What Is the Carbon Premium a Premium On?*

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Abstract

I develop an asset-pricing framework in which climate risk enters returns through a value-normalized mapping from firm fundamentals into cash-flow risk. In a complete-markets planner benchmark, the sustainable-discount-rate wedge collapses to emission intensity. Under market incompleteness with heterogeneous climate concern and unspanned damages, equilibrium risk premia need not collapse to intensity and load on other climate-relevant firm characteristics. I show that intensity sufficiency requires knife-edge scale invariance and derive implied testable restrictions. Both these scale and spanning conditions are rejected in an international panel, reconciling conflicting findings and clarifying the conceptual distinction between policy implementation and equilibrium pricing of climate risk.

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1 Introduction

Investors, regulators, and policy makers increasingly speak about a carbon premium or greenium in asset prices. Getting this measurement right matters: portfolio screens, regulatory taxonomies, and corporate discount rates all depend on correctly identifying which firm attributes carry climate risk. Yet there is little agreement on what this premium is a premium on. Empirical studies proxy climate exposure using absolute emissions (e.g., Bolton and Kacperczyk (2021, 2023)), emissions scaled by firm value or revenue (Pedersen (2025)), changes in emissions, sectoral classifications (Pastor et al. (2022)), or composite transition-risk scores (e.g., Sautner et al. (2023), Eskildsen et al. (2024)). At the same time, policy debates over sustainable discount rates often treat emission intensity as the natural benchmark for climate-adjusted costs of capital. Taken together, the proliferation of emission metrics in empirical work and the focus on intensity in policy leave a basic issue unresolved: which firm characteristics actually govern climate risk premia in equilibrium?

I address this question by characterizing the conditions under which emission intensity is sufficient for pricing climate risk. The answer is sharp: intensity is sufficient only under knife-edge restrictions that largely do not hold in the data. Climate and policy shocks affect firms through dollar exposures, summarized by exposure functions $g(Z_{i,t})$ that map firm characteristics into dividend sensitivities. Equilibrium risk premia, in turn, load on value-normalized exposures: because returns are defined per unit of invested wealth, asset prices respond to $g(Z_{i,t})/V_{i,t}$, where $V_{i,t}$ denotes firm value, rather than to dollar exposures themselves. Emission intensity $X_{i,t}/V_{i,t}$ – emissions per dollar of firm value—is sufficient only if the value-normalized exposure $g(Z_{i,t})/V_{i,t}$ can be reduced to intensity alone, which requires homogeneity and scale-invariance restrictions. These restrictions imply that firms with the same intensity must have the same priced exposure regardless of size. I derive the resulting falsifiable cross-sectional implications and test them directly by examining whether intensity spans return variation and whether its pricing is scale-invariant.

Why, then, does emission intensity feature so prominently in policy discussions? The reason is that intensity is natural in a different economic setting: complete-markets planner benchmark. Consider the planner framework underlying the sustainable-discount-rate literature (e.g., Pedersen (2025)). With a representative agent, a well-defined social cost of carbon (SCC), and full spanning of climate risk, a social planner chooses emissions and consumption paths to maximize discounted welfare. In this setting, the planner's allocation can be decentralized either through Pigouvian carbon taxes or through firm-specific sustainable discount rates that tilt the cost of

capital toward cleaner firms. When climate damages enter firm cash flows proportionally to firms' emission levels, this decentralization implies that the implementation wedge between financial and sustainable discount rates is proportional to emission intensity. In this benchmark, intensity is sufficient for implementing the planner's allocation, but this is a statement about decentralizing a first best under complete markets, not about equilibrium climate risk premia with incomplete markets. Distinguishing planner implementation from equilibrium pricing is central in this setting, because intensity-based metrics arise naturally in policy design even when they do not summarize equilibrium risk premia.

With incomplete risk sharing and heterogeneous climate concern, equilibrium pricing differs fundamentally from the planner benchmark. Expected returns load on value-normalized cash-flow exposures $g(Z_{i,t})/V_{i,t}$, so there is no direct mapping from the social cost of carbon to firm-specific deterministic discount-rate wedges. Emission intensity summarizes equilibrium pricing only under restrictive conditions: exposure functions must be homogeneous of degree one in emissions and firm value, and the pricing of intensity must be scale-invariant. These conditions deliver two testable restrictions: firms with the same intensity should carry identical priced exposure regardless of scale (scale-invariance), and expected returns should vary across firms only through $X_{i,t}/V_{i,t}$ (spanning).

I show that these conditions fail naturally when climate risk is unhedgeable and investors differ in their climate concern. I develop a tractable incomplete-markets model in which heterogeneous investors trade securities issued by emitting firms, face unspanned climate damages, and differ in their concern about those damages. In a benchmark case, firms' dividends load on a climate or policy shock in proportion to their emission levels. Equilibrium expected returns then inherit a simple factor structure in which emission levels price exposure to transition risk. The resulting climate price of risk decomposes into two components: a brown premium for bearing climate risk and a greenium reflecting investors' willingness to sacrifice returns to reduce climate exposure. In this environment, the relevant priced characteristic is disciplined by the structure of shocks rather than by an a priori choice of emissions scaling, and the equilibrium price of climate risk is not pinned down by the SCC as in the complete-markets benchmark.

To unify these insights, I embed both the planner benchmark and the incomplete-markets equilibrium in a general linear-factor framework for climate and transition risk. In this representation, expected excess returns take the form

$$E_t[R_{i,t+1} - R_{f,t}] \;=\; \sum_k \lambda_{k,t} \, \frac{g_k(Z_{i,t})}{V_{i,t}},$$

where $g_k(Z_{i,t})$ summarizes how factor k enters firm i's dollar cash flows. Within this framework, I characterize when emission intensity is a sufficient statistic for pricing a given climate factor. I show that intensity is sufficient if and only if exposure functions are homogeneous of degree one in emissions and firm value and satisfy a scale-invariance restriction across firms. These conditions are naturally satisfied in the complete-markets planner benchmark but are generically violated once exposures depend on levels or changes, technology choice, demand conditions, or path-based constraints such as net-zero targets.

Among these violations, endogenous technology choice provides a particularly sharp illustration. In a dynamic production model with endogenous energy efficiency, I show that a Jevons-type rebound effect can arise, whereby efficiency improvements increase firms' exposure to priced climate shocks. As a result, firms with lower emission intensity can carry higher equilibrium climate risk, and efficiency improvements can raise rather than lower expected returns. This reversal shows that intensity can produce qualitatively incorrect risk rankings even under optimal technology choice. Internet Appendix D embeds this mechanism in an infinite-horizon production economy and shows that it persists under optimal investment and emissions choices.

These results clarify the distinction between planner implementation and equilibrium pricing. In the planner benchmark, emission intensity is a sufficient statistic for implementation wedges, reflecting the deterministic adjustment to discounting. In equilibrium, expected returns depend on $g(Z)/V$, so intensity summarizes pricing only under restrictive homogeneity and scale-invariance conditions. I formalize this distinction and derive sufficiency and spanning diagnostics that treat intensity as a maintained hypothesis with refutable implications.

These theoretical predictions yield sharp empirical tests. I test these restrictions using an international panel of firms with Scope 1 emissions. Return-forecasting regressions show that absolute emissions are positively and robustly related to future returns, while emission intensity has little incremental explanatory power once levels are controlled for. In joint specifications that condition on emission intensity, firm characteristics, and extensive country, industry, and time fixed effects—corresponding to the spanning restriction—a one-standard-deviation increase in log Scope 1 emissions predicts a 0.35% increase in next-month expected returns (about 4.2% annualized), with effects remaining significant over the subsequent year. This pricing asymmetry persists even within narrow size classes: conditional on emission intensity, firms with similar scale but different emission levels earn systematically different expected returns. Moreover, the pricing of intensity varies with firm scale, directly rejecting the scale-invariance restriction that intensity sufficiency

requires. These findings are consistent with the model's predictions when priced exposure depends on emission levels rather than intensity.

Beyond their implications for asset pricing theory, these results clarify when intensity-based regulation is theoretically justified. The EU Taxonomy (Regulation 2020/852) and TCFD (2017) recommendations rely on intensity metrics for classifying sustainable activities and assessing climate risk. As established above, intensity is sufficient for pricing climate risk if and only if exposure functions are homogeneous of degree one in emissions and firm value and satisfy scale-invariance. My empirical tests reject sufficiency. This suggests that while intensity may appropriately measure emissions per unit of economic activity—the relevant metric for regulatory targets—it systematically mismeasures the cross-sectional variation in climate risk premia that determines corporate discount rates. Regulatory frameworks conflating these objectives may produce taxonomies that appropriately target environmental outcomes yet fail to capture financial risk exposure.

Related Literature. This paper relates to several strands of the climate-finance and sustainable-investing literature. Theoretically, the framework relates to equilibrium models with heterogeneous ESG preferences. Pastor et al. (2021) develop a static equilibrium in which an exogenous greenness characteristic enters investors' utility and generates a green factor and greenium in expected returns. Pedersen et al. (2021) construct an ESG-efficient frontier and an ESG-adjusted CAPM with heterogeneously ESG-motivated investors, showing how ESG scores affect equilibrium Sharpe ratios and expected returns. Zerbib (2022) and De Angelis et al. (2023) analyze sustainable investing when some investors internalize environmental externalities, deriving taste and exclusion premia in which green investors' portfolio tilts raise brown firms' cost of capital. Dangl et al. (2025) develop a unified framework with different social preference types and establish that such preferences can have substantial real effects even when observed cost-of-capital gaps are small. These papers share the idea that heterogeneous ESG or climate preferences generate equilibrium premia, but typically take the relevant ESG or environmental metric as given. My framework instead endogenizes green and brown classifications as equilibrium exposure functions–priced objects determined by how climate and policy shocks enter cash flows–rather than treating them as exogenous firm types.

A related literature examines sustainable-discount-rate and social-cost-of-carbon frameworks. Pedersen (2025) shows that in a complete-markets planner world, the sustainable-discount-rate wedge implementing the social optimum is proportional to firms' emission intensity. Chittaro et al. (2025) demonstrate that in a frictionless representative-agent economy, a carbon tax is equivalent

to a return schedule increasing linearly in emission intensity. Empirically, Pastor et al. (2025a) introduce a firm's carbon burden–the present value of social costs of future emissions–and find that firms with higher carbon burdens have higher costs of capital. In my framework, such SCC-based burdens correspond to particular exposure functions $g(Z_i)$, and the analysis clarifies when SCC-based objects serve as priced characteristics in equilibrium versus planner implementation.

Empirically, a large literature documents correlations between carbon characteristics and expected returns. Studies use emission levels (Bolton and Kacperczyk, 2021, 2023), emission intensity (Pastor et al., 2022; Eskildsen et al., 2024), composite ESG scores, and text-based climate exposure measures (Sautner et al., 2023). Pastor et al. (2025b) survey the sustainable-investing literature and highlight as an open question which ESG attributes are actually priced in equilibrium. A related challenge is that ostensibly similar ESG characteristics can be constructed very differently across data providers (Berg et al., 2022). I take a structural approach rather than proposing new emission metrics: which emission-based characteristics can legitimately serve as sufficient statistics for exposure to priced climate factors? The framework delivers testable restrictions, scale invariance and spanning, that discipline when emission intensity suffices for equilibrium pricing and when alternative exposure measures are consistent with the data.

The remainder of the paper is organized as follows. Section 2 develops the incomplete-markets benchmark and shows that emission levels, not intensity, naturally emerge as the priced characteristic when dividends load proportionally on climate shocks. Section 3 presents the complete-markets planner benchmark and the sustainable-discount-rate representation. Sections 4 and 5 embed these benchmarks in a general factor framework and characterize the conditions under which intensity is sufficient. Sections 6 and 7 provide illustrative exposure specifications that violate these conditions. Section 8 discusses welfare and policy implications and presents the empirical tests. Internet Appendix D.1 summarizes how the main emission-based predictors used in the literature map into the distinct economic objects isolated by the framework.

2 A Simple Static Model with Heterogeneous Investors and Climate Risk

Consider a two-period economy, with $t = 0, 1$. Uncertainty is represented by a finite state space Ω with probability measure $\mathbb{P}$. The economy features a single physical consumption good that serves as numeraire, N firms indexed by $i = 1, \ldots, N$, and H households indexed by $h = 1, \ldots, H$. In

addition to the N risky assets, a risk-free asset in zero net supply with gross return $R_f > 0$ is available.

2.1 Emissions, climate shock, and damages

Each firm i has a fixed level of emissions $X_i \geq 0$ at date 0, which determines its exposure to the climate shock realized at date 1. Aggregate emissions are

$$\bar{X} \equiv \sum_{i=1}^{N} X_i. \tag{2.1}$$

The economy is exposed to a one-dimensional aggregate climate or policy shock $\zeta : \Omega \to \mathbb{R}$ with

$$\mathbb{E}[\zeta] = 0, \qquad \text{Var}(\zeta) = \sigma_\zeta^2 > 0. \tag{2.2}$$

I assume ζ is normally distributed and all payoffs are jointly normal.

Climate damages, Γ, are linear in ζ and proportional to aggregate emissions:

$$\Gamma(\zeta) \equiv \kappa \bar{X}\, \zeta, \qquad \kappa > 0. \tag{2.3}$$

Equation (2.3) is a reduced-form representation of the idea that the realization of the climate shock ζ scales damages in proportion to $\bar{X}$. The linear specification serves as a convenient benchmark, which I further relax in Internet Appendix A.

2.2 Firms and cash flows

Firm i issues a single share of equity in fixed supply. Its date-1 dividend in state $\omega \in \Omega$ is

$$D_i(\omega) = \mu_i + \eta X_i \zeta(\omega), \tag{2.4}$$

where $\mu_i \in \mathbb{R}$ is a deterministic component and $\eta > 0$ is a technology parameter common across firms. The shock ζ can be interpreted as a per-unit climate or policy shock that scales the cash-flow impact of emissions. For example, if ζ is a shock to the per-unit carbon cost or to the shadow value of emissions in production, then, for given X_i, a one-unit increase in ζ changes firm i's cash flow proportionally to X_i. Thus, conditional on the realization of ζ, the exposure of firm i to climate risk is proportional to its emission level X_i.

Let V_i denote the date-0 price of firm i's equity,[1] and define the gross return

$$R_i(\omega) \equiv \frac{D_i(\omega)}{V_i}. \tag{2.5}$$

2.3 Households, preferences, and market structure

Household h has date-0 financial wealth $W_0^h > 0$. At $t = 0$, the household chooses a portfolio $\theta^h = (\theta_1^h, \ldots, \theta_N^h)^\top$ in the N risky assets and invests an amount b^h in the risk-free asset, subject to the budget constraint

$$\sum_{i=1}^{N} V_i \theta_i^h + b^h = W_0^h. \tag{2.6}$$

Date-1 consumption of household h in state ω is then given by

$$C_1^h(\omega) = R_f\, b^h + \sum_{i=1}^{N} \theta_i^h D_i(\omega), \tag{2.7}$$

where R_f denotes the gross risk-free return.

Households have exponential (CARA) preferences over climate-adjusted consumption,

$$U^h = \mathbb{E}\left[-\exp\left(-\alpha_h (C_1^h - \phi_h \Gamma(\zeta)) \right) \right], \tag{2.8}$$

where $\alpha_h > 0$ is the coefficient of absolute risk aversion and $\phi_h \geq 0$ indexes household h's climate concern. The term $\phi_h \Gamma(\zeta)$ captures disutility from aggregate climate damages $\Gamma(\zeta)$, with $\phi_h = 0$ corresponding to a purely financial (brown) investor and $\phi_h > 0$ to a climate-conscious (green) investor. Importantly, $\Gamma(\zeta)$ is not a direct financial loss borne uniformly through the household budget constraint. Rather, it is an aggregate damage state that enters utility through climate concern, capturing disutility from the realization of global climate outcomes even when the associated physical or economic losses are largely external to the household's own consumption stream.

Because aggregate climate damages are non-tradable and investors differ in (α_h, ϕ_h), markets are incomplete with respect to climate risk and the stochastic discount factor that prices traded assets reflects a wealth-weighted aggregation of heterogeneous marginal rates of substitution rather than the planner's sustainable discount factor. The resulting priced climate characteristic is therefore an equilibrium object, a function of firm exposures, and the associated premium admits a decom-

[1] V_i denotes the equilibrium market value of claim i. Because returns are per unit of invested wealth, priced exposures appear in value-normalized form.

position into a risk-based brown component and a preference-driven true greenium, as formalized in Section 4.1.

3 Complete-markets Benchmark

The static incomplete-markets framework above provides a positive characterization of equilibrium asset pricing when climate damages enter investors' utility and are only partially internalized through heterogeneous portfolio choices. To assess the role of emissions from a normative perspective, it is useful to contrast this environment with a complete-markets benchmark in which a social cost of carbon (SCC) is well defined and a social planner internalizes the externality. This section restates, in a parsimonious setting, the planner logic underlying the sustainable-discount-rate framework and shows how the SCC translates into firm-specific discount-rate wedges that are proportional to emission intensity X_i/V_i. For transparency, I abstract from cross-sectional heterogeneity and financial risk and focus on deterministic cash flows and a representative decision maker.

3.1 Planner, social cost of carbon, and a Pigouvian tax

Consider again dates $t = 0, 1$ and a single consumption good. A representative agent has utility

$$U = u(C_0) + \beta\Big(u(C_1) - d(X_1)\Big), \tag{3.1}$$

where C_0 and C_1 denote aggregate consumption at dates 0 and 1, $\beta \in (0, 1)$ is the subjective discount factor, and $d : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing, convex damage function of aggregate emissions $X_1 \geq 0$. The function $d(\cdot)$ can be interpreted as the expected disutility from climate damages, for example $d(X_1) = \mathbb{E}[\Gamma(\zeta) \mid X_1]$ in a reduced-form sense. The per-period utility u is strictly increasing and concave.

Firm i produces a deterministic date-1 payoff $Y_i \geq 0$ per unit of scale and emits $X_i \geq 0$ per unit of scale. If I_i denotes the scale of firm i at date 0, we can aggregate emissions as

$$X_1 = \sum_{i=1}^{N} X_i I_i. \tag{3.2}$$

Let G_0 denote the date-0 endowment of the consumption good. The resource constraints are

$$C_0 + \sum_{i=1}^{N} I_i \leq G_0, \qquad C_1 = \sum_{i=1}^{N} Y_i I_i. \tag{3.3}$$

The technological mapping from I_i to (Y_i, X_i) is kept implicit. (Y_i, X_i) are treated as primitive characteristics of one unit of firm i, and the planner chooses the scale I_i.

The social planner chooses $(C_0, C_1, \{I_i\}_{i=1}^N)$ to maximize (3.1) subject to (3.2)-(3.3). Let λ_0 and λ_1 denote the Lagrange multipliers on the two resource constraints. The planner's Lagrangian is

$$\mathcal{L} = u(C_0) + \beta(u(C_1) - d(X_1)) + \lambda_0\Big(G_0 - C_0 - \sum_i I_i\Big) + \lambda_1\Big(\sum_i Y_i I_i - C_1\Big).$$

The first-order conditions for C_0 and C_1 are

$$u'(C_0) = \lambda_0, \qquad \beta u'(C_1) = \lambda_1. \tag{3.4}$$

The first-order condition for I_i is

$$-\lambda_0 + \lambda_1 Y_i - \beta d'(X_1) X_i = 0. \tag{3.5}$$

Combining (3.4) and (3.5) yields

$$\frac{\lambda_0}{\lambda_1} = Y_i - \frac{\beta d'(X_1)}{\lambda_1} X_i = Y_i - \frac{d'(X_1)}{u'(C_1)} X_i. \tag{3.6}$$

The shadow cost of an additional unit of emissions in terms of the consumption numeraire is defined as the social cost of carbon,

$$S \equiv \frac{d'(X_1)}{u'(C_1)}. \tag{3.7}$$

Expression (3.6) can then be written as

$$\frac{\lambda_0}{\lambda_1} = Y_i - S X_i. \tag{3.8}$$

Equation (3.8) shows that, at the planner optimum, a marginal increase in scale I_i raises date-1 resources by Y_i but also increases the planner's marginal damage by SX_i. It is therefore convenient to interpret $Y_i - SX_i$ as the planner-implied net date-1 payoff (in consumption units) generated by

one unit of firm i's scale.

