A Smart-Contract to Resolve Multiple Equilibrium in an Intermediated Repo Trade

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Abstract

We construct an empirically founded model of a repo trade intermediated by two broker-dealers and prove there is multiple equilibrium. We then present a smart contract that resolves multiple equilibrium by imposing a structure whereby each broker-dealer reports its client demand schedule and its minimum rate spread constraint, and the smart contract selects the trade. We prove the existence and uniqueness of equilibrium at the joint profit maximizing volume of trade. The smart contract is a machine executed algorithm which eliminates the need for trust. Hardware and cryptography are used to prevent leakage of broker-dealer client trade schedules, and to enable privacy-protected auditing with zero-knowledge proofs of the integrity of computations. The outcome can be implemented by a myopic strategy where a broker-dealer truthfully reports its own variables without anticipating its counterparty's reports. This minimizes cognitive and computational complexity, thereby making our smart contract suitable for real-world deployment.

Keywords— smart contracts, multiple equilibrium, privacy, market design, coordination

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1 Introduction

Unreliable private information disclosure, limited commitment, multiple equilibrium and coordination problems are obstacles to exchange. In some abstract environments these obstacles can be overcome by the "central planner" of the mechanism design literature, which is a protocol that receives messages, selectively discloses information and allocates resources. However, concerns over trust in the planner and the desire to maintain privacy has limited the adoption of such protocols. We show how multiple equilibrium in a model intermediated trade can be resolved and joint profit maximization achieved with a smart-contract that acts as the planner.

We study a chain of exchanges in a financial market where trade between the ultimate seller and buyer, in the case of a market for final sales, or borrower and lender, in the case of a repo market (the "clients"), is intermediated by two broker-dealers. Each client trades with its own broker-dealer and the broker-dealers trade with each other in an interdealer market. We build a model of one particular type of trade, repo, where counterparties enter into two contracts, one for the purchase by one agent of a financial object today, and the other for the repurchase of the financial object at a later date by the same agent (Section 3). We construct our model from elements that are derived from the empirical and theoretical literature (Section 2). We demonstrate that repo trading results in multiple equilibrium (Section 4). Multiple equilibrium exists whether or not each broker-dealer observes the trade preferences of the counterparty's client. (Section 6 and Appendix C). One of the potential negative effects of multiple equilibrium is that the selected equilibrium can lie below a minimum spread of return that broker-dealer intermediaries must exceed to obtain the balance-sheet capacity to execute the trade. This could curtail trading.

To resolve multiple equilibrium and ensure the the minimum spread is exceeded, we propose a smart-contract protocol that selects a trade that exceeds the hurdle. It is a self-executing digital agreement with the terms of the agreement embedded in its code and protected form leakage by cryptography and hardware. These features enable the smart-contract to resolve the trust issue. However, the implementation of the smart-contract relies on several things; Deposits to enforce compliance and privacy of messaging; Privacy of messaging and computation to prevent front-running of trades or leakage of a broker-dealer's client's supply or demand schedules, and the ability to audit to ensure the correctness of the smart-contract computations. With these features the mechanism can operate without dependence on trust. The smart-contract is robust in three respects. First, it is invariant to any specification of the information held by agents. It always selects the outcome that maximizes the joint profit of the broker-dealers, subject to the hurdle constraint. Second, we show embodiments where the smart-contract is a computer server, interacting with banks where agents hold accounts, and another where the smart-contract is appended to a blockchain with agents trading tokenized objects. Third, it requires minimal cognitive engagement from agents. All that is required is for broker-dealers to provide client trade schedules and minimum spreads, both of which are dominant strategies.

Practical significance of the smart contract: Repo intermediation consumes bank balance-sheet capacity (Aronoff et al. [4]). Since the 2008 financial crisis, the balance sheets of major dealer banks have not expanded in line with the explosive growth of U.S. Treasuries outstanding, leaving intermediation capacity strained (Duffie [19]). When trading volumes spike, these balance-sheet constraints cap the amount of repo transactions dealers can intermediate, preventing broker-dealers from promptly absorbing order flow. The incapacity to scale up repo dealing in stress episodes has disrupted market liquidity and price efficiency, hampering the smooth allocation of capital. The March 2020 Treasury market turmoil revealed how constrained dealer capacity can amplify illiquidity and interfere with monetary policy implementation (Federal Reserve [23]; Duffie [19]). The risks of disruptions may increase as Treasury issuance and trading volumes are projected to climb further over the next decade (Duffie [19]). Expanding repo intermediation capacity is therefore critical for financial stability and the effective functioning of monetary policy.

Post-2008 financial crisis bank regulatory reforms – notably the Supplementary Leverage Ratio (SLR) – have raised the cost of balance-sheet usage and made leverage scarcer, especially for low-risk, low-yield positions (He et al. [31]). By applying a uniform capital charge regardless of asset risk, the SLR forces banks to ration internal capital more strictly across business lines, effectively *raising the minimum spread* (required return on equity) for each unit of balance sheet deployed. In this environment of heightened capital stringency, repo intermediation has suffered because it is a traditionally low-margin, "matched-book" business (Klingler and Sundaresan [43]; Duffie [18]). Banks are incentivized to allocate their limited balance-sheet capacity to higher-yield activities, which mitigates against allocations to repo trading. Duffie [18] formalizes this intuition, showing that under a binding leverage constraint, adding a Treasury repo can erode shareholder value (through "debt overhang") unless the intermediation spread is high enough to offset the cost of the extra equity capital required. Consistent with this, empirical evidence finds that dealers demand wide repo rate spreads before taking on additional repo volumes when balance sheet is tight – a phenomenon dubbed *"Treasury inconvenience yields"* that emerged once the SLR became binding (He et al. [31]), which has curtailed dealers' willingness to intermediate repos, contributing to the chronic under-provision of repo market-making capacity (Klingler and Sundaresan [43]; Duffie [18]).

Our smart contract loosens this constraint by inducing an equilibrium that maximizes the profit from intermediating repo trades. Increasing the profitability of repo intermediation makes it a more attractive use of internal capital, which increases the allocation of balance-sheet capacity to repo. More balance-sheet committed to repo intermediation means greater market-making capacity, which should alleviate the bottlenecks that have previously caused spikes in repo rates and dislocations in related markets. By boosting broker-dealer returns on repo intermediation, our smart contract can incentivize banks to expand their provision of repo market capacity – thereby mitigating the macroeconomic and policy risks associated with an overly constrained intermediation sector. [19] [23] [31] [43] [18]

Related literature. This paper builds on three related strands of literature that jointly inform our model of intermediated repo trade and the smart-contract implementation. The first concerns theoretical models of repo market intermediation. Duffie [17], Huh and Infante [40] and Gottardi et.al. [29] show how dealer balance-sheet constraints, collateral scarcity, and rehypothecation collectively determine repo rates and market liquidity. Their frameworks provide a microfoundation for the interdealer structure and hurdle-rate constraint which are resonant with our model. The second strand addresses coordination failures and multiplicity of equilibria in financial markets. Morris and Shin [48], Martin, Skeie, and von Thadden [46], and Cole and Kehoe [14] build models that resolve self-fulfilling runs or multiple equilibria through information structure or institutional design. We design a smart-contract that resolves multiple equilibrium through a design that coordinates trade and selects the equilibrium while being agnostic toward the information structure. The final strand examines how smart-contracts and blockchain technologies can implement financial market coordination. Cong and He [15] and Chiu and Koeppl [13] demonstrate that programmable contracts can replicate trust and enforcement functions traditionally performed by intermediaries, offering a theoretical basis for our algorithmic mechanism that selects the joint profit–maximizing equilibrium while preserving privacy and auditability.

Roadmap. The remainder of the paper is organized as follows. In Section 2 we describe the theoretical and empirical support for our modeling strategy. In Section 3 we present a repo model where intermediary broker-dealers have common knowledge of client shocks. In Section 4 we prove the existence and multiplicity of equilibrium it that setting. In Section 5 we prove that broker-dealer joint profit maximization is an equilibrium. In Section 6 and Appendix C we relax the common knowledge assumption and prove that multiple equilibrium exists when a broker-dealers does not observe the shock realizations of its counterparty's client. In Section8 we present a smart-contract that induces joint profit maximization as the unique outcome. Finally, in Section 9 we conclude.

2 Elements of The Model

In an over-the-counter (OTC) and other financial markets, trade between buyers and sellers is intermediated by broker-dealers[1]. Intermediaries trade with their buyer/seller clients and with each other. The largest trading market in the world, the US Treasuries repo market is intermediated. A repo trade involves two contracts entered into simultaneously whereby one counterparty initially purchases the financial asset (the "first-leg") and commits to repurchase the same financial asset at a later date (the "second-leg").[2] The estimated daily volume of US Treasuries repo is $12 Trillion (Hempel et.al. [36]). The market is intermediated by broker-

[1] In an over-the-counter ("OTC") market agents engage in bilateral exchanges, as opposed to a centralized exchange where e.g. agents submit limit orders from which a single market clearing price is determined.

[2] In a repo trade the volume of financial asset is the same at both legs. The money exchanged can differ.

dealers who conduit Treasuries from repo lenders - typically hedge funds, insurance companies and pension funds, to repo lenders, typically money market funds (Kahn et.al. [42]).

The revealed preference of this form of market organization suggest it is useful. However, intermediated trade creates a potential coordination problem which can result in inefficiently low trading due to the presence of multiple equilibrium. Multiple equilibrium is caused by a negative externality from the client transaction volume entered into by broker-dealers, which determines the volume of trade they will desire to offset in the interdealer market (e.g. if a broker-dealer purchases x units of the financial asset from its client, it will desire to sell x units to another broker-dealer). A low volume outcome can occur when a broker-dealer anticipates that its interdealer counterparty will arrive with a low volume of money or financial asset to trade and reduces the volume of trade with its client. In reaction the counterparty adjusts downward its client trade volume and so forth, since neither one wishes to carry excess inventory. This can lead to a self-reinforcing low-trade equilibrium.

In this section we state the key elements of our model and point to empirical theoretical studies that motivate and support our design choices.

2.1 Repo chains

Elements:
2 Clients; ultimate repo lender MM and ultimate repo borrower RM. 2 intermediary broker-dealers, BD_{mm} and BD_{rm}.
2 Traded objects; money M and Treasuries securities T.
2 Trading dates and contracts. First-leg, borrower sells T to lender today. Second-leg, borrower repurchases T from lender at a future date.

The setup is a chain of repo transactions composed as follows. There are 2 clients, a repo borrower who sells the financial asset, T, in exchange for money, M, at the first-leg and repurchases the at a later time called the second-leg. The repo borrower's supply function for the financial asset is $T = S_r$ and the repo lender's demand function for the financial asset is $T = D_r$, where r is the repo rate faced by the agent. $r = (p_2 - p_1)/p_1$, where p_1 is unit the price of the financial asset at the first-leg and p_2 is the unit price at the second-leg. There are 2 broker-dealers. Each client trades with a broker-dealer and each broker-dealer attempts to re-trade the financial object it acquired from its client with the counterparty broker-dealer (the "interdealer" trade); e.g at the first-leg the repo borrower's broker-dealer purchases the financial asset from its client and sells it to the counterparty broker-dealer. At the second-leg, that broker-dealer re-purchases the financial asset from the counterparty broker-dealer and re-sells it to its client. Figure 1 depicts the repo chain formed by these trades. At the first-leg T flows left to right, from the repo borrower RM - typically a hedge fund or a "risk manager" - to repo lender MM - typically a money market fund or "cash manager"

(Pozsar [52]). In between are the broker-dealers, and M flows in the opposite direction. The flows reverse at the second-leg.

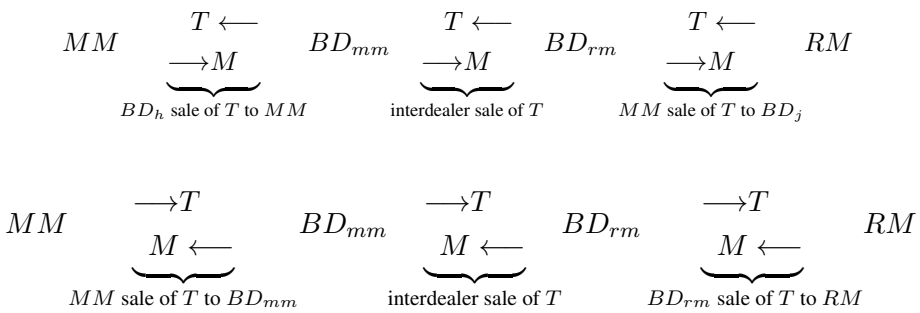


Figure 1: First and Second-Leg Repo Chain

Aronoff et.al. [3] decompose the US Treasuries repo network into chains and cycles of repo trades. The most prevalent configuration is the chain depicted in Figure 1.

2.2 Timing of repo trades

Elements:
Contracts for first-leg and second-leg transactions are entered into simultaneously at the first-leg date.
Contracts between broker-dealers and its clients are committed to before interdealer contracts between broker-dealers are committed to.

Broker-dealers typically commit first to client-facing transactions and subsequently offset these exposures through interdealer trades. Gardner and Huh [27] document that "DC-ID trades are instances in which customer trades are prearranged with offsetting interdealer trades," providing direct evidence that dealers arrange interdealer commitments only after securing client orders. Similarly, Huh and Infante [40] emphasize that "after dealers receive client orders, dealers transact in the interdealer cash and repo market," underscoring the sequencing of commitments across markets. At the microstructure level, Macchiavelli and Pettit [45] explain that a broker-dealer that extends a loan to a hedge fund client "typically rehypothecates some of these securities received as collateral in the repo markets," highlighting the practice of funding client trades ex post through wholesale interdealer borrowing. Consistent with these accounts, Hempel, Kahn, and Shephard [36] show that large dealer-banks operate matched books with "nearly offsetting repo and reverse repo positions," which arise precisely because client transactions are subsequently hedged through interdealer trades. Together, this evidence establishes that the sequence of commitments runs from client to dealer, then from dealer to interdealer, a timing structure that has important implications for market liquidity and risk transmission.

We model trading activity as taking place in two sequential steps. (1) clients and BD's agree to trades. (2) after that BD's trade together in a simultaneous move game. Figure 2 displays the repo market segments and the timing of repo trades in our model. Broker-dealer intermediaries trade with each other in the interdealer market and with their clients in the other two markets (Kahn and Olsen [42]). Broker-dealers typically lend to hedge-funds in the bilateral uncleared market and borrow from money market funds in the Tri-party market (Hempel et.al. [34] and Paddrik et.al. [49]).

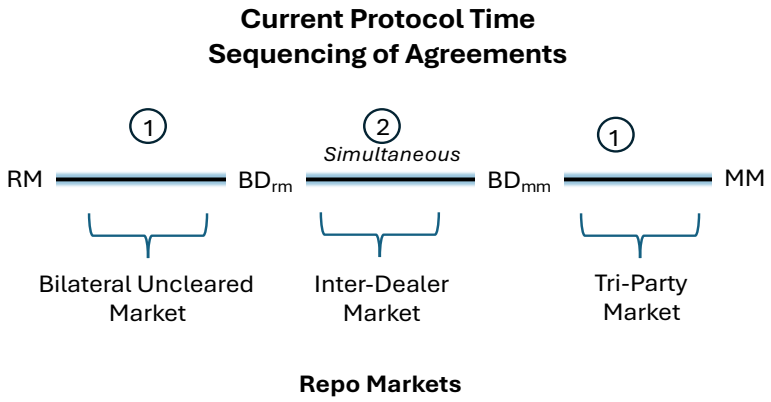


Figure 2: ***Timing under current protocol***. *Sequential ordering by number. First, client trades are agreed to. Second, the interdealer trade is agreed to.*

2.3 Broker-dealer price-setting in the client market

Elements: Broker-dealers are monopolist price-setters in the client markets.

