The Intangible Economy and the Decline of Corporate Leverage in Europe

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Abstract

Contrary to widespread concerns about rising indebtedness, corporate leverage has steadily declined over the past two decades across many western economies, and in particular in the EU. This paper documents this trend using novel microdata for the near-universe of firms across 21 European countries. Leverage fell throughout the period of loose monetary policy before 2020, at a similar rate across firms of different sizes, productivities, and dividend policies. Declining leverage has traditionally been associated with a tightening supply of external finance, but our evidence suggests an important role for falling credit demand, driven by in-house development of intangible capital. We build a dynamic model in which heterogeneous firms compete by investing into tangible and intangible assets. Firms that invest more into intangibles experience greater economies of scale, leading to higher profitability and accumulation of market power. However, intangibles face a higher external financing cost, and also a greater risk of expropriation by competitors that poach key human capital. Our model shows how firms save cash (and maintain low net leverage) due to both traditional precautionary reasons and a novel strategic motive to retain human capital. The model can be estimated to quantify the importance of each mechanism for explaining the decline in European corporate leverage.

Keywords. Corporate leverage, Technological change, intangible assets, secular trends.

JEL classifications. G32, G35, J24, J33

1. Introduction

Advanced economies experienced a historic expansion of financial intermediation during the post-World War II period, and corporate reliance on debt grew almost uninterruptedly for several decades. Jordà et al. (2022) document that debt-to-GDP ratios more than doubled between 1945 and 2000, for a sample of non-financial firms in 17 developed countries. The leverage ratio of U.S. public firms tripled over the same period (Graham et al., 2015). Corporate debt has also risen significantly in China, and is now higher than in most OECD economies (Beck et al., 2023). Current debt levels are eliciting concern from regulators and policymakers, since high corporate indebtedness has amplified recent financial crises (e.g., Kaplan, 2019). For example, highly levered European firms cut investment by larger amounts during the 2012 Eurozone crisis (Kalemli-Özcan et al., 2022).

However, a striking and largely unnoticed fact is that corporate leverage has been *steadily declining* for years among private firms—a reversal of the post-World War II trend, which began well before the 2008 financial crisis. In Europe, net leverage has continually fallen from 65.8% in 2002 to 18.9% in 2022, with only a minor increase during the crisis years of 2008–2012 (see Figure 1 in Section 3.2). The decline is similar across Europe's large and medium-sized economies (except Poland), as shown in Figure 2. This long-term trend is also taking place in the United States, where private firm leverage fell by 10 percentage points from 2004 to 2018 (Bellon et al., 2024), and in Japan (Khoo and Durand, 2017).

This structural break in leverage across the world's largest developed economies has been overshadowed by a focus on large publicly traded firms, whose balance sheets are much more readily available to researchers.[1] However, unlisted firms constitute a large share of overall GDP and the vast majority of total employment. Documenting and understanding the leverage choices of these firms is crucial for researchers to identify structural changes in modern economies, especially considering that public firm listings are declining rapidly and becoming less representative of overall corporate activity (Doidge et al., 2017, 2018).

[1] Moreover, the debt-to-GDP ratio may overstate corporate indebtedness, because it does not account for firms' repayment capacity and because the capital income share has risen across many economies (Rognlie, 2016). A more direct measure of financial strength is the leverage ratio, which links debt obligations to the productive assets backing them.

This paper's first contribution is to carefully document this generalized decrease, using novel data that represents both listed and unlisted firms across most of Europe. Our primary source is a micro-founded database compiled by the Competitiveness Research Network (CompNet), which has collaborated with the national central banks and statistical institutes to compile firm-level data from administrative sources such as tax records. There are 22 participating countries, including all large European Union members and Switzerland. CompNet's data include standard balance sheet and income statement items, obtained by sampling 60.5% of all firms in participating countries and weighted to ensure representativeness to the entire population. Each country aggregates the data into various statistical moments to preserve firm confidentiality. We supplement the CompNet data with firm-level information obtained from tax records covering all Dutch and French firms. Compared to other databases that contain private firms such as Orbis, a key advantage of our data is that each financial variable's definition and measurement is carefully harmonized across countries.

We use the CompNet data to document the 47 percentage-point (pp) decline in median net leverage across Europe over the past two decades. We further show that this trend is driven by both a decrease in gross leverage (from 69.5% in 2002 to 32.7% in 2022) and an increase in corporate cash balances (from 3.7% to 13%). We calculate these aggregates by weighting each individual country's median leverage or cash ratio by its share of total European corporate assets.

Existing research has identified several determinants of corporate credit usage, which could potentially lead to decreasing leverage over a short period of time. A broad strand of literature documents how a tightening of credit supply can prevent firms from raising debt (e.g., Ivashina and Scharfstein, 2010; Chodorow-Reich, 2014; Corbisiero and Faccia, 2020). Other common explanations are that secular stagnation has reduced the amount of profitable investment opportunities, especially in Europe (Gordon, 2012; Eichengreen, 2015; Jones and Philippon, 2016; Eggertsson et al., 2019) and that falling interest rates have reduced the value of debt tax shields (e.g., Dallari et al., 2018).

While these factors certainly explain part of the leverage decline in Europe, it is hard to reconcile them with both the steady decreases observed in the mid-2000s and in the years between the Eurozone crisis and COVID-19 pandemic. Interest rates were fairly low during both

periods, and the latter was characterized by exceptionally loose monetary policy in Europe. Notably, gross leverage fell from 46% to 41% during the post-crisis period, despite historically low borrowing costs.

We further document several facts that are at odds with explanations based on tightening credit supply. First, the share of credit-constrained firms (based on a measure from the ECB's SAFE survey) declined from 11.5% in 2013 to just 3.3% in 2021. This trend is similar using an alternative measure based on corporate investment activity. Importantly, credit constraints appear to be at historically low levels in Europe—using both measures, the share of constraints firms was significantly lower in the 2015–2019 period than in any year before the 2008 financial crisis. Dividend payments scaled by total assets have also increased from 3% in 2003 to 3.5% in 2020, including an almost continuous increase since the Eurozone crisis.

Moreover, leverage has fallen at a near-identical rate across different categories of firms that should have varying access to external financing. Across Europe, we document similar declines in net leverage for firms of different ages (younger or older than 5 years) and sizes (less than 50, 50–249, and 250 or more employees). We also turn to the Dutch and French microdata to classify firms along a greater range of dimensions. In both countries, we find remarkably similar declines in net leverage for dividend and non-dividend paying firms, and for firms with the highest and lowest productivity growth rates (measured by annual growth in sales per employee over the past three years). Importantly, the French microdata extend back to 1995, and reveal a persistent decrease in net leverage across an almost 30-year period.

The paper's second contribution is to offer a complementary *demand-driven* explanation for the leverage decline: Firms increasingly derive value from intangible assets, which require fewer upfront cash outlays to create than physical capital (and hence less debt financing). This explanation is distinct from the traditional view that intangible capital has low debt capacity due to its limited collateral value (e.g, Falato et al., 2022), or more recent research showing that intangibles allow firms to accumulate significant market power and thus may depress investment spending (Crouzet and Eberly, 2023).

Instead, our explanation is motivated by two defining characteristics of intangible assets. The first is that they are largely created by skilled employees, who contribute their human capital via

activities such as coding software, developing new product designs, collaborating on a marketing campaign, or analyzing large datasets. Unlike the purchase of physical property or equipment, such activities can be executed within significant cash spending by the firm. Skilled employees naturally must be rewarded for their human capital contributions, but their compensation is partly deferred (e.g., via unvested equity grants) to ensure retention until project completion (Sun and Xiaolan, 2019; Eisfeldt et al., 2023).

A second unique feature of intangible capital is that it can be easily expropriated by competitor firms, which creates a strategic motive for firms to hoard cash and maintain low leverage. Intangible assets can be deployed simultaneously across multiple divisions of a firm, allowing it to scale up quickly (Crouzet et al., 2024). This non-rivalry property also makes it easier for other firms to claim or copy the assets, a threat that can be particularly disruptive to the fastest-growing firms. However, accumulated liquidity can help firms to protect their intangible capital. For example, a firm with abundant cash holdings (and thus low net leverage) can more easily deter talent poaching, by matching the competing offers that its skilled employees receive. Döttling et al. (2025) show that established firms with a sufficiently conservative financial structure can deter young firms from entering the market and competing for human capital.

We formalize this explanation by constructing a dynamic discrete-time model of firm investment and production, which nests the two defining features of intangible capital alongside costly external financing and secular productivity shocks. Calibration of this model can quantify how much of the leverage decline in Europe is due to lower demand for debt driven by rising intangible investment, relative to a tightening of credit supply or decrease in investment opportunities. Another novel feature of our model is that market power arises endogenously from firms' development of intangible assets.

The framework models an economy with heterogenous firms operating under monopolistic competition, and a representative household of overlapping generations who supply labor and consume output (with preferences characterized by love of variety, as in Dixit and Stiglitz (1977)). Each firm uses tangible and intangible capital, complemented by skilled and unskilled labor, and is the sole producer of a single distinct good. At the start of each period, the firm hires workers from the household sector and builds capital. The firm must spend cash to invest

into tangible assets and (to a lessor extent) to create intangibles. This cash can come either from earnings retained from a previous period, or from costly external debt obtained from the household sector.

Firms differ in their ability to retain and enhance human capital, which is representing in the model by an investment cost parameter that varies across firms. This cost advantage allows some firms to build intangible capital more efficiently than others, and thus to scale up production more rapidly. Because each firm has some pricing power over the good that it produces, firms that can create intangible assets efficiently can also accumulate market power.

While such firms can increase profitability by investing heavily into intangible assets, the downside risk is that their capital is more prone to expropriation. In our model, an expropriation threat arises in each period following a Poisson process, and the arrival rate increases with the amount of intangible capital that the firm possesses. This feature represents that firms which create more intangible capital also signal that they have higher quality human capital, thus becoming more attractive targets of poaching. When a firm experiences expropriation, it loses all of its intangible capital and must exit the economy. However, a firm can deter expropriation by making a cash payment to the household, which can represent a cash bonus to match an employee's outside offer. In each period firms also face a risk of sudden depreciation of their tangible capital, but cash savings allow the firm to rebuild its physical assets prior to production.

Corporate cash holdings serve three distinct roles in our model. First, firms face a pecking order in which they prefer to fund investment needs with retained earnings rather than costly debt. Second, firms have a traditional precautionary motive to save cash, to rebuild physical capital that has depreciated. Third, cash holdings allow firms to prevent expropriation of intangible assets. In each period, the firm trades off investing its cash to create capital (thereby boosting current profits) versus saving it to protect existing capital or boost retained earnings (potentially increasing future profits).

