Optimal Monetary Policy and Rawls' Difference Principle*

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December 20, 2025

Abstract

We study optimal monetary policy when a central bank takes inequality concerns into account, in line with Rawls' Difference Principle. Using a tractable heterogeneous-agent New Keynesian model, we describe the optimal policy of a Rawlsian central bank that prioritises the welfare of hand-to-mouth households. Relative to a utilitarian central bank, the Rawlsian central bank places substantially greater weight on stabilising wage inflation, as wage dispersion increases firm profits that accrue mainly to savers, thereby widening inequality. Counterfactual simulations with a medium-scale version of the model estimated on U.S. data show, however, that the quantitative differences between the utilitarian and Rawlsian policy approaches are minor.

Keywords: Optimal monetary policy, Rawls' difference principle, Inequality
JEL Classification: E32, E52, E61

*The authors declare no sources of research support and no conflicts of interest. We are grateful to Kenza Benhima, Daeha Cho, Yoon Joo Jo, ShinHyuck Kang, Anastasios Karantounias, Mira Kim, Eunseong Ma, Riccardo Masolo, Taeha Oh, Yeonu Oh, Ricardo Reis, Keh-Kuan Sun, Sangjun Yea, and Jeonghwan Yun for insightful discussions and helpful suggestions. All errors are our own.

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1 Introduction

Social and economic inequalities are to be arranged so that they are to the greatest benefit of the least advantaged members of society. Rawls, John B. (1999). A Theory of Justice: Revised Edition. p.266

Economic inequality has been a central concern in both academic research and policy discussions. One of the fundamental questions in this area is how monetary policy influences inequality and, conversely, how inequality affects the transmission and effectiveness of monetary policy. As a result, a fast-growing literature (e.g., Kaplan et al., 2018; Auclert, 2019; Bilbiie, 2024) has studied the role of monetary policy through the lenses of heterogeneous agent New Keynesian (HANK) models, which capture the consequences of monetary policy on different households.

In most of the literature on monetary policy, the central bank's objective traditionally consists of a utilitarian welfare function that treats all agents' utility equally. However, this approach may not adequately address concerns related to inequality, as it implicitly assumes that the benefits of macroeconomic stabilisation are distributed proportionally across all households. An alternative perspective, inspired by Rawls' Difference Principle, suggests that policies should maximise the welfare of the least advantaged members of society.

We build on this idea by examining how central banks can incorporate inequality considerations into their monetary policy framework within a stylised, tractable HANK (THANK) model as in Bilbiie (2024).[1] The model consists of two types of households, savers and hand-to-mouth households, who have no access to financial markets. For this reason, they cannot smooth their consumption over time. Households switch from one type to another according to a Markov chain. Thus, a precautionary savings motive arises for savers who face an idiosyncratic risk of becoming hand-to-mouth. Importantly, we assume that wages are sticky, following Erceg et al. (2000), in which individual households set wages for their own differentiated labour types subject to Calvo frictions. This assumption implies that wage dispersion generates heterogeneity in hours worked across households: low-wage households supply more labour, while high-wage households sup-

[1] We do not study optimal fiscal policy under a Rawlsian planner because, in our framework, such a planner would use fiscal instruments to transfer virtually all profit dividends from savers to hand-to-mouth households, making it an excessively extreme and unrealistic scenario.

ply less. Firms reallocate demand towards cheaper labour types, thereby lowering their average marginal cost and increasing aggregate profits. Since savers own the firms, they receive the profits, while hand-to-mouth households rely primarily on labour income and government transfers.[2] Therefore, wage dispersion effectively causes a redistribution of hand-to-mouth households to savers, increasing inequality between the two types.

In such a context, we analyse the implications of a Rawlsian central bank, which deviates from the standard utilitarian objective as it places relatively more weight on the welfare of hand-to-mouth households than on savers. More specifically, we derive a second-order approximation of the welfare loss function under a non-utilitarian central bank. Due to the presence of sticky wages, the welfare loss function features three terms, namely squared price inflation, squared wage inflation and square output gap. We find that a Rawlsian welfare loss function has a larger weight on wage inflation and output gap stabilisation compared to a utilitarian welfare loss function. From a quantitative perspective, we find that, both under a Rawlsian and a utilitarian central bank, the weight on wage inflation is substantially larger than that on price inflation (one order of magnitude smaller) and on output gap (three orders of magnitude smaller). The reason a Rawlsian central bank focuses more strongly on mitigating wage fluctuations is that wage dispersion increases firms' profits, which accrue mainly to savers, thereby reducing the welfare of hand-to-mouth households whose primary source of income is labour.

We then test these theoretical predictions using a medium-scale THANK model, following Bilbiie et al. (2023), estimated on U.S. data with both standard macroeconomic series and measures of inequality. The model extends the baseline THANK framework by adding physical capital accumulation together with further nominal and real frictions. These elements are essential for capturing business cycle dynamics in line with the data and for assessing how monetary policy affects inequality. At the same time, the richer model remains tractable and provides a realistic setting for counterfactual policy experiments, allowing us to examine how the economy and inequality would evolve if either a Rawlsian or a utilitarian central bank followed the optimal policy under commitment. In line with our theoretical analysis, the numerical results suggest that a Rawlsian

[2]Transfers to hand-to-mouth households are fully financed by taxes on firm profits. However, because transfers are lump-sum and do not respond to the distributional consequences of wage dispersion, they do not fully offset the increase in inequality.

policy delivers lower wage volatility, though at the cost of greater variability in price inflation and the output gap. It also reduces the volatility of inequality measures. However, the counterfactual analysis also points out that, from a quantitative perspective, the differences in outcomes between the utilitarian and Rawlsian optimal policies are negligible, as both substantially reduce the volatility of macroeconomic aggregates and inequality measures relative to the actual data. We conclude that, provided monetary policy is conducted optimally, the conventional utilitarian approach does not lead to worse outcomes in either macroeconomic performance or inequality.

1.1 Related Literature

Our paper builds on three main strands of literature. The first strand focuses on the development of two-agent New Keynesian (TANK) and HANK models to understand how agent heterogeneity affects the transmission of business cycle shocks and economic policies, particularly monetary policy. Seminal contributions include McKay and Reis (2016), Kaplan et al. (2018), and Auclert (2019), who explore how household heterogeneity impacts macroeconomic dynamics. Iacoviello (2005) investigates credit constraints and house prices in monetary transmission in a model with savers, borrowers, and entrepreneurs. More recently, Debortoli and Galí (2024) showed that a well-calibrated TANK model, abstracting from idiosyncratic income risk, can approximate the aggregate effects of heterogeneity and its main transmission channels. Bilbiie et al. (2023) develop and estimate a medium-scale New Keynesian model with tractable household heterogeneity, showing that inequality amplifies economic downturns through a precautionary saving motive.

The second strand of literature studies optimal monetary policy within TANK and HANK models. This is a fast-growing field with many important contributions. Bilbiie (2008) examines optimal monetary policy in a TANK model where a fraction of agents are hand-to-mouth and have limited access to bond markets. More recent papers by Bilbiie (2020, 2024) build on this framework. Nisticò (2016) examines optimal monetary policy in a setting where all agents are non-Ricardian. The paper finds that under both discretionary and commitment-based policies, price stability may no longer be optimal. The seminal work of Bhandari et al. (2021) analyses optimal monetary and

fiscal policy in a quantitative HANK framework. Challe (2020) studies optimal monetary policy in a HANK model with labour market frictions and idiosyncratic unemployment risk. In this setting, contractionary cost-push or productivity shocks increase precautionary savings and reduce inflation, necessitating an accommodative monetary policy response.[3] Similarly, Bilbiie and Ragot (2021) argue that price stability is suboptimal when households self-insure against idiosyncratic risk using scarce liquid assets. Dávila and Schaab (2022) show that in HANK models, discretionary monetary policy exhibits an inflationary bias, as a utilitarian planner has an incentive to redistribute towards indebted households with higher marginal utility. Nuño and Thomas (2022) study optimal monetary policy under commitment in an economy with uninsurable idiosyncratic risk, long-term nominal bonds, and inflation costs. Their model introduces two key transmission channels: a Fisher channel, where inflation influences bond prices and redistributes wealth between debtors and creditors, and a liquidity channel. The paper finds that optimal policy involves front-loaded inflation and significant consumption and welfare redistribution compared to a zero-inflation benchmark. Acharya et al. (2023) study optimal monetary policy in an analytically tractable HANK model with rich cross-sectional heterogeneity. In their model, the planner's objective function explicitly includes reducing consumption inequality, alongside inflation and output stabilisation. When income risk is countercyclical, optimal policy mitigates output declines during recessions to curb rising inequality. McKay and Wolf (2023) study optimal policy rules in business cycle models with nominal rigidities and heterogeneous households. They argue that even a central bank with an explicit distributional mandate would not deviate much from conventional policy prescriptions. Bilbiie et al. (2024) highlight the interconnected nature of stabilisation and redistribution in models with heterogeneity, imperfect insurance, and nominal rigidity. Their analysis suggests that fiscal and monetary policies are inherently linked in aggregate demand management. Le Grand et al. (2024) also study whether monetary policy should care about redistribution in a HANK model with rich cross-sectional heterogeneity. They find that the importance monetary policy should place on redistribution crucially depends on fiscal policy. Bonciani and Oh (2025) analyse optimal monetary policy at the ZLB in a TANK framework where the central bank employs both quantitative easing and forward guidance.

[3] Bonciani and Oh (2021) examine a zero-liquidity HANK model akin to Challe (2020) and find that market incompleteness does not significantly amplify business cycle fluctuations at the zero lower bound.

Finally, from a methodological perspective, our analysis builds on established techniques for studying optimal policy in macroeconomic models (Rotemberg and Woodford, 1997; Khan et al., 2003; Woodford, 2003; Schmitt-Grohé and Uribe, 2004, 2007). In particular, we follow Benigno and Woodford (2004, 2005) to derive a second-order approximation of the welfare loss function under a non-utilitarian central bank. As in models with a distorted steady state, the assumption of a non-utilitarian planner introduces a non-zero linear term in the approximated welfare loss function. To eliminate this term, we employ a second-order approximation of the equilibrium condition linking inflation and output (i.e., the nonlinear New Keynesian Phillips curve). The resulting quadratic loss function retains the standard terms for inflation and output stabilisation and an additional "inequality-relevant" output gap term. Hence, we show that the Rawlsian central bank faces a different trade-off between inflation and output stabilisation compared to the canonical utilitarian central bank.

The remainder of the article is structured as follows. Sections 2 and 3 develop the baseline THANK model, derive the social welfare function, and characterise the optimal behaviour of a central bank that follows Rawls' difference principle. In Section 4, we extend this framework to a medium-scale HANK model. Section 5 presents the quantitative results from counterfactual analyses, comparing alternative monetary policies. Section 6 provides concluding remarks.

2 Baseline Heterogeneous Agent New Keynesian Model

This section introduces a baseline THANK model (see, e.g., Bilbiie, 2024) to study the interaction between monetary policy and inequality. Unlike Bilbiie (2024), we account for wage stickiness à la Erceg et al. (2000) in addition to price stickiness, which generates real wage dispersion under decentralised wage setting. Together with household heterogeneity, this setting allows us to capture the distributional consequences of monetary policy. Therefore, this framework enables us to study optimal monetary policy under a Rawlsian central bank.

2.1 Households

The economy consists of two types of households: savers (denoted S), who constitute a fraction $1-\lambda$ of the population, and hand-to-mouth households (denoted H), who make up the remaining fraction λ. Each household group $h \in \{S, H\}$ maximises its utility, which depends on consumption $C_{h,t}$ and labour supply $N_{h,t}$. The period utility function for a household of type h is given by:

$$U_{h,t}(j) = \frac{C_{h,t}(j)^{1-\sigma} - 1}{1-\sigma} - \varphi_h \frac{N_{h,t}(j)^{1+\nu}}{1+\nu}, \tag{1}$$

where σ is the coefficient of relative risk aversion, ν is the inverse of the Frisch elasticity of labour supply, and φ_h is a disutility parameter specific to each household type.

All intertemporal decisions are assumed to be made by a representative family head, who maximises the weighted average welfare of both household types, defined as:

$$\mathbb{W}_t = (1-\lambda) U_{S,t} + \lambda U_{H,t} + \beta E_t \mathbb{W}_{t+1}, \tag{2}$$

where β is the discount factor. This assumption simplifies the aggregation of preferences while preserving heterogeneity in consumption and saving behaviour. Moreover, it is assumed that the family head faces limits to risk sharing: they can freely transfer resources across members of the same household group (after idiosyncratic uncertainty is realised), but not across groups.

S-type households have access to financial markets and can smooth consumption by holding one-period nominal bonds $B_{S,t}$. Their budget constraint is:

$$P_t C_{S,t} + B_{S,t} = s R_{t-1} B_{S,t-1} + (1+\tau_w) \int_0^1 W_{S,t}(j) N_{S,t}(j)\, dj + \frac{1}{1-\lambda} P_t D_t + P_t T_{S,t}, \tag{3}$$

where P_t is the aggregate price level, R_t is the gross nominal interest rate, W_t is the nominal wage, D_t represents firm profits distributed to S-type households, and $T_{S,t}$ is a lump-sum transfer. s is the probability of an S-type household remaining an S-type in the next period. All savers share the interest income from the fraction s of time $t-1$ bonds. The remaining $1-s$ fraction of $t-1$ bonds is held by those H-type households who used to be S-type in $t-1$. Moreover, $\tau_w = \xi_w$ is a

subsidy that offsets steady-state distortions.

H-type households consume their entire income each period and do not participate in bond markets. Their budget constraint is:

$$P_t C_{H,t} = (1+\tau_w) \int_0^1 W_{H,t}(j) N_{H,t}(j) dj + P_t T_{H,t} + \frac{\lambda}{1-\lambda}(1-s) R_{t-1} B_{S,t-1}, \tag{4}$$

where $T_{H,t}$ is a lump-sum transfer.

Households supply differentiated labour varieties $N_{h,t}(j)$, facing demand:

$$N_{h,t}(j) = \left(\frac{W_{h,t}(j)}{W_{h,t}} \right)^{-\frac{1+\xi_w}{\xi_w}} N_{h,t}, \tag{5}$$

where $W_{h,t} = \left(\int_0^1 W_{h,t}(j)^{-\frac{1}{\xi_w}} dj \right)^{-\xi_w}$ is the aggregate wage index and ξ_w is the wage markup. Following Erceg et al. (2000), wages are set in a Calvo framework, with a fraction $1-\theta_w$ resetting optimally each period.

2.2 Firms

The production side of the economy consists of two types of firms: final goods firms and intermediate goods firms, operating under monopolistic competition with Calvo (1983)-style price stickiness.

Final goods firms' output Y_t by aggregating intermediate goods $Y_t(i)$, using a constant elasticity of substitutions technology:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\xi_p}} di \right)^{1+\xi_p}, \tag{6}$$

where ξ_p is the price markup. The demand for each intermediate good is:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\xi_p}{\xi_p}} Y_t \tag{7}$$

and the aggregate price index is:

$$P_t = \left(\int_0^1 P_t\left(i\right)^{-\frac{1}{\xi_p}} di \right)^{-\xi_p} \tag{8}$$

A continuum of intermediate goods firms produce output using labour as the sole input, with a linear production function:

$$Y_t\left(i\right) = A_t\left(L_{S,t}\left(i\right) + L_{H,t}\left(i\right)\right), \tag{9}$$

where $L_{h,t}\left(i\right)$ is the labour input hired by firm i. Firms face Calvo-type price stickiness, where each period a fraction $1-\theta_p$ of firms can reset their prices optimally, while the remaining θ_p keep their prices unchanged. An optimising firm chooses its reset price $P_t^{\#}\left(i\right)$ to maximise its expected discounted profits:

$$\max_{P_t^{\#}(i)} E_t \sum_{k=0}^{\infty} {\theta_p}^k \beta^k \Lambda_{S,t+k} \left(\left(1+\tau_p\right) P_t^{\#}\left(i\right) Y_{t+k}\left(i\right) - MC_{t+k} Y_{t+k}\left(i\right) - T_{t+k} \right), \tag{10}$$

subject to its demand (7). MC_t denotes the nominal marginal cost. $\tau_p = \xi_p$ is a subsidy offsetting steady-state distortions, and T_t are lump-sum taxes. The aggregate price level evolves as:

$$P_t = \left(\left(1-\theta_p\right) P_t^{\#-\frac{1}{\xi_p}} + \theta_p {P_{t-1}}^{-\frac{1}{\xi_p}} \right)^{-\xi_p}. \tag{11}$$

2.3 Government

The government balances its budget through lump-sum transfers $T_{S,t}$ and $T_{H,t}$ to S-type and H-type households, respectively, financed by taxing firm profits. The transfers are defined as:

$$T_{S,t} = -\frac{1}{1-\lambda}\tau_d D_t - \tau_w \frac{W_{S,t}}{P_t} N_{S,t}, \tag{12}$$

$$T_{H,t} = \frac{1}{\lambda}\tau_d D_t - \tau_w \frac{W_{H,t}}{P_t} N_{H,t}, \tag{13}$$

where τ_d is the tax rate on profits D_t. The government budget constraint ensures:

$$(1-\lambda)\, T_{S,t} + \lambda T_{H,t} = 0, \tag{14}$$

implying that transfers redistribute income between household types.

2.4 Market Clearing

The model is closed with market-clearing conditions for goods, labour, and bond markets.

The goods market-clearing condition is given by:

$$Y_t = (1-\lambda)\, C_{S,t} + \lambda C_{H,t}, \tag{15}$$

stating that output should equal aggregate consumption. The labour markets clear with labour supply from each household type:

$$(1-\lambda)\left(\int_0^1 N_{S,t}\,(j)^{\frac{1}{1+\xi_w}}\, dj\right)^{1+\xi_w} = (1-\lambda)\, N_{S,t} = L_{S,t} = \int_0^1 L_{S,t}\,(i)\, di, \tag{16}$$

$$\lambda\left(\int_0^1 N_{H,t}\,(j)^{\frac{1}{1+\xi_w}}\, dj\right)^{1+\xi_w} = \lambda N_{H,t} = L_{H,t} = \int_0^1 L_{H,t}\,(i)\, di. \tag{17}$$

It bears noting that due to price dispersion, induced by the Calvo pricing assumption, actual production is adjusted by a dispersion term Δ_t:

$$\Delta_t = \int_0^1 \left(\frac{P_t^{\#}\,(i)}{P_t}\right)^{-\frac{1+\xi_p}{\xi_p}} di. \tag{18}$$

Therefore, the aggregate production function becomes:

$$\Delta_t Y_t = A_t\left(L_{S,t} + L_{H,t}\right), \tag{19}$$

Finally, we assume that bonds are in zero net supply:

$$B_{S,t} = 0. \tag{20}$$

3 Analytical Instights

3.1 Welfare-Theoretic Loss Function

This section presents the non-utilitarian welfare loss function derived from the model. We start by defining the welfare function as a weighted average of the utilities of S-type and H-type households. Then, we derive a second-order approximation of the aggregate utility function (Woodford, 2003, Galí, 2015, and Benigno and Woodford, 2004, 2005):

$$\mathbb{W}_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left((1-\lambda^\star) U_{S,t} + \lambda^\star U_{H,t} \right) = -\frac{U_C C}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t + t.i.p. + O\left(||\xi||^3 \right), \tag{21}$$

where $\lambda^\star \geq \lambda$ denotes the weight that the central bank attaches to the H-type households' utility, $\mathbb{L}_t$ is the period t loss function, $t.i.p.$ are the terms independent of monetary policy, and $O\left(||\xi||^3\right)$ denotes all relevant terms that are of third or higher order. Under a utilitarian central bank, $\lambda^\star = \lambda$, so each household's utility is weighted by its population share. Under a Rawlsian central bank, $\lambda^\star > \lambda$, meaning that the central bank places relatively more weight on the welfare of hand-to-mouth households. We can then write the period-t welfare loss as follows:

$$\begin{aligned} \mathbb{L}_t = & \underbrace{\frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p} {\pi_{p,t}}^2 + \frac{\frac{1+\xi_w}{\xi_w}}{\gamma_w} {\pi_{w,t}}^2 + (\nu+\sigma)\, \tilde{y}_t^2}_{\text{RANK: Standard Volatility}} + \underbrace{\lambda\,(1-\lambda)\left(\sigma\,(c_{S,t}-c_{H,t})^2 + \nu\,(n_{S,t}-n_{H,t})^2\right)}_{\text{HANK (Utilitarian): Inequality Volatility } (A)} \\ & \underbrace{+2\,(\lambda^\star-\lambda)\left(c_{S,t}-c_{H,t}-n_{S,t}+n_{H,t}+\frac{1}{2}(1-\sigma)\left({c_{S,t}}^2-{c_{H,t}}^2\right)-\frac{1}{2}(1+\nu)\left({n_{S,t}}^2-{n_{H,t}}^2\right)\right)}_{\text{HANK (Rawlsian): Inequality Level } (B)}, \end{aligned} \tag{22}$$

where π_t, $\pi_{w,t}$, and $\tilde{y}_t$ denote the log-deviations of price inflation, wage inflation, and the output gap from their respective steady-state values. Similarly, lower-case variables (e.g., $c_{H,t}$) represent log-deviations from steady state.

Equation (22) highlights how the Rawlsian welfare loss function implied by our model relates to the standard formulations in both representative agent New Keynesian (RANK) and HANK settings. In a canonical RANK framework, the period-t welfare loss depends solely on the volatility of price inflation, wage inflation, and the output gap. In a HANK model with a utilitarian central bank, the welfare loss function is slightly more complex, as it also contains a term capturing the volatility of inequality between savers and hand-to-mouth households. This inequality term arises from fluctuations in the consumption gap and hours-worked gap between the two household types, as shown in Bilbiie (2024). Under a Rawlsian objective, the welfare loss function further includes a term reflecting the level of inequality itself. In other words, a Rawlsian central bank seeks not only to dampen cyclical fluctuations in inequality but also to reduce its level, thereby improving the welfare of the least advantaged households (i.e, the hand-to-mouth households).

As shown in Appendix A, the period loss function $\mathbb{L}_t$ can be rewritten as:

$$\mathbb{L}_t = \alpha_p {\pi_{p,t}}^2 + \alpha_w {\pi_{w,t}}^2 + \alpha_y \tilde{y}_t^2, \tag{23}$$

where the coefficients α_p, α_w, and α_y are defined as follows:

$$\alpha_p \equiv \frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}, \tag{24}$$

$$\alpha_w \equiv \frac{\frac{1+\xi_w}{\xi_w}}{\gamma_w}\left(1 + \frac{(\lambda - \tau_d)(\lambda^\star - \lambda)}{\lambda(1-\lambda)}(1+\nu)\right), \tag{25}$$

$$\alpha_y \equiv (\nu + \sigma)\left(1 + \frac{(\lambda - \tau_d)^2}{\lambda(1-\lambda)}\nu\sigma\left(1 + \frac{\lambda^\star - \lambda}{\lambda(1-\lambda)}\right)\right), \tag{26}$$

each representing the relative welfare cost of fluctuations in price inflation, wage inflation, and the output gap, respectively.[4]

From Equations (25) and (26), we notice that a Rawlsian central bank ($\lambda^\star > \lambda$) will place more

[4]Note that the additional variance terms of $\Phi_{w,t} \equiv \pi_{w,t} - \beta E_t \pi_{w,t+1}$ have a minor effect on welfare loss qualitatively, compared with the variance terms of price inflation, wage inflation, and the output gap. For simplicity, we ignore these terms.

