The Optimal Macro Tariff*

Oleg Itskhoki
itskhoki@fas.harvard.edu

Dmitry Mukhin
d.mukhin@lse.ac.uk

First draft: April, 2025

This draft: May 19, 2025

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https://itskhoki.com/papers/OptimalMacroTariff.pdf

Abstract

What is the optimal macroeconomic tariff when trade is imbalanced and the policy objectives go beyond social welfare and also include fiscal revenues, increasing the number of manufacturing jobs, and closing a trade deficit? We study these questions in an environment which allows for long-run bilateral and aggregate trade deficits that reflect the country's net foreign asset position and differential returns on foreign assets and liabilities (the "exorbitant privilege"). Only in special cases does the optimal tariff emerge as an increasing function of a trade deficit and for reasons unrelated to trade competitiveness. Instead, the planner trades off the conventional benefits of improved terms of trade with the costs from negative valuation effects on the country's *gross* financial position. In our model calibrated to the United States, its large external dollar liabilities reduce the optimal tariff three-fold, from 34% to 9%, and act as an effective hedge for its trade partners against a trade war. In contrast to the expenditure switching logic and Lerner symmetry, closing the trade imbalance calls for an appreciation of the dollar to a level that depends solely on the US external financial position, and not on trade shares or trade elasticities, and can be achieved by means of an import tariff or an export subsidy. Alternatively, financial and trade rebalancing may happen if a tariff war results in the loss of privilege and the associated dollar depreciation.

*We thank Rodrigo Adão, Mark Aguiar, Fernando Alvarez, Manuel Amador, Olivier Blanchard, Pol Antràs, Elhanan Helpman, Chang-Tai Hsieh, Kiminori Matsuyama, Marc Melitz, Brent Neiman, Mathieu Parenti, Esteban Rossi-Hansberg for comments and stimulating conversations. Dmitry Mukhin acknowledges the support from the Economic and Social Research Council (ESRC) [grant number ES/Y001540/1].

1 Introduction

What is the optimal macroeconomic tariff when trade is imbalanced? Can a tariff be used to permanently close an aggregate trade imbalance? We start with a standard two-country balanced-trade model, and study these questions in a sequence of its extensions to various environments that feature aggregate imbalances, valuation effects and convenience yields on gross foreign asset positions, and bilateral trade deficits in a multi-country model.

On the methodological side, we extend the analysis in Johnson (1950) by adopting a primal approach to the optimal policy problem (Lucas and Stokey 1983, Costinot, Lorenzoni, and Werning 2014). In particular, in Section 2, we introduce an implementability constraint for the home planner in the space of import and export quantities, characterizing the "trade possibilities frontier" for the country (TPF). This TPF combines the trade balance condition — or, more generally, the country budget constraint — with the optimal consumption and production decisions in the rest of the world. The optimal tariff corresponds to the elasticity of the TPF, which combines import demand and export supply elasticities of the rest of the world.

We use this TPF approach in Section 3 to derive the standard optimal tariff formula in a two-country model under balanced trade, as well as under various alternative objectives of the planner — namely, maximization of tariff revenues or of manufacturing employment. In particular, we show that it is a trade subsidy — whether on imports or exports — not a tax or tariff, that can counterbalance a "China shock" and offset the decline in the tradable-sector manufacturing employment. Finally, we use TPFs for home and the rest of the world to characterize the Nash equilibrium in a tariff war. We then compare the magnitudes of tariffs and welfare gains and losses under these different scenarios. Our tentative calculations using the model calibrated to the US suggest that the welfare-maximizing tariff is 34%, while the revenue-maximizing tariff is a lot higher and equal to 80%. The country benefits from both tariffs if they are imposed unilaterally (by 0.6% and 0.3% of aggregate consumption, respectively), but suffers significantly if it triggers a trade war Nash equilibrium (with a permanent loss of 2.7% of aggregate consumption).

Our main focus, however, is on models with trade imbalances: we study aggregate imbalances in static and dynamic environments in Section 4 and bilateral imbalances in a multi-country environment in Section 5. We extent the concept of TPF to these generalized environments and use it to derive the welfare-maximizing tariff, as well as study conditions when a tariff can be used to close a persistent trade imbalance.

Aggregate trade imbalances emerge when a country holds an imbalanced net foreign asset position, and the long-run trade deficit is shaped by the financial position of the country via its budget constraint. Unless a tariff affects the financial position of a country via valuation effects on gross foreign assets, it cannot change the long-run trade imbalance of a country. Nonetheless, tariffs do generally have valuation effects, and therefore can be used to close an aggregate

trade imbalance. The relevant criteria for this are neither trade share, nor trade elasticities, but rather the financial valuation effects of the tariff. In particular, under home currency debt, it is the currency appreciation induced by the tariff — as opposed to a depreciation — that allows to rebalance a trade deficit.

Furthermore, an import tariff and an export tax are, generally, no longer perfect substitutes — violating Lerner (1936) symmetry — in the presence of gross foreign asset positions. As a result, their combination may be used to engineer a pure transfer from the rest of the world by means of the associated valuation effects, and the planner would generally want to max out on the use of such tariffs.

Restricting focus to a single instrument, the planner now needs to balance the conventional optimal terms of trade manipulation in the goods market with the associated valuation effect in the asset market. An import tariff generally results in an exchange rate appreciation and, hence, a negative valuation effect for a country with gross international home-currency liabilities, which is the case for the US. As a result, the optimal tariff for such country is lower relative to the case of balanced trade under financial autarky. This also implies that holding foreign-currency assets against a major trade partner is a hedge against a trade war, as it both increases the cost of the war and reduces the size of the optimal tariff for the trade partner. Furthermore, in a world where countries hold large gross foreign asset positions against each other, optimal tariffs are low for everyone irrespective of net positions and trade deficits.

While an import tariff and an export tax are equivalent under Lerner symmetry, as both reduce gross trade quantities (imports and exports), there is no such equivalence when the objective is to close a trade deficit. Specifically, we show that the goal of closing a trade deficit in the presence of local-currency foreign liabilities requires either an import tariff or an export subsidy, as they both induce an exchange rate appreciation and a negative valuation effect.

A peculiar feature of the "Liberation Day" tariff announcement was a sharp devaluation of the dollar, unlike in the previous episodes of the tariff war. We argue that this can be only consistent with a model that feature convenience yields on the US liabilities which are eroded as a result of the tariff war. In such dynamic models, the on-impact valuation effects of gross international assets are combined with future excess returns that reflect "exorbitant privilege" for a lender that enjoys convenience yields on its issued debt. Such convenience yields and future expected excess returns may allow the country to run a persistent trade deficit even under circumstances of a negative net foreign asset position, as is arguably the case for the US. We show that, taking convenience yield as exogenous, this results in a larger optimal tariff. In contrast, the possibility of erosion of convenience yield from a trade war reduces the incentives to impose a tariff in the first place.

Finally, we consider a multi-country extension of the model where bilateral trade is not necessarily balanced even if the aggregate country-level net exports are equal to zero. We show that it is generally optimal to set different tariffs across trade partners. However, if

countries in the rest of the world can insure each other against tariffs (via financial-market risk sharing), the optimal country-specific tariff formula is exactly the same as in the baseline model with home and the rest of the world. In particular, a country would like to impose a higher tariff on trade partners that rely on its market as the main destination for exports, while the bilateral trade imbalances are irrelevant. More generally, when financial markets are incomplete and each country has to run a balanced aggregate trade, the home planner needs to take into account not only changes in bilateral terms of trade, but also income effects across its trade partners arising from movements in relative prices. Because of variation in bilateral trade shares, such implicit transfers change global demand for goods and have second-round effects consequential for the home terms of trade. This also explains why bilateral import and export tariffs are no longer isomorphic and it is optimal to use both sets of instruments.

Our work builds on the classic trade literature, including Lerner (1936), Baldwin (1948), Johnson (1950, 1953), Gros (1987), Jones (1967), Razin and Svensson (1983) and Bagwell and Staiger (1999), as well as Dixit and Norman (1980) and Helpman and Krugman (1989). Ossa (2016) and Caliendo and Parro (2022) provide surveys of the more recent optimal tariff literature. Other related work includes Lashkaripour and Lugovskyy (2023), Lloyd and Marin (2023), Auray, Devereux, and Eyquem (2024). There is an exposition of work on tariffs in the last weeks since the imposition of tariffs. The closest papers to ours in terms of the analysis of imbalances are Pujolas and Rossbach (2024) and Aguiar, Amador, and Fitzgerald (2025). We cite many of the related papers in the text of this draft below, and our reference list is to be completed.

2 Baseline Model with Balanced Trade

We begin our analysis using a static two-country exchange economy between home and the rest of the world. We then extend the model to feature production with labor allocated between tradable and non-tradable sectors. The key property of this setup is balanced trade between home and the rest of the world, which immediately implies that trade policies have no effect on the home's current account or trade deficit in the absence of foreign asset positions. The following sections extend the analysis to a multi-country setting with bilateral imbalances and to a dynamic model with international valuation effects and endogenous current account (im)balance at the country level.

2.1 Setup

We consider a two-country (or two-region) world consisting of the home economy (the US) and the rest of the world (a unified foreign economy, indexed by $*$). Home is contemplating to impose tariffs on imports and exports against the rest of the world. We then generalize the analysis to foreign retaliation and the trade war equilibrium.

Each region has an exogenous endowment of its own good that can be consumed locally or exported to the other economy:

$$Y = C_H + C_H^* \qquad \text{and} \qquad Y^* = C_F + C_F^*. \tag{1}$$

The preferences of households in two economies are given respectively by $u(C_H, C_F)$ and $u^*(C_H^*, C_F^*)$. For concreteness, we focus on CES utility, but allow for different elasticities of substitution and arbitrary home bias parameters:[1]

$$u = \left[(1-\gamma)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \qquad \text{and} \qquad u^* = \left[\gamma^{*\frac{1}{\eta}} C_H^{*\frac{\eta-1}{\eta}} + (1-\gamma^*)^{\frac{1}{\eta}} C_F^{*\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where $\gamma, \gamma^* \in [0, 1)$ are openness parameters, and θ, η are the elasticities of substitution, and we assume $\eta > 1$ to ensure that optimal tariffs are finite. Trade costs are not modeled explicitly and are implicitly captured by the home bias in preferences (when $\gamma^* < 1 - \gamma$).

2.2 Competitive equilibrium

For given trade policy chosen by a planner, the equilibrium prices and allocations are determined as follows. The ad valorem tariffs τ^I, τ^E create deviations from the law of one price across markets:

$$P_F = \tau^I P_F^* \qquad \text{and} \qquad P_H^* = \tau^E P_H, \tag{2}$$

where P_H, P_F denote consumer prices in the home economy and P_H^*, P_F^* are prices in the foreign economy. All prices are expressed in dollars and the law of one price holds for sellers even though consumer prices might be different across countries due to tariffs.[2]

Home households choose consumption that solves

$$\begin{aligned} \max_{C_H, C_F} \quad & u(C_H, C_F) \\ \text{s.t.} \quad & P_H C_H + P_F C_F = P_H Y + T, \end{aligned}$$

where a lump-sum transfer from the government reflects the revenue collected from tariffs

$$T = (\tau^I - 1) P_F^* C_F + (\tau^E - 1) P_H C_H^*.$$

[1] Our main results extend to arbitrary separable preferences and the method can be applied to derive results for even more general utility functions.

[2] We assume flexible prices and can adopt the normalization $P_H = 1$ taking the home good in the home market as numeraire. This also corresponds to an equilibrium in a monetary model in which the home monetary authority fully stabilizes the home good's prices $P_H = 1$, the foreign monetary authority fully stabilizes the foreign currency price of the foreign good $P_F^*/\mathcal{E} = 1$, thus letting the nominal exchange rate $\mathcal{E}$ track the relative price of the two goods in the two markets, $\mathcal{E} = P_F^*/P_H$.

The household optimality condition is given by:

$$\frac{u_F}{u_H} = \frac{P_F}{P_H}, \tag{3}$$

where u_i denotes a partial derivative, i.e., the marginal utility of good i. The problem of foreign households is symmetric and their demand for goods satisfies:

$$\frac{u_F^*}{u_H^*} = \frac{P_F^*}{P_H^*}. \tag{4}$$

The balanced trade condition follows from the foreign budget constraint and, by Walras law, can also be derived from the home budget constraint, fiscal balance, and market clearing conditions:

$$P_H^* C_H^* = P_F^* C_F. \tag{5}$$

In sum, given home tariffs (τ^I, τ^E), the prices (P_H, P_F, P_H^*, P_F^*) and consumption allocations (C_H, C_F, C_H^*, C_F^*) are determined by equilibrium conditions (1)-(5). Notice that there are seven equations and eight variables as only relative prices matter and we are free to choose a numeraire, e.g., $P_H = 1$. For brevity, it is also convenient to denote consumption aggregates (utility) in two countries with C, C^* and the corresponding ideal price indices with P, P^*.

2.3 Primal approach: trade possibilities frontier

The next section dicusses alternative Ramsey problems where the planner chooses tariffs to maximize different objectives subject to equilibrium conditions (1)-(5). Following the primal approach, the problem can be significantly simplified by eliminating some endogenous variables and constraints.

In particular, for any chosen allocation, household optimality conditions (3)-(4) determine relative prices in the two economies. Given their values, the side equation (2) pins down tariffs required to support this equilibrium.

$$\tau^I \tau^E = \frac{P_F/P_H}{P_F^*/P_H^*}. \tag{6}$$

The two trade instruments do not appear in any other equilibrium conditions and therefore, are isomorphic and can implement the same allocations, consistent with Lerner (1936) symmetry.

Lemma 1 *Lerner symmetry applies in this setting and the same allocations can be implemented with an import tariff τ^I or an export tax τ^E.*

Given this observation, we focus, without loss of generality, on a single tariff $\tau \equiv \tau^I \tau^E$ for the remainder of this section. However, the symmetry relies on the assumption that the tariffs are

uniform across all imported and exported goods and does not extend to environments with multiple countries or international asset positions as we discuss below.

Substitute (4) into the balanced-trade condition (5) to get rid of prices in the remaining equilibrium conditions. This leaves only one constraint in addition to market clearing conditions (1):

Lemma 2 *The planner can choose any combination of imports C_F and exports C_H^* that satisfies the implementability condition $C_H^* = g(C_F)$ implicitly defined by*

$$u_H^*(C_H^*, Y^* - C_F)C_H^* = u_F^*(C_H^*, Y^* - C_F)C_F. \tag{7}$$

Under CES preferences, the function $g(\cdot)$ is strictly increasing and strictly convex.

The implementability condition (7) defines a mapping $G(C_H^*, C_F) = 0$, which characterizes the feasible set of international allocations in terms of export and import quantities faced by the home planner. In contrast to the early literature that follows Johnson (1950), we call this mapping neither "foreign export supply", nor "foreign import demand" as it depends on foreign preferences u^*, foreign endowment Y^*, and more generally foreign production structure, and hence reflects both demand and supply sides of the foreign economy, as disciplined by the trade balance. Therefore, we refer to this mapping G, equivalently represented with $g(\cdot)$ in Lemma (7), as the *trade possibilities frontier* from the perspective of the home economy, or equivalently the foreign *offer curve*. We illustrate it with the red curve going through the endowment point E in the Edgeworth box in Figure 1 below.

Given the strict monotonicity of $g(\cdot)$, it is possible to define its inverse, $C_F = g^{-1}(C_H^*)$, which summarizes how many units of foreign good can be exchanged for C_H^* units of home goods. This can be interpreted as the trade "production function" from the perspective of the home planner (see Diamond and Mirrlees 1971). However, in some applications below — specifically, when C_F is a vector of import quantities from different source countries — it is more convenient to work with the original function $C_H^* = g(C_F)$ that summarizes the costs of obtaining C_F units of foreign good in terms of the home good.

In sum, the planner's problem boils down to maximizing its objective subject to the domestic resource constraint $Y = C_H + C_H^*$ and the implementability constraint $C_H^* = g(C_F)$ given by (7), which fully encodes all information about the rest of the world that is relevant for the optimal policy. This characterization applies for any objective of the planner provided that the policy instrument is a tariff. We make use of this trade possibilities frontier function $g(\cdot)$ in all our applications below, including the ones featuring trade imbalances.

3 The Optimal Tariff under Balanced Trade

This section characterizes the optimal tariff under alternative assumptions about the aims of the government policy in the baseline two-country model with balanced trade.

3.1 Welfare

Start with a classical problem of a benevolent local planner maximizing welfare of home households $u(C_H, C_F)$ subject to local resource constraint (1) and the implementability constraint (7), which can be written in compact form as:[3]

$$\max_{C_F} \; u\big(Y - g(C_F), C_F\big).$$

The first-order optimality condition for this problem is $u_H \cdot g' = u_F$, and we rewrite it as:

$$\frac{u_F}{u_H} = \frac{g'(C_F) \cdot C_F}{g(C_F)} \cdot \frac{C_H^*}{C_F}. \tag{8}$$

From the household demand condition (3), the ratio on the left-hand side is equal to the home relative price P_F/P_H, while the balanced trade condition (5) implies that the latter ratio on the right-hand side is equal to the foreign relative price P_F^*/P_H^*. Combining these observations with equation (2), we get the Johnson (1950) formula:

$$\tau^W = \varepsilon, \qquad \text{where} \qquad \varepsilon \equiv \frac{d \log C_H^*}{d \log C_F} = \frac{g'(C_F) \cdot C_F}{g(C_F)}. \tag{9}$$

Note that ε is the elasticity of the feasibility constraint faced by the planner which equals the percentage increase in export quantity C_H^* required to obtained an extra one percent of import quantity C_F. This elasticity reflects both foreign import demand, foreign export supply, and the requirement of trade balance. Thus, even though the mapping from model's parameters to optimal tariff can be quite complicated, the elasticity of the $g(\cdot)$ function is a sufficient statistic and no other information about home or foreign economies is required for the optimal tariff.

The optimal tariff characterization in (8) and (9) has the following immediate corollary. If a country is small and faces exogenous international relative prices $\mathcal{S} \equiv P_F^*/P_H^*$, then the $g(\cdot)$ function is simply $C_H^* = \mathcal{S} \cdot C_F$ and features a unitary elasticity, $\varepsilon = 1$. Indeed, this is the case when the country is a price taker and optimizes with respect to the country budget constraint with a given slope $\mathcal{S}$, which is the exogenous terms of trade of the country. As a result, the country cannot benefit from a tariff, and the optimal trade policy is laissez-faire with $\tau^W = 1$.

Note that the laissez-faire allocation is characterized by $u_H \cdot g = u_F \cdot C_F$ instead of the

[3] This note provides an alternative derivation without using the primal approach where the optimal tariff τ is chosen subject to the full set of decentralized equilibrium conditions (1)-(5).

