International Equity Flows, Monetary Policy, and Time Consistency *

Javier Bianchi
Federal Reserve Bank of Minneapolis

Alejandro Van der Ghote
European Central Bank

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Abstract

This paper studies optimal monetary policy in an environment with international equity flows. We show that foreign ownership of domestic equity introduces a time inconsistency problem that leads the monetary authority to deviate from the natural allocation. When foreign investors own domestic equity, the central bank has an incentive to keep real wages high to compress profits accruing to foreign investors. Anticipating lower returns, foreign investors purchase equity at discounted prices, reducing domestic households' wealth. We further show that households over-divest equity, generating multiple Pareto-ranked equilibria in which the bad equilibrium features depressed equity valuations and higher unemployment. Unlike existing theories of capital-flow management, our model provides a rationale for capital controls on equity inflows.

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1 Introduction

Over the past three decades, international equity flows have expanded markedly, reflecting the increasing depth and integration of global financial markets. As Figure 1 shows, the combined value of foreign direct investment (FDI) and foreign ownership of equity (FOE) has increased from roughly 20% of GDP in 1995 to nearly 80% in 2024. While a common view holds that cross-border equity positions promote risk-sharing and improve welfare, their growing scale raises important questions about their implications for macroeconomic policy and aggregate welfare.

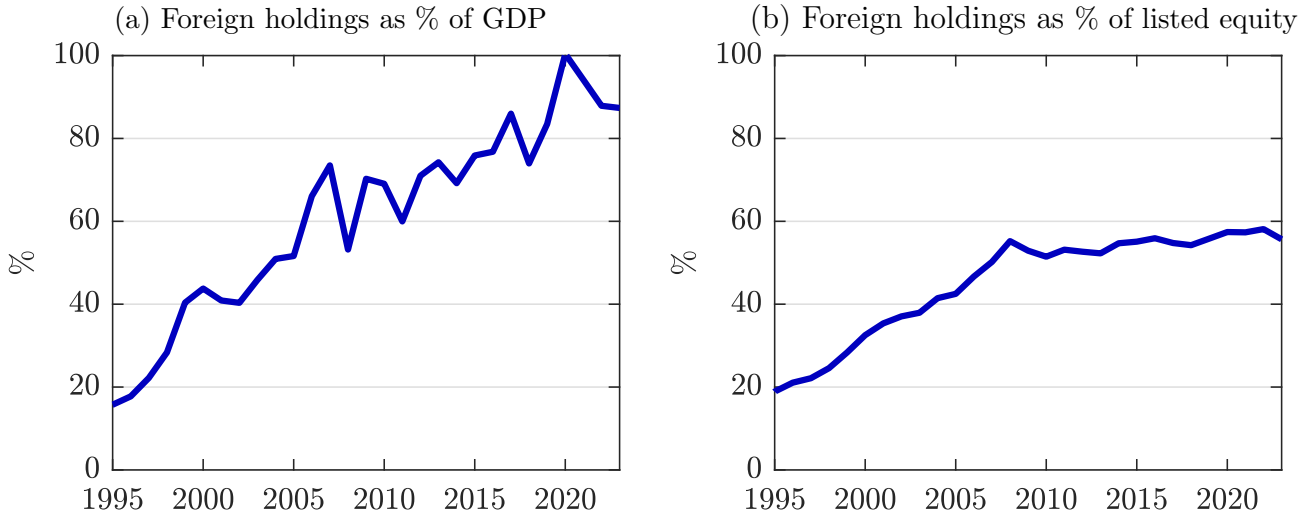


Figure 1: Foreign ownership of domestic equity

Notes: The figure shows the stock of domestic equity owned by foreigners as a share of GDP in panel (a) and as a share of the domestic equity market in panel (b). Foreign ownership of domestic equity is measured as the sum of foreign direct investment (FDI) and foreign ownership of non-financial and bank domestic equity stocks (FOE). The series represent a simple average across the following countries: Austria, Belgium, Brazil, Bulgaria, Canada, Chile, Colombia, Croatia, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Israel, Italy, Japan, South Korea, Mexico, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Turkey, United Kingdom, and the United States. Data source: External Wealth of Nations (EWN) database (Lane and Milesi-Ferretti, 2018) and OECD National Accounts Statistics.

This paper explores the implications of international equity flows for monetary policy and welfare. We show that foreign ownership of domestic equity introduces a time inconsistency problem for monetary policy, as the central bank has an incentive to depart ex post from the natural allocation in order to reduce firms' profits and erode the value of equity claims held by foreign investors. Moreover, we show that opening to international equity flows exposes the economy to a Pareto-inferior equilibrium in which households sell excessive amounts of domestic equity ex ante. This arises because households do not internalize that greater

foreign ownership exacerbates the central bank's ex-post incentive to reduce profits, which depresses the equity price at which they sell the equity claims.

We formalize these results in a tractable open-economy New Keynesian framework in which households trade domestic equity with foreign investors, in addition to foreign bonds. To transparently illustrate our results, the baseline model abstracts from uncertainty, imperfections in capital markets, and domestic heterogeneity. In the case where households own the entire share of domestic equity, optimal monetary policy attains full employment—equivalently, a zero labor wedge or zero output gap—by targeting a level of employment such that the marginal rate of substitution between consumption and leisure equals the marginal product of labor.

When households own the entire domestic equity stock, an increase in the real wage—holding employment fixed—has no effect on aggregate resources: the induced changes in labor income and profits exactly offset in the representative household's budget constraint. Hence, the split between labor income and profits is irrelevant for allocations. This irrelevance no longer holds when foreign investors own domestic equity. In that case, a higher real wage reduces the profits accruing to foreigners while increasing households' income. Although domestic households also receive less profits per their equity claims, their benefit from higher wages exceeds the loss in equity payouts since they only hold a fraction of the total equity. The split between labor income and profits now matters.

With nominal rigidities, monetary policy can affect real wages; thus, a central bank has an incentive to raise real wages when foreign investors own domestic equity. In our baseline model with nominal wage rigidities, we show that the central bank finds it optimal to over-appreciate the exchange rate relative to the full-employment allocation. The logic is that starting from an allocation with a zero labor wedge, there is a first-order gain from reducing employment because the disutility from working exceeds the marginal utility value of the additional resources, as a share of the output revenue is, in effect, leaked abroad. Accordingly, foreign ownership leads the central bank to appreciate the exchange rate to induce a positive labor wedge, and reallocate resources from foreign investors toward domestic households.

Our main result on the implications of international equity flows can be summarized in the following optimality condition for the monetary authority

$$-U_h = \lambda \left[F_h - \theta_1^* \Pi_h \right] \tag{1}$$

where U_h is the disutility from labor, λ is a shadow value from relaxing the country budget constraint, F_h is the marginal product of labor, and θ^* are foreign equity claims, and Π_h

denotes the partial derivative of equilibrium profits relative to employment. The case with full domestic ownership corresponds to $\theta_1^* = 0$, in which case we arrive to the standard efficiency condition that equates the marginal rate of substitution between labor and consumption to the marginal product of labor. However, when foreign investors own a positive fraction of the domestic equity, $\theta_1^* > 0$, condition (1) shows how the monetary authority perceives that a share of the benefits from the increase in production leaks effectively abroad.

The time-inconsistency problem arises because foreign investors anticipate the central bank's ex-post incentive to erode their profits and therefore purchase equity only at a discounted price. This lack of commitment reduces the wealth of domestic households. When households sell equity, they fail to internalize that greater foreign ownership intensifies the central bank's incentive to compress future profits, depressing equity valuations today and lowering welfare. As a result, households collectively sell too much equity, generating an ex-post distortion in output without delivering ex-ante benefits.

Moreover, we show that multiple equilibria arise quite naturally. When households sell more assets, this lowers future dividends and reduces the price of equity shares, to the extent that deep-pocket foreign investors take the other side in the market. While individual investors are indifferent between holding or not the equity, collectively, when they sell more equity, these sales reduce domestic wealth. Interestingly, the presence of multiple equilibria is not a result of fire sales and amplification through financial constraints, as it has been explored in the literature.

A key policy implication is that restricting foreign purchases of domestic equity is optimal. When domestic households initially own the entire equity stock, the optimal policy is to shut down all equity flows. When foreign investors already hold some equity, the optimal capital control trades off closing the output gap against capital gains accruing to foreigners. As a result, the optimal policy is to induce households to repurchase a limited share of the equity held by foreigners.

In one of the extensions of the model, we incorporate non-tradable goods. Our result that monetary policy departs from full employment remains unchanged, but now an important consideration is the impact of the relative price of non-tradables on foreigners' profits. While an appreciation lowers profits in firms in the non-tradable sector in units of domestic currency, the lower output of non-tradables raises its relative price and may increase the profits received by foreign investors in units of tradables. We show that the implications for monetary policy depend on the elasticity of substitution between tradables and non-tradables. When the elasticity is below one, the increase in the relative price of non-tradables may outweigh the benefits of higher real wages. Therefore, an appreciation is not necessarily optimal. On the

other hand, when the elasticity is above one, appreciating the exchange rate remains optimal because the increase the increase in the relative price of non-tradables is small relative to the increase in wages.

Related literature. Our paper belongs to the literature on monetary policy in open economies. A central theme in this literature, as in closed-economy models, is macro-stabilization: when prices are rigid, monetary policy can help align relative prices toward the full-employment or natural allocation (e.g., Friedman, 1953; Mundell, 1960; Schmitt-Grohé and Uribe, 2001). There are also studies that examine channels by which a departure from the natural allocation may be optimal in environments where trade in financial assets is restricted to bonds. One strand of this literature is concerned with terms of trade manipulation. When countries have market power in the tradable goods they produce, central banks can use monetary policy to tilt the terms of trade on their favor (e.g., Obstfeld and Rogoff, 1995 and Corsetti and Pesenti, 2001). Another strand of this literature considers the case when bonds are in domestic currency and analyzes how variations in exchange rates can generate valuation effects on the net foreign asset position. For instance, Fanelli (2023) shows how the optimal monetary policy under commitment balances macro-stabilization and insurance while Ottonello and Perez (2019), Du, Pflueger and Schreger (2020) and Engel and Park (2022) study the resulting inflationary bias when the government cannot commit. In contrast to these contributions, we consider a model with trade in equity claims and show how this gives rise to a novel source of time inconsistency with distinct implications for policy.

Our paper is also related to the literature on international portfolios.[1] A central theme in this literature is understanding the diversification of equity portfolios (e.g., Engel and Matsumoto, 2009; Coeurdacier, Kollmann and Martin, 2010; Kollmann, 2006; Baxter and Jermann, 1997; Devereux and Sutherland, 2006, Tille and Van Wincoop, 2010; Heathcote and Perri, 2013). Our paper takes a normative perspective and uncovers an aggregate welfare cost from such diversification. In addition, most of these studies are in the the context of real models, thus abstracting from the role of monetary policy. A notable exception is Engel and Matsumoto (2009) who show how a portfolio of home and foreign equities and a forward position in foreign exchange can achieve to a first-order the complete market allocations.

[1] See, for example, Engel and Matsumoto (2009), Coeurdacier, Kollmann and Martin (2010), Heathcote and Perri (2013); Kollmann (2006); Baxter and Jermann (1997) Devereux and Sutherland (2006), Tille and Van Wincoop (2010).

