Treating Inflation Targeting as a Natural Option Formula

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Abstract

I develop three novel analytical option formulas that measure inflation given a central bank has an inflation-targeting policy. Considering this policy commitment as a natural experiment, I establish the natural option class. Instead of measuring the dynamics via the risk-neutral or arbitrage-free market valuations, given the elements are independent of risk preferences, this option formula proposes the quantity theory of money as the underlying macroeconomic dynamics of prices. In the same manner, the quantity theory of money is shown to be a martingale. The methodology has two steps. In the first step, the pricing formula is derived by the multidimensional Itô's lemma. In the second step, when long memory is present, the multifractional multidimensional formulas are presented for constant H and nonconstant h Hurst exponents. Hence a new method of evaluating a natural experiment as an option is formed.

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1. Introduction

In this paper, I propose three option pricing formulas for an inflation expectation assuming an inflation targeting policy is followed by a central bank. Inflation targeting was first adoption as official policy in the 1990s and has since spread to many countries, (Bernanke and Mishkin, 1997; Svensson, 2010; Combes *et al.*, 2018; Mishkin, 2019). To construct these option formulas that address an inflation targeting policy, I propose a new definition of a natural option and introduce the first application of monetary economic theory, the quantity theory of money, to describe the option dynamics of inflation.

In general, in option pricing theory, the risk-neutral valuation (Black and Scholes, 1973), is the standard methodology of evaluating options, (Hull, 2022). More typically, for inflation option pricing there is a second macroeconomic methodological perspective presented by (Jarrow and Yildirim, 2003, 2023), where the Fisher equation describes an arbitrage-free market. The Fisher equation relates nominal interest rate to the real interest rate with an inflation expectation, for further discussion see (Dimand and Betancourt, 2012; Bilbiie, 2022; Holden, 2024; Bassetto and Miller, 2025). Whereas (Jarrow and Yildirim, 2003, 2023) describe the dynamics of inflation in terms of inflation securities as a function of nominal and real interest rates. Thus, both approaches to option pricing are dependent on interest rates. However, interest rates are not a direct cause of inflation. Another way to describe the dynamics of inflation expectations is through the quantity theory of money, (Friedman,

1956, 2018). The advantages of using the quantity theory of money are characterized by its ability to capture economic activity across all sectors of the economy and different monetary or fiscal policies, including interest rates. The principal significance of the theory for option pricing is found in its assertion of causal relationships within inflation dynamics. Therefore, this approach enhances the theoretical interpretation of economic dynamics for option pricing compared to the risk-neutral and the Fisher equation's arbitrage-free market approaches.

For the dynamics of an option to be valid in all states, the risk preference property must be neutral. Consumer prices describe consumption at every point in time; thus, one does not have an arbitrage opportunity to profit from one's consumption. Accordingly, (Fisher, 1911) popularized another approach to measure prices, the quantity theory of money states the macroeconomic conditions that affect consumer prices in an economy. These elements that are accounted for in any given economy are monetary aggregates, economic output, and velocity of money. Each element is independent of the risk preferences of a consumer. In consideration of the many measurement methods to adequately capture the effects of each element on inflation rates, none are influenced by risk preference, for example see (Reynard, 2023; Bordo and Duca, 2024; Castañeda and Cendejas, 2024; Horan, 2024; Stewart, 2024; Gao, Kulish and Nicolini, 2025). It follows, that this argument can be viewed as an analogy to risk neutral and arbitrage-free market methodologies.

Consequently, in the following section I prove that the quantity theory of money is a martingale and subsequently utilize it to derive the dynamics for an inflation targeting formula.