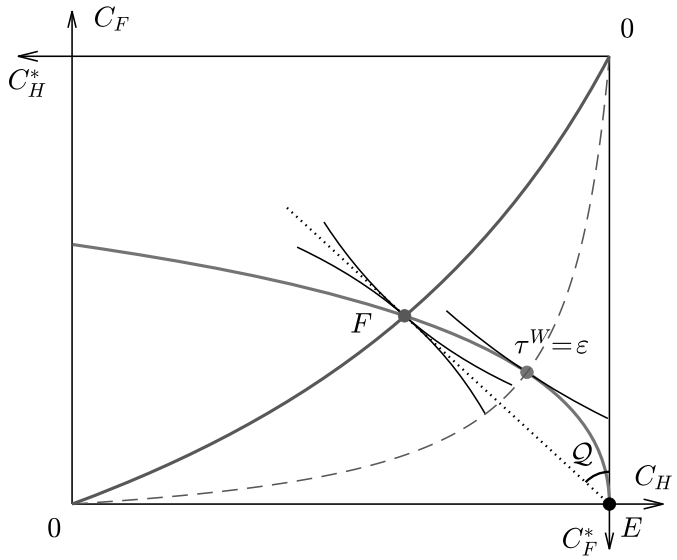


Figure 1: Laissez faire allocation and the optimal tariff

Note: The figure plots the Edgeworth box for the home economy and the rest of the world with $E = (Y, 0)$ denoting the home endowment point. The blue contract curve is the locus of all points such that $\frac{u_H}{u_F} = \frac{u_H^*}{u_F^*}$. The red curve is the implementability constraint for the home planner, $C_H + g(C_F) = Y$. The dotted black line is the budget constraint without tariffs, and the black dashed lines are the indifference curves. Blue dot F denotes the free-trade equilibrium, and red dot is the allocation under the optimal home unilateral tariff $\tau^W = \varepsilon > 1$.

optimal tariff condition $u_H \cdot g' = u_F$. That is, instead of (8), the laissez-faire allocation features:

$$\frac{u_F}{u_H} = \frac{C_H^*}{C_F}, \tag{10}$$

which is the result of the relative price equalization, $\frac{P_F}{P_H} = \frac{P_F^*}{P_H^*}$, and the trade balance (5) requirement that $\frac{P_F^*}{P_H^*} = \frac{C_H^*}{C_F}$. Whenever the planner faces non-constant relative prices of imports and exports, there is a motive for the optimal trade policy, as we illustrate in Figure 1. The figure plots the home planner's implementability constraint, $C_H + g(C_F) = Y$ (red curve), which crosses the contract curve (in solid blue) at the laissez-faire equilibrium point, and it illustrates the optimal tariff $\tau^W = \varepsilon$ when $\varepsilon > 1$. The optimal tariff lies on a distorted contract curve (dashed blue) along which the relative prices feature a wedge τ^W, namely $\frac{P_F}{P_H} = \tau^W \frac{P_F^*}{P_H^*}$ (see Aguiar, Itskhoki, and Mukhin 2025).

Further progress can be made in characterizing the optimal tariff if we leverage — for the first time in our analysis — the assumption of CES preferences in foreign economy.

Proposition 1 *The optimal tariff is positive and equal to:*

$$\tau^W = \varepsilon = 1 + \frac{1}{\eta - 1} \frac{1}{\Lambda^*} > 1, \tag{11}$$

where $\Lambda^* \equiv \frac{C_F^*}{Y^*} = \frac{P_F^* C_F^*}{P^* C^*}$ *is the foreign expenditure share on local goods, that is, an inverse measure of openness of the rest of the world to the home economy.*

Thus, it is generically optimal for home to impose a tariff to exploit the terms-of-trade externality and extract rents from the rest of the world, as illustrated in Figure 1. Formula (11) shows that the size of the optimal tariff depends on two sufficient statistics determining the elasticity of foreign demand and hence, the market power of the home economy. In particular, the optimal tariff is higher when the goods are less substitutable from the point of view of foreign consumers (η is low). This motive applies even when home country is small, i.e., $\Lambda^* = 1$, in which case the expression reduces to a standard markup formula $\tau^W = \frac{\eta}{\eta-1}$.[4] The optimal tariff is also higher when the home market accounts for a larger fraction of foreign sales, $\Lambda^* < 1$, giving the planner additional monopoly power. Notably, the latter sufficient statistic in a two-country model with trade balance is the same as the one that appears in the gains-from-trade formula of Arkolakis, Costinot, and Rodríguez-Clare (2012).[5]

We take the result in Proposition 1 as the baseline starting point for our analysis of the optimal tariff under alternative objectives and with bilateral and aggregate trade imbalances.

3.2 Fiscal revenues

An alternative goal of trade policy that has been frequently mentioned in recent debates is to raise additional fiscal revenues for the government. Although the standard Diamond and Mirrlees (1971) argument implies that it is suboptimal to tax differentially home and foreign producers to collect fiscal revenues, it might still be instructive to solve for the peak of the Laffer curve for tariffs.

To this end, suppose that the goal of the policy is to maximize fiscal revenues (in units of local goods). As Lerner symmetry applies not only to allocations, but also government revenues, it is without loss of generality to focus on import tariffs and maximize $(P_F - P_F^*)C_F/P_H$.[6] Using domestic demand (3) and balanced trade (5), the planner's problem can be expressed as

$$\max_{C_F} \qquad \frac{u_F(Y - g(C_F), C_F)}{u_H(Y - g(C_F), C_F)} C_F - g(C_F),$$

where we have substituted in the resource and implementability constraints. For separable

[4]Note that the planner would impose this tariff even if domestic firms set markups both in the home and the foreign markets, as it aims to distort the relative price between the two markets according to (6).

[5]See Gros (1987) for the optimal tariff in the Krugman model, Alvarez and Lucas (2007) in the Eaton-Kortum model, and Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung, and Larch (2013) for the optimal tariff in the Melitz model. Caliendo and Parro (2022) provide a survey of these and other related results in the literature, while Humphrey (1995) reviews the early contributions based on the geometric approach.

[6]Indeed, using the balanced trade condition, government revenues from import tariffs can be expressed as

$$\frac{(P_F - P_F^*)C_F}{P_H} = (\tau^I - 1)\frac{P_F^* C_F}{P_H} = (\tau^I - 1)\frac{P_H^* C_H^*}{P_H} = (\tau^I - 1)C_H^*.$$

At the same time, fiscal revenues from export tariffs are given by $\frac{(P_H^* - P_H)C_H^*}{P_H} = (\tau^E - 1)C_H^*$. Given that the allocation does not depend on the choice of the instrument, it follows that taxes collected are the same in two cases when $\tau^I = \tau^E$ (*cf.* Itskhoki and Mukhin 2022).

home preferences, the optimality condition can be written in terms of elasticities as

$$\frac{u_F}{u_H}\left(1+\frac{u_{FF}C_F}{u_F}+\frac{u_{HH}C_H}{u_H}\frac{g'C_F}{g}\frac{C_H^*}{C_H}\right)=\frac{g'C_F}{g}\frac{P_F^*}{P_H^*}.$$

Noticing that the implied import tariff can be inferred as $\tau^R=\frac{u_F/u_H}{P_F^*/P_H^*}$ and applying the formula to CES preferences from above, we get the following result:

Proposition 2 *The revenue-maximizing tariff is given by*

$$\tau^R=\frac{\theta\varepsilon}{\theta-1-\varepsilon\frac{1-\Lambda}{\Lambda}}\geq\varepsilon=\tau^W, \tag{12}$$

where $\varepsilon=1+\frac{1}{\eta-1}\frac{1}{\Lambda^*}$ *is the foreign elasticity as in* (11) *and* $\Lambda\equiv\frac{C_H}{Y}$ *is the domestic output share.*

In contrast to a benevolent planner from the previous section, a planner that aims to collect maximum fiscal revenues puts zero weight on the distortionary effects of tariffs, and hence $\tau^R\geq\tau^W$. The trade-off and the bounded solution arise from a standard Laffer-curve logic: a higher tariff decreases gross trade flows and therefore, diminishes the tax base. A higher home elasticity — determined by θ and Λ — means that households switch more easily to domestic goods when import prices go up due to tariffs and therefore, lower the optimal τ^R.[7]

Comparing formulae (11) and (12), it is clear that the tariff that maximizes fiscal revenues is, in general, higher than the optimal one from the welfare perspective, i.e., $\tau^R>\tau^W$. Intuitively, this is because the planner aims to extract rents from both home and foreign households in the former case and only from foreign households in the latter case.[8] In particular, when $\eta\to\infty$ and it becomes impossible to get rents from the term-of-trade manipulation, a benevolent planner does not use any tariffs $\tau^W=1$, while the revenue-maximizing planner still extracts rents from home households by setting $\tau^R\to\frac{\theta}{\theta-1/\Lambda}>1$ when $\theta>1/\Lambda$ and $\tau^R\to\infty$ otherwise. Symmetrically, when domestic elasticity between home and foreign goods goes up $\theta\to\infty$ and it becomes impossible to extract rents from local households, the optimal fiscal tariff converges to the welfare-maximizing one and depends only on foreign elasticity $\tau^R\to\varepsilon$.

Corollary 1 (a) *When home and foreign goods are perfect substitutes for home households,* $\theta\to\infty$*, the same optimal tariff maximizes welfare and fiscal revenues* $\tau^W,\tau^R\to\varepsilon$*, extracting surplus from foreigners.* (b) *When the foreign country features a perfectly elastic demand*

[7]Notice that in contrast to Λ^*, the home analog cannot be interpreted as a spending share from Arkolakis, Costinot, and Rodríguez-Clare (2012) as $\Lambda=\frac{P_H^*C_H}{P_H^*Y}\neq\frac{P_HC_H}{PC}$ due to tariffs.

[8]Mathematically, the difference between welfare and revenue maximization problems can be seen by leveraging properties of homothetic preferences and rewriting the planner's problem in two cases respectively as

$$\max_{C_F}\quad u_H(Y-g(C_F))+u_FC_F\qquad\text{vs.}\qquad\max_{C_F}\quad\frac{u_H(Y-g(C_F))+u_FC_F}{u_H}-Y.$$

for the home good, $\eta \to \infty$, the welfare-maximizing home tariff is zero, $\tau^W = \varepsilon = 1$, while the revenues maximizing tariff is positive, $\tau^R = \frac{\theta}{\theta - 1/\Lambda} > 1$, and finite when $\theta > 1/\Lambda \geq 1$, extracting surplus from domestic households.

Lastly, we quantify the difference between a welfare-maximizing and a revenue-maximizing tariff. We use the conventional value for the trade elasticities, $\theta = \eta = 4$, and calibrate the local shares Λ and Λ^* using data on GDP and trade flows between the US and the rest of the world prior to the tariff.[9] The welfare-maximizing tariff is then $\tau^W - 1 = 34\%$, while the revenue-maximizing tariff is a lot higher, $\tau^R - 1 = 80\%$. Interestingly, the home welfare increases in both cases — by 0.6% of aggregate consumption under τ^W, and by a half of that (0.3%) under τ^R. We discuss the case with retaliation below.

3.3 Manufacturing jobs

Another often discussed objective of trade policy is to bring home manufacturing jobs that have been outsourced to countries with lower labor costs. The argument is based on political reasons, security concerns and growth externalities.[10] What is the optimal tariff in this case?

To address this question, we consider an extension of the baseline model with endogenous production and an endowment of labor that is endogenously allocated between tradable and non-tradable sectors:

$$C_N = Y_N = F_N(L_N), \qquad Y = F_T(L_T), \qquad L_N + L_T = L,$$

where production functions exhibit decreasing returns to scale and satisfy the Inada condition. The tradable output is allocated between domestic consumption and exports in line with the market clearing condition (1). Households preferences are described by a nested CES utility:

$$u = \frac{\rho}{\rho - 1}\left(\kappa C_N^{\frac{\rho-1}{\rho}} + C_T^{\frac{\rho-1}{\rho}}\right), \qquad C_T = \left[(1-\gamma)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \qquad \rho \leq \theta,$$

where the separability between tradables and non-tradables is without loss of generality and the elasticity across tradables is higher than between tradables and non-tradables. Given a symmetric structure for the foreign economy, one can still derive the trade possibility frontier $C_H^* = g(C_F)$ from the implementability constraint (7).[11]

The main difference from the baseline model is the endogenous allocation of labor across the two sectors. The most efficient way of addressing this margin would be to use a sector-

[9] For consistency, we impose balanced trade and compute trade flows as the average of US imports and exports. The next section shows that imbalances have a first-order impact on the value of the optimal tariff.

[10] See Krugman (1987), Venables (1987), Rodrik (1998), Ossa (2011), Itskhoki and Moll (2019) and Benigno, Fornaro, and Wolf (2025).

[11] With foreign output endogenously produced according to $Y^* = F_T^*(L_T^*)$ and $Y_N^* = F_N^*(L^* - L_T^*)$, the elasticity of TPF $g(\cdot)$ depends not only on u_j^*s, but also on $F_T^{*\prime}$ and $F_N^{*\prime}$.

specific labor tax, but if such a policy tool is not available to the planner, he can use trade tariffs as a second-best instrument to reallocate workers to tradable sector subject to an additional implementability constraint. In particular, a free worker mobility equalizes wages across sectors, and firms' optimality conditions determine the allocation of labor as a function of relative prices of final goods:

$$\frac{P_H}{P_N} = \frac{W/F_T'}{W/F_N'} = \frac{F_N'(L - L_T)}{F_T'(L_T)}. \tag{13}$$

This condition can be interpreted as the relative supply of goods condition, and it shows that to maximize employment in the tradable sector, the planner needs to increase the relative price of the tradable good $p \equiv P_H/P_N$. Since $F_j'(\cdot)$ are both monotonic, we can invert this condition to express tradable employment as an increasing function of the this relative price, $L_T = f_S(p)$ such that $f_S'(\cdot) > 0$. This home relative price is a sufficient statistic for L_T in the sense that trade policy does not shift the $f_S(\cdot)$ schedule directly, only via its effect on equilibrium p.

The household optimization provides a corresponding relative demand curve:

$$\frac{P_H}{P_N} = \frac{u_H}{u_N} = \frac{u_H\big(F_T(L_T) - g(C_F), C_F\big)}{u_N\big(F_N(L - L_T)\big)}, \tag{14}$$

where the latter equality uses the separability of preferences to express u_H and u_N as functions of only tradable and non-tradable consumption, respectively, and substitutes in the trade possibility frontier for $C_H^* = g(C_F)$. Given the monotonicity of marginal utilities, we can invert this condition to express tradable employment as a decreasing function of $p = P_H/P_N$, $L_T = f_D(p; C_F)$ such that $\frac{\partial}{\partial p} f_D < 0$ and $\frac{\partial}{\partial C_F} f_D > 0$, and hence this schedule depends on the trade equilibrium directly as we emphasize with its dependence on C_F.[12]

The equilibrium relative price p ensures equality of relative supply and relative demand given trade policy, $f_S(p) = f_D(p; C_F)$, such that equilibrium p is an increasing function of C_F. As a result, the equilibrium tradable employment is also an increasing function of imported quantity C_F, as we illustrate in Figure 2. Greater imports C_F require greater exports according $C_H^* = g(C_F)$, and hence reduce the quantity of home tradables available for home consumption, increasing their price relative to non-tradables p, drawing more labor towards the tradable sector and away from non-tradable sector.

As we show more formally below, it follows that the planner's problem of using a tariff τ to maximize manufacturing employment L_T is equivalent to maximizing the quantity of imports C_F, or also the quantity of exports $C_H^* = g(C_F)$. This argument takes account of the fact that tradable employment is used both for import substitution and for exports, and shows that the latter effect generally dominates when the policy instrument is a uniform tariff. Intuitively, tradable employment is increasing in the size of the tradable sector, which is achieved by

[12] More generally, this schedule depends on both the point C_F and the trade possibility frontier $g(\cdot)$, such that an increase in foreign endowment Y^* also shifts this curve, as we discuss below.

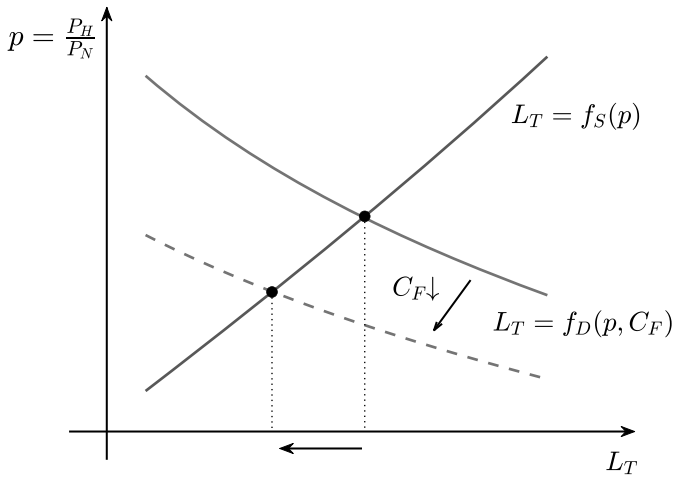


Figure 2: Tradable-sector employment

Note: The figure plots the relative supply (13) and relative demand (14) curves determining the equilibrium tradable-sector employment L_T and the relative price of home tradables $p = P_H/P_N$. Trade policy shifts import quantities C_F, which changes the relative demand for home tradables. Both a "China shock" ($Y^* \uparrow$) that shifts down $C_H^* = g(C_F; Y^*)$ and the tariff τ that reduces (C_H^*, C_F) along a given $C_H^* = g(C_F)$ result in lower tradable-sector employment.

subsidizing total trade, whether via import subsidy or export subsidy that are equivalent under Lerner symmetry. A trade subsidy expands tradable consumption around the world, and hence tradable employment, while a trade tax lowers it.

Proposition 3 *Holding fixed foreign preferences and productivity, the equilibrium tradable-sector employment L_T is monotonically increasing in the quantity of gross imports C_F and correspondingly gross exports $C_H^* = g(C_F)$. Therefore, an (additional) objective of increasing tradable-sector employment is achieved with a trade subsidy, $\tau^L < 1$ (or a lower trade tax).*

If the full objective is $\max\{u + \lambda L_T\}$, where λ is the planner's weight on tradable employment, then the optimal tariff τ^L maximizing this objective is strictly lower than the welfare maximizing tariff $\tau^W = \arg\max u$. This statement is true for any external objective: adding a weight on tradable employment reduces the optimal tariff. Indeed, condition (14) implies that $\partial L_T/\partial \tau < 0$ as an increase in τ reduces both imports C_F and exports $C_H^* = g(C_F)$, and hence the demand for the home tradable good. In the limiting case where the manufacturing employment L_T is the sole objective (i.e., when $\lambda \to \infty$), this goal is achieved with an infinite trade subsidy, $\tau^L \to 0$, which results in $C_N = C_H = 0$, $C_H^* = Y = F_T(L)$ and $C_F = g^{-1}(Y)$. In words, the planner shifts all employment into the tradable sector by means of maximizing export (and hence import) quantities. To conclude, it is counterproductive to impose an import tariff if the goal is increasing tradable manufacturing employment.[13]

[13]Note that our argument relies on the assumption of homogenous labor input and a uniform import tariff, but otherwise is robust to the details of production that allows for both exports and import substitution. Our emphasis is that the benchmark for expanding tradable employment must be a tradable-sector subsidy, not a tax or tariff, and the case for a tariff requires very specific circumstances that need to be spelled out.

Importantly, this does not mean that the model is inconsistent with the evidence on the effects of "China shock" (Autor, Dorn, and Hanson 2013) as a positive foreign productivity shock that increases Y^* and shifts down the trade possibility frontier function $C_H^* = g(C_F; Y^*)$, which pushes labor away from tradable and towards the non-tradable sector. Alternatively, and as we discuss in Section 4.3, the "China shock" can be a result of an increased foreign savings deposited in the US assets, in which case the shift in g is also associated with a trade imbalance. Irrespectively of the nature of this shock, Proposition 3 shows that trade policy that aims to preserve tradable employment does not try to undo the shock by raising prices of imported goods, but instead subsidizes trade (export and/or imports).

3.4 Retaliation

While the baseline model abstracts from foreign trade policy, our main results extend to a setting whether both countries choose optimal tariffs. To see this, allow for foreign import and export tariffs τ^{I*} and τ^{E*}. The pricing equation (2) is then replaced with

$$P_F = \tau^I \tau^{E*} P_F^* \qquad \text{and} \qquad P_H^* = \tau^E \tau^{I*} P_H, \tag{2'}$$

while the market clearing (1) and household demand (3)-(4) remain unchanged. It follows that the relative prices in two countries are distorted by both home and foreign tariffs

$$\frac{P_F}{P_H} = (\tau^I \tau^E)(\tau^{I*} \tau^{E*}) \frac{P_F^*}{P_H^*}.$$

Note that this tax wedges can also represent markup wedges and other sources of law of one price deviations.