Table 1: Baseline Calibration

Parameter	Description	Value
β	Discount factor	0.99
λ	Share of H-type households	0.2
σ	Risk aversion	1
ν	Inverse labour supply elasticity	1
s	Probability of remaining S-type households	0.96
ξ_p	Price markup	0.1
ξ_w	Wage markup	0.1
θ_p	Price stickiness	0.75
θ_w	Wage stickiness	0.75
τ_d	Tax rate on profits	0.1

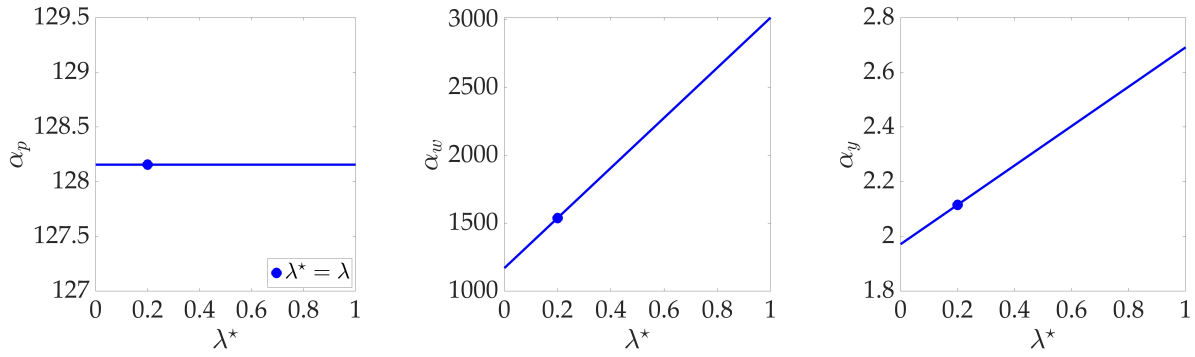
weight on stabilising wage inflation and the output gap compared to a utilitarian central bank. The weight on price inflation α_p (Equation (24)) is instead independent of $\lambda^\star$ and, hence, will be the same under Rawlsian or a utilitarian central bank.

3.2 Rawls' Difference Principle

To simplify the interpretation of the loss function's coefficients and get a sense of the orders of magnitude, Figure 1 shows how α_p, α_w and α_y change as we vary the welfare weight on hand-to-mouth households $\lambda^\star$. The results are obtained using the calibration shown in Table 1, in line with Bilbiie (2024).

As mentioned above, α_p (Panel 1(a)) does not depend on $\lambda^\star$. In contrast, α_w and α_y (Panels 1(b) and 1(c)) increase with $\lambda^\star$, meaning that the more weight the central bank places on the utility of hand-to-mouth households, the greater the weight it assigns to stabilising wage inflation and, to a lesser extent, the output gap.

The increasing weight on wage-inflation stabilisation arises from the real distortions introduced by nominal wage rigidity under decentralised wage setting. In our model, each household supplies a differentiated type of labour and sets its nominal wage subject to Calvo frictions. Because only a fraction of households can reset their wages in any given period, the resulting staggered wage setting leads to persistent real-wage dispersion. Firms, facing a CES labour aggregator, reallo-



(a) Price Inflation (b) Wage Inflation (c) Output Gap

Figure 1: Welfare Loss Coefficients and the Central Bank's Preference for H-type Households. $\lambda^\star$ is the social welfare weight attached to H-type households. α_p, α_w, and α_y are the coefficients in the welfare-loss function associated with price inflation, wage inflation, and the output gap. The analysis is conducted using the calibration in Table 1.

cate demand towards relatively cheaper labour types, which reduces their average marginal cost. This generates intratemporal misallocation, whereby the marginal rate of substitution between consumption and labour differs across households, even within the same type. At the second order of approximation, this wage dispersion also raises aggregate firm profits, as lower marginal costs translate into higher mark-ups. Crucially, these profits accrue mainly to savers, who own the firms, while hand-to-mouth households depend almost entirely on labour income. Consequently, higher wage dispersion redistributes income from workers to capital owners, widening inequality between the two household groups. As the Rawlsian planner attaches greater welfare weight to the least advantaged—namely the hand-to-mouth households—the planner internalises this unequal effect of wage-inflation volatility. Hence, stabilising wage inflation becomes essential not only to reduce allocative inefficiencies in the labour market but also to limit the income redistribution towards savers, which exacerbates inequality. It should also be noted that, while both coefficients α_w and α_y are increasing in $\lambda^\star$, the size of these coefficients is completely different. α_w is significantly larger than α_p and α_y. In particular, α_y is roughly 3 orders of magnitude smaller than α_w. Hence, we conclude that stabilising wage inflation is much more important than stabilising price inflation and the output gap.

4 Medium-Scale Heterogeneous Agent New Keynesian Model

This section extends the baseline THANK model presented in Section 2 into a medium-scale framework that incorporates additional real and nominal frictions, as well as richer household heterogeneity, to study optimal monetary policy under Rawls' difference principle. The medium-scale model builds on the two-agent structure with savers (S) and hand-to-mouth (H) households while introducing capital accumulation, investment adjustment costs, wage and price stickiness with indexation, and a monetary policy rule. This setup allows us to estimate the model using both aggregate and inequality data, following methodologies akin to those in Justiniano et al. (2013) and Bilbiie et al. (2023), and to evaluate the implications of Rawlsian monetary policy through counterfactual exercises.

The motivation for this extension is twofold. First, a medium-scale model captures key business cycle dynamics, such as the interaction between investment, labour supply, and consumption. This is critical for understanding and quantifying how monetary policy affects inequality. Second, it provides a tractable yet realistic framework to operationalise Rawls' difference principle, i.e., maximising the welfare of the least advantaged (here, the H-type households). Through the lens of this model, we are able to assess how countercyclical income risk and precautionary savings (key drivers of business cycle amplification) interact with a Rawlsian monetary policy stance.

4.1 Households

The economy consists of two household types: S-type households, with a population share $1-\lambda$, and H-type households, with a share λ. Each household type $h \in \{S, H\}$ maximises utility over consumption $C_{h,t}(j)$ and labour supply $N_{h,t}(j)$, where j indexes individual household members within each type. The period utility function is:

$$U_{h,t}(j) = b_t \left(\log\left(C_{h,t}(j) - \eta C_{h,t-1}\right) - \varphi_h \varphi_t \frac{N_{h,t}(j)^{1+\nu_h}}{1+\nu_h} \right), \tag{27}$$

where b_t is a preference shock, φ_t is a labour taste shock, η_h captures external habit formation specific to each type, ν_h is the inverse Frisch elasticity of labour supply, and φ_h is the disutility of labour. Habit formation generates persistence in consumption dynamics.

To maintain tractability, we assume a representative family head maximises the weighted intertemporal welfare of both household types:

$$\mathbb{W}_t = (1-\lambda)\, U_{S,t} + \lambda U_{H,t} + \beta E_t \mathbb{W}_{t+1}, \tag{28}$$

where β is the discount factor, and $U_{h,t} = \int_0^1 U_{h,t}(j)\, dj$ aggregates utility within each type. This family head assumption simplifies the analysis while preserving heterogeneity in consumption and saving behaviour across groups. This family head, however, faces limits to risk sharing: they can freely transfer resources across members of the same household group (after idiosyncratic uncertainty is realised), but not across groups.

S-type households have access to financial markets, allowing them to smooth consumption intertemporally via nominal bonds $B_{S,t}$ and physical capital $\bar{K}_{S,t}$. Their budget constraint is:

$$\begin{aligned} P_t C_{S,t} + P_t I_{S,t} + B_{S,t} = s_t \zeta_{t-1} R_{t-1} B_{S,t-1} + \int_0^1 W_t(j)\, N_{S,t}(j)\, dj + R_t^K u_t \bar{K}_{S,t-1} \\ - P_t a(u_t)\, \bar{K}_{S,t-1} + \frac{1}{1-\lambda} P_t D_t + P_t T_{S,t}, \end{aligned} \tag{29}$$

where P_t is the price level, $I_{S,t}$ is investment, R_t is the nominal interest rate, ζ_t is a risk premium shock, $W_t(j)$ is the nominal wage for labour variety j, R_t^K is the rental rate of capital, u_t is the capital utilisation rate with cost $a(u_t) = \frac{r^k}{\chi}\left(e^{\chi(u_t-1)} - 1\right)$, D_t is firm profits distributed to S-type households, and $T_{S,t}$ is a lump-sum transfer. The term $s_t = s_0 \left(\frac{X_t}{g_A{}^4 X_{t-4}}\right)^{\frac{s_1}{4}}$ governs the fraction of bond returns retained by S-type households, with X_t as GDP explained below, reflecting cyclical risk. $g_{A,t} \equiv \frac{A_t}{A_{t-1}}$ denotes the technology growth rate, and g_A is its steady-state value. Capital evolves according to:

$$\bar{K}_{S,t} = (1-\delta)\, \bar{K}_{S,t-1} + \mu_t \left(1 - \frac{\kappa_I}{2}\left(\frac{I_{S,t}}{I_{S,t-1}} - 1\right)^2\right) I_{S,t}, \tag{30}$$

where δ is depreciation, μ_t is an investment-specific technology shock, and κ_I captures adjustment

costs. Effective capital is as follows:

$$K_{S,t} = u_t \bar{K}_{S,t-1}. \tag{31}$$

H-type households consume all income each period, lacking access to bond or capital markets. Their budget constraint is:

$$P_t C_{H,t} = \int_0^1 W_t(j) N_{H,t}(j) dj + P_t T_{H,t} + \frac{\lambda}{1-\lambda}(1-s_t) R_{t-1} B_{S,t-1}, \tag{32}$$

where the last term reflects interest income from savers who transitioned to H, scaled by population shares. This setup captures the high marginal propensity to consume (MPC) of H-type households.

Households supply differentiated labour varieties $N_{h,t}(j)$, facing demand:

$$N_{h,t}(j) = \left(\frac{W_{h,t}(j)}{W_{h,t}}\right)^{-\frac{1+\xi_{w,t}}{\xi_{w,t}}} N_{h,t}, \tag{33}$$

where $W_{h,t} = \left(\int_0^1 W_{h,t}(j)^{-\frac{1}{\xi_{w,t}}} dj\right)^{-\xi_{w,t}}$ is the aggregate wage index, and $\xi_{w,t}$ is a time-varying wage markup shock. Wages are set in a Calvo framework, with a fraction $1-\theta_{w,h}$ resetting optimally each period, according to the indexation rule:

$$W_{h,t}(j) = W_{h,t-1}(j) \left(\Pi_{t-1} g_{A,t-1}\right)^{\iota_w} \left(\Pi g_A\right)^{1-\iota_w}, \tag{34}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is price inflation.

4.2 Firms

Firms operate in two sectors: final goods firms aggregate intermediate goods $Y_t(i)$ into output Y_t using a CES technology:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\xi_{p,t}}} di\right)^{1+\xi_{p,t}}, \tag{35}$$

yielding demand $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\xi_{p,t}}{\xi_{p,t}}} Y_t$ and price index $P_t = \left(\int_0^1 P_t(i)^{-\frac{1}{\xi_{p,t}}} di\right)^{-\xi_{p,t}}$.

Intermediate firms produce using a nested CES production function:

$$Y_t(i) = \left(\omega {A_t}^{\theta} L_{H,t}(i)^{\theta} + (1-\omega)\left(\alpha a_{K,t} K_t(i)^{\varrho} + (1-\alpha) {A_t}^{\varrho} L_{S,t}(i)^{\varrho}\right)^{\frac{\theta}{\varrho}}\right)^{\frac{1}{\theta}} - A_t F, \tag{36}$$

where $L_{h,t}(i)$ is labour input from type h, $K_t(i)$ is capital input, A_t and $a_{K,t}$ are technology shocks, F is a fixed cost, θ and ϱ govern the elasticity of substitution between inputs. Prices are set under Calvo pricing, with a fraction $1-\theta_p$ resetting optimally, according to the indexation rule:

$$P_t(i) = P_{t-1}(i) \Pi_{t-1}{}^{\iota_p} \Pi^{1-\iota_p}. \tag{37}$$

4.3 Government

The government balances its budget via lump-sum transfers and taxes:

$$(1-\lambda) T_{S,t} + \lambda T_{H,t} + G_t = 0, \tag{38}$$

with a transfer rule:

$$\frac{\Omega_{H,t} + T_{H,t}}{\Omega_{S,t} + T_{S,t}} = \tau_0 \left(\frac{\Omega_{H,t}}{\Omega_{S,t}} / \frac{\Omega_H}{\Omega_S}\right)^{\tau_1}, \tag{39}$$

where $\Omega_{h,t}$ is pre-tax income. This rule parametrises fiscal redistribution's impact on inequality.

Public spending is determined exogenously as a time-varying fraction of output:

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t, \tag{40}$$

where g_t is a government spending shock.

4.4 Central Bank

Monetary policy follows a Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left(\left(\frac{\prod_{k=0}^{3} \Pi_{t-k}}{\Pi_t^{*4}} \right)^{\frac{\phi_\pi}{4}} \left(\frac{X_t}{{g_A}^4 X_{t-4}} \right)^{\frac{\phi_X}{4}} \right)^{1-\rho_R} \xi_{R,t}, \tag{41}$$

where $\xi_{R,t}$ is a monetary policy shock, Π_t^* is an inflation target shock, and ϕ_π and ϕ_X reflect responses to inflation and GDP growth, where GDP is defined as:

$$X_t \equiv C_t + (1-\lambda)\, I_{S,t} + G_t. \tag{42}$$

4.5 Market Clearing

Goods market clearing is:

$$Y_t = C_t + (1-\lambda)\, I_{S,t} + G_t + (1-\lambda)\, a\left(u_t\right) \bar{K}_{S,t-1}, \tag{43}$$

with $C_t = (1-\lambda)\, C_{S,t} + \lambda C_{H,t}$. Capital and labour markets clear as follows:

$$(1-\lambda)\, K_{S,t} = \int_0^1 K_t\left(i\right) di, \tag{44}$$

$$(1-\lambda) \left(\int_0^1 N_{S,t}\left(j\right)^{\frac{1}{1+\xi_{w,t}}} dj \right)^{1+\xi_{w,t}} = (1-\lambda)\, N_{S,t} = L_{S,t} = \int_0^1 L_{S,t}\left(i\right) di, \tag{45}$$

$$\lambda \left(\int_0^1 N_{H,t}\left(j\right)^{\frac{1}{1+\xi_{w,t}}} dj \right)^{1+\xi_{w,t}} = \lambda N_{H,t} = L_{H,t} = \int_0^1 L_{H,t}\left(i\right) di. \tag{46}$$

Total hours worked N_t in the economy are defined as the population-weighted sum of hours supplied by two households:

$$N_t = (1-\lambda)\, N_{S,t} + \lambda N_{H,t}. \tag{47}$$

Moreover, the aggregate hourly wage W_t is computed as the weighted average of wages paid to each household type, normalised by total hours worked:

$$W_t = \frac{(1-\lambda)\, W_{S,t} N_{S,t} + \lambda W_{H,t} N_{H,t}}{N_t}. \tag{48}$$

Finally, the net supply of bonds is zero in equilibrium:

$$B_{S,t} = 0. \tag{49}$$

4.6 Exogenous Processes

The model contains eleven exogenous variables: technology growth ($g_{A,t}$), a preference shock (b_t), a risk premium (ζ_t), investment-specific technology shock (μ_t), capital technology shock ($a_{K,t}$), wage and price markup shocks ($\xi_{w,t}$ and $\xi_{p,t}$), government consumption (g_t), a monetary policy shock ($\xi_{R,t}$), an inflation target shock (Π_t^*), and labour taste shock (φ_t). These exogenous variables evolve according to the following processes:

$$\log g_{A,t} = \left(1-\rho_{g_A}\right) \log g_A + \rho_{g_A} \log g_{A,t-1} + \sigma_{g_A} e_{g_A,t}, \tag{50}$$

$$\log b_t = \rho_b \log b_{t-1} + \sigma_b e_{b,t}, \tag{51}$$

$$\log \zeta_t = \rho_\zeta \log \zeta_{t-1} + \sigma_\zeta e_{\zeta,t}, \tag{52}$$

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \sigma_\mu e_{\mu,t}, \tag{53}$$

$$\log a_{K,t} = \rho_{a_K} \log a_{K,t-1} + \sigma_{a_K} e_{a_K,t}, \tag{54}$$

$$\log\left(1+\xi_{w,t}\right) = \log\left(1+\xi_w\right) + \sigma_{\xi_w} e_{\xi_w,t}, \tag{55}$$

$$\log\left(1+\xi_{p,t}\right) = \left(1-\rho_{\xi_p}\right) \log\left(1+\xi_p\right) + \rho_{\xi_p} \log\left(1+\xi_{p,t-1}\right) + \sigma_{\xi_p} e_{\xi_p,t}, \tag{56}$$

$$\log g_t = \left(1-\rho_g\right) \log g + \rho_g \log g_{t-1} + \sigma_g e_{g,t}, \tag{57}$$

$$\log \xi_{R,t} = \sigma_{\xi_R} e_{\xi_R,t}, \tag{58}$$

$$\log \Pi_t^* = (1 - \rho_{\Pi^*}) \log \Pi^* + \rho_{\Pi^*} \log \Pi_{t-1}^* + \sigma_{\Pi^*} e_{\Pi^*,t}, \tag{59}$$

$$\log \varphi_t = \rho_\varphi \log \varphi_{t-1} + \sigma_\varphi e_{\varphi,t}. \tag{60}$$

5 Numerical Results

5.1 Bayesian Inference

In line with Bilbiie et al. (2023), we estimate the model using data on both aggregate economic indicators and measures of inequality over time. The aggregate variables include: (i) the growth rate of per capita real GDP, (ii) real consumption, (iii) real investment, (iv) the logarithm of per capita hours worked, (v) and (vi) two measures of the growth rate of real hourly wages, (vii) the (GDP deflator) inflation rate, and (viii) the federal funds rate. The inequality measures are the cross-sectional standard deviations of the logarithm of (ix) pre-tax labour earnings and (x) disposable income.

Notably, following Justiniano et al. (2013), we utilise two wage measures to filter out high-frequency wage fluctuations that are unrelated to macroeconomic trends. The data that we use are hourly employee compensation measures from the U.S. Bureau of Economic Analysis and average hourly earnings of employees from the U.S. Bureau of Labor Statistics. Since our model maps two distinct wage series to a single theoretical wage concept, we introduce a loading coefficient, Γ, for the second wage series. Further, we allow for measurement, *or* observation errors when matching these series to the model's theoretical wage construct as follows:

$$\begin{bmatrix} \Delta \log w_{1,t} \\ \Delta \log w_{2,t} \end{bmatrix} = \begin{bmatrix} 1 \\ \Gamma \end{bmatrix} \Delta \log w_t + \begin{bmatrix} \sigma_{g_{w_1}} e_{w_1,t} \\ \sigma_{g_{w_2}} e_{w_2,t} \end{bmatrix},$$

where $w_{1,t}$ and $w_{2,t}$ denote the two measures of real wages from the data, w_t refers to wage in our model, and $e_{w_1,t}$ and $e_{w_2,t}$ are i.i.d. observation errors with distributions $N(0,1)$. Similarly, we assume that our inequality series are observed with some noise – hence, we allow measurement

error in estimation.[5]

Series (i) through (v), (vii), and (viii) are available at a quarterly frequency from 1954Q3 to 2019Q4, while our additional measure of real hourly wages spans 1964Q2 to 2019Q4. The inequality series (ix) and (x) are only available at an annual frequency, from 1967 and 1978 respectively, to 2019. Appendix E provides detailed information on data sources and construction.

We estimate the posterior distribution of the parameters by combining the likelihood function of the log-linearised model with prior information. The likelihood function is evaluated using the Kalman filter, which can handle the mixed-frequency nature (quarterly and annual) of the aggregate and inequality data. Based on the observed data, the Kalman filter and smoother estimate the historical paths of the unobservable variables in the model, including potential and optimal output as well as the underlying shocks. These serve as key inputs in the counterfactual experiments regarding optimal policy in Section 5.2.

The population of the two types of households, as well as their transition from one type to the other, are governed by parameters that are assumed to be fixed. First, the share of H-type households (λ) is set to 0.2, consistent with the empirical consensus on the average marginal propensity to consume out of transitory income shocks (Kaplan et al., 2014; Bilbiie et al., 2023). The steady-state probability with which S agents stay S, s_0, is set to 0.987 to align with properties exhibited in the microdata on the annual changes in individuals' incomes (Bilbiie et al., 2023). Further, the quarterly depreciation rate of capital (δ) is fixed to 0.025 and the steady state ratio of government spending to GDP ($1 - 1/g$) to 0.2, corresponding to conventional values used in the literature.

The priors on the other parameters are reported in Table 2. They are generally diffuse and in line with the literature (e.g., Smets and Wouters, 2007; Justiniano et al., 2010, 2011, 2013, and Bilbiie et al., 2023). For each parameter, the table reports the mean, standard deviation (SD), mode, median, and the 5th and 95th percentiles of the posterior distribution. We assume beta prior distributions for the parameters between 0 and 1. These parameters include those that govern income shares in the production function α and ω (hereafter referred to as KORV (Krusell et al., 2000)

[5] As in Bilbiie et al. (2023), the variance of the measurement error is set to 1/16 of the variance of each series constructed from the data.

Table 2: Prior distributions and posterior parameter estimates

			Prior		Posterior			
Parameter	Description	Dist	Mean	SD	Mode	Median	5%	95%
α	KORV share parameter	B	0.5	0.25	0.1993	0.209	0.1809	0.2432
ω	KORV share parameter	B	0.5	0.25	0.0888	0.0978	0.067	0.131
$(1-\theta)^{-1}$	KORV elasticity	G	2.56	2	5.1286	5.492	4.6778	6.3611
$(1-\varrho)^{-1}$	KORV elasticity	G	2.56	2	0.9013	0.8726	0.8028	0.9297
$\bar{\xi}_p$	SS price markup	N	0.15	0.025	0.2499	0.2503	0.2118	0.2885
ι_p	Price indexation	B	0.5	0.15	0.045	0.053	0.0166	0.0961
θ_p	Price stickiness	B	0.66	0.15	0.6842	0.6833	0.64	0.7251
$100(\beta^{-1}-1)$	Discount factor	G	0.25	0.1	0.1291	0.1309	0.0491	0.2273
η_S	Habit formation (S)	B	0.5	0.1	0.8163	0.8094	0.7442	0.868
η_H	Habit formation (H)	B	0.5	0.1	0.5242	0.5	0.3574	0.6382
$100\log(L_{ss})$	SS log hours	N	0	0.5	-0.1029	-0.1054	-0.9163	0.7075
ν_S	Inverse Frisch (S)	G	3	2	2.2499	2.706	1.157	4.5571
ν_H	Inverse Frisch (H)	G	3	2	3.9737	5.3462	1.7956	9.6492
χ	Util. cost elasticity	G	5	1	3.9448	3.8827	2.321	5.5776
κ_I	Investment adjustment cost	G	4	1	2.1004	2.4556	1.5333	3.4916
$\frac{1}{2}+\frac{4}{5}s_1$	Risk elasticity	B	0.5	0.25	0.7985	0.5784	0.1555	0.9628
ι_w	Wage indexation	N	0.15	0.025	0.1486	0.1456	0.105	0.1864
$\theta_{w,S}$	Wage stickiness (S)	B	0.5	0.15	0.1014	0.1127	0.0449	0.1899
$\theta_{w,H}$	Wage stickiness (H)	B	0.66	0.15	0.6085	0.61	0.5219	0.6946
$100(\pi-1)$	SS inflation	B	0.66	0.15	0.9247	0.9203	0.8969	0.9422
ρ_R	Interest rate inertia	N	0.5	0.1	0.5041	0.5037	0.346	0.6618
ϕ_π	Monetary rule inflation reaction	B	0.6	0.2	0.7119	0.7224	0.65	0.788
ϕ_X	Monetary rule output gap reaction	N	1.7	0.3	2.3256	2.367	2.143	2.6027
τ_1	Cyclical redistribution	N	0.3	0.3	0.6343	0.6654	0.4895	0.8495
Γ	Loading coef.	U	1	0.58	0.2449	0.2594	0.1974	0.3252
$100g_A$	SS technology growth	U	1	0.58	0.2122	0.2124	0.1903	0.2356
ρ_{g_A}	Autocorr. neutral tech.	N	1	0.5	0.6443	0.6167	0.4707	0.7748
ρ_{a_K}	Autocorr. capital tech.	N	0.5	0.025	0.4102	0.4047	0.3571	0.4531
ρ_g	Autocorr. govt spending	B	0.4	0.2	0.4807	0.465	0.3234	0.6032
ρ_μ	Autocorr. investment	B	0.6	0.2	0.9251	0.8535	0.3926	1.107
ρ_ζ	Autocorr. risk premium	B	0.6	0.2	0.9967	0.9956	0.9909	0.9996
ρ_b	Autocorr. discount factor	B	0.6	0.2	0.862	0.829	0.75	0.9015
ρ_{ξ_p}	Autocorr. price markup	B	0.6	0.2	0.9788	0.9739	0.9452	0.996
ρ_φ	Autocorr. labor preference	B	0.6	0.2	0.1865	0.2161	0.047	0.4159
σ_{g_A}	Std neutral tech.	B	0.6	0.2	0.9945	0.9942	0.9885	0.9989
σ_{a_K}	Std capital tech.	B	0.6	0.2	0.972	0.9682	0.9456	0.9874
σ_{ξ_R}	Std monetary policy	IG1	1	1	0.6151	0.6203	0.5032	0.7398
σ_g	Std govt spending	IG1	0.5	1	0.1754	0.2042	0.1067	0.328
σ_μ	Std investment	IG1	0.15	1	0.1941	0.1959	0.1793	0.2131
σ_ζ	Std risk premium	IG1	1	1	0.3612	0.3638	0.3376	0.3908
σ_b	Std discount factor	IG1	1	1	3.7442	4.3141	3.0942	5.701
σ_{ξ_p}	Std price markup	IG1	0.5	1	0.1395	0.15	0.1137	0.1911
σ_{ξ_w}	Std wage markup	IG1	0.15	1	0.1531	0.1482	0.1092	0.1828
$\sigma_{\pi^\star}$	Std inflation target	IG1	0.15	1	0.1095	0.1094	0.0849	0.136
σ_φ	Std labor preference	IG1	0.15	1	0.0645	0.0789	0.0344	0.1349
$\sigma_{g_{w1}}$	Std measurement error 1	IG1	0.05	0.03	0.0498	0.0507	0.0375	0.065

Notes: B stands for Beta distribution, G for Gamma, N for Normal, U for Uniform and IG1 for Inverse Gamma.