Note that the inflation time-series commonly exhibits long memory, see for instance (Baillie, Chung and Tieslau, 1996; Comte and Renault, 1996; Goliński and Zaffaroni, 2016; Lovcha and Perez-Laborda, 2018). (Schennach, 2018) demonstrates that long memory is a natural phenomenon arising from the interaction of microeconomic sub-elements within a macroeconomic element, where the multifractional property emerges from multiple interactions. Moreover, (Baillie, Cho and Rho, 2024) find that the inflation process exhibits characteristics that transition between long memory and short memory properties at different time intervals, that strengthens the state-dependent effect found by (Teles, Uhlig and Valle e Azevedo, 2016; Ascari and Haber, 2022). Therefore, given the level of economic activity at different time intervals, Brownian motion will be in a different state, including fractional Brownian motion (fBm) and multifractional Brownian motion (mfBm).

The rest of the paper proceeds as follows. Section 2 presents the natural option class. Section 3 shows that the quantity theory of money is a martingale. Section 4 proceeds to derive an option formula based on Brownian motion with the quantity theory of money as the option's dynamics and Section 5 continues to derive two more option formulas based

on different properties of fractional Brownian motion. Section 6 discusses further applications of the option formulas. A summary is presented in Section 7.

2. Natural Option

It is not generally realized that policy commitments can be viewed as an option. Consider an inflation targeting policy, this is a commitment taken by the central bank in which an inflation rate target is announced for the following year. In addition, there is no dividend or coupon to be distributed, thus it is analogous to a European option. Thus, I introduce the natural option to price policy commitments. A natural option signifies that economic conditions represent natural experiments which are designed and taken upon by the participants themselves, this follows (Friedman, 2005; Lothian, 2011; Leatherdale, 2018; Ding *et al.*, 2024). Thus, this option pricing methodology is termed a natural option. Moreover, the lifetime duration of natural option is given by the chosen duration of a policy. Hence, I propose the following definition:

Natural Option Definition: A natural option is one's own indebtedness, but not the obligation, to achieve a predefined outcome for a specified time T and for a specified rate K.

In this example, the announcement or practice by a central bank of an inflation target rate falls under the definition of a natural option, since it satisfies both conditions that are required. The first condition is satisfied, when central banks elect to achieve a predefined

rate of inflation. The second condition is realized because they define the rate they wish to achieve for a specific time frame duration. For further inflation targeting policy discussion see (Benati, 2008; Nakamura *et al.*, 2018; Bernanke, 2020; Bianchi, Melosi and Rottner, 2021; Jia and Wu, 2023). Hence, the inflation target rate from a central bank perspective can be viewed as a natural option and can be specified as an option formula.

Accordingly, when applying the natural option to inflation policy commitments, the expected option payoffs differ from the payoff interpretation of profit from standard option pricing theory. Unlike a financial option which provides a right to exercise the option at the end of the lifetime of the option, a natural option does not. The exercise time *T* of the natural option occurs at the beginning of its lifetime. Hence, at the end of the lifetime of the natural option only the result of the policy is realized. Therefore, the natural option payoff interpretation is an evaluation of the performance of the implemented policy. Finally, the option function is defined as $max(P_T - K, 0)$, where P_T is realized inflation at time *T* and *K* is the target price.

When realized inflation P_T at the end of the lifetime of the option is greater than the inflation target rate *K*, then the natural option is called *over the target*. Thus, goods will be more expensive on average to buy than the specified increase of the inflation rate as indebted by the central bank.

On the other hand, if realized inflation is less than the inflation target rate, the natural option is called *under the target*. The natural option payoff provides three additional sub-payoffs in comparison to a financial option singular payoff. The first sub-payoff is when realized inflation is under the inflation target rate and positive. Thus, the price of goods on average increase but less than the target rate specified by the central bank. The second sub-payoff is when realized inflation is equal to zero. Thus, the price of goods on average remains unchanged over the option's lifetime. The third sub-payoff is when realized inflation is under the target and negative. Thus, the price of goods on average decrease over the option's lifetime.

Moreover, if realized inflation is equal to the inflation target rate, the natural option is called *on the target*. Thus, the price of goods will change on average in the magnitude of the specified inflation target rate chosen by the central bank.

