Regulating the Trade in Cap-and-Trade*

Jian Sun† Bart Zhou Yueshen‡

December 3, 2025

Abstract

We demonstrate that regulating the trade in cap-and-trade systems is as critical as enforcing an appropriate cap. Our model operates in an environment where firms can undertake socially costly actions to mitigate the cap-imposed regulatory costs of emission, rendering the first best inevitably unattainable. However, efficiency losses can be partially regained if policymakers *limit* firms' participation in exchanging emissions allowances, for example by raising trading fees or charging stamp duties. Such trade restrictions can further stimulate greater investment in abatement technologies.

Keywords: carbon trading, cap-and-trade, ETS, abatement technology

*This paper benefits tremendously from helpful discussions with Bo Becker, Dion Bongaerts, Jean-Edouard Colliard, Hans Degryse, Robin Döttling, Naveen Gondhi, Shiyang Huang, Joshua Mollner, Peter Koudijs, Yan Xiong, Liyan Yang, and Junyuan Zou; and the participants at Rotterdam School of Management, Central University of Finance and Economics, Lee Kong Chian School of Business. Southwestern University of Finance and Economics, 1st Finance Theory Group Asian Meeting (CUHK, 2024), and 9th SAFE Market Microstructure Conference, Ronn Toh Jun Ming and Tan Wan Yun contributed as research assistants to the early-stage works of this research. There are no competing financial interests that might be perceived to influence the analysis, the discussion, and/or the results of this article.

†Jian Sun (jiansun@smu.edu.sg) is affiliated with Lee Kong Chian School of Business, Singapore Management University, 50 Stamford Road, 178899 Singapore.

‡Bart Zhou Yueshen (b@yueshen.me) is affiliated with Lee Kong Chian School of Business, Singapore Management University, 50 Stamford Road, 178899 Singapore.

1 Introduction

Cap-and-trade systems have become increasingly popular over the past two decades and are now among the most prominent regulatory tools for reducing greenhouse gas emissions worldwide. For example, the European Union Emissions Trading System (EU ETS), launched in 2005, now covers approximately 40% of total EU emissions; the joint cap-and-trade program between California and Quebec, established in 2013, covers around 76% of their combined emissions; and China's national ETS, launched in 2021, currently covers about 60% of the country's emissions (ICAP, 2025). These are the largest cap-and-trade systems globally, but many others exist, including the New Zealand ETS, Korea ETS, and UK ETS.

In a nutshell, a cap-and-trade system stipulates the following compliance requirements: firms must monitor and report their emissions, the verified number of which must be met with sufficient carbon allowances (CAs) holding by the end of each compliance cycle. The regulator controls the total supply—the *cap*—of CAs and then distributes them to firms directly or through auctions. Firm can then *trade* their CAs to balance the above compliance requirements and their production needs.

Current discourse around cap-and-trade systems tends to focus primarily on the "cap": how it should be set in view of information frictions (Karp and Traeger, 2024), regulator's commitment to it (Biais and Landier, 2022), the overlapping policies across sectors (Perino, Ritz, and Van Benthem, 2019), and whether market reserves might help its stability (Borghesi et al., 2023). Instead, the "trade" aspect has received comparatively little attention, and this paper aims to fill the gap. In particular, we examine the extent of flexibility firms should have in exchanging CAs, the interaction between trade and cap regulations, and how CA trading affects investment in abatement technology.

Our main result is that regulating firms' trading of CAs, in addition to adjusting the cap, can be a fruitful policy avenue to improve efficiency. More concretely, we first present a benchmark where the cap-and-trade system (or the ETS for short) can achieve the first best. We then note that as soon as firms take costly private actions in response to the ETS compliance requirements, the first best can no longer

be achieved. Perhaps surprisingly, *limiting* firms' access to the carbon market (e.g., by increasing trading costs) may partially recover the lost efficiency. Limiting trading can further incentivize firms to invest in abatement technology. To the best of our knowledge, our work is the first to highlight the importance of regulating CA trading on the overall effectiveness of cap-and-trade systems.[1]

We now describe our model in detail and offer some intuition for our findings. Section 2 sets up a no-surprise baseline, where firms are subject to the ETS compliance requirement and can trade CAs. The analysis replicates the seminal ETS mechanism by, e.g., Montgomery (1972). Specifically, negative externality arises from firms' production because individually they fail to recognize the environmental damages of their emissions. Through the ETS, the planner requires firms hold sufficient CAs to offset their emission, and the market price of CAs therefore enters firms' optimization as their regulatory emissions costs. The planner then adjusts the supply of CAs, i.e., the cap, to align this regulatory cost with the social (marginal) cost of emissions, achieving the first best.

Section 3 introduces a realistic friction to the above baseline: in response to the ETS compliance requirement, firms have incentives—and the means—to engage in certain socially wasteful private actions to reduce their regulatory costs. Consider the following two concrete examples. First, firms may seek to shift production outside the ETS's regulatory scope, a prevalent concern known as "carbon leakage" (Aichele and Felbermayr, 2015), to avoid the ETS-imposed CA purchase costs. This strategy requires duplicate investments in infrastructure and/or supply chains to enable production beyond the ETS' reach, resulting in socially costly redundancy. Second, firms may misreport their emissions in view of the ETS compliance requirements, as reported by S&P Global Commodity Insights (2022); Reuters (2024). The regulator therefore must exert socially costly efforts to police firms' reporting, and such costs are ultimately born by the misreporting firms (e.g., via fines).[2]

[1] To sharpen our contribution, we narrow our analysis to how firms trade CAs in the secondary carbon markets, like the spot market of EU Allowance (EUA), the (physically settled) derivatives exchanges, and firms' internal carbon markets. We do not study CAs' primary market trading, like the EUA auctions (see, e.g., Hepburn et al., 2006), in which the CAs pass from the regulators to firms.

[2] It is worth emphasizing that the private actions are more general than the outright misconducts discussed above. Even ostensibly benign firm behavior, in response to the ETS, may lead to socially wasteful firm actions. Appendix A provides a concrete example where firms excessively minimize emissions uncertainty or measurement errors.

Because of firms' socially wasteful actions in response to ETS compliance requirement, we demonstrate that the first best is no longer implementable. This is because the first best would require firms not only to recognize their emissions costs but also refrain from *any* wasteful private actions. These two objectives are in direct conflict: as the planner tightens the cap, the implied regulatory emissions cost (the CA price) rises, but firms become increasingly motivated to mitigate it, thus incurring more wasteful actions. Consequently, with only one policy tool—the cap—the planner must balance the two conflicting objectives and accept a compromise between emissions deterrence and the avoidance of firms' wasteful actions.

We then demonstrate in Section 4 that the planner, facing the above friction, can enhance efficiency by regulating the "trade" component of cap-and-trade as an additional policy tool. We study the simplest form of trading regulation by letting the planner choose the fraction of firms that can access the CA market. That is, while all firms are still subject to the ETS compliance requirement, only a fraction of them can trade CAs.[3] Our main finding is that the planner can optimally *limit* firms' access to the CA market, or equivalently charge a non-trivial trading cost, to improve efficiency and, in particular, to lower overall emissions.

While it may seem intuitive that additional regulatory tools could enhance welfare, it is perhaps less immediately clear why *limiting* trading could be beneficial. To shed light on the mechanism, it is instructive to first understand how CAs are used in the presence of private actions. When a firm obtains an extra unit of CA, it only increases its emissions by a fraction $\alpha \in (0, 1)$. To see what happens to the remaining $1 - \alpha$ units, recall that the firm always takes some private action to reduce the regulatory emissions cost. The addition of CA, therefore, allows the firm to relieve such costly private action. That is, the $1 - \alpha$ units of CAs are used to substitute some existing private action (e.g., reducing the firm's carbon leakage or misreporting).

Now consider two otherwise identical firms, differing only in their abatement technology: the

[3] As a concrete implementation, we show in Appendix B that such a regulation can be achieved by properly adjusting firms' trading costs, such as exchange fees, stamp duties, etc.

*h*igh-emission one emits more than the *ℓ*ow-emission one for every marginal unit of production. When trading is allowed, intuitively and realistically, the cleaner ℓ-firm will want to sell its CAs to h-firm.[4] For every unit of CA traded, the ℓ-firm reduces its emissions by α_ℓ, while the h-firm increases by α_h. Importantly, in equilibrium, $\alpha_h > \alpha_\ell$. This is because, when the ETS is binding, the same marginal unit of production by an h-firm requires it to, for example, leak much more to be ETS-compliant than an ℓ-firm. Hence, the h-firm finds it more effective to use the acquired unit of CA in production, resulting in $\alpha_h > \alpha_\ell$. As a result, the aggregate emissions across firms increase with trading volume (at the rate of $\alpha_h - \alpha_\ell > 0$), exacerbating the environmental concern. Reversely, therefore, limiting trading reduces total emissions. Though at the cost of aggregate output, the reduction of emissions can still improve overall welfare, when the emissions externality is severe.

Section 5 allows firms to endogenously invest in their abatement technologies. Our model therefore speaks to the empirical debate regarding whether, and how, ETS curates innovation in clean technology (Kerr and Newell, 2003; Taylor, 2012; Calel and Dechezleprêtre, 2016). We find that limiting trading can actually incentivize firms' investment. Consequently, the planner might again want to limit firms' access to the CA market to maximize welfare. The intuition is as follows. A firm can comply with the ETS constraint by either buying more CAs or investing more into its abatement technology. In this sense, trading and technology investment are substitutes, but only the latter is socially beneficial from the planner's point of view. Therefore, by properly restricting trading, the planner can incentivize more firm investments and improve welfare.

Recent evidence applauds the success of ETS in curtailing greenhouse gas emission (e.g., Grubb et al., 2022; Colmer et al., 2024). Our results above add to such findings that, given the realistic friction that firms want to and can undo the ETS-imposed regulatory costs, the success of ETS could have been enhanced with the implementation of robust trading regulations. While regulators are paying

[4] For example, in 2023, Tesla, a leading electric vehicle company, reached record-high sales of $1.79 billion from carbon credits (CarbonCredits.com, 2023). One of the largest buyers of Tesla's carbon credits was Fiat Chrysler Automobiles (FCA), now part of Stellantis, which spent $2.4 billion on purchasing credits from Tesla between 2019 and 2021 (Sinbon, 2021). FCA has struggled with emission requirements; in 2019, FCA agreed to pay $305 million to resolve allegations of cheating on emission tests and failing to disclose unlawful defeat devices (U.S. Environmental Protection Agency, 2019).

close attention to the operation of carbon markets (see, e.g., ESMA, 2022, 2024), we argue that such monitoring should go beyond trading itself and consider also the impacts on firms' emissions and other reactions to ETSs. The trading of CAs is a multi-dimensional objective, and our analysis has only examined it from the arguably simplest angle of trading participation. Future research should explore other aspects of CA trading, drawing from existing studies on the operation of (financial) markets, and examine empirically the connection between trading design and the ETS effectiveness.

Contribution and related literature

Our paper contributes to several strands of the literature. First and foremost, we add to the debate regarding the design, the implementation, and the revision of the current cap-and-trade systems. Karp and Traeger (2024) discuss the design a "smart cap" to endogenously respond to firms' unobservable technology shocks. Biais and Landier (2022) highlight that the regulator faces a commitment problem in setting future caps (see also Acemoglu and Rafey, 2023 for an analysis on carbon tax). Montero (2005) emphasizes the implementation challenges of measuring and monitoring emission, as well as the associated cost of enforcement under cap-and-trade programs. Perino, Ritz, and Van Benthem (2019) study the overlapping environment policies across sectors, focusing on the so-called "waterbed effect," where the reduction of one sector's emission might be offset by that of another sector. The literature also debated the EU ETS's market stability reserve, which started operation in 2019 aiming to smoothen the CA supply (i.e., the cap); see, e.g., a review by Borghesi et al. (2023). Our analysis contributes to the discussion that regulating firms' trading is of equal importance and complements the cap design.

Second, as one concrete manifestation, our key friction, that firms can take private actions to mitigate or to avoid regulatory costs imposed by the cap-and-trade system, can be thought of as carbon leakage, i.e., firms' incentives to shifting pollution-intensive operations to less regulated regions. The risk of carbon leakage has been pointed out since Kyoto Protocol; see, e.g., Babiker (2005) among many others. Fischer and Fox (2012) study and compare various policies that might combat emission

leakages across borders. Böhringer, Rosendahl, and Storrøsten (2017) propose rebating and emission tax policies that are robust to carbon leakage. We emphasize that our model does not specifically address carbon leakage; and rather, we take it as *one of* the examples of our key friction. Indeed, as we discuss in Appendix A, there are other seemingly benign firm actions that nevertheless inflict socially costly redundancy. Also important is that while the private actions underlying our key friction help firms reduce their ETS-imposed regulatory costs, they do not reduce emissions per se. This differentiates our work from, e.g., Akey et al. (2024), who study how firms might take actions to voluntarily reduce carbon emission and how carbon trading might hurt that incentive.

Third, our model further speaks to how cap-and-trade systems might incentivize firms' technology investments. Goulder and Schneider (1999) study induced technological change (ITC) and highlight the different impacts of emission policies across different industries. Popp (2004) builds endogenous technological changes into the DICE model of Nordhaus (1994) to allow for induced technology innovation. Acemoglu et al. (2012) emphasize that in addition to regulations that control emissions, research subsidies should be used to direct R&D to green innovations. More generally, environmental regulations may affect firms' broader innovations, i.e., the Porter Hypothesis (1991); see Ambec et al. (2013) for a review. Our novelty lies in that the trade in cap-and-trade, not just the cap, can be—and perhaps should be—regulated to incentivize firms' investment in abatement technologies.

Fourth, there is a growing body of finance literature studying the role of (secondary market) equity trading in firms' environmental impacts. Goldstein et al. (2024) study how investors' green preferences affect firms' cost of capital, focusing on the information aggregation function of financial markets. Such preferences also shape how firms' adoption of green technology, as examined by Huang and Kopytov (2024). Piccolo, Schneemeier, and Bisceglia (2023) emphasize that concentration of green investors has market power and financing externalities on other firms. Given that such *financial* markets have nontrivial impacts on firms' production, emissions, and technology adoption, the effects from *carbon* market trading—our novel angle—should be more direct.

$t = 0$	$t = 1$	$t = 2$
Social planner: • allocates CAs to firms • regulates CA trading, *only in §4–5*	Firms: • invest in technology, *only in §5* • technology shocks occur	Firms: • produce output • trade CAs • take private actions, *only in §3–5*

Table 1: Timeline and overview. This timeline table gives an overview of the full model. The planner designs the ETS in $t = 0$, and the firms react accordingly in $t = 1$ (technology investment) and $t = 2$ (production, trading CAs, and taking private actions).

2 Baseline

This sections presents a baseline setting to highlight the firms' negative externality in their laissez-faire production, a utilitarian social planner's first best, and how an ETS can achieve the first best.

