# Automation in R&D Networks and the Singularity

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**Abstract**

AI labs are increasingly using AI itself to accelerate AI research, creating a feedback loop that could potentially lead to an "intelligence explosion". We develop a general semi-endogenous growth model with an innovation network, where research in one sector increases the productivity of research in other sectors, and derive a clean analytical condition under which growth becomes superexponential ("explosive"). The key intuition is that automation of research tasks both offsets diminishing returns to research and increases cross-sectoral research spillovers, making explosive growth more likely. Applying this model to a calibrated, AI-integrated economy, we demonstrate that the growth effects of automation may be slow initially but compound rapidly. In our benchmark calibration, automating the tasks required to double the long-run growth rate already achieves well over half of the automation level needed to generate explosive growth.

## 1 Introduction

The potential for artificial intelligence (AI) to fundamentally transform economic growth has been a topic of intense debate. While some claim that the effects of AI on growth

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will be a mere rounding error (Acemoglu, 2024), others argue that AI could lead to a technological "singularity" (Kurzweil, 2005, Bostrom, 2014). Central to this latter thesis is the concept of recursive self-improvement, where AI systems become increasingly capable of designing and improving themselves, with the resulting feedback loop leading to an "intelligence explosion" and potentially rapid economic growth (Good, 1966, Yudkowsky, 2013).

AI tools have already demonstrated significant potential in advancing scientific research. For example, the model AlphaFold successfully predicted the structures of previously unknown proteins, marking a breakthrough in molecular biology. Additionally, LLMs have become powerful coding assistants, enabling AI researchers to code more efficiently. Several recent studies have documented measurable productivity gains from such tools (Gambacorta et al., 2024, Song et al., 2024, Pandey et al., 2024), and Anthropic's CEO Dario Amodei has predicted that AI would be writing the large majority of their code imminently. These developments underscore AI's growing role not only as a scientific instrument but as a driver of innovation itself. The potential for AI to increase the productivity of AI research has critical implications for growth economics, which we consider in this paper by explicitly modeling AI as a distinct component of the growth process.

In this paper we focus on the conditions that give rise to explosive growth. We note that the essential component of any explosive growth story is amplifying feedback loops. Such feedback loops are absent in standard macroeconomic models, though notable exceptions include Malthusian or lab-equipment growth models; these models feature output being transformed back into the key driving force for additional growth (be it human labor or ideas). We label such forces *economic* feedback loops. Additionally, models could emphasize the importance of technological spillovers where progress in one technology can contribute to progress in another. We call these *technological* feed-

back loops since these don't pass through output channels.

The model developed here combines lab-equipment and technological spillovers to provide a clear characterization of how technological and economic feedback loops interact, and the conditions where these interactions can give rise to balanced or explosive growth. In particular, where the progress of components of these systems go to infinity in finite time; a 'Type II' singularity (Aghion et al., 2017), or 'hyperbolic' growth. We then use the insights of this model to illustrate how automation of both research and production can amplify existing feedback loops or spawn them where they did not exist before. This effect will be to accelerate growth, and in surprisingly moderate cases lead to hyperbolic growth. In particular, the model predicts hyperbolic growth will arise under the simple condition:

$$\boxed{f_S r_S + f_H r_H + \frac{1}{\alpha} \cdot f_A r_A + f_Y > 1}$$

where $r$ $(> 0)$ terms capture sectoral research productivity; $f \in [0, 1]$ terms capture the share of tasks in each sector that can be automated, and $S$, $H$, $A$, $Y$ denote software, hardware, TFP innovation and output respectively; and $\alpha$ is the labor share of output.

These results rest on strong assumptions. In particular, we impose Cobb-Douglas aggregation of task outputs at multiple stages in the model developed. Under this aggregation, output can be driven by explosive progress in a subset of tasks. This means that automation of some tasks is enough to kickstart explosive dynamics across the system. The hyperbolic growth threshold we arrive at is strikingly low. This suggests that one of the most important areas for additional research is understanding the degree of substitutability between economic tasks in a world with increasing automation. We discuss additional limitations of the model developed here in Section 6.

*Related Literature*– Several studies have investigated the possibility of AI and automation

leading to transformative growth. Aghion et al. (2017) characterize the conditions under which a system capable of recursive self-improvement exhibits hyperbolic growth. Trammell and Korinek (2025) extend this analysis by embedding such self-improving technologies in a macroeconomic setting where capital accumulation and technological progress reinforce each other. Jones (2025) constructs a richer environment on the deployment of better machines (including AI) to research and development to understand how improvements machines contributing to research could accelerate technological progress.

We advance this line of work in three main directions. First, we generalize the analysis to systems whose progress depends on a network of multiple technologies with heterogeneous returns to research effort, providing a unified condition for when growth becomes explosive. Second, we integrate technological heterogeneity with economic feedback loops, showing how output–technology complementarities amplify or dampen the possibility of hyperbolic growth. Third, we use this richer framework to derive a threshold for automation that marks the transition from balanced to hyperbolic growth and provide quantitative estimates under empirically grounded parameters.

Erdil et al. (2025) make a uniquely rich effort to model the growth implications of AI using an Integrated Assessment Model. Our simpler framework captures the same core dynamics—with AI displacing human labor in both research and production, thereby driving growth and accelerating further AI progress—while remaining analytically tractable. This tractability allows us to characterize the interaction between economic and technological feedback loops and to derive conditions under which growth becomes explosive.

Networked models of technological progress have also been applied to understand optimal R&D allocations (Liu and Ma, 2024), as well as the sources of heterogeneity in sectoral productivity growth (Ngai and Samaniego, 2011). Their analysis explicitly

rules out explosive growth by imposing constant returns to scale on cross-sector innovation spillovers, which ensures balanced growth in an endogenous-growth setting. This framework serves as a tractable model to understand the growth implications of automation, when AI is allowed to automate R&D across many sectors (including those that feed into AI progress). Specifically, by not imposing constant returns to scale, we demonstrate the conditions on the technology network that give rise to explosive growth, and how automation can strengthen the links in this network.

## 2 Motivating Example: Robotic Malthusian Growth

We motivate our full model with a simple illustrative example.[1] Suppose we have a Malthusian model for robots, where output is given by

$$Y_t = M^{1-\alpha}(\ell L_{AI,t})^{\alpha} \tag{1}$$

where $L_t$ is the available *robotic* labor at time $t$; $M$ is the supply of some fixed factor (potentially representing land, data, energy, human input); $\ell$ is the fraction of robots dedicated to producing output, where as the remaining robots are dedicated to producing more robots:

$$\dot{L}_t \;=\; n(Y_t)[(1-\ell)L_t]^{1-\beta} \tag{2}$$

we assume that $\beta > 0$ so that there is some diminishing returns to scale in increasing the stock of robot builders. Here $n(\cdot)$ is the (diminishing returns to scale-adjusted) replication rate of robots. We assume that this rate is a function of output since more output can be used to purchase the materials that can be used to construct more robots.

[1] Aghion et al. (2017) and Trammell and Korinek (2025) have considered similar thought experiments.

In the simplest case, suppose $n(Y_t) = Y_t^{\lambda}$. Therefore, we have the law of motion for robots

$$\dot{L}_t = Y_t^{\lambda}[(1-\ell)L_t]^{1-\beta} \tag{3}$$

$$\propto L_t^{\lambda\alpha-\beta+1} \tag{4}$$

and this autonomous differential equation exhibits hyperbolic growth if the exponent from equation 4 is greater than one. Equivalently, if

$$\frac{\lambda\alpha}{\beta} > 1. \tag{5}$$

We can see this condition is dictated by the strength of economic feedback between output and technology, $\lambda$, and technology and output, $\alpha$, as well as the strength of the technological 'self feedback' which is decreasing in $\beta$.

If we alternatively allowed $M$ to grow at rate $g_M$ then one can show that the balanced growth path of robots in this envrionment is given by

$$g_L^{BGP} = \frac{\frac{\lambda}{\beta}(1-\alpha)}{1-\frac{\lambda\alpha}{\beta}} g_M. \tag{6}$$

Inspecting the denominator, the condition that would prevent this balanced growth path from being well-defined (so the denominator is zero) is exactly the hyperbolic growth condition from inequality 5. This illustrative example motivates the main features of the model we introduce in the following section. In particular, it highlights three forces contributing to hyperbolic growth: i) the presence of accumulable factors ii) the strength of accelerating factors or spillovers leading factor accumulation and iii) the strength of dampening factors (such as diminishing returns) that weaken feedback loops.

