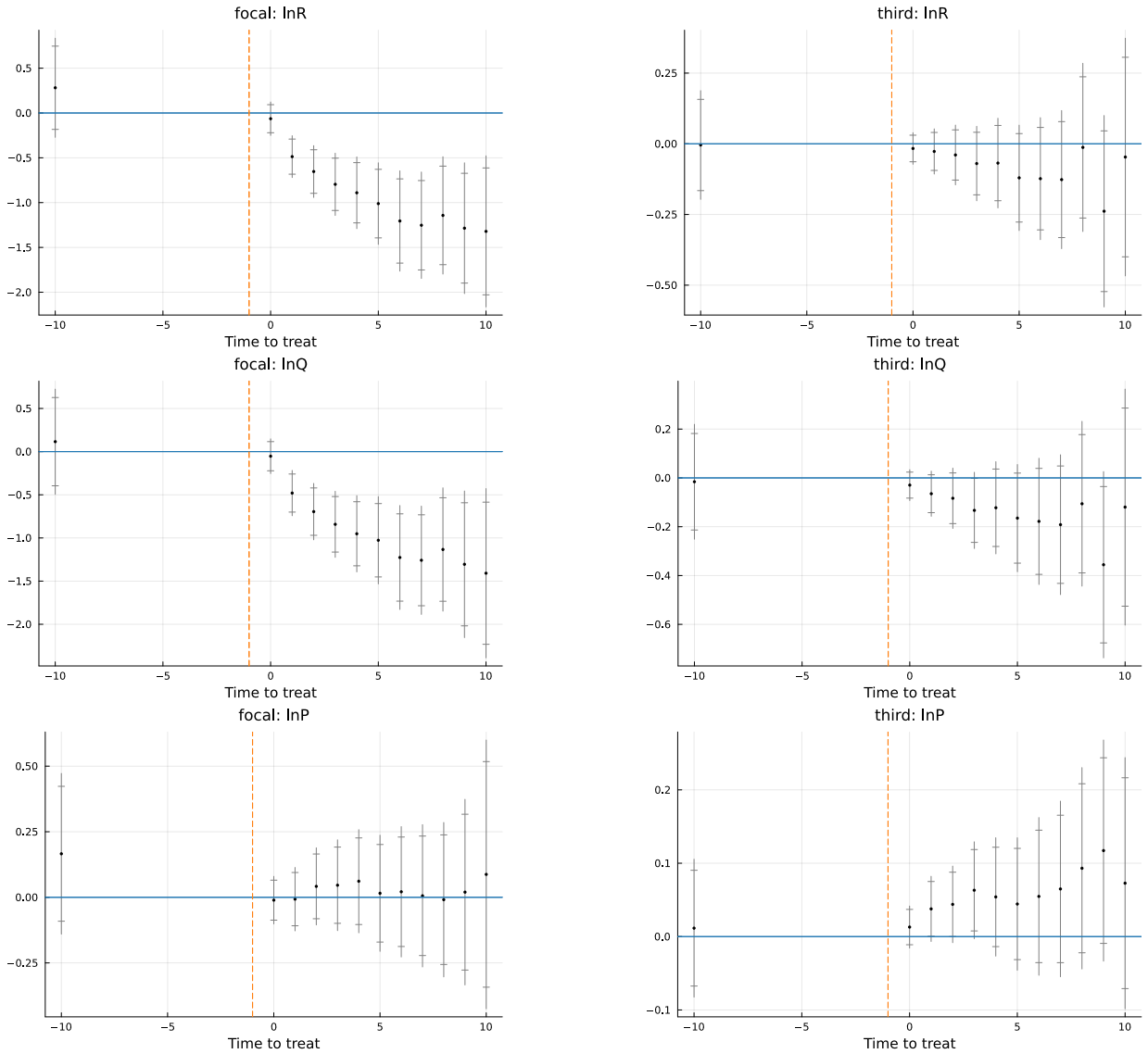
This stylized fact is evident in the first four panels of Figure 1 and continue to hold in the first four panels of Figure 2 when we restrict our AD cases and remove the pretrends. This finding is interesting, as it hints the existence of market interdependence (i.e., complementary markets) after the AD investigations are initiated. Specifically, both export value and quantity drop for about 120% in the focal market (around the fifth year after the initiation) while they drop for about 18% and 25% in the third market. These magnitudes combined imply that the increase in the unit price is nil in the focal market and about 7% in the third market.

Figure 1: Effect of AD investigation on revenue, quantities and prices. Event-study dynamic specifications.



Notes: All specifications include country-HS6-product and country-HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.

Figure 2: Effect of AD investigation on revenue, quantities and prices. Extrapolating pre-trends.



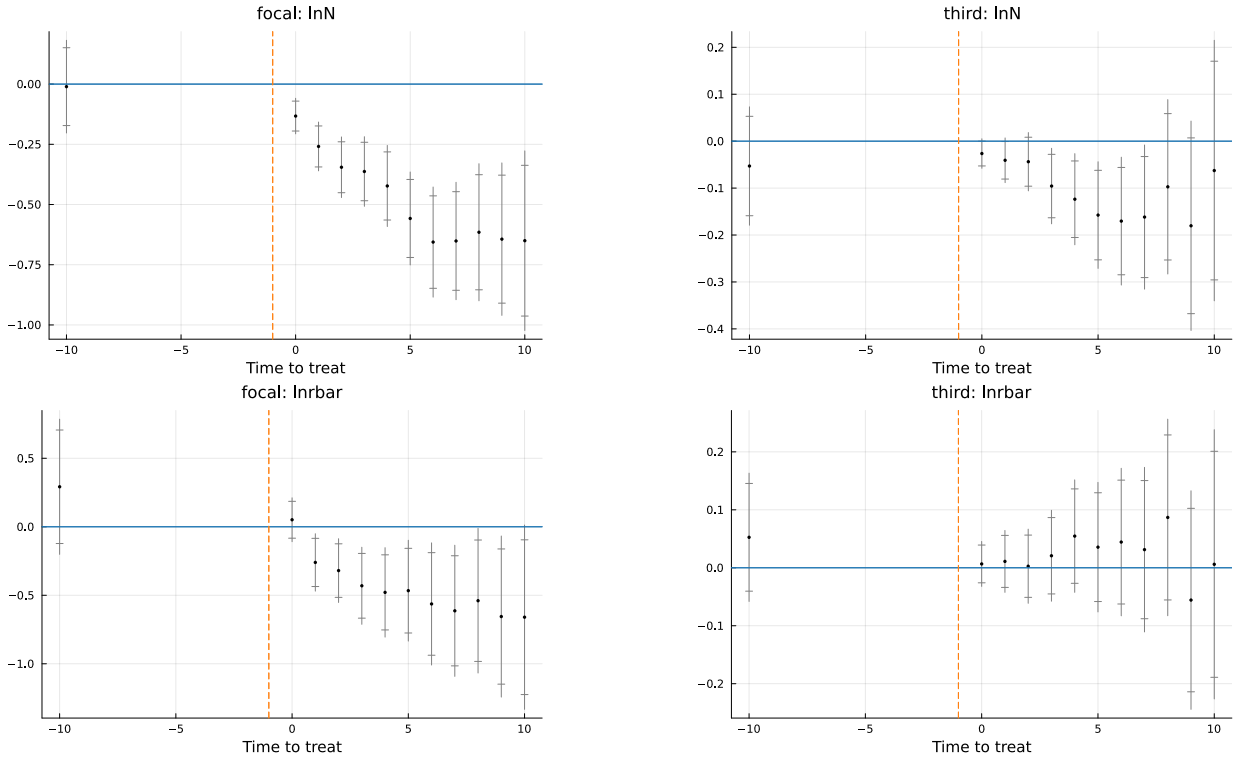
Notes: All specifications include country-HS6-product and country-HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.

Fact 2. Firms' entry and exit (i.e., the extensive margin) are the main driver of the simultaneous changes in aggregate exports *across markets*. With the initiation of the AD investigation, average revenue decreases in the focal markets but *barely changes* in the third market.

This stylized fact is evident in Figure 3 and Figure 6 in the appendix. This finding

is interesting, as it hints the importance of the extensive margin for explaining changes in aggregate trade values/volumes in a world with *interdependent markets*. Specifically, the number of exporting firms in the focal and third market drop simultaneously after the initiation of the AD investigation, while the magnitude of the drop is reasonably large in the focal market (65%) than in the third market (20%). For the focal market, both the intensive margin (the 55% drop in the firm-level average exports) and the extensive margin (the 65% drop in the number of firms) contribute to the overall drop in the total exported value. However, the drop at the extensive margin accounts for the entire drop in the total exported value to the third market. In a nutshell, the interdependence of drops in exported values between the two types of markets is entirely driven by the extensive margin. Moreover, the nil change in the firm-level exports in the third markets also exemplify the difference between the change in the total exported value and that that in the firm-level exports.

Figure 3: Effect of AD investigation at the market level: number of firms and average revenue.

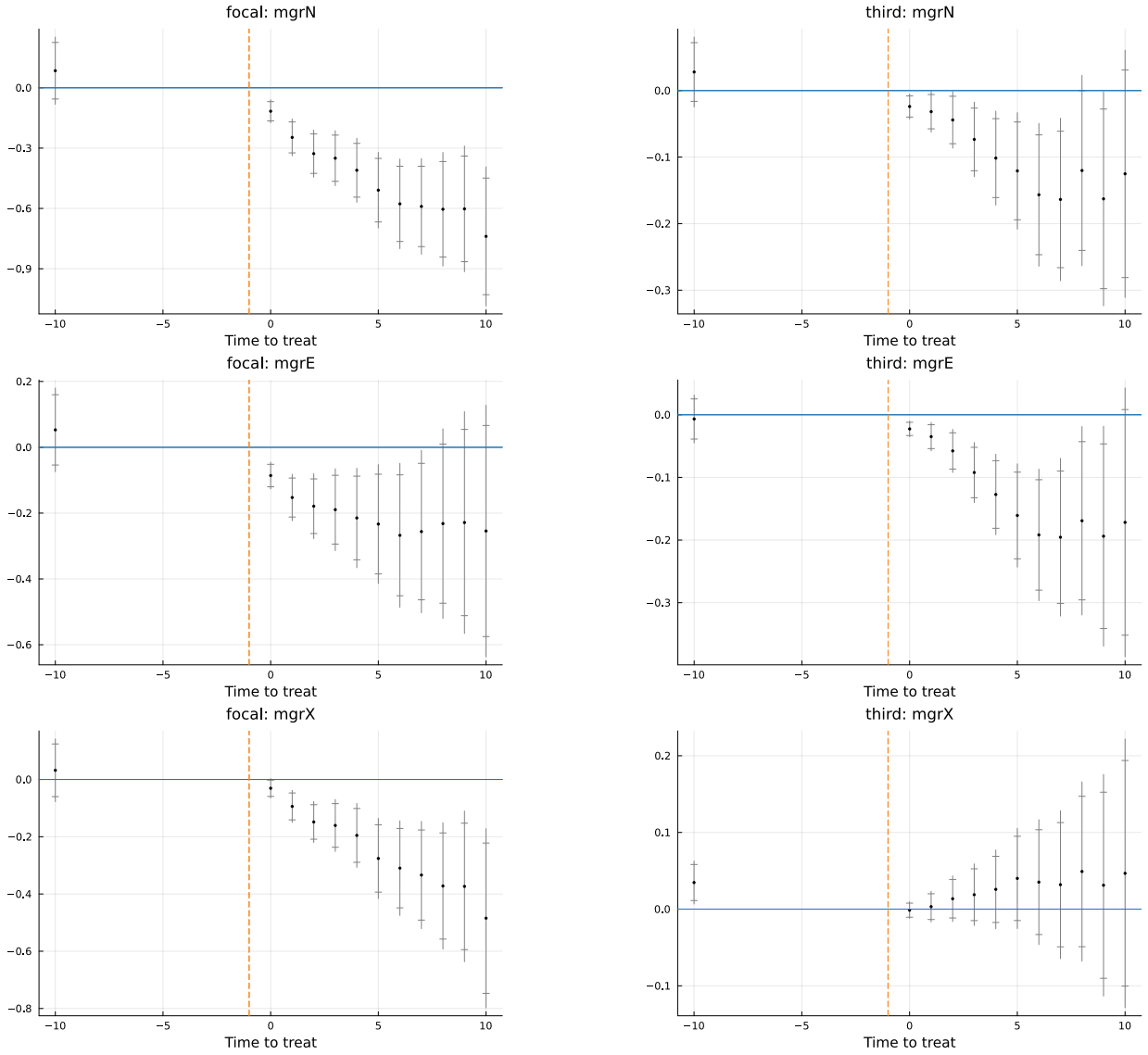


Notes: All specifications include country-HS6-product and country-HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.

Fact 3. The drop in firms' entry rate is the dominant driver of the growth rate at the extensive margin (i.e., the number of exporters) in the third market.

This stylize fact is evident in Figure 4, as the 65% drop in the growth rate of the number of exporters in the focal market is explained by the drop in the entry rate (25%) and that in the exit rate (40%) . In the third markets, the 20% drop in the growth rate of the number of exporters is entirely explained by the same level of the drop in the entry rate. In total, the entry margin is the key to explaining the interdependence at both the extensive margin and at the level of aggregate exports between the focal and the third markets.

Figure 4: Effect of AD investigation at the market level: mid-point growth in number of firms and decomposition into entrants and exiters.



Notes: All specifications include country-HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.

Table 3: Impact of AD in Focal and Third Markets, EU combined

	Level regressions					First differences		
	$\ln R$	$\ln Q$	$\ln P$	$\ln N$	$\ln \bar{r}$	$\frac{N_{iht}-N_{iht-1}}{(N_{iht}+N_{iht-1})/2}$	$\frac{E_{iht}}{(N_{iht}+N_{iht-1})/2}$	$\frac{-X_{iht-1}}{(N_{iht}+N_{iht-1})/2}$
focal, FEs: i-h, i-HS4-t	-1.241^a	-1.266^a	0.025	-0.643^a	-0.606^b	-0.622^a	-0.248^c	-0.375^a
	(0.309)	(0.340)	(0.151)	(0.134)	(0.242)	(0.135)	(0.145)	(0.112)
third, FEs: i-h, i-HS4-t	-0.110	-0.190	0.081	-0.134	0.023	-0.146^b	-0.184^b	0.039
	(0.143)	(0.166)	(0.065)	(0.092)	(0.081)	(0.071)	(0.077)	(0.061)

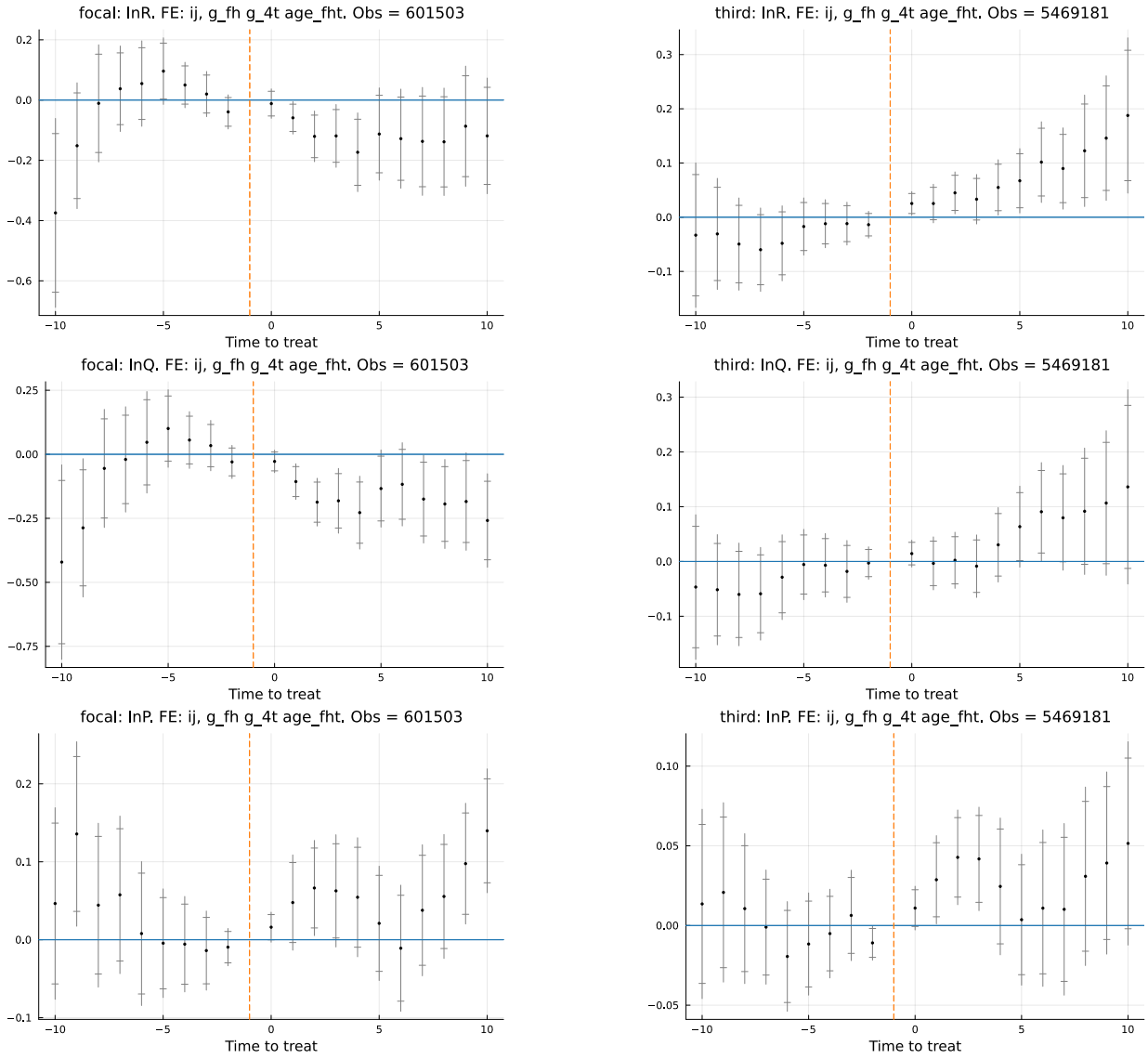
Notes: we calculate the average impact for at least six years after the treatment, i.e., $t \geq E_{ih} + 6$. It is calculated as $\frac{1}{K-5} \sum_{k \geq 6}^{K} \left(\hat{\beta}_k - \hat{\gamma}(k-1) \right)$ where $\hat{\beta}$ and γ are estimated from regression (2).

2.4 Firm-level responses to AD investigations

Fact 4. With the initiation of anti-dumping, firms' export value drops in the focal markets but *increases* in the third markets.

We now turn to the firm-level analysis. As discussed in Section 2.2, we aggregate the firm-country-product-year level data at the firm-market-product-year level and estimate the dynamic specification (4). The top two panels in Figure 5 present the results for the export value. Similar to the market-level results in Figure 1, we find a decline in the export value in the focal market, though the magnitude of the decline is much smaller at the firm level. However, the impact of AD on firms' export value in the third market is qualitatively different from the market-level analysis. We find that firm-level exports actually *increase* in the third market. Another difference is that we do not find much pretrends at the firm level, both in the focal and third markets. As is shown in the bottom two panels of Figure 5, AD investigations seem to raise the export prices in both markets. Though the estimates are only significantly different from zero at the 5% in certain years after the treatment, almost all of the point estimates are positive.

Figure 5: Effect of AD investigation at the firm level: Revenue, Quantity, and Price



Notes: All specifications include firm-HS6-product and HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.

3 Model

In this section, we rationalize the stylized fact using an industry equilibrium framework based on firms' simultaneous entry decisions across multiple markets. Section 3.1 describes the economic environment and highlights the key model assumptions. Section 3.2 presents our main analytical results in a special case with homogeneous firms. In Section 3.6, we

extend these results to the case of heterogeneous firms, under additional assumptions on the demand function.

3.1 Setup

There are $N+1$ countries, indexed by $i \in \{0, 1, \ldots, N\}$, where country 0 is the home country. We abstract our analysis from the home economy and focus on the N foreign economies: one focal market that changes its import tariff and $N-1$ third markets that do not change import tariffs. This focus is justified because AD tariffs are unlikely to generate general equilibrium effects in the domestic market, and firm-product-level data on domestic sales (within China) are unavailable. Moreover, we examine how firms from the home country (i.e., China) respond to the tariff shock, as similar data for firms from other countries are also unavailable.

Households In each foreign market i, goods are supplied by firms from all $N+1$ countries in the Armington fashion. We focus on an industry equilibrium in a single sector, where all goods are produced by firms from the home country. Specifically, the sectoral composite good Q_i, representing the total quantity supplied by home exporters in market i, is determined by the following inverse demand function:

$$P_i^I = D_i(Q_i) \Leftrightarrow E_i \equiv E_i(Q_i) = Q_i D_i(Q_i), \tag{5}$$

where D_i a demand shifter, and P_i^I and Q_i are the sectoral price and quantity respectively. Additionally, $\eta_i \equiv -\frac{\mathrm{d}\log Q_i}{\mathrm{d}\log P_i^I}$ is the elasticity of substitution between sectors (i.e., between goods produced by home exporters and those supplied by other N foreign countries). We assume that $\eta_i > 1$, so that $E_i'(Q_i) > 0$. A useful special case for the sectoral demand is the CES demand function:

$$P_i^I = D_i Q_i^{-\frac{1}{\eta_i}} \Leftrightarrow E_i = D_i Q_i^{1-\frac{1}{\eta_i}}, \quad \eta_i > 1,$$

where P_i^I is the *ideal price index* that are relevant for sectoral expenditure (or welfare) since $P^I Q = E$.

Firms from the home country produce a range of varieties in this sector, denoted by ω. We use Ω_i to denote the set of all varieties that are exported from the home country to foreign market i (i.e., exporting firms to country i), and Ω is the set of all exported varieties

(all exporting firms from home) from the home country.

For products produced by firms from the same country, consumers have a demand function that features homotheticity with direct implicit additivity (HDIA).[6] Specifically, we assume that consumers have the Kimball demand towards exported varieties ω given a sectoral composite Q_i (of varieties produced by home firms):

$$\int_{\Omega_i} \Upsilon\left(\frac{q_\omega^i}{Q_i}\right) \mathrm{d}\omega = 1, \tag{6}$$

The Kimball aggregator Υ satisfies that $\Upsilon' > 0, \Upsilon'' < 0$, and $\Upsilon(0) = 0$. The CES perference is a special case with $\Upsilon(x) = x^{\frac{\sigma-1}{\sigma}}$. We derive the inverse demand function for variety ω as[7]

$$\frac{p_\omega^i}{P_i} = \Upsilon'\left(\frac{q_\omega^i}{Q_i}\right), \tag{7}$$

in which the *price aggregator* P_i is defined as

$$P_i Q_i = \bar{\delta}_i \int_{\Omega_i} p_\omega^i q_\omega^i \mathrm{d}\omega \equiv \bar{\delta}_i E_i, \tag{8}$$

and the *demand index* $\bar{\delta}_i$ is defined as

$$\frac{1}{\bar{\delta}_i} = \int_{\Omega_i} \Upsilon'\left(\frac{q_\omega^i}{Q_i}\right) \frac{q_\omega^i}{Q_i} \mathrm{d}\omega. \tag{9}$$

Note that $P_i = P_i^I \bar{\delta}$ in the CES case. However, the ideal price index P_i^I does not coincide with the price aggregator P_i (up to a constant) in general, since $\bar{\delta}$ is also endogenous. In the special case, the demand system considered features a CES preference over all country-sector (or exporting-country specific) goods in the outer nest and a Kimball preference over varieties exported by the home country in the inner nest.

Entry and Exit In each period, firms make entry decisions in two stages. First, they pay an entry cost and choose an entry intensity (i.e., entry rate) for each foreign market, subject to a managerial capacity constraint. Next, firms draw productivities from an exogenous distribution, and entry opportunities are realized. Finally, firms decide whether to export to the markets that grant them access.

[6] See Matsuyama and Ushchev (2017) for details.

[7] See Appendix B for details.

The entry cost is denoted by f_e. Once the entry cost is paid, firms maximize the expected profits subject to the entry technology[8]:

$$\begin{aligned} \max_{J_i \geq 0} \quad & \sum_{i=0}^{N} J_i \bar{\pi}_i \\ \text{s.t.} \quad & \left[\sum_{i=0}^{N} \left(\frac{J_i}{h_i} \right)^{\frac{1}{1-\beta}} \right]^{1-\beta} = 1, \end{aligned}$$

in which h_i denoted the easiness of entering country i, $\bar{\pi}_i$ is the expected profit in country i, and $\beta \in [0, 1)$. Variable $\{J_i\}$ is the endogenous entry intensity chosen by the firm.

Solving this optimization problem gives the entry rate

$$J_i = h_i \left[\frac{(h_i \bar{\pi}_i)^{\frac{1}{\beta}}}{\sum_k (h_k \bar{\pi}_k)^{\frac{1}{\beta}}} \right]^{1-\beta} \tag{10}$$

The free entry condition implies that

$$\sum_i J_i \bar{\pi}_i = f_e \Leftrightarrow \left[\sum_k (h_k \bar{\pi}_k)^{\frac{1}{\beta}} \right]^{\beta} = f_e.$$

The parameter β is of key interest, as it captures the degree of market interdependence in firms' entry decisions. To illustrate its role, consider two extreme cases: $\beta = 0$ and $\beta \nearrow 1$. When $\beta = 0$, entry decisions across markets are independent, making different markets perfect substitutes. Firms will only enter the market(s) with the highest post-entry return; that is, $i \in \arg\max_k \{h_k \bar{\pi}_k\}$. Because firms must be indifferent when considering entry across multiple markets, the entry rate is determined by other equilibrium conditions. In this case, the free-entry condition implies that any market with positive entry must yield a constant

[8] Boehm and Oberfield (2020) consider a model extension with this entry technology in their online appendix. In our understanding, this constraint is fundamentally related to the *fixed* amount of managerial resources (e.g., time or capacity) that is use to promote successful entries into various export markets. Specifically, we normalize this amount to one and treat J_i as the endogenous entry probably into market i, and thus $J_i \in [0, 1]$. $\left(\frac{J_i}{h_i}\right)^{\frac{1}{1-\beta}}$ is the required convex managerial cost (capacity) that generates the entry probability of J_i in market i, as $\beta \in [0, 1)$ and $\left(\frac{1}{h_i}\right)^{\frac{1}{1-\beta}}$ is the cost shifter. As a result, the overall managerial cost across all markets is $\sum_{i=0}^{N} \left(\frac{J_i}{h_i}\right)^{\frac{1}{1-\beta}} = 1$ or $\left[\sum_{i=0}^{N} \left(\frac{J_i}{h_i}\right)^{\frac{1}{1-\beta}}\right]^{1-\beta} = 1$.

expected profit $\bar{\pi}_i = \frac{f_e}{h_i}$.[9] This property ensures complete insulation in the third markets when (tariff) shocks happen in the focal markets, i.e., markets are independent.

When $\beta \nearrow 1$, entry decisions across markets become perfect complements. In this case, firms enter all markets at exogenous rates, $J_i = h_i$. This corresponds to the standard entry technology in Melitz (2003): once the entry cost is paid, firms gain access to all foreign markets, although they may still choose not to export to some of them. Under this scenario, the free entry condition simplifies to the familiar form in which the sum of expected profits across all markets equals the entry cost: $\sum_i h_i \bar{\pi}_i = f_e$.

After entry, firms draw a productivity level z from an exogenous distribution G, and entry opportunities are realized. We use O to denote the set of markets to which a firm gains access. Each market features a fixed cost f_i, and firms export to this market if they make positive profit there. Incumbent firms exit each market randomly with a probability of ν_i. Both the fixed and entry costs are denominated in units of labor, which is chosen as the numéraire.

Production Each firm supplies a distinct variety ω and competes monopolistically. Labor is the only factor of production in the economy. The production function is linear: a firm with productivity z produces zl units of goods using l units of labor. Each country sets its own tariff rate, with t_i denoting the ratio between the consumer price and the producer price in country i. Additionally, τ_i represents the iceberg shipping cost from home country to country i, and $\tau_i \geq 1$.

Taking the residual demand given in equation (7), firms set prices in each market to maximize their profits there. Specifically, each firm solves the following profit maximization problem in each market:

$$\max_p \quad (p - \frac{\tau_i}{z}) q_i \left(t_i p\right),$$

where the explicit form of demand function is

$$q_i(p) = (\Upsilon')^{-1} \left(\frac{p}{P_i}\right) Q_i = \Psi \left(\frac{p}{P_i}\right) Q_i,$$

[9] To see this, let $k = h_i \bar{\pi}_i$ if $i \in \arg\max_k \{h_k \bar{\pi}_k\}$. Then,

$$k = k \sum_i \frac{J_i}{h_i} = \sum_i J_i \bar{\pi}_i = f_e.$$

and $\Psi \equiv (\Upsilon')^{-1}$. We assume that Kimball aggregator ensures a unique interior solution.[10] The usual Lerner formula gives

$$\frac{p - \tau/z}{p} = -\frac{q_i(t_i p)}{q_i'(t_i p) t_i p} \equiv \frac{1}{\sigma_i(t_i p)},$$

where $\sigma_i \equiv -\frac{\mathrm{d}\log q_i}{\mathrm{d}\log p}$ is the price elasticity. Noticing that σ_i could be written as a function of p/P_i or q/Q_i, i.e.,

$$\sigma_i(p) = \underbrace{-\frac{\Psi'\left(\frac{p}{P_i}\right)\frac{p}{P_i}}{\Psi\left(\frac{p}{P_i}\right)}}_{\sigma\left(\frac{p}{P_i}\right)} = \underbrace{-\frac{\Upsilon'\left(\frac{q_i}{Q_i}\right)}{\Upsilon''\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i}}}_{\sigma\left(\frac{q_i}{Q_i}\right)}$$

with $q_i \equiv q_i(p)$. In addition, let μ be the markup, i.e., the ratio between price and marginal cost. Then,

$$\mu_i = \frac{\sigma_i}{\sigma_i - 1},$$

and we define $\mu\left(\frac{p}{P_i}\right)$ and $\mu\left(\frac{q}{Q_i}\right)$ accordingly. As usual, we assume that $\sigma_i > 1$ so markup is positive.

We now summarize the set of variables related to firms' production. For a firm with productivity z, its price, markup, and quantity in country i are determined implicitly by

$$p_z^i = \frac{\mu_z^i \tau_i}{z}, \quad q_z^i = q_i(t_i p_z^i), \quad \text{and } \mu_z^i = \mu\left(\frac{t_i p_z^i}{P_i}\right) = \mu\left(\frac{q_z^i}{Q_i}\right),$$

Let $\tilde{P}_i = P_i/t_i$. Then, $\tilde{P}_i$ is the *producer price index* while P_i is the *consumer price index*. We have $\tilde{P}_i Q_i = R_i \bar{\delta}_i$, where $R_i = E_i/t_i$ is the aggregate export revenue in country i. Comparing to the CES case, markup is no longer constant, so the passthrough of tariff will be incomplete.

The per period variable profits and sales are as follows:

$$\pi_z^i = r_z^i\left(1 - \frac{1}{\mu_z^i}\right), \quad r_z^i = p_z^i q_z^i = \mu_z^i l_z^i.$$

Then, firms export to market i if $\pi_z^i \geq f_i$. Since the profit function is increasing and continuous, there exists a threshold z_i^* such that firms export to country i if and only if $z \geq z_i^*$. In

[10] This requires $\Upsilon'''(x)x + 2\Upsilon''(x) < 0$. That Υ''' exists will ensure that the profit function π_z^i is continuous in z.

the rest of the paper, we only consider the case in which z_i^* is within the range of possible productivity draws (i.e., within the interior of the distribution), i.e., $\pi^i_{z_i^*} = f_i$.

Finally, there is an exogenous exit shock ν_i in each country. Firms discount future profits at a rate of ρ. Then, the expected entry profit of country i satisfies that

$$\bar{\pi}_i = \frac{1}{\rho + \nu_i} \int_{z_i^*}^{\infty} [\pi_z^i - f_i] dG(z). \tag{11}$$

Equilibrium We consider a perfect foresight industry equilibrium in which the wage and tariffs $\{t_i\}$ are exogenous. In equilibrium, firms optimally decide on entry and production, taking as given the time series of sectoral prices and composite goods, which also clear all $N+1$ goods markets. For most of the analysis, we focus on a stationary equilibrium in which all aggregate variables remain constant. Let M be the number of entrants and N_i be the number of exporters to country i, then the law of motion on N_i is given by:

$$\dot{N}_i = J_i M[1 - G(z_i^*)] - \nu_i N_i.$$

In a stationary equilibrium, $\dot{N}_i = 0$ for all i.

Infra-marginal ratio of consumer surplus and cost passthrough Before, proceeding, we introduce two key endogenous variables that will be used intensively in our following analyses. First, the infra-marginal surplus ratio (as in Baqaee et al. (2023)) is given by:

$$\delta_i = \delta\left(\frac{q_i}{Q_i}\right) = \frac{\Upsilon\left(\frac{q_i}{Q_i}\right)}{\Upsilon'\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i}}$$

δ_i is one plus the ratio of consumer surplus to revenue evaluated at (p_i, q_i). Since Υ is concave, $\delta_i > 1$. Next, the firm-level passthrough rate of a cost on th price is defined as

$$\rho = -\frac{\mathrm{d}\log p}{\mathrm{d}\log z} = 1 - \frac{\mathrm{d}\log \mu}{\mathrm{d}\log z},$$

where the latter equality holds fixing τ_i and t_i. $\rho > 0$ due to the supermodularity of the profit function. Solving this equation renders that

$$\rho_i = \rho\left(\frac{q_i}{Q_i}\right) = \frac{1}{1 + \frac{\mu'\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i}}{\mu\left(\frac{q_i}{Q_i}\right)}\sigma\left(\frac{q_i}{Q_i}\right)},$$

which shows that the cost passthrough is related to the price elasticity and the elasticity of the markup (i.e., the elasticity of the price elasticity). Evidently, $\mu'\left(\frac{q_i}{Q_i}\right) = 0$ when the demand is CES, which implies that $\rho_i = 1$ (i.e., a complete passthrough). Intuitively, if the markup increases with the normalized quantity $\frac{q_i}{Q_i}$ faster (i.e., a larger elasticity), the cost passthorugh will be smaller implying that a given reduction in the marginal cost leads to a smaller reduction in the price. Accordingly, the producer's profit margin increases.