Model Setup 1: Timing. There are three periods (without discounting). In $t = 0$, the social planner designs the ETS. Then in $t = 1$, the firms' types realize. Finally, in $t = 2$, the firms trade CAs and produce. Table 1 provides an overview for both this baseline and subsequent extensions. ▲

Model Setup 2: Firms. A continuum of firms, indexed by $i \in [0, 1]$, produce the same numéraire good called "dollars" in $t = 2$. They have the same production technology: to produce y_i (≥ 0) dollars of output, a firm i incurs a cost of $k(y_i) = \frac{k}{2} y_i^2$.[5]

A firm i also emits $e_i(y_i) = e_i y_i$ units of pollutants, where we interpret the coefficient e_i (> 0) as the firm i's emission intensity, determined by some technology shocks at $t = 1$, and we refer to e_i as the firm's type. The types $\{e_i\}$ are i.i.d., randomly drawn at $t = 1$ from a two-type distribution:

$$\mathbb{P}[e_i = \ell] = \phi \in (0, 1) \text{ and } \mathbb{P}[e_i = h] = 1 - \phi, \text{ with } h > \ell > 0. \tag{1}$$

A firm with $e_i = h$ ($e_i = \ell$) is called a high-emission (low-emission) firm. The parameter ϕ is interpreted as the (average) level of the abatement technology. ▲

[5] The production technology can be alternatively modeled as $f(r_i)$, where $f(\cdot)$ is the production function that transfers a representative resource (e.g., labor) r_i, which can be acquired at a (weakly) convex cost $c(r_i)$, into output. The firm's profit is then $f(r_i) - c(r_i)$, which is equivalent to $y_i - k(y_i)$ if we write $y_i = f(r_i)$ and $k(y_i) = c(f^{-1}(y_i))$.

Remark 1 (Firm heterogeneity). It is important to have heterogeneous firms in our model, for otherwise the identical firms would not trade under the ETS (introduced formally in Section 2.3). While such heterogeneity may arise in many ways, we focus on firms' emission intensity e_i, because this conveniently bridges our analysis of firms' investment in abatement technology later in Section 5. Note that while firms' $\{e_i\}$ are random, the i.i.d. assumption, along with firms being infinitesimal, eliminates aggregate uncertainty in our model.

Model Setup 3: Pollution. The aggregate emission $E := \int_0^1 e_i(y_i)\mathrm{d}i$ inflicts on every firm a pollution damage of $g(E) = \frac{g}{2}E^2$, where g (> 0) measures the severity. We impose the following condition on g:

$$\frac{(1-\phi)(h-\ell)}{\phi\ell^2 + (1-\phi)h^2} \cdot \frac{k}{\ell} < g < \frac{k}{(h-\ell)\ell\phi} \tag{2}$$

to ensure interior production. ▲

Remark 2 (Pollution severity). Condition (2) is motivated by a social planner's tradeoff when designing optimal policies: the aggregate output vs. the pollution damage (detailed below in Section 2.3). If g is too large, the latter can be so severe that the planner wants to shut down the dirtier h-firms. On the other hand, if g is too small, the pollution concern could be too mild to warrant the ETS regulation. For simplicity and for realism, we avoid these corner scenarios by imposing (2), so that all firms in our setting shall be regulated by the ETS, but not to the extent that they need to be shut down. This assumption is consistent with the survey evidence from Martin et al. (2014), who document that managers of firms subject to the EU ETS do expect "downsizing risk" but only moderately.

Remark 3 (Pollution disutility). The functional form of $g(\cdot)$ is inessential for our results, as long as it is strictly monotone increasing and weakly convex in E. We adopt the quadratic form as opposed to the simpler linear $g(E) = gE$ only to emphasize the increasing marginal environmental cost, following, e.g., Nordhaus (2017), who assumes that the damage is quadratic in temperature change.

2.1 The laissez-faire outcome

In this no-regulation scenario, we only need to examine firms' $t = 2$ production decisions. Each firm i chooses its production y_i to maximize

$$y_i - k(y_i) - g(E).$$

Since every firm is small, it has no material impact on the aggregate emission E. Therefore, in choosing y_i, the firm effectively ignores its contribution to the pollution damage $g(E)$, generating negative externality. Under the quadratic $k(\cdot)$, the first-order condition determines the laissez-faire production:

$$1 - \dot{k}(y_i) = 1 - ky_i = 0 \implies y_i = \frac{1}{k},\ \forall i \in [0, 1]. \tag{3}$$

The resulting laissez-faire (LF) aggregate output and emission are

$$\Pi_{\text{LF}} := \int_0^1 (y_i - k(y_i))\mathrm{d}i = \frac{1}{2k} \text{ and } E_{\text{LF}} := \int_0^1 e_i(y_i)\mathrm{d}i = \frac{1}{k}\mathbb{E}[e_i], \tag{4}$$

where $\mathbb{E}[e_i] := \phi\ell + (1 - \phi)h$ follows (1). Given the firms' unity mass, these aggregates are also an average firm's surplus, Π_{LF}, and emission, E_{LF}.

2.2 The first best

The firms' aggregate well-being is:

$$W := \int_0^1 (y_i - k(y_i) - g(E))\mathrm{d}i = \int_0^1 (y_i - k(y_i))\mathrm{d}i - g\left(\int_0^1 e_i y_i \mathrm{d}i\right). \tag{5}$$

The first best is where the firms' productions $\{y_i\}$ jointly maximize welfare W. Specifically, each first-best y_i balances the firm's marginal contributions to the aggregate output $\int_0^1 (y_i - k(y_i))\mathrm{d}i$ and to the pollution damage $g(E)$:

$$\frac{\mathrm{d}W}{\mathrm{d}y_i} = \Big(1 - \dot{k}(y_i) - \dot{g}(E)\dot{e}_i(y_i)\Big)\mathrm{d}i = (1 - ky_i - gEe_i)\mathrm{d}i. \tag{6}$$

Compared to the firm's laissez-faire first-order derivative (3), the correction of $-\dot{g}(E)\dot{e}_i(y_i)$ in (6) accounts for the firm's otherwise ignored emission externality. The second-order derivative is strictly

negative, and so there is a unique first-best production level for each firm i. The first-best outcome is characterized as follows:

Proposition 1 (The first-best outcome). The first-best (1B) aggregate emission and output are

$$E_{1B} = \frac{\mathbb{E}[e_i]}{g\mathbb{E}[e_i^2] + k} < E_{LF} \text{ and } \Pi_{1B} = \frac{1}{2k}\left(1 - \frac{g^2\mathbb{E}[e_i^2]\mathbb{E}[e_i]^2}{\left(k + g\mathbb{E}[e_i^2]\right)^2}\right) < \Pi_{LF}, \tag{7}$$

with $\mathbb{E}[e_i] := \phi\ell + (1-\phi)h$ and $\mathbb{E}[e_i^2] := \phi\ell^2 + (1-\phi)h^2$; and each firm i produces $y_i = \frac{1}{k}(1 - e_i g E_{1B})$.

Note that the upper bound of (2) ensures that the pollution is not too severe, so that both types of firms always produce, i.e., $y_i > 0$ for both $e_i = \ell$ and $e_i = h$.

To achieve the first best, the most direct approach is for the planner to mandate each firm's production y_i according to Proposition 1. If this were feasible, there would be no need to design and implement the ETS, as seen in the real world in the last two decades. To motivate the ETS, therefore, we introduce some frictions in the planner's policymaking.

Model Setup 4: Constrained policymaking. The planner cannot mandate an individual firm i's behavior based on its type e_i. ▲

The constraint can arise from several real-world concerns: First, firms' true types are likely their private information, unobservable to the planner. Second, properly defining the planner's capacity to discriminate firms based on their types can be legally challenging. Third, as firms' types are likely evolving (due, e.g., to their investment in abatement technology), at the time of designing the policy, the planner is simply unable to condition on firms' future types. This last concern for timing is incorporated into the timeline in Table 1, where the planner design the ETS in $t = 0$, before firm types realize in $t = 1$.

2.3 Achieving the first best through an ETS

Despite the constraint on the planner's policy making, the first best can still be achieved via an ETS.

Model Setup 5: The ETS. The ETS has two components, the cap and the trade: In $t = 0$, the planner allocates q_i units of CAs to each firm i. In $t = 2$, each firm i not only produces outputs but also trades the CAs by choosing its demand x_i, taking as given the CA price p, which is determined via market clearing $\int_0^1 x_i \mathrm{d}i = \int_0^1 q_i \mathrm{d}i$. The ETS requires that no firm emits more than its CA holding, i.e.,

$$e_i(y_i) \leq x_i, \tag{8}$$

and so the aggregate emission $E = \int_0^1 e_i(y_i)\mathrm{d}i \leq \int_0^1 x_i \mathrm{d}i = \int_0^1 q_i \mathrm{d}i =: q$, where the aggregate CA supply q is referred to as the ETS cap, and we require $q > 0$. ▲

Remark 4 (Initial allocation). In reality, like under the EU ETS, firms receive their CAs either directly from the planner for free or from participating auctions. We do not distinguish these two methods of distributing CAs and simply refer to the planner's q_i choices at $t = 0$ as her "allocation." The allocation does not affect firms' equilibrium CA holdings because their trading always efficiently redistributes them (Goulder and Parry, 2008; Coase, 1960). Model Setup 4, however, implies that the allocation must be symmetric, i.e., $q_i = q$ for all i.

Remark 5 (Penalty on excess emission). We impose on firms a direct constraint of $e_i(y_i) \leq x_i$: their (ETS-covered) emission cannot exceed the CA holdings. This constraint endogenously arises when the penalty on the (marginal) excess emission, $e_i(y_i) - x_i$, is always more expensive than the market price p of acquiring a (marginal) unit of CA. For example, under the EU ETS, a firm with excess emission must not only recoup the missing CAs, effectively paying the CA price p, and is further subject to a penalty of €100 per ton (subject to inflation adjustment; see Article 16 of EU Directive, 2003/87). The total penalty can therefore be thought of as $p + €100 > p$. Indeed, from 2013 to 2021, only about 1% of the firms under the EU ETS were noncompliant (Marcu et al., 2022).

Bounds on the planner's cap. Although our ultimate objective is to study the optimal implementation of the ETS of the planner, as an intermediate step, it is necessary to analyze the equilibrium behavior of firms under a given ETS, in particular, the cap q. Therefore, when analyzing firm behavior

in $t = 2$ (and later also in $t = 1$ in Section 5), we assume

$$\left(1 - \frac{\ell}{h}\right)\frac{\ell}{k} < q < \frac{\ell}{k}. \tag{9}$$

This assumption echoes (2): we focus on the scenario where the ETS constraint (8) binds, but not too tightly, for all firms. The bounds ensure that the CA supply is neither too scarce nor too abundant, so that firms will curb their production but not to the extent of shutting down ($0 < y_i < y_{\text{LF}}$).

How the ETS achieves the first best. Consider a firm i's optimization problem in $t = 2$:[6]

$$\max_{x_i, y_i, \lambda_i} y_i - k(y_i) - (x_i - q_i)p - \lambda_i \cdot (e_i(y_i) - x_i), \tag{10}$$

where $\lambda_i \geq 0$ is the Lagrangian coefficient on the ETS constraint of (8), and we interpret it as the (marginal) *ETS-implied* emission cost. The first-order condition with respect to x_i is[7]

$$-p + \lambda_i = 0 \implies \lambda_i = p;$$

that is, because of the possibility to trade CAs, the regulatory emission cost λ_i is always aligned with the CA price p. The first-order derivative with respect to y_i is

$$1 - \dot{k}(y_i) - \lambda_i \dot{e}_i(y_i) = 1 - \dot{k}(y_i) - p\dot{e}_i(y_i),$$

which aligns with the planner's (6) in the first best, if $p = \dot{g}(E_{1\text{B}})$, i.e., if the CA price reflects the social marginal cost of emission in the first best. Indeed, the planner can carefully choose its the cap q (the aggregate CA supply) in $t = 0$ to achieve $p = \dot{g}(E_{1\text{B}})$ and, hence, the first best:

Proposition 2 (Achieving the first best through an ETS). Given an ETS cap q satisfying (9), the firm chooses $y_i = \frac{1}{k}(1 - e_i p)$ and $x_i = e_i y_i$, where the market price of CA is $p = \frac{\mathbb{E}[e_i] - qk}{\mathbb{E}[e_i^2]}$. The ETS constraint (8) binds for all firms with $\lambda_i = p > 0$. The first-best outcomes (7) are achieved if

[6] For clarity, the timeline in Table 1 sequentially lists the firm choices: y_i, x_i, and a_i (only after Section 3). However, the exact sequence in which a firm makes these choices does not matter, and below we solve them simultaneously.

[7] This first-order condition is *necessary* for optimality, because we admit $x_i \in \mathbb{R}$, which in particular means that firms may hold negative positions of CAs. While the EU ETS does not allow so, it is planned that the European Commission shall report to the European Parliament "how negative emissions [...] could be covered by emissions trading" (Article 1, (28)(d) of EU Directive, 2023/959). Indeed, if a firm does not emit but absorbs carbon dioxide in its production, i.e., if $e_i(y_i) < 0$, the corresponding ETS requirement of $e_i(y_i) \leq x_i$ would incentivize the firm to sell CAs until $x_i = e_i(y_i) < 0$.

and only if the planner chooses the aggregate CA supply to be $q := \int_0^1 q_i \mathrm{d}i = E_{1\mathrm{B}}$.

Three observations from the proposition are worth highlighting. First, that the CA price $p > 0$ is guaranteed by the upper bound of (9), so that $\lambda_i = p > 0$, which means that the ETS constraint (8) binds for all firms. Likewise, $y_i > 0$ is guaranteed by the lower bound of (9). The required cap $q = E_{1\mathrm{B}}$ also satisfies (9), thanks to (2).

Second, the reason that the planner can simply set $q = E_{1\mathrm{B}}$ to implement the first best is because the ETS constraint $e_i(y_i) = x_i$ is binding for all firms, so that the total CA supply is also the aggregate emission: $E = \int_0^1 e_i(y_i)\mathrm{d}i = \int_0^1 x_i \mathrm{d}i = q$, where the last equality holds by market clearing.

Third, note that the (marginal) regulatory cost $\lambda_i = p$ is the same for all firms. Hence, instead of the ETS, the first best can also be achieved via carbon tax: the planner can impose a flat tax rate at p dollars per unit of emission for all firms. Unsurprisingly, absent additional friction (e.g., aggregate uncertainty as in Weitzman, 1974), the planner can either regulate the supply q (the cap) or the price p (via carbon tax) to discipline firms' production. Next in Section 3 we shall see that neither ETS nor carbon tax achieves the first best when firms have incentive to "uncap" the cap (8) of the ETS.

3 Firms' incentive to uncap the ETS cap

Section 2 has demonstrated that the planner can achieve the first best via an ETS by imposing a proper cap q, without mandating firms' behaviors based on their types. We recognize that this is a standard result from seminal works by Montgomery (1972) and Roberts and Spence (1976), for example.