However, they do not consider optimal monetary policy.[2]

Our work also relates to recent studies examining the implications of the rise in cross-country asset ownership. Atkeson, Heathcote and Perri (2022) document that the U.S. "exorbitant privilege" from valuation effects largely disappeared post-2007. They show the deterioration is due to the surge in U.S. corporate equity values, driven by a rise in markups. Quadrini and Ríos-Rull (2024) analyze optimal taxation in a setting where multinationals engage in profit shifting. In contrast, our focus is on studying the implications of the rise in cross-country asset ownership for monetary policy.[3]

Our paper is also related to the literature on monetary unions, in particular, those emphasizing how joining a monetary union can deal with time inconsistency problems in monetary policy. In Alesina and Barro (2002), joining a monetary reduces the inflationary bias generated by the time inconsistency problem of monetary policy, as stressed by Barro and Gordon (1983). Fornaro (2022) presents an environment where the central bank has incentives to depreciate the exchange rate to reduce the real value of non-tradable collateral that foreign investors can seize from defaulting firms ex post. Because of this time inconsistency problem, he shows that joining a monetary union can help increase bond inflows. Different from these studies, the time inconsistency problem of monetary policy in this paper is related to international equity flows.

Our paper is also related to the literature on monetary policy with heterogeneous agents in closed economies. Much of the HANK literature assumes an equal share of profits between households or considers setups where equilibrium profits are zero, muting redistributive motives through equity holdings (e.g., Dávila and Schaab, 2023). Broer, Harbo Hansen, Krusell and Öberg (2020) analyze the role of the distribution of profits for the transmission mechanism (see also Bilbiie, 2008). Two papers that share a normative focus are Bhandari, Evans, Golosov and Sargent (2021) and Acharya, Challe and Dogra (2020), but examine optimal policy under commitment. Moreover, they assume that fiscal policies are in place that effectively equalize marginal utilities of consumption at the steady state. Thus, while their focus is on how monetary policy can provide insurance against aggregate shocks, we examine a redistribution motive that emerges naturally because the government is unable to tax foreign investors and does not place any weight on their welfare.

[2]Of course, models of optimal monetary policy with complete markets implicitly allow trade in equity claims. However, much of this literature (e.g., Clarida, Gali and Gertler, 2002; Devereux and Engel, 2003; Obstfeld and Rogoff, 1995; Corsetti, Dedola and Leduc, 2010) studies optimal policy under commitment and/or limited international cooperation, settings in which the mechanism we highlight is absent.

[3]Empirical contributions include Lane and Milesi-Ferretti (2005); Coppola, Maggiori, Neiman and Schreger (2021); Beck, Coppola, Lewis, Maggiori, Schmitz and Schreger (2024).

Finally, we are also related to the literature on capital controls and capital flow management policies. This literature has developed a rationale for taxes on debt flows based on pecuniary externalities (e.g., Bianchi, 2011) and aggregate demand externalities (e.g., Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016). See Bianchi and Lorenzoni (2021) for a review of the literature. In contrast, we provide a rationale for capital controls on equity flows.

Outline. The remainder of the paper is organized as follows. Section 2 introduces the environment. Section 3 studies the optimal exchange rate policy. Section 4 examines the role for capital controls. Section 5 analyzes extensions of the baseline framework. Section 6 concludes.

2 The Model

Consider a small open economy (SOE) with time indexed by $t \in \{0, 1, 2, \ldots\}$. The SOE is composed of a continuum of identical households and firms, and a monetary authority. There is an international financial market with a risk-free bond that pays an exogenous real return. Our key departure from the standard framework is that foreign investors may also trade equity claims on domestic firms.

We restrict attention to a deterministic environment where nominal rigidities will be present at $t = 1$. In addition, our baseline model assumes that households do not face any borrowing limits. This will allow us to highlight the key mechanism at play and characterize analytically the equilibrium. We next describe the decision problems of households, firms, and foreign investors before analyzing the competitive equilibrium and optimal policies.

2.1 Firms

There is a unit mass of identical firms that produce a single final good using labor according to a strictly increasing and strictly concave production function, $F(h)$. We assume that the production function satisfies $F(0) = 0, \lim_{h\to 0} F'(h) = \infty$, and $\lim_{h\to\infty} F'(h) = 0$.

We assume that the final good is tradable in the international market and the law of one price holds. Normalizing the foreign-currency price to one implies that the domestic-currency price of the good is $P_t = E_t$, where E_t denotes the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency.

The firms' problem consists of choosing employment to maximize profits:

$$\pi_t = \max_{h_t \geq 0} \left\{ E_t F\left(h_t\right) - W_t h_t \right\}, \tag{2}$$

where W_t is the nominal wage in domestic currency.

Profit maximization implies that labor demand satisfies

$$F'\left(h_t\right) = w_t \ , \ \forall t \geq 0. \tag{3}$$

where $w_t \equiv W_t / E_t$ is the real wage.

2.2 Households

There is a continuum of identical households of measure one. Their preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(\ell_t\right) \right],$$

where $\beta \in (0,1)$ is the discount factor, c_t denotes consumption, ℓ_t denotes hours worked. The utility function $u(\cdot)$ is strictly increasing and strictly concave, and the disutility from working is strictly increasing and convex. We assume the usual Inada conditions: $\lim_{c \to 0} u'(c) = \infty$, $\lim_{c \to \infty} u'(c) = 0$, $\lim_{\ell \to 0} v'(\ell) = 0$, and $\lim_{\ell \to \infty} v'(\ell) = \infty$.

Households receive labor income, consume, and trade domestic firms' shares and bonds in international financial markets.[4] We let $\theta_t \geq 0$ and b_t respectively denote the holdings of firms' shares and bonds carried to period t (chosen at $t-1$). Bonds are assumed to be one-period and denominated in foreign currency, or equivalently, units of consumption. Firms' shares entitle their owner to a fraction of the firm's profits. Given a constant amount of firms' shares, which we normalize to one, a household that owns θ_t units of equity receive $\pi_t \theta_t$ in period t.

Households' budget constraint is given by

$$c_t = w_t \ell_t + (\pi_t + q_t)\theta_t - q_t \theta_{t+1} + (1+r) b_t - b_{t+1}, \tag{4}$$

where q_t is the price of equity and r is the constant interest rate on bonds. We highlight that

[4] Our results would be unchanged if households were to hold foreign equities in addition to bonds. The reason is that a small open economy's policy does not affect outcomes in foreign markets and the fact that absent uncertainty, bonds and foreign equities are perfect substitutes.

while r is exogenously given, the price of equity q_t is endogenously determined in equilibrium. Moreover, we index the price by t to capture that aggregate states will fluctuate over time.

For our baseline model, we assume households do not face borrowing limits except for a no-Ponzi game condition:

$$\lim_{t\to\infty} \frac{b_{t+1} + q_t\theta_{t+1}}{(1+r)^t} \geq 0\,.$$

Wage rigidity. In period $t = 1$, the nominal wage W_1 is rigid. We assume that at that period labor is demand-determined, which means that households must supply the hours of labor that firms demand. We focus attention on the case where hours are evenly distributed across households. Using (3), we then have that the number of hours that each household work in equilibrium must be given by

$$\ell_1 = F'^{-1}\left(\frac{\overline{W}_1}{E_1}\right), \tag{5}$$

where $\overline{W}_1$ denotes the exogenous rigid wage.[5]

Household problem. At every period, households choose consumption, asset portfolio, and labor supply (except at period 1, where the number of labor hours is determined by (5)). We let $V_t(b_t, \theta_t)$ denote the value at the beginning of the period for a household with an initial individual asset portfolio (b_t, θ_t), where we index the value function by t because the aggregate portfolio and thus prices vary over time. The households' optimization problem can be represented recursively as follows:

$$V_t\left(b_t, \theta_t\right) = \max_{c_t, \ell_t, b_{t+1}, \theta_{t+1} \geq 0} \left\{u\left(c_t\right) - v\left(\ell_t\right) + \beta V_{t+1}\left(b_{t+1}, \theta_{t+1}\right)\right\}, \tag{6}$$

subject to the budget constraint

$$c_t = w_t\ell_t + (\pi_t + q_t)\theta_t - q_t\theta_{t+1} + (1+r)b_t - b_{t+1}\,,$$

the labor-supply schedule (5) for $t = 1$, and the no-Ponzi game.

Household optimality requires that

$$u'(c_t)w_t = v'(\ell_t)$$

[5]Our modeling of period-1 wage rigidity is motivated by expositional simplicity and can be extended to allow for wage setting in period 0 with differentiated labor, subject to pricing frictions.

for all $t \neq 1$ and the Euler equations

$$u'(c_t) = \beta(1+r)u'(c_{t+1}) \tag{7}$$

$$q_t\, u'(c_t) \geq \beta(\pi_{t+1} + q_{t+1})u'(c_{t+1}) \tag{8}$$

for all $t \geq 0$, where the second condition holds with equality if $\theta > 0$.

2.3 Foreign investors

The key departure from a standard framework is that foreign investors can buy and sell equity of domestic firms. We assume that foreign investors have deep pockets—i.e., they face no binding wealth or borrowing constraints—and that they face a no-short-selling constraint in domestic equity, so their holdings must be non-negative, $\theta^*_{t+1} \geq 0$. It follows from investors' optimality and (7)-(8), that for the market of equity to clear, it must be that the return on bonds equals the return on firms' shares.[6] That is,

$$1 + r = \frac{\pi_{t+1} + q_{t+1}}{q_t}. \tag{9}$$

Equation (9) implies that both households and foreign investors are indifferent about the composition of their portfolios between equity and bonds.

2.4 Competitive Equilibrium

We can now define a competitive equilibrium for a given exchange rate policy. We restrict attention to a symmetric equilibrium, in which all of the households choose the same portfolios.

Definition 1. Given an initial portfolio (b_0, θ_0), a world real interest rate r, a rigid wage $\overline{W}_1$, and an exchange rate policy, $\{E_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is a sequence of allocations $\{c_t, h_t\}_{t=0}^{\infty}$, portfolios $\{b_{t+1}, \theta_{t+1}, \theta^*_{t+1}\}_{t=0}^{\infty}$, profits π_t, and prices $\{W_t, q_t\}_{t=0}^{\infty}$ such that:

(i) Households optimize. That is, consumption, labor, and the portfolios solve problem (6).

(ii) Firms choose employment optimally. That is, (3) holds.

(iii) The price for firms' shares satisfies condition (9) and $\theta_{t+1} + \theta^*_{t+1} = 1$.

[6] Foreign investor's portfolio condition is $q_t \geq \beta(\pi_{t+1} + q_{t+1})$ with equality if $\theta^*_{t+1} > 0$. Likewise (7)-(8) yield $q_t \geq \beta(\pi_{t+1} + q_{t+1})$ with equality if $\theta > 0$. For $\theta_{t+1} + \theta^*_{t+1} = 1$, (9) must hold.

Profits, GNI, and balance of payment accounting. A key equilibrium variable in our analysis is firms' profits. Combining (2) and (3), equilibrium profits are a function of aggregate employment only:

$$\pi_t = F(h_t) - F'(h_t)h_t \equiv \Pi(h_t),$$

where the profit function $\Pi(\cdot)$ satisfies $\Pi(0) = 0$ and $\Pi'(h) = -F''(h)h > 0$ for $h > 0$.

