Fiscal Requirements for Price Stability When Households are not Ricardian

Stéphane Dupraz* Anna Rogantini Picco*
Banque de France ECB & CEPR

December 4, 2025

Abstract

Are restrictions on fiscal policy necessary for monetary policy to deliver price stability? When households are Ricardian, future fiscal surpluses must back the real value of public debt under stable prices. We show that when households are not Ricardian, fiscal requirements take a fundamentally different form: a limit on the debt-to-GDP ratio. Beyond that limit, the wealth effect of public debt on aggregate spending is so large that there is no natural rate of interest. To implement price stability when the debt-to-GDP limit holds, monetary policy must respond directly to the debt level, so that it tracks the natural rate.

Keywords: Monetary-Fiscal Interactions, Non-Ricardian Households, Price Stability.

JEL Classification: E31, E62.

*We thank our discussants Sara Casella, Wei Cui and Nigel McClung, as well as Jean Barthélémy, Marco Bassetto, Saroj Bhattarai, Francesco Bianchi, Nicolas Coeurdacier, Fabrice Collard, Bartosz Maćkowiak, Julien Matheron, Leonardo Melosi, Xavier Ragot, Ricardo Reis, Dmitriy Sergeyev and Sebastian Schmidt for very useful discussions, and seminar and conference participants at Banque de France, the ECB, Le Mans University, Sapienza Macro Days, the BSE forum, the SED, the CASFI Annual Research Conference, the DNB Annual Research Conference and Sciences Po for very useful comments. The views expressed herein are those of the authors and do not necessarily reflect those of the Banque de France, the ECB or the Eurosystem.

Introduction

Give the central bank a clear mandate of price stability. Grant it full independence from the government. And appoint at its head a steadfast governor who will not let anything but its mandate influence its policy. Does the central bank then have all it needs to deliver price stability? Or must requirements on the government's fiscal policy also be imposed? The question is a cornerstone of monetary-fiscal interactions, determining whether monetary policy has the power to insulate inflation from imprudent fiscal decisions, or is ultimately dependent on a well-behaved fiscal authority.

In policy circles fiscal requirements are typically seen as necessary, yet economic theory provides no uncontroversial basis for them. At the creation of the euro area, the convergence criteria of the Maastricht Treaty and then the Stability and Growth Pact introduced fiscal rules for national governments under the assumption they were necessary to allow the ECB to deliver on its price stability mandate. But the economic literature provides no consensus on whether such requirements are necessary. On the one hand, the monetarist view that the control of the price level is ultimately always in the realm of the central bank is embedded in most of the monetary models currently in use. On the other hand, Leeper (1991), Sims (1994), Woodford (1995, 2001) and the subsequent Fiscal Theory of the Price Level (FTPL) literature argue that for the central bank to be able to deliver price stability, current public debt must be backed by future fiscal surpluses.[1] Namely, the real value of public debt B_{t-1} at stable prices $P_t = P^*$ must be equal to the net present value (NPV) of future real fiscal surpluses T_{t+k},

$$\frac{B_{t-1}}{P^*} = \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} T_{t+k}, \tag{1}$$

where $\mathcal{R}_{t,t+k}$ is the real interest rate from t to $t+k$. If the government does not plan on sufficient future fiscal surpluses to make equation (1) hold, no equilibrium with stable prices exists. The central bank has therefore no chance of delivering it. To restore an equilibrium inflation must set in to erode the real value of public debt until it matches the level of real future surpluses.

Yet the NPV requirement (1) has remained controversial to this day. While early debates focused on the ability of the NPV equation (1) to determine the price level[2]—a central tenet of

[1] Distinct from the Fiscal Theory of the Price Level, the monetarist view was first challenged by Sargent and Wallace (1981)'s unpleasant monetarist arithmetic, which provides a distinct argument based on seigniorage revenues. In the present paper we stick to a cashless economy in which there are no seigniorage revenues from money holdings.

[2] See e.g. Kocherlakota and Phelan (1999); Buiter (2002); Niepelt (2004).

the FTPL—recent skepticism points out that the NPV requirement relies on the assumption of Ricardian households, when finite lives, financial frictions, or limited foresight are enough to make households non-Ricardian.[3] Whether fiscal policy can make monetary policy lose control over inflation when households are not Ricardian is heavily debated. Bassetto and Cui (2018) show that the NPV requirement can no longer determine inflation when households are not Ricardian and the real interest rate is below the growth rate of the economy.[4] Blanchard (2019) argues that when the real interest rate is below the growth rate of the economy the NPV equation no longer poses a constraint on fiscal policy. Reis (2021, 2022) and Brunnermeier, Merkel, and Sannikov (2020) argue that this does not imply there is no constraint on fiscal policy, but instead that the NPV requirement (1) needs to be augmented with a "bubble", or convenience yield, term. As this extra term is endogenous to monetary policy however, adding it to the NPV equation (1) leaves open the question of what fiscal policies are consistent with price stability and which ones are not.

In this paper, we show that when households are not Ricardian, fiscal requirements for price stability do exist, but that they reduce to the very different form of a limit on the real-debt-to-GDP ratio[5]

$$\frac{B_{t-1}}{P^* Y_t} - \frac{T_t}{Y_t} < d^*. \tag{2}$$

When the debt-to-GDP ratio is above the threshold d^* or projected to grow above this threshold in the future, no stable price equilibrium exists. The debt-to-GDP limit arises because above it, no interest rate however high can counterbalance the effect of higher debt on aggregate demand and bring it back in line with aggregate supply. Equivalently, the natural rate of interest—the real interest rate consistent with price stability—increases with the debt-to-GDP ratio, and at the limit d^* it becomes infinitely high.[6] To restore an equilibrium, inflation must necessarily set in to erode

[3] A note on terminology: In the FTPL, a key distinction is made between Ricardian and non-Ricardian *fiscal policies* (Woodford, 1995). A Ricardian fiscal policy is one that satisfies the intertemporal budget constraint of the government for any price level. Whether a fiscal policy is Ricardian or not is however different from whether households are Ricardian or not—they always are in the FTPL. To avoid confusion, in the paper we use the term *Ricardian* to refer to households only, and avoid using the term to refer to policies.

[4] Bassetto and Cui (2018) also show that the NPV equation remains a valid requirement when r is less than g because of high risk premia. On the role of risk premia in the NPV equation, see also Bohn (1995) and Jiang, Lustig, Nieuwerburgh, and Xiaolan (2024), who find that the NPV equation fails to hold in the data when the risk of future fiscal surpluses is priced through a realistic asset pricing model.

[5] The debt $B_{t-1}/P_t - T_t$ that appears on the left-hand side is the real quantity of debt the government needs to borrow at the end of period t. It is the difference between B_{t-1}/P_t, the maturity value of debt the government needs to repay at the beginning of period t (inclusive of interest rates accrued between $t-1$ and t), and taxes T_t raised at t.

[6] We use the definition of the natural rate as the real interest rate consistent with price stability. It is the short-term

the real value of public debt, lowering households' real wealth and therefore aggregate demand.

In our main result, we derive the fiscal requirement (2) in Blanchard (1985)'s model of perpetual youth, which breaks the Ricardian equivalence by assuming that households face a mortality risk.[7] We also show that fiscal requirements take the similar form of a limit on debt-to-GDP in a standard two-generation overlapping generations model. We focus on the perpetual-youth set-up because it has gained increasing appeal to study economies with non-Ricardian households, as households' mortality risk can be interpreted either literally as biological death—making it an overlapping generations model—or as the risk of hitting borrowing constraints—the financial frictions that are the focus of the HANK literature (e.g. Farhi and Werning, 2019; Wolf, 2025).

We derive condition (2) in two steps. First, following the derivation of the NPV requirement (1) when households are Ricardian, we consider what requirements arise from households' intertemporal budget constraints. In an equilibrium, all households' intertemporal budget constraints must hold with equality. Otherwise, households would be leaving cash on the table. Under a representative Ricardian household, the unique intertemporal budget constraint is the one of the unique representative household, and it is the mirror image of the one of the government. It must therefore hold with equality, imposing equation (1) as a fiscal requirement for price stability.

We show that when households are not Ricardian, households' intertemporal budget constraints impose only very weak requirements for a stable price equilibrium to exist. In particular, if, from any current level of public debt, the government plans on never raising any tax to repay it, this violates no household's intertemporal budget constraint. This does not mean individual households accumulate explosive amounts of public debt that they intend never to spend. Instead, they sell it to new generations. As a result, there is no need for inflation to make households' intertemporal budget constraints hold. Intertemporal budget constraints do impose a restriction on the path of future taxes and transfers, but it is a very weak one—never raising taxes satisfies it.

Second, we show this does not imply there exists no fiscal requirement for price stability. While intertemporal budget constraints no longer pose any significant constraint, a new requirement arises when households are not Ricardian. Higher public debt increases aggregate demand through a

natural rate that moves at business cycle frequency, and not the long-term (steady-state) natural rate. Note that some authors reserve "natural rate" for the latter and call the short-term natural rate the neutral rate (e.g. Obstfeld, 2023). Also note that here, the short-term natural rate does not necessarily correspond to the real interest rate that would obtain under flexible prices: With non-Ricardian households, the real interest rate that would obtain under flexible prices depends on the monetary and fiscal policies in place.

[7] Our results do not depend on the existence of a mortality risk however. We generalize Blanchard (1985)'s set-up to allow for population growth, which breaks the Ricardian equivalence even when households face no mortality risk.

wealth effect, putting upward pressure on inflation. This in itself poses no constraint on the ability of the central bank to maintain price stability. The central bank can counter the inflationary effect of higher debt with higher interest rates, just like it can counter any other inflationary shock with higher interest rates, retaining the ultimate control over inflation. At higher interest rates, households are willing to hold more public debt without spending their extra wealth, making aggregate demand in line with aggregate supply. In other words, to deliver price stability the central bank needs to deliver the natural rate of interest, which increases with the level of public debt when households are not Ricardian. Yet we show that there exists a threshold on the debt-to-GDP ratio above which even infinitely high interest rates are not enough to counter the wealth effect of public debt, resulting in the limit (2).

We show that the debt-to-GDP limit (2) can be both more and less stringent than the NPV requirement (1). On the one hand, the debt-to-GDP limit makes high levels of public debt inconsistent with price stability even when public debt is backed by future surpluses. This is in contrast to the NPV requirement, according to which only public debt that is not backed by future surpluses threatens price stability. On the other hand, when the interest rate is less than the growth rate of the economy ($r < g$), it is possible for public debt not to be backed by future surpluses without threatening price stability. However, even in this case there is a limit on how much debt the government can issue without planning on future fiscal surpluses. Because the real interest rate increases with the level of public debt, there exists a level of public debt above which r is necessarily greater than g, so that in the absence of fiscal surpluses debt-to-GDP necessarily ends up crossing the limit d^*.

We show that the level of d^* depends strongly on households' marginal propensity to consume. The higher households' MPC, the higher the fraction of their wealth they are willing to spend, and the faster we get to the point where even infinitely high interest rates are not enough to set aggregate demand in line with aggregate supply. When we interpret the model as an OLG model and calibrate the death probability accordingly, households' MPC is very small—0.01 quarterly—and as a result the debt-to-GDP limit d^* is very high— about 1700 times annual GDP. Yet, when we reinterpret the model as a HANK model and set the death probability to match the average MPC in the data—a quarterly MPC of 0.2 in line with empirical estimates—the debt-to-GDP limit is considerably lower, at 490% of GDP. We emphasize however that what matters is the MPC of households *that hold public debt*. To bring the distinction to life, we extend the model to allow for

a fraction of hand-to-mouth households. Any MPC of households that hold public debt can then be consistent with an average MPC of 0.2 by varying the share of hand-to-mouth households. We calibrate this extended model to match both the average MPC in the data and the sensitivity of the long-term interest rate to the debt-to-GDP ratio. Under this calibration strategy, the MPC of households that hold debt is 0.034 quarterly, and the debt-to-GDP limit d^* is 130 times GDP. This calibration approach therefore suggests that very high levels of public debt can be consistent with price stability provided the central bank finds a way to implement the stable price equilibrium.

How can the central bank implement the stable price equilibrium when it exists? The fiscal requirement (2) is intentionally derived abstracting from the question of implementation. We turn to it in Section 5. We first assume that the central bank sets nominal interest rates according to a standard Taylor rule, and the government sets taxes according to a similar feedback rule that responds to the level of public debt. In doing so we reconsider Leeper (1991)'s local version of the FTPL in the case of non-Ricardian households. We characterize analytically for which degree of responsiveness of fiscal and monetary policy there exists a unique bounded equilibrium, and when it exists whether inflation is insulated from fiscal shocks.

We show that under a standard Taylor rule where monetary policy responds to inflation only, fiscal shocks always affect inflation, however strong the response of monetary policy to inflation. It is no longer possible to distinguish between a monetary regime that insulates inflation from fiscal shocks and a fiscal regime that does not—the central result of Leeper (1991). This could suggest that price stability is never achievable when households are not Ricardian. Yet, we show that monetary policy can insulate inflation from fiscal shocks and implement the stable price equilibrium if, on top of reacting to inflation, it directly responds to the level of public debt—not just to the inflation that higher debt creates. Doing so allows it to track the natural rate, which depends on the level of public debt when households are not Ricardian. To insulate inflation from fiscal shocks, monetary policy must therefore monitor the level of public debt, in contrast to the idea that monetary dominance obtains when the central bank abstracts from fiscal developments.

By considering what the fiscal requirements for price stability are, this paper connects to the papers that have derived them in the case of Ricardian households, many of them associated to the FTPL literature developed by Leeper (1991), Sims (1994), Woodford (1995, 2001), Bassetto (2002), and Cochrane (2001, 2005).[8] Among these, we connect in particular to Woodford (2001), to which

[8]For reviews of the FTPL, see Canzoneri, Cumby, and Diba (2011), Leeper and Leith (2016) and Cochrane (2023), as well as Barthélémy, Mengus, and Plantin (2024). For a review of the FTPL with a special focus on empirical studies,

the title of the present paper is a reference. We depart from these by considering non-Ricardian households.

An important stream of the recent FTPL literature considers whether the dynamics of US inflation can be accounted for by models that feature elements of both monetary and fiscal dominance, either because the economy oscillates between regimes of fiscal and monetary dominance (Davig and Leeper, 2007; Bianchi and Ilut, 2017; Bianchi and Melosi, 2017, 2019; Schmidt, 2024), or because some fiscal shocks are funded while others are not (Cochrane, 2022; Bianchi, Faccini, and Melosi, 2023; Smets and Wouters, 2024)—see also Bassetto and Miller (2025) and Corsetti and Maćkowiak (2024) who show how sudden inflation can arise when the economy endogenously shifts to the fiscal regime.[9] We show that when households are not Ricardian the distinction between monetary and fiscal dominance is already blurred even absent regime switching or different reactions to different fiscal shocks. Elfsbacka-Schmoller and McClung (2024) consider the FTPL in a model with endogenous growth, a dimension from which we abstract. Jarocinski and Maćkowiak (2018), Bianchi, Melosi, and Rogantini Picco (2023) and Maćkowiak and Schmidt (2024) study a monetary union with several national fiscal authorities and study different fiscal-monetary configurations when common euro area debt is issued. We consider an economy with a unique fiscal authority. Barro and Bianchi (2023) analyze the drivers of the recent inflation surge in OECD countries through the lens of the FTPL. Hazell and Hobler (2024) use markets' reaction to the results of Georgia Senate runoff elections to assess how much deficits contributed to the inflation surge in the US.

A fast-growing literature analyzes fiscal-monetary interactions in models where the Ricardian equivalence breaks. A first stream considers models with financial frictions. Cui (2016) studies monetary-fiscal interactions in a model where public debt earns a liquidity premium because private assets face endogenous liquidity frictions. Interestingly, the shape of the determinacy region in his model is very similar to the one we find when monetary policy follows a standard Taylor rule. We show how a monetary rule that responds to public debt can restore the distinct regimes of monetary and fiscal dominance in Leeper (1991). Kaplan, Nikolakoudis, and Violante (2023) show that r can be less than g and the government can run permanent primary deficits in a HANK model. Mian, Straub, and Sufi (2022) consider a model with bonds-in-the-utility to quantify the

see Bianchi, Melosi, and Rogantini Picco (2024).

[9] On the comparison of the fiscal and monetary regimes and which prevailed when in the US, see also Bhattarai, Lee, and Park (2012, 2014).

maximum level of permanent deficits that the government can run. Mehrotra and Sergeyev (2021) analyze the determinants of the distribution of debt-to-GDP in a model with bonds-in-the-utility and where GDP is subject to risk. Campos, Fernández-Villaverde, Nuño, and Paz (2024) show that, in a HANK model, if the central bank fails to adjust its estimate of the long-term natural rate after a permanent increase in the level of public debt, inflation in the new steady-state is permanently above target. Our implementation result shows that fully insulating inflation from (temporary or permanent) increases in public debt is possible if the central bank's interest-rate rule tracks the short-term natural rate. Hagedorn (2024) contends that the FTPL fails in models with non-Ricardian households because markets are then incomplete.[10]

Another stream considers monetary-fiscal interactions in models with overlapping generations. Precursors are Bénassy (2003, 2005, 2008)—see also Bénassy (2007) for a review—and Leith and Wren-Lewis (2000). Farmer and Zabczyk (2023) study steady-state multiplicity and their determinacy properties in a quantitative OLG model where households live for 62 periods. In a perpetual-youth set-up similar to the one we consider, Brunnermeier, Merkel, and Sannikov (2020) show that the government can run permanent negative surpluses and that its intertemporal budget constraint needs to be augmented with a "bubble term". We show that fiscal requirements for price stability take the form of a debt-to-GDP limit. Aguiar, Amador, and Arellano (2023) analyze when an increase in public debt is Pareto-improving in a perpetual-youth set-up where the government issues real bonds. Consistent with our results, they show that, following an increase in public debt, monetary policy must increase interest rates with debt, otherwise steady-state inflation is permanently higher. Angeletos, Lian, and Wolf (2024a) consider how much deficits can finance themselves in a perpetual-youth model interpreted as a HANK model. Angeletos, Lian, and Wolf (2024b) compare the inflationary effects of deficits in such a model to the ones of the FTPL, under a specific monetary policy rule. Our implementation result derives what monetary policy rule can guarantee that deficits do *not* have an effect on inflation. Rachel and Ravn (2025) extend our determinacy results under a standard Taylor rule to a perpetual-youth economy with sticky prices. They show that under sticky prices as well, a monetary policy that responds only to inflation cannot insulate inflation from fiscal shocks. We show that monetary policy can if it responds to the level of public debt. Both Angeletos, Lian, and Wolf (2024b) and Rachel and Ravn (2025) consider the

[10] Although we deliberately stay away from normative statements and optimal policy, we also connect to papers that study optimal issuance of public debt in models with non-Ricardian households and/or where public debt provides liquidity to firms, such as Aiyagari and McGrattan (1998), Bhandari, Evans, Golosov, and Sargent (2017), Azzimonti and Yared (2019), Canzoneri, Cumby, and Diba (2016), Angeletos, Collard, and Dellas (2022) and Bassetto and Cui (2024).

linearized version of the perpetual-youth model. Our main result—the fiscal requirements for price stability—requires studying the non-linearized version of the model.[11]

Section 1 presents the main model of perpetual-youth with non-Ricardian households. Section 2 derives the fiscal requirements for price stability in this model, as well as in a standard two-generation OLG model. Section 3 discusses the new debt-to-GDP limit and compares it to the NPV requirement. Section 4 considers extensions of the model. Section 5 considers how the central bank can implement the stable price equilibrium once it exists.

