THE CYCLICALITY OF THE TERM STRUCTURE OF EXPECTED EQUITY RETURNS

by

ROBERTO C. SEVERINO

December, 2025

I would like to thank the faculty and staff of EDHEC Business School for their support throughout the development of this study. I am grateful to Emmanuel Jurczenko, Abraham Lioui, Raman Uppal, Nikolaos Tessaromatis, Gianpaolo Parise, and Mirko Rubin, as well as the workshop participants at the EDHEC Business School Research Presentation Series. I am particularly grateful to Ricardo Schroth for his thorough advice and guidance in preparing this paper. I also thank Michael Brandt from Duke University for his comprehensive review and comments, and John Campbell from Harvard University for his valuable feedback.

This research did not receive any external funding or grant support. It was conducted solely as part of my doctoral dissertation at EDHEC Business School. I have no financial relationships or other potential conflicts of interest to disclose.

© Copyright by Roberto Severino, 2025
All Rights reserved

THE CYCLICALITY OF THE TERM STRUCTURE OF EQUITY RETURNS

December 2025

ABSTRACT

There is ample consensus among scholars that the term structure of equity returns is countercyclical. However, whether a single change in the slope of the curve is sufficient to predict a recession, or whether those changes are directly associated with changes in an observable economic variable, remains unclear. This paper applies the vector autoregression method of Campbell and Viceira (2005) to extract the quarterly term structure of equity returns and analyzes its behavior across business cycles, its ability to predict the likelihood of a recession, and its relationship with investment growth. The first finding is that a shift from a downward- to an upward-sloping curve is not an immediate indication of a change in the business cycle toward a recession, but rather the steepness of the curve. The second finding is that a positive change in the slope of the curve predicts a decline in investment growth at least one year in advance. This prediction becomes even more significant when the slope of the curve predicts a higher likelihood of the economy entering a recession.

JEL Classification: D53, E32, E37, G01, G12, G17

Keywords: Financial markets, business cycles, financial cycles, asset pricing, financial forecasting.

1 Introduction

This study evaluates the behavior of the term structure of equity returns across business cycles and its relationship with investment growth. If stock prices are sensitive to new economic developments, the term structure of equity returns may incorporate information about expectations of changes in an economic variable, such as investment growth. Using the VAR modeling process proposed by Campbell and Viceira (2005), this study estimates the term structure of equity returns quarterly to examine whether changes in the slope of the curve predict the likelihood of recessions and future changes in the investment cycle.

The relationship between the term structure of equity and business cycles has been a long-standing subject of debate. Breugem et al. (2020), Gormsen (2021), and Gonçalves (2021) provide new evidence that the slope of the term structure of equity is countercyclical. They estimate the response of expected returns of investors on claims to dividends payable at a single future date over time as they approach the maturity of their investments to changes in the economy. Their findings relate to the facts of other asset classes, such as corporate bonds, and the sensitivity of the yield to maturity of these investments to changes in the business cycle, rather than evaluating the response of the holding period return of these investments. Although these studies have identified a general association between upward-sloping curves and recessions, questions remain about whether a single change in the slope of the curve, or the magnitude of that change, can predict a recession. Moreover, there is no conclusive evidence presented yet of a direct relationship between changes in the slope of the curve and an observable economic variable, such as investment growth. The aim of this study is to fill this gap in the term structure literature by providing concrete evidence of the interactions between an economic variable and the time variation of expected equity returns by investment horizon. This study contributes to the existing debate on the relationship between the term structure of equity returns and business cycles, documenting the slope of the curve under different economic conditions and whether changes in the slope of the curve predict changes in an observable economic variable.

This study estimates the term structure of equity returns using the methodology proposed by Campbell and Viceira (2005). Assuming the dynamics of asset returns and state variables follow a VAR(1) process, this study uses the lag of stock returns and dividend yield as the return forecasting variable. The VAR(1) process estimates equity returns over an investment horizon of up to 10 years on quarterly rolling windows. It begins with data from 1950:Q1 to 1960:Q1 and adds one quarter of new data with each quarterly VAR(1) estimate until 2023:Q2. Rolling windows are an appropriate

mechanism to incorporate market events into the sample data over multiple decades and evaluate the behavior of the term structure of expected equity returns under different economic conditions. The 10-year timeframe is not discretionary; it approximates the average business and financial cycle evidence presented by Claessens (2011) and Drehmann (2012), enabling testing expectations of equity returns that account for at least one change in the business cycle. Based on previous studies, investment growth is chosen as the economic variable to evaluate its relationship with the term structure of expected equity returns.[1] Changes in the slope of the curve are measured using equity return premiums. A positive equity return premium identifies an upward-sloping curve, a negative premium identifies a downward-sloping curve, and an equity premium close to zero identifies a flattener of the curve.

The analysis of the term structure of equity returns across business cycles consists of three sections. The first section evaluates the joint behavior of the investment growth rate and estimated equity return premium using a VAR model to determine the directional relationship between the two variables and their optimal lag length. The second section evaluates the behavior of the term structure of equity returns over periods to assess the conditions under which the slope of the curve predicts changes in the business cycle. A logit regression model is proposed to estimate the probability of recession as a function of the equity return premium. The third and final section examines how the relationship between investment growth rates and the term structure of equity returns varies under different economic conditions, conducting regression analyses conditionally and unconditionally on the business cycle.

The results reveal that the term structure of expected equity returns incorporates information about the expectations of future changes in business cycles and investment growth. The VAR estimates indicate that increases in the equity return premium precede declines in investment growth by about five quarters, with shocks to the equity return premium explaining approximately 34.2% of its variance. Analysis of the data from 1968:Q4 to 2023:Q2 further suggests that while the change to an upward-sloping curve does not immediately signal a shift in the economy to recession, it provides an early warning of such transitions, a finding confirmed by the significance of the logit regression results. Furthermore, the negative relationship between the equity return premium and investment growth becomes stronger when the regression model accounts only for positive equity return premiums, with the regression coefficient shifting from -0.07 to -0.22 and the explanatory power of the regression model rising from 0.17 to 0.47. Overall, the evidence indicates that changes in the slope

[1] Cochrane (1992) develops a production-based asset pricing model that identifies the relationship between expected returns and investment growth.

of the curve predict declines in investment growth at least one year in advance, more significantly when the slope of the curve predicts a higher probability of the economy entering a recession.

The remainder of this chapter is organized as follows: Section 2 reviews the existing term structure of equity returns literature; Section 3 presents the methodology used to extract the term structure of equity returns and defines the framework to evaluate the time series properties of the term structure across business cycles; Section 4 presents the main statistical results of this study; and Section 5 summarizes the main findings and presents the conclusion of this study.

2 Related Literature

A central debate in the study of the term structure of equity returns is its relationship with business cycles. Campbell and Cochrane (1999) and Bansal and Yaron (2000) are standard references to this debate. Campbell and Cochrane (1999) develop a consumption-based model to explain the long-horizon predictability of excess stock returns and the countercyclical variation in stock market volatility. Bansal and Yaron (2000) introduce an asset pricing model for growth rates that captures the decline in asset prices resulting from higher economic uncertainty and justifies a higher equity premium during recessions.

Studies of new modeling frameworks and empirical evidence regarding the countercyclical behavior of the term structure of equity returns have recently emerged. Breugem et al. (2020) use a survey of economic growth expectations and dividend strip data obtained from van Binsbergen et al. (2010) and Giglio et al. (2023) to develop a link between expected growth volatility and the cyclicality of the equity term premium. Gormsen (2021) analyze the one-period expected returns on dividend claims and associate a downward-sloping equity term structure with good times, consistent with the traditional macro-finance models proposed by Campbell and Cochrane (1999) and Bansal and Yaron (2000), but contrary to the findings presented by van Binsbergen et al. (2010). Gonçalves (2021) notes that the tradeoff between market and reinvestment risk explains the downward-sloping equity term structure at long maturities, and the time-variation in this risk tradeoff produces a countercyclical equity term structure.

The significant progress in the literature on the term structure of equity returns has provided new evidence of the interactions between the equity market and the economy. Although the countercyclical behavior of the term structure of equity returns resonates from previous studies, whether upward-sloping curves predict recessions, regardless of their magnitude, and whether changes in the slope of the curve relate to changes in an economic variable remain unclear. Furthermore, since the

last decade, the primary focus of research has been on estimating the yield to maturity of investments in dividend claims over the business cycle, rather than analyzing the holding period return of aggregated investments. Challenges in identifying a direct relationship between an economic variable and the term structure of equity returns prompt the research question of this study. Therefore, the aim of this study is to fill this gap in the term structure literature by providing concrete evidence of the interactions over the business cycle between an economic variable and the time variation of expected equity returns of the aggregate stock market over the investment horizon.

3 Methodology

This section presents the methodology used to extract the term structure of equity returns and the modeling framework used to evaluate the time series properties of the term structure of equity across business cycles.

3.1 Term Structure of Expected Equity Returns Model

The methodology used to estimate the term structure of equity returns is based on Campbell (1991) and Campbell and Viceira (2004, 2005). According to Campbell and Viceira (2005), any return dynamics of assets and state variables are defined as a VAR(1) model:

$$\mathrm{z}_{t+1} = \Phi_0 + \Phi_1 \mathrm{z}_t + \mathrm{v}_{t+1} \tag{1}$$

Equation 1 depicts the VAR(1) process, representing the equity return as a vector of the shocks to the returns of forecasted and forecasting variables, where Φ_0 is a vector of intercepts, Φ_1 is a vector of coefficients, z_{t+1} is a vector of $m \times 1$ asset returns. The vector v_{t+1} is the contemporaneous random shock of all variables that is normally distributed and serially uncorrelated, such as $\mathrm{v}_{t+1} \sim \mathcal{N}\big[0, Var_t(\mathrm{v}_{t+1})\big]$, where the matrix of contemporaneous variances and covariances $Var_t(\mathrm{v}_{t+1})$ is assumed to remain unchanged over time.