Combining household and government budget constraints with new prices (2′), the balanced trade condition can be expressed as

$$P_H^* C_H^* = (\tau^{I*} \tau^{E*}) P_F^* C_F. \tag{5'}$$

We consider a Nash equilibrium with each country setting its trade policy taking tariffs in the other economy as given.

Because tariffs only appear in the latter two equilibrium conditions, and import and export taxes enter together, Lerner symmetry still holds in this setting and it is sufficient to focus on $\tau = \tau^I \tau^E$ and $\tau^* = \tau^{I*} \tau^{E*}$. Furthermore, as before, equations (4) and (5′) result in the implementability constraint $C_H^* = g(C_F, \tau^*)$ for the home planner:

$$u_H^*(C_H^*, Y^* - C_F) C_H^* = \tau^* u_F^*(C_H^*, Y^* - C_F) C_F.$$

Since τ^* only affects home planner's problem through $g(\cdot)$, foreign elasticity ε remains a sufficient statistic for the optimal tariff best response, as shown by Johnson (1953). Furthermore,

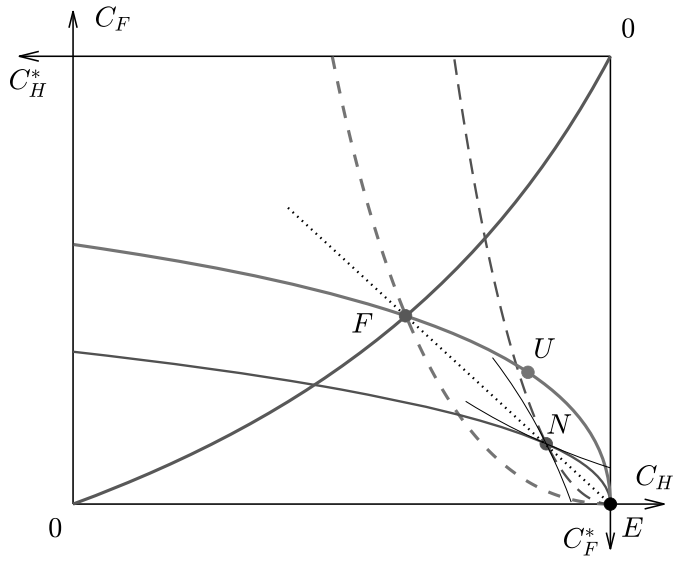


Figure 3: Free trade allocation and the tariff war Nash equilibrium

Note: see notes to Figure 1. The dashed red curve is the trade possibility frontier from the perspective of the foreign country when home tariff $\tau = 0$, i.e., $C_F = g^*(C_H^*; \tau = 0)$, while the solid red curve is still $C_H^* = g(C_F; \tau^* = 0)$. The intersection of these two red curves is the free trade equilibrium F. The corresponding purple curves plot $C_F = g^*(C_H^*; \tau = \varepsilon)$ and $C_H^* = g(C_F; \tau^* = \varepsilon^*)$, and their intersections corresponds to the trade war Nash equilibrium N, as discussed in the text. Point U corresponds to the unilateral home tariff.

once expressed in terms of sufficient statistics, the formula for optimal tariff remains exactly the same as before and, combined with a symmetric argument for the foreign economy, results in the following Nash equilibrium:

$$\tau = \varepsilon = 1 + \frac{1}{\eta - 1}\frac{1}{\Lambda} \qquad \text{and} \qquad \tau^* = 1 + \frac{1}{\theta - 1}\frac{1}{\Lambda^*}.$$

Of course, the equilibrium allocation and the values of tariffs are different when the foreign country also imposes a tariff. A positive foreign tariff implies that both economies become more closed, i.e., Λ, Λ^* increase, and the optimal tariff for home is lower. Thus, the best response functions in the space of tariffs are negatively slopped and the optimal tariffs in two countries are strategic substitutes.

We illustrate the trade war equilibrium in Figure 3 and summarize the results as follows:

Proposition 4 *The Nash equilibrium tariffs (τ, τ^*) have the same structure as the unilateral optimal tariffs characterized in Proposition 1, with $\tau = \varepsilon$ the elasticity of $g(\cdot)$ and $\tau^* = \varepsilon^*$ the elasticity of $g^*(\cdot)$, and additionally satisfy the fixed point:*

$$C_H^* = g(C_F, \tau^*) \qquad \textit{and} \qquad C_F = g^*(C_H^*, \tau),$$

where $g^*(\cdot)$ is the foreign trade possibility frontier given the home tariff τ. Under CES utility, these tariffs are smaller than the optimal unilateral tariffs, $\tau \leq \tau^W$ and $\tau^* \leq \tau^{W*}$ (with strict inequalities when both countries are large, $\Lambda, \Lambda^* < 1$), but the combined distortion is larger.

Finally, we provide a back-of-the-envelope quantification of the consequences of a trade war between the US and the rest of the world by following the approach outlined after Corollary 1 above. The Nash-equilibrium tariffs are marginally lower but quantitatively similar to the welfare-maximizing tariffs (33% versus 34%) emphasizing that a tariff is a dominant strategy irrespective of the strategy of the trade partner. However, the welfare consequences of equilibria with and without tariffs are vastly different: while a US unilateral tariff *raises* the US welfare by 0.6% of aggregate consumption, a trade war *lowers* the US welfare by 2.7%.

4 Global Imbalances

We return to the two-region framework (with home and the rest of the world), and consider the case with an aggregate trade imbalance. In particular, we first ask if and when a trade tariff can have an effect on the external balance. We then solve for the optimal long-run tariff in a model with trade imbalances, as well as characterize trade policy that closes the long-run trade deficit.

4.1 Long-run trade imbalance

We derive in this section a general restriction on the long-run trade (im)balance that must be satisfied in this model and is a consequence of the country budget constraint. We use this restriction as a guiding principle for the more specific analysis that follows.[14]

We focus on the home economy in a fully dynamic environment with a general portfolio choice. In any period t, home starts with vector of asset holdings B_{t-1}^j for $j \in J_{t-1}$ inherited from period $t-1$, where $B_{t-1}^j < 0$ means a liability. At t, asset j pays dividends D_t^j and its post-dividend price is denoted Q_t^j. Therefore, $R_t^j \equiv (Q_t^j + D_t^j)/Q_{t-1}^j$ is the realized one-period return on asset $j \in J_{t-1}$ at t. We denote with $\bar{R}_t$ the risk-free interest rate between periods t and $t+1$ (and known at t, hence the subscript). All asset returns, pay-outs and prices are quoted in the home currency (dollar) for concreteness, but the underlying assets can pay fixed income in foreign currency as well.

The new asset positions at time t are denoted with B_t^j for $j \in J_t$, and $\mathcal{B}_t \equiv \sum_{j \in J_t} Q_t^j B_t^j$ denotes the new net foreign asset position that is carried from period t into period $t+1$. We also write the pay-out on the entire net foreign asset position as $\mathcal{R}_t \mathcal{B}_{t-1} \equiv \sum_{j \in J_{t-1}} (Q_t^j + D_t^j) B_{t-1}^j$, where $\mathcal{R}_t$ can be thought as the realized rate of return on the entire NFA position.[15]

In general, the return $\mathcal{R}_t$ is different from the risk-free rate $\bar{R}_{t-1}$, both ex ante in expectation and ex post realization-by-realization. However, if there exists no arbitrage that can be

[14] This section is based on our note "Can a tariff be used to close a long-run trade deficit?" (posted 04/10/2025).

[15] Note that $\mathcal{R}_t$ is well-defined from the value of $\mathcal{R}_t \mathcal{B}_{t-1}$ whenever $\mathcal{B}_{t-1} \neq 0$, and otherwise we work directly with $\mathcal{R}_t \mathcal{B}_{t-1}$ which is always well-defined and can generally be non-zero even when $\mathcal{B}_{t-1} = 0$.

constructed using available returns $\{R^j_{t+1}\}_{j\in J_t}$, then the following result links $\mathcal{R}_{t+1}$ and $\bar{R}_t$:[16]

Lemma 3 *If there is no arbitrage for asset returns in J_t, then the pay out on the international asset portfolio $\mathcal{R}_{t+1}$ satisfies:*

$$\mathbb{E}_t\{\Theta_{t+1}(\mathcal{R}_{t+1} - \bar{R}_t)\} = 0, \tag{15}$$

where Θ_{t+1} is any stochastic discount factor that prices all assets $j \in J_t$.

This lemma shows that — in expectation and appropriately adjusted for risk — the return on the NFA position $\mathcal{R}_{t+1}$ is still linked to the risk-free rate $\bar{R}_t$, unless portfolio returns offer arbitrage opportunity. Of course, the latter possibility must not be excluded for a country like the United States which arguably enjoys an "exorbitant privilege" on its international portfolio (Gourinchas and Rey 2014).

With the general notation introduced above, the balance of payments requires that:

$$\mathcal{B}_t - \mathcal{R}_t\mathcal{B}_{t-1} = NX_t, \tag{16}$$

where NX_t is the dollar value of the country's trade balance, that is, the value of home exports minus the value of home imports in period t net of domestic tariffs (i.e., calculated using the rest-of-the world trade prices). Aggregating this flow country budget constraints and discounting using the risk-free interest rate, we obtain the intertemporal budget constraint that must hold along any future path of histories:

$$\mathcal{R}_t\mathcal{B}_{t-1} + \sum\nolimits_{s=t}^{\infty} q_{t,s}NX_s + \sum\nolimits_{s=t}^{\infty} q_{t,s+1}\left(\mathcal{R}_{s+1} - \bar{R}_s\right)\mathcal{B}_s - \lim_{s\to\infty} q_{t,s}\mathcal{B}_s = 0, \tag{17}$$

where $q_{t,t} \equiv 1$ and $q_{t,s} \equiv \left(\prod_{\ell=t}^{s-1}\bar{R}_\ell\right)^{-1}$ for $s \geq t+1$ is the risk-free discount factor based on linked one-period risk-free rates $\bar{R}_t$. If the risk-free rate is constant over time, $\bar{R}_s = 1/\beta$ for all s, then $q_{t,s} = \beta^{s-t}$ is geometric discounting. The last term $\lim_{s\to\infty} q_{t,s}\mathcal{B}_s = 0$ by the no-bubble condition on the international asset position (non-explosive NFA).[17]

We can rewrite the intertemporal budget constraint (17) resulting in:

[16] No arbitrage means that there exists no real vector $\{\alpha_{jt}\}_{j\in J_t}$ such that $\text{var}_t(\sum_j \alpha_{jt}R^j_{t+1}) = 0$ and $\mathbb{E}_t\sum_j \alpha_{jt}R^j_{t+1} > 0$. Then there exists a stochastic discount factor Θ_{t+1} that prices all assets $j \in J_t$ such that $\mathbb{E}_t\{\Theta_{t+1}R^j_{t+1}\} = 1$. In particular, the risk-free rate satisfies $\bar{R}_t = 1/\mathbb{E}_t\Theta_{t+1}$, where this is either a result if the risk-free rate is available at t (i.e., part of set J_t) or otherwise the definition of the shadow risk-free rate. See, for example, Campbell (2017).

[17] Some models may violate the no-bubble condition and then these deviation should be included in the financial position as part of the excess returns.

Proposition 5 *The long-run trade deficit (more generally, imbalance) is fully determined by the financial position of a country:*

$$\underbrace{-\sum_{s=t}^{\infty} q_{t,s} NX_s}_{\text{long-run trade deficit}} = \underbrace{\bar{R}_{t-1}\mathcal{B}_{t-1}}_{\substack{\text{exogenous} \\ \text{initial NFA}}} + \underbrace{(\mathcal{R}_t - \bar{R}_{t-1})\mathcal{B}_{t-1}}_{\substack{\text{on-impact} \\ \text{valuation effect}}} + \underbrace{\sum_{s=t+1}^{\infty} q_{t,s} \left(\mathcal{R}_s - \bar{R}_{s-1}\right) \mathcal{B}_{s-1}}_{\text{future realized excess returns}}, \quad (18)$$

that is, it does not change in response to a policy unless the policy results in a valuation effect on the existing net foreign asset position or a change in the future excess returns on the international portfolio of the country.

This is the sense in which the long run trade deficit is entirely determined by the financial position of the country — a combination of existing net foreign assets and the stream of valuation effects and excess returns on the gross assets and liabilities.

A corollary of equation (18) is that the long-run trade deficit can change only as a result of either the valuation effect on impact $\mathcal{R}_t$ on the pre-determined initial NFA position $\mathcal{B}_{t-1}$, or as a result of changes in future excess returns reflected in the terms $\left(\mathcal{R}_s - \bar{R}_{s-1}\right) \mathcal{B}_{s-1}$ for $s > t$. Equation (18) further simplifies when all assets j are priced by the same stochastic discount factor, as in Lemma 3, in which case the last term in (18) must be zero in expectation using this discount factor as there can be no expected risk-adjusted excess returns. As a result, the only change in the expected long-run trade deficit can come as a result of a surprise valuation effect on impact of the policy announcement at t.

Corollary 2 *If there is no arbitrage and all assets $j \in J_s$ can be priced by Θ_{s+1} for $s \geq t$, then:*

$$\underbrace{-\sum_{s=t}^{\infty} \mathbb{E}_t\{\Theta_{t,s} NX_s\}}_{\text{expected long-run trade deficit}} = \bar{R}_{t-1}\mathcal{B}_{t-1} + (\mathcal{R}_t - \bar{R}_{t-1})\mathcal{B}_{t-1}, \quad (19)$$

where $\Theta_{t,t} \equiv 1$ and $\Theta_{t,s} \equiv \prod_{\ell=t}^{s-1} \Theta_{\ell,\ell+1}$ for all $s \geq t+1$, hence $\mathbb{E}_t \Theta_{t,s} = q_{t,s}$. Therefore, the change in the expected long-run trade deficit at t equals the on-impact valuation effect $\mathrm{d}\mathcal{R}_t\mathcal{B}_{t-1}$.

Proposition 5 and Corollary 2 establish our claim that the long-run trade deficit is pinned down by the financial position of the country, not by relative prices in the goods market. As a result, unless the financial position of the country is affected by the tariff policy, the tariff has no effect on the long-run trade deficit.[18] Equations (18) and (19) describe the set of possible outcomes: the financial position of the country can change as a result of the valuation effect on the initial NFA position or the change in the future path of excess returns on international

[18] With the unchanged long-run trade imbalance (as we define it), the dynamic path of period-by-period trade deficits (and surpluses) can be affected by static or dynamic trade policies, as greater trade frictions reduce the magnitudes of equilibrium imbalances other things equal (Obstfeld and Rogoff 2001, Costinot and Werning 2025). See also Cuñat and Zymek (2024) on the effects of trade barriers on the bilateral and aggregate trade imbalances in the long run.

assets and liabilities. In models where there are no arbitrage opportunities, this latter channel is closed, and the expected long-run trade deficit only responds to on-impact valuation effects. If such valuation effects are absent, $\mathrm{d}\mathcal{R}_t\mathcal{B}_{t-1} = 0$, the long-run trade deficit is exogenous to the policy altogether.[19]

Tariffs do, in general, result in valuation effects on a country's international portfolio. These effects are, however, often independent from the goods market elasticities and terms of trade effects. In what follows, we study various environments where tariffs have such valuations effects, and characterize both the optimal tariff under such circumstances, as well as the possibility of using the tariff to close the long-run trade deficit.

4.2 Net foreign assets and valuation effects

Given the results of the previous section, we now consider a long-run (one-period) model with two countries and a given international asset position. In this model, we identify circumstances when tariffs have valuation effects on the international portfolio, and thus when they can be used to affect the equilibrium (long-run) trade imbalance. The static nature of the model implies that the valuation effect of a tariff must be on impact, and it excludes the possibility of the effect via future excess returns, to which we return in the following section.

We extend the baseline model from Section 2.1 by assuming that the home country holds an international portfolio $(-B, B^*)$ with total net value of $P_F^*B^* - P_H B$ in dollar terms. This permits two possible interpretations. First, note that the local value of home and foreign output is given by $P_H Y$ and $P_F^* Y^*$, respectively. Therefore, B^* can be interpreted as home holdings of foreign equity, and B as foreign holdings of home equity. Second, assuming that monetary policy in each economy stabilizes prices of local goods, B and B^* can represent positions in nominal bonds denominated in home and foreign currency, respectively.[20] We allow the asset positions to take negative values, and we abstract from defaults and expropriations.

The equilibrium conditions remain largely unchanged, including market clearing (1), the pricing block (2), and household optimization (3) and (4). However, the balanced-trade condition (5) needs to be replaced with a country budget constraint that takes into account its financial position:

$$\underbrace{P_H^* C_H^* - P_F^* C_F}_{NX} + \underbrace{P_F^* B^* - P_H B}_{NFA} = 0. \tag{20}$$

In our static setup, this condition is a counterpart to the dynamic equation (19), where the NFA term combines together the value of the initial net foreign asset position and the on-

[19] This, of course, does not mean that trade policy is inconsequential: the optimal tariff holding trade (im)balance constant (e.g., under balanced trade as studied in Section 3) optimizes over the terms of exchange of exports for imports in quantity terms with the unchanged net value of these flows.

[20] In this case, $P_H = 1$ is the home monetary target, while $P_F^* = \mathcal{E}$ equals the value of the floating nominal exchange rate (in units of home currency per one unit of foreign currency), while the foreign monetary authority stabilizes the foreign-currency price of the foreign good, $P_F^*/\mathcal{E} = 1$.

impact valuation effect from trade policy. Equation (20) then shows that a country can run a (long-run) trade deficit $NX < 0$ if it has a positive net foreign asset position or pays less on its foreign debt than it earns on its foreign assets, both of which result in $NFA > 0$.

In contrast to the balanced-trade model with $B = B^* = 0$, the main observation from equation (20) is that the terms of trade $\mathcal{S} \equiv P_F^*/P_H^*$ are no longer a sufficient relative price for trade policy, and now the real exchange rate $\mathcal{Q} \equiv P_F^*/P_H$ also matters, with the export tax τ^E being the wedge between the two according to (2).[21] Using the foreign consumer optimization (4), we generalize Lemma 1 and the home planner's implementability constraint in:

Lemma 4 *The planner can choose any combination of imports C_F and exports C_H^* that satisfies the implementability condition $C_H^* = g(C_F; B/\tau^E, B^*)$ implicitly defined by*

$$u_H^*(C_H^*, Y^* - C_F) \cdot \left(C_H^* - \frac{1}{\tau^E} B\right) = u_F^*(C_H^*, Y^* - C_F) \cdot \big(C_F - B^*\big). \tag{21}$$

When $B \neq 0$, the export tax τ^E does not drop out from the country's budget constraint conditional on the terms of trade, $P_F^*/P_H^* = u_F^*/u_H^*$. As a result, Lerner symmetry does not extend to a model with net foreign assets. While export and import tariffs can deliver the same terms of trade P_F^*/P_H^* and allocations under Lerner symmetry, the real exchange rate P_F^*/P_H appreciates in response to an import tariff and depreciates under an export tax. The latter difference is inconsequential per se when $B = B^* = 0$, but results in very different valuation effects and has far reaching consequences when countries hold foreign assets.[22]

In particular, when $B \neq 0$ and the country has access to both an import tariff and an export tax, it can combine them to engineer a pure transfer from the rest of the world. For example, when $B > 0$, home can "depreciate away" its local-currency debt B by imposing an export tariff $\tau^E > 1$ and an import subsidy $\tau^I < 1$, without constraining the overall trade wedge $\tau = \tau^E \tau^I$ or the resulting terms of trade P_F^*/P_H^*. In fact, as is clear from the implementability constraint (21), it is optimal to take $\tau^E \to \infty$ and thus fully eliminate all local-currency debt by means of a valuation effect. Conversely, a country with a positive asset position in its own currency $B < 0$ can appreciate its value with a combination of an export subsidy $\tau^E < 1$ and an import tariff $\tau^I > 1$. In fact, in this case, as $\tau^E \to 0$ and $\tau^I \to \infty$, it is is possible to generate an unbounded transfer from the rest of the world and immiserate it.