1 An Economy with Non-Ricardian Households

In this section, we lay out the main model of non-Ricardian households we rely on. It is a Blanchard-Yaari perpetual-youth set-up (Blanchard, 1985) in discrete time, under perfect foresight. Relative to the baseline version of the Blanchard-Yaari model, we add two features. Neither is necessary for our results, but we allow for them for generality. First, we allow for population growth at rate g, which breaks the Ricardian equivalence even if households are infinitely lived (Weil, 1989). We do so both to meaningfully talk about r and g and in order to stress that our results do not depend on the assumption that households die and buy life-insurance contracts. Second, we allow for individual incomes to shrink over time (Blanchard, 1985, Section III). We do so both for realism to capture the need to save for retirement, and in order to allow for negative interest rates. We assume that the supply-side of the economy is given by an exogenous path for Y_t. Under flexible prices, this exogenous path can be interpreted as the supply-determined level of GDP. Under sticky prices, it can be interpreted as the exogenous path for natural output.

1.1 Of Life and Death

The economy is populated by an infinity of households of different ages. Households face a probability λ of dying each period, independent of how long they have been alive. To insure them against the risk of dying with positive wealth, insurance companies provide them with actuarially fair insurance contracts. The wealth of households that die at t is redistributed to households still alive, in proportion to the financial wealth they held at the end of the previous period. The

[11] Papers using the perpetual-youth set-up also include Galí (2021), who studies bubble-driven fluctuations in a New-Keynesian perpetual-youth set-up, but without public debt and taxes.

redistributed amount λB_{t-1} is therefore redistributed to the savings $(1-\lambda)B_{t-1}$ of the surviving households. Each dollar of saving therefore receives $\lambda/(1-\lambda)$ dollars of annuity.

New households are born every period, with no wealth. We assume that at t a number $(\lambda + g)N_{t-1}$ of households are born, so the population grows at rate g. Since λ of households die every period regardless of their ages, the number of households of age n at t is:

$$N_t(n) = (1-\lambda)^n(\lambda+g)N_{t-n-1} = \frac{\lambda+g}{1+g}\left(\frac{1-\lambda}{1+g}\right)^n N_t. \tag{3}$$

The particular case of the Ricardian representative household arises when there is neither new births nor deaths, $\lambda = 0$ and $g = 0$, in which cases all households are identical. Either $\lambda > 0$ or $g > 0$ is enough to break the Ricardian equivalence. From now on, we denote this non-Ricardian case as $\lambda + g > 0$.

1.2 Households

A household i has preferences over its consumption path

$$\sum_{k=0}^{\infty}(\beta(1-\lambda))^k \log(C^i_{t+k}), \tag{4}$$

where β is the preference discount factor. The household effectively discounts the future at the stronger rate $\beta(1-\lambda)$ because it factors in the chance that it won't be there to enjoy consumption tomorrow.

Household i maximizes its utility (4) subject to its flow budget constraints and a No-Ponzi-scheme constraint. Having bought bonds with maturity value B^i_{t-1} in period $t-1$, it starts period t with wealth $1/(1-\lambda)B^i_{t-1}$. Its flow budget constraint is

$$C^i_t + \frac{1}{R_t}\frac{B^i_t}{P_{t+1}} = \frac{1}{1-\lambda}\frac{B^i_{t-1}}{P_t} + (Y^i_t - T^i_t), \tag{5}$$

where C^i_t is its real consumption, Y^i_t its real income, T^i_t the real taxes it has to pay, and where R_t is the real interest rate. Its No-Ponzi-scheme constraint is

$$\lim_{k\to\infty}\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}}\frac{B^i_{t+k}}{P_{t+k+1}} \geq 0, \tag{6}$$

where $\mathcal{R}_{t,t+k}$ is the compounded real rate over k periods

$$\mathcal{R}_{t,t+k+1} = \prod_{j=0}^{k} R_{t+j}. \tag{7}$$

Like in the flow budget constraint (5) the factor $(1-\lambda)$ enters the No-Ponzi-scheme condition. It takes into account that the wealth the household holds grows at a rate $R/(1-\lambda)$ and not just R since the household also receives the annuity on its wealth.

The optimal behavior of household i can be characterized in the usual way (see Appendix A for a derivation). Household i behaves optimally if and only if its consumption is given by the consumption function

$$C_t^i = \mu \left(\frac{1}{1-\lambda} \frac{B_{t-1}^i}{P_t} + H_t^i \right) \tag{8}$$

where $\mu = 1 - \beta(1-\lambda)$ is the household's marginal propensity to consume and H_t^i is its human capital

$$H_t^i = \sum_{k=0}^{\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} (Y_{t+k}^i - T_{t+k}^i), \tag{9}$$

and its No-Ponzi-scheme constraint (6) holds with equality

$$\lim_{k\to\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}} \frac{B_{t+k}^i}{P_{t+k+1}} = 0. \tag{10}$$

Since condition (10) states that the household leaves no cash on the table, we refer to it as the No-Cash-on-the-Table condition.

1.3 Income Distribution

We assume that a household's income decreases exponentially with its age, as the amount of labor it supplies decreases. This creates a desire to save for one's old age—present in the baseline two-period OLG model—that makes negative real interest rates possible. The household's income is

$$Y_t^i(n) = \kappa(1-\zeta)^n \left(\frac{Y_t}{N_t} \right), \tag{11}$$

where Y_t is aggregate income, $\zeta \in [0,1]$, and κ is a constant determined by the fact that individual incomes must sum to aggregate incomes Y_t. Given the age distribution in the population (3), the

constant is

$$\kappa = \frac{1+g-(1-\lambda)(1-\zeta)}{\lambda+g} \text{ if } \lambda+g>0. \tag{12}$$

The assumption that individual incomes decrease with age, $\zeta > 0$, is possible only in the non-Ricardian case $\lambda+g>0$. In the Ricardian case of the representative agent $\lambda+g=0$, all households necessarily get the same income, $Y_t^i = Y_t$. We can then only assume $\zeta = 0$ and $\kappa = 1$.

1.4 Fiscal Policy

We assume no government spending G. We do so to keep focus: We are interested in the effect of the financing of government expenditures through taxes for a fixed path for G which we set to zero to avoid burdening notations. We thus take fiscal policy to consist of an inherited debt level B_{-1} and a path for the level of aggregate taxes $(T_t)_{t\geq 0}$. The flow budget constraint of the government is

$$\frac{1}{R_t}\frac{B_t}{P_{t+1}} + T_t = \frac{B_{t-1}}{P_t}, \tag{13}$$

and determines the path for public debt B_t from the government's fiscal policy.

We assume that individual taxes are imposed proportionally on income, so that they follow the same age profile as income. A household of age n pays taxes

$$T_t^i(n) = \kappa(1-\zeta)^n \left(\frac{T_t}{N_t}\right). \tag{14}$$

1.5 Equilibrium and Stable Price Equilibrium

An equilibrium is defined in the standard way.

Definition 1. *For a given inherited level of public debt B_{-1}, distributed as an arbitrary $(B_{-1}^i)_i$ in the population, and a given fiscal policy $(T_t)_{t\geq 0}$, an equilibrium is an interest rate path $(R_t)_{t\geq 0}$ and an allocation $(C_t^i)_{t\geq 0,i}$ s.t.*

1. *All households behave optimally: Given interest rates $(R_t)_{t\geq 0}$, aggregate incomes $(Y_t)_{t\geq 0}$ and aggregate taxes $(T_t)_{t\geq 0}$—of which individual incomes and individual taxes depend according to (11) and (14)—each household is on its consumption function (8) and satisfies it No-Cash-on-the-Table condition (10).*

2. The goods market clears:

$$C_t = \int_i C_t^i di = Y_t. \tag{15}$$

Among all equilibria, we are interested in stable price equilibria. A stable price equilibrium is an equilibrium with on-target inflation. For convenience and without loss of generality, we assume that the inflation target is equal to zero, so that keeping inflation on target means keeping the price level constant to a level P^*.

Definition 2. *A stable price equilibrium is an equilibrium where the price level is constant at $P_t = P^*$ at all t.*

2 Fiscal Requirements for Price Stability

This section derives the conditions on fiscal policy for a stable price equilibrium to exist. We first rederive the NPV requirement in the case of Ricardian households then move to the main result of the paper: the debt-limit requirement in the case of non-Ricardian households. We then show that fiscal requirements take the similar form of a debt limit in a simple two-generation OLG model.

2.1 The Case of Ricardian Households

For further reference, we first rederive fiscal requirements for price stability in the traditional case where households are Ricardian (Leeper, 1991; Sims, 1994; Woodford, 1995, 2001).

Proposition 1. *Assume households are Ricardian $\lambda + g = 0$. Consider an initial level of debt B_{-1} and a future tax path $(T_t)_{t\geq 0}$. There exists a stable price equilibrium for this fiscal policy if and only if*

$$\frac{B_{t-1}}{P^*} = \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} T_{t+k}, \tag{16}$$

where the real interest rate is given by $R_t = \frac{Y_{t+1}}{\beta Y_t}$.

The necessity of condition (16) follows from the intertemporal budget constraint of the repre-

sentative household

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k} = \frac{B_{t-1}}{P_t} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k} - T_{t+k}), \tag{17}$$

which is obtained by combining its flow budget constraint (5) and No-Cash-on-the-Table constraint (10). Adding market-clearing (15) it implies (16) when prices are stable $P_t = P^*$. The value of the real interest rate follows from the Euler equation of the representative household. Appendix B proves that condition (16) is also sufficient for a stable price equilibrium to exist.

Proposition 1 states that in an equilibrium, the real value of public debt must be equal to the net present value of future fiscal surpluses. Because the intertemporal budget constraint of the representative household and the intertemporal budget constraint of the government coincide in the Ricardian case, the requirement (16) can be stated as a requirement on the intertemporal budget constraint of the government. The necessity of condition (16) as an equilibrium requirement comes from the optimality condition of the household however, not the government's. When the central bank stands ready to unconditionally buy government debt so that the government cannot default, nothing forces the government to satisfy any intertemporal budget constraint. But in an equilibrium, households cannot hold wealth that they plan never to spend. If the wealth they own in government bonds is not matched by future taxes to pay, households will spend it. Intuitively, the resulting inflation will dilute their real wealth on the left-hand-side of (16) until it equates the NPV of future surpluses.

2.2 The Case of Non-Ricardian Households

The following proposition states the main result of the paper.

Proposition 2. *Assume households are non-Ricardian $\lambda + g > 0$. Consider an initial level of debt B_{-1} and a future tax path $(T_t)_{t \geq 0}$. There exists a stable price equilibrium for this fiscal policy if and only if at all times t the following two conditions hold:*

$$\frac{B_{t-1}}{P^* Y_t} - \frac{T_t}{Y_t} < d^*, \tag{18}$$

where

$$d^* = \left(\frac{\mu}{1-\mu}\left(1 - \frac{(1-\lambda)(1-\zeta)}{(1+g)}\right)\right)^{-1}, \tag{19}$$

and

$$\lim_{k\to\infty} \sum_{j=1}^{k} \Omega(k,j) \frac{1}{\mathcal{R}_{t,t+j}} T_{t+j} = 0, \tag{20}$$

for coefficient $\Omega(k,j)$ given below in Lemma 3.

Proposition 2 states that when households are not Ricardian, fiscal requirements for price stability take the form of a limit (18) of the debt-to-GDP ratio. Technically there exists a second requirement in the form of condition (20), but it can be disregarded in practice. It only poses extremely weak constraints on fiscal policy—never raising taxes satisfies it.

The rest of this section derives the fiscal requirement of Proposition 2, explaining where they arise from. Section 3 discusses this new requirement and how it compares to the NPV requirement (16) that prevails when households are Ricardian.

2.2.1 Fiscal Requirement from Households' Intertemporal Budget Constraints

We first follow the logic of the case of Ricardian households and consider what fiscal requirements arise from households' intertemporal budget constraints, or equivalently their No-Cash-on-the-Table conditions (10). Because there is now an infinity of such constraints, we first need to determine the equilibrium debt holdings of an individual household. Appendix C shows the following lemma.[12]

Lemma 1. *Assume households are not Ricardian $\lambda + g > 0$.*

If all households are on their consumption function (8) and the goods market clears (15), then at time t a household of age $n \leq t$ has holdings of public debt

$$\frac{B_t^i(n)}{P_{t+1}} = \psi_B(n) \frac{B_t}{P_{t+1}N_t} - \sum_{k=0}^{n} \phi_T(n,k) \left(\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}}\right), \tag{21}$$

[12] Since we consider any arbitrary initial distribution of public debt (B_{-1}^i), expression (21) holds only for households born after period $t = 0$. For households born before $t = 0$, their debt holdings depend on their initial debt holdings B_{t-1}^i. If the initial distribution of public debt has been determined in the same way before $t = 0$ as after $t = 0$ however, expression (21) is valid for all households of any age.

where

$$\psi_B(n) = \kappa(1-\zeta)^n \mu \frac{1-x^{n+1}}{1-x}, \tag{22}$$

$$\phi_T(n,k) = \kappa(1-\zeta)^n x^k \left(1 - \mu \frac{1-x^{n-k+1}}{1-x}\right), \tag{23}$$

$$x = \frac{\beta(1+g)}{1-\zeta}. \tag{24}$$

The expression for the coefficient $\psi_B(n)$ on aggregate public debt shows that abstracting from the effect of the past path of taxes, the amount of public debt held by a household either monotonically increases with age to converge to a constant fraction of public debt (when $\zeta = 0$), or is a single-peaked function of age, peaking in middle-age to then gradually shrink to zero (when $\zeta > 0$).

We now determine a necessary and sufficient condition for the No-Cash-on-the-Table conditions of all households to be satisfied. Appendix D shows the following lemma.

Lemma 2. *Assume households are not Ricardian $\lambda + g > 0$.*

Assume all households consume according to the consumption function (8).

All the No-Cash-on-the-Table constraints (10) of all households are satisfied if and only if

$$\lim_{k\to\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}} \frac{\bar{B}_{t+k}}{P_{t+k+1}} = 0, \tag{25}$$

where $\bar{B}_{t+k}$ is the average debt held at $t+k$ by households alive at t and still alive at $t+k$.

Intuitively, Lemma 2 states that the No-Cash-on-the-Table conditions of all households are satisfied if and only if the No-Cash-on-the-Table condition of a fictitious average household is satisfied, where the fictitious average household has debt holdings equal to the average debt holding among households that were already alive at t.

Lemmas 1 and 2 can be combined to determine what requirements households' intertemporal budget constraints impose on fiscal policy. Appendix E shows that they impose condition (20) in Proposition 2.

Lemma 3. *Assume households are not Ricardian $\lambda + g > 0$.*

Consider an inherited debt level B_{-1} and a future tax path $(T_t)_{t\geq 0}$. In an equilibrium, condition

(20) *must hold, where*

$$\Omega(k,j) = (\beta(1-\lambda))^k \theta(j) - \xi(k), \tag{26}$$

$$\xi(k) = \frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right), \tag{27}$$

$$\theta(0) = 0, \tag{28}$$

$$\forall j \geq 1, \theta(j) = (\beta(1-\lambda))^{-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right]. \tag{29}$$

As mentioned above, condition (20) only puts a very weak requirement on fiscal policy, contrary to the NPV equation (16) in the case of Ricardian households. In particular, the current debt level B_{t-1} does not appear in condition (20). As a consequence, for any current level of public debt, the fiscal policy of never raising any surplus to repay public debt ($T_t = 0$ at all times) is consistent with all households' intertemporal budget constraints being satisfied. When households are not Ricardian, this does not mean individual households accumulate explosive amounts of debt they intend never to spend. Instead, they sell it to new generations. In addition, condition (20) does not feature inflation. Inflation does nothing to ease this constraint and restore the possibility of an equilibrium.

2.2.2 Fiscal Requirements from the IS Curve

When households are not Ricardian, households' intertemporal budget constraints no longer impose any significant constraint on fiscal policy for price stability. But other fiscal requirements arise, specific to the case of non-Ricardian households. They arise from the IS curve of the model, derived in Appendix F.

Lemma 4. *In an equilibrium, the following dynamic IS curve holds*

$$Y_t = \frac{1-\zeta}{\beta(1+g)}\frac{1}{R_t}Y_{t+1} + \chi\left(\frac{B_{t-1}}{P_t} - T_t\right), \tag{30}$$

where

$$\chi = \frac{\mu}{1-\mu}\left(1 - \frac{(1-\lambda)(1-\zeta)}{(1+g)}\right). \tag{31}$$

In the case of Ricardian households $\lambda + g = 0$ (and so $\zeta = 0$), the model reduces to the standard representative agent model and the dynamic IS curve reduces to the standard Euler equation where public debt does not appear, $\chi = 0$. When households are not Ricardian $\lambda + g > 0$, net aggregate wealth matters for aggregate consumption $\chi > 0$, and therefore for inflation.

The presence of aggregate wealth in the IS curve (30) does not in itself mean that the central bank loses control over inflation however. Many things can affect the level of aggregate demand, for instance exogenous shocks to households' preferences. The claim that monetary policy remains ultimately in control of the price level does not deny that other factors than monetary policy can affect inflation. It only argues that the central bank can use interest rates to counterbalance these shocks and deliver on-target inflation.

The same argument applies in principle to public debt: high public debt increases aggregate demand, but if it creates too much demand, the central bank can increase rates to bring it down. Specifically, if the central bank delivers the real interest rate

$$R_t = \frac{1-\zeta}{\beta(1+g)} \frac{Y_{t+1}}{Y_t} \left(1 - \chi \left(\frac{B_{t-1}}{P^* Y_t} - \frac{T_t}{Y_t}\right)\right)^{-1}, \tag{32}$$

in all periods, a stable price equilibrium obtains. The real interest rate (32) is the natural rate of interest—the real interest rate consistent with price stability—and delivering it delivers price stability.

But does there always exist an interest rate level that allows the central bank to bring down demand in line with supply? The following lemma shows there does not when the debt-to-GDP ratio is too high, delivering the fiscal requirement (18).

Lemma 5. *Assume households are not Ricardian $\lambda + g > 0$.*

Consider an inherited debt level B_{-1} and a future tax path $(T_t)_{t\geq 0}$. In a stable price equilibrium, the real-debt-to-GDP ratio $\frac{B_{t-1}}{P_t Y_t} - \frac{T_t}{Y_t}$ for on-target inflation $P_t = P^*$ must be below the threshold

$$d^* = \frac{1}{\chi} \tag{33}$$

at all periods, resulting in equation (18).

The debt-to-GDP limit follows from the fact that when the debt-to-GDP ratio is above the limit d^*, no interest rate, however large, can make the IS equation (30) hold. Even an infinitely large real

interest rate is not enough to counter the wealth effect of public debt on aggregate consumption and bring aggregate demand down to supply. The natural rate increases with the debt-to-GDP ratio, and becomes infinitely large when the debt-to-GDP ratio approaches d^*.[13]

Lemmas 3 and 5 together show that conditions (18) and (20) are necessary in a stable price equilibrium. Appendix G shows that they are also sufficient, proving the characterization of the stable price equilibrium in Proposition 2.

