Idea Rents and Firm Growth

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Abstract

Which firms drive aggregate productivity growth? A strong form of Gibrat's Law says that firm growth rates are iid, so that their expected contribution is proportional to their sales share. In contrast, we document that firms with high price-earnings ratios tend to see increases in their subsequent earnings relative to sales, which we interpret as rents from ideas. We construct an endogenous growth model with shocks to firm innovation step-sizes and R&D efficiency and calibrate it to match patterns in the data. The model implies that growth would be much lower, even with the same innovative effort, if firms had the same step sizes. The model can be used to infer expected growth contributions of individual firms (such as members of the Magnificent Seven) and individual sectors (such as AI firms). We find that the share of growth coming from the smallest listed firms substantially exceeds their 10% sales share, whereas the largest firms account for less than their 10% sales share.

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1 Introduction

How much do different firms contribute to aggregate productivity growth? A strong form of Gibrat's Law says that growth rates are iid across firms, so that a firm's expected contribution to aggregate growth is proportional to its current sales share. This is a common modeling ingredient, such as in the canonical Klette and Kortum (2004) model.

In contrast, Luttmer (2011) finds that iid firm growth cannot explain the young age of the largest firms in the U.S. He calculates that it should take firms centuries to become so large, whereas in the data the largest firms are less than a century old on average (e.g., Wal-Mart, Apple, or Amazon). Luttmer concludes some firms must have persistently high innovative opportunities.

In this paper we do three key things. First, we document that firms with high stock price-earnings ratios exhibit predictably faster subsequent earnings growth. This provides support for Luttmer's inference that firms must have predictably different growth rates. Second, we construct an endogenous growth model and calibrate it to match the empirical patterns. Third, we glean insights from our calibrated model about the contributions of different size firms to aggregate productivity growth.

On the empirical side, we use publicly-listed Compustat firms in all sectors from 1976 to 2013. In addition to faster earnings growth, higher P/E ratios predict faster growth in earnings *relative* to sales. This is true even when we look at earnings gross of R&D spending, which we can do for manufacturing. We find that research intensity (R&D spending relative to sales) is increasing in P/E ratios and decreasing in firm sales, respectively. The former is consistent with high P/E firms being more innovative. The latter suggests that smaller Compustat firms are more innovative. Intriguingly, we also find that sales growth is negatively related to sales. This contradiction of Gibrat's Law may partly reflect selection into the Compustat sample: for a firm with modest sales

to go public, it may need to exhibit high growth potential.[1]

Motivated by the facts we document, we construct an endogenous growth model in the spirit of Klette and Kortum (2004). Like their model, ours features multiproduct firms engaging in creative destruction through quality innovations. We deviate in three ways that are critical for mimicking the data: (i) stochastic idiosyncratic innovation step sizes across firms; (ii) stochastic R&D efficiency across firms; and (iii) innovation arrival rates that are a convex function of a firm's R&D expenditures alone.[2]

Step size shocks induce heterogeneity in P/E ratios, as price-cost markups are increasing in step sizes (Aghion and Howitt, 1992). Permanent differences in step sizes, however, would not generate heterogeneous P/E ratios in our model along a balanced growth path. Stochastic firm step sizes, in contrast, generate persistent but not permanent differences in P/E ratios across firms. High P/E ratio firms enjoy *temporarily* high step sizes, leading their sales to grow quickly for a while. In our model, high P/E ratios portend faster growth in earnings *relative* to sales. As firms add high-markup products, they increase the average markup in their portfolio of products. On top of this, the firm's future R&D spending will revert back down along with its innovation step size, boosting earnings net of R&D. Meanwhile, we find that differences in firm R&D efficiency are essential for generating realistic dispersion in firm size. The convexity of R&D costs otherwise constrains firms from differing dramatically in the level of their R&D investment.

Klette and Kortum (2004) assumed that R&D costs are increasing in a firm's innovation arrival rate, but decreasing in the firm's stock of products. We drop the stock of products from the firm R&D cost function, akin to Akcigit and Kerr

[1] The size and growth relationship may not all be due to selection. For all firms in the U.S. or for all U.S. manufacturers, including privately-held ones, Haltiwanger, Jarmin and Miranda (2011) and Akcigit and Kerr (2018) also find growth declining with size, where size is measured by employment.

[2] Berlingieri, De Ridder, Lashkari and Rigo (2025) find that bursts of creative destruction are also needed to fit the Pareto tail of firm size. They document that bursts of new product introductions are indeed observed in French manufacturing.

(2018), which makes it harder for larger firms to innovate. Doing so renders the model more tractable as a firm's number of products are no longer a state variable affecting its R&D decision. More importantly, it helps us to explain why R&D/Sales ratios are falling in firm size among U.S. listed firms, and why high sales firms exhibit slower sales growth on average.

We calibrate our model to match nine moments from the Compustat data: earnings growth per effective worker (a proxy for TFP growth); the average stochastic discount rate across firms in Compustat from David, Schmid and Zeke (2022); the share of predictable earnings growth from P/E ratios that is due to earnings/sales growth across firms; the share of predictable earnings/sales growth from P/E ratios that is due to gross-of-R&D earnings/sales growth across firms; total earnings divided by total sales; the gap between the 90th and 10th percentiles of predicted earnings growth based on P/E differences across firms; the gap between the 90th and 10th percentiles of predicted gross-of-R&D earnings growth based on P/E differences across firms; the projection of R&D/Sales on log sales across manufacturing firms; and the 90/50 ratio of sales among Compustat firms within years.

We choose values for nine parameters to match the nine moments: the curvature of the R&D cost function (R&D spending as a function of the innovation arrival rate); the household's discount factor; the drift, persistence, and volatility of firm-specific innovation step sizes and R&D costs; and the correlation between shocks to step sizes and shocks to R&D costs.

Our estimates imply that firms differ persistently in their step sizes and R&D costs. We find that the largest firms, who collectively account for 10% of sales, contribute only 1% of aggregate TFP growth over the subsequent five years. Conversely, the smallest firms accounting for 10% of sales are responsible for 60% of TFP aggregate growth. We find that growth would be 95% lower (0.08% rather than 1.5% per year) if aggregate innovative effort were the same but all firms with the same R&D costs shared the same step size. Thus heterogeneous step sizes are essential for understanding aggregate growth.

Finally, eliminating step size heterogeneity for given R&D costs reduces misallocation from markup dispersion and increases the level of TFP by 9%.

Related Literature

Our approach complements research using patents and R&D. This literature has uncovered rich insights, but patenting and R&D are less common outside manufacturing and software. Even within manufacturing, the propensity to patent may be low and vary across firms—see Argente, Baslandze, Hanley and Moreira (2020) for a comparison of products to patents in U.S. consumer packaged goods, and Aghion, Bergeaud, Boppart, Klenow and Li (2024) for the propensity to patent reported innovations in France. The methodology in the current paper can be applied to all firms with earnings and valuations, regardless of sector, patenting, or R&D reporting.

Like us, Kogan, Papanikolaou, Seru and Stoffman (2017) use stock prices to gauge the value of firm innovations. They examine changes in firm stock market capitalization in response to the announcement of firm patent grants. Our effort is distinct in not relying on patents and in focusing on the expected future growth contributions of firms rather than valuing individual innovations. Our paper relates to studies of Tobin's Q, such as Hall (1993), Hall (2001), and Crouzet and Eberly (2023). Tobin's Q has a backward-looking component (rents from past innovation), whereas the P/E ratio highlights the discounted value of future innovations not reflected in current earnings.

A vast literature explores the empirics of firm dynamics, in particular employment. Recent examples include Decker, Haltiwanger, Jarmin and Miranda (2016), Garcia-Macia, Hsieh and Klenow (2019), Sterk, Sedláček and Pugsley (2021), and Klenow and Li (2021). Our approach emphasizes the need to look at earnings data as well, to get at innovation rates and hence growth contributions. We find that fast-growing firms, also known as gazelles and Luttmer Rockets, are associated with rising earnings relative to sales in addition to growing sales and employment.

Studies using U.S. data that include private firms (not just publicly-listed firms) uncover systematic deviations from Gibrat's Law. Dunne, Roberts and Samuelson (1989), Haltiwanger, Jarmin and Miranda (2011), and Akcigit and Kerr (2018) find that firm or plant growth declines with size in the Census of Manufactures and the Longitudinal Business Database (LBD) from the U.S. Census Bureau, while Neumark, Wall and Zhang (2011) report the same qualitative pattern in the National Establishment Time Series data. Yeh (2025) finds that the variance of firm growth declines with size in the LBD, a deviation from the strong form of Gibrat's Law. We contribute to this area by documenting deviations from Gibrat's Law for publicly-listed U.S. firms and quantifying the economic significance of the deviation in terms of expected growth contributions.

Recent precursors to our quantitative modeling of firm dynamics and growth are Cao, Hyatt, Mukoyama and Sager (2017), Akcigit and Kerr (2018), Acemoglu, Akcigit, Alp, Bloom and Kerr (2018), Akcigit and Ates (2023), and Peters and Walsh (2025). Cao et al. model stochastic R&D efficiency (but not stochastic step sizes) in a model with variety creation and own-product innovation by incumbents. The middle three studies focus on firms who conduct R&D and patent. Peters and Walsh aim to understand the impact of declining population growth on firm dynamism and productivity growth.

Campbell and Shiller (1988) and Cochrane (2011) decompose variation in stock market-wide P/E ratios into time variation in the stochastic discount factor versus growth in earnings (or cash flow). Similarly, Atkeson, Heathcote and Perri (2022) and Atkeson, Heathcote and Perri (2024) stress that fluctuating rents from innovation show up in aggregate P/E ratio movements. They explore the implications of this for time-varying U.S. stock market valuations and the U.S. net international asset position. In contrast to these studies, we investigate the implications of cross-firm dispersion in P/E ratios for firm growth contributions.