To forecast long-horizon returns, it is necessary to estimate the cumulative τ-period of returns, resulting from the addition of one-period log returns over τ successive periods, such as.

$$\begin{aligned}\mathrm{z}_{t+1} + \mathrm{z}_{t+2} + \ldots + \mathrm{z}_{t+\tau} &= \Phi_0 + \Phi_1 \mathrm{z}_t + \mathrm{v}_{t+1} + \Phi_0 + \Phi_1 \mathrm{z}_{t+1} + \mathrm{v}_{t+2} + \ldots + \Phi_0 + \Phi_1 \mathrm{z}_{t+\tau-1} + \mathrm{v}_{t+\tau} \\ &= \sum_{i=1}^{\tau} \sum_{j=1}^{i} \Phi_0 \Phi_1^{i-j} + \sum_{i=1}^{\tau} \Phi_1^i \mathrm{z}_t + \sum_{i=1}^{\tau} \sum_{j=1}^{i} \Phi_1^{i-j} \mathrm{v}_{t+j}\end{aligned} \tag{2}$$

The parameter Φ_1 is bounded at ± 1 to ensure that the variables converge to their long-run means in a finite number of periods, excluding explosive (or nonstationary) behavior in these variables.

From Equation 2, the conditional cumulative mean of the asset return follows:

$$E_t(\mathrm{z}_{t+1} + \mathrm{z}_{t+2} + \ldots + \mathrm{z}_{t+\tau}) = \sum_{i=1}^{\tau} \sum_{j=1}^{i} \Phi_0 \Phi_1^{i-j} + \sum_{i=1}^{\tau} \Phi_1^i \mathrm{z}_t \tag{3}$$

It is now possible to extract the conditional mean per period of any asset from Equation 3 by selecting the portion of the VAR that contains the expected returns of the targeted asset.

The expected equity return at the τ-investment horizon measured at time t is:

$$r_{e,t}^{\tau} = \frac{1}{\tau} \mathbb{I}_r E_t(\mathrm{z}_{t+1} + \mathrm{z}_{t+2} + \ldots + \mathrm{z}_{t+\tau}) \tag{4}$$

the matrix $\mathbb{I}_r = [I_{(n+1)\times(n+1)} 0_{(m-n-1)\times(m-n-1)}]$ selects the 1^{st} element of the matrix that multiplies, where $I_{(n+1)\times(n+1)}$ is an identity matrix, m is the total number of variables, and $(m-n)$ is the number of state variables.

3.2 Changes in Expected Equity Returns Over Periods

To evaluate the change in the term structure of expected equity returns, Equation 1 uses the dividend yield as the only forecasting variable such as:

$$r_{e,t+1} = \phi_0 + \phi_e r_{e,t} + \phi_{ed} r_{d,t} + v_{e,t+1} \tag{5}$$

$$r_{d,t+1} = \phi_0 + \phi_d r_{d,t} + v_{d,t+1} \tag{6}$$

where ϕ_0 is the constant parameter, ϕ_e is the autorregressive coefficient of the equity, $\phi_{ed} r_{d,t}$ is the autorregresive coefficient of the equity and the dividend yield, $r_{e,t+1}$ is the equity return, $v_{e,t+1}$ is the unexpected equity returns, $\phi_d r_{d,t}$ is the autorregresive coefficient of the dividend, $r_{d,t+1}$ is the expected dividend yield, and $v_{d,t+1}$ is the unexpected dividend yield.

From equations 5 and 6, future single-period changes in equity returns follow:

$$
\begin{aligned}
r_{e,t+1} - E_t[r_{e,t+1}] &= v_{e,t+1} \qquad (7)\\
r_{e,t+2} - E_t[r_{e,t+2}] &= v_{e,t+2} + \phi_{ed}(r_{d,t+1} - E_t[r_{d,t+1}])\\
&= v_{e,t+2} + \phi_{ed} v_{d,t+1}\\
r_{e,t+3} - E_t[r_{e,t+3}] &= v_{e,t+3} + \phi_{ed} v_{d,t+2}\\
&\dots\\
r_{e,t+\tau} - E_t[r_{e,t+\tau}] &= v_{e,t+\tau} + \phi_{ed} v_{d,t+\tau-1} \qquad (8)
\end{aligned}
$$

Equation 8 is the general representation of any forecasted change other than a 1-period change. The τ-period change in expected equity returns depends on its own changes and the past changes in the dividend yield, adjusted by the autocorrelation between equity returns and the dividend yield. When the returns are not autocorrelated ($\phi_{ed} = 0$), the change of equity returns in each period is the same at any forecasting horizon, and the term structure of expected equity returns is flat. When the short-term return is below or above their long-term return, the term structure can be upward- or downward-sloping, respectively, and the dividend yield will induce expected equity returns to revert to their mean ($\phi_{ed} v_{d,t+\tau-1}$).

Based on Equation 4, and when there is only one state variable, the single-period variance and autocovariance of equity returns are:

$$
\begin{aligned}
\frac{1}{2} Var_t(r_{e,t}^2) &= \frac{1}{2} Var_t(r_{e,t+1} + r_{e,t+2})\\
&= \frac{1}{2} Var_t(r_{e,t+1}) + \frac{1}{2} Var_t(r_{e,t+2}) + Cov_t(r_{e,t+1}, r_{e,t+2}) \qquad (9)
\end{aligned}
$$

Replacing equations 7 and 8 into Equation 9, where $\tau = 2$, it follows:

$$
\begin{aligned}
\frac{1}{2} Var_t(r_{e,t}^2) &= \frac{1}{2}\sigma_t^2 + \frac{1}{2}(\sigma_t^2 \phi_{ed} + \sigma_t^2) + \phi_{ed}\sigma_{ed,t}^2\\
&= \sigma_t^2 + \frac{1}{2}\sigma_t^2 \phi_{ed} + \phi_{ed}\sigma_{es,t} \qquad (10)
\end{aligned}
$$

Equation 10 shows that the single-period variance of returns depends on the conditional change of equity returns and the previous changes in the dividend yield, which are autocorrelated with equity returns. The variance of equity returns is always positive, while the autocorrelation between the equity return and the dividend yield determines the direction of the change, which depends on

the sign of the product $\phi_{ed}\sigma_{es,t}$. As a result, the autocorrelation between equity returns and the dividend yield leads to mean reversion of expected equity returns over the investment horizon due to the process operated by the sign of the product $\phi_{ed}\sigma_{es,t}$. When the dividend yield forecast is positive ($\phi_{ed} > 0$), and the correlation between unexpected equity returns and changes in the dividend yield is negative ($\sigma_{es,t} < 0$), the process induces negative first-order autocovariance in one-period returns. Campbell et al. (1998) noted that the long-run behavior of the VAR system is dominated by the persistent movements of the log dividend-price ratio. Campbell and Viceira (2005) highlighted that the closer is ϕ_{ed} to one, that is, the more persistent is the forecasting variable, the longer it takes for its marginal contribution to disappear in the term structure.

3.3 Modeling Framework of the Time Series Properties of the Term Structure of Equity Returns

This section presents three linear regression models to evaluate the time-series properties of the slope of the term structure of equity returns across business cycles: (1) a logit regression to estimate the likelihood of a recession based on changes in the slope of the term structure equity returns, (2) a VAR model to identify coincident or lagging series of the economic variable on the equity return premium, and (3) a conditional regression on business cycles to evaluate changes in the relationship between the investment growth and the equity return premium. Therefore, the three main variables of interest are estimated equity returns, the recession indicator, and the investment growth.

Estimated equity returns follow Equation 4. Consequently, the term structure of expected equity returns is a vector of equity returns by investment horizon estimated from the VAR(1) model at time t, that is $\Lambda_{re,t} = [r^1_{e,t}, r^2_{e,t}, \ldots, r^T_{e,t}]$. It is possible to determine the change in the slope of the term structure by measuring the estimated equity return premium, resulting from the difference in returns between the n- and m-period investment horizons, such as:

$$\Lambda^{n,m}_{re,t} = r^n_{e,t} - r^m_{e,t} \tag{11}$$

where $n > m$. Equation 11 is the estimated equity return premium measured at time t and equal to the difference between two equity returns with an investment horizon of n and m estimated from the VAR model presented in Equation 4.

The variable Ψ_t is the economic recession indicator, and is binary, such that $\Psi_t = 1$ when the economy is in recession and $\Psi_t = 0$ when it is not.

The investment growth rate is defined as the change in the investment stock, such as:

$$\Delta I_t^{\tau} = \log(I_{t+\tau}/I_t) \tag{12}$$

where τ is the time period associated with the change in the investment stock.

3.3.1 Logit Regression Model

This section introduces a mechanism to evaluate the relationship between changes in business cycles and changes in the slope of the term structure of expected equity returns.

Breugem et al. (2020), Gormsen (2021), and Gonçalves (2021) provided new evidence that the term structure of expected equity returns is countercyclical. However, whether the single occurrence of an upward-sloping curve is sufficient to indicate a recession or how significantly the slope of the curve needs to change in order to predict one is unclear. To evaluate how changes in the slope of the term structure of expected equity returns incorporate expectations of economic expansions and contractions, a logit regression provides the setup for modeling the probability of changes in business cycles being in a given stage based on changes in the equity return premium.

The logit regression model proposed in this section associates positive changes in the equity return premium with a higher probability of the economy being in recession, and negative changes in the equity return premium with a higher probability of the economy being in expansion. It is possible to measure the slope of the curve using the equity return premium in equation 11. An upward-sloping curve is associated with a positive equity return premium, such that $\Lambda_{re,t}^{n,m} > 0$, a downward-sloping curve is associated with a negative premium, such that $\Lambda_{re,t}^{n,m} < 0$, and a flattener of the curve is associated with an equity premium close to zero, such that $\Lambda_{re,t}^{n,m} \approx 0$. The recession indicator is a binary variable. When the economy is in recession, the recession indicator is denoted as $\Psi_t = 1$, and when the economy is out of the recession is denoted as $\Psi_t = 0$. The recession indicator is a binary variable (Ψ_t). When the economy is in recession, it takes the value of 1, and when the economy is out of the recession, it takes the value of 0. The term structure of equity returns is measured at time $t-1$, or one period behind the reported period of the recession indicator (Ψ_t). Therefore, the probability of a change in the business cycle can be defined as follows:

$$P(\Psi_t|x_{t-1}) = \frac{1}{1+\exp(\theta_0 + \theta_1 x_{t-1})} \tag{13}$$

where $x_{t-1} = \max(0; \Lambda^{n,m}_{re,t-1})$ and $P(\Psi_t = 1|x_{t-1})$ is the probability when the economy is in recession, and $1 - P(\Psi_t = 1|x_{t-1})$ is the probability when the economy is not in recession.