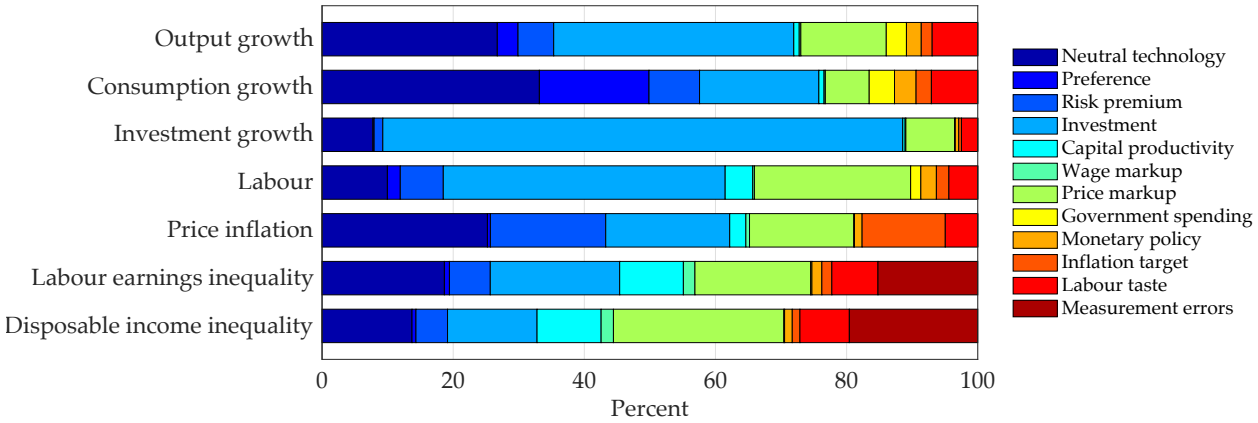


Figure 2: Unconditional variance decompositions of macroeconomic variables and inequality measures.

share parameters), price and wage indexations ι_p and ι_w, the Calvo parameters θ_p, $\theta_{w,S}$, $\theta_{w,H}$ and autocorrelation coefficients. For the parameters that govern the elasticity of substitution between H-type labour, capital, and labour from S-type households θ and ϱ (hereafter referred to as KORV elasticity parameters), we assume a gamma distribution as the prior. We assume the prior of τ_1, the pass-through from pre-tax to disposable income inequality, to be a uniform distribution on $[0, 2]$. We use inverse gamma prior distributions for the shocks' standard deviations, while for all other parameters, the priors are either normal or gamma.

Figure 2 shows the unconditional variance decomposition of key macroeconomic and inequality variables, highlighting the contribution of each structural shock to overall fluctuations. Investment-specific technology shocks account for a large share of the movements in output growth, investment growth, hours worked, price inflation and labour earnings inequality. Neutral technology shocks are also an important driver of consumption growth, output growth and price inflation. Price markup shocks are important for hours works and disposable income inequality.

5.2 Rawls' Difference Principle

Next, we carry out some numerical exercises using the estimated model and compare the outcomes to those under various counterfactual optimal policies, varying the weight that the central planner assigns to the utility of H-type households. To evaluate these counterfactual policies on

Table 3: Standard deviations

	Actual	Utilitarian (λ^*=0.2)	Rawlsian (λ^*=0.9)
Panel A. Macro variables			
Price inflation	0.566	0.508	0.515
Wage inflation	0.775	0.607	0.592
Output gap	4.791	3.850	3.883
Panel B. Inequality variables			
Consumption	5.210	3.724	3.647
Labour earnings	3.929	1.062	0.972
Disposable inc.	1.270	0.740	0.714

Notes: This table reports the standard deviation of the output gap, price inflation, and wage inflation series, the series of the consumption gap between H-type and S-type households, the labour earnings gap, and the disposable income gap under the actual policy and each respective transfer policy.

historical data, we use state estimates obtained from the Kalman smoother.

Panel A of Table 3 reports the standard deviations of the output gap, price inflation, and wage inflation under different monetary policies. The first column reports moments of the actual data. The various alternative policies that we consider include the utilitarian optimal policy, where $\lambda^\star$ equals the actual population share of H-type households ($\lambda = 0.2$), and two Rawlsian optimal policies, where $\lambda^\star$ is set to either 0.5 or 0.9.[6]

As shown, the Rawlsian optimal policies lead to greater volatility in the output gap and price inflation compared to the utilitarian case but lower volatility in wage inflation. The higher output gap volatility and lower wage inflation volatility are consistent with the small-scale model.

This result is in line with our theoretical predictions. As explained in Section 3.2, a Rawlsian central bank aims to limit wage volatility because higher wage inflation leads to greater wage dispersion, which raises firms' profits and widens inequality between savers and hand-to-mouth households. The mechanism arises because firms respond to wage dispersion by shifting labour demand towards lower-wage workers, which lowers their average marginal cost and boosts profits. Since these profits mainly go to savers, wage dispersion raises inequality between households, which calls for a more aggressive stance on wage inflation by the Rawlsian central bank.

[6]Setting $\lambda^\star$ higher than 0.9 leads to equilibrium indeterminacy in our model.

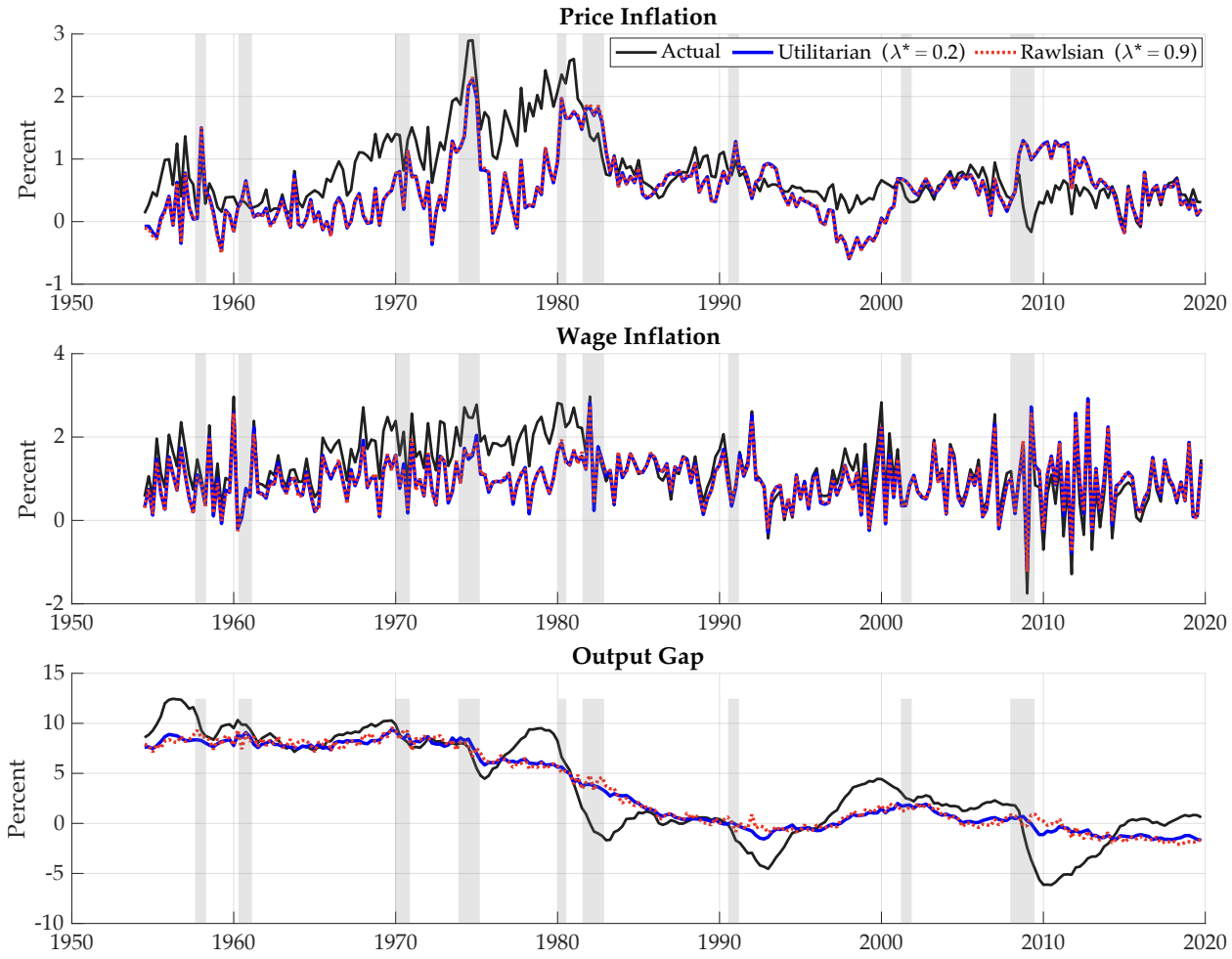


Figure 3: Macroeconomic variables. Actual, utilitarian, and Rawlsian output gap, price and wage inflation.

In line with Justiniano et al. (2013), who show that, in estimated medium-scale RANK models, optimal monetary policy features no meaningful trade-off between stabilising inflation and stabilising the output gap, our results point in the same direction. In particular, we also find that such a trade-off is quantitatively negligible.

Panel B of Table 3 presents similar information for the inequality variables. As expected, Rawlsian monetary policies lead to lower volatility in the inequality indicators. Although inequality is countercyclical and, as shown above, Rawlsian policies increase output gap volatility, they also reduce wage inflation variability, resulting in lower volatility of inequality indicators over time.

Next, we use our estimated model to run a set of counterfactual exercises and gain insights into

how the data would have looked if the central bank had followed the optimal policies discussed above. Results for the macroeconomic variables and the inequality indicators are displayed in Figures 3 and 4. In particular, each figure shows the actual data (black lines) alongside the counterfactual data under utilitarian (blue) and Rawlsian (dashed magenta and red dotted lines) optimal policies.

As shown also in the table above, the actual data, especially for the macroeconomic variables (Figure 3), are significantly more volatile than the counterfactual ones. The output gap (bottom panel) shows significant volatility in the actual data, particularly before the Great Moderation period, reflecting larger fluctuations in economic activity. Under the optimal policies, the output gap is more stable, with smaller swings during both expansions and contractions. Interestingly, while Table 3 showed the Rawlsian optimal policies to increase the output gap volatility compared to the utilitarian policy, Figure 3 displays that the two policies actually deliver an almost identical output gap path.

Similar results are found for price inflation and wage inflation. The actual data display greater fluctuations, especially prior to the 1990s, but the differences between the utilitarian and Rawlsian policies are quantitatively negligible.

Figure 4 presents the inequality variables (consumption inequality, labour earnings inequality, and disposable income inequality) again comparing actual data with counterfactuals. Consumption inequality in the actual data is strongly countercyclical, with a notable surge during the Great Recession. Compared to the actual data, the utilitarian and Rawlsian policies reduce both the average level and volatility of consumption inequality. This is particularly evident when looking at the smaller increase in inequality during the Great Recession.

Labour earnings inequality follows a similar countercyclical pattern. However, average inequality is lower in the actual data than in the counterfactuals. The counterfactual series under the optimal policies exhibit significantly lower volatility over the full sample. This is particularly evident in the late 1990s and early 2000s, when labour earnings inequality declined in the data, whereas under the optimal policies it would have remained somewhat higher. Conversely, these policies would have prevented inequality from rising after the Great Recession.

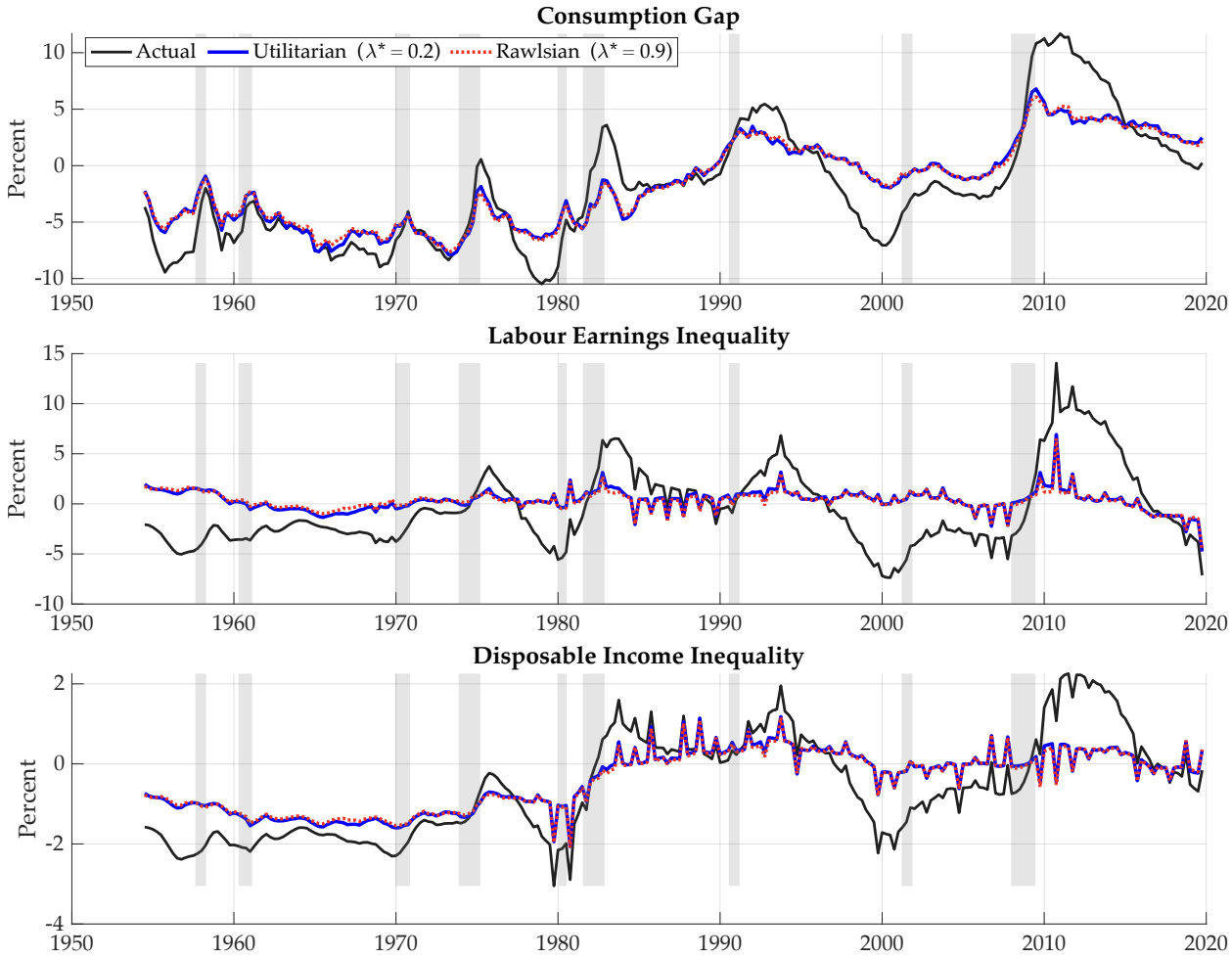


Figure 4: Inequality variables. Actual, utilitarian, and Rawlsian consumption gap, labour earnings inequality, and disposable income inequality.

Finally, disposable income inequality in the actual data remains fairly stable compared to inequality in consumption or labour earnings. Similarly as for the other measures of inequality, the counterfactual policies would have significantly mitigated the variability of inequality.

6 Conclusion

This paper studies how inequality considerations could shape monetary policy, drawing on Rawls' Difference Principle, which argues that policies should focus on improving the conditions of the least advantaged members of society. We explore this idea by analysing optimal monetary policy in a simple HANK model, where the central bank prioritises the welfare of hand-to-mouth house-

holds over that of savers. We derive a second-order approximation of the welfare loss function and show that a "Rawlsian" central bank would place greater emphasis on stabilising wage inflation rather than stabilising price inflation or the output gap. The rationale is that higher wage inflation, by generating greater wage dispersion, raises aggregate firm profits at the second order of approximation. Since these profits accrue mainly to savers, inequality between saver and hand-to-mouth households increases. Consequently, stabilising wage inflation helps to reduce both allocative inefficiencies and inequality.

We then extend the analysis to an estimated medium-scale HANK model to conduct a quantitative assessment of how a Rawlsian monetary policy affects both business cycles and inequality. The results confirm the theoretical insights from the simpler framework. In particular, the quantitative analysis shows that a Rawlsian policy reduces wage inflation volatility, but at the cost of higher variability in price inflation and the output gap. The numerical exercises further demonstrate that optimal Rawlsian and utilitarian monetary policies deliver nearly identical macroeconomic and distributional outcomes.

From a policy perspective, our results suggest that, within this relatively stylised environment, the optimal monetary policy chosen by a utilitarian central bank already aligns closely with the policy preferred under a Rawlsian objective. In this sense, placing explicit additional weight on hand-to-mouth households does not lead to large quantitative changes in policy. However, it should be noted that our framework abstracts from wealth inequality and focuses on income differences arising from wage dispersion. A richer HANK environment, in which households differ in asset holdings, may alter our policy implications. Moreover, we do not analyse optimal fiscal policy under a Rawlsian planner, as in our model such a planner would rely on fiscal instruments to transfer all profit dividends from savers to hand-to-mouth households, making it an excessively extreme and unrealistic scenario. We view our analysis as a first step in understanding how inequality considerations may shape optimal monetary policy, and extending the framework to incorporate wealth inequality as well as to study the joint design of monetary and fiscal policy constitutes an important direction for future work.

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Appendices

A Second-Order Approximation of the Aggregate Utility Function

We follow Woodford (2003) in deriving the utility-based loss function. We take a Taylor expansion of each term of the following utility function:

$$U_t = (1-\lambda^\star)\, U_{S,t} + \lambda^\star U_{H,t} = U\left(C_{S,t}, C_{H,t}\right) + V\left(N_{S,t}, N_{H,t}\right)$$
$$= (1-\lambda^\star)\left(\frac{{C_{S,t}}^{1-\sigma}-1}{1-\sigma}\right) + \lambda^\star\left(\frac{{C_{H,t}}^{1-\sigma}-1}{1-\sigma}\right) - (1-\lambda^\star)\left(\varphi_S \frac{\Delta_{w,t}{N_{S,t}}^{1+\nu}}{1+\nu}\right) - \lambda^\star\left(\varphi_H \frac{\Delta_{w,t}{N_{H,t}}^{1+\nu}}{1+\nu}\right). \tag{A.1}$$

Taking a second-order expansion around the steady state, we obtain

$$\begin{aligned} U\left(C_{S,t}, C_{H,t}\right) = U\left(C_S, C_H\right) + U_{C_S}\left(C_{S,t}-C_S\right) + \frac{1}{2}U_{C_SC_S}\left(C_{S,t}-C_S\right)^2 \\ + U_{C_H}\left(C_{H,t}-C_H\right) + \frac{1}{2}U_{C_HC_H}\left(C_{H,t}-C_H\right)^2 + t.i.p. + O\left(||\xi||^3\right), \end{aligned} \tag{A.2}$$

where $O\left(||\xi||^3\right)$ represents all relevant terms that are of third or higher order, and $t.i.p.$ denotes all the terms independent of monetary policy. Then, we take a second-order Taylor expansion of $C_{S,t}$ and $C_{H,t}$:

$$C_{S,t} = C_S\left(1 + c_{S,t} + \frac{1}{2}{c_{S,t}}^2\right) + O\left(||\xi||^3\right), \tag{A.3}$$

$$C_{H,t} = C_H\left(1 + c_{H,t} + \frac{1}{2}{c_{H,t}}^2\right) + O\left(||\xi||^3\right), \tag{A.4}$$

where $c_{S,t} \equiv \log C_{S,t} - \log C_S$ and $c_{H,t} \equiv \log C_{H,t} - \log C_H$. This implies:

$$C_{S,t} - C_S = C_S c_{S,t} + \frac{1}{2}C_S{c_{S,t}}^2 + O\left(||\xi||^3\right), \tag{A.5}$$

$$C_{H,t} - C_H = C_H c_{H,t} + \frac{1}{2}C_H{c_{H,t}}^2 + O\left(||\xi||^3\right). \tag{A.6}$$

Substituting Equations (A.5) and (A.6) into Equation (A.2) gives:

$$\begin{aligned} U\left(C_{S,t}, C_{H,t}\right) = U\left(C_S, C_H\right) + U_{C_S} C_S c_{S,t} + \frac{1}{2} U_{C_S} C_S {c_{S,t}}^2 + \frac{1}{2} U_{C_S C_S} {C_S}^2 {c_{S,t}}^2 \\ + U_{C_H} C_H c_{H,t} + \frac{1}{2} U_{C_H} C_H {c_{H,t}}^2 + \frac{1}{2} U_{C_H C_H} {C_H}^2 {c_{H,t}}^2 + t.i.p. + O\left(||\xi||^3\right) \end{aligned} \tag{A.7}$$

Note that $U\left(C_S, C_H\right)$ is independent of monetary policy. We rewrite (A.7) as:

$$\begin{aligned} U\left(C_{S,t}, C_{H,t}\right) = U_{C_S} C_S \left(c_{S,t} + \frac{1}{2} {c_{S,t}}^2 + \frac{1}{2} \frac{U_{C_S C_S} C_S}{U_{C_S}} {c_{S,t}}^2 \right) \\ + U_{C_H} C_H \left(c_{H,t} + \frac{1}{2} {c_{H,t}}^2 + \frac{1}{2} \frac{U_{C_H C_H} C_H}{U_{C_H}} {c_{H,t}}^2 \right) + t.i.p. + O\left(||\xi||^3\right). \end{aligned} \tag{A.8}$$

From the utility function, we have $\frac{U_{C_S C_S} C_S}{U_{C_S}} = \frac{U_{C_H C_H} C_H}{U_{C_H}} = -\sigma$. Thus, we obtain:

$$U\left(C_{S,t}, C_{H,t}\right) = U_{C_S} C_S \left(c_{S,t} + \frac{1}{2}\left(1-\sigma\right) {c_{S,t}}^2 \right) + U_{C_H} C_H \left(c_{H,t} + \frac{1}{2}\left(1-\sigma\right) {c_{H,t}}^2 \right) + t.i.p. + O\left(||\xi||^3\right). \tag{A.9}$$