We present analytical results showing that firms which can create intangible assets more efficiently also invest more overall into both types of capital. These firms charge higher markups for their good and thus accumulate a greater share of the market. We also show that the a firm's optimal cash policy is to retain earnings and build up cash savings until a buffer is reached, and

then to pay out all excess cash. This buffer rises with a firm's efficiency of creating intangible assets.

Finally, we show how the model can be calibrated to decompose the leverage decline in Europe based on different possible determinants. Our demand-driven explanation can be tested by increasing in the productivity of intangible capital relative to tangible assets. The traditional credit supply mechanism can be tested by increasing the cost of external debt, which rises by a larger amount in our model for firms that invest more into intangibles. The secular stagnation explanation can be tested by reducing the productivity factor in the firm's production function. The model can quantify the relative importance of the three channels, by conducting a counterfactual exercise that varies the model parameters one at a time.

The rest of the paper proceeds as follows. In Section 2 we present a review of the literature and position our contribution in light of previous work. In Section 3 we present our data and the empirical trends we observe across EU firms. In Section **??** we discuss our model and its implications. Finally, Section **??** concludes.

2. Literature Review

Our work is not the first one documenting declining corporate leverage. According to Beck et al. (2023), aggregate EU gross leverage declined modestly during the mid-2000s, surged briefly during the financial crisis, and then fell steadily from 36% in 2009 to 29% in 2021. Net leverage follows a similar pattern, particularly since 2016. Among individual EU states, leverage peaked in the mid-2000s in Germany, France, Austria, and the Netherlands. In periphery countries leverage declined substantially after 2009, and is now lower than in 1995. The primary exception is Central Europe (Poland, Slovakia, and the Czech Republic), where leverage rose from a low base as the region integrated into the EU. This prolonged decline in average leverage is matched in data on U.S. firms. Average US firm leverage has fallen by about 10 percentage points (pp) since 2004, with most of the decrease coming after the financial crisis (Bellon et al., 2024). Leverage has fallen by 12pp among Japanese listed firms since 1999 (Khoo and Durand, 2017). Since 2008 leverage fell by 10pp in Australia and by 6pp in the United Kingdom (Beck et al., 2023). These

trends suggest a structural shift in advanced economies rather than a European phenomenon.

Our study contributes to the literature via using firm level data to further document the phenomenon, linking it to other relevant trends we have observed over the same period.We consider three structural factors as possible drivers: financial regulation, secular stagnation and rising importance of intangible capital.

Declining leverage may in part reflect a rise in financial constraints, potentially due to tighter regulation (Almeida et al., 2004; Whited and Wu, 2006). A well-established result in finance research is that a firm may reduce debt usage due to a tightening of financial constraints. This arises when a firm would like to raise debt to pursue positive-NPV investment opportunities, but creditors are unwilling to provide sufficient financing at a reasonable cost of capital. In particular, negative macroeconomic shocks can exacerbate information asymmetry between a firm and its creditors about the quality of investment opportunities or the ability to repay debt. Prior work measures financial constraints using proxies for asymmetric information such as firm size, age, a commitment to pay out dividends, or the amount of tangible assets (Fazzari et al., 1988; Whited and Wu, 2006; Almeida and Campello, 2007; Hadlock and Pierce, 2010).Another strand of literature shows that shocks to credit providers can percolate through to the real economy, by reducing corporate access to external financing for investment. Fundamentally strong firms may struggle to raise external financing, if they rely on creditors whose balance sheets have deteriorated. This is a significant concern for EU firms, which rely heavily on bank loans instead of public debt or equity issuance. Kalemli-Özcan et al. (2022) compare EU firms that had pre-existing lending relationships to banks with high versus low sovereign bond holdings during the financial crisis (a proxy for balance sheet weakness). They show that more highly levered firms cut investment by a greater amount during the crisis, an effect amplified by a pre-existing relationship to a weakened bank.These findings suggest that declining leverage across the EU could be driven by weaknesses in firms and banks that arose during the financial and Eurozone crises. Declining gross leverage since 2008 could be caused by a reluctance or inability of banks to lend to firms, and net leverage could decrease due to firms conserving cash in anticipation of becoming financially constrained. Notably, in order for this channel to explain most of the decline in leverage across our sample period, it must be that firms' available supply of credit remained depressed

during the recovery period after the Eurozone crisis, despite record-low interest ratesIn the years after the crisis Kalemli-Özcan et al. (2022) show that EU firms with pre-existing relationships with hard-hit banks cut investment heavily. In addition, new EU bank capital norms under Basel III have clearly reduced bank incentives to lend to smaller firms since 2014. Yet the decline in leverage started before the 2008 crisis, and persists even once EU credit conditions had stabilized from the Euro crisis. ECB surveys document renewed bank willingness to provide credit (Beck et al., 2023). The decline in aggregate EU leverage accelerated after 2016. Strikingly this occurred at a time of loosening of monetary policy which reduced financing costs to historic lows.

We contribute to this literature by providing evidence against this interpretation, in fact in our data the incidence of firms that are credit or liquidity constrained is falling, while dividend payouts and profitability are increasing.

The steady decline in leverage coincides with two other major structural transformations in advanced economies. The first is a well-documented decline in business dynamism (e.g., Eggertsson et al., 2019; Eichengreen, 2015; Jones and Philippon, 2016; Gordon, 2012; Akcigit and Ates, 2023), which has contributed to an era of secular stagnation, even in periods of minimal interest rates. In a low-growth environment, firms may reduce borrowing due to weaker and more uncertain future prospects, while banks may also become more cautious in extending credit. This explanation aligns with research documenting rising secular stagnation (Eggertsson et al., 2019; Eichengreen, 2015; Jones and Philippon, 2016) and declining business dynamism across advanced economies (Gordon, 2012; Akcigit and Ates, 2023, 2021). These trends may be particularly pronounced for EU firms, which experienced prolonged economic weakness following the financial and Eurozone crises.

A second driver of declining leverage is the shift toward intangible capital, a defining feature of the digital transformation that began in the 1980s. Information technology has fundamentally changed corporate production and distribution, reducing reliance on physical assets and, consequently, the need for debt financing (Corrado and Hulten, 2010).

Unlike physical assets, intangible investments—such as software, brand value, and human capital—require different financing structures. Whereas physical capital often requires large upfront expenditures that can be debt-financed, intangible investment tends to be funded through

retained earnings or equity (Döttling et al., 2025; Eisfeldt et al., 2023). Furthermore, firms operating in intangible-intensive industries may experience increasing returns to scale, allowing a small set of successful firms to grow rapidly with minimal debt (Crouzet et al., 2024; Hsieh and Rossi-Hansberg, 2023; De Ridder, 2024).

Thus, the decline in leverage may not reflect a lack of investment opportunities but rather a shift in the nature of corporate investment. Advanced economies have seen a rapid increase in the share of intangibles relative to total assets (e.g., Corrado and Hulten, 2010; Falato et al., 2022). Our model contributes to this literature by providing a theoretical framework that accounts for these mechanisms and aligns with observed empirical trends.

Our work also advances the empirical literature on the relationship between debt financing and intangible capital. Prior studies argue that intangible assets' low transferability and lack of explicit property rights make them poor collateral for debt financing (Dell'Ariccia et al., 2021; Falato et al., 2022; Crouzet and Eberly, 2023). We build on this research by showing that firms with high intangible intensity do not further reduce leverage in response to tighter collateral constraints. This suggests that the decline in leverage among intangible-intensive firms is primarily driven by a lower demand for credit rather than by supply-side constraints.

3. Facts about European Leverage Trends

This section describes our two main sources of data on corporate leverage and other firm fundamental variables. It then presents stylized facts about leverage trends in Europe over the past two decades. We document trends aggregated across the vast majority of European firms, and also aggregated across different groups of firms that vary in the extent to which they are financially constrained.

3.1. Data sources

3.1.1. CompNet

Our first source is a micro-founded database compiled by the Competitiveness Research Network (CompNet). This research initiative is created through collaboration between national central

banks, statistical institutes, and research institutions from 22 European countries.[2] Each participating country gathers firm-level data from administrative sources such as business registries, tax records, and production surveys. These data include standard balance sheet and income statement line items. The panel of participating countries is unbalanced, but most enter the database by the mid-2000s and remain through 2020. Each country collects data from a wide range of listed and unlisted firms, such that its sample is representative of corporate activity across the entire economy. CompNet processes all firm-level data using a harmonized protocol, to ensure that variable definitions and sampling procedures are consistent across countries.

Next, CompNet aggregates each firm-level variable into a set of statistical moments, such as means, standard deviations, and various percentiles. This is done in order to preserve firm-level confidentiality and safeguard data privacy. Each variable is aggregated at several different levels, including (i) country and year; (ii) country, year, and age group (younger or older than 5 years); (iii) country, year, and size group (less than 50 employees, 50-249 employees, and 250 or more employees); and (iv) country, year, and industry (2-digit NACE Rev. 2 classification). CompNet constructs these aggregates using all sampled firms, and also using only firms with 20 or more employees. It also constructs variables that are aggregated directly from the source data, as well as variables that are aggregated after re-weighting data to to ensure that it represents the entire population of firms.[3]

We use aggregate measures that are based on the sample of firms with 20 or more employees, and constructed after weighting the underlying data. CompNet's internal analysis indicates that these data are more reliable than aggregates based on unweighted data or samples including tiny firms. Nevertheless, the sampled firms cover 60.5% of the total firm population in the participating countries, with a total count of 332,073 firms in the first year of the data coverage and 367,204 in the last. Manufacturing firms constitute approximately one-third of the sample,

[2]CompNet's primary mission is to foster collaboration between policymakers and researchers on competitiveness-related issues in Europe. Countries participating in the 9th or 10th vintages of CompNet are Belgium, Croatia, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Hungary, Italy, Latvia, Lithuania, Malta, Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, and Switzerland. The United Kingdom also participates in CompNet, but data is not available for most financial variables.

[3]For each individual country, weighted data are constructed by adjusting the composition of the country's sample to be consistent with its official firm counts, at the industry and size group level. These firm counts are obtained from Eurostat. For more information, see the CompNet 10th Vintage User Guide available at https://www.comp-net.org/data/10th-vintage.

yet their share declines from 35% in 2006 to 30% in 2018.

The CompNet database has been updated repeatedly, and the current version is the 10th vintage which extends through 2022. Some countries have not yet been included in this vintage, for example due to backlogs in processing of firm-level data by the national statistical offices. For these countries, we use data from the 9th vintage which ends in 2020. We omit observations from a few country-years in which the harmonization procedure changed significantly between the 9th and 10th vintages.[4] Table 1 lists the years in which data is available for each country, as well as the total size of each countries' sample and population of firms. Table 2 provides information on the aggregate industry composition of the CompNet sample and the total economies of the participating countries. Throughout the paper, when using CompNet data, we restrict the sample period to 2002–2022 to ensure that aggregates are based on data from a large number of countries.