3.3.2 VAR Model Order

The identification of a direct relationship between an economic variable and the slope of the term structure of equity returns is central to this study, and investment growth is an optimal candidate. Cochrane (1992) develops a partial equilibrium explanation of the forecastability of stock returns and the fact that stock returns forecast real variables including investment and gross national product. Cochrane (1992) suggests that firms will continue to adjust their investment and production plans until their investment returns are equal to their asset returns. If there is a relationship between investment growth and the term structure of equity returns, it is relevant to determine the timing of investment growth's response to changes in the slope of the term structure. If the joint dynamics of changes in investment growth and changes in the equity return premium follow an autoregressive process, it is possible to evaluate how investment growth responds to changes in the equity return premium. Therefore, two key questions follow: (1) What is the direction of the relationship between the two variables? (2) Do changes in the investment growth rate occur immediately after or follow changes in the slope of the term structure of equity returns? Assuming changes in investment growth and changes in the equity return premium follow an autoregressive process, such that:

$$\Delta I^{\tau}_t = \alpha_0 + \alpha_1 \Delta I^{\tau}_{t-1} + \alpha_2 \Delta I^{\tau}_{t-2} + \alpha_3 \Delta I^{\tau}_{t-3} + \ldots + \alpha_p \Delta I^{\tau}_{t-p} + \beta_1 \Lambda_{re,t-1} + \beta_2 \Lambda^{n,m}_{re,t-2} + \beta_3 \Lambda^{n,m}_{re,t-3} + \ldots + \beta_p \Lambda^{n,m}_{re,t-p} + \nu_t \tag{14}$$

where α_0 is a constant parameter, α_i is the investment growth autoregressive coefficient parameter, β_i is the autoregressive coefficient parameter of the equity return premium, the variable ΔI^{τ}_{t-i} is the investment growth rate with a lag of $t - i$, the variable $\Lambda^{n,m}_{re,t-i}$ is the equity return premium with a lag of $t - i$, and ν_t is a vector of random shocks of returns. $\forall\ i = 1, 2, \ldots, p$ and p is its optimal lag period.

Equation 14 defines the investment growth rate as a linear function of its lag value and the lag value of the estimated equity return premium. The optimal lag length of the VAR model can be obtained using an information criterion. The Bayesian information criterion (BIC) is commonly used as a fitting measurement to select the model type and VAR model order (p). The optimal VAR model order (p) is as follows:

$$BIC = k \ln n - 2 \ln(\hat{L}) \tag{15}$$

where n is the number of observations, k is the number of parameters, and $\hat{L}$ is the maximized value of the likelihood function of the model $M(\hat{L} = p(x|\hat{\theta}, M))$ in which $\hat{\theta}$ is the parameter value that maximizes the likelihood function, and x is the observed data point.

The BIC identifies the value that minimizes the negative log-likelihood function while penalizing the number of model parameters k and the sample size n, thus preventing the model from being overfitted to the data (see Equation 15).

3.3.3 Linear Regression Model

Recessions are a recurring phase of business cycles and may be a source of powerful economic disruptions. If the behavior of the term structure incorporates expectations of future changes in business cycles, the relationship between investment growth and the equity return premium must account for this association. Ignoring this effect in the regression may result in model misspecification and distort the distributional properties of the estimated changes in both the equity return premium and investment growth.

Therefore, the relationship between investment growth and the equity return premium can follow an unconditional and conditional dynamic to business cycles, such that:

$$E(\Delta I^{\tau}_{t+p}) = \theta_0 + \theta_1 \Lambda^{n,m}_{e,t} + \nu_t \tag{16}$$

$$E\left(\Delta I^{1y}_{t+p} | \Psi_t\right) = \theta_0 + \theta_1 \Lambda^{n,m}_{e,t,\Psi_t} + \nu_t \tag{17}$$

$$\Psi_t = \begin{cases} 1 & \text{when the economy is in recession; } \forall\ \Lambda^{n,m}_{e,t,\Psi_t=1} = \max\left(0; \Lambda^{n,m}_{e,t}\right) \\ 0 & \text{when the economy is in not in recession; } \forall\ \Lambda^{n,m}_{e,t,\Psi_t=0} = \min\left(0; \Lambda^{n,m}_{e,t}\right) \end{cases}$$

where Equation 16 is the unconditional investment growth rate regression, while Equation 17 is the investment growth rate regression conditional on business cycles. The variable ΔI^{τ}_{t+p} is the τ period growth of the investment observed at time $t + p$, p is the VAR model order minimized according to Equation 15, Ψ_t is the recession indicator, θ_0 is the constant parameter, and θ_1 is the

coefficient parameter. The term ν_t is the random shock of normally distributed returns such that $\nu_{t+1} \sim \mathcal{N}\big[0,\, Var_t(\nu_{t+1})\big]$. The variables $\Lambda^{n,m}_{e,t}$, $\Lambda^{n,m}_{e,t,\Psi_{t=1}}$, and $\Lambda^{n,m}_{e,t,\Psi_{t=0}}$ are the equity return premium in Equation 11 when the equity return premium is either unrestricted, restricted only to positive equity returns premiums, or restricted to only negative equity return premiums, respectively.

4 Empirical Results

This section presents the analysis of the time series properties of the term structure of equity returns across business cycles in three steps:

- Determination of whether there is a direct relationship between changes in the investment growth and changes in the slope of the term structure of equity returns.
- Examination of whether changes in the slope of the term structure of expected equity returns predict shifts in the business cycle.
- Analysis of the relationship between changes in the investment growth and changes in the slope of the term structure of equity returns under different business conditions.

4.1 Data

The time series of interest in the empirical analysis presented in this section includes investment growth, the recession indicator, and the equity return premium.

The time series of the investment growth uses quarterly gross private domestic investment (GPDI) data retrieved from the Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis.[2]. The time series of the recession indicator data is retrieved from the Federal Reserve Bank of St. Louis.[3] The recession indicator is based on the "Econbrowser Recession Indicator Index," a quarterly time series that assigns dates to U.S. recessions resulting from a mathematical model that differentiates recessions from expansions, developed by Chauvet and Hamilton (2005). The time series of the equity return and the dividend yield are based on the S&P Index and the dividend data available on Robert Shiller's website.[4] The measurement date of the equity return premium extracted from the VAR(1) process precedes both the measurement of the annual investment growth rate and the quarter when the GDP data is available and used for measuring the recession indicator.

[2]U.S. Bureau of Economic Analysis, gross private domestic investment [GPDI], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GPDI

[3]Hamilton, James, Dates of U.S. recessions as inferred by GDP-based recession indicator [JHDUSRGDPBR], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/JHDUSRGDPBR.

[4]To access Robert Shiller's data, please refer to http://www.econ.yale.edu/ shiller/data.htm

The time series of the investment growth starts from 1960:Q1 to 2023:Q2. The annual change in GPDI, denoted by ΔI_t^{1y}, represents the annual investment growth rate and is described by Equation 12. The time series of the recession indicator data starts from 1967:Q4 to 2023:Q2. The recession indicator, denoted by Ψ_t, is a binary condition that specifies when the economy is in recession ($\Psi_t = 1$) and when it is not ($\Psi_t = 0$). The variable $\Lambda_{re,t}^{10y,3m}$ is the 10-year equity return premium on the 3-month equity return, and follows Equation 11. The VAR(1) process estimates the equity return premium on quarterly rolling windows, starting with data from 1950:Q1 to 1960:Q1 and adding one quarter of new data with each quarterly VAR(1) estimate until 2023:Q2. The maximum 10-year investment horizon to estimate equity returns is not discretionary; it was chosen to incorporate at least one business cycle and approximate the evidence presented by some researchers and official publications. Drehmann et al. (2012) noted that the US business cycle involves frequencies from 1 to 8 years from a statistical range that distinguishes the cycle from the trend components in GDP. The data presented by NBER[5] reveals that since 1945, recessions have lasted an average of 10 months, whereas expansions have lasted from 12 to 128 months, 65 years on average, and 103 months on average since 1990. Similarly, Claessens et al. (2011) proposed that financial cycles are often more protracted than business cycles. They note that when observing downturns and upturns, excluding expansions, credit cycles take 13 quarters, whereas financial cycles take 29. Therefore, a maximum 10-year investment horizon and rolling windows are appropriate mechanisms to incorporate market events into the sample data over multiple decades and evaluate the behavior of the term structure of equity under different business conditions.

Once the term structure of equity returns is extracted from the VAR(1) model, equity return premium data are collected quarterly from 1960:Q1 to 2023:Q2.

4.2 The Term Structure of Equity Returns and the Investment Cycle

This section presents a bivariate time series analysis of the relationship between the investment growth rate and the equity return premium.

Figure 1 presents the quarterly time series of the annual investment growth rate (ΔI_t^{1y}) and the 10-year equity return premium on the 3-month equity return ($\Lambda_{e,t}^{10y,3m}$) from 1960:Q1 to 2023:Q2. Although both trend in a similar direction, the equity return premium moves frequently, whereas the investment growth rate moves gradually across periods. Moreover, the equity return premium moves earlier than the investment growth rate. The leading change in equity return premium relative to

[5]National Bureau of Economic Research, NBER. Chronology of US business cycles. Data source link: https://www.nber.org/research/business-cycle-dating

the investment growth rate is evident during high fluctuations of the equity return premium and the investment growth rate. For example, by the mid-1970s, the equity return premium increased by more than 50 basis points, and after 1 year, the annual investment growth rate decreased by 20%. Months later, the equity return premium reversed the change, decreasing more than 70 basis points, and after months, the annual investment growth rate increased by more than 20%. In the early 1980s, the equity return premium increased by approximately 5%, followed by a decrease of roughly 20% in the annual investment growth rate. Months later, the return premium decreased by 40%, followed by an increased annual investment growth rate of approximately 20%. This pattern was again observed in the mid-1980s, early 1990s, 2001, 2002, 2007-2010, and, more recently, 2020. However, the investment growth rate remains relatively stable as the term premium varies dramatically during other periods, such as those observed between 1997 and 1999 during the Asian crisis, which created global market instability. This misalignment between the US private investment growth rate and the equity return premium can be attributed to the international instability during the 1997 crisis, which had a more significant impact on the U.S. stock market than its economy. During the 1997 crisis, the U.S. economy remained resilient despite global market turmoil, which explains the more stable pattern of annual investment growth.[6]

4.2.1 Bivariate VAR Model Order Results

The VAR model presented in Equation 14 specifies the joint dynamics of the investment growth rates and the equity return premiums. The model includes one constant term, lag terms from the investment growth rate, and lag terms from the equity return premium. The lag length (the p^{th} order) is selected using the BIC method. The BIC identifies the value that minimizes the negative log-likelihood function, which penalizes the number of model parameters and sample size, thereby preventing the model from being overfitted to the data (see Equation 15).