In Sections 3 and 4 we depart from this illustrative Malthusian example, to one that incorporates economic feedback via lab-equipment and technological feedback both through 'self feedback' and through technological spillovers. Ultimately this model will provide the framework to evaluate conditions for explosive growth in the AI-economic environment developed in Section 5.

# 3 A General Framework for Hyperbolic Growth

In this section we first consider the necessary and sufficient conditions for hyperbolic growth in a network of technologies; with spillovers generating possibly feedback loops between technologies. We then embed this model of networked technological progress into an economic environment, demonstrating economic feedback loops that can be isolated analytically in their contribution balanced or explosive growth.

This model has (Section 3.1) a vector of technologies linked by spillovers, and (Section 3.2) output that depends on these technologies, and capital and labor inputs to R&D.

## 3.1 Technological Feedback Loops

Consider a general networked model of technological progress, featuring a set of distinct technologies, $I$, that number $N$. Allow technological levels in each sector $i \in I$ spillover to technological progress in other sectors:

$$\dot{A}_i = v_i R_i^{\lambda_i} \prod_{j \in I} A_j^{\phi_{i,j}} \tag{7}$$

where $R_i = L_i^{\gamma_i} K_i^{1-\gamma_i}$ is the aggregated capital and labor research inputs to sector $i$ and $v_i$ is a constant scaling parameter. Here $v$ are the only constant variables and are

ultimately unimportant for analysis, hence we drop time-subscripts entirely and note that all capitalized variables are growing over time. There are diminishing returns to parallel research input, $\lambda_i \in (0, 1]$, and sectoral spillovers, $\phi_{i,j} (\geq 0)$. We define $\phi_{i,i} = 1 - \beta_i$, where $\beta_i$ captures the degree of diminishing returns to research within a given sector introduced in Jones (1995). We impose $\beta_i > 0$ so ideas are getting harder to find; otherwise the system necessarily generates explosive growth.[2]

From equation 7, we can see for this system to exhibit balanced growth, for each technology we require

$$g_{A_i}^{BGP} = \frac{\lambda_i}{\beta_i} g_{R_i} + \sum_{j \in I \setminus i} \frac{\phi_{i,j}}{\beta_i} g_{A_j}^{BGP}. \tag{8}$$

That is, the growth rate of technology $i$ must be such that growth arising from research inputs and spillovers will offset the diminishing returns defined by $\beta_i$. We define relevant terms here:

$$r_i = \frac{\lambda_i}{\beta_i}, \qquad s_{i,j} = \frac{\phi_{i,j}}{\beta_i} \quad \text{for } i \neq j \tag{9}$$

The term $r_i$ is a sector specific measure of research productivity, capturing how technological progress responds to 'own sector' research effort. We also introduce an analogous term spillover term, $s_{i,j}$ capturing how technological progress in sector $j$ impacts technological progress in sector $i$. We can simplify this system further in matrix form. Defining the $N \times N$ technological feedback matrix, $\mathbf{F}^A$, such that

$$\mathbf{F}_{i,j}^A = \begin{cases} s_{i,j}, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}. \tag{10}$$

[2] Although, if $N = 1$, $\beta_i = 0$ would be insufficient for explosive growth (Aghion et al., 2017). However, we are generally interested in multi-technology systems where any one sector with $\beta_i = 0$ would be sufficient for explosive growth.

From here we define the central mathematical result with we subsequently apply to evaluate growth conditions under varying assumptions on the parameters defining the growth environment:

**Proposition 1.** *Take a (strictly positive) growth system defined by equations for each $i \in I$*

$$\dot{A}_i = v_i E_i^{\ell_i} A_i^{1-b_i} \prod_{j \in I/i} A_j^{p_{i,j}}$$

*with $b > 0$ and $p \geq 0$, and the (irreducible) matrix $\mathbf{F} \in \mathbb{R}_{\geq 0}^{N \times N}$ where $\mathbf{F}_{i,j} = p_{i,j}/b_i$ for $i \neq j$, $\mathbf{F}_{i,i} = 0$ and variables $E$ grow at a constant, exogenous rate $g_E \in \mathbb{R}_{\geq 0}^N$. The spectral radius of $\mathbf{F}$, $\rho(\mathbf{F})$, partitions the growth system into three cases:*

1. *$\rho(\mathbf{F}) < 1$. The system exhibits balanced growth along the path*

$$\boxed{g_A^{BGP} = \mathbf{\Psi} \mathbf{r} g_E} \tag{11}$$

   *where and $\mathbf{\Psi} = (\mathbf{I} - \mathbf{F})^{-1} \in \mathbb{R}_{\geq 0}^{N \times N}$ and $\mathbf{r} = diag(\ell_i/b_i, \ldots, \ell_N/b_N)$.*

2. *$\rho(\mathbf{F}) = 1$. The system growth at any time is bounded by double-exponential growth, and exponential in the purely endogenous growth case ($g_E = 0$).*

3. *$\rho(\mathbf{F}) > 1$. The system exhibits hyperbolic growth, with all variables growing to infinity in finite time.*

*Proof.* See Appendix A. □

This result implies that to understand the conditions that give rise explosive growth paths, we can generally limit our focus to the behavior of—or more specifically, the existence of—the balanced growth path. In our spillover model, the feedback matrix $\mathbf{F}^A$ conveniently provides (i) the balanced growth path via the (technological) Leontief

inverse, $\mathbf{\Psi_A} = (\mathbf{I} - \mathbf{F}^A)^{-1}$ and (ii) the conditions where the system in equation (7) exhibits hyperbolic growth. $\mathbf{\Psi}_A$ captures how progress in each sector feedback into all other sectors repeatedly along the balanced growth path; while $r$ captures the direct effect of research inputs within a sector. We can see that entries in $\mathbf{\Psi}_A$ will be increasing with returns to other sector research, $s_{i,j}$, and therefore increasing with spillover terms $\phi_{i,j}$ and decreasing with the strength of diminishing returns to research $\beta_i$.

This balanced growth path nests that of the standard semi-endogenous balanced growth path presented in Jones (1995); setting $\phi_{i,j} = 0$ (so $\mathbf{\Psi}_A = \mathbf{I}$) for $i \neq j$ and allowing research inputs to be proportional to population growth we have the vector of balanced growth paths

$$g_A^{BGP} = \mathbf{r}n.$$

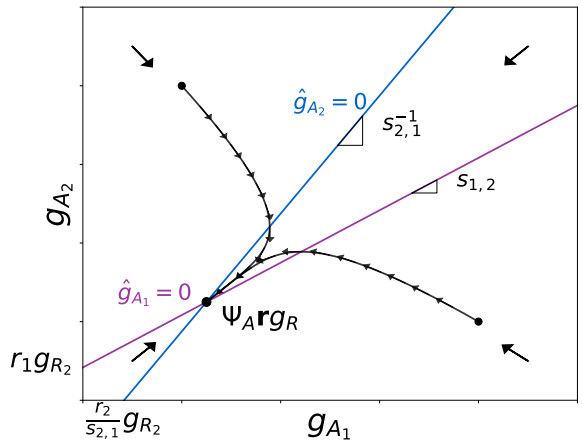
Liu and Ma (2024) also note that explosive growth can be inferred directly from the eigenvalues of the exponent matrix (of $\phi$ terms) in equation (7). However, the balanced-growth specification is particularly useful here since empirical estimations of returns to research effort that we ultimately use to calibrate a threshold from explosive growth from automation are calculated under the assumption of a balanced growth path. This aligns with the motivating example in Section 2: if the balanced growth path is not well defined then the system exhibits hyperbolic growth.

In Figure 1 we illustrate hyperbolic and balanced growth in phase diagrams of a two technology model. From this figure we can see stability of the balanced growth path—where $\hat{g}_A$, the growth in the growth rate of $A$, is zero—can be understood as a crossing condition on the isolines. In this case, the slope of the $\hat{g}_{A_1}$ isoline is $s_{1,2}$ and is $s_{1,2}^{-1}$ for the $\hat{g}_{A_2}$ isoline.[3] Therefore, lines cross whenever $s_{1,2}s_{2,1} > 1$. This is the exact case where the largest eigenvalue of $\mathbf{F}^A$ exceeds one and there is no well-defined $\mathbf{\Psi}_A$.