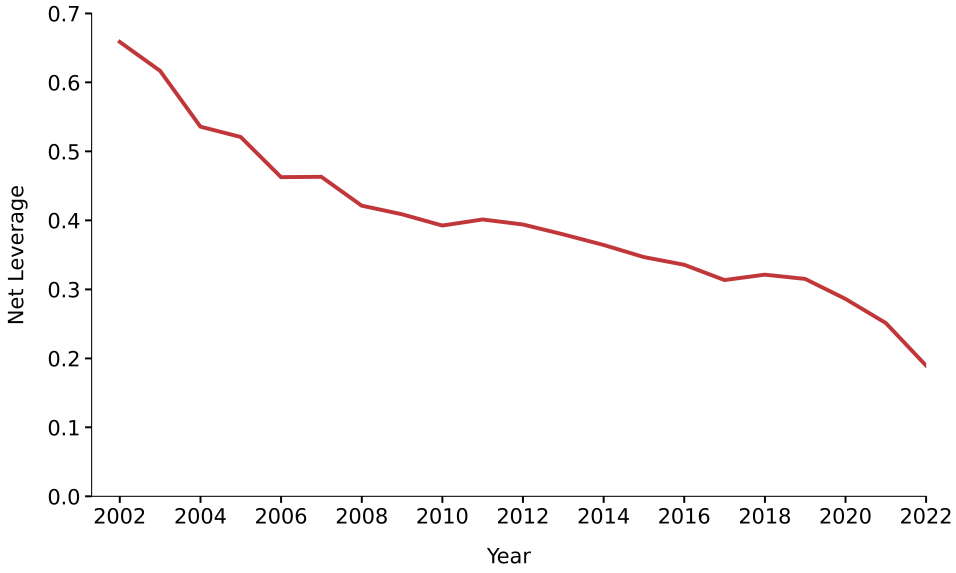
3.1.2. Administrative data from Netherlands and France

Our second data source is two national databases that contain financial data for the entire universe of firms in the Netherlands and France. The Dutch NFO database and French FICUS-FARE database are both constructed from tax records, and contain standard balance sheet and income statement variables at the firm-year level. The NFO database starts in 2005, while FICUS-FARE starts in 1995. Both datasets protect confidentiality by anonymizing firms. Therefore, it is not possible to merge these data with information on Dutch or French firms from other databases.

One key advantage of these microdata is that we can construct aggregated statistics on our own from the firm-level data, while CompNet only provides aggregates based on a limited amount of dimensions. For example, in Section 3.3.1 we measure each individual Dutch or French firm's sales growth, and then present leverage trends for different groups of firms based on this measure. In contrast, CompNet does not report any moments (for any variable) that are

[4]Belgium, Finland, Italy, Latvia, Sweden are in the 9th vintage but not yet in the 10th vintage, while Estonia is in the 10th vintage but not the 9th. For Sweden, we omit leverage aggregates from all years before 2007, when its calculation method appears to have changed. In France, the harmonization method changed significantly in the 10th vintage, and data from 2008 is currently missing. We fill in values for each variable based on a linear interpolation between the values in 2007 and 2009.

Figure 1: Time-Series Evolution of Corporate Net Leverage in Europe



aggregated based on sales, earnings, or productivity growth. However, a disadvantage is that these national databases are not representative of the overall European economy.[5] Throughout the paper, when using the national microdata, we restrict the sample to firms with five or more employees. This restriction ensures that we filter out inactive sole proprietorships.

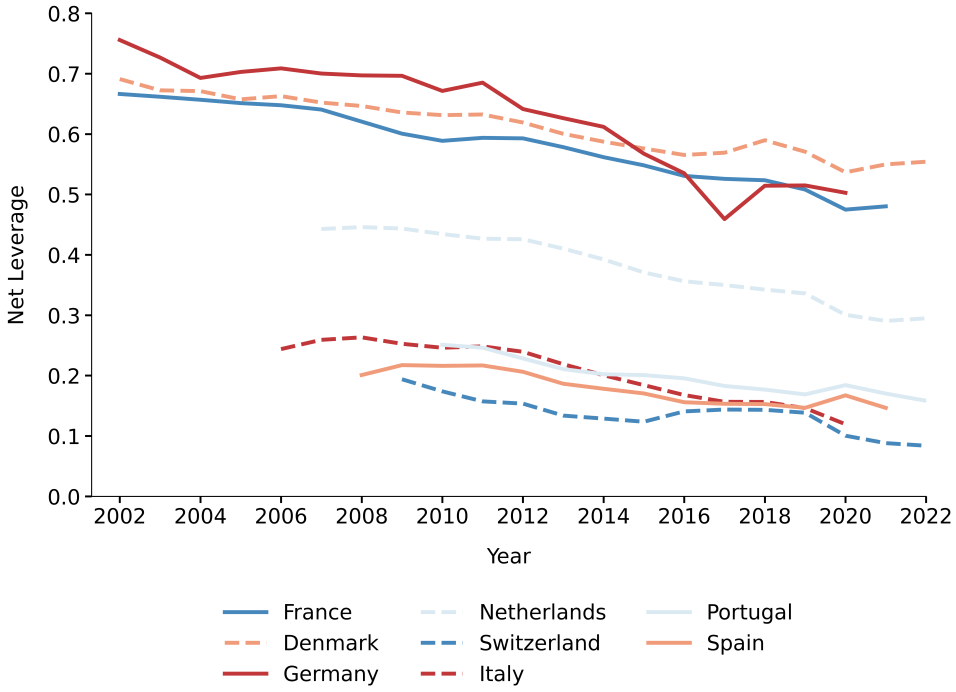
3.2. Stylized facts on aggregate leverage and cash holdings

Figure 1 documents median net leverage across all 22 European countries in the CompNet database. For each country, we measure net leverage as median gross leverage minus median cash holdings/total assets, where the sample medians are from CompNet. We then construct aggregate European net leverage by weighting each country's value using the total assets across the entire population of firms in the country. (We use this weighting measure in all figures that document aggregate European trends using CompNet data.)

The figure documents a remarkable decrease in net leverage across Europe, from 0.658 in 2002 to just 0.189 in 2022 (a 72% decline from the 2002 value). Notably, net leverage fell in almost

[5]We are currently in discussions with national statistical offices from four other European countries, to obtain access to their firm-level tax records.

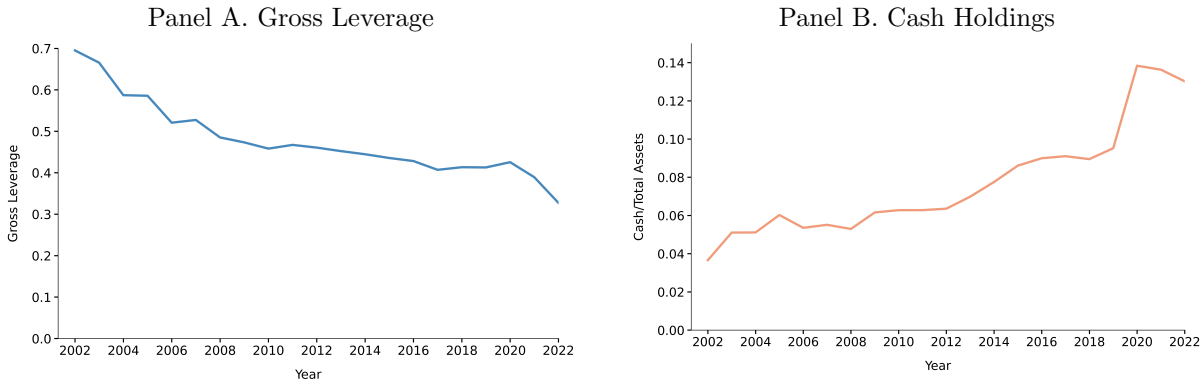
Figure 2: Evolution of Net Leverage, Select European Countries



all sample years, with slight increases of less than 0.01 occurring only in 2007, 2011, and 2018. Overall, net leverage appears to be in structural decline across Europe, as the trend preceded both the 2008 financial and 2012 Eurozone crises by several years.

Figure 2 presents trends in median net leverage separately for eight large and medium-sized European countries that participate in CompNet. The figure documents a persistent decline in leverage across all eight countries. In the three countries that are in the sample since 2002, net leverage declined gradually before the financial crisis. It fell from 0.756 in 2002 to 0.7 in 2007 in Germany, from 0.666 to 0.641 in France, and from 0.691 to 0.652 in Denmark. In each of these countries, the decline accelerated after 2008. Net leverage also has fallen persistently for firms that entered the CompNet sample in the mid- to late-2000s. Only in Italy was there a gradual increase from 0.244 in 2006 to 0.263 in 2008, and in Spain from 0.2 in 2008 to 0.217 in 2011. Overall, the figure shows that decreasing net leverage is a widespread phenomenon across many European countries.

Figure 3: Evolution of Gross Leverage and Cash Holdings in Europe



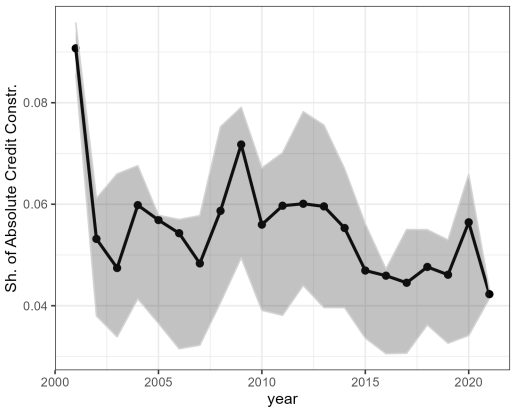
The decrease in net leverage results from both a decline in gross corporate indebtedness and an increase in cash balances. Figure 3, Panel A plots median gross leverage aggregated across the same sample of European firms as in Figure 1. It documents that gross leverage fell from 0.695 in 2002 to 0.327 in 2022, a decline of 37pp. Gross leverage has fallen by 16pp since 2008, but it also fell by a similar amount in the mid-2000s. Next, Panel B plots median cash holdings scaled by total assets. Cash balances across Europe rose steadily from 0.037 in 2002 to 0.095 in 2019, before jumping sharply at the start of the COVID-19 pandemic. Notably, the sharp drop in European net leverage during the pandemic is driven by both a reduction in debt issuance and an increase in cash savings.