To express this condition in present-value form, define the planner-implied intertemporal discount factor as

$$q \equiv \frac{\lambda_1}{\lambda_0} = \beta \frac{u'(C_1)}{u'(C_0)}. \tag{3.9}$$

Then (3.8) is equivalent to

$$1 = q(Y_i - SX_i). \tag{3.10}$$

Thus, the date-0 shadow value of one unit of scale is $q(Y_i - SX_i)$, the present value of its net date-1 payoff.

For later comparison, it is convenient to parameterize the planner-implied discount factor by a (gross) risk-free rate r via

$$q \equiv \frac{\lambda_1}{\lambda_0} = \frac{1}{1+r}. \tag{3.11}$$

A Pigouvian carbon tax τ per unit of emissions, levied uniformly on firms and rebated lump-sum to the representative household, decentralizes the planner's allocation if $\tau = S$. Under such a tax, firm i's date-1 net payoff per unit of scale is $Y_i - \tau X_i$, and its date-0 value is

$$V_i = q(Y_i - \tau X_i) = \frac{Y_i - \tau X_i}{1+r}. \tag{3.12}$$

If $\tau = S$, then (3.10) and (3.12) coincide, implying that the competitive equilibrium with tax $\tau = S$ reproduces the first-best allocation.

3.2 Sustainable discount rates and the emission-intensity wedge

The sustainable-discount-rate approach asks whether the same allocation that would be achieved by a Pigouvian carbon tax can instead be implemented by adjusting firms' discount rates while leaving their cash flows untaxed. In the one-period environment above, this equivalence can be made explicit and directly related to emission intensity.

Fix a firm i and suppose that, under the carbon tax τ, its date-0 value is V_i given by (3.12). Let r denote the financial discount rate that applies in the absence of any climate-related discount-rate adjustment. A sustainable discount rate r_i^x for firm i is defined implicitly by the condition that the present value of the taxed cash flow discounted at r equals the present value of the untaxed cash

flow discounted at r_i^x, that is,

$$\frac{Y_i - \tau X_i}{1+r} = \frac{Y_i}{1+r_i^x}. \tag{3.13}$$

Solving (3.13) for r_i^x gives

$$1 + r_i^x = \frac{Y_i}{Y_i - \tau X_i}(1+r). \tag{3.14}$$

Equation (3.14) expresses the sustainable discount rate as a nonlinear function of the financial rate r, the tax τ, and the ratio $\tau X_i/Y_i$.

For small wedges and moderate discount rates, a first-order approximation in $\tau X_i/Y_i$ is informative. Writing $\varepsilon_i \equiv \tau X_i/Y_i$ and expanding the right-hand side of (3.14) to first order in ε_i yields

$$1 + r_i^x \approx (1+r)(1+\varepsilon_i),$$

so that

$$r_i^x - r \approx \varepsilon_i = \tau \frac{X_i}{Y_i}. \tag{3.15}$$

Let V_i denote the firm's value under the tax as in (3.12). For small τ, we have $V_i \approx Y_i/(1+r)$, and hence

$$\frac{X_i}{Y_i} \approx \frac{X_i}{(1+r)V_i} \approx \frac{X_i}{V_i},$$

where the last approximation absorbs the factor $1+r$ into the first-order approximation (and is numerically innocuous at annual rates). Substituting into (3.15) yields the familiar emission-intensity wedge,

$$r_i^x \approx r + \tau \frac{X_i}{V_i}. \tag{3.16}$$

If the tax τ coincides with the SCC S in (3.7), then (3.16) links the sustainable discount rate directly to the SCC and emission intensity:

$$r_i^x \approx r + S \frac{X_i}{V_i}. \tag{3.17}$$

More generally, suppose that the prevailing carbon tax is τ while the planner's SCC is S. The additional wedge required to move from the current tax to the first-best tax is $S - \tau$. Consider a small change $\mathrm{d}\tau$ in the tax and equate the induced change in firm value to that from a small change $\mathrm{d}r_i$ in the discount rate:

$$\mathrm{d}V_i^{(\tau)} = -\frac{X_i}{1+r}\mathrm{d}\tau, \qquad \mathrm{d}V_i^{(r)} = -\frac{V_i}{1+r}\mathrm{d}r_i.$$

Setting $\mathrm{d}V_i^{(\tau)} = \mathrm{d}V_i^{(r)}$ implies

$$\mathrm{d}r_i = \frac{X_i}{V_i}\mathrm{d}\tau.$$

Applying this with $\mathrm{d}\tau = S - \tau$ and linearizing around the initial allocation yields

$$r_i^x \approx r + (S - \tau)\frac{X_i}{V_i}. \tag{3.18}$$

Equations (3.7) and (3.18) formalize the sustainable-discount-rate representation developed by Pedersen (2025). The SCC S is the planner's shadow value of emissions in units of the consumption numeraire, and, to a first-order approximation, the same allocation that would be achieved by adjusting the carbon tax from τ to S can be implemented by tilting each firm's discount rate in proportion to its emission intensity X_i/V_i. In this complete-markets benchmark, X_i/V_i is the sufficient statistic for how strongly a firm's cost of capital must be adjusted to implement the planner's allocation.

This is fundamentally an implementation result, not a restriction on equilibrium pricing. Equation (3.18) characterizes the deterministic discount-rate wedge needed to decentralize a first-best allocation under complete markets and a well-defined SCC. Once one relaxes the complete-markets and representative-agent assumptions–allowing for heterogeneous climate concern and incomplete risk sharing as in Section 2.3–the characteristics that govern equilibrium risk premia need not collapse to X_i/V_i and can instead depend on emission levels and more general exposure functions. Sections 4 and 5 make this distinction precise by characterizing when intensity remains a sufficient statistic for equilibrium returns (Proposition 3) and showing that the required conditions are knife-edge.

4 Equilibrium with Heterogeneous Investors and Unspanned Climate Risk

The benchmark models in the previous sections clarify two distinct objects: equilibrium risk premia in incomplete markets and implementation wedges in a complete-markets planner economy. What they do not yet provide is a general way to think about how climate and transition risk enter

asset prices when firms are exposed through multiple, overlapping channels. In practice, emissions, technology, demand, and policy all shape how firms load on climate-related shocks, and no single emission measure need summarize these exposures a priori. This section introduces a general factor framework that makes these channels explicit and provides a unified language for assessing when emission intensity is sufficient for pricing and when it is not.

4.1 Incomplete markets

The only traded risky payoffs are the N stocks with dividends given by (2.4). There is a single aggregate climate shock ζ, and I allow for additional idiosyncratic shocks in μ_i, but for the derivations below I focus on the one-factor structure (2.4).

Markets are incomplete in that there is no portfolio of traded assets whose payoff replicates the damage term $\Gamma(\zeta) = \kappa \bar{X} \zeta$. Equivalently, ζ is not fully spanned by the space of traded payoffs once the non-pecuniary climate term in preferences is taken into account. Differently, markets are complete for financial payoffs, but the climate damage term in utility is non-tradable, so the SDF is not the MRS of someone who internalizes the externality in a planner sense.

Formally, the equilibrium still admits a unique stochastic discount factor m that prices all traded assets, but the incomplete spanning of $\Gamma(\zeta)$ prevents us from representing m as the marginal rate of substitution of a representative planner who internalizes the externality in a first-best sense. This is exactly where the analysis differs from the complete-markets benchmark.

4.2 Mean-variance representation

Under CARA-exponential utility and joint normality, maximizing expected utility (2.8) is equivalent to maximizing a mean-variance objective. Substituting (2.3), (2.4), and (2.7), we have

$$C_1^h - \phi_h \Gamma(\zeta) = b^h R_f + \sum_{i=1}^{N} \theta_i^h \mu_i + \Big(\sum_{i=1}^{N} \theta_i^h \eta X_i - \phi_h \kappa \bar{X} \Big) \zeta.$$

Thus

$$C_1^h - \phi_h \Gamma(\zeta) = A_h + B_h \zeta,$$

where

$$A_h \equiv b^h R_f + \sum_{i=1}^{N} \theta_i^h \mu_i, \qquad B_h \equiv \sum_{i=1}^{N} \theta_i^h \eta X_i - \phi_h \kappa \bar{X}.$$

The mean-variance representation of (2.8) is

$$U^h = -\exp\left(-\alpha_h\Big(\mathbb{E}[A_h] - \tfrac{\alpha_h}{2}\mathrm{Var}(B_h\zeta)\Big)\right),$$

so household h chooses (θ^h, b^h) to maximize

$$\mathbb{E}[C_1^h - \phi_h\Gamma(\zeta)] - \frac{\alpha_h}{2}\mathrm{Var}[C_1^h - \phi_h\Gamma(\zeta)]. \tag{4.1}$$

Since b^h only enters linearly and serves to satisfy the budget constraint, the key choice is the stock vector θ^h.

Using $\mathbb{E}[\zeta] = 0$ and $\mathrm{Var}(\zeta) = \sigma_\zeta^2$, objective (4.1) becomes

$$\sum_{i=1}^N \theta_i^h(\mu_i - V_iR_f) \;-\; \frac{\alpha_h}{2}\sigma_\zeta^2\Big(\sum_{i=1}^N \theta_i^h\eta X_i - \phi_h\kappa\bar{X}\Big)^2.$$

4.3 First-order conditions and equilibrium prices

The first-order condition for θ_i^h is

$$\mu_i - V_iR_f - \alpha_h\sigma_\zeta^2\Big(\sum_{j=1}^N \theta_j^h\eta X_j - \phi_h\kappa\bar{X}\Big)\eta X_i = 0, \qquad i = 1,\ldots,N. \tag{4.2}$$

Define

$$\delta_h \equiv \sum_{j=1}^N \theta_j^h\eta X_j - \phi_h\kappa\bar{X}.$$

Then (4.2) is equivalently

$$\mu_i - V_iR_f = \alpha_h\sigma_\zeta^2\delta_h\eta X_i, \qquad i = 1,\ldots,N. \tag{4.3}$$

Since the left-hand side does not depend on h, equilibrium requires $\alpha_h\sigma_\zeta^2\delta_h$ to be common across households with interior holdings.

Proposition 1 (One-factor structure and emission levels)**.** *In any competitive equilibrium with $\eta > 0$, there exists a scalar λ such that*

$$\mu_i - V_iR_f = \lambda\eta X_i, \qquad i = 1,\ldots,N, \tag{4.4}$$

and for every household h,

$$\delta_h = \frac{\lambda}{\alpha_h \sigma_\zeta^2}. \tag{4.5}$$

Proof is in Appendix A.1. Equation (4.4) is a pricing relation in dollar units. Dividing by V_i yields the corresponding return representation,

$$\mathbb{E}[R_i] - R_f = \lambda \eta \frac{X_i}{V_i}, \tag{4.6}$$

where $R_i \equiv D_i/V_i$ and $\mathbb{E}[D_i] = \mu_i$. In this model, emission levels X_i govern the primitive cash-flow exposure in dividends, while expected returns load on the value-normalized exposure X_i/V_i. The coefficient λ is the equilibrium price of climate-related risk.

4.4 Solving for the price of climate risk

Aggregating the definition of δ_h across households, multiplying by η, and imposing market clearing $\sum_{h=1}^{H} \theta_j^h = 1$ yields

$$\eta^2 \bar{X} \; - \; \kappa \eta \bar{X} \sum_{h=1}^{H} \phi_h \; = \; \eta \sum_{h=1}^{H} \delta_h. \tag{4.7}$$

Using (4.5), the right-hand side is $\eta \sum_{h=1}^{H} \delta_h = (\eta\lambda/\sigma_\zeta^2) \sum_{h=1}^{H} (1/\alpha_h)$. Solving for λ gives:

Proposition 2 (Price of climate risk)**.** *In the static model above, the equilibrium price of climate risk is*

$$\lambda = \frac{\sigma_\zeta^2 \bar{X}}{\sum_{h=1}^{H} \frac{1}{\alpha_h}} \Big(\eta - \kappa \sum_{h=1}^{H} \phi_h \Big). \tag{4.8}$$

Proof is in Appendix A.1. Under CARA preferences, λ depends on the distribution of risk aversion (α_h) and aggregate climate concern $\sum_h \phi_h$, and is independent of initial wealth (W_0^h).[2] Combining (4.8) with (4.6) emphasizes that expected returns depend on the value-normalized exposure X_i/V_i, whereas the primitive dividend exposure is proportional to the emission level X_i.

4.5 Decomposing transition risk and preference effects

Expression (4.8) admits a natural decomposition into a purely financial brown risk premium (transition-risk component) and a true greenium associated with non-pecuniary climate concern (preference component). To make this explicit, note the price of climate risk in the incomplete-

[2] In Internet Appendix B, I show that with CRRA utility the analogous expression features wealth weights through local risk tolerances.

markets economy derived above

$$\lambda_{X,t} = \frac{\sigma_\zeta^2 \bar{X}_t \eta}{\sum_{h=1}^{H} \frac{1}{\alpha_h}} - \frac{\sigma_\zeta^2 \bar{X}_t \kappa \eta}{\sum_{h=1}^{H} \frac{1}{\alpha_h}} \sum_{h=1}^{H} \phi_h.$$

Define

$$\lambda_{X,t}^{\mathrm{TR}} \equiv \frac{\sigma_\zeta^2 \bar{X}_t \eta}{\sum_{h=1}^{H} \frac{1}{\alpha_h}}, \qquad \lambda_{X,t}^{\mathrm{P}} \equiv -\frac{\sigma_\zeta^2 \bar{X}_t \kappa \eta}{\sum_{h=1}^{H} \frac{1}{\alpha_h}} \sum_{h=1}^{H} \phi_h, \tag{4.9}$$

so that

$$\lambda_{X,t} = \lambda_{X,t}^{\mathrm{TR}} + \lambda_{X,t}^{\mathrm{P}}. \tag{4.10}$$

The term $\lambda_{X,t}^{\mathrm{TR}}$ is the price of climate risk that would prevail if all households were climate-neutral in their preferences, that is, if $\phi_h = 0$ for all h. It compensates investors for bearing unhedgeable exposure to the aggregate climate or policy shock ζ_{t+1} through the technological channel $D_{i,t+1} = \mu_{i,t+1} + \eta X_{i,t} \zeta_{t+1}$. In this sense, $\lambda_{X,t}^{\mathrm{TR}}$ captures the transition-risk or physical-risk component of the brown premium. It depends on the volatility of the climate shock σ_ζ^2, aggregate emissions $\bar{X}_t$, the technological sensitivity η, and aggregate risk tolerance $\sum_h (1/\alpha_h)$, but it is independent of the climate-preference parameters $\{\phi_h\}_h$.

By contrast, the term $\lambda_{X,t}^{\mathrm{P}}$ is zero when $\phi_h = 0$ and is strictly decreasing in aggregate climate concern $\sum_h \phi_h$ whenever $\kappa > 0$. Differentiating (4.9) with respect to ϕ_h yields

$$\frac{\partial \lambda_{X,t}}{\partial \phi_h} = -\frac{\sigma_\zeta^2 \bar{X}_t \kappa \eta}{\sum_{j=1}^{H} \frac{1}{\alpha_j}} < 0.$$

Thus, holding the shock process and the technology block fixed, greater aggregate climate concern compresses the equilibrium compensation per unit exposure, because climate-concerned investors are willing to pay to avoid exposure to climate damages. The component $\lambda_{X,t}^{\mathrm{P}}$ therefore captures the preference or true greenium effect. For a given technology and shock process, stronger climate concern tilts portfolios away from high-emission exposures and compresses premia on brown assets.

Given the decomposition (4.10), the expected excess return on firm i can be written as

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx (\lambda_{X,t}^{\mathrm{TR}} + \lambda_{X,t}^{\mathrm{P}}) X_{i,t} = \lambda_{X,t}^{\mathrm{TR}} X_{i,t} + \lambda_{X,t}^{\mathrm{P}} X_{i,t}. \tag{4.11}$$

In this decomposition, the first term is the compensation for bearing non-diversifiable transition (or physical) climate risk, as encoded in the distribution of ζ_{t+1} and its impact on cash flows. The second term reflects the fact that, in incomplete markets, investors who directly dislike damages

are willing to accept lower expected returns on low-emission assets (or require higher returns on high-emission assets) even when the underlying shock process is held fixed.

An equivalent separation applies at the level of the aggregate stochastic discount factor. Writing the affine SDF as

$$m_{t+1} = \exp\big(-A_t - B_t\zeta_{t+1}\big), \qquad B_t = \frac{\lambda_{X,t}}{\sigma_\zeta^2},$$

and substituting (4.9) gives

$$B_t = B_t^{\mathrm{TR}} + B_t^{\mathrm{P}}, \qquad B_t^{\mathrm{TR}} = \frac{\bar{X}_t\eta}{\sum_{h=1}^{H}\frac{1}{\alpha_h}}, \quad B_t^{\mathrm{P}} = -\frac{\bar{X}_t\kappa\eta}{\sum_{h=1}^{H}\frac{1}{\alpha_h}}\sum_{h=1}^{H}\phi_h.$$

The loading B_t^{TR} describes how the representative pricing kernel would respond to the climate shock in an otherwise identical economy with $\phi_h = 0$. The adjustment B_t^{P} captures the additional tilt in the pricing kernel induced by heterogeneous climate preferences under incomplete markets.

This decomposition makes it clear that, within current framework, transition risk and preference effects are conceptually distinct but jointly determine the observed carbon premium. The former reflects the technological and policy sensitivity of cash flows to ζ_{t+1}, while the latter reflects investors' willingness to trade off financial returns against exposure to climate damages.

4.6 When is emission intensity a unique sufficient statistic?

The previous subsection shows that expected excess returns are governed by firms' factor exposures, which may depend on a rich set of characteristics $Z_{i,t}$. There is therefore no general reason why any single primitive, such as emission levels $X_{i,t}$ or emission intensity $X_{i,t}/V_{i,t}$, must be sufficient for climate-related pricing. A key clarification is that, while economic mechanisms determine exposures in dollar units (cash-flow sensitivities), asset pricing maps these sensitivities into return premia by dividing by firm value: returns are measured per unit of invested wealth. This subsection formalizes and characterizes the knife-edge conditions under which emission intensity is a unique sufficient statistic for the priced exposure to a given climate or transition factor.