Following the standard balance of payment accounting, we define gross national income (GNI), the current account (CA), and the net foreign asset position (NFA) as

$$\begin{aligned} GNI_t &\equiv F(h_t) + rb_t - \theta_t^* \Pi(h_t), \\ CA_t &\equiv GNI_t - c_t, \\ NFA_t &\equiv b_t - q_t \theta_t^*. \end{aligned}$$

GNI is the income accruing to domestic residents: $F(h_t)$ is domestic output, $\theta_t^*\Pi(h_t)$ is profit income paid out to foreign shareholders, and rb_t is interest received on the country's net foreign bond position.[7] The CA measures the economy's net saving vis-a-vis the rest of the world—income accruing to residents minus domestic absorption. The NFA position is the value of foreign assets b_t net of the market value $q_t\theta_t^*$ of domestic equity held by foreigners, which represents a liability for the small open economy.

Using the household budget constraint, market clearing for equity claims $\theta_t^* = 1 - \theta_t$, and the expression for GNI yields the balance of payment condition: a current-account deficit must be financed either by reducing bond holdings b_t or by issuing additional domestic equity to foreign investors (raising θ_t^*).

$$CA_t = (b_{t+1} - b_t) - q_t(\theta_{t+1}^* - \theta_t^*).$$

Given this expression, our definition of the NFA, we can express its evolution as

$$NFA_t - NFA_{t-1} = CA_t - \theta_t^*(q_t - q_{t-1}).$$

That is, the change in the NFA position equals the current account plus a valuation term

[7] Notice that we can write the GNI equivalently as

$$GNI_t = \theta_t F(h_t) + (1 - \theta_t)F'(h_t)h_t + rb_t,$$

an average of total output and labor income (weighted by the fraction of equity owned by domestic households) plus interest on foreign bond holdings

$-\theta_t^*(q_t - q_{t-1})$, which captures capital gains and losses on the outstanding stock of equity claims held by foreign investors.

3 International Equity Flows and Exchange Rate Policy

In this section, we characterize equilibria and analyze the implications of international equity flows for the optimal exchange rate policy. To streamline the analysis, we assume $\beta(1+r) = 1$, so the real interest rate equals the households' discount rate and consumption is constant.

Roadmap. We proceed with our analysis as follows: First, we characterize the steady-state equilibrium for $t \geq 2$. Second, we analyze the competitive equilibrium at $t = 1$ and the optimal exchange rate policy. Finally, we study the scope for capital controls in period $t = 0$.

3.1 Steady-state equilibrium for $t \geq 2$

The assumption that nominal wages are flexible after $t \geq 2$ ensures monetary neutrality in this continuation equilibrium. Moreover, given that $\beta(1 + r) = 1$, we have that all allocations are constant. We use the variables c and h to denote steady-state values. Lemma 1 characterizes this continuation equilibrium for any given exchange rate policy and for any initial portfolio of assets and liabilities that characterizes the economy's NFA position (b_2, θ_2^*).

Lemma 1 (Steady-state equilibrium)**.** *Given an initial portfolio (b_2, θ_2^*), the continuation equilibrium for $t \geq 2$ features constant consumption and labor. In particular, the steady-state values, h and c, are given by*

$$\frac{v'(h)}{u'(c)} = F'(h)\,, \tag{10}$$

$$c = F(h) + rb_2 - \Pi(h)\theta_2^*\,, \tag{11}$$

and the sequence of portfolios $\{b_{t+1}, \theta_{t+1}^*\}_{t\geq 2}$ and the steady-state price of equity, q, satisfy

$$b_{t+1} - q\,\theta_{t+1}^* = b_2 - q\,\theta_2^* \tag{12}$$

and

$$q = \frac{F(h) - F'(h)h}{r}. \tag{13}$$

Proof. In Appendix A.1. □

Condition (10) is the standard condition equating the marginal rate of substitution between consumption and labor to the marginal product. For the analysis that follows, it will be useful to define $h = \mathcal{H}(c)$, as the level of employment consistent with (10), where $\mathcal{H}' < 0$, reflecting the standard wealth effect on labor supply.

Condition (11) says that in steady state—when the current account is balanced—consumption equals gross national income (GNI). Condition (12) states that any portfolio of bonds and domestic equity that keeps the NFA constant is consistent with equilibrium, reflecting that households are indifferent across portfolios that deliver the same level of consumption. Condition (13) states the asset price equals the present discounted value of profits.

3.2 Continuation equilibrium at period $t = 1$

In this section, we characterize the equilibrium starting from $t = 1$, given an initial portfolio (b_1, θ_1^*). Using the employment function $\mathcal{H}(\cdot)$ defined above, the following lemma characterizes the equilibrium outcome in period 1 for any given exchange rate E_1 and initial portfolio (b_1, θ_1^*).

Lemma 2 (Equilibrium for $t \geq 1$)**.** *Given an initial portfolio (b_1, θ_1^*), a rigid wage $\overline{W}_1$, an exchange rate E_1, the equilibrium for $t \geq 1$ is such that*

i) h_1 is given by

$$\frac{\overline{W}_1}{E_1} = F'(h_1) \tag{14}$$

ii) $h_t = \mathcal{H}(c)$ for $t > 1$, and $c_t = c$ for $t \geq 1$, with c given by

$$c = \frac{r}{1+r}\Big[F(h_1) + rb_1 - \Pi(h_1)\theta_1^*\Big] + \frac{1}{1+r}\Big[F\big(\mathcal{H}(c)\big) + rb_1 - \Pi(\mathcal{H}(c))\theta_1)\Big] \tag{15}$$

iii) $q_t = q$ with q given by (13) and the sequence of portfolios are such that

$$(b_2 - q\,\theta_2^*) \;=\; (b_1 - q\,\theta_1^*) + \frac{F(h_1) - \Pi(h_1)\theta_1^* - \big[F\left(\mathcal{H}(c)\right) - \Pi\left(\mathcal{H}(c)\right)\theta_1^*\big]}{1+r} \tag{16}$$

and $\left\{b_{t+1}, \theta_{t+1}^*\right\}_{t\geq 2}$ satisfy (12).

Proof. In Appendix A.5. □

Condition (14) captures that when the nominal wage is preset, the exchange rate pins down the real wage, and employment is determined by firms' demand at that real wage. Condition (15) reflects that consumption must be constant over time, given that $\beta(1+r)=1$ and that preferences between consumption and labor are separable. The expression for consumption follows from the intertemporal budget constraint: the two terms are the annuity value of period-1 GNI and of future GNI, respectively. Recall that returns on bonds and equity are equal in equilibrium, which implies that households are indifferent across any portfolio of bonds and domestic equity that supports the equilibrium level of consumption.[8] Condition (16) describes the evolution of the NFA. Notice that because in general period-1 GNI differs from the future GNI, the current account in period 1 is not necessarily balanced, hence the NFA varies between periods 1 and 2.

3.3 Optimal exchange rate policy

We now study the problem of a central bank that chooses monetary policy in period 1 to maximize households' welfare in the competitive equilibrium. Notice that by setting the period-1 exchange rate, the central bank can effectively choose labor in period 1. Using Lemma 4.1, this problem can be formulated as choosing a period-1 level of employment, h_1, a constant level of consumption, c, and an associated steady-state level of employment, $\mathcal{H}(c)$, subject a country budget constraint. That is, the optimal policy problem is

$$V_1(b_1,\theta_1^*)=\max_{c,h_1}\left\{u(c)-v(h_1)+\frac{u(c)-v\left(\mathcal{H}(c)\right)}{r}\right\},$$

subject to

$$\left(\frac{1+r}{r}\right)c=F(h_1)+rb_1-\Pi(h_1)\theta_1^*+\frac{F\left(\mathcal{H}(c)\right)+rb_1-\Pi\left(\mathcal{H}(c)\right)\theta_1^*}{r}. \tag{17}$$

Denoting by $\lambda>0$ the Lagrange multiplier on the intertemporal country budget constraint (15), we have the following optimality condition with respect to period-1 employment h_1:

$$v'(h_1)=\lambda\left[F'(h_1)-\Pi'(h_1)\theta_1^*\right], \tag{18}$$

Condition (18) equates the marginal disutility of working an additional unit of time in period

[8] Moreover, this also implies that the future GNI can also be expressed as $F(\mathcal{H}(c))+rNFA_1$.

to the marginal benefits, which are given by the additional period-1 resources times the marginal benefit of relaxing the country budget constraint. Crucially, the change in period-1 resources from a marginal increase in employment is the increase in *output net of the rise in profits*, that is $F'(h_1) - \theta_1^* \Pi'(h_1)$. The latter term captures the additional profits accruing to foreign shareholders, and its effect on domestic resources is

$$-\theta_1^* \Pi'(h_1) = \theta_1^* F''(h_1) h_1 \leq 0,$$

where the inequality is strict whenever $\theta_1^* > 0$ because the production function is strictly concave, $F''(h) < 0$. That is, higher employment raises output but lowers wages—as the central bank needs to depreciate the exchange rate to stimulate labor demand—and firms' profits. Because a share θ_1^* of firms' profits accrue abroad, a more stimulative policy generate an increase in the effective liabilities that the SOE faces. This is why stimulating employment in carries a higher marginal cost compared to the case without foreign ownership.

The optimality condition with respect to consumption c yields

$$u'(c) + \frac{1}{1+r}\Big[- v'\big(\mathcal{H}(c)\big) + \lambda\big(F'\big(\mathcal{H}(c)\big) - \Pi'\big(\mathcal{H}(c)\big)\theta_1^*\big)\Big]\mathcal{H}'(c) = \lambda. \tag{19}$$

Equation (19) states that the marginal benefit of increasing consumption must equal the shadow cost of the additional resources required to finance it. The first term on the left-hand side is the direct marginal utility of an additional permanent unit of consumption. The second term captures the indirect effect of higher consumption on welfare through its impact on steady-state employment: because $H'(c) < 0$, an increase in c reduces future hours, which in turn lowers both the marginal disutility from working, $v'\big(\mathcal{H}(c)\big)$, and the annuity value of future output and profits, $F'\left(\mathcal{H}(c)\right) - \Pi'\left(\mathcal{H}(c)\right)\theta_1^*$.

Combining optimality conditions (18) and (19), we obtain

$$\underbrace{F'(h_1)u'(c) - v'(h_1)}_{\text{Labor wedge}} = \left(\frac{\Pi'(h_1)u'(c) - \frac{1}{1+r}\Big[\Pi'(h_1)\, v'\big(\mathcal{H}(c)\big) - \Pi'(\mathcal{H}(c))\, v'(h_1)\Big]\mathcal{H}'(c)}{1 - \frac{1}{1+r}F'(\mathcal{H}(c))\mathcal{H}'(c)} \right) \theta_1^* \,. \tag{20}$$

The left-hand side of (20) represents the labor wedge. When foreign investors do not own equity claims on domestic firms, $\theta_1^* = 0$, the right-hand side of (20) is zero and the central bank sets the wedge to zero, replicating the flexible-wage (natural) allocation. By contrast, when $\theta_1^* > 0$, the right-hand side of (20) is generally different from zero, so the optimal policy features a non-zero labor wedge and the central bank finds it optimal to depart from the natural allocation. The intuition is that the central bank uses the exchange rate to

compress profits accruing to foreign shareholders, thereby reducing the value of the economy's foreign liabilities. As discussed above, a reduction in period-1 employment h_1 reduces contemporaneous profits, which induces the central bank to generate a positive labor wedge, as shown by the first term of the numerator on the right side. However, at the same time, a lower h_1 reduces period-1 total income and the net foreign asset position, which generates a negative wealth effect that induces households to supply more labor in steady state, raising long-run profits. Under the conditions stated below, this indirect effect is dominated by the contemporaneous benefit of higher wages in period-1, implying that the right-hand side of (20) is positive and that the optimal labor wedge is strictly positive whenever $\theta_1^* > 0$.