3. Martingale Proof

Given the identity of the quantity theory of money (1), (Friedman, 1956, 2018; Mishkin, 2019; Ireland, 2024), is a finite bounded Borel-measurable function defined as:

$$P_t = \frac{M_t V_t}{Y_t}, \qquad 0 \leq s \leq t \leq T, \tag{1}$$

where P is price, M is money supply, and Y is economic output within an economy. The Markov property, (Karlin and Taylor, 1975; Shreve, 2008; Øksendal, 2013), implies that:

$$E[P_T|\mathcal{F}_s] = E\left[E\left[\frac{M_T V_T}{Y_T}|\mathcal{F}_t\right]|\mathcal{F}_s\right] \tag{2}$$

$$= E\left[\frac{M_t V_t}{Y_t}|\mathcal{F}_s\right] \tag{3}$$

$$= \frac{M_s V_s}{Y_s}. \tag{4}$$

Hence, it is obtained that the quantity theory of money is a martingale. Moreover, in this manner it is equivalent to the risk-neutral and arbitrage-free market valuations that are not dependent on risk preferences. Furthermore, this result corresponds directly to an inflation process that possesses the Markov property (Ahlin and Shintani, 2007). ■

4. Brownian Motion Derivation

This study uses three stochastic processes that capture proportional change for each element of the quantity theory of money, following (Malliaris and Malliaris, 1995; Jarrow and Yildirim, 2023) studies on inflation. It follows that the three processes follow geometric Brownian motion:

$$dM_t = \mu_M M_t\, dt + \sigma_M M_t\, dB_M, \tag{5}$$

$$dV_t = \mu_V V_t\, dt + \sigma_V V_t\, dB_V, \tag{6}$$

$$dY_t = \mu_Y Y_t\, dt + \sigma_Y Y_t\, dB_Y. \tag{7}$$

Deriving the unifying process is established by the multidimensional Itô's Lemma:

$$dG = \left(\sum_{i=1}^{n} \frac{dG}{dx_i} a_i + \frac{dG}{dt} + \frac{1}{2}\sum_{i=1}^{n}\sum_{i=1}^{n} \frac{d^2G}{dx_i dx_j} b_i b_j \rho_{ij}\right) dt + \sum_{i=1}^{n} \frac{dG}{dx_i} b_i \, dB_i. \quad (8)$$

Applying the multidimensional Itô's Lemma (8) given the function of the price identity (1) and process elements (5), (6), (7), the derivation is as follows:

$$d\left(\frac{M_t V_t}{Y_t}\right) = \left(\mu_M M_t \frac{V_t}{Y_t} + \mu_V V_t \frac{M_t}{Y_t} - \mu_Y Y \frac{M_t V_t}{{Y_t}^2} + \sigma_M M_t \sigma_V V_t \rho_{MV} \frac{1}{Y_t} - \sigma_M M_t \sigma_Y Y_t \rho_{MY} \frac{V_t}{{Y_t}^2} - \right. \quad (9)$$

$$\left. \sigma_V V_t \sigma_Y Y_t \rho_{VY} \frac{M_t}{{Y_t}^2} + {\sigma_Y}^2 {Y_t}^2 \rho_{YY} \frac{M_t V_t}{{Y_t}^3}\right) dt + \left(\sigma_M M_t \frac{V_t}{Y_t} + \sigma_V V_t \frac{M_t}{Y_t} - \sigma_Y Y_t \frac{M_t V_t}{{Y_t}^2}\right) dB_i,$$

where zero terms are neglected and similar terms combined. This derivation provides the desired price properties as the elements are recomposed to the initial state. Note that the (Gould and Nelson, 1974; Gould *et al.*, 1978) hypothesis states that velocity follows a random walk, which results in interpreting velocity integrands as fluctuations of the other two elements. Hence, the following is the solution of the multidimensional Itô's Lemma for the quantity theory of money:

$$dP_t = P_t(\mu_M + \mu_V - \mu_Y + \sigma_M \sigma_V \rho_{MV} - \sigma_M \sigma_Y \rho_{MY} - \sigma_V \sigma_Y \rho_{VY} + \sigma_Y^2)\, dt + P_t(\sigma_M + \sigma_V - \sigma_Y)\, dB_i. \quad (10)$$