The rest of the paper is organized as follows. Section 2 documents empirical

patterns in Compustat. Section 3 builds an endogenous growth model to help interpret these patterns. Section 4 calibrates the model and characterizes firm growth contributions. Section 5 concludes.

2 Empirical patterns

2.1 Data source

Our main variables are stock prices from the Center for Research in Security Prices (CRSP), "street" earnings from the Institutional Brokers' Estimate System (I/B/E/S), and sales from Compustat.[3] Our data cover 15,627 unique firms and 162,075 firm-year observations from 1976 to 2023. While the data covers only publicly-listed firms, these firms account from roughly one-fourth of total private non-farm employment.[4]

We source the security-level share price on the last trading date of each month from CRSP. We multiply each firm's stock price by its number of outstanding shares to arrive at each firm's market capitalization. Our data covers securities listed on the NYSE, NYSE MKT, NASDAQ, and Arca (ETF) exchanges. We extract security-level fiscal year-end earnings per share from I/B/E/S. The earnings per share (EPS) is "street-level" in that I/B/E/S adjusts the accounting EPS for items that the majority of analysts include or exclude from their valuation calculations. We calculate the P/E ratio by dividing firm-level share prices by firm-level earnings per share. We construct firm-level earnings by multiplying EPS by the number of shares outstanding. Sales are from Compustat and for each firm's fiscal year.

We construct the growth of earnings h years ahead g_E^h and growth of the

[3]We use "street" earnings instead of Generally Accepted Accounting Principles (GAAP) earnings because the former is conceptually closer to firm performance and, according to Hillenbrand and McCarthy (2025), tracks in the aggregate GAAP earnings before deducting special items such as natural disasters. See Bradshaw and Sloan (2002) for more discussion.

[4]Employment reported in Compustat for firms in our sample relative to total private nonfarm employment from the BLS.

earnings-to-sales ratio $g^h_{E/S}$ as

$$g^h_E = \frac{1}{h}\log\left(\frac{E_{t+h}}{E_h}\right), \quad \text{and} \quad g^h_{E/S} = \frac{1}{h}\log\left(\frac{E_{t+h}/S_{t+h}}{E_t/S_t}\right).$$

We drop firms with negative earnings when calculating the growth rate of earnings or the earnings-to-sales ratio. Later on, we do the same in the model for purposes of comparing data and model moments.

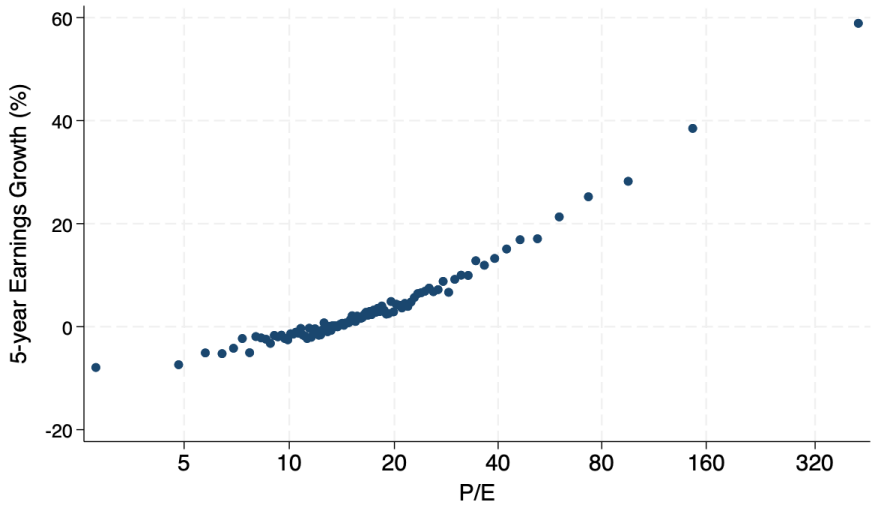
2.2 Motivating facts

Here we document several patterns in the data that will motivate our modeling choices. First, firms with high P/E ratios tend to have high subsequent earnings growth—even relative to sales growth. Second, within manufacturing where R&D data is available, high P/E ratio firms exhibit higher R&D/Sales ratios. Third, firms with higher sales tend to have lower R&D/Sales. subsequent earnings growth.

High P/E firms have higher subsequent growth of earnings and earnings/sales It is well known that P/E ratio varies significantly across firms (see discussion in Appendix Section A). This dispersion does not appear to be simply noise. Figure 1 plots the 5-year ahead earnings growth against current year P/E as a binned scatterplot. A high P/E ratio today predicts high subsequent earnings growth.

Markups (or profitability per $ of sale) may play an important role in the relationship between earnings growth and P/E ratios. Row 1 of Table 1 shows the coefficient from regressing h-year ahead annual earnings growth for $h = 1, 2, \ldots, 5$ on current $P/(E + R\&D)$ ratio while Figure 2 shows the underlying binned scatterplot. We use $P/(E + R\&D)$ instead of P/E to remove movements in E from the deduction of R&D expenditures (See appendix A for the analysis with P/E). The analysis focus on manufacturing firms where R&D data is better measured. At all horizons, higher price to earning ratios today predict higher

Figure 1: Earnings growth vs. P/E, all industry



Source: Compustat. The y-axis is average yearly earnings growth from year t to $t+5$. The x-axis is P/E ratio in t.

subsequent earnings growth. There is significant dispersion in the predicted earnings growth: the 90-10 range of predicted g_E is 9 percentage points.[5]

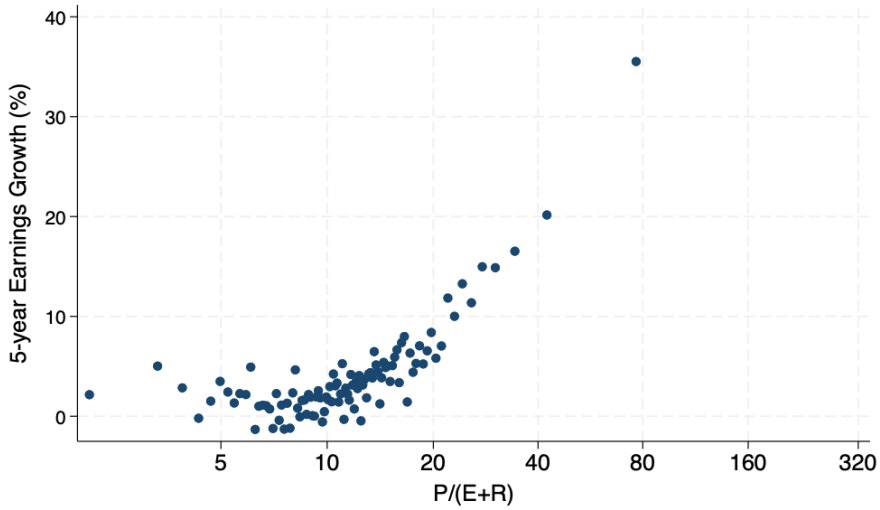
Rows 2, 3, and 4 decompose this relationship into growth of before R&D earnings relative to sales, growth of R&D relative to earnings, and sales growth, respectively. About half of the relationship ((0.028+0.003)/0.070 = 44% over a 5-year horizon) between earnings growth and $P/(E+R\&D)$ is accounted for by higher P/E ratios predicting higher growth in earnings *relative* to sales $g_{E/S}$ as opposed to predicting just higher sales growth g_S.[6] The contribution of $g_{E/S}$ comes mainly (0.028/(0.028+0.003) = 90%) from higher $P/(E+R\&D)$ predicting higher growth of profitability $g_{(E+R\&D)/S}$ as opposed to movements in R&D $g_{E/(E+R\&D)}$ which clearly motivates a markup channel to be operative.

Firms with higher P/E ratio have higher R&D intensity The predictability of earnings and profitability growth by P/E ratios may be related to firm's

[5]The range is 15 percentage points for growth of earnings before R&D $g_{E+R\&D}$.

[6]Appendix Table A1 shows the pattern is stronger when using P/E and including non-manufacturing firms: $g_{E/S}$ accounts for 77% of the relationship between g_E and P/E.

Figure 2: Earnings growth vs. $P/(E+R\&D)$, manufacturing



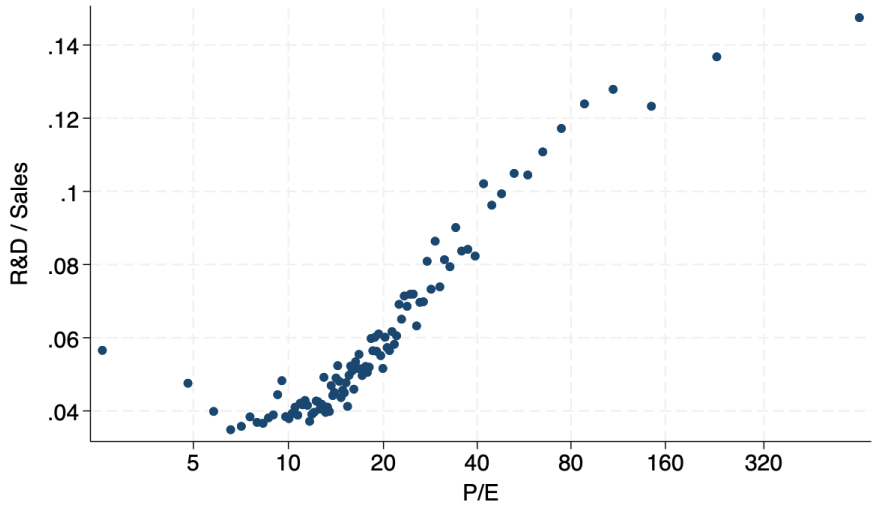
Source: Compustat. The y-axis is average yearly earnings growth from year t to $t+5$. The x-axis is P/E ratio in t. Manufacturing firms only.