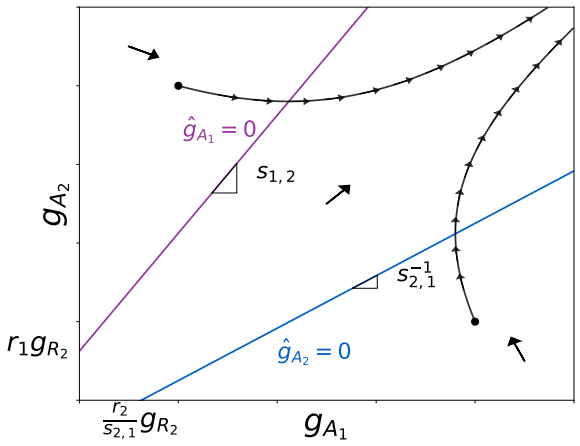
[3]One can arrive at this result by taking the time derivative of equation 7 and then setting the growth-in-growth rates, $\hat{g}_{A_1}$ and $\hat{g}_{A_2}$, to zero and rearranging $\lambda$, $\phi$ and $\beta$ terms into $s$ and $r$ terms by their definitions in equation 9.

Figure 1: Hyperbolic vs Balanced Growth in a Two Technology Network

(a) Stable, Balanced Growth ($s_{2,1}^{-1} > s_{1,2}$)




(b) Hyperbolic Growth ($s_{2,1}^{-1} < s_{1,2}$)




Intuitively, in the two-sector example, the explosive growth condition $s_{1,2}s_{2,1} > 1$ is more likely when either feedback term $s_{1,2}$ or $s_{2,1}$ is large. Recall that the feedback terms are defined as $s_{i,j} = \phi_{i,j}/\beta_i$, where $\phi_{i,j}$ measures the spillover of sector $j$ to growth in sector $i$, while $\beta_i$ measures the strength of the sector-$i$ "ideas-getting-harder-to-find" effect. Thus, explosive growth is more likely when either (1) spillovers are *large*, or (2) the ideas-getting-harder-to-find effect is *small*.

## 3.2 Economic and Technological Feedback Loops

Now we introduce the technological feedback expression in equation (7), to a broader economic environment. Specifically, instead of fixing the growth rate of research inputs as above, we endogenize this term through lab-equipment inputs and exogenous population growth. We also allow technological progress to increase productivity in the final goods sector. In turn, this will lead to faster capital accumulation and ultimately lead to faster technological progress.

Specifically, we investigate the following system

$$
\begin{aligned}
Y &= \bar{A} K_Y^{1-\alpha} L_Y^{\alpha} && (12)\\
\dot{A}_i &\propto (K_i^{1-\gamma_i} L_i^{\gamma_i})^{\lambda_i} A_i^{1-\beta_i} \prod_{j \in I \setminus i} A_j^{\phi_{i,j}} && (13)\\
\bar{A} &= \prod_{i \in I} A_i^{\tau_i} \quad \text{where} \quad \sum_{i \in I} \tau_i = 1 && (14)\\
\dot{K} &= aY - \delta K && (15)\\
K &= K_Y + \sum_{i \in I} K_i && (16)\\
L &= L_Y + \sum_{i \in I} L_i && (17)
\end{aligned}
$$

were capital and labor shares allocated to each technology and output remain constant and $a$ is the constant savings rate. From this environment we have the result:

**Corollary 1** (Economic and Technological Feedback)**.** *The balanced growth path of technologies the system described in equations (12)-(17) is given by*

$$\boxed{g_A^{BGP} = \mathbf{\Psi}_{A,Y} \mathbf{r} n} \tag{18}$$

*where* $\mathbf{\Psi}_{A,Y} = \left(\mathbf{I} - [\mathbf{F}^A + \mathbf{F}^Y]\right)^{-1}$ *and* $\mathbf{F}^A$ *and* $\mathbf{F}^Y$ *are the* $I \times I$ *technological and economic feedback matrices, with* $\mathbf{F}^A$ *and* $\mathbf{r}$ *defined above (equations (10) -(11) ) and*

$$\mathbf{F}^Y := \frac{1}{\alpha} [\mathbf{r}(\mathbf{1}_N - \gamma)]' \tau \tag{19}$$

*with* $(\mathbf{1}_N - \gamma)$ *and* $\tau$ *being the column vectors of capital contributions to research, the technology contributions to total factor productivity, and the sectoral returns to research effort.*

From this balanced growth definition, we can immediately see that including economic, alongside technological, feedback loops necessarily bring the system closer to

the hyperbolic tipping point; since $\mathbf{F}^Y$ is non-negative then $\mathbf{F}^Y + \mathbf{F}^A \geq \mathbf{F}^A$ entry-wise.

Note that entries in $\mathbf{F}^Y$ have a straightforward interpretation. The vector of terms, $\mathbf{r} \times (\mathbf{1}_N - \gamma)$, mediate the effect of output and hence capital on research inputs; on the balanced growth path (with $g_Y^{BGP} = g_K^{BGP}$), from equation (13) we can arrive at

$$g_{A_i}^{BGP} = r_i(1-\gamma)g_Y^{BGP} + \sum_{j \in I \setminus i} s_{i,j} g_{A_j}^{BGP} + r_i \gamma_i n.$$

Further, the vector of terms $\frac{1}{\alpha}\tau$ mediate the effect of technological progress on production; on the balanced growth path, from equation (12) we can arrive at

$$g_Y^{BGP} \quad = \quad \frac{1}{\alpha}\tau \cdot g_A^{BGP} + n.$$

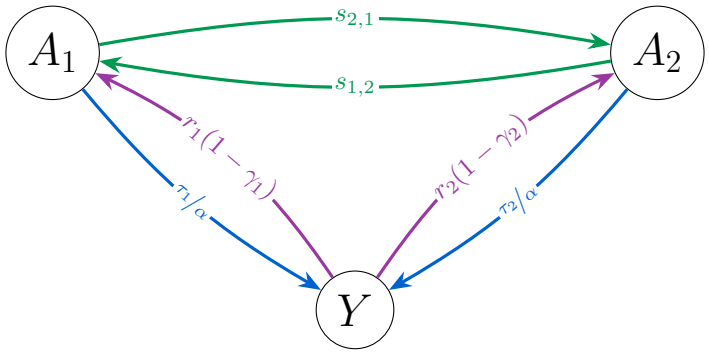
Therefore, we can observe that entries in the feedback matrix exhibit complementarity: intensifying feedback in one direction of the output-technology loop amplifies the impact of strengthening feedback in the opposite direction.

In Figure 2 we illustrate how feedback loops arise out of the, separate, output to technology vector (purple) and the technology to output vector (blue) from $\mathbf{F}^Y$, as well as the technology spillovers (green) from $\mathbf{F}^A$. Labor is also an input into each of these processes, but since it is non-accumulable, and hence cannot participate in feedback loops, we omit it from the figure.

The definition of the Leontief inverse, $\mathbf{\Psi}_{A,Y}$, provides a clear decomposition of the effects of technological and economic feedback loops to inform the balanced growth path of this economy. Further, since this definition of the balanced growth path has an identical structure to that in the technology-only case, we can similarly apply Proposition 1:

**Corollary 2.** *Supposing population growth, $n$, is exogenous, finite, and positive, a balanced*

Figure 2: Economic and Technological Feedback in a Two Technology Network




*Note*: Summarizing entries in $\mathbf{\Psi}_{A,Y}$, green lines represent the technology spillovers, $s_{i,j}$, that make up entries of $\mathbf{F}^A$; purple lines represent output-technology feedback mediated by the vector $\mathbf{r} \times (\mathbf{1}_N - \gamma)$ from $\mathbf{F}^Y$; blue lines represent the technology-output feedback mediated by the vector $\tau \times \alpha^{-1}$ from $\mathbf{F}^Y$.