We model the broker-dealer as a price-setting monopolist in the client market, consistent with extensive empirical and theoretical research (Infante [41], Avalos et al. [5], Eisenschmidt et al. [22], Duffie et al. [20], and Hempel et al. [35]). These studies provide evidence that broker-dealers often extract informational or relationship-based rents in client-facing trades. For example, Infante [41] models the dealer as a monopolist, illustrating how dealer pricing power can distort client outcomes. Avalos et al. [5] note that U.S. repo markets currently "rely heavily on [broker-dealer affiliates of] four banks as marginal lenders", implying that cash providers face limited counterparty options. Similarly, Eisenschmidt et al. [22] find that "dealers have market power over their customers, allowing them to charge OTC repo rates that differ from the interdealer rate," pointing to price discrimination across segments. A contributing factor enabling such behavior is the persistence of trading relationships: repeated interactions with hedge funds (Kruttli et al. [44], Financial Stability Board [1]) and money market funds (Han et al. [30], Hempel et al. [33]) facilitate dealer market power and enable the exercise of monopolistic pricing in the client market.

2.4 Concavity of money market fund demand and hedge fund supply

Elements:
The MM demand function (for acquiring T at the first-leg) is denoted $D_{r_{mm},\theta_{mm}}$ where r is the repo rate and θ_{mm} is a stochastic shock.
D is concave increasing - and twice differentiable - in r.
θ_{mm} shifts D monotonically.
The RM supply function (for selling T at the first-leg) is denoted $S_{r_{rm},\theta_{rm}}$ where r is the repo rate and θ_{rm} is a stochastic shock.
S is concave decreasing - and twice differentiable - in r.
θ_{rm} shifts S monotonically.

A body of empirical and theoretical work supports the view that both hedge fund borrowing demand and money market fund (MMF) repo supply are concave in relevant regions. On the borrower side, He, Nagel, and Song [32] show that "a simultaneous rise in reverse-repo rates charged by dealers reduces the attractiveness of a levered investment in Treasuries for (risk-averse) hedge funds," emphasizing that incremental demand falls disproportionately as funding costs increase. Relatedly, Banegas and Monin [6] document that "a 200 bp minimum haircut would reduce effective leverage from 56x to 25x," underscoring how small increases in margin requirements sharply reduce borrowing, consistent with a diminishing marginal willingness to lever. On the lender side, Huber [39] structurally estimates that MMFs exhibit "aversion to portfolio concentration and preference for stable lending," leading to a supply schedule in which incremental willingness to lend to a given dealer diminishes with position size. Complementing this, Hempel and Monin [37] highlight that the ON RRP facility "helps to set a floor on lending rates for MMFs," introducing a kink that flattens supply below the policy floor, while supply above the floor remains upward-sloping but with declining elasticity. Taken together, these findings provide both theoretical and empirical support for modeling hedge fund borrowing demand and MMF repo supply as concave functions.

2.5 Bargaining and surplus splitting in interdealer trades

Elements:
Broker-dealers split the surplus generated by their trade, where the surplus is the margin between the repo rate received from RM and the repo rate paid to MM, multiplied by T, the volume of trade between the broker-dealers.

$$Surplus = (r_{rm} - r_{mm})T$$

A literature models bilateral dealer-to-dealer transactions as the outcome of bargaining, with transaction

prices reflecting a division of surplus rather than price-taking. In the canonical OTC search model of Duffie, Gârleanu, and Pedersen [21], "prices are set through a bargaining process that reflects each investor's or marketmaker's alternatives to immediate trade," and dealers "capture part of the difference between the interdealer price and investors' reservation values." Building on this framework, Weill [57] shows that the generalized Nash bargaining solution yields prices that are "a convex combination of the counterparty's reservation value and the interdealer benchmark," making explicit the surplus-splitting logic of bilateral dealer trades. Empirical evidence corroborates these theoretical predictions: Hendershott, Li, Livdan, and Schürhoff [38] estimate that "dealers' bargaining power on the buy and sell sides is large, at 98% and 94%," implying that dealers capture nearly all of the surplus in bilateral negotiations. Similarly, Feldhütter [24] finds that dealers capture 97% of trade surpluses in corporate bonds, while Pinter, Üslü, and Wijnandts [51] show that such bargaining dynamics extend to government bond markets as well. Taken together, these studies provide both theoretical and empirical support for modeling interdealer transactions as surplus-splitting bargains, in which the price between two broker-dealers divides the surplus generated by their respective client repo trades.

2.6 Minimum spread in repo balance-sheet allocation

Elements:

$$1/2(r_{rm} - r_{mm}) \geq \underline{\pi}$$

Intermediating repo trades uses balance-sheet capacity in the sense that the broker-dealer is required to increase recorded assets between the first-leg and second-leg by the amount of cash it receives on the borrowing side of its trade, e.g trade with its right-side neighbor in Figure 1 (Aronoff et.al [4]). Empirical evidence indicates that US banks impose internal rate-of-return hurdles on the balance-sheet they allocate to their broker-dealer affiliates' repo activities. This reflected in the spread between the borrowing and lending rates. Surplus-splitting allocates half of the total spread between r_{rm} and r_{mm} to each BD (Equation 1). Andersen, Duffie, and Song [2] show that post-crisis leverage regulations (such as the Supplementary Leverage Ratio) create a debt-overhang cost that raises the required return (minimum spread) on low-risk, balance-sheet-intensive trades like repos, which implies that dealers will only allocate balance-sheet capacity for repo intermediation if the expected spread covers their internal cost of capital. Recent Federal Reserve data support this: Chabot et al. [12] show that the largest dealers demand a consistent positive spread (roughly 5–7 basis points on average) between the rates at which they borrow and lend in the Treasury repo market, effectively as compensation for using balance-sheet capacity. Such a spread can be viewed as an internal "hurdle" rate of return for repo business. More broadly, banks tend to set internal ROE targets in the low teens; for example, one industry analysis infers that banks apply an internal hurdle around 12%, whereas

holding low-risk Treasuries (via repo or outright) might only earn 4% returns [11]. This sizable gap helps explain why bank holding companies limit balance-sheet devoted to low-yield repo unless those activities can meet internal return requirements [11].

3 A Model of Intermediated Repo Trade

In this section we model the trade between two broker-dealers depicted in Figure 1. BD_{mm} has a client (or clients) MM from which it has entered into a repo trade to borrow money in exchange for collateral. BD_{rm} has a client (or clients) RM from which it has entered into a reverse-repo trade to lend money in exchange for collateral. We use the model elements stated in Section 2 for trade chain structure, timing of trades, broker-dealer price-setting in the client market, concavity of client supply and demand functions and surplus-splitting between broker-dealers.[3]

3.1 Broker-dealer objectives and information

Our baseline model assumes common knowledge of client shocks. This enables us to suppress the θ variables. When each broker-dealer observes the realization of its counterparty client's θ, the analysis of equilibrium existence and multiplicity, which is our subject, uses only the concavity of client supply and demand functions. We relax the common knowledge restriction in Section 6 and Appendix C, where we show that the results we derive here for existence and multiplicity of equilibrium continue to hold when there is asymmetric information about counterparty client preferences and those preferences are subjected to shocks. In Appendix A we relax the monopolistic price-setting assumption and allow broker-dealers to be market-makers who commit to trade minimum volumes with their clients.

3.2 The interdealer repo rate

BD_{mm} and BD_{rm} negotiate a split of the intermediation surplus $(r_{rm} - r_{mm})T$. The surplus split is reflected in the interdealer repo rate r_{bd}. The share of the surplus allocated to BD_{mm} is $(r_{bd} - r_{mm})T$, which is the spread between the rate at which BD_{mm} borrows from MM_{i_i} and lends to BD_{rm}. The share of the surplus allocated to BD_{rm} is $(r_{rm} - r_{bd})T$, which is the spread between the rate at which BD_{rm} borrows from BD_{mm} and lends to RM. We are agnostic on the allocation of the surplus between BD's or the bargaining process by which the allocation is determined. We posit an equal split for convenience, not necessity.[4]. The inter - dealer repo rate is

[3] We impose a hurdle rate in Section 5.1.

[4] The only properties of r_{bd} upon which our argument relies are that $\frac{\partial r_{bd}}{\partial r_{mm}} < 0$ and $\frac{\partial r_{bd}}{\partial r_{rm}} > 0$.

$$r_{bd} = \frac{1}{2}(r_{rm} - r_{mm}) + r_{mm} \tag{1}$$

3.3 Broker-dealer client pricing

Broker-dealers are price-setters in the client markets. For any volume of T traded in the inter-dealer market, a broker-dealer will set its client repo rate at the point the client demand or supply schedule intersects that volume. The reasoning is immediate. At each T the MM demand schedule lists the minimum repo rate at which it is willing to lend and the RM supply schedule lists the maximum repo rate at which it is willing to borrow. From Equation 1 it can be seen that BD_{mm} maximizes its profit by choosing the minimum r_{mm} compatible with volume T and BD_{rm} maximizes its profit by choosing the maximum r_{rm} compatible with volume T.

3.4 BD_{mm}'s problem

BD_{mm} sets its client repo rate r_{mm} in the morning at t_{1M}. In order to form a rational expectation of the interdealer repo rate r_{bd} and transaction volume that it will encounter afterwards, it must forecast BD_{rm}'s client repo rate (from which it can also forecast BD_{rm}'s client transaction volume) r_{rm}. Equation 2 is BD_{mm}'s problem, contingent on its forecast of r_{rm}.

$$\underset{r_{mm}}{\operatorname{argmax}}(r_{bd} - r_{mm})^{+}Q - \underbrace{(D_{r_{mm}} - Q)^{+}r_{mm}}_{gap(BD_{mm})} \tag{2}$$

Subject to $Q = \min\{D_{r_{mm}}, S_{r_{rm}})\}$ i.e. the amount of the interdealer trade. $r_{mm} \in [0, r_{rm}]$ is the domain of r_{mm}[5].

We can substitute in equation 1 and rewrite BD_{ii}'s problem as follows.

$$\underset{r_{mm}}{\operatorname{argmax}} \frac{1}{2}(r_{rm} - r_{mm})^{+}Q - gap(BD_{mm}) \tag{3}$$

In its client market, BD_{mm} conveys T to MM as collateral for a repo loan of M. BD_{mm} carries the M into the Inter-dealer trade. The first term of BD_{mm}'s problem is its profit from the interdealer repo trade. The second term, $gap(BD_{mm})$, represents BD_{mm}'s excess inventories; it is its loss incurred on borrowings of M in its client market that BD_{mm} is unable to trade with its BD_{rm} counterparty.

[5]We assume BD_{mm}'s profit is non-negative, which requires $r_{mm} \leq r_{rm}$. We assume there is a lower bound repo rate at which MM's demand for T is zero. We normalize this rate to be 0. This sets the bounds to the domain for r_{mm}.

To analyze BD_{mm}'s problem we partition it into two parts; (i) the case where BD_{mm} carries excess inventories and (ii) the case where BD_{mm} does not carry excess inventories.

3.4.1 BD_{mm} carries excess inventories of *M* into the interdealer trade. $gap_{B,Q} > 0$

Noting that $Q = S_{r_{rm}}$, BD_{mm}'s problem is

$$\underset{r_{mm}}{\operatorname{argmax}} \frac{1}{2}(r_{rm} - r_{mm})^{+} S_{r_{rm}} - gap(BD_{mm}) \tag{4}$$

The derivative of BD_{mm}'s problem is: $\underbrace{-\frac{1}{2}S_{r_{mm}}}_{<0}\underbrace{-gap(BD_{mm})'}_{<0} < 0$[6]. Therefore, BD_{mm} will choose a corner solution, setting its client repo rate r_{mm} to achieve $D_{r_{mm}} = S_{r_{rm}}$. This ensures BD_{mm} will not carry any inventories it cannot sell into the Inter-dealer trade.

3.4.2 BD_{mm} carries no excess inventories of *M* into the Inter-dealer trade. $gap_{B,Q} = 0$

The domain of the problem is $r_{mm} \in [0, r_{rm}]$ such that $D_{r_{mm}} \leq S_{r_{rm}}$. It is a compact interval [7]. BD_{mm}'s problem is

$$\underset{r_{mm}}{\operatorname{argmax}} \frac{1}{2}(r_{rm} - r_{mm}) D_{r_{mm}} \tag{5}$$

BD_{mm}'s problem is a continuous concave function on the compact interval $r_{mm} \in [0, r_{rm}]$. The concavity reflects the tension between the value of increasing transaction amount $D_{r_{mm}}$ against the increased cost, r_{mm}, required to obtain it, which reduces BD_{mm}'s profit margin $\frac{1}{2}(r_{rm} - r_{mm})$[8].

[6] When $gap(BD_{mm}) > 0$, then $\frac{\partial gap(BD_{mm})}{\partial r_{B,i}} := gap(BD_{mm})' = D' r_{mm} + (D_{r_{mm}} - Q) > 0$, since $D^{'} > 0$.

[7] Compactness is proved as follows. The derivative of $D_{r_{mm}}$ is strictly positive and continuous, therefore, by the Inverse Function Theorem, its inverse exists and is continuous (Rudin (1976) [53] Theorem 9.24, p. 221.). The domain of the inverse function, $[0, S_{r_{rm}}]$, is compact. Therefore, the range of the inverse function, which is the domain of BD_{mm}'s problem, is a compact interval (Rudin (1976) [53] Theorem 4.14, p. 89).

[8] We simplify notation by setting $\frac{dD_{r_{mm}}}{dr_{mm}} := D^{'}$ and $\frac{d^2 D_{r_{mm}}}{(dr_{mm})^2} := D^{"}$. Similarly for $S_{r_{rm}}$.

3.4.3 Characterizing BD_{mm}'s decision problem

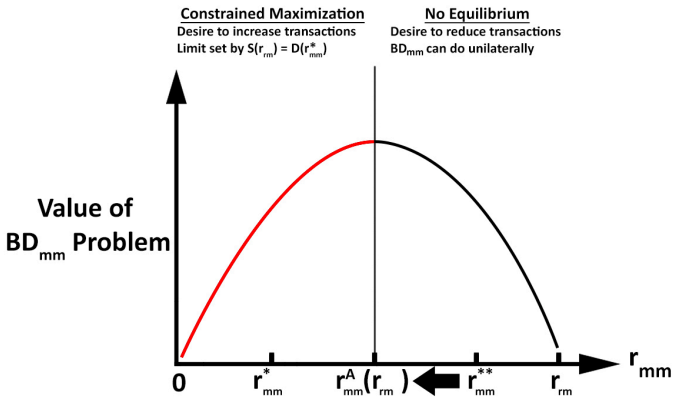


Figure 3: BD_{mm} Problem

Figure 3 shows the curve defining the value of BD_{mm}'s problem at different repo rates for a given value of r_{rm}.