2.3 Similar Characterization in a Standard Two-Generation OLG Model

Before discussing the new fiscal requirement of the debt-to-GDP ratio, we show that a similar characterization applies in a standard two-generation OLG model à la Samuelson (1958). Households live for two periods. In period t, households are thus divided between young households born at t and old households born at $t-1$. The population N_t grows at rate g, and is therefore divided between $N_t^{old} = \frac{1}{2+g} N_t$ households born at $t-1$ and $N_t^{young} = \frac{1+g}{2+g} N_t$ households born at t.

A household born at t has preferences over its consumption in periods t and $t+1$

$$\log(C_t^y) + \beta \log(C_{t+1}^o). \tag{34}$$

The household is born with no wealth and maximizes its utility (34) subject to the flow budget constraints

$$C_t^y + \frac{1}{R_t} \frac{B_t^y}{P_{t+1}} = Y_t^y - T_t^y, \tag{35}$$

$$C_{t+1}^o = \frac{B_t^y}{P_{t+1}} + Y_{t+1}^o - T_{t+1}^o. \tag{36}$$

We assume that young households receive collectively a share γ of the economy's total income while old households collectively receive the remaining share $1-\gamma$, $Y_t^y = \gamma \frac{Y_t}{N_t^y}$ and $Y_t^o = (1-\gamma)\frac{Y_t}{N_t^o}$. The government still sets aggregate taxes subject to the flow budget constraint (13) and taxes are still imposed proportionally to income, $T_t^y = \gamma \frac{T_t}{N_t^y}$ and $T_t^o = (1-\gamma)\frac{T_t}{N_t^o}$. An equilibrium and a stable-price equilibrium are still defined as in Definitions 1 and 2, where goods-market clearing now

[13] The limit d^* bears on the amount the government needs to borrow at the end of a period, $\frac{B_{t-1}}{P^*} - T_t = \frac{B_t}{P^* R_t}$. It does not bear on the maturity value the government needs to repay the following period, which includes interest rates. Appendix H shows that the latter can be arbitrarily large, but only because the government can face arbitrarily large interest rates. How much the government can borrow remains constrained by (18).

takes the form

$$N_t^y C_t^y + N_t^o C_t^o = Y_t. \tag{37}$$

When households live for two periods, their intertemporal budget constraints necessarily hold once they behave optimally. As a consequence, no fiscal requirement arises from intertemporal budget constraints, as pointed out by Bassetto and Cui (2018). The following proposition shows however that a real-debt-to-GDP limit similar to (18) characterizes the fiscal policies consistent with price stability (see Appendix L for a proof).

Proposition 3. *For a given initial level of debt B_{-1}, consider a future tax path $(T_t)_{t\geq 0}$.*

There exists a stable price equilibrium for this fiscal policy if and only if at all time t

$$\frac{B_{t-1}}{P^* Y_t} - \left(1 - \frac{\gamma\beta}{1+\beta}\right)\frac{T_t}{Y_t} \leq \frac{\gamma\beta}{1+\beta}. \tag{38}$$

Taxes now enter with a different coefficient than debt because young and old households have different MPCs in the standard OLG model. But otherwise the fiscal requirement for price stability takes the similar form of a limit of the debt-to-GDP ratio.

3 Discussion and Comparison to the NPV Requirement

In this section, we discuss the new debt-to-GDP limit (18) and compare it to the NPV equation (16), and answer common questions on the threat fiscal policy can pose to price stability through the lens of the model.

Note first that the debt-to-GDP limit (18) shares some of the flavor of the NPV requirement (16). Like in the NPV equation, in equation (18) inflation has the potential to restore an equilibrium when one does not exist, by decreasing the value of real debt. If condition (18) fails to be satisfied for stable prices $P_t = P^*$, then demand is greater than supply and inflation must arise to erode real public debt enough to equate demand and supply. If anything, this mechanism is more explicit in the case of Non-Ricardian households, since public debt explicitly increases aggregate demand through the IS equation (30). The idea of fiscal dominance as a situation in which the price level adjusts to stabilize the level of real public debt is therefore still present. We will return to this

point in Section 5 when studying implementation. For the moment, we compare both conditions as what they are in Propositions 1 and 2: characterizations of which fiscal policies are consistent with price stability.

3.1 Does it Mean the NPV Equation Never Needs to Hold?

No. In some cases the debt-to-GDP limit implies the NPV equation, so that any fiscal policy that delivers a stable price equilibrium must be such that the NPV equation holds. This is the case if the model parameters are such that, for any level of public debt, the real interest rate is necessarily above the growth rate of the economy, $R_t \geq Y_{t+1}/Y_t$ (see Appendix I for a proof). In turn, the real interest rate is above the growth rate of the economy for any level of public debt when the following conditions on parameters hold

$$\frac{1-\zeta}{\beta} > 1+g. \tag{39}$$

Condition (39) states that the population growth rate is not too large, and/or that households' income does not shrink too much with their age.

When condition (39) is satisfied, the NPV equation (16) necessarily holds in a stable price equilibrium.[14] Yet even in this case, the debt-to-GDP limit implies the NPV equation but is not equivalent to it. The debt-to-GDP limit is actually more stringent, as we explain in the next subsection.

3.2 Can Real-Debt-to-GDP Grow Unbounded yet Prices Remain Stable?

No, since there is a limit on it! But this is a big difference with the NPV requirement that applies when households are Ricardian. Indeed, the NPV requirement does not preclude a real-debt-to-GDP ratio that grows unbounded over time. Consider the case where households are Ricardian and assume for instance that from any positive debt position B_{-1} the government repays a share ψ_b of its public debt every period, with $0 < \psi_b < 1-\beta$. It implies that taxes at $t+k$ are

$$T_{t+k} = \psi_b(1-\psi_b)^k \mathcal{R}_{t,t+k} \frac{B_{t-1}}{P_t}, \tag{40}$$

[14] Angeletos, Lian, and Wolf (2024a,b) restrict to such cases by assuming a social fund makes transfers to new generations.

so that the NPV equation (16) is satisfied and there exists a stable price equilibrium. Yet the real-debt-to-GDP ratio is given by

$$\frac{B_{t+k}}{Y_{t+k}P_{t+k+1}} = \left(\frac{1}{\beta}(1-\psi_b)\right)^k \frac{B_{t-1}}{P_t Y_t}, \tag{41}$$

which diverges to infinity over time.

When households are not Ricardian, the debt-to-GDP limit precludes this. The NPV equation is not enough to guarantee price stability. This has implications for the interpretation of fiscal requirements for price stability as corresponding to fiscal backing of the currency, as we discuss in the next subsection.

3.3 Is Fiscal Backing of the Public Debt enough to Guarantee Price Stability?

Not if fiscal backing is understood as the NPV equation (16). When households are Ricardian, only high public debt unbacked by future fiscal surpluses threatens price stability. This is no longer the case when households are not Ricardian, as the case of an exploding real-debt-to-GDP ratio showed. The absolute level of the debt-to-GDP ratio is then a constraint on the existence of a stable price equilibrium. Obviously, future taxes still matter to assess the existence of a stable price equilibrium, since future taxes influence the future level of the debt-to-GDP ratio. But a plan to raise taxes in the future is not enough to guarantee the existence of a stable price equilibrium if future taxes are planned to be collected so far into the future that the debt-to-GDP ratio increases above the debt limit before taxes are collected. The debt-to-GDP limit can therefore be more stringent than the NPV equation. But it can also be less stringent, as we discuss in the next subsection.

3.4 When Can the NPV Equation Not Hold and Prices Remain Stable?

When r can be less than g, for at least some levels of public debt. This is the case that has attracted much attention recently (e.g. Blanchard, 2019; Reis, 2021; Brunnermeier, Merkel, and Sannikov, 2022). As these and other papers have pointed out, it is then sometimes possible for the government to run positive public debt that it never intends to repay, without threatening price stability. In this case, there is a stable price equilibrium yet the NPV equation (16) is not satisfied. However this does not mean that never repaying debt is never a threat to price stability, as we explain in the next subsection.

3.5 Can the Government Never Run Surpluses and Prices Remain Stable?

Only when the level of public debt is low enough. There is therefore a limit on the amount of spending the government can make without raising taxes nor threatening price stability.

It is *not* the case that the government does not need to run positive fiscal surpluses as long as public debt is below the threshold d^*. For a stable price equilibrium to exist, the debt-to-GDP limit (18) must hold in all future periods, not just today. Even if the debt-to-GDP ratio is less than d^* today, a tax path that lets the debt-to-GDP ratio inexorably increase will violate the debt limit at some point, making the fiscal policy inconsistent with price stability.

Whether public debt ends up crossing the threshold d^* at some future point depends on whether it is currently below a distinct threshold $\underline{d}$. Indeed, the dynamics of the real-debt-to-GDP ratio $b_{t-1} = B_{t-1}/(P^*Y_t)$ in the absence of future fiscal surpluses is determined by the flow budget constraint of the government (13)

$$b_t = \frac{R_t}{1+g} b_{t-1}, \tag{42}$$

combined with the interest rate (32) that must prevail under price stability. If, following this dynamics, the debt-to-GDP ratio becomes greater than d^* at any time, there exists no stable price equilibrium. As the left panel of Figure 1 shows, whether this happens depends on whether the current debt-to-GDP ratio b_{-1} is above or below a threshold $\underline{d}$.

The threshold $\underline{d}$ is the debt-to-GDP level at which the real interest rate becomes greater than the growth rate of the economy.[15] Indeed, if the debt-to-GDP ratio b_{-1} is already greater than $\underline{d}$ today, subsequent increases in b will only make r even greater than g (as plotted on the right panel of Figure 1), making b grow until it eventually reaches the debt-to-GDP limit d^*. Conversely, if b_{-1} is below $\underline{d}$ today, b will shrink back by itself, never crossing the debt-to-GDP limit d^*.

[15] Mian, Straub, and Sufi (2022) argue through a model with bonds-in-the-utility that the condition for a "free lunch" is not $r < g$, but the tighter condition $r < g - \phi$ where ϕ is the semi-elasticity of the real interest rate to debt. The standard $r < g$ condition we state is in no contradiction and is not due to the different model we use. The difference arises instead because we consider a different thought experiment. We consider under what condition the government can run *zero* fiscal surpluses indefinitely, $T_t = 0$ at all t. Mian, Straub, and Sufi (2022) consider under which condition the government can go from a permanent positive deficit to an even higher permanent positive deficit.

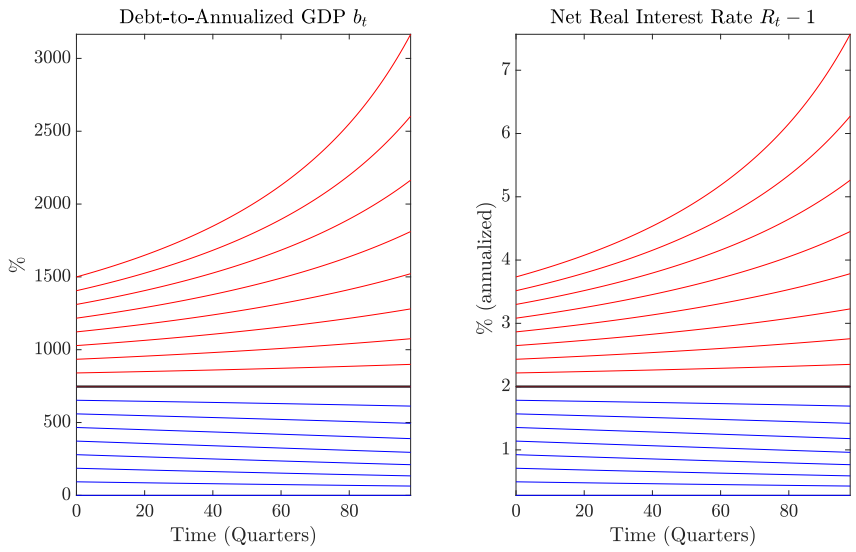


Figure 1: Debt Paths

Note: The left (right) panel plots the dynamics of the debt-to-annual-GDP ratio (the net real interest rate) over time when the government never sets positive fiscal surpluses, for different initial levels of the debt-to-annual-GDP ratio. For an initial debt-to-GDP ratio below $\underline{d}$, the debt-to-GDP ratio gradually shrinks back to zero (blue curves). For an initial debt-to-GDP ratio above $\underline{d}$, the debt-to-GDP ratio gradually increases and eventually diverges to infinity. The calibration is given in the first column of Table 1, OLG.

The threshold $\underline{d}$ can easily be obtained analytically. It is given by

$$\underline{d} = \left(1 - \frac{1-\zeta}{\beta(1+g)}\right) d^*. \tag{43}$$

Note that $\underline{d}$ is positive if and only if we are not in the case defined by equation (39). In that case, any level of public debt must be repaid with future surpluses, as discussed in Section 3.1.

3.6 What Determines the Debt-to-GDP Limit d^*?

The key determinant of d^* is households' marginal propensity to consume (MPC). The higher households' MPC is, the more of their wealth they want to spend. And the more of their wealth they want to spend, the quicker we reach the point where even infinitely high interest rates cannot prevent them from spending more than the natural level of output.

In turn, the key determinant of households' MPC $\mu = 1 - \beta(1-\lambda)$ is the death probability λ. Under a literal interpretation of the model as an OLG model, λ is the probability of biological death and cannot be set very high. As a result, households' MPC must be small. The first column of Table 1 considers such a calibration. We set $\lambda = 0.005$ on a quarterly frequency, i.e. a 2%

probability of dying every year. We set $\beta = 0.995$ and $g = 0.005$, i.e. an annual growth rate of 2%. We set the last parameter, the income-shrinking rate ζ, in order to match a steady-state real interest rate of $r_0 = 0.5\%$ at a debt-to-GDP level of 100%. This is the value of the natural rate in the US in 2016 (when debt-to-GDP was 100%) as estimated by Holston, Laubach, and Williams (2017). This gives $\zeta = 0.0043$.

Under this OLG calibration, households' MPC is very small, at 0.01 quarterly. As a result, the debt-to-GDP limit d^* is extremely high, at 1743 times annual GDP. The value of the threshold $\underline{d}$ is also very high, at $\underline{d} = 8.3$, or 830% of GDP.

Yet an MPC of 0.01 is much smaller than the average MPC found in the data. Higher calibrations of λ can be rationalized if the model is reinterpreted as a proxy for a HANK model (e.g. Del Negro, Giannoni, and Patterson, 2023; Farhi and Werning, 2019). Under this alternative interpretation, the death probability is to be interpreted as the probability for households to hit their borrowing constraints. This lets the model reproduce any value for the MPC.

As λ—and therefore households' MPC—increases, the debt-to-GDP limit d^* decreases considerably. The plain line of the left panel of Figure 2 shows how d^* decreases as λ increases from 0.005 to 0.196, the value that generates a quarterly MPC of 0.2, consistent with estimates in Fagereng, Holm, and Natvik (2021), keeping all other parameters constant. The debt-to-GDP limit decreases from 1743 to just 4.9 times GDP (Column LARGE-λ of Table 1).

Matching the average MPC in the data overlooks an important distinction however. What matters for the debt-to-GDP limit is the MPC of the households *that hold public debt*. In the perpetual-youth model all households have the same MPC so the distinction is irrelevant. But the importance of the distinction becomes clear once we allow for hand-to-mouth households who have a high MPC but do not hold public debt. We extend the model in this direction in the next section.

4 Extensions

In this section, we consider extensions of the model. In particular, we add hand-to-mouth households and calibrate the model to match the sensitivity of interest rates to the level of public debt.

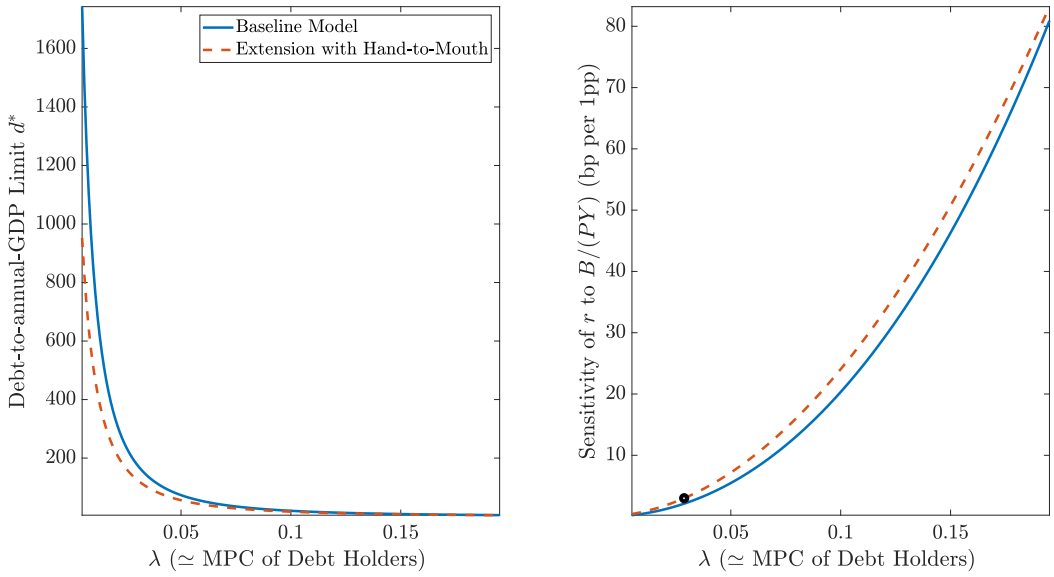


Figure 2: Dependence of the debt-to-GDP Limit in the MPC of Debt Holders

Note: The left panel plots the debt-to-GDP limit d^* (expressed in terms of annual GDP) as a function of the death probability λ, which is about equal to the MPC of non hand-to-mouth households $\mu = 1-\beta(1-\lambda)$. The right panel plots the sensitivity of the long-run real interest rate to the debt-to-GDP ratio as a function of λ. The black circle on the right panel is its empirical value of 3bp for a 1pp increase in debt-to-GDP, as estimated by Laubach (2009) and Engen and Hubbard (2004). In both panels, the relationship is plotted for two versions of the model. The plain lines are for the baseline model of Section 1. It is calibrated with $\beta = 0.995$, $g = 0.005$ and $\zeta = 0.0043$. The dashed lines are for the model's extension with hand-to-mouth households in Section 4.2. It is calibrated with $\beta = 0.995$, $g = 0.005$, $\zeta = 0.011$ and a share of hand-to-mouth households α_Y that is reset as λ varies so that the average MPC in the economy remains constant to 0.2.

4.1 Standard TANK

Before extending the perpetual youth model to allow for hand-to-mouth households, we first highlight the importance of considering hand-to-mouth households by deriving fiscal requirements for price stability in a standard Two-Agent New-Keynesian model (TANK).[16] In such TANK models, the only non-Ricardian households are hand-to-mouth households; others remain Ricardian. As we now show, distinguishing between different types of non-Ricardian households matters for fiscal requirements for price stability.