Table 1: Growth of earnings and earnings/sales in manufacturing

	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$
g_E on $\log(P/(E+R\&D))$	0.346	0.172	0.118	0.094	0.070
	(0.021)	(0.012)	(0.009)	(0.008)	(0.007)
$g_{(E+R\&D)/S}$	0.137	0.071	0.048	0.034	0.028
	(0.006)	(0.004)	(0.003)	(0.003)	(0.002)
$g_{E/(E+R\&D)}$	0.121	0.039	0.020	0.015	0.003
	(0.018)	(0.010)	(0.007)	(0.006)	(0.005)
g_S	0.087	0.063	0.051	0.044	0.040
	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)

Source: Compustat. The table focuses only on manufacturing firms. The first row displays the coefficient when regressing average yearly earnings growth from year t to $t+h$ on $P/(E+R\&D)$ ratio in t. The next three rows decompose the coefficient into growth in the ratio of earnings gross of R&D to sales, the ratio of earnings net of R&D to earnings gross of R&D, and sales.

Figure 3: R&D/Sales vs P/E, manufacturing



Source: Compustat. The y-axis is R&D over sales in year t. The x-axis is P/E ratio in t. The figure focuses on manufacturing firms only.

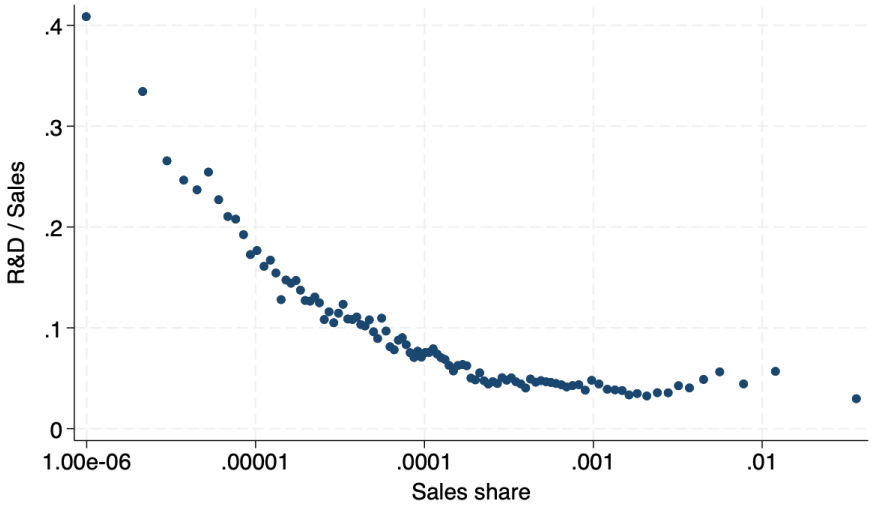
innovation. Among manufacturing firms for whom R&D is reliably reported, high P/E firms have higher R&D intensity, as shown in Figure 3.

Firms with higher sales have lower sales growth and lower R&D intensity In contrast, high sales firms tend to have lower R&D intensity, as shown in Figure 4. Larger firms also have lower sales growth as shown in Figure 5 for all firms and Figure 6 for manufacturing firms.

3 Model

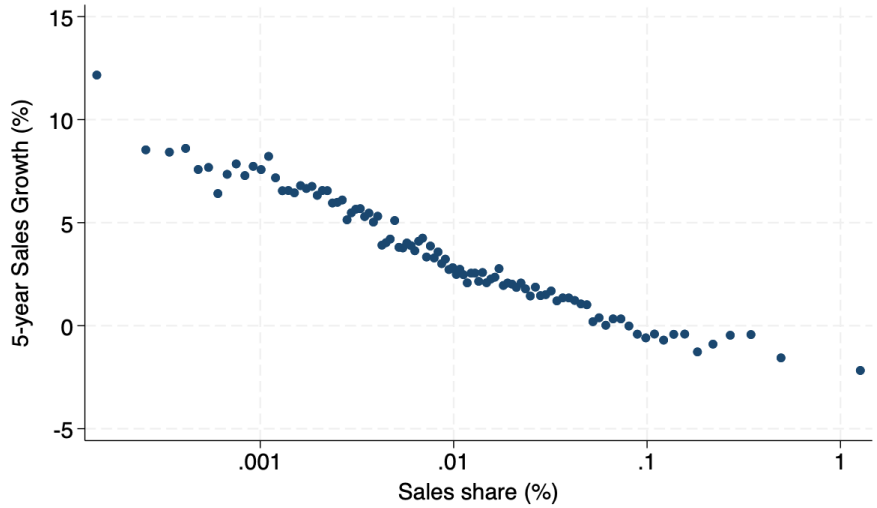
Based on the facts we documented in the previous section, we formulate an endogenous growth model with stochastic but persistent differences in innovation step sizes and R&D efficiencies across firms. Growth in earnings/sales (especially earnings gross of R&D expenditures) is suggestive of rising markups at high P/E firms. Thus high P/E ratio firms may have opportunities to invest in innovations with high quality steps that allow them

Figure 4: R&D/Sales vs. sales, manufacturing



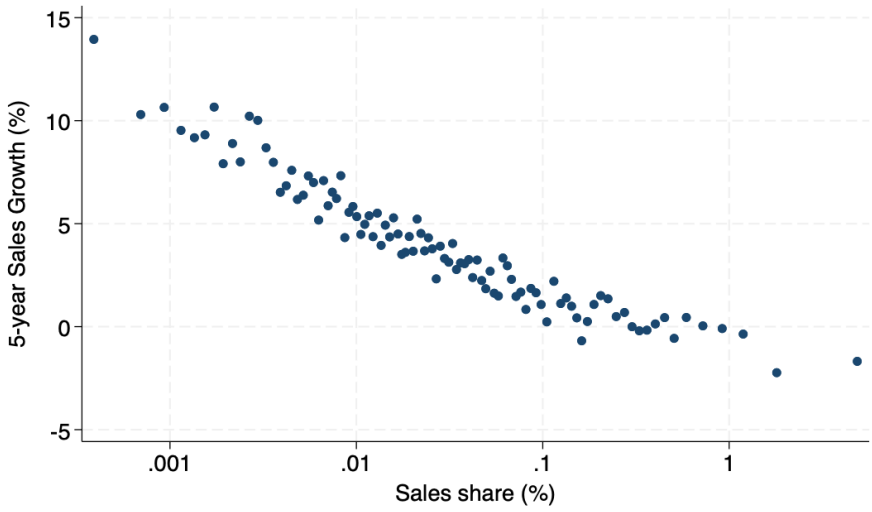
Source: Compustat. The y-axis is R&D over sales in year t. The x-axis is P/E ratio in t. The figure focuses on manufacturing firms only.

Figure 5: Sales growth vs. sales, all industry



Source: Compustat. The y-axis is average yearly growth of sales from year t to year $t+5$. The x-axis is share of total sales in t.

Figure 6: Sales growth vs sales, manufacturing



Source: Compustat. The y-axis is average yearly growth of sales from year t to year $t+5$. The x-axis is share of total sales in t. Manufacturing firms only.

to command high markups, fueling their fast earnings growth. A high P/E ratio could then reflect that investors have information about the firm's investment opportunities that they price into firm valuations. If firms have information about their opportunities as well, this could account for the high R&D intensity of high P/E ratio firms. Similarly, stochastic R&D efficiency can generate rising sales growth with P/E ratio because firms with temporarily good R&D efficiency spend more on R&D which lower earnings today but expands future sales.

Our model builds on Klette and Kortum (2004), which features endogenous quality growth and creative destruction by multiproduct firms. The key difference is that we allow each firm's innovation step size to evolve stochastically (and idiosyncratically) rather than being common or fixed over time. This will create markup dynamics and in turn heterogeneous P/E ratios to help the model match the patterns we found in the data.

3.1 Set up

Household A representative household inelastically supplies L units of production labor and maximizes its discounted utility

$$\max_{\{C_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(C_t),$$

subject to

$$A_{t+1} = A_t(1 + r_t) + w_t L - C_t,$$

and a standard no-Ponzi game condition. Here C denotes consumption, A the household's wealth from owning all of the firms in the economy, r the real interest rate, w the real wage, and $\beta \in (0,1)$ the discount factor. The household's Euler equation links along a balanced growth path the equilibrium (consumption) growth rate and interest rate to the discount factor:

$$1 + g = \beta(1 + r). \tag{1}$$

Final goods production Final output Y is a Cobb-Douglas bundle of a unit interval of intermediate goods with qualities $q(i)$:

$$Y = \exp\left(\int_0^1 \log\left(q(i)y(i)\right) di\right).$$

Final output can be used for consumption C or research expenditures R.

We assume the final goods market is competitive. So final output production can be characterized as the behavior of a representative firm solving

$$\max_{\{y(i)\}_{i=0}^{1}} P \cdot \exp\left(\int_0^1 \log\left(q(i)y(i)\right) di\right) - \int_0^1 p(i)y(i)\, di, \tag{2}$$

taking prices P and $p(i)$ as given. Due to the Cobb-Douglas structure, the solution to the final goods producer's problem features equal spending on

each product

$$p(i)y(i) = PY. \tag{3}$$

In the following, we normalize the price of the final good $P = \exp\left(\int_0^1 \log\left(p(i)/q(i)\right) di\right)$ to 1 in all periods.

Intermediate goods production The intermediate goods are produced by a large fixed number J of firms indexed by $j \in \{1, \ldots, J\}$. Each firm j has a blueprint to produce varieties i with quality $q(i, j)$. The firms needs one unit of production labor to produce one unit of the variety. More specifically, firm j can produce one unit of good i at quality $q(i, j)$ with technology $y(i, j) = l(i, j)$ where $l(i, j)$ is the labor input of firm j into producing good i.