*growth path exists for the system of equations in 12 - 17 iff* $\det\left(\mathbf{I} - [\mathbf{F}^A + \mathbf{F}^Y]\right) > 0$. *Otherwise, the system explodes in finite time.*

Just as with the balanced growth path under technological feedback loops alone, we can recover the standard balanced growth path from Jones (1995) if $\mathbf{\Psi}_{A,Y} = \mathbf{I}$. That is, there are i). no technology spillovers—so $\mathbf{F}^A_{i,j} = 0$ for all $i$ and $j$ ii). that no technology sector can simultaneously have contribute to research (so $\gamma_i < 1$ for all $i$) and contribute to total factor productivity (so $\tau_i = 0$ for all $i$)—in this case we'd have $\mathbf{F}^Y_{i,j} = 0$ for all $i$ and $j$.

In this model we assume that savings rates are constant, but arbitrary. Of course, optimal savings consumption paths will respond endogenously to increases in the growth rate. However, such responses would only invalidate the above results if savings were to decline to zero in response to transformative growth induced by AI. Trammell and Korinek (2025) presents a simple argument using the Euler equation, illustrating that in a similar environment savings declines ultimately will not preclude explosive growth.

In such a world, every unit of savings prior to the (Type II) singularity can bring arbitrarily large returns at a future date, hence incentives to save become increasingly high.

# 4 AI and Automation

The model above demonstrates the conditions for hyperbolic growth independent of automation. In this section we demonstrate that the networked-lab equipment model provides an inuative framework to understand how AI and automation can accelerate growth: by strengthening pre-existing feedback loops of generating them in places they did not exist before. To develop this model, we suggest that AI will *replace labor* some fraction of tasks in each research sector. Importantly, AI cognitive labor can accumulate in a way that human labor cannot; via both technological improvements and capital accumulation. We will see that by making assumptions on the process of automation we can calibrate the networked research model introduced above.

## 4.1 Automation and AI-Inputs

Here we adapt the automation framework from Zeira (1998), where *labor* outputs (be it from research or final production) are a Cobb-Douglas aggregation outputs from tasks in the set, $T$. In particular, we assume that *effective* labor working in a sector $i$ is given by

$$\hat{L}_i = p_i \prod_{q \in T} X_{i,q}^{\xi_q} \qquad \text{where } \sum_{q \in T} \xi_q = 1$$

where

$$X_{i,q} = \begin{cases} L_{i,q} & \text{if not automated} \\ C_{i,q} & \text{if automated} \end{cases}$$

where $L_i$ is human labor and $C_i$ is the human equivalent level of AI deployed on a task $i$ and $p_i$ is a productivity constant. I.e., we set $C_i$ such that one unit of AI deployment to a task is equivalent to one unit of human labor. Assuming optimal allocation of AI equivalent labor and human labor (where AI and labor stocks are spread evenly across tasks), as the number of tasks approach infinity we have effective labor given by

$$\hat{L}_i(C_i, L_i) = \tilde{p}_i(f_i) C_i^{f_i} L_i^{1-f_i} \quad \text{where} \quad \tilde{p}_i(f_i) = \frac{p_i}{f_i^{f_i}(1-f_i)^{1-f_i}} \tag{20}$$

where $f_i \in [0, 1]$ is the fraction of tasks, $q$, that are automated. Importantly, we treat $f_i$ as a constant to evaluate the growth implications of a given level of task automation. Hence, our starting point is downstream of questions related to optimal automation of output or research tasks which has received attention elsewhere (Jones, 2025, Acemoglu and Restrepo, 2018).

We separate AI capabilities, $C_i$, along two kinds of inputs:

- *Non-rivalrous*: progress in AI has been one of discovery of new, non-rivalrous, ideas. We assume that these non-rivalrous improvements result in some equivalent increase in AI equivalent labor. AI has emerged out of underlying progress in technologies such as compute hardware, algorithmic research or large-scale data collection.

- *Rivalrous*: the *use* of AI to complete tasks requires *inference compute* (compute to transform model inputs into productive work).

In this spirit, we define our sectoral AI inputs as a function levels of a subset of technology sectors, $\bar{I} \subset I$, as well as a specific kind of capital; computing hardware, $K^H$.

Like in the rest of the model, we take the AI index to be a Cobb–Douglas function of a subset of underlying technologies, multiplied by the number of copies that can be run, a linear function of the amount of inference compute available in a given sector:

$$C_i = K_i^H \times \prod_{j \in \bar{I}} A_j^{\sigma_j} \tag{21}$$

where we don't necessarily assume that $\sum_{i \in \bar{I}} \sigma_j = 1$ since standard replication arguments for constant returns to scale don't apply to non-rivalrous technologies, $A_i$. Implicitly, we assume that AI equivalent labor scales linearly with inference compute.

## 4.2 Hyperbolic Growth: Automated Research

Here we demonstrate that we can recover the same structure on balanced growth as the general technological feedback equation 7 for a baseline semi-endogenous technological law of motion with automation. Hence, we can apply Proposition 1 to determine the conditions for hyperbolic growth, in an automated research environment.

We begin by assuming that the only inputs to research are *effective* labor, combining both human labor and AI effective labor according to equation (21). Therefore the

technology laws of motion become

$$\begin{aligned}
\dot{A}_i &= \nu_i \hat{L}_i^{\lambda_i} A_i^{1-\beta_i} \prod_{j \in I \neq i} A_j^{\phi_{i,j}} \\
&= \nu_i \tilde{p}_i(f_i) C_i^{f_i \lambda_i} L_i^{(1-f_i)\lambda_i} A_i^{1-\beta_i} \prod_{j \in I \neq i} A_j^{\phi_{i,j}} \\
&\propto \underbrace{A_i^{1-\beta_i}}_{\text{Dim. returns}} \times \underbrace{[K_i^{H f_i} L_i^{(1-f_i)}]^{\lambda_i}}_{\text{Riv. inputs}} \times \underbrace{\prod_{j \in I} A_j^{f_i \lambda_i \sigma_j}}_{\text{AI feedback}} \times \underbrace{\prod_{j \in I \neq i} A_j^{\phi_{i,j}}}_{\text{Direct spillovers}}
\end{aligned} \tag{22}$$

where $\sigma_j = 0$ for $j \notin \bar{I}$. As a baseline, we assume that direct technological spillovers are non-existent ($\phi = 0$). In this case, we recover sectoral spillovers through research automation; a positive automation shock to a sector that feeds into AI progress accelerates research in all other sectors that have any automation. By matching terms we can calibrate our general network model with parameters we present the calibration in Table 1.

Table 1: AI-Parameterized R&D Network Model

| **General Model Input** | **AI-Parameterization** | **Description** |
|---|---|---|
| $\beta_i$ | $\beta_i - f_i \lambda_i \sigma_i$ | Diminishing returns (offset) |
| $\phi_{i,j}$ (for $i \neq j$) | $f_i \lambda_i \sigma_j$ | Automation spillovers |
| $\gamma_i$ | $f_i$ | Non-automated research task share |
| $\lambda_i$ | $\lambda_i$ | Parallelization penalty |

*Note*: This table summarizes how one can re-parameterize the networked technology model (equation (7)). Blue terms (first three lines) represent additional terms arising from technological feedback loops. The final line has the same interpretation as in the general model.

From this calibration, we make several observations:

- Diminishing returns to research within a sector, $\beta_i$, are directly offset by the ability

for AI to contribute to research in that sector. Introducing automated research is isomorphic to reducing diminishing returns.

- For progress in $j$ to spillover into progress in $i$, we require *both* automation in $i$ ($f_i > 0$) and $j$ to be relevant to AI progress ($\sigma_j > 0$). Therefore, a 'bilateral' feedback loop exists between $i$ and $j$ if both $f$ and $\sigma$ are greater than zero for both $i$ and $j$. An 'indirect' feedback loop exists if there is a chain of technologies with positive $f$ and $\sigma$ such that progress in one sector eventually reinforces itself after accelerating technological progress in one sector, which subsequently accelerates in another sector and so on until the original sector is accelerated.

- If we were to deviate from our baseline assumption of no non-AI spillovers ($\phi_{i,j} > 0$), those spillovers would just be additive to automation spillovers: $\hat{\phi}_{i,j} = \phi_{i,j} + f_i \lambda_i \sigma_j$. In this case automation amplifies existing spillovers.