The value of BD_{mm}'s problem is zero at $r_{mm} = 0$ and at $r_{mm} = r_{rm}$. There are no transactions at the former and no marginal profit at the latter. The concavity of the problem ensures that it is single-peaked between these two endpoints. This peak is depicted in Figure 3 as $r^A_{mm}(r_{rm})$. BD_{mm} will choose the maximum repo rate r_{mm} that (i) does not exceed r^A_{mm} and (ii) does not cause $gap(BD_{mm}) > 0$. To see (i), note that BD_{mm}'s problem is negatively sloped to the right of r^A_{mm} and it reduces inventory by reducing r_{mm}. Therefore, it will not reduce profit by reducing r_{mm}, since $gap_{r_{mm},Q}$ will shrink. (ii) reflects that BD_{mm} will not carry excess inventory in equilibrium.

To gain further insight we evaluate BD_{mm}'s decision at repo rates on either side of the maximum. Suppose r^*_{mm} in Figure 3 is the repo rate at which $gap(BD_{mm}) = 0$. BD_{mm} cannot increase r_{mm}, as doing so would cause $gap(BD_{rm}))$ to increase above zero. Since the value of BD_{mm}'s problem is increasing at r^*_{mm}, BD_{mm} maximizes its profit by choosing r^*_{mm}, which we will call a constrained equilibrium repo rate for BD_{mm}[9]. By contrast, at r^{**}_{mm} in Figure 3 BD_{mm}'s problem is negatively sloped, so BD_{mm} would reduce its repo rate (and its inventories) to attain the maximum value of its problem at $r^A_{mm}(r_{rm})$. We call this the unconstrained maximum repo rate for BD_{mm}.

3.5 BD_{rm}'s problem

BD_{rm}'s problem, in its timing and mathematical structure, is similar to BD_{mm}'s problem. Equation 6 is BD_{rm}'s problem, contingent on its forecast of r_{mm}.

[9]For BD_{mm}, any repo rate weakly below $r^A_{mm}(r_{rm})$ is a possible constrained outcome repo rate.

$$\underset{r_{rm}}{\operatorname{argmax}} \frac{1}{2}(r_{rm} - r_{mm})^{+}Q - gap(BD_{rm})) \tag{6}$$

Subject to $Q = \min\{D_{r_{mm}}, S_{r_{rm}}\}$. $r_{rm} \in [r_{mm}, r_b]$ is the domain of r_{rm}[10].

In its client market, BD_{rm} conveys M to RM as a repo loan secured by collateral of T. BD_{rm} carries the T into the Inter-dealer trade. The first argument of BD_{rm}'s problem is its profit from the interdealer repo trade. The second term, $gap(BD_{rm}))$, represents BD_{rm}'s excess inventories; is its loss incurred on borrowings of M from a bank used to extend the loan to RM in its client market, that it is unable to refinance in the Inter-dealer trade.

To analyze BD_{rm}'s problem we partition it into two parts; (i) the case where BD_{rm} carries excess inventories and (ii) the case where BD_{rm} does not carry excess inventories. We appeal to the demonstration for BD_{mm} in Section 3.4.1 as sufficient to show that BD_{rm} carries no excess inventory in equilibrium. Proposition 3.1 follows from the fact that neither BD will carry excess inventory into the Inter-dealer trade.

Proposition 3.1 (No excess inventory in Inter-dealer trade Equilibrium)**.** $D_{r_{mm}} = S_{r_{rm}}$ *in equilibrium, i.e. neither* BD *will carry excess inventories into the interdealer trade.*

3.5.1 BD_{rm} carries no excess inventories of T into the interdealer trade. $gap_{S,Q} = 0$

The domain of the problem is $r_{rm} \in [r_{mm}, r_b]$ such that $S_{r_{rm}} \leq D_{r_{mm}}$. It is a compact interval[11]. BD_{rm}'s problem is

$$\underset{r_{rm}}{\operatorname{argmax}} \frac{1}{2}(r_{rm} - r_{mm})S_{r_{rm}} \tag{7}$$

BD_{rm}'s problem is a continuous concave function. The concavity reflects the tension between the value of increasing transaction amount $S(r_{rm})$ against the decreased return, r_{rm}, required to obtain it, which reduces profit margin $\frac{1}{2}(r_{rm} - r_{mm})$.

[10] r_b is the upper bound of the domain of r_{rm} because it is the bank lending rate at which the RM's can borrow. The domain follows from the requirement that BD_{rm}'s profit is non-negative.

[11] The proof of compactness is nearly identical to the proof for the case of BD_{mm} above.

3.5.2 Characterizing BD_{rm}'s decision problem

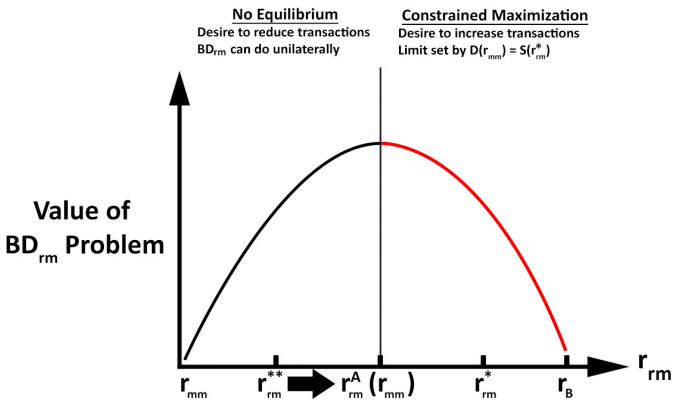


Figure 4: BD_{rm} Problem

Figure 4 shows the curve defining the value of BD_{rm}'s problem at different repo rates for a given value of r_{mm}.

The value if BD_{rm}'s problem is zero at $r_{rm} = r_{mm}$ and at $r_{rm} = r_b$. The concavity of the problem ensures that it is single-peaked between these two endpoints. This peak is depicted in Figure 4 as $r^A_{rm}(r_{mm})$. BD_{rm} will choose the maximum repo rate that (i) is not below $r^A_{rm}(r_{mm})$ and (ii) does not cause $gap(BD_{rm})) > 0$. To see (i), note that BD_{rm}'s problem is positively sloped to the left of $r^A_{rm}(r_{mm})$ and it reduces inventory by increasing r_{rm}. Therefore, it it will increase profit by increasing r_{rm}, since $gap(BD_{rm}))$ will shrink. (ii) reflects that BD_{mm} will not carry excess inventory in equilibrium.

To gain further insight we evaluate BD_{rm}'s decision at repo rates on either side of the maximum. Suppose r^*_{rm} in Figure 4 is the repo rate at which $gap(BD_{mm}) = 0$. BD_{rm} will not reduce r_{rm}, as doing so would cause $gap(BD_{rm}))$ to increase above zero. Since the value of BD_{rm}'s problem is decreasing at r^*_{mm}, BD_{rm} maximizes its profit by choosing r^*_{rm}, which we call a constrained outcome repo rate for BD_{rm}[12]. On the other hand, at r^{**}_{rm} in Figure 4 BD_{rm}'s problem is positively sloped, so BD_{rm} would increase its repo rate to $r^A_{rm}(r_{mm})$. We call this the unconstrained maximum repo rate for BD_{rm}.

3.5.3 Application to a final-sale market

The model presented in this section and discussed below can be applied to a final-sale market, such as the secondary market for US Treasuries. The translation is as follows. MM becomes the ultimate buyer. RM becomes the ultimate seller. r_{mm} becomes the unit price p_{mm} at which MM purchases T from BD_{mm}. r_{rm} becomes the unit price p_{rm} at which RM sells T to BD_{rm}. r_b becomes p_{bd} becomes the unit price at

[12]For BD_{rm}, any repo rate weakly above $r^A_{rm}(r_{mm})$ is a possible constrained equilibrium repo rate.

which BD_{mm} purchases T from BD_{rm}. It is left as an exercise for the reader to confirm that the analysis above applies to a final-sale market.

4 Multiple Equilibrium

In this section we prove there are multiple pure strategies equilibrium client repo rates chosen by broker-dealers. We first state definitions and concepts that are used in the proof of existence and multiplicity of equilibrium and then we state the proofs.

4.1 Unique balance and the definition of equilibrium

Restrictions are placed on the set of feasible equilibrium client repo rate pairs BD by the non-negative profit requirement and Proposition 3.1, which states that neither BD will carry excess inventories into the Inter-dealer trade in equilibrium. Possible equilibria are restricted to repo rate pairs $\{r_{mm}, r_{rm}\}$ such that (i) $r_{mm} \leq r_{rm}$ and (ii) $D_{r_{mm}} = S_{r_{rm}}$. We call the satisfaction of (i) and (ii) "balanced trade". Figure 5 displays the balanced trade surface (or manifold). The surface is downward sloping due to the fact that $D' > 0$ and $S' < 0$. The intuition from Figure 5 is that, when one repo rate is moved in one direction, up (down), the other repo rate must be moved in the opposite direction, down(up), to maintain balanced trade.

In Figure 5, $\hat{r}$ denotes the identical repo rate for BD_{mm} and BD_{rm} at which the amount of inter-dealer trade is maximized[13]. At any point on the curve that is below and to the right of $\hat{r}$, $r_{mm} > r_{rm}$. Both BD's would earn negative profit at any such repo rate pair. Therefore, equilibrium repo rate pairs occur only on the segment of the curve in Figure 5 lying to the left and above $\hat{r}$.

Definition 4.1 (Unique Balance)**.** *For each $r_{mm}(r_{rm})$ there is a unique $r_{rm}(r_{mm})$ for which $D_{r_{mm}} = S_{r_{rm}}$. We call this property unique balance.*

[13]The proof that trade is maximized at a common repo rate is as follows. We know from (i) that $r_{mm} \leq r_{rm}$. Suppose the inequality is strict. Let $r_{rm} - r_{mm} = 2\epsilon$. Since $D_{r_{mm}}$ and $S_{r_{rm}}$ are continuous functions respectively increasing and decreasing in their repo rates, there is some $\delta > 0$ such that $D_{r_{mm}+\epsilon} - D_{r_{mm}} \leq \delta$ and $S_{r_{rm}-\epsilon} - S_{r_{rm}} \leq \delta$. Therefore, there must be some $\gamma < \epsilon$ such that $D_{r_{mm}+\gamma} = S_{r_{rm}-\epsilon}$. The volume of trade is increased and balanced and $r_{mm} + \gamma \leq r_{rm} - \epsilon$.

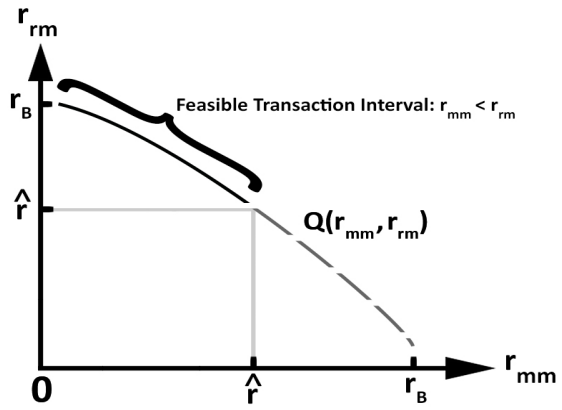


Figure 5: Balanced Inter-dealer Trade Surface

Definition 4.2 states the equilibrium concept that we use.

Definition 4.2 (Inter-dealer trade Equilibrium under perfect knowledge)**.** *A repo rate pair $\{r_{mm}, r_{rm}\}$ is an equilibrium if the following two conditions hold;*
(i) r_{mm} solves BD_{mm}'s problem, given its knowledge of the realizations of $\{\theta_b, \theta_s\}$ and BD_{rm}'s choice of r_{rm}.
(ii) r_{rm} solves BD_{rm}'s problem, given its knowledge of the realization of $\{\theta_b, \theta_s\}$ and BD_{mm}'s choice of r_{mm}.

4.2 Concepts antecedent to the proof of multiple equilibrium

In the section we present definitions and propositions that are used in the proof of multiple equilibrium.

4.2.1 Constrained equilibrium

Definition 4.3. *A 'constrained outcome' is a pair of repo rates $\{r_{mm}, r_{rm}\}$ such that*

(i) $D_{r_{mm}} = S_{r_{rm}}$ and

(ii) The derivative of BD_{mm}'s problem is is ≥ 0 and the derivative of BD_{rm}'s problem is ≤ 0 and

(iii) Either (a) the derivative of BD_{mm}'s problem is > 0, or (b) the derivative of BD_{rm}'s problem is < 0, or both.

Proposition 4.1. *A constrained outcome is an equilibrium*

Proof. Suppose the repo rate pair $\{r_{mm}, r_{rm}\}$ is a constrained outcome.

BD_{mm} will not deviate to a repo rate above r_{mm} , which would result in unsold inventory. BD_{mm} could lower its rate and reduce trade. If the derivative of BD_{mm}'s problem is positive at the constrained outcome, it will not deviate to a repo rate below r_{mm}, since the value of its problem decreases as it lowers its repo rate. Under these conditions BD_{mm} will not deviate.

BD_{rm} will not deviate to a repo rate below r_{rm}, which would result in unsold inventory. BD_{rm} could raise its rate and reduce trade. If the derivative of BD_{rm}'s problem is negative at the constrained outcome, then it will not deviate to a repo rate above r_{rm}, since the value of its problem decreases as it raises its repo rate. Under these conditions BD_{rm} will not deviate.

Therefore, the repo rate pair $\{r_{mm}, r_{rm}\}$ is an equilibrium. □

A BD is constrained if, in equilibrium, the derivative of its problem is not zero. An outcome where each BD attains the maximum of its unconstrained problem is an "unconstrained equilibrium". It is immediate that zero trade is an equilibrium.

4.2.2 Notional complimentarity

Proposition 4.2 (Notional complimentarity)**.** *In the BD problems, an increase in r_{rm} increases the unconstrained maximum repo rate of BD_{mm}'s problem (i.e. shifts r^A_{mm} in Figure 3 to the right). A decrease in r_{mm} reduces the unconstrained maximum repo rate of BD_{rm}'s problem (i.e. shifts r^A_{rm} in Figure 4 to the left).*

Proof. We first prove the proposition for BD_{mm}'s unconstrained problem. The FOC of the BD_{mm} problem is:

$f_{BD_{mm}} = \frac{D_{r^A_{mm}}}{D'} + r_{mm} - r_{rm} = 0$

By the Implicit Function Theorem, we obtain

$\frac{dr^A_{mm}}{dr_{rm}} = \frac{-\frac{\partial f}{\partial r_{rm}}}{\frac{\partial f}{\partial r^A_{mm}}} = \frac{1}{-\frac{D''D}{(D')^2}+\frac{D'}{D'}+1} = \frac{1}{2-\frac{D''D}{(D')^2}} > 0$

Therefore, $\uparrow r_{rm} \implies \uparrow r^A_{mm}$.

We next prove the proposition for BD_{rm}'s unconstrained problem. The FOC of the BD_{rm} problem is:

$f_{BD_{rm}} = r_{mm} - r_{rm} - \frac{S_{r^A_{rm}}}{S'} = 0$

By the Implicit Function Theorem, we obtain

$\frac{dr^A_{rm}}{dr_{mm}} = \frac{-\frac{\partial f}{\partial r_{mm}}}{\frac{\partial f}{\partial r_{rm}}} = \frac{-1}{-2+\frac{S''S}{(S')^2}} > 0.$

Therefore, $\downarrow r_{mm} \implies \downarrow r^A_{rm}$. □

We call the result of this proposition "notional" complimentarity because the solution to the unconstrained BD problems react like strategic compliments. An increase in BD_{rm}'s repo rate increases the repo rate spread and induces BD_{mm} to desire to increase its volume of trade by increasing its repo rate. Similarly, a reduction in BD_{mm}'s repo rate increases the repo rate spread and induces BD_{mm} to desire to increase its volume of trade by reducing its repo rate. However, the friction imposed by the zero inventory constraint prevents these adjustments from taking place. We state a definition derived from notional complementary that is used in the proof of equilibrium.