The interpretation of the dynamics of the solution is straightforward. The elements in the drift are as anticipated, price changes are the expected growth rates and covariances of each element, (Malliaris and Malliaris, 1995). The price deviation or diffusion dynamics are the anticipated risk measures of each price element. However, σ_Y^2 is an additional element

in the stochastic formulation in the integrand of the drift. This indicates that increased economic output uncertainty increases the price level.

Now define i and σ_i to be $i = \mu_M + \mu_V - \mu_Y + \sigma_M\sigma_V\rho_{MV} - \sigma_M\sigma_Y\rho_{MY} - \sigma_V\sigma_Y\rho_{VY} + \sigma_Y^2$, and $\sigma_i = \sigma_M + \sigma_V - \sigma_Y$. Observe that the average growth of prices and its volatility is rewritten as:

$$dP_t = P_t i\, dt + P_t \sigma_i\, dB, \tag{11}$$

which is in the familiar form of geometric Brownian motion. Consider that prices follow a lognormal distribution, where $G = ln\ P$, and further apply Itô's lemma (8), then the well-known solution is:

$$P_T = P_t e^{(i - {\sigma_i}^2/2)\tau - \sigma_i\sqrt{\tau}Z} \tag{12}$$

where $\tau = T - t$, given Brownian motion is a standardized normal distribution between two-time intervals. Furthermore, assume a central bank's inflation targeting policy commitment can be expressed as a natural option $max(P_T - K, 0)$ given the inflation target rate K. It follows immediately that taking the expectation of the process is:

$$P(p_t, t) = \tilde{E}[P_T - K, 0|\mathcal{F}_t] \tag{13}$$

and $\tilde{E}$ denotes the expected value given quantity theory of money dynamics. Then the natural option formula remains positive provided the following inequality holds:

$$Z < \frac{ln\left(\frac{P_t}{K}\right) + \left(\mu_M + \mu_V - \mu_Y + \sigma_M\sigma_V\rho_{MV} - \sigma_M\sigma_Y\rho_{MY} - \sigma_V\sigma_Y\rho_{VY} + \sigma_Y^2 - \frac{(\sigma_M + \sigma_V - \sigma_Y)^2}{2}\right)\tau}{(\sigma_M + \sigma_V - \sigma_Y)\sqrt{\tau}} = N(d_2) \tag{14}$$

where $N(\cdot)$ is the cumulative standard normal distribution function after resubstituting for i and σ_i. Therefore, the expectation of the inflation targeting formula is:

$$P(p_t,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{N(d_2)}\left(P_t e^{\left(\mu_M+\mu_V-\mu_Y+\sigma_M\sigma_V\rho_{MV}-\sigma_M\sigma_Y\rho_{MY}-\sigma_V\sigma_Y\rho_{VY}+\sigma_Y^2-\frac{(\sigma_M+\sigma_V-\sigma_Y)^2}{2}\right)\tau-(\sigma_M+\sigma_V-\sigma_Y)\sqrt{\tau}Z} - K\right)e^{-\frac{1}{2}Z^2}\,dZ \tag{15}$$

$$= P_t e^{(\mu_M+\mu_V-\mu_Y+\sigma_M\sigma_V\rho_{MV}-\sigma_M\sigma_Y\rho_{MY}-\sigma_V\sigma_Y\rho_{VY}+\sigma_Y^2)\tau}N(d_1) - KN(d_2), \tag{16}$$

and

$$N(d_1) = \frac{ln\left(\frac{P_t}{K}\right)+\left(\mu_M+\mu_V-\mu_Y+\sigma_M\sigma_V\rho_{MV}-\sigma_M\sigma_Y\rho_{MY}-\sigma_V\sigma_Y\rho_{VY}+\sigma_Y^2+\frac{(\sigma_M+\sigma_V-\sigma_Y)^2}{2}\right)\tau}{(\sigma_M+\sigma_V-\sigma_Y)\sqrt{\tau}}. \tag{17}$$