In summary, starting from a baseline of no sectoral spillovers and no contribution from lab-equipment, introducing research automation allows us to recover a networked, technological law of motion that conforms to a lab-equipment specification with spillovers. This means that after research automation, explosive conditions can emerge out of a model that precluded this possibility.

Substituting our recovered parameters into the balanced growth path described by equation (11) and denoting automation-adjusted parameters with hats, we have

$$g_A^{BGP} = \hat{\mathbf{\Psi}}_A \hat{\mathbf{r}} \hat{g}_R$$

where

$$
\begin{aligned}
\hat{g}_{R,i} &= f_i \times n + (1 - f_i) \times g_{H,i} \\
\hat{r}_i &:= \frac{\lambda_i}{\beta_i - f_i \lambda_i \sigma_i} \\
\hat{\mathbf{\Psi}}_A &= (\mathbf{I} - \hat{\mathbf{F}}_A)^{-1} \\
\hat{\mathbf{F}}^A_{i,j} &= \begin{cases} \hat{s}_{i,j} := \frac{f_i \lambda_i \sigma_j}{\beta_i - f_i \lambda_i \sigma_i} & \text{for } i \neq j \\ 0 \text{ if } i = j \end{cases}
\end{aligned}
$$

and black terms in $\hat{r}$ and $\hat{g}_R$ are those present in the baseline Jones (1995) model and while the blue terms in $\hat{r}$, $\hat{s}$ and $\hat{g}_R$ are those that enter through automated research channels.

We can apply Proposition 1 directly to this system. Yielding the result:

**Corollary 3.** *The automation-calibrated technology system* (*equation* (22)) *explodes in finite time when*

$$\sum_{i \in I} r_i f_i \sigma_i > 1 \quad .$$

Where this result comes directly from calculating $\det(\mathbf{I} - \hat{\mathbf{F}}^A)$ and rearranging.

## 4.3 Hyperbolic Growth: Automated Production and Research

Now we extend the AI-induced feedback loops captured in equation (22) to a more realistic setting where where AI can additionally automate some tasks in the production of goods, as well as allowing automated technological progress increase total factor productivity.

$$
\begin{aligned}
Y &= \bar{A} {K^Y}^{1-\alpha} \hat{L}_Y^{\alpha} && (23)\\
\bar{A} &= \prod_{i \in I} A_i^{\tau_i} \quad \text{where} \quad \sum_{i \in I} \tau_i = 1 && (24)\\
\dot{A}_i &\propto \hat{L}_i^{\lambda_i} A_i^{1-\beta_i} && (25)\\
\hat{L}_i &\propto L_i^{1-f_i} C_i^{f_i} \quad \text{where} \quad C_i = K_i^H \times \prod_{j \in I} A_j^{\sigma_j} && (26)\\
\dot{K}^Y &= a_Y Y - \delta_Y K^Y && (27)\\
\dot{K}^H &= a_H Y - \delta_H K^H && (28)\\
K^H &= K_Y^H + \sum_{i \in I} K_i^H && (29)\\
L &= L_Y + \sum_{i \in I} L_i && (30)
\end{aligned}
$$

where $a_H + a_Y \leq 1$ and allocations of hardware and labour across research sectors is constant over time. Solving for the balanced growth path, we have

$$
g_A^{BGP} = \hat{\mathbf{\Psi}}_{A,Y} \hat{\mathbf{r}} n \tag{31}
$$

where

$$
\begin{aligned}
\hat{\mathbf{\Psi}}_{A,Y} &= (\mathbf{I} - [\hat{\mathbf{F}}^A + \hat{\mathbf{F}}^Y])^{-1} && (32)\\
\hat{\mathbf{F}}^Y &:= \underbrace{[\hat{\mathbf{r}} f]'}_{dg_Y^{BGP}/dg_A^{BGP} \text{ from } 23} \underbrace{\frac{1}{1-f_Y}[\frac{1}{\alpha}\tau + f_Y \sigma]}_{dg_A^{BGP}/dg_Y^{BGP} \text{ from } 25} && (33)
\end{aligned}
$$

and $\hat{\mathbf{r}}$ and $\hat{\mathbf{F}}^A$ are defined as above.

In Table 2 we present how the core dynamics of the general R&D and production network model described by equations (12)-(17) can be calibrated to the automated

economy model in equations (23) - (30) by adjusting relevant parameters. From this, we can see that introducing automation into the production side of the model (in addition to the technological side) is equivalent to: decreasing the labor share of production by a factor of $(1 - f_Y)$; and increasing the contributions of each technology to the production of final goods by $f_Y \sigma$.

Table 2: AI-Parameterized R&D and Production Network Model

| **General Model Input** | **AI-Parameterization** | **Description** |
|---|---|---|
| $1 - \alpha$ | $1 - \alpha(1 - f_Y)$ | Capital share of output |
| $\tau_i$ | $\tau_i + \sigma_i f_Y$ | Technological contributions to output |
| $\beta_i$ | $\beta_i - f_i \lambda_i \sigma_i$ | Diminishing returns (offset) |
| $\phi_{i,j}$ (for $i \neq j$) | $f_i \lambda_i \sigma_j$ | Automation spillovers |
| $\gamma_i$ | $f_i$ | Non-automated research task share |
| $\lambda_i$ | $\lambda_i$ | Parallelization penalty |

*Note*: This table summarizes how one can re-parameterize the general model (equations (12)-(17)) to account changes in the parameter space due to automation. Purple terms (first two rows) represent additional terms arising from economic feedback loops while blue terms (middle three rows) represent additional terms arising from technological feedback loops. The final row has the same interpretation as in the general model.

Applying Proposition 1 to the balanced growth condition from equation (62), we have the following result:

**Corollary 4.** *With economic feedback loops, the automation-calibrated growth model (described in equations (23) - (30)) explodes in finite time if*

$$f_Y + \sum_{i \in I} r_i f_i \left(\frac{\tau_i}{\alpha} + \sigma_i\right) > 1 \, . \tag{34}$$

Note, in Appendix B we derive the equivalent condition for the case of a fixed factor

entering output, so output exhibits *diminishing* returns to capital and labor.

# 5 Calibrated Threshold: Hyperbolic Growth under Automation

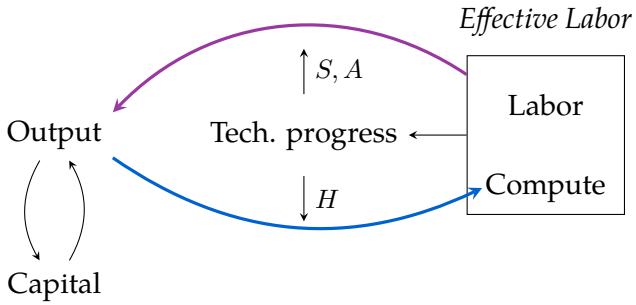
Above we introduce general models of networked growth, and motivate these models with application to the case of automation of both final goods production and research. Importantly, the balanced and hyperbolic growth conditions in Section 4 are stated in terms of parameters that can be directly calibrated; based on historical evidence of returns to research in software and hardware research and development as well as the labor share of output.

## 5.1 Simple Integrated AI-Economy

Here we integrate a simple model of AI progress within an economic envrionment. As in previous sections, we assume that AI counts for cognitive labor, replacing human labor in some fraction of tasks in each sector. The central assumption here is that AI progress stems from better algorithms and better computing hardware. Improved computing hardware allows us to run more computations for the same amount of capital investment, while improved algorithms make AI more capable of completing relevant tasks. We deliberately omit a number of additional components of the training process here to emphasize key feedback loops. For example, our model exclusively focuses on *inference* without modeling the relationship between training investment and inference capabilities. Erdil et al. (2025) considers a much richer AI-economic model, though such a framework makes it impossible to cleanly isolate the key feedback loops of interest.