Notice that in both cases, it is only home assets that can be used to generate valuation effects as the price of foreign assets is linked to the price of foreign goods, and hence their

[21] Note that $\mathcal{Q} \equiv P_F^*/P_H$ can be viewed as the producer-price real exchange rate, while the conventional CPI-based real exchange rate can be approximated as $\mathcal{Q}^C \equiv \frac{(P_F^*)^{1-\gamma^*}(P_H^*)^{\gamma^*}}{P_H^{1-\gamma} P_F^{\gamma}} = (P_F^*/P_H)^{1-\gamma-\gamma^*}(\tau^I)^{\gamma}(\tau^E)^{\gamma^*} = (P_F^*/P_H^*)^{1-\gamma-\gamma^*}(\tau^I)^{\gamma}(\tau^E)^{1-\gamma}$ (Itskhoki 2021). Irrespective of the definition used, tariffs (and, in particular, the export tax τ^E when γ is small) drive a wedge between the terms of trade $\mathcal{S} \equiv P_F^*/P_H^*$ and the real exchange rate.

[22] See related analysis of such valuation effects in Farhi, Gopinath, and Itskhoki (2014), Barbiero, Farhi, Gopinath, and Itskhoki (2019), Costinot and Werning (2019) and Itskhoki and Mukhin (2023).

international purchasing power does not change with tariff-induced valuations. Indeed, it is the ability of tariffs to change the purchasing power of home-currency assets B in terms of foreign goods that allows for international transfers, as captured by the implementability constraint (21). We summarize this discussion in:

Proposition 6 *Lerner symmetry between import and export tariffs holds under imbalanced trade if and only if the net foreign asset position is in terms of foreign assets B^* and not in home bonds or equity, $B = 0$. Otherwise, when $B > 0$, a combination of an unbounded export tax and import subsidy, or vice versa when $B < 0$, engineers a capital levy on the foreign asset position resulting in a maximum transfer from the rest of the world to home.*

4.2.1 The optimal import tariff

We now analyze the optimal import tariff $\tau = \tau^I$ in the absence of an export tax, that is, restricting $\tau^E = 1$. This is because Proposition 6 shows that either Lerner symmetry holds (when $B = 0$) and hence τ^E is redundant as in Section 3, or it fails (when $B \neq 0$) and the optimal policy involves unbounded trade taxes and subsidies if both instruments are available.

With $\tau^E = 1$, the implementability constraint (21) derived in Lemma 4 no longer depends on policy instruments, as was the case in earlier sections. As B and B^* are taken as given (along with endowments and preferences), we can write the trade possibility frontier simply as before, $C_H^* = g(C_F)$, and reproduce here the condition (21) that implicitly defines it as:

$$u_H^* \cdot (C_H^* - B) = u_F^* \cdot (C_F - B^*). \tag{22}$$

As before, the planner can choose any allocation that satisfies this condition together with resource constraints in (1).

Expressed in terms of function $g(\cdot)$, the home planner's problem remains the same as before, $\max_{C_F} u\big(Y - g(C_F), C_F\big)$. While the optimality condition, $u_H \cdot g' = u_F$, does not change and the optimal tariff is still related to the elasticity ε of the trade possibilities frontier $g(\cdot)$, the Johnson formula needs to account for the trade imbalance now:

$$\tau = \varepsilon \cdot \frac{EX}{IM}, \tag{23}$$

where $EX = P_H^* C_H^*$ and $IM = P_F^* C_F$ are home export and import values (in terms of their international purchasing power).[23] It may appear from (23) that the optimal tariff is decreasing in the trade deficit measured as IM/EX, but, of course, the value of elasticity ε is also endogenous to the trade balance.

[23]Note that with a trade imbalance, it is no longer the case that we can use the last equality in $C_F/g(C_F) = C_F/C_H^* = P_H^*/P_F^*$ to derive $\tau = \varepsilon$, as in the baseline Proposition 1. Appendix A provides a detailed derivation.

Solving for the elasticity ε using (22) under CES demand, the next proposition generalizes equation (11) from our baseline model and provides a simple characterization of the optimal tariff in terms of measurable sufficient statistics in the presence of imbalances. As before, the tariff is higher when the foreign elasticity of substitution η is low and the foreign economy is more open as measured by the inverse spending share $1/\Lambda^*$. However, in contrast to the earlier analysis, the optimal trade policy now also depends on foreign asset positions. In particular:

Proposition 7 *The optimal import tariff is given by:*

$$\tau = 1 + \frac{1}{\eta\left(1 + \frac{\bar{B}}{EX - \bar{B}}\right) - 1} \cdot \frac{1}{\Lambda^*}, \tag{24}$$

where $\bar{B} \equiv P_H^* B$ is the value of dollar debt and $\Lambda^* \equiv \frac{P_F^* C_F^*}{P^* C^*}$ is the foreign local spending share.

To what extent do trade deficits rationalize high import tariffs? Formula (24) sheds new light on the question at the heart of the policy debate. To build economic intuition, we next focus on three important limiting cases. Suppose, first, that only home assets are used in international transactions and, hence, there is no home investment in foregn bonds or equity, $B^* = 0$. The country's budget constraint then simplifies to $EX = IM + \bar{B}$ and the optimal tariff is given by $\tau - 1 = \frac{1}{\eta\left(1 + \frac{NX}{IM}\right) - 1} \frac{1}{\Lambda^*}$. In this special case, trade imbalances are indeed relevant for trade policy and the optimal tariff is increasing in trade deficits. In fact, assuming that the US market accounts for a small fraction of foreign sales $\Lambda^* \approx 1$ and taking the first-order approximation around $EX/IM = 1$, we get

$$\tau - 1 \approx \frac{1}{\eta - 1} + \frac{\eta}{\eta - 1} \frac{IM - EX}{IM},$$

an expression that closely resembles the (in)famous formula $\Delta\tau = \max\left\{10\%, \frac{1}{\varphi(\eta-1)} \frac{IM-EX}{IM}\right\}$ used by the Trump administration to rationalize the "Liberation Day" tariffs. While our model does not have incomplete pass-through calibrated by the administration to $\varphi \approx \frac{1}{\eta-1}$, the coefficient in front of trade imbalances is close to one when η is high.

This and other minor discrepancies notwithstanding, the formula has two major limitations. First, it is derived for a uniform tariff, not country-specific tariffs — a point we come back in a multi-country setting below. Second, the result relies on a counterfactual assumption that $B^* = 0$ and does not generalize to other structures of asset markets as we next show. On the one hand, it is possible to have imbalances that do not affect optimal tariffs. Indeed, in the opposite limiting case when US bonds and equity are traded only locally, $B = 0$, and all international positions are taken in foreign assets, the expression for optimal tariffs reduces to $\tau - 1 = \frac{1}{\eta-1} \frac{1}{\Lambda^*}$, just as in the baseline model with balanced trade. Thus, conditional on Λ^*, the optimal tariff is completely independent of whether the country runs a trade deficit or a

trade surplus. On the other hand, it is possible to have no imbalances and yet, the optimal tariff that depends on international portfolios. This happens when countries have significant gross positions $B, B^* \neq 0$, but the equilibrium NFA is zero, $P_F^* B^* - P_H^* B = 0$. Although there are no "imbalances" in the usual sense, the fact that $\bar{B} \neq 0$ implies that the optimal tariff will deviate from the baseline formula (11).

Corollary 3 *If only home assets are held internationally, $B^* = 0$, the optimal import tariff is increasing in trade deficits IM/EX. More generally, conditional on Λ^*, trade imbalances are neither necessary nor sufficient for international asset positions to affect the optimal tariff.*

The reason imbalances have such different implications for optimal tariffs is that it is not the trade balance per se that determines the optimal policy, but the additional valuation effects associated with gross asset positions. While the import tariff improves the terms of trade in the goods market (making foreign exports less valuable relative to foreign imports), it also increases the purchasing power of home assets held by foreigners (in terms of foreign exports). In other words, when $B > 0$, the favorable terms-of-trade effect in the goods market is partially offset by an unfavorable valuation effect on its foreign liabilities, and the optimal tariff from the perspective of the home economy is lower. This effect is absent for foreign assets B^* as their purchasing power in terms of foreign goods is independent of home tariffs, emphasizing the role a *numeraire* plays in the financial market even when prices are flexible.

This asymmetry between home and foreign assets explains why there is no one-to-one mapping between imbalances and optimal tariffs in general. Empirically, both US equity and dollar-denominated debt held by foreigners contribute to a large currency mismatch along with a negative NFA position, that is $B > B^* > 0$ (see e.g., Gourinchas and Rey 2014). Thus, despite trade deficit, the optimal tariff is lower relative to the balanced-trade case with no gross foreign assets and liabilities.[24]

Corollary 4 *Given the empirical structure of the US balance sheet, $B > B^* > 0$, the optimal import tariff is lower than under financial autarky with balanced trade ($B = B^* = 0$).*

To quantify the effects of the international asset position on the optimal tariff, we use again the data for the US and the rest of the world allowing now for differential values of imports and exports in the observed equilibrium without tariffs. In addition to trade shares and elasticities, we now calibrate $P_H B$ and $P_F^* B^*$ using two empirical moments about the US net and gross asset positions: the trade deficit of about 4% of GDP and the size of the US balance sheet (the sum of foreign assets and liabilities) of about 300% of GDP converted into flow terms using the annual interest rate of 4%. Under these circumstances, the optimal tariff according to (24)

[24]This conclusion constrasts with the results of Pujolas and Rossbach (2024) who focus on the special case of $B^* = 0$ and assume that $B < 0$ to reproduce US trade deficit. See also Ossa (2016) and Ignatenko, Lashkaripour, Macedoni, and Simonovska (2025). As we discuss below, a realistic calibration for the US incorporates convenience yield on B which allows the US to run trade deficits despite a negative NFA position with $B > B^* > 0$.

is equal to 9%, which is three times lower than in a model with balanced trade (34%). The implied welfare gains equal 0.1% of aggregate consumption and are an order of magnitude smaller than under balanced trade. The negative valuation effects due to the associated 5% appreciation of the exchange rate are quantitatively important and largely offset any gains from the improvement in the goods-market terms of trade.

Corollary 5 (a) *The US trade partners have incentives to accumulate US assets B as a hedge against a tariff and a trade war.* (b) *In a world with large foreign asset positions, the optimal tariff is lower for every country irrespective of its net asset position and trade imbalance.*

Interestingly, the insight above about the effects of foreign positions on the optimal import tariff suggests that accumulating the assets of the major trade partner also acts as a hedge against a potential trade war and reduces both the optimal tariff in case of such war, as well as incentives to begin the war altogether. In particular, this is the case when the rest of the world accumulates NFA vis-à-vis home in terms of home bonds, providing a rationale for FX reserves (*cf.* Dooley, Folkerts-Landau, and Garber 2004).

Furthermore, in a symmetric world where countries hold large gross foreign asset positions against each other, optimal import tariffs are low for everyone, and the probability of a trade war declines.[25] This further emphasizes the point that it is gross asset positions (which may be symmetric) rather than net positions and trade deficits (which are necessarily asymmetric) that matter for the optimal tariff. Note also that such portfolio structure makes the use of an export tax more appealing relative to an import tariff, as it can achieve the same terms of trade improvement, as well as a positive valuation effect. This leads to a puzzle why countries do not seem to favor export taxes, which in addition have lesser opposition from the WTO.

4.2.2 Closing the imbalance

Although our results above show that the optimal tariff is in general not directly linked to trade deficits, we nonetheless ask what trade policy, if any, can close the trade imbalance. We address this question in a context of our two-country two-asset model with the planner using only trade policy to close the imbalances.[26]

Consider the planner's problem of minimizing trade imbalances subject to the country's budget constraint and market clearing conditions, $\min(NX)^2$ subject to (1)–(4) and (20), where $NX \equiv P_H^* C_H^* - P_F^* C_F$. Of course, the most direct way of closing the imbalances would be to

[25] Gross foreign asset holding have increased around the world starting in 1980s (Gourinchas and Rey 2014), which may have contributed to the persistence of the low-tariff equilibrium and prevented the emergence of earlier trade wars (e.g., in response to large unemployment after the 2008 Global Financial Crisis). The 2025 trade war has significant international financial repercussions, as we are observing now.

[26] The analysis of this section is closely related to the contemporaneous work of Aguiar, Amador, and Fitzgerald (2025) who study trade balances implementable with import tariffs in the two countries. In contrast, we focus on unilateral policies and show that the effects of import and export tariffs are quite different in the presence of gross international positions.

use international transfers to eliminate the net foreign asset position, $NFA \equiv P_F^* B^* - P_H B$. Instead, we assume that such policy instrument is not available and the planner can only use trade tariffs.

The key observation that underlies this analysis is that to close trade imbalances, $NX = 0$, the planner needs to zero out the net valuation effects as well, $NFA = 0$. From (20), the former requires adjusting the terms of trade (ToT, $\mathcal{S}$) to equalize the value of imports and exports, while the latter is achieved by aligning the value of country's assets and liabilities via changes in the (PPI-based) real exchange rate (RER, $\mathcal{Q}$). By definition of NX and NFA in (20), we have:

Lemma 5 *Given the international asset position $(-B, B^*)$, a necessary and sufficient condition to close the equilibrium trade deficit, $NX = 0$, is that terms of trade and the real exchange rate take the following values:*

$$\mathcal{S} \equiv \frac{P_F^*}{P_H^*} = \frac{C_H^*}{C_F} \qquad \textit{and} \qquad \mathcal{Q} \equiv \frac{P_F^*}{P_H} = \frac{B}{B^*}. \tag{25}$$

The latter condition in (25) immediately implies that a zero financial position can only be achieved if $B/B^* \geq 0$, or with strict inequality if the real exchange rate must remain finite. Recall that B corresponds to "home-currency" liabilities and B^* to "foreign-currency" assets of the home economy. If the economy is a net borrower or a net saver again the other country in both currencies, no movements in relative prices of the two assets can bring the country's net asset position to zero, and additional policy tools — such as defaults and monetary inflation — are required to generate valuation effects to ensure $NFA = 0$.

Since it is highly unusual for economies to hold less than 0% and more than 100% of their own assets, we next restrict our analysis to the case when countries' portfolios are inside the Edgeworth box:

Assumption 1 $0 \leq B \leq Y, 0 \leq B^* \leq Y^*$, *that is countries take long position in foreign assets and do not short home output.*

We consider first the case when the planner can use both trade instruments and, therefore, according to Lemma 6, can generate substantial international valuation effects. In this case, the planner can implement any international allocation (C_H^*, C_F) such that $NX = 0$ with $P_F^*/P_H^* = u_F^*/u_H^*$; in other words, any allocation which satisfies $C_H^* = g(C_F)$ from Lemma 2:[27]

Lemma 6 *Suppose A1 holds and both import and export tariffs are available. Then the planner can unilaterally implement any balanced-trade equilibrium, including trade autarky.*

[27] **Proof**: Given the foreign household demand (4), the first equation in (25) is isomorphic to the balanced-trade implementability constraint (7) and describes the set of possibilities $C_H^* = g(C_F)$. For any value of imports C_F and exports C_H^* consistent with this condition, which ensures $NX = 0$, one can compute a corresponding export tariff $\tau^E \equiv \frac{P_H^*}{P_H} = \frac{P_F^*}{P_H} \cdot \frac{P_H^*}{P_F^*} = \frac{B}{B^*} \cdot \frac{C_F}{C_H^*}$, which ensures that the second equation in (25) also holds. By Lemma 5, this is sufficient to ensure that $NX = 0$. ■

Intuitively, even when Lerner symmetry holds, but also more generally (with arbitrary international asset positions), import and export tariffs have the same effects on country's terms of trade, but the opposite effects on its real exchange rate. As a result, the two trade instruments are sufficient to separately target the two prices in (25). Importantly, while there is a unique real exchange rate consistent with $NFA = 0$, namely $\mathcal{Q} = B/B^*$ for given B and B^*, there are many terms of trade satisfying $NX = 0$, namely $\mathcal{S} = C_H^*/C_F$ with the endogenous trade quantities C_H^* and C_F. As a result, the planner can pick any of the continuum of allocations consistent with $C_H^* = g(C_F)$ and market clearing conditions which ensure $NX = 0$. This includes a trade autarky equilibrium $C_F, C_H^* \to 0$ as a special case when home imposes infinite tariffs $\tau^I, \tau^E \to \infty$. The real exchange rate must remain finite and satisfy $\mathcal{Q} = P_F^*/P_H = B/B^*$ in this case, while the terms of trade $\mathcal{S} = \mathcal{Q}/\tau^E$ go to zero.[28]

Arguably, a more interesting and policy-relevant question is whether a balanced trade can be achieved with import tariffs alone. While the multiplicity of equilibria implementable with two instruments suggests a positive answer, it is less obvious how the resulting prices and allocations look like. Analytically, conditions (25) under $\tau^E = 1$ and hence $P_H = P_H^*$ imply that, if such policy exists, the balanced-trade equilibrium should satisfy:

$$\mathcal{S} = \mathcal{Q} = \frac{P_F^*}{P_H^*} = \frac{B}{B^*} = \frac{C_H^*}{C_F} = \frac{u_F^*(C_H^*, Y^* - C_F)}{u_H^*(C_H^*, Y^* - C_F)}, \tag{26}$$

where the last equality is the result of foreign household optimization (4). We start with the empirically relevant case when home (the US) runs a trade deficit and contrast it with the case of trade surpluses at the end of the section (for proof, see Lemma 7 below).

Proposition 8 *Suppose A1 holds and home runs a trade deficit $NX < 0$ under free trade, that is $NFA/P_H = (P_F^*/P_H) \cdot B^* - B > 0$ under the free trade real exchange $\mathcal{Q} = P_F^*/P_H$ given the international portfolio $(-B, B^*)$. Then there is always a unique balanced-trade equilibrium that the planner can implement with an import tariff, $\tau^I > 1$.*

The results can perhaps best be understood using the Edgeworth box shown in Figure 4. As before, the blue curve is the contract curve with marginal rates of substitution equalized across countries, $u_F/u_H = u_F^*/u_H^*$, and the red curve corresponds to the trade possibilities frontier under balanced-trade, $C_H^* = g(C_F^*)$. Point E denotes countries' initial endowments and P corresponds to their portfolios represented as claims on foreign goods. Under financial autarky, $B = B^* = 0$, and these points coincide ($P = E$), with the corresponding free-trade equilibrium in point A given by the intersection of the contract and balanced-trade curves. Line EA' corresponds then to the country's budget constraint and its slope (against a vertical line) is equal to the real exchange rate P_F^*/P_H, which coincides with the terms of trade P_F^*/P_H^*

[28]The proposition remains true in the limiting cases when only one asset is traded internationally, i.e. $B = 0$ or $B^* = 0$, although the implementation involves infinite tariffs and subsidies.