Finally,, we impose several conditions on the two elasticities before proceeding:

Assumption 1. *$\rho_i \leq 1$, $\sigma_i > \eta_i$ and $\delta_i < \frac{\eta_i}{\eta_i - 1}$*

Baqaee et al. (2023) argue that the first condition corresponds to the weak version of Marshall's second law of demand, which states that markups increase with productivity. This is equivalent to $\mu_i' \geq 0$. The second condition follows Atkeson and Burstein (2008), who assume that substitution between products is stronger within sectors than across sectors. In our context, this implies that substitution between goods exported from the same country is stronger than that between goods from different countries of origin. The third condition imposes diminishing returns to firm entry—that is, the number of firms in equilibrium must be finite. In general, firm entry generates both negative and positive externalities for incumbents. On one hand, it intensifies market competition, reducing profits for existing firms. On the other hand, when a variety effect is present, it boosts industry-level demand. This condition requires that the competition effect outweighs the aggregate demand effect, so that the post-entry profit of an entrant declines with the number of incumbent firms. Note that under the nested CES demand structure in Atkeson and Burstein (2008), the second condition is equivalent to the third. However, in general, the two conditions are not equivalent.

3.2 Homogeneous firms

We start with the case in which firms are homogeneous. As a result, the productivity distribution G degenerates to a single mass point at a given productivity level z. We assume

that in equilibrium, all markets have strictly positive measure of exporters.[11] To simplified the notation, we suppress the notation of the variety (ω) and use p_i and q_i to denote firm-level price and quantity in the case of homogeneous firms.

In what follows, we lay down three sets of equilibrium conditions. First, the Kimball preference with homogeneous firms implies that

$$N_i \Upsilon\left(\frac{q_i}{Q_i}\right) = 1. \tag{12}$$

The income accounting condition (or the balanced budget equation) implies that

$$E_i(Q_i) = N_i t_i p_i q_i \Leftrightarrow p_i^I(Q_i) = N_i t_i \mu\left(\frac{q_i}{Q_i}\right)\frac{\tau_i}{z}\frac{q_i}{Q_i}. \tag{13}$$

Equations (12) and (13) pin down $\frac{q_i}{Q_i}$ and Q_i as functions of N_i and model parameters such as t_i, z and τ_i. Therefore, we can write σ_i, η_i, ρ_i, δ_i as functions of N_i.

The next condition expresses the per period variable profits as a function of N_i, Q_i and model parameters:

$$\pi_i(N_i) = \left(1 - \frac{1}{\mu_i}\right) p_i q_i = \left(1 - \frac{1}{\mu_i}\right)\frac{E_i(Q_i)}{N_i t_i}. \tag{14}$$

equation (11) implies that $\bar{\pi}_i = \frac{\pi_i - f_i}{\rho + \nu_i}$. As the ratio of entry intensities is proportional to the ratio of expected post-entry profits:

$$\frac{J_i}{J_j} = \frac{h_i^{\frac{1}{\beta}} \bar{\pi}_i^{\frac{1-\beta}{\beta}}}{h_j^{\frac{1}{\beta}} \bar{\pi}_j^{\frac{1-\beta}{\beta}}},$$

we can use equation (11) to calculate the series of j_i. Finally, we have two sets of conditions for $(M, \{N_i\})$:

$$\sum_{i=0}^{N} (h_i \bar{\pi}_i)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}}, \quad \forall\, i$$

$$J_i M = \nu_i N_i,$$

where the first condition above is the free entry condition and the second set of conditions

[11] A sufficient condition could be $\liminf_{q/Q \to \infty} \mu(q/Q) = \underline{\mu} > 1$.

is the balanced flow of firms in each market. In total, there are $N+2$ equations for $N+2$ unknowns among all markets. Notice that with equation (10), the above equations imply that

$$J_i = h_i \left(\frac{h_i \bar{\pi}_i}{f_e}\right)^{\frac{1-\beta}{\beta}} = \frac{\nu_i N_i}{M} \quad \Rightarrow \quad \pi_i(N_i) = (\rho + \nu_i) \left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{\beta}{1-\beta}} \frac{f_e}{h_i} + f_i, \tag{15}$$

in which $\pi_i(N_i)$ is given implicitly in equation (14). Therefore, the second part of the above equations provides us an implicit function N_i in terms of M. We define such a mapping using $N_i = N_i(M)$. Inserting it back into the free entry condition, we pin down the equilibrium mass of entrants M as follows:

$$\sum_{i=0}^{N} \left(h_i \bar{\pi}_i(N_i(M))\right)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}}. \tag{16}$$

In this way, we have solved the stationary equilibrium. We show that a stationary equilibrium exists and is unique under Assumption 1:

Proposition 1. *Suppose Assumption 1 holds for all N_i and $\liminf_{N_i \to k} \frac{\eta_i}{\eta_i - 1} \frac{1}{\delta_i} > 1$ for $k \in \{0, +\infty\}$. Then, there exists a unique stationary equilibrium.*

Proof. See Appendix C.3. ■

3.2.1 Comparative statics: tariff shocks and market interdependence

We now analyze market-specific responses to tariff shocks $\{t_i\}$. Before presenting our results, it is helpful to clarify the model counterparts of the empirical patterns and to define several variables used in the empirical analysis. Specifically, we interpret *aggregate variables* in the model as corresponding to the product-country level in the data, and *firm variables* as corresponding to the firm-product-country level.

$$\mathbb{E}_x[y] = \frac{\int x(v) y(v) \mathrm{d}v}{\int y(v) \mathrm{d}v},$$

in which x and y are functions of v. Note that v could be both discrete and continuous. At the aggregate level in market i, we have total revenue R_i, total quantity $\tilde{Q}_i$, unit value p_i^m,

the number of exporters N_i, and average revenue r_i^m where

$$\tilde{Q}_i = \int_{\Omega_i} q_i \mathrm{d}\omega; \quad p_i^m = \frac{R_i}{\tilde{Q}_i} = \frac{\int_{\Omega_i} p_i q_i \mathrm{d}\omega}{\int_{\Omega_i} q_i \mathrm{d}\omega} = \mathbb{E}_q[p_i]; \quad r_i^m \equiv R_i / N_i.$$

At the firm level, we have revenue r_i, profit π_i, price p_i, and quantity q_i. In the case of homogeneous firms, unit price coincides with firm price, and average revenue coincides with firm revenue. In addition, we have the market-specific utility Q_i (which matters for welfare) and the total number of entrants M, both of which are unobservable from the data.[12]

We discuss how various price measures are related. There are four variables related to prices: firm price p_i, aggregate unit value p_i^m, (Kimball) price aggregator P_i, and the ideal price index P_i^I. First, the unit price p_i^m is the quantity weighted average of firm prices (p_i). In particular, $p_i = p_i^m$ in the case of homogeneous firms. Next, the Kimball price aggregator P_i follows the definition in equation (8), and the ideal price index P_i^I is defined in equation (5). They satisfy that $P_i = P_i^I \bar{\delta}_i$, where $\bar{\delta}_i$ is defined in equation (9). P_i^I is the aggregation of p_i adjusted by the number of varieties, while P_i is further adjusted by the infra-marginal surplus ratio. In the case of CES demand with homogeneous firms, we have $\bar{\delta}_i = \frac{\sigma_i}{\sigma_i - 1}$, $P_i^I = N_i^{\frac{1}{1-\sigma_i}} p_i$, and $P_i = \frac{\sigma_i}{\sigma_i - 1} N_i^{\frac{1}{1-\sigma_i}} p_i$.

The implications of tariffs We first present the following proposition that characterizes the response of M to a set of tariff shocks $\mathrm{d}\log t_i$. This is a central result since the transmission of tariff shocks across market works *solely* through firms' entry. All the results assume that the economy was in a stationary equilibrium before tariff changes.

Proposition 2. *Suppose assumption 1 holds, and firms have the same productivity z. In response to a set of tariff shocks $\mathrm{d}\log t_i$ where $i \in \{1, 2, 3, ..., N\}$, changes in the total number of entrants are given by*

$$\mathrm{d}\log M = -\frac{\sum_{i=0}^{N} s_i \xi_i \frac{\mathrm{d}\log t_i}{\theta_i}}{\sum_{i=0}^{N} s_i \xi_i} = -\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta_i}\right], \tag{17}$$

in which

$$\theta_i = \left(\frac{1}{\eta_i} - \frac{1}{\sigma_i}\right)\left(\frac{1}{\rho_i} - 1\right)\delta_i + 1 - \frac{\eta_i - 1}{\eta_i}\delta_i > 0, \tag{18}$$

[12] As we focus on a representative sector, we use the change in real consumption, $\mathrm{d}\log Q_i$, to measure the welfare impact in country i.

$$s_i = \frac{J_i \bar{\pi}_i}{\sum_k J_k \bar{\pi}_k}, \qquad \textit{and} \qquad \xi_i = \frac{\frac{\pi_i}{\pi_i - f_i}\eta_i \theta_i}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\eta_i \theta_i} \leq 1. \tag{19}$$

Proof. See Appendix C.2 ■

A tariff increase affects firm profits through two channels: a direct (negative) demand effect and an indirect (positive) competition effect. These effects must be offset by an adjustment in the mass of firms, as required by the free entry condition. The variable θ captures the necessary adjustment in firm mass to restore equilibrium, and this is reflected in the two terms on the right-hand side (RHS) of equation (18). The first term represents the pro-competitive effect arising from variable markups. This effect disappears when markups are constant (i.e., $\rho_i = 1$).[13] The second term (i.e., $1 - \frac{\eta_i - 1}{\eta_i}\delta_i$) captures the required firm mass adjustment due to the direct demand effect. This term becomes larger (i.e., the required adjustment becomes smaller) when expected post-entry profits exhibit stronger diminishing returns with respect to firm entry. Intuitively, when diminishing returns are strong, a smaller reduction in firm mass is sufficient to offset the profit loss and restore the free entry condition.[14]

When the free entry condition holds, the expected profit from entry remains constant. To preserve this equilibrium, entry must decline in response to adverse shocks. Proposition 2 shows that the required reduction in total entry equals the weighted average of the required entry reductions across individual markets. The weights, $s_i \xi_i$, are the product of the share of expected profits in market i, denoted s_i, and a wedge ξ_i, which captures the degree of entry interdependence across markets.

In this model, markets are independent of each other, except that they may import from the same set of firms. As a result, firm entry is the only channel through which shocks are transmitted across markets. In the following proposition, we show that the change in the total number of entrants, $\mathrm{d}logM$, serves as a sufficient statistic for capturing all market-level changes.

Proposition 3. *Suppose that Assumption 1 holds, and firms have the same productivity z. In response to a set of tariff shocks $\mathrm{d}\log t_i$ where $i \in \{1, 2, ..., N\}$, the following equations*

[13] Following a tariff shock, surviving firms raise their markups due to reduced competition, thereby softening competitive pressure. This pro-competitive effect is stronger—and the required firm mass reduction is therefore smaller—when the passthrough rate is lower.

[14] An example is the case of nested CES demand:

$$\pi_i \propto N_i^{\frac{\sigma-\eta}{1-\sigma}}, \quad \text{and} \quad \frac{\sigma_i - \eta_i}{1 - \sigma_i} = -\eta_i \left(1 - \frac{\eta_i - 1}{\eta_i}\frac{\sigma_i}{\sigma_i - 1}\right) \equiv -\eta_i \theta_i$$

, where the pro-competitive effect vanishes.

characterize changes of endogenous variables in equilibrium:

i) At the aggregate level,

$$\mathrm{d}\log N_i = -\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i \frac{\mathrm{d}\log t_i}{\theta_i}\right] = (1-\xi_i)\mathrm{d}\log M + \xi_i \frac{-\mathrm{d}\log t_i}{\theta_i}, \quad (20)$$

$$\mathrm{d}\log \tilde{Q}_i = \underbrace{\left(\delta_i\left(\frac{1}{\rho_i}-1\right) - \eta_i\theta_i + 1\right)}_{>0}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i,$$

$$\mathrm{d}\log R_i = \underbrace{\left[\left(\frac{1}{\rho_i}-1\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i - \eta_i\theta_i + 1\right]}_{>0}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i;$$

ii) At the firm level,

$$\mathrm{d}\log \pi_i = -\eta_i\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i,$$

$$\mathrm{d}\log p_i = -\left(\frac{1}{\rho_i}-1\right)\frac{\delta_i}{\sigma_i}\mathrm{d}\log N_i,$$

$$\mathrm{d}\log q_i = \left(\delta_i\left(\frac{1}{\rho_i}-1\right) - \eta_i\theta_i\right)\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i,$$

$$\mathrm{d}\log r_i = \left[\left(\frac{1}{\rho_i}-1\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i - \eta_i\theta_i\right]\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i;$$

$$\mathrm{d}\log Q_i = \underbrace{\left(\frac{\delta_i}{\rho_i} - \eta_i\theta_i\right)}_{>0}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i.$$

Proof. See Appendix C.2. ■

Proposition 3 provides a complete characterization of changes within each market. The number of firms in market i, denoted N_i, acts as a sufficient statistic for conducting comparative statics in that market. Specifically, changes in both firm-level and aggregate variables

are driven either by the local tariff shock or by the change in N_i. The change of N_i, $\mathrm{d}\log N_i$, is a weighted average of the change in the mass of all entrants ($\mathrm{d}\log M$) and the rescaled local tariff shock, $-\mathrm{d}\log t_i/\theta_i$. The latter term reflects the change in N_i that would occur in the absence of market interdependence. To align directly with the empirical analysis, we next consider a case in which only one (focal) market raises its tariff and derive the following result.

Corollary 1. *Suppose that assumption 1 holds, and firms have the same productivity z. Country j ("the focal market") raises its tariff, i.e., $\mathrm{d}\log t_j > 0$, while all other countries ("the third markets") retain previous tariffs, i.e., $\mathrm{d}\log t_i = 0$ for $i \neq j$. We have the following qualitative equilibrium changes:*

i) In the focal market j,

$$\mathrm{d}\log N_j < 0, \mathrm{d}\log \tilde{Q}_j < 0, \mathrm{d}\log R_j < 0, \mathrm{d}\log p_j > 0, \mathrm{d}\log q_j < 0, \mathrm{d}\log r_j < 0, \mathrm{d}\log Q_j < 0;$$

ii) In the third market $i \neq s$,

$$\mathrm{d}\log N_i < 0, \mathrm{d}\log \tilde{Q}_j < 0, \mathrm{d}\log R_i < 0, \mathrm{d}\log p_i > 0, \mathrm{d}\log Q_i < 0,$$

$$\mathrm{d}\log q_i > 0 \text{ if and only if } \frac{\eta_i}{\eta_i - 1} > \frac{\delta_i}{1 - \frac{\delta_i}{\sigma_i}\left(\frac{1}{\rho_i} - 1\right)},$$

$$\mathrm{d}\log r_i > 0 \text{ if and only if } \frac{\eta_i}{\eta_i - 1} > \delta_i \left[1 + \left(\frac{1}{\rho_i} - 1\right)\frac{1}{\sigma_i}\right].$$

Proof. See Appendix C.2 ■

The predictions in Corollary 1 align well with the empirical evidence. In particular, the results rationalize the simultaneous decline in aggregate trade value and quantity, as well as in the number of exporting firms, in both the focal and third markets. The corollary also predicts simultaneous increases in firm-level exports and unit prices across both markets—again, consistent with the empirical findings. Importantly, Corollary 1 allows for diverging effects at the firm and aggregate levels in third markets: firm-level export quantities or revenues may increase, even as aggregate export quantities or revenues decline. This divergence necessarily occurs under nested CES demand when $\rho = 1$.

The simultaneous decline in the number of exporters across both markets warrants further explanation. In the focal market, the tariff increase reduces the number of exporters due to two reinforcing effects: a decline in the total number of exporters and a reduction in the entry intensity into that market. In the third markets, although the entry intensity rises—as some firms reallocate their focus away from the focal market—the overall number of exporters still falls. This occurs because the decline in the total mass of entrants more than offsets the reallocation effect. Overall, the complementarity between markets—arising from the *shared* pool of exporting entrants—consistently outweighs the substitutability across markets that operates through the endogenous allocation of entry intensities.[15]

3.2.2 Some special cases

Nested CES demand : First, it is instructive to study the special case with nested CES preference. Namely, we have $\Upsilon_i(x) = x^{\frac{\sigma_i-1}{\sigma_i}}$, $D_i(x) = D_i x_i^{-\frac{1}{\eta_i}}$. Assumption 1 is equivalent to $\sigma_i > \eta_i$. Furthermore,

$$\mu_i = \delta_i = \frac{\sigma_i}{\sigma_i - 1}, \quad \eta_i\theta_i = \frac{\sigma_i - \eta_i}{\sigma_i - 1}, \quad \rho_i = 1.$$

We then restate proposition 3 as

Corollary 2. *Suppose Assumption 1 holds, and firms have the same productivity z. Furthermore, the preference is assumed to be nested CES. In response to a set of tariff shocks $\mathrm{d}\log t_i$ where $i \in \{1, 2, ..., N\}$, the following equations characterize changes in the equilibrium:*

i) At the aggregate level,

$$\mathrm{d}\log R_i = \mathrm{d}\log \tilde{Q}_i = \frac{\eta_i - 1}{\sigma_i - 1}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i;$$

ii) At the firm level,

$$\mathrm{d}\log p_i = 0, \quad \mathrm{d}\log \pi_i = \mathrm{d}\log r_i = \mathrm{d}\log q_i = -\frac{\sigma_i - \eta_i}{\sigma_i - 1}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i;$$

$$\mathrm{d}\log Q_i = \frac{\eta_i}{\sigma_i - 1}\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i.$$

[15] That is, an increase in expected profits in one market leads to a higher entry intensity into that market and lower entry intensities into all others.

The model with nested CES preference inherits all the properties of our general model except for the pro-competitive effect. With constant markups, profit, revenue, quantity co-move perfectly, and firm-level quantity and revenue always decrease with the number of firms in the third markets.

Independent markets: $\beta = 0$ Next, we consider the case where $\beta = 0$. In this case, $\xi_i = 1$. Then,

$$\mathrm{d}\log N_i = -\frac{\mathrm{d}\log t_i}{\theta_i}, \quad \mathrm{d}\log M = -s_i\frac{\mathrm{d}\log t_i}{\theta_i}.$$

As suggested by the name, only the focal market is affected by the tariff shock since firms make orthogonal entry decisions for each market. Comparing to equation (20), the focal market bears the full cost without any diversion into the third markets. The total number of entrants changes, only because the number of entrants into the focal market changes.

Common entry: $\beta \to 1$ In this case, entry is common across all markets up to some exogenous constants as in Melitz (2003). With exogenous exit, N_i is linear in M in the stationary equilibrium. Then, all markets witness the same change in the number of firms, which is given by the change in the number of total entrants. Formally,

$$\mathrm{d}\log N_i = \mathrm{d}\log M = -\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]; \quad \forall i \in \{1, 2, ..., N\},$$

in which $\xi_i = \frac{\pi_i\eta_i\theta_i}{\pi_i - f_i}$. In this way, the common entry technology standard in the literature implies a strong diversion force such that while tariff shocks may be asymmetric across markets, their impact on entry is always fully diversified across markets. All markets bear the same reduction in competition to jointly buffer a unilateral tariff shock.

3.2.3 Entry interdependence

We now discuss the role of entry interdependence (β) in the transmission of shocks. Note that the change in the number of active firms in market i ($\mathrm{d}\log N_i$) is a *sufficient statistic* to measure how β influences the shock transmission. As before, we consider only one country ("j") increases the tariff. Simple calculation shows

$$\frac{\mathrm{d}\log N_i}{\mathrm{d}\log t_j} = -\left[(1-\xi_i)\tilde{s}_j + \xi_i \mathbb{1}\{i=j\}\right]\frac{1}{\theta_j}$$

where $\tilde{s}_j = \frac{s_j \xi_j}{\sum_i s_i \xi_i}$ and $\xi_i = \frac{v_i}{v_i + \tilde{\beta}}$, in which $v_i = \frac{\pi_i}{\pi_i - f_i} \eta_i \theta_i$ and $\tilde{\beta} = \beta/(1-\beta)$. Since $\tilde{\beta}$ is a monotonic transformation of β, we consider the cross derivative of $\mathrm{d} \log N_i$ with respect to $\mathrm{d} \log t_j$ and $\tilde{\beta}$:

$$\frac{\partial}{\partial \tilde{\beta}} \frac{\mathrm{d} \log N_i}{\mathrm{d} \log t_j} = - \left[(1 - \xi_i) \tilde{s}_j \left[- \frac{\xi_j}{v_j} + \sum_i \tilde{s}_i \frac{\xi_i}{v_i} \right] - \frac{\xi_i^2}{v_i} \left(\mathbb{1}\{i = j\} - \tilde{s}_j \right) \right] \frac{1}{\theta_j}.$$

In general, the sign is ambiguous (i.e., not necessarily negative for the third markets), as an increase in β can either reduce or increase the importance of the focal market (i.e., $\tilde{s}_j$) in shock transmissions.[16] Nevertheless, we show in below that a larger β leads to more transmission of shocks unambiguously in cases where markets are either independent initially or symmetric.

Independent markets Markets are independent when $\beta = 0$, which implies $\xi_i = 1$ and $\tilde{s}_j = s_j$. Then, we have

$$\frac{\partial}{\partial \tilde{\beta}} \frac{\mathrm{d} \log N_i}{\mathrm{d} \log t_j} = \frac{1}{v_i} \left(\mathbb{1}\{i = j\} - s_j \right) \frac{1}{\theta_j} \quad \begin{cases} < 0, & \text{if } i \neq j, \\ > 0, & \text{if } i = j. \end{cases}$$

Now changes in $\tilde{s}_j$ do not matter (to the first order) when we vary β from the initial equilibrium. A tariff increase in market j leads to a larger (smaller) drop of the number of firm in the third (focal) market, when β increases. As a result, the tariff shock is more diversified with a bigger β which implies more interdependence across countries. In terms of the real consumption Q, the focal market suffers less while all the third markets suffer more when β is bigger.

Symmetric markets We consider another scenario in which all markets are identical in the initial equilibrium, i.e., $\xi_i = \xi$, $v_i = v$, $\theta_i = \theta$, $\tilde{s}_j = \frac{1}{N+1}$. Then, we have

$$\frac{\partial}{\partial \tilde{\beta}} \frac{\mathrm{d} \log N_i}{\mathrm{d} \log t_j} = \frac{\xi^2}{v} \left(\mathbb{1}\{i = j\} - \frac{1}{N+1} \right) \frac{1}{\theta} \quad \begin{cases} < 0, & \text{if } i \neq j, \\ > 0, & \text{if } i = j. \end{cases}$$

[16] When β is larger, markets are more interdependent but the influence of the shock $\tilde{s}_j$ can become smaller (the term of $-\frac{\xi_j}{v_j}$ in this equation). That is, even if all markets are more interdependent when β increases, the focal market can become relatively less important (among all markets) than before. For the focal market, the influence of shock $\tilde{s}_j$ can become *larger* (the term of $\sum_i \tilde{s}_i \frac{\xi_i}{v_i}$ in this equation) when β increases, which leads to an ambiguous sign for the focal market as well.

With symmetric markets, there is no first order change in the influence of shock $\tilde{s}_j$ for all ξ_i from the initial equilibrium. Therefore, the diversification force becomes the only equilibrium force. The distributional effect of a market-specific tariff shock across market becomes more pronounced with a larger β and is maximized when β approaches to 1.

3.3 Discussion of the entry margin

The delocation effect has traditionally been interpreted as evidence of increasing returns to scale (IRS). While our framework allows for industry-level IRS for generality, it is important to emphasize that the entry margin operates independently of the IRS assumption. Drawing on Hopenhayn (1992), we can construct an industry equilibrium model in which firms enter multiple markets under perfect competition and face decreasing returns to scale at the firm level. This alternative setup yields qualitatively similar responses to tariff shocks in both the focal and third markets, as described earlier.[17] Thus, the delocation effect does not necessarily indicate the presence of IRS. Instead, it can be fully explained by the extensive margin of firm entry.

3.4 General equilibrium effects

The baseline setup abstracts from general equilibrium effects—such as changes in relative wages—based on the view that antidumping (AD) tariffs are unlikely to generate sizable general equilibrium adjustments of this kind. In this section, we discuss how allowing for general equilibrium effects may alter the results. Because the home-country wage is chosen as the numéraire, we capture general equilibrium adjustments through shifts in foreign demand for the composite good. Specifically, demand for the composite good in market i is now given by

$$P_i^I = W_i D_i(Q_i) \Rightarrow \mathrm{d}\log P_i^I = \mathrm{d}\log W_i - \frac{1}{\eta_i}\mathrm{d}\log Q_i$$

A positive $\mathrm{d}\log W_i$ corresponds to an outward shift in demand for home exports—for instance, arising from a decline in the home-country wage, which is equivalent to an increase in the relative wage in the foreign country.

We summarize the discussion here and leave a complete exposition in Appendix E.1. In particular, we do not attempt to model the determination of W_i. Rather, we take $\mathrm{d}\log W_i$

[17] Materials available upon request.

as given and focus on its possible implications. First, we show all formulas hold if $\mathrm{d}\log t_i$ is replaced by

$$\mathrm{d}\log \tilde{t}_i = \mathrm{d}\log t_i - \mathrm{d}\log W_i,$$

the effective tariff shocks net of general equilibrium effect. Hence, in third markets with positive $\mathrm{d}\log W_i$, exporters are better off with negative effective tariffs.

Second, aggregate trade diversion can arise in third markets. In particular, the number of exporters serving a third market can increase when demand expands (large $\mathrm{d}\log W_i$) and the market is relatively insulated from the focal market (ξ_i is close to one). Moreover, home exporters' profitability in these third markets may increase even as the number of exporters rises: the demand expansion can be sufficiently strong to outweigh the intensified competition associated with additional entrants, $\mathrm{d}\log P_i^I$, is driven exclusively by changes in the number of exporters, $\mathrm{d}\log N_i$, and is independent of general equilibrium effects. Specifically, $\mathrm{d}\log P_i^I$ increases if and only if the number of exporters declines.

Our analysis shows that trade diversion primarily occurs toward third countries that are winners of the unilateral tariff. Specifically, these countries are less exposed to the tariff increase in the focal market (high ξ) and experience sizable real income gains (large $\mathrm{d}\log W_i$), reflecting strong terms-of-trade improvements. This interpretation contrasts with the conventional argument invoked by policymakers in third countries to justify the imposition of retaliatory or follow-up tariffs. The prevailing concern is that trade diversion intensifies competition in domestic markets, thereby harming local firms and necessitating protective measures to safeguard welfare. Our results suggest the opposite: trade diversion is a manifestation of relative wage gains in the third country rather than a precursor to welfare losses. Moreover, we show that unilateral tariffs not only protect firms from third countries in the focal market, but also benefit firms operating within those third countries.[18]

[18] Appendix E.1 explicitly models strategic interactions between home exporters and other sellers in destination markets to assess whether competition intensifies in third-country markets. In particular, we allow non-home firms in the third country to take the ideal price index of the home composite good into account when setting prices. This extension provides a microfoundation for the demand function for home goods, $(D_i(Q_i))$, so that all baseline results continue to apply. In partial equilibrium, an increase in the ideal price index of the home composite good, P_i^I, necessarily reduces the market share of home exporters: as home exporters raise prices following the tariff shock, non-home firms gain market share in the third-country market.

3.5 Firm-level non-constant returns to scale technology

The canonical trade diversion effect emphasized in the antidumping (AD) literature relies critically on the assumption that firms operate under increasing marginal costs (i.e., decreasing returns to scale). When exports to the targeted market are curtailed by AD tariffs, the resulting contraction in output lowers a firm's marginal cost, making it cheaper to serve third-country markets and thereby generating trade diversion.

Recent evidence by Almunia et al. (2021) supports this mechanism from a different angle. Studying the Spanish economy during the financial crisis—when domestic demand collapsed sharply—they document a surge in Spanish exports, a phenomenon they term the "venting-out" effect. This pattern is likewise consistent with firm-level production technologies featuring increasing marginal costs, as a decline in domestic sales reduces marginal costs and stimulates foreign sales. In this subsection, we introduce a non-constant-returns-to-scale production technology into the industry equilibrium and examine how this feature alters the model's predictions.

We summarize our key findings here and relegate technical details to Appendix E.2. First, the sufficient-statistics approach continues to apply: all equilibrium variables, as well as the (log) change in marginal cost, can still be expressed as functions of the tariff change and changes in the number of firms across markets. Second, we show that the model's extensive-margin predictions continue to hold in the case of symmetric markets (i.e., countries), despite the presence of increasing marginal costs at the firm level.[19] In particular, following a tariff shock in the focal market, both the number of entrants and the number of exporters decline in every market.

Finally, we prove that in a model with symmetric markets, when the pass-through of the cost shock, ρ, is sufficiently close to unity, aggregate export volume and value to third markets must decline regardless of the degree of decreasing returns to scale at the firm level. This result holds even when firm-level export volume and value to third markets increase after the tariff shock—that is, even in the presence of firm-level trade diversion. While the degree of decreasing returns to scale governs the increase in firm-level exports along the intensive margin, the pass-through parameter ρ plays a crucial role in determining the contraction along the extensive margin. Specifically, a higher ρ implies that markups—and hence producer prices—rise by less following the tariff shock, holding other factors constant. As a result, ex post profits (conditional on the number of firms) increase by less. Consequently, the free-entry condition requires a larger reduction in the number of entrants—and thus in the number

[19] This prediction also holds under CES demand with asymmetric markets.

of firms operating in each market—to restore the ex ante zero-profit condition following the tariff imposed in the focal market. The extensive-margin contraction therefore becomes more pronounced and more likely to dominate the intensive-margin expansion in firm-level exports when ρ is sufficiently large (i.e., close to unity).[20]

3.6 Heterogeneous firms

We now extend our benchmark model to an environment with heterogeneous firms for two main purposes. First, the heterogeneous-firm framework provides the foundation for our quantitative calibration. Second, we demonstrate that the key qualitative results derived under the homogeneous-firm setting continue to hold in the presence of firm heterogeneity.

3.6.1 Equilibrium conditions

We follow the same strategy to solve the equilibrium with heterogeneous firms. Compared to the case with homogeneous firms, there is one more equilibrium condition that determines the exit cutoff on firm productivity. This is the selection channel popularized by models of heterogeneous firms. Specifically, the zero-profit condition can be written as

$$\left(\mu_i(z_i^*)-1\right)\frac{\tau_i}{z_i^*}q_i(z_i^*)=\left(1-\frac{1}{\mu_i(z_i^*)}\right)p_i(z_i^*)q_i(z_i^*)=f_i, \tag{21}$$

where z_i^* is the exit cutoff and firms are indexed by their productivity draws and their export destination. In addition, the markup rule is pinned down by the pricing rule:

$$\frac{t_i}{P_i}\mu\left(\frac{q_i(z)}{Q_i}\right)\frac{\tau_i}{z}=\Upsilon'\left(\frac{q_i(z)}{Q_i}\right). \tag{22}$$

Given the price aggregator P_i and the real consumption Q_i, we can solve for $q_i(z)$ (and thus $\mu_i(z)$ and $\pi_i(z)$) as functions of P_i and Q_i.

The second set of equilibrium conditions uses the definition of the Kimball preference and the balance budget (or income accounting) equation to solve for P_i and Q_i. Specifically, we have

$$\frac{N_i}{1-G(z_i^*)}\int_{z_i^*}^{\infty}\Upsilon\left(\frac{q_i(z;P_i,Q_i)}{Q_i}\right)\mathrm{d}G(z)=1. \tag{23}$$

[20]The pass-through rate is itself endogenous and also affects the intensive-margin response of firm-level exports to the tariff shock, albeit to a lesser extent.

and

$$P_i = \bar{\delta}_i(P_i, Q_i)P_i^I = \bar{\delta}_i(P_i, Q_i)D_i(Q_i). \tag{24}$$

Together, they implicitly pin down P_i and Q_i (given N_i).

Third, the per-period variable profits can be expressed as a function of N_i, P_i and Q_i:

$$\bar{\pi}_i(N_i) = \frac{1}{\rho + \nu_i} \int_{z_i^*(N_i)}^{\infty} [\pi_i(z; N_i) - f_i] \, \mathrm{d}G(z), \tag{25}$$

as firm-level price and quantity depend on these three variables.

Finally, we have the two equilibrium conditions, the free entry condition and the balanced flow of firms, to determine the total number of entrants and the number of firms that export to each market $(M, \{N_i\})$:

$$\sum_{i=0}^{N} (h_i \bar{\pi}_i)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}},$$
$$J_i[1 - G(z_i^*)]M = \nu_i N_i.$$

There are $N + 2$ equations for $N + 2$ unknowns. Similar to the case of homogeneous firms, we can derive that

$$J_i = h_i \left(\frac{h_i \bar{\pi}_i}{f_e} \right)^{\frac{1-\beta}{\beta}} = \frac{\nu_i N_i}{M[1 - G(z_i^*)]} \quad \Rightarrow \quad \bar{\pi}_i(N_i) = \left(\frac{\nu_i N_i}{h_i M[1 - G(z_i^*)]} \right)^{\frac{\beta}{1-\beta}} \frac{f_e}{h_i}, \tag{26}$$

Combined with equation (25), the above equation provides us an implicit function N_i in terms of M. We define such mapping by $N_i = N_i(M)$. Inserting it back into the free entry equation, we can pin down the equilibrium mass of total entrants M as in the case of homogeneous firms:

$$\sum_{i=0}^{N} (h_i \bar{\pi}_i(N_i(M)))^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}},$$

3.6.2 Comparative Statics

We define $\lambda_i(z) = \frac{N_i t_i p_i(z) q_i(z)}{E_i}$, the sales density of firms with productivity z in country i. Then, $\mathbb{E}^i[\lambda_i] = 1$. We use $\mathbb{E}^i_x[y]$ to denote the x-weighted average of variable y:

$$\mathbb{E}^i_x[y] = \frac{\int_{z_i^*}^{\infty} x(z) y(z) \frac{g(z)}{1-G(z_i^*)} \mathrm{d}z}{\int_{z_i^*}^{\infty} x(z) \frac{g(z)}{1-G(z_i^*)} \mathrm{d}z}.$$

For example, a useful and easily verifiable identity is $\mathbb{E}^i_{\lambda_i}[\delta_i] = \bar{\delta}_i$. Additionally, we denote the number of potential entrants in market i by M_i^p, i.e., $M_i^p = J_i M$. With the balanced flow condition, $\mathrm{d}\log M_i^p = \mathrm{d}\log N + \varphi_i^* \mathrm{d}\log z_i^*$, in which $\varphi_i^* = \frac{z_i^* g(z_i^*)}{1-G(z_i^*)}$. In other words, φ_i^* is the elasticity of entrant share $1 - G(z_i^*)$ to threshold z_i^*, i.e., $\varphi_i^* = -\frac{\mathrm{d}\log[1-G(z_i^*)]}{\mathrm{d}\log z_i^*}$.