In Figure 3 we illustrate our environment for progress in AI and the integration of AI in the rest of the economy. In this environment we have independent technological

Figure 3: Channels for Impact of Software and Hardware Improvements




*Note:* This figure illustrates the presence of feedback loops through automation. Technologies (TFP, hardware quality and software quality) amplify feedback between output and effective labor. The purple arrow is amplified by software and TFP progress, whereas the blue arrow is amplified through hardware progress.

processes (TFP, software and hardware quality) which amplify feedback loops. Feedback from output to (effective) labor through accumulation of compute which supports the inference of AI models. This channel from output to compute is amplified through hardware progress; we assume hardware progress means more inference compute can be purchased for the same amount price over time. The capacity of effective labor to contribute to both research and production is amplified through software progress, which makes compute more effective at completing economic tasks. Here we define software in terms of productivity units of human labor and that doubling the software 'level' means that software can produce double the output on a specific task given the same amount of inference compute. The role of capital is unchanged from standard models, and labor grows exogenously.

We describe this growth system with the following equations:

$$Y = A\hat{L}_Y^{\alpha} K^{Y\,1-\alpha} \tag{35}$$

$$\hat{L}_i \propto L_i^{1-f_i} C_i^{f_i} \quad \text{where} \quad C_i = K_i^H S \tag{36}$$

$$\dot{S} \propto \hat{L}_{\dot{S}}^{\lambda_S} S^{1-\beta_S} \tag{37}$$

$$\dot{H} \propto \hat{L}_{\dot{H}}^{\lambda_H} H^{1-\beta_H} \tag{38}$$

$$\dot{A} \propto \hat{L}_{\dot{A}}^{\lambda_A} A^{1-\beta_A} \tag{39}$$

$$\dot{K}^H = a_H H Y - \delta_H K^H \tag{40}$$

$$\dot{K}^Y = a_Y Y - \delta_Y K^Y \tag{41}$$

$$K^H = K_Y^H + K_{\dot{H}}^H + K_{\dot{S}}^H + K_{\dot{A}}^H \tag{42}$$

$$L = L_Y + L_{\dot{H}} + L_{\dot{S}} + L_{\dot{A}} \tag{43}$$

Just as in Section 4.3 we can find the conditions for balanced and explosive growth by recognizing from equations (40) and (41) that along the balanced growth path we require $g_{K^Y} = g_{K^H} - g_H = g_Y$. Then we can use the equations for output and effective labor to solve for the balanced growth path of technologies as a function of fundamentals of the model. This is the same balanced growth condition for AI capabilities as in the general model from Section 4.3. Further, note that we only have one technology—TFP—feeding *directly* into increasing output productivity. Therefore, we can see that the system described in equations (37)-(43) is a three technology version of the general model from equations (23)-(30) technological contributions to output dictated by $\tau_z = \tau_h = 0$ and $\tau_A = 1$ and to AI progress dictated by $\sigma_z = \sigma_h = 1$ and $\sigma_A = 0$.[4] Given that the AI-economic model develop here is a specific case of the general model, we can

[4]The only difference between this calibrated model and the general model from Section 4.3 is that one of the technologies–hardware quality–scales compute accumulation rather than AI capabilities directly. Ultimately this does not effect the balance growth path calculation.

just calibrate Corollary 4 to derive the hyperbolic growth condition:

**Corollary 5.** *The AI-economic model described in equations (35)-(43) explodes in finite time if*

$$f_Y + f_S r_S + f_H r_H + \frac{1}{\alpha} f_A r_A > 1 \tag{44}$$

## 5.2 Calibration

We now turn to calibrating the relevant terms from equation (44). In Table 3 we report estimates of research productivity in software, hardware and aggregate TFP. Both software and (in particular) hardware research are significantly more productive than aggregate TFP.

Table 3: Parameter estimates

| Term | Parameter | Estimate | Source |
|---|---|---|---|
| Labor share | $\alpha$ | 0.6 | |
| Returns to research (software) | $r_S$ | 0.83 - 1.3 | Erdil et al. (2024) |
| Returns to research (hardware) | $r_H$ | 5 | Bloom et al. (2020) |
| Returns to research (TFP) | $r_A$ | 0.32 | Bloom et al. (2020) |

*Note:* The estimate ranges come from estimates over different software domains.

We use these estimates to calibrate the model. Table 4 presents the calibrated explosion conditions. We relegate discussion of the limitations of this calibration to Section 5.3.

Table 4 presents the calibrated condition for hyperbolic growth under heterogeneous automation in different sectors. We can see here that automating hardware progress

pushes the economy closer to hyperbolic growth much more than any sector due to the very high estimate of returns to research effort. We can see that in the full economy case, one hardware task offers about the same increase in the distance to the threshold as five tasks in software or final goods production and the same as ten tasks in the general TFP sector. Further, we can see that introducing a 10% fixed factor so that output is diminishing returns to scale in capital and effective labor, there is only a mild effect on the threshold since this effect doesn't pass through the hardware or software automation channels.[5] Appendix B provides the analytical balanced and hyperbolic growth conditions with a fixed factor.

Table 4: Applying historical estimates of research productivity to singularity conditions

| **Sectors with Automation** | **Calibrated Explosion Condition** |
|---|---|
| $S$ | $\sim 1f_S > 1$ |
| $H$ | $5f_h > 1$ |
| $S, H$ | $\sim 1f_S + 5f_h > 1$ |
| $H,\ Y$ | $5f_H + f_Y > 1$ |
| $S,\ Y$ | $\sim 1f_H + f_Y > 1$ |
| $S,\ H,\ Y$ | $\sim 1f_S + 5f_h + f_Y > 1$ |
| $S,\ H,\ A,\ Y$ | $\sim 1f_S + 5f_h + 0.53f_A + f_Y > 1$ |
| $S,\ H,\ A,\ Y$ (10% fixed factor) | $\sim 1f_z + 5f_h + 0.45f_A + 0.86f_Y > 1$ |

*Note:* Estimates of historical returns to research effort are taken from Bloom et al. (2020) for total factor productivity and hardware, and from Erdil et al. (2024) for software. Here, $f$ terms can be calibrated to the fraction of tasks that can be automated in research sectors; software ($S$), hardware ($H$), and general technology $A$.

In Table 5 use the calibrated $r$ terms to investigate, first, how much automation is re-

[5]Fixed factors can be introduced in the above model by redefining output shares on labor and capital according to $\alpha_L = 0.9 \times \alpha$ and $\alpha_K = 0.9 \times (1 - \alpha)$.

quired (assuming automation across all sectors is equal) to double the balanced growth path. We then repeat the same exercise investigating the fraction of automation required to achieve hyperbolic growth. Inspecting these columns we can see that in the full model, the automation required to double the balanced growth path is about three quarters of the automation necessary to achieve fully explosive growth. That is, the balanced growth path is initially very slow to respond to changes in automation, and then changes incredibly quickly. This just the result of the multiplication of spillovers through the Leontief inverse. While we do not attempt to model the rate of automation,

At the current pace of automation, we might expect that we could progress in the range of app of tasks fairly quickly potentially taking us from a fairly mild to a massive alteration of the growth path.

Table 5: Automation for Hyperbolic vs Balanced Growth

| **Automated Factors** | **Automation Threshold** | |
|---|---|---|
| | $2 \times g_Y^{BGP}$ | **Hyperbolic** |
| $S$ | – | $\sim 100\%$ |
| $H$ | – | $20\%$ |
| $S, H$ | – | $17\%$ |
| $H,\ Y$ | $12\%$ | $17\%$ |
| $S,\ Y$ | $16\%$ | $50\%$ |
| $S,\ H,\ Y$ | $11\%$ | $14\%$ |
| $S,\ H,\ A,\ Y$ | $8\%$ | $13\%$ |
| $S,\ H,\ A,\ Y$ (10% fixed factor) | $9\%$ | $14\%$ |

*Note:* Here we assume that $f = f_S = f_H = f_A = f_Y$. In the middle column we solve for the $f$ such that the balanced growth path doubles relative to no automation. In the right column we solve for the $f$ such that the conditions from Table 4 are satisfied. The first three balanced growth rows are empty since these are technology-only automation scenarios.

These quantitative results underscore two points. First is the importance of amplification of feedback loops. Under the simple model developed above, higher levels of automation – dialing up feedback loops – accumulates into rapid changes in the growth path quickly. Second is the emphasis that these are changes in the asymptotic balanced growth path. In reality it may take some time to converge to such a path and further, even under a hyperbolic growth path it may still take significant time to actually see radical changes in growth rates; hyperbolic growth does not necessarily imply trans formative growth in the short run. One can see this intuitively by recognizing that from the view point of the entire human history the current 2% annual growth *is* part of a long-run super-exponential trend.[6]

## 5.3 Calibration Limitations

Here we offer an illustrative calibration of the explosion threshold based on historical estimates of research productivity in different sectors. However, there are a number of reasons why these estimates might be inappropriate for this model.