Definition 4.4. *concavity/complimentarity From the strict concavity of the BD problems and the positive slope of the implicit functions in notional complimentarity, for each $r_{mm}(r_{rm})$ there is a unique $r^A_{rm}(r^A_{mm})$, i.e. unique peaks to the BD maximization problems. We call this property "concavity/ complimentarity".*

4.3 Proof of existence and multiplicity of equilibrium

We make use of the following relationships which are derived from the analysis of the BD problems in Section 3.[14]

(i) When r_{mm} is decreased to reduce inventory of M, BD_{rm}'s unconstrained optimal repo rate is pushed down, but it must do the opposite and increase r_{rm} in order to reduce its inventory of T to match the reduced volume of M. Hence, r_{rm} will increase rightward distance (or reduce leftward distance) relative to the peak, r^A_{rm}, of BD_{rm}'s problem.

(ii) When r_{rm} is increased to reduce inventory of T, BD_{mm}'s unconstrained optimal repo rate is pushed down, but it must do the opposite, and increase r_{mm} in order to reduce its inventory of M to match the reduced volume of T. Hence, r_{mm} will increase leftward distance (or reduce rightward distance) from the peak, r^A_{mm}, of BD_{mm}'s problem.

Theorem 4.1 (Existence of Inter-Dealer Pure Strategy Equilibrium Client repo rate Pairs)**.** *There is a mapping $r_{mm} \mapsto \{r_{mm}, r_{rm}\}$ such that any $r_{mm} \in [0, \hat{r}]$ maps to a repo rate pair $\{r'_{mm}, r'_{rm}\}$ that is on the Feasible Transaction Interval of the Balanced Trade Surface in Figure 5 and is an equilibrium.*[15]

Proof. We use an algorithm to find an equilibrium.

Step 1. Fix an arbitrary client money-market repo rate $r_{mm} \in [0, \hat{r}]$. Interpret this as the initial quote r_{mm} of dealer $BD_{r_{mm}}$. We will construct a pair $\{r'_{mm}, r'_{rm}\}$ and thereby a mapping $r_{mm} \mapsto \{r'_{mm}, r'_{rm}\}$ with

[14] Although we have shown that zero trade is an equilibrium, we provide another proof of existence of equilibrium and use the machinery employed in that proof to prove multiplicity.

[15] An equivalent theorem proves the mapping $r_{rm} \mapsto \{r_{mm}, r_{rm}\}$.

the desired properties.

Step 2. Choose a repo-market client rate r_{rm} such that money-market demand and repo-market supply balance,

$$D_{r_{mm}} = S_{r_{rm}}.$$

By the unique balance property, this condition defines a one-to-one mapping between r_{mm} and r_{rm}. At this point the pair (r_{mm}, r_{rm}) lies on the Balanced Trade Surface, as depicted in Figure 5. Steps 2–6 will place r_{mm} weakly to the left of its unconstrained maximum (see Figure 6).

Step 3. If r_{mm} is above the repo rate that maximizes BD_{1i}'s objective, we proceed to adjust r_{mm} as in Step 4. If instead r_{mm} is already on the constrained side of its maximizer (i.e., weakly to the left of the peak in Figure 6), then no adjustment to r_{mm} is needed and we may skip directly to Step 6.

Step 4. When r_{mm} is above its unconstrained maximizer, choose r_{mm} again so that

$$r_{mm} = r^A_{mm}(r_{rm}),$$

where $r^A_{mm}(r_{rm})$ denotes the argmax of $BD_{r_{mm}}$'s problem given r_{rm}. By concavity and complementarity, this map is one-to-one. Economically, this moves $BD_{r_{mm}}$'s client repo rate leftward to the peak of its payoff function. Since the money-market demand function $D_{r_{mm}}$ is increasing in r_{mm}, this leftward move reduces M, the inventory carried by $BD_{r_{mm}}$.

Step 5. Given the new r_{mm} from Step 4, choose r_{rm} again so that

$$D_{r_{mm}} = S_{r_{rm}}.$$

By the unique balance property, this mapping from r_{mm} to r_{rm} is again one-to-one. Because Step 4 reduced $BD_{r_{mm}}$'s inventory, we must reduce BD_rm's inventory correspondingly. This requires increasing r_{RM_j}, since the repo-market supply function $S_{r_{rm}}$ is decreasing in r_{rm}. By the notional complementarity property, this increase in r_{rm} raises r^A_{mm} while r_{mm} itself remains fixed. Hence r_{mm} remains in the constrained outcome region of $BD_{r_{mm}}$'s problem, on the left-hand side of the peak shown in Figure 6, while the balance condition keeps the pair on the Balanced Trade Surface.

Step 6. If r_{mm} was already in a constrained outcome position at Step 2, then Steps 3–5 are not applied, and r_{mm} is left unchanged. In either case, after Step 6 we have a pair (r_{mm}, r_{rm}) on the Balanced Trade Surface with $BD_{r_{mm}}$ in its constrained region (weakly to the left of its unconstrained maximizer).

Step 7. We now adjust r_{rm} while keeping r_{mm} in the constrained region. Steps 7–10 place r_{rm} weakly to the right of its unconstrained maximum (see Figure 7). If r_{rm} is below the repo rate that maximizes BD_{rm}'s

problem, we proceed to Step 8. If instead r_{rm} is already in the constrained maximization region (weakly to the right of its peak), we skip directly to Step 10.

Step 8. When r_{rm} is below its unconstrained maximizer, set

$$r_{rm} = r_{rm}^{A}(r_{mm}),$$

where $r_{rm}^{A}(r_{mm})$ is the argmax of BD_{rm}'s objective for given r_{mm}. By concavity and complementarity, this map is one-to-one. This step moves BD_{rm}'s client repo rate rightward to the peak of its problem. Since the supply function $S(\cdot)$ is decreasing in r_{rm}, raising r_{rm} reduces T, the inventory carried by BD_{rm}. By the notional complementarity property, this increase in r_{rm} raises r_{mm}^{A} while r_{mm} remains fixed. Thus r_{mm} is on the left-hand side of its peak with an excess inventory of M, while r_{rm} is at the unconstrained peak of BD_{rm}'s problem.

Step 9. Given the updated r_{rm} from Step 8, choose r_{mm} such that

$$D_{r_{mm}} = S_{r_{rm}}.$$

This balance condition again defines a one-to-one mapping by uniqueness. To offset the inventory reduction of BD_{rm} induced in Step 8, we decrease r_{mm} so as to reduce $BD_{r_{mm}}$'s inventory. By the notional complementarity property, this decrease in r_{mm} shifts r_{rm}^{A} leftward in BD_{rm}'s problem while leaving r_{rm} fixed. Consequently, r_{rm} remains in the constrained maximization region of BD_{rm} (on the right-hand side of the peak in Figure 7), and r_{mm}—already on the left-hand side of its peak—moves further away from its unconstrained maximizer. At this point, both $BD_{r_{mm}}$ and BD_{rm} are in their constrained optimization regions, and neither carries excess inventory. The pair $\{r_{mm}, r_{rm}\}$ is therefore an equilibrium point on the Balanced Trade Surface, and by the inventory constraints it lies on the Feasible Transaction Interval.

Step 10. If, in Step 7, r_{rm} was already in a constrained maximization position, then Steps 8–9 are not needed. The existing pair $\{r_{mm}, r_{rm}\}$ already has both dealers in their constrained regions with balanced inventories, so it is an equilibrium, again lying on the Feasible Transaction Interval of the Balanced Trade Surface.

Finally, denote by r'_{mm} and r'_{rm} the equilibrium client repo rates obtained at the end of this procedure for a given initial r_{mm}. The construction above defines a mapping $r_{mm} \mapsto \{r'_{mm}, r'_{rm}\}$ from $[0, \hat{r}]$ into equilibrium repo rate pairs on the Feasible Transaction Interval of the Balanced Trade Surface, which proves the theorem. □

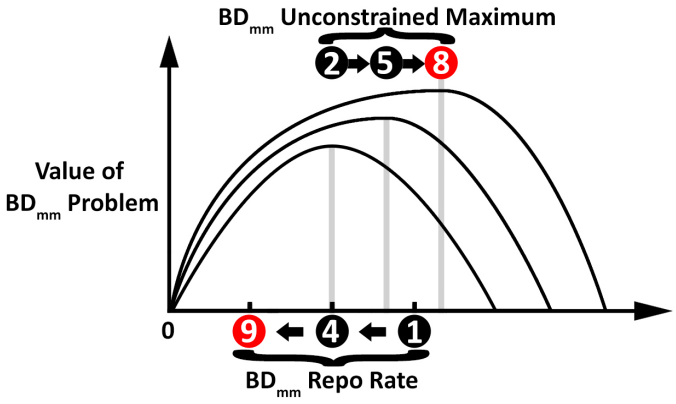


Figure 6: Sequence of r_{mm} steps

Figure 6 depicts the proof sequence of BD_{mm} argmax (notional) repo rates (top). They are identified by gray vertical lines and circled numbers at the top. It also depicts the proof sequence of actual repo rates (bottom). They ae identified by marks in the horizontal axis and circled numbers at the bottom. In the sequence of proof steps BD_{mm}'s argmax repo rate progressively increases and its actual repo rate progressively decreases. BD_{mm} is moved to its argmax repo rate in step 4 and stays on the LHS of its argmax repo rate in all subsequent proof steps. .

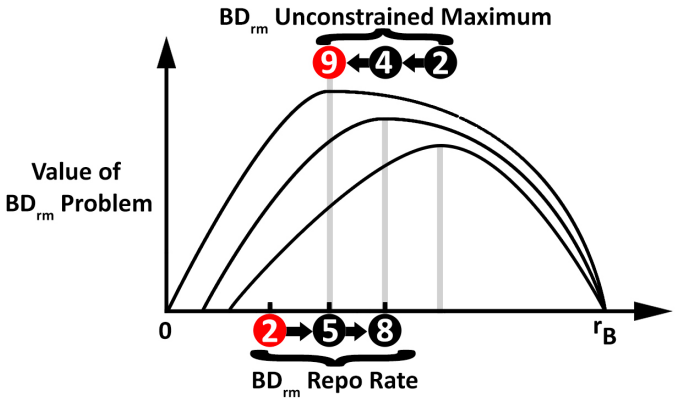


Figure 7: Sequence of r_{rm} steps

Figure 7 depicts the proof sequence of BD_{rm} argmax (notional) repo rates (top). They are identified by gray vertical lines and circled numbers at the top. It also depicts the proof sequence of actual repo rates (bottom). They ae identified by marks in the horizontal axis and circled numbers at the bottom. In the sequence of proof steps BD_{rm}'s argmax repo rate progressively decreases and its actual repo rate progressively increases. BD_{rm} is moved to its argmax repo rate in step 8 and stays on the RHS of its argmax repo rate in all subsequent proof steps.

The mapping $r_{mm} \implies \{r_{mm}, r_{rm}\}$, generated by the iterative process in the algorithm, is a composition of 1:1 mappings. Therefore, it is itself 1:1. This implies there is a multiplicity of equilibrium, since any value in the range $r_{mm} \in [0, \hat{r}]$ can be chosen at step 1 of the algorithmic proof of Theorem 4.1.

Theorem 4.2 (Multiplicity of Inter-Dealer Equilibrium repo rate Pairs)**.** *There are multiple equilibrium repo rate pairs* $\{r_{mm}, r_{rm}\}$.

Proof. Each of steps 2, 4, 5 8 and 9 of the algorithmic proof of the Existence of Equilibrium repo rate Pairs in Theorem 4.1 is a 1:1 mapping. The mapping $r_{mm} \implies \{r_{mm}, r_{rm}\}$ is a composition of those maps. The composition of 1:1 functions is a 1:1 function. Therefore, two distinct repo rates, r_{mm} and r^*_{mm}, selected in step 1 of the algorithm will each map to distinct repo rate pairs $\{r_{mm}, r_{rm}\}$. This proves there is more than one equilibrium repo rate pair. □

5 Joint Profit Maximization

In this section we demonstrate that broker-dealer joint profit maximization is an equilibrium. This results from the surplus allocation rule whereby each broker-dealer receives a fixed percentage of the surplus generated by the spread between the borrower and lender repo rates. Any deviation from the joint profit maximization point $\{r_{rm}, r_{mm}\}$ will reduce profit for both broker-dealers.

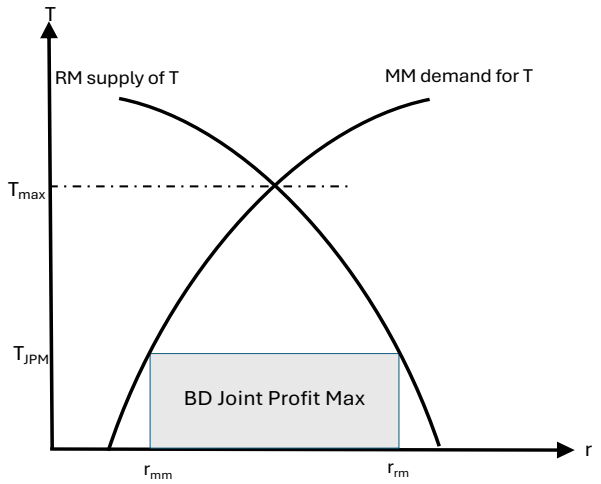


Figure 8: BD Joint Profit Maximization

From Figure 8 it can be seen that joint BD profit at any volume of trade T is the area of the rectangle with height T and sides $r_{mm} = D^{-1}(T)$ and $r_{rm} = S^{-1}(T)$. As T increases, the height of the rectangle rises and the width falls. At some volume of trade the area will reach its maximum. To analyze BD welfare,

we conceive of a planner who controls repo rates (and thereby the volume of trade), which is the same instrument the BD's control. Since BD's are risk neutral and their profit is their utility, BD utility is transferable. Therefore, maximizing BD welfare is equivalent to maximizing joint BD profits, which is represented by the area of the rectangle in Figure 8. The planner solves the following problem

$$\underset{r_{mm},r_{rm},T}{\operatorname{argmax}} (r_{rm} - r_{mm})T \text{ subject to } D(r_{mm}\theta_b) = T \text{ and } S(r_{rm}, \theta_s) = T \tag{8}$$

Noting that $r_{mm} = D^{-1}(T)$ and $r_{rm} = S^{-1}(T)$, we rewrite the problem in terms of T.

$$\underset{T}{\operatorname{argmax}}(S^{-1}(T) - D^{-1}(T))T \tag{9}$$

The FOC is

$$f(T) = \underbrace{S^{-1}(T) - D^{-1}(T)}_{repo\ rate\ spread} + \underbrace{T\{\frac{dS^{-1}(T)}{dT} - \frac{dD^{-1}(T)}{dT}\}}_{slope\ divergence} = 0 \tag{10}$$

$f(T)$ is continuous C2 function, since RM and D are C2 functions. D^{-1} and S^{-1} are, respectively, decreasing and increasing convex functions, since D and RM are, respectively, monotone increasing and monotone decreasing concave functions. These observations imply the following relationships.

$\frac{dS^{-1}(T)}{dT} < 0$ and $\frac{dD^{-1}(T)}{dT} > 0 \implies \frac{dS^{-1}(T)}{dT} - \frac{dD^{-1}(T)}{dT} < 0$

$D^{-1}(T_{max}) = S^{-1}(T_{max})$ (by definition).