Using the aggregate consumption $C_t = \left(1-\lambda\right) C_{S,t} + \lambda C_{H,t}$, we rewrite:

$$\frac{U\left(C_{S,t}, C_{H,t}\right)}{U_C C} = \frac{U_{C_S} C_S}{U_C C} \left(c_{S,t} + \frac{1}{2}\left(1-\sigma\right) {c_{S,t}}^2 \right) + \frac{U_{C_H} C_H}{U_C C} \left(c_{H,t} + \frac{1}{2}\left(1-\sigma\right) {c_{H,t}}^2 \right) + t.i.p. + O\left(||\xi||^3\right). \tag{A.10}$$

From the economy's resource constraint $Y_t = \left(1-\lambda\right) C_{S,t} + \lambda C_{H,t}$, we obtain:

$$y_t + \frac{1}{2} {y_t}^2 = \left(1-\lambda\right) \left(c_{S,t} + \frac{1}{2} {c_{S,t}}^2 \right) + \lambda \left(c_{H,t} + \frac{1}{2} {c_{H,t}}^2 \right) + O\left(||\xi||^3\right), \tag{A.11}$$

where $y_t \equiv \log Y_t - \log Y$. Substituting Equation (A.11), $\frac{U_{C_S} C_S}{U_C C} = 1 - \lambda^\star$, and $\frac{U_{C_H} C_H}{U_C C} = \lambda^\star$ into Equation (A.10) gives:

$$\begin{aligned} \frac{U\left(C_{S,t}, C_{H,t}\right)}{U_C C} = y_t + \frac{1}{2} {y_t}^2 - \frac{1}{2} \sigma \left(\left(1-\lambda\right) {c_{S,t}}^2 + \lambda {c_{H,t}}^2 \right) \\ - \left(\lambda^\star - \lambda\right) \left(c_{S,t} + \frac{1}{2}\left(1-\sigma\right) {c_{S,t}}^2 - c_{H,t} - \frac{1}{2}\left(1-\sigma\right) {c_{H,t}}^2 \right) + t.i.p. + O\left(||\xi||^3\right). \end{aligned} \tag{A.12}$$

Now, we also take a second-order Taylor expansion of $V\left(N_{S,t}, N_{H,t}, \Delta_{w,t}\right)$.

$$V\left(N_{S,t}, N_{H,t}, \Delta_{w,t}\right) = V\left(N_S, N_H, \Delta_w\right) + V_{\Delta_w}\left(\Delta_{w,t} - \Delta_w\right) + V_{N_S}\left(N_{S,t} - N_S\right) + \frac{1}{2}V_{N_S N_S}\left(N_{S,t} - N_S\right)^2 + V_{N_H}\left(N_{H,t} - N_H\right) + \frac{1}{2}V_{N_H N_H}\left(N_{H,t} - N_H\right)^2 + t.i.p. + O\left(||\xi||^3\right), \tag{A.13}$$

The second-order approximation of $\Delta_{w,t}$, $N_{S,t}$, and $N_{H,t}$ is:

$$\Delta_{w,t} = \Delta_w\left(1 + \delta_{w,t}\right) + O\left(||\xi||^3\right), \tag{A.14}$$

$$N_{S,t} = N_S\left(1 + n_{S,t} + \frac{1}{2}{n_{S,t}}^2\right) + O\left(||\xi||^3\right), \tag{A.15}$$

$$N_{H,t} = N_H\left(1 + n_{H,t} + \frac{1}{2}{n_{H,t}}^2\right) + O\left(||\xi||^3\right), \tag{A.16}$$

where $\delta_{w,t} \equiv \log\Delta_{w,t} - \log\Delta_w$, $n_{S,t} \equiv \log N_{S,t} - \log N_S$, and $n_{H,t} \equiv \log N_{H,t} - \log N_H$. This implies:

$$\Delta_{w,t} - \Delta_w = \Delta_w\delta_{w,t} + O\left(||\xi||^3\right), \tag{A.17}$$

$$N_{S,t} - N_S = N_S n_{S,t} + \frac{1}{2}N_S{n_{S,t}}^2 + O\left(||\xi||^3\right), \tag{A.18}$$

$$N_{H,t} - N_H = N_H n_{H,t} + \frac{1}{2}N_H{n_{H,t}}^2 + O\left(||\xi||^3\right). \tag{A.19}$$

Substituting Equations (A.17), (A.18), and (A.19) into Equation (A.13) gives:

$$V\left(N_{S,t}, N_{H,t}, \Delta_{w,t}\right) = V\left(N_S, N_H, \Delta_w\right) + V_{\Delta_w}\Delta_w\delta_{w,t} + V_{N_S}N_S n_{S,t} + \frac{1}{2}V_{N_S}N_S{n_{S,t}}^2 + \frac{1}{2}V_{N_S N_S}{N_S}^2{n_{S,t}}^2 + V_{N_H}N_H n_{H,t} + \frac{1}{2}V_{N_H}N_H{n_{H,t}}^2 + \frac{1}{2}V_{N_H N_H}{N_H}^2{n_{H,t}}^2 + t.i.p. + O\left(||\xi||^3\right). \tag{A.20}$$

Note that $V\left(N_S, N_H, \Delta_w\right)$ is independent of monetary policy. We rewrite (A.20) as:

$$V\left(N_{S,t}, N_{H,t}, \Delta_{w,t}\right) = V_{\Delta_w}\Delta_w\delta_{w,t} + V_{N_S}N_S\left(n_{S,t} + \frac{1}{2}{n_{S,t}}^2 + \frac{1}{2}\frac{V_{N_S N_S}N_S}{V_{N_S}}{n_{S,t}}^2\right) + V_{N_H}N_H\left(n_{H,t} + \frac{1}{2}{n_{H,t}}^2 + \frac{1}{2}\frac{V_{N_H N_H}N_H}{V_{N_H}}{n_{H,t}}^2\right) + t.i.p. + O\left(||\xi||^3\right). \tag{A.21}$$

Since $V_{\Delta_w} = -(1-\lambda^\star)\left(\varphi_S \frac{{N_S}^{1+\nu}}{1+\nu}\right) - \lambda^\star\left(\varphi_H \frac{{N_H}^{1+\nu}}{1+\nu}\right)$, $\frac{V_{N_S N_S} N_S}{V_{N_S}} = \frac{V_{N_H N_H} N_H}{V_{N_H}} = \nu$, we rewrite Equation (A.21) as:

$$V\left(N_{S,t}, N_{H,t}, \Delta_{w,t}\right) = -\left((1-\lambda^\star)\left(\varphi_S \frac{{N_S}^{1+\nu}}{1+\nu}\right) + \lambda^\star\left(\varphi_H \frac{{N_H}^{1+\nu}}{1+\nu}\right)\right)\delta_{w,t} + V_{N_S} N_S\left(n_{S,t} + \frac{1}{2}(1+\nu)\,{n_{S,t}}^2\right) + V_{N_H} N_H\left(n_{H,t} + \frac{1}{2}(1+\nu)\,{n_{H,t}}^2\right) + t.i.p. + O\left(||\xi||^3\right). \tag{A.22}$$

Now, recall that $N_S = N_H = N = Y = C = C_S = C_H = 1$ and $\varphi_S = \varphi_H = 1$. From the household's labour supply relations, we have:

$$(1-\lambda^\star)\left(\varphi_S \frac{{N_S}^{1+\nu}}{1+\nu}\right) + \lambda^\star\left(\varphi_H \frac{{N_H}^{1+\nu}}{1+\nu}\right) = \frac{1}{1+\nu} \tag{A.23}$$

$$-\frac{V_{N_S} N_S}{U_C C} = 1-\lambda^\star, \tag{A.24}$$

$$-\frac{V_{N_H} N_H}{U_C C} = \lambda^\star. \tag{A.25}$$

Therefore, we have:

$$\frac{V\left(N_{S,t}, N_{H,t}, \Delta_{w,t}\right)}{U_C C} = -\frac{1}{1+\nu}\delta_{w,t} - (1-\lambda^\star)\left(n_{S,t} + \frac{1}{2}(1+\nu)\,{n_{S,t}}^2\right) - \lambda^\star\left(n_{H,t} + \frac{1}{2}(1+\nu)\,{n_{H,t}}^2\right) + t.i.p. + O\left(||\xi||^3\right). \tag{A.26}$$

From the labour market clearing condition $N_t = (1-\lambda)\,N_{S,t} + \lambda N_{H,t}$, we obtain:

$$n_t + \frac{1}{2}{n_t}^2 = (1-\lambda)\left(n_{S,t} + \frac{1}{2}{n_{S,t}}^2\right) + \lambda\left(n_{H,t} + \frac{1}{2}{n_{H,t}}^2\right) + O\left(||\xi||^3\right). \tag{A.27}$$

Substituting Equation (A.27) into Equation (A.26) gives:

$$\begin{aligned}\frac{V\left(N_{S,t}, N_{H,t}, \Delta_{w,t}\right)}{U_C C} &= -\frac{1}{1+\nu}\delta_{w,t} - n_t - \frac{1}{2}{n_t}^2 - \frac{1}{2}\nu\left((1-\lambda)\,{n_{S,t}}^2 + \lambda {n_{H,t}}^2\right) \\ &+ (\lambda^\star - \lambda)\left(n_{S,t} + \frac{1}{2}(1+\nu)\,{n_{S,t}}^2 - n_{H,t} - \frac{1}{2}(1+\nu)\,{n_{H,t}}^2\right) + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{A.28}$$

From the production function $\Delta_{p,t} Y_t = A_t N_t$, we have:

$$n_t = \delta_{p,t} + y_t - a_t. \tag{A.29}$$

Substituting Equation (A.29) into Equation (A.28), we rewrite:

$$\begin{aligned}\frac{V\left(N_{S,t}, N_{H,t}, \Delta_{w,t}\right)}{U_C C} = &-\delta_{p,t} - \frac{1}{1+\nu}\delta_{w,t} - y_t - \frac{1}{2}\left({y_t}^2 - 2 y_t a_t\right) - \frac{1}{2}\nu\left(\left(1-\lambda\right){n_{S,t}}^2 + \lambda {n_{H,t}}^2\right) \\ &+ \left(\lambda^\star - \lambda\right)\left(n_{S,t} + \frac{1}{2}\left(1+\nu\right){n_{S,t}}^2 - n_{H,t} - \frac{1}{2}\left(1+\nu\right){n_{H,t}}^2\right) + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{A.30}$$

Combining Equations (A.12) and (A.30), we obtain:

$$\begin{aligned}\frac{U_t}{U_C C} \equiv \frac{U\left(C_{S,t}, C_{H,t}\right) + V\left(N_{S,t}, N_{H,t}, \Delta_{w,t}\right)}{U_C C} = &-\delta_{p,t} - \frac{1}{1+\nu}\delta_{w,t} + y_t a_t \\ &-\frac{1}{2}\sigma\left(\left(1-\lambda\right){c_{S,t}}^2 + \lambda {c_{H,t}}^2\right) - \frac{1}{2}\nu\left(\left(1-\lambda\right){n_{S,t}}^2 + \lambda {n_{H,t}}^2\right) \\ &- \left(\lambda^\star - \lambda\right)\left(c_{S,t} + \frac{1}{2}\left(1-\sigma\right){c_{S,t}}^2 - c_{H,t} - \frac{1}{2}\left(1-\sigma\right){c_{H,t}}^2\right) \\ &+ \left(\lambda^\star - \lambda\right)\left(n_{S,t} + \frac{1}{2}\left(1+\nu\right){n_{S,t}}^2 - n_{H,t} - \frac{1}{2}\left(1+\nu\right){n_{H,t}}^2\right) + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{A.31}$$

Then, we can rewrite:

$$\begin{aligned}\frac{U_t}{U_C C} = &-\delta_{p,t} - \frac{1}{1+\nu}\delta_{w,t} + y_t a_t - \frac{1}{2}\sigma\left(\left(1-\lambda\right){c_{S,t}}^2 + \lambda {c_{H,t}}^2\right) - \frac{1}{2}\nu\left(\left(1-\lambda\right){n_{S,t}}^2 + \lambda {n_{H,t}}^2\right) \\ &- \left(\lambda^\star - \lambda\right)\left(c_{S,t} + \frac{1}{2}{c_{S,t}}^2 - n_{S,t} - \frac{1}{2}{n_{S,t}}^2 - c_{H,t} - \frac{1}{2}{c_{H,t}}^2 + n_{H,t} + \frac{1}{2}{n_{H,t}}^2\right. \\ &\left. - \frac{1}{2}\left(\sigma\left({c_{S,t}}^2 - {c_{H,t}}^2\right) + \nu\left({n_{S,t}}^2 - {n_{H,t}}^2\right)\right)\right) + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{A.32}$$

From the households' budget constraints, we have:

$$\begin{aligned}c_{S,t} + \frac{1}{2}{c_{S,t}}^2 = w_t + \frac{1}{2}{w_t}^2 + n_{S,t} + \frac{1}{2}{n_{S,t}}^2 + w_t n_{S,t} - \frac{1-\tau_d}{1-\lambda}\left(w_t + \frac{1}{2}{w_t}^2 + \delta_{p,t} + w_t y_t - w_t a_t - y_t a_t\right) \\ + t.i.p. + O\left(||\xi||^3\right),\end{aligned} \tag{A.33}$$

$$\begin{aligned}c_{H,t} + \frac{1}{2}{c_{H,t}}^2 = w_t + \frac{1}{2}{w_t}^2 + n_{H,t} + \frac{1}{2}{n_{H,t}}^2 + w_t n_{H,t} - \frac{\tau_d}{\lambda}\left(w_t + \frac{1}{2}{w_t}^2 + \delta_{p,t} + w_t y_t - w_t a_t - y_t a_t\right) \\ + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{A.34}$$

Combining Equation (A.33) and (A.34), we have:

$$\begin{aligned} & c_{S,t} + \frac{1}{2}{c_{S,t}}^2 - n_{S,t} - \frac{1}{2}{n_{S,t}}^2 - c_{H,t} - \frac{1}{2}{c_{H,t}}^2 + n_{H,t} + \frac{1}{2}{n_{H,t}}^2 \\ = & w_t \left(n_{S,t} - n_{H,t}\right) - \frac{1 - \frac{\tau_d}{\lambda}}{1-\lambda}\left(w_t + \frac{1}{2}{w_t}^2 + \delta_{p,t} + w_t y_t - w_t a_t - y_t a_t\right) + t.i.p. + O\left(||\xi||^3\right). \end{aligned} \tag{A.35}$$

Substituting Equation (A.35) into Equation (A.32), we rewrite:

$$\begin{aligned} \frac{U_t}{U_C C} = & -\delta_{p,t} - \frac{1}{1+\nu}\delta_{w,t} + y_t a_t - \frac{1}{2}\sigma\left(\left(1-\lambda\right){c_{S,t}}^2 + \lambda {c_{H,t}}^2\right) - \frac{1}{2}\nu\left(\left(1-\lambda\right){n_{S,t}}^2 + \lambda {n_{H,t}}^2\right) \\ & - \left(\lambda^\star - \lambda\right)\left(w_t\left(n_{S,t} - n_{H,t}\right) - \frac{\lambda - \tau_d}{\lambda\left(1-\lambda\right)}\left(w_t + \frac{1}{2}{w_t}^2 + \delta_{p,t} + w_t y_t - w_t a_t - y_t a_t\right)\right. \\ & \left. - \frac{1}{2}\left(\sigma\left({c_{S,t}}^2 - {c_{H,t}}^2\right) + \nu\left({n_{S,t}}^2 - {n_{H,t}}^2\right)\right)\right) + t.i.p. + O\left(||\xi||^3\right). \end{aligned} \tag{A.36}$$

From the wage New Keynesian Phillips curves, we have:

$$w_t = \nu n_{S,t} + \sigma c_{S,t} - \frac{1}{\gamma_w}\Phi_{w,t}, \tag{A.37}$$

$$w_t = \nu n_{H,t} + \sigma c_{H,t} - \frac{1}{\gamma_w}\Phi_{w,t}, \tag{A.38}$$

where $\Phi_{w,t} \equiv \pi_{w,t} - \beta E_t \pi_{w,t+1}$. Combining Equations (A.27), (A.29), (A.37), and (A.38), we have:

$$\begin{aligned} w_t = & \nu\left(\delta_{p,t} + y_t - a_t + \frac{1}{2}\left({y_t}^2 - 2 y_t a_t - \left(1-\lambda\right){n_{S,t}}^2 - \lambda {n_{H,t}}^2\right)\right) \\ & + \sigma\left(y_t + \frac{1}{2}\left({y_t}^2 - \left(1-\lambda\right){c_{S,t}}^2 - \lambda {c_{H,t}}^2\right)\right) - \frac{1}{\gamma_w}\Phi_{w,t} + t.i.p. + O\left(||\xi||^3\right). \end{aligned} \tag{A.39}$$

Substituting Equation (A.39) into Equation (A.36), we rewrite:

$$\begin{aligned}\frac{U_t}{U_C C} = &-\delta_{p,t} - \frac{1}{1+\nu}\delta_{w,t} + y_t a_t - \frac{1}{2}\sigma\left((1-\lambda)\,{c_{S,t}}^2 + \lambda {c_{H,t}}^2\right) - \frac{1}{2}\nu\left((1-\lambda)\,{n_{S,t}}^2 + \lambda {n_{H,t}}^2\right)\\ &- (\lambda^\star - \lambda)\left(\left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right)\left(n_{S,t} - n_{H,t}\right)\right.\\ &-\frac{\lambda-\tau_d}{\lambda\,(1-\lambda)}\left(\nu\left(\delta_{p,t} + y_t + \frac{1}{2}\left({y_t}^2 - 2y_t a_t - (1-\lambda)\,{n_{S,t}}^2 - \lambda {n_{H,t}}^2\right)\right)\right.\\ &+\sigma\left(y_t + \frac{1}{2}\left({y_t}^2 - (1-\lambda)\,{c_{S,t}}^2 - \lambda {c_{H,t}}^2\right)\right) - \frac{1}{\gamma_w}\Phi_{w,t} + \frac{1}{2}\left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right)^2\\ &+\delta_{p,t} + \left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right) y_t - \left(\left(\nu+\sigma\right) y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right) a_t - y_t a_t\Big)\\ &\left.\left.-\frac{1}{2}\left(\sigma\left({c_{S,t}}^2 - {c_{H,t}}^2\right) + \nu\left({n_{S,t}}^2 - {n_{H,t}}^2\right)\right)\right) + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{A.40}$$

Then, we take the expected discounted sum over time, we obtain:

$$\begin{aligned}\frac{\mathbb{W}_0}{U_C C} &= E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t}{U_C C}\\ &= E_0 \sum_{t=0}^{\infty} \beta^t \left(-\delta_{p,t} - \frac{1}{1+\nu}\delta_{w,t} + y_t a_t - \frac{1}{2}\sigma\left((1-\lambda)\,{c_{S,t}}^2 + \lambda {c_{H,t}}^2\right) - \frac{1}{2}\nu\left((1-\lambda)\,{n_{S,t}}^2 + \lambda {n_{H,t}}^2\right)\right.\\ &\quad - (\lambda^\star - \lambda)\left(\left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right)\left(n_{S,t} - n_{H,t}\right)\right.\\ &\quad -\frac{\lambda-\tau_d}{\lambda\,(1-\lambda)}\left(\nu\left(\delta_{p,t} + y_t + \frac{1}{2}\left({y_t}^2 - 2y_t a_t - (1-\lambda)\,{n_{S,t}}^2 - \lambda {n_{H,t}}^2\right)\right)\right.\\ &\quad +\sigma\left(y_t + \frac{1}{2}\left({y_t}^2 - (1-\lambda)\,{c_{S,t}}^2 - \lambda {c_{H,t}}^2\right)\right) - \frac{1}{\gamma_w}\Phi_{w,t} + \frac{1}{2}\left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right)^2\\ &\quad +\delta_{p,t} + \left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right) y_t - \left(\left(\nu+\sigma\right) y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right) a_t - y_t a_t\Big)\\ &\quad \left.\left.-\frac{1}{2}\left(\sigma\left({c_{S,t}}^2 - {c_{H,t}}^2\right) + \nu\left({n_{S,t}}^2 - {n_{H,t}}^2\right)\right)\right)\right) + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{A.41}$$

When prices are staggered as in the discrete time Calvo (1983) fashion, Woodford (2003) shows that:

$$\sum_{t=0}^{\infty} \beta^t \delta_{p,t} = \frac{1}{2}\frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}\sum_{t=0}^{\infty} \beta^t {\pi_{p,t}}^2 + t.i.p. + O\left(||\xi||^3\right), \tag{A.42}$$

$$\sum_{t=0}^{\infty} \beta^t \delta_{w,t} = \frac{1}{2}\frac{(1+\nu)\,\frac{1+\xi_w}{\xi_w}}{\gamma_w}\sum_{t=0}^{\infty} \beta^t {\pi_{w,t}}^2 + t.i.p. + O\left(||\xi||^3\right). \tag{A.43}$$

where $\gamma_p \equiv \frac{\left(1-\theta_p\right)\left(1-\theta_p\beta\right)}{\theta_p}$ and $\gamma_w \equiv \frac{(1-\theta_w)(1-\theta_w\beta)}{\theta_w\left(1+\nu\frac{1+\xi_w}{\xi_w}\right)}$. Therefore, we can rewrite:

$$\begin{aligned}\frac{\mathbb{W}_0}{U_C C} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg(& -\frac{1}{2}\frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}{\pi_{p,t}}^2 - \frac{1}{2}\frac{\frac{1+\xi_w}{\xi_w}}{\gamma_w}{\pi_{w,t}}^2 + y_t a_t - \frac{1}{2}\sigma\left(\left(1-\lambda\right){c_{S,t}}^2 + \lambda {c_{H,t}}^2\right) - \frac{1}{2}\nu\left(\left(1-\lambda\right){n_{S,t}}^2 + \lambda {n_{H,t}}^2\right) \\ & - \left(\lambda^\star - \lambda\right)\left(\left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right)\left(n_{S,t} - n_{H,t}\right)\right. \\ & - \frac{\lambda - \tau_d}{\lambda\left(1-\lambda\right)}\left(\nu\left(\frac{1}{2}\frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}{\pi_{p,t}}^2 + y_t + \frac{1}{2}\left({y_t}^2 - 2y_t a_t - \left(1-\lambda\right){n_{S,t}}^2 - \lambda {n_{H,t}}^2\right)\right)\right. \\ & + \sigma\left(y_t + \frac{1}{2}\left({y_t}^2 - \left(1-\lambda\right){c_{S,t}}^2 - \lambda {c_{H,t}}^2\right)\right) - \frac{1}{\gamma_w}\Phi_{w,t} + \frac{1}{2}\left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right)^2 \\ & + \delta_{p,t} + \left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right) y_t - \left(\left(\nu + \sigma\right) y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right) a_t - y_t a_t \\ & \left.\left. - \frac{1}{2}\left(\sigma\left({c_{S,t}}^2 - {c_{H,t}}^2\right) + \nu\left({n_{S,t}}^2 - {n_{H,t}}^2\right)\right)\right)\Bigg) + t.i.p. + O\left(||\xi||^3\right). \end{aligned} \tag{A.44}$$

B Second-Order Approximation of the New Keynesian Phillips Curves

B.1 Price New Keynesian Phillips Curve

We follow Benigno and Woodford (2004, 2005). The price New Keynesian Phillips curve can be written as:

$$p_t^{\#} \equiv \frac{P_t^{\#}}{P_t} = \frac{1+\xi_p}{1+\tau_p}\frac{K_t}{F_t}, \tag{B.1}$$

where $\tau_p = \xi_p$ to eliminate the steady state distortion. The aggregate price level can be rewritten as:

$$p_t^{\#} = \left(\frac{1-\theta_p \Pi_{p,t}{}^{\frac{1}{\xi_p}}}{1-\theta_p}\right)^{-\xi_p}. \tag{B.2}$$

We can substitute Equation (B.2) into Equation (B.1):

$$\left(\frac{1-\theta_p \Pi_{p,t}{}^{\frac{1}{\xi_p}}}{1-\theta_p}\right)^{-\xi_p} = \frac{K_{p,t}}{F_{p,t}}. \tag{B.3}$$

Taking logs gives us:

$$-\xi_p \log\left(\frac{1-\theta_p \Pi_{p,t}{}^{\frac{1}{\xi_p}}}{1-\theta_p}\right) = \log K_{p,t} - \log F_{p,t}. \tag{B.4}$$