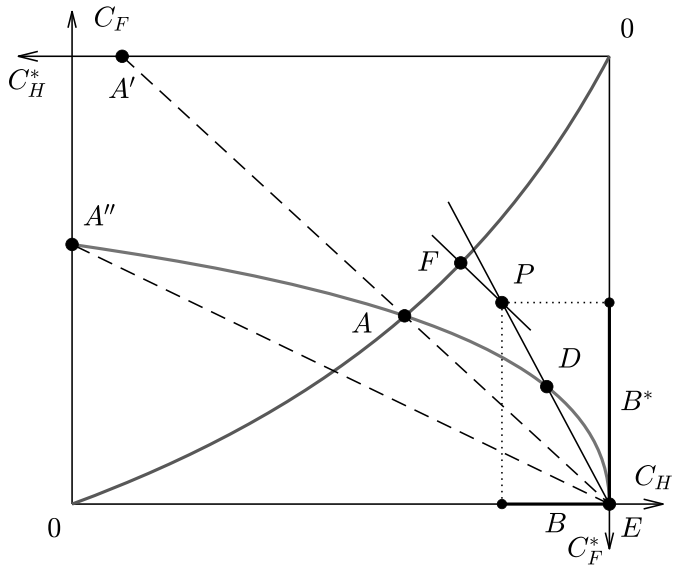


Figure 4: Closing imbalances with an import tariff

Note: Point $E = (Y, 0)$ is endowment, while point $P = (Y - B, B*)$ is endowment combined with international portfolio. The blue curve is the frictionless contract curve and the red curve is the balanced trade frontier $C_H^* = g(C_F)$, as in Figure 1. A is the free-trade equilibrium when $B = B^* = 0$ and F is the free trade equilibrium for the portfolio in P. D is the only balanced trade equilibrium feasible under an import tariff.

under laissez faire. Notice that any portfolio P that lies above (below) the EA' line results in a free-trade equilibrium F on a contract curve above (below) point A and therefore, implies that home runs a trade deficit (surplus) in the absence of government interventions. Thus, we can restrict our attention to P to the northeast from EA' with home country having relatively small liabilities B and large foreign assets B^* to enjoy $NFA > 0$ and $NX < 0$.

Suppose next that the planner aims to close imbalances in both goods and asset markets. Graphically, the new equilibrium D can be found as the intersection of the balanced trade curve and the ray EP that shows the set of points that satisfy the equality $C_H^*/C_F = B/B^*$ in (26). According to Lemma 2, the red line is strictly monotonic, convex and goes through point E, which guarantees that a zero-imbalance equilibrium always exists and is unique (more on why point E is not an equilibrium below). The move from a free-trade equilibrium to a balanced trade is achieved by imposing large enough import tariffs that shift the contract curve to go through point D. Because the new equilibrium lies to the southeast from point A, the tariff required to close imbalances is unambiguously positive, i.e. $\tau^I > 1$.

The real exchange rate plays the crucial role in the transmission of trade policy and has several surprising features. In particular, the imbalances can only be closed if the real exchange rate appreciates. Indeed, a positive initital net foreign asset position implies that $P_F^*/P_H > B/B^*$ in a free-trade equilibrium and thus, P_F^*/P_H needs to go down (corresponding to an appreciation) to close the financial imbalance. Graphically, the initial real exchange rate, $P_F^*/P_H = P_F^*/P_H^*$, is given by the slope of PF (against the vertical axis), which is always higher than the slope of ED — the real exchange rate after tariffs are imposed. This obser-

vation goes very much against a standard expenditure-switching logic that a depreciation of the exchange rate is required to improve trade balance.[29] This partial equilibrium intuition ignores the general equilibrium budget constraint and the fact that it is not possible to increase net exports without generating negative valuation effects. The latter means that the value of home liabilities needs to go up relative to the value of home foreign assets.

Moreover, this effect is completely independent of any trade shares or trade elasticities — including whether the Marhsall-Lerner condition holds — and is fully determined by international asset positions. Indeed, given the standard structure of international portfolios A1, this corresponds to an increase in P_H relative to P_F^* with the new equilibrium real exchange rate equal to B/B^*. This suggests that the exchange rate might provide a more robust target for policymakers aiming at closing the imbalances than the level of the optimal tariff, which depends on structural parameters of the models and might be hard to compute.

Corollary 6 *When initial $NFA > 0$, the real exchange rate needs to appreciate to close the trade deficit, and its new level is determined solely by financial positions, $\mathcal{Q} = B/B^*$, not trade shares or trade elasticities.*

Of course, in practice there are many asset prices other than the exchange rate that can generate a negative valuation effect. In particular, a fall in US stock prices and a rise in treasury yields decrease the value of home assets held by foreign investors, and additional valuation effects are generated by changes in future returns and interest rates. In other words, in a dynamic setting, the value of the B, B^* position can adjust through other mechanisms and generate international transfers without an appreciation, as we discuss in the next section. What our analysis shows, however, is that it would be wrong to cite the observed depreciation of the dollar — gradual since the beginning of 2025 and sharp after the "Liberation Day" — as evidence of trade policies achieving their goal of closing the trade deficit.[30]

Alternative implementations The limitations of the partial equilibrium and balanced-trade intuition are even more evident when considering the implementation of closing the imbalances. As mentioned above, a positive import tariff that increases C_F^* and lowers u_F^*/u_H^* is required to appreciate the real exchange rate and achieve $NFA = 0$. A standard Lerner symmetry argument that the same outcome can be achieved with export taxes, however, does not apply when $B \neq 0$ (recall Lemma 6). Indeed, imposing $\tau^E > 1$ lowers home exports,

[29] Note that without the export tax, $\tau^E = 1$, the terms of trade moves in lockstep with the real exchange rate, $\mathcal{S} \equiv P_F^*/P_H^* = P_F^*/P_H \equiv \mathcal{Q}$, and both appreciate (go down). This compels the foreigners to buy less of the home good C_H^* relative to their local good C_F^*, ensuring $u_F^*/u_H^* = P_F^*/P_H^*$. The import tariff, $\tau^I > 1$, in turn ensures that home households face a higher relative prices, $P_F/P_H > P_F^*/P_H^*$, and hence shift away from consuming C_F relative to C_H, to satisfy $u_F/u_H = P_F/P_H$. Thus, both countries become more home biased due to the trade distortion. Nonetheless, C_F/C_H^* increases (since EP is steeper than EF), but by less than P_F^*/P_H^* declines, to ensure $NX = P_H^* C_H^* - P_F^* C_F = 0$ under the tariff.

[30] Indeed, empirical evidence from Gourinchas and Rey (2007) shows that the depreciation of the dollar generates positive valuation effects for the US economy.

increases C_H and *depreciates* the real exchange rate $\mathcal{Q} = P_F^*/P_H = P_F/P_H = u_F/u_H$. This further increases country's net asset position $P_F^* B^* - P_H B$ and deteriorates the trade balance. It follows that an export subsidy, not a tax, is required to close the imbalances.

Intuitively, Lerner symmetry under balanced trade relies on import tariffs and export taxes resulting in the same terms of trade, $\mathcal{S} = P_F^*/P_H^*$, which is a sufficient statistic for allocations. The trade is balanced and the real exchange rate $\mathcal{Q}$ is largely irrelevant in this case, as it moves in opposite directions under the two trade policies. Instead, in a model with international asset positions, the real exchange rate is a sufficient statistic for trade imbalances. This turns Lerner symmetry on its head, making import tariffs similar to export subsidies in terms of their implications for trade deficits.

Corollary 7 *In contrast to Lerner symmetry of an import tariff and an export subsidy under balanced trade, closing trade deficits requires an import tariff $\tau^I > 1$ or an export subsidy $\tau^E < 1$.*

Of course, the terms of trade are still relevant for the allocation and the two trade policies result in different equilibria on the balanced-trade curve. Figure 5 illustrates this point showing allocations implementable with import tariffs $C_H^* = g^I(C_F)$ and allocations implementable with export taxes $C_H^* = g^E(C_F)$.[31] Under financial autarky $B = B^* = 0$, the two sets coincide with the balanced-trade curve, $g^I = g^E = g$. Furthermore, as follows from Lemma 6, $B = 0$ is sufficient for Lerner symmetry to hold, and the two instruments are equivalent (i.e., $g^I = g^E$) when home assets are not traded internationally for any value of B^* (shifting from the balanced trade curve g up for $B^* > 0$ and down for $B^* < 0$).

More generally, however, import and export tariffs result in very different allocations and the implied equilibria coincide only under free trade $\tau^I = \tau^E = 1$ (in point F). While the two tariffs result in the same shift of the contract curve (recall (6)), τ^E drives a wedge between the real exchange rate and the terms of trade, generating additional valuation effects in (20).[32] Therefore, as can be seen from the intersections of function g with g^I and g^E, respectively, the balanced-trade equilibrium implemented with an import tariff $\tau^I > 1$ features lower exports C_H^* and imports C_F than the balanced-trade allocation achieved with an export subsidy $\tau^E < 1$ (*cf.* point G and point D, which are on opposite sides of point A along the g curve in the left panel of Figure 5).[33]

Interestingly, an infinite tariff fails to eliminate trade imbalance, as in general it fails to achieve financial balance, that is, $NX, NFA \not\to 0$ as either τ^I or τ^E tend to ∞ or zero. When

[31] Note that $C_H^* = g^I(C_F)$ is defined implicitly by equation (22), while $C_H^* = g^E(C_F)$ is derived from (21) by substituting $\frac{1}{\tau^E} = \frac{u_F^*/u_H^*}{u_H/u_F}$ yielding $u_H^* C_H^* = u_F^*\big(C_F - B^* + \frac{u_H}{u_F} B\big)$, where u_js are evaluated at $(Y - C_H^*, C_F)$.

[32] In particular, this explains why point P with consumption of each good equal to corresponding dividends can be implemented as an equilibrium outcome with import tariffs, but not export tariffs. Indeed, while the implementability condition under the import tariff in (22) always permits allocation $C_H^* = B^*$ and $C_F = B$, this is not the case under the export tariff which results in $P_F^*/P_H^* \neq P_F^*/P_H$ in (20).

[33] In contrast, the optimal tariff from Proposition 7 implements allocation where the home indifference curve is tangent to g^I. Similarly, the optimal export tax ensures that the home indifference curve is tangent to g^E.

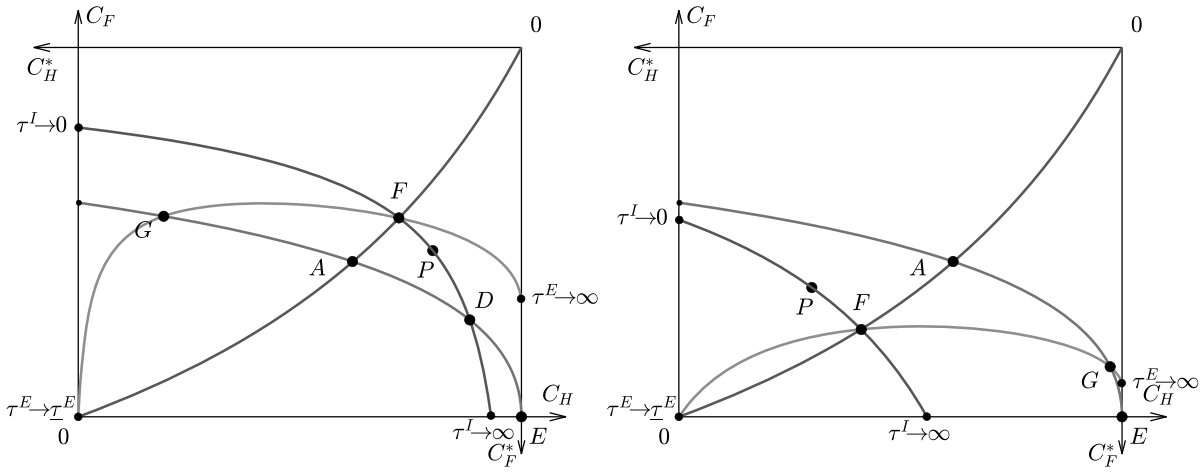


Figure 5: Effects of import and export tariffs under imbalances

Note: The blue curve is the contract curve and the red curve is $C_H^* = g(C_F)$, the set of implementable allocation under balanced trade when $B = B^* = 0$ (see Figure 4). The portfolio point P in the left and right panels features portfolio (B, B^*) such that $NFA \gtrless 0$, respectively. Purple (green) curve corresponds to g^I (g^E), i.e., the set of implementable allocations for a given portfolio (B, B^*) using an import tariff (export tax). See Lemma 7.

a prohibitive import tariff is imposed, $\tau^I \to \infty$, the resulting equilibrium features zero imports $C_F = 0$, but positive exports $C_H^* > 0$. The appreciation of the exchange rate beyond point D results in $NFA < 0$, which has to be supported by $NX > 0$. In contrast, an infinite export tax $\tau^E \to \infty$ depreciates the real exchange rate and increases $NFA > 0$ resulting in zero exports $C_H^* = 0$ and positive imports $C_F > 0$. The following lemma provides a formal characterization of all the limiting cases, illustrated in Figure 5, with the proof contained in the appendix (which also implies the result in Proposition 8):

Lemma 7 (a) *Export and import quantities, C_H^* and C_F, are decreasing in the import tariff τ^I and converge to $C_H^* \in (0, B)$, $C_F = 0$ such that $NX > 0$ when $\tau^I \to \infty$, and to $C_H^* = Y^*$, $C_F \in (B^*, Y^*)$ such that $NX < 0$ when $\tau^I \to 0$.* (b) *In contrast, if $B > 0$, export and import quantities converge to $C_H^* = Y^*$, $C_F = 0$ such that $NX < 0$ when the export tax $\tau^E \to \infty$, and to $C_H^* = Y^*$, $C_F \in (B^*, Y^*)$ such that $NX > 0$ when $\tau^E \to \underline{\tau}^E$, where $\underline{\tau}^E \in (0, 1)$ is the export subsidy that results in complete home immiserization (due to a negative valuation effect).*

Motivated by the US experience, the previous analysis focuses on the case of initial trade deficit (the left panel of Figure 5). Yet, symmetric results apply to countries running a trade surplus and aiming to close it with a tariff. For any initial portfolio below EA' and above EA'' in Figure 4, an import subsidy or an export tax can be used to depreciate the real exchange rate and generate positive valuation effects to close the imbalances. Interestingly, if the initial asset position is below the EA'', i.e., the country's external liabilities significantly exceed its assets ($B \gg B^*$), an import subsidy cannot close the imbalance, and any feasible equilibrium features $NX > 0$, as depicted in the right panel of Figure 5. Indeed, even as $\tau^I \to 0$, the exchange

rate depreciation is not sufficiently large, so that even in this limit $NFA < 0$ and $NX > 0$.[34] In contrast, an export tax that results in a sufficiently large exchange rate depreciation can always eliminate the imbalances in this case (point G).

Corollary 8 *A country running a trade surplus, $NX > 0$ under initial $NFA < 0$, can always achieve a balanced trade using an export tariff, $\tau^E > 1$. Only if the initial imbalances are not too large, there also exists an import subsidy, $\tau^I < 1$, that can close the imbalances.*

4.3 Dynamic model with convenience yield

The important limitation of the static model discussed above is that it cannot distinguish stocks from flows and, as a result, does not separate on-impact valuation effects and future excess returns emphasized in equation (18). As we discuss below, it also struggles to make sense of the depreciation of the dollar and the rise in US interest rates in response to trade tariffs introduced on the "Liberation Day" (see Jiang, Krishnamurthy, Lustig, Richmond, and Xu 2025). To address these issues, this section offers a simple dynamic model that can match the salient properties of the US external imbalance: a negative NFA position, persistent trade deficit, and positive valuation effects. We argue that only one type of shocks can explain the dollar depreciation in response to import tariffs — namely, a (partial) loss of convenience yield on the US assets. We also show, depending on the types of assets traded, when the model and the implied optimal tariff collapse to the static case from the previous section.

Setup Consider a dynamic extension of the baseline two-country model. Households have CRRA preferences across periods with inverse elasticity σ and the same CES preferences between home and foreign goods as before, with $\eta = \theta$. In each period, the resource constraints are given by (1), and the law of one price holds. There are home and foreign internationally traded assets with prices Q_t and Q_t^* paying P_{Ht} and P_{Ft}^* in all future period until maturity, which happens randomly every period with probability $1 - \delta$.[35] This payoff structure allows us to nest as special cases one-period local-currency bonds with $\delta = 0$, equity with $\delta = 1$, and long-term bonds with duration controlled by $\delta \in (0, 1)$, as in the sovereign debt literature (see Aguiar and Amador 2021).

[34] Formally, Figure 4 provides the partition of the space of initial portfolios $(-B, B^*)$ for which trade balance is implementable with an import tariff, or an import subsidy, or not at all. Specifically, line EA' is the locus of initial portfolios that result in $NX = NFA = 0$ under free trade (point A on this line). Any portfolio above this line features $NFA > 0$ and $NX < 0$ under free trade, and $NFA = NX = 0$ is implementable with an import tariff, $\tau^I > 1$, resulting in points below A on the trade balance curve. All portfolios that lie between EA' and EA'' feature $NFA < 0$ and $NX > 0$ under free trade, and $NFA = NX = 0$ is implementable with an import subsidy, $\tau^I < 1$, resulting in points above A on the trade balance curve. All portfolios below EA'' feature a larger $NFA < 0$ under free trade, and there exists no import subsidy (or tariff) that can implement $NFA = NX = 0$ from such initial conditions (line EA'' itself corresponds to the locus of portfolios that require infinite subsidy, $\tau^I = 0$, to achieve $NFA = NX = 0$ in point A'').

[35] Compare with the notion of home and foreign assets in Itskhoki and Mukhin (2025).

Two assumptions substantially simplify the analysis allowing us to focus on the main mechanism of interest. First, home assets are only held by households from the other country, and, second, asset supply is exogenously determined and constant over time. These assumptions guarantee that we do not need to solve complicated portfolio choice problems at the aggregate level, and there is no transition dynamics. This can be rationalized by a high exposure of households to local risk due to their ownership of the domestic endowment, and hence their incentives to diversify portfolios toward foreign assets, which are, however, in limited supply determined by the securitization capacity of the country and by the supply of government debt (Caballero, Farhi, and Gourinchas 2008). As long as agents abroad cannot take short positions, the equilibruim features a corner solution with home household acquiring all foreign bonds and equity. Second, motivated by the result from Corollary 2, we focus on a deterministic setup and introduce excess returns via convenience yields (Krishnamurthy and Vissing-Jorgensen 2012).

Home household solve a standard consumption-saving problem:

$$\begin{aligned} \max_{\{C_t, B_t^*\}} \quad & \sum_{t=0}^{\infty} \beta^t u(C_t) \\ \text{s.t.} \quad & Q_t^* B_t^* = (P_{Ft}^* + \delta Q_t^*) B_{t-1}^* + P_{Ht} Y_t - P_t C_t + T_t, \end{aligned}$$

where B_t^* are the holdings of foreign assets, C_t is the aggregate consumption, P_t is the consumer price index, and T_t is the consolidated lump-sum transfer from firms and the government that issue assets and collect revenues from tariffs:

$$T_t = Q_t B_t - (P_{Ht} + \delta Q_t) B_{t-1} + (\tau_t^I - 1) P_{Ft}^* C_{Ft} + (\tau_t^E - 1) P_{Ht} C_{Ht}^*.$$

Taking the optimality conditions, we get a standard no-arbitrage for foreign bonds

$$Q_t^* = \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} (P_{Ft+1}^* + \delta Q_{t+1}^*), \tag{27}$$

which determines the price Q_t^* and the return $R_{t+1}^* \equiv \frac{P_{Ft+1}^* + \delta Q_{t+1}^*}{Q_t^*}$ on the foreign asset given its exogenous supply B_t^*.

The problem of foreign households is symmetric except for the convenience yields:

$$\begin{aligned} \max_{\{C_t^*, B_t\}} \quad & \sum_{t=0}^{\infty} \beta^t \Big(u(C_t^*) + v_t(B_t) \Big) \\ \text{s.t.} \quad & Q_t B_t = (P_{Ht} + \delta Q_t) B_{t-1} + P_{Ft}^* Y_t^* - P_t^* C_t^* + T_t^*. \end{aligned}$$

Time index in $v_t(\cdot)$ allows for exogenous demand shocks. The aggregate transfer from foreign

government and asset-issuing firms is given by

$$T_t^* = Q_t^* B_t^* - (P_{Ft}^* + \delta Q_t^*) B_{t-1}^*.$$

The resulting optimality condition for home assets B_t has an extra term that reflects convenience yield:

$$Q_t = \beta \frac{u'(C_{t+1}^*)}{u'(C_t^*)} \frac{P_t^*}{P_{t+1}^*} (P_{Ht+1} + \delta Q_{t+1}) + \frac{v_t'(B_t)}{u'(C_t^*)/P_t^*}, \tag{28}$$

and therefore increases its equilibrium price Q_t and reduces its return $R_{t+1} \equiv \frac{P_{Ht+1}+\delta Q_{t+1}}{Q_t}$.