$T = 0 \implies r_{mm} = 0$ and $r_{rm} = r_B \implies S^{-1}(0) - D^{-1}(0) = r_B > 0$

These facts enable us to state a proposition concerning BD joint profit maximization.

Proposition 5.1 (BD Joint Profit Maximization Repo Rates - Existence)**.** *There is a repo rate pair which attains BD joint profit maximization.*

Proof. See Appendix B □

We can prove that joint profit maximization is an equilibrium.

Corollary 5.1.1 (BD Joint Profit Maximization Equilibrium)**.** *There is a joint profit maximizing equilibrium.*

Proof. Proposition 5.1 states that there is a repo rate pair that achieves BD joint profit maximization. Consider a proposed equilibrium at that repo rate pair. It is immediate that since BD's split the surplus generated by their client transactions, neither BD will deviate and change its client repo rate. Consider BD_{rm}. If it deviates and increases r_{mm} it will reduce the spread and it cannot increase transaction volume, so it will carry excess inventories. Therefore, it will not deviate upward. Consider a deviation that reduces r_{mm} to $r^{'}_{mm}$. The transaction volume is reduced and the spread is increased. But it is increased by weakly less than if BD_{mm} increases its client repo rate to $r^{'}_{rm}$ to reduce its inventories to match BD_{rm}'s inventory level. Yet, by assumption the profit at $\{r_{mm}, r_{rm}\}$ cannot exceed joint profit maximization. So there is no possible gain to BD_{rm} from deviating. A similar argument applies to BD_{mm}.[16] □

5.1 Minimum spread

Section 2.6 cited empirical evidence for the use by banks of a minimum spread in allocating balance-sheet to repo intermediation. Suppose each BD faces a hurdle marginal profit rate $\pi \geq \underline{\pi}(BD)$ that it must achieve to enter into an inter-dealer trade. WLOG let BD_{rm} face the higher minimum spread. This contracts the feasible transaction interval on the balanced trade surface of Figure 5 away from $\hat{r}$ to $1/2(r_{rm} - r_{mm}) \geq \underline{\pi}(BD_{rm})$ and pushes down T_{max} in Figure 8. This limits the range of possible equilibria, but it does not otherwise change the problem. There is at least one joint profit maximization volume of T. It is immediate that the proofs of Proposition 5.1 and Corollary 5.1.1 apply to the contracted interval of feasible trade when there is a minimum spread imposed. We state this result in Corollary 5.1.2

Corollary 5.1.2 (Joint Profit Maximization Equilibrium with a minimum spread)**.** *When a minimum spread $1/2(r_{rm} - r_{mm}) \geq \underline{\pi}$ is imposed, there is a joint profit maximization equilibrium.*

We refer to this as the "constrained joint profit maximization" equilibrium.

6 Equilibrium Under Asymmetric Information

Our model can be extended to the case where the client shock of one broker is unobserved by the other broker. Each BD chooses its repo rate after observing its own client shock, but does not know the shock realization of its counterparty's client. In Appendix C we prove there are multiple equilibria when this information pattern holds. We define an asymmetric equilibrium as follows.

Definition 6.1 (Equilibrium under Asymmetric Information)**.** *In a model where each BD sets its repo rate*

[16] It is not possible to prove there is a unique repo rate pair that achieves BD joint profit maximization. This can be seen by examining the properties of $f(T)$. The repo rate spread is decreasing in T, since r_{mm} increases and r_{rm} decreases in T. The concavity of the demand and supply functions ensure the slope divergence is decreasing in T, but slope divergence is multiplied by T, making it impossible to pin down the sign of the second term without knowing the parameters of the functions.

after observing its own client shock, but not its counterparty's client shock, and the distribution of clients shocks is common knowledge, a pair of strategies S_1, S_2 is an equilibrium if the following holds;

$S_1 : \theta_{mm} \implies r_{mm}$, *and*

$S_2 : \theta_{rm} \implies r_{rm}$

such that S_1 and S_2 are equilibrium strategies.

Appendix C contains proofs of the existence and multiplicity of equilibrium under asymmetric information.

Theorem 6.1 (Existence of Asymmetric Equilibrium Strategies)**.** *There is an inventory T such that the strategies of setting repo rates to achieve T volume of transaction is an equilibrium.*

$S_1 : \theta_{mm} \implies r_{mm}$ *such that* $D_{r_{mm},\theta_{mm}} = T$

$S_2 : \theta_{rm} \implies r_{rm}$ *such that* $S_{r_{rm},\theta_{rm}} = T$

Proof. See Appendix C □

Theorem 6.2 (Multiplicity of Asymmetric Equilibrium Strategies)**.** *There are multiple equilibrium strategies under asymmetric information involving inventory volumes of T, i.e. there are at least two values T and $T^{'}$ such that (i) $T \neq T^{'}$ and (ii) there is an equilibrium in which each BD transacts T inventory with it client and there is another equilibrium in which each BD transacts $T^{'}$ inventory with its client.*

Proof. See Appendix C □

The proof is intricate, but the key insight is that BD's can coordinate on inventories. There are multiple values of T such that it is an equilibrium for BD_{mm} to set r_{mm} to convey T to its client and for BD_{rm} to set r_{rm} to receive T from its client, even though neither BD knows the counterparty repo rates required to achieve that outcome.

7 Balance-Sheet Costs and Trade Restrictions

In this section we evaluate the effect of balance-sheet cost incurred by broker-dealers in restricting the volume of repo trade they can intermediate. The constraint arises from the interaction of accounting rules and bank capital regulations. A broker-dealer engages in a matched-book trade where it passes on at each leg the money and collateral provided by the clients, retaining only its profit. A broker-dealer does not need to hold an inventory of money or collateral between the first-leg and second-leg. Nevertheless, GAAP accounting rules require the broker-dealer to increase its recorded assets by the full first-leg sale price of the T (since repurchase agreements are treated as secured borrowings rather than sales under GAAP) (Bowman

et.al. [9]). At the same time, bank capital regulations – notably the supplementary leverage ratio (SLR) – limit the volume of assets a bank can record on its balance-sheet, as the leverage ratio serves as a non-risk-based constraint on overall leverage (BIS [7]). This creates a shadow price on the allocation of balance-sheet assets – even where there is no underlying inventory.[17] Denote c the marginal shadow cost of balance-sheet. This equates to $c = p_1$, where p_1 is the first-leg unit price of T. Let $p_1 = p_i^M - h$ for all trades, where p_1^M is the market price of T at the first-leg date and h is the haircut.[18] Each broker-dealer makes one purchase of T, so its shadow price is c per unit of T. We assume the bank allocates capacity to its broker-dealer affiliate prior to the commencement of trading, in which case the allocation decision is based on the ex-ante expected profit of trading. Equation 11 is the hurdle rate of return for each broker-dealer, expressed as a constraint on the client repo rate spread.

$$\mathbb{E}\left[\frac{1}{2}(r_{rm} - r_{mm})\right] \geq c \tag{11}$$

The feasible range of the spread is $[0, r_B]$ (Figure 5), which extends below the hurdle rate. An ex-ante distribution over multiple equilibrium repo rate pairs may yield an expectation that is below the hurdle rate, even though there are equilibria that exceed the hurdle. For example, suppose there are two equilibrium repo rate pairs $\{\{12, 6\}, \{8, 6\}\}$, $c = 2.5$ and an equal ex-ante probability of each equilibrium. The ex-ante expected unit profit is 2, which is below the shadow price, even as one of the equilibria has a spread of 3. In this case no trade will occur even though there is an equilibrium that generates a profit.

8 A Smart Contract That Resolves Multiple Equilibrium

In this section, we describe a mechanism that resolves multiple equilibrium by selecting the constrained joint profit maximization equilibrium. It coordinates between all four participants, clients, and broker-dealers, to select a trade that satisfies the minimum spread of return and reflects the incentives of broker-dealers to maximize intermediation profit. There are three additional attractive features of the smart-contract.

- There is no leakage of client schedules to the counterparty BD, or to any other party.
- Truthful reporting to the smart-contract by each BD of its client schedule and minimum spread is a dominant strategy for the former and a weakly dominant strategy for the latter. This minimizes the computational and knowledge burden imposed on the BD's to implement their strategies.

[17] See Aronoff et al. [4] for a detailed explanation of repo accounting rules and capital regulations and an analysis of their interaction.

[18] This assumption reflects empirical U.S. Treasury repo market practice, where most such repo transactions are executed with very low or zero haircuts (Hempel et.al. [34]).

- The smart-contract achieves these results without requiring trust between agents or the smart-contract. It does so by use of cryptographic methods and computer hardware that protect privacy while enabling the smart-contract to be audited.

Finally, not only does the smart-contract maximize welfare of broker-dealers, it also improves welfare of the clients in a limited sense. The smart-contract ensures that trade will occur if there is at least one equilibrium that satisfies the minimum spread of each broker-dealers. This is not assured in the absence of the smart-contract.

8.1 Smart contract preliminaries

The broker-dealers split surplus and are price-setters in their respective client markets. We modify Equation 8 to express the constrained joint profit objective.

$$\underset{T}{\operatorname{argmax}} \underbrace{(S^{-1}(T) - D^{-1}(T))}_{\text{intermediation profit}} - \underbrace{\underline{\pi} T}_{\text{shadow cost}} \tag{12}$$

Where $\underline{\pi}$ is the highest minimum spread between the two broker-dealers. The solution is to select the joint profit maximization equilibrium in the region of repo rate pairs that exceeds the hurdle. Corollary 5.1.2 showed that such a joint profit maximization configuration of trades exists. Our smart-contract achieves this outcome without imposing any informational requirements on any of the agents. Not only can it handle asymmetric information on client schedules, it achieves joint profit maximization when the broker-dealers are ignorant of any feature of the counterparty's client supply or demand schedule for T. A smart-contract is used to achieve this result. Notably, agents interacting with a smart-contract do not rely on trust in a individual or an organization, but rather they rely on the correct execution of the code.

Definition 8.1. *A smart contract is a self-executing digital agreement with the terms of the agreement directly written into lines of code, stored on a single server or a blockchain. When predefined conditions are met, the contract automatically executes the corresponding actions, which can include moving objects between accounts.*

State space: Agents agree to trades on a grid of T-volumes $\mathbb{T} = \{T_1, .., .T_i, ..., T_I\}$.

First-leg and second-leg The smart-contract generates two things; (i) the joint profit maximization tuple of repo rates and trade volume $\{r_{rm,*}, r_{mm,*}, T^*\}$ and (ii) the first and second-leg client prices, $\{p_{1,rm}, p_{1,mm}$ and $\{p_{2,rm}, p_{2,mm}\}$. This implies an first-leg spread of $\frac{1}{2}(p_{1,rm} - p_{1,mm})T$ and similarly for the second-leg.

Messages and objects The smart-contract receives messages from broker-dealers containing client trade schedules and minimum spreads and sends first-leg and second-leg contracts to agents. When traded objects

M and T are appended to blockchains and held in agent wallets, the smart-contract operates an escrow that receives and sends first-leg objects with agents. When objects are held as accounts at banks, the first-leg escrow is held by the agent's bank. The banks message the smart-contract the receipt of objects in escrow and the smart-contract messages the banks with instructions for moving objects between banks and agent accounts.

No collusion. We place one restriction on agent behavior. We assume that client broker pairs trade at the repo rate reported to the smart-contract. This prohibits e.g. BD_{mm} from reporting at volume T an r'_{mm} which is above the r_{mm} on the client schedule. This could be implemented with a side payment where BD_{mm} obtains from MM a more favorable surplus split on $(r'_{mm} - r_{mm})T$ than it can obtain from BD_{rm} (the area denoted by A in Figure 9).

8.2 Smart-contract protocol

The smart-contract protocol works in the following sequential order.

1. Each client- broker-dealer pair agrees to a trade schedule $\{r_i, T_i\}_{i=1}^{T_{max}}$ repo-rate: security pairs it is willing to trade, one for each value of T (the first-leg and second-leg prices are agreed to and embedded in the repo rates).

2. - Each BD reports the smart-contract its client trade schedule, signed by itself and its client.
   - The smart-contract selects one BD to send its minimum spread of return (the "first-mover").
   - Each client sends its deposit to the smart-contract.

   ▷ (Default) If a client fails to send its required object the contract is terminated and the client's deposit is distributed to the non-defaulting agents.

   ▷ The protocol is compatible with any formula for distributing the deposit.

3. The smart-contract sends the first-mover's minimum spread to the other BD (the "second-mover").

4. The second-mover either accepts the first-mover's minimum spread, or reports a new minimum spread above the first-mover's minimum spread.

   ▷ The final minimum spread is the "spread-constraint" used by the smart-contract.

5. the smart-contract computes the spread for each trade volume and discards any T_i for which the spread is below the spread-constraint.

   ▷ If all $\mathbb{T}$ are discarded the trade is aborted and deposits are returned.

6. The smart-contract compares the profit at each remaining trade volume, $(r_{rm,i} - r_{mm,i})T_i$ and selects the trade volume T^* that achieves joint profit maximization.

7. The first-leg and second-leg contracts between each client and its BD automatically fix at the prices associated with T^* from each client trade schedule, which construct $(r_{rm,*}, T^*)$ and $(r_{mm,*}, T^*)$ respectively. The interdealer repo rate is fixed at $r_{bd} = \frac{1}{2}(r_{rm,*} + r_{mm,*})$.

   ▷ The smart-contract reports the contract terms to each agent, which is binding.

8. (Execution step): The smart-contract causes objects to be sent to each agent in accordance with the first-leg contracts.

   ▷ Any excess deposit is refunded to the clients.

8.3 Equilibrium in the smart contract game

Here we prove existence and uniqueness of the constrained joint profit maximization equilibrium under the smart-contract.

8.3.1 Deposits and margin

The smart-contract is a sequence of steps. There is an interval of time between the commitment to contracts (Step 1) and the execution of trade (Step 5). During this gap a client might decide to defect. MM could find a higher return and RM could find a lower borrowing cost in other markets due to shocks that occur in the time interval.[19] The first instantiation of the deposit is designed to prevent this from occurring.

Client compliance at the first-leg is assured (or becomes irrelevant) if the maximum volume of objects are placed into escrow ("max deposit"). RM sends T_I into escrow and MM sends the maximum first-leg price, $M = p_{1,mm,i}T_i$ on its client schedule. In that case there is no subgame where a broker-dealer is required to send an object to meet its inter-dealer obligation because there are sufficient committed objects to complete any trade. [20]

Compliance of all agents at the second-leg is assured by methods already in place in the repo market, namely haircuts and margin. The former is a subsidy on the first-leg price to over-collateralize the loan made by MM (in the form of the first-leg purchase). Margin is an additional escrow payment made periodically when the time lag between the two legs is beyond one day. The basic idea of margin is to place

[19] A counter-argument is that the time interval may be so small as to practically eliminate the possibility of a new outside option appearing. In that case no deposit would be necessary.

[20] An initial deposit is required to ensure execution at the first-leg because the legal penalty for defaulting on the contract before execution under e.g. the SIFMA master repurchase agreement (1996) paragraph 11 "Default and Remedies", is typically the counterparties lost profit, to the extent it cannot be mitigated. This is likely to be a small amount when the contract is negotiated in a competitive market. See e.g. the SIFMA master repurchase agreement (1996) paragraph 11 "Events of Default" `https://www.sifma.org/documents/master-repurchase-agreement-mra-2/`.

the counterparties in a position where each is unaffected by a default by the other. For the current margin formula in the cleared US Treasuries repo market see [25].