We discuss equilibrium changes in key variables in what follows. In the homogeneous firms case, the change in the number of firms (i.e., $\mathrm{d}\log N_i$) is a sufficient statistic that characterizes various changes in the equilibrium. Now, either $\mathrm{d}\log N_i$ or $\mathrm{d}\log z_i^*$ can be a sufficient statistic. We decide to focus on changes in z_i^* rather than N_i for cleaner expositions, which leads to the following proposition.

Proposition 4. *In response to a set of tariff shocks* $\mathrm{d}\log t_i$*, the following equations characterize changes in the equilibrium:*

$$\begin{aligned}
\mathrm{d}\log M &= -\frac{\sum_i s_i \xi_i^t \mathrm{d}\log t_i}{\sum_i s_i \xi_i^M} \\
\mathrm{d}\log z_i^* &= z_i^M \mathrm{d}\log M + z_i^t \mathrm{d}\log t_i \\
\mathrm{d}\log M_i^p &= (1 - \xi_i^M)\mathrm{d}\log M - \xi_i^t \mathrm{d}\log t_i,
\end{aligned}$$

where

$$z_i^M = -\frac{1}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)}, \quad z_i^t = \frac{N_i^t - \frac{1-\beta}{\beta}\bar{\pi}_i^t}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)},$$

$$\xi_i^M = \frac{\frac{1-\beta}{\beta}\bar{\pi}_i^z}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)}, \quad \xi_i^t = -\frac{\frac{1-\beta}{\beta}\left[\bar{\pi}_i^z N_i^t - (\varphi_i^* + N_i^z)\bar{\pi}_i^t\right]}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)}, \tag{27}$$

and s_i *is the expected profit share as in equation* (19)*. We define* $\bar{\pi}_i^z$*,* $\bar{\pi}_i^t$*,* N_i^z*,* N_i^t *in Appendix D.2.*

Compared to the expression in equation (19) from the homogeneous-firm case, the weights ξ_i^M and ξ_i^t in the heterogeneous-firm setting include additional terms that capture the effects of firm selection and its interaction with variable markups. To illustrate this distinction, we consider the case of nested CES demand. Under this specification, the difference arises solely from the selection effect, and the weights ξ_i^t and ξ_i^M take the following forms:

$$\xi_i^M = \frac{\frac{\hat{\pi}_i}{\hat{\pi}_i - f_i}\eta_i\theta_i}{\frac{\hat{\pi}_i}{\hat{\pi}_i - f_i}\eta_i\theta_i + \frac{\beta}{1-\beta}\left(1 + \frac{\eta_i\theta_i}{\sigma_i - 1}\lambda_i^*\varphi_i^*\right)}, \qquad \text{and} \qquad \xi_i^t = \xi_i^M \frac{1}{\theta_i},$$

where θ_i is defined in equation (18) and $\hat{\pi}_i \equiv \int_{z_i^*}^{\infty} \pi_i \frac{g(z)}{1-G(z_i^*)} \mathrm{d}z$ is the average profit. Relative to equation (19), the weight of entry interdependence, $\frac{\beta}{1-\beta}$, is augmented by the weight of selection cutoff $\lambda_i^*\varphi_i^*$. Intuitively, the impact on number of total entrants is smaller, when the productivity thresholds are more responsive to the tariff shock(s). In other words, the movement of thresholds absorbs the impact of the tariff shock(s) to some extent. Unfortunately, we are unable to prove the results that we have obtained in the homogeneous firms case in general. In order to obtain sharper predictions on the comparative statics, we introduce the following additional assumption.

Assumption 2. *We assume the following conditions hold:*

i) The strong second Marshall law of demand: $\rho_i \leq 1$, $\rho_i' \leq 0$;

ii) Excessive entry: $\bar{\delta}_i \leq \mathbb{E}_{\lambda_i}^i[\mu_i^{-1}]^{-1}$;

iii) Under-selection: $\bar{\delta}_i \geq \delta_i^*$.

iv) Firms' profit decreases on entry threshold z_i^*: $\frac{\mathrm{d}\log \pi_i}{\mathrm{d}\log z_i^*} < 0$.

Compared to Assumption 1, the first assumption above presents a stronger version of Marshall's second law of demand: not only is there incomplete passthrough, but the passthrough rate also decreases with firm productivity. The second and third assumptions are drawn from Baqaee et al. (2023). The second implies that consumer welfare improves when resources used for firm entry are reallocated to production. Similarly, the third suggests that consumer welfare increases when the exit threshold rises.

The final assumption governs the relationship between the exit threshold and firm-level profitability.[21] Mathematically, the above assumption assures that both ξ_i^M and ξ_i^t defined in equation (27) are strictly positive.

[21] This assumption holds trivially under CES demand, where the profit ratio between two firms is propor-

The following proposition summarizes our comparative statics results in the case of heterogeneous firms:

Proposition 5. *Suppose that assumption 1 and 2 hold. Country j ("the focal market") raises its tariff, i.e., $\mathrm{d}\log t_j > 0$, while all other countries ("the third markets") do not change tariffs, i.e., $\mathrm{d}\log t_i = 0$ for $i \neq j$. Then, $\xi_i^t > 0$, $\xi_i^M > 0$, $z_i^t > 0$, $z_i^M > 0$. We have the following equilibrium changes:*

i) In all markets:

$$\mathrm{d}\log M < 0, \quad \mathrm{d}\log Q_i < 0, \quad \mathrm{d}\log M_i^p < 0, \quad \mathrm{d}\log p_i > 0, \quad \mathrm{d}\log \tilde{Q}_i < 0, \quad \log R_i < 0$$

ii) In the third market $i \neq j$:

$$\mathrm{d}\log z_i^* < 0, \quad \mathrm{d}\log p_i^m > 0$$

iii) In the focal market:

$$\mathrm{d}\log z_j^* = \underbrace{z_j^M}_{>0}\Bigg[\underbrace{\left(\frac{s_j\xi_j^M}{\sum_i s_i\xi_i^M} - 1\right)}_{<0}\underbrace{N_j^t}_{<0} + \underbrace{\bar{\pi}_j^t}_{<0}\underbrace{\left(\frac{1-\beta}{\beta} - \frac{s_j\xi_j^M}{\sum_i s_i\xi_i^M}\frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z}\right)}_{>0}\Bigg]\mathrm{d}\log t_j, \tag{28}$$

where we define $\bar{\pi}_i^z$, $\bar{\pi}_i^t$, N_i^z, N_i^t in Appendix D.2.

iv) Across firms in the same market:

$$\frac{\partial\,\mathrm{d}\log\mu_i}{\partial\log z} > 0, \quad \frac{\partial\,\mathrm{d}\log\pi_i}{\partial\log z} < 0, \quad \frac{\partial\,\mathrm{d}\log r_i}{\partial\log z} < 0, \quad \frac{\partial\,\mathrm{d}\log q_i}{\partial\log z} < 0$$

Proposition 5 demonstrates that most of the qualitative results established in the homogeneous-firm case continue to hold under heterogeneity, provided Assumption 2 is satisfied. In particular, both the total number of entrants and the number of potential entrants into each market decline in response to a tariff increase. However, a key distinction arises due to firm

tional to their productivity ratio (e.g., z_i^* and z_i), and the marginal firm earns profits equal to the fixed operating cost. However, under variable markups, intensified competition (i.e., an increase in z_i^*) reduces firm profits, though this reduction is partially offset by changes in markups due to shifts in firm-level market shares. As a result, the assumption does not hold trivially in the general case.

selection: we are unable to guarantee that the number of actual entrants—and thus the number of active firms in each market—decreases with the tariff. This ambiguity stems from the selection effect, whereby the cutoff productivity level decreases, potentially allowing more low-productivity firms to enter despite reduced aggregate entry.

Turing to the selection effect, we show that threshold z_i^* must decrease in third markets, while it changes ambiguously in the focal market, as indicated by equation (28). In third markets, competition softens as the number of entrants declines. Holding the cutoff constant, all surviving firms earn higher profits due to increased market shares. This rise in firm-level profitability lowers the exit threshold z_i^*, allowing less productive firms to survive.

In the focal market, the dynamics are more complex. Surviving firms raise their prices and markups, with more productive firms increasing their markups disproportionately. This anti-competitive effect benefits lower-productivity firms more and thus tends to reduce the exit threshold. However, the focal market also experiences a direct negative shock to firm profits from the tariff increase. Whether the exit threshold rises or falls depends on the relative strength of these two forces: the anti-competitive markup effect versus the direct tariff effect. Hence, the direction of change in z_j^* in the focal market is generally ambiguous. An exception arises under nested CES demand, where markups are constant. In that case, the anti-competitive effect disappears, and the exit threshold in the focal market increases unambiguously due to the direct impact of the tariff.[22]

Proposition 5 also shows that the consumption of the composite good Q_i decreases in all markets. In third markets, this loss is intuitive: there are fewer varieties, lower aggregate productivity, and higher markups, all of which reduce consumer welfare. In the focal market, the result indicates that even if the exit threshold increases—potentially allowing only more productive firms to survive—it is insufficient to fully counteract the negative effects of reduced variety and higher markups. A similar logic applies to the decline in total quantity $\tilde{Q}_i$. While the number of actual entrants might rise in some cases, these firms sell significantly less in equilibrium to benefit from higher markups. Consequently, total market quantity declines despite possible increases in firm counts.

With heterogeneous firms, the model does not yield definite predictions about changes in firm-level profit, revenue, or quantity. Nevertheless, it delivers two unambiguous predictions regarding firm-level behavior:

[22]There are two cases where the exit cutoff is more likely to increase in the focal market: 1) the focal market has a large weight $\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M}$, which may due to a large profit share s_j; 2) markets are very independent: β is close to zero.

First, all surviving firms raise their markups and prices in both the focal and third markets. This outcome stems from reduced firm entry, which increases the market power of incumbents. In third markets, the exit threshold always declines, implying that the average productivity of firms falls post-shock. As a result, the unit price increases more than the average price increase among continuing firms due to a compositional effect—less productive firms now comprise a larger share of the market. In contrast, in the focal market, the exit threshold may rise, allowing only more productive firms to enter post-shock. Consequently, the unit price may increase less than the average firm-level price increase because new, highly productive entrants exert downward pressure on the average.

Second, conditional on survival, low-productivity firms gain relative to high-productivity firms in terms of profits, sales, and quantities. This is because, under the strong form of Marshall's second law of demand, more productive firms increase their prices and markups more aggressively in response to the tariff shock. That is, more productive firms are more exposed to the negative impact of the tariff because they reduce their quantities more in pursuit of higher markups. In contrast, less productive firms raise prices less and thus benefit more from the anti-competitive effects induced by reduced competition. Together, these insights provide another rationale for why the exit threshold may fall in the focal market: marginal (less productive) firms can experience rising profits, while more productive ones may see their profits decline.

CES demand We relegate the full equilibrium analysis under CES demand to Appendix D.2. Notably, CES demand satisfies Assumption 2. Compared to the general case, this special case yields the clear prediction that $\mathrm{d}\log z_j^* > 0$ in the focal market. It also simplifies the comparative statics. However, the analytical tractability offered by CES demand is insufficient to justify its use for quantitative exercises. Due to the constant markup assumption, CES demand rules out firm-level price adjustments, implying that changes in quantity and sales must move one-to-one. This stands in contrast to our empirical findings, which document significant price changes at both the firm and aggregate levels.

Entry Interdependence As in the homogeneous case, it suffices to study the impact of β on the equilibrium threshold z_i^*, which serves as a sufficient statistic for changes in other endogenous variables. For the same reason, we focus on the symmetric case and omit the country index. The following analysis confirms that the implications of entry interdependence derived in the homogeneous case carry over to the heterogeneous firm environment.

First, $\mathrm{d}\log M$ is independent of β, as

$$\mathrm{d}\log M = -\frac{1}{N+1}\frac{\xi^t}{\xi^M}\mathrm{d}\log t_j = -\frac{1}{N+1}\left((\varphi^* + N^z)\frac{\bar{\pi}^t}{\bar{\pi}^z} - N^t\right)\mathrm{d}\log t_j < 0,$$

in which $\bar{\pi}^z < 0$, $\bar{\pi}^t < 0$, $N^z + \varphi^* > 0$, $N^t < 0$, and they are independent of β. In third markets, we have

$$\mathrm{d}\log z_i^* = z^M \mathrm{d}\log M = \frac{1}{(N^z + \varphi^*) - \frac{1-\beta}{\beta}\bar{\pi}^z}\mathrm{d}\log M, \tag{29}$$

where z^M is the exit cutoff in the initial equilibrium. Thus, $\mathrm{d}\log z_i^*$ decreases with β. We show in Appendix D.2 that $\mathrm{d}\log Q_i$ increases with $\mathrm{d}\log z_i^*$ in both the focal and third markets. Therefore, there is a larger reduction in the consumption of the sectoral composites in the third markets when β increases. For the focal market,

$$\mathrm{d}\log z_j^* = \frac{\frac{N}{N+1}N^t\frac{1}{N+1}\frac{\bar{\pi}^t}{\bar{\pi}^z}(\varphi^* + N^z) - \bar{\pi}^t\frac{1-\beta}{\beta}}{\frac{1-\beta}{\beta}\bar{\pi}^z - (N^z + \varphi^*)}\mathrm{d}\log t_j.$$

We show in Appendix D.2 that $\frac{\partial \mathrm{d}\log z_j^*}{\partial \beta} > 0$ and $\mathrm{d}\log Q_j$ increases with $\mathrm{d}\log z_j^*$. Therefore, the impact of a bigger β on the consumption of the sectoral composites in the focal market is always positive, despite that $\mathrm{d}\log z_j^*$ can be either positive or negative. Namely, the focal market suffers less from the tariff shock, when markets are more interdependent via firms' entry.

3.7 Homothetic demand with a single aggregator and tariff uncertainty

In this final extension, we reformulate the model using a homothetic demand system with a single aggregator (HSA), following Matsuyama and Ushchev (2017). The HSA demand system differs from the HDIA demand system in two key respects. First, under HSA demand, firms choose prices relative to a single price aggregator, whereas under HDIA demand firms choose quantities. Second, a firm's market share depends on its relative price rather than its relative quantity.

This extension serves two purposes. First, we show that all qualitative results derived under the HDIA demand system—and the sufficient-statistics approach we employ—continue to hold under HSA demand. Second, and more importantly, the HSA framework allows us to

study the effects of (unrealized) expected tariffs in third markets on firm-level and aggregate trade flows. A detailed analysis is provided in Appendix F.

Tariff uncertainty is closely related to the so-called tariff-echoing effect. Specifically, an AD tariff imposed by a focal country on a given product is likely to be followed by similar tariffs imposed by third countries on the same product exported from the same home country. In this sense, AD actions "echo" across countries against the same exporter–product pair rather than occurring in isolation. As a result, the investigation or imposition of an AD tariff in the focal market may generate expectations of future tariffs in third markets on the same product, which can depress trade flows to those markets even before any tariffs are realized. Although we show that our empirical findings are robust to excluding the top ten AD-investigating countries against China—those most likely to induce exporters to anticipate follow-on tariffs—we remain interested in the potential effects of such expected tariffs.

We summarize the main findings on tariff uncertainty here and relegate the detailed analysis to Appendix F.3. An expected—but unrealized—tariff in a third country reduces firms' expected profits from entry. Consequently, both the mass of entrants and the number of exporters to each market decline relative to the case without tariff uncertainty in the third country. This excessive decrease leads to larger increases in firm-level and aggregate prices in the third country, even in the absence of tariff realization. As a result, total export value and volume to the third country decline, with the contraction being larger under tariff uncertainty than in the benchmark case. Overall, tariff uncertainty alone can generate price increases and aggregate trade reductions in the third country, even without the imposition of an actual tariff.

One key lesson from this exercise is that tariff uncertainty—absent tariff realization—affects the responses of endogenous variables to a tariff shock only through its impact on firm entry and the number of exporting firms in each market. In other words, tariff uncertainty does not play an independent role beyond this extensive-margin channel. By contrast, a realized tariff in the focal market has an independent effect on outcomes in that market, over and above its impact on the number of firms. As a result, our sufficient-statistics approach continues to apply in this setting. It is possible that part of the observed decline in the number of entrants and exporting firms is driven by expected tariffs in a third market. Separately identifying the realized tariff in the focal country and the expected tariff in the third country would require sufficiently rich data (i.e., enough data moments). Nonetheless, the sufficient-statistics approach introduced in this paper can still be applied to achieve this goal.[23]

[23]Introducing tariff uncertainty into the model would alter the calibrated structural parameters (e.g., β, η

4 Model Parameterization (work-in-progress)

In this section, we use the theoretical results derived above to calibrate the model and conduct a counterfactual analysis that highlights the role of β in transmitting tariff shocks across countries. Typically, estimating structural parameters such as η and β requires fully solving the model. However, by log-linearizing around the steady state, we can infer these structural parameters without fully solving the model. The goal of this quantitative exercise is to demonstrate how key parameters in the model can be cleanly identified from empirical moments.

To ensure transparency in parameter identification, we begin by analyzing the case of homogeneous firms in symmetric markets. That is, we abstract from heterogeneity across firms and countries. Consequently, both the focal and third markets share identical model parameters. Next, we assume the fixed cost of exporting, f, is zero. This assumption simplifies the mapping from β to the model moments, making it more transparent. Finally, we adopt a specific functional form for the demand function: the Klenow-Willis version of the Kimball demand (see Klenow and Willis (2016)). Specifically, the Kimball aggregator, $\Upsilon(\cdot)$, is defined as

$$\Upsilon(x) = 1 + (\bar{\sigma} - 1) \exp\left(\frac{1}{\varepsilon}\right) \varepsilon^{(\bar{\sigma}/\varepsilon)-1} \left(\Gamma\left(\frac{\bar{\sigma}}{\varepsilon}, \frac{1}{\varepsilon}\right) - \Gamma\left(\frac{\bar{\sigma}}{\varepsilon}, \frac{x^{\varepsilon/\bar{\sigma}}}{\varepsilon}\right)\right),$$

where $x \equiv \frac{q(\omega)}{Q}$, and $\Gamma(.)$ is the incomplete Gamma function defined as

$$\Gamma(u, z) = \int_z^{\infty} s^{u-1} e^{-s} ds.$$

The two key parameters in this specification are $\bar{\sigma}$ and ε, which together determine the so-called "super-elasticity", $\frac{\varepsilon}{\bar{\sigma}}$—that is, the elasticity of the elasticity. This term governs how the price elasticity varies with the relative quantity, $\frac{q(\omega)}{Q}$. Note that the price elasticity, the

and the demand system etc.).

markup, and the cost passthrough are calculated as follows:

$$\begin{aligned}
\sigma(x) &= \bar{\sigma} x^{-\varepsilon/\bar{\sigma}}, \\
\epsilon(x) &\equiv \frac{\mathrm{d}\sigma(x)}{\mathrm{d}\log x} = -\sigma^{'}(x)x = 1 - \frac{\Upsilon'(x)}{x\Upsilon''(x)} - \frac{\Upsilon'(x)\Upsilon'''(x)}{\Upsilon''(x)^2} = \varepsilon x^{-\varepsilon/\bar{\sigma}} \qquad (30) \\
\mu(x) &= \left(1 - \frac{1}{\bar{\sigma}} x^{\varepsilon/\bar{\sigma}}\right)^{-1}, \\
\rho(x) &= \left(1 + \frac{\varepsilon}{\bar{\sigma}}\mu(x)\right)^{-1}, \qquad (31)
\end{aligned}$$

where $\epsilon(x)$ measures how the price elasticity decreases with (rescaled) firm-level output $x = \frac{q(\omega)}{Q}$, and the key parameter is the super-elasticity (i.e., $\frac{\varepsilon}{\bar{\sigma}}$). Importantly, the CES demand emerges as a special case of the Klenow-Willis specification of Kimball demand when $\varepsilon = 0$.

4.1 Simple Calibration

We calibrate the model using the responses of aggregate variables to the tariff shock in both the focal and third markets. Specifically, we utilize the following: $\mathrm{d}\log R_j$, $\mathrm{d}\log R_i$, $\mathrm{d}\log \tilde{Q}_j$, $\mathrm{d}\log \tilde{Q}_i$, $\mathrm{d}\log N_j$, $\mathrm{d}\log N_i$ where j denotes the focal market and i is the third market. Under the assumptions of homogeneous firms and symmetric countries, these six responses are governed by five parameters or equilibrium variables—given a tariff change in the focal market: $\beta, \eta, \sigma, \rho, \delta$. Note that β, η are structural parameters, while σ, ρ, δ are endogenous equilibrium variables. Moreover, Proposition 3 implies that $\frac{\mathrm{d}\log R_j - \mathrm{d}\log \tilde{Q}_j}{\mathrm{d}\log N_j} = \frac{\mathrm{d}\log R_i - \mathrm{d}\log \tilde{Q}_i}{\mathrm{d}\log N_i}$, which means that one of the six responses is redundant under symmetry. Therefore, we use five of the responses to identify the five relevant parameters and equilibrium variables.

The five moments we use are $\hat{n} \equiv \frac{\mathrm{d}\log N_i}{\mathrm{d}\log N_j}$, $\mathrm{d}\log N_j$, $\hat{r}_j \equiv \frac{\mathrm{d}\log R_j}{\mathrm{d}\log N_j}$, $\hat{r}_i \equiv \frac{\mathrm{d}\log R_i}{\mathrm{d}\log N_i}$, $\hat{q}_j \equiv \frac{\mathrm{d}\log \tilde{Q}_j}{\mathrm{d}\log N_j}$. The moment excluded is $\mathrm{d}\log \tilde{Q}_i$. Given that the average tariff applied in anti-dumping (AD) cases against China during 2000–2011 was approximately 60%, we set $\mathrm{d}\log t_j = 0.5$, corresponding to a post-shock tariff level of $t_j^{after} = 1.65$. Additionally, we assume $D_i = D_j = 1$, $\tau_i = \tau_j = 1$, $t_j^{before} = t_i^{before} = t_i^{after} = 1$. We consider eight symmetric foreign countries, one of which is the focal market (i.e., $N = 8$).[24]

Our identification proceeds in three steps. First, calculation shows

$$\hat{r}_j - \hat{r}_i = -\eta \frac{\mathrm{d}\log t_j}{\mathrm{d}\log N_j} \Rightarrow 0.9431 = 0.7289\eta,$$

[24]We choose this value because the data suggest that the ratio of sales in the third markets to those in the focal market is approximately seven.

from which we can back out $\eta = 1.2937$, as $\hat{r}_j$, $\hat{r}_i$ and $\log N_j$ are the data moments and $\mathrm{d}\log t_j = 0.5$. Next, we back out β. Note that

$$\mathrm{d}\log N_i = -\frac{1-\xi}{N}\frac{\mathrm{d}\log t_j}{\theta};\ \mathrm{d}\log N_j = -\left(\frac{1-\xi}{N}+\xi\right)\frac{\mathrm{d}\log t_j}{\theta};\ \hat{n} = \frac{0.172}{0.686} = 0.2507.$$

Therefore, we have

$$\hat{n} = \frac{\frac{1}{N}}{\frac{1}{N}+\frac{\xi}{1-\xi}} \Rightarrow \frac{\xi}{1-\xi} = \frac{1-\hat{n}}{\hat{n}N} = 0.3736;\ \ -\frac{\mathrm{d}\log t_j/\theta}{\mathrm{d}\log N_j} = \frac{1}{\frac{1-\xi}{N}+\xi} = ((N-1)\hat{n}+1) = 2.7549.$$

Having recalled that $\hat{r}_j - \hat{r}_i = -\eta\frac{\mathrm{d}\log t_j}{\mathrm{d}\log N_j}$, we arrive at

$$\eta\theta = \frac{\hat{r}_j-\hat{r}_i}{\frac{-\mathrm{d}\log t_j/\theta}{\mathrm{d}\log N_j}} = \frac{\hat{r}_j-\hat{r}_i}{(N-1)\hat{n}+1} = \frac{0.9431}{7*0.2507+1} = 0.3423,\ \ \theta = \frac{\theta\eta}{\theta} = \frac{0.3423}{1.2937} = 0.2646,$$

which can be calculated using the data moments. Also note that

$$\xi = \frac{\eta\theta}{\frac{\beta}{1-\beta}+\eta\theta} \Rightarrow \frac{\xi}{1-\xi} = \eta\theta\frac{1-\beta}{\beta}.$$

Therefore, we have

$$\frac{\xi}{1-\xi} = \eta\theta\frac{1-\beta}{\beta} \Rightarrow \frac{1-\hat{n}}{N\hat{n}} = \frac{\hat{r}_j-\hat{r}_i}{(N-1)\hat{n}+1}\frac{1-\beta}{\beta} \Rightarrow 0.3736 = 0.3423\frac{1-\beta}{\beta},$$

from which we calculate $\beta = 0.4781$.

Finally, we can use two remaining moments $\hat{r}_i, \hat{q}_j$, θ and η to back out values of σ, ρ, δ. Specifically, we have

$$\delta\left(\frac{1}{\rho}-1\right) = \hat{q}_j - \hat{r}_j + \hat{r}_i + \eta\theta - 1 = 0.07/0.686 + 0.171/0.172 + 0.3423 - 1 = 0.4385,$$

$$\hat{r}_j - \hat{q}_j = -0.07/0.686 = -0.102 = -\frac{\delta}{\sigma}\left(\frac{1}{\rho}-1\right),$$

from which we derive $\sigma = 0.4385/0.102 = 4.2990$. Additionally, we have

$$\theta = 0.2646 = 0.4385\left(\frac{1}{1.2937}-\frac{1}{4.2990}\right)+1-\frac{1.2937-1}{1.2937}\delta = 1.2372-0.2270\delta \Rightarrow \delta = 4.2849.$$

The passthrough rate is calculated as

$$\rho = \frac{1}{\frac{0.4385}{\delta} + 1} = 0.9076.$$

Table 4 reports the calibrated values of the model parameters and equilibrium variables. First, the calibrated parameter values satisfy the conditions outlined in Assumption 1. Notably, the calibrated value of the entry-dependence parameter, β, lies neither near zero nor close to one. This supports the importance of allowing for a general form of the entry technology, as specified in equation (10). Moreover, the calibrated passthrough rate of a cost shock is close to the range estimated by Corsetti et al. (2023) (0.93-0.95).Finally, we can infer the implied relative output and the corresponding parameters in the Klenow-Willis demand specification as follows: $(\frac{q_i(\omega)}{Q_i}, \varepsilon, \bar{\sigma}) = (0.0058, 0.2247, 2.8759)$ and $\Upsilon\left(\frac{q_i(\omega)}{Q_i}\right) = 0.071$, The calibrated super elasticity in our model, $\frac{\varepsilon}{\bar{\sigma}}$, is comparable to the estimate from Edmond et al. (2023).

Table 4: Calibration Results

η	ξ	β	σ	δ	ρ
1.2937	0.2720	0.4781	4.2990	4.2849	0.9076

4.2 The Global Impact of Anti-Dumping Tariffs: the Role of Entry Interdependence

We now examine how the parameter β influences the transmission of a tariff shock across markets. Specifically, we consider two polar cases: $\beta = 0$, representing no interdependence in firm entry, and $\beta = 1$, corresponding to full interdependence, as in Melitz (2003). To isolate the impact of β on the transmission mechanism, we compare the responses to a market-specific tariff shock across three alternative environments. To ensure comparability, we adjust the entry efficiency parameters, h_i, so that all three scenarios share the same initial equilibrium allocation prior to the tariff shock.

Table 5 presents changes in various equilibrium variables as a function of β. Since only five parameters/variables are calibrated, the model cannot match the sixth moment, $\tilde{Q}_i$. In the calibrated model where $\beta = 0.4782$, the majority of the reduction in the number of exporters comes from the third markets ($63.7\% = 0.172 \times 7/(0.686 + 0.172 \times 7)$).[25] Interestingly, profits

[25] Alternatively, only 36.3% of the reduction in the number of exporting firms across all markets originates

from the focal market (π_j) and the third market (π_i) move in opposite directions, as the free entry condition must hold.

The model substantially overpredicts the spillover effect to the third market and underpredicts it for the focal market in the case of Melitz (2003), where $\beta = 1$. This discrepancy is primarily driven by changes in the number of exporting firms. Naturally, the spillover effect disappears when $\beta = 0$. In summary, entry interdependence plays a significant role in the transmission of tariff shocks across markets.

Table 5: Entry Interdependence and Spillovers across Markets

	Focal Market						Third Markets					
β	N_j	$\tilde{Q}_j$	R_j	Q_j	π_j	$\%t$	N_i	$\tilde{Q}_i$	R_i	Q_i	π_i	$\%t$
0.4781	-0.6860	-1.3990	-1.3290	-3.6490	-0.4122	36.3	-0.1720	-0.1886	-0.1710	-0.7527	0.0589	9.1
0.0000	-1.8900	-2.7189	-2.5260	-8.9177	-0.0000	100.0	-0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0
1.0000	-0.2362	-0.9060	-0.8819	-1.6808	-0.5661	12.5	-0.2362	-0.2590	-0.2349	-1.0338	0.0809	12.5

$\%t$: tariff incidence measured by exposure to joint entry.

5 Conclusions

In this paper, we document both aggregate-level and firm-level empirical evidence of market interdependence by examining the effects of antidumping (AD) tariffs against Chinese firms. We find that the affected products experience a decline in total sales in both the focal market and third markets, indicating a global trade depression effect. A decomposition exercise further reveals that the simultaneous export declines across countries are primarily driven by a reduction in the number of entrants/exporting firms. In contrast, individual exporters' sales decrease in the focal markets but increase in third markets, providing evidence of trade deflection at the firm level.

To rationalize these findings, we develop an industry equilibrium framework based on firms' interdependent entry decisions across multiple markets. We show that the entry margin alone can generate cross-market policy spillovers and heterogeneous tariff responses consistent with the data. Finally, we calibrate the model and find that the entry interdependence parameter plays a quantitatively significant role in the transmission of tariff shocks across markets.

Much work remains to be done. First, extending the model to a general equilibrium

from the focal market.

framework with endogenous wages and optimal firm behavior in both the focal and third markets is essential. This would allow for a deeper understanding of the global impact of large-scale bilateral trade conflicts, such as the U.S.-China trade war. Second, incorporating production fragmentation and firms' foreign direct investment (FDI) decisions into the industry equilibrium model is also important, given existing evidence of tariff-jumping behavior—such as during the U.S.-Japan trade tensions in the 1980s. Third, extending the model to a dynamic setting with sunk market-entry costs would help capture the role of trade policy uncertainty in shaping policy spillovers, particularly in terms of overall trade flows and extensive margin responses. Finally, developing a unified framework that can quantify both firm-level spillover channels (e.g., production fragmentation and the venting-out effect) and aggregate-level spillover mechanisms—such as the one emphasized in this paper—is a crucial direction for future research. We leave these exciting avenues for future work.