This section introduces a novel, realistic friction: firms have incentives to avoid or to mitigate the ETS-implied regulatory emissions costs—to "uncap" the cap constraint (8). To the extent that the effort of doing so is socially wasteful, we show that the ETS can no longer achieve the first best. This is because, as we explain in detail below, the CA cap as the only policy tool cannot simultaneously minimize firms' wasteful actions and align their emission costs with the pollution damage—the planner is unable to kill two birds with one stone. We concretize this intuition by augmenting the baseline

model as follows.

Model Setup 6: Private actions. A firm i can take a private action a_i (≥ 0) in $t = 2$, incurring a quadratic *deadweight* cost $c(a_i) = \frac{c}{2}a_i^2$, to relax the ETS constraint (8) to

$$e_i(y_i) - a_i \leq x_i. \tag{11}$$

To emphasize, the firm's actual emission remains as $e_i(y_i)$ and is unaffected by the *private* action a_i, which is unobservable by the planner. ▲

Remark 6 (Interpretation of a_i). The private action a_i can represent a number of not mutually-exclusive firm behaviors. One concrete example is carbon leakage. Aichele and Felbermayr (2015) provide evidence that Kyoto Protocol has induced committed countries to increase their carbon emboided imports from non-committed countries.[8] Critical for our result is that the costs of such actions, $c(a_i)$, are deadweight social losses. Indeed, redirecting production outside the ETS reach implies duplicating infrastructure and logistics costs, and such investments are likely socially unnecessary, especially considering the loss of the economies of scale. We note that in addition to firms' foul plays like carbon leakage, there are also seemingly innocuous and even environment-benign actions that can lead to similar deadweight losses, and we discuss them in Appendix A. While the ETS constraint (8) can also be relaxed by lowering the actual emissions $e_i(y_i)$, we defer such a possibility to Section 5, where firms' (ex-ante) investments in abatement technology can lower their emissions.

[8] Other more recent evidence of carbon leakage include, for example, Känzig, Marenz, and Olbert (2024), who document leakage from Europe to Africa; Ben-David et al. (2021), who evidence leakage from strict to weak policy countries; and Bartram, Hou, and Kim (2022), who show leakage within the US. states. While some studies, for example, by Colmer et al. (2024), find little evidence of carbon leakage in Phase I and II of the EU ETS, the review by Grubb et al. (2022) argues that this is largely due to "the generous free allocations [of CAs] with the express aim of preventing leakage" and that the literature evidence "does not rule out the possibility of higher leakage [...] in the absence of free allocation." Indeed, the Phase III and IV of the EU ETS have steered away from free allocation, targeting an eventual 100% pahse-out by 2034 (EU Parliament, 2022). Carbon leakage therefore may still pose as a threat to the success of the EU ETS. Last but not least, carbon leakage also manifests in how firms disguise their emission in less or unregulated greenhouse gases, as reported by Financial Times (2024).

3.1 Firm behavior and the trading equilibrium

Each firm i, taking the CA price p as given, solves

$$\max_{a_i, x_i, y_i, \lambda_i} y_i - k(y_i) - c(a_i) - (x_i - q_i)p - \lambda_i \cdot (e_i(y_i) - a_i - x_i), \tag{12}$$

which generalizes (10) by allowing for the private action a_i.

Proposition 3 (The trading equilibrium). Given an ETS cap q satisfying (9), the CA price is $p = \frac{c\mathbb{E}[e_i] - cqk}{k + c\mathbb{E}[e_i^2]} > 0$, and each firm i faces a binding emission constraint (11) with the same Lagrange multiplier $\lambda_T = p > 0$, produces $y_T(e_i, p) = \frac{1}{k}(1 - e_i p) \in \left(0, \frac{1}{k}\right)$, takes private action $a_T(p) = \frac{p}{c} > 0$, and holds $x_T(e_i, p) = e_i(y_T(e_i, p)) - a_T(p)$ units of CAs.

Note that we subscript the above equilibrium objects with "T" to emphasize that they are obtained when firms can trade. (This is to differentiate against the no-trading ("N") scenario studied later.)

We next highlight some features of this equilibrium. First, a lower c—easier for firms to take private actions—indicates more severe deadweight losses.

Corollary 1 (The special case of no private action). A firm's private action $a_T(p)$ and its cost $c(a_T(p))$ both monotonically *decrease* in c. In particular, when $c \to \infty$, no firm takes any private action, i.e., $a_T(p) \to 0$, there is no deadweight loss, i.e., $c(a_T(p)) \to 0$, and the other equilibrium objects $\{p, y_T(e_i, p), x_T(e_i, p)\}$ converge to those described in Proposition 2.

That is, the setup in this section nests the baseline model in Section 2 as a special case when the private action cost $c \to \infty$. In this limiting case, there is no private action in equilibrium ($a_T(p) = 0$ $\forall i$), and the associated deadweight loss is also eliminated. As c decreases, private actions become more economically attractive, prompting firms to adjust their CA purchase, production, and emission strategies accordingly. Figure 1 illustrates the pattern.

Second, it is natural to expect that the clean ℓ-firms should sell CAs to the dirty h-firms, because the former need not emit as intensely as the latter and would therefore demand less CAs to cover their emission. However, Corollary 2 below caveats that the above heuristic is incomplete, and an additional

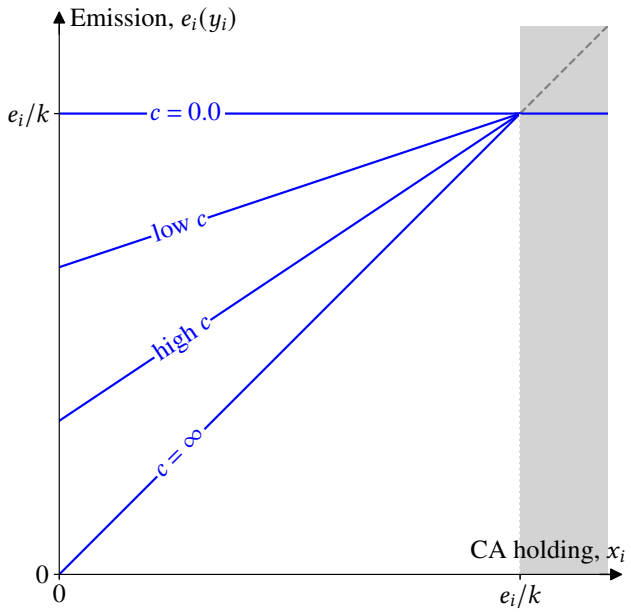


Figure 1: How private actions relax the ETS constraint. This figure illustrates how a firm i's holding x_i of CAs, exogenously varied on the horizontal axis, affects its emission $e_i(y_i)$, shown on the vertical axis, for various levels of costs of private action c. The 45-degree line, which overlaps with the case of $c = \infty$, is the case where the ETS constraint (8) exactly binds. This is the case until $x_i > e_i/k$, the unconstrained optimal amount of emission by the firm (Section 2.1), beyond which the shaded area shows that the ETS constraint is no longer binding. When $c < \infty$, the firm engages in some private action a_i to relax the ETS constraint according to (11), thus raising the emission $e_i(y_i)$ above the 45-degree line; and the distance between is the amount of $a_i = e_i(y_i) - x_i$.

condition is needed.

Corollary 2 (The buying and selling of CAs). The ℓ-firms sell CAs to the h-firms, i.e., $x_T(\ell, p) < x_T(h, p)$, if and only if

$$k > \frac{\ell h c}{1 + (\ell + h)cq}. \tag{13}$$

To see why (13) is needed to ensure ℓ-firms selling CAs to h-firms, consider how the emission intensity e_i affects a firm's CA demand (using the expressions from Proposition 3):

$$\frac{\mathrm{d}x_T}{\mathrm{d}e_i} = y_T(e_i, p) + e_i \frac{\mathrm{d}y_T}{\mathrm{d}e_i} \lesseqgtr 0.$$

The first term $y_T(e_i, p)$ is positive, and it reflects the above heuristic that if a firm emits more, it needs to purchase more CAs to cover its higher emission. The second term is negative ($\frac{\mathrm{d}y_T}{\mathrm{d}e_i} = -\frac{p}{k} < 0$). It is a

countervailing force through production: a higher emission intensity drives down a firm's production and hence also its CA demand. Corollary 2 clarifies that this production channel becomes inconsequential if production is very costly—large k as in (13)—because firms produce relatively little in the first place, and a further reduction in their already limited production has little impact on emissions. We note that ℓ-firms selling CAs to h firms is not only intuitive but also realistic. For example, Abrell, Ndoye Faye, and Zachmann (2011) document that firms in the power sector were the dominant buyers of CAs under the EU ETS. In addition, in 2023, Tesla, a leading electric vehicle company, reached record-high sales of $1.79 billion from carbon credits (CarbonCredits.com, 2023); and one of the largest buyers of Tesla's carbon credits was Fiat Chrysler Automobiles (FCA), which spent $2.4 billion on purchasing credits from Tesla between 2019 and 2021 (Sinbon, 2021).

Third, we introduce some notations for firms' surplus and emission for later analysis. Because of market clearing, an average firm's trading profit (or loss), $(x_{\mathrm{T}}(e_i, p) - q_i)p$, is always zero. Let $\pi_{\mathrm{T}}(e_i, p) := y_{\mathrm{T}}(e_i, p) - k(y_{\mathrm{T}}(e_i, p)) - c(a_{\mathrm{T}}(p))$ denote the firm's surplus excluding CA trading profits. Then an average firm's surplus and emission—when trading—are

$$\begin{aligned} \Pi_{\mathrm{T}}(q) &:= \int_0^1 \pi_{\mathrm{T}}(e_i, p)\mathrm{d}i = \phi\pi_{\mathrm{T}}(\ell, p) + (1-\phi)\pi_{\mathrm{T}}(h, p) \\ E_{\mathrm{T}}(q) &:= \int_0^1 e_i y_{\mathrm{T}}(e_i, p)\mathrm{d}i = \phi\ell y_{\mathrm{T}}(\ell, p) + (1-\phi)h y_{\mathrm{T}}(h, p) \end{aligned} \tag{14}$$

where we note that the price p is a monotone decreasing function in q. The planner's welfare objective, therefore, can be written as

$$W(q) = \Pi_{\mathrm{T}}(q) - g(E_{\mathrm{T}}(q)) = \int_0^1 \big(y_{\mathrm{T}}(e_i, p) - k(y_{\mathrm{T}}(e_i, p)) - c(a_{\mathrm{T}}(p))\big)\mathrm{d}i - g\left(\int_0^1 e_i y_{\mathrm{T}}(e_i, p)\mathrm{d}i\right), \tag{15}$$

which, compared to (5), highlights the deadweight losses $c(a_i)$.

3.2 The first best is no longer achievable

Since the private action is socially wasteful, it only reduces welfare. Formally, $\frac{\mathrm{d}W}{\mathrm{d}a_i} = -\dot{c}(a_i)\mathrm{d}i = -ca_i\mathrm{d}i \leq 0$. Therefore, the first-best outcome requires $a_i = 0$, and the first-best outcome remains the same as in Proposition 1, even when private actions $\{a_i\}$ are feasible. However, the first best is no longer attainable via the ETS in this case.

Proposition 4 (The ETS cap cannot achieve the first best). As long as firms' private actions are feasible, i.e., if $c < \infty$, then $W(q_{\mathrm{T}}^*) < W_{1\mathrm{B}} = \Pi_{1\mathrm{B}} - g(E_{1\mathrm{B}})$, where $q_{\mathrm{T}}^* = \dfrac{\theta(c)\mathbb{E}[e_i]}{g\mathbb{E}[e_i^2] + \theta(c)k}$ is the planner's optimal cap, with $\theta(c) := 1 - \dfrac{g\mathbb{E}[e_i^2]}{k + c\mathbb{E}[e_i^2]}$.

We use the subscript "T" to emphasize that q_{T}^* is the optimal supply "with trading." Note that when private actions become infeasible, i.e., when $c \to \infty$, then $\theta(c) \to 1$ and $q_{\mathrm{T}}^* \to E_{1\mathrm{B}}$ as defined in (7), achieving the first best (Proposition 2).

Proposition 4 highlights the conundrum facing the planner, also illustrated in Figure 2. She has only one policy tool, namely the cap q, to achieve two conditions underpinning the first best: the firms should not emit too much ($e_i(y_i)$ is low; the dot on the solid line) and should not take any private actions ($a_i = 0$; the dot on the dashed line). However, as seen from Proposition 3, firms' production y_i—hence also their emission $e_i(y_i)$—decreases in CA price p but their private action a_i increases in p. Therefore, if the planner lowers the cap q to increase the emission cost p, production and emission will reduce, but only to incentivize the firm to take more wasteful actions a_i to avoid regulatory costs. Therefore, the planner has to accept a compromise between emissions deterrence and the avoidance of firms' wasteful actions, unable to reach the first best. (The same logic applies to why carbon tax also fails: a higher tax rate only reduces emissions but incentivizes more wasteful private actions.)

More formally, suppose the planner aims for no private actions. Given the concavity of the firm objective (12) in a_i, this requires that, in equilibrium,

$$\left.\frac{\mathrm{d}\pi_i}{\mathrm{d}a_i}\right|_{a_i=0} = \Big(-\dot{c}(a_i) + \lambda_i \Big)\Big|_{a_i=0} = \lambda_i \leq 0,$$

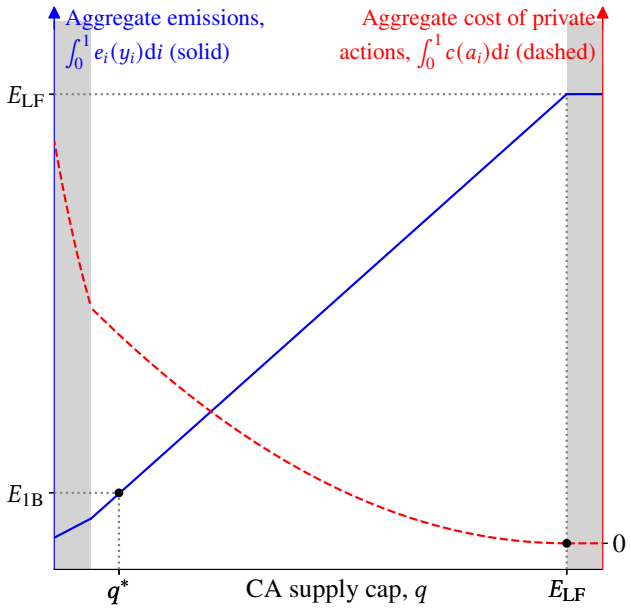


Figure 2: The first best is no longer achievable. This figure illustrates why, when the firms can take private actions to relax the ETS constraint, the first best is no longer achievable by choosing only the optimal CA supply cap q, which is varied exogenously on the horizontal axis. The left and the right shaded areas indicate, respectively, too little or too much CA supply, as discussed in relation to (9). In between, it can be seen that the planner can always lower the cap q to tighten the ETS constraint, raise the CA price p, and lower the aggregate emission (solid line, left axis) until it reaches the first-best level E_{1B}. However, as q lowers, the firms have stronger incentive to undo the tighter and tighter ETS constraint, thereby taking more and more socially wasteful private action (dashed line, right axis). There is not a single q that simultaneously achieves the first-best emissions $E_{1\mathrm{B}}$ and eliminates private actions.

which means that the ETS emission constraint (11) must be non-binding. Second, as shown in Section 2.3, the first best is achieved only if $\lambda_i = p = \dot{g}(E_{1\mathrm{B}}) > 0$, but this is incompatible with the above $\lambda_i \leq 0$. That is, as the planner attempts to kill two birds ($a_i = 0$ and $p = \dot{g}(E_{1\mathrm{B}})$) with only one stone (the ETS cap q), she inevitably fails both. Only if $c \to \infty$, which ensures $a_i \to 0$ (Corollary 1), can the cap q be used to align p with the marginal social cost $\dot{g}(E_{1\mathrm{B}})$, as seen in Proposition 2.