Define the equilibrium labor wedge under optimal exchange rate policy $E_1(b_1, \theta_1^*) = \overline{W}_1 / F'\left(h_1(b_1, \theta_1^*)\right)$ as $\zeta_1(b_1, \theta_1^*)$, with

$$\zeta_1(b_1, \theta_1^*) \equiv u'(c(b_1, \theta_1^*))F'(h_1(b_1, \theta_1^*)) - v'(h_1(b_1, \theta_1^*))\,,$$

where consumption $c(b_1, \theta_1^*)$ and period-1 employment $h_1(b_1, \theta_1^*)$ jointly solve (17) and (20) for any given international portfolio (b_1, θ_1^*). The below proposition establishes conditions under which $\zeta_1(b_1, \theta_1^*) > 0$ when $\theta_1^* > 0$.

Proposition 1 (Positive labor wedge)**.** *Suppose that either one of the following two conditions hold:*

1. *Wealth effects on labor supply are arbitrarily small. Formally:*

$$-\mathcal{H}'(c) = -u''(c)\frac{F'\big(\mathcal{H}(c)\big)}{v''\big(\mathcal{H}(c)\big) - u'(c)F''\big(\mathcal{H}(c)\big)} \leq \varepsilon \;\; \forall c > 0,$$

   where $\varepsilon > 0$ is positive, but arbitrarily small.

2. *$\{u(\cdot), v(\cdot), F(\cdot)\}$ are isoelastic. That is:*

$$u(c) = \frac{1}{1-\frac{1}{\sigma}}c^{1-\frac{1}{\sigma}}\,, \quad v(\ell) = \chi\frac{1}{1+\psi}\ell^{1+\psi}\,, \quad F(h) = h^{\alpha}\,, \tag{21}$$

   with $\sigma > 0$, $\chi > 0$, $\psi > 0$ and $\alpha \in (0,1)$ being parameters.

Then, $\zeta_1(b_1, \theta_1^*) \geq 0$, and $\zeta_1(b_1, \theta_1^*) > 0$ if $\theta_1^* > 0$.

Proof. In Appendix A.3 □

Wealth effects on labor supply are arbitrarily small if—for instance—utility of consumption is arbitrarily close to be linear. Isoleastic functional forms guarantee that, as far as

determination of the optimal labor wedge is concerned, wealth effects on labor supply are of second-order importance relative to contemporaneous effects on firm profits.

Next, we study how the equilibrium labor wedge responds to marginal changes in the share of domestic equities owned by foreign investors. Naturally, everything else fixed, changes in foreign ownership of domestic equities affect the wealth of the SOE and, through a resulting wealth effect, the labor wedge. To abstract from those wealth effects away, we restrict attention to perturbations to the international portfolio that absent changes in the price of domestic equities keep the NFA fixed. Formally, define the equilibrium price of domestic equities in period 0 given international portfolio (b_1, θ_1^*) as $q_0(b_1, \theta_1^*)$, with

$$q_0(b_1, \theta_1^*) \equiv \frac{1}{1+r}\left[\Pi(h_1(b_1, \theta_1^*)) + \frac{1}{r}\Pi(\mathcal{H}(c(b_1, \theta_1^*)))\right].$$

The considered perturbations are constructed as $\hat{\theta}_1^* = \theta_1^* + \varepsilon$ and $\hat{b}_1 = b_1 + q_0\left(b_1, \theta_1^*\right)\varepsilon$, where $\varepsilon > 0$ is positive, but arbitrarily small. Note that $NFA_1 = \hat{b}_1 - q_0\left(b_1, \theta_1^*\right)\hat{\theta}_1^* = b_1 - q_0\left(b_1, \theta_1^*\right)\theta_1^*$ for any ε. The below proposition establishes conditions under which $\zeta_1(\hat{b}_1, \hat{\theta}_1^*) > \zeta_1(b_1, \theta_1^*)$. The proof to the proposition also shows that under the stated conditions $h_1(\hat{b}_1, \hat{\theta}_1^*) < h_1(b_1, \theta_1^*)$ and $c(\hat{b}_1, \hat{\theta}_1^*) < c(b_1, \theta_1^*)$.

Proposition 2 (Increasing labor wedge)**.** *Suppose that either one of the following two conditions hold:*

1. *Wealth effects on labor supply are arbitrarily small and $F(\cdot)$ is isoelastic.*
2. *$\{u(\cdot), v(\cdot), F(\cdot)\}$ are isoelastic.*

Then,

$$\frac{\partial \zeta_1}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial \zeta_1}{\partial \theta_1^*} > 0\,.$$

Proof. In Appendix A.4 □

Lastly, we discuss the case of an isoelastic production function, to illustrate that foreign ownership of domestic equities could be interpreted from the perspective of the SOE to a reduction in productivity. Formally, under $F(h) = h^\alpha$ with $\alpha \in (0,1)$, GNI_t can be expressed as

$$GNI_t = \left[1 - (1-\alpha)\theta_t^*\right] F\left(h_t\right) + r b_t\,,$$

where $(1-\alpha)\theta_t^*$ is akin from the perspective of the SOE to a productivity loss. Since the monetary authority internalizes that a share of firms' output accrues to foreign investors,

it finds it optimal if $\theta_t^* > 0$ to induce a lower level of employment relative to the natural allocation, mimicking the effect of lower productivity. Naturally, this also reduces the level of consumption, creating a labor wedge or increasing it.

3.4 Quantitative results

In this section, we provide a simple quantification to assess the role of international equity flows in shaping the optimal exchange rate policy and the welfare implications. We parametrize $\{u(\cdot), v(\cdot), F(\cdot)\}$ using the isoelastic functional forms in (21), interpreting σ as intertemporal elasticity of substitution of consumption, χ as the disutility weight of labor supply, ψ as the inverse of the Frisch elasticity of labor supply, and α as the return to scale in production.

The time period is a year. We set the intertemporal elasticity of substitution to $\sigma = 1$, the inverse labor supply elasticity to $\psi = 1$, the labor share to $\alpha = 0.65$, and the gross interest rate $R = 1.04$. These values are standard in the literature. We also normalize the disutility from labor χ so that output equals 1 under flexible prices.

We set the initial NFA to -60% of GDP, roughly the U.S. value in 2024. A reference value for θ, following Figure 1, is $\theta = 0.5$. In the simulations below, we consider values for $\theta \in [0, 1]$ and adjust the initial bond holdings so that the net foreign asset position remain constant in the flexible price allocation. In particular, we vary b_1 so that $b_1 - \tilde{q}\,\theta_1^* = -60\%$, where $\tilde{q}$ denotes the price of equity under flexible wages).

Figure 2 presents the results for the optimal exchange rate policy at $t = 1$. The top panels plot employment, the labor wedge, and welfare (relative to the natural allocation); the bottom panels display the exchange rate, consumption, and profits. We express the exchange rate, output, consumption, and profits as percentage deviation from an economy without foreign ownership.

When $\theta_1^* = 0$, the optimal policy sets the labor wedge to zero (panel b), reproducing the flexible-wage allocation. As foreign holdings increase, this strengthens the motive to raise real wages and compress profits, and therefore the labor wedge increases and the exchange rate appreciates. For $\theta_1^* = 0.5$, the monetary authority appreciates the exchange rate by 4 percent; labor income and profits each fall 6 percent; hours and output fall 10 and 4 percent, and consumption declines 0.15 percent relative to the case with $\theta_1^* = 0$.

Panel (f) reports the welfare gains from implementing the optimal exchange rate policy relative to a policy that would keep a zero labor wedge. As the figure shows, there are sizable gains from departing from the natural allocation. When foreign investors own the entire

share of firms, households are willing to give up 1.75% of period 1 consumption to shift to the optimal policy.

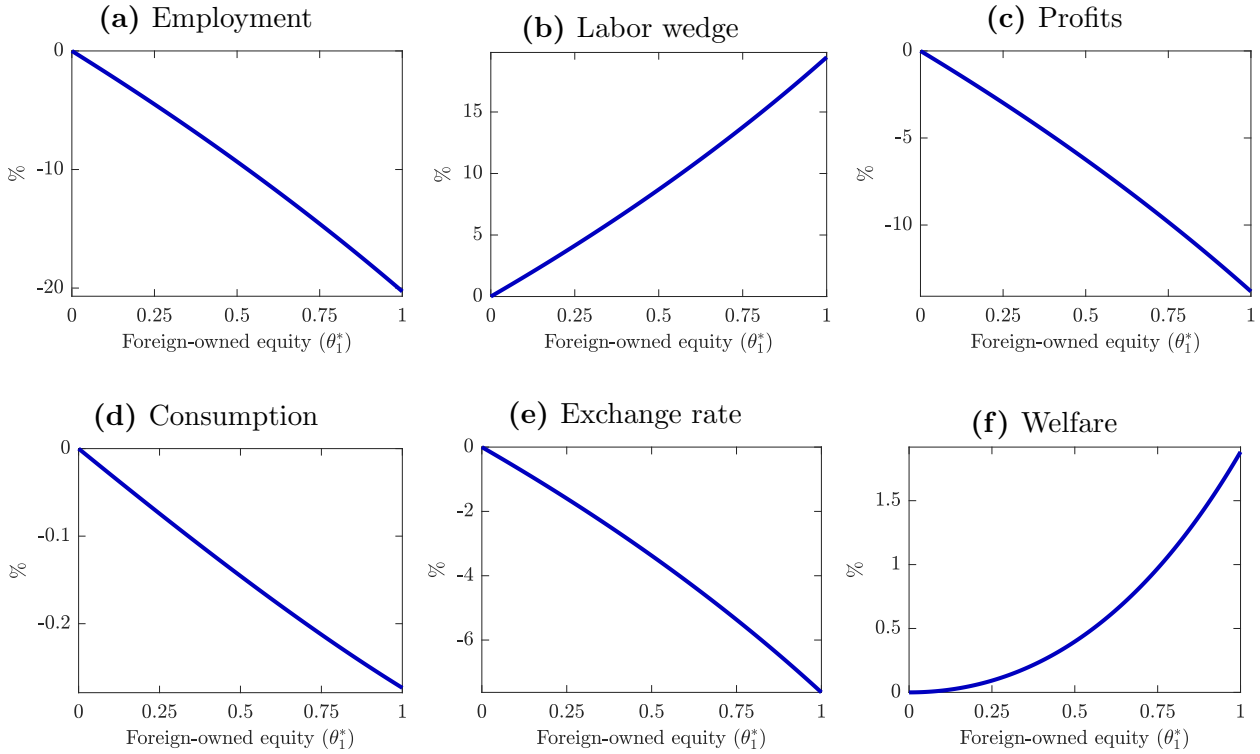


Figure 2: Continuation equilibrium at period $t = 1$ under the optimal exchange rate

Notes: The figure plots the continuation equilibrium at period $t = 1$ under the optimal exchange rate as a function of the foreign-owned fraction of domestic firms' shares. For any given θ_1^* external debt is such that $b_1 = \tilde{q}\,\theta_1^*$, where $\tilde{q}$ is the price of equity under flexible wages. Variables are expressed as deviations from a continuation equilibrium with flexible nominal wages. Welfare represents the welfare gain of the optimal policy relative to a policy that keeps the labor wedge at zero, and is expressed in terms of period-1 consumption equivalence.