Finally, combining the flow of funds for households and the government in either country, we get the country's budget constraint

$$Q_t^* B_t^* - Q_t B_t = (P_{Ft}^* + \delta Q_t^*) B_{t-1}^* - (P_{Ht} + \delta Q_t) B_{t-1} + (P_{Ht}^* C_{Ht}^* - P_{Ft}^* C_{Ft}), \tag{29}$$

where the last term is home's net exports NX_t. This equation is the special case of the general flow budget constraint (16). In particular, we can use R_t^* as the baseline rate of return (which does not feature convenience yield) to rewrite (29) as:

$$\mathcal{B}_t = R_t^* \mathcal{B}_{t-1} + (R_t^* - R_t) Q_{t-1} B_{t-1} + NX_t,$$

where $\mathcal{B}_t \equiv Q_t^* B_t^* - Q_t B_t$ is the NFA position going forward, $R_t^* \mathcal{B}_{t-1}$ is the payout on the previous NFA position without the convenience yield, and $(R_t^* - R_t) Q_{t-1} B_{t-1}$ is the convenience yield on the gross home liabilities.

Valuation effects Assume that asset supply is constant, $B_t = B$ and $B_t^* = B^*$, and that all shocks are fully unanticipated and permanent. Then there is no transition dynamics and the economy immediately jumps into a stationary equilibrium, which we characterize next dropping time subscripts. The Euler equations (27) and (28) imply that foreign assets earn gross return $1/\beta$, while returns on home asset are dampened and the asset price is inflated by the convenience yield:

$$Q^* = \frac{\beta}{1 - \beta\delta} P_F^* \qquad \text{and} \qquad Q = \frac{1}{1 - \beta\delta} \left(\beta P_H + \frac{v'(B)}{u'(C^*)/P^*} \right),$$

such that $R^* \equiv \frac{P_F^* + \delta Q^*}{Q^*} = \frac{1}{\beta}$ and $R \equiv \frac{P_H + \delta Q}{Q} < \frac{1}{\beta}$ when $v'(B) > 0$.

Using the expressions for asset prices Q and Q^*, we can rewrite the steady-state version of the flow budget constraint (29) as follows:

$$NX + (1 - \beta) \Big((P_F^* + \delta Q^*) B^* - (P_H + \delta Q) B \Big) + \frac{v'(B) B}{u'(C^*)/P^*} = 0, \tag{30}$$

or in net present value terms if divided through by $(1 - \beta)$. This expression is a direct counter-

part of equation (18) in flow terms: the first term corresponds to a long-run trade (im)balance, the second term reflects returns of the initial net foreign position and the on-impact valuation effects, while the last term summarizes future excess returns. It follows that in contrast to the static model above, it is now possible for a country to have a negative net foreign asset position ($NFA = Q^*B^* - QB < 0$) and simultaneously run a persistent trade deficit ($NX < 0$), as arguably is the case in the US With high enough convenience yield, $v'(B)B > 0$, a country pays a lower return on its liabilities than it receives on its holdings of foreign assets, earning an "exorbitant privilege".

It is important to emphasize, however, that convenience yield affect not only the excess returns, but also the prices of assets, i.e., both the second and the third terms in the budget constraint (30). To evaluate the net effect, we substitute solution for Q and Q^* from above and get the following result.

Lemma 8 *The intertemporal budget constraint is equivalent to*

$$NX + \frac{1-\beta}{1-\beta\delta}(P_F^*B^* - P_H B) + \frac{1-\delta}{1-\beta\delta} \cdot \frac{v'(B)B}{u'(C^*)/P^*} = 0. \tag{31}$$

The valuation effects of convenience yield are zero for equity with $\delta = 1$ and are highest for short-term bonds with $\delta = 0$.

The first two terms in (31) are the same as in the static model. In particular, the net dividends on international financial positions summarized by the second terms depend solely on exogenous supply of assets B, B^* and endogenous exchange rate P_F^*/P_H. In contrast, the last term in (31) is new, and shows that the valuation effects from convenience yields depend crucially on the maturity structure of assets.

The case in which countries hold only equity, $\delta = 1$, is particularly instructive as it implies that the last term in (31) is equal to zero, and the dynamic model effectively collapses to a static one analyzed above. The reason is that in this case the on-impact valuation effects and future excess returns perfectly offset each other. For example, if foreigners lose interest in the US equity, Q collapses generating a positive on-impact valuation effect for the US However, the required returns on the US equities increase and the future convenience yields go down. More generally, demand shocks for assets generate non-zero valuation effects for assets with finite maturity, $\delta < 1$. Indeed, in the opposite extreme case of short-term bonds with $\delta = 0$, the on-impact valuation effects are zero as all bond holdings expire, and an increase in convenience yield translates exclusively into lower interest on country's liabilities in all future periods.

Another important advantage of the dynamic model is that it helps to make sense of the depreciation of the dollar in the first months of 2025 despite the introduction of US import tariffs. Indeed, the previous section shows that in a static model that features only the first two terms from equation (31), the exchange rate must appreciate in response to this shock.

Furthermore, the same applies to most other shocks that likely accompany higher tariffs: a recession that lower supply of home goods Y or a fall in payouts on home assets B that generate a *positive* on-impact valuation effect (*cf.* Atkeson, Heathcote, and Perri 2022). This leaves a fall in future convenience yields under $\delta < 1$ as the only likely source of negative valuation effects that can lead to a dollar depreciation.[36] This mechanism is also consistent with a fall in asset prices and increase in yields on the "Liberation Day".

Corollary 9 *An import tariff can depreciate the real exchange rate $\mathcal{Q} = P_F^*/P_H$ only if it triggers negative valuation effects due to a reduction in convenience yield $v'(B)$ in* (31).

Optimal tariff What is the optimal trade policy in the presence of exorbitant privilege? As before, the implementability constraint $C_H^* = g(C_F)$ combines the country's budget constraint with foreign optimality conditions and the market clearing condition. The difference from the previous cases comes from extra valuation effects when $\delta < 1$. As can be seen from the last term in equation (31), the latter in general depend on marginal utility $u'(C^*)$ and aggregate price index P^* complicating the analysis. However, in the special case of equal intertemporal and intra-temporal elasticities ($\sigma\eta = 1$), the valuation effects due to convenience yield depend only on B and hence are exogenous from the point of view of the home planner, substantially simplifying the optimal policy. We prove in appendix:

Proposition 9 *Assume $\sigma\eta = 1$. Then the optimal import tariff is given by*

$$\tau = 1 + \frac{1}{\eta\left(1 + \frac{\bar{B}}{EX - \bar{B}}\right) - 1} \cdot \frac{1 + (1 - \Lambda^*)\frac{CY}{EX - \bar{B}}}{\Lambda^*}, \tag{32}$$

where as before $\Lambda^* \equiv \frac{P_F^* C_F^*}{P^* C^*}$ and $EX \equiv P_H^* C_H^*$, $\bar{B}^* \equiv \frac{1-\beta}{1-\beta\delta} P_F^* B^*$ and $\bar{B} \equiv \frac{1-\beta}{1-\beta\delta} P_H B$ are flow cash payouts on home assets and liabilities, and $CY \equiv \frac{1-\delta}{1-\beta\delta} \frac{v'(B)B}{u'(C^*)/P^*}$ is the flow value of convenience yield, such that $NX + (\bar{B}^* - \bar{B}) + CY = 0$ is the country budget constraint (31).

This proposition generalizes formula (24) for the optimal tariff without convenience yield, i.e., when the term $CY = 0$, and hence net exports are financed via net returns on the asset position, $NX + (\bar{B}^* - \bar{B}) = 0$. When convenience yield is positive, and we have $IM - \bar{B}^* > EX - \bar{B}$ where $IM = P_F^* C_F$, the optimal import tariff is higher than what is prescribed by (24), other things equal. Convenience yield allows to sustain larger imports for a given value of the asset position, and if convenience yield $v'(B)B$ is exogenous, this justifies a larger tax. Of course, if $v'(B)B$ were endogenous to the tariff, this would affect the optimal tariff formula, as we discuss below.[37]

[36] Note that a decrease in $v'(B)$ can be also thought of as a loss of productivity in the US financial service sector.

[37] Also note that both the budget constraint (31) and the corresponding optimal tariff (32) are obtained under the assumption that import tariffs are introduced unexpectedly. If, instead, agents anticipate changes in trade policy, they preemptively re-optimize their portfolios, changing equilibrium prices and convenience yields. In turn, this changes the benefits of introducing tariffs and their optimal level.

Proposition 9 also clarifies that it is net flows on assets and liabilities that are relevant for the optimal policy, not stocks. Indeed, for foreign assets it can be shown that $\bar{B}^* = (P_F^* - (1-\delta)Q^*)B^* = (R^* - 1)Q^*B^*$, which corresponds to net cash payouts on the country's position abroad. This object can be directly measured in the data. Things are more complicated for home assets as their returns need to be calculated excluding convenience yield. One option is to calculate asset payoffs and discount them using foreign returns R^*. Alternatively, if both types of assets have similar maturity structure δ, one can use the following expression to evaluate $\bar{B} = (R^* - 1)\frac{Q^*}{P_F^*}P_H B$. Given $\bar{B}$, the flow value of convenience yield can be evaluated from the country budget constraint as $CY = -(NX + NFA)$, where $NFA \equiv \bar{B}^* - \bar{B}$ is flow cash value from net foreign assets. Given these calculations, formula (32) still describes the optimal tariff in terms of measurable sufficient statistics.

To conclude, we briefly discuss the case where convenience yield $v'(B)$ is endogenous to the trade policy. If a trade war causes a loss of confidence in the US assets by foreign investors, this may eliminate all or part of the convenience yield of the US assets from the perspective of foreign investors, and result in the loss of privilege of the US, along with the depreciation of the dollar. We capture this with $\Delta v'(B) < 0$ conditional on the start of the trade war. In this case, the (per-period) welfare benefit of the optimal tariff relative to status quo (free trade) needs to be weighted against the loss of excess returns equal to $\frac{1-\delta}{1-\beta\delta}\Delta v'(B)B$. When $v'(B)B$ is sufficiently large, a trade war may be too costly even if it is welfare-increasing in a static model. This may be the economic reason why the US did not engage in a trade war over an extended period of trade and financial imbalances that opened up after mid-1990s.

5 Multiple Countries and Bilateral Trade Imbalances

So far, we focused on a uniform tariff on all imported (or exported) goods and assumed that the only source imbalances between the two countries are international financial positions. Another source of bilateral imbalances is heterogeneity across economies with a typical country running a trade deficit with some of its trade partners and trade surpluses with the other ones. Is it optimal to impose different import tariffs across countries of origin in this case? What is the optimal tariff and in particular, should it target bilateral imbalances? Are there benefits from using both import and export tariffs? This section addresses these questions extending the baseline model to a multicountry setting.

5.1 Setup

Consider home country indexed by $i = 0$ and its trade partners indexed by $i = 1...N$. Households in region i are endowed with Y_i units of country-specific good and have CES preferences $u^i = C_i = \left(\sum_{j=0}^{N} \gamma_{ji}^{\frac{1}{\theta}} C_{ji}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$. To simplify the analysis, we assume the same elasticity of substitution θ in all countries. As before, all prices are expressed in dollars, the law of one price

holds for sellers, and any effects of trade costs are captured by the home bias in preferences. The set of instruments available to the home planner potentially includes country-specific import tariffs $\{\tau_i^I\}_{i=1}^N$ and export taxes $\{\tau_i^E\}_{i=1}^N$, which for brevity, we denote respectively with $\tau_i \equiv \tau_i^I$ and $\varsigma_i \equiv 1/\tau_i^E$.

Given home trade policy, the global competitive equilibrium is determined as follows. The law of one price holds for sellers, while consumer price of good j in country i is equal

$$P_{ji} = P_{jj} \text{ for } j, i \neq 0, \qquad P_{j0} = P_{jj}\tau_j, \qquad P_{0i} = \frac{P_{00}}{\varsigma_i}, \tag{33}$$

where $\tau_0^I = \tau_0^E = 1$. The optimal demand of households in country i for good j is given by

$$C_{ji} = \gamma_{ji} P_{ji}^{-\theta} \lambda_i, \qquad \text{where } \lambda_i = P_i^{\theta} C_i$$

is the aggregate demand shifter in country i and $P_i = \left(\sum_{j=0}^N \gamma_{ji} P_{ji}^{1-\theta}\right)^{\frac{1}{1-\theta}}$ is the ideal price index. Aggregating demand across countries, we get the system of market clearing conditions

$$\sum_{i=0}^{N} \gamma_{ji} P_{ji}^{-\theta} \lambda_i = Y_j, \qquad j = 0...N. \tag{34}$$

While economies can run bilateral surpluses and deficits, the trade is assumed to be balanced at a country level:

$$\sum_{j=0}^{N} \gamma_{ji} P_{ji}^{1-\theta} \lambda_i = P_{ii} Y_i, \qquad i = 1...N. \tag{35}$$

Because of the Walras Law, the budget constraint for the home economy is automatically satisfied given the other budget constraints and market clearing conditions. In sum, any allocation implementable with home trade instruments should satisfy conditions (33)-(35).

Finally, we define trade shares that are the key sufficient statistics in our analysis below. In particular, let α_{ji} denote market i share in country j's total output and let s_{ji} denote the spending share of country i on good j:

$$\alpha_{ji} \equiv \frac{C_{ji}}{Y_j} \qquad \text{and} \qquad s_{ji} \equiv \frac{P_{ji} C_{ji}}{P_i C_i},$$

with $\sum_{j=0}^N \alpha_{ij} = \sum_{j=0}^N s_{ji} = 1$ for every i. Of course, these shares are determined endogenously and depend on home's trade policy. We next characterize the optimal policy relegating all proofs to Appendix A.

5.2 Complete markets

To build economic intuition, we start with a simplified version of the multi-country economy that assumes complete asset markets between the RoW countries. This allows countries to diversify the risk of home tariffs between themselves, even though they cannot hedge against them in aggregate as there are no financial contracts with home agents that they can sign. The optimal transfers between foreign economies imply that their budget constraints can be aggregated and the system (35) is replaced with one constraint on the home planner. Furthermore, we focus on import tariffs and allow only for a uniform export tax, which according to Lerner symmetry is isomorphic to a uniform import tariff. The preferences of households in the RoW can then be aggregated into a representative-agent CES utility function $u^*\big(\{C_j^*\}_{i=0}^N\big)$ over $C_j^* \equiv \sum_{i=1}^N C_{ji}$.[38] Thus, the environment closely resembes the baseline model, except for the presence of multiple goods.[39]

The home planner's problem then boils down to

$$\max_{\{C_j^*\}} \quad u\big(\{Y_j - C_j^*\}_{j=0}^N\big) \qquad \text{s.t.} \quad \sum\nolimits_{j=0}^N u_j^*\big(\{C_j^*\}\big)(Y_j^* - C_j^*) = 0.$$

where $\{C_j^*\}$ is the vector for $j = 0...N$, $u \equiv u^0$ and $Y_j^* = Y_j$ for $j = 1...N$ and $Y_0^* = 0$ for brevity.

Proposition 10 *In a multi-country model with complete markets between the RoW economies, the optimal bilateral tariff is given by*

$$\tau_j = 1 + \frac{1}{\theta - 1}\frac{1}{\Lambda_j^*}, \tag{36}$$

where $\Lambda_j^* \equiv \frac{C_j^*}{Y_j}$ is the share of endowment of country j consumed by the RoW.

The proposition generalizes formula (11) to a multi-country setting and shows that once expressed in terms of sufficient statistics, the optimal bilateral tariff has exactly the same structure as in a two-country case. Remarkably, despite the general equilibrium spillovers across economies, the optimal tariff on individual country is a function of home's share in its total sales and independent of any other trade flows. In particular, a higher tariff should be imposed on countries that rely on the US as the main market and conditional on this statistic, the bilateral trade imbalances are irrelevant. On the one hand, home might run a trade deficit against country j, but account for a small fraction of its total sales — either because country j is relatively closed or exports most of its output to other destinations — in which case the optimal

[38] In a dynamic model, this requires an additional assumption that intertemporal elasticity of substitution is equal to the intra-temporal one. Alternatively, one can assume that households in all countries have the same static preferences across goods.

[39] The structure of the problem is similar to that in Costinot, Lorenzoni, and Werning (2014) with the difference that j indexes trade partners rather than periods, and we are solving for optimal tariffs, not capital controls.

tariff is low. On the other hand, home might have a balanced trade with country i, but be the main market of destination for i's goods, in which case the optimal tariff is high.[40]

5.3 Optimal tariffs

We next drop the assumption of complete markets and discuss two ways of solving for optimal import tariffs — the first one extends Johnson (1950) to a multicountry setting and offers a simple reduced-form formula, while the second approach provides a more explicit system of equations in terms of optimal tariffs and measurable statistics — and then extend analysis to the case when the planner can use both import and export tariffs and there are international asset positions.

Johnson formula Similarly to a two-country setting, one can think of home trade policy choosing a vector of imported quantities $\{C_{j0}\}_{j=1}^{N}$ and the system of market clearing conditions (34) and countries' budget constraints (35) determining home's exports $\sum_{i=0}^{N} C_{0i}$. The resulting mapping $Y_0 - C_{00} = g\big(\{C_{j0}\}\big)$ summarizes the trade-off between importing more and exporting less and is the same as in the baseline model except that $g(\cdot)$ is now a function of N variables rather than one. The planner's problem can then be parsimoniously written as

$$\max_{\{C_{j0}\}} \qquad u\Big(Y_0 - g\big(\{C_{j0}\}\big), \{C_{j0}\}\Big),$$

where $\{C_{j0}\}$ is the vector for $j = 1...N$ and we denote $u \equiv u^0$ for brevity. Rewrite the first-order condition $u_0 g_j = u_j$ as

$$\frac{u_j}{u_0} = \frac{g_j C_{j0}}{g} \cdot \frac{P_{00} g}{P_{jj} C_{j0}} \cdot \frac{P_{jj}}{P_{00}},$$

and use household optimization together with equations (33) to obtain a Johnson-type formula.

Lemma 9 *The optimal tariff on imports from country j is equal*

$$\tau_j = \frac{\varepsilon_j}{z_j}, \tag{37}$$

where $\varepsilon_j \equiv \frac{\partial \log g(\{C_{i0}\})}{\partial \log C_{j0}}$ is the elasticity of exports w.r.t. imports from j and $z_j \equiv \frac{P_{jj} C_{j0}}{\sum_{i=1}^{N} P_{ii} C_{i0}}$ is the value share of imports from j in total imports of home.

The intuition behind the formula is the same as before, and the scaling factor z_j corrects for the fact that elasticity ε_j is mechanically lower for goods accounting for a small fraction

[40] This result that the country exploits its monopsony, but not monopoly power contrasts with the optimality of a uniform import tariff and sector-specific export subsidies in Costinot, Donaldson, Vogel, and Werning (2015) and is partly due to the assumption of uniform export tariffs. The next sections show that in general, it is optimal to use both instruments.

of country's trade. It is easy to see that in case of one trade partner $N = 1$, we get $z_j = 1$ and the formula simplifies to the baseline result from Section 3.1. More generally, equation (37) shows that one needs to estimate partial trade elasticities ε_j and measure import shares z_j to compute optimal tariffs. As before, conditional on those statistics, no information about home preferences is required.