Table 1 presents the VAR results. The VAR coefficient parameter is significant and negative at 0.0294. The optimal VAR model order is minimized when BIC is -8.47, or at a lag of five quarters ($p = 5$). Table 1 lists the results of the forecast error variance decomposition (FEVD). The results indicate that the forecast error variance of the annual investment growth rate is explained by 34.2% of the shocks of the equity return premium at a five-quarter lag. Figure 3 shows the results of the VAR model order structure. The estimated BIC criterion begins to decrease immediately after the

[6]Harrigan, James, (2000), The impact of the Asia crisis on U.S. industry: an almost-free lunch? Economic Policy Review, issue Sep, p. 71-81, https://EconPapers.repec.org/RePEc:fip:fednep:y:2000:i:sep:p:71-81:n:v.6no.3.

first lag and is minimized when the lag is five quarters. Once the BIC reaches the minimum lag, it begins to increase regularly.

As discussed earlier, the equity return premium is estimated using the dividend yield as the primary state variable. The equity return premium is estimated using alternative state variables to evaluate its bivariate relationship with the annual investment growth rate (Appendix A). The results include the VAR statistic, BIC values, and FEVD results. The evidence in Appendix A confirms that the VAR results for the joint dynamics of the investment growth rate and the equity return premium are more robust when equity returns are estimated using the dividend yield as the primary state variable.

4.2.2 Investment Growth Rates and Equity Return Premium Correlation

The previous section confirmed that the most robust specification of the vector autoregression analysis of the investment growth and the estimated equity return is a model order of five quarters. This section analyzes how the relationship between the annual investment growth rate and the equity return premium correlates over time, applying a lag of five quarters between the measurement of each of the variables. The correlation coefficient between the annual investment growth rate and the equity return premium is defined as follows:

$$\rho(\Delta I^{1y}_{t+5}, \Lambda^{10y,3m}_{re,t}) = \frac{\sum_{i=0}^{20} \left(\Delta I^{1y}_{t+5+i} - \overline{\Delta I}^{1y}_{t+5}\right)\left(\Lambda^{10y,3m}_{re,t+i} - \overline{\Lambda}^{10y,3m}_{re,t,20}\right)}{\sqrt{\sum_{i=0}^{20} \left(\Delta I^{1y}_{t+5+i} - \overline{\Delta I}^{1y}_{t+5}\right)^2 \sum_{i=0}^{20} \left(\Lambda^{10y,3m}_{re,t+i} - \overline{\Lambda}^{10y,3m}_{re,t,20}\right)^2}} \tag{18}$$

where $i = 0, 1, \ldots, 20$ quarters, the variable ΔI^{1y}_{t+5+i} is the annual investment growth rate observed five quarters after time $t + i$, following Equation 12, and the variable $\Lambda^{10y,3m}_{re,t+i}$ is the 10-year equity return premium on the 3-month equity return from the VAR(1) model measured at time $t + i$, following Equation 11. The mean of the investment growth rate is $\overline{\Delta I}^{1y}_{t+5}$ and the mean of the equity return premium is $\overline{\Lambda}^{10y,3m}_{re,t,20}$. The correlation coefficients are estimated over 20 quarters of data on a quarterly rolling basis, from 1960:Q1 to 2023:Q2.

Given the VAR model order of five quarters, the equity return premium is estimated five quarters prior to the measurement of the annual investment growth rate. Although the VAR model order contributes to minimizing the systematic relationship between the errors by achieving a good bias-variance of the variables and can improve the overall predictive accuracy, it does not eliminate the measurement error. The attenuation of the expected correlation coefficient as a function of the error components will vary depending on the measurement error present in each of the sam-

ples. Therefore, under the estimation process presented in Equation 18, the correlation coefficient $\rho(\Delta I^{1y}_{t+5}, \Lambda^{10y,3m}_{re,t})$ is always biased towards zero and expected to be weaker than the true correlation coefficient $\rho(\Delta I^{*}_{t+5}, \Lambda^{*}_{re,t})$.

Figure 2 presents a histogram of the correlation coefficients between the annual investment growth rate and the 10-year equity return premium on the 3-month equity return. The correlation coefficient of the annual investment growth rate and equity return premium tends to be negative over time (Figure 2). The correlation coefficients are positively skewed, with the highest frequency ranging from -0.3 to -0.2 (Figure 2). However, there are exceptional periods when the correlation coefficients are positive. Between 2000 and 2009, changes in the equity return premium ($\Lambda^{10y,3m}_{re,t}$) correlate positively with the annual investment growth rate (ΔI^{1y}_{t+5}), with most observations concentrating at 0.2 correlation.

The results presented in this section confirm that the relationship between the equity return premium and the investment growth rate, using a five-quarter lag, is negative. When the slope of the term structure moves clockwise (the equity return premium is negative or $\Lambda^{10y,3m}_{re,t} < 0$), the annual investment growth rate increases ($\Delta I^{1y}_{t+5} > 0$). Conversely, when the slope of the term structure moves counterclockwise (the equity return premium is positive or $\Lambda^{10y,3m}_{re,t} > 0$), the annual investment growth rate is expected to decrease ($\Delta I^{1y}_{t+5} < 0$).

4.3 Economic Cycles and the Term Structure of Equity Returns

This section evaluates the term structure of expected equity returns, measuring the frequency of upward- and downward-sloping curves under different business cycles.

Figure 4 presents the time series of the 10-year equity return premium on the 3-month equity return ($\Lambda^{10y,3m}_{re,t}$), the annual investment growth rate (ΔI^{1y}_{t+5}), and the recession indicator index (Ψ_t). The data cover the period from 1968:Q4 to 2023:Q2. The equity return premium is estimated at the reported date of the recession indicator on quarterly intervals using data from the previous 10 years. The annual investment growth rate is measured five quarters after the equity return premium is calculated ($t+5$). Recessions occurred twice per decade until 1980 (Figure 4); however, from then on, most recessions occurred in cycles of approximately 10 years. The equity return premium moves in the opposite direction to the variations of the investment growth rate. The relationship between changes in equity return premium and changes in investment growth rate is more evident during periods of dramatic change than in more stable periods. These drastic changes coincide with shifts in the business cycle to recession.

Table 2 documents the number of positive and negative equity return premiums when the economy is in recession and when it is not from 1968:Q4 until 2023:Q2. Nevertheless, recessions and expansions can be subdivided into periods of positive and negative investment growth rates. The U.S. economy has been in recession in 41 out of 218 months and in expansion in 177 out of 218 months (Table 2). Downward-sloping curves are usually observed when the economy is not in recession. In 121 out of 177 months, the term structure of equity returns was downward-sloping when the economy was not in recession. Upward-sloping curves are not exclusive to recessions; they also occur when the economy is out of recession. There are more observations with upward-sloping curves when the economy is out of recession than when it is in recession. In 56 out of 75 months, the term structure of equity was upward-sloping when the economy was not in recession. A normal economy lasts much longer than a recession, which may explain the symmetry of the number of observations of upward-sloping curves for each business cycle. Therefore, investigation of the relative number of upward-sloping curves over the total number of observations when the economy is in recession and when it is not can provide a more assertive view than examining the absolute value: the ratio of upward-sloping curves over the total number of quarters in a recession is much higher than when the economy is out of recession. During recessions, the term structure of equity returns was upward-sloping in 19 out of 75 months (25.3%) and downward-sloping in 22 out of 144 months (15.4%).

The empirical evidence in Table 2 suggests that when the economy is in recession, months with negative equity return premiums are predominantly observed when investment growth rates are positive. Conversely, months with positive equity return premiums predominantly occur when investment growth rates are negative. During recessions, in 17 out of 22 months, the investment growth rate was positive ($\Delta I^{1y}_{t+5} > 0$), and the term structure was downward-sloping ($\Lambda^{10y,3m}_{re,t} < 0$), while However, in 16 out of 19 months, the investment growth rate was negative ($\Delta I^{1y}_{t+5} < 0$), and the term structure was upward-sloping ($\Lambda^{10y,3m}_{re,t} > 0$). When the economy is not in recession, months with negative equity return premiums predominantly occur when investment growth rates are positive. However, the direction of the investment growth rate is less apparent when the equity return premium is positive. When the economy is not in recession, in 97 out of 122 months, the investment growth rate is positive ($\Delta I^{1y}_{t+5} > 0$) while the term structure is downward-sloping ($\Lambda^{10y,3m}_{re,t} < 0$). In 22 out of 55 months, the investment growth rate is negative ($\Delta I^{1y}_{t+5} > 0$) while the term structure is upward-sloping ($\Lambda^{10y,3m}_{re,t} > 0$). The evidence presented in Table 2 above shows that upward sloping curves are not events exclusively observed during recessions. Besides, it seems that the relationship

between changes in the investment growth and the slope of the term structure is more evident when the economy is in recession.

4.3.1 Logit Regression Results

This section evaluates whether changes in the slope of the term structure of expected equity returns can predict the likelihood of changes in business cycles, following the logit regression modeling framework presented in Section 3.1. To achieve a more comprehensive understanding of the relationship between changes in the slope of the term structure of expected equity returns and business cycles, the logit regression analysis examines both immediate and smoothed impacts in order to evaluate a more sustainable effect of the equity return premium, such that:

- $P(\Psi_t = 1|\max(0; \Lambda_{re,t}^{10y,3m}))$ evaluates the probability that the economy will enter a recession, that is $\Psi_t = 1$, given a positive equity return premium. The variable $\Lambda_{re,t}^{10y,3m}$ follows Equation 11 and is the 10-year equity return premium on the 3-month equity return extracted from the VAR(1) model.

- $P(\Psi_t = 1|\max(0; \overline{\Lambda}_{re,t,1y}^{10y,3m}))$ evaluates the probability that the economy will enter a recession, that is $\Psi_t = 1$, given a positive average equity return premium. The variable $\overline{\Lambda}_{re,t,1y}^{10y,3m}$ is the annual average of the 10-year equity return premium on the 3-month equity return extracted from the VAR(1) model.