Takeaway. This section highlighted the implications of foreign ownership of domestic equities for the determination of the optimal exchange rate. The key insight is that the central bank has incentives to over-appreciate the exchange rate to boost real wages and reduce the profits of firms accrued by foreign investors.

4 International Equity Flows and Capital Controls

We now turn to the determination of the initial portfolio $\{b_1, \theta_1^*\}$ and explore the implications of foreign ownership for the desirability of capital controls.

4.1 Equilibria for $t \geq 0$

We study the equilibrium for $t \geq 0$ when the monetary authority is expected to follow at period 1 the optimal exchange rate policy $E_1(b_1, \theta_1^*)$. The proposition below shows that there is a continuum of equilibria with different welfare implications.

Given an international portfolio (b_0, θ_0^*) and the continuation allocation $\{c(b_1, \theta_1^*), h_1(b_1, \theta_1^*)\}$ associated with the period-1 optimal exchange rate policy $E_1(b_1, \theta_1^*)$, there exists a continuum of Pareto ranked competitive equilibria indexed by (b_1, θ_1^*), where (b_1, θ_1^*) satisfies

$$c(b_1, \theta_1^*) = F(\mathcal{H}(c(b_1, \theta_1^*))) - \Pi(\mathcal{H}(c(b_1, \theta_1^*)))\theta_0^* + q_0(b_1, \theta_1^*)\,(\theta_1^* - \theta_0^*) + (1+r)b_0 - b_1\,. \quad (22)$$

Proof. In Appendix A.5. □

As in the continuation equilibria in Section 3, households are indifferent across portfolios of bonds and domestic firms' shares that deliver the level of permanent consumption consistent with the intertemporal budget constraint. However, while the indeterminacy in the portfolio was inconsequential in the continuation equilibria, the indeterminacy at period 0 gives rise to multiple equilibria with different prices and allocations. As equation (**??**) shows, the portfolio chosen in period 0 determines the asset price q_0, and, in turn, the level of permanent consumption, per equation (22). When households choose to sell more of their equity claims, the central bank chooses in period 1 a more appreciated exchange rate, which boosts real wages, but also reduces working hours as well as lowers firms' profits. Because investors perfectly anticipate the central bank decision, they are only willing to buy the equity at a lower price, which generates a negative wealth effect in the SOE and a reduction in consumption.

The existence of multiple Pareto ranked equilibria reflects a coordination failure, which emerges fundamentally because the monetary authority cannot commit to future monetary policy. Consider a situation where households at period 0 own all domestic equity, $\theta_0^* = 0$. In this case, if households were to keep the entire domestic equity, $\theta_1^* = 0$, the central bank would implement, at $t = 1$, a level of output consistent with the natural level. By contrast, if households were to choose to sell any positive share of domestic equity, foreign investors would buy the equity at a lower price—since they would anticipate an over-appreciated exchange rate—and this would result in a lower wealth for households. Naturally, both situations constitute a competitive equilibrium.

The multiplicity of equilibria is illustrated in Figure 3, where use the same parameterization

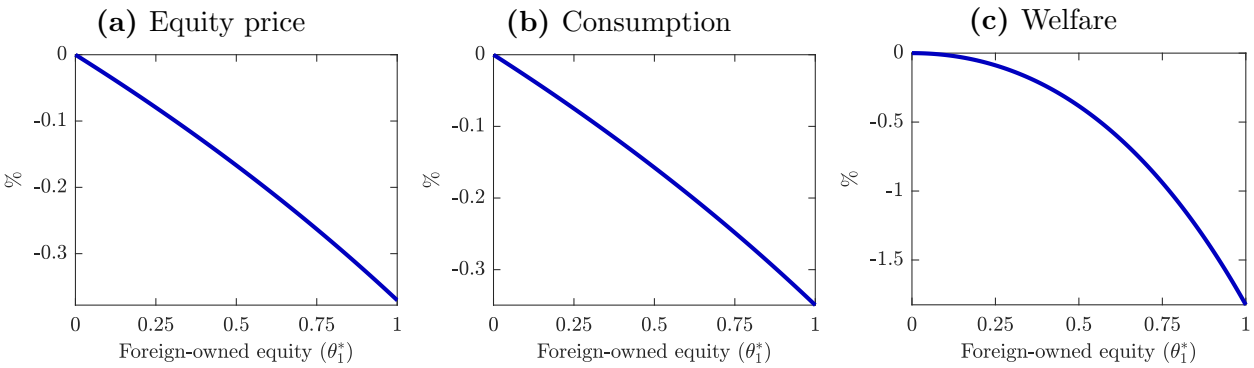


Figure 3: Equilibrium at period $t = 0$ as a function of the capital control

Notes: The figure plots the equilibrium at period $t = 0$ as a function of θ_1^*. The initial bond holdings are such that $b_0 = \theta_0^* \tilde{q}$. The initial foreign ownership is set to $\theta_0^* = 0$.

as in subsection 3.4, and set $\theta_0^* = 0$. Panels (a) and (b) present the equity price and consumption as a function of the domestic equity that households choose to sell. Panel (c) presents the welfare losses relative to the equilibrium where households continue to hold all the domestic equity. The worst equilibrium is the one where households choose to sell all the equity. In that case, as the figure illustrates, the equity price falls significantly, which significantly lowers consumption since households obtain a lower price for their sales of equity.

Time inconsistency problem. At the heart of the coordination failure is a lack of commitment by the central bank. If the central bank were able to commit to a certain exchange rate in period 1, this would break the link between the portfolio composition and the price of equity, and would render a unique equilibrium. Moreover, if the monetary authority were to start from $\theta_0^* = 0$, the optimal period-1 exchange rate policy would implement the natural allocation in period 1. Intuitively, if foreign investors do not own any initial domestic equity, a departure from the natural allocation would result in a period-1 distortion without any benefits. Because investors equate the expected return on equity to bonds, a commitment to lowering employment at $t = 1$ would result in a lower return for the SOE's holdings of domestic equity.

The proposition below characterizes the equilibrium in an alternative scenario in which the central bank at period 0 has commitment and the coordination failure can thus be prevented.

Proposition 3. *Suppose that the central bank at $t = 0$ can commit to a period-1 exchange rate E_1. Then, given initial portfolios (b_0, θ_0^*), the competitive equilibrium features unique values of consumption and labor in all periods. Moreover, if $\theta_0^* = 0$, the optimal period-1 exchange rate policy implements the natural allocation in period 1.*

Proof. In Appendix A.6. □

4.2 Capital control on international equity flows

The coordination failure discussed above suggests that introducing controls on equity flows would be desirable. We assume that a domestic financial regulatory authority that can set capital controls on international equity flows, taking into account that monetary policy is set in a time-consistent manner in period 1. In particular, we assume that the regulatory authority chooses θ_1 on behalf of households and let households choose consumption, labor, and bonds. Because households continue to choose their bonds, this implies that they will continue to smooth consumption over time, for any policy chosen by the regulatory authority.[9]

The regulatory authority's problem can be written as

$$V_0(b_0, \theta_0^*) = \max_{c_0, b_1, \theta_1^*} \left\{ u(c_0) - v(\mathcal{H}(c_0)) + \frac{1}{1+r} V_1(b_1, \theta_1^*) \right\}, \tag{23}$$

subject to:

$$c_0 = F(\mathcal{H}(c_0)) - \Pi(\mathcal{H}(c_0))\theta_0^* + q_0(b_1, \theta_1^*)(\theta_1^* - \theta_0^*) + (1+r)b_0 - b_1,$$

$$c_0 = c(b_1, \theta_1^*).$$

The below proposition establishes conditions under which the regulatory authority finds it optimal to restrict international equity flows.

Proposition 4. *Suppose that either one of the following two conditions holds:*

1. *Wealth effects on labor supply are arbitrarily small and $F(\cdot)$ is isoelastic.*
2. *$\{u(\cdot), v(\cdot), F(\cdot)\}$ are isoelastic.*

Then, the solution to problem (23) features $\theta_1^* < \theta_0^*$ if $\theta_0^* > 0$ and $\theta_1^* = 0$ otherwise.

Proof. In Appendix **??**. □

Condition $\partial \mathcal{L}_0 / \partial b_1 = 0$ guarantees that the marginal costs of reducing one unit of resources in period 0 equals the marginal benefits from the reduction accrued in period 1. The important characteristic of the derivative in the lemma is its sign. The first term on its

[9] We have also considered a case where the regulatory authority can control both bond flows and equity flows, which has similar implications.

right-hand side can be interpreted as the marginal change in the Lagrangian associated with the country-resource constraint while the other term can be interpreted as the corresponding change associated with the consumption-smoothing constraint. The effect of a marginal change in the foreign holdings of domestic equities on the resources of the country can be summarized by the effect on the transaction value of the infra-marginal units in the change in the holdings. The latter can be decomposed in two terms: (i) the change in the quantity held of domestic equities by foreigners itself—as captured by $\theta_1^* - \theta_0^*$—and (ii) the change in the market price of the holdings. The latter in turn can be decomposed into a direct effect—as measured by $\partial q_0/\partial \theta_1^*$—and an indirect one—as captured by $(\partial q_0/\partial b_1)\, q_0(b_1, \theta_1^*)$. The direct effect isolates the change in the price resulting from the change in the foreign holdings while keeping the wealth of the SOE constant. The indirect effect encompasses the additional change in the price resulting from the generated change $q_0(b_1, \theta_1^*)$ in the SOE's wealth. Assuming $\lambda_0 > 0$, everything else the same, the regulatory authority is willing to reduce the foreign holdings of domestic equities below their initial level θ_0^*, provided that the market value of the holdings falls as a consequence.

5 Extensions

[TO BE REVISED]

In this section, we extend the theory in two directions. First, we allow for binding financial constraints. Second, we incorporate a non-tradable good in the model.

5.1 The case with a binding debt limit

In our baseline model, the SOE does not face financial constraints. This implies that households are able to perfectly smooth consumption over time independently of the equity inflows they receive. We now consider a situation where the economy may face a binding borrowing limit.

Specifically, we assume that households borrowing must be such that $b_{t+1} \geq -\underline{b}$ and focus on a situation where the initial debt level is $b_0 < -\underline{b}$. We interpret this situation as a sudden unwillingness of foreign investors of rolling over the bonds. To repay its debt obligations and smooth consumption, the small open economy therefore has incentives to sell some of its domestic equity.

In this context, the optimal capital control policy is no longer to prevent all equity inflows.

The government, however, still imposes controls on equity inflows, because households could end up selling too much equity from a social point of view. Indeed, by selling a sufficiently large amount of equity, households may still be able to perfectly smooth consumption.

Figure 4 presents the optimal capital control and the resulting equilibrium outcome as a function of external debt $-b_0$ (for the case of $\theta_0 = 1$). We can see in the figure (panel [a]) that for $b_0 \geq -\underline{b}$), the government still prevents any sales of domestic equity—all equity inflows are banned. As b_0 falls strictly below $\underline{b}$, the borrowing constraint becomes binding (panel [b]) and the government allows for some equity inflows. In this case, the central bank faces a trade-off between smoothing consumption, on the one hand, and preventing the emergence of an output gap at $t = 1$ and lower equity prices at $t = 0$, on the other.

The key takeaway is that even in a situation where equity inflows are socially desirable, the government still has incentives to reduce the amount of inflows relative to the unregulated competitive equilibrium. Notice that the optimal equity inflows restriction helps support equity prices, which indeed reduces the SOE's needs to sell domestic equity as well as mitigates a vicious cycle—usually referred to as "fire sales"—between drops in equity prices and further equity selling needs. Importantly, in this environment, equity prices fall not because of inherent differences in valuations between domestic and foreign investors, but because of distortionary incentives induced by equity inflows to the future exchange rate policy.