Assume that a fraction α_Y of aggregate income Y_t goes to hand-to-mouth households that consume their income every period $C_t^i = Y_t^i$. Assume that the remaining households are Ricardian permanent-income households ($\lambda + g = 0$). We assume that hand-to-mouth households collectively pay $\alpha_T \times T_t$ in taxes, with α_T possibly distinct from α_Y. Appendix J shows the following

[16] Bhattarai, Lee, and Yang (2023) use a TANK model under fiscal dominance to study fiscal transfers during the Covid crisis.

Table 1: Calibration and Debt Limit

β		0.995		
g		0.005		
	OLG	LARGE-λ	OLG-HTM	MAIN
ζ	0.0043	0.0043	0.0043	0.011
λ	0.005	0.196	0.005	0.029
α_Y	0	0	0.19	0.17
Average MPC	0.01	0.20	0.20	0.20
Sensitivity of r to $B/(PY)$	0.23	82	0.29	3
d^*	1742.8	4.9	1408.3	133.8
$\underline{d}$	7.5	0.02	6.1	1.5

Note: The table gives the average MPC, sensitivity of interest rates to debt, and debt thresholds d^* and $\underline{d}$ in three calibrations of the model: the OLG calibration presented in Section 3.6, the OLG+HTM calibration presented in Section 4.2, and the main calibration presented in Section 4.3. The average MPC is the quarterly average MPC. The sensitivity of r to $B/(PY)$ is the derivative of the annualized real interest rate with respect to the debt-to-annual-GDP ratio in a steady-state of the model, expressed in basis points for percentage point increase in the debt-to-annual-GDP ratio. The debt thresholds d^* and $\underline{d}$ are expressed as debt to annual GDP.

generalization of Proposition 1.

Proposition 4. *Consider a standard TANK model where non hand-to-mouth households are Ricardian $\lambda + g = 0$. Consider an initial level of debt B_{-1} and a future tax path $(T_t)_{t\geq 0}$.*

There exists a stable price equilibrium for this fiscal policy if and only if the NPV equation (16) holds for the interest rate path $R_t = (Y_{t+1} + \alpha_T/(1-\alpha_Y)T_{t+1})/(\beta(Y_t + \alpha_T/(1-\alpha_Y)T_t))$.

Proposition 4 states that in a standard TANK model the NPV requirement still characterizes the fiscal policies that are consistent with price stability. The only difference is that the equilibrium real interest rate takes a different value than in the absence of hand-to-mouth households.

A TANK model breaks the Ricardian equivalence yet still results in the NPV requirement (16). The result that fiscal requirements for price stability take the form of a limit on the debt-to-GDP ratio when households are not Ricardian therefore needs a more precise statement. What is required for fiscal requirements to take the form of the debt-to-GDP limit is that the households *that hold public debt* are not Ricardian. In a TANK model, the hand-to-mouth households are not Ricardian but they do not hold public debt, so their presence does not change the result that fiscal requirements take the form of the NPV equation.

4.2 Extension with Hand-to-Mouth Households

We now extend the perpetual youth model to allow for a share of hand-to-mouth households, as in Wolf (2025). Assume once again that a fraction α_Y of aggregate income Y_t goes to hand-to-mouth households that consume their income every period $C_t^i = Y_t^i$, and that hand-to-mouth households collectively pay $\alpha_T \times T_t$ in taxes.[17] We continue to assume that among non hand-to-mouth households, taxes are imposed proportional to income, as per (14), where T_t is to be replaced by $(1 - \alpha_T)T_t$. Appendix K shows the following generalization of Proposition 2.

Corollary 1. *Consider the model with a share of hand-to-mouth households. Assume the non hand-to-mouth households are not Ricardian* $\lambda + g > 0$.

Consider an initial level of debt B_{-1} *and a future tax path* $(T_t)_{t\geq 0}$. *There exists a stable price equilibrium for this fiscal policy if and only if are satisfied both condition* (20) *and the amended condition*

$$\frac{B_{t-1}}{P^* Y_t} - \left(\frac{\chi + \alpha_T}{\chi}\right)\frac{T_t}{Y_t} < d^*, \tag{44}$$

where

$$d^* = \frac{1 - \alpha_Y}{\chi}. \tag{45}$$

Corollary 1 states that the requirements of Proposition 1 are robust to the addition of hand-to-mouth households. The debt-to-GDP limit now takes a different value, but fiscal requirements still take the form of a limit on the debt-to-GDP ratio.

In addition, for a given calibration, hand-to-mouth households make little change to the quantitative value of the debt-to-GDP limit. Hand-to-mouth households decrease it, but not by much. The third column OLG-HTM of Table 1 reports the debt-to-GDP limit when the model is calibrated as in the OLG calibration of Section 3.6, but with enough hand-to-mouth households (19%) to match an average MPC of 0.20. The debt-to-GDP limit is only slightly lower, to 1408 times annual GDP. The value of $\underline{d}$ is similarly only slightly lower, to 6.1 times annual GDP.

Hand-to-mouth households do not affect d^* much because fiscal requirements for price stability

[17] As far as the aggregate economy is concerned, how the incomes of hand-to-mouth households is distributed among themselves, e.g. according to age, does not matter.

primarily depend on the behavior of the households that hold public debt, and hand-to-mouth households hold none. Hand-to-mouth households do lower the value of the threshold (45) on public debt, by a factor $(1-\alpha_Y)$. But this is *not* because hand-to-mouth households increase the average MPC. Because hand-to-mouth households do not own wealth, their MPC does not directly matter for fiscal requirements. They affect the debt-to-GDP limit only indirectly. Because they consume a share α_Y of income, the consumption of the non hand-to-mouth households must now be lower. This puts a lower threshold on the maximum level of public debt they can hold in a stable price equilibrium.

Yet, although hand-to-mouth households do not change the debt-to-GDP limit much for a given calibration, they matter because they disentangle the MPC of debt-holders from the average MPC. This allows to match the average MPC in the data for any calibration of λ. This opens the door to alternative approaches to calibrating λ, as we now turn to.

4.3 Matching the Sensitivity of Interest Rates to Public Debt

To calibrate λ in the extended model, we add an additional target beyond the average MPC: the sensitivity of interest rates to the level of public debt. This builds on the calibration strategy Mian, Straub, and Sufi (2022) use to calibrate a bonds-in-the-utility model. Mian, Straub, and Sufi (2022) review estimates of this sensitivity in the literature and find that long-term interest rates increase by about 1 to 5 basis points for every percentage point increase in the debt-to-annual-GDP ratio. In particular, Laubach (2009) reports a value of 3 to 4bp and Engen and Hubbard (2004) of 3bp, both based on the US.[18]

Both the OLG calibration of Section 3.6 and the OLG-HTM calibration of Section 4.2 give much smaller values. Relying on steady-state relationships, the sensitivity of the interest rate to a one percentage point increase in debt-to-GDP is 0.23bp in the former and 0.29bp in the latter, an order of magnitude smaller than in the data.[19] This matters for the value of the debt-to-GDP limit: A calibration that understates how much the natural rate increases with the level of public debt will overstate the maximum amount of public debt consistent with a finite natural rate.

[18] Mian, Straub, and Sufi (2022) report estimates for both the sensitivity of interest rates and the sensitivity of the convenience yield on public debt. There is no convenience yield in our perpetual-youth model so we focus on Laubach (2009) and Engen and Hubbard (2004) who report the former. The estimates Mian, Straub, and Sufi (2022) report for the convenience yield are of the same order of magnitude as the estimates in Laubach (2009) and Engen and Hubbard (2004) for interest rates.

[19] The steady-state relationship between r and $B/(PY)$ is linear, so that the derivative does not depend on the level of debt—see Appendix M for a derivation.

In our main calibration—calibration MAIN in Table 1—we target both the average MPC in the data and the sensitivity of interest rates to the debt-to-GDP ratio. Like Mian, Straub, and Sufi (2022), we consider the steady-state of our model and assume that taxes/transfers to hand-to-mouth are nil, $\alpha_T = 0$. We still set $\beta = 0.995$ and $g = 0.005$ and calibrate the three remaining parameters ζ, α_Y and λ to match three statistics: a real interest rate at 100% debt-to-annual-GDP of $r_0 = 0.5\%$ in steady-state (as in Section 4.2); an average marginal propensity to consume of 0.2 (as in Section 4.2); and a derivative of the real interest rate to the debt-to-annual-GDP ratio in steady-state of 0.03 from Laubach (2009) and Engen and Hubbard (2004).

The resulting value for the quarterly death probability is $\lambda = 0.029$, i.e. an MPC of households that hold public debt of $\mu = 0.034$. This is higher than in the OLG and OLG-HTM calibrations of the model, which produce too low a sensitivity of interest rates to debt. Yet this is smaller than the value $\lambda = 0.196$ needed to match an average MPC of 0.2 in the absence of hand-to-mouth households. Such a high value of λ this time *overstates* the sensitivity of interest rates to debt, as shown by the dashed line of the right panel of Figure 2. It plots the interest-rate sensitivity to debt as a function of λ keeping all other parameters constant at their value in our main calibration—except for α_Y that is reset to still match an average MPC of 0.2 for all values of λ. For $\lambda = 0.196$, the long-term real interest rate increases by 83bp for every percentage point increase in the debt-to-GDP ratio. This is more than an order of magnitude larger than in the data. If the households that hold public debt have a high MPC, the real interest rate becomes extremely sensitive to the level of debt.[20]

In our main calibration, the debt-to-GDP limit d^* is 134 times GDP. It is an order of magnitude smaller than in the OLG calibrations of the model, but much larger than when calibrating debt holders to have an MPC of 0.2.[21] Under our main calibration, even though the debt-to-GDP limit excludes many fiscal policies as inconsistent with price stability, very high levels of public debt remain consistent with price stability provided the central bank increases interest rates as needed.

[20] An alternative calibration strategy of the perpetual youth model with hand-to-mouth households is to set λ and α_Y to match the whole profile of intertemporal MPC (iMPC) (Auclert, Rognlie, and Straub, 2018), as in Wolf (2025) and Angeletos, Lian, and Wolf (2024a). Doing so, Wolf (2025) and Angeletos, Lian, and Wolf (2024a) find a value of $\lambda = 0.135$. Such a high value, while less than $\lambda = 0.196$, still produces a sensitivity of the interest rate to debt that is an order of magnitude larger than in the data, as apparent on the right panel of Figure 2.

[21] The value of $\underline{d}$ is 150% of GDP, close to the value of 160% that Bayer, Born, and Lütticke (2023) calculate in a HANK model with illiquid capital. It is a bit lower than the 218% of GDP that Mian, Straub, and Sufi (2022) find for their bonds-in-the-utility model. The difference is partly due to the higher r_0 we target: +0.5% based on the estimate of r^* in Holston, Laubach, and Williams (2017) instead of -0.5% in Mian, Straub, and Sufi (2022) based on the value of the real interest rate in 2019. When we target $r_0 = -0.5\%$ as they do, we find a value of 180%.

4.4 Blanchard-Yaari Bewley Model

A value of $\lambda = 0.028$ as in our main calibration is better justified through the HANK interpretation of the model. Yet a possible caveat in the HANK interpretation of the model is that it takes the probability of hitting one's borrowing constraint as exogenous, when in a full-fledged HANK model it is endogenous and depends on the level of public debt. Appendix N highlights this point by considering Woodford (1990)'s alternating-endowment model, a particular case of a HANK model which is tractable enough to be solved analytically. As Woodford (1990) shows, the alternating-endowment model is exactly equivalent to the two-generation OLG model of Section 2.3 when the borrowing constraint of low-endowment households binds. This buttresses the case for using the perpetual-youth model in its HANK interpretation. But when public debt is high enough, borrowing constraints never bind even for low-endowment households, as there is enough liquidity in the economy for everyone to self-insure. The model is then equivalent to one with a representative Ricardian household. The real interest rate is constant to $1/\beta$ and no longer increases with public debt. The property that the equilibrium interest rate asymptotes to $1/\beta$ at high levels of public debt is a general property of HANK models (e.g. Chamberlain and Wilson, 2000). If higher and higher levels of public debt do not require higher and higher interest rates, there does not exist a level of public debt beyond which even infinitely high interest rates cannot convince households to hold debt.

Yet in practice households face both borrowing constraints and death. Appendix O considers a model that combines both OLG and HANK departures from the representative agent: a Blanchard-Yaari Bewley model. As in a standard Bewley (1977) model, households face idiosyncratic income risk and a borrowing constraint, and can only borrow and save through non-contingent debt. In the Blanchard-Yaari Bewley model, when the debt-to-GDP ratio increases the real interest rate does not asymptote to a finite value. Instead it approaches the behavior of the real interest rate in the Blanchard-Yaari model with overlapping generations alone, since households then have enough liquid wealth to self-insure. The debt-to-GDP limit d^* then applies.

5 Implementation

Proposition 2 derived fiscal requirements for price stability abstracting from how the central bank can then implement price stability. In this section, we turn to the implementation question. In

doing so, we reconsider Leeper (1991)'s analysis of fiscal and monetary dominance in the case of non-Ricardian households.

5.1 Local Dynamics

We assume that prices are flexible, as in Leeper (1991)'s original analysis. The dynamics of the aggregate economy reduces to only two equations: the IS curve (30) and the flow budget constraint of the government (13), which can be written in per capita terms

$$y_t = \frac{1-\zeta}{\beta}\frac{\pi_{t+1}}{I_t}y_{t+1} + \chi\left(\frac{b_{t-1}}{\pi_t} - \tau_t\right), \tag{46}$$

$$b_t\frac{1+g}{I_t} + \tau_t = \frac{b_{t-1}}{\pi_t}. \tag{47}$$

where $b_{t-1} = B_{t-1}/(P_{t-1}N_t)$, $y_t = Y_t/N_t$ and $\tau_t = T_t/N_t$ are real public debt per capita, GDP-per-capita, and taxes per capita, π_t is the inflation rate and I_t is the nominal interest rate.

Following Leeper, we take the exogenous level of GDP under flexible prices to be constant, and consider the log-linearized version of these two equations around a steady state with real interest rate R and debt to GDP ratio b/y,[22]

$$\hat{i}_t - \hat{\pi}_{t+1} = \eta\left(\hat{b}_{t-1} - \frac{b}{y}\hat{\pi}_t - \hat{\tau}_t\right), \tag{48}$$

$$\hat{b}_t = \frac{R}{1+g}\left(\hat{b}_{t-1} - \hat{\tau}_t\right) + \frac{b}{y}\left(\hat{i}_t - \frac{R}{1+g}\hat{\pi}_t\right), \tag{49}$$

where

$$\eta = \frac{\chi}{\left(\frac{1-\zeta}{\beta R}\right)}. \tag{50}$$

The analysis therefore reduces to the same two-equation system in b_t and π_t as in Leeper (1991). The only difference is that when households are not Ricardian $\eta > 0$, public debt now enters the Fisher equation (48), as higher public debt increases the real interest rate.

Following Leeper, we assume monetary and fiscal policies are conducted according to feedback

[22] When loglinearizing we define $\hat{b}_t = db_t/y^*$ to allow for the possibility of a zero level of public debt in steady-state.

rules. Monetary policy follows a standard Taylor rule and taxes respond to the level of public debt

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \nu_t^i, \tag{51}$$

$$\hat{\tau}_t = \psi_b \hat{b}_{t-1} - \nu_t^g. \tag{52}$$

Plugging in the policy equations (51) and (52) into the system (48)-(49) gives a 2-by-2 system in $\hat{\pi}_t$ and $\hat{b}_t$.

5.2 Blurred Lines

Appendix P extends the result of Leeper (1991) on equilibrium determinacy to the case of non-Ricardian households. It shows that when households are not Ricardian, the sharp distinction between a monetary and fiscal regime is no longer applicable.

Proposition 5. *Assume monetary and fiscal policy are given by the feedback rules* (51) *and* (52)*.*

The economy has a unique bounded equilibrium if and only if

$$\Big(1-\phi_\pi\Big)\left(1-\frac{R}{1+g}(1-\psi_b)\right)+\eta\frac{b}{y}\Big((1-\psi_b)\phi_\pi-1\Big)<0. \tag{53}$$

- *When households are Ricardian $\eta = 0$ or when steady-state public debt is zero $b/y = 0$, then when a unique bounded equilibrium exists the economy is either in a monetary regime $\phi_\pi > 1, \psi_b > 1 - 1/R$ where fiscal shocks ν_t^g have no effect on inflation, or in a fiscal regime $\phi_\pi < 1, \psi_b < 1 - 1/R$ where they do.*
- *When households are not Ricardian $\eta > 0$ and when steady-state public debt is positive $b/y > 0$, then when a unique bounded equilibrium exists fiscal shocks ν_t^g always affect inflation.*

The first item of Proposition 5 is Leeper's classical result. When households are Ricardian $\eta = 0$ (and so $g = 0$), condition (53) becomes

$$\Big(1-\phi_\pi\Big)\Big(1-R(1-\psi_b)\Big)<0. \tag{54}$$

This determines two regions, represented graphically in the (ϕ_π, ψ_b) plane on the left panel of Figure 3. The first case $\phi_\pi > 1, \psi_b > 1 - (1+g)/R$ is what Leeper calls a regime of monetary dominance. In this case fiscal shocks ν_t^g have no effect on inflation. Indeed, the unique bounded solution can

then be obtained by iterating forward equations (48)-(51) to give equilibrium inflation as

$$\hat{\pi}_t = -\sum_{k=0}^{\infty} \left(\frac{1}{\phi_\pi}\right)^{k+1} \nu^i_{t+k}, \tag{55}$$

which is independent of the fiscal shocks ν^g_t. The second case $\phi_\pi < 1, \psi_b < 1 - (1+g)/R$ is what Leeper calls a regime of fiscal dominance. In this case fiscal shocks ν^g_t affect inflation. For instance, when the interest rate is fully pegged $i_t = 0$, iterating equations (49)-(52) forward gives

$$(1-\psi_b)\hat{b}_{t-1} - \frac{b}{y}\hat{\pi}_t = -\sum_{k=0}^{\infty} \left(\frac{1}{R(1-\psi_b)}\right)^{k} \nu^g_{t+k}, \tag{56}$$

which gives the level of inflation necessary to restore the intertemporal budget constraint of the government.

The second item of Proposition 5 considers how the Leeper result changes once we move away from the case of Ricardian households. The determinacy space in this case is represented in the middle panel of Figure 3. It no longer consists of two distinct regions so that there is no straightforward way to distinguish between two regimes. Interestingly, the shape of the determinacy region when households are not Ricardian is similar to the one found by Cui (2016) in a model where firms face endogenous liquidity frictions and public debt is valued for its liquidity (see Figure 2 in Cui, 2016).

The absence of a fiscal and monetary regime is clear when looking at the inflationary effect of fiscal shocks. Appendix P shows that when households are not Ricardian, inflation is given by

$$\hat{\pi}_t = \mu\hat{b}_{t-1} + \sum_{k=0}^{\infty} \left(\frac{1}{\upsilon}\right)^{k+1} \left(\left(\mu\frac{b}{y} - 1\right)\nu^i_{t+k} + \left(\mu\frac{R}{1+g} + \eta\right)\nu^g_{t+k}\right), \tag{57}$$

whenever a unique equilibrium exists, where υ is the root of the economy that is greater than 1 and μ is a positive constant given in Appendix P. Fiscal shocks ν^g now always affect inflation.