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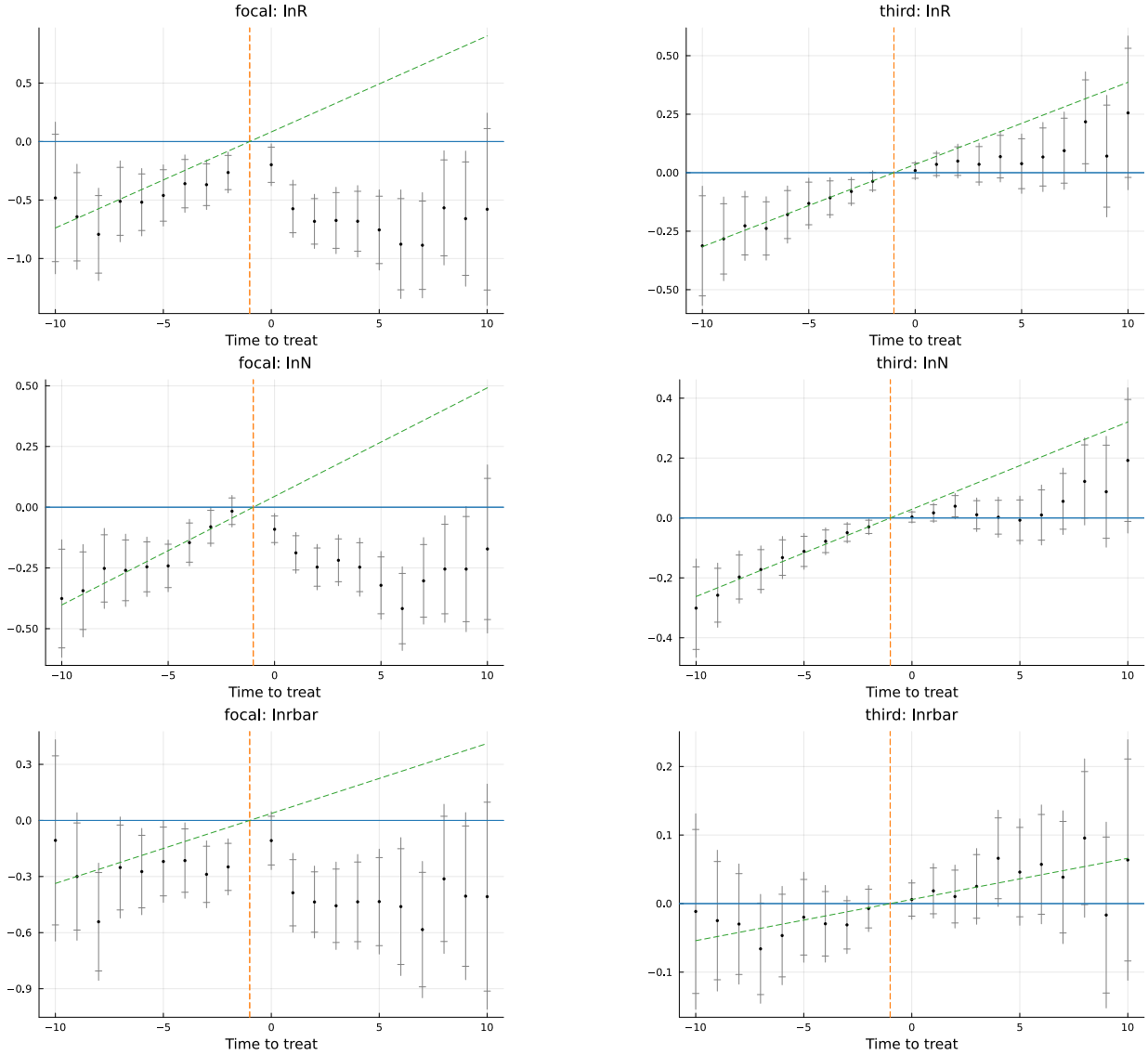
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A Additional Empirical Results

A.1 Market-level Analysis

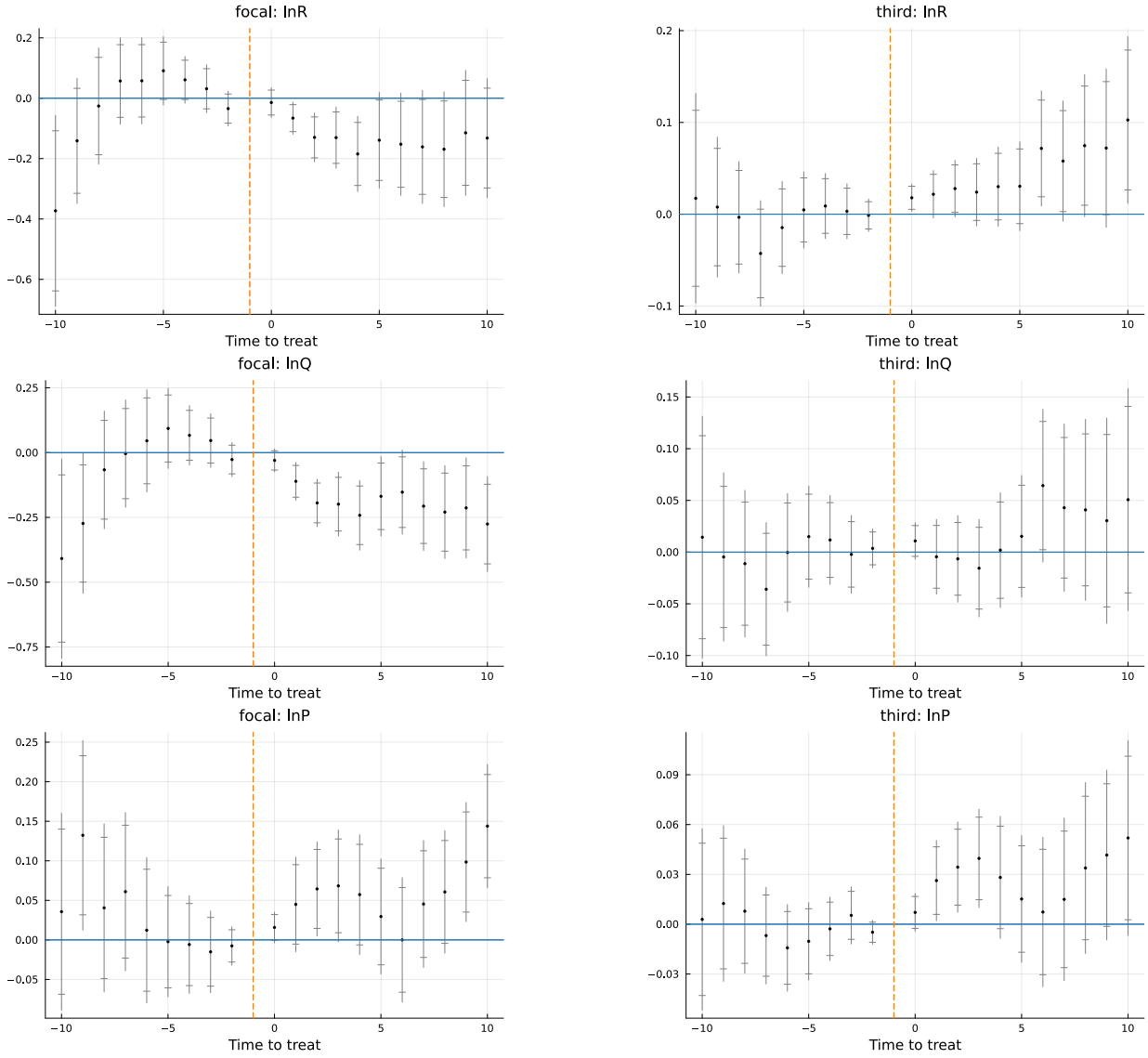
Figure 6: Effect of AD investigation at the market level: number of firms and average revenue.

Notes: All specifications include country-HS6-product and country-HS4-product-year fixed effects. Standard errors are clustered at the HS6 product level.



A.2 Firm-level Analysis

Figure 7: Effect of AD investigation at the firm-country-product-year level.



B Derivations

Derivation of demand function equation (7) Households solve the following expenditure minimization problem holding Q:

$$\begin{aligned} E = \min_{q_\omega} \quad & \int_\Omega p_\omega q_\omega \mathrm{d}\omega \\ \text{s.t.} \quad & \int_\Omega \Upsilon\left(\frac{q_\omega}{Q}\right) \mathrm{d}\omega = 1. \end{aligned}$$

Let λ be the Lagrange multiplier of the utility constraint. The first order condition is then

$$p_w = \lambda \Upsilon'\left(\frac{q_w}{Q}\right)\frac{1}{Q} \Rightarrow \frac{p_w}{P} = \Upsilon'\left(\frac{q_w}{Q}\right),$$

in which P is defined such that $PQ \equiv \lambda$. Multiplying both sides with $\frac{q_w}{Q}$, and summing them over ω, we obtain that

$$\frac{1}{PQ}\int_\Omega p_\omega q_\omega \mathrm{d}\omega = \int_\Omega \Upsilon'\left(\frac{q_\omega}{Q}\right)\frac{q_w}{Q}\mathrm{d}\omega \equiv \frac{1}{\bar{\delta}} \Rightarrow \lambda = \bar{\delta}E.$$

C Propagation and aggregation equations with homogeneous firms

C.1 Equilibrium conditions

We expand the equilibrium equations presented in Section 3.2 to the first order in the shocks.

Welfare

$$\delta_i \mathrm{d}\log N_i + \mathrm{d}\log\frac{q_i}{Q_i} = 0 \tag{32}$$

Budget balance

$$\begin{aligned} \mathrm{d}\log P_i^I =& \mathrm{d}\log N_i + \mathrm{d}\log t_i + \mathrm{d}\log\mu_i + \mathrm{d}\log\frac{\tau_i}{z} + \mathrm{d}\log\frac{q_i}{Q_i} \\ \Rightarrow \quad -\frac{1}{\eta_i}\mathrm{d}\log Q_i =& \mathrm{d}\log N_i + \mathrm{d}\log t_i + \mathrm{d}\log\frac{\tau_i}{z} + \left[\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+1\right]\mathrm{d}\log\frac{q_i}{Q_i} \end{aligned} \tag{33}$$

Markup

$$\mathrm{d}\log\mu = \left(\frac{1}{\rho_i} - 1\right)\frac{1}{\sigma_i}\mathrm{d}\log\frac{q_i}{Q_i}, \tag{34}$$

in which

$$\rho_i = -\frac{\partial \log p_i}{\partial \log z} = \frac{1}{1 + \frac{\mu'\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i}}{\mu\left(\frac{q_i}{Q_i}\right)}\sigma_i}.$$

Variable profit

$$\mathrm{d}\log\pi_i = \frac{1}{\mu_i - 1}\mathrm{d}\log\mu_i + \mathrm{d}\log P_i^I + \mathrm{d}\log Q_i - \mathrm{d}\log N_i - \mathrm{d}\log t_i \tag{35}$$

Balanced flow

$$\frac{\pi_i \mathrm{d}\log\pi_i - f_i \mathrm{d}\log f_i}{\pi_i - f_i} = \mathrm{d}\log(\rho + \nu_i) + \frac{\beta}{1-\beta}\left[\mathrm{d}\log\nu_i + \mathrm{d}\log N_i - \mathrm{d}\log h_i - \mathrm{d}\log M\right] + \mathrm{d}\log f_e - \mathrm{d}\log h_i \tag{36}$$

Free entry

$$\frac{1}{1-\beta}\sum_{i=0}^{N}\frac{\left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{1}{1-\beta}}}{\sum_k\left(\frac{\nu_k N_k}{h_k M}\right)^{\frac{1}{1-\beta}}}\left[\mathrm{d}\log N_i + \mathrm{d}\log\nu_i - \mathrm{d}\log h_i - \mathrm{d}\log M\right] = 0$$

$$\Rightarrow \quad \sum_{i=0}^{N} s_i\left[\mathrm{d}\log N_i + \mathrm{d}\log\nu_i - \mathrm{d}\log h_i\right] = \mathrm{d}\log M, \tag{37}$$

in which $s_i = \frac{J_i\bar{\pi}_i}{\sum_k J_k\bar{\pi}_k} = \frac{\left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{1}{1-\beta}}}{\sum_k\left(\frac{\nu_k N_k}{h_k M}\right)^{\frac{1}{1-\beta}}}$.

C.2 Proof of Proposition 2, 3 and Corollary 1

Changes in M and N_i We consider only $\mathrm{d}\log t_i$ and set $\mathrm{d}\log\frac{\tau_i}{z} = \mathrm{d}\log\nu_i = \mathrm{d}\log h_i = \mathrm{d}\log f_i = \mathrm{d}\log f_e = 0$. Then, we obtain the following system of equations

$$\begin{aligned}
\delta_i \mathrm{d}\log N_i + \mathrm{d}\log\frac{q_i}{Q_i} &= 0 \\
-\frac{1}{\eta_i}\mathrm{d}\log Q_i = \mathrm{d}\log N_i + \mathrm{d}\log t_i + \left[\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+1\right]\mathrm{d}\log\frac{q_i}{Q_i} \\
\mathrm{d}\log\pi_i = \frac{1}{\rho_i}\mathrm{d}\log\frac{q_i}{Q_i} + \mathrm{d}\log Q_i \\
\frac{\pi_i}{\pi_i - f_i}\mathrm{d}\log\pi_i = \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M\right] \\
\sum_{i=0}^{N} s_i \mathrm{d}\log N_i = \mathrm{d}\log M
\end{aligned}$$

Combining the first three equations, we obtain that

$$\mathrm{d}\log\pi_i = -\eta_i\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i,$$

in which

$$\theta_i = \underbrace{\left(\frac{1}{\eta_i}-\frac{1}{\sigma_i}\right)\left(\frac{1}{\rho_i}-1\right)\delta_i}_{\geq 0} + \underbrace{1-\frac{\eta_i-1}{\eta_i}\delta_i}_{>0} > 0.$$

The first bracket is non-negative since $\sigma_i > \eta_i$ and $\rho_i \leq 1$. The second bracket is positive since $\delta_i < \frac{\eta_i}{\eta_i - 1}$.

Then, we obtain that

$$\begin{aligned}
&\frac{\pi_i}{\pi_i - f_i}\left[-\eta_i\theta_i \mathrm{d}\log N_i - \eta_i \log t_i\right] = \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M\right] \\
&\Rightarrow \mathrm{d}\log N_i = \frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\eta_i\theta_i}\mathrm{d}\log M - \frac{\eta_i\frac{\pi_i}{\pi_i - f_i}}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\eta_i\theta_i}\mathrm{d}\log t_i
\end{aligned}$$

Using the free entry condition,

$$\mathrm{d}\log N_i - \mathrm{d}\log M = -\xi_i \mathrm{d}\log M - \xi_i\frac{\mathrm{d}\log t_i}{\theta_i},$$

where

$$\xi_i = \frac{\frac{\pi_i}{\pi_i - f_i}\eta_i\theta_i}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\eta_i\theta_i} \leq 1,$$

then

$$\sum_{i=0}^{N} s_i \mathrm{d}\log N_i = \mathrm{d}\log M \Rightarrow \mathrm{d}\log M = -\frac{\sum_{i=0}^{N} s_i\xi_i\frac{\mathrm{d}\log t_i}{\theta_i}}{\sum_{i=0}^{N} s_i\xi_i} = -\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right].$$

Hence, we can re-write $\mathrm{d}\log N_i$ as

$$\mathrm{d}\log N_i = (1-\xi_i)\mathrm{d}\log M - \xi_i\frac{\mathrm{d}\log t_i}{\theta_i} = -\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right]$$

Changes in other variables Similarly,

$$\begin{aligned}
\mathrm{d}\log\pi_i &= -\eta_i\theta_i\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i \\
&= \eta_i\theta_i\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right] - \eta_i\mathrm{d}\log t_i \\
&= \eta_i(1-\xi_i)\theta_i\left\{\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] - \frac{\mathrm{d}\log t_i}{\theta_i}\right\}
\end{aligned}$$

The relative quantity has the following expression:

$$\mathrm{d}\log\frac{q_i}{Q_i} = -\delta_i\mathrm{d}\log N_i = \delta_i\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right]$$

The utility change:

$$\begin{aligned}
\mathrm{d}\log Q_i &= -\eta_i\left(\mathrm{d}\log N_i + \mathrm{d}\log t_i + \left[\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+1\right]\mathrm{d}\log\frac{q_i}{Q_i}\right) \\
&= -\eta_i\left\{1 - \left[\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+1\right]\delta_i\right\}\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i \\
&= -\eta_i\left[\underbrace{(1-\delta_i)}_{<0} - \underbrace{\delta_i\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}}_{>0}\right]\mathrm{d}\log N - \eta_i\mathrm{d}\log t_i
\end{aligned}$$

Notice that

$$\begin{aligned}\eta_i\theta_i &= \left(1-\frac{\eta_i}{\sigma_i}\right)\left(\frac{1}{\rho_i}-1\right)\delta_i+\eta_i-(\eta_i-1)\delta_i\\ &= -\frac{\eta_i}{\sigma_i}\left(\frac{1}{\rho_i}-1\right)\delta_i+\eta_i(1-\delta_i)+\frac{\delta_i}{\rho_i},\end{aligned}$$

then $\eta_i\theta_i-\frac{\delta_i}{\rho_i}<0$. Plugging it into the formula for $\mathrm{d}\log Q_i$ and using the expression for $\mathrm{d}\log N_i$,

$$\begin{aligned}\mathrm{d}\log Q_i &= \left(\eta_i\theta_i-\frac{\delta_i}{\rho_i}\right)\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]+\xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right]-\eta_i\theta_i\frac{\mathrm{d}\log t_i}{\theta_i}\\ &= \underbrace{-\left(\frac{\delta_i}{\rho_i}-\eta_i\theta_i\right)(1-\xi_i)}_{<0}\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]\underbrace{-\left[\xi_i\frac{\delta_i}{\rho_i}+(1-\xi_i)\,\eta_i\theta_i\right]}_{<0}\frac{\mathrm{d}\log t_i}{\theta_i}\end{aligned}$$

To obtain the individual quantity change, we have

$$\begin{aligned}\mathrm{d}\log q_i =&\mathrm{d}\log\frac{q_i}{Q_i}+\mathrm{d}\log Q_i\\ =&\delta_i\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]+\xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right]-\left(\frac{\delta_i}{\rho_i}-\eta_i\theta_i\right)(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]\\ &-\left[\xi_i\frac{\delta_i}{\rho_i}+(1-\xi_i)\,\eta_i\theta_i\right]\frac{\mathrm{d}\log t_i}{\theta_i}\\ =&\left(\delta_i\left(1-\frac{1}{\rho_i}\right)+\eta_i\theta_i\right)(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]+\left[\xi_i\delta_i\left(1-\frac{1}{\rho_i}\right)-(1-\xi_i)\eta_i\theta_i\right]\frac{\mathrm{d}\log t_i}{\theta_i}\\ =&\left(\delta_i\left(1-\frac{1}{\rho_i}\right)+\eta_i\theta_i\right)(1-\xi_i)\left(\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]-\frac{\mathrm{d}\log t_i}{\theta_i}\right)-\underbrace{\delta_i\left(\frac{1}{\rho_i}-1\right)}_{>0}\frac{\mathrm{d}\log t_i}{\theta_i}.\end{aligned}$$

Observe that

$$\begin{aligned}\delta_i\left(1-\frac{1}{\rho_i}\right)+\eta_i\theta_i &= \delta_i\left(1-\frac{1}{\rho_i}\right)+\left(1-\frac{\eta_i}{\sigma_i}\right)\left(\frac{1}{\rho_i}-1\right)\delta_i+\eta_i-(\eta_i-1)\delta_i\\ &= -\frac{\eta_i}{\sigma_i}\left(\frac{1}{\rho_i}-1\right)\delta_i+\eta_i-(\eta_i-1)\delta_i\\ &= \eta_i\delta_i\Bigg[\underbrace{-\frac{1}{\sigma_i}\left(\frac{1}{\rho_i}-1\right)}_{<0}+\underbrace{\frac{1}{\delta_i}-\frac{\eta_i-1}{\eta_i}}_{>0}.\Bigg]\end{aligned}$$

Hence, it is negative with ρ_i is small[26] and $\delta_i \approx \frac{\eta_i}{\eta_i - 1}$ It becomes positive when passthrough rate is close to complete. For example, with CES demand, $\rho = 1$, and this term is always positive. The following condition gives the precise signing condition:

$$\delta_i \left(1 - \frac{1}{\rho_i}\right) + \eta_i \theta_i > 0 \iff \frac{\eta_i}{\eta_i - 1} > \frac{\delta_i}{1 - \frac{\delta_i}{\sigma_i}\left(\frac{1}{\rho_i} - 1\right)}.$$

The GE effect on firms' quantity is ambiguous. This is intuitive: while market share increases with firms' exit ($\mathrm{d}\log \frac{q_i}{Q_i} > 0$), the aggregate demand also goes down with less variety ($\mathrm{d}\log Q_i < 0$). However, if $\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] - \frac{\mathrm{d}\log t_i}{\theta_i} < 0$ and $\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] > 0$, $\mathrm{d}\log q_i < 0$.

The total quantity $\tilde{Q}_i = N_i q_i$. Then,

$$\begin{aligned}
\mathrm{d}\log \tilde{Q}_i &= \mathrm{d}\log N_i + \mathrm{d}\log q_i \\
&= \underbrace{\left(\delta_i \left(1 - \frac{1}{\rho_i}\right) + \eta_i \theta_i - 1\right)}_{<0} (1 - \xi_i)\, \mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \underbrace{\left[\xi_i \delta_i \left(1 - \frac{1}{\rho_i}\right) - (1 - \xi_i)\eta_i \theta_i - \xi_i\right]}_{<0} \frac{\mathrm{d}\log t_i}{\theta_i}.
\end{aligned}$$

The first coefficient is negative since

$$\begin{aligned}
\delta_i \left(1 - \frac{1}{\rho_i}\right) + \eta_i \theta_i - 1 &= -\frac{\eta_i}{\sigma_i}\left(\frac{1}{\rho_i} - 1\right)\delta_i + \eta_i - (\eta_i - 1)\delta_i - 1 \\
&= \underbrace{-\frac{\eta_i}{\sigma_i}\left(\frac{1}{\rho_i} - 1\right)\delta_i}_{\leq 0} + \underbrace{(\eta_i - 1)(1 - \delta_i)}_{<0}
\end{aligned}$$

We then turn to prices. With homogeneous firms, the aggregate unit value, $\frac{N_i p_i q_i}{N_i q_i}$, is the same as individual price, p_i. Furthermore, the price change is equivalent to the markup change with only tariff shocks. Therefore,

$$\begin{aligned}
\mathrm{d}\log p_i = \mathrm{d}\log \mu_i &= \left(\frac{1}{\rho_i} - 1\right)\frac{1}{\sigma_i}\mathrm{d}\log \frac{q_i}{Q_i} \\
&= \left(\frac{1}{\rho_i} - 1\right)\frac{\delta_i}{\sigma_i}\left[(1 - \xi_i)\, \mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i \frac{\mathrm{d}\log t_i}{\theta_i}\right].
\end{aligned}$$

[26] If the strong version of Marshall's second law of demand in Baqaee et al. (2023) holds, passthrough is decreasing with productivity. Hence, a lower ρ is associated with a lower σ (higher markup μ), making this term more negative.

Finally, we discuss individual sales, $r_i = p_i q_i$ and aggregate sales, $R_i = p_i \tilde{Q}_i$.

$$
\begin{aligned}
\mathrm{d}\log r_i =& \mathrm{d}\log p_i + \mathrm{d}\log q_i \\
=& \left(\delta_i\left(1-\frac{1}{\rho_i}\right)+\eta_i\theta_i\right)(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \left[\xi_i\delta_i\left(1-\frac{1}{\rho_i}\right)-(1-\xi_i)\eta_i\theta_i\right]\frac{\mathrm{d}\log t_i}{\theta_i} \\
&+\left(\frac{1}{\rho_i}-1\right)\frac{\delta_i}{\sigma_i}\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right]+\xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right] \\
=& \left[\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i+\eta_i\theta_i\right](1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] \\
&+\underbrace{\left[\xi_i\delta_i\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)-(1-\xi_i)\eta_i\theta_i\right]}_{<0}\frac{\mathrm{d}\log t_i}{\theta_i} \\
=& \left[\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i+\eta_i\theta_i\right](1-\xi_i)\,\mathbb{E}_{s\xi}\left(\left[\frac{\mathrm{d}\log t}{\theta}\right]-\frac{\mathrm{d}\log t_i}{\theta_i}\right) \\
&-\delta_i\left(\frac{1}{\rho_i}-1\right)\left(1-\frac{1}{\sigma_i}\right)\frac{\mathrm{d}\log t_i}{\theta_i}
\end{aligned}
$$

Even though $\mathrm{d}\log p_i$ is always positive to tariff shocks, the impact of $\mathbb{E}_{s\xi}[\mathrm{d}\log t/\mathrm{d}\theta]$ on $\mathrm{d}\log r_i$ is similar to that in $\mathrm{d}\log q_i$ and remains ambiguous. Note that

$$
\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i+\eta_i\theta_i = (\eta_i-1)\delta_i\left[-\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+\frac{\eta_i}{\eta_i-1}\frac{1}{\delta_i}-1\right],
$$

then we obtain the following sign condition:

$$
\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i+\eta_i\theta_i>0 \iff \frac{\eta_i}{\eta_i-1}>\delta_i\left[1+\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}\right].
$$

We do similar exercises for the aggregate revenue

$$
\begin{aligned}
\mathrm{d}\log R_i =& \mathrm{d}\log r_i + \mathrm{d}\log N_i \\
=& \left[\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i + \eta_i\theta_i\right](1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] \\
&+ \left[\xi_i\delta_i\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right) - (1-\xi_i)\eta_i\theta_i\right]\frac{\mathrm{d}\log t_i}{\theta_i} - \left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \xi_i\frac{\mathrm{d}\log t_i}{\theta_i}\right] \\
=& \underbrace{\left[\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i + \eta_i\theta_i - 1\right]}_{<0}(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] \\
&+ \underbrace{\left[\xi_i\delta_i\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right) - (1-\xi_i)\eta_i\theta_i - \xi_i\right]}_{<0}\frac{\mathrm{d}\log t_i}{\theta_i} \\
=& \left[\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i + \eta_i\theta_i - 1\right](1-\xi_i)\,\mathbb{E}_{s\xi}\left(\left[\frac{\mathrm{d}\log t}{\theta}\right] - \frac{\mathrm{d}\log t_i}{\theta_i}\right) \\
&- \left[\delta_i\left(\frac{1}{\rho_i}-1\right)\left(1-\frac{1}{\sigma_i}\right)+1\right]\frac{\mathrm{d}\log t_i}{\theta_i}
\end{aligned}
$$

The signing follows that

$$
\begin{aligned}
\left(1-\frac{1}{\rho_i}\right)\left(1-\frac{1}{\sigma_i}\right)\delta_i + \eta_i\theta_i - 1 &= -\frac{\eta_i-1}{\sigma_i}\left(\frac{1}{\rho_i}-1\right)\delta_i + \eta_i - (\eta_i-1)\delta_i - 1 \\
&= \underbrace{-\frac{\eta_i-1}{\sigma_i}\left(\frac{1}{\rho_i}-1\right)\delta_i}_{\leq 0} + \underbrace{(\eta_i-1)(1-\delta_i)}_{<0} < 0.
\end{aligned}
$$

As the aggregate quantity, the aggregate revenue must decline in all markets despite that prices go up.

The case with $\beta \to 1$ With fixed entry h_i, the balanced flow condition is now $h_i M = \nu_i N_i$. Then

$$\mathrm{d}\log N_i = \mathrm{d}\log M.$$

Totally differentiating the free entry condition $\sum_i h_i\bar{\pi}_i = f_e$,

$$\sum_i \frac{h_i}{\rho+\nu_i}\mathrm{d}\pi_i = 0 \Rightarrow \sum_i \frac{h_i\bar{\pi}_i}{\sum_k h_k\bar{\pi}_k}\frac{\pi_i}{\pi_i - f_i}\mathrm{d}\log\pi_i = 0.$$

Recall that $\mathrm{d}\log\pi_i = -\eta_i\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i$,

$$\sum_i s_i\xi_i \mathrm{d}\log M = -\sum_i s_i\xi_i \frac{\mathrm{d}\log t_i}{\theta_i} \Rightarrow \mathrm{d}\log M = -\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right],$$

in which $\xi_i = \frac{\pi_i\eta_i\theta_i}{\pi_i - f_i}$.

C.3 Proof of proposition 1

Totally differentiating the system, we have shown that

$$\frac{\mathrm{d}\log\pi_i}{\mathrm{d}\log N_i} = -\eta_i\theta_i < 0 \quad \text{and} \quad \frac{\mathrm{d}\log N_i}{\mathrm{d}\log M} = 1 - \xi_i \geq 0,$$

where that assumption 1 holds for all N_i ensures $\theta_i > 0$.

Therefore, $\pi_i(M)$ is decreasing in M, and so is $\frac{N_i(M)}{M}$. The left hand side of equation (16) is also decreasing in M. In that sense, the equilibrium will be unique if it exists. From equation (15), we must have $N_i(M) \to \infty$ if $M \to \infty$ and $N_i(M) \to 0$ if $M \to 0$. It suffices to have $\pi_i(N_i)$ satisfy the Inada condition that

$$\lim_{N_i\to 0} \pi_i(N_i) = \infty, \quad \lim_{N_i\to\infty} \pi_i(N_i) = 0.$$

Note that for any $N_1 > N_0 > 0$,

$$\frac{\pi_i(N_1)}{\pi_i(N_0)} = \exp\left(-\int_{N_0}^{N_1} \frac{\eta_i(x)\theta_i(x)}{x}\mathrm{d}x\right).$$

The integral diverges at $N_1 = \infty$ or $N_0 = 0$ since the limit conditions ensures that

$$\min\left[\liminf_{N_i\to 0} \eta_i\theta_i, \liminf_{N_i\to\infty} \eta_i\theta_i\right] > 0.$$

Hence, we have verified the Inada condition and established the existence of the equilibrium.

D Propagation and aggregation equations with heterogeneous firms

D.1 Equilibrium conditions

We expand the equilibrium equations presented in Section 3.6 to the first order in the shocks.

Pricing Totally differentiating the pricing equation (22)

$$\mathrm{d}\log\mu_i - \mathrm{d}\log P_i + \mathrm{d}\log\tau_i + \mathrm{d}\log t_i = -\frac{1}{\sigma_i}\mathrm{d}\log\frac{q_i}{Q_i}. \tag{38}$$

We suppress firm productivity z for brevity. Similarly, μ_i is an abbreviation for $\mu\left(\frac{q_i(z)}{Q_i}\right)$.

Selection Totally differentiating the selection equation (21),

$$\left.\frac{\partial\log\left(\mu_i(z)-1\right)\frac{\tau_i}{z}q_i(z)\right)}{\partial\log z}\right|_{z=z^*}\mathrm{d}\log z_i^* + \mathrm{d}\left(\log(\mu_i(z_i^*)-1)\frac{\tau_i}{z_i^*}q_i(z_i^*)\right) = \mathrm{d}\log f_i.$$

The first partial derivative has the following form:

$$\frac{\partial\log\left(\mu_i(z)-1\right)\frac{\tau_i}{z}q_i(z)\right)}{\partial\log z} = \frac{\mu_i(z)}{\mu_i(z)-1}\frac{\partial\log\mu_i(z)}{\partial\log z} - 1 + \frac{\partial\log q_i(z)}{\partial\log z} = \sigma_i(z) - 1.$$

Since $\frac{\mu_i\tau_i}{z} = P_i\Upsilon'\left(\frac{q_i}{Q_i}\right)$,

$$\frac{\partial\log\mu_i(z)}{\partial\log z} - 1 = \frac{\Upsilon''\left(\frac{q_i(z)}{Q_i}\right)\frac{q_i(z)}{Q_i}}{\Upsilon'\left(\frac{q_i(z)}{Q_i}\right)}\frac{\partial\log q_i(z)}{\partial\log z} = -\frac{1}{\sigma_i(z)}\frac{\partial\log q_i(z)}{\partial\log z}.$$

Plugging it into the equation, we obtain the RHS equals to $\sigma_i - 1$ with $\frac{\mu_i}{\mu_i-1} = \sigma_i$.[27] Finally, we rewrite the selection condition as follows:[28]

$$(\sigma_i(z_i^*)-1)\mathrm{d}\log z_i^* + \mathrm{d}\log\pi_i(z_i^*) = \mathrm{d}\log f_i. \tag{39}$$

[27] This is straightforward with the CES preference: $\pi \propto z^{\sigma-1} \Rightarrow \frac{\partial\log\pi}{\partial\log z} = \sigma - 1$,

[28] The notation $\mathrm{d}\log x_i \equiv \mathrm{d}\log x_i(z)$, the log change in x_i holding z constant.

Welfare Totally differentiating equation (23) and using that $\Upsilon'\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i} = \frac{t_i p_i q_i}{E_i \bar{\delta}_i}$, we obtain that

$$\bar{\delta}_i \mathrm{d}\log N_i + \left(\bar{\delta}_i - \delta_i(z_i^*)\lambda_i(z_i^*)\right)\frac{g(z_i^*)}{1-G(z_i^*)}\mathrm{d}z^* + \mathbb{E}^i_{\lambda_i}\left[\mathrm{d}\log\frac{q_i}{Q_i}\right] = 0 \tag{40}$$

Price index (budget balance) Note that

$$\frac{1}{\bar{\delta}_i} = \frac{N_i}{1-G(z_i^*)}\int_{z^*}^{\infty}\Upsilon'\left(\frac{q_i(z)}{Q}\right)\frac{q_i(z)}{Q}g(z)\mathrm{d}z$$

Totally differentiating $\bar{\delta}_i$,

$$-\mathrm{d}\log\bar{\delta}_i = \mathrm{d}\log N_i + (1-\lambda_i(z_i^*))\frac{g(z_i^*)}{1-G(z_i^*)}\mathrm{d}z_i^* + \mathbb{E}^i_{\lambda_i}\left[\left(1-\frac{1}{\sigma_i}\right)\mathrm{d}\log\frac{q_i}{Q_i}\right]. \tag{41}$$

Then, we have

$$\mathrm{d}\log P_i + \frac{1}{\eta_i}\mathrm{d}\log Q_i = \mathrm{d}\log\bar{\delta}_i. \tag{42}$$

Markup

$$\mathrm{d}\log\mu_i = \left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}\mathrm{d}\log\frac{q_i}{Q_i}, \tag{43}$$

in which

$$\rho_i = -\frac{\partial\log p_i}{\partial\log z} = \frac{1}{1+\frac{\mu'\left(\frac{q_i}{Q_i}\right)\frac{q_i}{Q_i}}{\mu\left(\frac{q_i}{Q_i}\right)}\sigma_i}.$$

Variable and expected profit

$$\mathrm{d}\log\pi_i = \sigma_i\mathrm{d}\log\mu_i + \mathrm{d}\log\tau_i + \mathrm{d}\log q_i. \tag{44}$$

$$\mathrm{d}\log\bar{\pi}_i = -\mathrm{d}\log(\rho+\nu_i) + \mathbb{E}^i_{\pi_i - f_i}\left[\frac{\pi_i\mathrm{d}\log\pi_i - f_i\mathrm{d}\log f_i}{\pi_i - f_i}\right]. \tag{45}$$

Balanced flow Totally differentiating equation (26),

$$\frac{1}{\beta}\log h_i + \frac{1-\beta}{\beta}\left[\log \bar{\pi}_i - \log f_e\right] = \log \nu_i + \log N_i - \log M + \frac{g(z_i^*)}{1-G(z_i^*)}\mathrm{d}z_i^*. \tag{46}$$

Free entry

$$\sum_{i=0}^{N} s_i \left[\mathrm{d}\log h_i + \mathrm{d}\log \bar{\pi}_i\right] = \mathrm{d}\log f_e, \tag{47}$$

in which $s_i = \left(\frac{h_i \bar{\pi}_i}{f_e}\right)^{\frac{1}{\beta}} = \frac{J_i \bar{\pi}_i}{\sum_k J_k \bar{\pi}_k}$.