3.3. Does decreasing credit supply fully explain European trends?

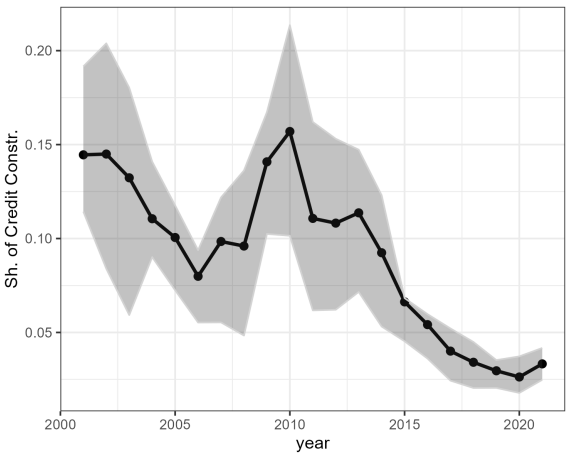
One possible explanation for the decrease in net leverage is that European banks have become progressively less willing to supply credit to firms over the past two decades. Undoubtedly, many European banks have reduced their balance sheets since 2008, and also face a wide range of new regulations that increase the cost of lending to some firms. Since most European firms rely primarily on bank loans for external financing, a reduction in supply of bank credit could lead to a decrease in gross leverage. A closely related explanation is that cash balances have risen over time because firms have anticipated increased tightening of bank credit, and engaged

Figure 4: Time-Series Evolution of Share of Credit-Constrained Firms

Panel A. Investment-based Measure



Panel B. SAFE Survey-based Measure



in precautionary cash savings to avoid becoming financially constrained. This could explain the decreasing trend in net leverage.

A reduction in credit supply likely cannot explain the persistent decreases in gross and net leverage across the entire 2002–2022 period. The European Central Bank's fixed interest rate was a constant 2% from mid-2003 through the end of 2005, and then was held at 0% from mid-2016 through mid-2022. The ECB's quantitative easing and extraordinarily loose monetary policy after the Eurozone crisis translated to historically low borrowing rates for many European firms, however aggregate gross leverage decreased by 10pp during this time period.

Figure 4 documents that the share of credit-constrained European firms has not increased since the mid-2000s, with the exception of a brief increase around the 2008 financial crisis. Panel A reports a CompNet measure that classifies firms as constrained when they report positive investment while simultaneously reducing debt and capital, or if they reduce investment while maintaining a positive financing gap. The figure plots the weighted average across all countries of the fraction of sample firms that are classified as constrained. With the exception of 2008, the share of constrained firms has fluctuated between 0.047 and 0.061 from 2002 through 2020. There was no clear trend in the share of constrained firms before the financial crisis, and a

marked decrease after the end of the Eurozone crisis.

Panel B plots a second measure of credit constraints, based on the ECB's Survey on Access to Finance of Enterprises (SAFE).[6] CompNet does not have access to the SAFE survey data, so it estimates credit constraints at the firm level based on loan rejections, limited credit approvals, prohibitive borrowing costs, and discouraged borrowing. It then defines a threshold for credit constraints that varies across years and countries, such that the fraction of firms above the threshold in each country matches the share reported in the SAFE survey.[7] In this plot, the share of credit-constrained firms has declined significantly over the past decade, from 0.155 in 2008 to 0.033 in 2021.

Importantly, both panels show that the fraction of credit-constrained firms was lower in the period between the Eurozone crisis and the COVID-19 pandemic 2015–2019 period than in any year before the financial crisis. In other words, firms recently appeared to be less financially constrained than in the early to mid-2000s, yet also maintained much lower leverage. This indicates that a reduction in credit supply likely does not explain the decrease in net leverage, at least in the post-crisis period.

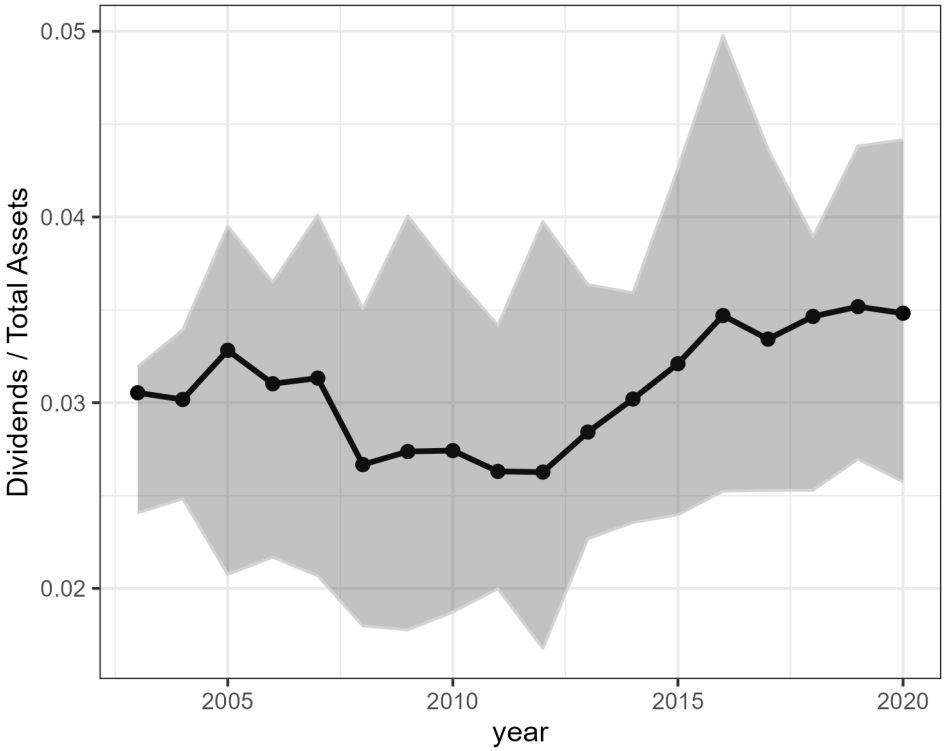
Further supporting this conclusion, Figure 5 documents that dividend payouts as a share of total assets have increased from 3% to 3.5% over the whole sample period. Payouts have also risen consistently since the end of the crisis period. This trend suggests that firms' financial flexibility has increased in recent years, as they have been able to distribute a greater portion of the cash holdings to shareholders rather than retaining precautionary liquidity or repaying debt. In contrast, credit-constrained firms would typically prioritize liquidity preservation over returning capital to shareholders.

To further examine whether leverage trends can be explained by a decrease in credit supply, we plot net leverage aggregated over separate groups of firms that likely vary in exposure to credit constraints. Figure 6 reports separate plots for firms incorporated within the past 5 years and for firms incorporated more than 5 years ago. Younger firms are often considered to have

[6] The SAFE is conducted by the ECB jointly with the European Commission twice per year. The survey assesses the financial conditions of firms in the euro area, and is also conducted for some non-euro area countries.

[7] See Ferrando and Ruggieri (2018) for more details on the variables included and the steps of the estimation process.

Figure 5: Time-Series Evolution of Dividends



more difficulty raising external financing than mature firms, in part because information asymmetry is higher between creditors and firms with a not-yet-established business model. Thus, a reduction in credit supplied by European banks should have a greater impact on the leverage of young than old firms. However, the figure documents a near-identical trend across both groups of firms until the COVID-19 pandemic. Net leverage fell by 34pp from 2002 through 2019 for older firms, and by 38pp over the same period for younger firms. (During the pandemic, net leverage fell significantly faster for young than old firms.)

Similarly, Figure 7 reports separate plots for firms in three different size classes, based on number of employees. Smaller firms are often considered to be more financially constrained than large firms. Yet as with firm age, the trend in net leverage is almost identical for firms in each size class. Net leverage fell by 35pp from 2002 through 2019 for firms with fewer than 50 employees, by 34pp for firms with 50 to 249 employees, and by 28pp for firms with 250 or more employees.

Figure 6: Evolution of Net Leverage by Firm Age

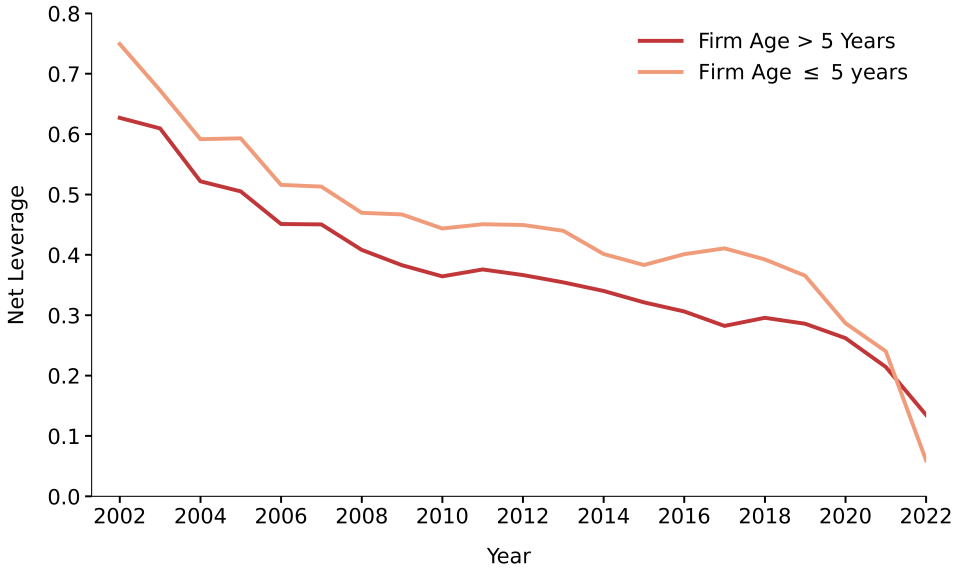
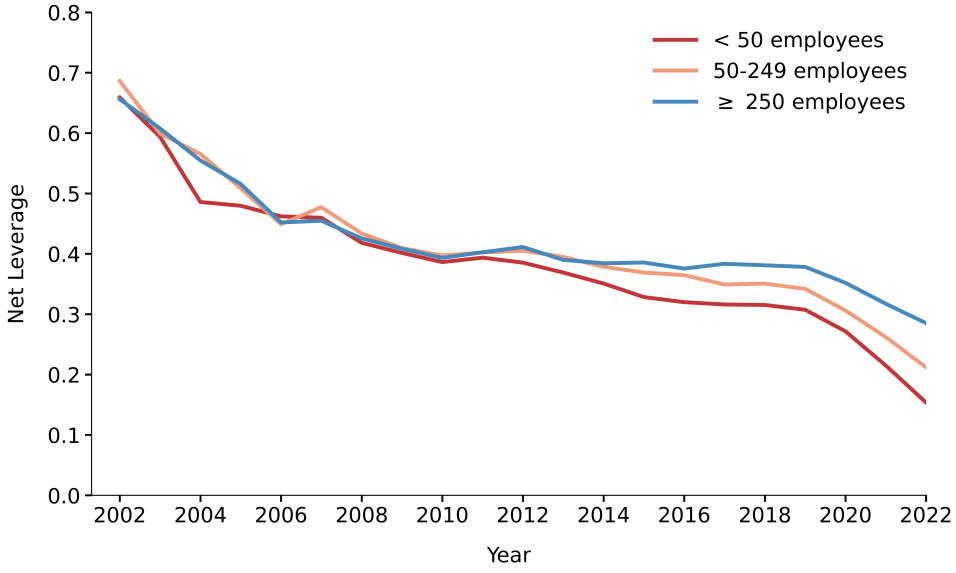