4 Trading as a policy tool

That the first best is not feasible under the ETS suggests room for improving welfare, but how? We approach this question from the *trading* of CAs. Indeed, as the name suggests, a cap-and-trade system

should not be only about the cap q but also about the trading. While there are many aspects of trading, for concreteness, we consider how the planner might regulate firms' access to the CA market, for its ease of implementation and adjustment (see Appendix B for details).

Model Setup 7: Regulating CA trading. The planner can choose in $t = 0$, in addition to the cap q, the fraction $\mu \in [0, 1]$ of firms who can trade:

- When a firm can trade, it behaves the same as in Section 3.1, solving (12).
- When a firm cannot trade, it is constrained by the planner's initial CA allocation of $x_i = q_i$ but can still choose its production y_i and private action a_i accordingly in $t = 1$.

Note that since firms are homogeneous at $t = 0$ (and because the planner cannot observe firms' private types $\{e_i\}$), we assume that the initial allocations are the same $q_i = q$. This symmetry follows Model Setup 4. If the planner can allocate CAs according to firm types (e.g., after they realize in $t = 2$), this additional flexibility will permit further welfare improvement, thus strengthening our call for regulating CA trading. ▲

Remark 7 (Regulating firms' entry to trading). The planner can influence firms' entry probability μ by, for example, imposing trading costs, charging stamp duty, or regulating firms' internal carbon markets, etc. Appendix B provides a concrete microfoundation, in which each firm plays a mixed strategy to endogenously access the market with probability μ, by paying an entry cost set by the planner.

4.1 Firm behavior

Under the trading regulation, each firm has access to the CA market only with probability μ. By $t = 2$, therefore, there are two types of firms: those who can trade ("T") and those who cannot ("N").

With trading. With access to the CA market, T-firms behave the same as in Proposition 3. In particular, because of their restricted access to the market, the market clearing condition becomes $\mu \int_0^1 x_i \mathrm{d}i = \mu q$, and so the equilibrium objects in Proposition 3 are not affected by μ. An average trading firm's $\Pi_{\mathrm{T}}(q)$ and $E_{\mathrm{T}}(q)$, as given by (14), remain only as functions of q but not of μ,

No trading. An N-firm takes the initial allocation $q_i = q$ as given and solves

$$\max_{a_i, y_i, \lambda_i} y_i - k(y_i) - c(a_i) - \lambda_i \cdot (e_i(y_i) - a_i - q), \tag{16}$$

which is similar to (12) but with $x_i = q$, because it cannot trade.

Proposition 5 (The no-trading firms). Given an ETS cap q satisfying (9), each non-trading firm i produces $y_{\mathrm{N}}(e_i, q) = \dfrac{1 + qce_i}{k + ce_i^2}$, takes private action $a_{\mathrm{N}}(e_i, q) = e_i(y_{\mathrm{N}}(e_i, q)) - q = \dfrac{e_i - qk}{k + ce_i^2}$, and faces a binding emission constraint (11) with Lagrange multiplier $\lambda_{\mathrm{N}}(e_i, q) = ca_{\mathrm{N}}(e_i, q) > 0$.

To distinguish with the trading firms' equilibrium behavior (Proposition 3), we subscript the equilibrium objects above with "N." Similar to (14), we can define a type-e_i firm's surplus as $\pi_{\mathrm{N}}(e_i, q) = y_{\mathrm{N}}(e_i, q) - k(y_{\mathrm{N}}(e_i, q)) - c(a_{\mathrm{N}}(e_i, q))$, so that an average firm's surplus and emission are

$$\begin{aligned} \Pi_{\mathrm{N}}(q) &:= \phi \pi_{\mathrm{N}}(\ell, q) + (1 - \phi)\pi_{\mathrm{N}}(h, q) \\ E_{\mathrm{N}}(q) &:= \phi \ell y_{\mathrm{N}}(\ell, q) + (1 - \phi) h y_{\mathrm{N}}(h, q) \end{aligned} \tag{17}$$

which are again only functions of the initial allocation q but not of μ, because firms, unable to trade, optimize without interacting with each other, making μ irrelevant.[9]

4.2 Main result

In such an economy, an average firm's surplus is $\int_0^1 (y_i - k(y_i) - c(a_i))\mathrm{d}i = \mu\Pi_{\mathrm{T}}(q) + (1 - \mu)\Pi_{\mathrm{N}}(q)$, while its emission is $E = \mu E_{\mathrm{T}}(q) + (1-\mu)E_{\mathrm{N}}(q)$. The planner then chooses μ and q to maximize welfare

$$W(\mu, q) = \mu\Pi_{\mathrm{T}}(q) + (1 - \mu)\Pi_{\mathrm{N}}(q) - g\big(\mu E_{\mathrm{T}}(q) + (1 - \mu)E_{\mathrm{N}}(q)\big). \tag{18}$$

As a special case, the optimal q_{T}^* found in Proposition 4 is the solution when $\mu = 1$, i.e., when all firms are trading, hence the subscript "T." We next state our main result.

[9] The "no trading" scenario may appear similar to the planner's "command and control (CAC)" that each firm shall emit no more than q units of pollution. It should be noted, however, such CAC only applies to an endogenous fraction of $1 - \mu$ of the firms. The literature has argued that market-based approaches, like cap-and-trade and ETS, are superior compared to CAC (Ellerman, 2003; Schmalensee and Stavins, 2017), suggesting that the optimal μ is $\mu = 1$. Our analysis below, however, highlights that the planner possibly should go for a blend, i.e., $\mu < 1$, between firms that can and cannot access the CA market. This nuanced result arises from the novel friction studied Section 3.

Proposition 6 (Optimal trading restriction). Let (μ^*, q^*) maximize (18). There exist thresholds $\underset{\sim}{g}$ and $\hat{c}$ (given in the proof), such that the optimal $\mu^* < 1$ if and only if $g > \underset{\sim}{g}$ and $c < \hat{c}$.

That is, the planner may want to restrict firms' access to the CA market to improve welfare, and this happens when the pollution externality is severe (high g) and when firms prevalently take private actions (low c).

4.3 Discussion

Below we provide a heuristic discussion regarding the main result, Proposition 6. The first thing to note is that restricting trading always hurts a firm's surplus: $\Pi_{\mathrm{T}}(q^*) > \Pi_{\mathrm{N}}(q^*)$, which follows the First Welfare Theorem. Therefore, for trading restrictions ($\mu^* < 1$) to improve welfare W, there must be significant emission reduction, i.e., $E_{\mathrm{N}}(q^*) < E_{\mathrm{T}}(q^*)$.[10] How does this happen?

Substitution between CA holdings and private action. Recall from Propositions 3 and 5 that the ETS constraint (11) always binds, i.e., $e_i(y_i) - a_i = x_i$, and hence $\mathrm{d}e_i(y_i) = \mathrm{d}x_i + \mathrm{d}a_i$. This is because, intuitively, a firm's additional emission $\mathrm{d}e_i(y_i)$ must be met by a combination of acquiring more CAs ($\mathrm{d}x_i$) and taking more private action ($\mathrm{d}a_i$)—x_i and a_i are substitutes.

Because of such substitution, an increase in a firm's CA holding x_i will result in (i) more production and more emission, i.e., higher $e_i(y_i)$; and (ii) lower private action, i.e., lower a_i. This can be seen from, e.g., the lines of "low c" or "high c" in Figure 1: as x_i increases, the emissions increase at a rate less than one—the lines are flatter than the 45-degree line. That is, only a fraction of the marginal unit of CA is used to expand production and emission as it allows, while the rest is used to offset the costly private actions. More concretely, from the firm's first-order condition with respect to y_i, $1 - \dot{k}(y_i) - \lambda_i \dot{e}_i(y_i) = 0$, we have, by the implicit function theorem,

$$\alpha_i := \frac{\mathrm{d}e_i(y_i)}{\mathrm{d}x_i} = \frac{\dot{e}_i(y)^2 \ddot{c}(e_i(y) - x)}{\dot{e}_i(y)^2 \ddot{c}(e_i(y) - x) + \ddot{e}_i(y)\dot{c}(e_i(y) - x) + \dddot{k}(y)} = \frac{e_i^2 c}{e_i^2 c + k} \in (0, 1);$$

[10] More formally, $W(1, q^*) - W(\mu^*, q^*) < 0$ means that $0 < (1 - \mu^*)\big(\Pi_{\mathrm{T}}(q^*) - \Pi_{\mathrm{N}}(q^*)\big) < g\big(E_{\mathrm{T}}(q^*)\big) - g\big(\mu^* E_{\mathrm{T}}(q^*) + (1 - \mu^*)E_{\mathrm{N}}(q^*)\big)$, which, given that $g(\cdot)$ is increasing, requires $E_{\mathrm{N}}(q^*) < E_{\mathrm{T}}(q^*)$.

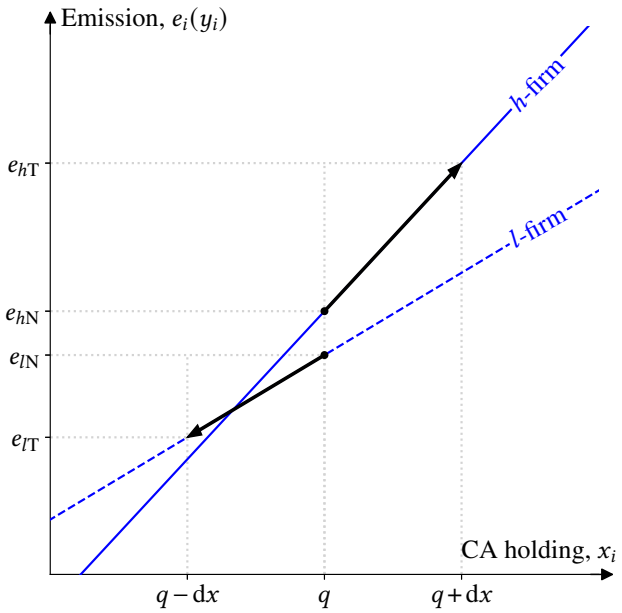


Figure 3: CA trading can lead to more emissions. This figure illustrates how total emissions might increase when an ℓ-firm sells CAs to an h-firm. The two firms' emission-to-CA lines (as in Figure 1) are plotted. They are both initially endowed with q units of CA. Without trading, the two firms produce and emit $e_{\ell\mathrm{N}}$ and $e_{h\mathrm{N}}$, respectively. When the ℓ-firm sells $\mathrm{d}x$ units to the h-firm, their emissions diverge to $e_{\ell\mathrm{T}}$ and $e_{h\mathrm{T}}$, respectively. Because the h-firm's emission is more sensitive to its CA holdings (steeper line), the trade raises total emissions.

and, hence complementarily, $\frac{\mathrm{d}a_i}{\mathrm{d}x_i} = \alpha_i - 1 = -\frac{k}{e_i^2 c + k} \in (-1, 0)$.

***h*-firms' emission is more sensitive than ℓ-firms' to their CA holdings.** Importantly, the sensitivity α_i is higher for the dirtier h-firms than for the cleaner ℓ-firms:

$$\alpha_h := \left.\frac{\mathrm{d}e_i(y_i)}{\mathrm{d}x_i}\right|_{e_i=h} = \frac{h^2 c}{h^2 c + k} > \frac{\ell^2 c}{\ell^2 c + k} = \left.\frac{\mathrm{d}e_i(y_i)}{\mathrm{d}x_i}\right|_{e_i=\ell} =: \alpha_\ell.$$

Intuitively, this is because h-firms' production is, by definition, more expensive in terms of emissions: for each unit of the production, the h-firm requires more allowances to comply with the cap.

Such disparity in the sensitivity is key to the understanding of why ETS trading might lead to higher emission. Following Corollary 2, consider the case where an h-firm buys one unit of CA from an ℓ-firm. The h-firm's emission will increase by $\frac{h^2 c}{h^2 c + k}$, which is more than the reduction of $\frac{\ell^2 c}{\ell^2 c + k}$ by the ℓ-firm, therefore resulting in a net increase in emission. While CA trading is zero-sum, because of the heterogeneous emission sensitivities of the firms, the resulting emission change is not, as illustrated in

Figure 3. The following corollary gives states this result formally.

Corollary 3 (Emission with vs. without trading). Fix an exogenous cap q. An average non-trading firm's emission is lower than an average trading firm's, i.e., $E_{\mathrm{N}}(q) < E_{\mathrm{T}}(q)$, if and only if ℓ-firms sell CAs to h-firms when they trade, i.e., if and only if (13) holds.

Private action must be affordable. Proposition 6 further requires $c < \hat{c}$. This is to achieve two purposes. First, as Corollary 3 notes, $E_{\mathrm{N}}(q) < E_{\mathrm{T}}(q)$ only if ℓ-firms sell to h-firms, i.e., only if (13) holds. In fact, (13) can be rewritten as an upper bound of c, and $c < \hat{c}$ ensures so.

Second, observe that if private action is very costly, the emission-to-CA sensitivity

$$\alpha_i \xrightarrow{c\to\infty} 1,$$

irrespective of the emission intensity e_i. Intuitively, if private action is prohibitive, the firms can only use CAs to meet the ETS requirement (8), matching CAs and emission one-to-one. In this case, CA transfers among firms is equivalent to transferring emissions rights, and the emission increase from an h-firm is exactly offset by the reduction in an ℓ-firm. The upper bound $\hat{c}$ in Proposition 6, therefore, also ensures that c is sufficiently low so that the above substitution effect is strong enough.

Pollution must be severe. Lastly, Proposition 6 emphasizes that the pollution severity g cannot be too low. This is because the planenr is ultimately trading off lower surplus against lower emission. Intuitively, if g is too low, meaning pollution is not a significant concern for the planner, the benefit of restricting trading—lower emissions—will be limited. Restricting trading in this case mainly hurts firm surplus and therefore is suboptimal.