Despite its important advantages, Johnson approach does not provide much information about the heterogeneity of import tariffs across trade partners.[41] Furthermore, it is not straightforward to adapt it to the case of both import and export tariffs. For this reason, we next consider a complementary way of deriving optimal tariffs and on the way provide their alternative characterization.

Import tariffs Leveraging the fact that $\varsigma_j = 1$, substitute out bilateral prices (33) from other equilibrium conditions. The planner then chooses equilibrium prices of goods $\{P_{jj}\}$ and demand shifters $\{\lambda_j\}$ in other economies that determine home residual consumption $\{C_{j0}\}$ subject to countries' budget constraints:

$$\max_{\{P_{jj},\lambda_j\}_{j=1}^N} \quad u\left(\left\{Y_j - P_{jj}^{-\theta}\sum_{i=1}^N \gamma_{ji}\lambda_i\right\}_{j=0}^N\right) \qquad \text{s.t. } \lambda_i \sum_{j=0}^N \gamma_{ji} P_{jj}^{1-\theta} = P_{ii}Y_i, \;\; i = 1...N.$$

Although no closed-form solution is available in general case, the next proposition characterizes optimal trade policy in terms of sufficient statistics and reveals that the only required information to compute tariffs are trade shares and the elasticity of substitution. It also shows that the implied tariffs are in general heterogenous across countries and depend not only on local characteristics of a trade partner but also the whole network of trade flows.

Proposition 11 *The optimal country-specific import tariffs satisfy the following system*

$$\tau_j = \frac{1}{\Lambda_j^*}\left[\frac{1}{\theta}\bar{\tau}_j + \frac{\theta-1}{\theta}\sum_{i=1}^N \alpha_{ji}\bar{\tau}_i\right], \qquad \textit{where} \quad \bar{\tau}_i \equiv \sum_{j=0}^N s_{ji}\tau_j \tag{38}$$

is the average tariff imposed on countries selling in market i and $\Lambda_j^* \equiv 1 - \alpha_{j0}$ is the total share of good j consumed by the rest of the world.

[41] In principle, elasticities ε_j can be recovered from the full structural model differentiating equations (34)-(35):

$$\textstyle\sum_{i=1}^N \alpha_{ji}\left(\theta \mathrm{d}\log P_{jj} - \mathrm{d}\log\lambda_i\right) = \alpha_{j0}\mathrm{d}\log C_{j0}, \qquad \sum_{j=0}^N s_{ji}\left(\mathrm{d}\log\lambda_i - (\theta-1)\mathrm{d}\log P_{jj}\right) = \mathrm{d}\log P_{ii}.$$

Given parameter θ and trade shares $\{\alpha_{ji}, s_{ji}\}$, this system of $N+1$ market clearing conditions and N budget constraints can be solved for N demand shifters λ_i, N prices P_{jj} and one $\mathrm{d}\log C_{00}$ as functions of N imports $\mathrm{d}\log C_{j0}$ allowing to compute the vector of elasticities ε_j.

Intuition What is the economic intuition behind Proposition 11? When setting optimal tariffs in a multi-country world, the planner needs to take into account not only changes in bilateral terms of trade, but also income effects across its trade partners arising from movements in relative prices. Because of heterogenous preferences, such implicit transfers change global demand for goods and have second-round effects consequential for the home economy (*cf.* Keynes 1929, Ohlin 1929). Clearly, such effects are absent in a simple model with complete markets or homogeneous RoW preferences discussed above expaining why sales share $1-\alpha_{j0}$ alone is a sufficient statistic for τ_j in that case.

We next provide a more detailed discussion of each element in system (38). It is convenient to normalize US prices $P_{00} = u_0$, so that dollars have a marginal utility interpretation. It follows then from household optimality conditions that an import tariff $\tau_j = u_j/P_{jj}$ can be interpreted as home marginal utility from spending extra dollar on imports from country j. In contrast to the case of free trade when households equalize it across all goods, a planner creates wedges by making local prices deviate from global ones.

The logic behind system (38) can then be understood in two steps. First, consider the home shadow value of taking away one dollar from country i and spending it in the same proportion across goods to keep global demand unchanged. Given country i's spending shares s_{ji} and home marginal utilities τ_j, home gains from such transfer are equal $\bar{\tau}_i \equiv \sum_{j=0}^{N} s_{ji}\tau_j$. Second, while home cannot directly transfer dollars from other economies, it takes into account the implicit redistribution associated with its trade policy.

In particular, suppose the planner imposes a tariff and decreases imports of good j by one unit. The marginal costs associated with lower consumption are proportional to τ_j, the left hand side of equation (38). The marginal benefits from such move come from redistribution and are two fold. First, a fall in demand C_{j0} decreases the price of good j by $\mathrm{d}\log P_{jj} = \frac{\alpha_{j0}}{\theta(1-\alpha_{j0})}\mathrm{d}\log C_{j0}$ from market clearing condition (34). Country j's income $P_{jj}Y_{jj}$ falls proportionately, and the first term on the right hand side of equation (38) reflects the corresponding benefits for home. Second, a lower P_{jj} makes other countries increase their spendings on good j by $\mathrm{d}\log(P_{jj}C_{ji}) = (1-\theta)\mathrm{d}\log P_{jj}$. This expenditure switching decreases demand for other goods with the corresponding benefits for home summarized by the last term in formula (38). The optimal policy then increases tariffs up to the point where marginal costs and marginal benefits are equalized.

To gain better understanding, it is instructive to consider again the special case of two countries $N = 1$. Since $\tau_0^I = 1$, the shadow value of one dollar transfered from the RoW is given by

$$\bar{\tau}_j = (1 - s_{jj}) + s_{jj}\tau_j.$$

From market clearing condition $(1-\gamma^*)P_{jj}^{-\theta}\lambda_j = Y_j - C_{j0}$, it follows that conditional on λ_j, the elasticity of import spendings $P_{jj}C_{j0}$ with respect to consumption of foreign good C_{j0} is

equal $1+\frac{1}{\theta}\frac{1-\alpha_{jj}}{\alpha_{jj}}$. Given the marginal costs of decreasing consumption of foreign good τ_j and the marginal gains of saving dollars (and hence, home goods) $\bar{\tau}_j$, the optimal tariff is set at

$$\tau_j = \left[1+\frac{1}{\theta}\frac{1-\alpha_{jj}}{\alpha_{jj}}\right]\bar{\tau}_j.$$

Thus, we get a fixed point problem: given $\bar{\tau}_j = 1$, the planner imposes import tariff $\tau_j > 1$ to economize dollars it pays to foreigners. This elevates the marginal value of foreign goods and the shadow value of foreign spendings $\bar{\tau}_j$ making the planner even more eager to redistribute income from foreign via higher tariffs τ_j. Iterating the system, we converge to formula (11) from the baseline model.

Import and export tariffs Does Lerner symmetry hold in a multi-country world or are there additional gains from using unrestricted export tariffs? To answer this question, allow the planner to choose country-specific $\{\tau_j.\varsigma_j\}$. While this gives planner additional N instruments, it does not eliminate any constraints. Instead, the market clearing condition for home goods is now different and allows the planner to shift exports across destinations:

$$\begin{aligned}\max_{\{P_{jj},P_{0j},\lambda_j\}_{j=1}^N} \quad & u\left(Y_0-\sum\nolimits_{i=1}^N \gamma_{0i}\lambda_i P_{0i}^{-\theta}, \left\{Y_j - P_{jj}^{-\theta}\sum\nolimits_{i=1}^N \gamma_{ji}\lambda_i\right\}_{j=1}^N\right)\\ \text{s.t.} \quad & \lambda_i\left[\sum\nolimits_{j=1}^N \gamma_{ji}P_{jj}^{1-\theta}+\gamma_{0i}P_{0i}^{1-\theta}\right] = P_{ii}Y_i, \qquad i=1...N.\end{aligned}$$

Proposition 12 *The optimal import tariffs and export subsidies satisfy the following system*

$$\left(\frac{\theta}{\theta-1}-s_{0i}\right)\varsigma_i = \sum_{j=1}^N s_{ji}\tau_j \qquad \textit{and} \qquad \Lambda_j^*\tau_j = \frac{1}{\theta-1}\varsigma_j + \sum_{i=1}^N \alpha_{ji}\varsigma_i, \tag{39}$$

where $\Lambda_j^* \equiv 1-\alpha_{j0}$ *and* $\tau_0^I = \tau_0^E = 1$.

System (39) is homogeneous of degree one and the aggregate level of import tariffs and export subsidies is not uniquely determined. However, it is generically optimal to use both import and export tariffs and their values are vary across countries.

Financial portfolios To better understand the implications of bilateral vs. aggregate imbalances for the trade policy, consider next a multi-country model with international financial positions. We allow for fairly general portfolios with matrix $\{B_{ji}\}$ summarizing the holdings of equity/bond j by country i. As before, asset j pays in units of good of economy j and, without loss of generality, the net supply of assets can be normalized so that

$$\sum\nolimits_{i=0}^N B_{ji} = Y_j,$$

and the share of country i in total supply of asset j is given by $b_{ji} \equiv \frac{B_{ji}}{Y_j}$. The pricing block (33) and the goods market clearing (34) are not affected by international portfolios. Instead, the

transfers created by asset holdings appear in countries' budget constraints

$$\sum_{j=0}^{N} P_{ji} C_{ji} = \sum_{j=0}^{N} P_{jj} B_{ji}.$$

We abstract from capital levy on foreign positions by focusing on import tariffs and follow the primal approach to solve the home planner's problem

$$\max_{\{P_{jj},\lambda_j\}_{j=1}^{N}} u\left(\left\{Y_j - P_{jj}^{-\theta}\sum_{i=1}^{N}\gamma_{ji}\lambda_i\right\}_{j=0}^{N}\right) \qquad \text{s.t.} \quad \lambda_i \sum_{j=0}^{N}\gamma_{ji}P_{jj}^{1-\theta} = \sum_{j=0}^{N} P_{jj}B_{ji}, \;\; i = 1...N.$$

The next formula generalizes the result from Proposition 11, which can be obtained as a special case when countries hold only local stocks, i.e. $b_{jj} = 1$ and $b_{ji} = 0$ for $j \neq i$ (see Appendix A for the proof).

Proposition 13 *The optimal country-specific import tariffs satisfy*

$$\tau_j = \frac{1}{\theta}\frac{1}{\Lambda_j^*}\sum_{i=1}^{N}\Big[(\theta-1)\alpha_{ji} + b_{ji}\Big]\bar{\tau}_i, \qquad \textit{where } \bar{\tau}_i \equiv \sum_{j=0}^{N} s_{ji}\tau_j \tag{40}$$

is the average tariff imposed on countries selling in market i and $\Lambda_j^* \equiv 1 - \alpha_{j0}$ is the total share of good j consumed by the rest of the world.

5.4 Special cases

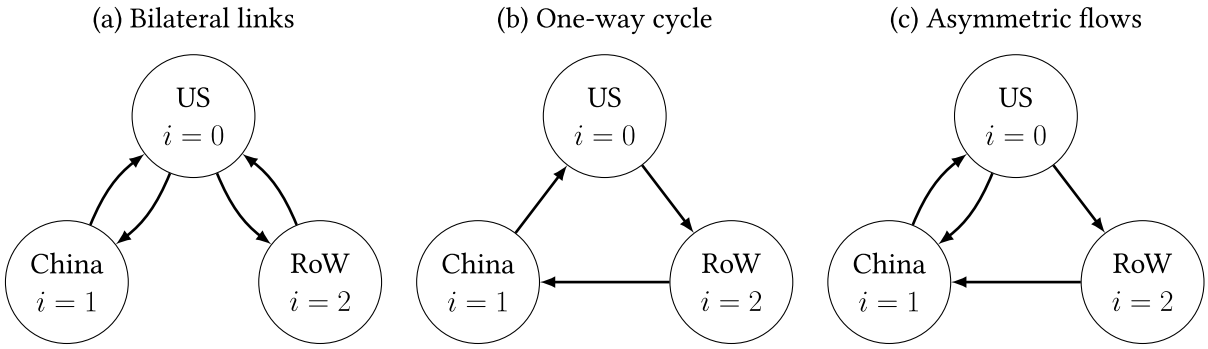
To build economic intuition, we next apply derived formulae in a few special cases illustrated in Figure 6 that include balanced trade with each partner and the opposite extreme when the US (home) imports only from China and exports only to the RoW. In all cases, we assume that trade is balances at a country level.

Bilateral links Consider first the case from Figure 6a when country j is trading only with the US $i = 0$ and therefore, trade between the two is always balanced. Substituting $s_{kj} = 0$ and $\alpha_{jk} = 0$ for $k \neq 0, j$, $\alpha_{jj} = s_{jj}$ and $\tau_0 = 1$ into formula (38) and solving the resulting equation, we get

$$\tau_j = 1 + \frac{1}{\theta - 1}\frac{1}{\alpha_{jj}}.$$

Thus, the optimal tariff takes exactly the same form as in Proposition 11 from the baseline two-country model. This shows that what matters for the optimal tariff is not the number of trade partners, but rather the substitution across bilateral trade flows. At the same time, system (39) is degenerate with equations for τ_j and ς_j being collinear: the export tariffs are redundant in this setting because Lerner symmetry applies to each individual trade partner.

Figure 6: Trade flows across three regions



One-way cycle The next example shown in Figure 6b assumes that the US $i = 0$ imports goods only from China $i = 1$, which imports from the RoW $i = 2$, which in turn imports from the US There is no trade in the opposite direction providing an extreme example of unbalanced trade between any pair of countries. This corresponds to the following trade shares

$$\alpha_{20} = \alpha_{12} = s_{12} = s_{01} = 0, \quad \alpha_{10} = 1 - \alpha_{11} = 1 - s_{11} = s_{21}, \quad \alpha_{21} = 1 - \alpha_{22} = 1 - s_{22} = s_{02}.$$

Since Lerner symmetry holds for the aggregate import and export tariffs, we normalize $\tau_2 = 1$, which seems to be a natural assumption given no imports from the RoW to the US Substituting trade shares and normalization into system (39), we get optimal import and export tariffs:

$$\tau_2 = \varsigma_1 = 1, \qquad \tau_1 = 1 + \frac{1}{\theta - 1}\frac{1}{\alpha_{11}}, \qquad \frac{1}{\varsigma_2} = 1 + \frac{1}{\theta - 1}\frac{1}{\alpha_{22}}.$$

It is intuitive that the US taxes imports from China $\tau_1 > 1$ and exports to the RoW $\varsigma_2 < 1$, but does not use other instruments given zero exports to China and imports from the RoW. Furthermore, despite bilateral imbalances, the optimal tariffs take exactly the same form as in a two-country model.

Interestingly, the same optimal policy and allocation can be implemented with import tariffs alone. This is a surprising observation: the policy above uses two instruments, but with import tariffs alone and no imports from the RoW, the US effectively has just one instrument — a tariff on Chinese goods. The argument relies on Lerner symmetry that allows rescaling all tariffs as follows:

$$\tilde{\varsigma}_2 = 1, \qquad \tilde{\tau}_2 = \tilde{\varsigma}_1 = 1/\varsigma_2, \qquad \tilde{\tau}_1 = \tau_1/\varsigma_2 = \left(1 + \frac{1}{\theta - 1}\frac{1}{\alpha_{11}}\right)\left(1 + \frac{1}{\theta - 1}\frac{1}{\alpha_{22}}\right).$$

The key observation that $\tilde{\tau}_2$ and $\tilde{\varsigma}_1$ are irrelevant because the corresponding trade flows are zero and hence, $\tilde{\tau}_1$ is sufficient to replicate the same allocation. Indeed, solving system (38) for optimal import tariffs, we get the same results:

$$\tau_1 = \left(1 + \frac{1}{\theta - 1}\frac{1}{\alpha_{11}}\right)\tau_2, \qquad \tau_2 = 1 + \frac{1}{\theta - 1}\frac{1}{\alpha_{22}}.$$

Asymmetric flows Although these results do not generalize to the case of arbitrary flows between the three countries, they do extended to the case when the US simultaneously imports and export to China, while the RoW only imports from the US and exports only to China. The trade shares then satisfy $s_{12} = \alpha_{12} = s_{20} = \alpha_{20} = 0$.

Start again with the case of both import and export tariffs and substitute parameter values together with normalization $\tau_2 = 1$ into system (39). It is straightforward to show that the solution is exactly as before and, conditional on domestic trade shares, the optimal tariffs do not depend on additional trade flows between the US and China:

$$\tau_2 = \varsigma_1 = 1, \qquad \tau_1 = 1 + \frac{1}{\theta - 1}\frac{1}{\alpha_{11}}, \qquad \frac{1}{\varsigma_2} = 1 + \frac{1}{\theta - 1}\frac{1}{\alpha_{22}}.$$

Thus, despite two-way trade flows between the US and China, one trade instrument is still sufficient as long as the US can impose a separate export tariff on the RoW.

However, it must be clear that implementing the same allocation with only import tariffs is no longer feasible as both ς_1 and ς_2 matter in this case and cannot simultaneously be set to one. Solving system (39), we get

$$\tau_2 = 1 + \frac{1}{(\theta - 1)\alpha_{22} + \theta\frac{s_{01}}{\alpha_{01}}}, \qquad \tau_1 = \left(1 + \frac{1}{\theta - 1}\frac{1}{\alpha_{11}}\right)\left(1 + \frac{1 - \frac{s_{01}}{\alpha_{10}}}{(\theta - 1)\alpha_{22} + \frac{s_{01}}{\alpha_{10}}}\right).$$

Thus, relative to the one-way cycle, the added trade flows between the U.S and China summarized by $s_{01} \neq 0$ distort the optimal tariffs. While τ_2 is still irrelevant, the value of τ_1 is lower relative to the benchmark case due to $s_{01} > 0$.

6 Conclusion

We characterize the optimal tariff with and without trade imbalanced, as well as trade policy that can close such imbalances. We do so in a sequence of models focusing on either bilateral imbalances or aggregate imbalances that reflect the country's net foreign assets and liabilities. A dynamic model with international portfolios and convenience yields that give rise to excess returns on international assets relative to liabilities (the "exorbitant privilege") reconciles the imbalanced US trade and portfolio position, namely where a negative NFA position can be consistent with a persistent trade deficit. This also provides a framework in which an import tariff can trigger an exchange rate depreciation, of the kind observed after the April 2, 2025 announcement, provided that this shock eliminates (a part) of the convenience yield of the US. We leave for future work the analysis of an environment with a complete international portfolio choice to endogenize the international (safe) asset demand to the risk of a trade war.

A Proofs

Proofs of Propositions 1 and 7 With a general net foreign asset position (B, B^*) and no export tax, $\tau^E = 1$, the implementability condition that follows from Lemma 4 is given by:

$$u_H^*(C_H^*, Y^* - C_F) \cdot (C_H^* - B) = u_F^*(C_H^*, Y^* - C_F) \cdot \big(C_F - B^*\big), \tag{41}$$

which derives from the foreign optimality condition (4), $u_F^*/u_H^* = P_F^*/P_H^*$, with $C_F^* = Y - C_F$ substituted from the resource constraint (1), and the budget constraint (20), which we rewrite as:

$$C_H^* - B = \frac{P_F^*}{P_H^*} \cdot (C_F - B^*)$$

after using the fact that $P_H^* = P_H$ when $\tau^E = 1$. Note that (41) reduces to (2) in Lemma 1 when $B = B^* = 0$.