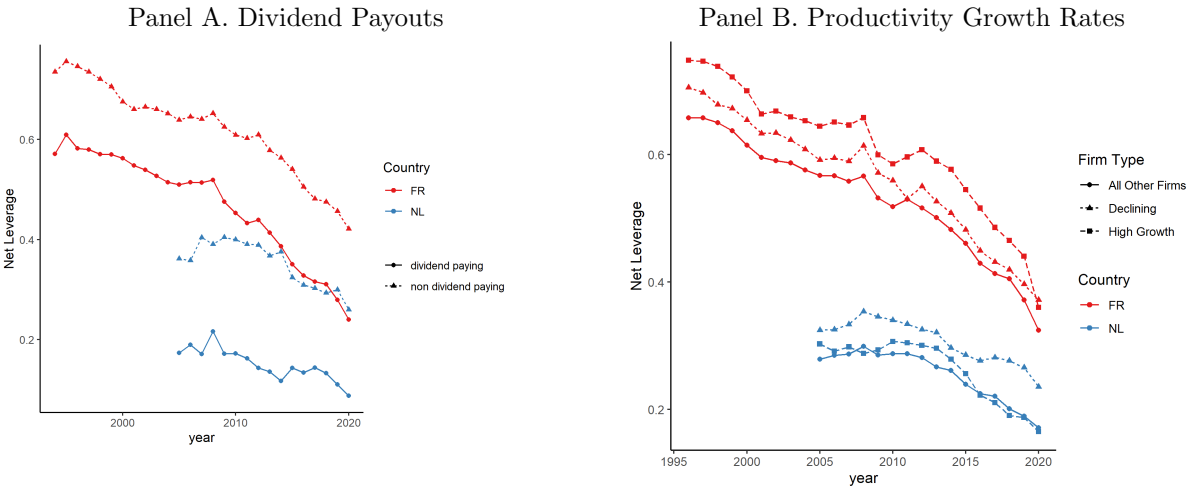
Figure 7: Evolution of Net Leverage by Firm Size



3.3.1. Firm-level evidence from Netherlands and France

We further use the national Dutch and French microdata to fully examine the credit supply explanation. As discussed in Section 3.1, a key advantage of these data is that we can aggregate

Figure 8: Net Leverage Trends in Netherlands and France



net leverage across a wider range of firm types than in CompNet.

Figure 8 plots net leverage trends in the Netherlands and France, separating firms along two additional dimensions related to financial constraints. Panel A plots trends separately for dividend and non-dividend paying firms in each country. In both the Netherlands and France, dividend-paying firms consistently maintain lower net leverage than non-dividend paying firms. However, the trend in leverage across both sets of firms, in each country, is almost identical.

Panel B plots trends for firms with different growth rates, measured as the three-year average of annual sales growth divided by total employees. High-growth firms are those in the top quintile of this measure in each country, while Declining firms are those with negative values for the measure. Remarkably, net leverage fell by almost the same amount for High-growth as Declining firms in both the Netherlands and France.

One further advantage of the national microdata is that financial data is available for France since 1995. The figures thus document that leverage has been declining consistently in this country since the mid-1990s.

4. Model of Intangible Investment and Leverage Demand

This model has the purpose of understanding how technological trends happening at the firm level can have aggregate implications for macro-financial trends.

We start from the assumption that advanced economies are withstanding a technological long term trend characterized by increasing relevance of intangible capital in production. We conceptualize this asset as something that is developed in-house by firms and skilled workers and is a key driver of their performance. Examples of such trends are digitalization and rise of e-commerce, as well as increasing value of brand and other types of intellectual property or finally high impact of marketing on sales growth (Corrado and Hulten, 2010). We therefore lay down a framework centered around a firm level production function that can capture such features. This production function includes both types of capital and is meant to capture two basic features of intangible capital that make it distinguishable from tangible capital. The first one is internal economies of scale, that unevenly benefit firms that are able to deploy intangibles.[8] As an example, this could be due to their higher ability to tap into the potential of better quality human capital, hired in the form of skilled workers (Döttling et al., 2025), or in their higher efficiency at using ICT technologies. One the asset is created, the marginal cost of production is lower than for traditional capital. As an example, think of the cost of executing an additional operation for a software vs assembling an extra good for a machinery. In most applications, the former is lower than the second by a considerable amount. The second assumption is limited appropriability: by definition, due to their immaterial nature it is harder to exclude other firms from using intangible capital. As an example, consider R&D or other forms of knowledge developed as a co-investment between firms and skilled workers. These key employees retain a part of the value of the asset, posing a potential spillover risk for companies in case they are poached by a competitor (similarly to Crouzet et al. (2024) and Döttling et al. (2025)). Similarly, consider a successful brand: this can be imitated by new entrants, diluting the exclusive ability of the original owner to appropriate the benefits. In both examples, the firm may need liquidity to enforce its ability to exclusively benefit from the asset, either in the

[8] For other modeling approaches of this competitive advantage given by intangible capital see for instance (Crouzet et al., 2024; Hsieh and Rossi-Hansberg, 2023; De Ridder, 2024).

form of wage negotiations or strategic acquisitions.

We embed these mechanisms into a dynamic heterogeneous-firm model designed to study long-run trends in corporate leverage, cash holdings, and the rising importance of intangible capital. Firms differ in their ability to accumulate intangible capital, face financial frictions that interact with their investment decisions, and operate under a constant-elasticity-of-substitution (CES) demand system. Survival risk depends on intangible intensity, and entry is governed by a free-entry condition featuring a cutoff in firm types.

4.1. Setup

Time, goods, and industries. Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The economy is populated by a unit mass of firms by $i \in [0, 1]$. Each firm is the monopolistic producer of a distinct good y_{it}.

Government. The government levies a proportional tax on firm profits at an effective rate $\tau_{it} \in [0, \bar{\tau}]$ that depends on the firm's leverage (defined below). All tax revenue is rebated lump-sum to the representative household as a consumption subsidy. The government runs a balanced budget each period, so tax payments are transfers and do not absorb real resources.

Household and final demand. The representative household has love for variety (Dixit and Stiglitz, 1977) and aggregates individual goods according to a CES utility function:

$$U = \left[\int_0^1 y_{it}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

with $\sigma > 0$ the elasticity of substitution across varieties. The household has three sources of income to finance consumption. First, it owns the firms and therefore receives all dividends that are distributed in each period. Second, it receives lump-sum transfers from the Government and provides firms with the investment goods needed to build capital (see below). We denote total per period income as Y_t.

From a standard utility maximization problem we can derive the familiar demand function for each variety i:

$$y_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\sigma} \frac{Y_t}{P_t}. \tag{2}$$

where the aggregate price index P_t is:

$$P_t = \left(\int_0^1 p_{it}^{1-\sigma}, di \right)^{\frac{1}{1-\sigma}}. \tag{3}$$

To improve tractability we impose $P_t = 1$ in equilibrium, so firms perceive a constant demand curve:

$$y_{it}(p_{it}) = p_{it}^{-\sigma} Y_t. \tag{4}$$

Financial intermediaries. Firms can borrow one–period debt D_{it} from competitive lenders depending on the quality of their collateral. Lenders are risk-neutral and require an exogenous gross rate $1 + r$ in expectation. Debt issuance is capped by collateral, which is provided by the firm's tangible capital, as intangible capital is only weakly pledgeable.

Firm heterogeneity and entry. There is a continuum of potential entrants. Each entrant draws a firm-specific parameter ϕ from a Pareto distribution $G(\phi)$ with lower bound $\underline{\phi} > 0$ and shape parameter $\xi > 1$. The parameter ϕ governs the efficiency of intangible investment. Entrants pay no sunk entry cost (we set $f_e = 0$) but start with zero cash holdings, $Q_{i0} = 0$. Only firms whose entry value is non-negative choose to operate. In the stationary equilibrium, this gives rise to a cutoff type ϕ^* such that $V(0, \phi^*) = 0$ and only firms with $\phi \geq \phi^*$ are active.

4.2. Technology, costs, and frictions

Production technology. Each firm i produces its variety using tangible capital K_{it} and intangible capital H_{it} according to a CES production function

$$y_{it} = A \left[\alpha K_{it}^{\rho} + (1 - \alpha) H_{it}^{\rho} \right]^{\frac{1}{\rho}}, \tag{5}$$

where $A > 0$ is TFP, $\alpha \in (0, 1)$ governs the share of tangible capital, and $\rho \leq 1$ controls the elasticity of substitution between K and H.

Both K_{it} and H_{it} fully depreciate within the period: there is no carry-over of the capital stock. Intangible capital should be interpreted broadly to include organization capital, software, data, brand, and other non-physical assets. Moreover, in this model we consider labour as a produc-

tion factor that is embedded in capital, so that K and H should be considered as tangible and intangible "bundles" (capital and labour operating it) that needs to be reset each period.

Investment costs. Installing capital is costly. For given type ϕ_i, the period investment costs are convex in K and H:

$$c_K(K_{it}) = \frac{\omega_K}{c_K} K_{it}^{c_K}, \tag{6}$$

$$c_H(H_{it}; \phi_i) = \frac{\omega_H}{c_H\, \phi_i} H_{it}^{c_H}, \tag{7}$$

with $\omega_K, \omega_H > 0$ and $c_K, c_H > 1$. The parameter ϕ_i scales the cost of intangible investment: high-ϕ firms can accumulate H more cheaply, reflecting better organizational capabilities or higher quality of intangible opportunities.

Collateral and borrowing costs. At the beginning of period t, firm i enters with cash holdings $Q_{it} \geq 0$ and chooses new investment (K_{it}, H_{it}) and debt $D_{it} \geq 0$. The firm must satisfy a financing constraint:

$$Q_{it} + D_{it} \;\geq\; c_K(K_{it}) + c_H(H_{it}; \phi_i). \tag{8}$$

The interest rate on debt D_{it} is set at r_t, but firms face a collateral constraint in the form of:

$$D_{it} \leq \theta_K K_{it} \tag{9}$$

where $\theta_K > 0$ measures credit tightness.