5 Trading and technology investment

So far in our model the abatement technology—represented by the parameter ϕ—is exogenously given. In reality, cap-and-trade systems have nontrivial impacts on firms' innovation activities. While some empirical evidence suggests that they do not encourage or may even reduce innovation in clean tech-

nology (Kerr and Newell, 2003; Taylor, 2012), other more recent studies document contrasting results (Calel and Dechezleprêtre, 2016; Calel, 2020). This section contributes to this debate from the novel "trade" angle of cap-and-trade. Specifically, we endogenize the ϕ in Model Setup 2 as follows:

Model Setup 8: Firms' technology investments. At the beginning of $t = 1$, each firm i chooses $\phi_i \in [0, 1]$ by incurring an investment cost of $z\phi_i$, where z (> 0) is an exogenous parameter reflecting how costly such R&D is. The firm's emission intensity then realizes at the end of $t = 1$ to be $e_i = \ell$ with probability ϕ_i and $e_i = h$ otherwise. We will look for a symmetric-strategy equilibrium where all firms choose the same ϕ, in line with the two-type firm distribution (1). ▲

Before proceeding with our analysis, we introduce two parameter assumptions. First, we are interested in the equilibrium where firms are heterogeneous in $t = 2$ so that there are gains from trading CAs. That is, the equilibrium ϕ needs to be interior, which is guaranteed by the following:

$$\frac{k - gh\ell}{k + gh^2} < \zeta < \frac{k - gh\ell}{k + g\ell^2}, \text{ where } \zeta := \sqrt{1 - \frac{2k(h + \ell)z}{h - \ell}} \in [0, 1]. \tag{19}$$

Since ζ is monotone decreasing in z, the above effectively imposes upper and lower bounds for the technology cost z. Intuitively, the upper (lower) constraint of ζ—a sufficiently cheap (expensive) technology z—bounds ϕ away from 1 (from 0).

Second, because the previously exogenous ϕ is now endogenous, we replace (2) with:

$$\frac{c}{1 + \frac{(h+\ell)\ell c}{(1-\zeta)k}} < g < \frac{c}{1 + \frac{(h^2-\ell^2)\ell c}{(1-\zeta)hk}}, \tag{20}$$

to ensure firms' interior production. That is, we focus on the scenario where all firms shall be regulated but not to the extent that they need to be shut down (see Remark 2).

5.1 Firms' problems

A firm i's behavior in $t = 2$ remains the same as characterized by Proposition 3 if it can trade and by Proposition 5 if it cannot trade. The only caveat is that if it can trade, its behavior depends on the CA

price p, which in turn depends on

$$\mathbb{E}[e_i] = \phi_{-i}\ell + (1-\phi_{-i})h \text{ and } \mathbb{E}[e_i^2] = \phi_{-i}\ell^2 + (1-\phi_{-i})h^2,$$

where ϕ_{-i} is the symmetric technology level chosen by all other firms at the beginning of $t = 1$. As such, we write the p from Proposition 3 as $p(\phi_{-i}, q)$. Therefore, before her type e_i realizes, following (14), a trading firm with technology ϕ_i expects a surplus of

$$\Pi_{\mathrm{T}}\big(\phi_i, p(\phi_{-i}, q)\big) := \phi_i \pi_{\mathrm{T}}(\ell, p(\phi_{-i}, q)) + (1-\phi_i)\pi_{\mathrm{T}}(h, p(\phi_{-i}, q));$$

while a non-trading firm, following (17), expects

$$\Pi_{\mathrm{N}}(\phi_i, q) := \phi_i \pi_{\mathrm{T}}(\ell, q) + (1-\phi_i)\pi_{\mathrm{T}}(h, q).$$

Next, consider the firm's investment ϕ_i at the beginning of $t = 1$. Given the planner's choice of (μ, q) at time 0, a firm i chooses its ϕ^* to solve:

$$\phi^* = \arg\max_{\phi_i \in [0,1]} \mu\Pi_{\mathrm{T}}\big(\phi_i, p(\phi_{-i}, q)\big) + (1-\mu)\Pi_{\mathrm{N}}(\phi_i, q) - z\phi_i \tag{21}$$

Since we consider symmetric equilibria, we also require

$$\phi^* = \phi_{-i}. \tag{22}$$

We then obtain the following result regarding the equilibrium level of abatement technology:

Lemma 1 (Endogenous abatement technology). Given any μ and q chosen by the planner, there exists a unique symmetric level of abatement technology $\phi^*(\mu, q)$ that solves (21) and (22). Moreover, $\phi^*(\mu, q)$ is decreasing in μ if and only if $\phi^*(1, q) \geq \frac{1}{1+\sqrt{\frac{k+c\ell^2}{k+ch^2}}}$.

Lemma 1 provides a theoretical explanation that might help reconcile the mixed empirical findings regarding how cap-and-trade systems affect innovation in abatement technology (Taylor, 2012; Calel, 2020). Notably, we see that introducing trading regulation (lowering μ) on top of a given cap might improve technology innovation. This happens when the innovation level under the cap-only, full-trading system is already high. With full trading ($\mu = 1$) and if $\phi^*(1, q)$ is very high from firms'

investments in $t = 1$, many will become ℓ-type and sell CAs, and few h-type will buy, in $t = 2$. If putatively the planner restricted trading ($\mu < 1$) in $t = 0$, h-firms in $t = 2$ would be significantly hurt because they would no longer have access to such a buyer's market. They would wish that they had invested more in the abatement technology in $t = 1$. On the other hand, ℓ-firms in $t = 1$ would not be very much affected, because of the buyer's market. On average, therefore, restricting trading in $t = 0$ incentivizes firms to invest in $t = 1$.

5.2 The planner's problem.

Given the firms' optimal investment $\phi^*(\mu, q)$, the planner at $t = 0$ understands that the equilibrium level of emission will be

$$E = \mu E_{\mathrm{T}}\big(\phi^*(\mu,q), p(\phi^*(\mu,q),q)\big) + (1-\mu)E_{\mathrm{N}}\big(\phi^*(\mu,q),q\big),$$

where $E_{\mathrm{T}}(\cdot)$ and $E_{\mathrm{N}}(\cdot)$ follow (14) and (17), respectively, and we write them in addition as functions of the now endogenous $\phi^*(\mu, q)$. The planner then chooses μ and q to maximize welfare, which is the average firm's objective (21) less the pollution damage $g(E)$:

$$W(\mu,q;\phi^*) = \mu\Pi_{\mathrm{T}}\big(\phi^*, p(\phi^*,q)\big) + (1-\mu)\Pi_{\mathrm{N}}\big(\phi^*,q\big) - z\phi^* - g\big(\mu E_{\mathrm{T}}\big(\phi^*, p(\phi^*,q)\big) + (1-\mu)E_{\mathrm{N}}\big(\phi^*,q\big)\big),$$

where we abbreviate the dependence of ϕ^* on μ and on q. Denote the solution by $(\mu^*_{\mathrm{A}}, q^*_{\mathrm{A}})$, where we use the subscript "A" to emphasize that these are equilibrium objects when Abatement technology is endogenous. The main result from this section is as follows.

Proposition 7 (Optimal trading restriction, with technology investment). There exist thresholds $\hat{z}_{\mathrm{A}}$ and $\hat{c}_{\mathrm{A}}$ (given in the proof), such that the optimal $\mu^*_{\mathrm{A}} < 1$ if $c < \hat{c}_{\mathrm{A}}$ and $z < \hat{z}_{\mathrm{A}}$.

To see the intuition, note that restricting trading has two effects on welfare W:

$$\frac{\mathrm{d}W}{\mathrm{d}\mu} = \frac{\partial W}{\partial \mu} + \frac{\partial W}{\partial \phi}\frac{\partial \phi}{\partial \mu}.$$

The direct effect, $\frac{\partial W}{\partial \mu}$, has been our focus in Section 4, and the main result Proposition 6 highlights that

when the private action cost is low, $\frac{\partial W}{\partial \mu} < 0$, i.e., it is better to restrict trading to improve welfare. This is guaranteed by $c < \hat{c}_{\mathrm{A}}$ in this proposition. (Note that $\hat{c}_{\mathrm{A}}$ differs from the $\hat{c}$ in Proposition 6 due to the endogenous ϕ.)

The indirect effect, $\frac{\partial W}{\partial \phi}\frac{\partial \phi}{\partial \mu}$, works through firms' endogenous investment ϕ in abatement technology. Intuitively, a better abatement technology always benefits environment and hence welfare, implying $\frac{\partial W}{\partial \phi} > 0$. Lemma 1 has shown that restricting trading may increase ϕ, i.e., $\frac{\partial \phi}{\partial \mu} < 0$, especially when the full-trading level of abatement technology is already high. The condition $z < \hat{z}_{\mathrm{A}}$—technology investment being affordable—guarantees this result and, hence, also that $\frac{\partial W}{\partial \phi}\frac{\partial \phi}{\partial \mu} < 0$.

In summary, as both the direct and the indirect effects can be negative in firms' accessibility to CA trading, the planner may find it optimal to restrict trading, i.e., setting $\mu^*_{\mathrm{A}} < 1$. Adding to Proposition 6, this result highlights a novel channel of limiting CA trading: it can incentivize firms' investment in abatement technology.

6 Conclusion

This paper underscores the pivotal role of regulating the "trade" of cap-and-trade systems, an area that has received less attention compared to the setting of the "cap." We begin by introducing a realistic and prevalent friction to a standard cap-and-trade or ETS framework: firms have incentive to take socially wasteful actions to mitigate or to avoid regulatory costs imposed by the ETS. We show that such a friction prevents policymakers from achieving the first-best outcome solely through cap adjustments. Instead, regulating firms' trading of CAs can significantly enhance overall welfare.

In particular, contrary to conventional wisdom, our findings suggest that *reducing* firms' access to the carbon market—e.g., by raising exchange trading fees—can improve the efficiency tradeoff between productivity and environmental damages. Our analysis further reveals that restricting CA trading may also incentivize firms to invest in abatement technologies. Again, such additional investment leads to a reduction in emissions that might not be attainable through cap adjustments alone.

By highlighting the importance of trading regulations, we contribute a novel perspective to the discourse on cap-and-trade systems, emphasizing that effective environmental policy must consider both the cap and the mechanisms of trade to optimize outcomes. We encourage further research into how the trade in cap-and-trade should be designed and regulated, what other trading frictions are there, and how trading efficiency relates to the overall production-environment efficiency. Such investigations would further inform the development of more effective policies aimed at reducing greenhouse gas emissions while balancing economic efficiency.

Appendix

A A benign view of firms' private actions

We have primarily interpreted a firm's private action a_i as carbon leakage (Model Setup 6). However, the friction that results in deadweight losses and prevents the first-best outcome is more general, including some benign firm activities in response to the ETS. As a concrete example, we draw from two additional real-world features of ETS (and ignore the possibility of leakage).

(i) Excess emission penalty: The EU ETS technically allows excess emission but imposes a flat per-ton penalty of €100, so that it incurs a marginal cost of $p + €100$ (Remark 5).

(ii) Emission uncertainty: Firms do not know how much they will emit with full certainty. Intrinsic production fluctuation aside, the scrutiny by accredited verifiers and other regulatory bodies inflicts additional risks (see, e.g., Bundesrepublik Deutschland v. Nordzucker AG, 2015).

These two features lead to the following form of *expected* penalty, given a firm i's production y_i and CA holding x_i:

$$(p + €100) \cdot \mathbb{E}[\max\{0, e_i(y_i) + \varepsilon_i - x_i\}]$$

where ε_i represents the firm's idiosyncratic, unpredictable randomness in emission. For concreteness,

assume ε_i is normally distributed with mean zero and variance τ_i^{-1}. Then

Lemma A.1 (Expected penalty). A firm i's expected penalty $(p+€100)\cdot\mathbb{E}[\max\{0, e_i(y_i) + \varepsilon_i - x_i\}]$ is a function of the mean an the precision of the emissions: $f(e_i(y_i) - x_i, \tau_i)$, where $f(\cdot,\cdot)$ is strictly increasing (decreasing) in the first (second) argument.

Therefore, the firm will have incentive to reduce the *uncertainty* τ_i^{-1} of its emission to lower the expected penalty.[11] Suppose it can do so by choosing $a_i = \tau_i$ at a convex cost $c(a_i)$. Then the firm objective becomes $y_i - k(y_i) - (x_i - q_i)p - f(e_i(y_i) - x_i, a_i) - c(a_i) - g(E)$. On the other hand, the planner's welfare objective is

$$\int_0^1 (y_i - k(y_i) - c(a_i))\mathrm{d}i - g\left(\int_0^1 e_i(y_i)\mathrm{d}i\right), \tag{A.1}$$

because the expected penalty $f(\cdot)$ is transfer from firms to the planner and because the firms' trading profit $(x_i - q_i)p$ always aggregates to zero, and so neither enters the objective.

It can be seen that firms' seemingly benign private actions of reducing their idiosyncratic emission uncertainty leads to the same welfare objective as seen earlier in (15). In other words, from the planner's point of view, such efforts are, again, wasteful. This is because, the planner can not care less about the *diversifiable* idiosyncratic noises in firms' emissions—the resulting fines are transfers. She would like the firms to spend their effort in reducing emission levels, not the dispersions. Each individual firm, however, cares very much about its own emission's dispersion, because of the convexity in the penalty structure of $\max\{0, e_i(y_i) + \varepsilon_i - x_i\}$.

In summary, there are frictions other than firms' carbon leakage that also prevent the ETS from implementing the first best. The overarching economic force is that firms have incentive to take private actions to uncap the cap, i.e., to reduce the ETS-imposed regulatory costs. As long as such private actions lead to deadweight social costs, the first best is not achievable via an ETS with only the cap in the planner's policy menu: Whenever she lowers the cap, the regulatory emission cost goes up, thus

[11] The firm's incentive here is to reduce the regulatory cost $f(\cdot)$ by reducing the second moment of its total emission $e_i(y_i) + \varepsilon_i$. To compare, in our earlier analysis in Section 3, the firm's incentive can be thought of as to reduce the first moment of the ETS-covered emission from $e_i(y_i)$ to $e_i(y_i) - a_i$.

incentivizing firms to take more private actions, raising deadweight losses.

B Firms' endogenous entry to trading

Section 4 assumes that the planner can directly choose firms' access rate μ to the CA market (Model Setup 7). This appendix illustrates that this can be equivalently achieved by letting the planner set the market access fee f and by letting firms endogenously choose whether to trade. Specifically, we make the following two changes to Model Setup 7:

- At $t = 0$, instead of μ, the planner chooses the market access fee f (in addition to the cap q).
- At the beginning of $t = 1$, each firm i choose whether to pay f for the later access to the CA market in $t = 2$ by choosing the probability $\mu_i \in [0, 1]$ of paying the fee.[12]

Below we look for a symmetric strategy equilibrium, where all firms choose the same $\mu_i = \mu$, and show that the equilibrium μ decreases in f. This way, the planner can choose f to control firms' access to the CA market. The same holds when firms endogenously invest in abatement technology as in Section 5, but for simplicity this appendix takes ϕ as exogenously given.

The planner. The planner chooses both q and f to maximize the same welfare objective (18), where f determines firms' endogenous access rate μ. Note that f does not enter welfare because firms' access fees are transfers. This means that the optimal solution (q^*, μ^*) remains the same as in Proposition 6, and the planner only needs to choose the right f to implement $\mu(f) = \mu^*$.