8.3.2 Reporting client schedules

Proposition 8.1. *It is a dominant strategy for each BD to truthfully report client schedules in the simultaneous move game of the smart-contract protocol.*

Proof. We refer to the surplus-splitting rule for allocating fees $1/2(r_{rm} - r_{mm})T$, Figure 9 below and the assumption of no collusion between broker-dealers and their clients. We first address BD_{mm}. There are two cases to consider; At some volume T, BD_{mm} reports a client repo rate above the rate on the MM client schedule, or BD_{mm} reports a client repo rate below the rate on the MM client schedule.

Report repo rate higher than the MM schedule at some T: In this case BD_{mm} reports a client repo rate r'_{mm} that is above the repo rate r_{mm} on its client MM schedule for T. The MM will agree to deliver T since its demand is increasing in T. The surplus splitting rule implies that BD_{mm}'s profit will be 1/2(A +B) when it reports the client schedule repo rate r_{mm}, and it will drop to 1/2(B) when it reports the higher repo rate r'_{mm}. In general, BD_{mm}'s profit from reporting a higher client repo rate will be lower *for any client repo rate reported by BD_{rm}*. This proves that BD_{mm} will never report a client repo rate above its client schedule at any T.

Report repo rate lower than the client schedule at T: This is ruled out by 'no -collusion'. The client is unwilling to fund the trade at r''_{mm}. so it will not report a trade at that volume.

The conclusion is that it is a dominant strategy for BD_{mm} to report the repo rate on its client schedule for all values of T. The same reasoning applies to BD_{rm}. □

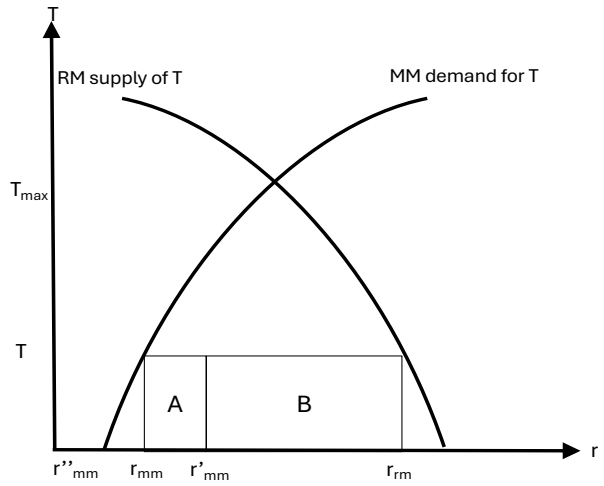


Figure 9: Feasible Trade Graph

8.3.3 Reporting minimum spreads

Proposition 8.2. *Any equilibrium will be consistent with setting the minimum spread at the maximum of the true minimum spreads of the BDs and truthful reporting is a weakly dominant strategy..*

Proof. It suffices to show the following:

(1) That truthful reporting is an equilibrium.

(2) That there cannot be an equilibrium where the spread-constraint used by the smart-contract either excludes a feasible joint profit maximization trade, or allows an infeasible joint profit maximization trade.

Suppose, for concreteness, that BD_{mm} is the first-mover and that the minimum spreads are denoted in Figure 10 by H, the high minimum spread, and L, the low minimum spread.

1. Equilibrium with truthful reporting:

The second-mover, BD_{rm} can either accept the first-mover minimum spread, or report a higher one, which will be the minimum minimum spread constraint on the smart-contract. If its minimum spread is L and it chooses not to make a false report at a spread above H, it will not rule out any joint profit maximization trade in region I. If its minimum spread is H and it chooses not to report a lower rate, it will enable the smart-contract selection of an infeasible trade in region I. If its minimum spread is H and it chooses not to report a higher rate, it not exclude a joint profit maximization trade in the feasible trade in region II.

The first-mover cannot increase joint profit maximization by deviating and reporting an untruthful minimum spread, since, given the reaction of BD_{rm}, truthful reporting will cause the smart-contract to choose the joint profit maximization in the feasible trade in region II.

Consequently, truthful reporting is an equilibrium strategy for the both BD's.

2. Optimality of the spread-constraint

Spread-constraint above the maximum minimum spread: Suppose the smart-contract's spread constraint exceeds H, at H'. If the BD reporting H' believes there is any possibility the joint profit maximization lies between the two, it is profitable to deviate and report a lower minimum spread (bounded by H).

Spread-constraint below the maximum minimum spread: Suppose the smart-contract's spread constraint is below the maximum BD minimum spread, at H". If the BD reporting H" believes there is any possibility the joint profit maximization lies between H and H", it is profitable to deviate and report a higher minimum spread (bounded by H).

Consequently, there cannot be a spread-constraint that rules out a feasible joint profit maximization trade in equilibrium. □

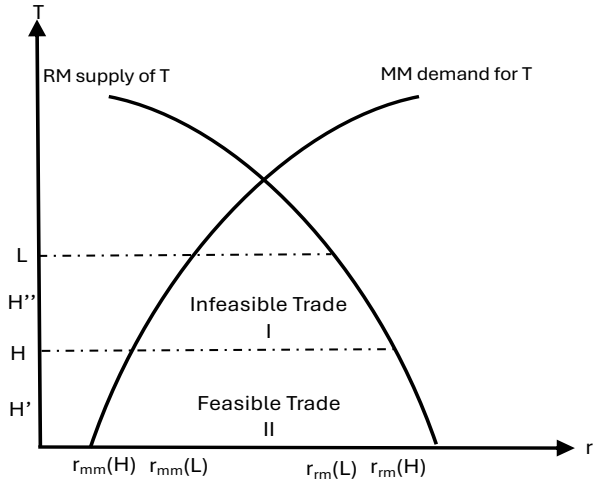


Figure 10: Spread Constraint

A notable feature of these equilibrium results is that they can be implemented by myopic strategies where a BD's reports its own client schedule and minimum spread, without need to consider any competitive response by its counterparty. In a very real sense, this renders the smart-contract cognitively implementable.

8.4 Integrity, privacy and auditability

This section formalizes the cryptographic guarantees delivered by the smart-contract described above. Two broker–dealers, $BD_{r_{rm}}$ and $BD_{r_{mm}}$, privately report schedules of trades. The smart-contract computes and delivers to each BD the single trade $\{r_{rm,*}, r_{mm,*}, T^*\}$ that maximizes joint profit under the pre-programmed

objective in (Equation 12), while revealing nothing else about either schedule. Correctness is publicly auditable via a succinct zero-knowledge proof (zk-SNARK/NIZK), and liveness/fairness are enforced by hash-timelocks with cancellation on timeout. These guarantees are obtained in either of two implementations: (i) a generic hardware protected trusted-execution environment (TEE) and (ii) a public-chain smart contract that verifies zero-knowledge proofs on-chain. In both cases the computation is performed on a server with a TEE. Figure 11 depicts the encrypted incoming messages, the decryption and execution inside the TEE, the recording of the transcript, the re-encryption and sending of the computed output to the parties.

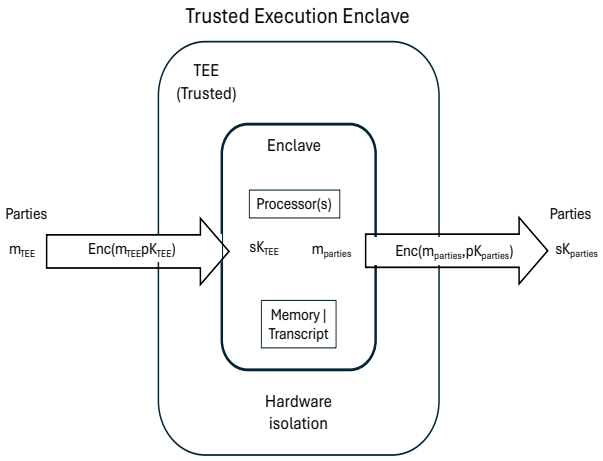


Figure 11: Trusted Execution Enclave

Cryptographic primitives (both implementations). Each BD forms a binding, hiding vector commitment to its schedule (e.g., a Merkle/Pedersen root): C_{rm} commits to $S_{r_{rm}}$ and C_{mm} to $D_{r_{mm}}$. [21] A hash-timelock establishes phased deadlines: COMMIT $\rightarrow$ PROVE/VERIFY $\rightarrow$ DELIVER. If a deadline is missed, the protocol aborts without disclosure. The smart-contract must output only $\{r_{rm,*}, r_{mm,*}, T^*\}$ (encrypted separately to each BD's public key), together with a publicly verifiable proof that $\{r_{rm,*}, r_{mm,*}, T^*\}$ is admissible and profit-maximizing with respect to the posted commitments and the fixed code implementing the smart-contract protocol.[22]

Zero-knowledge statement (high level). The proof attests that: (i) $T^* \in S_{r_{rm}}$ and $T^* \in D_{r_{mm}}$ and satisfies Equation 12 (via commitment - membership proofs against C_{rm}, C_{mm}); (ii) $\Pi(T^*) \geq \Pi(T)$ for every admissible candidate T encoded by the bounded schedule grid $\mathbb{T}$; and (iii) the published ciphertexts $E_{rm} = ENC_{pk_{rm}}(T^*)$ and $E_{mm} = ENC_{pk_{mm}}(T^*)$ are correct encryptions of the same T^*. The ground truth is the transcript inside the TEE. No plaintext schedule elements are revealed.[23]

[21] Catalano and Fiori [10], Merkle [47] and Pederson [50] describe ector commitments.
[22] See Todd [56] and Friedenbach and Lombrozo [26].
[23] Thaler [55] is a standard reference for ZKP methods.

Outcomes

- Integrity. The only accepted outcome is one for which a valid proof links $\{r_{rm,*}, r_{mm,*}, T^*\}$ to the committed schedules and the fixed objective (Equation 12); thus the selected trade is the joint-profit maximizer encoded by the program.
- Privacy. Beyond the delivery of the actual trade, $\{r_{rm,*}, r_{mm,*}, T^*\}$ (privately, as E_{rm}, E_{mm}), no information about $S_{r_{rm}}$ or $D_{r_{mm}}$ is revealed; the proof is zero-knowledge.
- Auditability. Verification of π (and of attestation in the TEE case) lets any BD—or third party—confirm correctness without seeing inputs.
- Fairness/liveness. Hash-timelocks deliver ordering and bounded execution; missed deadlines trigger cancellation without compromising privacy.

Appendix E contains the implementation details.

9 Conclusion

We presented a model of intermediated trade in the U.S. Treasuries repo market. We focused on a repo chain composed of two broker-dealers flanked on either side by, respectively, an ultimate repo borrower and an ultimate repo lender. Each element of our model - from the length of the repo chain to the timing of trading to the functional forms of supply and demand schedules to the bargaining power of agents - is derived from the empirical and theoretical literature. We demonstrated that strategic interactions between agents leads to multiple equilibrium.

We then introduced a smart-contract protocol that resolves the coordination and hurdle challenge by selecting the joint profit maximization volume of trade as an equilibrium, which is implemented by a dominant strategy of broker-dealer truthfully reporting its client schedule. The smart-contract also selects a spread-constraint (between borrowing and lending rates) , which bounds the region of feasible trades. This constraint reflects the shadow-price of allocating balance-sheet capacity to the broker-dealers. We demonstrate that the optimal spread-constraint can be implemented by a weakly dominant strategy of broker-dealer truthfully reporting its true hurdle-constraint.

The smart-contract is a self-executing digital agreement with the terms of the agreement embedded in the code. Agent trust resides in the correctness of the code – which can be audited — rather than the conduct of an individual or entity. Broker-dealers send messages containing contingent trading schedules - repo rates associated with each volume of trade - with their clients and their internal minimum spread of return. the

smart-contract selects the trade that maximizes broker-dealer joint profit, which is an equilibrium. Agent compliance with the rules are enforced by deposits posted by the clients.

Privacy is maintained by encrypting messages between agents and the smart-contract and protecting leakage from the computations. We explored two implementations. In one the computation and auditing takes place inside secure hardware. In the other the encrypted output is appended to a public blockchain for auditing. In both cases privacy of the auditing is achieved by using zero knowledge proofs.

The smart-contract has practical significance.

- It ensures that trading will not be truncated by equilibria that do not meet the minimum spread. This is important in the US Treasuries repo market where there has been concern among regulators and academics that broker-dealers are allocating an insufficient amount of balance-sheet capacity to support the market.
- It can be implemented by the myopic strategy of truthful reporting, which minimizes the cognitive burden on agents by eliminating the need to assess knowledge of counterparties and to simulate counterparty responses.

Finally, this paper contributes to the understanding of how cryptography, hardware and smart-contract technologies can be integrated into market designs to overcome obstacles of trust and coordination and improve economic outcomes.

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A Funding Liquidity

It is remarked by market participants that a feature of the relationship formed between a broker-dealer and its client is that the broker-dealer will guarantee to transact a certain volume. Our model can be extended to the case where a broker-dealer commits to trade a fixed volume with its client in certain client states. Funding commitments induce a feasible trade surface of client repo rates that can result in excess inventories. In this appendix, we prove that the main results of the analysis in the text continue to hold for the case of funding liquidity.

A.1 Funding liquidity equilibrium

We prove that there is a unique equilibrium when a broker-dealer makes a funding commitment to its client.

Proposition A.1 (Existence of Equilibrium with Client Funding Commitments)**.** *There is at least one equilibrium interdealer repo rate pair when a broker-dealer commits to trade a minimum volume of financial asset with its client.*

Proof. **Existence of Equilibrium**

There are three cases to consider. In the first case, each BD makes a funding commitment to its client and both client repo rates are fixed. If $r_{rm}^{fc} \geq r_{mm}^{fc}$ interdealer trade is profitable up to the minimum volume of the two client trades. Otherwise, interdealer trade is not profitable and will not occur. In the remaining two cases one of the BD's makes a funding commitment and fixes its client repo rate, which implies there is no strategic interaction. The other BD chooses a the client repo rate that maximizes its profit.

$\underline{BD_{rm} \text{ makes a funding commitment}}$. In case the feasible transaction interval is the segment of the horizontal line extending from r_{rm}^{fc} on the vertical axis of Figure 12 to its intersection with the vertical line extending from r_{rm}^{fc} on the horizontal axis (the "RM funding commitment transaction interval"). This reflects that the MM client repo rate must be below r_{rm}^{fc} for interdealer trade to be profitable.

$\underline{BD_{mm} \text{ makes a funding commitment}}$. In case the feasible transaction interval is the segment of the vertical line extending from r_{mm}^{fc} on the horizontal axis of Figure 12 that is above r_{mm}^{fc} on the vertical axis, bounded above by r_b (the "MM funding commitment transaction interval"). This reflects that the RM client repo rate must be above r_{mm}^{fc} for interdealer trade to be profitable.