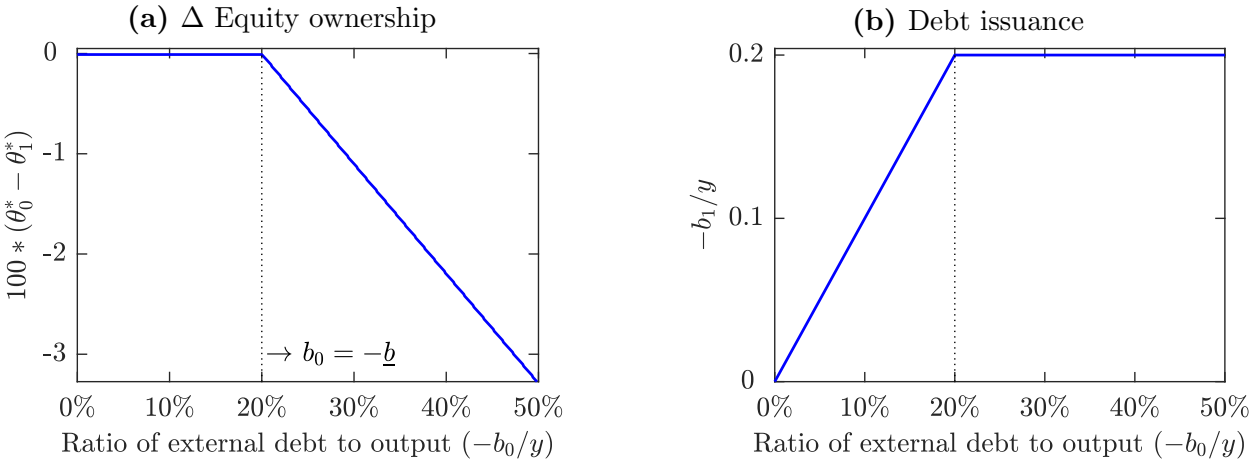


Figure 4: Optimal Capital Control when the SOE can be financially constrained

Notes: The figure plots the equilibrium outcome under the constrained optimal capital control as a function of external debt $-b_0$ for the case of $\theta_0^* = 0$. The debt limit is such that $\underline{b} = -0.2\tilde{y}$, where $\tilde{y}$ is the output level in the benchmark equilibrium. All other parameters are as in the baseline calibration.

5.2 Model with tradable and non-tradable goods

Until now, we have considered an environment with a single final good, assumed to be tradable internationally. In this section, we extend the baseline model by introducing a non-tradable good and we analyze the implications of international equity flows of non-tradable firms for monetary policy.

Households. Preferences are given again by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(h_t\right) \right],$$

where now $c_t = c(c_t^T, c_t^N)$ is a linearly homogeneous aggregator of the tradable good, c_t^T, and the non-tradable good, c_t^N. Equivalently, we will use $U(c_t^T, c_t^N)$ to denote the utility as a function of the two consumption goods, and γ_t the elasticity of substitution between the two goods. In addition, h_t is the total number of hours households work in the tradable and non-tradable sector. That is, $h_t = h_t^T + h_t^N$.

The budget constraint expressed in units of tradables is given by

$$c_t^T + p_t^N c_t^N + b_{t+1} + \sum_j q_t^j \theta_{t+1}^j = w_t h_t + (1+r) b_t + \sum_j \left(\pi_t^j + q_t^j\right) \theta_t^j$$

where the sums are indexed by $j \in \{N, T\}$, q_t^j is the price of equity of firms in sector j in units of foreign currency, and p_t^N is price of the non-tradable good in foreign currency as well.

In addition to the optimality conditions presented in Section 2, we have a condition equating the marginal rate of substitution between the goods to their relative price:

$$p_t^N = \frac{U_N(c_t^T, c_t^N)}{U_T(c_t^T, c_t^N)}.$$

where U_T and U_N respectively denote the marginal utility with respect to the tradable and the non-tradable good.

We continue to assume that the nominal wage in units of domestic currency is fixed at $\bar{W}$ at $t = 1$.[10] This implies that hours worked in both sectors are determined by firms' labor demand.

[10] An analysis with sticky prices for non-tradables is available upon request.

Firms. Firms in each sector choose labor hours to maximize profits. That is, firms solve

$$\pi_t^j \equiv \max_{h_t^j \geq 0} \left\{ p_t^j F^j \left(h_t^j \right) - w_t h_t^j \right\} \ , \text{ for } j \in \{N, T\} \ .$$

where recall that by the law of one price and the fact that the foreign price is normalized to one, we have $p_t^T = 1$.

Competitive equilibrium. We define now a competitive equilibrium in this environment.

Definition 2. Given initial conditions (b_0, θ_0), a rigid wage $\overline{W}_1$, and an exchange rate policy, $\{E_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is a sequence of allocations $\{c_t^T, c_t^N, h_t^T, h_t^N\}_{t=0}^{\infty}$, portfolios $\{b_{t+1}, \theta_{t+1}^T, \theta_{t+1}^N\}_{t=0}^{\infty}$ and prices $\left\{w_t, p_t^N, q_t^T, q_t^N\right\}_{t=0}^{\infty}$ such that:

(i) Households and firms optimize.

(ii) Market for non-tradable clears. That is, $c_t^N = F^N \left(h_t^N \right)$.

(iii) Prices for firms' shares satisfy

$$(1+r) q_t^j = \pi_{t+1}^j + q_{t+1}^j \ , \text{for } j \in \{T, N\} \ .$$

Crucially, relative to the definition of competitive equilibrium in Section 2, an equilibrium also requires the market for the non-tradable good to clear domestically.

Profits for non-tradable firms. Assume that the production function for non-tradables is given by $F^N(h) = h^{\alpha^N}$, with $\alpha^N \in (0, 1)$. Then, in any continuation equilibrium at period $t = 1$, profits in the non-tradable sector are given by

$$\pi_t^N = \underbrace{\frac{1-\omega}{\omega} \left(\frac{F^N \left(h_t^N \right)}{c_t^T} \right)^{-1/\gamma}}_{= p_t^N} \left(1 - \alpha^N \right) F^N \left(h_t^N \right), \tag{24}$$

Equation (24) suggests that whether the central bank will seek to target a higher or lower employment in non-tradable sector depends on whether $-1/\gamma + 1$ is smaller or greater than zero.

If $\gamma = 1$, non-tradable employment does not affect firm profits in the sector. This is because lower employment lower wages but the price of non-tradables raises in the same proportion, muting any effects on profits in units of tradables. In a situation where households

own all shares in the tradable sector, that is, $\theta^T = 1$, the above indicates that the central bank will not depart from a zero labor wedge when $\gamma = 1$.

If $\gamma < 1$ (i.e., tradable and non-tradable goods are complements), the relative price responds more than proportionally to a change in the ratio of the quantities consumed. Thus, everything else the same, declines in non-tradable production boosts firm profits in the sector, whereas increases depress them. For given labor hours in the tradable sector h_1^T and h^T, it may be optimal for the central bank to depreciate the currency relative to the case where the small own economy own all shares in the non-tradable sector.

Finally, if $\gamma > 1$ (i.e., the goods are substitutes), the relative price responds less than proportionally to a change in the ratio of the quantities consumed. In this situation, the central bank has incentives to over-appreciate the currency, as in the baseline model with only tradables.

Optimal exchange rate policy In what follows, we assume that $U(c^T, c^N) = \omega^T u(c^T) + \omega^N u(c^N)$, with $\omega^T + \omega^N = 1$. Together with the assumption that $\beta(1+r) = 1$, this ensures a stationary continuation equilibrium at period $t = 2$ and constant tradable consumption over time.

The proposition below provides a necessary condition for exchange rate optimality at period 1.

Proposition 5 (Necessary condition for optimality)**.** *The optimal exchange rate policy satisfies*

$$\sum_j \frac{\mathcal{P}_1^j}{E_1}\left[F'^j\left(\mathcal{H}_1^j\right) - \frac{v'\left(\mathcal{H}_1^j + \mathcal{H}_1^{-j}\right)}{\omega^j u'\left(\mathcal{C}_1^j\right)}\right]\frac{\partial \mathcal{H}_1^j}{\partial E_1} = \sum_j \left(1-\theta_1^j\right)\left[\frac{\partial\left(\pi_1^j/E_1\right)}{\partial E_1} + \frac{1}{R-1}\frac{\partial\left(\pi^j/E\right)}{\partial E_1}\right].$$

As in Section 2, the optimal exchange rate induces a labor wedge that depends on how changes in labor affects profits received by foreign investors. The difference is that now the analysis considers how profits in both sectors change.

As we saw above, a depreciation may indeed affect profits in the two sectors in different directions. An implication of this result is that there may exist multiple combinations of θ^T and θ^N such that the government does not have incentives to depart from a zero labor wedge. This set is formally given by

$$\Gamma \equiv \left\{\left(\theta_1^N, \theta_1^T\right) \in [0,1]^2 : \sum_j \left(1-\theta_1^j\right)\left[\frac{\partial\left(\pi_1^j/E_1\right)}{\partial E_1} + \frac{1}{R-1}\frac{\partial\left(\pi^j/E\right)}{\partial E_1}\right] = 0\right\}.$$

Figure 5 illustrates the set for different values for the elasticity. In all cases, we consider the case where the two goods are complements. We can see that as the share of equity increases in one sector, the share of equity in the other sector needs to increase to mute the incentives to deviate from a zero labor wedge.

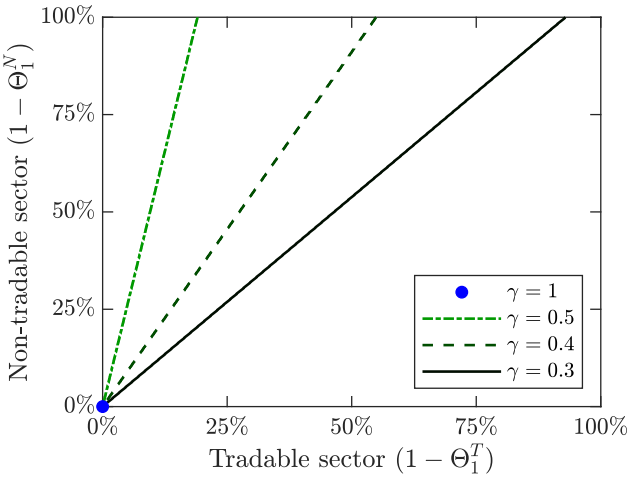


Figure 5: International equity portfolios that induce a non-distortionary exchange rate

Notes: The figure plots the set of international equity portfolios that induce a non-distortionary exchange. Parameter values are $\chi = \alpha^N$, $\psi = 0$, $\alpha^T = 0.25$, $\alpha^N = 0.75$, $R = 1.04$, $\gamma = \{0.3, 0.4, 0.5, 1\}$ and $\omega^T = \{0.06, 0.1, 0.13, 0.25\}$. Condition $\chi = \alpha^N$ is set to normalize non-tradable output to 1 under flexible nominal wages. Parameter ω^T is adjusted so that under flexible nominal wages the share of tradable output in total output equals 0.20.

An important takeaway from this section is that the composition of equity flows across sectors is crucial for monetary policy. In particular, a central bank may want to depreciate or appreciate depending on which equity shares investors hold. Moreover, a central bank that seeks to reduce its incentives to reduce investors' rents ex-post can still stimulate equity inflows to the extent that they are balanced appropriately across sectors.