First, is that returns to research effort in software from Erdil et al. (2024) have been estimated in software domains other than frontier AI research. Specifically, these estimates come from chess engines and computer vision. Estimating $r_S$ directly from the rate of progress at frontier AI labs is difficult not just because of challenges associated with estimating research inputs, but also because $r_S$ is estimated as returns to research along the balanced growth path. Given the boom in progress in AI it is impossible to tell if we are anywhere close to balanced growth software research. Ho and Whitfill (2025) make some attempt to make this calculation directly, finding $r_S$ in the range of 1.2 - 1.8. Though, given limitations in this approach, we rely on relatively conservative

[6] Hanson (2000) makes this point through a theory-free projection of growth by fitting historical growth rates to a series of increasing exponential functions until 1998 which predicted growth rates larger than 20% by 2040.

estimates from Erdil et al. (2024).

Second, we assume that these parameters are foundational to the knowledge discovery process, rather than the *human* knowledge discovery process. For example, that diminishing returns to research ($\beta$)or parallelization of research ($\lambda$) are independent of whether it is humans of AI completing that scientific research. Trammell and Korinek (2025) suggest reasons why both of these parameters may be different under AI R&D. For example, as identified by Ekerdt and Wu (2025), increasing the researcher share of population may result in declines in returns to research effort as the average quality of researcher declines; in the case of AI we might expect constant effective researcher quality, increasing $r$ for AI researchers relative to human researchers.

Third, we take estimates from Bloom et al. (2020) and Erdil et al. (2024) as given. Specifically, the estimate of $r_A$ from Bloom et al. (2020) is an aggregate estimate of returns to research across the whole economy. However, we should expect that the research that has contributed to hardware and software progress have in fact contributed to aggregate TFP. Since we separate out software and hardware progress from TFP we should also adjust $r_A$ to be the returns to research in TFP, net of software and hardware sectors. We do not have a good estimate of the share of aggregate technological progress that has come from software and hardware ($\tau_S$ and $\tau_H$ terms from above) hence we just assume these are small and calibrate $r_A$ as the aggregate economy estimate.

# 6 Discussion

The above analysis employs a simple model to derive a critical threshold for the automation of tasks: once this threshold is crossed, intelligence and output are projected to grow super-exponentially, ultimately reaching infinity within finite time. Crucially, the possibility of a singularity hinges on two assumptions: constant research produc-

tivity up to the singularity and correctly specified research production functions, which may be satisfied soon, even though they may fail to hold in the long run.

Regarding constant research productivity, our findings indicate that automating hardware quality research is a significant contributor to meeting the explosion condition, largely due to the extreme pace of Moore's law over the past half-century. However, physical constraints on increasing transistor density in chips that support AI training and inference may limit this progress. Current state-of-the-art chips already feature transistors as small as 2 nanometers, which may approach the boundaries of physical feasibility. Such challenges have fostered skepticism regarding the continued validity of Moore's law, which has historically predicted consistent advancements in transistor density (Leiserson et al., 2020).

Regarding the specification of the production function, we have assumed that the production factors -— human labor and AI cognitive labor —- are substitutes. In the short run, this assumption may hold; for instance, AI is already capable of performing coding tasks semi-autonomously, replacing human programmers. However, as automation advances, human and AI cognitive labor may appear complementary over some time periods. For example, while AI may excel in chip architecture, manufacturing, and testing, it might remain unable to conduct the physical experiments necessary for chip design before sufficiently advanced robots are developed, thereby relying on human researchers. Such considerations receive substantial consideration in Jones (2025). In such cases, the pace of hardware improvement could be constrained by the availability of human experimenters. By alleviating reliance on this scarce resource, these efforts could delay or mitigate the stalling of progress caused by bottlenecks. While such adaptive responses may not eliminate bottlenecks entirely, they could extend the period of rapid advancements before fundamental constraints take hold.

Although limitations are likely to emerge in the long run, we argue that assuming

(roughly) constant research productivity and substitutable labor provides a reasonable approximation of the production process in the short term. Thus, a more realistic interpretation of the explosion conditions derived above is that they signify a threshold for temporary super-exponential growth take-off in economic output and AI capabilities. Crucially, this super-exponential growth is driven not by resource reallocation within the economy but by changes in the fundamental drivers of economic and technological progress.

# 7 Conclusion

This paper develops a framework for understanding how advances in artificial intelligence could fundamentally transform economic growth dynamics. By modeling the interconnected roles of hardware, software, and general technological progress, we show that automation of research and development activities could generate powerful feedback effects leading to rapid growth acceleration. Our calibration using historical estimates of research productivity suggests that these effects could be substantial, particularly through the automation of semiconductor research.

These findings have important implications for economic policymakers. First, they suggest that standard growth models may need significant revision to account for the potential of recursive technological progress. Traditional frameworks that treat technological advancement as an exogenous or smoothly evolving process may not capture the possibility of sudden acceleration in growth rates driven by AI automation of research activities.

Second, our results highlight the strategic importance of semiconductor research and development. The high historical productivity of hardware research, combined with growing automation capabilities in chip design, suggests that this sector could

play a crucial role in determining the pace of overall technological progress. This raises important questions about both the market concentration and geographic concentration of semiconductor research and production capabilities (Korinek and Vipra, 2025).

Third, our analysis suggests that monitoring automation levels in research and development activities may be as important as tracking traditional macroeconomic indicators. The extent of automation in key research sectors could serve as an early warning system for potential growth acceleration.

Several promising directions for future research emerge from our analysis. Researchers could develop systematic metrics to measure automation levels across different research domains. Empirical work might test for the presence and strength of feedback loops between AI advances and technological progress, particularly in semiconductor and software research. Future studies could investigate the degree of complementarity between human and AI researchers in different activities. Analysis of how existing concentrations of AI and semiconductor research capabilities might influence regional growth patterns would be valuable. Finally, work is needed to better characterize the physical, computational, and economic constraints that might prevent growth acceleration.

Understanding these dynamics is crucial for economists and economic policymakers. If AI progress can indeed generate self-reinforcing technological acceleration, this has profound implications for economic planning, research funding priorities, and international economic coordination. While our analysis suggests this scenario is plausible given historical patterns of research productivity, much work remains to be done in understanding how these dynamics might unfold in practice.

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# A Deriving Conditions for Hyperbolic Growth

*Proof of Proposition 1.* First we divide equation (7) by $A_i$ to get technology growth rates and then take the logs and time derivative to get the rate of change in growth rates given by the vector

$$\dot{g}_A = diag(g_A)[(\mathbf{S} - \mathbf{B})g_A + diag(\lambda)g_R]$$

where $\mathbf{S}_{i,j} = \phi_{i,j}$ for $i \neq j$ and zero for diagonal elements, and $\mathbf{B} = diag(\beta_1, \beta_2, \ldots, \beta_N)$. We define the 'exponent matrix', $\mathbf{\Omega} := \mathbf{S} - \mathbf{B}$. Therefore, We can relate the (balanced growth path) spillover matrix, $\mathbf{F}$ and the exponent matrix $\mathbf{\Omega}$ according to $\mathbf{\Omega} = \mathbf{B}(\mathbf{F} - \mathbf{I})$. Further, let $u \gg 0$ be the Perron–Frobenius right eigenvector of $\mathbf{F}$, $\mathbf{F}u = \rho(\mathbf{F})u$, and set $\mu := \rho(\mathbf{F}) - 1$. Then

$$(\mathbf{F} - \mathbf{I})u = \mu u, \qquad \mathbf{\Omega}u = \mathbf{B}(\mathbf{F} - \mathbf{I})u = \mu\,\mathbf{B}u \quad (\gg 0).$$

Further, we define the scalars

$$\bar{h}(t) := \frac{g_{A_{\bar{i}}}(t)}{u_{\bar{i}}} \quad (> 0), \quad \text{where} \quad \bar{i} = \arg\max_i \frac{g_{A,i}(t)}{u_i}$$
$$\underline{h}(t) := \frac{g_{A_{\underline{i}}}(t)}{u_{\underline{i}}} \quad (> 0), \quad \text{where} \quad \underline{i} = \arg\min_i \frac{g_{A,i}(t)}{u_i}$$