Figure 4 gives the inflationary effect of a contemporaneous fiscal shock ν^g_t as a function of ϕ_π for fixed values of ψ_b. The upper panel gives the impact effect of the shock on inflation. As monetary policy becomes more reactive, it decreases. But it does so gradually and without ever reaching zero, approaching it only as its asymptotic limit where ϕ_π tends toward infinity. This is in contrast to the Ricardian case, where an active monetary policy $\phi_\pi > 1$ is enough to perfectly insulate inflation from fiscal shocks. In addition, the *cumulative* impact on inflation can be much larger, and even

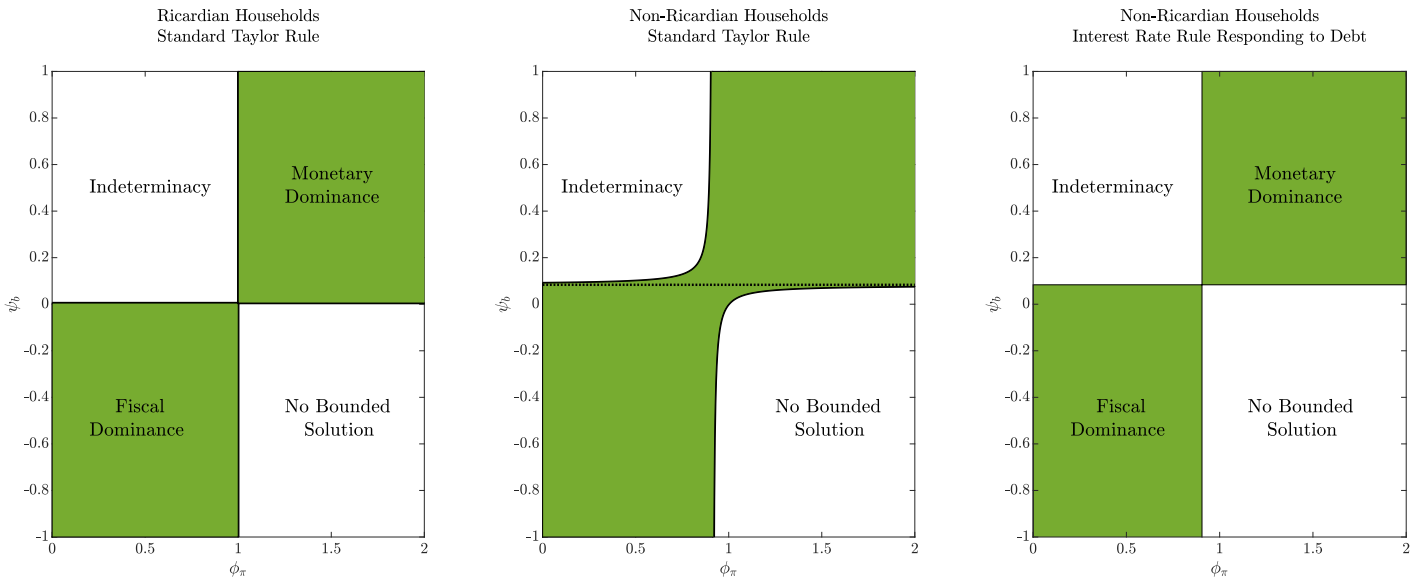


Figure 3: Condition for Equilibrium Uniqueness

Note: The left and middle panels of the figure illustrate Proposition 5; the right panel illustrates Proposition 6. On all panels, in green is the region of parameters (ϕ_π, ψ_b) for which there exists a unique bounded equilibrium: in the economy with Ricardian households and a standard Taylor rule (51)(left panel), in the economy with non-Ricardian households and a standard Taylor rule (51) (middle panel), and in the economy with non-Ricardian households and a monetary policy rule that responds to public debt on top of inflation (58) (right panel). The calibration is for a high $\lambda = 0.10$, $\beta = 0.995$, $g = 0.005$, $b/y = 1 \times 4$ (100% of annual GDP), and sets ζ such that the steady-state real interest rate is 0.5% ($\zeta = 0.09$).

increasing in ϕ_π, as shown on the bottom panel of Figure 4. This is because increasing ϕ_π increases the backward-looking root of the economy and therefore the persistence of inflation, as shown in the middle panel of Figure 4.

5.3 Implementing Price Stability by Letting Monetary Policy Respond to Debt

The analysis so far would conclude that it is impossible for monetary policy to insulate inflation from fiscal shocks. No degree of responsiveness to inflation ϕ_π in the Taylor rule (51) can fully prevent fiscal shocks from affecting inflation.

Yet this is only if one restricts monetary policy to respond only to inflation. We now show that monetary policy can implement the stable price equilibrium if it ventures beyond a simple Taylor rule (51) and follows a policy rule that responds to public debt in addition to inflation. If the reaction function of the central bank is

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \eta(\hat{b}_{t-1} - \hat{\tau}_t), \tag{58}$$

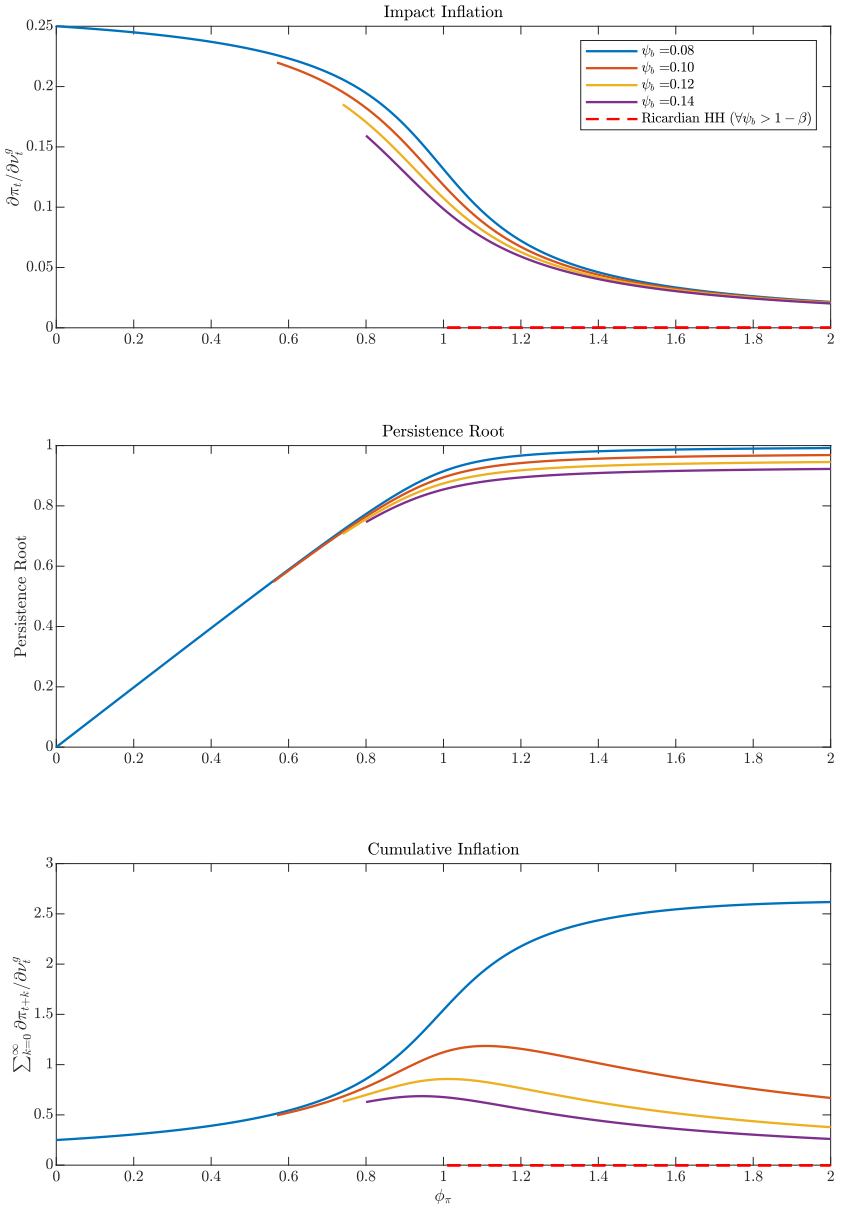


Figure 4: Inflationary Effect of Contemporaneous Fiscal Shock

Note: The figure plots the impact inflationary effect of a fiscal shock, the persistence root, and the cumulative effect of a fiscal shock, all three as a function of the reactivity of the Taylor rule ϕ_π, and for a parameter in the fiscal rule $\psi_b = 1 - \frac{1}{\frac{R}{1+g}+\eta\frac{b}{y}}$ or higher. For this first value of ψ_b there exists an equilibrium under all values of ϕ_π in the case of non-Ricardian households. In the case of Ricardian households, there exists an equilibrium only for $\phi_\pi > 0$. The calibration is for a high $\lambda = 0.10$, $\beta = 0.995$, $g = 0.005$, $b/y = 1 \times 4$ (100% of annual GDP), and sets ζ such that the steady-state real interest rate is 0.5% ($\zeta = 0.09$).

then monetary policy can again insulate inflation from fiscal shocks. This recovers the notion of a regime of monetary dominance—in the sense defined by the following proposition.

Proposition 6. *Assume monetary and fiscal policy are given by the feedback rules* (52) *and* (58)*.*

The economy has a unique bounded equilibrium iff it is in either of the two following regimes

- *Monetary regime* $\phi_\pi > 1 - \eta\frac{b}{y}$ *and* $\psi_b > 1 - \frac{1}{\frac{R}{1+g}+\frac{b}{y}\eta}$*. Fiscal shocks* ν_t^g *have then no effect on inflation.*
- *Fiscal regime* $\phi_\pi < 1 - \eta\frac{b}{y}$ *and* $\psi_b < 1 - \frac{1}{\frac{R}{1+g}+\frac{b}{y}\eta}$*. Fiscal shocks* ν_t^g *then affect inflation.*

See Appendix Q for a proof. The two regimes are plotted in the $(\phi_\pi,\ \psi_b)$ plane on the right panel of Figure 3. Just as in the case of Ricardian households (54), in the monetary regime the unique bounded solution can be obtained by solving forward equations (48)-(58), to give

$$\hat{\pi}_t = -\sum_{k=0}^{\infty}\left(\frac{1}{\phi_\pi + \eta\frac{b}{y}}\right)^{k+1}\nu_{t+k}^i. \tag{59}$$

which is independent of the fiscal shocks ν_t^g. Because it insulates inflation from fiscal shocks, monetary policy can be said to be active again. However, this flips on its head the definition of an active monetary policy in Leeper (1991):

> *"I couch active and passive policy in terms of the constraints a policy authority faces. An active authority pays no attention to the state of government debt and is free to set its control variable as it sees fit. A passive authority responds to government debt shocks. Its behavior is constrained by private optimization and the active authority's actions."*

In contrast, the monetary policy (58) delivers monetary dominance *because* it reacts to public debt. Having monetary policy react to public debt is not a standard specification of monetary policy.[23] Yet it follows naturally from the fact that the natural rate of interest—the real interest rate consistent with stable prices—depends on the level of public debt once households are not Ricardian—equation (32). The monetary rule (58) is a Taylor rule augmented with the natural

[23] It can be found in some specification of monetary policy however. Bayer, Born, and Lütticke (2023) estimate a Taylor rule that responds to public debt as part of the global Bayesian estimation of a HANK model, and find a positive coefficient.

rate as intercept

$$\hat{i}_t = \hat{r}_t^n + \phi_\pi \hat{\pi}_t, \tag{60}$$

where in log-linearized form the natural rate is the function of public debt

$$\hat{r}_t^n = \eta(\hat{b}_{t-1} - \hat{\tau}_t). \tag{61}$$

Having the natural rate of interest as an intercept in the Taylor rule is a standard feature in specifications of monetary policy.

Responding directly to public debt restores a notion of monetary dominance, but for it to obtain, the conditions on monetary and fiscal policy are not the same as in the Ricardian case. The threshold on ψ_b is now $\psi_b^* = 1 - \frac{1}{\frac{R}{1+g} + \frac{b}{y}\eta}$. It increases with η—with how much non-Ricardian households are—and with the level of public debt. Why ψ_b is higher is best understood by considering the dynamics of debt under price stability. Combining the flow budget constraint of the government (49) and the dependence of the natural rate on debt (61) gives the dynamics of debt as

$$\hat{b}_t = \left(\frac{R}{1+g} + \eta\frac{b}{y}\right)(\hat{b}_{t-1} - \hat{\tau}_t). \tag{62}$$

Absent adjustment in taxes, public debt increases at the rate $\frac{R}{1+g} + \eta\frac{b}{y}$, so fiscal surpluses must increase by at least as much or debt explodes. The term $\eta\frac{b}{y}$ captures that when households are not Ricardian higher public debt increases the natural rate, accelerating how fast debt diverges.

How much does this increase the threshold ψ_b^* for monetary dominance? Here again, the answer depends heavily on the calibration of the death probability λ. When $\lambda = 0.10$ as in Figure 3, the threshold is $\psi_b^* = 0.08$ at a debt-to-annual-GDP ratio of 100%, versus about $\psi_b^* = 0.0025$ when households are Ricardian. When $\lambda = 0.15$, the threshold is higher still, at $\psi_b^* = 0.30$. For such values, monetary dominance requires the government to stabilize debt extremely quickly, arguably unrealistically so. Yet, such high calibrations of λ greatly exaggerate the sensitivity of the natural rate to the level of public debt, as discussed in Section 4.3. When matching a sensitivity of the natural rate to the debt-to-GDP ratio of about 3bp/pp as found by Laubach (2009) and Engen and Hubbard (2004)—which requires a much lower λ of about 0.03—the term $\eta\frac{b}{y}$ only adds about 0.0075 to the threshold ψ_b^*. This is a significant increase, but one that remains consistent with a fiscal policy which only slowly brings debt down following a fiscal expansion.

Of course, the effect of non-Ricardian households on the threshold ψ_b^* is only the shadow, once restricting to the model's local dynamics and to a particular class (52) of fiscal rules, of the full constraints on fiscal policy imposed by price stability derived in Section 2. When considering the global dynamics of the model and any path of future taxes, fiscal requirements for price stability take the form of the debt-to-GDP limit (18). As the local analysis emphasizes however, respecting the debt-to-GDP limit at all times requires to take into account that the natural rate of interest increases with the level of public debt.

6 Conclusion

Can the central bank deliver price stability whatever fiscal policy the government sets? The NPV requirement that current government debt needs to be backed by future fiscal surpluses has remained controversial since its inception and relies on the strong assumption of Ricardian households. In this paper, we have argued that moving away from the particular case of Ricardian households provides a less controversial—and also more intuitive—answer to this question. With non-Ricardian households, higher public debt makes households richer. As households spend their higher wealth, aggregate demand increases, putting inflationary pressure on the economy. While monetary policy can increase interest rates to counter the inflationary effects of public debt, it can no longer do so when public debt is too high. For the central bank to keep control over the price level, fiscal policy must be such that it never lets debt reach that level. Provided this fiscal requirement is satisfied, monetary policy can deliver price stability. But to do so, it needs to increase its policy rate with the level of public debt—not just increase it in response to the inflation that higher public debt generates.

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Fiscal Requirements for Price Stability When Households are not Ricardian

Supplementary Online Appendix

Stéphane Dupraz* Anna Rogantini Picco

Banque de France ECB & CEPR

A Derivation of Individual Optimality

Maximizing (4) subject to the flow budget constraint (5) and the No-Ponzi-scheme constraint (6) gives the Euler equation

$$C_t^i = (\beta R_t)^{-1} C_{t+1}^i \tag{A.1}$$

and the No-Ponzi-scheme condition holding with equality, i.e. the No-Cash-on-the-Table condition (10). Iterating the Euler equation (A.1) forward gives for all $k \geq 0$

$$C_{t+k}^i = (\beta^k \mathcal{R}_{t,t+k}) C_t^i. \tag{A.2}$$

Combining the FBC (5) and the No-Cash-on-the-Table condition (10), the intertemporal budget constraint is

$$\sum_{k=0}^{\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} C_{t+k}^i = \frac{1}{1-\lambda} \frac{B_{t-1}^i}{P_t} + H_t^i \tag{A.3}$$

where

$$H_t^i = Y_t^i - T_t^i + \frac{1-\lambda}{R_t} H_{t+1}^i \tag{A.4}$$

is the household's intertemporal wealth, or human capital. Iterated forward, it writes as equation (9).

Injecting (A.2) into the IBC (A.3) gives the consumption function (8). Conversely, equations

(8) and (10) are sufficient for individual optimality.

B Proof of Proposition 1

The text already showed that condition (16) is necessary for a stable price equilibrium. We show that it is sufficient. The proof is constructive. Given an initial level of public debt B_{-1} distributed as $(B^i_{-1})_i$ in the population, consider a path for taxes $(T_t)_{t\geq 0}$. Define the following path for the interest rate

$$R_t = \frac{1}{\beta}\frac{Y_{t+1}}{Y_t}. \tag{B.1}$$

For this interest rate path, define individual consumption allocations through the consumption function (8), which in combination with the FBC (5) of household i defines an entire path for its consumption and debt holdings.

We show that this allocation is a stable price equilibrium.

Market-clearing

Start with market-clearing. Since by construction all households are on the consumption function (8), aggregate consumption is given by the aggregate consumption function (F.1), which writes since $\lambda = g = \zeta = 0$ in the Ricardian case

$$C_t = (1-\beta)\left(B^d_{t-1} + \sum_{k=0}^{\infty}\frac{1}{\mathcal{R}_{t,t+k}}(Y_{t+k} - T_{t+k})\right). \tag{B.2}$$

Given this interest rate path, we have that

$$\frac{Y_{t+k}}{\mathcal{R}_{t,t+k}} = \frac{Y_{t+k}}{R_{t+k}\mathcal{R}_{t,t+k-1}} = \beta\frac{Y_{t+k-1}}{\mathcal{R}_{t,t+k-1}}, \tag{B.3}$$

and by iteration

$$\frac{Y_{t+k}}{\mathcal{R}_{t,t+k}} = \beta^k Y_t. \tag{B.4}$$

The consumption function (F.1) therefore implies

$$C_t = Y_t + (1-\beta)\left(B^d_{t-1} - \sum_{k=0}^{\infty} \frac{T_{t+k}}{\mathcal{R}_{t,t+k}}\right). \tag{B.5}$$

Starting from $t=0$, given that $B^d_{-1} = B_{-1}$ initially, the NPV equation (16) at $t=0$ guarantees market clearing in the goods market at $t=0$. It therefore implies market clearing in the debt market at $t=1$, $B^d_0 = B_0$. Continuing by induction, it proves market clearing in all periods.

Individual Optimality

We now check individual optimality, which is characterized by the consumption function (8) and No-Cash-on-the-Table condition (10). Condition (8) is satisfied by construction. Combined with the flow budget constraint of the representative household (5), its No-Cash-on-the-Table constraint is equivalent to its intertemporal budget constraint

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k} = B^d_{t-1} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k} - T_{t+k}). \tag{B.6}$$

Since the goods market clears in all periods, it is equivalent to condition (16), which is satisfied by assumption.

C Proof of Lemma 1

Preliminary: Market-Clearing in the Debt Market

We first simply prove Walras's Law: the assumption of market-clearing in the goods market (15) implies market-clearing in the debt market when combined with flow budget constraints, i.e.

$$\int_i B^i_t di = B_t, \tag{C.1}$$

where the integral is over households alive at t.