The quality levels at which a firm produces change endogenously over time as a result of R&D activity. Each firm has access to a R&D technology with heterogeneous and stochastic step sizes and R&D efficiency. That is, a firm j can randomly draw x_j number of lines to innovate upon by spending

$$R_{jt} = \phi_{jt} x_{jt}^{\nu} Y_t \tag{4}$$

units of final output. The parameter ϕ_{jt} is a R&D cost shifter of firm j at time t and the R&D efficiency of the firm j is given by $1/\phi_{jt}$. For the x_{jt} lines drawn by firm j in period t, the firms can improve the highest quality in the line by a factor of $\gamma_{jt} > 1$ and can produce at this higher quality level from next period onward. In each period, the firms also lose products at rate $\bar{x}$ that is endogenous in general equilibrium, but is taken as given by the firms. Along a balanced growth path $\bar{x}$ is constant.

The step size and R&D cost of a firm (γ_{jt}, ϕ_{jt}) is stochastic and follows a N-state Markov chain. It can take on N values: $(\gamma_1, \phi_2), \ldots, (\gamma_N, \phi_N)$. The transition

probabilities are given by

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1N} \\ \vdots & \vdots & \vdots & \vdots \\ m_{N1} & m_{N2} & \cdot & m_{NN} \end{bmatrix}, \tag{5}$$

where $m_{kk'}$ denotes the probability of a firm with step size and R&D cost k in t moving to step size and R&D cost state k' in $t+1$. The Markov chain is ergodic.

We assume that the intermediate inputs markets feature Bertrand competition. Under this, the highest quality producer in each line sets the price to the quality-adjusted marginal cost of the second-highest quality producer $p(i) = \frac{w}{q(i,j'(i))}$ while its own marginal cost is $\frac{w}{q(i,j(i))}$. Hence, markup in each line is determined by the quality of the best relative to the second-best producer

$$\mu(i) = \frac{q(i,j(i))}{q(i,j'(i))}. \tag{6}$$

Since the highest-quality producer is the firm that last innovated on the line, the markup in a line is determined by the quality step size of the last innovator *at the time it acquired the line*. This is in general different from the current step size of the producer. Going forward, we will use $\gamma(i)$ to denote the markup on the line i. Thus the profit relative to aggregate output Y of the firm producing in line i is

$$\pi(\gamma(i)) = \left(1 - \frac{1}{\gamma(i)}\right). \tag{7}$$

Let $\mathcal{N}_j$ be the portfolio of lines where firm j is the highest-quality producer. The measure of $\mathcal{N}_j$ is n_j. Firm j's total earnings relative to output Y before expensing R&D in a period are

$$\int_{i \in \mathcal{N}_j} \pi(\gamma(i))\, di.$$

Since there are N types of step size, we can write these earnings before R&D as

$$\tilde{E}(\{\omega_k^j\}, n_j) = n_j \sum_k \omega_k^j \pi(\gamma_k), \tag{8}$$

where ω_k^j is the share of firm j's portfolio where the firm had step size γ_k when it acquired the line and n_j is the number of lines in the portfolio. Earnings after R&D (again expressed relative to aggregate output) are

$$E(\{\omega_k^j\}, n_j, x_j, \phi_j) = \tilde{E}(\{\omega_k^j\}, n_j) - \phi_j x_j^\nu. \tag{9}$$

In each period, the firm sees its current step size γ_{jt} and R&D cost ϕ_{jt} and then chooses the number of lines to draw x_j to maximize the present discounted value of the firm. The value of the firm (before dividend) is the present discounted value of its earnings stream

$$\begin{aligned} V_{j,t} \quad = \quad & \max_{\{x_{j,t+\tau}, n_{j,t+\tau+1}\}_{\tau=0}^{\infty}} \left\{ E(\{\omega_{k,t}^j\}, n_{j,t}, x_{j,t}) Y_t \right. && (10) \\ & \left. + \sum_{\tau=1}^{\infty} \left(\prod_{s=1}^{\tau} \frac{1}{1+r_{t+s}} \right) E(\{\omega_{k,t+\tau}^j\}, n_{j,t+\tau}, x_{j,t+\tau}) Y_{t+\tau} \right\}, && (11) \end{aligned}$$

subject to

$$n_{j,\tau+1} = n_{j,\tau}(1-\bar{x}) + x_{j,\tau}, \; \forall \tau.$$

The earnings stream in turn depends on the history of the firm's step size and R&D cost draws, on how fast the firm loses its products due to creative destruction, and on its innovation decisions:

$$E(\{\omega_{k,t+1}^j\}, n_{j,t+1}, x_{j,t+1}) = \tilde{E}(\{\omega_{kt}^j\}, n_{jt})(1-\bar{x}_t) + x_{jt}\pi(\gamma_{jt}) - \phi x_{j,t+1}^\nu. \tag{12}$$

We consider a balanced growth path with an equilibrium path along which the interest rate r, the rate of creative destruction $\bar{x}$ and the growth rate of $g_t = Y_t/Y_{t-1} - 1 = g$ are constant $\forall t$. Then, the problem of the firm simplifies to

solving

$$V(\{\omega_{kt}^j\}, n_{jt}, \gamma_{jt}, \phi_{jt}) = \tilde{E}(\{\omega_{kt}^j\}, n_{jt}) \frac{1+r}{1+r-(1+g)(1-\bar{x})} \tag{13}$$
$$+ \max_{\{x_{j,t+\tau}\}_{\tau=0}^{\infty}} \mathbb{E}_{\gamma_{jt},\phi_{jt}} \left\{ \sum_{\tau=0}^{\infty} \left(\frac{1+g}{1+r} \right)^{\tau} \left(x_{j,t+\tau} v(\gamma_{j,t+\tau}) \frac{1+g}{1+r} - \phi_{j,t+\tau} x_{j,t+\tau}^{\nu} \right) \right\}.$$

The first line is the firm value from the portfolio of products that the firm acquired from past innovations, whereas the second line is firm value from expect earnings from future innovation. Here $v(\gamma)$ denotes the present discounted value of a line aquired with step size γ relative to output Y, i.e.,

$$v(\gamma(i)) = \pi(\gamma(i)) \frac{1+r}{1+r-(1+g)(1-\bar{x})}. \tag{14}$$

In the above firm problem, the optimal choice of x_{jt} only depends on the current γ_{jt} and ϕ_{jt}. Hence, we can write the R&D decision as choosing a policy function $x(\gamma, \phi)$ that solves

$$\max_{x \geq 0} \left\{ x v(\gamma) \frac{1+g}{1+r} - \phi x^{\nu} \right\}. \tag{15}$$

The solution to this is

$$x(\gamma, \phi) = \left(\frac{1+g}{1+r} v(\gamma) \frac{1}{\nu \phi} \right)^{1/(\nu-1)}, \tag{16}$$

which says that firms with higher current step sizes and higher R&D efficiency (lower ϕ) draw more lines. A high quality step and high R&D present a window of opportunity with superior innovation technology. The R&D spending is

$$\phi x(\gamma, \phi)^{\nu} = \left(\frac{1}{\phi} \right)^{\frac{1}{\nu-1}} \left(\frac{1+g}{1+r} v(\gamma) \frac{1}{\nu} \right)^{\nu/(\nu-1)}. \tag{17}$$

which increases with R&D efficiency and step size. The additional firm value

generated by today's R&D net of the R&D cost is

$$\left(\frac{1+g}{1+r}v(\gamma)\right)^{\nu/(\nu-1)} \cdot \left(1-\frac{1}{\nu}\right)\left(\frac{1}{\nu\phi}\right)^{1/(\nu-1)}. \tag{18}$$

Hence, firms with higher step sizes and higher R&D efficiency spend more on R&D and add more value.

Substitute the optimal R&D decision into the firm's value function yields the value of a firm

$$\begin{aligned} V\left(\{\omega_{kt}^j\}, n_{jt}, \gamma_{jt}, \phi_{jt}\right) &= \tilde{E}(\{\omega_{kt}^j\}, n_{jt})\frac{1+r}{1+r-(1+g)(1-\bar{x})} \\ + \quad \left(1-\frac{1}{\nu}\right) &\cdot \mathbb{E}_{\gamma_{jt},\phi_{jt}} \sum_{\tau=0}^{\infty}\left(\frac{1+g}{1+r}\right)^{\tau}\left(\frac{1+g}{1+r}v(\gamma_{j,t+\tau})\right)^{\nu/(\nu-1)}\left(\frac{1}{\nu\phi_{j,t+\tau}}\right)^{1/(\nu-1)}. \end{aligned} \tag{19}$$

The ratio of price to earnings before R&D is then

$$\begin{aligned} \frac{V(\{\omega_{kt}^j\}, n_{jt}, \gamma_{jt}, \phi_{jt})}{\tilde{E}(\{\omega_{kt}^j\}, n_{jt})} &= \frac{1+r}{1+r-(1+g)(1-\bar{x})} \\ &+ \left(1-\frac{1}{\nu}\right)\cdot\frac{\left(\frac{1+g}{1+r}v(\gamma_{j,t})\right)^{\nu/(\nu-1)}\left(\frac{1}{\nu\phi_{j,t}}\right)^{1/(\nu-1)}}{\tilde{E}(\{\omega_{kt}^j\}, n_{jt})} \\ &+ \left(1-\frac{1}{\nu}\right)\cdot\mathbb{E}_{\gamma_{jt},\phi_{jt}}\frac{\sum_{\tau=1}^{\infty}\left(\frac{1+g}{1+r}\right)^{\tau}\left(\frac{1+g}{1+r}v(\gamma_{j,t+\tau})\right)^{\nu/(\nu-1)}\left(\frac{1}{\nu\phi_{j,t+\tau}}\right)^{1/(\nu-1)}}{\tilde{E}(\{\omega_{kt}^j\}, n_{jt})}. \end{aligned}$$

The first term does not depend on firm size because all firms face the same rate of creative destruction. The second term depends on the current step size and R&D efficiency of the firm. Firms with higher step size and R&D efficiency innovate more and add value in the next period, raising the value of the firm relative to current earnings. The third term depends on the stochastic process of the step size and R&D efficiency. For example, when step sizes and R&D efficiency are iid, expected future earnings is the same across firms and hence this third term declines with today's earnings. On the other hand, the relationship is ambiguous if step size and/or R&D efficiency are positively

correlated.