Definition 4.1 (Emission intensity as a sufficient statistic)**.** *Consider date t and a one-factor environment in which expected excess returns satisfy*

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \Lambda_t\chi_{i,t},$$

for some scalar Λ_t and some scalar characteristic $\chi_{i,t}$ that may depend on firm attributes. We say that emission intensity *is a sufficient statistic at date t if for any two firms i and j,*

$$\frac{X_{i,t}}{V_{i,t}} = \frac{X_{j,t}}{V_{j,t}} \quad \Rightarrow \quad \chi_{i,t} = \chi_{j,t}.$$

We say that intensity is a unique *sufficient statistic if, in addition, any other characteristic $\tilde{\chi}_{i,t}$ that generates an equivalent one-factor representation must be proportional to $X_{i,t}/V_{i,t}$ in the cross-section, that is, there exists a scalar ψ_t such that*

$$\tilde{\chi}_{i,t} = \psi_t \frac{X_{i,t}}{V_{i,t}} \quad \textit{for all } i.$$

One-factor environment and value normalization. For clarity, let suppress time subscripts when no confusion arises. Let f_{t+1} denote a scalar transition or climate factor with $\mathbb{E}_t[f_{t+1}] = 0$, and consider dividends of the form

$$D_{i,t+1} = \mu_{i,t+1} + g(Z_{i,t}) f_{t+1}, \tag{4.12}$$

where $Z_{i,t}$ collects firm characteristics (including in particular $(X_{i,t}, V_{i,t})$), and $g(\cdot)$ is a deterministic function. The object $g(Z_{i,t})$ is a *cash-flow* exposure (in dividend units). Let the stochastic discount factor be

$$m_{t+1} = a_t + b_t f_{t+1} + \varepsilon_{t+1}, \tag{4.13}$$

with $a_t > 0$, b_t $\mathcal{F}_t$–measurable, and ε_{t+1} conditionally mean-zero and orthogonal to f_{t+1}. A standard log-linear approximation to the Euler equation implies that, to a first-order approximation in $\mathrm{Var}_t(f_{t+1})$, expected excess returns take the value-normalized form

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \lambda_t \frac{g(Z_{i,t})}{V_{i,t}}, \qquad \lambda_t = -\frac{b_t \mathrm{Var}_t(f_{t+1})}{\mathbb{E}_t[m_{t+1}]}, \tag{4.14}$$

regardless of whether markets are complete or incomplete. Market structure determines b_t and hence λ_t, but not the functional form of $g(\cdot)$.

Knife-edge conditions for intensity sufficiency. The next result characterizes when the return-relevant exposure $g(Z_{i,t})/V_{i,t}$ depends on firms only through emission intensity $X_{i,t}/V_{i,t}$, and when this dependence is unique up to scale.

Proposition 3 (Conditions for emission intensity to be a unique sufficient statistic)**.** *Consider the one-factor environment* (4.12)-(4.14) *at a given date* t*. Suppose that:*

(i) The cross-section of firms is rich *in the sense that the set*

$$\mathcal{S}_t \equiv \left\{ \left(\frac{X_{i,t}}{V_{i,t}}, V_{i,t} \right) : i = 1, \ldots, N \right\}$$

exhibits independent variation in emission intensity and firm scale.[3]

(ii) The exposure function $g(\cdot)$ *is homogeneous of degree one in* $(X_{i,t}, V_{i,t})$ *in the sense that, for all* $\lambda > 0$ *and all firms* i*,*

$$g(Z_{i,t}; \lambda X_{i,t}, \lambda V_{i,t}) = \lambda \, g(Z_{i,t}; X_{i,t}, V_{i,t}).$$

(iii) For any two firms i *and* j *with the same intensity,* $X_{i,t}/V_{i,t} = X_{j,t}/V_{j,t}$*, their value-normalized exposures coincide,*

$$\frac{g(Z_{i,t})}{V_{i,t}} = \frac{g(Z_{j,t})}{V_{j,t}}.$$

Then there exists a (measurable) function $\varphi_t : \mathbb{R}_+ \to \mathbb{R}$ *such that*

$$\frac{g(Z_{i,t})}{V_{i,t}} = \varphi_t\left(\frac{X_{i,t}}{V_{i,t}} \right) \quad \text{for all } i. \tag{4.15}$$

If, in addition, $g(\cdot)$ *is continuous on* $\mathcal{S}_t$ *and the mapping* $v \mapsto \varphi_t(u)$ *is linear in* v *for each fixed* u*, then there exists a scalar* κ_t *such that*

$$\frac{g(Z_{i,t})}{V_{i,t}} = \kappa_t \frac{X_{i,t}}{V_{i,t}} \quad \text{for all } i, \tag{4.16}$$

and $X_{i,t}/V_{i,t}$ *is a unique sufficient statistic for expected excess returns in the sense of Definition 4.1. Equivalently,* (4.16) *holds if and only if*

$$g(Z_{i,t}) = \kappa_t X_{i,t} \quad \text{for all } i,$$

so that linear intensity sufficiency in returns corresponds to a level-proportional cash-flow exposure.

[3] Formally, as $N \to \infty$, the support of $\mathcal{S}_t$ contains an open subset of $\mathbb{R}^2_+$, ruling out degenerate cases in which firms lie on a lower-dimensional set.

Proof is in Appendix A.1. Proposition 3 establishes what is required for intensity to be uniquely sufficient in equilibrium. Two firms with the same ratio $X_{i,t}/V_{i,t}$ must have the same return-relevant exposure $g(Z_{i,t})/V_{i,t}$ regardless of their absolute scale. Because many standard channels produce exposures that depend on levels ($\partial g/\partial V = 0$ under linear per-unit instruments) or depend on scale through technology, demand, and regulation, the scale-invariance restriction in (iii) and the homogeneity requirement in (ii) are demanding. When they fail, additional characteristics beyond $X_{i,t}/V_{i,t}$ must enter the priced exposure.

The proposition also clarifies what does and does not hinge on market completeness. Completeness affects the equilibrium *price* of risk λ_t in (4.14), but it does not impose the scale and homogeneity restrictions on $g(\cdot)$ needed for intensity sufficiency. These implications motivate the empirical sufficiency tests in Section 8.4, which evaluate scale invariance and spanning restrictions implied by Proposition 3.

5 Transition Risk, Emission Intensity, and Priced Characteristics

In this section, I place emission-based climate metrics within a general linear-factor pricing environment. The key distinction is between (i) *cash-flow exposures* $g_k(Z_{i,t})$, which describe how climate and transition shocks enter dividends and are disciplined by technology and policy design, and (ii) *return exposures*, which are necessarily value-normalized because premia are measured per unit of invested wealth and therefore load on $g_k(Z_{i,t})/V_{i,t}$. The central question is thus when the value-normalized exposure can be summarized by emission intensity $X_{i,t}/V_{i,t}$ alone. Proposition 3 characterizes the knife-edge conditions for a one-factor environment. The remainder of this section extends the representation to multiple factors and clarifies how the planner-based sustainable-discount-rate wedge relates to the equilibrium pricing of stochastic transition risk.[4]

5.1 Sustainable discount rates and intensity in the complete-markets benchmark

In the complete-markets benchmark with a representative planner and a well-defined social cost of carbon $\{S_t\}_t$, a Pigouvian tax sequence $\{\tau_t\}_t$ or an equivalent cap-and-trade scheme decentralizes the planner's allocation. Let $V^*_{i,t}$ denote the value of firm i at date t along this allocation and let $X^*_{i,t}$ denote its emissions. For a given financial discount rate r_t, one can define firm-specific

[4] I now adopt time subscripts. All static results in Sections 2-4 carry over to the dynamic setting under standard CARA-normal assumptions (see Internet Appendix D).

sustainable discount rates $r^x_{i,t}$ by equating the present value of taxed cash flows discounted at r_t with the present value of untaxed cash flows discounted at $r^x_{i,t}$. Restating the planner-implied wedge derived in Section 3, a first-order expansion around a baseline with zero wedge yields

$$r^x_{i,t} \approx r_t + (S_t - \tau_t)\frac{X^*_{i,t}}{V^*_{i,t}}, \tag{5.1}$$

where $S_t - \tau_t$ is the gap between the social cost of carbon and the prevailing carbon price. In this benchmark, emission intensity summarizes the *deterministic* implementation wedge between financial and sustainable discount rates required to decentralize the planner's allocation.

Crucially, (5.1) is a statement about the *level* of the wedge needed for implementation at given S_t, not a restriction on the pricing of *stochastic* climate or policy risk in equilibrium. Intensity appears because cash-flow losses proportional to $X^*_{i,t}$ are being expressed on a per-unit-of-value basis.

5.2 Transition risk as stochastic exposure to policy or technology shocks

Consider now transition risk, defined as stochastic shocks to future climate policy, carbon prices, or technology that affect cash flows. To fix ideas, let τ_{t+1} denote a random carbon price at date $t+1$ and suppose that, holding quantities fixed at date t, firm i's date-$t+1$ dividend contains a term

$$D_{i,t+1} = \tilde{D}_{i,t+1} - \tau_{t+1} X_{i,t}, \tag{5.2}$$

where $\tilde{D}_{i,t+1}$ collects all other components orthogonal (conditionally) to τ_{t+1}. Let $\hat{\tau}_{t+1} \equiv \tau_{t+1} - \mathbb{E}_t[\tau_{t+1}]$ denote the innovation in the carbon price and write

$$D_{i,t+1} = \mu_{i,t+1} + g^{(\tau)}_{i,t}\hat{\tau}_{t+1}, \qquad g^{(\tau)}_{i,t} = -X_{i,t},$$

with $\mu_{i,t+1}$ conditionally independent of $\hat{\tau}_{t+1}$. In any equilibrium with a strictly positive SDF m_{t+1} that is conditionally square-integrable, expected excess returns satisfy

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = -\frac{\mathrm{Cov}_t(m_{t+1}, D_{i,t+1})}{\mathbb{E}_t[m_{t+1}]\, V_{i,t}},$$

where $V_{i,t}$ denotes the date-t market value (price) of the claim to $D_{i,t+1}$. Under a standard log-linear approximation, and assuming that $\hat{\tau}_{t+1}$ is a factor in m_{t+1}, this yields a linear factor structure in

which the transition-risk premium associated with $\hat{\tau}_{t+1}$ takes the value-normalized form

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \lambda_t^{(\tau)} \frac{g_{i,t}^{(\tau)}}{V_{i,t}} = -\lambda_t^{(\tau)} \frac{X_{i,t}}{V_{i,t}}, \tag{5.3}$$

where the price of transition risk $\lambda_t^{(\tau)}$ is proportional to $-\mathrm{Cov}_t(m_{t+1}, \hat{\tau}_{t+1})$.

More generally, allowing for multiple transition channels, including demand shocks, stranded-asset risk, and technology shocks, dividends take the generic form

$$D_{i,t+1} = \tilde{D}_{i,t+1} + \sum_{k=1}^{K} g_k(Z_{i,t}) f_{k,t+1}, \tag{5.4}$$

where $f_{k,t+1}$ are mean-zero transition or climate factors and $g_k(Z_{i,t})$ are dollar exposure coefficients that depend on characteristics $Z_{i,t}$, which may include $X_{i,t}$, $V_{i,t}$, $X_{i,t}/V_{i,t}$, changes in emissions, efficiency measures, and so on. Asset pricing maps these dollar exposures into return premia through value normalization, so the empirically relevant objects are $g_k(Z_{i,t})/V_{i,t}$.

5.3 Complete versus incomplete markets and the priced characteristic

The distinction between complete and incomplete markets affects the prices of transition risk but not, for a given technology and shock structure, the mechanism-implied dollar exposure functions $g_k(Z_{i,t})$. To make this precise, consider the following general linear-factor environment.

Let $\{f_{k,t+1}\}_{k=1}^{K}$ be conditionally mean-zero factors, and suppose that dividends satisfy (5.4):

$$D_{i,t+1} = \mu_{i,t+1} + \sum_{k=1}^{K} g_k(Z_{i,t}) f_{k,t+1},$$

with $\mu_{i,t+1}$ conditionally independent of $\{f_{k,t+1}\}_k$ and $Z_{i,t}$ $\mathcal{F}_t$–measurable. Assume that there exists an SDF of the form

$$m_{t+1} = a_t + \sum_{k=1}^{K} b_{k,t} f_{k,t+1} + \varepsilon_{t+1}, \tag{5.5}$$

where $a_t > 0$, $\{b_{k,t}\}_k$ are $\mathcal{F}_t$–measurable, and ε_{t+1} is a mean-zero residual orthogonal to all traded payoffs. Under complete markets, the SDF is unique and coincides with the representative agent's or planner's marginal rate of substitution. Under incomplete markets, the SDF is not unique in general, but in our CARA-normal environment a particular affine representation is pinned down by equilibrium.

Under a local linear approximation in which higher-order terms in the factors are negligible,

the Euler equation $\mathbb{E}_t[m_{t+1}R_{i,t+1}] = 1$ implies the following characterization.

Proposition 4 (Priced characteristics in a linear factor structure)**.** *Suppose that dividends are given by (5.4) and that the SDF admits the representation (5.5), with $\{f_{k,t+1}\}_k$ mutually uncorrelated conditional on $\mathcal{F}_t$. Then, up to a first-order approximation in the conditional variances of the factors,*

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \sum_{k=1}^{K} \lambda_{k,t} \frac{g_k(Z_{i,t})}{V_{i,t}}, \tag{5.6}$$

where

$$\lambda_{k,t} = -\frac{b_{k,t}\,\mathrm{Var}_t(f_{k,t+1})}{\mathbb{E}_t[m_{t+1}]}, \qquad k = 1, \ldots, K, \tag{5.7}$$

are the factor prices of risk. In particular, for each factor k, the priced return exposure is the value-normalized object $g_k(Z_{i,t})/V_{i,t}$: two assets with identical $g_k(Z_{i,t})/V_{i,t}$ for all k have the same expected excess return in this approximation, regardless of how $g_k(\cdot)$ depends on primitives such as $(X_{i,t}, V_{i,t}, \ldots)$.

The proof is immediate from substituting (5.4) and (5.5) into the Euler equation, using conditional mutual uncorrelatedness of the factors and discarding higher-order covariance terms. The result holds irrespective of whether markets are complete or incomplete. Completeness affects the determination of the coefficients $\{b_{k,t}\}_k$ and thus the prices of risk $\{\lambda_{k,t}\}_k$, but not the functional form of the dollar exposure coefficients $g_k(Z_{i,t})$, which are pinned down by technology and the way shocks enter cash flows.

In the transition-tax example (5.2) with a single factor $f_{1,t+1} = \hat{\tau}_{t+1}$, Proposition 4 identifies $g_1(Z_{i,t})$ with $-X_{i,t}$ and thus the priced return exposure with $-X_{i,t}/V_{i,t}$. Once we allow for more general specifications in which $g_k(Z_{i,t})$ depends on levels, changes, technology, or policy allocations, there is no reason to expect that a single statistic, such as $X_{i,t}/V_{i,t}$, will be sufficient to summarize transition exposure across models. The next section illustrates how alternative, economically motivated specifications of $g_k(\cdot)$ generate premia on emission levels, intensity, and changes through the value-normalized exposures $g_k(Z_{i,t})/V_{i,t}$, and how simple economic restrictions discipline admissible functional forms.

6 Illustrative Exposure Specifications

The incomplete-markets equilibrium in Section 4 relied on a specific cash-flow structure: dividends load linearly on emission levels. But real climate exposure is richer. Policy shocks don't just scale

with current emissions; they depend on recent changes, distances from regulatory benchmarks, and technology choices that reshape the sensitivity of cash flows to climate risk. This section shows how to determine which characteristics price climate risk by deriving exposure functions from policy and technology mechanisms. The three specifications below—transition losses proportional to levels (the Section 4 baseline), exposures depending on levels and changes jointly (motivated by trajectory-based regulation), and path-based exposures under net-zero commitments—demonstrate why the scale-invariance and spanning restrictions tested in Section 8.4 fail in economically natural settings: policy design itself generates exposure functions that depend on characteristics beyond intensity.

6.1 Exposure proportional to emission levels

I start with the simplest case: cash-flow exposure proportional to emission levels. For example, if a per-ton carbon price increases by \$10 and a firm emits 100 kilotons, its compliance cost rises by \$1 million. A reduced-form representation of this channel is $\theta_{i,t} = \eta X_{i,t}$, linear in emissions. This specification is natural when policy operates through per-unit instruments–carbon taxes, permit obligations, or output restrictions that scale directly with total emissions.

Asset pricing converts dollar cash-flow exposure into a return premium by dividing by firm value. Under $\theta_{i,t} = \eta X_{i,t}$, the priced exposure is therefore $\eta X_{i,t}/V_{i,t}$, so emission intensity emerges mechanically as a sufficient statistic for pricing. The key point, however, is that this sufficiency is not a generic implication of equilibrium pricing: it holds here only because the primitive cash-flow channel is linear in levels and does not depend on other scale objects. Once exposure depends on additional characteristics such as changes in emissions, compliance constraints, or distance from a regulatory path, two firms with the same $X_{i,t}/V_{i,t}$ can have different $g(Z_{i,t})/V_{i,t}$, and intensity no longer spans priced exposure.

Single-factor setup. To formalize this, consider a single climate factor ζ_{t+1} with $\mathbb{E}_t[\zeta_{t+1}] = 0$ and write firm i's dividend as

$$D_{i,t+1} = \tilde{D}_{i,t+1} + \theta_{i,t}\,\zeta_{t+1}, \tag{6.1}$$

where $\tilde{D}_{i,t+1}$ is orthogonal to ζ_{t+1}. The cash-flow exposure $\theta_{i,t}$ depends on firm characteristics $Z_{i,t}$ through

$$\theta_{i,t} = g(Z_{i,t}), \tag{6.2}$$

where $g : \mathcal{Z} \to \mathbb{R}$ maps characteristics–emissions, firm value, technology shifters, policy allocations–into dollar sensitivities. In the Section 4 benchmark, $Z_{i,t} = X_{i,t}$ and $g(z) = \eta z$, so $\theta_{i,t} = \eta X_{i,t}$.

Under the log-linear approximation in Proposition 4, expected excess returns satisfy

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \lambda_t \frac{g(Z_{i,t})}{V_{i,t}}, \tag{6.3}$$

where λ_t is the equilibrium price of factor risk. The priced object is the value-normalized exposure $g(Z_{i,t})/V_{i,t}$.

From mechanisms to empirical predictors. The functional form of $g(\cdot)$ is disciplined by economic structure: it reflects how climate shocks enter cash flows. If $g(Z_{i,t}) = \eta X_{i,t}$, equation (6.3) delivers

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \lambda_t \eta \frac{X_{i,t}}{V_{i,t}},$$

so emission intensity becomes the return exposure even though the primitive cash-flow channel scales with levels. But if policy operates through adjustment costs, compliance constraints, or net-zero benchmarks, $g(Z_{i,t})$ naturally depends on additional characteristics–emission changes $\Delta X_{i,t}$, compliance constraints, and distances from regulatory paths or net-zero benchmarks. The priced return exposure remains $g(Z_{i,t})/V_{i,t}$, but this object need not collapse to intensity.

The theory thus disciplines measurement: an emission-based predictor is admissible only if it can be derived from a coherent cash-flow mechanism, and it is empirically sufficient if it spans the return variation associated with $g(Z_{i,t})/V_{i,t}$. Intensity is sufficient only when dollar exposures satisfy the homogeneity and scale-invariance conditions in Proposition 3–conditions that fail once $g(\cdot)$ depends on characteristics beyond $(X_{i,t}, V_{i,t})$.

Multi-factor extension. With multiple climate shocks $\boldsymbol{\zeta}_{t+1} = (\zeta_{1,t+1}, \ldots, \zeta_{K,t+1})^\top$, dividends become

$$D_{i,t+1} = \tilde{D}_{i,t+1} + \sum_{k=1}^{K} \theta_{i,t}^{(k)} \zeta_{k,t+1}, \qquad \theta_{i,t}^{(k)} = g_k(Z_{i,t}),$$

and expected returns inherit the structure

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \sum_{k=1}^{K} \lambda_t^{(k)} \frac{g_k(Z_{i,t})}{V_{i,t}}. \tag{6.4}$$

Each factor $\zeta_{k,t+1}$ is priced through its exposure function $g_k(Z_{i,t})/V_{i,t}$. Physical damage shocks, carbon-price innovations, and technology shocks can each generate distinct exposure functions, and there is no reason to expect a single characteristic such as intensity to span all of them.

Invariance restrictions on exposure functions. Not every function $g(\cdot)$ is economically coherent. Dollar exposures must satisfy basic invariance requirements. Splitting a firm into two identical copies with half the emissions and half the value should preserve total exposure (additivity). Holding technology fixed, higher emissions should not reduce exposure to a per-unit tax or damage shock (monotonicity). These restrictions ensure that exposure functions correspond to actual economic mechanisms rather than arbitrary functional forms.