D.2 Proof of proposition 4 and 5

D.2.1 Solving for $\mathrm{d}\log M$ and $\mathrm{d}\log z_i^*$

We consider only $\mathrm{d}\log t_i$ and set $\mathrm{d}\log \tau_i = \mathrm{d}\log \nu_i = \mathrm{d}\log h_i = \mathrm{d}\log f_i = \mathrm{d}\log f_e = 0$. Also, let $\sigma_i^* = \sigma_i(z_i^*)$, $\varphi_i^* = \frac{z_i^* g(z_i^*)}{1-G(z_i^*)}$, $\pi_i^* = \pi_i(z_i^*)$, $\delta_i^* = \delta_i(z_i^*)$, and $\lambda_i^* = \lambda_i(z_i^*)$. Then, we obtain the following system of equations:

$$\begin{aligned}
\mathrm{d}\log P_i &= \frac{1}{\rho_i \sigma_i}\mathrm{d}\log\frac{q_i}{Q_i} + \mathrm{d}\log t_i, \\
(\sigma_i^* - 1)\mathrm{d}\log z_i^* + \mathrm{d}\log \pi_i^* &= 0, \\
\bar{\delta}_i \mathrm{d}\log N_i + \left(\bar{\delta}_i - \delta_i^* \lambda_i^*\right)\varphi_i^* \mathrm{d}\log z_i^* + \mathbb{E}^i_{\lambda_i}\left[\mathrm{d}\log\frac{q_i}{Q_i}\right] &= 0, \\
-\mathrm{d}\log P_i - \frac{1}{\eta_i}\mathrm{d}\log Q_i = \mathrm{d}\log N_i + (1-\lambda_i^*)\varphi_i^* \mathrm{d}\log z_i^* + \mathbb{E}^i_{\lambda_i}&\left[\left(1 - \frac{1}{\sigma_i}\right)\mathrm{d}\log\frac{q_i}{Q_i}\right], \\
\mathrm{d}\log \pi_i &= \frac{1}{\rho_i}\mathrm{d}\log\frac{q_i}{Q_i} + \mathrm{d}\log Q_i, \\
\mathbb{E}^i_{\pi_i - f_i}\left[\frac{\pi_i \mathrm{d}\log \pi_i}{\pi_i - f_i}\right] &= \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M + \varphi_i^* \mathrm{d}\log z_i^*\right], \\
\sum_{i=0}^{N} s_i \left[\mathrm{d}\log N_i + \varphi_i^* \mathrm{d}\log z_i^*\right] &= \mathrm{d}\log M.
\end{aligned}$$

First, we write $\mathrm{d}\log\frac{q_i}{Q_i} = \rho_i\sigma_i[\mathrm{d}\log P_i - \mathrm{d}\log t_i]$. Then

$$\mathrm{d}\log \pi_i = \sigma_i[\mathrm{d}\log P_i - \mathrm{d}\log t_i] + \mathrm{d}\log Q_i \Longrightarrow (\sigma_i^* - 1)\mathrm{d}\log z_i^* = -\sigma_i^*[\mathrm{d}\log P_i - \mathrm{d}\log t_i] - \mathrm{d}\log Q_i.$$

Combining conditions on welfare and price index,

$$
\begin{aligned}
-\frac{\bar{\delta}_i}{\eta_i}\mathrm{d}\log Q_i =&\bar{\delta}_i\mathrm{d}\log P_i - \left(\bar{\delta}_i - \delta_i^*\lambda_i^*\right)\varphi_i^*\mathrm{d}\log z_i^* - \mathbb{E}^i_{\lambda_i}\left[\mathrm{d}\log\frac{q_i}{Q_i}\right]\\
&+\bar{\delta}_i\left(1-\lambda_i^*\right)\varphi_i^*\mathrm{d}\log z_i^* + \bar{\delta}_i\,\mathbb{E}^i_{\lambda_i}\left[\left(1-\frac{1}{\sigma_i}\right)\mathrm{d}\log\frac{q_i}{Q_i}\right],\\
-\frac{\bar{\delta}_i}{\eta_i}\mathrm{d}\log Q_i =&\bar{\delta}_i\mathrm{d}\log P_i - (\bar{\delta}_i-\delta_i^*)\lambda_i^*\varphi_i^*\mathrm{d}\log z_i^* + \mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right]\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right].
\end{aligned}
$$

We obtain the following relationship between $\mathrm{d}\log Q_i$ and $\mathrm{d}\log z_i^*$:

$$
\begin{aligned}
&\left\{\frac{1}{\sigma_i^*}\left(\bar{\delta}_i + \mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right]\right) - \frac{\bar{\delta}_i}{\eta_i}\right\}\mathrm{d}\log Q_i\\
&\quad= -\left\{\frac{\sigma_i^*-1}{\sigma_i^*}\left(\bar{\delta}_i + \mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right]\right) + (\bar{\delta}_i-\delta_i^*)\lambda_i^*\varphi_i^*\right\}\mathrm{d}\log z_i^* + \bar{\delta}_i\mathrm{d}\log t_i\\
\Rightarrow\quad \mathrm{d}\log Q_i &= -\left[(\sigma_i^*-1)\left(1+\frac{\bar{\delta}_i}{\eta_i\gamma_i}\right) + (\bar{\delta}_i-\delta_i^*)\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* + \frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i,
\end{aligned}
$$

in which $\gamma_i = \frac{1}{\sigma_i^*}\left(\bar{\delta}_i + \mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right]\right) - \frac{\bar{\delta}_i}{\eta_i}$. It is helpful to take a pause here and comment on the signs of these coefficients. Before provoking Assumption 2, we have

$$
\begin{aligned}
\bar{\delta}_i + \mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right] =&\bar{\delta}_i\left(1+\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i]\right) - \mathbb{E}^i_{\lambda_i}[\sigma_i\rho_i]\\
&\geq 1+\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i] - \mathbb{E}^i_{\lambda_i}[\sigma_i\rho_i] = 1-\mathbb{E}^i_{\lambda_i}[\rho_i]\geq 0,
\end{aligned}
$$

where we have used that $\rho_i \leq 1$, the weak second Marshall law of demand, and $\bar{\delta}_i \geq 1$. Thus, we have $\gamma_i \geq -\frac{\bar{\delta}_i}{\eta_i}$. Under Assumption 2, we further show that

$$
\begin{aligned}
\mathbb{E}^i_{\lambda_i}\left[\left(\bar{\delta}_i(\sigma_i-1)-\sigma_i\right)\rho_i\right] \leq&\,\mathbb{E}^i_{\lambda_i}[\mu_i^{-1}]^{-1}\,\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i] - \mathbb{E}^i_{\lambda_i}[\sigma_i\rho_i]\\
=&\,\mathbb{E}^i_{\lambda_i}[\mu_i^{-1}]^{-1}\left(\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i] - \mathbb{E}^i_{\lambda_i}[\mu_i^{-1}]\,\mathbb{E}^i_{\lambda_i}[\sigma_i\rho_i]\right)\\
=&-\mathbb{E}^i_{\lambda_i}[\mu_i^{-1}]^{-1}\mathrm{Cov}^i_{\lambda_i}\left(\sigma_i\rho_i, \frac{1}{\mu_i}\right) < 0.
\end{aligned}
$$

The second equality uses that $\sigma_i - 1 = \frac{\sigma_i}{\mu_i}$. With the strong second Marshall law of demand, both $\sigma_i\rho_i$ and $1/\mu_i$ decrease on z. Then, $\mathrm{Cov}^i_{\lambda_i}\left(\sigma_i\rho_i, \frac{1}{\mu_i}\right) > 0$. Since $\sigma_i^* > \eta_i$ and $\bar{\delta}_i > \delta_i^*$,

$$
-\frac{\bar{\delta}_i}{\eta_i} \leq \gamma_i < 0, \quad Q_i^z \equiv -\left[(\sigma_i^*-1)\left(1+\frac{\bar{\delta}_i}{\eta_i\gamma_i}\right) + (\bar{\delta}_i-\delta_i^*)\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right] > 0.
$$

That is, holding tariffs, welfare increases with productivity threshold. With the CES demand, we have

$$\gamma_i = \frac{\eta_i - \sigma_i}{\eta_i(\sigma_i - 1)} < 0, \quad Q_i^z = \frac{\eta_i(\sigma_i - 1)}{\sigma_i - \eta_i} > \eta_i > 1.$$

We then turn to the welfare condition. Note that

$$\begin{aligned}
&\bar{\delta}_i \mathrm{d}\log N_i + \left[\left(\bar{\delta}_i - \delta_i^* \lambda_i^*\right)\varphi_i^* - \frac{\sigma_i^* - 1}{\sigma_i^*}\mathbb{E}_{\lambda_i}^i[\rho_i \sigma_i]\right]\mathrm{d}\log z_i^* = \frac{1}{\sigma_i^*}\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]\mathrm{d}\log Q_i \\
\Rightarrow\quad &\bar{\delta}_i \mathrm{d}\log N_i + \left[\left(\bar{\delta}_i - \delta_i^* \lambda_i^*\right)\varphi_i^* - \frac{\sigma_i^* - 1}{\sigma_i^*}\mathbb{E}_{\lambda_i}^i[\rho_i \sigma_i]\right]\mathrm{d}\log z_i^* \\
&= \frac{1}{\sigma_i^*}\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]\left\{-\left[(\sigma_i^* - 1)\left(1 + \frac{\bar{\delta}_i}{\eta_i\gamma_i}\right) + (\bar{\delta}_i - \delta_i^*)\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* + \frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i\right\} \\
\Rightarrow\quad &\bar{\delta}_i \mathrm{d}\log N_i = \left\{-\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*\gamma_i}\left[(\sigma_i^* - 1)\frac{\bar{\delta}_i}{\eta_i} + (\bar{\delta}_i - \delta_i^*)\lambda_i^*\varphi_i^*\right] - \left(\bar{\delta}_i - \delta_i^*\lambda_i^*\right)\varphi_i^*\right\}\mathrm{d}\log z_i^* + \frac{\bar{\delta}_i\,\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*\gamma_i}\mathrm{d}\log t_i \\
\Rightarrow\quad &\mathrm{d}\log N_i = \left\{-\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*\gamma_i}\left[(\sigma_i^* - 1)\frac{1}{\eta_i} + (1 - \frac{\delta_i^*}{\bar{\delta}_i})\lambda_i^*\varphi_i^*\right] - \left(1 - \frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^*\right)\varphi_i^*\right\}\mathrm{d}\log z_i^* + \frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*\gamma_i}\mathrm{d}\log t_i
\end{aligned}$$

Also,

$$\begin{aligned}
\mathrm{d}\log \pi_i &= \frac{\sigma_i}{\sigma_i^*}\left(-(\sigma_i^* - 1)\mathrm{d}\log z_i^* - \mathrm{d}\log Q_i\right) + \mathrm{d}\log Q_i \\
&= -\sigma_i\frac{\sigma_i^* - 1}{\sigma_i^*}\mathrm{d}\log z_i^* + \frac{\sigma_i^* - \sigma_i}{\sigma_i^*}\mathrm{d}\log Q_i \\
&= -\sigma_i\frac{\sigma_i^* - 1}{\sigma_i^*}\mathrm{d}\log z_i^* + \frac{\sigma_i^* - \sigma_i}{\sigma_i^*}\left\{-\left[(\sigma_i^* - 1)\left(1 + \frac{\bar{\delta}_i}{\eta_i\gamma_i}\right) + (\bar{\delta}_i - \delta_i^*)\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* + \frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i\right\} \\
&= -\sigma_i\frac{\sigma_i^* - 1}{\sigma_i^*}\mathrm{d}\log z_i^* - (\sigma_i^* - \sigma_i)\left[\frac{(\sigma_i^* - 1)}{\sigma_i^*}\left(1 + \frac{\bar{\delta}_i}{\eta_i\gamma_i}\right) + \frac{\bar{\delta}_i - \delta_i^*}{\sigma_i^*}\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* + \frac{\sigma_i^* - \sigma_i}{\sigma_i^*}\frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i \\
&= -(\sigma_i^* - 1)\mathrm{d}\log z_i^* - (\sigma_i^* - \sigma_i)\left[\frac{(\sigma_i^* - 1)}{\sigma_i^*}\frac{\bar{\delta}_i}{\eta_i\gamma_i} + \frac{\bar{\delta}_i - \delta_i^*}{\sigma_i^*}\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* + \frac{\sigma_i^* - \sigma_i}{\sigma_i^*}\frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i.
\end{aligned}$$

Comparing to the CES case, tariffs t_i have direct effect on firms' profit since markups are distorted. While aggregate demand shock (the third term) has direct impact on firms' market power, the redistribution of firms (the second term) imposes an additional one. Notice that average variable profit $\bar{\pi}_i$ and revenue $\hat{r}_i$ have the following form:

$$\hat{\pi}_i = \int_{z_i^*}^{\infty}\pi_i\frac{g(z)}{1 - G(z_i^*)}\mathrm{d}z, \quad \hat{r}_i = \int_{z_i^*}^{\infty}p_iq_i\frac{g(z)}{1 - G(z_i^*)}\mathrm{d}z = \int_{z_i^*}^{\infty}\sigma_i\pi_i\frac{g(z)}{1 - G(z_i^*)}\mathrm{d}z$$

Then,

$$\begin{aligned}
\mathbb{E}^i_{\pi_i-f_i}\left[\frac{\pi_i \mathrm{d}\log\pi_i}{\pi_i-f_i}\right] =& -(\sigma_i^*-1)\,\mathbb{E}^i_{\pi_i-f_i}\left[\frac{\pi_i}{\pi_i-f_i}\right]\mathrm{d}\log z_i^* \\
&- \mathbb{E}^i_{\pi_i-f_i}\left[\frac{\pi_i(\sigma_i^*-\sigma_i)}{\pi_i-f_i}\right]\left[\frac{(\sigma_i^*-1)}{\sigma_i^*}\frac{\bar\delta_i}{\eta_i\gamma_i}+\frac{\bar\delta_i-\delta_i^*}{\sigma_i^*}\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\mathrm{d}\log z_i^* \\
&+ \mathbb{E}^i_{\pi_i-f_i}\left[\frac{\pi_i(\sigma_i^*-\sigma_i)}{\pi_i-f_i}\right]\frac{\bar\delta_i}{\sigma_i^*\gamma_i}\mathrm{d}\log t_i \\
=& -\left\{\frac{\hat\pi_i(\sigma_i^*-1)}{\hat\pi_i-f_i}+\frac{\sigma_i^*\hat\pi_i-\hat r_i}{\hat\pi_i-f_i}\left[\frac{(\sigma_i^*-1)}{\sigma_i^*}\frac{\bar\delta_i}{\eta_i\gamma_i}+\frac{\bar\delta_i-\delta_i^*}{\sigma_i^*}\frac{\lambda_i^*\varphi_i^*}{\gamma_i}\right]\right\}\mathrm{d}\log z_i^* \\
&+ \frac{\sigma_i^*\hat\pi_i-\hat r_i}{\hat\pi_i-f_i}\frac{\bar\delta_i}{\sigma_i^*\gamma_i}\mathrm{d}\log t_i
\end{aligned}$$

We define that

$$\begin{aligned}
\bar\pi_i^z &= -\left\{\frac{\hat\pi_i(\sigma_i^*-1)}{\hat\pi_i-f_i}+\bar\pi_i^t\left[\frac{\sigma_i^*-1}{\eta_i}+\left(1-\frac{\delta_i^*}{\bar\delta_i}\right)\lambda_i^*\varphi_i^*\right]\right\}<0, \quad \bar\pi_i^t=\frac{\sigma_i^*\hat\pi_i-\hat r_i}{\hat\pi_i-f_i}\frac{\bar\delta_i}{\sigma_i^*\gamma_i}\le 0 \\
N_i^z &= -N_i^t\left[\frac{\sigma_i^*-1}{\eta_i}+(1-\frac{\delta_i^*}{\bar\delta_i})\lambda_i^*\varphi_i^*\right]-\left(1-\frac{\delta_i^*}{\bar\delta_i}\lambda_i^*\right)\varphi_i^*, \quad N_i^t=\frac{\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]}{\sigma_i^*\gamma_i}<0,
\end{aligned}$$

as $\gamma_i < 0$ and Assumption 2 holds. Moreover, since $\gamma_i < 0$ and $N_i^t < 0$, we must have $\varphi_i^* + N_i^z > 0$. Under the CES demand, the above terms become

$$\bar\pi_i^z = -\frac{\hat\pi_i(\sigma_i-1)}{\hat\pi_i-f_i}, \quad \bar\pi_i^t=0, \quad N_i^z=\frac{(\sigma_i-1)^2}{\sigma_i-\eta_i}-(1-\lambda_i^*)\,\varphi_i^* \quad N_i^t=\frac{1}{\gamma_i}=\frac{\eta_i(\sigma_i-1)}{\eta_i-\sigma_i}$$

Then, we write the balanced flow condition,

$$\begin{aligned}
&\frac{1-\beta}{\beta}\left[\bar\pi_i^z\mathrm{d}\log z_i^*+\bar\pi_i^t\mathrm{d}\log t_i\right]=(N_i^z+\varphi_i^*)\,\mathrm{d}\log z_i^*+N_i^t\mathrm{d}\log t_i-\mathrm{d}\log M \\
\Rightarrow\quad &\left[\frac{1-\beta}{\beta}\bar\pi_i^z-(N_i^z+\varphi_i^*)\right]\mathrm{d}\log z_i^*=\left[N_i^t-\frac{1-\beta}{\beta}\bar\pi_i^t\right]\mathrm{d}\log t_i-\mathrm{d}\log M \\
\Rightarrow\quad &\mathrm{d}\log z_i^*=z_i^M\mathrm{d}\log M+z_i^t\mathrm{d}\log t_i,
\end{aligned}$$

where

$$z_i^M=-\frac{1}{\frac{1-\beta}{\beta}\bar\pi_i^z-(N_i^z+\varphi_i^*)}, \quad \text{and } z_i^t=\frac{N_i^t-\frac{1-\beta}{\beta}\bar\pi_i^t}{\frac{1-\beta}{\beta}\bar\pi_i^z-(N_i^z+\varphi_i^*)}.$$

Therefore,

$$\begin{aligned}\mathrm{d}\log N_i - \mathrm{d}\log M + \varphi_i^* \mathrm{d}\log z_i^* =& \frac{1-\beta}{\beta}\left[\bar{\pi}_i^z \mathrm{d}\log z_i^* + \bar{\pi}_i^t \mathrm{d}\log t_i\right] \\ =& -\frac{\frac{1-\beta}{\beta}\bar{\pi}_i^z}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)}\mathrm{d}\log M \\ &+ \frac{\frac{1-\beta}{\beta}\left[\bar{\pi}_i^z N_i^t - (\varphi_i^* + N_i^z)\bar{\pi}_i^t\right]}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)}\mathrm{d}\log t_i \\ \equiv& -\xi_i^M \mathrm{d}\log M - \xi_i^t \mathrm{d}\log t_i.\end{aligned}$$

Then, we have solved for $\mathrm{d}\log M$:

$$\mathrm{d}\log M = -\frac{\sum_i s_i \xi_i^t \mathrm{d}\log t_i}{\sum_i s_i \xi_i^M}.$$

Note that $\varphi_i^* + N_i^z > 0$, $\bar{\pi}_i^z < 0$, $\bar{\pi}_i^t \leq 0$ and $N_i^t < 0$, we must have

$$\bar{\pi}_i^z N_i^t - (\varphi_i^* + N_i^z)\bar{\pi}_i^t = -\frac{\hat{\pi}_i(\sigma_i^* - 1)}{\hat{\pi}_i - f_i} N_i^t - \frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* \varphi_i^* \bar{\pi}_i^t > 0 \tag{48}$$

which implies that $\xi_i^t > 0$. Moreover, since $\bar{\pi}_i^z < 0$ and $\varphi_i^* + N_i^z > 0$, we must have $0 < \xi_i^M < 1$. Therefore, $\mathrm{d}\log M < 0$ when the tariff in market i increases.

With the CES demand, ξ_i^t and ξ_i^M take the following expressions:

$$\begin{aligned}\xi_i^M =& \frac{\frac{1-\beta}{\beta}\bar{\pi}_i^z}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)} = \frac{\frac{\hat{\pi}_i}{\hat{\pi}_i - f_i}\eta_i\theta_i}{\frac{\hat{\pi}_i}{\hat{\pi}_i - f_i}\eta_i\theta_i + \frac{\beta}{1-\beta}\left(1 + \frac{\eta_i\theta_i}{\sigma_i - 1}\lambda_i^*\varphi_i^*\right)}, \\ \xi_i^t =& -\frac{\frac{1-\beta}{\beta}\left[\bar{\pi}_i^z N_i^t - (\varphi_i^* + N_i^z)\bar{\pi}_i^t\right]}{\frac{1-\beta}{\beta}\bar{\pi}_i^z - (N_i^z + \varphi_i^*)} = -\xi_i^M N_i^t = \xi_i^M \frac{1}{\theta_i},\end{aligned}$$

where we have used that $\eta_i\theta_i = \frac{\sigma_i - \eta_i}{\sigma_i - 1}$ from Section 3.2.2.

Signs of $\mathrm{d}\log z_i^*$ We then go back to the case with $\mathrm{d}\log z_i^*$. Since $\bar{\pi}_i^z < 0$ and thus $z_i^M > 0$. Then, $\mathrm{d}\log z_i^* < 0$ in the third market unambiguously. The case with the focal market is more complicated since z_i^t can be either positive (when β is close to one) or negative (when β is close to zero). In general, we cannot derive an unambiguous result on the sign of $\mathrm{d}\log z_j^*$.

In other words, the productivity threshold of exiting may fall or increase in the focal market.

$$\mathrm{d}\log z_j^* = z_j^M \left[\left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t + \bar{\pi}_j^t \left(\frac{1-\beta}{\beta} - \frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} \frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z} \right) \right] \mathrm{d}\log t_j \tag{49}$$

With the CES demand (and thus $\bar{\pi}_j^t = 0$), $\mathrm{d}\log z_j^* = z_j^M \left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t \log t_j > 0$, as $z_j^M > 0$, $N_j^t < 0$ and $\mathrm{d}\log t_j$. In other words, the exit cutoff increases in the focal market unambiguously.

Entry Interdependence in the focal market Let $\tilde{\beta} = \frac{1-\beta}{\beta}$. Partially differentiating equation (29) with respect to $\tilde{\beta}$, we obtain that

$$\begin{aligned} \frac{\partial \mathrm{d}\log z_j^*}{\partial \tilde{\beta}} &= \frac{\bar{\pi}^t (N^z + \varphi^*) + \bar{\pi}^z \left(-\frac{N}{N+1} N^t - \frac{1}{N+1} \frac{\bar{\pi}^t}{\bar{\pi}^z} (\varphi^* + N^z) \right)}{\left[(N^z + \varphi^*) - \frac{1-\beta}{\beta} \bar{\pi}^z \right]^2} \\ &= \frac{\frac{N}{N+1} \left(\bar{\pi}^t (N^z + \varphi^*) - \bar{\pi}^z N^t \right)}{\left[(N^z + \varphi^*) - \frac{1-\beta}{\beta} \bar{\pi}^z \right]^2} < 0, \end{aligned}$$

as $\varphi_i^* + N_i^z > 0$, $\bar{\pi}_i^z < 0$, $\bar{\pi}_i^t \leq 0$ and $N_i^t < 0$. Since $\frac{\mathrm{d}\tilde{\beta}}{\mathrm{d}\beta} < 0$, we obtain $\frac{\partial \mathrm{d}\log z_j^*}{\partial \beta} > 0$.

Changes in M_i^p For the potential entrants M_i^p,

$$\mathrm{d}\log M_i^p = (1 - \xi_i^M) \mathrm{d}\log M - \xi_i^t \mathrm{d}\log t_i.$$

Therefore, $\mathrm{d}\log M_i^p < 0$ in both focal and third markets.

Changes in Q_i Finally, we consider the welfare change:

$$\mathrm{d}\log Q_i = Q_i^z \mathrm{d}\log z_i^* + \frac{\bar{\delta}_i}{\gamma_i} \mathrm{d}\log t_i.$$

In the third market, $\mathrm{d}\log Q_i < 0$ since $\mathrm{d}\log z_i^* < 0$ and $Q_i^z > 0$. In the focal market j,

$$\mathrm{d}\log Q_j = \left\{ Q_j^z z_j^M \left[\left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t + \bar{\pi}_j^t \left(\frac{1-\beta}{\beta} - \frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} \frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z} \right) \right] + \frac{\bar{\delta}_j}{\gamma_j} \right\} \mathrm{d}\log t_j.$$

We show that the bracket is negative by showing that

$$Q_j^z z_j^M \left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t + \frac{\bar{\delta}_j}{\gamma_j} = \frac{1}{\gamma_j} \left[Q_j^z z_j^M \left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} + \bar{\delta}_j \right] < 0,$$

since the left term is negative. Note that $z_j^M \leq \frac{1}{N_i^z + \varphi_i^*}$,

$$\bar{\delta}_i + Q_j^z z_j^M \left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} \geq \bar{\delta}_i - Q_j^z z_j^m \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} \geq \bar{\delta}_i - \frac{Q_j^z}{N_i^z + \varphi_i^*} \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*}.$$

With $\gamma_j < 0$, it suffices to show that $\bar{\delta}_j > \frac{Q_j^z}{N_i^z + \varphi_i^*} \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*}$. We can rewrite $N_j^z + \varphi_j^*$ as follows

$$N_j^z + \varphi_j^* = \frac{1}{\bar{\delta}_j} \left(\frac{\mathbb{E}_{\lambda_j}^j [\rho_i \sigma_j]}{\sigma_j^*} \left[(\sigma_j^* - 1) + Q_j^z \right] + \delta_j^* \lambda_j^* \varphi_j^* \right)$$

Then,

$$\frac{Q_j^z}{N_i^z + \varphi_i^*} \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} = \bar{\delta}_j \frac{\frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} Q_j^z}{\frac{\mathbb{E}_{\lambda_j}^j [\rho_i \sigma_j]}{\sigma_j^*} \left[(\sigma_j^* - 1) + Q_j^z \right] + \delta_j^* \lambda_j^* \varphi_j^*} < \bar{\delta}_j.$$

We then complete the proof that $\mathrm{d} \log Q_j < 0$. Regardless of threshold changes, welfare loss in the focal market is certain.

Changes in P_i It is useful to note that in the focal market,

$$\begin{aligned} &(\sigma_j^* - 1) \mathrm{d} \log z_j^* + \mathrm{d} \log Q_j \\ &= \left\{ \left((\sigma_j^* - 1) + Q_j^z \right) z_j^M \left[\left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t + \bar{\pi}_j^t \left(\frac{1 - \beta}{\beta} - \frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} \frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z} \right) \right] + \frac{\bar{\delta}_j}{\gamma_j} \right\} \mathrm{d} \log t_j < 0, \end{aligned}$$

where the negative sign comes from the same arguments that prove $\mathrm{d} \log Q_j < 0$. Then, $(\sigma_i^* - 1) \mathrm{d} \log z_i^* + \mathrm{d} \log Q_i < 0$ for all country i. Recall that

$$\mathrm{d} \log P_i - \mathrm{d} \log t_i = -\frac{1}{\sigma_i^*} \left[(\sigma_i^* - 1) \mathrm{d} \log z_i^* + \mathrm{d} \log Q_i \right].$$

Then, we observe $\mathrm{d} \log P_i > 0$ in both focal and third markets.

Aside: Conditions on the sign of $\bar{\pi}_i^z$ Note that

$$\bar{\pi}_i^z = -\frac{\hat{\pi}_i(\sigma_i^* - 1)}{\hat{\pi}_i - f_i} + \frac{\sigma_i^*\hat{\pi}_i - \hat{r}_i}{\sigma_i^*(\hat{\pi}_i - f_i)}\left[(\sigma_i^* - 1) + Q_i^z\right] = -\frac{\hat{r}_i(\sigma_i^* - 1)}{\sigma_i^*(\hat{\pi}_i - f_i)} + \frac{(\sigma_i^*\hat{\pi}_i - \hat{r}_i)Q_i^z}{\sigma_i^*(\hat{\pi}_i - f_i)}.$$

Then

$$\bar{\pi}_i^z < 0 \iff \frac{\frac{\sigma_i^*\hat{\pi}_i}{\hat{r}_i} - 1}{\sigma_i^* - 1}Q_i^z < 1.$$

That is, whenever $\frac{\sigma_i^*\hat{\pi}_i}{\hat{r}_i}$ is small (close to one), $\bar{\pi}_i^z < 0$. In the CES case, $\frac{\sigma_i^*\hat{\pi}_i}{\hat{r}_i} = 1$.

D.2.2 Changes in firm-level variables

Changes in π_i and $\bar{\pi}_i$ For profit,

$$\begin{aligned}\mathrm{d}\log\pi_i &= \sigma_i\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right] + \mathrm{d}\log Q_i \\ &= -\frac{\sigma_i}{\sigma_i^*}\left[(\sigma_i^* - 1)\mathrm{d}\log z_i^* + \mathrm{d}\log Q_i\right] + \mathrm{d}\log Q_i.\end{aligned}$$

Then, $\mathrm{d}\log\pi_i$ always increases with σ_i. That is, conditional on survival, small firms (high σ_i) gain relative to large firms (low σ_i) in log term. We now consider the expected profit $\bar{\pi}_i$:

$$\mathrm{d}\log\bar{\pi}_i = \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M + \varphi_i^*\mathrm{d}\log z_i^*\right] = \frac{\beta}{1-\beta}\xi_j^t\left[\frac{s_j\xi_i^M}{\sum_k s_k\xi_k^M} - \mathbb{1}\{i=j\}\right]\mathrm{d}\log t_j$$

The expected profit drops in the focal market while increases in the third market. Whereas, the sign of average profit $\hat{\pi}_i$ is rather ambiguous. Observe that

$$\mathrm{d}\log\hat{\pi}_i = \frac{\hat{\pi}_i - f_i}{\hat{\pi}_i}\left(\varphi_i^*\mathrm{d}\log z_i^* + \mathrm{d}\log\bar{\pi}_i\right) = \frac{\hat{\pi}_i - f_i}{\hat{\pi}_i}\left((\varphi_i^* + \bar{\pi}_i^z)\mathrm{d}\log z_i^* + \bar{\pi}_i^t\mathrm{d}\log t_i\right),$$

where the sign of $\varphi_i^* + \bar{\pi}_i^z$ is undetermined. Thus, the average profit may fall or increase even with the CES demand.

Changes in p_i and $\frac{q_i}{Q_i}$ We can discuss the implication on average revenue and quantity. Note that

$$\begin{aligned}
\mathrm{d}\log\frac{q_i}{Q_i} &= \rho_i\sigma_i\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right] > 0, \\
\mathrm{d}\log\mu_i = \left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}\mathrm{d}\log\frac{q_i}{Q_i} &= (1-\rho_i)\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right] > 0.
\end{aligned}$$

Therefore, markup μ_i increases among all firms and in both focal and third markets. So does the price p_i, and larger firms respond more in prices.

Changes in r_i For sales,

$$\begin{aligned}
\mathrm{d}\log r_i &= -\frac{1}{\mu_i - 1}\mathrm{d}\log\mu_i + \mathrm{d}\log\pi_i \\
&= \left[1+\rho_i(\sigma_i-1)\right]\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right] + \mathrm{d}\log Q_i \\
&= -\frac{1+\rho_i(\sigma_i-1)}{\sigma_i^*}\left[(\sigma_i^*-1)\mathrm{d}\log z_i^* + \mathrm{d}\log Q_i\right] + \mathrm{d}\log Q_i \\
&= -\frac{1+\rho_i(\sigma_i-1)}{\sigma_i^*}(\sigma_i^*-1)\mathrm{d}\log z_i^* + \frac{(\sigma_i^*-1)-\rho_i(\sigma_i-1)}{\sigma_i^*}\mathrm{d}\log Q_i
\end{aligned}$$

In third market, the first term is positive and the second negative. In the focal market, $\mathrm{d}\log r_j < 0$ if $\mathrm{d}\log z_j^* > 0$. But we do not know the sign when $\mathrm{d}\log z_j^* < 0$. With the CES demand, $\mathrm{d}\log r_i = -(\sigma_i^*-1)\mathrm{d}\log z_i^*$. That is, sales increases in third markets while decreases in the focal market.

Changes in q_i For quantity,

$$\begin{aligned}
\mathrm{d}\log q_i &= \rho_i\sigma_i\left[\mathrm{d}\log P_i - \mathrm{d}\log t_i\right] + \mathrm{d}\log Q_i \\
&= -\frac{\rho_i\sigma_i}{\sigma_i^*}(\sigma_i^*-1)\mathrm{d}\log z_i^* + \frac{\sigma_i^*-\rho_i\sigma_i}{\sigma_i^*}\mathrm{d}\log Q_i
\end{aligned}$$

The analysis is identical to that on sales. With the CES demand, $\mathrm{d}\log q_i = -(\sigma_i^*-1)\mathrm{d}\log z_i^*$. In both quantity and sales, small firms gain relative to large firms.

D.2.3 Changes in other aggregate variables

We use f_x^i to denote the density function of variables in market i, i.e., $f_x^i(z) = \frac{x(z)\frac{g(z)}{1-G(z)}}{\int_{z_i^*}^{\infty} x(z)\frac{g(z)}{1-G(z)}\mathrm{d}z}$. In particular, the welfare density of firms with productivity z in market i is given by

$$f_\Upsilon^i(z) = \frac{\Upsilon\left(\frac{q_i(z)}{Q_i}\right)\frac{g(z)}{1-G(z)}}{\int_{z_i^*}^{\infty}\Upsilon\left(\frac{q_i(z)}{Q_i}\right)\frac{g(z)}{1-G(z)}\mathrm{d}z} = N_i\Upsilon\left(\frac{q_i(z)}{Q_i}\right),$$

where we have used the definition of Kimball aggregation. Furthermore, we define that $f_x \succcurlyeq f_y$ if the distribution of x first-order stochastically dominates that of y. Finally, we let $\bar{x}_i = \int_{z_i^*}^{\infty} x(z)\frac{g(z)}{1-G(z)}\mathrm{d}z$. The following lemmas turn out to be useful in later analysis.

Lemma 1. *In any market i, $f_q^i \succcurlyeq f_r^i$, and $f_q^i \succcurlyeq f_\Upsilon^i$.*

Proof. It is well known that the monotone likelihood ratio property (MLRP) implies first order stochastic dominance. Namely, if $\frac{f_x}{f_y}$ increases in z, then $f_x \succcurlyeq f_y$. Note that $\frac{r_i(z)}{q_i(z)} = p_i(z)$ and p is decreasing on z, then $f_q^i \succcurlyeq f_r^i$. Since Υ is concave, $\Upsilon\left(\frac{q_i}{Q_i}\right)/q_i$ decreases on z, $f_q^i \succcurlyeq f_\Upsilon^i$. ■

Lemma 2. *In any market i, $\lambda_i^* = \frac{r_i^*}{\bar{r}_i} > \frac{q_i^*}{\bar{q}_i}$, and $\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* > \frac{q_i^*}{\bar{q}_i}$.*

Proof. Consider x and y such that $\frac{f_x}{f_y}$ satisfies the MLRP. Then,

$$\frac{x_i^*}{\bar{x}_i} - \frac{y_i^*}{\bar{y}_i} = \frac{1}{\bar{x}_i\bar{y}_i}\left(\int_{z_i^*}^{\infty}(x_i^* y(z) - y_i^* x(z))\frac{g(z)}{1-G(z)}\mathrm{d}z\right) < 0$$

since $\frac{x_i^*}{y_i^*} < \frac{x(z)}{y(z)}$ for $z > z^*$. Using this relationship, we obtain $\lambda_i^* > \frac{q_i^*}{\bar{q}_i}$, and $N_i\Upsilon\left(\frac{q_i(z)}{Q_i}\right) > \frac{q_i^*}{\bar{q}_i}$. Also,

$$N_i\Upsilon\left(\frac{q_i(z)}{Q_i}\right) = N_i\delta_i\Upsilon'\left(\frac{q_i^*}{Q_i}\right)\frac{q_i^*}{Q_i} = \frac{N_i\delta_i t_i r_i^*}{\bar{\delta}_i E_i} = \frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^*$$

■

Changes in R_i Aggregate sales drop in all markets. To see this,

$$\mathrm{d}\log R_i = \left(1 - \frac{1}{\eta_i}\right)\mathrm{d}\log Q_i - \mathrm{d}\log t_i < 0.$$

Since N_i may become larger or smaller, the sign average revenue r_i is ambiguous. This will also be true even with the CES demand. In the third market, while firms may export more value, small exporters also enter.