By assumption, market-clearing held in period $t=-1$. We show that market-clearing holds in the debt market in all periods by showing that if it holds at $t-1$ then it holds at t. Summing the

households' FBC (5) across households alive at t gives

$$C_t + \frac{1}{R_t}\frac{\int_i B_t^i di}{P_{t+1}} = \frac{B_{t-1}}{P_t} + Y_t - T_t, \tag{C.2}$$

where we used the fact that $\int_i B_{t-1}^i di = (1-\lambda)B_{t-1}$, since the remaining λB_{t-1} belonging to households that passed away has been taken by the insurance fund and redistributed to remaining households as annuities.

Combining (C.2) with goods market-clearing condition (15) gives

$$\frac{1}{R_t}\frac{\int_i B_t^i di}{P_{t+1}} = \frac{B_{t-1}}{P_t} - T_t, \tag{C.3}$$

which combined with the FBC of the government (13) implies

$$\int_i B_t^i di = B_t, \tag{C.4}$$

which ends the proof.

Preliminary: Defining Aggregate Human Capital

Define aggregate human capital as the sum of individual human capitals

$$H_t = \sum_{k=0}^{\infty} N_t(k) H_t^i(k). \tag{C.5}$$

The human capital (9) of a household of age n can then be written as a function of aggregate human capital per capita

$$H_t^i(n) = \kappa(1-\zeta)^n \left(\frac{H_t}{N_t}\right), \tag{C.6}$$

where aggregate human capital per capita can be written

$$\frac{H_t}{N_t} = \sum_{k=0}^{\infty} \frac{((1-\lambda)(1-\zeta))^k}{\mathcal{R}_{t,t+k}} \left(\frac{Y_{t+k} - T_{t+k}}{N_{t+k}}\right), \tag{C.7}$$

or recursively

$$\frac{H_t}{N_t} = \left(\frac{Y_t - T_t}{N_t}\right) + \frac{(1-\lambda)(1-\zeta)}{R_t}\left(\frac{H_{t+1}}{N_{t+1}}\right). \tag{C.8}$$

Aggregate human capital solves the recursion

$$H_t = (Y_t - T_t) + \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t} H_{t+1}. \tag{C.9}$$

An Intermediary Lemma

We now move to the core of the proof. To prove Lemma 1, we first prove the following lemma:

Lemma C.1. *If all households are on their consumption function (8) and the goods market clears (15), then at time t, a household of age $n \leq t$ has holdings of public debt*

$$\frac{B_t^i(n)}{P_{t+1}} = \kappa(1-\zeta)^n \left[\mu \frac{1-x^{n+1}}{1-x} \frac{\mathcal{R}_{t-n,t+1}}{(1+g)^{n+1}} \frac{B_{t-(n+1)}}{N_{t-(n+1)} P_{t-n}} - \sum_{k=0}^{n} \left(x^k + \mu \frac{1-x^k}{1-x} \right) \frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}} \right], \tag{C.10}$$

where

$$x = \frac{\beta(1+g)}{1-\zeta}. \tag{C.11}$$

Proof. The proof is by induction. For future abundant use, first notice that combining the FBC (5) and the consumption function (8) gives the debt-holding function of a household at t, i.e. how many bonds to hold as a function of inherited bonds and future and present incomes

$$\frac{B_t^i}{P_{t+1}} = R_t \left(\beta \frac{B_{t-1}^i}{P_t} + Y_t^i - T_t^i - \mu H_t^i \right). \tag{C.12}$$

Base Case(s)

To initialize the recursion, we determine the debt holdings of households of age 0 and of age 1. (The inductive step will show the property for $n+1$ assuming it holds for n and $n-1$, so it requires a double-initialization.) Applying (C.12) to households of age 0, who inherit no financial wealth from the previous period $B_{t-1}^i(0) = 0$, gives

$$\frac{B_t^i(0)}{P_{t+1}} = R_t \kappa \left(\frac{Y_t - T_t - \mu H_t}{N_t} \right). \tag{C.13}$$

Aggregating equation (C.12) across all households alive at t and using market-clearing in the debt market gives

$$\frac{B_t}{P_{t+1}} = R_t \left(\beta(1-\lambda) \frac{B_{t-1}}{P_t} + Y_t - T_t - \mu H_t \right). \tag{C.14}$$

Taking the difference between (C.14) and κ/N_t times (C.13) gives

$$\frac{B_t^i(0)}{P_{t+1}} - \kappa\frac{B_t}{N_t P_{t+1}} = -\kappa\frac{\beta(1-\lambda)}{(1+g)}R_t\frac{B_{t-1}}{P_t N_{t-1}}. \tag{C.15}$$

The government's flow budget constraint (13) can be rewritten in per-capita terms as

$$\frac{B_t}{N_t P_{t+1}} = R_t\left(\frac{1}{1+g}\frac{B_{t-1}}{N_{t-1}P_t} - \frac{T_t}{N_t}\right). \tag{C.16}$$

Using (C.16) to replace $B_t/(N_t P_{t+1})$, equation (C.15) can be written

$$\frac{B_t^i(0)}{P_{t+1}} = \kappa\left(\mu\frac{R_t}{(1+g)}\frac{B_{t-1}}{N_{t-1}P_t} - R_t\frac{T_t}{N_t}\right), \tag{C.17}$$

which proves the base case $n = 0$. The base case $n = 1$ is shown very similarly to the inductive step below, noting that a household of age 0 at t had no debt at $t-1$.

Inductive Step

Assume the property holds for households of age n and $n-1$. We show it then holds for households of age $n+1$. Taking the difference between equation (C.12) applied to a household of age $n+1$ and $(1-\zeta)$ times equation (C.12) applied to a household of age n gives

$$\frac{B_t^i(n+1)}{P_{t+1}} - (1-\zeta)\frac{B_t^i(n)}{P_{t+1}} = R_t\beta\left(\frac{B_{t-1}^i(n)}{P_t} - (1-\zeta)\frac{B_{t-1}^i(n-1)}{P_t}\right). \tag{C.18}$$

We now use the fact that the property holds for n and $n-1$ to rewrite the term on the right-hand side of (C.18) as

$$\begin{aligned}\frac{B_{t-1}^i(n)}{P_t} - (1-\zeta)\frac{B_{t-1}^i(n-1)}{P_t} = \kappa(1-\zeta)^n\Bigg[\mu\Bigg(&\frac{1-x^{n+1}}{1-x}\frac{\mathcal{R}_{t-1-(n+1),t}}{(1+g)^{n+1}}\frac{B_{t-1-(n+1)}}{N_{t-1-(n+1)}P_{t-(n+1)}}\\ &-\frac{1-x^n}{1-x}\frac{\mathcal{R}_{t-1-n,t}}{(1+g)^n}\frac{B_{t-1-n}}{N_{t-1-n}P_{t-n}}\Bigg)\\ &-\left(x^n+\mu\frac{1-x^n}{1-x}\right)\frac{\mathcal{R}_{t-1-n,t}}{(1+g)^n}\frac{T_{t-1-n}}{N_{t-1-n}}\Bigg].\end{aligned} \tag{C.19}$$

Using the government flow budget constraint (C.16) to replace $B_{t-1-n}/(N_{t-1-n}P_{t-n})$, this rewrites

$$\frac{B_{t-1}^i(n)}{P_t} - (1-\zeta)\frac{B_{t-1}^i(n-1)}{P_t} = \kappa(1-\zeta)^n\left[\mu x^n\frac{\mathcal{R}_{t-(n+2),t}}{(1+g)^{n+1}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - x^n\frac{\mathcal{R}_{t-(n+1),t}}{(1+g)^n}\frac{T_{t-(n+1)}}{N_{t-(n+1)}}\right]. \tag{C.20}$$

Multiplying by βR_t,

$$\beta R_t \left(\frac{B^i_{t-1}(n)}{P_t} - (1-\zeta)\frac{B^i_{t-1}(n-1)}{P_t} \right) = \kappa(1-\zeta)^{n+1} \left[\mu x^{n+1} \frac{\mathcal{R}_{t-(n+2),t+1}}{(1+g)^{n+2}} \frac{B_{t-(n+2)}}{N_{t-(n+2)}} - x^{n+1} \frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+1}} \frac{T_{t-(n+1)}}{N_{t-(n+1)}} \right]. \tag{C.21}$$

Meanwhile, the term $(1-\zeta)B^i_t(n)/P_{t+1}$ on the left-hand side of equation (C.18) can be rewritten, using (C.10) and the government flow budget constraint (C.16) to eliminate $B_{t-(n+1)}/(N_{t-(n+1)}P_{t-n})$, as

$$(1-\zeta)\frac{B^i_t(n)}{P_{t+1}} = \kappa(1-\zeta)^{n+1} \left[\mu \frac{1-x^{n+1}}{1-x} \frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+2}} \frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \mu \frac{1-x^{n+1}}{1-x} \frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+1}} \frac{T_{t-(n+1)}}{N_{t-(n+1)}} - \sum_{k=0}^{n} \left(x^k + \mu \frac{1-x^k}{1-x} \right) \frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}} \right]. \tag{C.22}$$

Injecting equations (C.21) and (C.22) into equation (C.18) gives

$$\frac{B^i_t(n+1)}{P_{t+1}} = \kappa(1-\zeta)^{n+1} \left[\mu \left(\frac{1-x^{n+1}}{1-x} + x^{n+1} \right) \frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+2}} \frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \left(x^{n+1} + \mu \frac{1-x^{n+1}}{1-x} \right) \frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+1}} \frac{T_{t-(n+1)}}{N_{t-(n+1)}} - \sum_{k=0}^{n} \left(x^k + \mu \frac{1-x^k}{1-x} \right) \frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}} \right]. \tag{C.23}$$

It rewrites

$$\frac{B^i_t(n+1)}{P_{t+1}} = \kappa(1-\zeta)^{n+1} \left[\mu \left(\frac{1-x^{n+2}}{1-x} \right) \frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+2}} \frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \sum_{k=0}^{n+1} \left(x^k + \mu \frac{1-x^k}{1-x} \right) \frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}} \right], \tag{C.24}$$

which ends the proof. □

Finishing the Proof

To get from Lemma C.1 to Lemma 1, we just need to express $B^i_t(n)$ as a function of B_t instead

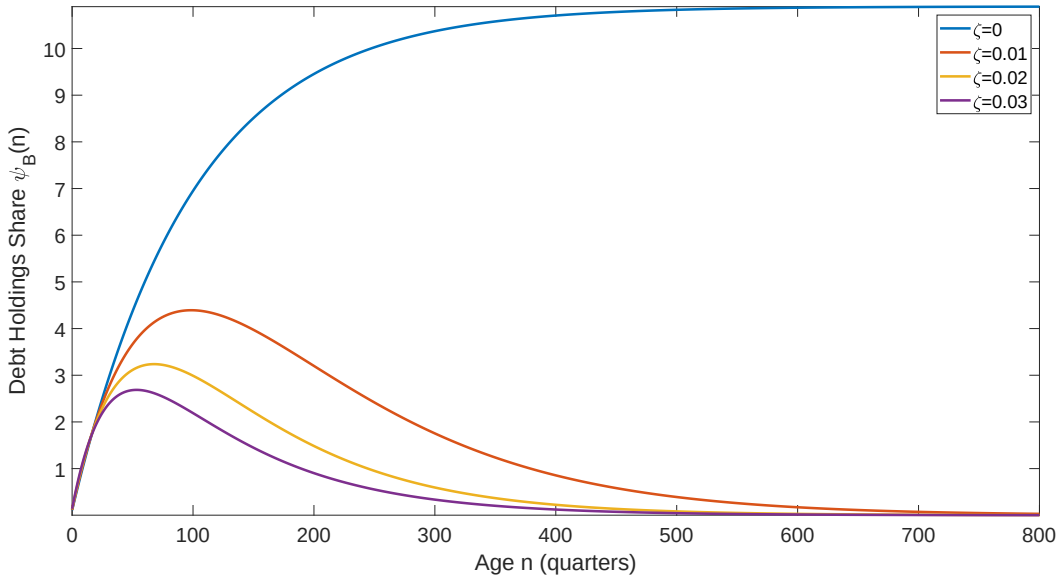


Figure C.1: Individual Debt Holdings as a Function of Age

Note: The figure represents the coefficient $\psi_B(n)$ in the expression of individual debt holdings (21) as a function of the household's age, for different values of the parameter ζ defined in equation (11). The three other parameters are set to $\beta = 0.995$, $\lambda = 0.005$ and $g = 0.005$.

of B_{t-n}. Iterate the government's flow budget constraint (C.16) from $t-(n+1)$ to t to obtain

$$\frac{B_t}{N_t P_{t+1}} = \frac{\mathcal{R}_{t-n,t+1}}{(1+g)^{n+1}} \frac{B_{t-(n+1)}}{N_{t-(n+1)} P_{t-n}} - \sum_{k=0}^{n} \frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}}. \tag{C.25}$$

Injecting it in equation (C.10) gives (21).

Figure C.1 plots the coefficient $\psi_B(n)$ as a function of age n for different values of ζ.

Sanity Check

Note as a sanity check that we have

$$\sum_{n=0}^{\infty} \psi_B(n) \frac{N_t(n)}{N_t} = 1, \tag{C.26}$$

$$\forall k \in [\![0, n]\!], \sum_{n=k}^{\infty} \phi_T(n,k) N_t(n) = 0, \tag{C.27}$$

so that when enough time has passed so that all the debt holdings of households of all ages is given by (21), the sum of debt holdings by all households sums to B_t.

D Proof of Lemma 2

If all households alive at t satisfy their No-Cash-on-the-Table condition (10), then (25) necessarily holds by taking the average over all households alive at t. The lengthier part is to show that (25) is also sufficient.

Let i be a household alive at t, and $n \geq 0$ its age at time t. It is therefore of age $n+k$ at $t+k$. We can decompose its debt holding at $t+k$ into the two following terms

$$\frac{B^i_{t+k}(n+k)}{P_{t+k+1}} = \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} + \left(\frac{B^i_{t+k}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} \right). \tag{D.1}$$

It follows that

$$\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} \frac{B^i_{t+k}(n+k)}{P_{t+k+1}} = \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} + \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} \left(\frac{B^i_{t+k}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} \right). \tag{D.2}$$

The proof amounts to showing that the second term in (D.2) necessarily tends to zero as $k \to \infty$. Condition (25) then implies that the first term tends to zero since $\kappa(1-\zeta)^n$ is a constant.

We denote with the superscript *old* the variables aggregated over all households already alive at t. Let $k \geq 0$. Aggregating (C.12) at period $t+k$ across households already alive at t gives

$$\frac{B^{old}_{t+k}}{P_{t+k+1}} = R_{t+k} \left(\beta(1-\lambda) \frac{B^{old}_{t+k-1}}{P_{t+k}} + Y^{old}_{t+k} - T^{old}_{t+k} - \mu H^{old}_{t+k} \right), \tag{D.3}$$

where

$$\begin{aligned} Y^{old}_{t+k} - T^{old}_{t+k} - \mu H^{old}_{t+k} &= \sum_{n=k}^{\infty} N_{t+k}(n) (Y^i_{t+k}(n) - T^i_{t+k}(n) - \mu H^i_{t+k}(n)) \\ &= \kappa \left(\frac{(1-\zeta)(1-\lambda)}{(1+g)} \right)^k (Y_{t+k} - T_{t+k} - \mu H_{t+k}). \end{aligned} \tag{D.4}$$

Injecting (D.4) into (D.3)

$$\frac{B^{old}_{t+k}}{P_{t+k+1}} = R_{t+k} \left(\beta(1-\lambda) \frac{B^{old}_{t+k-1}}{P_{t+k}} + \kappa \left(\frac{(1-\zeta)(1-\lambda)}{(1+g)} \right)^k (Y_{t+k} - T_{t+k} - \mu H_{t+k}) \right). \tag{D.5}$$

The number of households alive at t that are still alive at $t+k$ is $(1-\lambda)^k N_t$. Dividing (D.5)

by this to get the average debt holding:

$$\frac{\bar{B}_{t+k}}{P_{t+k+1}} = R_{t+k}\left(\beta\frac{\bar{B}_{t+k-1}}{P_{t+k}} + \kappa(1-\zeta)^k\left(\frac{Y_{t+k}}{N_{t+k}} - \frac{T_{t+k}}{N_{t+k}} - \mu\frac{H_{t+k}}{N_{t+k}}\right)\right). \tag{D.6}$$

Applying equation (C.12) at $t+k$ to household i gives

$$\frac{B^i_{t+k}(n+k)}{P_{t+k+1}} = R_{t+k}\left(\beta\frac{B^i_{t+k-1}(n+k-1)}{P_{t+k}} + \kappa(1-\zeta)^{n+k}\left(\frac{Y_{t+k}}{N_{t+k}} - \frac{T_{t+k}}{N_{t+k}} - \mu\frac{H_{t+k}}{N_{t+k}}\right)\right). \tag{D.7}$$

Taking the difference between (D.7) and $\kappa(1-\zeta)^n$ times (D.6) gives

$$\frac{B^i_{t+k}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n\frac{\bar{B}_{t+k}}{P_{t+k+1}} = \beta R_{t+k}\left(\frac{B^i_{t+k-1}(n+k-1)}{P_{t+k}} - \kappa(1-\zeta)^n\frac{\bar{B}_{t+k-1}}{P_{t+k}}\right) \tag{D.8}$$

Iterating backward:

$$\frac{B^i_{t+k}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n\frac{\bar{B}_{t+k}}{P_{t+k+1}} = \beta^k\mathcal{R}_{t,t+k+1}\left(\frac{B^i_{t-1}(n)}{P_t} - \kappa(1-\zeta)^n\frac{\bar{B}_{t-1}}{P_t}\right) \tag{D.9}$$

This implies that the second term in (D.2) can be written as

$$\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}}\left(\frac{B^i_{t+k}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n\frac{\bar{B}_{t+k}}{P_{t+k+1}}\right) = (\beta(1-\lambda))^k\left(\frac{B^i_{t-1}(n)}{P_t} - \kappa(1-\zeta)^n\frac{\bar{B}_{t-1}}{P_t}\right), \tag{D.10}$$

which tends to zero as $k\to\infty$. This ends the proof.