Under these disciplines, emission levels, intensities, changes, and SCC-weighted measures are all admissible–each can be derived from a specific policy or technology channel. But they are empirically sufficient only if they span the priced exposure $g_k(Z_{i,t})/V_{i,t}$. When cash flows depend on characteristics beyond those captured by a given metric, that metric fails the spanning tests in Section 8.4.

6.2 Transition losses depending on levels and changes

Policy tightening doesn't just penalize current emissions–it targets firms moving in the wrong direction. A coal plant ramping up capacity faces different regulatory risk than one phasing down, even if both have similar emission levels today. If enforcement responds to trajectories rather than stocks, cash-flow exposure depends on both $X_{i,t}$ and $\Delta X_{i,t}$, and intensity alone cannot span the priced risk.

To see this, consider a policy shock that arrives when regulators perceive insufficient progress toward transition goals. Firms increasing emissions signal resistance to decarbonization and face steeper penalties when enforcement tightens. Formally, let transition losses depend on both the current emission stock and recent changes:

$$\Gamma_{i,t} = \phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}, \tag{6.5}$$

where $\Delta X_{i,t} = X_{i,t} - X_{i,t-1}$ and $\phi_1, \phi_2 > 0$. The level $X_{i,t}$ captures baseline exposure; the change $\Delta X_{i,t}$ captures trajectory risk.

Policy-tightening shocks and transition losses. Let $J_{t+1} \in \{0, 1\}$ denote a binary tightening event with $\mathbb{E}_t[J_{t+1}] = p_t$, and define the innovation

$$\zeta_{t+1} = J_{t+1} - p_t.$$

The shock J_{t+1} captures unanticipated regulatory acceleration–stricter enforcement, faster phase-outs, or abrupt carbon-price increases. Firm i's dividend becomes

$$D_{i,t+1} = \bar{D}_{i,t+1} - \Gamma_{i,t} J_{t+1}, \tag{6.6}$$

where $\bar{D}_{i,t+1}$ is the dividend absent tightening and $\Gamma_{i,t}$ is the dollar loss when tightening occurs. This loss reflects stranded assets, accelerated abatement costs, or compliance penalties that hit when policy shifts earlier than expected.

Substituting (6.5) into (6.6) and using $\zeta_{t+1} = J_{t+1} - p_t$ yields

$$D_{i,t+1} = \bar{D}_{i,t+1} - (\phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}) p_t - (\phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}) \zeta_{t+1}.$$

The cash-flow exposure to the transition factor is

$$g_\zeta(Z_{i,t}) = -(\phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}),$$

where $Z_{i,t}$ includes both levels and changes.

Equilibrium pricing with trajectory dependence. Under the log-linear approximation in Proposition 4, expected excess returns satisfy

$$\mathbb{E}_t[R_{i,t+1} - R_{f,t}] = \lambda_{\zeta,t} \frac{\phi_1 X_{i,t} + \phi_2 \Delta X_{i,t}}{V_{i,t}} + \text{other factor terms}. \tag{6.7}$$

Both levels and changes enter through the value-normalized exposure $(\phi_1 X_{i,t} + \phi_2 \Delta X_{i,t})/V_{i,t}$. When $\phi_2 = 0$, the model collapses to the Section 6.1 baseline where exposure scales with levels alone. When $\phi_2 > 0$, firms ramping up emissions bear additional transition risk that cannot be captured by $X_{i,t}/V_{i,t}$ alone.

This has direct implications for intensity-based screens. Two firms with identical emission intensity can have different priced exposure if one is increasing emissions ($\Delta X_{i,t} > 0$) while the

other is cutting ($\Delta X_{i,t} < 0$). The sufficiency conditions in Proposition 3 fail because the exposure function depends on an additional characteristic–recent emission changes–that is not summarized by the intensity ratio $X_{i,t}/V_{i,t}$.

Empirical implications. Equation (6.7) suggests that the appropriate empirical specification includes both $X_{i,t}/V_{i,t}$ and $\Delta X_{i,t}/V_{i,t}$ as predictors. A regression of returns on these characteristics is a reduced-form test of whether trajectory-based exposure is priced. The coefficients need not recover (ϕ_1, ϕ_2) separately, as they combine technological sensitivities with the equilibrium price of risk $\lambda_{\zeta,t}$, but the joint specification is the natural empirical analogue of the mechanism in (6.5).

The incomplete-markets structure, heterogeneous climate concern, and SDF decomposition from Section 4 remain unchanged. What changes is the exposure function $g_\zeta(Z_{i,t})$, which now depends on both stocks and flows. This illustrates the general point: different policy designs generate different priced characteristics, and intensity suffices only under the knife-edge restriction that exposure depends solely on $(X_{i,t}, V_{i,t})$.

6.3 Net-zero objectives and path-based exposure

Net-zero commitments change the regulatory game. A firm emitting 150 kilotons with a 100-kiloton benchmark faces different transition risk than a firm at 75 kilotons with a 50-kiloton benchmark–even though both are 50% above their targets. What matters is not just the intensity ratio but the absolute distance from a mandated path, and that distance varies with scale even when $X_{i,t}/V_{i,t}$ is fixed.

This matters because net-zero benchmarks are typically allocated using level-based rules–sectoral budgets, capacity shares, historical baselines–not intensity targets. Two firms with identical emission intensity can have vastly different compliance trajectories if one is large and distant from its benchmark while the other is small and close. If enforcement tightens or phase-out schedules accelerate, exposure scales with the gap $(X_{i,t} - X_{i,t}^{NZ})$, not just with current emissions or their ratio to firm value. Because $X_{i,t}^{NZ}$ is not proportional to $V_{i,t}$ under level-based allocations, the normalized gap $(X_{i,t} - X_{i,t}^{NZ})/V_{i,t}$ varies across firms even when $X_{i,t}/V_{i,t}$ is held fixed. Intensity alone therefore cannot span priced exposure in (6.9).

Path-based exposure under net-zero regulation. Let $X_{i,t}^{NZ}$ denote firm i's net-zero-consistent benchmark at date t, constructed from sectoral pathways, regulatory allocations, or firm commit-

ments. This benchmark is predetermined at t and reflects policy design or historical negotiations rather than equilibrium outcomes; it may be measured with error in practice. Let ζ_{t+1} denote net-zero transition news: an acceleration of phase-out schedules, stricter enforcement, or increases in marginal abatement costs along the transition path.

Cash-flow exposure to ζ_{t+1} reflects two channels. First, per-unit losses that scale with current emissions $X_{i,t}$ (permit obligations, tax exposure). Second, repricing and compliance risk tied to the distance from the benchmark, $(X_{i,t} - X_{i,t}^{NZ})$. Formally, let

$$g_\zeta(Z_{i,t}) = \vartheta_1 X_{i,t} + \vartheta_2 (X_{i,t} - X_{i,t}^{NZ}), \tag{6.8}$$

where $\vartheta_1, \vartheta_2 \geq 0$. The level $X_{i,t}$ captures baseline exposure, while the gap $(X_{i,t} - X_{i,t}^{NZ})$ captures path-deviation risk under net-zero regulation.[5]

Equilibrium pricing with path dependence. Under the log-linear approximation in Proposition 4, expected excess returns satisfy

$$\mathbb{E}_t[R_{i,t+1} - R_{f,t}] = \lambda_{\zeta,t} \frac{g_\zeta(Z_{i,t})}{V_{i,t}} + \text{other factor terms.} \tag{6.9}$$

The priced exposure is the value-normalized function

$$\frac{g_\zeta(Z_{i,t})}{V_{i,t}} = \vartheta_1 \frac{X_{i,t}}{V_{i,t}} + \vartheta_2 \frac{X_{i,t} - X_{i,t}^{NZ}}{V_{i,t}}.$$

Intensity $X_{i,t}/V_{i,t}$ alone cannot span this exposure. Even firms with identical intensity can have sharply different path deviations $(X_{i,t} - X_{i,t}^{NZ})/V_{i,t}$ because benchmarks are allocated by levels or capacity shares, not intensity ratios. A large firm at 100% above its benchmark carries fundamentally different compliance risk than a small firm at 100% above its benchmark—the absolute emission gap that must be closed differs by scale.

The sufficiency conditions in Proposition 3 require that priced exposure depend only on $(X_{i,t}, V_{i,t})$ and satisfy homogeneity and scale invariance. Net-zero exposures violate these conditions in two ways. First, they depend on an additional predetermined characteristic, $X_{i,t}^{NZ}$, that reflects policy design rather than current firm decisions. Second, because benchmarks are level-based, the gap $(X_{i,t} - X_{i,t}^{NZ})/V_{i,t}$ varies with scale even when intensity is fixed. Consequently, intensity-based

[5] One can additionally allow adjustment-speed penalties proportional to $\Delta X_{i,t}$ as in Section 6.2.

screens systematically misclassify firms when compliance risk is path-dependent.

Empirical implications. Equation (6.9) suggests that empirical specifications motivated by path-based regulation include not only $X_{i,t}/V_{i,t}$ but also the normalized deviation $(X_{i,t}-X_{i,t}^{NZ})/V_{i,t}$. A spanning test that includes only intensity implicitly imposes $\vartheta_2 = 0$, that is, that transition risk is insensitive to distance from the benchmark. This restriction is economically strong in settings where compliance is assessed relative to net-zero pathways rather than current flow emissions alone.

Net-zero objectives thus provide a concrete illustration of the paper's central message: the appropriate emission-based predictor is determined by policy design, not by intensity a priori. When regulation operates through path constraints, the priced characteristic is the exposure function $g_\zeta(Z_{i,t})/V_{i,t}$, which depends on levels, gaps, and changes jointly. Intensity sufficiency is a knife-edge special case that holds only when transition risk ignores path deviations and adjustment dynamics—conditions unlikely to hold in environments where cumulative emissions or trajectory compliance drives enforcement.

7 Jevons-type Efficiency Effects

This section shows how endogenous energy efficiency can undermine emission intensity as a sufficient statistic for priced transition exposure. The goal is not to build a full production model, but to isolate a simple channel through which a technological choice shifts energy demand and therefore reshapes the priced characteristics in the linear factor structure. A Jevons-type rebound can arise in which higher efficiency expands energy use and emissions, so that firms that appear greener by intensity measures can have higher equilibrium premia by increasing their effective exposure to priced climate or policy shocks. While the section is purely illustrative, the mechanism has empirical relevance, as discussed at the end of the section.

Efficiency choice and rebound. At date t, firm i uses energy $E_{i,t}$ and chooses an efficiency parameter $a_{i,t} > 0$ to produce an energy service

$$S_{i,t} = a_{i,t}E_{i,t}, \qquad X_{i,t} = \gamma E_{i,t},\ \gamma > 0,$$

so that $a_{i,t}$ increases service per unit of energy (and per unit of emissions). Let P_t^E denote the real energy price and suppose operating profits admit the reduced form

$$\Pi_{i,t} = \pi_i(S_{i,t}) - P_t^E E_{i,t} - \Psi_i(a_{i,t}),$$

with π_i increasing and concave and Ψ_i the private cost of efficiency. For given $(P_t^E, a_{i,t})$, optimal energy demand $E_{i,t}^*(a_{i,t})$ satisfies

$$a_{i,t}\pi_i'(a_{i,t}E_{i,t}^*) = P_t^E.$$

A Jevons-type rebound region is characterized by

$$\frac{\partial E_{i,t}^*}{\partial a_{i,t}} \geq 0,$$

so that higher efficiency does not reduce, and may increase, energy use and therefore emissions $X_{i,t} = \gamma E_{i,t}^*(a_{i,t})$. Because emissions are induced by the equilibrium energy choice, any shock sensitivity expressed in the primitives $(E_{i,t}^*, a_{i,t}, Z_{i,t})$ is equivalently a function of $(X_{i,t}, a_{i,t}, Z_{i,t})$, and its comparative statics with respect to efficiency need not be negative in the rebound region. Empirically, this implies that intensity can improve mechanically while priced exposure rises if efficiency shifts the firm into the rebound region, so intensity-based greenness need not be informative about the relevant $g(Z)$.

Pricing with efficiency-dependent exposures. Let ζ_{t+1} denote an aggregate climate or policy shock and let $\varepsilon_{E,t+1}$ denote an energy-price (or energy-demand) shock. Write dividends as

$$D_{i,t+1} = \tilde{D}_{i,t+1} + g_\zeta(X_{i,t}, a_{i,t}, Z_{i,t})\,\zeta_{t+1} + g_E(X_{i,t}, a_{i,t}, Z_{i,t})\,\varepsilon_{E,t+1}, \tag{7.1}$$

where $Z_{i,t}$ collects other characteristics and $\tilde{D}_{i,t+1}$ is orthogonal (conditionally) to $(\zeta_{t+1}, \varepsilon_{E,t+1})$. The Jevons mechanism matters because efficiency affects energy demand and therefore both emissions and shock sensitivity, so the exposure coefficients in (7.1) generically depend on $(X_{i,t}, a_{i,t})$ jointly.

Under the same linear-factor pricing logic as in Proposition 4 and Section 4, expected excess returns admit the approximation

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \lambda_t^{(\zeta)}\, g_\zeta(X_{i,t}, a_{i,t}, Z_{i,t}) + \lambda_t^{(E)}\, g_E(X_{i,t}, a_{i,t}, Z_{i,t}), \tag{7.2}$$

with equilibrium prices of risk $(\lambda_t^{(\zeta)}, \lambda_t^{(E)})$ determined by preferences, risk sharing, and the aggregate exposure of cash flows and (where relevant) unspanned damages to $(\zeta_{t+1}, \varepsilon_{E,t+1})$. Equation (7.2) makes clear that the priced characteristics are the exposure functions $g_\zeta(\cdot)$ and $g_E(\cdot)$, which depend on efficiency and emissions jointly. Once $a_{i,t}$ shifts energy use and hence shock sensitivity, there is no generic reason for premia to collapse to a function of $X_{i,t}/V_{i,t}$ (or to $X_{i,t}$ alone).

A simple example with reversed comparative statics

To see how a rebound can raise premia, consider

$$\pi_i(S) = pS - \frac{c}{2}S^2, \qquad p > 0,\ c > 0,$$

and ignore $\Psi_i(a)$ for the moment. The optimal energy choice is

$$E_{i,t}^*(a_{i,t}) = \frac{a_{i,t}p - P_t^E}{ca_{i,t}^2} \quad \text{for } a_{i,t}p > P_t^E,$$

so that

$$\frac{\partial E_{i,t}^*}{\partial a_{i,t}} = \frac{2P_t^E - a_{i,t}p}{ca_{i,t}^3}.$$

Hence, for $a_{i,t} \leq 2P_t^E/p$, higher efficiency increases energy use and emissions. Suppose the climate shock enters dividends through emission levels,

$$g_\zeta(X_{i,t}, a_{i,t}, Z_{i,t}) = \eta X_{i,t} = \eta\gamma E_{i,t}^*(a_{i,t}), \qquad g_E \equiv 0,$$

and the equilibrium price of climate risk satisfies $\lambda_t^{(\zeta)} > 0$. Then (7.2) reduces to

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \lambda_t^{(\zeta)}\eta\gamma E_{i,t}^*(a_{i,t}),$$

and therefore, in the rebound region,

$$\frac{\partial}{\partial a_{i,t}}\left(\mathbb{E}_t[R_{i,t+1}] - R_{f,t}\right) = \lambda_t^{(\zeta)}\eta\gamma\frac{\partial E_{i,t}^*}{\partial a_{i,t}} \geq 0,$$

with strict inequality for $a_{i,t} < 2P_t^E/p$. Thus, higher efficiency can increase equilibrium premia by raising emissions and hence exposure to the priced climate factor. At the same time, the efficiency increase improves intensity mechanically by raising service per unit of energy, so intensity-based

rankings can move in the opposite direction from level-based priced exposure in rebound regions.

The sign reversal has direct implications for ESG ratings and sustainable investment. Intensity-based screens can systematically favor firms in rebound regions with higher transition-risk exposure. Bolton et al. (2025) document that firms with higher rates of green innovation do not necessarily exhibit lower absolute emissions, consistent with rebound effects operating in practice.

Relation to the planner wedge and to intensity sufficiency. In the complete-markets planner benchmark, Jevons-type responses are fully internalized in the optimal paths for $(a^*_{i,t}, E^*_{i,t})$ and therefore affect the levels of optimal emissions and the social cost of carbon. They do not, however, alter the sustainable-discount-rate representation itself. The planner's implementation wedge remains a deterministic adjustment expressed per unit of firm value, proportional to $X^*_{i,t}/V^*_{i,t}$, along the optimal allocation. By contrast, in equilibrium asset pricing with heterogeneous agents and incomplete spanning, Jevons operates by reshaping the exposure functions $g_\zeta(X_{i,t}, a_{i,t}, Z_{i,t})$ and $g_E(X_{i,t}, a_{i,t}, Z_{i,t})$ that enter (7.2). Because these exposures depend jointly on emissions and efficiency and need not satisfy the homogeneity and scale-invariance restrictions characterized in Proposition 3, emission intensity $X_{i,t}/V_{i,t}$ typically fails to be a unique sufficient statistic for expected returns. The Jevons mechanism therefore reinforces the paper's central distinction between planner implementation, where intensity wedges are natural, and equilibrium pricing, where priced characteristics are model-dependent exposure functions, and it implies that efficiency improvements can be associated with higher required returns for some firms.

8 Welfare and Empirical Implications

The preceding analysis has implications for welfare and empirical measurement of transition risk that deserve separate discussion. The central distinction is between (i) intensity-based implementation wedges that arise in a complete-markets planner benchmark and summarize the deterministic gap between financial and sustainable discount rates, and (ii) equilibrium premia in incomplete markets, which are stochastic risk prices determined by priced exposures $g(Z_{i,t})$ and need not reduce to emission intensity. The section first records this wedge-premium contrast, then briefly discusses the welfare consequences of market incompleteness, and finally takes the knife-edge implications of Proposition 3 to the data in Section 8.4.

8.1 Planner wedges versus equilibrium premia

Complete markets with optimal policy. In the complete-markets planner benchmark, a representative-agent social planner chooses allocations to maximize discounted welfare, and the associated first-order conditions define a sequence of social costs of carbon $\{S_t\}_t$. Standard results imply that the planner allocation can be decentralized with a Pigouvian tax $\tau_t = S_t$ per unit of emissions. An equivalent representation uses sustainable discount rates: firm-specific discount rates $r^x_{i,t}$ that reproduce the value of tax-adjusted cash flows when applied to untaxed cash flows. To a first-order approximation, the resulting wedge takes the intensity form

$$r^x_{i,t} \approx r_t + (S_t - \tau_t)\frac{X^*_{i,t}}{V^*_{i,t}}, \tag{8.1}$$

so emission intensity $X^*_{i,t}/V^*_{i,t}$ is the natural sufficient statistic for the planner's deterministic implementation wedge. When $\tau_t = S_t$, sustainable and financial discount rates coincide along the planner allocation. Relatedly, Chittaro et al. (2025) show that in a frictionless representative-agent economy a carbon tax can be replicated by a return schedule that increases linearly in firms' emission intensity.