Changes in $\tilde{Q}_i$ The total quantity $\tilde{Q}_i = N_i \int_{z_i^*}^{\infty} q_i \frac{g(z)}{1-G(z_i^*)} \mathrm{d}z$. Then,

$$\mathrm{d}\log \tilde{Q}_i = \mathrm{d}\log N_i + \varphi_i^* \mathrm{d}\log z_i^* - \frac{q_i^*}{\bar{q}_i}\varphi_i^* \mathrm{d}\log z_i^* + \mathbb{E}_{q_i}^i[\mathrm{d}\log q_i],$$

where

$$\mathbb{E}_{q_i}^i[\mathrm{d}\log q_i] = -\mathbb{E}_{q_i}^i[\rho_i\sigma_i]\frac{\sigma_i^*-1}{\sigma_i^*}\mathrm{d}\log z_i^* + \left(1 - \frac{\mathbb{E}_{q_i}^i[\rho_i\sigma_i]}{\sigma_i^*}\right)\mathrm{d}\log Q_i,$$

and $\bar{q}_i$ is the average quantity. Using the welfare condition and the expression of $\mathrm{d}\log \frac{q_i}{Q_i}$, we obtain

$$\begin{aligned}
\mathrm{d}\log \tilde{Q}_i &= \left(\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* - \frac{q_i^*}{\bar{q}_i}\right)\varphi_i^* \mathrm{d}\log z_i^* + \left(-\frac{1}{\bar{\delta}_i}\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i] + \mathbb{E}_{q_i}^i[\rho_i\sigma_i]\right)[\mathrm{d}\log P_i - \mathrm{d}\log t_i] + \mathrm{d}\log Q_i \\
&\leq \left(\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* - \frac{q_i^*}{\bar{q}_i}\right)\varphi_i^* \mathrm{d}\log z_i^* - \left(1 - \frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*}[(\sigma_i^*-1)\mathrm{d}\log z_i^* + \mathrm{d}\log Q_i] + \mathrm{d}\log Q_i \\
&= \left(\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* - \frac{q_i^*}{\bar{q}_i}\right)\varphi_i^* \mathrm{d}\log z_i^* + \left\{-\left(1 - \frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*}[(\sigma_i^*-1) + Q_i^z] + Q_i^z\right\}\mathrm{d}\log z_i^* \\
&\quad + \left\{-\left(1 - \frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*} + 1\right\}\frac{\bar{\delta}_i}{\gamma_i}\mathrm{d}\log t_i,
\end{aligned}$$

where the first inequality uses that $\mathbb{E}_{q_i}^i[\rho_i\sigma_i] \leq \mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]$, a result of Lemma 1, and the second equality uses the expression of $\mathrm{d}\log Q_i$ in $\mathrm{d}\log z_i^*$.

We show first that in third market, $\mathrm{d}\log \tilde{Q}_i < 0$. It suffices to show that the coefficient of $\mathrm{d}\log z_i^*$ is positive. As Lemma 2 shows that $\frac{\delta_i^*}{\bar{\delta}_i}\lambda_i^* > \frac{q_i^*}{\bar{q}_i}$, we only need that

$$f(\bar{\delta}_i) \equiv -\left(1 - \frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}_{\lambda_i}^i[\rho_i\sigma_i]}{\sigma_i^*}[(\sigma_i^*-1) + Q_i^z] + Q_i^z > 0.$$

Using the definition of Q_i^z,

$$\begin{aligned}\gamma_i f(\bar{\delta}_i) =& \left(1-\frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]}{\sigma_i^*}\left((\sigma_i^*-1)\bar{\delta}_i+(\bar{\delta}_i-\delta_i^*)\lambda_i^*\varphi_i^*\right)\\ &-\frac{\sigma_i^*-1}{\sigma_i^*}\left(\bar{\delta}_i+\bar{\delta}_i\,\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i]-\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right)-(\bar{\delta}_i-\delta_i^*)\lambda_i^*\varphi_i^*\\ =&-\frac{\sigma_i^*-1}{\sigma_i^*}\left[\bar{\delta}_i\left(1+\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i]-\left(1-\frac{1}{\bar{\delta}_i}\right)\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right)-\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right]\\ &-\left[1-\left(1-\frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]}{\sigma_i^*}\right](\bar{\delta}_i-\delta_i^*)\lambda_i^*\varphi_i^*.\end{aligned}$$

Note that

$$\begin{aligned}&-\frac{\sigma_i^*-1}{\sigma_i^*}\left[\bar{\delta}_i\left(1+\mathbb{E}^i_{\lambda_i}[(\sigma_i-1)\rho_i]-\left(1-\frac{1}{\bar{\delta}_i}\right)\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right)-\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right]\\ =&-\frac{\sigma_i^*-1}{\sigma_i^*}\bar{\delta}_i\left(1+\mathbb{E}^i_{\lambda_i}[(\rho_i-)\sigma_i]-\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]\right)\\ =&-\frac{\sigma_i^*-1}{\sigma_i^*}\bar{\delta}_i\left(1-\mathbb{E}^i_{\lambda_i}[\rho_i]\right)<0,\end{aligned}$$

and

$$\underbrace{-\left[1-\left(1-\frac{1}{\bar{\delta}_i}\right)\frac{\mathbb{E}^i_{\lambda_i}[\rho_i\sigma_i]}{\sigma_i^*}\right]}_{>0}\underbrace{(\bar{\delta}_i-\delta_i^*)}_{>0}\lambda_i^*\varphi_i^*<0.$$

Then, $\gamma_i f(\bar{\delta}_i)<0$, and $f(\bar{\delta}_i)>0$.

Next, we show that $\mathrm{d}\log\tilde{Q}_j<0$ in the focal market. From the last step, we obtain

$$\mathrm{d}\log\tilde{Q}_j\leq A_j\mathrm{d}\log z_j^*+B_j\mathrm{d}\log t_j,$$

where

$$\begin{aligned}A_j=&\left(\frac{\delta_j^*}{\bar{\delta}_j}\lambda_j^*-\frac{q_j^*}{\bar{q}_j}\right)\varphi_j^*-\left(1-\frac{1}{\bar{\delta}_j}\right)\frac{\mathbb{E}^j_{\lambda_j}[\rho_j\sigma_j]}{\sigma_j^*}\left[(\sigma_j^*-1)+Q_j^z\right]+Q_j^z\\ B_j=&\left\{-\left(1-\frac{1}{\bar{\delta}_j}\right)\frac{\mathbb{E}^i_{\lambda_j}[\rho_i\sigma_j]}{\sigma_j^*}+1\right\}\frac{\bar{\delta}_j}{\gamma_j}.\end{aligned}$$

We shown in the third market case that $A_j \geq 0$. Using equation (49),

$$A_j \mathrm{d}\log z_j^* + B_j \mathrm{d}\log t_j = N_j^t \mathrm{d}\log t_j \left[A_j z_j^M \left[\left(\frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} - 1 \right) N_j^t + \bar{\pi}_j^t \left(\frac{1-\beta}{\beta} - \frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} \frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z} \right) \right] + \frac{B_j}{N_j} \right],$$

Since $\frac{\bar{\pi}_j^t}{N_j^t} \left(\frac{1-\beta}{\beta} - \frac{s_j \xi_j^M}{\sum_i s_i \xi_i^M} \frac{\varphi_j^* + N_j^z}{\bar{\pi}_j^z} \right) > 0$, it suffices to show that $-A_j z_j^M + \frac{B_j}{N_j} > 0$. Note that

$$\begin{aligned} -A_j z_j^M + \frac{B_j}{N_j} = & - \frac{(N_j^z + \varphi_j^*) - \frac{q_j^*}{\bar{q}_j} \varphi_j^* - (\sigma_j^* - 1) + \left(1 - \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} \right) \left[(\sigma_j^* - 1) + Q_j^z \right]}{(N_j^z + \varphi_j^*) - \frac{1-\beta}{\beta} \bar{\pi}_i^z} \\ & + 1 + \left(\frac{\sigma_j^*}{\mathbb{E}_{\lambda_j}^j [\rho_i \sigma_i]} - 1 \right) \bar{\delta}_j > \frac{\frac{q_j^*}{\bar{q}_j} \varphi_j^* + (\sigma_j^* - 1) - \left(1 - \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} \right) \left[(\sigma_j^* - 1) + Q_j^z \right]}{N_j^z + \varphi_j^*} \\ & + \left(\frac{\sigma_j^*}{\mathbb{E}_{\lambda_j}^j [\rho_i \sigma_i]} - 1 \right) \bar{\delta}_j. \end{aligned}$$

Using the expression of $N_j^z + \varphi_j^*$ in $(\sigma_j^* - 1) + Q_j^z$,

$$\begin{aligned} \left(\frac{\sigma_j^*}{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]} - 1 \right) \bar{\delta}_j (N_j^z + \varphi_j^*) &= \left(\frac{\sigma_j^*}{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]} - 1 \right) \bar{\delta}_j \frac{1}{\bar{\delta}_j} \left(\frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} \left[(\sigma_j^* - 1) + Q_j^z \right] + \delta_j^* \lambda_j^* \varphi_j^* \right) \\ &> \left(1 - \frac{\mathbb{E}_{\lambda_j}^j [\rho_j \sigma_j]}{\sigma_j^*} \right) \left[(\sigma_j^* - 1) + Q_j^z \right]. \end{aligned}$$

Therefore,

$$-A_j z_j^M + \frac{B_j}{N_j} > \frac{\frac{q_j^*}{\bar{q}_j} \varphi_j^* + (\sigma_j^* - 1)}{N_j^z + \varphi_i^*} > 0.$$

We then complete the proof that $\mathrm{d}\log \tilde{Q}_j < 0$.

Changes in p_i^m Finally, we consider the aggregate price level. Recall that $p_i^m = \frac{R_i}{\bar{Q}_i} = \frac{\int_{z_i^*}^{\infty} r_i g(z)\mathrm{d}z}{\int_{z_i^*}^{\infty} q_i g(z)\mathrm{d}z}$, then

$$\begin{aligned}\mathrm{d}\log p_i^m &= \left(-\frac{r_i^*}{\bar{r}_i} + \frac{q_i^*}{\bar{q}_i}\right)\varphi_i^* \mathrm{d}\log z_i^* + \mathbb{E}_{r_i}^i[\mathrm{d}\log r_i] - \mathbb{E}_q[\mathrm{d}\log q_i] \\ &= \left(-\frac{r_i^*}{\bar{r}_i} + \frac{q_i^*}{\bar{q}_i}\right)\varphi_i^* \mathrm{d}\log z_i^* + \left(1 - \mathbb{E}_{r_i}^i[\rho_i] + \mathbb{E}_{r_i}^i[\rho_i\sigma_i] - \mathbb{E}_{q_i}^i[\rho_i\sigma_i]\right)(\mathrm{d}\log P_i - \mathrm{d}\log t_i)\end{aligned}$$

Lemma 2 shows that $-\frac{r_i^*}{\bar{r}_i} + \frac{q_i^*}{\bar{q}_i} < 0$, and Lemma 1 implies that $\mathbb{E}_{r_i}^i[\rho_i\sigma_i] > \mathbb{E}_{q_i}^i[\rho_i\sigma_i]$.

With $\mathbb{E}_{r_i}^i[\rho_i] \leq 1$ and $\mathrm{d}\log P_i - \mathrm{d}\log t_i > 0$, we proved that $\mathrm{d}\log p_i^m > 0$ if $\mathrm{d}\log z_i^* < 0$. That is, average price rises at the aggregate level in third markets. In the focal market, the sign is ambiguous. For example, we have shown that the average price will go up if goods are homogeneous. Whereas, it is immediate that the average price will go down with the CES demand ($\mathrm{d}\log z_i^* > 0$).

E Model Extensions

E.1 General equilibrium effect

Since we normalize the domestic wage to one, the general equilibrium effect can be captured by a change in the wage of third countries. As a result, the foreign demand for domestic products in (33) becomes

$$\begin{aligned}\mathrm{d}\log P_i^I =& \mathrm{d}\log N_i + \mathrm{d}\log t_i + \mathrm{d}\log \mu_i + \mathrm{d}\log\frac{\tau_i}{z} + \mathrm{d}\log\frac{q_i}{Q_i} \\ \Rightarrow \quad \mathrm{d}\log W_i - \frac{1}{\eta_i}\mathrm{d}\log Q_i =& \mathrm{d}\log N_i + \mathrm{d}\log t_i + \mathrm{d}\log\frac{\tau_i}{z} + \left[\left(\frac{1}{\rho_i} - 1\right)\frac{1}{\sigma_i} + 1\right]\mathrm{d}\log\frac{q_i}{Q_i}\end{aligned}$$

Then, the system of equations that characterize the equilibrium become as follows:

$$\begin{aligned}
\delta \mathrm{d}\log N_i + \mathrm{d}\log\frac{q_i}{Q_i} &= 0 \\
\log W_i - \frac{1}{\eta_i}\mathrm{d}\log Q_i = \mathrm{d}\log N_i + \mathrm{d}\log t_i + \left[\left(\frac{1}{\rho_i}-1\right)\frac{1}{\sigma_i}+1\right]\mathrm{d}\log\frac{q_i}{Q_i} \\
\mathrm{d}\log\pi_i &= \frac{1}{\rho_i}\mathrm{d}\log\frac{q_i}{Q_i} + \mathrm{d}\log Q_i \\
\frac{\pi_i}{\pi_i - f_i}\mathrm{d}\log\pi_i &= \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M\right] \\
\sum_{i=0}^{N} s_i \mathrm{d}\log N_i &= \mathrm{d}\log M
\end{aligned}$$

Let $\mathrm{d}\log\tilde{t}_i = \mathrm{d}\log t_i - \mathrm{d}\log W_i$. We then have exactly the same set of equations as the baseline case in Appendix C. Immediately, we obtain

$$\mathrm{d}\log M = -\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log\tilde{t}}{\theta}\right] = -\,\mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log t}{\theta}\right] + \mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log W}{\theta}\right] \tag{50}$$

$$\mathrm{d}\log N_i = (1-\xi_i)\mathrm{d}\log M - \xi_i\frac{\mathrm{d}\log t_i}{\theta_i} + \xi_i\frac{\mathrm{d}\log W_i}{\theta_i} \tag{51}$$

For notational brevity, we use $\bar{x} = \mathbb{E}_{s\xi}\left[\frac{\mathrm{d}\log x}{\theta}\right]$, $\hat{x}_i = \mathrm{d}\log x_i/\theta_i$, for $x \in \{x, W\}$. Then, $\mathrm{d}\log M = \bar{W} - \bar{t}$, and $\mathrm{d}\log N_i = (1-\xi_i)(\bar{W}-\bar{t}) + \xi_i(\hat{W}_i - \hat{t}_i)$. In what follows, we discuss the implications of general equilibrium effects through real wage changes.

First, $\mathrm{d}\log M > 0$ if the real wage effect dominates the negative demand effect due to higher tariffs. Even if we focus on the more likely case with $\mathrm{d}\log M < 0$ ($\bar{t} > \bar{W}$),

$$\mathrm{d}\log N_i > 0 \quad \Leftrightarrow \quad \xi_i(\hat{W}_i - \hat{t}_i) > (1-\xi_i)(\bar{t} - \bar{W}).$$

In third markets ($\hat{t}_i = 0$), there will be an increase in N_i if (1) the real wage effect is strong (a large and positive $\hat{W}_i$), and/or (2) the degree of interdependence across markets is weaker (i.e., ξ_i is close to 1).

Second, profitability in third markets does not always increase. Notice that

$$\mathrm{d}\log\pi_i = -\eta_i\theta_i\mathrm{d}\log N_i - \eta_i\mathrm{d}\log\tilde{t}_i = \eta_i\theta_i(1-\xi_i)\left[(\hat{W}_i - \hat{t}_i) - (\bar{W}-\bar{t})\right]$$

In third markets, profitability improves if and only if $\hat{W}_i > \bar{W} - \bar{t}$. With $\mathrm{d}\log M < 0$, this condition is satisfied whenever there is a positive real wage effect, $\hat{W}_i > 0$. Moreover,

improved profitability no longer implies a reduction in the number of firms or total exports. Consider the case in which $\mathrm{d}\log M < 0$ and $\mathrm{d}\log N_i > 0$. From the discussion above, we must have $\hat{W}_i > 0$, and then $\mathrm{d}\log \pi_i > 0$. Moreover, recall that the change in total export in third market i ($\mathrm{d}\log t_i = 0$) is

$$\mathrm{d}\log R_i = \underbrace{\left[\left(\frac{1}{\rho_i} - 1\right)\left(1 - \frac{1}{\sigma_i}\right)\delta_i - \eta_i\theta_i + 1\right]}_{>0}\mathrm{d}\log N_i + \eta_i\theta_i\frac{\mathrm{d}\log W_i}{\theta_i} > 0.$$

We then obtain a scenario in which $\mathrm{d}\log \pi_i > 0$, $\mathrm{d}\log N_i > 0$, and $\mathrm{d}\log R_i > 0$. This possibility sharply contrasts with the partial equilibrium result that improved firm profitability necessarily implies a reduction in the number of firms and total exports. Intuitively, increased aggregate demand thanks to the relative wage gains can dominate fiercer competition among home exporters, leading to higher individual profits.

Finally, it is noteworthy that the relationship between price and the number of firms remain the unchanged, regardless of the general equilibrium effect. For individual prices, we have

$$\mathrm{d}\log p_i = -\left(\frac{1}{\rho_i} - 1\right)\frac{\delta_i}{\sigma_i}\mathrm{d}\log N_i,$$

For the ideal price index of composite Q_i,

$$\mathrm{d}\log P_i^I = \mathrm{d}\log W_i - \frac{1}{\eta_i}\mathrm{d}\log Q_i = \underbrace{-\frac{1}{\eta_i}\left(\frac{\delta_i}{\rho_i} - \eta_i\theta_i\right)}_{<0}\mathrm{d}\log N_i + \mathrm{d}\log t_i. \tag{52}$$

As $\mathrm{d}\log t_i = 0$ in third markets, the firm-level price, p_i, and the ideal price index, P_i^I, increase in the third market if and only if the number of exporters decreases, i.e., $\mathrm{d}\log N_i < 0$. The same intuition from the partial equilibrium analysis applies here: an increase in the number of firms leads to both more competition and varieties, both of which reduces the price of the composite.

A micro-foundation of $D_i(P_i^I)$ We consider a micro-foundation for $D_i(P_i^I)$ in the baseline model to study competition in the destination market. We follow Atkeson and Burstein (2008) to assume an CES nest between the composite good from the home country and that from

the foreign country in market i. Formally,

$$D_i(P_i^I, P_i^h, E_i) = E_i \frac{(P_i^I)^{-\eta}}{(P_i^I)^{1-\eta} + (P_i^h)^{1-\eta}},$$

where η is a constant elasticity of substitution. Totally differentiating it,

$$\begin{aligned}
\mathrm{d}\log D_i(P_i^I, P_i^h, E_i) &= \mathrm{d}\log E_i - \eta \mathrm{d}\log P_i^I - \mathrm{d}\log\left((P_i^I)^{1-\eta} + (P_i^h)^{1-\eta}\right) \\
&= \mathrm{d}\log E_i - \eta \mathrm{d}\log P_i^I - (1-\eta)(s_i \mathrm{d}\log P_i^I + (1-s_i)\mathrm{d}\log P_i^h) \\
&= \mathrm{d}\log E_i - \mathrm{d}\log P_i^I + (\eta-1)(1-s_i)\left(\mathrm{d}\log P_i^h - \mathrm{d}\log P_i^I\right)
\end{aligned}$$

Notice that the price of firms from countries other than the home country, P_i^h, will also respond to changes in P_i^I and E_i. Hence, we write it as follows:

$$\mathrm{d}\log P_i^h = \eta_i^c \mathrm{d}\log P_i^I + \eta_i^e \mathrm{d}\log E_i,$$

where η_i^c captures the cross-elasticity between home exports and other products, and η_i^e captures the returns to scale. $\eta_i^e \leq 0$ if there are constant or increasing returns to scale. We give a micro foundation of η_i^c and η_i^e later. Plugging it back into the above equation, we have

$$\begin{aligned}
\mathrm{d}\log D_i(P_i^I, P_i^h, E_i) &= \mathrm{d}\log E_i - \mathrm{d}\log P_i^I + (\eta-1)(1-s_i)\left(\eta_i^c \mathrm{d}\log P_i^I + \eta_i^e \mathrm{d}\log E_i - \mathrm{d}\log P_i^I\right) \\
&= -\left(1 + (\eta-1)(1-s_i)(1-\eta_i^c)\right)\mathrm{d}\log P_i^I + \left(1 + (\eta-1)(1-s_i)\eta_i^e\right)\mathrm{d}\log E_i
\end{aligned}$$

Then,

$$\eta_i \equiv -\frac{\partial \log D_i(P_i^I, P_i^h, E_i)}{\partial \log P_i^I} = 1 + (\eta-1)(1-s_i)(1-\eta_i^c).$$

Therefore, $\eta_i > 1$ if and only if $\eta_i^c < 1$. In a well-defined equilibrium in which there are positive exports, $\eta_i > 1$, and we must have $\eta_i^c < 1$.[29] As $\mathrm{d}\log P_i^I - \mathrm{d}\log P_i^h = (1-\eta_i^c)\mathrm{d}\log P_i^I - \eta_i^e \mathrm{d}\log E_i$, the price of home composite good always increases more than the price of other products whenever there is an increase in P_i^I in the partial equilibrium. This force is even stronger in the general equilibrium in which $\mathrm{d}\log E_i > 0$. This result shows that the import competition in the third market is reduced with a tariff imposed in the focal market. Namely, there will be more protection for the domestic firms and other exporters in the third country.

[29] If $\eta_i < 1$, $P_i^I D_i(P_i^I)$ increases holding E_i constant.

Finally, the general equilibrium effect in this micro-founded model is captured by

$$\mathrm{d}\log W_i = (1 + (\eta - 1)(1 - s_i)\eta_i^e)\,\mathrm{d}\log E_i,$$

which includes both the change in the relative wage $\mathrm{d}\log E_i$ and the equilibrium change in the pricing of producers from other countries that originates from the change in the relative wage in equilibrium, $(\eta - 1)(1 - s_i)\eta_i^e \mathrm{d}\log E_i$.

E.2 Firm-level non-constant returns to scale production function

In this subsection, we consider how a decreasing returns to scale production function at firm level might change the results. In particular, we assume that the marginal cost (c) is a function of total quantity, i.e.,

$$c = c(q^f), \quad \text{s.t. } q^f = \sum_{i=0}^{N} q_i.$$

That $c' < 0$ captures increasing returns to scale (IRS) and $c' > 0$ decreasing returns to scale (DRS) at the firm-level . We then denote $\psi(q) = -\frac{c'(q)q}{c(q)}$. To simplify the analysis, we abstract away from the general entry technology defined in equation (10) and assumes the Melitz-type entry (i.e., $\beta = 1$). Since firms are homogeneous, this simplification then implies that all firms export to all markets.[30]

We proceed as follows. First, the equilibrium conditions are now given by

$$\begin{aligned}
&N_i \Upsilon\left(\frac{q_i}{Q_i}\right) = 1, \\
&E_i(Q_i) = N_i t_i p_i q_i \Leftrightarrow p_i^I(Q_i) = N_i t_i \mu\left(\frac{q_i}{Q_i}\right) c\tau_i \frac{q_i}{Q_i}, \\
&\pi_i(N_i) = \left(1 - \frac{1}{\mu_i}\right) p_i q_i = \left(1 - \frac{1}{\mu_i}\right) \frac{E_i(Q_i)}{N_i t_i}, \\
&\sum_{i=0}^{N} \bar{\pi}_i = f_e, \quad M = \nu_i N_i,
\end{aligned}$$

[30] If entries happen with certain probabilities which happens when $\beta \neq 1$, firms' marginal cost will be different even conditional on productivity z since it now depends on the realized market entries. Then aggregation becomes significantly more complicated as we need to integrate over the distribution of the markets each firm enters ex post. We focus on this simple case to better illustrate the role played by the decreasing returns to scale technology.

where $\bar{\pi}_i = \frac{\pi_i - f_i}{\rho + \nu_i}$. Compared to the benchmark model, the marginal cost c is determined in the equilibrium and changes with the tariffs. Totally differentiating these conditions, we obtain

$$\begin{aligned}
&\delta \mathrm{d}\log N_i + \mathrm{d}\log\frac{q_i}{Q_i} = 0,\\
&-\frac{1}{\eta_i}\mathrm{d}\log Q_i = \mathrm{d}\log N_i + \mathrm{d}\log t_i + \mathrm{d}\log c + \left[\left(\frac{1}{\rho_i} - 1\right)\frac{1}{\sigma_i} + 1\right]\mathrm{d}\log\frac{q_i}{Q_i},\\
&\mathrm{d}\log\pi_i = \frac{1}{\rho_i}\mathrm{d}\log\frac{q_i}{Q_i} + \mathrm{d}\log Q_i + \mathrm{d}\log c,\\
&\sum_{i=0}^{N} s_i^{\pi}\mathrm{d}\log\pi_i = 0, \quad \mathrm{d}\log N_i = \mathrm{d}\log M, \quad \mathrm{d}\log c = -\psi(q^f)\sum_{i=0}^{N} s_i^{q}\mathrm{d}\log q_i,
\end{aligned}$$

in which $s_i^{\pi} = \frac{\pi_i/(\rho+\nu_i)}{\sum_k \pi_k/(\rho+\nu_k)}$ and $s_i^{q} = q_i/(\sum_i q_i)$. As before, we obtain that

$$\mathrm{d}\log\pi_i = -\eta_i\theta_i\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i - (\eta_i - 1)\mathrm{d}\log c, \tag{53}$$

$$\mathrm{d}\log q_i = \left(\delta_i\left(\frac{1}{\rho_i} - 1\right) - \eta_i\theta_i\right)\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i - \eta_i\mathrm{d}\log c. \tag{54}$$

Substituting equations (53) and (54) into the $\sum_{i=0}^{N} s_i^{\pi}\mathrm{d}\log\pi_i = 0$, $\mathrm{d}\log N_i = \mathrm{d}\log M$ and $\mathrm{d}\log c = -\psi(q^f)\sum_{i=0}^{N} s_i^{q}\mathrm{d}\log q_i$ leads to the following the system of two equations:

$$-\left(\sum_i s_i^{\pi}\eta_i\theta_i\right)\mathrm{d}\log M - \left(\sum_i s_i^{\pi}(\eta_i - 1)\right)\mathrm{d}\log c = \sum_i s_i^{\pi}\eta_i\mathrm{d}\log t_i \tag{55}$$

$$\left(\sum_i s_i^{q}\left(\delta_i\left(\frac{1}{\rho_i} - 1\right) - \eta_i\theta_i\right)\right)\mathrm{d}\log M + \left(\frac{1}{\psi} - \sum_i s_i^{q}\eta_i\right)\mathrm{d}\log c = \sum_i s_i^{q}\eta_i\mathrm{d}\log t_i \tag{56}$$

When $\psi = 0$, we obtain the baseline equations such that $\mathrm{d}\log c = 0$ and $\mathrm{d}\log M = -\frac{\sum_i s_i^{\pi}\eta_i\mathrm{d}\log t_i}{\sum_i s_i^{\pi}\eta_i\theta_i} < 0$. We now focus on the general case with $\psi \in (-\infty, 1)$.[31]

[31] To see this range, consider a production function $q = l^{\alpha}$ and wage w. Then, $TC(q) = wq^{\frac{1}{\alpha}}$, $c(q) = TC'(q) = \frac{w}{\alpha}q^{\frac{1}{\alpha}-1}$, and $\psi = -\frac{\mathrm{d}\log c}{\mathrm{d}\log q} = 1 - \frac{1}{\alpha} \in (-\infty, 1)$ for $\alpha \in (0, +\infty)$.

Now, we invert the above two linear equations to

$$\mathrm{d}\log M = \frac{\left(\frac{1}{\psi} - \sum_i s_i^q \eta_i\right) \sum_i s_i^\pi \eta_i \mathrm{d}\log t_i + \left(\sum_i s_i^\pi (\eta_i - 1)\right) \sum_i s_i^q \eta_i \mathrm{d}\log t_i}{\left(-\sum_i s_i^\pi \eta_i \theta_i\right) \left(\frac{1}{\psi} - \sum_i s_i^q \eta_i\right) + \left(\sum_i s_i^\pi (\eta_i - 1)\right) \left(\sum_i s_i^q \left(\delta_i \left(\frac{1}{\rho_i} - 1\right) - \eta_i \theta_i\right)\right)},$$

$$\mathrm{d}\log c = \frac{-\sum_i s_i^q \left(\delta_i \left(\frac{1}{\rho_i} - 1\right) - \eta_i \theta_i\right) \sum_i s_i^\pi \eta_i \mathrm{d}\log t_i - \sum_i s_i^\pi \eta_i \theta_i \sum_i s_i^q \eta_i \mathrm{d}\log t_i}{\left(-\sum_i s_i^\pi \eta_i \theta_i\right) \left(\frac{1}{\psi} - \sum_i s_i^q \eta_i\right) + \left(\sum_i s_i^\pi (\eta_i - 1)\right) \left(\sum_i s_i^q \left(\delta_i \left(\frac{1}{\rho_i} - 1\right) - \eta_i \theta_i\right)\right)}.$$

Without further restrictions, we can obtain all possible combinations in terms of the signs of $\mathrm{d}\log M$ and $\mathrm{d}\log c$. If $s_i^\pi \approx s_i^q$ (which is the case when we start from the case of symmetric countries before the tariff shock) and $\psi < 0$, the common denominator of the above two expressions must be positive, as

$$\left(-\sum_i s_i^\pi \eta_i \theta_i\right) \left(\frac{1}{\psi} - \sum_i s_i^q \eta_i\right) + \left(\sum_i s_i^\pi (\eta_i - 1)\right) \left(\sum_i s_i^q \left(\delta_i \left(\frac{1}{\rho_i} - 1\right) - \eta_i \theta_i\right)\right),$$

which equals

$$\left(\sum_i s_i^q \eta_i \theta_i \left(1 - \frac{1}{\psi}\right)\right) + \left(\sum_i s_i^\pi (\eta_i - 1)\right) \left(\sum_i s_i^q \left(\delta_i \left(\frac{1}{\rho_i} - 1\right)\right)\right) > 0.$$

The numerator of d$logM$ must be negative as

$$\left(\frac{1}{\psi} - \sum_i s_i^q \eta_i\right) \sum_i s_i^\pi \eta_i \mathrm{d}\log t_i + \left(\sum_i s_i^\pi (\eta_i - 1)\right) \sum_i s_i^q \eta_i \mathrm{d}\log t_i = -(1 - \frac{1}{\psi}) \sum_i s_i^q \eta_i \mathrm{d}\log t_i < 0.$$

The numerator of d$logc$ must be negative as well since

$$-\sum_i s_i^q \left(\delta_i \left(\frac{1}{\rho_i} - 1\right) - \eta_i \theta_i\right) \sum_i s_i^\pi \eta_i \mathrm{d}\log t_i - \sum_i s_i^\pi \eta_i \theta_i \sum_i s_i^q \eta_i \mathrm{d}\log t_i,$$

which equals

$$-\sum_i s_i^q \left(\delta_i \left(\frac{1}{\rho_i} - 1\right)\right) \sum_i s_i^\pi \eta_i \mathrm{d}\log t_i < 0.$$

Consequently, $\mathrm{d}\log M < 0$, and $\mathrm{d}\log c < 0$. This makes sense, as $\psi < 0$ implies that overall output has to go down due to the DRS technology at the firm level and the depressed worldwide demand. Intuitively, the depressed worldwide demand (due to the tariff in the

focal market) incentivizes the firm to scale back its global production, as the marginal cost increases with output. It also disincentivize potential entrants to start exporting, leading to a drop in the number of (potential) entrants ($\mathrm{d}\log M < 0$).

We now discuss two special cases for sharper characterizations. As the literature tends to find a DRS production function at the firm level (e.g., Almunia et al. (2021)), we focus on the case where $\Psi < 0$ in what follows.

CES demand and $\nu_i = \nu$ With these conditions, we have $s_i^\pi = s_i^q = s_i$. Subtracting (56) using (55), we obtain

$$\left(\sum_i s_i \delta_i \left(\frac{1}{\rho_i} - 1\right)\right) \mathrm{d}\log M + \left(\frac{1}{\psi} - 1\right) \mathrm{d}\log c = 0$$

Since $\rho_i = 1$ for CES demand, we have $\mathrm{d}\log c = 0$ and $\mathrm{d}\log M < 0$. In other words, with CES demand, the results will be the same regardless of the nature of the production at the firm level (i.e, $\psi(q)$). Specifically, the total quantity (and sales) remain the same at the firm level. The firm-level export volume to market i equals

$$\begin{aligned}
\mathrm{d}\log q_i =& \left(\delta_i \left(\frac{1}{\rho_i} - 1\right) - \eta_i \theta_i\right) \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i - \eta_i \mathrm{d}\log c \\
=& -\eta_i \theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i \\
=& \sum_j s_j^q \eta_j \mathrm{d}\log t_j - \eta_i \mathrm{d}\log t_i,
\end{aligned}$$

Thus, it fall in the focal market and increases in the third market (as only $\mathrm{d}\log t_i = 0$ in all third markets). Since the markup is constant, firm-level sales in the focal and third markets follow the same changes. Finally, aggregate export volume to any market i, $\tilde{Q}_i$, equals

$$\begin{aligned}
\mathrm{d}\log \tilde{Q}_i =& \left(\delta_i \left(\frac{1}{\rho_i} - 1\right) - \eta_i \theta_i + 1\right) \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i - \eta_i \mathrm{d}\log c \\
=& (1 - \eta_i \theta_i) \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i \\
=& (\eta_i - 1)(\delta_i - 1) \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i < 0,
\end{aligned}$$

as $\eta_i > 1$, $\delta_i > 1$, $\mathrm{d}\log N_i < 0$ and $\mathrm{d}\log t_i \geq 0$ (and strictly positive in the focal market). Since the markup is constant, aggregate export value in the focal and third markets also fall. In summary, the nature of the firm-level production does not matter for the model's

predictions when the demand function is CES.