Taxes and leverage. Let pre–tax operating profits be

$$\Pi_{it}^{\text{pre}} = p_{it}\, y_{it} - c_K(K_{it}) - c_H(H_{it}; \phi_i) - r_t D_{it}. \tag{10}$$

The effective profit tax rate is decreasing in the firm's leverage ratio

$$\ell_{it} = \frac{D_{it}}{K_{it} + \max\left\{0,\, Q_{it} - c_K(K_{it})\right\}}, \tag{11}$$

which captures the idea that interest payments on debt are tax-deductible and that tangible

assets plus residual cash provide a tax base. Specifically,

$$\tau(D_{it}, K_{it}, Q_{it}) = \frac{\bar{\tau}}{1 + \ell_{it}^{\psi}}, \qquad \bar{\tau} > 0,\ \psi > 0. \tag{12}$$

After–tax profits are therefore

$$\Pi_{it} = [1 - \tau(D_{it}, K_{it}, Q_{it})]\, \Pi_{it}^{\text{pre}}. \tag{13}$$

We assume that firms prefer internal cash to external borrowing. In other words, even if debt incurs a convex tax-shield distortion, this does never compensates the interest burden $r \cdot D$.

Exit risk and survival. Each period, firm i faces the possibility of being hit by a idiosyncratic shock to its assets. If the shock hits, the asset value is depleted and the firm must repay part of its original investment R_t to stay in the economy, where $R_t \leq c_K(K_{it}) + c_H(H_{it}; \phi_i)$. One example of such shock could be damage to physical capital, poachers targeting key human capital or potential competitors trying to replicate the firm's business model. Paying R_t is feasible only if the firm has enough liquid resources; thus cash plays a strategic role. This creates a novel motive for precautionary liquidity: preventing the loss of the very asset that drives productivity and market power. If the firm does not have liquidity to sustain the cost it cannot produce and exits the economy due to default [9]. The shock arrival rate follows a Poisson process with hazard rate $\varphi(H_{it}) = \delta + \lambda(H_{it})$, where

$$\lambda(H_{it}) = \lambda_0 \frac{H_{it}^{\lambda_1}}{\lambda_1}, \qquad \lambda_0 > 0,\ \lambda_1 > 1. \tag{14}$$

We interpret $\lambda(H)$ as the risk that the firm's intangible capital is depleted or imitated by competitors. This risk is increasing in H.

Conditional on being hit by the exit risk, a firm needs to repay an amount of cash determined by:

$$R_t \equiv R_t(H_{it}, K_{it}, \phi_i) = \gamma_K \cdot c_K(K_{it}) + \gamma_H \cdot c_H(H_{it}; \phi_i) \tag{15}$$

[9] Think of it as a shock that occurs in between investment and production.

where $\gamma_K, \gamma_H \in (0, 1]$ represent the average severity of the asset depletion between tangible and intangible capital, respectively.

If the firm does not have enough liquidity to pay back R_t, it exits and its continuation value is zero (due to limited liability); new entrants arrive to keep the population of potential firms stationary. It follows that at the end of each period, the firm faces a trade-off between distributing its profits in the form of dividends and building up cash reserves to increase its continuation value. This trade-off is the key mechanism behind the dynamic problem faced by the firm.

4.3. Timing

In each period t the representative firm is characterized by its cash holdings Q_t and its productivity type ϕ. Production inputs for the period, physical capital K_t and intangible capital H_t[10], are chosen anew every period and do not carry over as state variables, as well as debt issued D_t.

The sequence of events within a period is as follows.

1. **Beginning–of–period cash.** At the start of period t the firm observes its cash position Q_t and its type ϕ. It takes aggregate conditions (the demand shifter Y, the equilibrium price index P and the distribution of competitors) as given.

2. **Static production and pricing problem.** Conditional on Q_t and ϕ, the firm chooses intraperiod inputs and prices:

$$(K_t, H_t, D_t, p_t) \in \mathcal{S}(Q_t, \phi),$$

where K_t is tangible capital, H_t is intangible capital, D_t is one period debt, and p_t is the firm specific price. Given the CES demand system and the collateral constraint

$$D_t \ \leq \ \theta_K K_t,$$

[10] In the baseline model both tangible and intangible capital are fully depreciated within the period. The only intertemporal state at the firm level is cash. Persistent "organizational" or relational capital is captured in reduced form by the rebuild cost process described below.

the firm solves a static profit maximization problem. Let $\pi(Q_t;\phi)$ denote the optimal after–tax profit net of investment and interest costs,

$$\pi(Q_t;\phi) \equiv (1-\tau(\ell_t))\Big[p_t y_t - \mathrm{CI}(K_t, H_t;\phi) - rD_t\Big],$$

where y_t is demand at price p_t, $\mathrm{CI}(\cdot)$ is the cost of tangible and intangible inputs, and $\tau(\ell_t)$ is the effective corporate tax rate as a function of leverage ℓ_t.

3. **Choice of cash buffer and dividends.** After solving the static problem and before the rebuild shock is realized, the firm chooses next–period cash holdings Q_{t+1} and commits to distriobuting the residual as dividends:

$$d_t \;=\; Q_t + \pi(Q_t;\phi) - Q_{t+1}.$$

The choice of Q_{t+1} is thus the *dynamic* control: a higher cash buffer today reduces current dividends but improves the firm's ability to withstand future shocks.

4. **Rebuild shock and survival.** Given (Q_t, ϕ) and the chosen intraperiod scale (K_t, H_t), the firm faces a Poisson–type shock to its total capital. With probability

$$\varphi(Q_t, \phi) \;=\; \delta + \lambda(H_t;\phi),$$

a *rebuild event* occurs and the firm must pay a rebuild cost

$$R(K_t, H_t;\phi) \;=\; \gamma_K \mathrm{c}_K K_t \;+\; \gamma_H \mathrm{c}_H H_t,$$

If no rebuild event occurs, the firm can produce, collect revenues, pay back investment costs and debt - including interest - and implements the dividend policy it committed to in step (3). It then survives into $t+1$ without further payments. If a rebuild event occurs, survival depends on the cash buffer Q_{t+1} chosen in step (3)[11]:

[11] Notice that Q_{t+1} depends on both start of the period cash and end of the period profits. We assume that even if profits are not yet accrued by the firm when the shock hits, the firm can receive an emergency interest-free

- if $Q_{t+1} \geq F_t$, the firm pays F_t, survives, can collect profits as in the case of no rebuilding event and enters next period with cash

$$Q_{t+1}^{+} \;=\; Q_{t+1} - F_t;$$

- if $Q_{t+1} < F_t$, the firm cannot finance the rebuild and exits.

5. **Depreciation and next period.** At the end of the period, physical and intangible capital are fully depreciated, in the sense that K_{t+1} and H_{t+1} are chosen anew in the next static problem. The only state carried into $t+1$ is the surviving firm's cash position

$$Q_{t+1} = \begin{cases} Q_{t+1}, & \text{if no rebuild event occurs,} \\ Q_{t+1} - F_t, & \text{if rebuild event occurs and is paid,} \\ \text{exit}, & \text{if rebuild event occurs and cannot be paid.} \end{cases}$$

If the firm exits, it is replaced by a potential entrant that draws a type ϕ from the exogenous distribution G and decides whether to enter based on the free entry condition.

Under this timing, the firm chooses a savings rule $Q_{t+1} = g(Q_t, \phi)$ that trades off current payouts d_t against the option value of surviving future rebuild shocks. The intertemporal problem can be represented in recursive form we introduce in the next section.

4.4. Dynamic firm problem

The firm's key state variable is its beginning-of-period cash stock Q_{it}. Idiosyncratic type ϕ_i is drawn at entry and remains fixed thereafter. Given aggregate objects (Y_t, P_t), a firm of type ϕ chooses in each period t a policy for investment (K_{it}, H_{it}), debt D_{it}, price p_{it}, dividends d_{it} and next-period cash $Q_{i,t+1}$.

We focus on a stationary equilibrium in which aggregate variables are constant over time, so we can drop time subscripts on aggregate objects (Y, P, p) and write the firm's problem in

loan that can anticipate the expected profits.

recursive form. Given (Q, K, H, D, p), after–tax profits are $\Pi(Q; K, H, D, p; \phi)$ as in (13). Cash evolves according to

$$Q' = Q + \Pi(Q; K, H, D, p; \phi) - d, \tag{16}$$

with non–negativity constraints $Q' \geq 0$ and $d \geq 0$. Equivalently, one can treat Q' as a control and recover dividends residually as

$$d(Q, Q') = Q + \Pi(Q; K, H, D, p; \phi) - Q', \qquad 0 \leq Q' \leq Q + \Pi(Q; K, H, D, p; \phi). \tag{17}$$

Firms cannot be net creditors (we do not allow $D < 0$) and must satisfy the financing constraint (8). As described in the previous section, each period the firm faces a potential exit risk due to the damage R. If the shock hits, the firm is left with $Q' - R$, otherwise Q'. This implies that at the end of each period the firm needs to decide whether to distribute the cashflow collected from operating profits or store it as a reserve buffer for the next period.

Taking (Y, P, p) as given, the value function of a firm with type ϕ and cash Q solves:

$$V(Q, \phi) = \max_{K,H,D,p,Q'} \left\{ d(Q, Q') + \beta \left[(1 - \varphi(H)) V(Q', \phi) + \varphi(H) \mathbf{I}_{\{Q' \geq R\}} V(Q' - R, \phi) \right] \right\} \tag{18}$$

subject to the constraints described above. Here Q' denotes next-period cash holdings and $d(Q, Q')$ are dividends paid to the household in the current period.

To solve the firm problem we proceed in two steps: first, we solve a cost minimization conditional on $y(p)$ and (Q, ϕ) to derive conditional equilibrium levels of $K^*(p, Q, \phi), H^*(p, Q, \phi), D^*(p, Q, \phi)$. Second, we solve for the optimal intertemporal cash policy $Q'^*(Q, \phi)$.

Static cost minimization. For a given output level $y(p)$ and cash Q, the firm chooses (K, H, D) to minimize the after–tax total cost of production:

$$\min_{K,H,D} \left\{ [1 - \tau(D, K, Q)] \big(c_K(K) + c_H(H; \phi) + r_t D \big) \right\} \tag{19}$$

subject to

$$y = A\left[\alpha K^{\rho} + (1-\alpha)H^{\rho}\right]^{1/\rho}, \qquad Q + D \geq c_K(K) + c_H(H;\phi), \qquad D \geq 0. \tag{20}$$

This problem pins down the cost-minimizing ratio between tangible and intangible capital and the optimal level of debt for a given output. In numerical work we reduce the dimensionality of the problem by parameterizing the capital mix via the ratio $s \equiv K/H$ and solving a one–dimensional minimization problem in s (see Appendix A.2 for additional details).

Demand and pricing. From the household problem, we know that the firm faces a constant perceived elasticity of demand equal to σ . Given (K, H, D) and Q, the firm's problem in p is to choose a markup over its after-tax marginal cost. This yields a standard Lerner rule of the form

$$p = \frac{\sigma}{\sigma - 1}\widetilde{MC}(Q, \phi), \tag{21}$$

where $\widetilde{MC}(Q, \phi)$ is the after–tax marginal cost of supplying an additional unit of y given optimal input choices and financing. In practice, this marginal cost is computed from the cost minimization problem for given y and (Q, ϕ) and depends on how changes in y affect optimal (K, H, D) and the associated tax and interest wedges.