Market clearing and aggregates Since we have a unit measure of lines, the aggregate rate of creative destruction is equal to

$$\bar{x} = \sum_{j=1}^{J} x(\gamma_{jt}, \phi_{jt}) = \sum_{k=1}^{N} J s_k x(\gamma_k, \phi_k), \tag{20}$$

where s_k is the stationary share of firms (γ_k, ϕ_k) according the the Markov transition probabilities M. It is the share of firms with step size γ_k and R&D cost ϕ_k in each period.

The equilibrium growth rate of aggregate output (holding aggregate labor input fixed) is

$$1 + g = \prod_{k=1}^{N} (\gamma_k)^{x(\gamma_k, \phi_k) \cdot s_k J}. \tag{21}$$

Equations (20), (21), and the household's Euler equation (1) jointly pins down equilibrium $\bar{x}$, r and g.

We also have the labor market clearing condition

$$\int_0^1 l(i,j)di = L,$$

the asset market clearing condition

$$\sum_{j=1}^{J} V_{j,t} = A_t,$$

and goods market clearing

$$\sum_j R_j = R = Y - C. \tag{22}$$

Finally, we have the number of lines operated by the firms summing to 1, i.e.,

$$1 = \sum_{j=1}^{J} n_j.$$

Equilibrium definition, BGP definition A decentralized equilibrium consists of a sequence of quantities and prices that jointly solve the problems of the final producer, the intermediate producer, the household and is consistent with the market clearing conditions and the aggregate constraints. As mentioned above, we define a balanced growth path (BGP) in the standard way, i.e., as a path along which all quantities grow at constant rates.

Model mechanisms The model is tractable but can still generate dispersions in P/E ratios, expected growth, R&D intensities across firms and relatively rich dynamics in firm dynamics and markups.

As each firm has a set of product lines of measure n_j in its portfolio a firm never loses all its leading market positions due to creative destruction and there is no exit and entry in the model (as in Aghion, Bergeaud, Boppart, Klenow and Li (2023)). Relatedly, in our model, the size of a firm is fully determined by its history of all the ϕs and γs.

In Klette and Kortum (2004) the expected innovation rate is along a balanced growth path unrelated to size as the the R&D cost function is scaled appropriately by the number of lines a firm is active in. In contrast, in (4) we do not do such a scaling. As a consequence our model can potentially generate a very strong deviation from Gibrat's law. At the same time, as illustrated below, with fully persistent ϕ and γ differences, our model can still generate a case in which the expected (sales) growth rate is unrelated to size.

Our model generates a rich interaction between past ϕs and γs, firm size and future growth potential of a firm. Today's R&D investment only depend on the current ϕ and γ states. The firm size depends on the history of these states as high step size and low R&D cost firms innovated more in the past and grew

faster. Similarly, the average markup a firm can charge is informative about the past step sizes it had. In contrast the P/E ratio is forward looking taking into account that if there is some persistence in the states that firms with superior R&D technology today are expected to have good R&D potential in future too.

To build intuition it is helpful to consider the extreme cases of either iid ϕ and γ draws or the case of full persistent heterogeneity. In the iid case, the history of states and therefore firm size is unrelated to today's states. In this case the expected R&D activity and (absolute) growth contribution is the same for all firms irrespective whether they large or small. This gives rise to a case in which the firm earnings or sales growth rate is strongly decreasing in size and a strong violation of Gibrat's law. More persistence in the ϕ and γ states will make the relationship between firm size and sales growth flatter. In the opposite extreme with fully persistent states, the (expected) sales growth rate is unrelated to size.

The dispersion of the γ states are informative about the share of predicted earning growth that comes from increasing markups as opposed to sales. In contrast, a low ϕ draw allows a firm to grow faster too but without being systematically related to changes in markups. Finally, the corelation between ϕ and γ determines how the average firm level markup and firm size are related. If ϕ and γ negatively correlated big firms have added a lot of their product lines with large step sizes and therefore show a high (average) markup. If ϕ and γ are strongly positively related it may even be the case that larger firms have lower (average) markups.

4 Calibration and consequences

In this section, we calibrate the model and calculated the growth contribution of firms. To keep the model parsimonious we approximate the Markov chain in the model with a VAR(1) process using the method described in Gospodinov

and Lkhagvasuren (2014). Namely, we calibrate parameters of the process

$$\begin{pmatrix} \log(\gamma_t - 1) \\ \log \phi_t \end{pmatrix} = \begin{pmatrix} \rho_\gamma \log(\gamma_{t-1} - 1) \\ \rho_\phi \log \phi_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{\gamma,t} \\ \varepsilon_{\phi,t} \end{pmatrix}$$

$$\begin{pmatrix} \epsilon_{\gamma,t} \\ \epsilon_{\phi,t} \end{pmatrix} \overset{iid}{\sim} N \left(\begin{pmatrix} \mu_\gamma \\ \mu_\phi \end{pmatrix}, \begin{bmatrix} \sigma_\gamma^2 & \sigma_{\gamma,\phi} \\ \sigma_{\gamma,\phi} & \sigma_\phi^2 \end{bmatrix} \right).$$

We use this method to generate $N = 625$ discrete states and the associated transition matrix for the Markov process of step size and R&D cost.

We calibrate 9 parameters to 9 moments. The parameters are the curvature ν of the R&D cost function, the drift (μ_γ, μ_ϕ), persistence (ρ_γ, ρ_ϕ) and volatility $(\sigma_\gamma, \sigma_\phi, \sigma_{\gamma,\phi})$ of the step size and R&D cost VAR(1) process, and the discount factor β. The nine moments are the growth rate of aggregate earnings net of labor input growth and growth in the number of firms, the average stochastic discount factor from David et al. (2022), the contribution of the earnings-to-sales ratio to the relationship between the average 5-years earnings growth and initial $P/(E + R\&D)$, the contribution of earnings before R&D to sales ratio to the relationship between 5 year earnings growth and $P/(E + R\&D)$ aggregate earnings to aggregate sales ratio, the dispersion of predicted average 5 year growth in earnings and pre-R&D earnings, the relationship between R&D intensity and sales, and sales dispersion.

The model is non-linear and all nine parameters jointly determine the moments. However, some moments are more disciplining for some parameters. The aggregate earnings/sales and growth in aggregate earnings together are informative of the drift μ_γ and μ_ϕ. Given the aggregate growth moment, the discount rate pins down β. The dispersion in pre-R&D earnings growth is informative of the volatility parameter σ_γ while the dispersion of sales is informative of σ_ϕ. The growth of net earnings is less dispersed than the

growth of gross earnings, indicating that high step draws incur higher R&D cost and hence a positive covariance between ϕ and γ. The contribution of earnings/sales is informative of the curvature parameter. When curvature is higher, firms R&D choice and hence the number of products they innovate upon vary less with the step size and R&D cost draws. As a result, a firm's earnings growth comes less from variation in the number of product lines it adds and more from growth in earnings/sales via different step sizes or R&D cost. The contribution of pre-R&D earnings to sales ratio is informative of the persistence of step size draws ρ_γ. The R&D intensity relationship with sales is informative of the persistence ρ_ϕ. All else equal, higher ρ_ϕ generates more dispersion in sales and a flatter relationship between R&D/sales with sales.

Table 2 displays the target and data moments while Table 3 displays the calibrated parameters.[7] The overall fit is good, although we generate less aggregate earnings/sales and sales dispersion than in the data. The implied cost weighted price-cost markup is 1.22. We find that the step size is quite persistent with $\rho_\gamma = 0.93$ while the R&D cost is has a persistence $\rho_\phi = 0.82$. Step size and R&D cost have a correlation of 0.54. The model needs high variance of R&D cost to generate the empirical sales dispersion.

4.1 Firm's contribution to growth

Using the calibrated model, we can calculate firms contribution to growth. For a group of firms I, contribution to growth is

$$m(I) \cdot \frac{\sum_k x(\gamma_k, \phi_k) \cdot s_k^i \cdot \log(\gamma_k)}{\sum_k x(\gamma_k, \phi_k) \cdot s_k \cdot \log(\gamma_k)}$$

where s_k^i is the share for firms with step size k and $m(I)$ is the fraction of firms in I. The so-called Gibrat's Law refers to the special case when contribution to

[7] Since we use R&D data to calculate some of the moments, we restrict the sample to manufacturing firms. In the sample of all firms, we find stronger contribution (77% vs 44%) of $g_{E/S}$ to the predictability of P/E ratio for earnings growth, larger dispersion in the predicted growth rate of earnings (0.18 vs 0.09), and lower 90/50 ratio of sales (14.8 vs 17.6).