Symmetric markets With symmetric markets, we have $s_i^q = s_i^\pi = \frac{1}{N+1}$, and all the elasticities are the same across markets. We suppress the subscripts and obtain the following equation:

$$\delta\left(\frac{1}{\rho}-1\right)\mathrm{d}\log M+\left(\frac{1}{\psi}-1\right)\mathrm{d}\log c=0 \tag{57}$$

Substituting it into equation (55) or (56), we obtain

$$\mathrm{d}\log M=-\frac{s\eta\sum_i \mathrm{d}\log t_i}{\frac{-(\eta-1)}{1/\psi-1}\delta\left(\frac{1}{\rho}-1\right)+\eta\theta}<0,$$

as $\psi<0$ and $\mathrm{d}\log t_i\geq 0$. Moreover, since $\frac{1}{\psi}-1<0$, $\mathrm{d}\log c<0$. I.e., the firm reduces its global output.

We now study the changes in firm-level export volume, q_i, and total export volume, $\tilde{Q}_i$ in third markets where $\eta_i \mathrm{d}\log t_i=0$. First, note that

$$\begin{aligned}\mathrm{d}\log q_i&=\left(\delta_i\left(\frac{1}{\rho_i}-1\right)-\eta_i\theta_i\right)\mathrm{d}\log N_i-\eta_i\mathrm{d}\log t_i-\eta_i\mathrm{d}\log c\\&=\left(\eta\left(\frac{1}{\sigma}+\frac{1}{1/\psi-1}\right)\delta\left(\frac{1}{\rho}-1\right)-\eta+(\eta-1)\delta\right)\mathrm{d}\log M,\end{aligned}$$

where we have utilized the symmetry assumption. Thus, we have

$$\frac{\mathrm{d}\log q_i}{\mathrm{d}\log M}>0\Longleftrightarrow\psi_q^*\equiv\underbrace{\frac{\frac{\eta-1}{\eta}\delta-1}{\delta}}_{<0}\frac{\rho}{1-\rho}+\frac{1}{\sigma}>\frac{\psi}{\psi-1}. \tag{58}$$

In addition, we have

$$\begin{aligned}\mathrm{d}\log\tilde{Q}_i&=\left(\delta_i\left(\frac{1}{\rho_i}-1\right)-\eta_i\theta_i+1\right)\mathrm{d}\log N_i-\eta_i\mathrm{d}\log t_i-\eta_i\mathrm{d}\log c\\&=\left(\delta\left(\frac{1}{\rho}-1\right)-\eta\theta+1+\frac{\eta}{1/\psi-1}\delta\left(\frac{1}{\rho}-1\right)\right)\mathrm{d}\log M.\end{aligned}$$

Note that

$$\begin{aligned}
&\delta\left(\frac{1}{\rho}-1\right)-\eta\theta+1+\frac{\eta}{1/\psi-1}\delta\left(\frac{1}{\rho}-1\right)\\
=&(1+\frac{\eta}{1/\psi-1})\delta\left(\frac{1}{\rho}-1\right)-\left(1-\frac{\eta}{\sigma}\right)\delta\left(\frac{1}{\rho}-1\right)-\eta+(\eta-1)\delta+1\\
=&\eta\left(\frac{1}{\sigma}+\frac{1}{1/\psi-1}\right)\delta\left(\frac{1}{\rho}-1\right)+(\delta-1)(\eta-1).
\end{aligned}$$

Therefore, we have overall trade depression if and only if

$$\frac{\mathrm{d}\log\tilde{Q}_i}{\mathrm{d}\log M}>0\Longleftrightarrow\psi^*_{\tilde{Q}}\equiv\frac{1}{\sigma}+\frac{\delta-1}{\delta}\frac{\eta-1}{\eta}\frac{\rho}{1-\rho}>\frac{\psi}{\psi-1}. \tag{59}$$

Two things are worth mentioning here. First, if the condition in equation (58) is satisfied, then equation (59) must hold. In other words, if we have firm-level depression, then aggregate-level trade depression must be true.[32] Second, the condition in equation (59) must hold if ρ is close enough to one (i.e., a complete passthrough). In particular, it does not depend on the nature of firm-level production technology. This implies that the extensive margin drop in the number of firms must dominate any potential firm-level diversion, leading to an overall drop in the export volume to third markets.

Lastly, we consider the consumption Q_i:

$$\mathrm{d}\log Q_i=\left(\frac{\delta_i}{\rho_i}-\eta_i\theta_i\right)\mathrm{d}\log N_i-\eta_i\mathrm{d}\log t_i-\eta_i\mathrm{d}\log c.$$

Since $\mathrm{d}\log t_i=0$ in the third market, we have

$$\begin{aligned}
\mathrm{d}\log Q_i&=\left(\frac{\delta}{\rho}-\eta\theta+\frac{\eta}{1/\psi-1}\delta\left(\frac{1}{\rho}-1\right)\right)\mathrm{d}\log M\\
&=\left[\eta\left(\frac{1}{\sigma}+\frac{1}{1/\psi-1}\right)\delta\left(\frac{1}{\rho}-1\right)+(\delta-1)\eta\right]\mathrm{d}\log M.
\end{aligned}$$

Therefore,

$$\frac{\mathrm{d}\log Q_i}{\mathrm{d}\log M}>0\Longleftrightarrow\psi^*_{Q}\equiv\frac{1}{\sigma}+\frac{\delta-1}{\delta}\frac{\rho}{1-\rho}>\frac{\psi}{\psi-1}. \tag{60}$$

Again, this condition must hold if ρ is close enough to one.

[32] This has to be true, as we have proven that $\mathrm{d}logN_i=\mathrm{d}logM<0$.

F Homothetic demand with a single aggregator (HAS)

In this section, we rewrite our model using a homothetic demand function with a single aggregator (HAS) which is introduced by Matsuyama and Ushchev (2017). We will show that all the qualitative results we have obtained using the Kimball demand continue to hold in the HSA case.

F.1 Setup

Households There are $N+1$ countries indexed by i, and $i \in \{0, 1, \ldots, N\}$ in which country 0 is the home country. There are also many sectors in the economy, but our analysis focuses on an industry equilibrium of one sector. The home country produces many varieties in this sector which are denoted by ω. We use Ω_i to denote the set of all varieties that are exported to country i, and Ω the set of all varieties in the home country.

In country i, consumers have HSA preference towards exported varieties ω given a sectoral composite Q_i:

$$\int_{\Omega_i} s\left(\frac{p_\omega^i}{A_i}\right) \mathrm{d}\omega = 1,$$

where A_i is price aggregate that differs from the ideal price and $s(.)$ is the (revenue) market share function. Note that

$$\frac{d \log P_i^I}{d \log p_\omega^i} = s\left(\frac{p_\omega^i}{A_i}\right).$$

The ideal price index can be calculated

$$\log\left(P_i^I\right) = C + \log(A_i) - \int_{\omega \in \Omega_i} \int_{\frac{p_\omega^i}{A_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta d\omega, \tag{61}$$

See Matsuyama and Ushchev (2017) for the detail.

Production We first solve the firm's optimal pricing. First, the industry-level demand function is

$$P_i^I = D_i(Q_i) = D_i Q_i^{-\frac{1}{\eta_i}} \Leftrightarrow E_i \equiv E_i(Q_i) = Q_i D_i(Q_i) = D_i Q_i^{1-\frac{1}{\eta_i}} = D_i^{\eta_i} {P_i^I}^{1-\eta_i}.$$

Thus, the objective function of the firm can be written as

$$\max_{p_\omega^i} D_i^{\eta_i} P_i^{1-\eta_i} s\left(\frac{t_i p_\omega^i}{A_i}\right) \frac{p_\omega^i - \frac{\tau_i}{z}}{t_i p_\omega^i},$$

where p_ω^i is the FOB price and $t_i p_\omega^i$ is the CIF price. Note that $\frac{D_i^{\eta_i} P_i^{1-\eta_i} s\left(\frac{t_i p_\omega^i}{A_i}\right)}{t_i p_\omega^i}$ is the output supplied by firm ω. The FOC is

$$s^{'}\left(\frac{t_i p_\omega^i}{A_i}\right) \frac{t_i}{A_i} \frac{p_\omega^i - \frac{\tau_i}{z}}{p_\omega^i} + s\left(\frac{t_i p_\omega^i}{A_i}\right) \frac{\frac{\tau_i}{z}}{\left(p_\omega^i\right)^2} = 0,$$

which is equivalent to

$$s^{'}\left(\frac{t_i p_\omega^i}{A_i}\right) \frac{t_i p_\omega^i}{A_i} \left(p_\omega^i - \frac{\tau_i}{z}\right) + s\left(\frac{t_i p_\omega^i}{A_i}\right) \frac{\tau_i}{z} = 0,$$

leads to the optimal pricing rule as

$$t_i p_\omega^i = \frac{\sigma(\upsilon)}{\sigma(\upsilon) - 1} \frac{t_i \tau}{z}$$

where $\upsilon \equiv \frac{t_i p_\omega^i}{A_i}$. Note that we have used the definition of the elasticity of substitution:

$$\sigma(\upsilon) = 1 - \frac{\upsilon s^{'}(\upsilon)}{s(\upsilon)}.$$

In addition, let μ be the markup, i.e., the ratio between price and marginal cost. Then,

$$\mu_i = \frac{\sigma_i}{\sigma_i - 1},$$

and we define $\mu\left(\frac{p_\omega^i}{A}\right)$ accordingly. As usual, we assume that $\sigma_i > 1$ so markup is positive.

We now summarize the set of variables related to firms' production. For a firm with productivity z, its price, markup, and quantity in country i are determined implicitly by

$$t_i p_z^i = \frac{\mu_z^i t_i \tau_i}{z}, \quad q_z^i = q_i\left(\frac{t_i p_z^i}{A_i}\right) \equiv q_i\left(\frac{p_z^i}{P_i}\right), \quad \text{and } \mu_z^i = \mu\left(\frac{t_i p_z^i}{A_i}\right) \equiv \mu\left(\frac{p_z^i}{P_i}\right).$$

where $P_i \equiv \frac{A_i}{t_i}$ is the (producer) price aggregator while A_i is the (consumer) price aggregator. $R_i = E_i / t_i$ is the aggregate export revenue in country i. Comparing to the CES case, markup

is no longer constant, so the passthrough of tariff will be incomplete.

The per period variable profits and sales are as follows

$$\pi_z^i = r_z^i \left(1 - \frac{1}{\mu_z^i}\right), \quad r_z^i = p_z^i q_z^i = \mu_z^i l_z^i,$$

as the wage rate is normalized to a constant in various markets. Then, firms export to market i if

$$\pi_z^i \geq f_i$$

Since the profit function is increasing and continuous, there exists a threshold z_i^* such that firms export to country i if and only if $z \geq z_i^*$. In the rest of the paper, we only consider the case in which z_i^* is interior, i.e., $\pi_{z_i^*}^i = f_i$.

Finally, there is an exogenous exit shock ν_i in each country. Firms discount future profits at a rate of ρ. Then, the expected entry profit of country i is defined as in equation (11), which is the same as in the HDIA case.

Entry and Exit This part is the same as in the Kimball case.

Equilibrium This part is the same as in the Kimball case.

Infra-marginal surplus ratio and passthroughs The infra-marginal surplus ratio in the HSA case is defined as:

$$\delta_i = \delta\left(\frac{p_i}{P_i}\right) = 1 + \frac{\int_{\frac{p_i}{P_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta}{s\left(\frac{p_i}{P_i}\right)} > 1,$$

where $P_i = \frac{A_i}{t_i}$ and A_i is the (consumer) price aggregator. δ_i is the ratio of consumer surplus to revenue evaluated at (p_i, q_i). In the case of homogeneous firms where $s\left(\frac{p_i}{P_i}\right) = \frac{1}{N_i}$, we have

$$\delta_i = 1 + \int_{\omega \in \Omega_i} \int_{\frac{p_i}{P_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta d\omega = 1 + C + \log(A_i) - \log\left(P_i^I\right),$$

where we have used equation (61). We define the individual passthrough of a variety as the elasticity of its price to its productivity:

$$\rho = -\frac{\mathrm{d}\log p}{\mathrm{d}\log z} = 1 - \frac{\mathrm{d}\log \mu}{\mathrm{d}\log z},$$

in which the latter equality holds fixing τ_i and t_i. $\rho > 0$ due to the supermodularity of the profit function. Note that

$$\mu_i = 1 + \frac{s(\upsilon)}{-\upsilon s'(\upsilon)}.$$

Therefore, we have

$$\rho_i = \rho\left(\frac{p_i}{P_i}\right) = \frac{1}{1 - \frac{\mu'\left(\frac{p_i}{P_i}\right)\frac{p_i}{P_i}}{\mu\left(\frac{p_i}{P_i}\right)}}.$$

In addition, we continue to assume Assumption 1 holds.

F.2 Homogeneous firms

F.2.1 Solving the equilibrium

We start with the case of homogeneous firms. In such a case, the productivity distribution G degenerates into a single mass point at z. We assume that in equilibrium, all markets have strictly positive measure of exporters.

First, we compute the profit in each market π_i. The HSA preference implies that

$$N_i s\left(\frac{p_i}{P_i}\right) = 1, \tag{62}$$

where $P_i \equiv \frac{A_i}{t_i}$. Note that

$$E_i = D_i^{\eta_i} {P_i^I}^{1-\eta_i}$$

and

$$\log\left(P_i^I\right) = C + \log(P_i) + \log(t_i) - N_i \int_{\frac{p^i}{P_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta. \tag{63}$$

Balanced budget in country i implies

$$E_i = D_i^{\eta_i} P_i^{I\,1-\eta_i} = N_i t_i p_i q_i \Leftrightarrow D_i^{\eta_i} P_i^{I\,1-\eta_i} = N_i t_i \mu \left(\frac{t_i p_i}{A_i}\right) \frac{s\left(\frac{t_i p_i}{A_i}\right) D_i^{\eta_i} P_i^{I\,1-\eta_i}}{t_i p_i} \frac{\tau_i}{z} \Leftrightarrow 1 = \mu \left(\frac{p_i}{P_i}\right) \frac{\frac{\tau_i}{z}}{p_i}. \tag{64}$$

Equations (62), (63) and (64) explicitly determine $\frac{p_i}{P_i}$, P_i^I and p_i as functions of N_i. Therefore, we may write σ_i, η_i, ρ_i, δ_i as functions of N_i. Similarly, we express profit π_i as a function of N_i with the following equation

$$\pi_i(N_i) = \left(1 - \frac{1}{\mu_i}\right) p_i q_i = \left(1 - \frac{1}{\mu_i}\right) \frac{E_i}{N_i t_i}. \tag{65}$$

(11) implies that $\bar{\pi}_i = \frac{\pi_i - f_i}{\rho + \nu_i}$.

Next, we solve for the equilibrium $(M, \{N_i\})$ using the free entry condition and balanced flow of firms. Namely,

$$\sum_{i=0}^{N} (h_i \bar{\pi}_i)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}},$$

$$J_i M = \nu_i N_i.$$

There are $N+2$ equations for $N+2$ unknowns. Noticing that with (10), the above equations imply that

$$J_i = h_i \left(\frac{h_i \bar{\pi}_i}{f_e}\right)^{\frac{1-\beta}{\beta}} = \frac{\nu_i N_i}{M} \quad \Rightarrow \quad \pi_i(N_i) = (\rho + \nu_i) \left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{\beta}{1-\beta}} \frac{f_e}{h_i} + f_i, \tag{66}$$

in which $\pi_i(N_i)$ is given implicitly in (65). Therefore, this equation gives us an implicit function N_i of M. We define such mapping by $N_i = N_i(M)$. Inserting it into the free entry equation, we pin down the equilibrium mass of entrants M as follows:

$$\sum_{i=0}^{N} \left(h_i \bar{\pi}_i(N_i(M))\right)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}}. \tag{67}$$

In this way, we have solved the stationary equilibrium. We can show that a stationary equilibrium exists and is unique under Assumption 1 as in the Kimball case.

F.2.2 Propagation and aggregation equations with homogeneous firms

We expand the equilibrium equations presented in Section F.2 to the first order in the shocks.

Welfare

$$\mathrm{d}\log N_i - \left(\sigma_i\left(\frac{p_i}{P_i}\right) - 1\right)\mathrm{d}\log\frac{p_i}{P_i} = 0. \tag{68}$$

Definition of price aggregator

$$\mathrm{d}\log P_i^I = \mathrm{d}\log P_i + \mathrm{d}\log t_i + N_i s\left(\frac{p_i}{P_i}\right)\mathrm{d}\log\frac{p_i}{P_i} - \mathrm{d}\log N_i\left(\delta_i - 1\right).$$

Note that

$$\frac{\mathrm{d}\log\mu_i}{\mathrm{d}\log\frac{p_i}{P_i}} = -\left(\frac{1}{\rho_i} - 1\right),$$

$$N_i s\left(\frac{p_i}{P_i}\right) = 1,$$

and

$$\frac{\sigma_i - 1}{\frac{p_i}{P_i}} = -\frac{s^{'}(\upsilon)}{s(\upsilon)}$$

Therefore, the above equation can be simplified to

$$\mathrm{d}\log P_i^I = \mathrm{d}\log P_i + \mathrm{d}\log t_i + \mathrm{d}\log\frac{p_i}{P_i} - \mathrm{d}\log N_i\left(\delta_i - 1\right) \Rightarrow \mathrm{d}\log P_i^I = \mathrm{d}\log p_i + \mathrm{d}\log t_i - \mathrm{d}\log N_i\left(\delta_i - 1\right). \tag{69}$$

Markup

$$\mathrm{d}\log\mu_i = -\left(\frac{1}{\rho_i} - 1\right)\mathrm{d}\log\frac{p_i}{P_i}, \tag{70}$$

in which

$$\rho_i = \rho\left(\frac{p_i}{P_i}\right) = \frac{1}{1 - \frac{\mu'\left(\frac{p_i}{P_i}\right)\frac{p_i}{P_i}}{\mu\left(\frac{p_i}{P_i}\right)}}.$$

Pricing Pricing implies that

$$\mathrm{d}\log p_i = \mathrm{d}\log \mu_i = -\left(\frac{1}{\rho_i} - 1\right)\mathrm{d}\log\frac{p_i}{P_i},$$

where the fist part comes from the pricing rule and the second part comes from the definition of the passthrough.

Variable profit

$$\mathrm{d}\log \pi_i = \frac{1}{\mu_i - 1}\mathrm{d}\log \mu_i + (1-\eta_i)\mathrm{d}\log P_i^I - \mathrm{d}\log N_i - \mathrm{d}\log t_i$$

We can simplify the above expression to

$$\mathrm{d}\log \pi_i = -\left(\frac{1}{\rho_i} - 1\right)(\sigma_i - 1)\mathrm{d}\log\left(\frac{p_i}{P_i}\right) - (\eta_i - 1)\left[\mathrm{d}\log p_i + \mathrm{d}\log t_i - \mathrm{d}\log N_i\,(\delta_i - 1)\right] - \mathrm{d}\log N_i - \mathrm{d}\log t_i \tag{71}$$

Balanced flow

$$\frac{\pi_i \mathrm{d}\log \pi_i - f_i \mathrm{d}\log f_i}{\pi_i - f_i} = \mathrm{d}\log(\rho + \nu_i) + \frac{\beta}{1-\beta}\left[\mathrm{d}\log \nu_i + \mathrm{d}\log N_i - \mathrm{d}\log h_i - \mathrm{d}\log M\right] + \mathrm{d}\log f_e - \mathrm{d}\log h_i. \tag{72}$$

Free entry

$$\begin{aligned} &\frac{1}{1-\beta}\sum_{i=0}^{N}\frac{\left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{1}{1-\beta}}}{\sum_k\left(\frac{\nu_k N_k}{h_k M}\right)^{\frac{1}{1-\beta}}}\left[\mathrm{d}\log N_i + \mathrm{d}\log \nu_i - \mathrm{d}\log h_i - \mathrm{d}\log M\right] = 0 \\ \Rightarrow \quad &\sum_{i=0}^{N} s_i\left[\mathrm{d}\log N_i + \mathrm{d}\log \nu_i - \mathrm{d}\log h_i\right] = \mathrm{d}\log M, \end{aligned} \tag{73}$$

where $s_i = \frac{J_i \bar{\pi}_i}{\sum_k J_k \bar{\pi}_k} = \frac{\left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{1}{1-\beta}}}{\sum_k\left(\frac{\nu_k N_k}{h_k M}\right)^{\frac{1}{1-\beta}}}$.

F.2.3 Proof of comparative statics results when firms are homogeneous

In this subsection, we prove that all the results established in Proposition 2, 3 and Corollary 1 are still true when the demand function features HSA. Specifically, the change in M and

that of N_i take similar forms as in the Kimball case and $\mathrm{d}\log N_i$ is still the sufficient statistic for measuring changes in other variables in market i. The signs of changes established in Corollary 1 continue to hold now.

Changes in M and N_i We consider only $\mathrm{d}\log t_i$ and set $\mathrm{d}\log\frac{\tau_i}{z} = \mathrm{d}\log\nu_i = \mathrm{d}\log h_i = \mathrm{d}\log f_i = \mathrm{d}\log f_e = 0$. Then, we obtain the following system of equations

$$\begin{aligned}
\mathrm{d}\log N_i =& \left(\sigma_i\left(\frac{p_i}{P_i}\right)-1\right)\mathrm{d}\log\frac{p_i}{P_i}\\
\mathrm{d}\log P_i^I =& \mathrm{d}\log P_i + \mathrm{d}\log t_i + \mathrm{d}\log\frac{p_i}{P_i} - \mathrm{d}\log N_i\,(\delta_i-1)\\
\mathrm{d}\log\pi_i =& -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log N_i - (\eta_i-1)\left[\mathrm{d}\log p_i + \mathrm{d}\log t_i - \mathrm{d}\log N_i\,(\delta_i-1)\right]\\
& - \mathrm{d}\log N_i - \mathrm{d}\log t_i\\
\mathrm{d}\log p_i =& \mathrm{d}\log\mu_i = -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i}\\
\frac{\pi_i}{\pi_i-f_i}\mathrm{d}\log\pi_i =& \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M\right]\\
\sum_{i=0}^{N} s_i\mathrm{d}\log N_i =& \mathrm{d}\log M
\end{aligned}$$

Combining the first, third and fourth equations, we have

$$\mathrm{d}\log\pi_i = -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log N_i - (\eta_i-1)\left[-\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i} + \mathrm{d}\log t_i - \mathrm{d}\log N_i\,(\delta_i-1)\right] - \mathrm{d}\log N_i - \mathrm{d}\log t_i$$

or

$$\mathrm{d}\log\pi_i = -\left(\frac{1}{\rho_i}-1\right)\frac{\sigma_i-\eta_i}{\sigma_i-1}\mathrm{d}\log N_i - \left[1-(\eta_i-1)(\delta_i-1)\right]\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i,$$

where $1-(\eta_i-1)(\delta_i-1) > 1-(\eta_i-1)\frac{1}{\eta_i-1} = 0$. Therefore, we have

$$\mathrm{d}\log\pi_i = -\theta_i\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i, \tag{74}$$

where

$$\theta_i \equiv \left(\frac{1}{\rho_i}-1\right)\frac{\sigma_i-\eta_i}{\sigma_i-1} + \left[1-(\eta_i-1)(\delta_i-1)\right] > 0.$$

Then, we obtain that

$$\frac{\pi_i}{\pi_i - f_i}\left[-\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i\right] = \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M\right]$$
$$\Rightarrow \mathrm{d}\log N_i = \frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\theta_i}\mathrm{d}\log M - \frac{\eta_i \frac{\pi_i}{\pi_i - f_i}}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\theta_i}\mathrm{d}\log t_i$$

Using the free entry condition,

$$\mathrm{d}\log N_i - \mathrm{d}\log M = -\xi_i \mathrm{d}\log M - \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i,$$

where

$$\xi_i = \frac{\frac{\pi_i}{\pi_i - f_i}\theta_i}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\theta_i} \leq 1,$$

Then, we have

$$\sum_{i=0}^{N} s_i \mathrm{d}\log N_i = \mathrm{d}\log M \Rightarrow \mathrm{d}\log M = -\frac{\sum_{i=0}^{N} s_i \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i}{\sum_{i=0}^{N} s_i \xi_i} = -\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right].$$

Hence, we can re-write $\mathrm{d}\log N_i$ as

$$\begin{aligned}\mathrm{d}\log N_i &= (1-\xi_i)\mathrm{d}\log M - \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i \\ &= -\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right] + \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right]\end{aligned}$$

Therefore, both the total number of entrants and the number of active firms in each market decline.

Changes in other variables

Firm-level profit π_i Similarly, we have

$$\begin{aligned}\mathrm{d}\log \pi_i &= -\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i, \\ &= \theta_i\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right] + \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right] - \eta_i \mathrm{d}\log t_i \\ &= \theta_i(1-\xi_i)\left\{\mathbb{E}_{s\xi}\left[\frac{\eta \mathrm{d}\log t}{\theta}\right] - \frac{\eta_i \mathrm{d}\log t_i}{\theta_i}\right\}\end{aligned}$$

Therefore, firm-level profit drops in the focal market and *increases* in the third market.

Relative price $\frac{p_i}{P_i}$ The relative price has the following expression:

$$\mathrm{d}\log\frac{p_i}{P_i}=\frac{\mathrm{d}\log N_i}{\sigma_i-1},$$

which declines in both the focal and the third markets (as the number of firms decreases). Moreover, $\frac{p_i}{A_i}$ decreases in every market, since $A_i=P_it_i$. Of course, this is consistent with the increasing market share $s(\frac{p_i}{A_i})$ of surviving firms in both the focal and third markets.

Firm-level price p_i Let us look at the mark-up pricing rule:

$$p_i=\mu_i\frac{\tau_i}{z},$$

where $\mu_i\equiv\frac{\sigma_i}{\sigma_i-1}$ and

$$\sigma_i=1-\frac{\upsilon s^{'}(\upsilon)}{s(\upsilon)}$$

We assume that the demand elasticity (or markup) increases (or decreases) with the relative price $\frac{p_i}{P_i}$. Since we have

$$\mathrm{d}\log\frac{p_i}{P_i}=\frac{\mathrm{d}\log N_i}{\sigma_i-1}<0,$$

and

$$\mathrm{d}\log p_i=\mathrm{d}\log\mu_i=-\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i}=-\left(\frac{1}{\rho_i}-1\right)\frac{\mathrm{d}\log N_i}{\sigma_i-1},$$

we must have

$$\mathrm{d}\log p_i>0.$$

Price aggregator A_i, the ideal price index P_i^I Based on the discussion in the previous step, we must have

$$\mathrm{d}\log\frac{p_i}{P_i}=\frac{\mathrm{d}\log N_i}{\sigma_i-1}<0,\ \ \mathrm{d}\log p_i=\mathrm{d}\log\mu_i>0\Rightarrow\mathrm{d}\log P_i>\mathrm{d}\log p_i>0,\ \ \mathrm{d}\log A_i=\mathrm{d}\log P_i+\mathrm{d}\log t_i>0,$$

where $\mathrm{d}\log t_i = 0$ in third markets. Therefore, we must have that the price aggregator increases in all markets. Moreover, we have

$$\mathrm{d}\log P_i^I = \mathrm{d}\log p_i + \mathrm{d}\log t_i - \mathrm{d}\log N_i\left(\delta_i - 1\right) = -\left(\frac{1}{\rho_i} - 1\right)\frac{\mathrm{d}\log N_i}{\sigma_i - 1} + \mathrm{d}\log t_i - \mathrm{d}\log N_i\left(\delta_i - 1\right).$$

Since $\mathrm{d}\log t_i \geq 0$ and $\mathrm{d}\log N_i < 0$ and $\mathrm{d}\log p_i > 0$ for all markets. The ideal price index must increase in all markets.

Total export value E_i and real consumption Q_i Since the ideal price index must increase in all markets, the real consumption $Q_i = D_i^{\eta_i} {P_i^I}^{-\eta_i}$ and total expenditure on Chinese goods $E_i = Q_i P_i^I = D_i^{\eta_i} {P_i^I}^{1-\eta_i}$ drop in all markets. Specifically, we have

$$\mathrm{d}\log Q_i = -\eta_i \mathrm{d}\log P_i^I = \eta_i\left[\left(\frac{1}{\rho_i} - 1\right) + \frac{\mathrm{d}\log N_i}{\sigma_i - 1}\mathrm{d}\log N_i\left(\delta_i - 1\right) - \mathrm{d}\log t_i\right] < 0,$$

and

$$\mathrm{d}\log R_i = (1-\eta_i)\mathrm{d}\log P_i^I = (\eta_i - 1)\left[\left(\frac{1}{\rho_i} - 1\right) + \frac{\mathrm{d}\log N_i}{\sigma_i - 1}\mathrm{d}\log N_i\left(\delta_i - 1\right) - \mathrm{d}\log t_i\right] < 0.$$

Note that $\mathrm{d}\log t_i = 0$ in third markets.

Firm-level output q_i Recall that average firm output equals

$$q_i = \frac{s\left(\frac{p_i}{P_i}\right) D^{\eta_i} {P_i^I}^{1-\eta_i}}{t_i p_i}.$$

Therefore, we must have

$$\mathrm{d}\log q_i = \mathrm{d}\log s_i + (1-\eta_i)\mathrm{d}\log P_i^I - \mathrm{d}\log t_i - \mathrm{d}\log p_i,$$

which is equivalent to

$$\mathrm{d}\log q_i = -\left[1 - (\eta_i - 1)(\delta_i - 1)\right]\mathrm{d}\log N_i - \eta_i\left(\mathrm{d}\log p_i + \mathrm{d}\ \log t_i\right),$$

where $1-(\eta_i-1)(\delta_i-1)>0$. Substituting $\mathrm{d}\log p_i = -\left(\frac{1}{\rho_i}-1\right)\frac{\mathrm{d}\log N_i}{\sigma_i-1}$ into the above equation, we have

$$\mathrm{d}\log q_i = -\left[1-(\eta_i-1)(\delta_i-1)-\frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i,$$

which leads to

$$\mathrm{d}\log q_i = \left[1-(\eta_i-1)(\delta_i-1)-\frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right]+\xi_i\frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right]-\eta_i\mathrm{d}\log t_i \tag{75}$$

where the first term reflects the GE effect of the change in the number of firms on firm output (which can be positive or negative) while the second term captures the direct effect coming from the tariff change.

There are two cases to consider. First, when ρ_i is very small (i.e., close to zero), output level drops in all markets as $\mathrm{d}\log t_i \geq 0$ in all markets and strictly positive in one market. $\rho \approx 0$ implies that there are substantial increases in *the price level* p_i in all markets (due to the GE feedback from $\mathrm{d}\log N_i$) after a market-specific tariff shock. As a result, output q_i drops in all markets.

Next, when ρ_i is close to one (meaning that $1-(\eta_i-1)(\delta_i-1)-\frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)>0$), output level increases in the third market unambiguously. In such a case, we can write out $\mathrm{d}\log q_j$ in the focal market as

$$\begin{aligned}
& \mathrm{d}\log q_j \\
= & \left[1-(\eta_j-1)(\delta_j-1)-\frac{\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)\right] \\
& \left[(1-\xi_j)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right]+\frac{\xi_j\eta_j}{1-(\eta_j-1)(\delta_j-1)+\frac{\sigma_j-\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}\mathrm{d}\log t_j\right]-\eta_j\mathrm{d}\log t_j \\
< & \frac{1-(\eta_j-1)(\delta_j-1)-\frac{\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}{1-(\eta_j-1)(\delta_j-1)+\frac{\sigma_j-\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}\eta_j\mathrm{d}\log t_j-\eta_j\mathrm{d}\log t_j \\
< & 0,
\end{aligned}$$

as $\mathrm{d}\log t_j > 0$. This implies that quantity sold in the focal market goes down in all possible cases. When $\rho \approx 1$, there is almost no price (and markup) change after the tariff shock. Moreover, the market share per firm in the third market increases, and there is no tariff

shock. Therefore, output has to go up in the third market. For the focal market, the impact is negative, as there is a direct negative impact on the output coming from the tariff shock.

Firm-level revenue r_i Based on the above derivation, we can write out the change in firm-level (tariff-exclusive) revenue as

$$\mathrm{d}\log r_i = \mathrm{d}\log p_i + \mathrm{d}\log q_i = -\left[1-(\eta_i-1)(\delta_i-1)\right]\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i + (1-\eta_i)\mathrm{d}\log p_i,$$

where $1-(\eta_i-1)(\delta_i-1)>0$. Since

$$\mathrm{d}\log p_i = \mathrm{d}\log \mu_i = -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i} = -\left(\frac{1}{\rho_i}-1\right)\frac{\mathrm{d}\log N_i}{\sigma_i-1},$$

we have

$$\mathrm{d}\log r_i = \left[1-(\eta_i-1)(\delta_i-1)-\frac{\eta_i-1}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right]+\xi_i\frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right]-\eta_i\mathrm{d}\log t_i \tag{76}$$

Equation (76) resembles equation (75). Therefore, a similar argument applies here. There are two cases to consider. First, when ρ_i is very small (i.e., close to zero), output level drops in all markets as $\mathrm{d}\log t_i \geq 0$ in all markets and strictly positive in one market. $\rho \approx 0$ implies that there are substantial increases in *the price level* p_i in all markets (due to the GE feedback from $\mathrm{d}\log N_i$) after a market-specific tariff shock. As a result, both output q_i and revenue $p_i q_i$ drop in all markets (as the price elasticity is always bigger than unity).