Uniqueness of Equilibrium The concavity of S_{rm} and D_{mm} ensure there is a single repo rate that maximizes profit in each case, as explained in Section 3 □

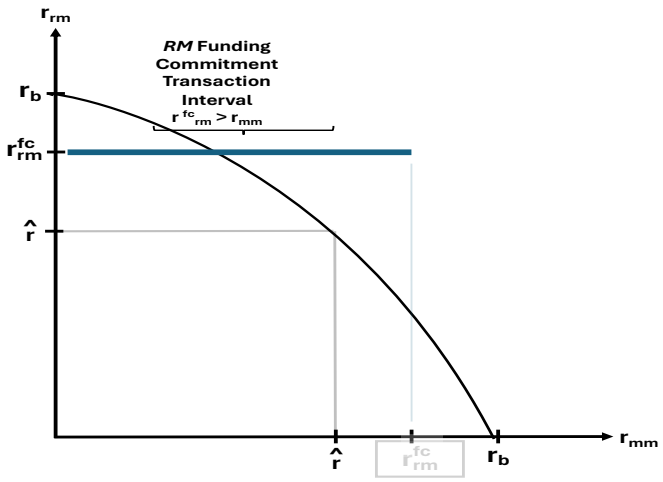


Figure 12: Funding Liquidity BD_{rm} lock

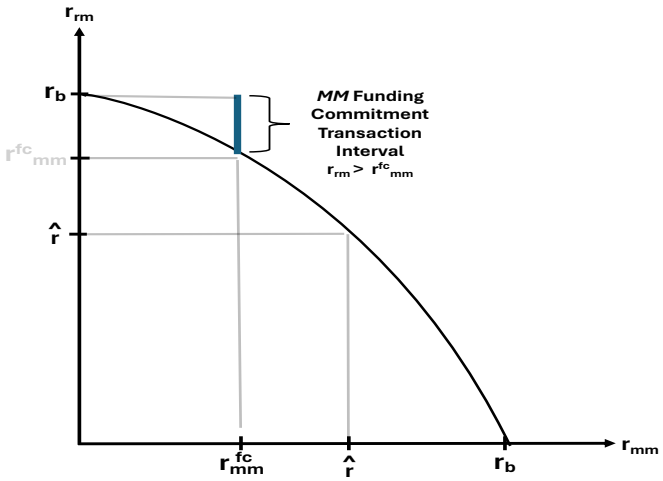


Figure 13: Funding Liquidity BD_{mm} lock

A key takeaway from Proposition A.1 is that a broker-dealer may carry excess inventories in equilibrium due to a commitment to provide funding liquidity to its client. In the case of a commitment to RM, BD_{rm} will carry excess inventories if r_{mm} lies to the left of the intersection of the RM funding commitment interval with the balance transaction surface. In the case of a commitment to MM, BD_{mm} will carry excess inventories if r_{rm} lies below the intersection of the MM funding commitment interval with the balance transaction surface.

B Proof of Existence of Joint Profit Maximization Equilibrium

Proof. Consider the interval $[0, T_{max}]$ of transaction volumes. To prove existence we establish that f is greater than zero at the left endpoint of the interval and less than zero at the right endpoint of the interval.

$f(T_{max}) = T\{\frac{dS^{-1}(T_{max})}{dT} - \frac{dD^{-1}(T_{max})}{dT}\} < 0$ since, by assumption, the price spread $= 0$ at T_{max}.

$f(0) = S^{-1}(0) - D^{-1}(0) = r_{BD} > 0$, since the slope divergence $= 0$ when $T = 0$.

It follows from the continuity of f that there is at least one $T^* \in [0, T_{max}]$ at which $f(T^*) = 0$[24]. This proves existence. □

C Proof of Multiple Equilibrium Under Asymmetric Information

In this appendix we prove the existence of multiple equilibria in the case where each BD chooses its price *after* observing the state to its client demand function. An asymmetric equilibrium is defined as follows.

Definition C.1 (Equilibrium under Asymmetric Information)**.** .
In a model where each BD sets its price after observing its own client state, but not its counterparty's client state, and the distribution of clients states is common knowledge, a pair of strategies S_1, S_2 is an equilibrium if the following obtains;

$S_1 : \theta_{mm} \implies r_{mm}$, *and*

$S_2 : \theta_{rm} \implies r_{rm}$

such that S_1 and S_2 are Bayesian Nash equilibrium strategies.

We assume the support for BD states are intervals over positive values $\theta_{mm} \in [\underline{\theta}_B, \overline{\theta}_B]$ and $\theta_{rm} \in [\underline{\theta}_S, \overline{\theta}_S]$. We present a constructive proof that maps a perfect information equilibrium price pair into an asymmetric equilibrium strategy. First, select an equilibrium price pair conditioned on the minimal state terms $\{\underline{\theta}_B, \underline{\theta}_S\}$. Next, identify the inventory volume T implied by this volume. The claim is that T is an equilibrium repo volume and the repo rates necessary to induce T - for any realization is states - are equilibrium strategies under asymmetric information.

The intuition for the proof of equilibrium is as follows. Take BD_{rm}. Suppose MM's state is $\theta_{mm} > \underline{\theta}_B$. BD_{rm} will not increase its inventory in an equilibrium where BD_{mm} is carrying T into the interdealer market. The remaining question is whether BD_{rm} would reduce its inventory below T. For this to occur

[24] Rudin (1976) [53] Theorem 4.23, p.93.

there must be at least one possible outcome of the proposed strategy in which the price that induces MM to demand T under perfect information is above (to the right of) BD_{rm}'s problem. We know that, at state $\underline{\theta}_B$ the price is weakly below (to the left of) the peak of BD_{rm}'s unconstrained problem. Any increase in state value, which increases MM demand at any price, requires that BD_{rm} reduce its price to induce T. This moves r_{mm} further to the left of BD_{rm}'s initial peak. Moreover, an increase in r_{rm}, which is required for BD_{mm} to induce T for any realization of θ_{rm} above its minimum, will push the peak of BD_{rm}'s unconstrained problem to the right (as demonstrated by the notional complimentary proposition). So far, so good. However, as we demonstrate below, an increase in θ_{mm} shifts the peak of BD_{rm}'s problem to the left. Thus, in order for our proposed strategy to be an equilibrium, we must show that the r_{mm} required to induce T is always weakly less than the peak of BD_{rm}'s problem. In that case, under perfect information, if BD_{rm} observed r_{rm} it would set a price that induced T *for any price chosen by* BD_{mm} *that induces* T.

This implies that BD_{rm} will not deviate from its strategy of carrying T inventories. BD_{rm} does not need to observe r_{rm} because it would choose its strategy no matter what values of r_{rm} occur (if it knew them) in this asymmetric equilibrium. By an analogous argument, we prove that BD_{mm} will not deviate.

To prove there is a multiplicity of equilibrium strategies, note that, for the state pair $\{\underline{\theta}_B, \underline{\theta}_S\}$, we have proven there are multiple perfect knowledge equilibrium price pairs. Since $D(r_{mm}, \underline{\theta}_B)$ is a monotonic function, each equilibrium price pair corresponds to a unique inventory volume T. Therefore, there are multiple strategies, centered on T that can be equilibria under asymmetric information.

A sufficient condition for our proposed strategy to be an equilibrium is that (i) an increase in θ_{mm} induces a leftward movement of r_{mm} that is weakly greater than the leftward movement of r^A_{mm} and (ii) an increase in θ_{rm} induces a rightward movement of r_{rm} that is weakly greater than the rightward movement of r^A_{rm}.

C.1 Economic Intuition

For BD_{rm} an increase in θ_{mm} implies increased MM demand for T at any price. Thus, in order to hold T constant, the price must be reduced. However, in determining its unconstrained maximum, BD_{rm} balances the increased profit from lowering its borrowing rate r_{mm} against the gain from increasing the volume of trade. This tension is reflected in the profit function $\frac{1}{2}(r_{rm} - r_{mm})^+ D(r_{mm}, \theta_{mm})$. The spread term pushes BD_i to lower its borrowing cost, while the demand function pushes it to increase volume, which requires increasing its borrowing cost. Balancing these opposing forces will cause it to make a less than full adjustment and choose a somewhat higher borrowing cost in exchange for a larger trade volume. As a result, the unconstrained maximum price r^A_{mm} will shift downward by a lesser amount than what is required to hold T constant.

An analogous argument rationalization applies to the BD_{mm} adjustment.

C.2 Effect of own client state on the BD Problems Under Perfect Knowledge

Proposition C.1 (Client states on BD Problems)**.** *In the unconstrained BD problems, an increase in θ_{mm} reduces the solution to BD_{rm}'s problem (i.e. shifts r^A_{mm} to the left). An increase in θ_{rm} increases the solution to BD_{mm}'s problem (i.e. shifts r^A_{rm} to the right).*

Proof. We first prove the proposition for BD_{rm}'s problem.

The MM demand function is [25] $D(r_{mm}, \theta_{mm}; r_D, \overline{M}) = T = \frac{1}{r_D}[1 + \overline{M}\theta_{mm}] - \frac{1}{r_{mm}}$, where $\frac{dD}{dr_{mm}} = \frac{1}{(r_{mm})^2}$ and $\frac{dD}{d\theta_{mm}} = \frac{\overline{M}}{r_D}$. The FOC of BD_{rm}'s problem is

$f_1(r_{mm}; r_{rm}, \theta_{mm}) = (r_{rm} - r_{mm})^+ \frac{dD}{dr_{mm}} - D(r_{mm}, \theta_{mm}) = 0,$
which evaluates to$(r_{rm} - r_{mm})^+ \frac{1}{(r_{mm})^2} - \frac{1}{r_D}[1 + \overline{M}\theta_{mm}] + \frac{1}{r_{mm}} = 0.$

By the Implicit Function Theorem we obtain

$$\frac{dr^A_{mm}}{d\theta_{mm}} = \frac{-\frac{\partial f_1}{\partial \theta_{mm}}}{\frac{\partial f_1}{\partial r^A_{mm}}} = -\frac{-\frac{\overline{M}}{r_D}}{-2(r_{rm}-r_{mm})^+\frac{1}{(r_{mm})^3} - \frac{2}{(r_{mm})^2}} = -\frac{\frac{\overline{M}}{r_D}}{\frac{2p_s}{(r_{mm})^3}} = -\frac{\overline{M}(r_{mm})^3}{2r_D r_{rm}} < 0.$$

Therefore $\uparrow \theta_{mm} \implies \downarrow r^A_{mm}$.

We next prove the proposition for BD_{mm} problem. The RM supply function is $S(r_{rm}, \theta_{rm}; r_{mm}) = T = (\frac{\theta_{rm}}{r_{rm}} - 1)$[26]

where $\frac{\partial S}{\partial r_{rm}} = -\frac{\theta_{rm}}{(r_{rm})^2}$ and $\frac{\partial S}{\partial \theta_{rm}} = \frac{1}{r_{rm}}$. The FOC of BD_{mm}'s problem is

$f_2(r_{rm}; r_{mm}, \theta_{rm}) = (r_{rm} - r_{mm})^+ \frac{\partial S}{\partial \theta_{rm}} + S(r_{mm}, \theta_{rm}) = 0$, which evaluates to $-(r_{rm} - r_{mm})^+ \frac{\theta_{rm}}{(r_{rm})^2} + \frac{\theta_{rm}}{r_{rm}} - 1 = 0.$

By the Implicit Function Theorem we obtain

$$\frac{dr^A_{rm}}{d\theta_{rm}} = \frac{-\frac{\partial f_2}{\partial \theta_{rm}}}{\frac{\partial f_2}{\partial r^A_{rm}}} = -\frac{\frac{r_{mm}}{(r_{rm})^2}}{\frac{\theta_{rm}}{(r_{rm})^2}\left[\frac{2(r_{rm}-r_{mm})}{r_{rm}} - 2\right]} = \frac{\frac{r_{mm}}{\theta_{rm}}}{2\left[1 - \frac{(r_{rm}-r_{mm})}{r_{rm}}\right]} = \frac{r_{rm}}{2\theta_{rm}} > 0.$$

For the sign of the denominator to be negative, the expression inside the bracket evaluates to $r_{rm} - r_{mm} < r_{rm}$, which is true by definition.

Therefore $\uparrow \theta_{rm} \implies \uparrow r^A_{mm}$. □

Proposition C.2 (Client states on Demand/Supply Functions)**.** *An increase in θ_{mm} reduces the r_{mm} required to induce MM demand $M = T$. An increase in θ_{rm} increases the r_{rm} required to induce RM supply of T.*

[25] We normalize $p_T = 1$.

[26] The support of repo rates is $[0, r_{mm}]$, so r_{mm} does not appear in BD_{mm}'s supply function.

Proof. We first prove the proposition for BD_{rm}.
The BD_{rm} client price required to induce $M = T$ is derived from $D(r_{mm}, \theta_{mm}) = \frac{}{r_D}\left[1 + \overline{M}\theta_{mm}\right] - \frac{1}{r_{mm}} = T$. Rearranging terms yields $\hat{D} = \frac{1}{r_D}\left[1 + \overline{M}\theta_{mm}\right] - \frac{1}{r_{mm}} - T = 0$.

By the Implicit Function Theorem we obtain

$$\frac{dr_{B|D=T}}{d\theta_{mm}} = \frac{-\frac{\partial \hat{D}}{\partial \theta_{mm}}}{\frac{\partial \hat{D}}{\partial r_{mm}}} = -\frac{\frac{\overline{M}}{r_D}}{\frac{1}{(r_{mm})^2}} = -\frac{\overline{M}(r_{mm})^2}{r_D} < 0$$

Therefore $\uparrow \theta_{mm} \implies \downarrow p_{b|D=T}$

We next prove the proposition for BD_{mm}. The BD_{mm} client price required to induce T is derived from $S(r_{rm}, \theta_{rm}) = \frac{1}{b}\left[a - \frac{r_{rm}}{\theta_{rm}}\right]^{\frac{1}{2}} = T$. Rearranging terms yields $\hat{S} = \frac{1}{b}\left[a - \frac{r_{rm}}{\theta_{rm}}\right]^{\frac{1}{2}} - T = 0$

By the Implicit Function Theorem we obtain

$$\frac{dr_{S|S=T}}{d\theta_{rm}} = \frac{-\frac{\partial \hat{S}}{\partial \theta_{rm}}}{\frac{\partial \hat{S}}{\partial r_{rm}}} = -\frac{\frac{1}{2b}\left[a - \frac{r_{rm}}{\theta_{rm}}\right]^{-\frac{1}{2}} \frac{r_{rm}}{(\theta_{rm})^2}}{\frac{1}{2b}\left[a - \frac{r_{rm}}{\theta_{rm}}\right]^{-\frac{1}{2}}\left(-\frac{1}{\theta_{rm}}\right)} = \frac{-\frac{r_{rm}}{(\theta_{rm})^2}}{-\frac{1}{\theta_{rm}}} = \frac{r_{rm}}{\theta_{rm}} > 0$$

Therefore $\uparrow \theta_{rm} \implies \uparrow p_{s|S=T}$ □

We state a corollary that proves that, for a fixed inventory T, (i) an increase in θ_{mm} decreases BD_{rm}'s price by more than it decreases the peak to BD_{rm}'s problem and (ii) an increase in θ_{rm} increases BD_{mm}'s price by more than it increases the peak to BD_{mm}'s problem

Corollary C.2.1. *An increase in client states moves the price required to induce an inventory of T by weakly more than it moves the peak of the BD problem. specifically,*
(i) $|\frac{dr_{B|D=T}}{d\theta_{mm}}| \geq |\frac{dr^A_{mm}}{d\theta_{mm}}|$, *and*
(ii) $\frac{dr_{S|S=T}}{d\theta_{rm}} \geq \frac{dr^A_{rm}}{d\theta_{rm}}$

Proof. We first prove (i).

$|\frac{dp_{b|D=T}}{d\theta_{mm}}| = \frac{\overline{M}(r_{mm})^2}{r_D} = W$

$|\frac{dr^A_{mm}}{d\theta_{mm}}| = \frac{\overline{M}(r_{mm})^3}{2r_D r_{rm}} = X$

Multiplying X and Y by $\frac{p_D}{\overline{M}(r_{mm})^2}$ modifies the expressions to $W = 1$, $X = \frac{r_{mm}}{2p_s}$, which is less than 1 , since by assumption $r_{mm} < r_{rm}$. Therefore, $W > X$, which proves (i).