E Proof of Lemma 3

In an equilibrium, all households must be on their consumption functions (8) so we can apply Lemma 2. In an equilibrium the No-Cash-on-the-Table conditions of all households must be satisfied, so from Lemma 2 condition (25) must hold. We show that it implies Proposition 3. To derive $\bar{B}_{t+k}$, we calculate B^{old}_{t+k} by calculating its complement: the quantity of public debt that is held at $t+k$ by households born at $t+1$ or later. We denote it B^{young}_{t+k}. It is equal to the sum of the debt held by households of age $0\leq n\leq k-1$ at $t+k$. Since in an equilibrium all households are on their consumption function and the goods-market clears, we can use the expression for debt holdings in

Lemma 1 to calculate it.

$$\begin{aligned}\frac{B_{t+k}^{young}}{P_{t+k+1}} &= \sum_{n=0}^{k-1} \frac{B_{t+k}^{i}(n)}{P_{t+k+1}} N_{t+k}(n) \\ &= \left[1 - \frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right)\right]\frac{B_{t+k}}{P_{t+k+1}} \\ &- \sum_{j=1}^{k}(\beta(1-\lambda))^{k-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right] \\ &\times \mathcal{R}_{t+j,t+k+1} T_{t+j}. \end{aligned} \tag{E.1}$$

We can therefore express $\bar{B}_{t+k}$ as

$$\begin{aligned}\frac{\bar{B}_{t+k}}{P_{t+k+1}} &= \frac{B_{t+k} - B_{t+k}^{young}}{P_{t+k+1}(1-\lambda)^k N_t} \\ &= \frac{1}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right)\frac{B_{t+k}}{N_t P_{t+k+1}} \\ &+ \frac{1}{(1-\lambda)^k}\sum_{j=1}^{k}(\beta(1-\lambda))^{k-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right] \\ &\times \mathcal{R}_{t+j,t+k+1}\frac{T_{t+j}}{N_t}. \end{aligned} \tag{E.2}$$

Condition (25) therefore implies that

$$\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}}\frac{\bar{B}_{t+k}}{P_{t+k+1}} = \xi(k)\frac{B_{t+k}}{N_t\mathcal{R}_{t,t+k+1}P_{t+k+1}} + (\beta(1-\lambda))^k\left(\sum_{j=1}^{k}\theta(j)\frac{1}{\mathcal{R}_{t,t+j}}\frac{T_{t+j}}{N_t}\right) \tag{E.3}$$

must tend to zero as $k \to \infty$, where

$$\xi(k) = \frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right), \tag{E.4}$$

$$\theta(j) = (\beta(1-\lambda))^{-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right]. \tag{E.5}$$

Using the budget constraint of the government

$$\frac{1}{\mathcal{R}_{t,t+k+1}}\frac{B_{t+k}}{P_{t+k+1}} = \frac{B_{t-1}}{P_t} - \sum_{j=0}^{k}\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j} \tag{E.6}$$

to replace B_{t+k}/P_{t+k+1}, and multiplying by N_t, this implies equivalently that

$$\xi(k)\frac{B_{t-1}}{P_t} + \left(\sum_{j=1}^{k}\left((\beta(1-\lambda))^k\theta(j) - \xi(k)\right)\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j}\right) - \xi(k)T_t \tag{E.7}$$

must tend to zero as $k \to \infty$.

Since $\lambda > 0$ or $g > 0$ in the non-Ricardian case, $\xi(k)$ tends to zero as $k \to \infty$, and so the term in B_{t-1} tends to zero. The condition is therefore equivalent to

$$\lim_{k\to\infty}\sum_{j=1}^{k}\left((\beta(1-\lambda))^k\theta(j) - \xi(k)\right)\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j} - \xi(k)T_t = 0. \tag{E.8}$$

Defining

$$\Omega(k,j) = (\beta(1-\lambda))^k\theta(j) - \xi(k) \text{ for } j \geq 1, \tag{E.9}$$

$$\Omega(k,0) = -\xi(k) \text{ for } j = 0, \tag{E.10}$$

this ends the proof.

F Proof of Lemma 4

In an equilibrium, all households are on their consumption function (8). Aggregating the consumption function (8) across households gives the aggregate consumption function

$$C_t = \mu\left(\frac{B^d_{t-1}}{P_t} + H_t\right). \tag{F.1}$$

where we denote by B^d_t the aggregate demand for public debt at t, $B^d_t = \int_i B^i_t di$ so that it does not assume market-clearing in the debt market, and where we used the fact that $\int_i B^i_{t-1}di = (1-\lambda)B^d_{t-1}$ since only $(1-\lambda)$ of household who demanded debt at $t-1$ are still alive at t.

Differentiate the aggregate consumption function (F.1), using $(1-\lambda)(1-\zeta)/((1+g)R_t)$ as the discount factor:

$$C_t - \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t}C_{t+1} = \mu\left[\left(\frac{B^d_{t-1}}{P_t} - \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t}\frac{B^d_t}{P_{t+1}}\right) + Y_t - T_t\right]. \tag{F.2}$$

It simplifies using the aggregate flow budget constraint (C.2) into

$$C_t = \frac{1}{\beta R_t}\left(\frac{1-\zeta}{1+g}\right) C_{t+1} + \chi \frac{B_t^d}{R_t P_{t+1}}, \tag{F.3}$$

where χ is defined in equation (31). Note that equation (F.3) is a purely decision-theoretic object. It relies on no assumption of market clearing.

In equilibrium, market clearing holds in the debt market $B_t^d = B_t$. Replacing B_t by B_{t-1} using the flow budget of the government (13) gives equation (30).

G Proof of Proposition 2

Proposition 3 and 5 already showed that conditions (20) and (18) are necessary for a stable price equilibrium. We show that they are sufficient. The proof is constructive. Given an initial level of public debt B_{-1} distributed as $(B_{-1}^i)_i$ in the population, consider a path for taxes $(T_t)_{t\geq 0}$. Define recursively the following path for the interest rate

$$R_t = \frac{\frac{1-\zeta}{\beta(1+g)} Y_{t+1}}{Y_t - \chi\left(B_{t-1} - T_t\right)}, \tag{G.1}$$

which in combination with the FBC (13) of the government defines an entire path for interest rates and public debt. Because condition (18) is satisfied, equation (G.1) defines a positive finite interest rate. For this interest rate path, define individual consumption allocations through the consumption function (8), which in combination with the FBC (5) of household i defines an entire path for its consumption and debt holdings.

We show that this allocation is a stable price equilibrium.

Market-Clearing

Start with market-clearing. Since by construction all households are on the consumption function (8), aggregate consumption is given by the aggregate consumption function (F.1), which writes injecting the definition of aggregate human capital (C.7)

$$C_t = \mu\left(\frac{B_{t-1}^d}{P_t} + \sum_{k=0}^{\infty} \frac{a^k}{\mathcal{R}_{t,t+k}}(Y_{t+k} - T_{t+k})\right), \tag{G.2}$$

where

$$a = \frac{(1-\lambda)(1-\zeta)}{1+g}. \tag{G.3}$$

Define

$$Z_{t+k} = a^k \frac{Y_{t+k}}{\mathcal{R}_{t,t+k}}. \tag{G.4}$$

Using the definition of the interest rate path (G.1) which can be written

$$\frac{Y_{t+1}}{R_t} = \frac{\beta(1+g)}{1-\zeta}\left(Y_t - \chi \frac{B_t}{R_t P_{t+1}}\right), \tag{G.5}$$

Z_{t+k} satisfies the recursion

$$Z_{t+k} = bZ_{t+k-1} - \chi b \frac{a^{k-1}}{\mathcal{R}_{t,t+k}} B_{t+k-1}, \tag{G.6}$$

where

$$b = \beta(1-\lambda). \tag{G.7}$$

By iteration, it gives

$$Z_{t+k} = b^k Y_t - \chi \left(\sum_{j=0}^{k-1} b^{k-j} a^j \frac{B_{t+j}}{\mathcal{R}_{t,t+j+1}} \right). \tag{G.8}$$

It follows that (after permuting a double sum)

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu}\left(Y_t - \chi\beta(1-\lambda)\left(\sum_{j=0}^{k-1} a^j \frac{B_{t+j}}{\mathcal{R}_{t,t+j+1}} \right)\right). \tag{G.9}$$

Using the budget constraint of the government

$$\frac{B_{t+j}}{P_{t+j+1}\mathcal{R}_{t,t+j+1}} = \frac{B_{t-1}}{P_t} - \sum_{i=0}^{j} \frac{1}{\mathcal{R}_{t,t+i}} T_{t+i}, \tag{G.10}$$

it can be rewritten (after permuting a double sum again)

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu}\left(Y_t - \frac{\chi\beta(1-\lambda)}{1-a}\left(\frac{B_{t-1}}{P_t} - \sum_{i=0}^{\infty} a^i \frac{T_{t+i}}{\mathcal{R}_{t,t+i}}\right)\right), \tag{G.11}$$

or noticing that $\chi\beta(1-\lambda)/(1-a) = \mu$,

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu}Y_t - \left(\frac{B_{t-1}}{P_t} - \sum_{i=0}^{\infty} a^i \frac{T_{t+i}}{\mathcal{R}_{t,t+i}}\right). \tag{G.12}$$

Injecting equation (G.12) into the aggregate consumption function (G.2) implies

$$C_t = Y_t + \mu\left(\frac{B^d_{t-1}}{P_t} - \frac{B_{t-1}}{P_t}\right). \tag{G.13}$$

Because $B^d_{-1} = B_{-1}$, this implies market clearing in the goods market at $t = 0$, and therefore in the debt market, and so on at all periods by induction.

Individual Optimality

We now check individual optimality, which is characterized by the consumption function (8) and No-Cash-on-the-Table condition (10). Condition (8) is satisfied by construction. We need to show that the No-Cash-on-the-Table constraints of all households are satisfied. Since all households behave according to the consumption function (8), we can apply Lemma 2 and simply show that condition (25) is satisfied. Since all households are on their consumption function and the goods market clears, we can use Lemma 1. That $\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}}\frac{\bar{B}_{t+k}}{P_{t+k+1}}$ tends to zero as $k \to \infty$ can therefore be rewritten as equation (E.8), which holds by assumption from condition (18).

H Absence of a Limit on the Maturity Value of Public Debt

Using the flow budget constraint of the government (13), the debt-to-GDP limit can be rewritten as a constraint on the maturity value of debt,

$$\frac{B_t}{P^* Y_t} \leq d^* R_t. \tag{H.1}$$

Since the real interest rate R_t can potentially be arbitrarily large, this does not necessarily put an upper-bound on the maturity value of debt in a stable price equilibrium. We show by way of an

example that the maturity value of debt can indeed take arbitrarily high values in a stable price equilibrium. Assume that the government sets its fiscal policy to maintain a constant level of the maturity value of debt relative to GDP, $b = B_t/(P^*Y_{t+1})$, where b is an arbitrarily large level. That is, it sets taxes each period as $T_t = b(Y_t - Y_{t+1}/R_t)$.

Equation (32) gives the real interest rate consistent with this fiscal policy,

$$R_t = \frac{1-\zeta}{\beta(1+g)}\frac{Y_{t+1}}{Y_t} + \chi(1+g)b. \tag{H.2}$$

This real rate always exists for any level of the maturity value of debt b. This shows that it is possible for b to be arbitrarily large in a stable price equilibrium. Yet the maturity value of debt is then arbitrarily large only because the real interest rate (H.2) is then arbitrarily large. The government is still unable to borrow a principal larger than d^*, but it can repay an arbitrarily large amount because it repays arbitrarily large interest rates on its debt.

I Proof of the Implication in Section 3.1

Write the budget constraint of the government (13) in per-GDP terms, and iterate it forward to give

$$\frac{B_{t-1}}{P_tY_t} = \sum_{k=0}^{\infty}\left(\Pi_{i=0}^{k}\frac{\frac{Y_{t+i+1}}{Y_{t+i}}}{R_{t+i}}\right)\frac{T_{t+k}}{Y_{t+k}} + \lim_{k\to\infty}\left(\Pi_{i=0}^{k}\frac{\frac{Y_{t+i+1}}{Y_{t+i}}}{R_{t+i}}\right)\frac{B_{t+k}}{P_{t+k+1}Y_{t+k+1}}. \tag{I.1}$$

Assume that the debt-to-GDP limit (18) is satisfied and that the real interest rate is necessarily above the growth rate of the economy, $R_t \geq Y_{t+1}/Y_t$. The last bubble term is then the product of a term that tends to zero and of a bounded term, so it tends to zero. Multiplying what is left of equation (I.1) by Y_t gives the NPV equation (16).

That condition (39) implies that $R_t \geq Y_{t+1}/Y_t$ for all levels of public debt follows from the dynamic IS curve (30).

J Proof of Proposition 4

The intertemporal budget constraint of non hand-to-mouth households, denoted with the superscript PI for permanent-income households, is

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k}^{PI} = \frac{B_{t-1}}{P_t} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k}^{PI} - T_{t+k}^{PI}). \tag{J.1}$$

Injecting the expression for income $Y_t^{PI} = (1-\alpha_Y)Y_t$ and taxes $T_t^{PI} = (1-\alpha_T)T_t$, as well as the equilibrium requirement that $C_t^{PI} = Y_t - C_t^{HTM} = (1-\alpha_Y)Y_t + \alpha_T T_t$, gives the intertemporal budget constraint of the government (16).

The proof of sufficiency follows the proof of Proposition 1.

K Proof of Corollary 1

Condition (20) can be derived following exactly the same steps as in the case without hand-to-mouth households. Indeed, in the proof, taxes only intervene through the flow budget constraint of the government.

We show that condition (44) is necessary. Following the same steps as without hand-to-mouth households, one can show that equation (F.3) holds for the aggregate consumption of non hand-to-mouth households,

$$C_t^{PI} = \frac{1}{\beta R_t} \left(\frac{1-\zeta}{1+g} \right) C_{t+1}^{PI} + \chi \frac{B_t^d}{R_t P_{t+1}}. \tag{K.1}$$

Since non hand-to-mouth households collectively hold all of public debt, assuming the debt market clears and using the flow budget constraint of the government (13) gives

$$C_t^{PI} = \frac{1}{\beta R_t} \left(\frac{1-\zeta}{1+g} \right) C_{t+1}^{PI} + \chi \left(\frac{B_{t-1}}{P_t} - T_t \right). \tag{K.2}$$

Using the fact that in equilibrium $C_t^{PI} = Y_t - C_t^{HTM} = (1-\alpha_Y)Y_t + \alpha_T T_t$ gives the IS curve

$$Y_t = \frac{1}{\beta R_t} \left(\frac{1-\zeta}{1+g} \right) \left(Y_{t+1} + \frac{\alpha_T}{1-\alpha_Y} T_{t+1} \right) + \frac{\chi}{1-\alpha_Y} \frac{B_{t-1}}{P_t} - \frac{\alpha_T + \chi}{1-\alpha_Y} T_t. \tag{K.3}$$

Condition (44) is obtained when R_t is set to infinity.

The proof of sufficiency follows the proof of Proposition 2.

L Proof of Proposition 3

The two flow budget constraints (35) and (36) can be combined into the intertemporal budget constraint

$$C_t^y + \frac{1}{R_t} C_{t+1}^o = Y_t^y - T_t^y + \frac{1}{R_t}(Y_{t+1}^o - T_{t+1}^o). \tag{L.1}$$

Combining it with the household's first-order condition, the optimal behavior of households is characterized by the following consumption function,

$$C_t^y = \frac{1}{1+\beta}\left((Y_t^y - T_t^y) + \frac{1}{R_t}(Y_{t+1}^o - T_{t+1}^o)\right), \tag{L.2}$$

$$C_t^o = \frac{B_{t-1}^y}{P_t} + (Y_t^o - T_t^o). \tag{L.3}$$

Given that households have finite lives and leave no wealth when they die, there is no extra No-Cash-on-the-Table constraint as part of individual optimality.

Necessary Condition: We first show that condition (38) is necessary in equilibrium. Summing up (L.2) and (L.3), and using the fact that the debt market cleared in period $t-1$, $B_{t-1} = N_{t-1}^{young} B_{t-1}^{young}$, total aggregate consumption is given by

$$C_t = \frac{1}{1+\beta}\left(\gamma(Y_t - T_t) + \frac{1}{R_t}(1-\gamma)(Y_{t+1} - T_{t+1})\right) + \frac{B_{t-1}}{P_t} + (1-\gamma)(Y_t - T_t). \tag{L.4}$$

Imposing market clearing on the goods market, we get the dynamic IS curve

$$Y_t = \frac{1-\gamma}{\gamma\beta}\frac{1}{R_t}(Y_{t+1} - T_{t+1}) + \frac{1+\beta}{\gamma\beta}\frac{B_{t-1}}{P_t} - \left(\frac{1+\beta}{\gamma\beta} - 1\right) T_t. \tag{L.5}$$

Setting the interest rate to infinity gives the condition (38).

Sufficient Condition: If condition (38) is satisfied in all periods, define a path for the real interest

rate through

$$R_t = \frac{\frac{1-\gamma}{\gamma\beta}(Y_{t+1} - T_{t+1})}{Y_t - \frac{1+\beta}{\gamma\beta}\frac{B_{t-1}}{P_t} + \left(\frac{1+\beta}{\gamma\beta} - 1\right)T_t} \tag{L.6}$$

and the government flow budget constraint (13). For this interest rate path, define the consumption of young and old households through the consumption function (L.2) and (L.3). Individual optimality is satisfied by construction. Rewriting the expression of aggregate consumption (L.4) using the expression (L.6) for R_t shows market clearing $C_t = Y_t$. This ends the proof.

M Details on the Calibration Strategies of Sections 3.6, 4.2, 4.3

For both the OLG calibration of Section 3.6 and the OLG-HTM calibration of Section 4.2, we set values for β, g and λ, which gives $\mu = 1 - \beta(1-\lambda)$. For the OLG calibration, we set $\alpha_Y = 0$. For the OLG-HTM calibration, we set $\alpha_Y = (AMPC - \mu)/(1-\mu)$, where $AMPC$ is the average MPC target of 0.2. In both calibrations, we then set ζ to match a steady-state real interest rate of 0.5% at a debt-to-GDP ratio of 100%. From the IS curve (K.3) with $\alpha_T = 0$, the steady-state relationship between the annualized net real interest rate r and the debt-to-annual-GDP ratio $B/(4PY)$ is

$$r_0 = 4\left(\frac{1-\zeta}{\beta} - 1\right) + \frac{16\chi(1+g)}{1-\alpha_Y}\frac{B}{4PY}, \tag{M.1}$$

where we calculate the debt-to-annual-GDP ratio using public debt at its maturity value B, in line with public sector debt statistics. The value of ζ can be obtained by solving (M.1) in ζ, knowing χ depends on ζ through (31).

For the calibration in Section 4.3, we set values for β and g and calibrate the three remaining parameters α_Y, λ and ζ to match three statistics: an average MPC of $AMPC = 0.2$, a derivative of the real interest rate to the debt-to-annual-GDP ratio in steady-state of $\epsilon = 0.03$, and a annualized net real interest rate at 100% debt-to-annual-GDP in steady-state of $r_0 = 0.5\%$. The three equations

relating α_Y, λ and ζ to $AMPC$, ϵ and r_0 are

$$AMPC = \alpha_Y + (1-\alpha_Y)(1-\beta(1-\lambda)), \tag{M.2}$$

$$r_0 = 4\left(\frac{1-\zeta}{\beta} - 1\right) + \frac{16\chi(1+g)}{1-\alpha_Y}, \tag{M.3}$$

$$\epsilon = \frac{\partial r}{\partial(B/4PY)} = \frac{16\chi(1+g)}{1-\alpha_Y}, \tag{M.4}$$

where χ depends on λ and ζ through (31). We solve them for α_Y, λ and ζ.

Equation (M.4) can be used to calculate the sensitivity of the real interest rate to the debt-to-GDP ratio in other calibrations of the model, as reported in Table 1.