The recession indicator developed by Chauvet and Hamilton (2005) is an inference based on the advance GDP estimates available one quarter after the reported date, and the series by definition is never revised[7]. The measurement date of the equity return premium estimated from the VAR(1) process precedes the quarter when the GDP data is available and used for measuring the recession indicator. The recession indicator's earliest data starts in the fourth quarter of 1967. Therefore, the equity return premium and the recession indicator data range from the fourth quarter of 1968 to the second quarter of 2023, aligning with the data range of the twelve-month average of the equity return premium.

Figure 5 presents the relationship between business cycles and the slope of the term structure of expected equity returns based on a sample of 218 observations. Panel 5a shows a scatter plot of the recession indicator (Ψ_t) and the equity return premiums ($\Lambda_{i,t}^{10y,3m}$ and its average $\overline{\Lambda}_{e,t,1y}^{10y,3m}$). Data points for the equity return premium ($\Lambda_{i,t}^{10y,3m}$) are plotted along the recession indicator when the economy is in recession ($\Psi_t = 1$) and when it is not ($\Psi_t = 0$). When the economy is in recession,

[7]For more information about this series, visit http://econbrowser.com/recession-index.

equity return premiums concentrate in the lower range of -0.45 and the upper range of 0.5. However, when it is not in recession, they concentrate in the lower -0.75 and upper 0.5 range of the equity return premium. Similarly, the data points of the average equity return premium ($\overline{\Lambda}^{10y,3m}_{re,t,1y}$) are plotted along the recession indicator, more compacted than the equity return premium, $\Lambda^{10y,3m}_{i,t}$. When the economy is in recession, the average equity return premium observations concentrate in the lower range of -0.25 and the upper range of 0.5. In contrast, when it is not in recession, equity return premiums concentrate in the lower -0.5 and upper 0.25 range.

Panel 5b presents the logit regression results of the probability of a recession when the equity return premium and its average are positive, such as $P(\Psi_t|\max(0;\Lambda^{10y,3m}_{i,t}))$ and $P(\Psi_t|\max(0;\overline{\Lambda}^{10y,3m}_{re,t,1y}))$, respectively. The logit regression $P(\Psi_t|\max(0;\Lambda^{10y,3m}_{i,t}))$ results present a negative constant parameter of -1.9816, a positive coefficient parameter of 5.4621 with a significant t-statistic of 4.702. The logit regression $P(\Psi_t|\max(0;\overline{\Lambda}^{10y,3m}_{re,t,1y}))$ results includes a negative constant parameter of -2.216, a positive coefficient parameter of 16.7432 with a significant t-statistic of 5.753. The probability of recession increases when the equity return premium and its average increase. It reaches the highest value when the equity return premium is exceptionally high compared with its average. When the equity return premium exceeds 0.50, the expected probability of recession is 68%, and when the equity return premium is 0.80, the probability of recession is 92%. When the one-year average equity return premium exceeds 0.20, the expected probability of recession is 75%, and when the average equity return premium exceeds 0.80, the expected probability of recession is 100%. The recession probability substantially decreases when the slope of the term structure of equity returns moves clockwise, such as when the equity return premium decreases to 0.20, the expected probability of recession is not higher than 29%, and when the average equity return premium is down to 0.10, the expected probability of recession is not higher than 37%. The expected probability of recession is as low as 13% when both the equity return premium and its average are 0.0207.

The empirical evidence presented in this section confirms that the relationship between future changes in business cycles and changes in the slope of the term structure of equity returns is significant. Extreme economic conditions are not frequently observable events; they occur in cycles, and the frequency of upward- and downward-sloping curves predicts the stage of the business cycle in which the economy will be. Although the change from a downward to an upward-sloping curve is not an immediate indication of a shift in the business cycle to recession, the steepness of the curve is a sign of its likelihood. Consistently, the logit regression model results indicate that the slope of the term structure of equity returns predicts the probability of a recession. The source of a steep upward-sloping term structure of expected equity returns has an economic explanation. When the

economy performs well over the cycle, investors shift their attention to news other than the economy. However, when economic conditions deteriorate, investors become more sensitive to news about the impact of a recession on future cash flow generation. Consequently, steeper upward-sloping curves predict a high probability of a recession.

4.4 Conditional Term Structure of Expected Equity Returns

The previous sections discussed the relationship between the slope of the term structure of expected equity returns and the economic cycle. The expectations of a high likelihood of recession are associated with a steeper term structure of expected equity returns. Therefore, the relationship between investment growth and equity return premium should account for this association. This section examines the relationship between investment growth and the term structure of equity returns conditional and unconditional to business cycles, using data from 1969:Q4 to 2023:Q2. Following the linear regression methodology defined in Subsection 3.3, the assessment of the relationship between investment growth rates and equity return premiums includes three distinct regressions:

i. The investment growth rate regression on the equity return premium is conditional on recession, such as::

$$E(\Delta I_{t+5}^{1y}|\Psi_t = 1) = \theta_0 + \theta_1 \Lambda_{re,t,\Psi_t=1}^{10y,3m}$$

where $\forall\ \Lambda_{re,t,\Psi_t=1}^{10y,3m} = \max(0; \Lambda_{re,t}^{10y,3m}) \Leftrightarrow \Psi_t = 1$. The equity return premium is positive or floored to zero when negative, if and only if the U.S. economy is in recession.

ii. The investment growth rate regression on the equity return premium is conditional on expansion, such as:

$$E(\Delta I_{t+5}^{1y}|\Psi_t = 0) = \theta_0 + \theta_1 \Lambda_{re,t,\Psi_t=0}^{10y,3m} \quad .$$

where $\forall\ \Lambda_{re,t,\Psi_t=0}^{10y,3m} = \min(0; \Lambda_{re,t}^{10y,3m}) \Leftrightarrow \Psi_t = 0$. The equity return premium is negative or capped to zero when positive, if and only if the U.S. economy is not in recession.

iii. The investment growth rate regression on the equity return premium is not conditional on the business cycle, such as:

$$E(\Delta I_{t+5}^{1y}) = \theta_0 + \theta_1 \Lambda_{re,t}^{10y,3m}$$

Under perfect foresight, the equity return premium, denoted by $\Lambda_{re,t}^{n,m}$, perfectly predicts the investment growth rate, denoted by ΔI_{t+p}^{τ}, such that the coefficient parameter is negative and one ($\theta_1 = -1$). However, similar to Section 4.2, when discussing the correlation coefficient between the investment growth rate and the equity return premium, the presence of measurement errors in both the estimated equity return premium and the investment growth rate does not cancel out, even as the number of samples increases. Therefore, the estimation of the regression parameters implies that the effect is the attenuation of the slope coefficients toward zero.

Table 3 presents the regression results. The equity return premium is measured at time t, while the measurement of the annual investment growth rate is at time $t + 5$ quarters after. The first column, $E(\Delta I_{t+5}^{1y})$, presents the results of the unconditional regression for predicting the investment growth rate without considering changes in the business cycle. The second column, $E(\Delta I_{t+5}^{1y}|\Psi_t = 0)$, presents the results of the conditional regression for predicting the investment growth rate when the U.S. economy is not in a recession. The last column, $E(\Delta I_{t+5}^{1y}|\Psi_t = 1)$, presents the results of the conditional regression for predicting the investment growth rate when the US economy is in a recession. The coefficient parameters of the three regressions are negative and significant, confirming that the slope of the term structure of expected equity returns is countercyclical to investment growth. The unconditional regression coefficient is -0.13, with a coefficient of determination of 0.17. The coefficient parameter of the conditional regression when the economy is not in recession is close to zero at -0.07; however, its coefficient of determination is 0.05. When the economy is in a recession, the coefficient parameter of the conditional regression is -0.22, and its coefficient of determination is 0.47, almost three times higher than the coefficient of determination of the unconditional regression.

The empirical evidence in this section confirms that the relationship between investment growth rates and the term structure of equity returns changes over the business cycle. The regression results show that the relationship between investment growth and the equity return premium is stronger when the slope of the term structure of expected equity returns predicts a high probability of recession, and it weakens when the forecast of the likelihood of recession is very low. When the term structure of expected equity returns is upward-sloping, a positive change in the equity return premium indicates a higher likelihood that the economy will enter a recession and forecasts a decline in annual investment growth at least one year in advance. When the term structure of expected equity returns is downward-sloping, it indicates a low probability that the economy will enter a recession, and the relationship between investment growth and the term structure of expected equity returns weakens.

5 Conclusion

This study evaluates the relationship between changes in the slope of the term structure of equity returns, business cycles, and changes in the investment growth from 1960:Q1 to 2023:Q2. The measurement of the slope of the term structure of expected equity returns was based on the 10-year equity return premium on the 3-month equity return estimated from the VAR(1) process at quarterly intervals, using the dividend yield as the primary forecasting variable. The analysis of the time series properties of the term structure of equity returns across business cycles included three steps: (1) a vector autoregression model to evaluate the relationship between changes in investment growth and changes in the equity return premium; (2) a logit regression model to estimate the expected probability of a recession based on the equity return premium, and (3) a conditional regression model to examine how the relationship between investment growth rate and the equity return premium changes depending on the stage of the business cycle.

This study presents three main findings. The first finding identifies a direct and negative relationship between changes in investment growth and changes in the equity return premium. The analysis of the data from 1960:Q1 to 2023:Q2 shows that the correlation coefficient between the annual investment growth rate and the equity return premium is positively skewed. In addition, the specification of the vector autoregression analysis of the investment growth and the estimated equity return premium is more robust at capturing the dynamics of their relationship at one year and three months. The variance decomposition indicates that shocks to the equity return premium explain approximately 34.2% of the forecast error variance in investment growth. The second finding confirms that changes in the slope of the curve can predict the likelihood of changes in the business cycle. The significance of the logit regression results indicates that more positive equity return premiums predict an increase in the likelihood of the economy entering a recession, while negative equity return premiums, or less positive, predict a decrease in its likelihood. However, changes in the slope of the curve must be analyzed carefully: The empirical evidence confirms that the change from a downward to an upward-sloping curve is not an immediate indication of a shift in the business cycle to recession. More importantly, it provides an early warning of such transitions such that a steep term structure of expected equity returns signals expectations of a high probability of a severe and prolonged economic downturn. The third and final finding reveals that the relationship between investment growth and the equity return premium is conditional on the business cycle. When the regression of the investment growth accounts for negative equity return premiums and changes for positive equity return premiums, the regression coefficient shifts from -0.07 to -0.22, and the ex-

planatory power of the regression model rises from 0.05 to 0.47. The empirical evidence indicates that when the slope of the term structure of expected equity returns predicts a high probability of recession, and the curve is steeper, it precedes a corresponding decline in the investment cycle at least 1 year in advance. In contrast, when the slope of the curve predicts a very low likelihood of recession, it is less certain which direction investment growth will take following a change in the slope. Overall, these findings highlight the importance of accounting for the role of business cycles in the relationship between changes in investment growth and in the equity return premium, in order to better understand their cyclical dynamics and more accurately forecast transitions in the investment growth cycle.