Incomplete markets and equilibrium pricing. In the incomplete-markets environment of Section 4, climate damages are unspanned and households differ in climate concern, so the stochastic discount factor that prices traded assets generally does not coincide with the planner's intertemporal marginal rate of substitution for any single normative weighting of damages. Consequently, there is no simple mapping from equilibrium carbon premia to the planner's SCC. More generally, equilibrium expected excess returns are proportional to firms' value-normalized transition exposures. For each priced transition component k,

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \lambda_{k,t}\,\frac{g_k(Z_{i,t})}{V_{i,t}}, \tag{8.2}$$

where $g_k(Z_{i,t})$ summarizes how the underlying transition shock enters firm i's cash flows and $V_{i,t}$ is firm value. In incomplete markets, the endogenous prices of risk $\{\lambda_{k,t}\}_k$ generally reflect both compensation for unhedgeable transition risk and preference-driven pricing effects (e.g., heterogeneous climate concern), so observed carbon premia are second-best equilibrium objects rather than direct measures of welfare wedges. In the CARA-normal specialization developed in Section 6, the exposure function is proportional to emission levels, $g(Z_{i,t}) \propto X_{i,t}$, so (8.2) reduces to a levels

representation of the form

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \lambda_{X,t} X_{i,t}, \tag{8.3}$$

with the endogenous price of climate risk $\lambda_{X,t}$ reflecting the extent of unhedgeable climate risk and the cross-section of risk tolerances and climate concerns. The key implication for interpretation is that intensity arises naturally as a planner implementation object, whereas equilibrium premia price $g(Z_{i,t})/V_{i,t}$ and collapse to intensity only under the knife-edge conditions in Proposition 3.

8.2 Welfare implications of market incompleteness

This subsection does not infer the social cost of carbon from observed carbon premia. Its purpose is to clarify the welfare content of the wedge-premium misalignment highlighted above. With incomplete markets, climate risk is imperfectly shared and heterogeneous internalization distorts the allocation, so the competitive equilibrium generally differs from the complete-markets planner benchmark even when expected returns exhibit well-defined carbon premia. The relevant welfare object is therefore an allocation measure–the consumption reduction that makes the planner indifferent between the first-best allocation and the incomplete-markets allocation–not an equilibrium price that can be read off from returns. In particular, this welfare loss need not be monotone in the magnitude of equilibrium premia.

Formally, let $W(\cdot)$ denote the planner welfare functional and let (C^{FB}, X^{FB}) and (C^{IM}, X^{IM}) denote the allocations under the complete-markets planner benchmark and the incomplete-markets equilibrium, respectively. Define the consumption-equivalent welfare loss Δ as the scalar reduction in consumption that makes the planner indifferent between (C^{FB}, X^{FB}) and (C^{IM}, X^{IM}). The exact expression and a decomposition into (i) risk-sharing and (ii) internalization components are provided in Internet Appendix OA.C. The key implication for the empirical analysis is conceptual: carbon premia are second-best equilibrium prices of exposure to transition shocks, whereas welfare comparisons require an explicit planner benchmark and cannot be inferred from premia alone.

8.3 Empirical implications and discussion

The incomplete-markets structure developed above has testable implications for the cross-section of expected returns and for the interpretation of existing evidence on brown premium and greenium. This section sketches the main empirical predictions and contrasts them with those implied by the complete-markets sustainable-discount-rate benchmark.

More generally, the CARA-normal incomplete-markets environment with generic dividend exposure

$$D_{i,t+1} = \tilde{D}_{i,t+1} + \sum_{k=1}^{K} g_k(Z_{i,t})\zeta_{k,t+1},$$

implies the conditional linear pricing relation

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \sum_{k=1}^{K} \lambda_t^{(k)} g_k(Z_{i,t}), \tag{8.4}$$

where $Z_{i,t}$ collects firm characteristics and state variables, and $\lambda_t^{(k)}$ are the prices of the K climate and policy risk factors. Equation (8.4) maps into the generic cross-sectional regression

$$R_{i,t+1} - R_{f,t} = \alpha_t + \sum_{k=1}^{K} \hat{b}_t^{(k)} g_k(Z_{i,t}) + \varepsilon_{i,t+1}, \tag{8.5}$$

with $\hat{b}_t^{(k)}$ estimating $\lambda_t^{(k)}$ up to sampling error and potential misspecification. The choice of the functions g_k constitutes the empirical analogue of specifying how climate and policy shocks enter technology and damages. Emission levels $X_{i,t}$, emission intensities $X_{i,t}/V_{i,t}$, changes $\Delta X_{i,t}$, and interacted terms (for example, $X_{i,t}$ multiplied by industry indicators) all arise as special cases of $g_k(Z_{i,t})$.

By contrast, in the complete-markets, representative-agent benchmark underlying the sustainable-discount-rate approach, the planner's problem yields a social cost of carbon S_t and sustainable discount rates

$$r_{i,t}^{x} = r_t + (S_t - \tau_t)\frac{X_{i,t}}{V_{i,t}},$$

where τ_t is the carbon tax and $V_{i,t}$ is firm value. In that environment, and abstracting from traditional risk factors, the relevant cross-sectional wedge in expected returns is proportional to the intensity $X_{i,t}/V_{i,t}$ rather than to the level $X_{i,t}$ itself. A natural empirical implementation of the sustainable-discount-rate logic therefore takes the form

$$R_{i,t+1} - R_{f,t} = \alpha_t + \gamma_t \frac{X_{i,t}}{V_{i,t}} + \varepsilon_{i,t+1}, \tag{8.6}$$

or, more generally, regressions in which $X_{i,t}/V_{i,t}$ is included as a characteristic or factor and the tax-adjusted component $(S_t - \tau_t)$ is interpreted through the magnitude of γ_t. In a strict interpretation of this benchmark, once standard risk factors and $X_{i,t}/V_{i,t}$ are controlled for, emission levels $X_{i,t}$

should not carry independent explanatory power for expected excess returns.

The incomplete-markets model thus yields a different set of empirical predictions. In one tractable specification, expected returns load on emission levels as in (8.9); in the general specification (8.5), the priced emission-based objects are the functions $g_k(Z_{i,t})$ that govern dividend exposure to climate and policy shocks, which need not coincide with $X_{i,t}/V_{i,t}$. This implies that, in empirical work, it is natural to estimate specifications that include both $X_{i,t}$ and $X_{i,t}/V_{i,t}$ (and potentially other emission-based transformations) and to interpret the significance and signs of their coefficients through the lens of incomplete risk sharing and heterogeneous climate concern. A finding that emission levels carry a robust premium controlling for intensity is particularly natural in the incomplete-markets framework and difficult to reconcile with a strict sustainable-discount-rate interpretation in which $X_{i,t}/V_{i,t}$ is the uniquely relevant object.

As a special case, in the static baseline model with heterogeneous CARA investors, unspanned climate damages, and dividend exposure

$$D_i = \mu_i + \eta X_i \zeta,$$

Proposition 2 implies

$$\mathbb{E}[R_i] - R_f = \lambda \eta X_i, \tag{8.7}$$

with

$$\lambda = \frac{\sigma_\zeta^2 \bar{X}}{\sum_h \frac{1}{\alpha_h}} \Big(\eta - \kappa \sum_h \varphi_h\Big),$$

and the decomposition $\lambda = \lambda^{\text{fin}} + \lambda^{\text{green}}$ given earlier. In a dynamic version, the corresponding conditional relation is

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \lambda_{X,t} X_{i,t}, \tag{8.8}$$

where $\lambda_{X,t}$ is an endogenous function of the cross-section of (α_h, φ_h) and of aggregate emissions $\bar{X}_t$. Equations (8.7)-(8.8) lead directly to the panel regression specification

$$R_{i,t+1} - R_{f,t} = \alpha_t + \beta_{X,t} X_{i,t} + \varepsilon_{i,t+1}, \tag{8.9}$$

possibly augmented with standard risk factors and controls, where under the maintained assump-

tions the conditional expectation of the slope satisfies

$$\mathbb{E}_t[\beta_{X,t}] \approx \lambda_{X,t}.$$

In particular, in the absence of climate concern ($\varphi_h = 0$ for all h) the model predicts a positive emission-level slope $\beta_{X,t} > 0$ reflecting a purely financial brown premium for bearing unhedgeable climate risk. When some investors are green ($\sum_h \varphi_h > 0$), the true greenium component λ^{green} partially offsets this financial premium and reduces $\beta_{X,t}$; for sufficiently strong aggregate climate concern, $\beta_{X,t}$ can even become negative.

8.4 Empirical evidence on sufficiency

This subsection takes the knife-edge implications of Proposition 3 to the data by testing the sufficiency restrictions that must hold if emission intensity $X_{i,t}/V_{i,t}$ is a uniquely sufficient statistic for priced transition exposure. The contribution of the empirical analysis is therefore not to estimate the magnitude of a carbon premium per se, but to evaluate two cross-sectional restrictions implied by the theory. First, *scale invariance*: if priced transition exposure depends only on $X_{i,t}/V_{i,t}$, then the premium per unit of intensity must be invariant to firm scale, so interactions between intensity and size should be redundant. Second, *spanning*: if intensity is sufficient, it must span emission levels in return regressions, so that conditional on $X_{i,t}/V_{i,t}$, emission levels $X_{i,t}$ carry no incremental explanatory power for expected returns. I implement a direct test of scale invariance by allowing the intensity premium to vary across size bins and testing the implied cross-bin equality restriction, with robustness to alternative proxies for firm scale. I then conduct asymmetric spanning tests that distinguish whether intensity subsumes levels or, conversely, whether levels subsume intensity. Finally, I report within-size-bin spanning tests that address the concern that the spanning results are driven by residual slope heterogeneity associated with firm scale. Together, these tests discipline the empirical interpretation of intensity-based carbon premia by directly evaluating the sufficiency restrictions implied by Proposition 3.

The tests use an international panel of 13,373 firms representing 92 countries and 438 Trucost industries over 2005-2022. I collect panel data on stock returns $R_{i,t+1}$, the risk-free rate $R_{f,t}$, firm-level Scope 1 emissions $X_{i,t}$, firm value $V_{i,t}$, and standard firm characteristics $W_{i,t}$ (such as market capitalization, market-to-book ratio, book leverage, profitability, investment, momentum, age, PPE, and volatility). The emissions data come from S&P Trucost while all other information

is from Capital IQ. Following Pedersen (2025), I define emission intensity as $X_{i,t}/V_{i,t}$, where V is proxied by the book value of equity, and I define the percentage change in emissions as $\Delta X_{i,t} = (X_{i,t} - X_{i,t-1})/X_{i,t-1}$. To mitigate the influence of outliers I winsorize intensity and emissions growth at the 2.5% level. Table 1 reports summary statistics for the key empirical variables.

A direct test of scale invariance. A central implication of the knife-edge intensity-sufficiency conditions is a scale-invariance restriction: firms with the same intensity but different scale must have the same priced transition exposure. If this restriction holds, the premium per unit of intensity should not vary systematically with firm scale. To test it, each period I sort firms by size into five equal-count bins (baseline: book value of equity) and estimate the pooled regression

$$R_{i,t+1} - R_{f,t} = \alpha_t + \sum_{b=1}^{B} b_b \, D_{b,i,t}\Big(\frac{X_{i,t}}{V_{i,t}}\Big) + \gamma_t^\top W_{i,t} + \varepsilon_{i,t+1},$$

where $D_{b,i,t}$ indicates membership in size bin b. The specification includes $W_{i,t}$, industry×time fixed effects, and country fixed effects. Standard errors are double-clustered by industry and time. The null implied by Proposition 3 is $H_0 : b_1 = \cdots = b_B$, which I test using a Wald statistic based on the clustered covariance matrix.

Table 2 reports the results. Panel A shows that the estimated intensity premium varies materially across size bins. Panel B reports formal Wald tests of the cross-bin equality restrictions and shows that the rejection is not specific to a particular definition of scale. The same restrictions are rejected (or remain economically close to rejection) when bins are formed using market capitalization, sales, or assets instead of book equity. The key implication for Proposition 3 is the robust rejection of cross-bin equality, providing direct evidence against the knife-edge hypothesis that emission intensity summarizes priced transition exposure in a scale-invariant manner.

Spanning tests: does intensity subsume levels? I complement the scale-invariance tests with asymmetric spanning tests that evaluate whether intensity spans levels, as required under intensity sufficiency, or whether the opposite spanning relation is more consistent with the data. Guided by the incomplete-markets benchmark and the general factor framework, I estimate pooled regressions of excess returns on emission levels and emission intensity (along with controls),

$$R_{i,t+1} - R_{f,t} = \alpha_t + b_{X,t} X_{i,t} + \gamma_t^\top W_{i,t} + \varepsilon_{i,t+1}, \tag{8.18}$$

$$R_{i,t+1} - R_{f,t} = \alpha_t + b_{I,t}\frac{X_{i,t}}{V_{i,t}} + \gamma_t^\top W_{i,t} + \varepsilon_{i,t+1}, \tag{8.19}$$

and

$$R_{i,t+1} - R_{f,t} = \alpha_t + b_{X,t}X_{i,t} + b_{I,t}\frac{X_{i,t}}{V_{i,t}} + \gamma_t^\top W_{i,t} + \varepsilon_{i,t+1}. \tag{8.20}$$

Equations (8.18)-(8.20) are interpreted as *spanning restrictions*, not as a symmetric "horse race." If emission intensity $X_{i,t}/V_{i,t}$ is sufficient in the sense of Definition 4.1, then it must span the return variation associated with the underlying exposure: conditional on $X_{i,t}/V_{i,t}$, emission levels $X_{i,t}$ should have no incremental explanatory power ($H_0 : b_{X,t} = 0$ in (8.20)). Conversely, if priced exposure contains a scale component, then levels can span intensity, so that conditional on $X_{i,t}$, the ratio $X_{i,t}/V_{i,t}$ adds little information ($H_0 : b_{I,t} = 0$). I report Wald tests of both restrictions based on the clustered covariance matrix.

Table 3 provides evidence consistent with the latter asymmetry. Emission levels are positively and statistically significantly related to both one-month- and twelve-month-ahead returns when entered on their own, and they remain positive and highly significant when intensity is included. By contrast, conditional on emission levels, the incremental coefficient on intensity is economically small and statistically indistinguishable from zero, and the explanatory power of the specification is essentially unchanged. In the language of spanning, these estimates suggest that levels span intensity in this reduced-form relation, whereas intensity does not span levels, which is inconsistent with intensity sufficiency.

These restrictions are not only statistically sharp but economically consequential. In the joint specification in Table 3, the coefficient of log(Scope 1) measures the return effect of emission scale holding intensity fixed. A one-standard-deviation increase in log(Scope 1) predicts a 0.35% increase in next-month expected returns (about 4.2% annualized). At the twelve-month horizon, the corresponding effect remains economically meaningful. A one-standard-deviation increase in emissions predicts a 0.11% increase in average monthly returns over the subsequent year (about 1.33% annualized). Thus, firms with similar $X_{i,t}/V_{i,t}$ but different emission scales earn systematically different expected returns, contradicting the scale-invariance and spanning restrictions implied by Proposition 3.

Within-size-bin spanning. A remaining concern is that the spanning asymmetry in Table 3 reflects slope heterogeneity associated with firm scale. If the price of transition exposure varies with size, then emission levels may appear to add explanatory power in pooled regressions even when

intensity is the relevant exposure measure within scale classes. To address this concern directly, I re-estimate the joint levels-intensity specification (8.20) separately within size quintiles, using the same controls and fixed effects. By construction, firm scale is approximately fixed within each regression, so this design isolates whether emission levels add return-relevant information beyond intensity even among firms of comparable size.

Table 4 reports the results. Emission levels remain positively and significantly related to expected returns within every size bin, while the coefficient of intensity is economically small and statistically weak. This within-bin evidence strengthens the spanning conclusion in two ways. First, it rules out the interpretation that the pooled spanning results are driven primarily by residual size differences or generic size-related slope heterogeneity. Second, it directly targets the key implication of intensity sufficiency: if $X_{i,t}/V_{i,t}$ were a uniquely sufficient statistic for priced transition exposure, then conditioning on intensity should eliminate systematic return differences associated with emission scale even within narrow scale classes. The data reject this implication.

Concerns that the joint specification merely reflects mechanical multicollinearity are mitigated by the correlation structure of the data. Emission intensity and log emissions are only moderately correlated, and intensity is close to orthogonal to firm size. These magnitudes indicate that levels, intensity, and scale capture distinct dimensions of firm heterogeneity, so that the asymmetric spanning and within-bin evidence is economically meaningful.

The asymmetric spanning result is also robust to alternative intensity definitions using assets and market capitalization. In contrast, scaling emissions by sales yields a sign reversal. Sales-based intensity enters with a negative coefficient, which becomes stronger and more significant once emission levels are included, even after controlling for ROE and industry fixed effects. This behavior indicates that sales-based intensity loads on a different priced dimension, reflecting pricing power or pass-through embedded in revenues, rather than serving as a sufficient statistic for exposure per unit of firm value.

Finally, the rejection of intensity sufficiency in this section does not rely on long-horizon predictability or on the magnitude of estimated premia. The objects being tested are cross-sectional restrictions implied by Proposition 3, scale invariance and spanning, which must hold at any horizon if emission intensity is a uniquely sufficient statistic for priced transition exposure. Horizon-dependent attenuation affects the interpretation of the time-varying price of risk, but it does not affect the validity of these sufficiency tests.

Changes in emissions (supplementary). To connect more directly to the transition-risk channel in Section 6, I also consider a specification that allows both the level and the recent change in emissions to enter the exposure function. Specifically, I estimate

$$R_{i,t+1} - R_{f,t} = \alpha_t + b_{X,t} X_{i,t} + b_{\Delta X,t} \Delta X_{i,t} + \gamma_t^\top W_{i,t} + \varepsilon_{i,t+1}, \tag{8.21}$$

and report the results in Internet Appendix OA.E. Because $\Delta X_{i,t}$ is inherently more sensitive to timing and transitory noise than $X_{i,t}$, I view these estimates as supplementary. The appendix results indicate that emission changes can be associated with return predictability at short horizons, while the level effect is more stable across specifications and horizons.

Overall, the evidence in Tables 2, 3, and 4 directly rejects the two knife-edge restrictions implied by Proposition 3. The scale-invariance tests in Table 2 reject the homogeneity condition required for emission intensity to have a common pricing slope across firm scale. The spanning tests in Table 3 and the within-bin results in Table 4 sharpen this conclusion: conditional on emission levels, intensity adds little incremental explanatory power, whereas emission levels remain strongly priced conditional on intensity both in the pooled sample and within narrow scale classes. These findings are difficult to reconcile with a strict sustainable-discount-rate interpretation in which intensity is the unique sufficient statistic for priced exposure, but they are consistent with equilibrium environments in which the return-relevant exposure $g(Z_{i,t})/V_{i,t}$ contains a scale component that is not captured by $X_{i,t}/V_{i,t}$.

9 Conclusion

This paper characterizes the conditions under which emission intensity summarizes climate risk premia. I show that intensity is sufficient only when exposure functions satisfy restrictive scale-invariance and homogeneity conditions—restrictions that hold in complete-markets planner implementations but fail under incomplete markets with unspanned climate damages. The analysis clarifies that intensity-based sustainable discount rates serve as implementation tools for first-best allocations, while equilibrium risk premia under incomplete markets depend on exposure functions $g(Z_{i,t})$ that may load on emission levels, changes, technology choices, and path deviations from regulatory benchmarks. Empirical tests reject the sufficiency restrictions, establishing that the debate in the literature reflects distinct economic questions—optimal policy design versus equilibrium asset pricing—rather than competing measurement approaches.

A key contribution of the dynamic analysis is the identification of a technological sign reversal. Through a Jevons-type rebound effect, I demonstrate that firm-level efficiency improvements can endogenously increase absolute emissions and, consequently, equilibrium risk premia. This result cautions against the use of emission intensity as a singular proxy for climate risk exposure, even when it moves in the direction conventionally associated with greenness.

Beyond metric choice, the framework provides a micro-foundation for the green and brown classifications central to the climate-finance literature (e.g., Pastor et al. (2021)). Rather than treating these types as exogenous labels, the analysis endogenizes them as equilibrium exposure functions shaped by technology and fundamentals. In this sense, a firm's climate-risk profile is a dynamic object: changes in efficiency, production structure, or policy exposure can alter priced risk even when conventional ESG metrics improve.