A second-order Taylor series for the left-hand side of Equation (B.4) can be written as:

$$\log\left(\frac{1-\theta_p \Pi_{p,t}{}^{\frac{1}{\xi_p}}}{1-\theta_p}\right) = -\frac{\theta_p}{(1-\theta_p)\,\xi_p}\left(\pi_{p,t} + \frac{1}{2}\frac{1}{(1-\theta_p)\,\xi_p}{\pi_{p,t}}^2\right) + O\left(||\xi||^3\right). \tag{B.5}$$

It remains to derive similar second-order approximations for $\log K_{p,t}$ and $\log F_{p,t}$ on the right-hand side. The definitions of $K_{p,t}$ and $F_{p,t}$ imply the following second-order expansions:

$$k_{p,t} + \frac{1}{2}{k_{p,t}}^2 = (1-\theta_p\beta)\,E_t \sum_{T=t}^{\infty} (\theta_p\beta)^{T-t}\left(\widehat{k}_{p,t,T} + \frac{1}{2}\widehat{k}^2_{p,t,T}\right) + O\left(||\xi||^3\right), \tag{B.6}$$

$$f_{p,t} + \frac{1}{2}{f_{p,t}}^2 = (1-\theta_p\beta)\,E_t \sum_{T=t}^{\infty} (\theta_p\beta)^{T-t}\left(\widehat{f}_{p,t,T} + \frac{1}{2}\widehat{f}^2_{p,t,T}\right) + O\left(||\xi||^3\right), \tag{B.7}$$

where $\widehat{k}_{p,t,T}$ and $\widehat{f}_{p,t,T}$ are given by:

$$\widehat{k}_{p,t,T} \equiv \widehat{k}_{p,T} + \frac{1+\xi_p}{\xi_p} \sum_{s=t+1}^{T} \pi_{p,s}, \tag{B.8}$$

$$\widehat{f}_{p,t,T} \equiv \widehat{f}_{p,T} + \frac{1}{\xi_p} \sum_{s=t+1}^{T} \pi_{p,s}, \tag{B.9}$$

where $\widehat{k}_T$ and $\widehat{f}_T$ are defined as:

$$\widehat{k}_{p,T} \equiv -\sigma c_{S,T} + y_T + w_T - a_T, \tag{B.10}$$

$$\widehat{f}_{p,T} \equiv -\sigma c_{S,T} + y_T. \tag{B.11}$$

Substituting Equation (B.5) into Equation (B.4) yields:

$$\pi_{p,t} + \frac{1}{2} \frac{1}{(1-\theta_p)\,\xi_p} {\pi_{p,t}}^2 = \frac{1-\theta_p}{\theta_p} \left(k_{p,t} - f_{p,t}\right) + O\left(||\xi||^3\right). \tag{B.12}$$

where $k_{p,t} \equiv \log K_{p,t} - \log K_p$ and $f_{p,t} \equiv \log F_{p,t} - \log F_p$. We can use Equations (B.6) and (B.7) to obtain a second-order expansion for the right-hand side of Equation (B.12). Subtracting Equation (B.7) from Equation (B.6), we obtain:

$$k_{p,t} - f_{p,t} = \left(1-\theta_p\beta\right) E_t \sum_{T=t}^{\infty} \left(\theta_p\beta\right)^{T-t} \left(\widehat{k}_{p,t,T} - \widehat{f}_{p,t,T} + \frac{1}{2}\left(\widehat{k}^2_{p,t,T} - \widehat{f}^2_{p,t,T}\right)\right) - \frac{1}{2}\left({k_{p,t}}^2 - {f_{p,t}}^2\right) + O\left(||\xi||^3\right). \tag{B.13}$$

We rewrite:

$$k_{p,t} - f_{p,t} = \left(1-\theta_p\beta\right) E_t \sum_{T=t}^{\infty} \left(\theta_p\beta\right)^{T-t} \left(\widehat{k}_{p,t,T} - \widehat{f}_{p,t,T} + \frac{1}{2}\left(\widehat{k}^2_{p,t,T} - \widehat{f}^2_{p,t,T}\right)\right) - \frac{1}{2}\left(k_{p,t} - f_{p,t}\right)\left(k_{p,t} + f_{p,t}\right) + O\left(||\xi||^3\right). \tag{B.14}$$

Using Equation (B.12), to first order, we have:

$$k_{p,t} - f_{p,t} = \frac{\theta_p}{1-\theta_p}\pi_{p,t} + O\left(||\xi||^2\right). \tag{B.15}$$

Moreover, using Equations (B.6) and (B.7), to first order, we have:

$$k_{p,t} + f_{p,t} = \left(1 - \theta_p \beta\right) \Theta_{p,t} + O\left(||\xi||^2\right), \tag{B.16}$$

where:

$$\Theta_{p,t} \equiv E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \left(\widehat{k}_{p,t,T} + \widehat{f}_{p,t,T}\right). \tag{B.17}$$

Therefore, substituting Equations (B.15) and (B.16) into Equation (B.14) yields:

$$k_{p,t} - f_{p,t} = \left(1 - \theta_p \beta\right) E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \left(\widehat{k}_{p,t,T} - \widehat{f}_{p,t,T} + \frac{1}{2}\left(\widehat{k}^2_{p,t,T} - \widehat{f}^2_{p,t,T}\right)\right) - \frac{1}{2}\left(1 - \theta_p \beta\right) \frac{\theta_p}{1 - \theta_p} \pi_{p,t} \Theta_{p,t} + O\left(||\xi||^3\right). \tag{B.18}$$

We can use the definitions of $\widehat{k}_{p,t,T}$ and $\widehat{f}_{p,t,T}$ to further expand the first term on the right-hand side of Equation (B.18). We obtain:

$$E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \left(\widehat{k}_{p,t,T} - \widehat{f}_{p,t,T}\right) = E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \left(\widehat{k}_{p,T} - \widehat{f}_{p,T}\right) + E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \sum_{s=t+1}^{T} \pi_{p,s}. \tag{B.19}$$

Then, we rewrite:

$$E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \left(\widehat{k}_{p,t,T} - \widehat{f}_{p,t,T}\right) = E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \left(\widehat{k}_{p,T} - \widehat{f}_{p,T}\right) + \frac{1}{1 - \theta_p \beta} \Omega_{p,t}. \tag{B.20}$$

where:

$$\Omega_{p,t} \equiv E_t \sum_{T=t+1}^{\infty} \left(\theta_p \beta\right)^{T-t} \pi_{p,T}. \tag{B.21}$$

Moreover, we obtain:

$$\frac{1}{2} E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \left(\widehat{k}^2_{p,t,T} - \widehat{f}^2_{p,t,T}\right) = \frac{1}{2} E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \left(\widehat{k}^2_{p,T} - \widehat{f}^2_{p,T}\right) + E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \left(\frac{1 + \xi_p}{\xi_p} \widehat{k}_{p,T} - \frac{1}{\xi_p} \widehat{f}_{p,T}\right) \sum_{s=t+1}^{T} \pi_{p,s} + \frac{1}{2}\left(\frac{2 + \xi_p}{\xi_p}\right) E_t \sum_{T=t}^{\infty} \left(\theta_p \beta\right)^{T-t} \left(\sum_{s=t+1}^{T} \pi_{p,s}\right)^2. \tag{B.22}$$

We can rewrite:

$$\frac{1}{2}E_t\sum_{T=t}^{\infty}\left(\theta_p\beta\right)^{T-t}\left(\widehat{k}_{p,t,T}^2-\widehat{f}_{p,t,T}^2\right)=\frac{1}{2}E_t\sum_{T=t}^{\infty}\left(\theta_p\beta\right)^{T-t}\left(\widehat{k}_{p,T}^2-\widehat{f}_{p,T}^2\right)+E_t\sum_{T=t+1}^{\infty}\left(\theta_p\beta\right)^{T-t}\pi_{p,T}\Gamma_{p,T}$$
$$+\frac{1}{2}\frac{\frac{2+\xi_p}{\xi_p}}{1-\theta_p\beta}E_t\sum_{T=t+1}^{\infty}\left(\theta_p\beta\right)^{T-t}\pi_{p,T}\left(\pi_{p,T}+2\Omega_{p,T}\right), \tag{B.23}$$

where:

$$\Gamma_{p,t}\equiv E_t\sum_{T=t}^{\infty}\left(\theta_p\beta\right)^{T-t}\left(\frac{1+\xi_p}{\xi_p}\widehat{k}_{p,T}-\frac{1}{\xi_p}\widehat{f}_{p,T}\right). \tag{B.24}$$

Substituting Equations (B.20) and (B.23) into Equation (B.18), we obtain:

$$k_{p,t}-f_{p,t}=\left(1-\theta_p\beta\right)E_t\sum_{T=t}^{\infty}\left(\theta_p\beta\right)^{T-t}\left(\widehat{k}_{p,T}-\widehat{f}_{p,T}+\frac{1}{2}\left(\widehat{k}_{p,T}^2-\widehat{f}_{p,T}^2\right)\right)$$
$$+E_t\sum_{T=t+1}^{\infty}\left(\theta_p\beta\right)^{T-t}\pi_{p,T}+\left(1-\theta_p\beta\right)E_t\sum_{T=t+1}^{\infty}\left(\theta_p\beta\right)^{T-t}\pi_{p,T}\Gamma_T$$
$$+\frac{1}{2}\left(\frac{2+\xi_p}{\xi_p}\right)E_t\sum_{T=t+1}^{\infty}\left(\theta_p\beta\right)^{T-t}\pi_{p,T}\left(\pi_{p,T}+2\Omega_{p,T}\right)-\frac{1}{2}\left(1-\theta_p\beta\right)\frac{\theta_p}{1-\theta_p}\pi_{p,t}\Theta_{p,t}+O\left(||\xi||^3\right). \tag{B.25}$$

This can be written recursively as:

$$k_{p,t}-f_{p,t}+\frac{1}{2}\left(1-\theta_p\beta\right)\frac{\theta_p}{1-\theta_p}\pi_{p,t}\Theta_{p,t}=\left(1-\theta_p\beta\right)\left(\widehat{k}_{p,t}-\widehat{f}_{p,t}+\frac{1}{2}\left(\widehat{k}_{p,t}^2-\widehat{f}_{p,t}^2\right)\right)$$
$$+\theta_p\beta E_t\pi_{p,t+1}+\left(1-\theta_p\beta\right)\theta_p\beta E_t\pi_{p,t+1}\Gamma_{p,t+1}+\frac{1}{2}\left(\frac{2+\xi_p}{\xi_p}\right)\theta_p\beta E_t\pi_{p,t+1}\left(\pi_{p,t+1}+2\Omega_{p,t+1}\right) \tag{B.26}$$
$$+\theta_p\beta E_t\left(k_{p,t+1}-f_{p,t+1}+\frac{1}{2}\left(1-\theta_p\beta\right)\frac{\theta_p}{1-\theta_p}\pi_{p,t+1}\Theta_{p,t+1}\right)+O\left(||\xi||^3\right).$$

Using Equation (B.12), we rewrite:

$$\pi_{p,t}+\frac{1}{2}\frac{1}{\left(1-\theta_p\right)\xi_p}{\pi_{p,t}}^2+\frac{1}{2}\left(1-\theta_p\beta\right)\pi_{p,t}\Theta_{p,t}=\gamma\left(\widehat{k}_{p,t}-\widehat{f}_{p,t}+\frac{1}{2}\left(\widehat{k}_{p,t}^2-\widehat{f}_{p,t}^2\right)\right)$$
$$+\left(1-\theta_p\right)\beta E_t\pi_{p,t+1}+\left(1-\theta_p\right)\left(1-\theta_p\beta\right)\beta E_t\pi_{p,t+1}\Gamma_{p,t+1}+\frac{1}{2}\left(\frac{2+\xi_p}{\xi_p}\right)\left(1-\theta_p\right)\beta E_t\pi_{p,t+1}\left(\pi_{p,t+1}+2\Omega_{p,t+1}\right)$$
$$+\theta_p\beta E_t\left(\pi_{p,t+1}+\frac{1}{2}\frac{1}{\left(1-\theta_p\right)\xi_p}{\pi_{p,t+1}}^2+\frac{1}{2}\left(1-\theta_p\beta\right)\pi_{p,t+1}\Theta_{t+1}\right)+O\left(||\xi||^3\right). \tag{B.27}$$

where $\gamma_p \equiv \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}$. We can furthermore eliminate $\Gamma_{p,t}$ by observing that Equation (B.24) implies that:

$$\begin{aligned}\Gamma_{p,t} \equiv E_t \sum_{T=t}^{\infty} (\theta_p\beta)^{T-t} \left(\frac{1+\xi_p}{\xi_p}\widehat{k}_{p,T} - \frac{1}{\xi_p}\widehat{f}_{p,T}\right) &= \frac{1}{2}E_t \sum_{T=t}^{\infty} (\theta_p\beta)^{T-t} \left(\widehat{k}_{p,T} + \widehat{f}_{p,T} + \frac{2+\xi_p}{\xi_p}\left(\widehat{k}_{p,T} - \widehat{f}_{p,T}\right)\right) \\ &= \frac{1}{2}E_t \sum_{T=t}^{\infty} (\theta_p\beta)^{T-t} \left(\widehat{k}_{p,t,T} + \widehat{f}_{p,t,T} + \frac{2+\xi_p}{\xi_p}\left(\widehat{k}_{p,t,T} - \widehat{f}_{p,t,T}\right)\right) - \frac{\frac{2+\xi_p}{\xi_p}}{1-\theta_p\beta}\Omega_{p,t} \\ &= \frac{1}{2}\Theta_{p,t} + \frac{1}{2}\frac{\frac{2+\xi_p}{\xi_p}}{\gamma_p}\pi_{p,t} - \frac{\frac{2+\xi_p}{\xi_p}}{1-\theta_p\beta}\Omega_{p,t}.\end{aligned} \tag{B.28}$$

By substituting Equation (B.28) into Equation (B.27), we obtain:

$$\begin{aligned}\pi_{p,t} + \frac{1}{2}\frac{1}{(1-\theta_p)\,\xi_p}{\pi_{p,t}}^2 + \frac{1}{2}\left(1-\theta_p\beta\right)\pi_{p,t}\Theta_{p,t} = \gamma_p\left(\widehat{k}_{p,t} - \widehat{f}_{p,t} + \frac{1}{2}\left(\widehat{k}_{p,t}^2 - \widehat{f}_{p,t}^2\right)\right) \\ +\beta E_t\pi_{p,t+1} + \frac{1}{2}\beta\frac{1}{(1-\theta_p)\,\xi_p}E_t{\pi_{p,t+1}}^2 + \frac{1}{2}\beta\left(1-\theta_p\beta\right)E_t\pi_{p,t+1}\Theta_{p,t+1} + \frac{1}{2}\frac{1+\xi_p}{\xi_p}\beta E_t{\pi_{p,t+1}}^2 + O\left(||\xi||^3\right).\end{aligned} \tag{B.29}$$

This can be integrated forward:

$$\mathbb{V}_{p,0} = \gamma_p E_0\sum_{t=0}^{\infty}\beta^t\left(\widehat{k}_{p,} - \widehat{f}_{p,t} + \frac{1}{2}\left(\widehat{k}_{p,t}^2 - \widehat{f}_{p,t}^2\right)\right) + \frac{1}{2}\frac{1+\xi_p}{\xi_p}E_0\sum_{t=0}^{\infty}\beta^t{\pi_{p,t}}^2 + O\left(||\xi||^3\right), \tag{B.30}$$

where:

$$\mathbb{V}_{p,t} \equiv \pi_{p,t} + \frac{1}{2}\frac{1}{(1-\theta_p)\,\xi_p}{\pi_{p,t}}^2 + \frac{1}{2}\left(1-\theta_p\beta\right)\pi_{p,t}\Theta_{p,t} + \frac{1}{2}\frac{1+\xi_p}{\xi_p}{\pi_{p,t}}^2. \tag{B.31}$$

Using Equations (B.10) and (B.11), we have:

$$\widehat{k}_{p,t} - \widehat{f}_{p,t} = w_t - a_t, \tag{B.32}$$

$$\widehat{k}_{p,t} + \widehat{f}_{p,t} = 2\left(-\sigma c_{S,t} + y_t\right) + w_t - a_t. \tag{B.33}$$

Therefore, we can have:

$$\widehat{k}_{p,t} - \widehat{f}_{p,t} + \frac{1}{2}\left(\widehat{k}_{p,t}^2 - \widehat{f}_{p,t}^2\right) = w_t - a_t + \frac{1}{2}\left(w_t - a_t\right)\left(2\left(-\sigma c_{S,t} + y_t\right) + w_t - a_t\right). \tag{B.34}$$

Substituting Equation (B.34) into Equation (B.30), we finally obtain:

$$\begin{aligned}\mathbb{V}_{p,0} = \gamma_p E_0 \sum_{t=0}^{\infty} \beta^t \left(w_t - a_t + \frac{1}{2} \left(w_t - a_t \right) \left(2 \left(-\sigma c_{S,t} + y_t \right) + w_t - a_t \right) \right) \\ + \frac{1}{2} \frac{1+\xi_p}{\xi_p} E_0 \sum_{t=0}^{\infty} \beta^t {\pi_{p,t}}^2 + t.i.p. + O\left(||\xi||^3 \right).\end{aligned} \tag{B.35}$$

Then, we rewrite:

$$\mathbb{V}_{p,0} = \gamma_p E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2} \frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p} {\pi_{p,t}}^2 + w_t - a_t + \frac{1}{2} \left(w_t - a_t \right) \left(2 \left(-\sigma c_{S,t} + y_t \right) + w_t - a_t \right) \right) + t.i.p. + O\left(||\xi||^3 \right), \tag{B.36}$$

The expression above can be reformulated as:

$$\mathbb{V}_{p,t} \equiv \pi_{p,t} + \frac{1}{2} \left(\frac{1}{\left(1 - \theta_p \right) \xi_p} + \frac{1+\xi_p}{\xi_p} \right) {\pi_{p,t}}^2 + \frac{1}{2} \left(1 - \theta_p \beta \right) \pi_{p,t} \Theta_{p,t}, \tag{B.37}$$

where:

$$\Theta_{p,t} \equiv E_t \sum_{T=t}^{\infty} \left(\theta_p \beta \right)^{T-t} \left(2 \left(-\sigma c_{S,T} + y_T \right) + w_T - a_T + \sum_{s=t+1}^{T} \pi_{p,s} \right). \tag{B.38}$$

B.2 Wage New Keynesian Phillips Curve

We follow Benigno and Woodford (2004, 2005). The wage New Keynesian Phillips curve can be written as:

$$w_{h,t}^{\#} \equiv \frac{W_{h,t}^{\#}}{W_{h,t}} = \left(\frac{1+\xi_w}{1+\tau_w} \frac{K_{w,h,t}}{F_{w,h,t}} \right)^{\frac{1}{1+\nu \frac{1+\xi_w}{\xi_w}}}, \tag{B.39}$$

where $\tau_w = \xi_w$ to eliminate the steady state distortion. The aggregate wage level can be rewritten as:

$$w_t^{\#} = \left(\frac{1 - \theta_w \Pi_{w,t}^{\frac{1}{\xi_w}}}{1 - \theta_w} \right)^{-\xi_w}. \tag{B.40}$$

We can substitute Equation (B.40) into Equation (B.39):

$$\left(\frac{1 - \theta_w \Pi_{w,t}^{\frac{1}{\xi_w}}}{1 - \theta_w} \right)^{-\xi_w} = \left(\frac{K_{w,h,t}}{F_{w,h,t}} \right)^{\frac{1}{1+\nu \frac{1+\xi_w}{\xi_w}}}. \tag{B.41}$$

Taking logs gives us:

$$-\xi_w \log\left(\frac{1-\theta_w \Pi_{w,t}^{\frac{1}{\xi_w}}}{1-\theta_w}\right) = \frac{1}{1+\nu\frac{1+\xi_w}{\xi_w}}\left(\log K_{w,h,t} - \log F_{w,h,t}\right). \tag{B.42}$$

A second-order Taylor series for the left-hand side of Equation (B.42) can be written as:

$$\log\left(\frac{1-\theta_w \Pi_{w,t}^{\frac{1}{\xi_w}}}{1-\theta_w}\right) = -\frac{\theta_w}{(1-\theta_w)\,\xi_w}\left(\pi_{w,t} + \frac{1}{2}\frac{1}{(1-\theta_w)\,\xi_w}{\pi_{w,t}}^2\right) + O\left(||\xi||^3\right). \tag{B.43}$$

It remains to derive similar second-order approximations for $\log K_{w,h,t}$ and $\log F_{w,h,t}$ on the right-hand side. The definitions of $K_{w,h,t}$ and $F_{w,h,t}$ imply the following second-order expansions:

$$k_{w,h,t} + \frac{1}{2}{k_{w,h,t}}^2 = (1-\theta_w\beta)\,E_t\sum_{T=t}^{\infty}(\theta_w\beta)^{T-t}\left(\widehat{k}_{w,h,t,T} + \frac{1}{2}\widehat{k}^2_{w,h,t,T}\right) + O\left(||\xi||^3\right), \tag{B.44}$$

$$f_{w,h,t} + \frac{1}{2}{f_{w,h,t}}^2 = (1-\theta_w\beta)\,E_t\sum_{T=t}^{\infty}(\theta_w\beta)^{T-t}\left(\widehat{f}_{w,h,t,T} + \frac{1}{2}\widehat{f}^2_{w,h,t,T}\right) + O\left(||\xi||^3\right), \tag{B.45}$$

where $\widehat{k}_{w,h,t,T}$ and $\widehat{f}_{w,h,t,T}$ are given by:

$$\widehat{k}_{w,h,t,T} \equiv \widehat{k}_{w,h,T} + (1+\nu)\,\frac{1+\xi_w}{\xi_w}\sum_{s=t+1}^{T}\pi_{w,s}, \tag{B.46}$$

$$\widehat{f}_{w,h,t,T} \equiv \widehat{f}_{w,h,T} + \frac{1}{\xi_w}\sum_{s=t+1}^{T}\pi_{w,s}, \tag{B.47}$$

where $\widehat{k}_{w,h,T}$ and $\widehat{f}_{w,h,T}$ are defined as:

$$\widehat{k}_{w,h,T} \equiv (1+\nu)\,n_{h,T}, \tag{B.48}$$

$$\widehat{f}_{w,h,T} \equiv -\sigma c_{h,T} + w_T + n_{h,T}. \tag{B.49}$$

Substituting Equation (B.43) into Equation (B.42) yields:

$$\pi_{w,t} + \frac{1}{2}\frac{1}{(1-\theta_w)\,\xi_w}{\pi_{w,t}}^2 = \frac{1-\theta_w}{\theta_w}\frac{1}{1+\nu\frac{1+\xi_w}{\xi_w}}\left(k_{w,h,t} - f_{w,h,t}\right) + O\left(||\xi||^3\right). \tag{B.50}$$

where $k_{w,h,t} \equiv \log K_{w,h,t} - \log K_{w,h}$ and $f_{w,h,t} \equiv \log F_{w,h,t} - \log F_{w,h}$. We can use Equations (B.44) and (B.45) to obtain a second-order expansion for the right-hand side of Equation (B.50). Subtracting Equation (B.45) from Equation (B.44), we obtain:

$$\begin{aligned} k_{w,h,t} - f_{w,h,t} = (1-\theta_w\beta)\, E_t \sum_{T=t}^{\infty} (\theta_w\beta)^{T-t} \left(\widehat{k}_{w,h,t,T} - \widehat{f}_{w,h,t,T} + \frac{1}{2}\left(\widehat{k}^2_{w,h,t,T} - \widehat{f}^2_{w,h,t,T} \right) \right) \\ -\frac{1}{2}\left({k_{w,h,t}}^2 - {f_{w,h,t}}^2 \right) + O\left(||\xi||^3 \right). \end{aligned} \tag{B.51}$$

We rewrite:

$$\begin{aligned} k_{w,h,t} - f_{w,h,t} = (1-\theta_w\beta)\, E_t \sum_{T=t}^{\infty} (\theta_w\beta)^{T-t} \left(\widehat{k}_{w,h,t,T} - \widehat{f}_{w,h,t,T} + \frac{1}{2}\left(\widehat{k}^2_{w,h,t,T} - \widehat{f}^2_{w,h,t,T} \right) \right) \\ -\frac{1}{2}\left(k_{w,h,t} - f_{w,h,t} \right)\left(k_{w,h,t} + f_{w,h,t} \right) + O\left(||\xi||^3 \right). \end{aligned} \tag{B.52}$$