Suppose the planner takes μ as given. Her optimal choice of the cap q is determined by

$$q(\mu) = \arg\max_q \mu\Pi_{\mathrm{T}}(q) + (1-\mu)\Pi_{\mathrm{N}}(q) - g\big(\mu E_{\mathrm{T}}(q) + (1-\mu)E_{\mathrm{N}}(q)\big),$$

where the objective follows (18). It is easy to verify that the problem is strictly concave and, therefore,

[12] We assume that the firm needs to pay the access fee f before its emission intensity e_i realizes. This way, firms are homogeneous and their choices of μ_i do not depend on e_i. The analysis is similar if firms make their market entry decisions after seeing their e_i, although the fractions of ℓ- and h-firms in the market become endogenous.

the first-order condition with respect to q uniquely determines the (interior) solution $q(\mu)$:

$$\mu\dot{\Pi}_{\mathrm{T}}(q) + (1-\mu)\dot{\Pi}_{\mathrm{N}}(q) - \dot{g}\Big(\mu E_{\mathrm{T}}(q) + (1-\mu)E_{\mathrm{N}}(q)\Big)\Big(\mu\dot{E}_{\mathrm{T}}(q) + (1-\mu)\dot{E}_{\mathrm{N}}(q)\Big) = 0, \tag{B.1}$$

and it has the following property:

Lemma B.1. Suppose (13) holds (so that ℓ firms sell CAs to h firms). Then the planner's cap decreases in firms' market access rate. That is, the $\dot{q}(\mu) < 0$ as implied by (B.1).

Firms. The firms take the ETS cap q and the access fee f as given. If a firm pays the fee, it can trade in $t = 2$ and expects $\Pi_{\mathrm{T}}(q) - f$ following Proposition 3. If a firm does not pay the fee, it does not trade in $t = 2$ and expects $\Pi_{\mathrm{N}}(q)$ following Proposition 5. Therefore, the firm's optimal μ_i choice is:

$$\mu_i = \begin{cases} 1, & \text{if } \Pi_{\mathrm{T}}(q) - f > \Pi_{\mathrm{N}}(q); \\ \in [0,1], & \text{if } \Pi_{\mathrm{T}}(q) - f = \Pi_{\mathrm{N}}(q); \\ 0, & \text{if } \Pi_{\mathrm{T}}(q) - f < \Pi_{\mathrm{N}}(q). \end{cases} \tag{B.2}$$

Clearly, if $\Pi_{\mathrm{T}}(q) - f \neq \Pi_{\mathrm{N}}(q)$, all firms will choose the same $\mu \in \{0, 1\}$. If instead $\Pi_{\mathrm{T}}(q) - f = \Pi_{\mathrm{N}}(q)$, then they are indifferent, and we assume that they will choose the same common μ. We shall write this symmetric solution as $\mu(q, f)$.

Lemma B.2. Suppose (13) holds (so that ℓ firms sell CAs to h firms). Then the surplus gap between a trading and a non-trading firm widens with the cap. That is, $\frac{\mathrm{d}}{\mathrm{d}q}\big(\Pi_{\mathrm{T}}(q) - \Pi_{\mathrm{N}}(q)\big) > 0$.

Intuitively, firms always prefer being able to trade (the First Welfare Theorem). In particular, with a more relaxed ETS, i.e., when q increases (within the bounds of (2)), the benefit of exchanging CAs becomes even higher, thus widening the above gap.

Equilibrium. The equilibrium is a fixed point at which the planner's optimal $q(\mu)$, as in (B.1), is consistent with the firms' symmetric access rate $\mu = \mu(q, f)$, as in (B.2). In particular, when the firms are indifferent of its access to trading, the following must hold: $\Pi_{\mathrm{T}}(q(\mu)) - \Pi_{\mathrm{N}}(q(\mu)) = f$. Thanks to Lemmas B.1 and B.2, therefore, $\dfrac{\mathrm{d}\mu}{\mathrm{d}f} = -\dfrac{(-1)}{\big(\dot{\Pi}_{\mathrm{T}}(q(\mu)) - \dot{\Pi}_{\mathrm{N}}(q(\mu))\big)\dot{q}(\mu)} < 0$; that is, the equilibrium

μ must be monotone decreasing in the fee f. Finally, note that if $f \to 0$, then $\Pi_{\mathrm{T}}(q) - f > \Pi_{\mathrm{N}}(q)$ always holds, resulting in $\mu = 1$, whereas if $f \to \infty$, then $\Pi_{\mathrm{T}}(q) - f < \Pi_{\mathrm{N}}(q)$ always holds (since both firm surpluses are obviously bounded), resulting in $\mu = 0$. Therefore, there exist unique thresholds $0 < \underset{\sim}{f} < \hat{f} < \infty$ such that firms' symmetric equilibrium access rate μ satisfies $\mu(f) = 1$ if $f < \underset{\sim}{f}$; $\mu(f) = 0$ if $f > \hat{f}$; and $\mu(f)$ monotonically decreases from 1 to 0 as f increases on $[\underset{\sim}{f}, \hat{f}]$. The planner, therefore, can choose f to adjust firms' access rate μ to the optimal μ^* given in Proposition 6.

C Proofs

Proposition 1

Proof. The planner mandates every firm's production y_i according to the first-order condition of setting (6) to zero. This yields $y_i = \frac{1}{k}\left(1 - gEe_i\right)$. In aggregate, the total emission is $E = \int_0^1 e_i y_i \mathrm{d}i = \frac{1}{k}(1 - gE\mathbb{E}[e_i])$, solving which yields the $E_{1\mathrm{B}}$ stated in (7). Directly evaluating (5) and adding back $g(E_{1\mathrm{B}})$, we get the $\Pi_{1\mathrm{B}}$ in (7). Finally, the inequalities in (7) can be verified by directly comparing with (4). □

Proposition 2

Proof. The proof proceeds in the following steps.

Conjecture. We begin by conjecturing that each firm i chooses interior production, i.e., $y_i \in (0, \frac{1}{k})$, where the upper bound is the laissez-faire level y_{LF} shown in (4); and that $\lambda_i > 0$, i.e., the ETS constraint (8) always binds. We then verify these below.

Firm behavior. Following the optimization problem (10), each firm i solves its first-order conditions (taking the price p as given): $1 - ky_i - \lambda_i e_i = 0$, $-p + \lambda_i = 0$, and $-e_i y_i + x_i = 0$. The linear system has a unique solution of $\{x_i, y_i, \lambda_i\}$ as stated in the proposition. The solution is both sufficient and necessary for the optimality because the objective function is concave (quadratic) and because of the above conjecture. The market clearing condition $\int_0^1 x_i \mathrm{d}i = q$ then determines the equilibrium price p,

as stated in the proposition. Direct computation of the equilibrium aggregate emission E and the aggregate output Π verifies that the first-best outcome (7) is achieved if $q = E_{1B}$.

Verifying the conjecture. We now verify that $0 < y_i < \frac{1}{k}$ and that $\lambda_i = p > 0$. To do so, we directly use the above closed-form expressions of y_i and λ_i. In particular, y_i is decreasing in p, which in turn is monotone decreasing in q. Thanks to such monotonicity, it suffices to evaluate y_i and $\lambda_i = p$ at the bounds of q as assumed in (9). Doing so verifies the conjecture.

Planner choice of q. Now we examine how the planner chooses the optimal cap q to maximize welfare

$$W = \int_0^1 (y_i - k(y_i) - (x_i - q_i)p)\mathrm{d}i - g(E),$$

where $E = \int_0^1 e_i(y_i)\mathrm{d}i$ and $\{x_i, y_i, p\}$ are interior solutions out of firms' optimizations as stated in the proposition. Therefore, directly computing the first-order derivative yields

$$\frac{\mathrm{d}W}{\mathrm{d}q} = \frac{\mathbb{E}[e_i]}{\mathbb{E}[e_i^2]} - \left(\frac{k}{\mathbb{E}[e_i^2]} + g\right)q,$$

which ensures that W is strictly concave in q and the maximum is achieved at $q = E_{1B}$ (> 0). □

Propositions 3 and 4

Proof. The proof proceeds in the following steps.

Conjecture. We begin by conjecturing that each firm i chooses interior production, i.e., $y_i \in (0, \frac{1}{k})$, where the upper bound is the laissez-faire level y_{LF} shown in (4); that private action $a_i > 0$; and that $\lambda_i > 0$, i.e., the ETS constraint (11) always binds.

Firm behavior. The firm objective (12) is quadratic in $\{a_i, y_i\}$ and linear in $\{x_i, \lambda_i\}$. It can be verified that it is strictly concave. Then given the conjecture, the first-order conditions, $-ca_i+\lambda_i = 0$, $-p+\lambda_i = 0$, $1 - ky_i - \lambda_i e_i = 0$, and $-e_i y_i + a_i + x_i = 0$, form a linear equation system that determine the unique solution to (12) as stated in Proposition 3. The market clearing condition $\int_0^1 x_i\mathrm{d}i = q$ determines the equilibrium price p, also as stated.

Verifying the conjecture. To do so, we directly use the above closed-form expressions of $y_i = y_{\mathrm{T}}$, $\lambda_i = \lambda_{\mathrm{T}}$, and $a_i = a_{\mathrm{T}}$. In particular, y_{T} is decreasing in p, which in turn is monotone decreasing in q. Thanks to such monotonicity, it suffices to evaluate y_{T}, $\lambda_{\mathrm{T}} = p$, and a_{T} at the bounds of q as assumed in (9). Doing so verifies the conjecture.

Planner choice of q. The planner's objective $W(q)$ as in (15) can be computed using the closed-form expressions from Proposition 3. Directly computing its first-order derivative with respect to q yields

$$\frac{\mathrm{d}W}{\mathrm{d}q} = \frac{c\mathbb{E}[e_i]((c-g)\mathbb{E}[e_i^2]+k)}{(c\mathbb{E}[e_i^2]+k)^2} - \frac{ck^2 + ck\mathbb{E}[e_i^2] + \left(\mathbb{E}[e_i^2]\right)^2 c^2 g}{(c\mathbb{E}[e_i^2]+k)^2} q,$$

which is monotone decreasing in q. In other words, W is concavely quadratic in q. Therefore, the maximum welfare is reached at q_{T}^*, which uniquely solves the first-order condition $\frac{\mathrm{d}W}{\mathrm{d}q} = 0$, as stated in Proposition 4. Using this q_{T}^*, direct computation also confirms that $W(q_T^*) < W_{\mathrm{1B}}$.

□

Corollary 1

Proof. The statements in the corollary can be directly verified using the closed-form expressions from Proposition 3 by taking the limit of $c \to \infty$. □

Corollary 2

Proof. Using the closed-form expression of $x_{\mathrm{T}}(e_i, p)$ from Proposition 3, we directly compare $x_{\mathrm{T}}(\ell, p)$ and $x_{\mathrm{T}}(h, p)$ at the equilibrium p and obtain that $x_{\mathrm{T}}(\ell, p) < x_{\mathrm{T}}(h, p)$ if and only if (13). □

Proposition 5

Proof. The proof proceeds in the following steps.

Conjecture. We begin by conjecturing that each firm i chooses interior production, i.e., $y_i \in (0, \frac{1}{k})$, where the upper bound is the laissez-faire level y_{LF} shown in (4); that private action $a_i > 0$; and

that $\lambda_i > 0$, i.e., the ETS constraint (11) always binds.

Firm behavior. The firm objective (16) is quadratic in $\{a_i, y_i\}$ and linear in λ_i. It can be verified that it is strictly concave. Then given the conjecture, the first-order conditions (taking q as given), $-ca_i + \lambda_i = 0$, $1 - ky_i - \lambda_i e_i = 0$, and $-e_i y_i + a_i + q = 0$, form a linear equation system that determine the unique solution to (16) as stated in Proposition 5.

Verifying the conjecture. To do so, we directly use the above closed-form expressions of $y_i = y_{\mathrm{N}}$, $\lambda_i = \lambda_{\mathrm{N}}$, and $a_i = a_{\mathrm{N}}$. In particular, y_{N} is increasing and a_{N} is decreasing in q. Thanks to such monotonicity, it suffices to evaluate y_{N}, λ_{N}, and a_{N} at the bounds of q as assumed in (9). Doing so verifies the conjecture. □

Proposition 6

Proof. First note from (18) that $W(\cdot)$ is strictly concave in μ: $\frac{\mathrm{d}^2 W}{\mathrm{d}\mu^2} = -\ddot{g}\big(\mu E_{\mathrm{T}}(q)+(1-\mu)E_{\mathrm{N}}(q)\big)\big(E_{\mathrm{T}}(q)-E_{\mathrm{N}}(q)\big)^2 < 0$. Therefore, for $\mu^* < 1$ to hold, it is both necessary and sufficient to require

$$\left.\frac{\partial W}{\partial \mu}\right|_{\mu=1} = \Pi_{\mathrm{T}}(q_{\mathrm{T}}^*) - \Pi_{\mathrm{N}}(q_{\mathrm{T}}^*) - \dot{g}(E_{\mathrm{T}}(q_{\mathrm{T}}^*))\big(E_{\mathrm{T}}(q_{\mathrm{T}}^*) - E_{\mathrm{N}}(q_{\mathrm{T}}^*)\big) < 0,$$

where we can evaluate the left-hand side of the inequality using the closed-form expressions derived in (14) and (17) at q_{T}^* given in Proposition 4. After simplifying, we obtain $g > \underset{\sim}{g}$ and $c < \hat{c}$ where the thresholds are given by

$$\underset{\sim}{g} := \frac{k}{2(\ell + h)\mathbb{E}[e_i]} \quad \text{and} \quad \hat{c} := \frac{k^2 - 2(\ell + h)\mathbb{E}[e_i]kg\mathbb{1}_{\{g\ell h < k\}}}{(g\ell h - k)\mathbb{E}[e_i^2]}.$$

In the knife-edge case of $g\ell h = k$, we define $\hat{c} = \infty$, i.e., any $c > 0$ together with $g = \frac{k}{\ell h}$ suffices for $\mu^* < 1$. □

Corollary 3

Proof. Using the closed-form expressions from (17) and (14), we obtain

$$E_{\mathrm{N}}(q) - E_{\mathrm{T}}(q) = -\frac{(k - ch\ell + ck(h+\ell)q)(h-\ell)^2(h+\ell)(1-\phi)\phi c}{(k+ch^2)(k+c\ell^2)(k+c\mathbb{E}[e_i^2])},$$

and hence the sign depends on whether $k - ch\ell + ck(h+\ell)q \lesseqgtr 0$, which is indeed equivalent to (13). □

Lemma 1

Proof. Since both $\Pi_T(\phi_i, \phi_{-i}, q)$ and $\Pi_N(\phi_i, \phi_{-i}, q)$ are linear in ϕ_i, the first order derivative of the firm's objective (21) is

$$\begin{aligned} &\mu(\pi_{\mathrm{T}}(\ell,p) - \pi_{\mathrm{T}}(h,p)) + (1-\mu)(\pi_{\mathrm{N}}(\ell,q) - \pi_{\mathrm{N}}(h,q)) - z \\ =&\pi_{\mathrm{N}}(\ell,q) - \pi_{\mathrm{N}}(h,q) + \mu\cdot[(\pi_{\mathrm{T}}(\ell,p) - \pi_{\mathrm{T}}(h,p)) - (\pi_{\mathrm{N}}(\ell,q) - \pi_{\mathrm{N}}(h,q))] - z. \end{aligned} \tag{C.1}$$

The price p on the right-hand side depends on ϕ^* in equilibrium, due to the symmetry $\phi^* = \phi_{-i}$. Note that $\pi_{\mathrm{N}}(\ell,q) - \pi_{\mathrm{N}}(h,q)$ is independent of ϕ^*.