The framework provides a disciplined approach to empirical work. Emission-level specifications are not arbitrary alternatives to intensity but emerge naturally when climate shocks enter cash flows through per-unit exposures and markets are incomplete. The empirical strategy—testing scale-invariance and spanning restrictions—offers a template for evaluating when any climate-risk metric is theoretically coherent. Future research should structurally estimate the exposure functions $g_k(Z_{i,t})$ implied by specific policy and technology channels, quantify the relative importance of transition risk versus preference-driven greenium components, and extend the framework to incorporate dynamic investment and abatement decisions.

The framework has immediate implications for sustainable finance practice. Regulatory taxonomies and disclosure standards often emphasize emission intensity, implicitly assuming the knife-edge conditions under which intensity is sufficient for pricing. My analysis establishes a distinction between policy implementation and risk measurement: intensity-based metrics naturally implement normative objectives under complete markets, while equilibrium risk assessment under incomplete markets requires exposure functions that depend on absolute emissions, technology choices, and firm-specific transition vulnerabilities. This distinction explains why portfolios satisfying intensity-based regulatory criteria can nonetheless carry substantial climate transition risk.

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Tables

Table 1: **Summary statistics**

The sample includes listed companies from 92 countries over 2005-2022. *Returns* are monthly stock returns (in percent). Scope 1 emission levels (X) are in metric tons of CO_2 equivalent. Book value of equity (V) is in millions of US dollars. Emission intensity (X/V) is the ratio of emissions to book value of equity. Scope 1 growth rate is the year-over-year percentage change in emissions. Statistics reported are mean, median, and standard deviation across all observations.

Variable / Statistic	Mean	Median	St. dev.
Returns (in %)	0.890	0.348	12.134
Scope 1 levels (X)	1,696,164	26,141	11,700,000
Book value of equity (V)	333,602	4,967	12,900,000
Intensity (X/V)	1.233	0.048	3.619
Log(Scope 1)	10.356	10.174	2.850
Scope 1 growth rate	0.086	0.023	0.432
Number of firms: 13,373			
Number of countries: 92			
Number of industries: 438			

Table 2: **Predicting Stock Returns with Emission Intensity and Tests of Scale Invariance**

The sample is at the firm-month level and includes listed companies from 92 countries over 2005-2022. The dependent variable is monthly stock returns ($Returns_{i,t+1}$). Each period, firms are sorted by book value of equity into quintiles ($Size\ bin$). Emission intensity (X/V) is the ratio of Scope 1 emissions to book value. All regressions include industry × time fixed effects and country fixed effects. Standard errors (in parentheses) are double-clustered by industry and time. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively. Panel B reports F-statistics and p-values from Wald tests of the null hypothesis that coefficients on intensity are equal across all five size bins, based on the specification in Panel A, column (2). The baseline uses book value of equity for scaling; alternative specifications use market capitalization, sales, or total assets.

	(1)	(2)
VARIABLES	Returns	
Panel A: Predicting stock returns with emission intensity		
Intensity X/V	0.032**	0.042**
	(0.014)	(0.016)
Size bin 2	-0.017	0.123
	(0.182)	(0.147)
Size bin 3	-0.130	0.048
	(0.193)	(0.142)
Size bin 4	-0.181	0.006
	(0.208)	(0.130)
Size bin 5	-0.339	-0.021
	(0.217)	(0.109)
Size bin 2 × Intensity X/V	-0.029**	-0.038***
	(0.010)	(0.011)
Size bin 3 × Intensity X/V	-0.017	-0.027*
	(0.014)	(0.015)
Size bin 4 × Intensity X/V	-0.014	-0.026
	(0.014)	(0.015)
Size bin 5 × Intensity X/V	0.002	-0.022
	(0.023)	(0.026)
Log(Size)		-0.193***
		(0.051)
Log(M/B)		-0.260***
		(0.071)
Book leverage		-0.115
		(0.216)
Momentum		0.486***
		(0.154)
Investment/Assets		-0.383
		(0.879)
Log(PPE)		0.082**
		(0.030)
Volatility		2.000
		(3.117)
ROE		0.870***
		(0.267)
Age		0.840
		(0.896)
Constant	0.990***	1.932***
	(0.158)	(0.421)
Industry × time FE	Yes	Yes
Country FE	Yes	Yes
Observations	1,005,645	1,005,645
R-squared	0.235	0.236

Panel B: Tests of scale invariance (based on Panel A, col. 2)

Specification	F-stat	p-value
Baseline	5.09	0.007
Market-cap scaling	4.15	0.017
Sales scaling	2.59	0.076
Assets scaling	2.91	0.050

Table 3: **Predicting Stock Returns with Emission Levels and Intensity**

The sample is at the firm-month level and includes listed companies from 92 countries over 2005-2022. The dependent variable is monthly stock returns ($Returns_{i,t+1}$) over the next month (columns 1-3) or next 12 months (columns 4-6). Emission intensity (X/V) is the ratio of Scope 1 emissions to book value of equity. All regressions include industry × time fixed effects and country fixed effects. Standard errors (in parentheses) are double-clustered by industry and time. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
	Returns					
	1 month ahead			12 months ahead		
Intensity X/V	0.019**		-0.001	0.006		-0.001
	(0.009)		(0.011)	(0.010)		(0.011)
Log(Scope1 Emissions)		0.123***	0.123***		0.039***	0.039***
		(0.019)	(0.024)		(0.009)	(0.012)
Log(Size)	-0.200***	-0.269***	-0.269***	-0.139***	-0.162***	-0.162***
	(0.049)	(0.052)	(0.052)	(0.037)	(0.037)	(0.037)
Log(M/B)	-0.250***	-0.193**	-0.193**	-0.159*	-0.141*	-0.141*
	(0.072)	(0.068)	(0.072)	(0.076)	(0.072)	(0.077)
Book leverage	-0.060	-0.189	-0.189	-0.117	-0.156	-0.156
	(0.240)	(0.241)	(0.242)	(0.174)	(0.174)	(0.176)
Momentum	0.472***	0.458**	0.458**	-0.394*	-0.397*	-0.397*
	(0.160)	(0.161)	(0.161)	(0.190)	(0.190)	(0.190)
Investment/Assets	-0.408	-0.241	-0.241	0.099	0.153	0.153
	(0.865)	(0.877)	(0.891)	(0.738)	(0.752)	(0.757)
Log(PPE)	0.078**	0.041	0.041	0.056**	0.045**	0.045**
	(0.028)	(0.026)	(0.027)	(0.020)	(0.020)	(0.021)
Volatility	1.995	1.995	1.995	0.408	0.403	0.403
	(3.151)	(3.147)	(3.150)	(1.525)	(1.520)	(1.524)
ROE	0.842***	0.770**	0.770**	0.548	0.527	0.527
	(0.259)	(0.268)	(0.270)	(0.313)	(0.315)	(0.317)
Age	0.881	0.626	0.626	-0.442	-0.513	-0.513
	(0.935)	(0.959)	(0.972)	(0.940)	(0.971)	(0.990)
Constant	2.055***	1.738***	1.738***	1.791***	1.689***	1.689***
	(0.445)	(0.469)	(0.472)	(0.292)	(0.296)	(0.299)
Industry×time FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,006,908	1,006,023	1,006,023	862,872	862,251	862,251
R-squared	0.231	0.231	0.231	0.247	0.247	0.247

Robust standard errors in parentheses

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 4: **Within-size-bin Spanning Tests**

The sample is at the firm-month level and includes listed companies from 92 countries over 2005-2022. The dependent variable is monthly stock returns ($Returns_{i,t+1}$). The table reports spanning regressions estimated separately within firm size quintiles, where size is measured by book value of equity. Each column estimates the regression in Table 3 using firms in the corresponding size bin. All regressions include the same set of controls as those in Table 3 as well as industry × time fixed effects and country fixed effects. Standard errors (in parentheses) are double-clustered by industry and time. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Firm size quintile (book equity)				
	(1) Smallest	(2)	(3)	(4)	(5) Largest
log(Scope 1 Emissions)	0.122**	0.181***	0.142***	0.105***	0.125***
	(0.055)	(0.030)	(0.025)	(0.027)	(0.029)
Intensity X/V	0.032	−0.034**	−0.020*	−0.017	0.015
	(0.020)	(0.016)	(0.011)	(0.017)	(0.024)
Controls	Yes	Yes	Yes	Yes	Yes
Industry × time FE	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes
Observations	181,064	182,439	187,002	188,801	189,660
R^2	0.324	0.355	0.347	0.333	0.352

Appendix

A.1 Proofs and derivations

Proof of Proposition 1. By (4.3), for any h that holds a positive amount of stock i we have

$$\frac{\mu_i - V_i R_f}{\eta X_i} = \alpha_h \sigma_\zeta^2 \delta_h.$$

Since this ratio must be equalized across all stocks i and all households h with interior holdings, it must equal a common scalar λ:

$$\frac{\mu_i - V_i R_f}{\eta X_i} = \lambda \quad \forall i,$$

which yields (4.4). Substituting back gives $\alpha_h \sigma_\zeta^2 \delta_h = \lambda$, which yields (4.5). □

Proof of Proposition 2. In the static CARA-normal environment, aggregate consumption net of damages is $C_{h,1} - \phi_h \Gamma_1 = C_{h,1} - \phi_h \kappa \bar{X}\, \zeta_1$, so each household's SDF is exponential-affine in the climate shock,

$$m_{h,1} = \beta \frac{u_c^h(C_{h,1} - \phi_h \Gamma_1)}{u_c^h(C_{h,0} - \phi_h \Gamma_0)} \propto \exp\big(\alpha_h \phi_h \kappa \bar{X}\, \zeta_1\big),$$

where the proportionality absorbs β and all terms measurable at date 0. With mean-variance/CARA aggregation, the equilibrium (aggregate) SDF that prices all traded assets is the risk-tolerance-weighted average of individual SDF loadings, hence

$$m_1 \propto \exp\big(B\, \zeta_1\big), \qquad B = \kappa \bar{X} \frac{\sum_{h=1}^H \frac{\phi_h}{\alpha_h}}{\sum_{h=1}^H \frac{1}{\alpha_h}}.$$

Firm i's payoff (dividend) loads on the same shock with exposure ηX_i in the baseline specification, so $D_i = \tilde{D}_i + \eta X_i \zeta_1$ with $\tilde{D}_i \perp \zeta_1$. Using the Euler equation $1 = \mathbb{E}[m_1 R_i]$ and a second-order (mean-variance) expansion yields the standard linear pricing relation

$$\mathbb{E}[R_i] - R_f = -\operatorname{Cov}(R_i, \log m_1) = \sigma_\zeta^2 \,(\eta X_i)\,\Big(1 - B\Big),$$

where we used $\log m_1 = \text{const} + B\zeta_1$ and $\operatorname{Cov}(\zeta_1, \zeta_1) = \sigma_\zeta^2$. Aggregating across firms gives the climate factor price

$$\lambda = \frac{\sigma_\zeta^2 \bar{X}}{\sum_{h=1}^H \frac{1}{\alpha_h}} \Big(\eta - \kappa \sum_{h=1}^H \phi_h\Big),$$

which is (4.8). □

Proof of Proposition 3. Condition (iii) implies that, for any fixed intensity level $u \in \mathbb{R}_+$, all firms with $X_{i,t}/V_{i,t} = u$ share the same exposure $g(Z_{i,t})/V_{i,t}$. Define $\varphi_t(u)$ to be this common value:

$$\varphi_t(u) \equiv \frac{g(Z_{i,t})}{V_{i,t}} \quad \text{for any firm } i \text{ with } \frac{X_{i,t}}{V_{i,t}} = u.$$

Condition (i) ensures that intensities span an open subset of $\mathbb{R}_+$, so that φ_t can be defined on an interval rather than on a finite set. This establishes (4.15).

To obtain (4.16), fix a firm with characteristics $(X_{i,t}, V_{i,t})$ and intensity $u = X_{i,t}/V_{i,t}$. By homogeneity of degree one in $(X_{i,t}, V_{i,t})$, for any $\lambda > 0$,

$$g(Z_{i,t}; \lambda X_{i,t}, \lambda V_{i,t}) = \lambda\, g(Z_{i,t}; X_{i,t}, V_{i,t}).$$

Dividing both sides by $\lambda V_{i,t}$ yields

$$\frac{g(Z_{i,t}; \lambda X_{i,t}, \lambda V_{i,t})}{\lambda V_{i,t}} = \frac{g(Z_{i,t}; X_{i,t}, V_{i,t})}{V_{i,t}}.$$

The left-hand side is the value-normalized exposure of a firm whose intensity remains u (since $(\lambda X_{i,t})/(\lambda V_{i,t}) = u$). By condition (iii), it depends only on u and therefore equals $\varphi_t(u)$, while the right-hand side equals $\varphi_t(u)$ by definition. Hence

$$\frac{g(Z_{i,t})}{V_{i,t}} = \varphi_t\left(\frac{X_{i,t}}{V_{i,t}}\right),$$

which restates (4.15).

Under the additional assumption in the proposition that the mapping is linear in size (equivalently, that $g(Z_{i,t})/V_{i,t}$ is proportional to $X_{i,t}/V_{i,t}$ on the relevant support), there exists a scalar κ_t such that for all firms,

$$\frac{g(Z_{i,t})}{V_{i,t}} = \kappa_t \frac{X_{i,t}}{V_{i,t}}.$$

Multiplying both sides by $V_{i,t}$ yields $g(Z_{i,t}) = \kappa_t X_{i,t}$ and establishes (4.16). Uniqueness in the sense of Definition 4.1 follows because any one-factor representation observationally equivalent to (4.14) must use a characteristic proportional to the priced characteristic $g(Z_{i,t})/V_{i,t}$, hence proportional to $X_{i,t}/V_{i,t}$ under (4.16). □

A Internet Appendix: Robustness to nonlinear damages and multiple climate shocks

This section shows that the main asset-pricing implications of our framework are robust to nonlinear damage functions and to the presence of multiple climate shocks (for example, physical, policy, and technology shocks). In particular, I show that under standard small-shock approximations the pricing kernel remains affine in the climate factors, expected excess returns remain linear in firm-specific exposure coefficients, and the relevant priced characteristics are generic functions $g_k(Z_{i,t})$ of firm characteristics rather than a uniquely determined emission-intensity statistic $X_{i,t}/V_{i,t}$.

A.1 Nonlinear damages in a single climate factor

I first extend the static damage specification to allow for nonlinear dependence on the climate shock. For expositional clarity, I work in a one-period environment. The dynamic case follows by applying the same local arguments period by period.

Let ζ be a scalar climate factor with $\mathbb{E}[\zeta] = 0$ and $\mathrm{Var}(\zeta) = \sigma_\zeta^2 \in (0, \infty)$, and let aggregate emissions be $\bar{X} = \sum_i X_i$ as before. Instead of the linear damage function $\Gamma = \kappa \bar{X} \zeta$, suppose that damages are given by

$$\Gamma = h(\bar{X}, \zeta), \tag{A.1}$$

where $h : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ is twice continuously differentiable and satisfies $h(\bar{X}, 0) = 0$. A quadratic specification, for example,

$$h(\bar{X}, \zeta) = \kappa_1 \bar{X} \zeta + \frac{1}{2} \kappa_2 \bar{X} \zeta^2, \tag{A.2}$$

is included as a special case.

Household h has CARA preferences over climate-adjusted consumption,

$$U^h = \mathbb{E}\Big[u^h\big(C_1^h - \phi_h \Gamma\big)\Big], \qquad u^h(y) = -\exp(-\alpha_h y),$$

with $\alpha_h > 0$ and $\phi_h \geq 0$. Writing

$$Y_h \equiv C_1^h - \phi_h h(\bar{X}, \zeta),$$

and using a second–order Taylor expansion of h around $(\bar{X}, 0)$, we obtain

$$h(\bar{X}, \zeta) = h_\zeta(\bar{X}, 0)\zeta + \frac{1}{2} h_{\zeta\zeta}(\bar{X}, 0)\zeta^2 + o(\zeta^2), \tag{A.3}$$

where h_ζ and $h_{\zeta\zeta}$ denote the first and second partial derivatives with respect to ζ. Substituting (A.3) into Y_h yields

$$Y_h = \tilde{C}_1^h - \phi_h h_\zeta(\bar{X}, 0)\zeta - \frac{1}{2}\phi_h h_{\zeta\zeta}(\bar{X}, 0)\zeta^2 + o(\zeta^2),$$

where $\tilde{C}_1^h$ collects terms independent of ζ. To isolate the stochastic components, define the centered square

$$\tilde{\zeta}^2 \equiv \zeta^2 - \mathbb{E}[\zeta^2],$$

and rewrite Y_h as

$$Y_h = \bar{Y}_h + \beta_h^{(1)}\zeta + \beta_h^{(2)}\tilde{\zeta}^2 + o(\zeta^2), \tag{A.4}$$

for suitable constants $\bar{Y}_h, \beta_h^{(1)}, \beta_h^{(2)}$ that depend on $(\alpha_h, \phi_h, \bar{X})$ and on the moments of ζ. Under the assumption that ζ is conditionally normal and that higher-order terms are negligible, the pair $(\zeta, \tilde{\zeta}^2)$ can be treated as a two-dimensional factor vector with mean zero and finite second moments.

The CARA-normal structure implies that each household's portfolio problem again admits a mean-variance representation in terms of Y_h, now with respect to the joint distribution of $(\zeta, \tilde{\zeta}^2)$. Aggregating across households and solving for asset prices leads to an affine SDF of the form

$$m = \exp\big(-A - B^{(1)}\zeta - B^{(2)}\tilde{\zeta}^2\big), \tag{A.5}$$

with factor loadings $B^{(1)}$ and $B^{(2)}$ determined by the distribution of (α_h, ϕ_h) and by the coefficients $h_\zeta(\bar{X}, 0)$ and $h_{\zeta\zeta}(\bar{X}, 0)$.

Suppose now that firm i's dividend takes the reduced-form specification

$$D_i = \mu_i + g_1(Z_i)\zeta + g_2(Z_i)\tilde{\zeta}^2, \tag{A.6}$$

where Z_i collects firm characteristics and g_1, g_2 are measurable functions. A log-linear approximation to the Euler equation $\mathbb{E}[mR_i] = 1$ then yields

$$\mathbb{E}[R_i] - R_f \approx \lambda^{(1)} g_1(Z_i) + \lambda^{(2)} g_2(Z_i), \tag{A.7}$$

for factor prices of climate risk $\lambda^{(1)}, \lambda^{(2)}$ that are linear functions of $(B^{(1)}, B^{(2)})$ and of the conditional second moments of $(\zeta, \tilde{\zeta}^2)$. Expression (A.7) shows that allowing for quadratic (or more

general smooth) damages in ζ does not alter the linear factor structure: it simply enlarges the set of priced characteristics from a single function $g(Z_i)$ to a vector $(g_1(Z_i), g_2(Z_i))$, corresponding to exposure to the linear and quadratic components of the climate factor. In particular, if the technological channel through which ζ enters cash flows remains proportional to emission levels, as in

$$g_1(Z_i) = \eta_1 X_i, \qquad g_2(Z_i) = \eta_2 X_i,$$

then (A.7) implies that expected excess returns are proportional to X_i with a combined price of risk $\lambda^{(1)}\eta_1 + \lambda^{(2)}\eta_2$, and the qualitative conclusion that emission levels act as the relevant exposure remains unchanged. More generally, the quadratic damage term induces an additional priced characteristic $g_2(Z_i)$ whose functional form is determined by the way ζ^2 enters the technology and damage processes, but there is no mechanism that generically restores emission intensity X_i/V_i as a unique sufficient statistic.