In conclusion, the results of this study confirm that changes in the slope of the curve are neither random nor disentangled from economic developments; they predict the probability at which stage the business cycle is, as well as future changes in investment growth. When measured across multiple business cycles, changes in the slope of the curve incorporate information about the equity stock market and economic developments. The empirical evidence of this study suggests that the term structure of equity returns is a valuable tool for investors to assess the timing and horizon of their investment decisions over different business cycles. Moreover, it provides new evidence of the relationship between the equity stock market and investment growth. Future research could further explore this relationship, identifying new economic and state variables that may improve the forecasting power of the term structure of equity returns.

References

Bansal, R. and Yaron, A. (2000). Risks for the long run: A potential resolution of asset pricing puzzles. NBER Working Papers 8059, National Bureau of Economic Research, Inc.

Breugem, M., Colonnello, S., Marfe, R., and Zucchi, F. (2020). Dynamic equity slope. Carlo Alberto Notebooks 626, Collegio Carlo Alberto.

Campbell, J. Y. (1991). A variance decomposition for stock returns. *Economic Journal*, 101(405):157–79.

Campbell, J. Y. and Cochrane, J. H. (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. Scholarly articles, Harvard University Department of Economics.

Campbell, J. Y., Lo, A., MacKinlay, A. C., and Whitelaw, R. F. (1998). The econometrics of financial markets. *Macroeconomic Dynamics*, 2(4):559–562.

Campbell, J. Y. and Shiller, R. J. (1986). The dividend-price ratio and expectations of future dividends and discount factors. NBER Working Papers 2100, National Bureau of Economic Research, Inc.

Campbell, J. Y. and Viceira, L. M. (2004). Long-horizon mean-variance analysis: A user guide. *Manuscript, Harvard University, Cambridge, MA*.

Campbell, J. Y. and Viceira, L. M. (2005). The term structure of the risk-return tradeoff. NBER Working Papers 11119, National Bureau of Economic Research, Inc.

Chauvet, M. and Hamilton, J. (2005). Dating business cycle turning points. NBER Working Papers 11422, National Bureau of Economic Research, Inc.

Claessens, S., Kose, A., and Terrones, M. (2011). Financial cycles: What? how? when? CEPR Discussion Papers 8379, C.E.P.R. Discussion Papers.

Cochrane, J. (1992). A cross-sectional test of a production-based asset pricing model. NBER Working Papers 4025, National Bureau of Economic Research, Inc.

Drehmann, M., Borio, C., and Tsatsaronis, K. (2012). Characterising the financial cycle: don't lose sight of the medium term! BIS Working Papers 380, Bank for International Settlements.

Giglio, S., Kelly, B. T., and Kozak, S. (2023). Equity term structures without dividend strips data. NBER Working Papers 31119, National Bureau of Economic Research, Inc.

Gonçalves, A. S. (2021). Reinvestment risk and the equity term structure. *Journal of Finance*, 76(5):2153–2197.

Gormsen, N. (2021). Time variation of the equity term structure. *Journal of Finance*, 76(4):1959–1999.

van Binsbergen, J., Brandt, M. W., and Koijen, R. (2010). On the timing and pricing of dividends. NBER Working Papers 16455, National Bureau of Economic Research, Inc.

(a) Time Series of the Annual Investment Growth Rate and the Equity Return Premium

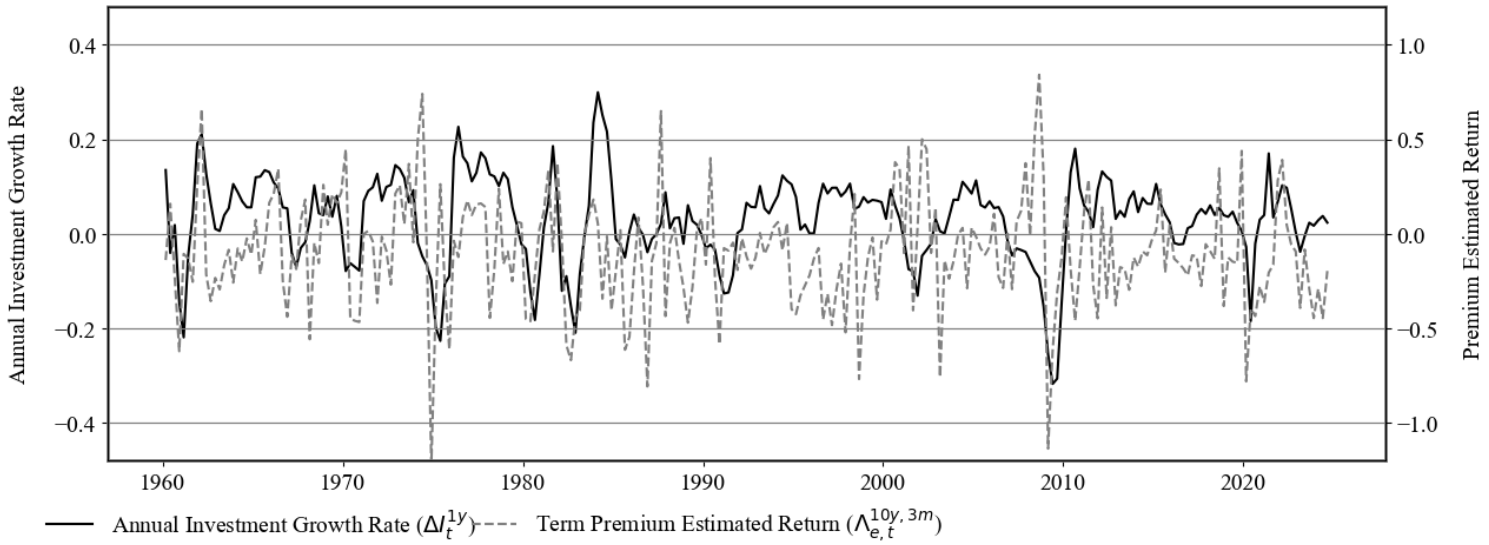


Figure 1: Time Series of the Annual Investment Growth Rate and the Equity Return Premium Estimated From the VAR(1) Model. Figure 4 presents the time series of the annual Investment Growth Rate and the equity return premium estimated from the VAR(1) model, from 1960-Q1 until 2023-Q1. The annual Investment Growth Rate, ΔI_t^{1y}, follows Equation 12 and is measured from the quarterly Gross Private Domestic Investment from the U.S. Bureau of Economic Analysis and retrieved from FRED of the Federal Reserve Bank of St. Louis. The equity return premium, $\Lambda_{re,t}^{10y,3m}$, follows Equation 11 and is the difference between the 10-year and the 3-month equity returns estimated from the VAR(1) model.