Table 2: Calibration data vs. model

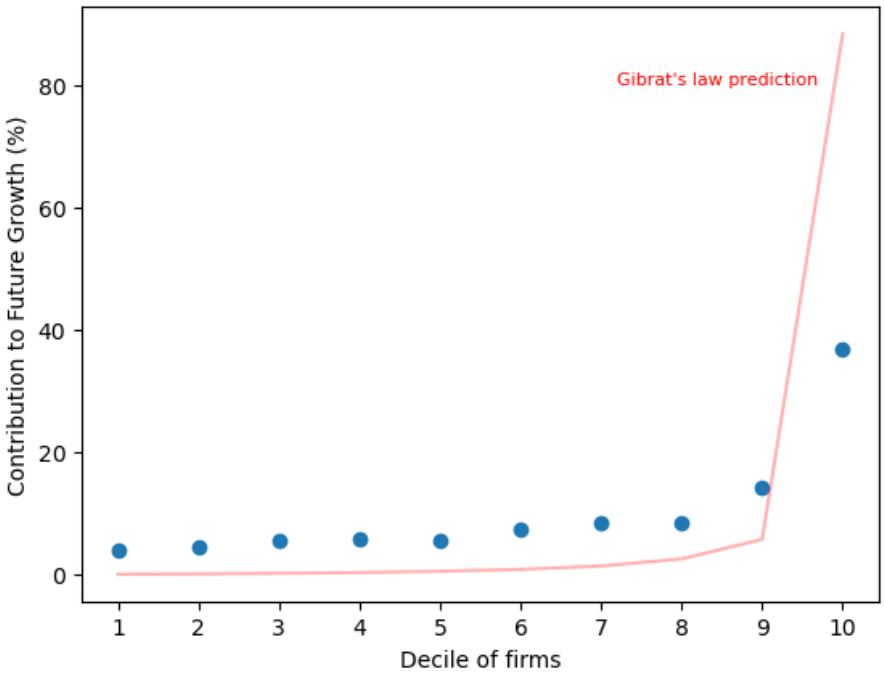
Targets	Data	Model
1. Growth in aggregate earnings, net of labor input growth, $g^\star$	1.3%	1.5%
2. Average stochastic discount factor, $r^\star$	9.7%	9.9%
3. Contribution of $g_{E/S}$ to g_E and $P/(E+R\&D)$ relationship	44%	42%
4. Contribution of $g_{(E+R\&D)/S}$ to $g_{E/S}$ and $P/(E+R\&D)$ relationship	90%	94%
5. Aggregate earnings/aggregate sales	0.28	0.18
6. 90–10 range of fitted g_E	0.09	0.08
7. 90–10 range of fitted $g_{E+R\&D}$	0.15	0.13
8. R&D/sales projected on log sales	-0.028	-0.032
9. 90:50 sales ratio	17.6	14.5

Source: 1. earnings from I/B/E/S and labor input growth from the BLS. 2. average of firm-specific discount rate from David et al. (2022). 3 to 9 are manufacturing firms from CRSP, I/B/E/S, and Compustat.

Table 3: Calibrated parameters

Parameter		Value
Curvature of R&D cost function	ν	3.57
Persistence of γ process	ρ_γ	0.93
Mean of shocks to γ process	μ_γ	-0.05
Variance of shocks to γ process	σ^2_γ	1.14
Persistence of ϕ process	ρ_ϕ	0.82
Mean of shocks to ϕ process	μ_ϕ	4.64
Variance of shocks to ϕ process	σ^2_ϕ	41.2
Covariance between shocks	$\sigma_{\gamma\phi}$	3.73
Discount factor	β	0.92

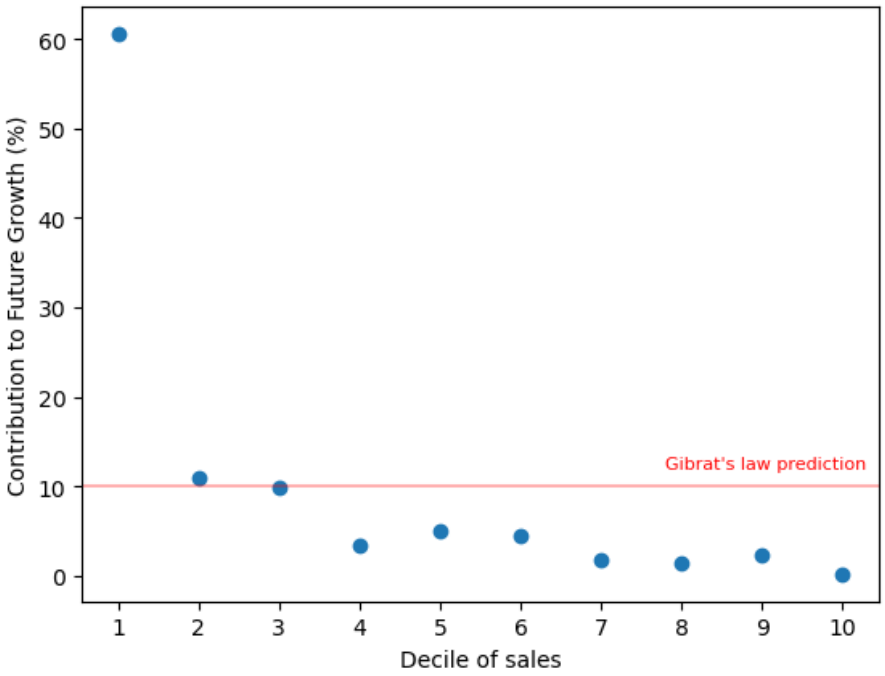
Figure 7: Contribution to growth by firm decile



growth is equal to the sales share of the firms. In contrast, when step sizes are independent, the contribution is 1% for all size percentiles.

Using this metric, we find that large firms (higher sales) do contribute more to growth but not as much as what Gibrat's Law predicts. Figure 7 ranks the firms by sales and show the contribution of firms in each firm decile. If the shocks were iid the contribution would be 10% for each firm decile. The largest 10% of firms contribute to more than 10% of growth. This comes from the high persistence in the shocks. However, the contribution is smaller than the contribution according to Gibrat's Law. Figure 8 shows this more clearly by plotting the growth contribution by sales deciles. The largest firms that account for 10% of sales contribute much less than 10% of growth. While the smallest firms that account for 10% of sales contribute 60% of growth.

Figure 8: Contribution to growth by sales decile



4.2 Effect of step size heterogeneity on growth

Our model also highlights the importance of heterogeneity in step sizes. To show this, we calculate a counterfactual growth rate and allocative efficiency where we give all firms the same constant step size conditional on R&D cost ϕ

$$\gamma^c(\phi) = \exp(\mathbb{E}[\log(\gamma - 1)|\log\phi]) + 1$$

This removes step size heterogeneity conditional on ϕ but preserves the covariance between step size and R&D cost across firms.[8] We keep all the other parameters the same and set the stochastic process of R&D cost to have the

[8]The counterfactual removed most of the dispersion in step size. The dispersion in current step size across firms can be decomposed into dispersion conditional on ϕ and dispersion across ϕ:

$$\underbrace{Var(\log(\gamma-1))}_{8.634} = \underbrace{\mathbb{E}[Var(\log(\gamma-1)|\log\phi)]}_{6.645} + \underbrace{Var(\mathbb{E}[\log(\gamma-1)|\log\phi])}_{1.990}$$

same drift, persistence, and variance as the baseline model. To focus on the effect of removing γ heterogeneity on growth, we implement a subsidy τ on R&D to keep the aggregate R&D to aggregate output ratio the same as the baseline. Namely, the firm's R&D decision x^c in the counterfactual solves

$$\max_{x^c \geq 0} \left\{ x^c v(\gamma^c) \frac{1+g^c}{1+r^c} - (1-\tau)\phi (x^c)^\nu \right\} \tag{23}$$

given the counterfactual equilibrium growth rate g^c and interest rate r^c. We set the subsidy rate τ such that total R&D relative to output is the same in both economies

$$\sum_{k=1}^{N} s_k \phi_k \left((x_k^c)^\nu - x_k^\nu \right) = 0. \tag{24}$$

Table 4 compares the baseline growth rate and allocative efficiency to the counterfactuals. The allocative efficiency is the ratio of the geometric relative to the arithmetic average of the inverse markups across lines, or formally

$$\text{Allocative efficiency } = \frac{\exp\left(\int_0^1 \log \frac{1}{\mu(i)} \, di \right)}{\int_0^1 \frac{1}{\mu(i)} \, di} \leq 1. \tag{25}$$

This term is smaller than one due to markup dispersion across lines in the decentralized equilibrium. In the first counterfactual, the growth rate reduces from 1.5% to 0.08%. This is because high step size firms spend more on R&D conditional on ϕ in the baseline. On the other hand, misallocation due to markup dispersion lowers the level of output by 9.3% in baseline model while it has negligible effect in the counterfactual. As aforementioned, this is because in the baseline calibration most of the step size γ dispersion is dispersion conditional on ϕ.

Almost 80% (6.645/8.634) of the dispersion in step size comes from dispersion conditional on R&D cost. The covariance of step size and R&D cost $Cov(\log(\gamma - 1), \log \phi)$ is 15.85 in both the baseline and the counterfactual.

Table 4: Effect of removing step size heterogeneity

	Growth rate	Allocative efficiency
Baseline	1.5%	0.907
$\gamma^c(\phi) = \exp(\mathbb{E}[\log(\gamma - 1) \mid \log \phi]) + 1$	0.08%	1.000

4.3 Growth contribution of firms

We can use our model to estimate the expected contribution of individual firms to future growth in the data. Although we do not observe the step size or R&D efficiency of a company's current innovations, we derive a proxy by regressing in the calibrated model a firm's expected growth contribution over the next 5 years on the firm's market capitalization, sales, earnings (net of R&D), allowing the relationship to differ depending on whether the firm has positive current earnings. Table 5 displays the regression results. It shows that a firm's expected contribution to growth loads positively on its market capitalization and negatively on its current sales. A firm's market capitalization represents the presented discounted value of its future earnings from past and future innovations, while current earnings and sales depends only on past innovations. A firm's expected growth contribution therefore increases with its market capitalization, controlling for current sales and earnings. The fitted relationship accounts for almost 60% of variation in expected growth contributions in the model simulated data.