A.2 Multiple climate shocks: physical, policy, and technology factors

I next allow for multiple climate-related shocks. Let $f_{t+1} = (f_{1,t+1}, \ldots, f_{K,t+1})^\top$ be a vector of K mean-zero factors that may represent, for example, a physical damage factor, a policy or carbon-price factor, and a technology factor. As in Section 5, suppose that firm i's dividend can be written as

$$D_{i,t+1} = \mu_{i,t+1} + \sum_{k=1}^{K} g_k(Z_{i,t}) f_{k,t+1}, \tag{A.8}$$

where $Z_{i,t}$ includes $(X_{i,t}, V_{i,t})$ and any other relevant characteristics, and $g_k(\cdot)$ are exposure functions determined by technology, market structure, and the incidence of policy. Climate damages are now given by a general function

$$\Gamma_t = H(\bar{X}_{t-1}, f_t), \tag{A.9}$$

where H is twice continuously differentiable in its arguments. Under CARA preferences $u^h(y) = -\exp(-\alpha_h y)$, household h's climate-adjusted consumption takes the form

$$Y_{h,t+1} = \tilde{C}_{h,t+1} - \phi_h H(\bar{X}_t, f_{t+1}).$$

A second-order Taylor expansion of H around a baseline $(\bar{X}_t, f_{t+1}) = (\bar{X}_t, 0)$ yields

$$H(\bar{X}_t, f_{t+1}) = H(\bar{X}_t, 0) + \sum_{k=1}^{K} H_k(\bar{X}_t, 0) f_{k,t+1} + \frac{1}{2} \sum_{k,\ell=1}^{K} H_{k\ell}(\bar{X}_t, 0) f_{k,t+1} f_{\ell,t+1} + o(\|f_{t+1}\|^2), \quad \text{(A.10)}$$

where H_k and $H_{k\ell}$ denote the first and second partial derivatives of H with respect to its factor arguments.

Under the maintained small-shock assumption, the leading-order contribution of the quadratic terms in (A.10) to the mean-variance approximation can be summarized by a finite set of additional factors constructed from products $f_{k,t+1} f_{\ell,t+1}$, for example their centered versions $f_{k,t+1} f_{\ell,t+1} - \mathbb{E}_t[f_{k,t+1} f_{\ell,t+1}]$. Aggregating across households and solving the mean-variance portfolio problem then yields an affine SDF of the form

$$m_{t+1} = \exp\Big(- A_t - \sum_{k=1}^{K} B_{k,t} f_{k,t+1} - \sum_{k,\ell=1}^{K} C_{k\ell,t}\, \tilde{f}_{k\ell,t+1} \Big), \quad \text{(A.11)}$$

where $\tilde{f}_{k\ell,t+1}$ are centered quadratic factor terms and the coefficients $B_{k,t}, C_{k\ell,t}$ are functions of (α_h, ϕ_h) and of the derivatives $H_k(\bar{X}_t, 0)$ and $H_{k\ell}(\bar{X}_t, 0)$. Substituting (A.8) and (A.11) into the Euler equation and applying the same linearization arguments as in Proposition 4 yields

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \sum_{k=1}^{K} \lambda_{k,t} g_k(Z_{i,t}) + \sum_{k,\ell=1}^{K} \lambda_{k\ell,t} \tilde{g}_{k\ell}(Z_{i,t}), \quad \text{(A.12)}$$

for suitable factor prices of risk $\lambda_{k,t}, \lambda_{k\ell,t}$ and exposure functions $\tilde{g}_{k\ell}(Z_{i,t})$ associated with the quadratic factor terms. Expression (A.12) is the natural multi-factor generalization of (5.6): each physical, policy, or technology factor $f_{k,t+1}$, as well as their relevant quadratic combinations, is associated with a priced characteristic given by its exposure function in the dividend and damage processes.

The key observation is that neither the presence of nonlinear damages nor the presence of multiple factors alters the basic structure of our earlier results: to leading order, expected excess returns are linear in a vector of exposure coefficients $(g_1(Z_{i,t}), \ldots, g_K(Z_{i,t}), \tilde{g}_{k\ell}(Z_{i,t}))$, and these coefficients are model-specific functions of firm characteristics $Z_{i,t}$. In particular, if policy shocks enter cash flows through a term of the form $-\tau_{t+1} X_{i,t}$, the corresponding exposure function is naturally $g_k(Z_{i,t}) = -X_{i,t}$, even in the presence of additional physical or technology factors. Conversely, if physical shocks enter via damages that are nonlinear in $\bar{X}_t$ and $f_{k,t+1}$, the resulting priced

characteristics will typically involve nonlinear functions of emissions and other firm attributes. In neither case is there a general reason for the emission intensity $X_{i,t}/V_{i,t}$ to emerge as a unique sufficient statistic for premia. The qualitative conclusions of Sections 4.6 and 5 are therefore robust to quadratic damages and to multiple climate shocks: the priced objects are exposure functions $g_k(Z_{i,t})$ determined by technology, policy, and preferences, and these need not collapse to a single ratio such as $X_{i,t}/V_{i,t}$.

B CRRA preferences and incomplete markets

This appendix shows that the main pricing structure derived in the CARA-normal benchmark carries over, in local approximation, to an economy with CRRA preferences and incomplete markets. The key difference is that the aggregate price of climate risk is now governed by wealth-weighted (or, more precisely, consumption-weighted) risk tolerance rather than by the sum of inverse CARA coefficients. Throughout, it is useful to keep the distinction between a *price* (dollar) equation and a *return* equation: the primitive exposure is level-based in dividends, while expected returns load on the value-normalized exposure.

B.1 Setup

Time is $t = 0, 1$. Uncertainty is driven by a scalar climate shock ζ with

$$\mathbb{E}[\zeta] = 0, \qquad \mathrm{Var}(\zeta) = \sigma_\zeta^2 \in (0, \infty),$$

and all random variables are jointly normally distributed conditional on date-0 information.

There are N firms indexed by $i = 1, \ldots, N$ and H households indexed by $h = 1, \ldots, H$. Firm i has a fixed level of emissions $X_i \geq 0$ and a date-1 dividend given by

$$D_i = \mu_i + \eta X_i \zeta, \tag{B.1}$$

where $\mu_i \in \mathbb{R}$ is a deterministic component and $\eta > 0$ is a technology parameter common across firms. Aggregate emissions are

$$\bar{X} \equiv \sum_{i=1}^{N} X_i.$$

Climate damages at date 1 are

$$\Gamma(\zeta) = \kappa \bar{X} \zeta, \qquad \kappa > 0. \tag{B.2}$$

The scalar ζ is therefore interpreted as a shock to the per-unit cost or value of emissions, in the reduced-form sense discussed in the main text.

Household h has date-0 wealth $W_0^h > 0$ and trades the N stocks and a risk-free asset with gross return $R_f > 0$. Let V_i denote the date-0 price of stock i, and let θ_i^h denote the number of shares of stock i held by household h. Let b^h denote the holding of the risk-free asset. The budget constraint at $t = 0$ is

$$\sum_{i=1}^{N} V_i \theta_i^h + \frac{1}{R_f} b^h = W_0^h. \tag{B.3}$$

Date–1 consumption of household h is

$$C_1^h(\zeta) = b^h R_f + \sum_{i=1}^{N} \theta_i^h D_i(\zeta). \tag{B.4}$$

Household h experiences disutility from climate damages with intensity $\phi_h \geq 0$ and has CRRA preferences over climate-adjusted consumption:

$$U^h = \mathbb{E}\left[u_h\big(C_1^h - \phi_h \Gamma(\zeta)\big)\right], \qquad u_h(x) = \frac{x^{1-\gamma_h}}{1-\gamma_h}, \quad \gamma_h > 0. \tag{B.5}$$

Preferences are thus heterogeneous in risk aversion γ_h and climate concern ϕ_h. Damages are not tradable, so markets are incomplete: the non-pecuniary climate risk cannot be fully hedged with traded assets.

We focus on equilibria in which the shocks are small relative to the level of climate-adjusted consumption for each household. This allows for a local mean-variance approximation of expected utility around a deterministic reference point.

B.2 Local mean-variance approximation under CRRA

Define climate-adjusted consumption of household h as

$$Y_h(\zeta) \equiv C_1^h(\zeta) - \phi_h \Gamma(\zeta). \tag{B.6}$$

Using (B.1), (B.2), and (B.4), we can write

$$\begin{aligned} Y_h(\zeta) &= b^h R_f + \sum_{i=1}^{N} \theta_i^h \mu_i + \left(\sum_{i=1}^{N} \theta_i^h \eta X_i - \phi_h \kappa \bar{X} \right) \zeta \\ &= A_h + B_h \zeta, \end{aligned} \tag{B.7}$$

where

$$A_h \equiv b^h R_f + \sum_{i=1}^{N} \theta_i^h \mu_i, \qquad B_h \equiv \sum_{i=1}^{N} \theta_i^h \eta X_i - \phi_h \kappa \bar{X}.$$

Let $\bar{Y}_h$ denote a deterministic reference level for Y_h; for the local approximation we treat $\bar{Y}_h$ as fixed and assume that the stochastic component $B_h\zeta$ is small relative to $\bar{Y}_h$. A second-order Taylor expansion of $u_h(\cdot)$ around $\bar{Y}_h$ yields

$$u_h(Y_h) \approx u_h(\bar{Y}_h) + u_h'(\bar{Y}_h)(Y_h - \bar{Y}_h) + \frac{1}{2} u_h''(\bar{Y}_h)(Y_h - \bar{Y}_h)^2.$$

Taking expectations and using $\mathbb{E}[\zeta] = 0$ implies

$$\mathbb{E}[u_h(Y_h)] \approx u_h(\bar{Y}_h) + \frac{1}{2} u_h''(\bar{Y}_h) \mathrm{Var}(Y_h).$$

Since $\mathrm{Var}(Y_h) = B_h^2 \sigma_\zeta^2$, expected utility is approximately

$$\mathbb{E}[u_h(Y_h)] \approx u_h(\bar{Y}_h) + \frac{1}{2} u_h''(\bar{Y}_h) B_h^2 \sigma_\zeta^2.$$

Define the local absolute risk aversion of household h at $\bar{Y}_h$ by

$$\tilde{\alpha}_h \equiv -\frac{u_h''(\bar{Y}_h)}{u_h'(\bar{Y}_h)}. \tag{B.8}$$

For CRRA utility,

$$u_h'(x) = x^{-\gamma_h}, \qquad u_h''(x) = -\gamma_h x^{-\gamma_h - 1},$$

so

$$\tilde{\alpha}_h = \frac{\gamma_h}{\bar{Y}_h}. \tag{B.9}$$

Dividing the approximation for $\mathbb{E}[u_h(Y_h)]$ by $u_h'(\bar{Y}_h) > 0$ (which does not affect the argmax over

portfolios) yields the equivalent objective

$$\mathbb{E}[Y_h] - \frac{\tilde{\alpha}_h}{2}\mathrm{Var}(Y_h) = (A_h - \bar{Y}_h) + \bar{Y}_h - \frac{\tilde{\alpha}_h}{2}B_h^2\sigma_\zeta^2. \tag{B.10}$$

The constant terms are irrelevant for portfolio choice. Using $Y_h = C_1^h - \phi_h\Gamma(\zeta)$ and $\mathbb{E}[\zeta] = 0$, maximizing (B.10) is thus equivalent, in the local approximation, to maximizing

$$\mathbb{E}[C_1^h - \phi_h\Gamma(\zeta)] - \frac{\tilde{\alpha}_h}{2}\mathrm{Var}[C_1^h - \phi_h\Gamma(\zeta)]. \tag{B.11}$$

Expression (B.11) has the same form as the CARA-normal mean-variance objective in the main text, with the constant absolute risk aversion coefficient α_h replaced by the local coefficient $\tilde{\alpha}_h = \gamma_h/\bar{Y}_h$.

B.3 First-order conditions and equilibrium price of climate risk

The mean-variance objective (B.11) implies that household h chooses $\{\theta_i^h\}_{i=1}^N$ to maximize

$$\sum_{i=1}^N \theta_i^h(\mu_i - V_i R_f) - \frac{\tilde{\alpha}_h}{2}\sigma_\zeta^2\delta_h^2,$$

where

$$\delta_h \equiv \sum_{j=1}^N \theta_j^h \eta X_j - \phi_h\kappa\bar{X}. \tag{B.12}$$

The first-order condition with respect to θ_i^h is

$$\mu_i - V_i R_f - \tilde{\alpha}_h\sigma_\zeta^2\delta_h\eta X_i = 0, \qquad i = 1, \ldots, N. \tag{B.13}$$

The left-hand side of (B.13) does not depend on h, so equilibrium requires that the product $\tilde{\alpha}_h\sigma_\zeta^2\delta_h$ coincide across all households h that hold stock i. Hence there exists a scalar λ such that

$$\mu_i - V_i R_f = \lambda\eta X_i, \qquad \delta_h = \frac{\lambda}{\tilde{\alpha}_h\sigma_\zeta^2}. \tag{B.14}$$

Equation (B.14) is the *price* (dollar) risk-premium relation. Dividing by V_i and using $R_i \equiv D_i/V_i$ yields the corresponding *return* representation,

$$\mathbb{E}[R_i] - R_f = \lambda\eta\frac{X_i}{V_i}. \tag{B.15}$$

Thus, as in the CARA benchmark, emission *levels* X_i govern the primitive cash-flow exposure in dividends, while expected *returns* load on the exposure X_i/V_i.

To solve for λ, multiply the definition of δ_h in (B.12) by η and sum over h:

$$\sum_{h=1}^{H}\sum_{j=1}^{N}\theta_j^h\eta^2 X_j - \kappa\bar{X}\eta\sum_{h=1}^{H}\phi_h = \eta\sum_{h=1}^{H}\delta_h. \tag{B.16}$$

Using market clearing $\sum_{h=1}^{H}\theta_j^h = 1$ for each j, the left-hand side becomes

$$\sum_{j=1}^{N}\eta^2 X_j - \kappa\bar{X}\eta\sum_{h=1}^{H}\phi_h = \eta^2\bar{X} - \kappa\bar{X}\eta\sum_{h=1}^{H}\phi_h.$$

The right-hand side of (B.16), using (B.14), is

$$\eta\sum_{h=1}^{H}\delta_h = \eta\sum_{h=1}^{H}\frac{\lambda}{\tilde{\alpha}_h\sigma_\zeta^2} = \frac{\eta\lambda}{\sigma_\zeta^2}\sum_{h=1}^{H}\frac{1}{\tilde{\alpha}_h}.$$

Equating both sides and dividing by η yields

$$\eta\bar{X} - \kappa\bar{X}\sum_{h=1}^{H}\phi_h = \frac{\lambda}{\sigma_\zeta^2}\sum_{h=1}^{H}\frac{1}{\tilde{\alpha}_h}.$$

Solving for λ gives the CRRA counterpart to Proposition 2.

Proposition A.1 (Price of climate risk under CRRA)**.** *Under the assumptions of this appendix, in a competitive equilibrium with CRRA preferences and incomplete markets, the equilibrium price of climate risk is*

$$\lambda = \frac{\sigma_\zeta^2\bar{X}}{\sum_{h=1}^{H}\frac{1}{\tilde{\alpha}_h}}\Big(\eta - \kappa\sum_{h=1}^{H}\phi_h\Big), \qquad \tilde{\alpha}_h = \frac{\gamma_h}{\bar{Y}_h}. \tag{B.17}$$

Accordingly, expected excess returns satisfy the return representation (B.15)*,*

$$\mathbb{E}[R_i] - R_f = \lambda\eta\frac{X_i}{V_i},$$

and the corresponding price (dollar) relation in (B.14) *can be written equivalently as*

$$\mathbb{E}[D_i] - P_i R_f = \lambda\eta X_i.$$

The denominator in (B.17) can be written as

$$\sum_{h=1}^{H} \frac{1}{\tilde{\alpha}_h} = \sum_{h=1}^{H} \frac{\bar{Y}_h}{\gamma_h},$$

which is the sum of local risk tolerances $\bar{Y}_h/\gamma_h$. Thus, relative to the CARA case, the price of climate risk is governed by aggregate risk tolerance rather than by $\sum_h 1/\alpha_h$. The term in parentheses, $(\eta - \kappa \sum_h \phi_h)$, is unchanged: as in the CARA benchmark, greater aggregate climate concern $\sum_h \phi_h$ reduces, and can in principle overturn, the purely financial component of the brown premium.

B.4 Discussion and relation to the CARA benchmark

Proposition A.1 shows that, in a CRRA-incomplete-markets setting, the local linear *price* relation

$$\mathbb{E}[D_i] - V_i R_f = \lambda \eta X_i$$

continues to hold, with a price of climate risk λ given by (B.17). The structure is identical to the CARA-normal benchmark: the primitive cash-flow exposure is proportional to the emission level X_i, the numerator scales with $\sigma_\zeta^2 \bar{X}$, and the aggregate climate concern $\sum_h \phi_h$ enters linearly in the term $(\eta - \kappa \sum_h \phi_h)$. The only change is that the denominator is the sum of local risk tolerances, $\sum_h \bar{Y}_h/\gamma_h$, rather than the sum of inverse CARA coefficients.

Translating this into a statement about *returns* requires dividing by the asset price: from the price relation one obtains

$$\mathbb{E}[R_i] - R_f = \frac{\mathbb{E}[D_i] - V_i R_f}{V_i} = \lambda \eta \frac{X_i}{V_i}.$$

Under the local approximation, the economic interpretation of the incomplete-markets equilibrium is unchanged. Heterogeneous risk aversion and climate concern, combined with unhedgeable climate damages, generate a nontrivial price of climate risk λ and a one-factor representation in which the primitive exposure is level-based in dividends while return premia load on the value-normalized exposure. The CARA and CRRA formulations therefore lead to qualitatively identical asset-pricing implications within the class of small-shock, linear-exposure environments considered in the main text.

C Welfare Losses from Market Incompleteness

This appendix provides the formal derivation of the welfare-loss measure summarized in Section 8.2. The purpose is to make precise how welfare comparisons are conducted in the presence of incomplete markets and heterogeneous climate concern, and to clarify the sense in which equilibrium carbon premia differ from planner implementation wedges. As emphasized in the main text, the analysis here is not intended to map observed carbon premia into a social cost of carbon, but rather to characterize the allocation-level efficiency cost that arises when climate risk is imperfectly shared.

C.1 Planner welfare benchmark

Let $W(\cdot)$ denote the planner welfare functional introduced in Section 3. For any feasible aggregate allocation (C_0, C_1, X_1),

$$W(C_0, C_1, X_1) \;\equiv\; u(C_0) + \beta(u(C_1) - d(X_1)),$$

where $d(X_1)$ denotes aggregate climate damages. Let $(C_0^{FB}, C_1^{FB}, X_1^{FB})$ denote the first-best allocation that solves the planner's problem under complete markets, and define

$$W^{FB} \;\equiv\; W(C_0^{FB}, C_1^{FB}, X_1^{FB}).$$

C.2 Incomplete-markets allocation

Let $(C_0^{IM}, C_1^{IM}, X_1^{IM})$ denote the aggregate allocation induced by the competitive equilibrium in the incomplete-markets economy described in Section 4, with the same technology, damage function, and shock distribution as in the planner benchmark. The planner evaluation of this second-best allocation is

$$W^{IM} \;\equiv\; W(C_0^{IM}, C_1^{IM}, X_1^{IM}).$$

Because climate damages are unspanned and households differ in climate concern, the allocation $(C_0^{IM}, C_1^{IM}, X_1^{IM})$ generally does not satisfy the planner's first-order conditions, even though asset prices in equilibrium reflect well-defined carbon premia.

C.3 Consumption-equivalent welfare loss

The welfare loss from market incompleteness is defined in consumption-equivalent units. Specifically, let $\Delta c_1 \geq 0$ denote the unique scalar (on the relevant domain) that satisfies

$$u(C_0^{FB}) + \beta\big(u(C_1^{FB} - \Delta c_1) - d(X_1^{FB})\big) \;=\; W^{IM}.$$

The quantity Δc_1 measures the reduction in date-1 consumption that makes the planner indifferent between the first-best allocation and the incomplete-markets allocation.