Optimal Cash Policy Once we have solved for the static problem, we turn to the dynamic problem. We take the first order condition with respect to Q' and derive the Euler equation:

$$1 = \beta\left[(1-\varphi)V_Q(Q', \phi) + \varphi V_Q(Q' - R, \phi)\right] \tag{22}$$

which equates the immediate cost of forgoing one unit of cash on the left hand side - 1:1 reduction in dividends - to the marginal increase in continuation value, due to increase probability of survival and lower cost of investment in the next period.

At the boundaries, we get the following solutions:

$$
\begin{aligned}
1 &\geq \beta\, V_Q(0, \phi) \\
1 &\leq \beta\, V_Q(Q + \Pi, \phi)
\end{aligned}
\tag{23}
$$

At $Q' = 0$, the marginal value of saving is too low and the firm prefers to pay out everything in dividends. Conversely if the firm does not have enough liquidity to ensure (18), then it uses all available cash to build up reserves. This implies that in equilibrium the firm accumulates cash until its marginal continuation value equals 1. Beyond this point, the firm distributes excessive cash in the form of dividends. Whenever cash reserves are depleted due to the arrival of a shock, the firm will start rebuilding them until it has restored its target buffer, refraining from distributing dividends in the meantime.

4.5. Free entry and stationary equilibrium

Free entry and cutoff in ϕ. Potential entrants draw ϕ from the Pareto distribution $G(\phi)$ with lower bound $\underline{\phi}$ and shape ζ. Upon entry they start with $Q_0 = 0$ and value $V(0, \phi)$. Since we impose no sunk entry cost and firms are free to choose whether to operate, free entry implies that there exists a cutoff type ϕ^* such that

$$
V(0, \phi^*) = 0, \tag{24}
$$

and only firms with $\phi \geq \phi^*$ enter and operate. In the stationary equilibrium, the inflow of entrants with $\phi \geq \phi^*$ and $Q_0 = 0$ equals the outflow of exiting firms implied by the survival process $\varphi(H)$.

Stationary distribution of firms. Let $\mu(Q, \phi)$ denote the stationary joint distribution of active firms over cash Q and type ϕ. Given optimal policy functions ($K(Q, \phi)$, $H(Q, \phi)$, $D(Q, \phi)$, $p(Q, \phi)$, $Q'(Q, \phi)$ and survival probabilities $\varphi(H(Q, \phi), Q'(Q, \phi))$, the distribution μ is the fixed point of the law of motion induced by these policies and the entry process. Intuitively, firms with given (Q, ϕ) survive with probability $\varphi(H(Q, \phi), Q'(Q, \phi))$ and move to $Q' = Q'(Q, \phi)$; exiting firms are replaced by entrants with $(Q, \phi) = (0, \tilde{\phi})$, $\tilde{\phi} \sim G(\cdot)$ truncated

at ϕ^*. We characterize μ numerically in the quantitative analysis.

Definition 4.1 (Stationary equilibrium)**.** A stationary equilibrium is a set of aggregate objects (Y, P), firm value and policy functions $\{V(Q,\phi), K(Q,\phi), H(Q,\phi), D(Q,\phi), p(Q,\phi), Q'(Q,\phi)\}$, a cutoff type ϕ^*, and a stationary distribution of active firms $\mu(Q,\phi)$ such that:

(i) Given (Y, P), the value function (18) and associated policy functions solve the firm's dynamic problem for all (Q,ϕ).

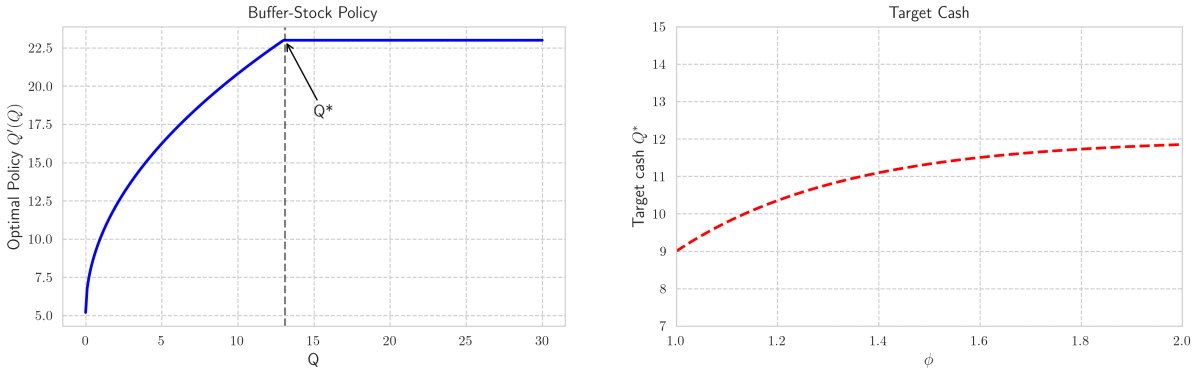
(ii) Potential entrants follow the cutoff rule $\phi \geq \phi^*$ and ϕ^* satisfies $V(0,\phi^*) = 0$.

(iii) The distribution $\mu(Q,\phi)$ is stationary under the law of motion induced by the policy functions and the entry and exit process.

(iv) The household budget constraint and the goods market clearing condition hold, with taxes rebated to the household as lump-sum transfers.

In the quantitative section we characterize this equilibrium numerically, discuss the implied stationary distribution of leverage, cash holdings, and intangible intensity across firms, and use the model to study counterfactual changes in taxes, collateralizability, and intangible technology.

5. Current state of the Project



(a) Graphical representation of policy function. Optimal level of cash accumulation Q' given Q.

(b) Steady state cash holdings as a function of ϕ.

Figure 9: Policy function and steady state cash holdings.

Figure 9a shows graphically the buffer-stock policy function presented in the previous section. The solution of the dynamic problem is implicitly defined by the Euler equation for any $Q < Q^*$, while is constant for any higher value. The firm will therefore retain cash whenever its reserves are below the optimal level Q^* at the beginning of the period, while it is free to either store or distribute all the excess in the alternative scenario. It follows that since a firm with higher value of ϕ can generate more cash flow, it also meets more easily its buffer target, thereby reducing the probability of exiting the economy following one of the shocks to its total capital stock.

Numerical simulations confirm that a reduction in the value of the parameter α (relative productivity of intangibles) strengthen the dynamics presented in the previous section, making the economy more reliant on intangibles, better able to generate cash and less reliant on debt. This result though is not mechanically built in and depends on the parameter set. We are currently working on the model calibration at the beginning and at the end of the sample horizon, and plan to decompose the decline in leverage based on a set of different channels:

- Increasing relative productivity of intangibles ($\alpha \downarrow$).
- Secular stagnation ($A \downarrow$).
- Bank credit tightening ($\psi \downarrow$).

- Reduction in corporate taxation ($\bar{\tau} \downarrow$).

Our model allows to shut down one or more channels at the time, by keeping the value of the related parameters to the level of the first years of the sample, therefore allowing for the construction fo a counterfactual. This allows to conclude the relative weight of the increasing relevance of intangibles for the overall decline in corporate leverage across EU countries.

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Table 1: Country Coverage

	Years	Number of Firms (CompNet)		Number of Firms (Population)	
Country		First Year	Last Year	First Year	Last Year
Belgium	2000-2020	6915	11197	15200	16054
Croatia	2002-2023	3872	6070	6462	6834
Czech Republic	2005-2022	15375	9808	21216	21586
Denmark	2001-2022	10288	9755	15198	12670
Estonia	2004-2023	1432	1324	-	-
Finland	1999-2020	6044	9471	8473	10611
France	1995-2021	79801	86442	107491	97193
Germany	2001-2020	-	-	36607	186987
Hungary	2003-2022	11957	11936	16267	15347
Italy	2006-2020	53001	51432	94578	75769
Latvia	2007-2019	4155	4271	5765	4466
Lithuania	2000-2022	4409	6183	5788	7067
Netherlands	2007-2022	19506	23399	28977	27022
Poland	2002-2022	27733	31747	38656	48972
Portugal	2004-2022	18231	19261	18598	20982
Romania	2005-2023	23654	21788	25172	25447
Slovakia	2000-2022	3987	13160	5521	14877
Slovenia	2002-2023	2717	3380	3209	4418
Spain	2008-2021	20187	23846	77145	70137
Sweden	2003-2020	12635	15669	16999	19370
Switzerland	2009-2022	5214	5088	-	-

Note: The table presents the number of firms in the sample by country and year. The "CompNet" columns represent firms from the Competitiveness Network dataset, while "Population" columns refer to the full population of firms in the country. Missing values in Germany are indicated by a dash (-) and are due to confidentiality requirements.

Table 2: Macro-Sector Coverage

	Number of Firms (CompNet)		Number of Firms (Population)	
Macrosector	First Year	Last Year	First Year	Last Year
Manufacturing	92618	110545	197383	175831
Construction	33890	41176	53560	71740
Wholesale & Retail Trade	60685	84292	95686	144618
Transportation	18147	29701	32974	52765
Accommodation & Food Serv.	12619	28290	25958	66096
ICT	10096	17844	17763	29820
Real Estate	3933	4708	6488	8530
Professional Services	14060	26133	41614	50530
Admin & Support Services	15125	30847	32595	60398

Note: The table presents the number of firms in the sample by macrosector (1 digit level of NACE REv.2 classification) and year. The "CompNet" columns represent firms from the Competitiveness Network dataset, while "Population" columns refer to the full population of firms in the country.

Internet Appendix for "A Structural Break in EU Corporate Leverage Trends"

A. Additional details and derivations

This appendix collects derivations and technical details for the model in Section 4.1. We first derive the demand system and elasticities implied by the nested CES structure, then characterize the firm's static cost minimization problem, and finally discuss the stationary firm distribution and the free entry cutoff.

A.1. Demand system and elasticities

The representative household maximizes utility (1) subject to its budget each period:

$$\max\ U \quad \text{s.t.} \int_0^1 p_{it} y_{it}\ di \leq Y_t \tag{25}$$

Total income is determined by the sum of dividends, firms' investment costs and the tax transfers:

$$Y_t = \int_0^1 d_{it} + c_K(K_{it}) + c_H(H_{it}; \phi_i)\ di + T_t \tag{26}$$

Upon introducing the price index (3), we can solve the utility maximization with respect to y_{it} and retrieve the familiar individual good demand (2).

A.2. Static cost minimization

For a given output level y and beginning-of-period cash holdings Q, a firm of type ϕ solves the static cost minimization problem (19) subject to (20). Here we describe a convenient reparameterization that reduces the dimensionality of the problem.