Proving $\rho(\mathbf{F}) > 1 \implies$ hyperbolic growth. Suppose $\rho(\mathbf{F}) > 1$ so $\mu > 0$. Take the rate of change in the growth rate of $A_{\underline{i}}$ at a specific time $t$.

$$
\begin{aligned}
\dot{g}_{A,\underline{i}}(t) &= g_{A,\underline{i}}(t)(\mathbf{\Omega} g_A + diag(\lambda) g_R)_{\underline{i}} && (45)\\
&\geq g_{A,\underline{i}}(t)(\mathbf{\Omega} g_A)_{\underline{i}} && (46)\\
&= g_{A,\underline{i}}(t)(\sum_{j \neq \underline{i}} \mathbf{\Omega}_{j,\underline{i}} g_{A,j} + \mathbf{\Omega}_{\underline{i},\underline{i}} g_{A,\underline{i}}(t)) && (47)\\
&\geq g_{A,\bar{i}}(t)(h(t) \sum_{j \neq \underline{i}} \mathbf{\Omega}_{j,\underline{i}} u_j + \mathbf{\Omega}_{\underline{i},\underline{i}} g_{A,\underline{i}}(t)) && (48)\\
&= \underline{h}(t)^2 u_{\underline{i}} (\mathbf{\Omega} u)_{\underline{i}} && (49)
\end{aligned}
$$

where 48 comes from the fact that by definition $g_{A,j}(t) \geq h(t) u_j$ as well as that $\Omega_{j,\underline{i}} \geq 0$ and 49 comes from the fact that this holds with equality when $j = \underline{i}$. Next, since $\dot{\underline{h}}(t) = \dot{g}_{A,\underline{i}}(t)/u_{\underline{i}}$ then we have

$$
\begin{aligned}
\dot{\underline{h}}(t) &\geq \frac{\underline{h}(t)^2}{u_{\bar{i}}} (\mathbf{\Omega} u)_{\bar{i}} && (50)\\
&= \frac{\underline{h}(t)^2}{u_{\bar{i}}} (\mu \mathbf{B} u)_{\bar{i}} && (51)\\
&= \underline{h}(t)^2 \mu \beta_{\underline{i}} u_{\underline{i}} && (52)
\end{aligned}
$$

which implies $\underline{h}(t)$ grows hyperbolically, which in turn implies that $g_{A_i}$ grows hyperbolically for all $i$ by definition of $h(t)$, which implies $A_i$ grows hyperbolically $\forall i$.

Next, proving the $\rho(F) \leq 1 \implies$ no hyperbolic growth. Assuming $\rho(\mathbf{F}) \leq 1$ gives

$\mu \leq 0$. Following a similar procedure as above, we have

$$\begin{aligned}
\dot{g}_{A,\bar{i}} &= g_{A,\bar{i}}(t)(\mathbf{\Omega} g_A(t) + diag(\lambda) g_R)_{\bar{i}} && (53)\\
&= g_{A,\bar{i}}(t)(\mathbf{\Omega} g_A)_{\bar{i}} + g_{A,\bar{i}}(t)\lambda_i g_R && (54)\\
&= g_{A,\bar{i}}(t)(\sum_{j \neq \bar{i}} \mathbf{\Omega}_{\bar{i},j} g_{A,j} + \mathbf{\Omega}_{\bar{i},\bar{i}} g_{A,\bar{i}}) + g_{A,\bar{i}}(t)\lambda_{\bar{i}} g_R && (55)\\
&\leq g_{A,\bar{i}}(t)(\sum_{j \neq \bar{i}} \mathbf{\Omega}_{\bar{i},j} \bar{h}(t) u_j + \mathbf{\Omega}_{\bar{i},\bar{i}} g_{A,\bar{i}}) + g_{A,\bar{i}}(t)\lambda_{\bar{i}} g_R && (56)\\
&= g_{A,\bar{i}}(t)\bar{h}(t)(\sum_{j \neq \bar{i}} \mathbf{\Omega}_{\bar{i},j} u_j + \mathbf{\Omega}_{\bar{i},\bar{i}} u_{\bar{i}}) + \bar{h}(t) u_{\bar{i}} \lambda_i g_R && (57)\\
&= \bar{h}(t)^2 u_{\bar{i}} (\mathbf{\Omega} u)_{\bar{i}} + \bar{h}(t) u_{\bar{i}} \lambda_{\bar{i}} g_R && (58)\\
&= \bar{h}(t)^2 u_{\bar{i}} \mu (\mathbf{B} u)_{\bar{i}} + \bar{h}(t) u_{\bar{i}} \lambda_{\bar{i}} g_R && (59)
\end{aligned}$$

and since $\dot{\bar{h}}(t) u_{\bar{i}} = \dot{g}_{A,\bar{i}}$ then we can upper bound by the logistic differential equation

$$\dot{\bar{h}}(t) \leq \bar{h}(t)^2 \mu \beta_{\bar{i}} u_{\bar{i}} + \bar{h}(t) u_{\bar{i}} \lambda_{\bar{i}} g_R \tag{60}$$

and since $\mu < 0$ the quadratic part of the expression dominates as $\bar{h}$ grows so $\bar{h}(t)$ remains finite for all $t$. Further in the case of $\mu = 0$ the inequality reduces to $\dot{\bar{h}}(t) \leq \bar{h}(t)\lambda_{\bar{i}} g_R$, yielding at most exponential growth in $\bar{h}$. Finally, we know that no explosive growth in $\bar{h}$ implies there is no explosive growth in $g_{A,i}$ for all $i \in I$ (nor in $A_i$).

Finally we prove fully endogenous balanced growth along $g_A = cu$. When $g_E = 0$ we have the the motion of technology growth rates $g_A^{BGP} = \mathbf{F} g_A^{BGP}$ and therefore, $g_A^{BGP} = \mathbf{F} g_A^{BGP}$

□

# B Balanced and Hyperbolic Growth with a Fixed Factor

Here we take the growth model from Section 4.3, but allow for the inclusion of a fixed factor, $M$, into output. Where output is constant returns to scale in capital, labor and the fixed factor

$$Y = ZK_Y^{\alpha_K}\hat{L}_Y^{\alpha_L}M^{1-\alpha_K-\alpha_L}. \tag{61}$$

We maintain equations (25) - (30) to describe law of motion of technology, hardware, capital and resource constraints.

Solving for the balanced growth path under the assumption that output is constant returns to scale in capital and labor we have

$$g_A^{BGP} = \hat{\mathbf{\Psi}}_{A,Y} \times \left(\hat{\mathbf{r}} \times \frac{\mathbf{1}_n - f - \alpha_K(\mathbf{1}_n - f) - \alpha_L(\mathbf{1}_n \times f_Y - f)}{1 - \alpha_K - \alpha_L f_Y}\right) \times n \tag{62}$$

where

$$\hat{\mathbf{\Psi}}_{A,Y} = (\mathbf{I} - [\hat{\mathbf{F}}^A + \hat{\mathbf{F}}^Y])^{-1} \tag{63}$$

$$\hat{\mathbf{F}}^Y := \underbrace{[\hat{\mathbf{r}} \times f]'}_{dg_Y^{BGP}/dg_A^{BGP} \text{ from 61}} \times \underbrace{\frac{\alpha_L}{1 - \alpha_K - \alpha_L f_Y} \times [\frac{1}{\alpha_L}\tau + f_Y\sigma]}_{dg_A^{BGP}/dg_Y^{BGP} \text{ from tech L.O.M}} \tag{64}$$

and $\hat{\mathbf{r}}$ and $\hat{\mathbf{F}}^A$ are defined as in Section 4.2.

Then, deriving the explosive growth condition, we get

**Corollary 6.** *The automation-calibrated growth model with a fixed factor (described in equations (61) and (25) - (30)) explodes in finite time if*

$$\frac{\alpha_L}{1 - \alpha_K} f_Y + \sum_{i \in I} f_i r_i (\frac{\tau_i}{1 - \alpha_K} + \sigma_i) > 1 \tag{65}$$