Using Equation (B.50), to first order, we have:

$$k_{w,h,t} - f_{w,h,t} = \frac{\theta_w}{1-\theta_w}\left(1 + \nu \frac{1+\xi_w}{\xi_w} \right) \pi_{w,t} + O\left(||\xi||^2 \right). \tag{B.53}$$

Moreover, using Equations (B.44) and (B.45), to first order, we have:

$$k_{w,h,t} + f_{w,h,t} = (1-\theta_w\beta)\, \Theta_{w,h,t} + O\left(||\xi||^2 \right), \tag{B.54}$$

where:

$$\Theta_{w,h,t} \equiv E_t \sum_{T=t}^{\infty} (\theta_w\beta)^{T-t} \left(\widehat{k}_{w,h,t,T} + \widehat{f}_{w,h,t,T} \right). \tag{B.55}$$

Therefore, substituting Equations (B.53) and (B.54) into Equation (B.52) yields:

$$\begin{aligned} k_{w,h,t} - f_{w,h,t} = (1-\theta_w\beta)\, E_t \sum_{T=t}^{\infty} (\theta_w\beta)^{T-t} \left(\widehat{k}_{w,h,t,T} - \widehat{f}_{w,h,t,T} + \frac{1}{2}\left(\widehat{k}^2_{w,h,t,T} - \widehat{f}^2_{w,h,t,T} \right) \right) \\ -\frac{1}{2}\left(1-\theta_w\beta\right) \frac{\theta_w}{1-\theta_w}\left(1 + \nu \frac{1+\xi_w}{\xi_w} \right) \pi_{w,t}\Theta_{w,h,t} + O\left(||\xi||^3 \right). \end{aligned} \tag{B.56}$$

We can use the definitions of $\widehat{k}_{w,h,t,T}$ and $\widehat{f}_{w,h,t,T}$ to further expand the first term on the right-hand side of Equation (B.56). We obtain:

$$\begin{aligned} E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} \left(\widehat{k}_{w,h,t,T} - \widehat{f}_{w,h,t,T} \right) &= E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} \left(\widehat{k}_{w,h,T} - \widehat{f}_{w,h,T} \right) \\ &+ \left((1+\nu) \frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right) E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} \sum_{s=t+1}^{T} \pi_{w,s}. \end{aligned} \tag{B.57}$$

Then, we rewrite:

$$E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} \left(\widehat{k}_{w,h,t,T} - \widehat{f}_{w,h,t,T} \right) = E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} \left(\widehat{k}_{w,h,T} - \widehat{f}_{w,h,T} \right) + \frac{(1+\nu) \frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w}}{1 - \theta_w \beta} \Omega_{w,h,t}. \tag{B.58}$$

where:

$$\Omega_{w,h,t} \equiv E_t \sum_{T=t+1}^{\infty} (\theta_w \beta)^{T-t} \pi_{w,T}. \tag{B.59}$$

Moreover, we obtain:

$$\begin{aligned} \frac{1}{2} E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} \left(\widehat{k}^2_{w,h,t,T} - \widehat{f}^2_{w,h,t,T} \right) &= \frac{1}{2} E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} \left(\widehat{k}^2_{w,h,T} - \widehat{f}^2_{w,h,T} \right) \\ &+ E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} \left((1+\nu) \frac{1+\xi_w}{\xi_w} \widehat{k}_{w,h,T} - \frac{1}{\xi_w} \widehat{f}_{w,h,T} \right) \sum_{s=t+1}^{T} \pi_{w,s} \\ + \frac{1}{2} \left((1+\nu) \frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right) \left((1+\nu) \frac{1+\xi_w}{\xi_w} + \frac{1}{\xi_w} \right) E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} &\left(\sum_{s=t+1}^{T} \pi_{w,s} \right)^2. \end{aligned} \tag{B.60}$$

We can rewrite:

$$\begin{aligned} \frac{1}{2} E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} \left(\widehat{k}^2_{w,h,t,T} - \widehat{f}^2_{w,h,t,T} \right) &= \frac{1}{2} E_t \sum_{T=t}^{\infty} (\theta_w \beta)^{T-t} \left(\widehat{k}^2_{w,h,T} - \widehat{f}^2_{w,h,T} \right) + E_t \sum_{T=t+1}^{\infty} (\theta_w \beta)^{T-t} \pi_{w,T} \Gamma_{w,h,T} \\ &+ \frac{1}{2} \frac{\left((1+\nu) \frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right) \left((1+\nu) \frac{1+\xi_w}{\xi_w} + \frac{1}{\xi_w} \right)}{1 - \theta_w \beta} E_t \sum_{T=t+1}^{\infty} (\theta_w \beta)^{T-t} \pi_{w,T} \left(\pi_{w,T} + 2\Omega_{w,h,T} \right), \end{aligned} \tag{B.61}$$

where:

$$\Gamma_{w,h,t} \equiv E_t \sum_{T=t}^{\infty} (\theta_p \beta)^{T-t} \left((1+\nu) \frac{1+\xi_w}{\xi_w} \widehat{k}_{w,h,T} - \frac{1}{\xi_w} \widehat{f}_{w,h,T} \right). \tag{B.62}$$

Substituting Equations (B.58) and (B.61) into Equation (B.56), we obtain:

$$\begin{aligned} k_{w,h,t} - f_{w,h,t} = (1-\theta_w\beta)\, E_t \sum_{T=t}^{\infty} (\theta_p\beta)^{T-t} \left(\widehat{k}_{w,h,T} - \widehat{f}_{w,h,T} + \frac{1}{2}\left(\widehat{k}^2_{w,h,T} - \widehat{f}^2_{w,h,T} \right) \right) \\ + \left((1+\nu)\frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right) E_t \sum_{T=t+1}^{\infty} (\theta_w\beta)^{T-t} \pi_{w,T} + (1-\theta_w\beta)\, E_t \sum_{T=t+1}^{\infty} (\theta_w\beta)^{T-t} \pi_{w,T}\Gamma_{w,h,T} \\ + \frac{1}{2}\left((1+\nu)\frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right)\left((1+\nu)\frac{1+\xi_w}{\xi_w} + \frac{1}{\xi_w} \right) E_t \sum_{T=t+1}^{\infty} (\theta_w\beta)^{T-t} \pi_{w,T}\left(\pi_{w,T} + 2\Omega_{w,h,T} \right) \\ - \frac{1}{2}(1-\theta_w\beta)\frac{\theta_w}{1-\theta_w}\left(1 + \nu\frac{1+\xi_w}{\xi_w} \right) \pi_{w,t}\Theta_{w,h,t} + O\left(||\xi||^3 \right). \end{aligned} \tag{B.63}$$

This can be written recursively as:

$$\begin{aligned} k_{w,h,t} - f_{w,h,t} + \frac{1}{2}(1-\theta_w\beta)\frac{\theta_w}{1-\theta_w}\left(1 + \nu\frac{1+\xi_w}{\xi_w} \right)\pi_{w,t}\Theta_{w,t} = (1-\theta_w\beta)\left(\widehat{k}_{w,h,t} - \widehat{f}_{w,h,t} + \frac{1}{2}\left(\widehat{k}^2_{w,h,t} - \widehat{f}^2_{w,h,t} \right) \right) \\ + \left((1+\nu)\frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right)\theta_w\beta E_t\pi_{w,t+1} + (1-\theta_w\beta)\,\theta_w\beta E_t\pi_{w,t+1}\Gamma_{w,h,t+1} \\ + \frac{1}{2}\left((1+\nu)\frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right)\left((1+\nu)\frac{1+\xi_w}{\xi_w} + \frac{1}{\xi_w} \right)\theta_w\beta E_t\pi_{w,t+1}\left(\pi_{w,t+1} + 2\Omega_{w,h,t+1} \right) \\ + \theta_w\beta E_t\left(k_{w,h,t+1} - f_{w,h,t+1} + \frac{1}{2}(1-\theta_w\beta)\frac{\theta_w}{1-\theta_w}\left(1 + \nu\frac{1+\xi_w}{\xi_w} \right)\pi_{w,t+1}\Theta_{w,h,t+1} \right) + O\left(||\xi||^3 \right). \end{aligned} \tag{B.64}$$

Using Equation (B.50), we rewrite:

$$\begin{aligned} \pi_{w,t} + \frac{1}{2}\frac{1}{(1-\theta_w)\,\xi_w}{\pi_{w,t}}^2 + \frac{1}{2}(1-\theta_w\beta)\,\pi_{w,t}\Theta_{w,h,t} = \gamma_w\left(\widehat{k}_{w,h,t} - \widehat{f}_{w,h,t} + \frac{1}{2}\left(\widehat{k}^2_{w,h,t} - \widehat{f}^2_{w,h,t} \right) \right) \\ + (1-\theta_w)\,\beta E_t\pi_{w,t+1} + \frac{(1-\theta_w)(1-\theta_w\beta)}{1+\nu\frac{1+\xi_w}{\xi_w}}\beta E_t\pi_{w,t+1}\Gamma_{w,h,t+1} \\ + \frac{1}{2}\left((1+\nu)\frac{1+\xi_w}{\xi_w} + \frac{1}{\xi_w} \right)(1-\theta_w)\,\beta E_t\pi_{w,t+1}\left(\pi_{w,t+1} + 2\Omega_{w,h,t+1} \right) \\ + \theta_w\beta E_t\left(\pi_{w,t+1} + \frac{1}{2}\frac{1}{(1-\theta_w)\,\xi_w}{\pi_{w,t+1}}^2 + \frac{1}{2}(1-\theta_w\beta)\,\pi_{w,t+1}\Theta_{w,h,t+1} \right) + O\left(||\xi||^3 \right). \end{aligned} \tag{B.65}$$

where $\gamma_w \equiv \frac{(1-\theta_w)(1-\theta_w\beta)}{\theta_w\left(1+\nu\frac{1+\xi_w}{\xi_w}\right)}$. We can furthermore eliminate $\Gamma_{w,t}$ by observing that Equation (B.62) implies that:

$$\begin{aligned}
\Gamma_{w,h,t} &\equiv E_t \sum_{T=t}^{\infty} (\theta_w\beta)^{T-t} \left((1+\nu)\frac{1+\xi_w}{\xi_w}\widehat{k}_{w,h,T} - \frac{1}{\xi_w}\widehat{f}_{w,h,T} \right) \\
&= \frac{1}{2}E_t \sum_{T=t}^{\infty} (\theta_w\beta)^{T-t} \left(\left((1+\nu)\frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right)\left(\widehat{k}_{w,h,T} + \widehat{f}_{w,h,T}\right) + \left((1+\nu)\frac{1+\xi_w}{\xi_w} + \frac{1}{\xi_w} \right)\left(\widehat{k}_{w,h,T} - \widehat{f}_{w,h,T}\right) \right) \\
&= \frac{1}{2}E_t \sum_{T=t}^{\infty} (\theta_w\beta)^{T-t} \left(\left((1+\nu)\frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right)\left(\widehat{k}_{w,h,t,T} + \widehat{f}_{w,h,t,T}\right) + \left((1+\nu)\frac{1+\xi_w}{\xi_w} + \frac{1}{\xi_w} \right)\left(\widehat{k}_{w,h,t,T} - \widehat{f}_{w,h,t,T}\right) \right) \\
&\quad - \frac{\left((1+\nu)\frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right)\left((1+\nu)\frac{1+\xi_w}{\xi_w} + \frac{1}{\xi_w} \right)}{1-\theta_w\beta}\Omega_{w,h,t} \\
&= \frac{1}{2}\left((1+\nu)\frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right)\Theta_{w,h,t} + \frac{1}{2}\frac{\theta_w\left((1+\nu)\frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right)\left((1+\nu)\frac{1+\xi_w}{\xi_w} + \frac{1}{\xi_w} \right)}{(1-\theta_w)(1-\theta_w\beta)}\pi_{w,t} \\
&\quad - \frac{\left((1+\nu)\frac{1+\xi_w}{\xi_w} - \frac{1}{\xi_w} \right)\left((1+\nu)\frac{1+\xi_w}{\xi_w} + \frac{1}{\xi_w} \right)}{1-\theta_w\beta}\Omega_{w,h,t}.
\end{aligned} \tag{B.66}$$

By substituting Equation (B.66) into Equation (B.65), we obtain:

$$\begin{aligned}
\pi_{w,t} + \frac{1}{2}\frac{1}{(1-\theta_w)\xi_w}{\pi_{w,t}}^2 + \frac{1}{2}(1-\theta_w\beta)\pi_{w,t}\Theta_{w,h,t} &= \gamma_w\left(\widehat{k}_{w,h,t} - \widehat{f}_{w,h,t} + \frac{1}{2}\left(\widehat{k}^2_{w,h,t} - \widehat{f}^2_{w,h,t} \right) \right) \\
&\quad + \beta E_t\pi_{w,t+1} + \frac{1}{2}\beta\frac{1}{(1-\theta_w)\xi_w}E_t{\pi_{w,t+1}}^2 + \frac{1}{2}\beta(1-\theta_w\beta)E_t\pi_{w,t+1}\Theta_{w,h,t+1} \\
&\quad + \frac{1}{2}(1+\nu)\frac{1+\xi_w}{\xi_w}\beta E_t{\pi_{w,t+1}}^2 + O\left(||\xi||^3\right).
\end{aligned} \tag{B.67}$$

This can be integrated forward:

$$\mathbb{V}_{w,h,0} = \gamma_w E_0\sum_{t=0}^{\infty}\beta^t\left(\widehat{k}_{w,h,t} - \widehat{f}_{w,h,t} + \frac{1}{2}\left(\widehat{k}^2_{w,h,t} - \widehat{f}^2_{w,h,t} \right) \right) + \frac{1}{2}(1+\nu)\frac{1+\xi_w}{\xi_w}E_0\sum_{t=0}^{\infty}\beta^t{\pi_{w,t}}^2 + O\left(||\xi||^3\right), \tag{B.68}$$

where:

$$\mathbb{V}_{w,h,t} \equiv \pi_{w,t} + \frac{1}{2}\frac{1}{(1-\theta_w)\xi_w}{\pi_{w,t}}^2 + \frac{1}{2}(1-\theta_w\beta)\pi_{w,t}\Theta_{w,h,t} + \frac{1}{2}(1+\nu)\frac{1+\xi_w}{\xi_w}{\pi_{w,t}}^2. \tag{B.69}$$

Using Equations (B.48) and (B.49), we have:

$$\widehat{k}_{w,h,t} - \widehat{f}_{w,h,t} = \nu n_{h,t} + \sigma c_{h,t} - w_t, \tag{B.70}$$

$$\widehat{k}_{w,h,t} + \widehat{f}_{wh,t} = 2\left(-\sigma c_{h,t} + n_{h,t}\right) + \nu n_{h,t} + \sigma c_{h,t} + w_t. \tag{B.71}$$

Therefore, we can have:

$$\begin{aligned} \widehat{k}_{w,h,t} - \widehat{f}_{w,h,t} + \frac{1}{2}\left(\widehat{k}^2_{w,h,t} - \widehat{f}^2_{w,h,t}\right) &= \nu n_{h,t} + \sigma c_{h,t} - w_t \\ + \frac{1}{2}\left(\nu n_{h,t} + \sigma c_{h,t} - w_t\right)\left(2\left(-\sigma c_{h,t} + n_{h,t}\right) + \nu n_{h,t} + \sigma c_{h,t} + w_t\right). & \end{aligned} \tag{B.72}$$

Substituting Equation (B.72) into Equation (B.68), we finally obtain:

$$\begin{aligned} \mathbb{V}_{w,h,0} = \gamma_w E_0 \sum_{t=0}^{\infty} \beta^t \Bigg(\nu n_{h,t} + \sigma c_{h,t} - w_t + \frac{1}{2}\left(\nu n_{h,t} + \sigma c_{h,t} - w_t\right)\left(2\left(-\sigma c_{h,t} + n_{h,t}\right) + \nu n_{h,t} + \sigma c_{h,t} + w_t\right) \Bigg) \\ + \frac{1}{2}\left(1+\nu\right)\frac{1+\xi_w}{\xi_w} E_0 \sum_{t=0}^{\infty} \beta^t {\pi_{w,t}}^2 + t.i.p. + O\left(||\xi||^3\right). \end{aligned} \tag{B.73}$$

Then, we rewrite:

$$\begin{aligned} \mathbb{V}_{w,h,0} = \gamma_w E_0 \sum_{t=0}^{\infty} \beta^t \Bigg(\frac{1}{2}\frac{\left(1+\nu\right)\frac{1+\xi_w}{\xi_w}}{\gamma_w} {\pi_{w,t}}^2 + \nu n_{h,t} + \sigma c_{h,t} - w_t \\ + \frac{1}{2}\left(\nu n_{h,t} + \sigma c_{h,t} - w_t\right)\left(2\left(-\sigma c_{h,t} + n_{h,t}\right) + \nu n_{h,t} + \sigma c_{h,t} + w_t\right) \Bigg) + t.i.p. + O\left(||\xi||^3\right), \end{aligned} \tag{B.74}$$

The expression above can be reformulated as:

$$\mathbb{V}_{w,h,t} \equiv \pi_{w,t} + \frac{1}{2}\left(\frac{1}{\left(1-\theta_w\right)\xi_w} + \left(1+\nu\right)\frac{1+\xi_w}{\xi_w}\right){\pi_{w,t}}^2 + \frac{1}{2}\left(1-\theta_w\beta\right)\pi_{w,t}\Theta_{w,h,t}, \tag{B.75}$$

where:

$$\Theta_{w,h,t} \equiv E_t \sum_{T=t}^{\infty} \left(\theta_w\beta\right)^{T-t} \left(2\left(-\sigma c_{h,T} + n_{h,T}\right) + \nu n_{h,T} + \sigma c_{h,T} + w_T + \sum_{s=t+1}^{T} \pi_{w,s}\right). \tag{B.76}$$

Finally, we aggregate two wage New Keynesian Phillips curves using $mrs_t = \nu n_{S,t} + \sigma c_{S,t} = \nu n_{H,t} + \sigma c_{H,t}$:

$$\begin{aligned}\mathbb{V}_{w,0} &= (1-\lambda)\,\mathbb{V}_{w,S,0} + \lambda \mathbb{V}_{w,H,0} \\ &= \gamma_w E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2} \frac{(1+\nu)\,\frac{1+\xi_w}{\xi_w}}{\gamma_w} {\pi_{w,t}}^2 + mrs_t - w_t + \frac{1}{2}\left(mrs_t - w_t\right)\left(2\left(-\sigma c_t + n_t\right) + mrs_t + w_t\right) \right) \\ &\quad + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{B.77}$$

The expression above can be reformulated as:

$$\mathbb{V}_{w,t} \equiv \pi_{w,t} + \frac{1}{2}\left(\frac{1}{(1-\theta_w)\,\xi_w} + (1+\nu)\,\frac{1+\xi_w}{\xi_w} \right) {\pi_{w,t}}^2 + \frac{1}{2}\left(1-\theta_w\beta\right)\pi_{w,t}\Theta_{w,t}, \tag{B.78}$$

where:

$$\Theta_{w,t} \equiv E_t \sum_{T=t}^{\infty} \left(\theta_w\beta\right)^{T-t} \left(2\left(-\sigma c_T + n_T\right) + mrs_T + w_T + \sum_{s=t+1}^{T} \pi_{w,s} \right). \tag{B.79}$$

B.3 Aggregation

We recall the second-order price and wage New Keynesian Phillips curves:

$$\mathbb{V}_{p,0} = \gamma_p E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2} \frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p} {\pi_{p,t}}^2 + w_t - a_t + \frac{1}{2} (w_t - a_t) (2 (-\sigma c_{S,t} + y_t) + w_t - a_t) \right) + t.i.p. + O\left(||\xi||^3 \right), \tag{B.80}$$

$$\mathbb{V}_{w,0} = \gamma_w E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2} \frac{(1+\nu) \frac{1+\xi_w}{\xi_w}}{\gamma_w} {\pi_{w,t}}^2 + mrs_t - w_t + \frac{1}{2} (mrs_t - w_t) (2 (-\sigma c_t + y_t - a_t) + mrs_t + w_t) \right)$$
$$+ t.i.p. + O\left(||\xi||^3 \right). \tag{B.81}$$

We combine Equation (B.80) and (B.81):

$$\mathbb{V}_0 = \frac{1}{\gamma_p} \mathbb{V}_{p,0} + \frac{1}{\gamma_w} \mathbb{V}_{w,0}$$
$$= E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2} \frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p} {\pi_{p,t}}^2 + \frac{1}{2} \frac{(1+\nu) \frac{1+\xi_w}{\xi_w}}{\gamma_w} {\pi_{w,t}}^2 + mrs_t - a_t + \frac{1}{2} (mrs_t - a_t) (2 (-\sigma c_{S,t} + y_t) + mrs_t - a_t) \right.$$
$$\left. - \sigma (mrs_t - w_t) (c_t - c_{S,t}) \right) + t.i.p. + O\left(||\xi||^3 \right) \tag{B.82}$$

Using Equation (A.39), we have

$$\mathbb{V}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2} \frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p} {\pi_{p,t}}^2 + \frac{1}{2} \frac{(1+\nu) \frac{1+\xi_w}{\xi_w}}{\gamma_w} {\pi_{w,t}}^2 \right.$$
$$+ \nu \left(\frac{1}{2} \frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p} {\pi_{p,t}}^2 + y_t + \frac{1}{2} \left({y_t}^2 - 2 y_t a_t - (1-\lambda) {n_{S,t}}^2 - \lambda {n_{H,t}}^2 \right) \right) + \sigma \left(y_t + \frac{1}{2} \left({y_t}^2 - (1-\lambda) {c_{S,t}}^2 - \lambda {c_{H,t}}^2 \right) \right)$$
$$\left. + \frac{1}{2} \left((\nu + \sigma) y_t - (1+\nu) a_t \right) (2 (-\sigma c_{S,t} + y_t) + (\nu + \sigma) y_t - (1+\nu) a_t) - \frac{\sigma}{\gamma_w} \Phi_{w,t} (y_t - c_{S,t}) \right) + t.i.p. + O\left(||\xi||^3 \right) \tag{B.83}$$

C Fully Quadratic Utility-Based Welfare Criterion

In this Appendix, we derive a fully quadratic expression of the welfare function. Recall Equations (A.44) and (B.83):

$$\begin{aligned}\frac{\mathbb{W}_0}{U_C C} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg(& -\frac{1}{2}\frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}{\pi_{p,t}}^2 - \frac{1}{2}\frac{\frac{1+\xi_w}{\xi_w}}{\gamma_w}{\pi_{w,t}}^2 + y_t a_t - \frac{1}{2}\sigma\left((1-\lambda){c_{S,t}}^2 + \lambda {c_{H,t}}^2\right) - \frac{1}{2}\nu\left((1-\lambda){n_{S,t}}^2 + \lambda {n_{H,t}}^2\right) \\ & - (\lambda^\star - \lambda)\left(\left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right)\left(n_{S,t} - n_{H,t}\right)\right. \\ & - \frac{\lambda - \tau_d}{\lambda(1-\lambda)}\left(\nu\left(\frac{1}{2}\frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}{\pi_{p,t}}^2 + y_t + \frac{1}{2}\left({y_t}^2 - 2y_t a_t - (1-\lambda){n_{S,t}}^2 - \lambda {n_{H,t}}^2\right)\right)\right. \\ & + \sigma\left(y_t + \frac{1}{2}\left({y_t}^2 - (1-\lambda){c_{S,t}}^2 - \lambda {c_{H,t}}^2\right)\right) - \frac{1}{\gamma_w}\Phi_{w,t} + \frac{1}{2}\left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right)^2 \\ & + \frac{1}{2}\frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}{\pi_{p,t}}^2 + \left(\nu\left(y_t - a_t\right) + \sigma y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right) y_t - \left((\nu+\sigma) y_t - \frac{1}{\gamma_w}\Phi_{w,t}\right) a_t - y_t a_t \\ & \left.\left. - \frac{1}{2}\left(\sigma\left({c_{S,t}}^2 - {c_{H,t}}^2\right) + \nu\left({n_{S,t}}^2 - {n_{H,t}}^2\right)\right)\right)\Bigg) + t.i.p. + O\left(||\xi||^3\right). \end{aligned} \tag{C.1}$$

$$\begin{aligned}\mathbb{V}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg(& \frac{1}{2}\frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}{\pi_{p,t}}^2 + \frac{1}{2}\frac{(1+\nu)\frac{1+\xi_w}{\xi_w}}{\gamma_w}{\pi_{w,t}}^2 \\ & + \nu\left(\frac{1}{2}\frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}{\pi_{p,t}}^2 + y_t + \frac{1}{2}\left({y_t}^2 - 2y_t a_t - (1-\lambda){n_{S,t}}^2 - \lambda {n_{H,t}}^2\right)\right) + \sigma\left(y_t + \frac{1}{2}\left({y_t}^2 - (1-\lambda){c_{S,t}}^2 - \lambda {c_{H,t}}^2\right)\right) \\ & + \frac{1}{2}\left((\nu+\sigma) y_t - (1+\nu) a_t\right)\left(2\left(-\sigma c_{S,t} + y_t\right) + (\nu+\sigma) y_t - (1+\nu) a_t\right) - \frac{\sigma}{\gamma_w}\Phi_{w,t}\left(y_t - c_{S,t}\right)\Bigg) + t.i.p. + O\left(||\xi||^3\right). \end{aligned} \tag{C.2}$$