N OLG vs. HANK Interpretations and Woodford (1990) Model

We show the importance of the feedback of the level of public debt on the probability of hitting borrowing constraints through Woodford (1990)'s alternating-endowment model, a particular case of a HANK model which is tractable enough to be solved analytically. The economy is populated by two types of infinitely-lived households which cannot borrow. They face idiosyncratic risk because their individual incomes alternate between a low and a high endowment. In odd periods, type-A households receive the high endowment Y_t^h and pay taxes T_t^h while type-B households receive the low endowment Y_t^l and pay taxes T_t^l, and conversely in even periods. Both types of households have the same preferences

$$\sum_{k=0}^{\infty} \beta^k \log(C_{t+k}^i), \tag{N.1}$$

which they maximize subject to their flow budget constraint

$$\frac{1}{R_t}\frac{B_t^i}{P_{t+1}} + C_t^i = \frac{B_{t-1}^i}{P_t} + Y_t^i - T_t^i, \tag{N.2}$$

and the constraint that they cannot borrow

$$B_t^i \geq 0. \tag{N.3}$$

As Woodford (1990) shows, the model is exactly equivalent to the two-generation OLG model of Section 2.3 when the borrowing constraint of the low-endowment household is always binding.

Indeed in this case, the low-endowment household holds no debt so the high-endowment household holds all of public debt. Since the high-endowment household holds positive public debt its borrowing constraint is not binding. At t, it decides how much to save facing the t and $t+1$ budget constraints

$$C_t^h + \frac{1}{R_t}\frac{B_t}{P_{t+1}} = Y_t^h - T_t^h, \tag{N.4}$$

$$C_{t+1}^l = \frac{B_t}{P_{t+1}} + Y_t^l - T_t^l, \tag{N.5}$$

where C_t^h and C_t^l are the consumption of high and low-endowment households and B_t is aggregate public debt. This is exactly the same program as in the OLG model of Section 2.3, with high-income households in the stead of young households and low-income households in the stead of old households (with no population growth, $g = 0$).

The exact equivalence between the alternating-endowment model and the two-generation OLG model buttresses the case for using the perpetual-youth model in its HANK interpretation. However, the equivalence only holds when the borrowing constraint of the low-endowment households is binding. Yet whether it is binding depends on the level of public debt. If public debt is high enough there is enough liquidity in the economy for households to self-insure and avoid ever being up against their borrowing constraint in the low-endowment state. The model is then no longer equivalent to the OLG model.

To see it, consider a steady-state where the high and low endowments Y^h and Y^l are constant, and where the government maintains a constant level of public debt B through constant taxes $T = (1 - 1/R)B$, which it splits between $T^h = Y^h/Y \times T$ and $T^l = Y^l/Y \times T$. We look for a steady-state equilibrium, i.e. values for R, C^h and C^l.

In both the OLG and the HANK models, we always have

$$R = \frac{1}{\beta}\frac{C^l}{C^h}. \tag{N.6}$$

In the OLG model, we always have in addition that

$$C^h = Y^h - T^h - \frac{1}{R}B, \tag{N.7}$$

$$C^l = B + Y^l - T^l. \tag{N.8}$$

Together, the three equations (N.6)-(N.7)-(N.8) determine an equation in R that gives the equilibrium real rate.

In the HANK model, the equilibrium is characterized by equations (N.6)-(N.7)-(N.8) only up to the level of public debt such that $C^l = C^h$ and $R = 1/\beta$. Beyond this level, the low-endowment household is no longer up against its borrowing constraint, and is instead on its Euler equation

$$R = \frac{1}{\beta}\frac{C^h}{C^l}, \tag{N.9}$$

so that $C^l = C^h$ and $R = 1/\beta$. There is perfect self-insurance.

Figure N.1 illustrates the difference between the OLG and HANK models by plotting the steady-state relationship between the real interest rate and the real-debt-to-GDP ratio in both the alternating-endowment model and the OLG model. Up to a threshold on the debt-to-GDP ratio, the borrowing constraint of the low-endowment households is binding and the two models are exactly equivalent. But once public debt is above this threshold, borrowing constraints are no longer binding in the alternating-endowment model. The model is then equivalent to one with a representative Ricardian household. As a result, the real rate is constant to $1/\beta$ and no longer increases with public debt. In the OLG model in contrast, the real interest rate keeps increasing with public debt.

O Blanchard-Yaari Bewley Model

We consider a model that combines both OLG and HANK departures from the representative agent: a Blanchard-Yaari Bewley model. As in a standard Bewley (1977) model, households face idiosyncratic income risk and a borrowing constraint, and can only borrow and save through non-contingent debt. As in the Blanchard-Yaari model of Section 1, households are born with no wealth, die with probability λ each period and insure against this mortality risk through life-insurance contracts. For simplicity, we abstract from population growth and from incomes that shrink with age ($g = \zeta = 0$).

A household i maximizes its utility (4) subject to its flow budget constraint (5) and No-Ponzi-

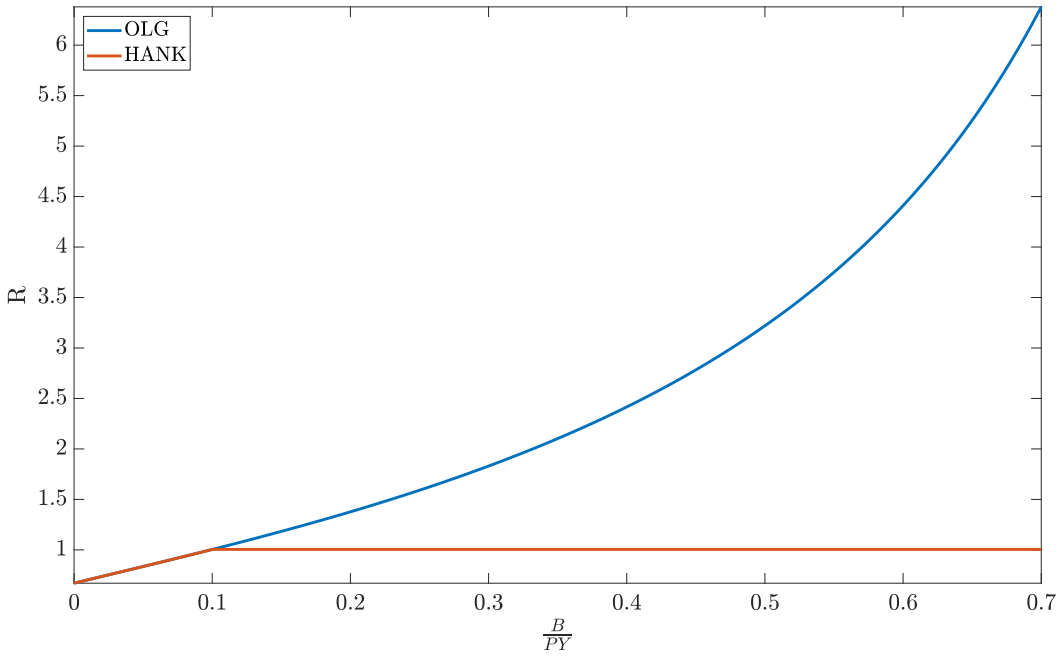


Figure N.1: Steady-State Relationship between Interest Rate and Debt-to-GDP, HANK vs. OLG

Note: The figure plots the steady-state relationship between the real interest rate R and the real debt-to-GDP ratio $B/(PY)$ in the OLG model of Section 2.3 and the alternating-endowment HANK model of Woodford (1990). The models are calibrated with $\beta = 0.995$ and, in the alternating-endowment HANK model, a difference between the high and low endowments equal to 20% of the average endowment. Taxes are distributed proportional to income.

scheme constraint (6), but it now faces the additional borrowing constraint

$$B_t^i \geq 0, \tag{O.1}$$

and its income is now given by an AR(1) stochastic process in log

$$\log(Y_t^i) = \rho \log(Y_{t-1}^i) + \varepsilon_t^i, \tag{O.2}$$

where the income processes are independent across households. Taxes are proportional to income. Fiscal policy is still given by a path for aggregate taxes, which still translates into a path for public debt through the flow budget constraint of the government (13). Goods market clearing still imposes equation (15).

We consider the relationship between the real interest rate and debt-to-GDP ratio in a steady-state with stable prices, and compare it to the relationship in a standard Bewley model without

Blanchard-Yaari overlapping generations. We calibrate the model quarterly with $\beta = 0.99$, $\rho = 0.98$, a standard deviation of the Gaussian innovations of shocks $\sigma_\varepsilon = 0.12$, and, in the case of the Blanchard-Yaari Bewley model, $\lambda = 0.005$.[1] The relationships between the interest rate and the debt-to-GDP ratio are plotted on Figure O.1. Because the debt-to-GDP limit d^* bears on the amount $D = B/R$ the government borrows and not on the amount B it repays, we plot the figure for the debt-to-GDP ratio $D/(PY)$.

Two results are apparent. First, in the Blanchard-Yaari Bewley model the real interest rate does not asymptote to a finite value when debt-to-GDP increases, contrary to the Bewley model absent overlapping generations. At high levels of debt, the Blanchard-Yaari Bewley model resembles the Blanchard-Yaari model with overlapping generations alone, since households then have enough liquid wealth to self-insure. The debt-to-GDP limit d^* then applies.

Second however, financial frictions matter for the quantification of the debt limit d^*. At low to medium levels of public debt, financial frictions lower the real interest rate and steepen the slope of the relationship between interest rates and debt. As the debt level increases, the effect of financial frictions fades while the effect of overlapping generations remains. The sensitivity of interest rates to debt therefore decreases. Since what matters for the quantification of the debt limit is the sensitivity at high levels of debt, using the sensitivity at current levels of debt as we did in our main calibration in the third column of Table 1 may understate the debt-to-GDP limit d^*.

This is illustrated on Figure O.1 through the tangent to the curve of the Blanchard-Yaari Bewley model at a debt-to-GDP ratio of 100%. The slope of the tangent is 1.7bp for every 1pp increase in debt-to-annual-GDP, close to the value of 3bp in the data that we match in our main calibration. Yet because this slope encompasses the effect of financial frictions it is higher than the sensitivity of the interest rate to debt at the asymptote determined by the Blanchard-Yaari model absent financial frictions. The latter is an order of magnitude lower, at 0.12bp.[2]

[1] We approximate the idiosyncratic income process through Rouwenhorst's method with 11 grid points, and use 500 grid points on the asset dimension.

[2] It is a bit lower than in the OLG and OLG-HTM calibrations of the model in the first two columns of Table 1 where it is 0.23bp and 0.29bp because we set $\zeta = g = 0$. But all these are an order of magnitude smaller than in our main calibration in the third column of Table 1.

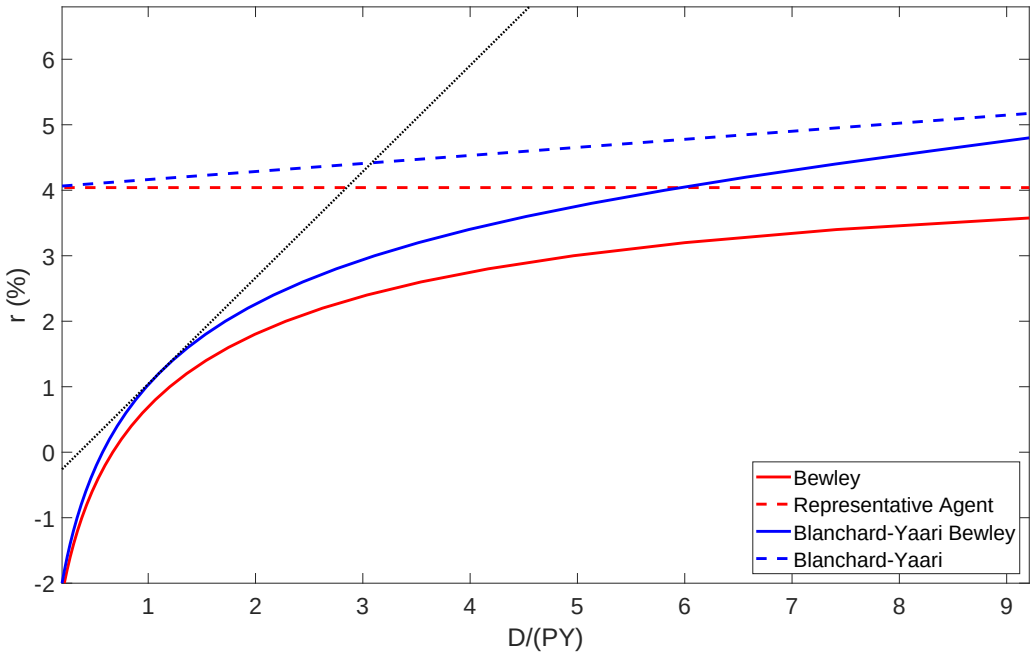


Figure O.1: Steady-State Relationship between Interest Rates and Debt-to-GDP, HANK vs. HANK+OLG

Note: The figure plots the steady-state relationship between the net real interest rate $r = R - 1$ (annualized) and the real debt-to-annual-GDP ratio in four models: the Blanchard-Yaari Bewley model of Section O, the Blanchard-Yaari model of Section 1, a standard Bewley model without overlapping generations, and a Representative Agent model. The debt-to-GDP ratio $D/(PY)$ is for the amount the government borrows $D = B/R$—what the debt limit d^* bears on. The models are calibrated with $\beta = 0.99$, $\rho = 0.98$, a standard deviation of innovations to the income process $\sigma_\varepsilon = 0.12$, and in the case of the Blanchard-Yaari HANK-OLG model, $\lambda = 0.005$. The dotted black line is the tangent to the curve of the Blanchard-Yaari Bewley model at a debt-to-annual-GDP ratio of 100%.

P Proof of Proposition 5

Plugging in the policy equations (51) and (52) into the system (48)-(49) gives the 2-by-2 system in $\hat{\pi}_t$ and $\hat{b}_t$

$$\left(\phi_\pi + \eta\frac{b}{y}\right)\hat{\pi}_t + \nu_t^i = \hat{\pi}_{t+1} + \eta\left(\left(1-\psi_b\right)\hat{b}_{t-1} + \nu_t^g\right), \tag{P.1}$$

$$\hat{b}_t = \frac{R}{1+g}\left(\left(1-\psi_b\right)\hat{b}_{t-1} + \nu_t^g\right) + \frac{b}{y}\left(\left(\phi_\pi - \frac{R}{1+g}\right)\hat{\pi}_t + \nu_t^i\right). \tag{P.2}$$

The system (P.1)-(P.2) can be written in matrix form

$$A \begin{bmatrix} \hat{\pi}_t \\ \hat{b}_{t-1} \end{bmatrix} = \begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{b}_t \end{bmatrix} + B \begin{bmatrix} \nu_t^i \\ \nu_t^g \end{bmatrix}, \tag{P.3}$$

where

$$A = \begin{bmatrix} \phi_\pi + \eta \frac{b}{y} & -\eta(1-\psi_b) \\ \frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right) & \frac{R}{1+g}(1-\psi_b) \end{bmatrix} \tag{P.4}$$

$$B = \begin{bmatrix} -1 & \eta \\ -\frac{b}{y} & -\frac{R}{1+g} \end{bmatrix} \tag{P.5}$$

The economy has a unique bounded solution if and only if the matrix A has one eigenvalue outside the unit circle and one within. Calculating its trace and determinant, its roots are the solution to the quadratic equation

$$P(\upsilon) = \upsilon^2 - \left(\frac{R}{1+g}(1-\psi_b) + \phi_\pi + \eta\frac{b}{y}\right)\upsilon + (1-\psi_b)\phi_\pi\left(\frac{R}{1+g} + \eta\frac{b}{y}\right). \tag{P.6}$$

Necessary Condition: If the economy has a unique equilibrium, then one root is outside the unit circle and one is inside. If so, the roots are necessarily real (complex roots have the same modulus). If roots are real, they are both positive, so having one inside the unit circle and the other outside is equivalent to having one root greater than one and the other smaller than one. In turn, this is equivalent to $P(1) < 0$, which is condition (53).

Sufficient Condition: Conversely, if condition (53) is satisfied, then it means $1 - tr + det < 0$. In this case the determinant of the quadratic equation $\Delta = tr^2 - 4det > (det+1)^2 - 4det = (det-1)^2 > 0$, so both roots are positive. Given that condition (53) is satisfied, it means one root is greater than one and the other outside. Hence there is a unique equilibrium.

When there is a unique equilibrium, let υ be the root of A that is greater than one and let $(1, -\mu)$ be its left eigenvector. The constant μ can be solved to be

$$\mu = \frac{\eta(1-\psi_b)}{\upsilon - \frac{R}{1+g}(1-\psi_b)}. \tag{P.7}$$

Since $\upsilon > \frac{R}{1+g}(1-\psi_b)$, the constant μ is positive. Note that the expression for μ is ill-defined in

the case of fiscal dominance of the Ricardian case, $\eta = 0$ and $\upsilon = \frac{R}{1+g}(1-\psi_b)$. In this case, the constant μ can be expressed as

$$\mu = \frac{\phi_\pi - \frac{R}{1+g}(1-\psi_b)}{\frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right)}. \tag{P.8}$$

Equation (P.3) implies

$$\upsilon(\hat{\pi}_t - \mu\hat{b}_{t-1}) = (\hat{\pi}_{t+1} - \mu\hat{b}_t) + \left(\left(\mu\frac{b}{y} - 1\right)\nu_t^i + \left(\eta + \mu\frac{R}{1+g}\right)\nu_t^g\right). \tag{P.9}$$

Iterating forward gives equations (57).

Finally, we show that in the particular case $\psi_b = 0$, the impact effect of fiscal shocks on inflation is the same as in the Ricardian case. When $\psi_b = 0$, we have that the two roots of the polynomial (P.6) are ϕ_π and $\upsilon = \frac{R}{1+g} + \eta\frac{b}{y}$. It follows that $\mu = 1/(b/y)$. Replacing the expression for μ in equation (57) gives

$$b_{t-1} - \frac{b}{y}\hat{\pi}_t = -\sum_{k=0}^{\infty}\left(\frac{1}{\frac{R}{1+g} + \eta\frac{b}{y}}\right)^k \nu_{t+k}^g, \tag{P.10}$$

which implies that in particular $\frac{\partial \pi_t}{\partial \nu_t^g} = \frac{1}{\frac{b}{y}}$.

Q Proof of Proposition 6

Equations (P.1)-(P.2) are now

$$\left(\phi_\pi + \eta\frac{b}{y}\right)\hat{\pi}_t + \nu_t^i = \hat{\pi}_{t+1}, \tag{Q.1}$$

$$\hat{b}_t = \left(\frac{R}{1+g} + \frac{b}{y}\eta\right)\left(\left(1-\psi_b\right)\hat{b}_{t-1} + \nu_t^g\right) + \frac{b}{y}\left(\left(\phi_\pi - \frac{R}{1+g}\right)\hat{\pi}_t + \nu_t^i\right). \tag{Q.2}$$

The system (Q.1)-(Q.2) can be written as in equation (P.3) but replacing matrix A with the matrix

$$A' = \begin{bmatrix} \phi_\pi + \eta\frac{b}{y} & 0 \\ \frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right) & \left(\frac{R}{1+g} + \frac{b}{y}\eta\right)(1-\psi_b) \end{bmatrix} \tag{Q.3}$$

The two roots of A' are $\phi_\pi + \eta\frac{b}{y}$ and $\left(\frac{R}{1+g} + \frac{b}{y}\eta\right)(1-\psi_b)$, from which the result follows.