Now let us analyze $\pi_{\mathrm{T}}(\ell,p) - \pi_{\mathrm{T}}(h,p) - (\pi_{\mathrm{N}}(\ell,q) - \pi_{\mathrm{N}}(h,q))$. Under the parameter region we consider (Equation (9)), we have

$$\pi_{\mathrm{T}}(e,p) = y_{\mathrm{T}}(e,p) - \frac{k}{2}y_{\mathrm{T}}(e,p)^2 - \frac{c}{2}(ey_{\mathrm{T}}(e,p) - x_{\mathrm{T}}(e,p))^2 - (x_{\mathrm{T}}(e,p) - q)p.$$

Substitute the expressions for y_{T} and x_{T} from Proposition 3, we get

$$\begin{aligned} \pi_{\mathrm{T}}(e,p) &= \frac{1+(2e-kx_{\mathrm{T}})cx_{\mathrm{T}}}{2(k+ce^2)} - px_{\mathrm{T}} + pq = \frac{1+cex_{\mathrm{T}}}{2(k+ce^2)} + \frac{(e-kx_{\mathrm{T}})cx_{\mathrm{T}}}{2(k+ce^2)} - px_{\mathrm{T}} + pq \\ &= \frac{1}{2(k+ce^2)} + pq + \frac{x_{\mathrm{T}}}{2}\left[\frac{ce}{(k+ce^2)} - p\right] = \frac{1+kcx_{\mathrm{T}}{}^2}{2(k+ce^2)} + pq. \end{aligned}$$

Then

$$\pi_{\mathrm{T}}(\ell,p) - \pi_{\mathrm{T}}(h,p) = \frac{1}{2(k+c\ell^2)} - \frac{1}{2(k+ch^2)} + \frac{kcx_{\mathrm{T}}{}^2(\ell,p)}{2(k+c\ell^2)} - \frac{kcx_{\mathrm{T}}{}^2(h,p)}{2(k+ch^2)}.$$

Besides, using the results from Proposition 5, we know

$$\pi_{\mathrm{N}}(e,q)=\frac{1+(2e-kq)cq}{2(k+ce^2)}.$$

Then

$$\begin{aligned}\pi_{\mathrm{N}}(\ell,q)-\pi_{\mathrm{N}}(h,q)&=\frac{1+(2\ell-ks)cq}{2(k+c\ell^2)}-\frac{1+(2h-kq)cq}{2(k+ch^2)}\\&=\left(\frac{c\ell}{k+c\ell^2}-\frac{ch}{k+ch^2}\right)q+\frac{1-kcq^2}{2}\left(\frac{1}{k+c\ell^2}-\frac{1}{k+ch^2}\right).\end{aligned}$$

Then

$$\begin{aligned}&(\pi_{\mathrm{T}}(\ell,p)-\pi_{\mathrm{T}}(h,p))-(\pi_{\mathrm{N}}(\ell,q)-\pi_{\mathrm{N}}(h,q))\\=&\frac{\frac{kc{x_{\mathrm{T}}}^2(\ell)}{2}}{k+c\ell^2}-\frac{\frac{kc{x_{\mathrm{T}}}^2(h)}{2}}{k+ch^2}-\left[\frac{c\ell q-\frac{kcq^2}{2}}{k+c\ell^2}-\frac{chq-\frac{kcq^2}{2}}{k+ch^2}\right]\\=&\frac{\frac{kc}{2}(x_{\mathrm{T}}(\ell)-q)^2+cs(kx_{\mathrm{T}}(\ell)-\ell)}{k+c\ell^2}-\frac{\frac{kc}{2}(x_{\mathrm{T}}(h)-q)^2+cs(kx_{\mathrm{T}}(h)-h)}{k+ch^2}\\=&\frac{kc}{2}\left[\frac{(x_{\mathrm{T}}(\ell)-q)^2}{k+c\ell^2}-\frac{(x_{\mathrm{T}}(h)-q)^2}{k+ch^2}\right]\\=&\frac{kc}{2}(x_{\mathrm{T}}(h)-q)^2\left[\left(\frac{1-\phi^*}{\phi^*}\right)^2\frac{1}{k+c\ell^2}-\frac{1}{k+ch^2}\right]\\=&\frac{kc}{2}\frac{\left(\frac{1-\phi^*}{\phi^*}\right)^2\frac{1}{k+c\ell^2}-\frac{1}{k+ch^2}}{\left(\frac{1-\phi^*}{\phi^*}\frac{1}{k+c\ell^2}+\frac{1}{k+ch^2}\right)^2}\left(\frac{\frac{h-kq}{k+ch^2}-\frac{\ell-kq}{k+c\ell^2}}{k}\right)^2.\end{aligned}$$

The above expression is a decreasing function of ϕ^*, because the derivative of the above expression with respect to ϕ^* is

$$-\frac{kc}{2}\left(\frac{\frac{h-kq}{k+ch^2}-\frac{\ell-kq}{k+c\ell^2}}{k}\right)^2\frac{2\left(k+ch^2\right)^2\left(k+c\ell^2\right)^2}{\left(k+ch^2(1-\phi^*)+\phi c\ell^2\right)^3}<0.$$

Then Equation (C.1) is a monotonically decreasing function of ϕ. Besides, the right-hand side is increasing in μ if and only if $\left(\frac{1-\phi^*}{\phi^*}\right)^2\frac{1}{k+c\ell^2}-\frac{1}{k+ch^2}$ is positive. Let $\bar{\phi}$ be the solution of

$$\left(\frac{\bar{\phi}}{1-\bar{\phi}}\right)^2=\frac{k+ch^2}{k+c\ell^2},$$

then $(\pi_{\mathrm{T}}(\ell, p) - \pi_{\mathrm{T}}(h, p)) - (\pi_{\mathrm{N}}(\ell, q) - \pi_{\mathrm{N}}(h, q)) \geq 0 \Longleftrightarrow \phi^* \leq \bar{\phi}$.

Based on the above results, if $\phi^*(1, q) \geq \bar{\phi} = \frac{1}{1+\sqrt{\frac{k+c\ell^2}{k+ch^2}}}$, $\phi^*(\mu, q)$ will be decreasing in μ, so $\phi^*(\mu, q) \geq \bar{\phi} = \frac{1}{1+\sqrt{\frac{k+c\ell^2}{k+ch^2}}}$ for all $\mu \in [0, 1]$. Similarly, if $\phi^*(1, q) \leq \bar{\phi} = \frac{1}{1+\sqrt{\frac{k+c\ell^2}{k+ch^2}}}$, $\phi^*(\mu, q)$ will be increasing in μ, so $\phi^*(\mu, q) \leq \bar{\phi} = \frac{1}{1+\sqrt{\frac{k+c\ell^2}{k+ch^2}}}$ for all $\mu \in [0, 1]$. Then we conclude that $\phi^*(\mu, q)$ is decreasing in μ if and only if $\phi^*(1, q) \geq \frac{1}{1+\sqrt{\frac{k+c\ell^2}{k+ch^2}}}$. □

Proposition 7

Proof. For convenience, we introduce the planner's optimal CA supply when choosing $\mu = 1$, denoted as $q^*_{\mathrm{T,A}}$, which solves $\max_q W(1, q; \phi^*(1, q))$. We consider how planner's payoff W changes with μ at $\mu = 1$ and $q = q^*_{T,A}$. Let's conjecture that first, $q^*_{T,A}$ must satisfy Equation (9), so all firms have positive production when trading CA is possible, and all CAs are utilized when trading is not possible; second, the equilibrium ϕ solved by Equation (21) is interior. We will show later that these conjectures are guaranteed by Equation (19) and Equation (20).

We know

$$\left.\frac{dW}{d\mu}\right|_{\mu=1} = W_\phi \left.\frac{d\phi^*}{d\mu}\right|_{\mu=1} + W_\mu\big|_{\mu=1}.$$

To show that $\mu^* < 1$, it is sufficient to show that $W_\phi \left.\frac{d\phi^*}{d\mu}\right|_{\mu=1} + W_\mu\big|_{\mu=1} < 0$.

First, let's assume that $c < \hat{c}_{A,1} = \frac{k}{hl}$, then when $\mu = 1$, the interior ϕ solved by Equation (21) satisfies

$$\phi(q) = \frac{ch^2 + k - \frac{k - chl + ck(h+l)q}{\zeta}}{c(h-l)(h+l)}.$$

Substitute this into the planner's problem, we can get the optimal $q^*_{T,A}$ and $\phi^*(\mu = 1, q^*_{T,A})$ under the case $\mu = 1$ are

$$q^*_{T,A} = \frac{c - g - c\zeta + g\zeta}{cg(h+l)},$$

and

$$\phi^*(\mu = 1, q^*_{T,A}) = \frac{k(1-\zeta) + gh(l + h\zeta)}{g(h-l)(h+l)\zeta}.$$

It's clear that

$$\frac{d\phi^*}{d\zeta} = \frac{k - ghl}{g(h^2 - l^2)\zeta^2}.$$

Equation (19) guarantees that $\frac{d\phi^*}{d\zeta} > 0$, and since ζ is decreasing in z, there must exists $\hat{z}_A$ satisfying

$$\phi^*(\mu = 1, q^*_{T,A}) = \frac{k(1 - \zeta(z)) + gh(l + h\zeta(z))}{g(h - l)(h + l)\zeta(z)} \geq \frac{1}{1 + \sqrt{\frac{k + c\ell^2}{k + ch^2}}}.$$

for all $z \leq \hat{z}_A$. By Lemma 1, we know

$$\left.\frac{\partial \phi^*(\mu, q^*_{T,A})}{\partial \mu}\right|_{\mu=1} \leq 0 \quad \text{when } z \leq \hat{z}_A.$$

Besides, since ϕ^* is an interior solution of Equation (21),

$$W_\phi\big|_{\mu=1} = \left.\frac{\partial \Pi_{\mathrm{T}}}{\partial \phi_{-i}}\right|_{\mu=1} - g'(E)\left.\frac{\partial E}{\partial \phi^*}\right|_{\mu=1}.$$

Lemma C.1. $\frac{\partial \Pi_{\mathrm{T}}}{\partial \phi_{-i}} = 0$.

Proof.

$$\frac{\partial \Pi_{\mathrm{T}}}{\partial \phi_{-i}} = \left[\phi^* \frac{\partial \pi_{\mathrm{T}}(\ell, p)}{\partial p} + (1 - \phi^*)\frac{\partial \pi_{\mathrm{T}}(h, p)}{\partial p}\right]\frac{\partial p}{\partial \phi_{-i}}.$$

Note that

$$\pi_{\mathrm{T}}(e, p) = \frac{1 + kc{x_{\mathrm{T}}}^2}{2(k + ce^2)} + pq^*_{T,A},$$

then

$$\begin{aligned}
\phi^* \frac{\partial \pi_{\mathrm{T}}(\ell, p)}{\partial p} + (1 - \phi^*)\frac{\partial \pi_{\mathrm{T}}(h, p)}{\partial p} =& \frac{\partial \mathbb{E}[\pi_{\mathrm{T}}(e, p)]}{\partial p} \\
=& \mathbb{E}\left[\frac{kcx_{\mathrm{T}}}{(k + ce^2)}\frac{\partial x_{\mathrm{T}}}{\partial p}\right] + q^*_{T,A} \\
=& -\mathbb{E}\left[\frac{kcx_{\mathrm{T}}}{(k + ce^2)}\frac{k + ce^2}{kc}\right] + q^*_{T,A} \\
=& 0.
\end{aligned}$$

So $\frac{\partial \Pi_{\mathrm{T}}}{\partial \phi_{-i}} = 0$. □

The total emission is

$$\begin{aligned} E &= \Pi_{\mathrm{N}} + \mu(\Pi_{\mathrm{T}} - \Pi_{\mathrm{N}}) \\ &= \left(q^*_{T,A} + \frac{h - kq^*_{T,A}}{k + ch^2}\right) + \left[\mu \frac{\frac{ch^2}{k+ch^2} - \frac{c\ell^2}{k+c\ell^2}}{k\left[\frac{1}{k+ch^2}\frac{1}{1-\phi^*} + \frac{1}{\phi^*}\frac{1}{k+c\ell^2}\right]} - \phi^*\right]\left(\frac{h - kq^*_{T,A}}{k + ch^2} - \frac{\ell - kq^*_{T,A}}{k + c\ell^2}\right). \end{aligned}$$

Then

$$\left.\frac{\partial E}{\partial \phi^*}\right|_{\mu=1} = \left[\frac{\left(k + c\ell^2\right)\left(k + ch^2\right)}{\left(k + ch^2(1-\phi^*) + c\ell^2\phi^*\right)^2}\right]\left(\frac{\ell - kq^*_{T,A}}{k + c\ell^2} - \frac{h - kq^*_{T,A}}{k + ch^2}\right).$$

When $c < \hat{c}_{A1} = \frac{k}{h\ell}$, we must have $\frac{\ell - kq^*_{T,A}}{k+c\ell^2} < \frac{h - kq^*_{T,A}}{k+ch^2}$, and thus $\left.\frac{\partial E}{\partial \phi^*}\right|_{\mu=1} < 0$. Since $g'(E)$ is always positive and $\frac{\partial \Pi_{\mathrm{T}}}{\partial \phi_{-i}} = 0$, we must have $W_\phi\big|_{\mu=1} > 0$. In summary, we show that when $c < \hat{c}_{A1} = \frac{k}{h\ell}$, and $z < \hat{z}_A$, we must have $\left.W_\phi \frac{d\phi^*}{d\mu}\right|_{\mu=1} \leq 0$.

Next let's examine W_μ. Similar to our analysis in the proof of Proposition 6, we can show that the partial derivative $W_\mu\big|_{\mu=1}$ is negative if $gk(3\zeta - 2) + c(k - ghl - k\zeta) \leq 0$. Equation (19) ensures that $k - ghl - k\zeta > 0$, then this condition becomes

$$c \leq \hat{c}_{A,2} = \frac{gk(3\zeta - 2)}{ghl + k(\zeta - 1)}.$$

In summary, when $c < \hat{c}_{A2}$, we must have $W_\mu\big|_{\mu=1} < 0$.