We can multiply Equation (C.2) by $\frac{(\lambda-\tau_d)(\lambda^\star-\lambda)}{\lambda(1-\lambda)}$ and subtract from Equation (A.41), and use the following equations:

$$c_{S,t} = \left(1 - \frac{\nu(\lambda-\tau_d)}{1-\lambda}\right) y_t + \frac{\nu(\nu+1)(\lambda-\tau_d)}{(1-\lambda)(\nu+\sigma)} a_t + \frac{\nu(\lambda-\tau_d)}{(1-\lambda)\gamma_w(\nu+\sigma)}\Phi_{w,t}, \tag{C.3}$$

$$c_{H,t} = \left(1 + \frac{\nu(\lambda-\tau_d)}{\lambda}\right) y_t - \frac{\nu(\nu+1)(\lambda-\tau_d)}{\lambda(\nu+\sigma)} a_t - \frac{\nu(\lambda-\tau_d)}{\lambda\gamma_w(\nu+\sigma)}\Phi_{w,t}, \tag{C.4}$$

$$n_{S,t} = \left(1 + \frac{\sigma(\lambda-\tau_d)}{1-\lambda}\right) y_t - \left(1 + \frac{\sigma(1+\nu)(\lambda-\tau_d)}{(1-\lambda)(\nu+\sigma)}\right) a_t - \frac{\sigma(\lambda-\tau_d)}{(1-\lambda)\gamma_w(\nu+\sigma)}\Phi_{w,t}, \tag{C.5}$$

$$n_{H,t} = \left(1 - \frac{\sigma(\lambda - \tau_d)}{\lambda}\right) y_t - \left(1 - \frac{\sigma(1+\nu)(\lambda - \tau_d)}{\lambda(\nu+\sigma)}\right) a_t + \frac{\sigma(\lambda - \tau_d)}{\lambda \gamma_w (\nu+\sigma)} \Phi_{w,t}. \tag{C.6}$$

Moreover, note that the variance terms of $\Phi_{w,t}$ have a minor effect on utility qualitatively, compared with the variance terms of price inflation, wage inflation, and the output gap. Thus, we omit these terms. Accordingly, the welfare function can be rewritten as:

$$\begin{aligned}\frac{\mathbb{W}_0}{U_C C} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg(& -\frac{1}{2} \frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p} {\pi_{p,t}}^2 - \frac{1}{2} \frac{\frac{1+\xi_w}{\xi_w}}{\gamma_w} {\pi_{w,t}}^2 - \frac{1}{2} \left(1 + \frac{(\lambda - \tau_d)^2}{\lambda(1-\lambda)} \nu\sigma\right) \left((\nu+\sigma) {y_t}^2 - 2(1+\nu) y_t a_t\right) \\ & - \frac{1}{2} \frac{(\lambda - \tau_d)(\lambda^\star - \lambda)}{\lambda(1-\lambda)} \frac{(1+\nu)\frac{1+\xi_w}{\xi_w}}{\gamma_w} {\pi_{w,t}}^2 - \frac{1}{2} \frac{(\lambda - \tau_d)^2 (\lambda^\star - \lambda)}{\lambda^2 (1-\lambda)^2} \nu\sigma \left((\nu+\sigma) {y_t}^2 - 2(1+\nu) y_t a_t\right) \Bigg) \\ & + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{C.7}$$

Therefore, we can rewrite:

$$\begin{aligned}\frac{\mathbb{W}_0}{U_C C} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg(& \frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p} {\pi_{p,t}}^2 + \frac{\frac{1+\xi_w}{\xi_w}}{\gamma_w} \left(1 + \frac{(\lambda - \tau_d)(\lambda^\star - \lambda)}{\lambda(1-\lambda)} (1+\nu)\right) {\pi_{w,t}}^2 \\ & + (\nu+\sigma) \left(1 + \frac{(\lambda - \tau_d)^2}{\lambda(1-\lambda)} \nu\sigma \left(1 + \frac{\lambda^\star - \lambda}{\lambda(1-\lambda)}\right)\right) \tilde{y}_t^2 \Bigg) + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{C.8}$$

where $\tilde{y}_t \equiv y_t - y_t^e$ and $y_t^e \equiv \frac{1+\nu}{\nu+\sigma} a_t$. Finally, we can simplify the expression as:

$$\frac{\mathbb{W}_0}{U_C C} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t + t.i.p. + O\left(||\xi||^3\right). \tag{C.9}$$

The average period t welfare loss is thus given as:

$$\mathbb{L}_t \equiv \alpha_p {\pi_{p,t}}^2 + \alpha_w {\pi_{w,t}}^2 + \alpha_y \tilde{y}_t^2, \tag{C.10}$$

where:

$$\alpha_p \equiv \frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}, \tag{C.11}$$

$$\alpha_w \equiv \frac{\frac{1+\xi_w}{\xi_w}}{\gamma_w} \left(1 + \frac{(\lambda - \tau_d)(\lambda^\star - \lambda)}{\lambda(1-\lambda)} (1+\nu)\right), \tag{C.12}$$

$$\alpha_y \equiv (\nu+\sigma)\left(1+\frac{(\lambda-\tau_d)^2}{\lambda(1-\lambda)}\nu\sigma\left(1+\frac{\lambda^\star-\lambda}{\lambda(1-\lambda)}\right)\right). \tag{C.13}$$

$$\begin{aligned}&-\frac{1}{2}\left(\frac{\frac{1+\xi_p}{\xi_p}}{\gamma_p}{\pi_{p,t}}^2+\frac{\frac{1+\xi_w}{\xi_w}}{\gamma_w}{\pi_{w,t}}^2+(\nu+\sigma)\left(1+\frac{(\lambda-\tau_d)^2}{\lambda(1-\lambda)}\nu\sigma\right){x_t}^2\right.\\&\left.+\frac{(\lambda-\tau_d)^2\nu\sigma}{\lambda(1-\lambda)\gamma_w(\nu+\sigma)}\left(\frac{1}{\gamma_w}\left(\pi_{w,t}-\beta E_t\pi_{w,t+1}\right)^2-2(\nu+\sigma)x_t\left(\pi_{w,t}-\beta E_t\pi_{w,t+1}\right)\right)\right)\end{aligned} \tag{C.14}$$

D Medium-Scale Model

Detailed derivations of the medium-scale model follow Bilbiie et al. (2023). Below, we report the equilibrium conditions and the welfare criterion.

D.1 Nonstationary Equilibrium Conditions

$$\lambda_{S,t} = \frac{b_t}{C_{S,t} - \eta_S C_{S,t-1}} \tag{D.1}$$

$$\lambda_{S,t} = \beta \zeta_t E_t \left(s_{t+1} \lambda_{S,t+1} + (1 - s_{t+1}) \lambda_{H,t+1} \right) \frac{R_t}{\Pi_{t+1}} \tag{D.2}$$

$$s_t = s_0 \left(\frac{X_t}{X_t^\star} \right)^{s_1} \tag{D.3}$$

$$\Omega_{S,t} = w_{S,t} N_{S,t} + \left(r_t^k u_t - \frac{r^k}{\chi} \left(e^{\chi(u_t - 1)} - 1 \right) \right) \bar{K}_{S,t-1} + D_{S,t} \tag{D.4}$$

$$(1 - \lambda) D_{S,t} = Y_t - (1 - \lambda) r_t^k u_t \bar{K}_{S,t-1} - (1 - \lambda) w_{S,t} N_{S,t} - \lambda w_{H,t} N_{H,t} \tag{D.5}$$

$$w_{S,t}^{\#} = \frac{w_{S,t}^n}{w_{S,t}^d} \tag{D.6}$$

$$\begin{aligned} w_{S,t}^n = {} & \varphi_S \left(1 + \xi_{w,t} \right) b_t \varphi_t \left(\frac{w_{S,t}^{\#}}{w_{S,t}} \right)^{-(1+\nu_S)\frac{1+\xi_{w,t}}{\xi_{w,t}}} {N_{S,t}}^{1+\nu_S} \\ & + \theta_{w,S} \beta E_t \left(\frac{w_{S,t}^{\#}}{w_{S,t+1}^{\#}} \frac{\left(\Pi g_A \right)^{1-\iota_w} \left(\Pi_t g_{A,t} \right)^{\iota_w}}{\Pi_{t+1}} \right)^{-(1+\nu_S)\frac{1+\xi_{w,t}}{\xi_{w,t}}} w_{S,t+1}^n \end{aligned} \tag{D.7}$$

$$w_{S,t}^d = \lambda_{S,t} \left(\frac{w_{S,t}^{\#}}{w_{S,t}} \right)^{-\frac{1+\xi_{w,t}}{\xi_{w,t}}} N_{S,t} + \theta_{w,S} \beta E_t \left(\frac{w_{S,t}^{\#}}{w_{S,t+1}^{\#}} \right)^{-\frac{1+\xi_{w,t}}{\xi_{w,t}}} \left(\frac{\left(\Pi g_A \right)^{1-\iota_w} \left(\Pi_t g_{A,t} \right)^{\iota_w}}{\Pi_{t+1}} \right)^{-\frac{1}{\xi_{w,t}}} w_{S,t+1}^d \tag{D.8}$$

$${w_{S,t}}^{-\frac{1}{\xi_{w,t}}} = \left(1 - \theta_{w,S} \right) {w_{S,t}^{\#}}^{-\frac{1}{\xi_{w,t}}} + \theta_{w,S} \left(w_{S,t-1} \frac{\left(\Pi g_A \right)^{1-\iota_w} \left(\Pi_{t-1} g_{A,t-1} \right)^{\iota_w}}{\Pi_t} \right)^{-\frac{1}{\xi_{w,t}}} \tag{D.9}$$

$$1 = q_t \mu_t \left(1 - \kappa_I \left(\frac{I_{S,t}}{I_{S,t-1}} - g_A \right) \frac{I_{S,t}}{I_{S,t-1}} - \frac{\kappa_I}{2} \left(\frac{I_{S,t}}{I_{S,t-1}} - g_A \right)^2 \right) + \beta \kappa_I E_t \frac{\lambda_{S,t+1}}{\lambda_{S,t}} q_{t+1} \mu_{t+1} \left(\frac{I_{S,t+1}}{I_{S,t}} - g_A \right) \left(\frac{I_{S,t+1}}{I_{S,t}} \right)^2 \tag{D.10}$$

$$r^k e^{\chi(u_t - 1)} = r_t^k \tag{D.11}$$

$$q_t = \beta E_t \frac{\lambda_{S,t+1}}{\lambda_{S,t}} \left(r^k_{t+1} u_{t+1} - \frac{r^k}{\chi} \left(e^{\chi(u_{t+1}-1)} - 1 \right) + (1-\delta) q_{t+1} \right) \tag{D.12}$$

$$\bar{K}_{S,t} = (1-\delta) \bar{K}_{S,t-1} + \mu_t \left(1 - \frac{\kappa_I}{2} \left(\frac{I_{S,t}}{I_{S,t-1}} - g_A \right)^2 \right) I_{S,t} \tag{D.13}$$

$$K_{S,t} = u_t \bar{K}_{S,t-1} \tag{D.14}$$

$$\lambda_{H,t} = \frac{b_t}{C_{H,t} - \eta_H C_{H,t-1}} \tag{D.15}$$

$$\Omega_{H,t} = w_{H,t} N_{H,t} \tag{D.16}$$

$$w^{\#}_{H,t} = \frac{w^n_{H,t}}{w^d_{H,t}} \tag{D.17}$$

$$\begin{aligned} w^n_{H,t} = {} & \varphi_H \left(1 + \xi_{w,t}\right) b_t \varphi_t \left(\frac{w^{\#}_{H,t}}{w_{H,t}} \right)^{-(1+\nu_H)\frac{1+\xi_{w,t}}{\xi_{w,t}}} {N_{H,t}}^{1+\nu_H} \\ & + \theta_{w,H} \beta E_t \left(\frac{w^{\#}_{H,t}}{w^{\#}_{H,t+1}} \frac{\left(\Pi g_A\right)^{1-\iota_w} \left(\Pi_t g_{A,t}\right)^{\iota_w}}{\Pi_{t+1}} \right)^{-(1+\nu_H)\frac{1+\xi_{w,t}}{\xi_{w,t}}} w^n_{H,t+1} \end{aligned} \tag{D.18}$$

$$w^d_{H,t} = \lambda_{H,t} \left(\frac{w^{\#}_{H,t}}{w_{H,t}} \right)^{-\frac{1+\xi_{w,t}}{\xi_{w,t}}} N_{H,t} + \theta_{w,H} \beta E_t \left(\frac{w^{\#}_{H,t}}{w^{\#}_{H,t+1}} \right)^{-\frac{1+\xi_{w,t}}{\xi_{w,t}}} \left(\frac{\left(\Pi g_A\right)^{1-\iota_w} \left(\Pi_t g_{A,t}\right)^{\iota_w}}{\Pi_{t+1}} \right)^{-\frac{1}{\xi_{w,t}}} w^d_{H,t+1} \tag{D.19}$$

$${w_{H,t}}^{-\frac{1}{\xi_{w,t}}} = \left(1 - \theta_{w,H}\right) {w^{\#}_{H,t}}^{-\frac{1}{\xi_{w,t}}} + \theta_{w,H} \left(w_{H,t-1} \frac{\left(\Pi g_A\right)^{1-\iota_w} \left(\Pi_{t-1} g_{A,t-1}\right)^{\iota_w}}{\Pi_t} \right)^{-\frac{1}{\xi_{w,t}}} \tag{D.20}$$

$$C_{H,t} = w_{H,t} N_{H,t} + T_{H,t} \tag{D.21}$$

$$\Delta^p_t Y_t = \left(\omega {A_t}^{\theta} \left(\lambda N_{H,t}\right)^{\theta} + (1-\omega) \left(\alpha {a_{K,t}}^{\varrho} \left(\left(1-\lambda\right) K_{S,t} \right)^{\varrho} + \left(1-\alpha\right) {A_t}^{\varrho} \left(\left(1-\lambda\right) N_{S,t} \right)^{\varrho} \right)^{\frac{\theta}{\varrho}} \right)^{\frac{1}{\theta}} - A_t F \tag{D.22}$$

$$r^k_t = \left(1-\omega\right) \alpha m c_t \left(\Delta^p_t Y_t + A_t F \right)^{1-\theta} \left(\alpha {a_{K,t}}^{\varrho} \left(\left(1-\lambda\right) K_{S,t} \right)^{\varrho} + \left(1-\alpha\right) {A_t}^{\varrho} \left(\left(1-\lambda\right) N_{S,t} \right)^{\varrho} \right)^{\frac{\theta}{\varrho}-1} {a_{K,t}}^{\varrho} \left(\left(1-\lambda\right) K_{S,t} \right)^{\varrho-1} \tag{D.23}$$

$$\begin{aligned} w_{S,t} = {} & \left(1-\omega\right) \left(1-\alpha\right) m c_t \left(\Delta^p_t Y_t + A_t F \right)^{1-\theta} \left(\alpha {a_{K,t}}^{\varrho} \left(\left(1-\lambda\right) K_{S,t} \right)^{\varrho} + \left(1-\alpha\right) {A_t}^{\varrho} \left(\left(1-\lambda\right) N_{S,t} \right)^{\varrho} \right)^{\frac{\theta}{\varrho}-1} \\ & {A_t}^{\varrho} \left(\left(1-\lambda\right) N_{S,t} \right)^{\varrho-1} \end{aligned} \tag{D.24}$$

$$w_{H,t} = \omega m c_t \left(\Delta^p_t Y_t + A_t F \right)^{1-\theta} {A_t}^{\theta} \left(\lambda N_{H,t}\right)^{\theta-1} \tag{D.25}$$

$$p^{\#}_t = \frac{p^n_t}{p^d_t} \tag{D.26}$$

$$p_t^n = \left(1+\xi_{p,t}\right) mc_t Y_t + \theta_p \beta E_t \frac{\lambda_{S,t+1}}{\lambda_{S,t}} \left(\frac{\Pi^{1-\iota_p}\Pi_t{}^{\iota_p}}{\Pi_{t+1}}\right)^{-\frac{1+\xi_{p,t}}{\xi_{p,t}}} p_{t+1}^n \tag{D.27}$$

$$p_t^d = Y_t + \theta_p \beta E_t \frac{\lambda_{S,t+1}}{\lambda_{S,t}} \left(\frac{\Pi^{1-\iota_p}\Pi_t{}^{\iota_p}}{\Pi_{t+1}}\right)^{-\frac{1}{\xi_{p,t}}} p_{t+1}^d \tag{D.28}$$

$$1 = \left(1-\theta_p\right) p_t^{\#-\frac{1}{\xi_{p,t}}} + \theta_p \left(\frac{\Pi^{1-\iota_p}\Pi_{t-1}{}^{\iota_p}}{\Pi_t}\right)^{-\frac{1}{\xi_{p,t}}} \tag{D.29}$$

$$\Delta_t^p = \left(1-\theta_p\right) p_t^{\#-\frac{1+\xi_{p,t}}{\xi_{p,t}}} + \theta_p \left(\frac{\Pi^{1-\iota_p}\Pi_{t-1}{}^{\iota_p}}{\Pi_t}\right)^{-\frac{1+\xi_{p,t}}{\xi_{p,t}}} \Delta_{t-1}^p \tag{D.30}$$

$$\left(1-\lambda\right) T_{S,t} + \lambda T_{H,t} + G_t = 0 \tag{D.31}$$

$$\frac{\Omega_{H,t} + T_{H,t}}{\Omega_{S,t} + T_{S,t}} = \tau_0 \left(\frac{\Omega_{H,t}/\Omega_{S,t}}{\Omega_H/\Omega_S}\right)^{\tau_1} \tag{D.32}$$

$$C_t = \left(1-\lambda\right) C_{S,t} + \lambda C_{H,t} \tag{D.33}$$

$$C_t + \left(1-\lambda\right) I_{S,t} + G_t + \left(1-\lambda\right) \frac{r^k}{\chi} \left(e^{\chi(u_t-1)} - 1\right) \bar{K}_{S,t-1} = Y_t \tag{D.34}$$

$$X_t \equiv C_t + \left(1-\lambda\right) I_{S,t} + G_t \tag{D.35}$$

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t \tag{D.36}$$

D.2 Welfare

D.2.1 Savers

$$V_{S,t}\left(i\right) = b_t \log\left(C_{S,t} - \eta_S C_{S,t-1}\right) - \varphi_S b_t \varphi_t \frac{N_{S,t}\left(i\right)^{1+\nu_S}}{1+\nu_S} + \beta E_t V_{S,t+1}\left(i\right) \tag{D.37}$$

$$\mathbb{W}_{S,t} = b_t \log\left(C_{S,t} - \eta_S C_{S,t-1}\right) - \varphi_S b_t \varphi_t \int_0^1 \frac{N_{S,t}\left(i\right)^{1+\nu_S}}{1+\nu_S} di + \beta E_t \mathbb{W}_{S,t+1} \tag{D.38}$$

$$\mathbb{W}_{S,t} = b_t \log\left(C_{S,t} - \eta_S C_{S,t-1}\right) - \varphi_S b_t \varphi_t \frac{N_{S,t}{}^{1+\nu_S}}{1+\nu_S} \int_0^1 \left(\frac{w_{S,t}\left(i\right)}{w_{S,t}}\right)^{-\left(1+\nu_S\right)\frac{1+\xi_{w,t}}{\xi_{w,t}}} di + \beta E_t \mathbb{W}_{S,t+1} \tag{D.39}$$

$$\mathbb{W}_{S,t} = b_t \log\left(C_{S,t} - \eta_S C_{S,t-1}\right) - \varphi_S b_t \varphi_t \Delta_{S,t}^w \frac{N_{S,t}{}^{1+\nu_S}}{1+\nu_S} + \beta E_t \mathbb{W}_{S,t+1} \tag{D.40}$$

$$\Delta_{S,t}^{w} = (1-\theta_{w,S})\left(\frac{w_{S,t}^{\#}}{w_{S,t}}\right)^{-(1+\nu_S)\frac{1+\xi_{w,t}}{\xi_{w,t}}} + \theta_{w,S}\left(\frac{w_{S,t-1}}{w_{S,t}}\frac{\left(\Pi g_A\right)^{1-\iota_w}\left(\Pi_{t-1}g_{A,t-1}\right)^{\iota_w}}{\Pi_t}\right)^{-(1+\nu_S)\frac{1+\xi_{w,t}}{\xi_{w,t}}}\Delta_{S,t-1}^{w} \tag{D.41}$$

D.2.2 Hand-to-Mouth

$$\mathbb{W}_{H,t} = b_t \log\left(C_{H,t} - \eta_H C_{H,t-1}\right) - \varphi_H b_t \varphi_t \Delta_{H,t}^{w}\frac{{N_{H,t}}^{1+\nu_H}}{1+\nu_H} + \beta E_t \mathbb{W}_{H,t+1} \tag{D.42}$$

$$\Delta_{H,t}^{w} = (1-\theta_{w,H})\left(\frac{w_{H,t}^{\#}}{w_{H,t}}\right)^{-(1+\nu_H)\frac{1+\xi_{w,t}}{\xi_{w,t}}} + \theta_{w,H}\left(\frac{w_{H,t-1}}{w_{H,t}}\frac{\left(\Pi g_A\right)^{1-\iota_w}\left(\Pi_{t-1}g_{A,t-1}\right)^{\iota_w}}{\Pi_t}\right)^{-(1+\nu_H)\frac{1+\xi_{w,t}}{\xi_{w,t}}}\Delta_{H,t-1}^{w} \tag{D.43}$$

D.2.3 Aggregate

$$\begin{aligned}\mathbb{W}_t &\equiv (1-\lambda)\mathbb{W}_{S,t} + \lambda \mathbb{W}_{H,t} \\ &= (1-\lambda) b_t \left(\log\left(C_{S,t} - \eta_S C_{S,t-1}\right) - \varphi_S \varphi_t \Delta_{S,t}^{w}\frac{{N_{S,t}}^{1+\nu_S}}{1+\nu_S}\right) \\ &\quad + \lambda b_t \left(\log\left(C_{H,t} - \eta_H C_{H,t-1}\right) - \varphi_H \varphi_t \Delta_{H,t}^{w}\frac{{N_{H,t}}^{1+\nu_H}}{1+\nu_H}\right) + \beta E_t \mathbb{W}_{t+1}\end{aligned} \tag{D.44}$$