5. Fractional Brownian Motion Derivations

Now suppose that the price time series possess long memory, which is evaluated, for example, by test statistics provided by (Hurst, 1951; Giraitis et al., 2003; Hurvich, Moulines and Soulier, 2005; Hurvich, Moulines and Hurvich, 2008; Rooch, Zelo and Fried, 2019), that confirm the existence of fractional Brownian motion. Thus, a multifractional as well as a fractional inflation targeting formula to address these properties are derived. The derivation procedure will be similar to the previous formula; however, Ito's Lemma is now adjusted to reflect the multidimensional multifractional Brownian motion properties. Therefore, the

following Itô lemma for a multidimensional multifractional Brownian motion is stated, as a correspondent of the non-fractional Itô lemma, and a natural expansion to the multidimensional Itô lemma.

Multifractional Multidimensional Itô Lemma: Given function $G \in C^2(R)$ and X_i is a stochastic process defined for a known set of dimensions as generated by multifractional multidimensional Brownian motion:

$$dX_i = a(x,t)_i\, dt + b(x,t)_i d\mathcal{B}(h)_i$$

Then the following is the multifractional multidimensional Itô formula:

$$dG = \left(\sum_{i=1}^{n} \frac{dG}{dx_i} a_i + \frac{dG}{dt} + \sum_{i=1}^{n}\sum_{i=1}^{n} \frac{d^2G}{dx_i dx_j} b_i b_j \rho_{ij}\, [h'(t)ln(t) + t^{-1}h(t)]t^{2h(t)}\right) dt + \sum_{i=1}^{n} \frac{dG}{dx_i} b_i d\mathcal{B}(h)_i. \quad (18)$$

This Itô type lemma is the analogue extension that is in line with the literature of (Araneda, 2024; Bender, Lebovits and Lévy Véhel, 2024). Note that for a constant Hurst exponent $h = H$, the fractional multidimensional Itô lemma is recovered; and when $h = 1/2$, the multidimensional Itô lemma is recovered. Accordingly, the multifractional Brownian motion quantity theory of money process is:

$$dP_t = P_t \left(\mu_M + \mu_V - \mu_Y + 2(\sigma_M\sigma_V\rho_{MV} - \sigma_M\sigma_Y\rho_{MY} - \sigma_V\sigma_Y\rho_{VY} + \sigma_Y^2)[h'(t)ln(t) + t^{-1}h(t)]t^{2h(t)}\right) dt + P_t(\sigma_M + \sigma_V - \sigma_Y) d\mathcal{B}(h)_i. \quad (19)$$

It is interesting to note for the economic interpretation that only the price risk measures are affected by multifractional Brownian motion. Further, assume that prices have a single

source of uncertainty and substitute $\bar{\mu}_h = \mu_M + \mu_V - \mu_Y + 2(\sigma_M\sigma_V\rho_{MV} - \sigma_M\sigma_Y\rho_{MY} - \sigma_V\sigma_Y\rho_{VY} + \sigma_Y^2)\left[h'(t)ln(t) + t^{-1}h(t)\right]t^{2h(t)}$ and $\bar{\sigma}_h = (\sigma_M + \sigma_V - \sigma_Y)$. Consequently, the option formula takes a known form of a multifractional geometric Brownian motion:

$$dP_t = \bar{\mu}_h P_t dt + \bar{\sigma}_h P_t d\mathcal{B}(h). \tag{20}$$

Therefore, the solution obtained assuming prices follow a lognormal distribution, where $G = ln\, P$, from the multifractional multidimensional Itô lemma (18) is:

$$dG = \left(\bar{\mu}_h - \left(h'(t)ln(t) + t^{-1}h(t)\right)t^{2h(t)}\bar{\sigma}_h{}^2\right)dt + \bar{\sigma}_h d\mathcal{B}(h). \tag{21}$$