6 Conclusion

In this paper, we have examined the implications of international equity flows for monetary policy in a small open economy. We develop a tractable New Keynesian framework in which foreign investors can hold domestic equity claims and the central bank sets the exchange rate under nominal wage rigidity. We show that when foreign investors own domestic equity, the central bank has an incentive to deviate from the natural allocation by appreciating the exchange rate, raising real wages, and compressing the profits that accrue to foreign investors. In effect, foreign ownership acts like a reduction in productivity: from the perspective of the

domestic planner, a share of the output leaks in the form of dividends paid to foreigners, and the central bank responds by choosing lower employment than in the flexible-wage allocation.

Foreign ownership creates a novel source of time inconsistency for monetary policy. Anticipating lower returns, foreign investors purchase equity at discounted prices, reducing domestic households' wealth and giving rise to multiple Pareto-ranked equilibria in which the bad equilibrium features larger equity inflows, depressed asset prices, and higher unemployment. To mitigate this time inconsistency problem, we show that it is optimal to manage equity flows. In particular, it is optimal to limit equity inflows ex ante, to reduce the temptation of the central bank to over-appreciate the exchange rate ex post.

Taken together, our results highlight that the globalization of equity portfolios has important implications for monetary policy and for capital flow management. To highlight the key mechanisms at play, we have presented a parsimonious model without risk, frictions on financial intermediaries, heterogeneity, and with a simple temporary nominal rigidity. For future work, it would be valuable to extend the framework along these dimensions and to confront it with data, in order to further analyze the normative implications of foreign equity ownership.

References

Acharya, Sushant, Edouard Challe, and Keshav Dogra, "Optimal monetary policy according to HANK," 2020.

Alesina, A. and R. Barro, "Currency Unions," *Quarterly Journal of Economics*, 2002, *117* (2), 409–436.

Atkeson, Andrew, Jonathan Heathcote, and Fabrizio Perri, "The end of privilege: A reexamination of the net foreign asset position of the united states," Technical Report, National Bureau of Economic Research 2022.

Barro, R. and D. Gordon, "Rules, discretion and reputation in a model of monetary policy," *Journal of Monetary Economics*, 1983, *12* (1), 101–121.

Baxter, Marianne and Urban J. Jermann, "The International Diversification Puzzle Is Worse Than You Think," *American Economic Review*, 1997, *87* (1), 170–180.

Beck, Roland, Antonio Coppola, Angus J Lewis, Matteo Maggiori, Martin Schmitz, and Jesse Schreger, "The geography of capital allocation in the Euro Area," Technical Report, National Bureau of Economic Research 2024.

Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J Sargent, "Inequality, business cycles and monetary-fiscal policy," *Econometrica*, 2021, *89* (6), 2559–2599.

Bianchi, Javier, "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 2011, *101* (7), 3400–3426.

_ **and Guido Lorenzoni**, "The Prudential Use of Capital Controls and Foreign Currency Reserves," 2021. Handbook of International Economics, Volume V, edited by Gita Gopinath, Elhanan Helpman and Kenneth Rogoff, North Holland.

Bilbiie, Florin O., "Limited asset markets participation, monetary policy and (inverted) aggregate demand logic," *Journal of Economic Theory*, 2008, *140* (1), 162–196.

Broer, Tobias, Niels-Jakob Harbo Hansen, Per Krusell, and Erik Öberg, "The New Keynesian transmission mechanism: A heterogeneous-agent perspective," *The Review of Economic Studies*, 2020, *87* (1), 77–101.

Clarida, Richard, Jordi Gali, and Mark Gertler, "A Simple Framework for International Monetary Policy Analysis," *Journal of Monetary Economics*, 2002, *49* (5), 879–904.

Coeurdacier, Nicolas, Robert Kollmann, and Philippe Martin, "International portfolios, capital accumulation and foreign assets dynamics," *Journal of International Economics*, 2010, *80* (1), 100–112.

Coppola, Antonio, Matteo Maggiori, Brent Neiman, and Jesse Schreger, "Redrawing the map of global capital flows: The role of cross-border financing and tax havens," *The Quarterly Journal of Economics*, 2021, *136* (3), 1499–1556.

Corsetti, Giancarlo and Paolo Pesenti, "Welfare and Macroeconomic Interdependence," *Quarterly Journal of Economics*, 2001, *116* (2), 421–445.

_ , Luca Dedola, and Sylvain Leduc, "Optimal monetary policy in open economies," in "Handbook of monetary economics," Vol. 3, Elsevier, 2010, pp. 861–933.

Dávila, Eduardo and Andreas Schaab, "Optimal Monetary Policy with Heterogeneous Agents: Discretion, Commitment, and Timeless Policy," Technical Report, National Bureau of Economic Research 2023.

Devereux, Michael B. and Alan Sutherland, "Solving for Country Portfolios in Open Economy Macro Models," Technical Report 5966, Centre for Economic Policy Research 2006.

Devereux, Michael B and Charles Engel, "Monetary policy in the open economy revisited: Price setting and exchange-rate flexibility," *The review of economic studies*, 2003, *70* (4), 765–783.

Du, Wenxin, Carolin E Pflueger, and Jesse Schreger, "Sovereign debt portfolios, bond risks, and the credibility of monetary policy," *The Journal of Finance*, 2020, *75* (6), 3097–3138.

Engel, Charles and Akito Matsumoto, "The international diversification puzzle when goods prices are sticky: it's really about exchange-rate hedging, not equity portfolios," *American economic Journal: macroeconomics*, 2009, *1* (2), 155–188.

_ and JungJae Park, "Debauchery and original sin: The currency composition of sovereign debt," *Journal of the European Economic Association*, 2022, *20* (3), 1095–1144.

Fanelli, Sebastian, "Monetary policy, capital controls, and international portfolios," 2023. Mimeo, MIT.

Farhi, Emmanuel and Iván Werning, "A theory of macroprudential policies in the presence of nominal rigidities," *Econometrica*, 2016, *84* (5), 1645–1704.

Fornaro, L., "Monetary union and financial integration," 2022. Forthcoming, Review of Economic Studies.

Friedman, M., "The Case for Flexible Exchange RatesV in Essays in Positive Economics," *Chicago, Il. Univ. of Chicago Press*, 1953, *157*, 203.

Heathcote, Jonathan and Fabrizio Perri, "The international diversification puzzle is not as bad as you think," *Journal of Political Economy*, 2013, *121* (6), 1108–1159.

Kollmann, Robert, "International portfolio equilibrium and the current account," 2006.

Lane, Philip R and Gian Maria Milesi-Ferretti, "The international equity holdings of euro area investors," 2005.

_ **and** _ , "The external wealth of nations revisited: international financial integration in the aftermath of the global financial crisis," *IMF Economic Review*, 2018, *66*, 189–222.

Mundell, R., "The Monetary Dynamics of International Adjustment under Fixed and Flexible Exchange Rates," *Quarterly Journal of Economics*, 1960, *74* (2), 227–257.

Obstfeld, Maurice and Kenneth Rogoff, "Exchange Rate Dynamics Redux," *Journal of Political Economics*, 1995, *103* (3), 624–660.

Ottonello, Pablo and Diego J Perez, "The currency composition of sovereign debt," *American Economic Journal: Macroeconomics*, 2019, *11* (3), 174–208.

Quadrini, Vincenzo and José-Víctor Ríos-Rull, "International tax competition with rising intangible capital and financial globalization," *Journal of Monetary Economics*, 2024, *141*, 101–120.

Schmitt-Grohé, S. and M. Uribe, "Stabilization policy and the costs of dollarization," *Journal of money, credit and banking*, 2001, pp. 482–509.

Schmitt-Grohé, Stephanie and Martin Uribe, "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 2016, *124* (5), 1466–1514.

Tille, Cedric and Eric Van Wincoop, "International capital flows," *Journal of international Economics*, 2010, *80* (2), 157–175.

A Proofs

A.1 Proof of Lemma 1

Proof. Consider the problem (6). Together with the envelope conditions, the first-order conditions with respect to $c_t, l_t, b_{t+1}, \theta_{t+1}$ yield

$$w_t = \frac{v'(l_t)}{u'(c_t)}, \tag{A.1}$$

$$u'(c_t) = \beta(1+r)\,u'(c_{t+1}) \tag{A.2}$$

$$u'(c_t)q_t = \beta(\pi_{t+1} + q_{t+1})u'(c_{t+1}) \tag{A.3}$$

where the latter condition already incorporates that any $\theta_{t+1} \geq 0$ is optimal, a result that follows directly from (A.2) and (9). Using that $\beta(1+r) = 1$ we obtain from (A.2) that $c_t = c_{t+1} = c$ for all t. From (A.2),(A.3) and (9), it is also immediate that the portfolio between bonds and equity is undetermined.

Given $c_t = c$, equations (A.1) and (3), imply that $\ell_t = h_t = h$ and $w_t = w$ for all $t \geq 2$. We thus have that profits are constant as well and given by $\pi = F(h) - F'(h)h$. The expression (13) in the lemma then follows by iterating forward on (9) and using a no-bubble condition.

Iterating forward on the budget constraint (4) and using No-Ponzi, we obtain

$$c = w\ell + \pi\theta_2 + rb_2,$$

Combining this expression with (3) and market clearing conditions $h_t = \ell_t$ and $\theta_t + \theta_t^* = 1$ we arrive to (11). Finally, replacing c in the budget constraint (4), we obtain (12).

□

A.2 Proof of Lemma 4.1

Proof. First, note that for $t \geq 2$, the economy is in a steady-state equilibrium. Therefore, we have labor, wages, profits, and prices for $t \geq 2$, as characterized in Lemma 1.

At $t = 1$, households solve

$$\begin{aligned} V_1(b_1, \theta_1) &= \max_{c_1, b_2, \theta_2 \geq 0} \left\{ u(c_1) - v(\ell_1) + \frac{1}{1+r} V(b_2, \theta_2) \right\} \\ &\quad \text{subject to:} \\ &\quad c_1 + b_2 + q\,\theta_2 = \frac{\bar{W}_1}{E_1}\ell_1 + (1+r)b_1 + (\pi_1 + q)\,\theta_1 \,, \end{aligned}$$

where we have used that $q_1 = q$ and that the value function is constant for $t \geq 2$, a result that follows because in a steady-state equilibrium, wages, profits, and equity prices are constant.

The value for $t \geq 2$ is given by

$$V(b_2, \theta_2) = \frac{1+r}{r}\left[u(c) - v(\ell)\right] \,,$$

with $c = w\ell + \pi\theta_2 + rb_2$ and $w = v'(\ell)/u'(c)$.

From optimality with respect to b_2 and using the envelope condition, we obtain $c_1 = c$. Using that consumption, wages, and hours are constant and that q satisfies (9), we obtain by iterating forward on the budget constraint that

$$c = \frac{r}{1+r}\left(\frac{\bar{W}_1}{E_1}\ell_1 + \pi_1\theta_1\right) + \frac{1}{1+r}\left(w\ell + \pi\theta_1\right) + rb_1 \ .$$

Combining this expression with (3), market clearing conditions $\ell_t = h_t$ and $\theta_t + \theta_t^* = 1$, and mapping $h = \mathcal{H}(c)$, we arrive to (15). Finally, replacing c_1 together with (3), $\ell_t = h_t$, $\theta_t + \theta_t^* = 1$, and $h = \mathcal{H}(c)$ in the budget constraint (A.5), we obtain (16).