D.3 Stationary Equilibrium Conditions

$g_{A,t} \equiv \frac{A_t}{A_{t-1}}$, $\hat{C}_{S,t} \equiv \frac{C_{S,t}}{A_t}$, $\hat{C}_{H,t} \equiv \frac{C_{H,t}}{A_t}$, $\hat{I}_{S,t} \equiv \frac{I_{S,t}}{A_t}$, $\hat{\bar{K}}_{S,t} \equiv \frac{\bar{K}_{S,t}}{A_t}$, $\hat{K}_{S,t} \equiv \frac{K_{S,t}}{A_t}$, $\hat{D}_{S,t} \equiv \frac{D_{S,t}}{A_t}$, $\hat{Y}_t \equiv \frac{Y_t}{A_t}$, $\hat{X}_t \equiv \frac{X_t}{A_t}$, $\hat{G}_t \equiv \frac{G_t}{A_t}$, $\hat{T}_{S,t} \equiv \frac{T_{S,t}}{A_t}$, $\hat{T}_{H,t} \equiv \frac{T_{H,t}}{A_t}$, $\hat{\Omega}_{S,t} \equiv \frac{\Omega_{S,t}}{A_t}$, $\hat{\Omega}_{H,t} \equiv \frac{\Omega_{H,t}}{A_t}$, $\hat{w}_{S,t} \equiv \frac{w_{S,t}}{A_t}$, $\hat{w}_{H,t} \equiv \frac{w_{H,t}}{A_t}$, $\hat{w}_{S,t}^{\#} \equiv \frac{w_{S,t}^{\#}}{A_t}$, $\hat{w}_{H,t}^{\#} \equiv \frac{w_{H,t}^{\#}}{A_t}$, $\hat{p}_t^n \equiv \frac{p_t^n}{A_t}$, $\hat{p}_t^d \equiv \frac{p_t^d}{A_t}$, $\hat{\lambda}_{S,t} \equiv \lambda_{S,t}A_t$, $\hat{\lambda}_{H,t} \equiv \lambda_{H,t}A_t$, $\hat{w}_{S,t}^d = w_{S,t}^d A_t$, $\hat{w}_{H,t}^d = w_{H,t}^d A_t$

$$\hat{\lambda}_{S,t} = \frac{b_t}{\hat{C}_{S,t} - \eta_S \frac{\hat{C}_{S,t-1}}{g_{A,t}}} \tag{D.45}$$

$$\hat{\lambda}_{S,t} = \beta\zeta_t E_t\left(s_{t+1}\frac{\hat{\lambda}_{S,t+1}}{g_{A,t+1}} + (1-s_{t+1})\frac{\hat{\lambda}_{H,t+1}}{g_{A,t+1}}\right)\frac{R_t}{\Pi_{t+1}} \tag{D.46}$$

$$s_t = s_0\left(\frac{\hat{X}_t}{\hat{X}_t^{\star}}\right)^{s_1} \tag{D.47}$$

$$\hat{\Omega}_{S,t} = \hat{w}_{S,t} N_{S,t} + \left(r_t^k u_t - \frac{r^k}{\chi} \left(e^{\chi(u_t - 1)} - 1 \right) \right) \frac{\hat{\bar{K}}_{S,t-1}}{g_{A,t}} + \hat{D}_{S,t} \tag{D.48}$$

$$(1-\lambda)\, \hat{D}_{S,t} = \hat{Y}_t - (1-\lambda)\, r_t^k u_t \frac{\hat{\bar{K}}_{S,t-1}}{g_{A,t}} - (1-\lambda)\, \hat{w}_{S,t} N_{S,t} - \lambda \hat{w}_{H,t} N_{H,t} \tag{D.49}$$

$$\hat{w}_{S,t}^{\#} = \frac{w_{S,t}^n}{\hat{w}_{S,t}^d} \tag{D.50}$$

$$\begin{aligned} w_{S,t}^n = {} & \varphi_S \left(1 + \xi_{w,t}\right) b_t \varphi_t \left(\frac{\hat{w}_{S,t}^{\#}}{\hat{w}_{S,t}} \right)^{-(1+\nu_S)\frac{1+\xi_{w,t}}{\xi_{w,t}}} {N_{S,t}}^{1+\nu_S} \\ & + \theta_{w,S} \beta E_t \left(\frac{\hat{w}_{S,t}^{\#}}{\hat{w}_{S,t+1}^{\#} g_{A,t+1}} \frac{\left(\Pi g_A\right)^{1-\iota_w} \left(\Pi_t g_{A,t}\right)^{\iota_w}}{\Pi_{t+1}} \right)^{-(1+\nu_S)\frac{1+\xi_{w,t}}{\xi_{w,t}}} w_{S,t+1}^n \end{aligned} \tag{D.51}$$

$$\begin{aligned} \hat{w}_{S,t}^d = {} & \hat{\lambda}_{S,t} \left(\frac{\hat{w}_{S,t}^{\#}}{\hat{w}_{S,t}} \right)^{-\frac{1+\xi_{w,t}}{\xi_{w,t}}} N_{S,t} \\ & + \theta_{w,S} \beta E_t \left(\frac{\hat{w}_{S,t}^{\#}}{\hat{w}_{S,t+1}^{\#} g_{A,t+1}} \right)^{-\frac{1+\xi_{w,t}}{\xi_{w,t}}} \left(\frac{\left(\Pi g_A\right)^{1-\iota_w} \left(\Pi_t g_{A,t}\right)^{\iota_w}}{\Pi_{t+1}} \right)^{-\frac{1}{\xi_{w,t}}} \frac{\hat{w}_{S,t+1}^d}{g_{A,t+1}} \end{aligned} \tag{D.52}$$

$$\hat{w}_{S,t}{}^{-\frac{1}{\xi_{w,t}}} = \left(1 - \theta_{w,S}\right) \hat{w}_{S,t}^{\#}{}^{-\frac{1}{\xi_{w,t}}} + \theta_{w,S} \left(\frac{\hat{w}_{S,t-1}}{g_{A,t}} \frac{\left(\Pi g_A\right)^{1-\iota_w} \left(\Pi_{t-1} g_{A,t-1}\right)^{\iota_w}}{\Pi_t} \right)^{-\frac{1}{\xi_{w,t}}} \tag{D.53}$$

$$\begin{aligned} 1 = {} & q_t \mu_t \left(1 - \kappa_I \left(\frac{\hat{I}_{S,t} g_{A,t}}{\hat{I}_{S,t-1}} - g_A \right) \frac{\hat{I}_{S,t} g_{A,t}}{\hat{I}_{S,t-1}} - \frac{\kappa_I}{2} \left(\frac{\hat{I}_{S,t} g_{A,t}}{\hat{I}_{S,t-1}} - g_A \right)^2 \right) \\ & + \beta \kappa_I E_t \frac{\hat{\lambda}_{S,t+1}}{\hat{\lambda}_{S,t} g_{A,t+1}} q_{t+1} \mu_{t+1} \left(\frac{\hat{I}_{S,t+1} g_{A,t+1}}{\hat{I}_{S,t}} - g_A \right) \left(\frac{\hat{I}_{S,t+1} g_{A,t+1}}{\hat{I}_{S,t}} \right)^2 \end{aligned} \tag{D.54}$$

$$r^k e^{\chi(u_t - 1)} = r_t^k \tag{D.55}$$

$$q_t = \beta E_t \frac{\hat{\lambda}_{S,t+1}}{\hat{\lambda}_{S,t} g_{A,t+1}} \left(r_{t+1}^k u_{t+1} - \frac{r^k}{\chi} \left(e^{\chi(u_{t+1} - 1)} - 1 \right) + \left(1 - \delta\right) q_{t+1} \right) \tag{D.56}$$

$$\hat{\bar{K}}_{S,t} = \left(1 - \delta\right) \frac{\hat{\bar{K}}_{S,t-1}}{g_{A,t}} + \mu_t \left(1 - \frac{\kappa_I}{2} \left(\frac{\hat{I}_{S,t} g_{A,t}}{\hat{I}_{S,t-1}} - g_A \right)^2 \right) \hat{I}_{S,t} \tag{D.57}$$

$$\hat{K}_{S,t} g_{A,t} = u_t \hat{\bar{K}}_{S,t-1} \tag{D.58}$$

$$\hat{\lambda}_{H,t} = \frac{b_t}{\hat{C}_{H,t} - \eta_H \frac{\hat{C}_{H,t-1}}{g_{A,t}}} \tag{D.59}$$

$$\hat{\Omega}_{H,t} = \hat{w}_{H,t} N_{H,t} \tag{D.60}$$

$$\hat{w}^{\#}_{H,t} = \frac{w^{n}_{H,t}}{\hat{w}^{d}_{H,t}} \tag{D.61}$$

$$\begin{aligned} w^{n}_{H,t} = {} & \varphi_H \left(1+\xi_{w,t}\right) b_t \varphi_t \left(\frac{\hat{w}^{\#}_{H,t}}{\hat{w}_{H,t}}\right)^{-(1+\nu_H)\frac{1+\xi_{w,t}}{\xi_{w,t}}} {N_{H,t}}^{1+\nu_H} \\ & + \theta_{w,H}\beta E_t \left(\frac{\hat{w}^{\#}_{H,t}}{\hat{w}^{\#}_{H,t+1} g_{A,t+1}} \frac{\left(\Pi g_A\right)^{1-\iota_w}\left(\Pi_t g_{A,t}\right)^{\iota_w}}{\Pi_{t+1}}\right)^{-(1+\nu_H)\frac{1+\xi_{w,t}}{\xi_{w,t}}} w^{n}_{H,t+1} \end{aligned} \tag{D.62}$$

$$\hat{w}^{d}_{H,t} = \hat{\lambda}_{H,t}\left(\frac{\hat{w}^{\#}_{H,t}}{\hat{w}_{H,t}}\right)^{-\frac{1+\xi_{w,t}}{\xi_{w,t}}} N_{H,t} + \theta_{w,H}\beta E_t \left(\frac{\hat{w}^{\#}_{H,t}}{\hat{w}^{\#}_{H,t+1} g_{A,t+1}}\right)^{-\frac{1+\xi_{w,t}}{\xi_{w,t}}} \left(\frac{\left(\Pi g_A\right)^{1-\iota_w}\left(\Pi_t g_{A,t}\right)^{\iota_w}}{\Pi_{t+1}}\right)^{-\frac{1}{\xi_{w,t}}} \frac{\hat{w}^{d}_{H,t+1}}{g_{A,t+1}} \tag{D.63}$$

$$\hat{w}_{H,t}{}^{-\frac{1}{\xi_{w,t}}} = \left(1-\theta_{w,H}\right)\hat{w}^{\#}_{H,t}{}^{-\frac{1}{\xi_{w,t}}} + \theta_{w,H}\left(\frac{\hat{w}_{H,t-1}}{g_{A,t}} \frac{\left(\Pi g_A\right)^{1-\iota_w}\left(\Pi_{t-1} g_{A,t-1}\right)^{\iota_w}}{\Pi_t}\right)^{-\frac{1}{\xi_{w,t}}} \tag{D.64}$$

$$\hat{C}_{H,t} = \hat{w}_{H,t} N_{H,t} + \hat{T}_{H,t} \tag{D.65}$$

$$\Delta^{p}_{t}\hat{Y}_t = \left(\omega\left(\lambda N_{H,t}\right)^{\theta} + \left(1-\omega\right)\left(\alpha a_{K,t}\left(\left(1-\lambda\right)\hat{K}_{S,t}\right)^{\varrho} + \left(1-\alpha\right)\left(\left(1-\lambda\right)N_{S,t}\right)^{\varrho}\right)^{\frac{\theta}{\varrho}}\right)^{\frac{1}{\theta}} - F \tag{D.66}$$

$$r^{k}_{t} = \left(1-\omega\right)\alpha m c_t \left(\Delta^{p}_{t}\hat{Y}_t + F\right)^{1-\theta}\left(\alpha a_{K,t}\left(\left(1-\lambda\right)\hat{K}_{S,t}\right)^{\varrho} + \left(1-\alpha\right)\left(\left(1-\lambda\right)N_{S,t}\right)^{\varrho}\right)^{\frac{\theta}{\varrho}-1} a_{K,t}\left(\left(1-\lambda\right)\hat{K}_{S,t}\right)^{\varrho-1} \tag{D.67}$$

$$\hat{w}_{S,t} = \left(1-\omega\right)\left(1-\alpha\right) m c_t \left(\Delta^{p}_{t}\hat{Y}_t + F\right)^{1-\theta}\left(\alpha a_{K,t}\left(\left(1-\lambda\right)\hat{K}_{S,t}\right)^{\varrho} + \left(1-\alpha\right)\left(\left(1-\lambda\right)N_{S,t}\right)^{\varrho}\right)^{\frac{\theta}{\varrho}-1}\left(\left(1-\lambda\right)N_{S,t}\right)^{\varrho-1} \tag{D.68}$$

$$\hat{w}_{H,t} = \omega m c_t \left(\Delta^{p}_{t}\hat{Y}_t + F\right)^{1-\theta}\left(\lambda N_{H,t}\right)^{\theta-1} \tag{D.69}$$

$$p^{\#}_{t} = \frac{\hat{p}^{n}_{t}}{\hat{p}^{d}_{t}} \tag{D.70}$$

$$\hat{p}^{n}_{t} = \left(1+\xi_{p,t}\right) m c_t \hat{Y}_t + \theta_p \beta E_t \frac{\hat{\lambda}_{S,t+1}}{\hat{\lambda}_{S,t} g_{A,t+1}}\left(\frac{\Pi^{1-\iota_p}\Pi_t{}^{\iota_p}}{\Pi_{t+1}}\right)^{-\frac{1+\xi_{p,t}}{\xi_{p,t}}} \hat{p}^{n}_{t+1} g_{A,t+1} \tag{D.71}$$

$$\hat{p}^{d}_{t} = \hat{Y}_t + \theta_p \beta E_t \frac{\hat{\lambda}_{S,t+1}}{\hat{\lambda}_{S,t} g_{A,t+1}}\left(\frac{\Pi^{1-\iota_p}\Pi_t{}^{\iota_p}}{\Pi_{t+1}}\right)^{-\frac{1}{\xi_{p,t}}} \hat{p}^{d}_{t+1} g_{A,t+1} \tag{D.72}$$

$$1 = \left(1-\theta_p\right) p^{\#}_{t}{}^{-\frac{1}{\xi_{p,t}}} + \theta_p \left(\frac{\Pi^{1-\iota_p}\Pi_{t-1}{}^{\iota_p}}{\Pi_t}\right)^{-\frac{1}{\xi_{p,t}}} \tag{D.73}$$

$$\Delta^{p}_{t} = \left(1-\theta_p\right) p^{\#}_{t}{}^{-\frac{1+\xi_{p,t}}{\xi_{p,t}}} + \theta_p \left(\frac{\Pi^{1-\iota_p}\Pi_{t-1}{}^{\iota_p}}{\Pi_t}\right)^{-\frac{1+\xi_{p,t}}{\xi_{p,t}}} \Delta^{p}_{t-1} \tag{D.74}$$

$$(1-\lambda)\,\hat{T}_{S,t}+\lambda\hat{T}_{H,t}+\hat{G}_t=0 \tag{D.75}$$

$$\frac{\hat{\Omega}_{H,t}+\hat{T}_{H,t}}{\hat{\Omega}_{S,t}+\hat{T}_{S,t}}=\tau_0\left(\frac{\hat{\Omega}_{H,t}/\hat{\Omega}_{S,t}}{\Omega_H/\Omega_S}\right)^{\tau_1} \tag{D.76}$$

$$\hat{C}_t=(1-\lambda)\,\hat{C}_{S,t}+\lambda\hat{C}_{H,t} \tag{D.77}$$

$$\hat{C}_t+(1-\lambda)\,\hat{I}_{S,t}+\hat{G}_t+(1-\lambda)\,\frac{r^k}{\chi}\left(e^{\chi(u_t-1)}-1\right)\frac{\hat{\bar{K}}_{S,t-1}}{g_{A,t}}=\hat{Y}_t \tag{D.78}$$

$$\hat{X}_t\equiv\hat{C}_t+(1-\lambda)\,\hat{I}_{S,t}+\hat{G}_t \tag{D.79}$$

$$\hat{G}_t=\left(1-\frac{1}{g_t}\right)\hat{Y}_t \tag{D.80}$$

$$\mathbb{W}_{S,t}=b_t\log\left(\hat{C}_{S,t}-\eta_S\frac{\hat{C}_{S,t-1}}{g_{A,t}}\right)-\varphi_S b_t\varphi_t\Delta^w_{S,t}\frac{{N_{S,t}}^{1+\nu_S}}{1+\nu_S}+\beta E_t\mathbb{W}_{S,t+1} \tag{D.81}$$

$$\Delta^w_{S,t}=(1-\theta_{w,S})\left(\frac{\hat{w}^{\#}_{S,t}}{\hat{w}_{S,t}}\right)^{-(1+\nu_S)\frac{1+\xi_{w,t}}{\xi_{w,t}}}+\theta_{w,S}\left(\frac{\hat{w}_{S,t-1}}{\hat{w}_{S,t}g_{A,t}}\frac{(\Pi g_A)^{1-\iota_w}\left(\Pi_{t-1}g_{A,t-1}\right)^{\iota_w}}{\Pi_t}\right)^{-(1+\nu_S)\frac{1+\xi_{w,t}}{\xi_{w,t}}}\Delta^w_{S,t-1} \tag{D.82}$$

$$\mathbb{W}_{H,t}=b_t\log\left(\hat{C}_{H,t}-\eta_H\frac{\hat{C}_{H,t-1}}{g_{A,t}}\right)-\varphi_H b_t\varphi_t\Delta^w_{H,t}\frac{{N_{H,t}}^{1+\nu_H}}{1+\nu_H}+\beta E_t\mathbb{W}_{H,t+1} \tag{D.83}$$

$$\Delta^w_{H,t}=(1-\theta_{w,H})\left(\frac{\hat{w}^{\#}_{H,t}}{\hat{w}_{H,t}}\right)^{-(1+\nu_H)\frac{1+\xi_{w,t}}{\xi_{w,t}}}+\theta_{w,H}\left(\frac{\hat{w}_{H,t-1}}{\hat{w}_{H,t}g_{A,t}}\frac{(\Pi g_A)^{1-\iota_w}\left(\Pi_{t-1}g_{A,t-1}\right)^{\iota_w}}{\Pi_t}\right)^{-(1+\nu_H)\frac{1+\xi_{w,t}}{\xi_{w,t}}}\Delta^w_{H,t-1} \tag{D.84}$$

$$\mathbb{W}_t\equiv(1-\lambda)\,\mathbb{W}_{S,t}+\lambda\mathbb{W}_{H,t} \tag{D.85}$$

E Data

We estimate our model using ten data series. Below, we present a detailed description of the sources and definitions of each series. With the exception of the two real wage series, the source and construction of the series closely follow Bilbiie et al. (2023). We take the eight aggregate macroeconomic indicators from the FRED database of the Federal Reserve Bank of St. Louis. The series' availability spans from 1954Q3 to 2019Q4. Details on the raw data are reported in Table E.1.

Table E.1: Raw Data Sources

Name	Source	Code[a]	Label
Macro Data			
Gross Domestic Product	FRED (BEA)	GDP	GDP
PCE: Nondurable Goods	FRED (BEA)	PCESV	Services
PCE: Nondurable Goods	FRED (BEA)	PCND	Nondurables
PCE: Durable Goods	FRED (BEA)	PCDG	Durables
Gross Private Domestic Investment	FRED (BEA)	GPDI	Investment
GDP Implicit Price Deflator	FRED (BEA)	GDPDEF	Deflator
Effective Federal Funds Rate	FRED	FEDFUNDS	FFR
Civilian Non-Institutional Population	FRED (BLS)	CNP16OV	
Total Economy: Compensation of Employees	FRED (BEA)	W209RC1Q027SBEA	Comp 1
Hourly Earnings of Employees	FRED (BEA)	AHETPI	Comp 2
Total Economy: Hours of All Persons	BLS	*Link: BLS*	Total Hours
Data to Construct Inequality Measures			
Salary	ASEC (CPS)	INCWAGE	Salary
Non-Farm Business Income	ASEC (CPS)	INCBUS	Non-Farm Income
Farm Business Income	ASEC (CPS)	INCFARM	Farm Income
Total Income	ASEC (CPS)	INCTOT	Total Income
Federal Income Tax Liability Before Credits	ASEC (CPS)	FEDTAX	Fed Tax
State Income Tax Liability Before Credits	ASEC (CPS)	STATETAX	State Tax
Social Security Retirement Payroll Deduction	ASEC (CPS)	FICA	Soc. Sec. Ret. Pay. Deduct.
Earned Income Tax Credit	ASEC (CPS)	EITCRED	Income Tax Credit
Tax Data before 1991			
Federal Taxes	TAXSIM	FIITAX	
State Taxes	TAXSIM	SIITAX	
Social Security Retirement Payroll Deduction	TAXSIM	FICA	

Notes: *a*) For data used to construct inequality measures, *Codes* refer to IPUMS acronyms.

The population series (labelled as "Population Trend") we use to express the macroeconomic aggregates in per capita terms is the Hodrick-Prescott trend (with smoothing parameter equal to 1600) of the logarithm of the Civilian Non-institutional Population series. The series of hours worked comes directly from the Total U.S. Economy Hours & Employment data file, available on the Bureau of Labor Statistics website.

We then apply the following transformations to the raw data:

1. Growth rate of real GDP per capita:

$$100 \times \Delta \log \left(\frac{\text{GDP}}{\text{Poulation Trend} \times \text{Deflator}} \right)$$

2. Growth rate of real consumption per capita:

$$100 \times \Delta \log \left(\frac{\text{Nondurables + Services}}{\text{Population Trend} \times \text{Deflator}} \right)$$

3. Growth rate of real investment per capita:

$$100 \times \Delta \log \left(\frac{\text{Investment + Durables}}{\text{Population Trend} \times \text{Deflator}} \right)$$

4. Log of per capita hours:

$$100 \times \log \left(\frac{\text{Total Hours}}{\text{Population Trend}} \right)$$

5. Growth rate of real hourly wages 1:

$$100 \times \Delta \log \left(\frac{\text{Comp 1}}{\text{Total Hours} \times \text{Deflator}} \right)$$

6. Growth rate of real hourly wages 2:

$$100 \times \Delta \log \left(\frac{\text{Comp 2}}{\text{Deflator}} \right)$$

7. Inflation rate:

$$100 \times \Delta \log \text{Deflator}$$

8. Short-term nominal interest rate expressed as a quarterly rate:

$$\frac{\text{FFR}}{4}.$$

To estimate our model, we also include two measures of inequality in labour earnings and disposable income. We construct these using data from the Annual Social and Economic (ASEC) supplement of the Current Population Survey, as reported by *IPUMS*. As in Heathcote et al. (2020) and Bilbiie et al. (2023), we drop households with any member in the armed forces, or having zero or negative ASEC weight, or reporting no hours worked but non-zero earnings, and households with no reference person or without any member between 25 and 60 years of age. We then

construct the series of household labour earnings as (using the labelling described in Table E.1):

$$\frac{\text{Salary} + (2/3) \times [\text{Non-Farm Income} + \text{Farm Income}]}{\text{Number of Adult Equivalents}}$$

and disposable income:

$$\frac{\text{Total Income} - \text{Tax Liabilities}}{\text{Number of Adult Equivalents}}$$

where the "Number of adult equivalents" follows the definition in Heathcote et al. (2010). Tax liabilities after 1991 are calculated as:

$$\text{Fed Tax} + \text{State Tax} + \text{Soc. Sec. Ret. Pay. Deduct.} - \text{Income Tax Credit.}$$

Prior to 1991, IPUMS does not provide tax information. For this reason, we compute tax liabilities using *Taxsim*, which estimates Federal Taxes (FIITAX), State Taxes (SIITAX, available starting in 1977), and Social Security Retirement Payroll Deduction (FICA). The details closely follow the procedure of Bilbiie et al. (2023).

The two inequality series are defined as the standard deviation of per-adult-equivalent pretax labour earnings and disposable income for each survey year. To construct these series, we discretise the labour earnings distribution using its 15^{th} (P15), 35^{th} (P35), 45^{th} (P45), 55^{th} (P55), 65^{th} (P65), 75^{th} (P75), 85^{th} (P85), and 95^{th} (P95) percentiles. We then take the logarithm of these percentiles and compute their weighted standard deviation for each year. Since the series display an upward trend, which the model cannot account for, we apply a band-pass filter to isolate fluctuations with periodicities of 30 years or less. Our results align closely with those of Bilbiie et al. (2023).