Let $\hat{c}_A = \min\{\hat{c}_{A1}, \hat{c}_{A2}\}$, then when $z < \hat{z}_A$ and $c < \hat{c}_A$, we have $\left.W_\phi \frac{d\phi^*}{d\mu}\right|_{\mu=1} \leq 0$ and $W_\mu\big|_{\mu=1} < 0$, and thus

$$\frac{dW}{d\mu} = \Bigg|_{\mu=1} < 0.$$

The last step is to verify that that under Equation (20), $q^*_{T,A}$ must satisfy Equation (9) and under Equation (19), ϕ^* is also interior. It can be shown that

$$\frac{(h-l)l}{hk} < \frac{c(1-\zeta) + g(\zeta - 1)}{cg(h+l)} < \frac{l}{k}$$

is equivalent to Equation (20) and

$$0 < \frac{k(1-\zeta) + gh(l + h\zeta)}{g(h-l)(h+l)\zeta} < 1$$

is equivalent to Equation (19). □

Lemma A.1

Proof. Denote by $m_i := e_i(y_i) - x_i$ as a shorthand for the (expected) amount of excess emission. Then

$$f(m_i, \tau_i) = (p + 100) \int_{-m_i}^{\infty} (m_i + \varepsilon_i) \sqrt{\frac{\tau_i}{2\pi}} e^{-\frac{\tau_i \varepsilon_i^2}{2}} \mathrm{d}i.$$

Direct computation verifies the claimed monotonicity: $\frac{\mathrm{d}f}{\mathrm{d}m_i} = (p + 100)\mathbb{P}[\varepsilon_i > -m_i] > 0$; and $\frac{\mathrm{d}f}{\mathrm{d}\tau_i} = -\frac{p+100}{2\tau\sqrt{2\pi\tau}} e^{-\frac{\tau m^2}{2}} < 0$. □

Lemma B.1

Proof. For notation simplicity, denote the planner's objective function as $W(q; \mu)$, taking μ as an exogenous parameter. Then the first-order condition (B.1) can be written as $\frac{\mathrm{d}W}{\mathrm{d}q} = 0$, which implies a function of $q(\mu)$. By the implict function theorem, $\dot{q}(\mu) = -\frac{\partial^2 W}{\partial q \partial \mu} / \frac{\partial^2 W}{\partial q^2}$. Given the concavity of W in q, the denominator is strictly negative (Corollary 3), and, hence, the sign of $\dot{q}(\mu)$ is the same as that of

$$\frac{\partial^2 W}{\partial q \partial \mu} = -2(E_{\mathrm{N}} - E_{\mathrm{T}})\big(\dot{E}_{\mathrm{N}} - \dot{E}_{\mathrm{T}}\big)g\mu + gE_{\mathrm{N}}\big(2\dot{E}_{\mathrm{N}} - \dot{E}_{\mathrm{T}}\big) - gE_{\mathrm{T}}\dot{E}_{\mathrm{N}} - \dot{\Pi}_{\mathrm{N}} + \dot{\Pi}_{\mathrm{N}},$$

where we abbreviate the argument of q wherever possible to lessen notational complexity. Note that $\frac{\partial^2 W}{\partial q \partial \mu}$ is linear in μ, with coefficient $-2(E_{\mathrm{N}} - E_{\mathrm{T}})\big(\dot{E}_{\mathrm{N}} - \dot{E}_{\mathrm{T}}\big)g < 0$. Below we show that at the optimal $q(\mu)$ (satisfying (B.1)), $\frac{\partial^2 W}{\partial q \partial \mu}\Big|_{\mu=0} = gE_{\mathrm{N}}\big(2\dot{E}_{\mathrm{N}} - \dot{E}_{\mathrm{T}}\big) - gE_{\mathrm{T}}\dot{E}_{\mathrm{N}} - \dot{\Pi}_{\mathrm{N}} + \dot{\Pi}_{\mathrm{N}} < 0$, and therefore $\dot{q}(\mu) < 0$.

Note that, at $\mu = 0$, $\frac{\mathrm{d}W}{\mathrm{d}q} = 0$ can be rewritten as $g = \frac{\dot{\Pi}_{\mathrm{N}}}{E_{\mathrm{N}}\dot{E}_{\mathrm{N}}}$, which we substitute into $\frac{\partial^2 W}{\partial q \partial \mu}\Big|_{\mu=0}$ and then divide it by $\dot{\Pi}_{\mathrm{N}}$ to get that the sign we are interested in is the same as that of

$$1 - \frac{\dot{E}_{\mathrm{T}}}{\dot{E}_{\mathrm{N}}} - \frac{E_{\mathrm{T}}}{E_{\mathrm{N}}} + \frac{\dot{\Pi}_{\mathrm{T}}}{\dot{\Pi}_{\mathrm{N}}}.$$

The three fractions can be directly evaluated using the closed-form expressions of (14) and (17). Carefully evaluation of the above leads to that it is strictly negative. □

Lemma B.2

Proof. Direct computation using (14) and (17) yields

$$\frac{\mathrm{d}}{\mathrm{d}q}\big(\Pi_{\mathrm{T}}(q) - \Pi_{\mathrm{N}}(q)\big) = \frac{c^2(1-\phi)\phi(h-\ell)^2(h+\ell)((h+\ell)ckq - ch\ell + k)}{(ch^2+k)(c\ell^2+k)\big(c\mathbb{E}[e_i^2]+k\big)}.$$

Note that under (13), $q > \frac{ch\ell-k}{(h+\ell)ck}$, which is both sufficient and necessary for the above to be positive. □

References

Abrell, Jan, Anta Ndoye Faye, and Georg Zachmann. 2011. "Assessing the impact of the EU ETS using firm level data." Tech. rep., Bruegel working paper.

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous. 2012. "The environment and directed technical change." *American economic review* 102 (1):131–166.

Acemoglu, Daron and Will Rafey. 2023. "Mirage on the horizon: Geoengineering and carbon taxation without commitment." *Journal of Public Economics* 219:104802.

Aichele, Rahel and Gabriel Felbermayr. 2015. "Kyoto and carbon leakage: An empirical analysis of the carbon content of bilateral trade." *Review of Economics and Statistics* 97 (1):104–115.

Akey, Pat, Ian Appel, Aymeric Bellon, and Johannes Klausmann. 2024. "Do carbon markets undermine private climate initiatives?" Tech. rep.

Ambec, Stefan, Mark A Cohen, Stewart Elgie, and Paul Lanoie. 2013. "The Porter hypothesis at 20: can environmental regulation enhance innovation and competitiveness?" *Review of environmental economics and policy* 7:2–22.

Babiker, Mustafa H. 2005. "Climate change policy, market structure, and carbon leakage." *Journal of International Economics* 65 (2):421–445.

Bartram, Söhnke M, Kewei Hou, and Sehoon Kim. 2022. "Real effects of climate policy: Financial constraints and spillovers." *Journal of Financial Economics* 143 (2):668–696.

Ben-David, Itzhak, Yeejin Jang, Stefanie Kleimeier, and Michael Viehs. 2021. "Exporting pollution: where do multinational firms emit CO2?" *Economic Policy* 36 (107):377–437.

Biais, Bruno and Augustin Landier. 2022. "Emission caps and investment in green technologies." Tech. rep.

Böhringer, Christoph, Knut Einar Rosendahl, and Halvor Briseid Storrøsten. 2017. "Robust policies to mitigate carbon leakage." *Journal of Public Economics* 149:35–46.

Borghesi, Simone, Michael Pahle, Grischa Perino, Simon Quemin, and Maximilian Willner. 2023. "The market stability reserve in the EU emissions trading system: a critical review." *Annual Review of Resource Economics* 15 (1):131–152.

Bundesrepublik Deutschland v. Nordzucker AG. 2015. URL https://www.bailii.org/eu/cases/EUECJ/2015/C14814.html. ECLI:EU:C:2015:261.

Calel, Raphael. 2020. "Adopt or innovate: Understanding technological responses to cap-and-trade." *American Economic Journal: Economic Policy* 12 (3):170–201.

Calel, Raphael and Antoine Dechezleprêtre. 2016. "Environmental policy and directed technological change: evidence from the European carbon market." *Review of economics and statistics* 98 (1):173–191.

CarbonCredits.com. 2023. "Tesla Hits Record High Sales from Carbon Credits at $1.79 Billion." URL https://carboncredits.com/tesla-hits-record-high-sales-from-carbon-credits-at-1-79b/. Accessed: 2024-11-05.

Coase, Ronald H. 1960. "The Problem of Social Cost." *Journal of Law and Economics* 3:1.

Colmer, Jonathan, Ralf Martin, Mirabelle Muûls, and Ulrich J Wagner. 2024. "Does Pricing Carbon Mitigate Climate Change? Firm-Level Evidence from the European Union Emissions Trading System." *Review of Economic Studies* :rdae055.

Ellerman, A Denny. 2003. "Are cap-and-trade programs more effective in meeting environmental goals than command-and-control alternatives?" Tech. rep.

ESMA. 2022. "Final Report on Emission Allowances and Associated Derivatives." Tech. rep., European Securities and Markets Authority. ESMA70-445-38.

———. 2024. "Market Report on EU Carbon Markets 2024." Tech. rep., European Securities and Markets Authority. ESMA50-43599798-10379.

EU Directive. 2003/87. "Establishing a scheme for greenhouse gas emission allowance trading within the Community and amending Council Directive 96/61/EC." URL http://data.europa.eu/eli/dir/2003/87/oj.

———. 2023/959. "Amending Directive 2003/87/EC establishing a system for greenhouse gas emission allowance trading within the Union and Decision (EU) 2015/1814 concerning the establishment and operation of a market stability reserve for the Union greenhouse gas emission trading system (Text with EEA relevance)." URL https://eur-lex.europa.eu/eli/dir/2023/959/oj.

EU Parliament. 2022. "Climate change: Deal on a more ambitious Emissions Trading System (ETS)."

Financial Times. 2024. "How oil and gas companies disguise their methane emissions." URL https://www.ft.com/content/374e78dc-69ae-419e-a583-0842b3833d9d.

Fischer, Carolyn and Alan K Fox. 2012. "Comparing policies to combat emissions leakage: Border carbon adjustments versus rebates." *Journal of Environmental Economics and management* 64 (2):199–216.

Goldstein, Itay, Alexandr Kopytov, Lin Shen, and Haotian Xiang. 2024. "On ESG investing: Heterogeneous preferences, information, and asset prices." Tech. rep., National Bureau of Economic Research.

Goulder, Lawrence H and Ian WH Parry. 2008. "Instrument choice in environmental policy." *Review of environmental economics and policy* .

Goulder, Lawrence H and Stephen H Schneider. 1999. "Induced technological change and the attractiveness of CO2 abatement policies." *Resource and energy economics* 21 (3-4):211–253.

Grubb, Michael, Nino David Jordan, Edgar Hertwich, Karsten Neuhoff, Kasturi Das, Kaushik Ranjan Bandyopadhyay, Harro Van Asselt, Misato Sato, Ranran Wang, William A Pizer et al. 2022. "Carbon leakage, consumption, and trade." *Annual Review of Environment and Resources* 47 (1):753–795.

Hepburn, Cameron, Michael Grubb, Karsten Neuhoff, Felix Matthes, and Maximilien Tse. 2006. "Auctioning of EU ETS phase II allowances: how and why?" *Climate Policy* 6 (1):137–160.

Huang, Shiyang and Alexandr Kopytov. 2024. "Sustainable finance under regulation." Tech. rep., Available at

SSRN 4231723.

ICAP. 2025. "Compare ETS." URL https://icapcarbonaction.com/en/compare/43/55/45. Retrieved on August 7, 2025.

Känzig, Diego R, Julian Marenz, and Marcel Olbert. 2024. "Carbon Leakage to Developing Countries." *Available at SSRN 4833343* .

Karp, Larry and Christian Traeger. 2024. "Smart cap." *Journal of the European Economic Association* :jvae030.

Kerr, Suzi and Richard G Newell. 2003. "Policy-induced technology adoption: evidence from the US lead phasedown." *The Journal of Industrial Economics* 51 (3):317–343.

Marcu, Andrei, Juan Fernando López Hernández, Elena Bonfiglio, and Anita Vollmer. 2022. "ERCST Explanatory Note on EU ETS Compliance 2022." Tech. rep.

Martin, Ralf, Mirabelle Muûls, Laure B De Preux, and Ulrich J Wagner. 2014. "Industry compensation under relocation risk: A firm-level analysis of the EU emissions trading scheme." *American Economic Review* 104 (8):2482–2508.

Montero, Juan-Pablo. 2005. "Pollution markets with imperfectly observed emissions." *RAND Journal of Economics* :645–660.

Montgomery, W David. 1972. "Markets in licenses and efficient pollution control programs." *Journal of economic theory* 5 (3):395–418.

Nordhaus, William D. 1994. *Managing the Global Commons: The economics of the greenhouse effect*. MIT Press, Cambridge, MA.

———. 2017. "Revisiting the social cost of carbon." *Proceedings of the National Academy of Sciences* 114 (7):1518–1523.

Perino, Grischa, Robert A Ritz, and Arthur Van Benthem. 2019. "Overlapping climate policies." Tech. rep., National Bureau of Economic Research.

Piccolo, Alessio, Jan Schneemeier, and Michele Bisceglia. 2023. "Externalities of responsible investments." Tech. rep., Available at SSRN 4183855.

Popp, David. 2004. "ENTICE: endogenous technological change in the DICE model of global warming." *Journal of Environmental Economics and management* 48 (1):742–768.

Porter, Michael. 1991. "America's green strategy." *Scientific America* 264:168.

Reuters. 2024. "China to crack down on emissions data fraud as CO_2 market expansion nears." Tech. rep. URL https://www.reuters.com/sustainability/boards-policy-regulation/china-crack-down-emissions-data-fraud-co2-market-expansion-nears-2024-02-05/. Reporting by Beijing newsroom and David Stanway in Singapore; Editing by Louise Heavens and Christopher Cushing.

Roberts, Marc J and Michael Spence. 1976. "Effluent charges and licenses under uncertainty." *Journal of Public Economics* 5 (3-4):193–208.

Schmalensee, Richard and Robert N Stavins. 2017. "Lessons learned from three decades of experience with cap and trade." *Review of Environmental Economics and Policy* .

Sinbon. 2021. "How Tesla Made Great Profits from Regulatory Credits." URL https://www.sinbon.com/solution/integrated-solutions/how-tesla-made-great-profits-from-regulatory-credits. Accessed: 2024-11-05.

S&P Global Commodity Insights. 2022. "China's emissions fraud cases signal challenges in carbon market rollout." Tech. rep. URL https://www.spglobal.com/commodity-insights/en/news-research/latest-news/energy-transition/031822-chinas-emissions-fraud-cases-signal-challenges-in-carbon-market-rollout. Market Specialist – Energy Transition.

Taylor, Margaret R. 2012. "Innovation under cap-and-trade programs." *Proceedings of the National Academy of Sciences* 109 (13):4804–4809.

U.S. Environmental Protection Agency. 2019. "Fiat Chrysler Automobiles Clean Air Act Civil Settlement Information Sheet." URL https://www.epa.gov/enforcement/fiat-chrysler-automobiles-clean-air-act-civil-settlement-information-sheet. Accessed: 2024-11-05.

Weitzman, Martin L. 1974. "Prices vs. Quantities." *Review of Economic Studies* 41 (4):477–491.