We next prove (ii).

$\frac{dp_{s|S=T}}{d\theta_{rm}} = \frac{r_{rm}}{\theta_{rm}} = Y$, and
$\frac{dr^W_{rm}}{d\theta_{rm}} = \frac{r_{rm}}{2\theta_{rm}} = Z$

Multiplying Y and Z by $\frac{\theta_{rm}}{r_{rm}}$ modifies the expressions to $Y = 1$, $Z = \frac{1}{2}$, which proves (ii).

□

C.3 Proof of Multiple Equilibrium under Asymmetric Information

Theorem C.1 (Existence of Asymmetric Equilibrium Strategies)**.** *There is an inventory T such that the strategies of setting repo rates to to induce T is an equilibrium.*

$S_1 : \theta_{mm} \implies r_{mm}$ *such that* $D_{r_{mm},\theta_{mm}} = T$

$S_2 : \theta_{rm} \implies r_{rm}$ *such that* $S_{r_{rm},\theta_{rm}} = T$

Proof. We state an algorithm to find an equilibrium.

Initialize by setting the client states at their respective minimums.

choose $\theta_{mm} = \underline{\theta}_B$ and $\theta_{rm} = \underline{\theta}_S$

Select an equilibrium price pair. We proved the existence of such a pair in the equilibrium Theorem 4.1 under perfect information.

choose an equilibrium $\{r_{mm}, r_{rm}\}$

Denote by T the inventory induced by the price pair.

In the proof of the equilibrium Theorem 4.1 under perfect information, we demonstrated the mapping $\{r_{mm}, r_{rm}\} \implies T$ is 1:1.
Note that $r_{mm} \leq r^A_{mm}$ and $r_{rm} \geq r^A_{rm}$. We evaluate movements in the repo rates in response to increases in the state terms.

choose $\theta_{mm} > \underline{\theta}_b$ and $\theta_{rm} > \underline{\theta}_s$

First, consider the adjustment to BD_{rm}'s price and problem solution. Holding r_{rm} unchanged, $p_{b|D=T}$ is below the new peak of BD_{rm}'s problem r^A_{mm}. An increase in r_{rm} moves the peak further upward without moving $p_{b|D=T}$. Therefore $p_{b|D=T}$ is below the peak of BD_{rm}'s problem and is an optimal choice, conditional on BD_{mm} choosing inventory T.

Next, consider the adjustment to BD_{mm}'s price. Holding r_{mm} unchanged, Corollary C.2.1 $p_{s|S=T}$ is above the new peak of BD_{mm}'s problem r^A_{rm}. A decrease in r_{mm} moves the peak further downward without moving $p_{s|S=T}$. Therefore $p_{s|S=T}$ is above the peak of BD_{mm}'s problem and is an optimal choice, conditional on BD_{mm} choosing inventory T.

Note that the monotonicity of the expressions $\frac{dr_{B|D=T}}{d\theta_{mm}}$ and $\frac{dr_{S|S=T}}{d\theta_{rm}}$ demonstrated in the proof to Proposition C.2, imply that the maps from the initial repo rates to the rates that induce T after the change in state value, are 1:1.

Thus, the pair $\{p_{b|D=T}, p_{s|S=T}\}$, which is a function of each BD's own client state, is a perfect knowledge constrained equilibrium for every realization of client states; i.e. they are the repo rates the BD's would choose if they had perfect knowledge. It follows that the strategy of choosing the repo rates that induce inventory T for each client state is an equilibrium strategy for each BD.

□

Figure 14 tracks the movement of BD_{mm} price and unconstrained problem peak in the proof. The algorithm is initialized at the lower bound state $\underline{\theta}_B$ and the price pair $\{r_{mm}, r_{rm}\}$. The labels ① indicate the positions of r_{mm} (which induces the MM demand for T) and the peak of BD_{rm}'s problem, which is weakly to the right of the price. Labels ② indicates positions after the MM state is increased. Notably, the price declines by more than the peak. Finally, label ③ indicates the movement of the peak - to the right- after BD_{mm} increases r_{rm} to induce supply T in response to an increase in the RM state.

The interpretation of Figure 15, which tracks the movement of price and problem peak for BD_{rm} has a similar interpretation.

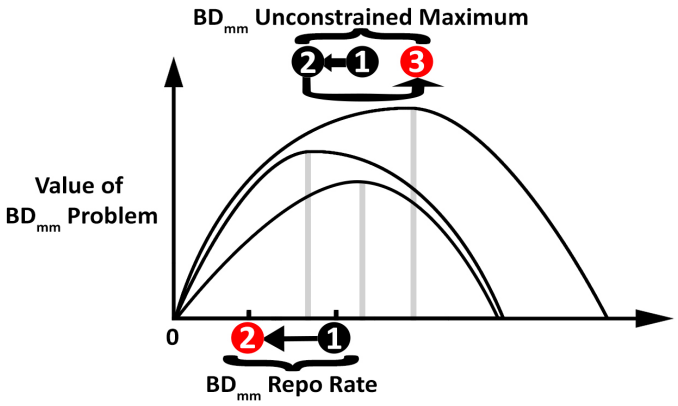


Figure 14: BD_{mm} Moves in Proof

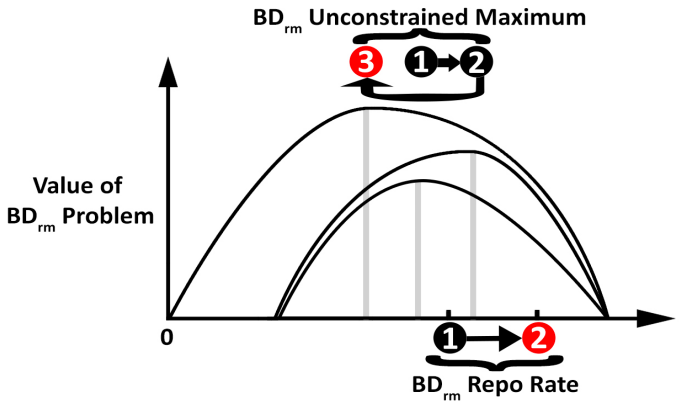


Figure 15: BD_{rm} Moves in Proof

Next, we show there is an infinite number of asymmetric equilibrium strategies.

Theorem C.2 (Multiplicity of Asymmetric Equilibrium Strategies)**.** *There are multiple equilibrium strategies under asymmetric information involving inventory volumes of T, i.e. there are at least two values T and $T^{'}$ such that (i) $T \neq T^{'}$ and (ii) there is an equilibrium in which each BD transacts T inventory with it client and there is another equilibrium in which each BD transacts $T^{'}$ inventory with its client.*

Proof. The multiplicity of equilibrium under perfect information established that, for any combination of states there are multiple equilibrium price pairs. The monotonicity of the client demand and supply functions ensure there is a unique T associated withe each equilibrium pair.

Therefore, we can initialize the proof of existence of asymmetric equilibrium strategies Theorem with two distinct price pairs corresponding to two distinct values of T. Each initialization corresponds to a distinct strategy to set the price to induce its unique T. This prove there are multiple equilibrium strategies. □

D Note: concavity of hedge fund (RM) borrowing demand and MM money supply

This appendix provides microfoundations under which (i) hedge fund borrowing demand and (ii) money market fund (MMF) repo supply are concave in the relevant pricing margin. The primitives mirror the frictions documented empirically and used theoretically in the cited literature—risk aversion and funding/margin constraints for leveraged investors [32, 6, 28] and concentration costs plus an outside option (the ON RRP floor) for MMFs [39, 37].

Hedge funds (borrowing demand as a function of the repo rate). Let a hedge fund choose leverage $L \geq 0$ on an arbitrage (or carry) opportunity with expected excess return α over the secured financing rate, financed at repo rate r. The fund is risk-averse with mean–variance preferences and faces convex funding/liquidity costs that increase with scale (capturing balance–sheet usage, margin frictions, and liquidity risk), consistent with [32, 28]. Consider the static problem

$$\max_{S \geq 0} \underbrace{S(\alpha - r)}_{\text{gross carry}} - \underbrace{\tfrac{1}{2}(\gamma\sigma^2 + k)\, S^2}_{\text{risk + quadratic funding cost}} - \underbrace{\tfrac{m}{3}\, S^3}_{\text{scale/liquidity convexity}},$$

with $\gamma > 0$, $\sigma^2 > 0$, $k \geq 0$, $m > 0$. The FOC for interior L is

$$\alpha - r \;=\; (\gamma\sigma^2 + k)\, S \;+\; m\, S^2 \;\equiv\; A\, S + mL^2, \quad A := \gamma\sigma^2 + k.$$

Solving for the Treasury suply $S(r)$ gives the positive root

$$S(r) \;=\; \frac{-A + \sqrt{A^2 + 4m(\alpha - r)}}{2m} \qquad \text{for } r < \alpha,$$

and $S(r) = 0$ otherwise. Differentiating,

$$\frac{dS}{dr} \;=\; -\frac{1}{A + 2mS} \;<\; 0, \qquad \frac{d^2S}{dr^2} \;=\; -\frac{2m}{(A + 2mS)^3} \;<\; 0.$$

Hence $S(r)$ is decreasing and concave in r on $(-\infty, \alpha)$. This aligns with the empirical evidence that higher funding costs or margins sharply reduce leverage (e.g., a 200bp minimum haircut reducing effective leverage from $56\times$ to $25\times$ [6]) and with margin–based asset–demand theories in which binding constraints flatten supply at the margin [28]. Moreover, when haircuts or margins introduce a binding constraint, $S(r)$ becomes piecewise defined with a kink (as in [32]), reinforcing curvature.

MMFs (demand for Treasuries from a given dealer as a function of the dealer's offered rate). Fix a single dealer–MMF relationship (the per–dealer schedule relevant for dealer market power). Let the MMF choose exposure $D \geq 0$ to this dealer at offered rate r, with an outside option r_0 (e.g., the ON RRP floor) that creates a rate floor/kink [37]. Following the structural evidence on aversion to portfolio concentration and preference for stable lending [39], model concentration costs by a convex function $C(x) = \frac{a}{2}x^2 + \frac{b}{3}x^3$ with $a > 0$, $b > 0$. The MMF solves

$$\max_{D \geq 0} \; (r - r_0)\, x \;-\; \frac{a}{2}x^2 \;-\; \frac{b}{3}x^3,$$

implying the FOC (for $r \geq r_0$)

$$r - r \;=\; aD + bD^2.$$

The per–dealer supply function is the positive root

$$D(r) \;=\; \frac{-a+\sqrt{a^2+4b(r-r_0)}}{2b} \qquad \text{for } r \geq \underline{r},$$

and $D(r) = 0$ for $r < \underline{r}$. By differentiation,

$$\frac{dD}{dr} \;=\; \frac{1}{a+2b\,D} \;>\; 0, \qquad \frac{d^2D}{dr^2} \;=\; -\frac{2b}{(a+2b\,D)^3} \;<\; 0.$$

Thus, above the policy floor $\underline{r}$ the MMF's per–dealer demand is increasing and concave in the offered rate, with a kink at $\underline{r}$ consistent with the ON RRP floor's role in shaping the lending schedule [37]. Concavity captures the diminishing marginal willingness to concentrate exposure in a single counterparty documented in [39].

Discussion. Taken together, these primitives deliver concave Treasury supply on the leveraged–investor, RM, side (due to risk aversion and convex funding/liquidity costs) and concave per–dealer demand on the MMF, MM, side (due to concentration costs and an outside option). Both features are consistent with the empirical patterns reported in [32, 6, 39, 37] and provide a parsimonious foundation for the curvature assumed in the main text.

E Details for Smart Contract Implementation

This appendix discusses the cryptography required to implement the smart contract functions on a stand alone server and a blockchain.

Implementation A: smart-contract inside a TEE (hardware-backed privacy, ZK auditability). The comparison/optimization code that parses $S_{r_{rm}}, D_{r_{mm}}$, computes Equation 12, selects $\{r_{rm,*}, r_{mm,*}, T^*\}$, and runs the proof generator is loaded into a TEE. Remote attestation binds a code hash (and the verifier/proving keys) to the enclave. After verifying attestation, each BD transmits its full schedule over the attested secure channel (or sends schedule elements sufficient for membership checking). Inside the enclave, the smart-contract computes $T^* = \arg\max_T \Pi(T)$ over the admissible set (e.g., intersection/feasible pairs induced by the two schedules and hurdle-rate filter), then outputs: (a) E_{rm}, E_{mm}—the per-BD encryptions of T^*—and (b) a zk-SNARK/NIZK π proving membership and optimality of $\{r_{rm,*}, r_{mm,*}, T^*\}$relative to C_{rm}, C_{mm} and the attested program. Each BD verifies the attestation and π; if either check fails or a deadline expires, the protocol aborts. Privacy rests on enclave isolation; auditability rests on the public verification of π (not on trust in the operator).[27]

[27] Costan and Devadas [16], Bauman et.al. [8] and Schuster et.al [54] are descriptions of TEE implementations.

Implementation B: smart-contract as a public-chain contract (trustless verification, on-chain audit). The contract records C_{rm}, C_{mm} and enforces the phase timeouts. Off-chain, a prover computes $\{r_{rm,*}, r_{mm,*}, T^*\}$from the private schedules and generates a zk-SNARK/NIZK π for the statement above (with schedule-size bounds fixed ex ante). On-chain, the contract verifies π against (C_{rm}, C_{mm}) and, if valid before the deadline, publishes only $E_{rm} = Enc_{pk_{rm}}(T^*)$ and $E_{mm} = ENC_{pk_{mm}}(T^*)$. The plaintext $\{r_{rm,*}, r_{mm,*}, T^*\}$never appears on-chain; each BD decrypts locally. If commitments or proof are not posted by their respective deadlines, the contract cancels without leakage. Anyone can re-verify π, yielding public auditability.

The off-chain phase runs either as a two-party computation between BD_{rm} and BD_{mm} on their own machines or via a small set of independent multiparty computation ("MPC") servers. In all cases, each BD's schedule is secret-shared (or garbled) so that no single processor ever observes the other's plaintext input. The prover jointly computes $\{r_{rm,*}, r_{mm,*}, T^*\}$ and produces a zk-proof π that (i) the private inputs used open the on-chain commitments (C_{rm}, C_{mm}), (ii) $\{r_{rm,*}, r_{mm,*}, T^*\}$ is admissible and maximizes the objective (Equation 12) over the bounded grid, and (iii) the posted ciphertexts CT_{rm}, CT_{mm} are valid encryptions of the same trades. Thus privacy derives from MPC (simulation-based), integrity from input-binding to commitments, and auditability from on-chain verification of π; only the encrypted $\{r_{rm,*}, r_{mm,*}, T^*\}$ is delivered to each BD, and missed deadlines trigger cancellation.

Remarks. Circuit size and proving cost scale with schedule bounds and the complexity of $\Pi(\cdot)$; using vector commitments (Merkle/polynomial) and bounded grids $\mathbb{T}$ renders the "max" check implementable via batched pairwise comparisons inside the circuit. The approach is agnostic to the concrete proof system (zk-SNARK or NIZK) and to platform: any TEE with remote attestation or any chain with a verifier suffices. In both implementations, BD_{rm} and BD_{mm} learn only the same $\{r_{rm,*}, r_{mm,*}, T^*\}$, sent privately by the smart-contract, and can independently verify that it is exactly the trade the mechanism should select under Equation 12.