□

A.3 Proof of Proposition 1

Proof. Consider first a case in which condition 1 holds. Then, taking limits on both sides of (20) as $\varepsilon \to 0$ approaches zero, one gets:

$$\lim_{\varepsilon \to 0} \zeta_1(b_1, \theta_1^*) = \Pi'(h_1(b_1, \theta_1^*))u'(c(b_1, \theta_1^*))\theta_1^* \,,$$

which confirms that for $\varepsilon > 0$ positive, but arbitrarily small both $\zeta_1(b_1, \theta_1^*) \geq 0$ and $\zeta_1(b_1, \theta_1^*) > 0$ if $\theta_1^* > 0$ hold.

Consider now a case in which condition 2 holds. From (10) it follows that

$$\mathcal{H}(c) = \left(\frac{\alpha}{\chi} c^{-1/\sigma}\right)^{\frac{1}{1+\psi-\alpha}}, \tag{A.6}$$

and from (20) one gets that

$$h_1 = z(c)\,\mathcal{H}(c), \tag{A.7}$$

with

$$z(c) \equiv \left[1 - (1-\alpha)\,\theta_1^* \frac{(1+\psi-\alpha)\,c}{(1+\psi-\alpha)\,c + \frac{1}{\sigma}\frac{1}{1+r}\left[1-(1-\alpha)\,\theta_1^*\right]\alpha\,(\mathcal{H}(c))^{\alpha}}\right]^{\frac{1}{1+\psi-\alpha}}.$$

Note that $z(c) = 1\,\forall c > 0$ if $\theta_1^* = 0$ and that $z(c) < 1\,\forall c > 0$ if $\theta_1^* > 0$. This implies that $h_1 = \mathcal{H}(c)$ if $\theta_1^* = 0$ and $h_1 < \mathcal{H}(c)$ if $\theta_1^* > 0$, which in turn implies that both $\zeta_1(b_1, \theta_1^*) \geq 0$ and $\zeta_1(b_1, \theta_1^*) > 0$ if $\theta_1^* > 0$ hold.

A.4 Proof of Proposition 2

Proof. Applying the Implicit Function Theorem to {(17),(20)} to derive the partial derivatives of $\{h_1(b_1, \theta_1^*), c(b_1, \theta_1^*)\}$ with respect to b_1 and θ_1^*, one obtains:

$$\begin{aligned}
\frac{\partial h_1}{\partial b_1} q_0 + \frac{\partial h_1}{\partial \theta_1^*} &= -\frac{\mathcal{A}_{\theta_1^*}}{\Delta}\mathcal{Z}_c, \\
\frac{\partial c}{\partial b_1} q_0 + \frac{\partial c}{\partial \theta_1^*} &= \frac{\mathcal{A}_{\theta_1^*}}{\Delta}\mathcal{Z}_{h_1},
\end{aligned}$$

with

$$\mathcal{A}_{\theta_1^*} = -\Pi'(h_1)u'(c) - \frac{1}{1+r}\Big(-\Pi'(h_1)v'(\mathcal{H}(c)) + \Pi'(\mathcal{H}(c))v'(h_1)\Big)\mathcal{H}'(c)\,,$$

$$\mathcal{Z}_c = \frac{1}{1+r}\Big[F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*\Big]\,\mathcal{H}'(c) - 1\,,$$

$$\mathcal{Z}_{h_1} = \frac{r}{1+r}\Big[F'(h_1) - \Pi'(h_1)\theta_1^*\Big]\,,$$

$$\begin{aligned}\mathcal{A}_c = {} & [F'(h_1) - \Pi'(h_1)\theta_1^*]u''(c) + \\ & - \frac{1}{1+r}\Bigg[[F'(h_1) - \Pi'(h_1)\theta_1^*]\,v''(\mathcal{H}(c))[\mathcal{H}'(c)]^2 - [F''(\mathcal{H}(c)) - \Pi''(\mathcal{H}(c))\theta_1^*]\,v'(h_1)[\mathcal{H}'(c)]^2 + \\ & + \Big[[F'(h_1) - \Pi'(h_1)\theta_1^*]v'(\mathcal{H}(c)) - [F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*]v'(h_1)\Big]\,\mathcal{H}''(c)\Bigg]\,,\end{aligned}$$

$$\begin{aligned}\mathcal{A}_{h_1} = {} & [F''(h_1) - \Pi''(h_1)\theta_1^*]u'(c) - v''(h_1) + \\ & - \frac{1}{1+r}\Bigg[[F''(h_1) - \Pi''(h_1)\theta_1^*]\,v'(\mathcal{H}(c)) - [F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*]\,v''(h_1)\Bigg]\mathcal{H}'(c)\,,\end{aligned}$$

$$\Delta = \mathcal{A}_{h_1}\mathcal{Z}_c - \mathcal{A}_c\mathcal{Z}_{h_1}\,.$$

Note that

$$\frac{\partial\zeta_1}{\partial b_1}q_0 + \frac{\partial\zeta_1}{\partial\theta_1^*} = [F''(h_1)u'(c) - v''(h_1)]\left(\frac{\partial h_1}{\partial b_1}q_0 + \frac{\partial h_1}{\partial\theta_1^*}\right) + F'(h_1)u''(c)\left(\frac{\partial c}{\partial b_1}q_0 + \frac{\partial c}{\partial\theta_1^*}\right)\,.$$

Note also that $F(h) = h^\alpha$ implies that $\Pi(h) = (1-\alpha)F(h)$.

Assume condition 1 holds. Then,

$$\begin{aligned}&\lim_{\varepsilon\to 0}\mathcal{A}_{\theta_1^*} = -\Pi'(h_1)u'(c) < 0\,,\\ &\lim_{\varepsilon\to 0}\mathcal{Z}_c = -1 < 0\,,\\ &\lim_{\varepsilon\to 0}\mathcal{Z}_{h_1} = \frac{r}{1+r}\Big[1 - (1-\alpha)\theta_1^*\Big]F'(h_1) > 0\,,\\ &\lim_{\varepsilon\to 0}\Delta \geq -[1 - (1-\alpha)\theta_1^*]F''(h_1)u'(c) + v''(h_1) > 0\,,\end{aligned}$$

which implies that

$$\lim_{\varepsilon\to 0}\left\{\frac{\partial h_1}{\partial b_1}q_0+\frac{\partial h_1}{\partial \theta_1^*}\right\}<0\,,\quad \lim_{\varepsilon\to 0}\left\{\frac{\partial c}{\partial b_1}q_0+\frac{\partial c}{\partial \theta_1^*}\right\}<0\,,\quad \lim_{\varepsilon\to 0}\left\{\frac{\partial \zeta_1}{\partial b_1}q_0+\frac{\partial \zeta_1}{\partial \theta_1^*}\right\}>0\,.$$

Assume now condition 2 holds. Then, (A.6) implies that $\mathcal{H}''(c)>0$ and (A.7) implies that $-\Pi'(h_1)v'(\mathcal{H}(c))+\Pi'(\mathcal{H}(c))v'(h_1)<0$ and $[F'(h_1)-\Pi'(h_1)\theta_1^*]v'(\mathcal{H}(c))-[F'(\mathcal{H}(c))-\Pi'(\mathcal{H}(c))\theta_1^*]v'(h_1)>0$. These results guarantee that $\mathcal{A}_{\theta_1^*}<0$ and $\mathcal{A}_c<0$. Note that $\mathcal{Z}_c<0$, $\mathcal{Z}_{h_1}>0$ and $\mathcal{A}_{h_1}<0$. Thus,

$$\frac{\partial h_1}{\partial b_1}q_0+\frac{\partial h_1}{\partial \theta_1^*}<0\,,\quad \frac{\partial c}{\partial b_1}q_0+\frac{\partial c}{\partial \theta_1^*}<0\,,\quad \frac{\partial \zeta_1}{\partial b_1}q_0+\frac{\partial \zeta_1}{\partial \theta_1^*}>0\,.$$

□

A.5 Proof of Lemma 4.1

Proof. At $t=0$, households solve

$$\begin{aligned} V_0(b_0,\theta_0) \;=\; & \max_{c_0,b_1,\theta_1\geq 0}\left\{u(c_0)-v(\ell_0)+\frac{1}{1+r}V_1(b_1,\theta_1)\right\} \\ & \text{subject to:} \\ & c_0+b_1+q_0\theta_1=w_0\ell_0+(1+r)b_0+(\pi_0+q_0)\theta_0\,, \end{aligned}$$

where $q_0=\frac{1}{1+r}(\pi_1+q)$ already uses $q_1=q$ and where recall that for $t\geq 2$ the economy is in a steady-state equilibrium. From problem (A.4) it follows that

$$\frac{\partial V_1}{\partial b_1}=1+r\;\text{ and }\;\frac{\partial V_1}{\partial \theta_1}=\pi_1+q\,.$$

These conditions together with optimality at $t=0$ imply that $c_0=c$. Iterating forward on the budget constraint we arrive to

$$c=\frac{r}{1+r}(w_0\ell_0+\pi_0\theta_0)+\frac{r}{(1+r)^2}\left(\frac{\bar{W}_1}{E_1}\ell_1+\pi_1\theta_0\right)+\frac{1}{(1+r)^2}(w\ell+\pi\theta_0)+rb_0\,.$$

Combining this expression with (3), market clearing conditions $\ell_t=h_t$ and $\theta_t+\theta_t^*=1$, and mapping $h=\mathcal{H}(c)$, we arrive to (22).

□

A.6 Proof of Proposition 3

Proof. The proof has two parts. For the first part, observe first that (7) implies that consumption is constant, and in addition, we have $h_t = h = \mathcal{H}(c)$ for all $t \neq 1$. Using that

$$q_0 = \frac{1}{1+r}\left[\Pi\left(h_1\right) + \frac{1}{r}\Pi\left(\mathcal{H}(c_0)\right)\right].$$

and iterating forward on the budget constraint, and using the transversality condition we have that c solves

$$\left(\frac{1+r}{r}\right)c = F(\mathcal{H}(c)) + (1+r)b_0 - q_0\theta_0^* + \frac{F(h_1)}{1+r} + \frac{F\big(\mathcal{H}(c)\big)}{(1+r)^2} \tag{A.9}$$

where h_1 is given by

$$\frac{\overline{W}}{E_1} = F'\big(h_1\big)\,,$$

The fact that $\mathcal{H}'(\cdot) < 0$ implies the right-hand side of (A.9) is decreasing in c. Because the left-hand side of (A.9) is increasing in c, the level of c must be unique, and so is h.

For the second part, we use that c satisfies (A.9) for any period-1 exchange rate policy and using that θ_0, we can express the central bank problem as

$$V_0(b_0, 0) = \max_{c, b_1, \theta_1^*, h_1} \left\{ u(c) - v(\mathcal{H}(c)) + \frac{u(c) - v\left(h_1\right)}{1+r} + \frac{1}{(1+r)^2}\frac{u(c) - v\left(\mathcal{H}(c)\right)}{r} \right\},$$

subject to:

$$\left(\frac{1+r}{r}\right)c \leq F(\mathcal{H}(c)) + (1+r)b_0 + \frac{F(h_1)}{1+r} + \frac{F\big(\mathcal{H}(c)\big)}{(1+r)^2}$$

Optimality with respect to h_1 and c result in

$$v'(h_1) = u'(c)F'(h_1)$$

which implies a zero labor wedge in period 1. □