We can apply the fitted relationship in Table 5 to actual firms in the Compustat sample to infer their expected contribution to growth implied by our calibrated model. In Table 6, we do so for the top 7 growth contributors in 2024, which turn out to be the Magnificent Seven. These firms are 30% of total market value and have higher earnings to sales than average. They are expected to contribute over 75% of growth (among Compustat firms) over the

Table 5: Regression of expected growth contribution on market capitalization, sales and earnings in the model

$\log(\text{Market Cap.}) \times \mathbb{1}\{\text{Earnings} > 0\}$	2.23
$\log(\text{Market Cap.}) \times (1 - \mathbb{1}\{\text{Earnings} > 0\})$	1.38
$\log(\text{Sales}) \times \mathbb{1}\{\text{Earnings} > 0\}$	−0.14
$\log(\text{Sales}) \times (1 - \mathbb{1}\{\text{Earnings} > 0\})$	−0.02
$\log(\text{Earnings}) \times \mathbb{1}\{\text{Earnings} > 0\}$	−0.47
$\mathbb{1}\{\text{Earnings} > 0\}$	−4.55
R^2	0.58

next five years, more than eight times larger than their 9% share of 2024 sales.[9]

Table 7 contrasts some of the Magnificent 7 firms to other large firms in their industry. Intel has a sales share similar to Nvidia's but is expected to account for only 0.03% of all growth versus 6.95% for Nvidia. Walmart has a market share close to that of Amazon, but is expected to contribute much less to growth. General Motors and Ford are expected to contribute less to growth than Tesla despite having almost double the sales and comparable earnings as Telsa. These examples underscore that innovation giants of the past are not necessarily innovation leaders in the future.

For manufacturing firms in Compustat, Figure 9 illustrates a strong positive association between a firm's expected growth contribution and its share of aggregate R&D. This provides some reassurance that our measure of a firm's growth contribution is indeed related to its innovation efforts. One might think that this relationship is a simply a byproduct of Gibrat's Law holding in the data but Table 5 shows that our measure of expected growth is negatively related to sales conditional on market capitalization.

[9]Our sample of public firms accounts for approximately one fourth of total private nonfarm employment. One can multiply our growth contribution by one fourth to infer the contribution to aggregate growth. However, this estimate may be conservative if public firms are selected on their growth contribution conditional on their sales.

Figure 10 shows the share of expected growth by the top 7 contributing firms versus the sales share of those same firms, for each year from 1976 to 2024. The identity of the seven firms differs over time (see Table 8). Their sales share has been fairly stable, while their expected growth contribution is much higher than their sales share and varies more over time. Notice a local peak in the contribution of the top 7 firms in 2000. The contribution in recent years is even higher than during the dot-com era, with a surge in the last ten years.

Table 6: Expected contribution of the top 7 contributors in 2024

Name	Share of Market Cap	Share of Earnings	Share of Sales	Growth Contribution	Relative Contribution
Microsoft	6.27	4.24	1.15	20.7	17.9
Apple	6.65	5.01	1.84	20.4	11.1
Amazon	4.36	2.85	3.00	9.68	3.23
Tesla	2.45	0.41	0.46	8.66	18.9
Alphabet	4.08	4.82	1.64	7.08	4.31
Nvidia	2.86	1.56	0.29	6.95	24.3
Meta	2.41	3.00	0.77	3.03	3.92
Total	29.1	21.9	9.15	76.5	8.36

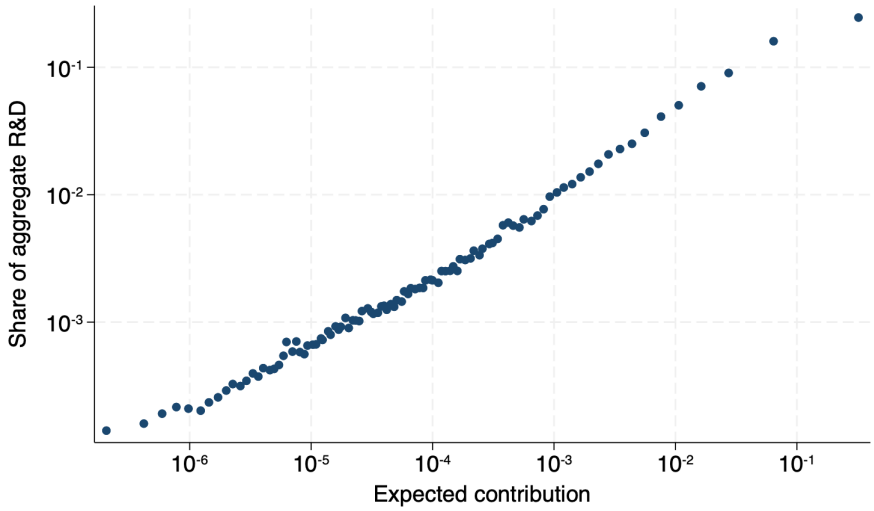
Source: I/B/E/S, CRSP, Compustat. The first three columns show each firms' percent share of market capitalization, earnings, and sales relative to the total across all firms in the sample in 2024. The fourth column estimates expected growth contribution for each firm using the coefficients in Table 5, where total expected growth contribution across all firms in the sample is normalized to 1. The final column shows a firm's expected contribution to growth relative to its sale share.

Table 7: Comparison of expected firm contributions in 2024

Name	Share of Market Cap	Share of Earnings	Share of Sales	Growth Contribution	Relative Contribution
Nvidia	2.86	1.56	0.29	6.95	24.3
AMD	0.37	0.26	0.12	0.19	1.56
Intel	0.16	-0.03	0.25	0.03	0.14
Amazon	4.36	2.85	3.00	9.68	3.23
Walmart	0.84	0.87	3.03	0.43	0.14
Tesla	2.45	0.41	0.46	8.66	18.9
GM	0.11	0.58	0.88	0.007	0.008
Ford	0.07	0.36	0.87	0.003	0.004

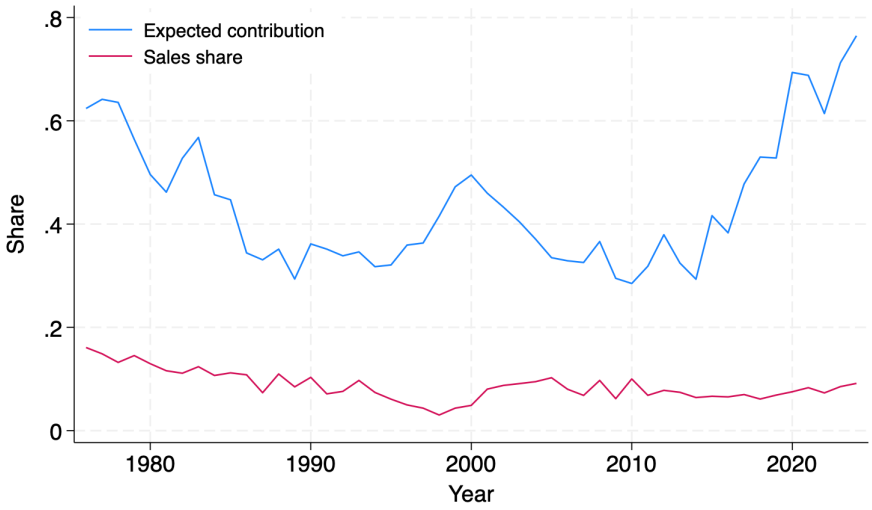
Source: See notes to Table 6.

Figure 9: Share of annual R&D vs. expected growth contribution



Source: I/B/E/S, CRSP, Compustat. Manufacturing firms only. The y-axis is R&D expenditure relative to total R&D across manufacturing firms in a year. The x-axis is estimated expected growth contribution.

Figure 10: Expected Growth Contribution of Top 7 Contributors over Time



Source: I/B/E/S, CRSP, Compustat. The blue line shows the combined expected contribution of the top 7 contributors in each year between 1976 and 2024. The red line shows the combined sales share of these firms.

Table 8: Top 7 Firms by Expected Contribution by Decade

	1980	1990	2000	2010
1	IBM	General Motors	Cisco	ExxonMobil
2	Getty Oil	Exxon	Microsoft	Apple
3	AT&T	IBM	General Electric	General Electric
4	Exxon	Philip Morris	Pfizer	Microsoft
5	General Motors	General Electric	Oracle	Wal-Mart
6	Shell Oil	Bristol Myers Squibb	AIG	Coca-Cola
7	ARCO	Merck	Wal-Mart	IBM

Source: I/B/E/S, CRSP, Compustat. Each column estimates expected growth contribution for each firm using the coefficients in Table 5 in the headline year, where total expected growth contribution across all firms in the sample is normalized to 1.

5 Conclusion

This paper constructs an endogenous growth model with shocks to firm innovation step-size and R&D costs and calibrates it to match the relationship between P/E ratios, earnings, earnings growth, and earnings-to-sales growth across firms. Shocks to the step-size and R&D cost of firms generates differences in future earnings and hence differences in valuation relative to current earnings. Our calibrated model implies significant deviation from the Gibrat's Law with the largest (smallest) firms contributing much less (more) than their share of sales.

Our model suggests that a combination of market capitalization, earnings, and sales could be a good indicator of firm's contribution to growth. There are potential policy uses for real-time estimates of firm contributions to future productivity growth. Because innovations build on each other, large step-size firms confer larger knowledge spillovers. Aghion, Bergeaud, Boppart, Klenow and Li (2024) highlight potential growth gains from reallocation research effort from low to high step-size firms. If governments offer enhanced research subsidies, however, then firms might have an incentive to manipulate their market capitalization or earnings.