And the multifractional inflation targeting option pricing formula after resubstituting of $\bar{\mu}_h$ and $\bar{\sigma}_h$ is:

$$P(p_t, t) = P_t e^{\left(\mu_M+\mu_V-\mu_Y+2\left(\sigma_M\sigma_V\rho_{MV}-\sigma_M\sigma_Y\rho_{MY}-\sigma_V\sigma_Y\rho_{VY}+\sigma_Y^2\right)\left(h'(t)ln(t)+t^{-1}h(t)\right)t^{2h(t)}\right)\tau}N(d_{1h}) - KN(d_{2h}), \tag{22}$$

where the multifractional inflation targeting formula dynamics are:

$$N(d_{2h}) = \frac{ln\left(\frac{P_t}{K}\right) + \left(\mu_M+\mu_V-\mu_Y+\left(2\left(\sigma_M\sigma_V\rho_{MV}-\sigma_M\sigma_Y\rho_{MY}-\sigma_V\sigma_Y\rho_{VY}+\sigma_Y^2\right) - \bar{\sigma}_h{}^2\right)\left(h'(t)ln(t)+t^{-1}h(t)\right)t^{2h(\tau)}\right)\tau}{\sqrt{(\sigma_M+\sigma_V-\sigma_Y)^2t^{2h(t)}}} \tag{23}$$

and

$$N(d_{1h}) = \frac{ln\left(\frac{P_t}{K}\right) + \left(\mu_M+\mu_V-\mu_Y+\left(2\left(\sigma_M\sigma_V\rho_{MV}-\sigma_M\sigma_Y\rho_{MY}-\sigma_V\sigma_Y\rho_{VY}+\sigma_Y^2\right) + \bar{\sigma}_h{}^2\right)\left(h'(t)ln(t)+t^{-1}h(t)\right)t^{2h(t)}\right)\tau}{\sqrt{(\sigma_M+\sigma_V-\sigma_Y)^2t^{2h(t)}}}. \tag{24}$$

However, given $h = H$ is a constant the fractional inflation targeting option pricing formula is:

$$P(p_t,t) = P_t e^{(\mu_M+\mu_V-\mu_Y+2(\sigma_M\sigma_V\rho_{MV}-\sigma_M\sigma_Y\rho_{MY}-\sigma_V\sigma_Y\rho_{VY}+\sigma_Y^2)Ht^{2H-1})\tau}N(d_{1H}) - KN(d_{2H}), \quad (25)$$

then the fractional inflation targeting formula dynamics are:

$$N(d_{2H}) = \frac{ln\left(\frac{P_t}{K}\right)+(\mu_M+\mu_V-\mu_Y+(2(\sigma_M\sigma_V\rho_{MV}-\sigma_M\sigma_Y\rho_{MY}-\sigma_V\sigma_Y\rho_{VY}+\sigma_Y^2)-(\sigma_M+\sigma_V-\sigma_Y)^2)Ht^{2H-1})\tau}{\sqrt{(\sigma_M+\sigma_V-\sigma_Y)^2t^{2H}}}, \quad (26)$$

and

$$N(d_{1H}) = \frac{ln\left(\frac{P_t}{K}\right)+(\mu_M+\mu_V-\mu_Y+(2(\sigma_M\sigma_V\rho_{MV}-\sigma_M\sigma_Y\rho_{MY}-\sigma_V\sigma_Y\rho_{VY}+\sigma_Y^2)+(\sigma_M+\sigma_V-\sigma_Y)^2)Ht^{2H-1})\tau}{\sqrt{(\sigma_M+\sigma_V-\sigma_Y)^2t^{2H}}}. \quad (27)$$

Given the different properties of Brownian motion, a corresponding formula is derived to reflect each property. Consequently, the properties reflecting each state of all three-option pricing formulas are satisfied. Thus, testing the existence of inflation targeting and prediction under inflation targeting regime are established.