Condition (41) defines a mapping $C_H^* = g(C_F)$ which is strictly increasing and concave when $B, B^* > 0$. The planners problem of maximizing $u(C_H, C_F)$ subject to the implementability and resource constraints reduces then to $\max_{C_F} u\big(Y - g(C_F), C_F\big)$, which has the necessary and sufficient optimality condition $u_H \cdot g' = u_F$. Recall that the import tariff (when $\tau^E = 1$) can be expressed as $\tau = \tau^I = (P_F/P_H)/(P_F^*/P_H^*)$ from (2) and $P_F/P_H = u_F/u_H$ from the home optimality (3). Therefore, the optimal tariff equals, as in (23) in the text:

$$\tau = \frac{u_F}{u_H} \cdot \frac{P_H^*}{P_F^*} = \underbrace{\frac{g'(C_F)C_F}{g(C_F)}}_{\equiv \varepsilon} \cdot \underbrace{\frac{P_H^* C_H^*}{P_F^* C_F}}_{EX/IM},$$

where the second equality substitutes in the tariff optimality condition and multiplies and divides the expression by $C_F/g(C_F) = C_F/C_H^*$. Under balanced trade, $P_H^* C_H^* = P_F^* C_F$, and the optimal tariff satisfies $\tau = \varepsilon$.

Finally, the elasticity ε of the trade possibilities frontier function $g(\cdot)$ derives from the full differential of (41) and is given by:

$$\varepsilon = \frac{\mathrm{d}\log C_H^*}{\mathrm{d}\log C_F} = \frac{\eta \frac{C_F}{C_F - B^*} + \frac{C_F}{Y^* - C_F}}{\eta \frac{C_H^*}{C_H^* - B} - 1},$$

where we used the CES functional form for $u^*(C_H^*, C_F^*) = \frac{\eta}{\eta - 1}\Big[{\gamma^*}^{\frac{1}{\eta}} {C_H^*}^{\frac{\eta-1}{\eta}} + (1-\gamma^*)^{\frac{1}{\eta}} {C_F^*}^{\frac{\eta-1}{\eta}}\Big]$. Substituting this expression into the optimal tariff, we have:

$$\tau = \frac{\eta \frac{C_F}{C_F - B^*} + \frac{C_F}{Y^* - C_F}}{\eta \frac{C_H^*}{C_H^* - B} - 1} \frac{P_H^* C_H^*}{P_F^* C_F} = \frac{\eta \frac{P_H^* C_H^*}{P_F^* C_F - P_F^* B^*} + \frac{P_H^* C_H^*}{P_F^* Y^* - P_F^* C_F}}{\eta \frac{P_H^* C_H^*}{P_H^* C_H^* - P_H^* B} - 1} = 1 + \frac{1 + \frac{P_H^* C_H^*}{P_F^* C_F^*}}{\eta \frac{P_H^* C_H^*}{P_H^* C_H^* - P_H B} - 1},$$

where the last equality uses the fact that $P_F^* C_F - P_F^* B^* = P_H^* C_H^* - P_H B$ by the budget constraint (20) and that $P_H = P_H^*$ when $\tau^E = 1$. Finally, we note that

$$1 + \frac{P_H^* C_H^*}{P_F^* C_F^*} = \frac{P_H^* C_H^* + P_F^* C_F^*}{P_F^* C_F^*} = \frac{P^* C^*}{P_F^* C_F^*} = \frac{1}{\Lambda^*},$$

where Λ^* is the foreign local expenditure share. With this, we write the optimal import tariff as:

$$\tau = 1 + \frac{1}{\eta\left(1 + \frac{P_H B}{P_H^* C_H^* - P_H B}\right) - 1} \cdot \frac{1}{\Lambda^*}.$$

When $B = 0$, this corresponds to (11) in Proposition 1. When $B \neq 0$, this corresponds to (24) in Proposition 7, where $\bar{B} \equiv P_H B$. ■

Proofs of Lemma 7 **Part a**: For import tariffs, notice that the set of implementable allocations is determined by two conditions: the household demand

$$\frac{u_F}{u_H} = \tau^I \frac{u_F^*}{u_H^*}$$

and the country's budget constraint

$$u_F^*(C_F - B^*) = u_H^*(C_H^* - B).$$

When $\tau^I \to \infty$, there are two possibilities. Suppose first that u_F/u_H stays finite. Then the former condition requires $u_F^*/u_H^* \to 0$ and hence, $u_H^* \to \infty$ and $C_H^* \to 0$. This is inconsistent with the budget constraint. The only other possibility is that $u_F/u_H \to \infty$, which implies $u_F \to \infty$ and $C_F \to 0$. Substituting into the budget constraint, we get

$$\Phi(C_H^*) \equiv \left(\frac{1-\gamma^*}{\gamma^*}\frac{C_H^*}{Y^*}\right)^{\frac{1}{\eta}} - \frac{B - C_H^*}{B^*} = 0.$$

As long as $B > 0$, function $\Phi(\cdot)$ is continuous, strictly increasing with $\Phi(0) < 0$ and $\Phi(B) > 0$, which guarantees a unique solution $0 < C_H^* < B$. Since $C_F = 0$, we get $NX > 0$. And as $B \to 0$, the limiting point converges to autarky, $C_H^* = C_F = 0$.

Suppose $\tau^I \to 0$. If $u_F/u_H \not\to 0$, then $u_F^* \to \infty$ and $C_F^* \to 0$, $C_F \to Y^*$, which is inconsistent with the budget constraint. It follows that $u_F/u_H \to 0$ and hence, $C_H \to 0$, $C_H^* \to Y$. The equilibrium value of C_F is then uniquely pinned down by the budget constraint

$$\left(\frac{1-\gamma^*}{\gamma^*}\frac{Y}{Y^* - C_F}\right)^{\frac{1}{\eta}} = \frac{Y - B}{C_F - B^*}$$

and satisfies $B^* < C_F < Y^*$. Both imports and exports are positive in this limit with $NX < 0$.

Indeed, monotonicty proven below implies that C_H^* rises and C_F falls relative to a free-trade equilibrium, which depreciates the real exchange rate, i.e., reduces $P_F^*/P_H^* = u_F^*/u_H^*$. Thus, as long as $NFA > 0$ under free trade, the trade balance remains negative in the limit of $\tau^I \to 0$.

To prove monotonicity, substitute marginal utilities into the budget constraint, take logs, differentiate and simplify:

$$\frac{Y^* - B^* + (\eta - 1)(Y^* - C_F)}{(Y^* - C_F)(C_F - B^*)} dC_F = \frac{B + (\eta - 1)C_H^*}{(C_H^* - B)C_H^*} dC_H^*.$$

Notice that $C_F - B^*$ and $C_H^* - B$ always have the same sign. Given that $\eta > 1$ and $C_F, B^* \leq Y^*$, all other terms are positive and we get $dC_H^*/dC_F > 0$. As τ^I changes from zero to infinity, the economy moves along the budget constraint and, given previous limiting cases, both C_F and C_H^* decrease.

Part b: For export tariffs, the equilibrium conditions are given by household demand

$$\frac{u_H^*}{u_F^*} = \tau^E \frac{u_H}{u_F}$$

and the budget constraint

$$u_F(C_F - B^*) = u_H(\tau^E C_H^* - B).$$

Suppose $\tau^E \to 0$. The budget constraint implies $\frac{u_H}{u_F} = \frac{B^* - C_F}{B} < \infty$ and hence, from the former condition $u_H^*/u_F^* \to 0$ and $C_F^* \to 0$. But then $C_F \to Y^*$ and $B^* - C_F < 0$, which is inconsistent with $u_H/u_F > 0$. Thus, there is no equilibrium for $\tau^E \to 0$ and there is a maximum feasible subsidy $\underline{\tau}^E$.

Consider next $\tau^E \to \underline{\tau}^E$. The allocation must then converge to the boundaries of the Edgeworth box with either $C_F = Y^*$ or $C_H = 0$. If $C_F \to Y^*$, then $C_F^* \to 0$ and $u_F^* \to \infty$. Given that $\tau^E u_H/u_F \not\to 0$, the optimal demand requires $u_H^* \to \infty$ and $C_H^* \to 0$. This is inconsistent with the budget constraint $\frac{u_F}{u_H} = -\frac{B}{Y^* - B^*} < 0$. Suppose then $C_H \to 0$ and $C_H^* \to Y$. It follows from household demand $u_F/u_F^* \to \infty$ and hence, $C_F \to 0$ and $C_F^* \to Y^*$. Substitute into the budget constraint

$$\frac{u_F^*}{u_H^*} = \left(\frac{1 - \gamma^*}{\gamma^*} \frac{Y}{Y^*}\right)^{\frac{1}{\eta}} = \frac{B/\tau^E - Y}{B^*},$$

which determines $\underline{\tau}^E$. As long as $B < Y$, we get $0 < \underline{\tau}^E < 1$. The fact that $C_H^* \to Y$ and $C_F \to 0$ implies that $NX > 0$.

Finally, suppose $\tau^E \to \infty$. If $u_H^*/u_F^* \not\to \infty$, then $u_F \to \infty$ and $C_F \to 0$, $C_F^* \to Y^*$, which is inconsistent with the budget constraint. It follows that $u_H^*/u_F^* \to \infty$ and $C_H^* \to 0$,

$C_H \to Y$. The budget constraint implies

$$\left(\frac{\gamma}{1-\gamma}\frac{Y}{C_F}\right)^{\frac{1}{\theta}} = \frac{B}{B^* - C_F}$$

and thus, there is a unique solution $0 < C_F < B^*$. There is zero exports, positive imports and $NX < 0$ in this limit. ■

Proof of Proposition 8 Note that monotonicity of $C_H^* = g^I(C_F)$ proven in part (a) of Lemma 7 implies the result in Proposition 8: starting at the free trade point with $NFA > 0$ and $NX < 0$, there always exists a unique $\tau^I > 1$ such that in the equilibrium with import tariff $NX = NFA = 0$. For larger values of τ^I, the resulting $NX > 0$, as is the case in the limit of $\tau^I \to \infty$. ■

Proof of Proposition 9 (model with convenience yield) Given no export tax, τ^E, we can substitute $P_F^*/P_H = P_F^*/P_H^* = u_F^*/u_H^*$ into the budget constraint (31):

$$\left(C_H^* - \hat{B}\right) - \frac{u_F^*}{u_H^*}\left(C_F - \hat{B}^*\right) + \frac{P^*}{P_H^*}\frac{v'(B)\tilde{B}}{u'(C^*)} = 0,$$

where $\hat{B} \equiv \frac{1-\beta}{1-\beta\delta}B$ and $\hat{B}^* \equiv \frac{1-\beta}{1-\beta\delta}B^*$, and $\tilde{B} \equiv \frac{1-\delta}{1-\beta\delta}B$ for brevity, and the foreign CES price index can be expressed as:

$$\frac{P^*}{P_H^*} = P^*\left(1, \frac{u_F^*}{u_H^*}\right) = \left(\gamma^* + (1-\gamma^*)\left(\frac{1-\gamma^*}{\gamma^*}\frac{C_H^*}{C_F^*}\right)^{\frac{1-\eta}{\eta}}\right)^{\frac{1}{1-\eta}} = \left(\frac{C_H^*}{\gamma^* C^*}\right)^{\frac{1}{\eta}},$$

using the definition of $C^* \equiv \left[\gamma^{*\frac{1}{\eta}} C_H^{*\frac{\eta-1}{\eta}} + (1-\gamma^*)^{\frac{1}{\eta}} C_F^{*\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$.

For CRRA preferences, $u'(C^*) = (C^*)^{-\sigma}$, and if $\sigma\eta = 1$, it follows $u'(C^*)P_H^*/P^* = (\gamma^*/C_H^*)^{\frac{1}{\eta}}$. Thus, the new implementability constraint $C_H^* = g(C_F)$ is implicitly defined by

$$\gamma^{*\frac{1}{\eta}} C_H^{*-\frac{1}{\eta}}(C_H^* - \hat{B}) - (1-\gamma^*)^{\frac{1}{\eta}} C_F^{*-\frac{1}{\eta}}(C_F - \hat{B}^*) + v'(B)\tilde{B} = 0,$$

where $v'(B)\tilde{B}$ is taken by the planner as exogenous constant. Note that it is the same as (21) in Lemma 4 when $v'(B)\tilde{B} = 0$.

The planner's problem is still to maximize $u(Y - g(C_F), C_F)$, which results in the first-order condition $u_H g' = u_F$. Differentiating the implementability constraint and using the

properties of the CES utility, we get

$$g' = \frac{\mathrm{d}C_H^*}{\mathrm{d}C_F^*} = \frac{u_F^*}{u_H^*} \frac{\eta + \frac{C_F - \hat{B}^*}{C_F^*}}{\eta - \frac{C_H^* - \hat{B}}{C_H^*}}.$$

Substitute into the expression for the import tariff, $\tau = \frac{P_F/P_H}{P_F^*/P_H^*} = \frac{u_F/u_H}{u_F^*/u_H^*} = \frac{g'}{u_F^*/u_H^*}$, we have:

$$\tau - 1 = \frac{1 + \frac{P_H^* C_H^*}{P_F^* C_F^*} \frac{P_F^*(C_F - \hat{B}^*)}{P_H^*(C_H^* - \hat{B})}}{\eta \frac{P_H^* C_H^*}{P_H^*(C_H^* - \hat{B})} - 1} = \frac{1 + \frac{1-\Lambda^*}{\Lambda^*} \frac{IM - \bar{B}^*}{EX - \bar{B}}}{\eta \frac{EX}{EX - \bar{B}} - 1},$$

where we used notation $EX = P_H^* C_H^*$, $IM = P_F^* C_F$, $1 - \Lambda^* = 1 - \frac{P_F^* C_F^*}{P^* C^*} = \frac{P_H^* C_H^*}{P^* C^*}$, and $\bar{B} = P_H^* \hat{B}$ and $\bar{B}^* = P_F^* \bar{B}^*$ as defined in the proposition. Finally, note that with this notation, the budget constraint (31) implies $(IM - EX) - (\bar{B}^* - \bar{B}) = \frac{v'(B)\tilde{B}}{u'(C^*)P^*} \equiv CY$. Using this, we can rewrite the formula above as (32) ■

Multi-country model (Section 5)

Proof of Proposition 10 Denote the Lagrange multiplier with μ and take the first-order conditions:

$$u_j = \mu u_j^* \left[1 - \sum_{i=0}^{N} \frac{u_{ij}^*}{u_j^*} (Y_i^* - C_i^*) \right].$$

Use the properties of CES utility to get

$$\frac{u_j}{u_0} = \frac{u_j^*}{u_0^*} \frac{\theta + (Y_j^* - C_j^*)/C_j^*}{\theta - 1}$$

and substitute in household demand function $u_j/u_0 = \tau_j u_j^*/u_j^0$ to get equation (36). ■

Proof of Proposition 11 Denote the Lagrange multipliers on countries' budget constraints with μ_i and take the first-order conditions with respect to

$$\begin{aligned}
\lambda_i: \quad & -\sum_{j=0}^{N} u_j P_{jj}^{-\theta} \gamma_{ji} + \mu_i \sum_{j=0}^{N} \gamma_{ji} P_{jj}^{1-\theta} = 0, \\
P_{jj}: \quad & \theta u_j P_{jj}^{-\theta-1} \sum_{i=1}^{N} \gamma_{ji} \lambda_i + \left[(1-\theta) P_{jj}^{-\theta} \sum_{i=1}^{N} \mu_i \lambda_i \gamma_{ji} - \mu_j Y_j \right] = 0.
\end{aligned}$$

Simplify and rewrite in terms of import tariffs using household demand $\tau_j = \frac{u_j/u_0}{P_{jj}/P_{00}}$, $P_{00} = 1$:

$$u_0 \sum_{j=0}^{N} \gamma_{ji} P_{jj}^{1-\theta} \tau_j = \mu_i \sum_{j=0}^{N} \gamma_{ji} P_{jj}^{1-\theta},$$
$$\theta u_0 \tau_j P_{jj}^{-\theta} \sum_{i=1}^{N} \gamma_{ji} \lambda_i = \mu_j Y_j + (\theta - 1) P_{jj}^{-\theta} \sum_{i=1}^{N} \mu_i \lambda_i \gamma_{ji}.$$

The former condition can be used to express the Lagrange multiplier

$$\mu_i = u_0 \frac{\sum_{j=0}^{N} \gamma_{ji} P_{jj}^{1-\theta} \tau_j}{\sum_{j=0}^{N} \gamma_{ji} P_{jj}^{1-\theta}} = u_0 \sum_{j=0}^{N} s_{ji} \tau_j \equiv u_0 \bar{\tau}_i$$

where we use the fact that spending shares can be expressed as $s_{ji} = \gamma_{ji} \left(\frac{P_{jj}}{P_i} \right)^{1-\theta}$. Substitute μ_i and $\lambda_i = P_i^{\theta} C_i$ into the second optimality condition to get

$$\theta \tau_j (1 - \alpha_{j0}) = \bar{\tau}_j + (\theta - 1) \sum_{i=1}^{N} \alpha_{ji} \bar{\tau}_i.$$

This is isomorphic to formula (38). ■

Proof of Proposition 12 Denote Lagrange multipliers with μ_i and take the first-order conditions with respect to

$$\lambda_i : \quad u_0 \gamma_{0i} P_{0i}^{-\theta} + \sum_{j=1}^{N} u_j P_{jj}^{-\theta} \gamma_{ji} = \mu_i \left[\sum_{j=1}^{N} \gamma_{ji} P_{jj}^{1-\theta} + \gamma_{0i} P_{0i}^{1-\theta} \right],$$
$$P_{jj} : \quad \theta u_j P_{jj}^{-\theta-1} \sum_{i=1}^{N} \gamma_{ji} \lambda_i = (\theta - 1) P_{jj}^{-\theta} \sum_{i=1}^{N} \gamma_{ji} \mu_i \lambda_i + \mu_j Y_j,$$
$$P_{0i} : \quad \theta u_0 \gamma_{0i} P_{0i}^{-\theta-1} \lambda_i = (\theta - 1) \mu_i \lambda_i \gamma_{0i} P_{0i}^{-\theta}.$$

Normalize $P_{00} = 1$ and substitute in $\varsigma_i = \frac{1}{P_{0i}}$ and $\tau_j = \frac{u_j^0}{u_0^0} \frac{1}{P_{jj}}$. The latter FOC implies then

$$\mu_i = u_0 \frac{\theta}{\theta - 1} \varsigma_i.$$

Substitute together with $\lambda_i = P_i^\theta C_i$ into other optimality conditions

$$\gamma_{0i} P_{0i}^{1-\theta} \varsigma_i + \sum_{j=1}^{N} \tau_j \gamma_{ji} P_{jj}^{1-\theta} = \frac{\theta}{\theta - 1} \varsigma_i P_i^{1-\theta},$$
$$\tau_j P_{jj}^{-\theta} \sum_{i=1}^{N} \gamma_{ji} P_i^{-\theta} C_i = P_{jj}^{-\theta} \sum_{i=1}^{N} \gamma_{ji} \varsigma_i P_i^{-\theta} C_i + \frac{1}{\theta - 1} \tau_j^E Y_j$$

and rewrite using demand functions to get system (39). ■

Proof of Proposition 13 Denote the Lagrange multipliers on countries' budget constraints with μ_i and follow the same steps as in the proof of Proposition 11. The optimality condition with respect to λ_i is unchanged and, rewritten in terms of susfficient statistics, implies that $\mu_i = u_0 \bar{\tau}_i$. The first-order condition with respect to P_{jj} is given by

$$\theta u_j P_{jj}^{-\theta-1} \sum_{i=1}^{N} \gamma_{ji} \lambda_i = (\theta - 1) P_{jj}^{-\theta} \sum_{i=1}^{N} \gamma_{ji} \mu_i \lambda_i + \sum_{i=1}^{N} \mu_i B_{ji}.$$

Combine it with household optimality conditions $\tau_j = \frac{u_j / u_0}{P_{jj} / P_{00}}$ and definitions of trade shares α_{ji} and portfolio shares b_{ji} to obtain equation (40). ■

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