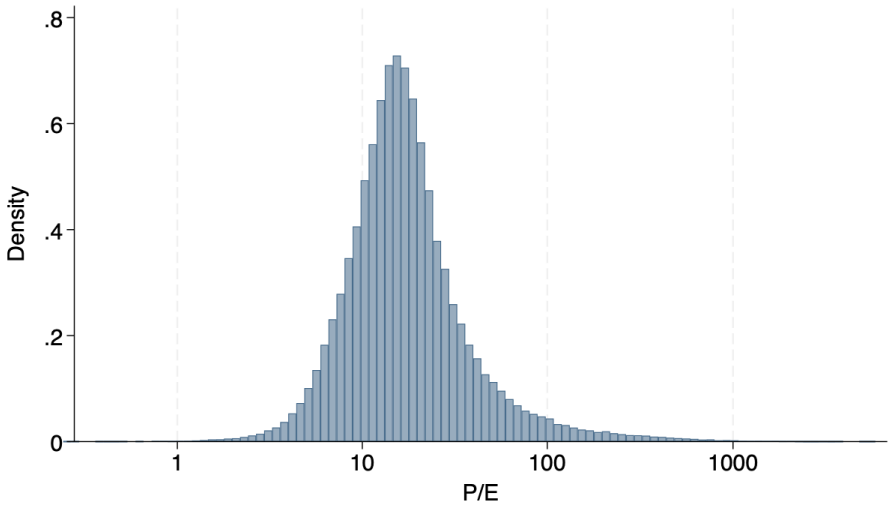
Appendix

A Empirics

Significant dispersion in P/E ratios across firms. Figure A1 shows the distribution of raw P/E ratios pooled across firm-years. Figure A2 shows the standard deviation and interquartile range across firms within each year of our sample. The degree of dispersion fluctuates noticeably, but is nontrivial in all years. Figures A3 and A4 show significant dispersion after taking out year fixed effects and fiscal-year end month effects or 4-digit SIC industry fixed effects.

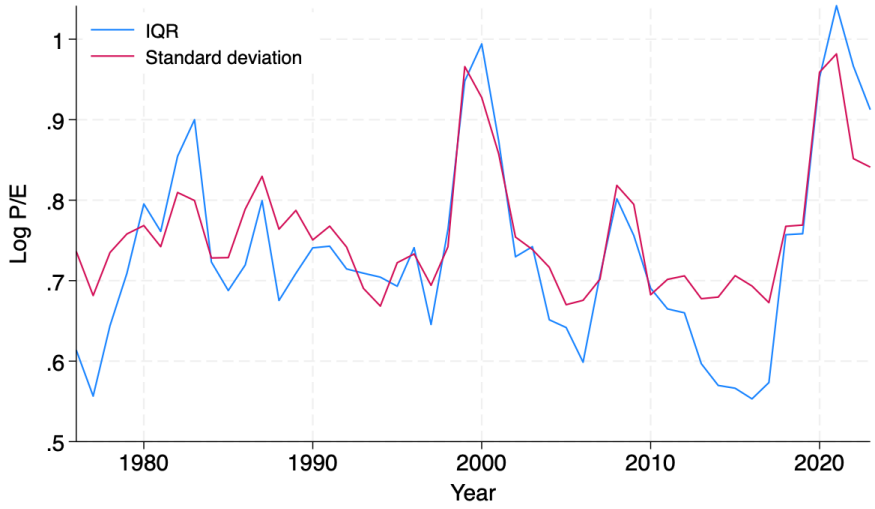
Differences in P/E ratios across firms could be due to firm-specific discount rates. However, the cross-sectional dispersion is not just about dispersion in firm-specific risks. We calculate the residual of $\frac{P}{E}_{it} - \frac{1}{r_{it}}$ for each firm i and year t, where r_{it} is the firm-specific risk measured by David, Schmid and Zeke (2022) using Fama-French factor models. According to the Gordon growth formula, the $\frac{1}{r_{it}}$ is a proxy for variation in the $\frac{P}{E}_{it}$ ratio if all firms have the same constant earnings growth rate. Dispersion in the residual thus reflects variation in the expected growth rate of rents. We find significant dispersion in the residual (see Figure A5). This is consistent with the findings of Vuolteenaho (2002) and De La O, Han and Myers (2025) that differences in firm-level stock returns are partly driven by changes in cash-flow expectations as opposed to entirely reflecting changes in the discount rate.

Figure A1: Distribution of P/E



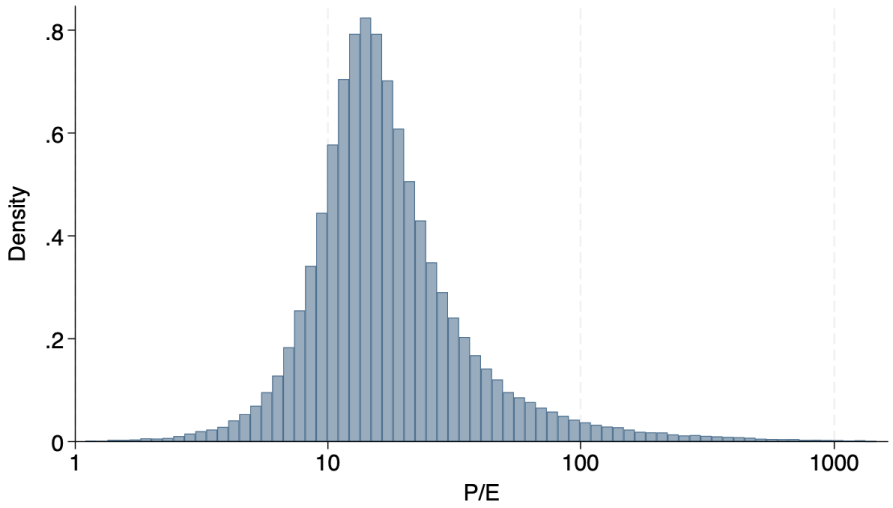
Source: CRSP and I/B/E/S. Figure trims the top and bottom 0.1% observations each year.

Figure A2: Cross-firm dispersion of P/E ratios over time



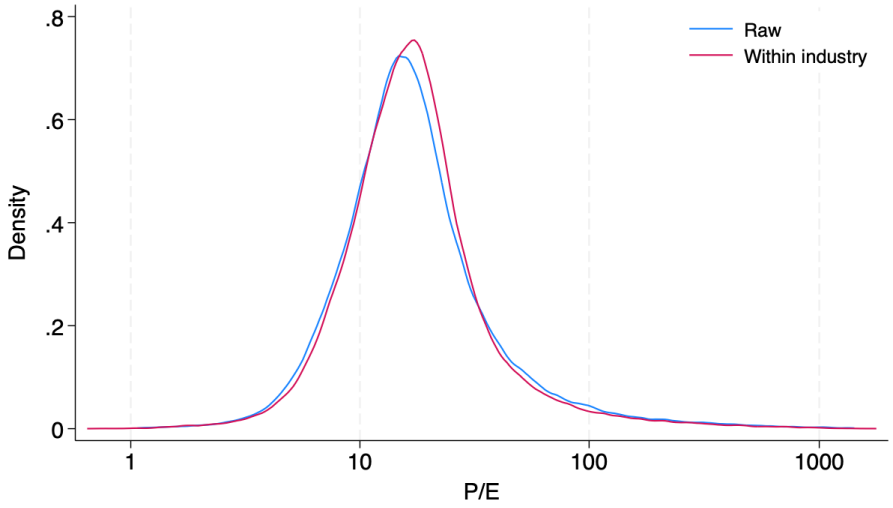
Source: CRSP and I/B/E/S.

Figure A3: Distribution of P/E, removing year and fiscal month FE



Source: CRSP and I/B/E/S.

Figure A4: Distribution of P/E, removing industry FE



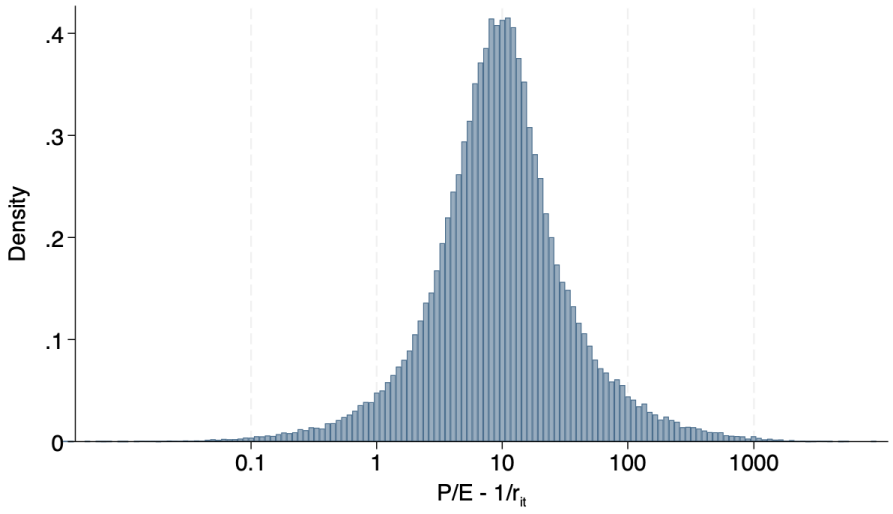
Source: CRSP and I/B/E/S.

Table A1: Growth of earnings and earnings/sales vs P/E ratio

	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$
g_E on $\log(P/E)$	0.422	0.262	0.189	0.150	0.127
	(0.005)	(0.003)	(0.002)	(0.002)	(0.002)
$g_{E/S}$	0.355	0.213	0.151	0.117	0.098
	(0.005)	(0.003)	(0.003)	(0.002)	(0.002)
g_S	0.067	0.049	0.039	0.033	0.029
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)

Source: Compustat. The first row displays the coefficient when regressing average yearly earnings growth from year t to $t+h$ on P/E ratio in t. The second and third row decomposes the coefficient into growth in the ratio of earnings to sales and sales.

Figure A5: Distribution of P/E, removing firm-specific risk



Source: CRSP and I/B/E/S. r_{it} from David et al. (2022)

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