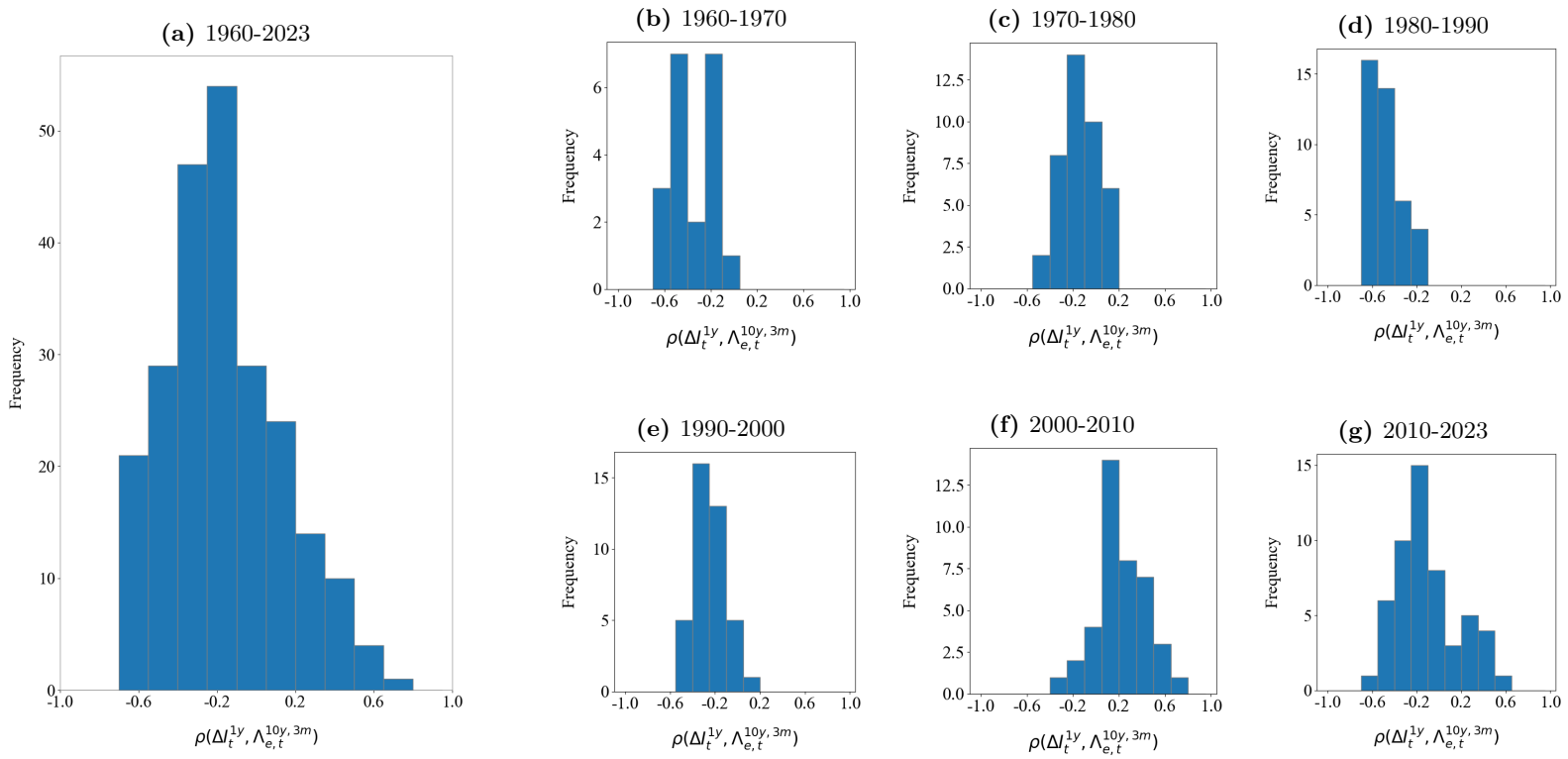


Figure 2: Correlation Coefficient Histogram Between Annual Investment Growth Rate and the Equity Return Premium Extracted From the VAR(1) Model. This figure shows the histogram of the correlation coefficient between the annual investment growth rate and the equity return premium, denoted as $\rho(\Delta I_{t+p}^{1y}, \Lambda_{e,t}^{10y,3m})$ and measured in quarterly rolling windows, following Equation 18. The annual investment growth rate, ΔI_{t+p}^{1y}, is measured at time $t+p$, where $p = 5$ quarters. The subscript p is the VAR model order obtained in Section 4.2.1. The equity return premium, $\Lambda_{re,t}^{10y,3m}$, is the 10-year equity return premium on the 3-month equity return of the term structure of equity extracted from the VAR(1) process at time t.

Table 1: VAR Descriptive Statistics For ΔI_t^{1y}. This table presents the vector of autoregression results of the investment growth rate, ΔI_t^{1y}. The lagging variables are the investment growth rate, ΔI_{t-p}^{1y} and the equity return premium, $\Lambda_{re,t-p}^{10y,3m}$. p is the lag length and $p = 1, 2, ..., n$. The sample period is from 1960-Q1 to 2023-Q1. The significance of the t-statistics is reported in parentheses. The three stars symbol ** indicates significance at the 1% confidence level. The autoregressive model follows Equation 14. The last two columns correspond to the Bayesian Information Criterion (BIC) and the Forecast Error Variance Decomposition (FEVD).

Period	observations	Lag	Coefficient	Standard Error	t-stat	BIC	FEVD
1960-2023	248	5	-0.0307	0.01	-2.915**	-8.42	0.342

(a) VAR Model Order Structure Results of Bayesian Information Criterion (BIC)

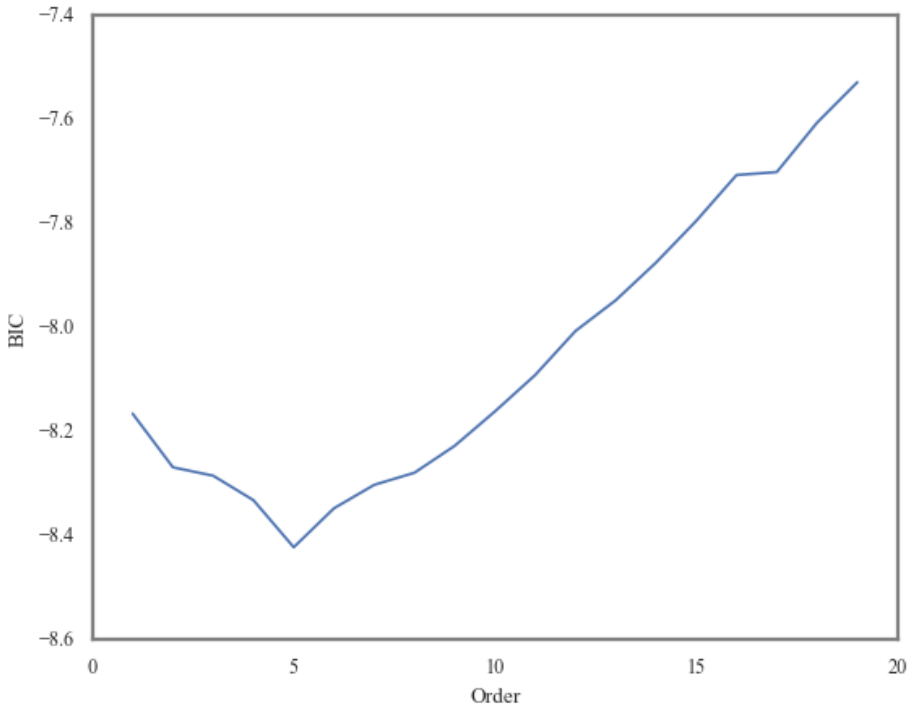


Figure 3: VAR Model Order Structure Results - Bayesian Information Criterion. This figure shows the vector of the autoregression model order structure based on the Bayesian Information Criterion (BIC) of the investment growth rate, $\Lambda_{e,t}^{10y,3m}$. The lagging variables are the investment growth rate, ΔI_{t-p}^{1y}, and the equity return premium estimated from the VAR(1) model, $\Lambda_{e,t-p}^{10y,3m}$. Figure 3 presents the negative values of the BIC for each possible prediction from Equation 14. The Bayesian Information Criterion (BIC) is a fitting measurement described in Equation 15, and it is commonly used to select the model type and the VAR model order (p). Lower values of the BIC indicate a better fit of the vector of the autoregression model.

(a) US Recessions, Annual Investment Growth Rate, and the Term Premium Estimated Return

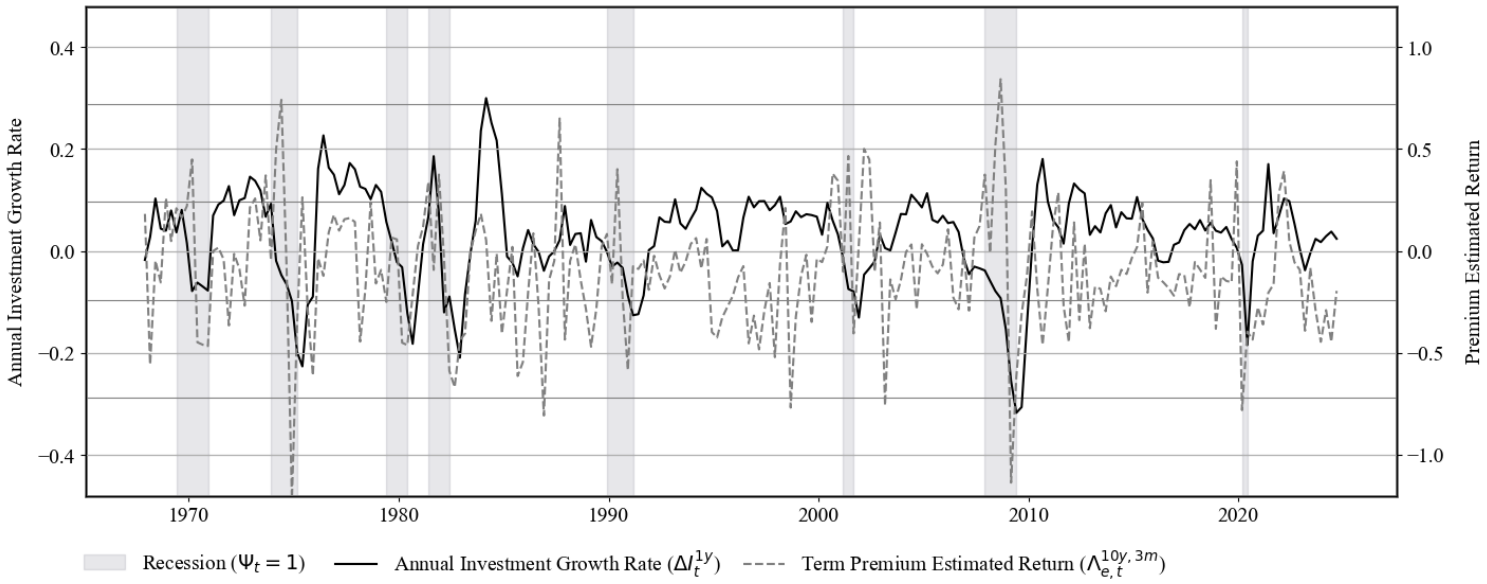


Figure 4: Time Series of U.S. Recessions, Investment Growth Rates, and Equity Return Premium from the VAR(1) model. This figure presents the time series of the recession indicator index, the annual investment growth rate, and the equity return premium estimated from the VAR(1) model from 1968:Q4 until 2023:Q2. The annual Investment Growth Rate, ΔI_t^{1y}, follows Equation 12 and is measured from the quarterly Gross Private Domestic Investment from the U.S. Bureau of Economic Analysis and retrieved from FRED of the Federal Reserve Bank of St. Louis. The equity return premium, $\Lambda_{re,t}^{10y,3m}$, follows Equation 11 and is the difference between the 10-year and the 3-month equity returns estimated from the VAR(1) model. The recession indicator index is retrieved from FRED, Federal Reserve Bank of St. Louis.[8]

Table 2: Number of Downward- and Upward-Sloping Curves Across Business and Investment Cycles. This table documents the upward- and downward-sloping term structure of equity returns across business and investment cycles from 1968:Q4 until 2023:Q2. The slope of the curve is measured using the equity return premium extracted from the VAR(1) model in quarterly rolling windows. Downward-sloping curves are associated with negative equity return premiums ($\Lambda_{e,t}^{10y,3m} < 0$), and upward-sloping curves are associated with positive equity return premiums ($\Lambda_{e,t}^{10y,3m} > 0$). Positive and negative equity return premiums are subdivided when the business cycle is in recession ($\Psi_t = 1$) and when it is not ($\Psi_t = 0$). The business cycle, on the other side, is subdivided when investment growth rates are positive ($\Delta I_{t+p}^{1y} > 0$) and when they are negative ($\Delta I_{t+p}^{1y} < 0$). The investment growth is measured at time $t+p$, where $p = 5$ quarters. The time series data of the equity return premium from the VAR(1) model, investment growth rate, and economic index are retrieved according to Section 4.1. The equity return premium from the VAR(1) model follows Equation 11, and it is measured on quarterly rolling windows at the economic index reporting date. The annual investment growth rates are measured five quarters after the measurement date of the equity return premium.