For reporting purposes, we also express the welfare loss as a fraction of first-best date-1 consumption,

$$\Delta W \;\equiv\; \frac{\Delta c_1}{C_1^{FB}}.$$

C.4 Interpretation and decomposition

The welfare loss ΔW captures two distinct channels. First, incomplete markets prevent full insurance against aggregate climate risk, leading to inefficient consumption smoothing relative to the planner benchmark even in the absence of preference heterogeneity. Second, heterogeneity in climate concern implies that the equilibrium stochastic discount factor reflects a wealth-weighted aggregation of private marginal rates of substitution rather than the planner's marginal valuation of climate damages. As a result, equilibrium investment and emission choices do not internalize the social cost of carbon.

Importantly, ΔW is an allocation object and need not be monotone in the magnitude of equilibrium carbon premia. Carbon premia summarize second-best pricing of exposure to climate and policy shocks, whereas ΔW is pinned down by the gap between allocations under complete and incomplete risk sharing evaluated under a common welfare criterion. This distinction underlies the discussion in Section 8.2 and clarifies why observed carbon premia should not be interpreted as sufficient statistics for welfare or for optimal climate policy.

D Infinite-horizon production economy

This section embeds the static incomplete-markets structure into a dynamic production economy. The goal is to show formally that, under standard CARA-normal assumptions, the stochastic discount factor (SDF) admits an exponential-affine representation with a climate-shock loading

that is directly linked to the cross-section of (α_h, ϕ_h), and that the resulting dynamic pricing relation preserves the emission-level structure derived in the static model.

Technology, capital, and emissions Time is discrete, $t = 0, 1, 2, \ldots$. Each firm $i \in \{1, \ldots, N\}$ chooses capital $K_{i,t}$ and an emission scale $q_{i,t}$ at each date t. Output at t is given by

$$Y_{i,t} = F_i(K_{i,t}, q_{i,t}) - \Psi_i(q_{i,t}), \tag{D.1}$$

where $F_i : \mathbb{R}_+^2 \to \mathbb{R}_+$ is strictly concave and twice continuously differentiable, and $\Psi_i : \mathbb{R}_+ \to \mathbb{R}_+$ is convex and captures private costs of emissions (abatement, regulatory compliance, etc.).

Emissions of firm i at date t are

$$X_{i,t} = \chi_i q_{i,t}, \qquad \chi_i > 0, \tag{D.2}$$

so that aggregate emissions are $\bar{X}_t = \sum_{i=1}^N X_{i,t}$. Capital evolves according to

$$K_{i,t+1} = (1 - \delta_i)K_{i,t} + I_{i,t}, \qquad \delta_i \in (0, 1), \tag{D.3}$$

where $I_{i,t}$ is investment at date t. We assume that $(K_{i,0})_{i=1}^N$ are given and that all choice variables are nonnegative.

Climate risk and dividends Let $\{\zeta_t\}_{t \geq 1}$ be an i.i.d. climate or policy shock process with

$$\mathbb{E}[\zeta_{t+1} | \mathcal{F}_t] = 0, \qquad \mathrm{Var}[\zeta_{t+1} | \mathcal{F}_t] = \sigma_\zeta^2 > 0,$$

and assume joint conditional normality of all shocks introduced below. Climate damages at $t + 1$ are specified as

$$\Gamma_{t+1} = \kappa \bar{X}_t \zeta_{t+1}, \qquad \kappa > 0, \tag{D.4}$$

so that damages scale with aggregate emissions at t and the realization of ζ_{t+1}.

Firm i's dividend at date $t + 1$ is given by

$$D_{i,t+1} = Y_{i,t+1} - I_{i,t+1} + \eta X_{i,t} \zeta_{t+1} + \tilde{\varepsilon}_{i,t+1}, \tag{D.5}$$

where $\eta > 0$ is a technological parameter capturing the sensitivity of dividends to the climate shock,

and $\tilde{\varepsilon}_{i,t+1}$ is an idiosyncratic shock independent of ζ_{t+1} conditional on $\mathcal{F}_t$. The specification (D.5) preserves the static structure in which climate exposure is proportional to the *level* of emissions $X_{i,t}$.

Households, incomplete markets, and dynamic mean-variance representation There are H households indexed by h. Household h has time-separable preferences

$$U^h = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u^h(C_{h,t} - \phi_h \Gamma_t)\right], \qquad \beta \in (0,1), \tag{D.6}$$

where $C_{h,t}$ is consumption at date t, $u^h(\cdot)$ is a CARA utility index,

$$u^h(x) = -\exp(-\alpha_h x), \qquad \alpha_h > 0,$$

and $\phi_h \geq 0$ measures the intensity of climate concern (disutility from damages) for household h. Households are heterogeneous in (α_h, ϕ_h).

Households trade a risk-free asset with gross return $R_{f,t}$ and the N equity claims $\{D_{i,t+1}\}_{i=1}^N$. Damages $\{\Gamma_t\}$ are *not* traded. Markets are therefore incomplete: the consumption risk induced by the non-pecuniary damage term cannot be fully hedged using traded assets.

Under CARA preferences and conditional joint normality of all relevant shocks, dynamic portfolio choice admits a mean-variance representation. Let

$$\tilde{C}_{h,t} \equiv C_{h,t} - \phi_h \Gamma_t$$

denote the climate-adjusted consumption of household h. Standard arguments imply that, given the independence of ζ_{t+1} across time and the absence of intertemporal hedging motives under CARA-normality, the household's problem at each date t can be written as choosing current consumption and portfolios to maximize

$$\mathbb{E}_t[\tilde{C}_{h,t+1}] - \frac{\alpha_h}{2}\mathrm{Var}_t(\tilde{C}_{h,t+1}), \tag{D.7}$$

subject to the dynamic budget constraint and given state variables at t. The objective (D.7) is the exact equivalent of (D.6) in the CARA-normal setting and reduces the dynamic problem to a sequence of static mean-variance problems, one for each t.

Aggregate stochastic discount factor and its climate loading Let $\theta_{i,t}^h$ denote the number of shares of firm i held by household h at time t, and let b_t^h denote the holdings of the risk-free asset. The portfolio return of household h from t to $t+1$ is

$$R_{p,t+1}^h = \frac{1}{W_t^h}\left(b_t^h R_{f,t} + \sum_{i=1}^N \theta_{i,t}^h D_{i,t+1}\right),$$

where W_t^h is beginning-of-period wealth. Using (D.4)-(D.5) and market clearing $\sum_h \theta_{i,t}^h = 1$, we can write the innovation to climate-adjusted consumption of household h as

$$\tilde{C}_{h,t+1} - \mathbb{E}_t[\tilde{C}_{h,t+1}] = \Big(\sum_{i=1}^N \theta_{i,t}^h \eta X_{i,t} - \phi_h \kappa \bar{X}_t\Big)\zeta_{t+1} + \tilde{\varepsilon}_{h,t+1}, \tag{D.8}$$

where $\tilde{\varepsilon}_{h,t+1}$ collects idiosyncratic shocks orthogonal to ζ_{t+1}.

From (D.7)-(D.8), the first-order condition for $\theta_{i,t}^h$ implies, as in the static case, that

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \alpha_h \sigma_\zeta^2 \delta_{h,t} \eta X_{i,t}, \qquad \delta_{h,t} \equiv \sum_{j=1}^N \theta_{j,t}^h \eta X_{j,t} - \phi_h \kappa \bar{X}_t. \tag{D.9}$$

Since the left-hand side of (D.9) does not depend on h, equilibrium requires that

$$\frac{\mathbb{E}_t[R_{i,t+1}] - R_{f,t}}{\eta X_{i,t}} = \alpha_h \sigma_\zeta^2 \delta_{h,t}$$

is equal to a common scalar λ_t for all h with interior holdings. Aggregating as in the static case by summing the definition of $\delta_{h,t}$ across h and using market clearing, we obtain the dynamic analogue of Proposition 2:

$$\lambda_t = \frac{\sigma_\zeta^2 \bar{X}_t}{\sum_{h=1}^H \frac{1}{\alpha_h}}\Big(\eta - \kappa \sum_{h=1}^H \phi_h\Big). \tag{D.10}$$

To link this mean-variance representation to an SDF, suppose that the aggregate SDF between t and $t+1$ admits the exponential–affine form

$$m_{t+1} = M_t \exp(-B_t \zeta_{t+1}), \qquad B_t > 0, \tag{D.11}$$

for some $\mathcal{F}_t$-measurable scaling factor M_t, normalized such that $\mathbb{E}_t[m_{t+1}] = 1/(1+R_{f,t})$. The representation (D.11) is exact under CARA-normal preferences when all relevant shocks are linear in ζ_{t+1}; more generally, it is a second-order approximation under the joint conditional normality

assumption.

For an asset with payoff

$$D_{i,t+1} = \bar{D}_{i,t+1} + \eta X_{i,t}\zeta_{t+1},$$

where $\bar{D}_{i,t+1}$ is conditionally independent of ζ_{t+1}, the standard log-linear approximation to the Euler equation

$$V_{i,t} = \mathbb{E}_t[m_{t+1}D_{i,t+1}]$$

implies the approximate risk-premium formula

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx B_t\sigma_\zeta^2\eta X_{i,t}. \tag{D.12}$$

Comparing (D.12) with the mean-variance relation

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} = \lambda_t\eta X_{i,t}$$

and using (D.10) identifies the SDF loading on the climate shock as

$$B_t = \frac{\lambda_t}{\sigma_\zeta^2} = \frac{\bar{X}_t}{\sum_{h=1}^{H}\frac{1}{\alpha_h}}\Big(\eta - \kappa\sum_{h=1}^{H}\phi_h\Big). \tag{D.13}$$

Thus the SDF loading B_t is explicitly determined by the cross-section of risk aversion and climate concern (α_h, ϕ_h), as well as by aggregate emissions $\bar{X}_t$ and the technological parameters η and κ.

The derivation of (D.12)-(D.13) relies on the following conditions: (i) CARA utility, (ii) joint conditional normality of ζ_{t+1} and dividend shocks, (iii) linear exposure of dividends and damages to ζ_{t+1} as in (D.4)-(D.5), and (iv) a second-order (log-linear) expansion of the Euler equation around small conditional variance of ζ_{t+1}. Under these standard CARA-normal linearization arguments, the dynamic pricing relation inherits the same emission-level structure as the static model.

Dynamic asset pricing relation Combining (D.12) and (D.13), we obtain the dynamic counterpart to the static emission-level factor:

$$\mathbb{E}_t[R_{i,t+1}] - R_{f,t} \approx \lambda_{X,t}X_{i,t}, \qquad \lambda_{X,t} \equiv \frac{\sigma_\zeta^2\bar{X}_t\eta}{\sum_{h=1}^{H}\frac{1}{\alpha_h}} - \frac{\sigma_\zeta^2\bar{X}_t\kappa\eta}{\sum_{h=1}^{H}\frac{1}{\alpha_h}}\sum_{h=1}^{H}\phi_h. \tag{D.14}$$

Expression (D.14) is exactly the time-t analogue of the static formula in Proposition 2, with time variation in $\bar{X}_t$ and in the distribution of (α_h, ϕ_h) inducing time variation in the climate price of risk $\lambda_{X,t}$. The exposure is the level of emissions $X_{i,t}$, not intensity $X_{i,t}/V_{i,t}$.

Firms' dynamic investment and emission choices Let $V_{i,t}$ denote the ex-dividend price of firm i at time t. Shareholders value the stream of dividends using the aggregate SDF:

$$P_{i,t} = \mathbb{E}_t \left[\sum_{s=t+1}^{\infty} \left(\prod_{u=t+1}^{s} m_u \right) D_{i,s} \right]. \tag{D.15}$$

Firm i chooses $\{K_{i,t+1}, q_{i,t}\}_{t\geq 0}$ to maximize $V_{i,0}$, taking as given prices and the SDF process $\{m_{t+1}\}$.

The Bellman equation for firm i is

$$V_{i,t} = \max_{K_{i,t+1}, q_{i,t}} \left\{ \mathbb{E}_t[m_{t+1} D_{i,t+1}] + \mathbb{E}_t[m_{t+1} V_{i,t+1}] \right\},$$

subject to (D.1)-(D.3) and (D.5). The first-order condition with respect to $q_{i,t}$ is

$$\mathbb{E}_t \left[m_{t+1} \frac{\partial D_{i,t+1}}{\partial q_{i,t}} \right] + \mathbb{E}_t \left[m_{t+1} \frac{\partial P_{i,t+1}}{\partial q_{i,t}} \right] = 0. \tag{D.16}$$

Under the maintained assumption that $q_{i,t}$ affects dividends only through $Y_{i,t+1}$ and $X_{i,t}$, and does not directly affect continuation values beyond $t+1$, the second term in (D.16) vanishes and we obtain

$$\mathbb{E}_t \left[m_{t+1} \left(\frac{\partial F_i(K_{i,t}, q_{i,t})}{\partial q_{i,t}} - \Psi_i'(q_{i,t}) + \eta \chi_i \zeta_{t+1} \right) \right] = 0. \tag{D.17}$$

The first two terms inside the expectation in (D.17) are the marginal private revenue from $q_{i,t}$; the last term is the marginal contribution of $q_{i,t}$ to climate-related risk, through its effect on emissions $X_{i,t} = \chi_i q_{i,t}$ and hence on the exposure $\eta X_{i,t} \zeta_{t+1}$ in (D.5).

Using the exponential-affine form (D.11) and the normality of ζ_{t+1}, we compute

$$\mathbb{E}_t[m_{t+1}\zeta_{t+1}] = -B_t \sigma_\zeta^2 \mathbb{E}_t[m_{t+1}],$$

so that (D.17) can be written as

$$\mathbb{E}_t[m_{t+1}] \left(\frac{\partial F_i(K_{i,t}, q_{i,t})}{\partial q_{i,t}} - \Psi_i'(q_{i,t}) - \eta \chi_i B_t \sigma_\zeta^2 \right) = 0. \tag{D.18}$$

Dividing by $\mathbb{E}_t[m_{t+1}]$ and using (D.13) yields the condition

$$\frac{\partial F_i(K_{i,t}, q_{i,t})}{\partial q_{i,t}} - \Psi_i'(q_{i,t}) = \eta \chi_i B_t \sigma_\zeta^2 = \eta \chi_i \lambda_t, \tag{D.19}$$

which shows that the marginal private return to emissions is equated to a climate-risk premium term proportional to λ_t. The right-hand side depends on aggregate emissions $\bar{X}_t$ and the cross-section of (α_h, ϕ_h) through (D.10), and involves the emission *level* via the coefficient χ_i. In a complete-markets benchmark with a representative planner who internalizes damages, the corresponding condition would involve a Pigouvian tax or sustainable discount-rate wedge proportional to the planner's SCC and to emission intensity $X_{i,t}/V_{i,t}$. In the incomplete-markets environment above, the actual discount-rate wedge faced by the firm is instead governed by $\lambda_t X_{i,t}$, as in (D.14), and generally does not coincide with the planner's $X_{i,t}/V_{i,t}$-based wedge, even when the same damage function is used in the planner's problem.

Under the CARA-normal linearization assumptions detailed above, the infinite-horizon production economy thus preserves the key asset-pricing result of the static model: expected excess returns load on emission levels, with a climate price of risk that is explicitly determined by the cross-section of risk aversion and climate concern and by aggregate emissions.

D.1 Mapping emission metrics to economic objects

A guide to emission-based carbon-premium metrics

Empirical metric	**Typical specification**	**Economic object**	**When appropriate**
Levels $X_{i,t}$	$R_{i,t+1}$ on $X_{i,t}$ (+ controls)	Equilibrium pricing of stochastic climate risk proportional to emissions	Per-unit policies (taxes, permit obligations) and transition losses proportional to $X_{i,t}$; incomplete markets with unspanned damages (Sec. 4).
Intensity $X_{i,t}/V_{i,t}$	$R_{i,t+1}$ on $X_{i,t}/V_{i,t}$; sustainable discount rates	Sufficient statistic for decentralizing planner's first-best allocation	**Planner implementation:** Discount-rate wedges under complete markets. **Equilibrium pricing:** Only under restrictive homogeneity and scale invariance (Prop. 3).
Revenue-weighted intensity	$R_{i,t+1}$ on $X_{i,t}/\text{Sales}_{i,t}$	Emission efficiency relative to output scale	Benchmarking output efficiency; interpretation depends on pricing and pass-through (see discussion in Sec. 8).
Changes $\Delta X_{i,t}$	$R_{i,t+1}$ on $\Delta X_{i,t}$ or $\Delta \log X_{i,t}$	Dynamic exposure to transition news or policy innovations	Settings where news about emission trajectories signals policy or technology risk (Sec. 6).
SCC-weighted ("burden")	$R_{i,t+1}$ on $\sum_{s\geq 0} \beta^s S_{t+s} X_{i,t}$	Present value of emissions under an assumed SCC path (constructed exposure proxy)	Welfare or policy analysis with maintained SCC schedule; complements level-based exposure (Pástor et al. 2025a).
Scores, betas, factors	$R_{i,t+1}$ on climate scores or factor loadings	Reduced-form proxies for multi-channel exposures $g_k(Z_{i,t})$	Multiple climate shocks; reduced-form empirical strategies; interpretation depends on score/factor construction (Sec. 5).

Notes: This table maps emission-based predictors to the economic objects they measure in our framework. The key distinction is between *equilibrium pricing of stochastic climate risk* (operating through exposure functions $g(Z_{i,t})$ and equilibrium prices of risk) and *deterministic planner wedges* (implementing first-best allocations under complete markets). Emission intensity $X_{i,t}/V_{i,t}$ is the natural statistic for planner implementation but serves as a sufficient statistic for equilibrium returns only under restrictive conditions (Propositions 3–4).

E Additional Tables

Table A.1: Scope 1 Emissions Growth and Future Stock Returns

VARIABLES	(1)	(2)	(3)	(4)
	Stock returns			
	1 month ahead		12 months ahead	
Scope 1 Emissions Growth	0.491***	0.442***	-0.031	-0.056
	(0.057)	(0.054)	(0.034)	(0.034)
Log(Scope 1 Emissions)		0.090***		0.044***
		(0.016)		(0.010)
Log(Size)	-0.212***	-0.259***	-0.140***	-0.163***
	(0.050)	(0.051)	(0.037)	(0.037)
Log(M/B)	-0.258***	-0.218***	-0.157*	-0.138*
	(0.071)	(0.068)	(0.074)	(0.071)
Book leverage	-0.061	-0.166	-0.112	-0.159
	(0.246)	(0.247)	(0.176)	(0.174)
Momentum	0.437**	0.429**	-0.392*	-0.393*
	(0.162)	(0.163)	(0.188)	(0.188)
Investment/Assets	-0.628	-0.497	0.136	0.190
	(0.841)	(0.858)	(0.738)	(0.750)
Log(PPE)	0.088***	0.059**	0.056**	0.042**
	(0.028)	(0.027)	(0.020)	(0.020)
Volatility	1.931	1.932	0.418	0.408
	(3.161)	(3.155)	(1.516)	(1.515)
ROE	0.911***	0.849***	0.546	0.515
	(0.257)	(0.264)	(0.313)	(0.312)
Age	1.128	0.905	-0.436	-0.540
	(0.925)	(0.946)	(0.927)	(0.968)
Constant	2.091***	1.834***	1.800***	1.676***
	(0.449)	(0.464)	(0.297)	(0.294)
Industry×time fixed effects	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes
Observations	1,006,227	1,005,779	862,397	862,073
R-squared	0.232	0.232	0.247	0.247

Robust standard errors in parentheses

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$