6. Discussion Further Applications

The formulas have applications beyond the natural option's ability to detect inflation targeting. The natural option formula can serve as a policy analysis tool that assesses the implications of policy commitments. At equilibrium, expected inflation equals the inflation target rate. Hence, the formula dynamics for the inflation expectation and the target rate are set accordingly:

$$P_t e^{(\mu_M+\mu_V-\mu_Y+2(\sigma_M\sigma_V\rho_{MV}-\sigma_M\sigma_Y\rho_{MY}-\sigma_V\sigma_Y\rho_{VY}+:\sigma_Y^2)(h'(t)ln(t)+t^{-1}h(t))t^{2h(t)})\tau}N(d_{1h}) = KN(d_{2h}). \quad (28)$$

Consequently, mapping the corresponding dynamics obtains a solution that assesses the paths available to a central bank. However, a more general theory is required to account for aggregate economic activity. The money element (Berentsen, Huber and Marchesiani, 2018; Friedman, 2018; Kugler and Reynard, 2022) expands into a function of $M = f(M_S, M_D)$, where M_S denotes money supply and M_D denotes money demand. Money supply $M_S = f_S(Y, r_S,)$ is a function of national income Y and an interest rate (opportunity cost of money) r_S. Money demand is more intricate since it resides at the intersection of inflation expectations *P* and economic activity that affects money demand. The money demand function f_D is composed of $M_D = P\ f_D(r_b, r_e, r_m, Y_n, W_n, o_D)$, where r_b denotes the expected interest on fixed income assets, r_e denotes the expected return on productivity, r_m denotes the expected nominal interest rate on money, Y_n denotes the sum of fractional single wealth holders forming the national income, W_n denotes total property wealth, o_D denotes a vector of other factors that affect money demand. Therefore, a policy proposal can be extracted while accounting for the principal macroeconomics impacts. This mapping yields a recommendation for the theoretical-optimal policy based on the natural option, whose dynamics follow the quantity theory of money.

Furthermore, these formulas apply to price inflation-adjusted securities by substituting the inflation target rate of the natural option with the market strike price. This substitution implies the standard option formula and payoff definitions. Thus, market participants benefit from comprehensive macroeconomic insights that do not rely solely on interest rate

dynamics. The full-element formulation of the natural option's dynamics incorporates commercial activity alongside fiscal and monetary policies (including interest rates) into the dynamics of inflation options. Given the varying levels of economic activity, fractional and multifractional Brownian motion capture additional layers of uncertainty and economic regime changes in the underlying processes that determine prices. Accordingly, the pricing of policy commitments and the practical implications of pricing of inflation-adjusted securities are unified.

7. Summary

In this paper, I introduce a new option formula class, the natural option. The natural option measures natural experiments thereby it permits to price or detect a policy commitment. To formulate a reasonable expectation the theory underlying the dynamics must represent the causal relationships implied by the policy commitment. Thus, if the natural option price deviates substantially from the stated commitment, it implies that an alternative policy is being implemented. In addition, policy-proposals estimates provide guidance on probable outcomes and thereby provide further insight into the proposed policy's effectiveness.

Moreover, I introduce a dynamic method to detect the existence of inflation targeting by a central bank. This method is based on a formulation of the quantity theory of money as dynamics of an option pricing formula. Thus, independent macroeconomic elements

characterize the dynamics of the natural option which are independent of risk preferences. I develop three option formulas, where the first formula assumes prices follow Brownian motion; the second assumes fractional Brownian motion; and the third assumes multifractional Brownian motion. Consequently, this approach effectively captures the distinct states of Brownian motion and provides interpretable macroeconomic dynamics for expected inflation. This establishes the formulas' ability to distinguish the inflation state. Hence, the natural option provides a method to price policy commitments and to test whether inflation targeting is, in fact, the central bank's policy.

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