Next, when ρ_i is close to one (meaning that $1-(\eta_i-1)(\delta_i-1)-\frac{\eta_i-1}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)>0$), output level increases in the third market unambiguously. In such a case, we can write out $\mathrm{d}\log q_j$ in the focal market as

$$\begin{aligned}
& \mathrm{d}\log r_j \\
= & \left[1-(\eta_j-1)(\delta_j-1)-\frac{\eta_j-1}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)\right] \\
& \left[(1-\xi_j)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right]+\frac{\xi_j\eta_j}{1-(\eta_j-1)(\delta_j-1)+\frac{\sigma_j-\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}\mathrm{d}\log t_j\right]-\eta_j\mathrm{d}\log t_j \\
< & \frac{1-(\eta_j-1)(\delta_j-1)+\frac{1-\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}{1-(\eta_j-1)(\delta_j-1)+\frac{\sigma_j-\eta_j}{\sigma_j-1}\left(\frac{1}{\rho_j}-1\right)}\eta_j\mathrm{d}\log t_j-\eta_j\mathrm{d}\log t_j \\
< & 0,
\end{aligned}$$

as $\theta_i \equiv \left(\frac{1}{\rho_i}-1\right)\frac{\sigma_i-\eta_i}{\sigma_i-1}+[1-(\eta_i-1)(\delta_i-1)]>0$ and $\mathrm{d}\log t_j>0$. This implies export revenue in the focal market goes down in all possible cases. When $\rho\approx 1$, there is almost no price (and markup) change after the tariff shock. Moreover, the market share per firm in the third market increases, and there is no tariff shock. Therefore, both output and revenue go up in the third market. For the focal market, the impact is negative, as there is a direct negative impact on revenue coming from the tariff shock.

Total export quantity $\tilde{Q}_i$ Total (physical) quantity of output across firms is $\tilde{Q}_i = N_i q_i$. Thus, we must have

$$\mathrm{d}\log\tilde{Q}_i = \mathrm{d}\log N_i + \mathrm{d}\log q_i,$$

which

$$\mathrm{d}\log\tilde{Q}_i = \mathrm{d}\log N_i \mathrm{d}\log q_i = \left[(\eta_i-1)(\delta_i-1)+\frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i < 0, \tag{77}$$

as $\mathrm{d}\log N_i<0$ and $\mathrm{d}\log t_i\geq 0$ in all markets. In summary, the action at the extensive margin drive the change in aggregated export quantity/value (even if firm-level export value increases).

F.3 Tariff uncertainty

We study the effect of expected tariffs in third markets in this subsection of the appendix. In an uncertainty world, the objective function of the firm can be written as

$$\max_{p^i_\omega}\mathbb{E}\left[D_i^{\eta_i}{P_i^I}^{1-\eta_i}s\left(\frac{t_i p^i_\omega}{A_i}\right)\frac{p^i_\omega-\frac{\tau_i}{z}}{t_i p^i_\omega}\right], \tag{78}$$

where p^i_ω is the FOB price and $t_i p^i_\omega$ is the CIF price. Note that $\frac{D_i^{\eta_i}{P_i^I}^{1-\eta_i}s\left(\frac{t_i p^i_\omega}{A_i}\right)}{t_i p^i_\omega}$ is the output supplied by firm ω. Since $P_i=\frac{A_i}{t_i}$ and A_i is the (consumer) price aggregator, we can transform the above optimization problem into

$$\max_{p^i_\omega}\mathbb{E}\left[\frac{D_i^{\eta_i}{P_i^I}^{1-\eta_i}}{t_i}s\left(\frac{p^i_\omega}{P_i}\right)\frac{p^i_\omega-\frac{\tau_i}{z}}{p^i_\omega}\right], \tag{79}$$

The key things are p^i_ω, P, N_i and M are all determined before the actual tariff is realized. As a result, $s(.)$ (which equals $\frac{1}{N_i}$) and $s^{'}(.)$ are not random variables. The FOC is

$$\mathbb{E}\left[\frac{D_i^{\eta_i} P_i^{I^{1-\eta_i}}}{t_i}\right]\left[s^{'}\left(\frac{p^i_\omega}{P_i}\right)\frac{1}{P_i}\frac{p^i_\omega - \frac{\tau_i}{z}}{p^i_\omega} + s\left(\frac{p^i_\omega}{P_i}\right)\frac{\frac{\tau_i}{z}}{\left(p^i_\omega\right)^2}\right] = 0, \tag{80}$$

which is equivalent to

$$\mathbb{E}\left[s^{'}\left(\frac{p^i_\omega}{P_i}\right)\frac{p^i_\omega}{P_i}\left(p^i_\omega - \frac{\tau_i}{z}\right) + s\left(\frac{p^i_\omega}{P_i}\right)\frac{\tau_i}{z}\right] = 0,$$

leads to the optimal pricing rule as

$$p^i_\omega = \mathbb{E}\left[\frac{\sigma(\upsilon)}{\sigma(\upsilon)-1}\right]\frac{\tau}{z}$$

where $\upsilon \equiv \frac{p^i_\omega}{P_i}$. Note that we have used the definition of the elasticity of substitution:

$$\sigma(\upsilon) = 1 - \frac{\upsilon s^{'}(\upsilon)}{s(\upsilon)}. \tag{81}$$

In addition, let μ be the markup, i.e., the ratio between price and marginal cost. Then,

$$\mu_i = \frac{\sigma_i}{\sigma_i - 1},$$

and we can define $\mu\left(\frac{p^i_\omega}{P_i}\right)$ accordingly. As usual, we assume that $\sigma_i > 1$ so markup is positive.

F.3.1 Equilibrium Conditions

$$N_i s\left(\frac{p_i}{P_i}\right) = 1, \tag{82}$$

where $P_i \equiv \frac{A_i}{t_i}$. Note that

$$E_i = D_i^{\eta_i} P_i^{I^{1-\eta_i}}$$

and

$$\log\left(P_i^I\right) = C + \log(P_i) + \log(t_i) - N_i \int_{\frac{p_i}{P_i}}^{\infty} \frac{s(\zeta)}{\zeta} d\zeta. \tag{83}$$

Balanced budget in country i implies

$$E_i = D_i^{\eta_i} P_i^{I^{1-\eta_i}} = N_i t_i p_i q_i \Leftrightarrow D_i^{\eta_i} P_i^{I^{1-\eta_i}} = N_i t_i \mu \left(\frac{p_i}{P_i}\right) \frac{s\left(\frac{p_i}{P_i}\right) D_i^{\eta_i} P_i^{I^{1-\eta_i}}}{t_i p_i} \frac{\tau_i}{z} \Leftrightarrow 1 = \mu \left(\frac{p_i}{P_i}\right) \frac{\frac{\tau_i}{z}}{p_i}. \tag{84}$$

Note that the above condition is about the realized national income and thus does not have an expectation operator. Also note that the markup in firm's mind is the same as the realized markup as

$$\sigma\left(s\left(\frac{t_i p_i}{A_i}\right)\right) = \sigma\left(s\left(\frac{p_i}{P_i}\right)\right).$$

Equations (82), (83) and (84) explicitly determine $\frac{p_i}{P_i}$, P_i^I and P_i as functions of N_i. Therefore, we may write σ_i, η_i, ρ_i, δ_i as functions of N_i. Similarly, we can write the profit function, $\mathbb{E}\,\pi_i$, as a function of N_i with the following equation:

$$\mathbb{E}\,\pi_i(N_i) = \left(1 - \frac{1}{\mu_i}\right) p_i q_i = \left(1 - \frac{1}{\mu_i}\right) \frac{1}{N_i} \mathbb{E}\left(\frac{E_i}{t_i}\right) = \left(1 - \frac{1}{\mu_i}\right) \frac{1}{N_i} \mathbb{E}\left(\frac{D_i^{\eta_i} P_i^{I^{1-\eta_i}}}{t_i}\right). \tag{85}$$

The key difference under tariff uncertainty is that the firm-level profit and revenue now have the expectation operator, as the firm tries to forecast the realized tariff and thus the overall demand for exports from Chines. However, since both the firm-level price and the number of firms are determined before the realization of the ex post tariff, $\frac{p_i}{P_i}$ and thus the markup rate are also determined before the tariff. Equation (11) implies that $\bar{\pi}_i = \frac{\mathbb{E}\,\pi_i - f_i}{\rho + \nu_i}$. Note that $\pi_i(N_i)$ here is the profit in firm's mind. I.e., even when the realized tariff in the third country is zero, the expected tariff in the firm's mind is still positive.

Next, we solve for the equilibrium $(M, \{N_i\})$ using the free entry condition and balanced flow of firms. Namely,

$$\sum_{i=0}^{N} (h_i \bar{\pi}_i)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}},$$

$$J_i M = \nu_i N_i.$$

There are $N+1$ equations for $N+1$ unknowns. Noticing that with (10), the above equations

imply that

$$J_i = h_i \left(\frac{h_i \bar{\pi}_i}{f_e} \right)^{\frac{1-\beta}{\beta}} = \frac{\nu_i N_i}{M} \quad \Rightarrow \quad \pi_i(N_i) = (\rho + \nu_i) \left(\frac{\nu_i N_i}{h_i M} \right)^{\frac{\beta}{1-\beta}} \frac{f_e}{h_i} + f_i,$$

in which $\pi_i(N_i)$ is given implicitly in (65). Therefore, this equation gives us an implicit function N_i of M. We define such mapping by $N_i = N_i(M)$. Inserting it into the free entry equation, we pin down the equilibrium mass of entrants M as follows:

$$\sum_{i=0}^{N} \left(h_i \bar{\pi}_i (N_i(M)) \right)^{\frac{1}{\beta}} = f_e^{\frac{1}{\beta}}.$$

F.3.2 Propagation and aggregation equations with homogeneous firms and tariff uncertainty

We expand the equilibrium equations presented in previous subsection to the first order in the shocks.

Welfare

$$\mathrm{d} \log N_i - \left(\sigma_i \left(\frac{p_i}{P_i} \right) - 1 \right) \mathrm{d} \log \frac{p_i}{P_i} = 0. \tag{86}$$

Definition of price aggregator

$$\mathrm{d} \log P_i^I = \mathrm{d} \log P_i + \mathrm{d} \log t_i + N_i s \left(\frac{p_i}{P_i} \right) \mathrm{d} \log \frac{p_i}{P_i} - \mathrm{d} \log N_i \left(\delta_i - 1 \right) \tag{87}$$

Note that

$$\frac{\mathrm{d} \log \mu_i}{\mathrm{d} \log \frac{p_i}{P_i}} = - \left(\frac{1}{\rho_i} - 1 \right),$$

$$N_i s \left(\frac{p_i}{P_i} \right) = 1,$$

and

$$\frac{\sigma_i - 1}{\frac{p_i}{P_i}} = - \frac{s^{'}(\upsilon)}{s(\upsilon)}$$

Therefore, the above equation can be simplified to

$$\mathrm{d}\log P_i^I = \mathrm{d}\log P_i + \mathrm{d}\log t_i + \mathrm{d}\log\frac{p_i}{P_i} - \mathrm{d}\log N_i\left(\delta_i - 1\right) \Rightarrow \mathrm{d}\log P_i^I = \mathrm{d}\log p_i + \mathrm{d}\log t_i - \mathrm{d}\log N_i\left(\delta_i - 1\right). \tag{88}$$

Note that $\mathrm{d}\log t_i = 0$ in the third country, as the realized tariff is zero in the third country.

Markup

$$\mathrm{d}\log\mu_i = -\left(\frac{1}{\rho_i} - 1\right)\mathrm{d}\log\frac{p_i}{P_i}, \tag{89}$$

in which

$$\rho_i = \rho\left(\frac{p_i}{P_i}\right) = \frac{1}{1 - \frac{\mu'\left(\frac{p_i}{P_i}\right)\frac{p_i}{P_i}}{\mu\left(\frac{p_i}{P_i}\right)}}.$$

Pricing Pricing implies that

$$\mathrm{d}\log p_i = \mathrm{d}\log\mu_i = -\left(\frac{1}{\rho_i} - 1\right)\mathrm{d}\log\frac{p_i}{P_i},$$

where the fist part comes from the pricing rule and the second part comes from the definition of the passthrough.

Variable profit

$$\mathbb{E}\,\mathrm{d}\log\pi_i = \frac{1}{\mu_i - 1}\mathrm{d}\log\mu_i + \mathbb{E}(1-\eta_i)\mathrm{d}\log P_i^I - \mathbb{E}\,\mathrm{d}\log t_i - \mathrm{d}\log N_i$$

We can simplify the above expression to

$$\begin{aligned}\mathbb{E}\,\mathrm{d}\log\pi_i = &-\left(\frac{1}{\rho_i} - 1\right)(\sigma_i - 1)\mathrm{d}\log\left(\frac{p_i}{P_i}\right)\\ &-(\eta_i - 1)\left[\mathrm{d}\log p_i + \mathbb{E}\,\mathrm{d}\log t_i - \mathrm{d}\log N_i\left(\delta_i - 1\right)\right] - \mathrm{d}\log N_i - \mathbb{E}\,\mathrm{d}\log t_i.\end{aligned} \tag{90}$$

Note that although the realized tariff is zero in the third market, the expected tariff (in firm's mind) is positive in the third market (i.e., $\mathbb{E}\,\mathrm{d}\log t_i > 0$). In turn, the tariff variation (on expectation) also leads to different changes in δ_i and P_i^I which are forecasted by the rational

firm (in various scenarios).

Balanced flow

$$\frac{\pi_i \mathrm{d}\log\pi_i - f_i \mathrm{d}\log f_i}{\pi_i - f_i} = \mathrm{d}\log(\rho+\nu_i) + \frac{\beta}{1-\beta}\left[\mathrm{d}\log\nu_i + \mathrm{d}\log N_i - \mathrm{d}\log h_i - \mathrm{d}\log M\right] + \mathrm{d}\log f_e - \mathrm{d}\log h_i. \tag{91}$$

Free entry

$$\frac{1}{1-\beta}\sum_{i=0}^{N}\frac{\left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{1}{1-\beta}}}{\sum_k\left(\frac{\nu_k N_k}{h_k M}\right)^{\frac{1}{1-\beta}}}\left[\mathrm{d}\log N_i + \mathrm{d}\log\nu_i - \mathrm{d}\log h_i - \mathrm{d}\log M\right] = 0$$

$$\Rightarrow \quad \sum_{i=0}^{N} s_i\left[\mathrm{d}\log N_i + \mathrm{d}\log\nu_i - \mathrm{d}\log h_i\right] = \mathrm{d}\log M, \tag{92}$$

in which $s_i = \frac{J_i\bar{\pi}_i}{\sum_k J_k\bar{\pi}_k} = \frac{\left(\frac{\nu_i N_i}{h_i M}\right)^{\frac{1}{1-\beta}}}{\sum_k\left(\frac{\nu_k N_k}{h_k M}\right)^{\frac{1}{1-\beta}}}$.

F.3.3 Proof of Comparative Statics Results in the Homogeneous Firms Case

Changes in M and N_i We consider only $\mathrm{d}\log t_i$ and set $\mathrm{d}\log\frac{\tau_i}{z} = \mathrm{d}\log\nu_i = \mathrm{d}\log h_i = \mathrm{d}\log f_i = \mathrm{d}\log f_e = 0$. In the focal market, we obtain the following system of equations

$$\begin{aligned}
\mathrm{d}\log N_i =& \left(\sigma_i\left(\frac{p_i}{P_i}\right) - 1\right)\mathrm{d}\log\frac{p_i}{P_i}\\
\mathrm{d}\log P_i^I =& \mathrm{d}\log P_i + \mathrm{d}\log t_i + \mathrm{d}\log\frac{p_i}{P_i} - \mathrm{d}\log N_i\left(\delta_i - 1\right)\\
\mathrm{d}\log\pi_i =& -\left(\frac{1}{\rho_i} - 1\right)\mathrm{d}\log N_i - (\eta_i - 1)\Big[\mathrm{d}\log p_i + \mathrm{d}\log t_i\\
& - \mathrm{d}\log N_i\left(\delta_i - 1\right)\Big] - \mathrm{d}\log N_i - \mathrm{d}\log t_i\\
\mathrm{d}\log p_i =& \mathrm{d}\log\mu_i = -\left(\frac{1}{\rho_i} - 1\right)\mathrm{d}\log\frac{p_i}{P_i}\\
\frac{\pi_i}{\pi_i - f_i}\mathrm{d}\log\pi_i =& \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i - \mathrm{d}\log M\right]\\
\sum_{i=0}^{N} s_i\mathrm{d}\log N_i =& \mathrm{d}\log M.
\end{aligned}$$

In the third market, we have

$$\begin{aligned}
\mathrm{d}\log N_i =& \left(\sigma_i\left(\frac{p_i}{P_i}\right)-1\right)\mathrm{d}\log\frac{p_i}{P_i}\\
\mathrm{d}\log P_i^I =& \mathrm{d}\log P_i+\mathrm{d}\log\frac{p_i}{P_i}-\mathrm{d}\log N_i\left(\delta_i-1\right)\\
\mathbb{E}\,\mathrm{d}\log\pi_i =& -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log N_i-\left(\eta_i-1\right)\Big[\mathrm{d}\log p_i+\mathbb{E}\,\mathrm{d}\log t_i\\
&-\mathrm{d}\log N_i\left(\delta_i-1\right)\Big]-\mathrm{d}\log N_i-\mathbb{E}\,\mathrm{d}\log t_i\\
\mathrm{d}\log p_i =& \mathrm{d}\log\mu_i=-\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i}\\
\frac{\pi_i}{\pi_i-f_i}\,\mathbb{E}\,\mathrm{d}\log\pi_i =& \frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i-\mathrm{d}\log M\right]\\
\sum_{i=0}^{N}s_i\mathrm{d}\log N_i =& \mathrm{d}\log M.
\end{aligned}$$

Note that the second and third equations are changed from the case without the tariff uncertainty, as there is no realize tariff in the third market and the profit that determined entry is the profit in firm's mind. In the focal market, we can combine the first, third and fourth equations to obtain

$$\begin{aligned}
\mathrm{d}\log\pi_i &= -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log N_i-\left(\eta_i-1\right)\left[-\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i}+\mathrm{d}\log t_i-\mathrm{d}\log N_i\left(\delta_i-1\right)\right]\\
&\quad -\mathrm{d}\log N_i-\mathrm{d}\log t_i\\
&= -\left(\frac{1}{\rho_i}-1\right)\frac{\sigma_i-\eta_i}{\sigma_i-1}\mathrm{d}\log N_i-\left[1-(\eta_i-1)(\delta_i-1)\right]\mathrm{d}\log N_i-\eta_i\mathrm{d}\log t_i,
\end{aligned}$$

where $1-(\eta_i-1)(\delta_i-1)>1-(\eta_i-1)\frac{1}{\eta_i-1}=0$. Therefore, we have

$$\mathrm{d}\log\pi_i=-\theta_i\mathrm{d}\log N_i-\eta_i\mathrm{d}\log t_i, \tag{93}$$

where

$$\theta_i\equiv\left(\frac{1}{\rho_i}-1\right)\frac{\sigma_i-\eta_i}{\sigma_i-1}+\left[1-(\eta_i-1)(\delta_i-1)\right]>0.$$

In the third market where the realized tariff is zero, we have

$$\begin{aligned}
& \mathbb{E}\,\mathrm{d}\log\pi_i \\
= & -\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log N_i-(\eta_i-1)\left[-\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i}+\mathbb{E}\,\mathrm{d}\log t_i-\mathrm{d}\log N_i\,(\delta_i-1)\right] \\
& -\mathrm{d}\log N_i-\mathbb{E}\,\mathrm{d}\log t_i \\
= & -\left(\frac{1}{\rho_i}-1\right)\frac{\sigma_i-\eta_i}{\sigma_i-1}\mathrm{d}\log N_i-[1-(\eta_i-1)(\delta_i-1)]\,\mathrm{d}\log N_i-\eta_i\,\mathbb{E}\,\mathrm{d}\log t_i \\
= & -\theta_i\mathrm{d}\log N_i-\eta_i\,\mathbb{E}\,\mathrm{d}\log t_i,
\end{aligned}$$

where the expected tariff in the third market, $\mathbb{E}\,\mathrm{d}\log t_i>0$, negatively affects the expected profit obtained from the third market (as the overall demand for Chinese exports drops under a positively expected tariff). Then, we obtain that

$$\frac{\pi_i}{\pi_i-f_i}\left[-\theta_i\mathrm{d}\log N_i-\eta_i\mathrm{d}\log t_i\right]=\frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i-\mathrm{d}\log M\right]$$

for the focal market and

$$\frac{\pi_i}{\pi_i-f_i}\left[-\theta_i\mathrm{d}\log N_i-\eta_i\,\mathbb{E}\,\mathrm{d}\log t_i\right]=\frac{\beta}{1-\beta}\left[\mathrm{d}\log N_i-\mathrm{d}\log M\right]$$

for the third market. Note that since we use the initial values as the weights to implement the first-order expansion, There is no expectation operator in front of π_i. Consequently, we have

$$\mathrm{d}\log N_i=\frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta}+\frac{\pi_i}{\pi_i-f_i}\theta_i}\mathrm{d}\log M-\frac{\eta_i\frac{\pi_i}{\pi_i-f_i}}{\frac{\beta}{1-\beta}+\frac{\pi_i}{\pi_i-f_i}\theta_i}\mathrm{d}\log t_i$$

for the focal market and

$$\mathrm{d}\log N_i=\frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta}+\frac{\pi_i}{\pi_i-f_i}\theta_i}\mathrm{d}\log M-\frac{\eta_i\frac{\pi_i}{\pi_i-f_i}}{\frac{\beta}{1-\beta}+\frac{\pi_i}{\pi_i-f_i}\tilde{\theta}_i}\,\mathbb{E}\,\mathrm{d}\log t_i$$

for the third market. Using the free entry condition, we have

$$\mathrm{d}\log N_j-\mathrm{d}\log M=-\xi_j\mathrm{d}\log M-\xi_j\frac{\eta_j}{\theta_j}\mathrm{d}\log t_j,$$

for the market and

$$\xi_j=\frac{\frac{\pi_j}{\pi_j-f_j}\theta_j}{\frac{\beta}{1-\beta}+\frac{\pi_j}{\pi_j-f_j}\theta_j}\leq 1.$$

For the third market, we have

$$\mathrm{d}\log N_i - \mathrm{d}\log M = -\xi_i \mathrm{d}\log M - \xi_i \frac{\eta_i}{\theta_i}\,\mathbb{E}\,\mathrm{d}\log t_i,$$

where

$$\xi_i = \frac{\frac{\pi_i}{\pi_i - f_i}\theta_i}{\frac{\beta}{1-\beta} + \frac{\pi_i}{\pi_i - f_i}\theta_i} \leq 1.$$

In total, we have

$$\sum_{i=0}^{N} s_i \mathrm{d}\log N_i = \mathrm{d}\log M \Rightarrow \mathrm{d}\log M = -\frac{\sum_{i=0}^{N} s_i \xi_i \frac{\eta_i}{\theta_i} \mathrm{d}\log t_i}{\sum_{i=0}^{N} s_i \xi_i} = -\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right],$$

where the tariffs here include both the realized tariff in the focal market and the expected tariffs in the third market. Hence, we can re-write $\mathrm{d}\log N_i$ as

$$\begin{aligned}
\mathrm{d}\log N_j &= (1-\xi_j)\mathrm{d}\log M - \xi_j \frac{\eta_j}{\theta_j}\mathrm{d}\log t_j \\
&= -\left[(1-\xi_j)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right] + \xi_j \frac{\eta_j}{\theta_j}\mathrm{d}\log t_j\right]
\end{aligned}$$

for the focal market and

$$\begin{aligned}
\mathrm{d}\log N_i &= (1-\xi_i)\mathrm{d}\log M - \xi_i \frac{\eta_i}{\theta_i}\,\mathbb{E}\left(\mathrm{d}\log t_i\right) \\
&= -\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right] + \xi_i \frac{\eta_i}{\theta_i}\,\mathbb{E}\left(\mathrm{d}\log t_i\right)\right]
\end{aligned}$$

for the third market. Therefore, both the total number of entrants and the number of active firms in each market decline. Importantly, we can see that even an expected tariff (without realization) in the third market can lead to a drop in the number of firms in all market.

F.3.4 Changes in other variables

Firm-level profit π_i Similarly, we have

$$\begin{aligned}\mathrm{d}\log\pi_i &= -\theta_i \mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i, \\ &= \theta_i \left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right] + \xi_i \frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right] - \eta_i \mathrm{d}\log t_i \\ &= \theta_i(1-\xi_i)\left\{\mathbb{E}_{s\xi}\left[\frac{\eta \mathrm{d}\log t}{\theta}\right] - \frac{\eta_i \mathrm{d}\log t_i}{\theta_i}\right\}\end{aligned}$$

For the focal market, we have

$$\mathrm{d}\log\pi_j = \theta_j(1-\xi_j)\left\{\mathbb{E}_{s\xi}\left[\frac{\eta \mathrm{d}\log t}{\theta}\right] - \frac{\eta_i \mathrm{d}\log t_j}{\theta_j}\right\} < 0,$$

as long as the increase in the expected tariff in all third markets is small than the actual tariff in the focal market: $\mathrm{d}\log t_j > \mathbb{E}\,\mathrm{d}\log t_i$. For the third market, the realized tariff is zero. Therefore, the change in the *realized profit* is

$$\mathrm{d}\log\pi_i = \theta_i(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta \mathrm{d}\log t}{\theta}\right] > 0.$$

Relative price $\frac{p_i}{P_i}$ The relative price has the following expression:

$$\mathrm{d}\log\frac{p_i}{P_i} = \frac{\mathrm{d}\log N_i}{\sigma_i - 1},$$

which declines in both the focal and the third markets (as the number of firms decreases). Moreover, $\frac{t_i p_i}{A_i}$ decreases in every market. Of course, this is consistent with the increasing market share $s(\frac{p_i}{A_i})$ of surviving firms in both the focal and third markets.

Firm-level price p_i Let us look at the mark-up pricing rule:

$$p_i = \mu_i \frac{\tau_i}{z},$$

where $\mu_i \equiv \frac{\sigma_i}{\sigma_i - 1}$ and

$$\sigma_i = 1 - \frac{\upsilon s^{'}(\upsilon)}{s(\upsilon)}$$

We assume that the demand elasticity (or markup) increases (or decreases) with the relative price $\frac{p_i}{P_i}$. Since we have

$$\mathrm{d}\log\frac{p_i}{P_i}=\frac{\mathrm{d}\log N_i}{\sigma_i-1}<0,$$

$$\mathrm{d}\log p_i=\mathrm{d}\log\mu_i=-\left(\frac{1}{\rho_i}-1\right)\mathrm{d}\log\frac{p_i}{P_i}=-\left(\frac{1}{\rho_i}-1\right)\frac{\mathrm{d}\log N_i}{\sigma_i-1},$$

we must have

$$\mathrm{d}\log p_i>0.$$

Price aggregator A_i, the ideal price index P_i^I Based on the discussion in the previous step, we must have

$$\mathrm{d}\log\frac{p_i}{P_i}=\frac{\mathrm{d}\log N_i}{\sigma_i-1}<0,\ \ \mathrm{d}\log p_i=\mathrm{d}\log\mu_i>0\Rightarrow\mathrm{d}\log P_i>\mathrm{d}\log p_i>0,\ \ \mathrm{d}\log A_i=\mathrm{d}\log P_i+\mathrm{d}\log t_i>0,$$

where $\mathrm{d}\log t_i=0$ in third markets. Therefore, we must have that the price aggregator increases in all markets. Moreover, we have

$$\mathrm{d}\log P_i^I=\mathrm{d}\log p_i+\mathrm{d}\log t_i-\mathrm{d}\log N_i\left(\delta_i-1\right)=-\left(\frac{1}{\rho_i}-1\right)\frac{\mathrm{d}\log N_i}{\sigma_i-1}+\mathrm{d}\log t_i-\mathrm{d}\log N_i\left(\delta_i-1\right).$$

Since $\mathrm{d}\log t_i\geq 0$ and $\mathrm{d}\log N_i<0$ and $\mathrm{d}\log p_i>0$ for all markets. The ideal price index must increase in all markets.

Total export value E_i and real consumption Q_i Since the ideal price index must increase in all markets, the real consumption $Q_i=D_i^{\eta_i}{P_i^I}^{-\eta_i}$ and total expenditure on Chinese goods $E_i=Q_iP_i^I=D_i^{\eta_i}{P_i^I}^{1-\eta_i}$ drop in all markets. Specifically, we have

$$\mathrm{d}\log Q_i=-\eta_i\mathrm{d}\log P_i^I=\eta_i\left[\left(\frac{1}{\rho_i}-1\right)+\frac{\mathrm{d}\log N_i}{\sigma_i-1}\mathrm{d}\log N_i\left(\delta_i-1\right)-\mathrm{d}\log t_i\right]<0,$$

and

$$\mathrm{d}\log R_i=(1-\eta_i)\mathrm{d}\log P_i^I=(\eta_i-1)\left[\left(\frac{1}{\rho_i}-1\right)+\frac{\mathrm{d}\log N_i}{\sigma_i-1}\mathrm{d}\log N_i\left(\delta_i-1\right)-\mathrm{d}\log t_i\right]<0.$$

Note that $\mathrm{d}\log t_i=0$ in third markets.

Firm-level output q_i Recall that average realized firm output equals

$$q_i = \frac{s\left(\frac{p_i}{P_i}\right) D^{\eta_i} P_i^{I\,1-\eta_i}}{t_i p_i}.$$

Therefore, we must have

$$\mathrm{d}\log q_i = \mathrm{d}\log s_i + (1-\eta_i)\mathrm{d}\log P_i^I - \mathrm{d}\log t_i - \mathrm{d}\log p_i,$$

which is equivalent to

$$\mathrm{d}\log q_i = -\left[1-(\eta_i-1)(\delta_i-1)\right]\mathrm{d}\log N_i - \eta_i\left(\mathrm{d}\log p_i + \mathrm{d}\ \log t_i\right),$$

where $1-(\eta_i-1)(\delta_i-1) > 0$. Substituting $\mathrm{d}\log p_i = -\left(\frac{1}{\rho_i}-1\right)\frac{\mathrm{d}\log N_i}{\sigma_i-1}$ into the above equation, we have

$$\mathrm{d}\log q_i = -\left[1-(\eta_i-1)(\delta_i-1) - \frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\mathrm{d}\log N_i - \eta_i \mathrm{d}\log t_i, \tag{94}$$

which leads to

$$\mathrm{d}\log q_i = \left[1-(\eta_i-1)(\delta_i-1) - \frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right] + \xi_i\frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right] - \eta_i \mathrm{d}\log t_i$$

where the first term reflects the GE effect of the change in the number of firms on firm output (which can be positive or negative) while the second term captures the direct effect coming from the tariff change.

There are two cases to consider. First, when ρ_i is very small (i.e., close to zero), output level drops in all markets as $\mathrm{d}\log t_i \geq 0$ in all markets. $\rho \approx 0$ implies that there are substantial increases in *the price level* p_i in all markets (due to the GE feedback from $\mathrm{d}\log N_i$) after a market-specific tariff shock. As a result, output q_i drops in all markets.

Next, when ρ_i is close to one (meaning that $1-(\eta_i-1)(\delta_i-1) - \frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right) > 0$ which is true when $\rho_i = 1$), output level increases in the third market unambiguously, while it it is unclear how firm-level output changes in the focal market. $\rho \approx 1$ implies that there is almost no price (and markup) change after the tariff shock. Moreover, the market share per firm in the third market increases and there is no tariff shock. Therefore, output has to go up. For the focal market, the impact is unclear as there is a direct negative effect coming form the tariff increase.

Firm-level revenue r_i Based on the above derivation, we can write out the change in firm-level (tariff-exclusive) revenue as

$$\mathrm{d}\log r_i = \mathrm{d}\log p_i + \mathrm{d}\log q_i = -\left[1-(\eta_i-1)(\delta_i-1)\right]\mathrm{d}\log N_i - \eta_i\left(\mathrm{d}\log p_i + \mathrm{d}\ \log t_i\right) + \mathrm{d}\log p_i,$$

where $1-(\eta_i-1)(\delta_i-1)>0$ and

$$\mathrm{d}\log q_i = \left[1-(\eta_i-1)(\delta_i-1)-\frac{\eta_i-1}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\left[(1-\xi_i)\,\mathbb{E}_{s\xi}\left[\frac{\eta}{\theta}\mathrm{d}\log t\right]+\xi_i\frac{\eta_i}{\theta_i}\mathrm{d}\log t_i\right]-\eta_i\mathrm{d}\log t_i \tag{95}$$

Therefore, the same argument as in the previous subsection applies here.

Total export quantity $\tilde{Q}_i$ Total (physical) quantity of output across firms is $\tilde{Q}_i = N_i q_i$. Thus, we must have

$$\mathrm{d}\log\tilde{Q}_i = \mathrm{d}\log N_i + \mathrm{d}\log q_i,$$

which

$$\mathrm{d}\log\tilde{Q}_i = \mathrm{d}\log N_i + \mathrm{d}\log q_i = \left[(\eta_i-1)(\delta_i-1)+\frac{\eta_i}{\sigma_i-1}\left(\frac{1}{\rho_i}-1\right)\right]\mathrm{d}\log N_i - \eta_i\mathrm{d}\log t_i < 0, \tag{96}$$

as $\mathrm{d}\log N_i < 0$ and $\mathrm{d}\log t_i \geq 0$ in all markets. In summary, the action at the extensive margin drive the change in aggregated export quantity/value (even if firm-level export value increases).

The key lesson is that the tariff uncertainty affects various variables in all markets through the channel of $\mathrm{d}\log N_i$ in the case of homogeneous firms (as there is no actual tariff realized in the third market). In particular, $\mathrm{d}\log N_i$ determines $\mathrm{d}\log p_i$, $\mathrm{d}\log\mu_i$, $\mathrm{d}\log q_i$, $\mathrm{d}\log P_i$, $\mathrm{d}\log P_i^I$, $\mathrm{d}\log E_i$, $\mathrm{d}\log Q_i$, $\mathrm{d}\log\tilde{Q}_i$. The expected tariff does not have an independent effect on all these variables other than affecting N_i.