Using the production function (5), we can parameterize the capital mix by the ratio $s \equiv K/H > 0$. For any given (y, ϕ), production implies

$$\left(\frac{y}{A}\right)^{\rho} = \alpha K^{\rho} + (1-\alpha) H^{\rho} = [\alpha s^{\rho} + (1-\alpha)] H^{\rho}. \tag{27}$$

Therefore,

$$H(s; y) = \frac{y}{A}\Big[\alpha s^{\rho} + (1-\alpha)\Big]^{-1/\rho}, \qquad K(s; y) = s\, H(s; y). \tag{28}$$

Substituting these into the cost functions (6)–(7) yields

$$c_K(s;y) = \frac{\omega_K}{c_K} K(s;y)^{c_K}, \tag{29}$$

$$c_H(s;y;\phi) = \frac{\omega_H}{c_H\,\phi} H(s;y)^{c_H}. \tag{30}$$

Total investment cost is $C_I(s;y,\phi) = c_K(s;y) + c_H(s;y;\phi)$.

The financing constraint (8) binds whenever $C_I(s;y,\phi) > Q$ and then determines debt as

$$D(s;y,Q,\phi) = \max\left\{0,\, C_I(s;y,\phi) - Q\right\}. \tag{31}$$

The tangible capital share is

$$K_\text{share}(s) = \frac{K}{K+H} = \frac{s}{1+s}, \tag{32}$$

The leverage ratio used in the tax function (12) is

$$\ell(s;y,Q,\phi) = \frac{D(s;y,Q,\phi)}{K(s;y) + \max\{0, Q - c_K(s;y)\}}, \tag{33}$$

and the effective tax rate is

$$\tau(s;y,Q,\phi) = \frac{\bar{\tau}}{1+\ell(s;y,Q,\phi)^{\psi}}. \tag{34}$$

After–tax total cost (including interest) as a function of the ratio s is

$$J(s;y,Q,\phi) = \left[1 - \tau(s;y,Q,\phi)\right]\left(C_I(s;y,\phi) + rD(s;y,Q,\phi)\right). \tag{35}$$

The static cost minimization problem (19) reduces to a one–dimensional minimization:

$$s^*(y,Q,\phi) = \arg\min_{s>0} J(s;y,Q,\phi). \tag{36}$$

Given s^*, the optimal choice of (K,H,D) for a given (y,Q,ϕ) is

$$K^*(y,Q,\phi) = K\big(s^*;y\big), \quad H^*(y,Q,\phi) = H\big(s^*;y\big), \quad D^*(y,Q,\phi) = D\big(s^*;y,Q,\phi\big). \tag{37}$$

These static policies are used inside the dynamic problem to compute after–tax profits and

marginal cost.

A.3. Stationary distribution and free entry

Let $\mu_t(Q, \phi)$ denote the cross-sectional distribution of active firms over (Q, ϕ) at the beginning of period t. Given the policy functions $Q'(Q, \phi)$ and $H^*(Q, \phi)$ and the survival probabilities $\varphi(H^*(Q, \phi))$, the law of motion for μ_t is

$$\mu_{t+1}(B) = \iint \varphi\big(H^*(Q, \phi)\big)\mathbf{1}\{Q'(Q, \phi) \in B\}\, d\mu_t(Q, \phi) + \iint \mathbf{1}\{Q_0 \in B\}\, d\mu_t^{\text{entry}}(\phi), \quad (38)$$

for any Borel set B in the support of Q, where μ_t^{entry} is the measure of entrants in period t and $Q_0 = 0$ is the entry cash level. In steady state the distribution is constant over time, $\mu_{t+1} = \mu_t = \mu$, and the mass of entrants equals the mass of exiting firms implied by the survival process.

Under our free entry condition with zero entry cost, only firms with $\phi \geq \phi^*$ enter, where ϕ^* satisfies $V(0, \phi^*) = 0$ and $V(0, \phi) > 0$ for $\phi > \phi^*$. The stationary distribution $\mu(Q, \phi)$ is then supported on $\{(Q, \phi) : Q \geq 0, \phi \geq \phi^*\}$ and is characterized numerically through an iterative procedure that applies the transition implied by (Q', φ) to an initial guess for μ until convergence.

In the quantitative analysis we discretize the (Q, ϕ) state space and approximate the transition of μ using standard non-stochastic interpolation on the Q-grid. This yields a stationary distribution of firms that we use to compute model moments and to conduct counterfactual experiments.

A.4. Equivalence of timing structures

For the numerical solution we adopt a formulation in which the only firm–level state is cash on hand Q. The firm solves a one–dimensional dynamic program with control Q', the cash buffer carried into the next period. In this appendix we show that the specific sequencing of intraperiod events—"timing A" versus "timing B" in the main text—is immaterial for the recursive representation, provided that (i) intraperiod decisions can be summarized by current net payouts and (ii) the rebuild shock affects only the transition of cash between Q' and next period's state.

Timing A. Under timing A the within–period events can be summarized as:

1. The firm enters period t with cash Q_t.

2. It chooses intraperiod inputs (K_t, H_t, D_t, p_t), produces, repays investment and debt, and obtains after–tax profit $\Pi(Q_t; \phi)$.

3. It chooses next–period cash Q_{t+1} and dividends $d_t = Q_t + \Pi(Q_t; \phi) - Q_{t+1}$.

4. A rebuild shock arrives with probability $\varphi(Q_t, \phi)$, requiring a payment $R(Q_t, \phi)$ to preserve the firm's organizational capital.

   - If the shock does not arrive, the firm enters $t+1$ with cash Q_{t+1}.
   - If the shock arrives and $Q_{t+1} \geq R(Q_t, \phi)$, the firm pays $R(Q_t, \phi)$ and enters $t+1$ with cash $Q_{t+1} - R(Q_t, \phi)$.
   - If the shock arrives and $Q_{t+1} < R(Q_t, \phi)$, the firm exits.

Given this timing, the recursive problem is

$$V_A(Q; \phi) \;=\; \max_{Q' \in [0, Q + \Pi(Q;\phi)]} \left\{ Q + \Pi(Q; \phi) - Q' + \beta\, \Gamma(Q'; Q, \phi) \right\}, \tag{39}$$

where

$$\Gamma(Q'; Q, \phi) \;=\; \big[1 - \varphi(Q, \phi)\big] V_A(Q'; \phi) \;+\; \varphi(Q, \phi)\, \mathbf{1}_{\{Q' \geq R(Q,\phi)\}} V_A\big(Q' - R(Q, \phi); \phi\big). \tag{40}$$

If the firm cannot pay the rebuild cost it exits and receives continuation value 0.

Timing B. In the main text we describe an alternative narrative (timing B) in which the firm chooses its savings policy before the realization of the shock and before the full resolution of intraperiod cash flows. The sequence of events is:

1. The firm starts with cash Q_t and type ϕ.

2. It chooses (K_t, H_t, D_t, p_t) and *commits* to a savings rule Q_{t+1}, equivalently choosing dividends $d_t = Q_t + \Pi_t - Q_{t+1}$ based on expected intraperiod profits $\tilde{\Pi}_t$.

3. A rebuild shock is realized with probability $\varphi(Q_t, \phi)$; if the shock occurs and $Q_{t+1} < R(Q_t, \phi)$ the firm exits; otherwise it survives.

4. Conditional on survival, production is completed, revenues are realized, and the firm ends the period with cash Q_{t+1} if no rebuild occurs or $Q_{t+1} - R(Q_t, \phi)$ if the rebuild cost is paid.

The crucial observation is that under timing B the firm still faces the same trade–off between current payouts and the distribution of next–period cash conditional on Q_{t+1}. All intraperiod variables (K_t, H_t, D_t, p_t) affect continuation only through three reduced–form objects:

- after–tax profit $\Pi(Q_t; \phi)$, which pins down total resources $Q_t + \Pi(Q_t; \phi)$ available for dividends plus next–period cash,
- shock arrival probability $\varphi(Q_t, \phi)$, and
- rebuild cost $R(Q_t, \phi)$.

Given (Q_t, ϕ) and the choice of Q_{t+1}, the law of motion for the next–period cash state is identical to that under timing A:

$$Q_{t+1} = \begin{cases} Q_{t+1}, & \text{with prob. } 1 - \varphi(Q_t, \phi), \\ Q_{t+1} - R(Q_t, \phi), & \text{with prob. } \varphi(Q_t, \phi)\, \mathbf{1}_{\{Q_{t+1} \geq R(Q_t, \phi)\}}, \\ \text{exit}, & \text{otherwise.} \end{cases}$$

Therefore the Bellman equation under timing B is

$$V_B(Q; \phi) \;=\; \max_{Q' \in [0, Q + \Pi(Q;\phi)]} \left\{ Q + \Pi(Q; \phi) - Q' + \beta\, \Gamma(Q'; Q, \phi) \right\}, \tag{41}$$

with the same continuation function $\Gamma(\cdot)$ as in (40).

Proposition 1 (Equivalence of timing A and B)**.** Suppose that (i) all intraperiod choices (K, H, D, p) are optimally chosen conditional on cash Q and type ϕ and do not carry over as state variables, and (ii) the rebuild shock affects only the transition from Q' to next period's cash via $(\varphi(Q, \phi), R(Q, \phi))$. Then $V_A(\cdot; \phi)$ and $V_B(\cdot; \phi)$ defined in (39) and (41) coincide, and the optimal savings policies are identical:

$$V_A(Q; \phi) = V_B(Q; \phi), \qquad g_A(Q, \phi) = g_B(Q, \phi) \quad \forall Q, \phi.$$

Proof. The Bellman operators associated with timing A and timing B map any bounded con-

tinuous function V into

$$(\mathcal{T}V)(Q;\phi) \;=\; \max_{Q'\in[0,Q+\Pi(Q;\phi)]} \left\{Q+\Pi(Q;\phi)-Q'+\beta\Gamma_V(Q';Q,\phi)\right\},$$

where

$$\Gamma_V(Q';Q,\phi) \equiv \big[1-\varphi(Q,\phi)\big]V(Q';\phi)+\varphi(Q,\phi)\,\mathbf{1}_{\{Q'\geq R(Q,\phi)\}}V(Q'-R(Q,\phi);\phi).$$

Under assumptions (i) and (ii) the mapping Γ_V is the same under both timings, because the distribution of next–period cash conditional on (Q,Q') is identical. Hence the Bellman operators for timing A and B coincide:

$$\mathcal{T}_A = \mathcal{T}_B \equiv \mathcal{T}.$$

Since $\mathcal{T}$ is a contraction on the space of bounded functions (given discount factor $\beta\in(0,1)$), it admits a unique fixed point V^*, and the value functions induced by either timing must satisfy $V_A = V_B = V^*$. The associated optimal policies are generated by the same argmax of the same objective, and therefore also coincide. □

Proposition 1 justifies using the simpler single–state recursive formulation in the numerical solution, while interpreting the economic timing in the richer narrative described in Section 4.3.