Term Structure Slope	Recession ($\Psi_t = 1$)			Out of Recession ($\Psi_t = 0$)		
	$\Delta I_{t+p}^{1y} < 0$	$\Delta I_{t+p}^{1y} > 0$	Total	$\Delta I_{t+p}^{1y} < 0$	$\Delta I_{t+p}^{1y} > 0$	Total
Downward ($\Lambda_{e,t}^{10y,3m} < 0$)	5	17	22	25	97	122
Upward ($\Lambda_{e,t}^{10y,3m} > 0$)	16	3	19	22	33	55

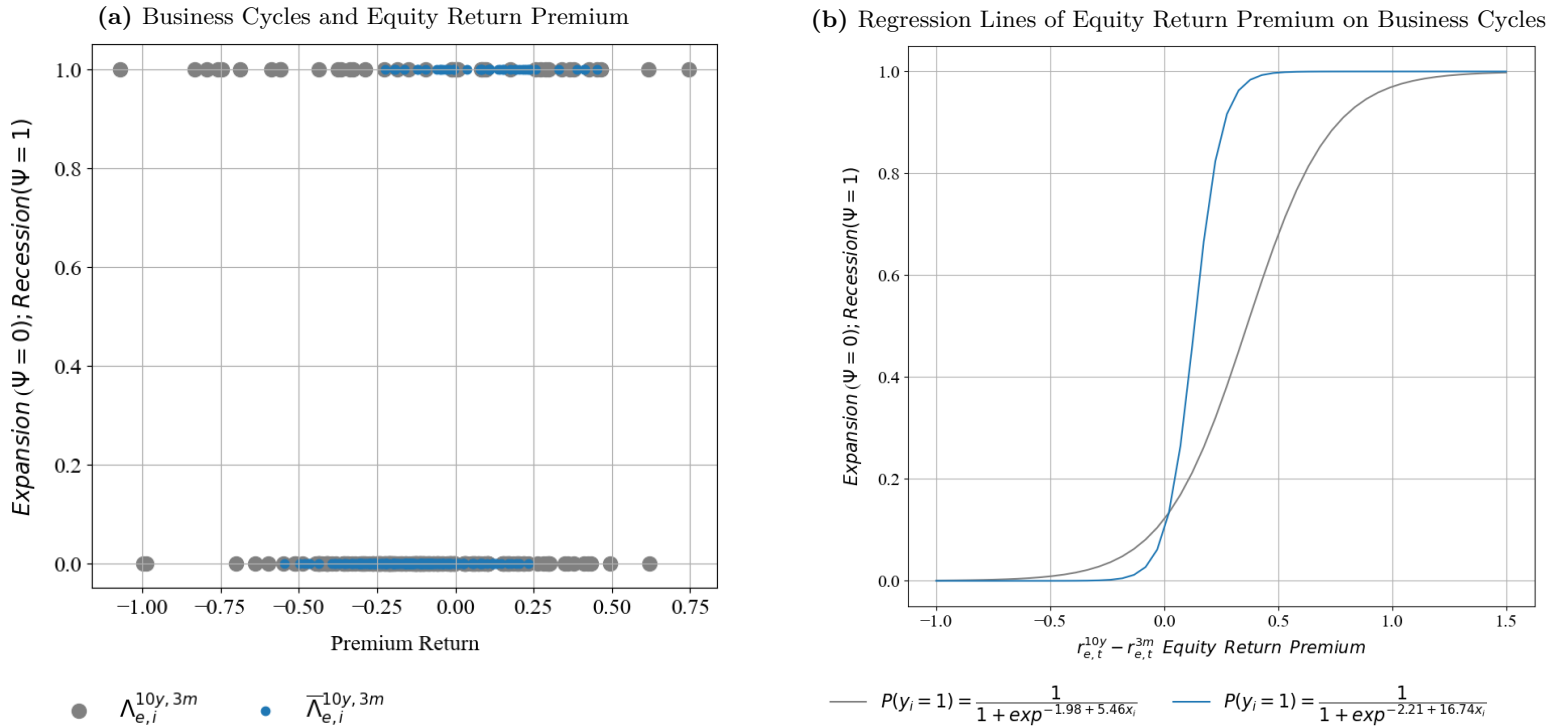


Figure 5: Economic Cycles Drive the Change in the Slope of the Term Structure of Equity Returns. Panel 5a shows the scatter plots of the equity return premium, $\Lambda_{e,t}^{10y,3m}$, its average, $\overline{\Lambda}_{e,t}^{10y,3m}$, and the recession indicator, Ψ, using data from 1968:Q4 to 2023:Q2. Blue dots represent equity return premiums, and gray dots represent the one-year average of equity return premiums. Panel 5a shows the results of two logit regression lines: the gray line is recession indicator, Ψ_t on the estimated equity return premium, $\Lambda_{e,t}^{10y,3m}$; the blue line is the recession indicator, Ψ_t, on the average estimated equity return premium, $\overline{\Lambda}_{e,t,1}^{10y,3m}$. The regression sample is quarterly from 1968:Q4 to 2023:Q2. The recession indicator is denoted by Ψ and is a binary variable. The term structure of expected equity returns is estimated on a quarterly rolling window, using the stock market data available one month earlier than the release date of the data used by the Recession Indicator. The variable $\Lambda_{re,t}^{10y,3m}$ follows Equation 11 and is the 10-year equity return premium on the 3-month equity return extracted from the VAR(1) model. The variable $\overline{\Lambda}_{re,t,1y}^{10y,3m}$ is the annual average of the equity return premium.

Table 3: Predicting Investment Cycles Using the Term Structure of Expected Equity Returns. This table presents the unconditional and conditional regression results of the annual investment growth rate on the equity return premium from the VAR(1) model. The table includes the coefficient parameters, constant parameters, and the coefficient of determination. t-statistics are in parentheses. The conditional regression follows Equation 17 and the unconditional regression follows Equation 16. The unconditional regression takes any equity return premium regardless of the stage of the business cycle. The conditional regression takes any negative value of the equity return premium when the economy is not in recession such as $\Psi_t = 0$, flooring it at zero when it is positive. Conversely, it takes any positive value of the equity return premium when the economy is in recession, such as $\Psi_t = 1$, flooring it at zero when it is negative. The regression sample covers the period from 1968:Q4 until 2023-Q1 on a quarterly basis. The first column of Table 3 presents the unconditional regression results. The second column presents the conditional regression results when the economy is not in recession, and the third column presents the results when it is in recession.

	$E(\Delta I^{1y}_{t+5})$	$E(\Delta I^{1y}_{t+5} \mid \Psi_i = 0)$	$E(\Delta I^{1y}_{t+5} \mid \Psi_i = 1)$
$\Lambda^{10y3m}_{t,\Psi_i}$	-0.13**		
	(0.02)		
$\Lambda^{10y3m}_{t,\Psi_i} = 1$		-0.07**	
		(0.02)	
$\Lambda^{10y3m}_{t,\Psi_i} = 0$			-0.22**
			(0.04)
Constant	0.01*	0.03**	-0.05**
	(0.01)	(0.01)	(0.02)
Test	Unconditional	Conditional to Expansion	Conditional to Recession
Obs	218	177	41
Adj. R^2	0.17	0.05	0.47

(*) indicates significance at the 5% confidence level
(**) indicates significance at the 1% confidence level

Appendices

A VAR Models Alternative Variables Results

This appendix extends Section 4.2 to evaluate the contribution of alternative state variables to the joint autoregressive process of annual investment growth and the equity return premium, as extracted from the VAR(1) model.

The equity return premium follows the definitions provided in Section 3.1. Previous research has identified other than the dividend yield as equity return predictors, such as the short-term interest rate and the yield spread between long-term and short-term bonds.[9] In addition to these state variables, this appendix incorporates the Consumer Sentiment Index as a forecasting variable. Starting with the dividend yield, the other three combinations of state variables are as follows:

- The dividend yield (d_y).
- The dividend yield and the consumer sentiment index (d_y, c_s).
- The short-term rate, the dividend yield, and the yield spread (r_0, d_y, s_0).
- The short-term rate, the dividend yield, the yield spread, and the consumer sentiment index (r_0, d_y, s_0, c_s).

The short-term nominal interest rate is the 3-month T-Bill. The yield spread is the difference between the log yield on a zero-coupon 5-year Treasury bond and the yield on a 3-month T-Bill. The Standard and Poor's Index (S&P), the S&P dividend yield, and the 5-year Treasury bond data are retrieved from Robert Shiller's website.[10] The 3-month T-Bill[11] and the Consumer Sentiment Index[12] are retrieved from the Federal Reserve Bank of St. Louis.

B VAR Model Results

Equation 14 defines investment growth as a linear function of the lag value of its own and the equity return premium. The data-generating process of the VAR model includes the annual investment growth rate (ΔI_t^{1y}) and the 10-year equity return premium on the 3-month equity return. The

[9] See Campbell and Shiller (1986), Campbell (1991), and Campbell and Viceira (2005)

[10] Please refer to http://www.econ.yale.edu/ shiller/data.htm

[11] Board of Governors of the Federal Reserve System (U.S.), 3-Month Treasury Bill Secondary Market Rate, Discount Basis [TB3MS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/TB3MS

[12] U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCNS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CPIAUCNS, June 20, 2022.

equity return premium from the VAR(1) model, ${}^{i}\Lambda_{re,t}^{10y,3m}$, uses alternative state variables defined at the introduction of this section and differentiated with the superscript i, such as: ${}^{1}\Lambda_{e,t-p}^{10y,3m} = f(d_y)$, ${}^{2}\Lambda_{e,t-p}^{10y,3m} = f(d_y, c_s)$, ${}^{3}\Lambda_{e,t-p}^{10y,3m} = f(r_0, d_y, s_0)$, and ${}^{4}\Lambda_{e,t-p}^{10y,3m} = f(r_0, d_y, s_0, c_s)$.

Table 4 presents the estimated VAR(5) model for the investment growth rate using data from 1960:Q1 to 2023:Q2. The model contains one constant term, the investment growth rate, and the equity return premium. The VAR model results show that the statistic parameters are significant at five quarters lag when the equity return premium is estimated using the dividend yield as forecasting variables, such ${}^{1}\Lambda_{e,t-p}^{10y,3m}$, or the dividend yield and the consumer sentiment index, such ${}^{2}\Lambda_{e,t-p}^{10y,3m}$. The VAR results are not significant at five quarter lags when the equity return premium is estimated using the short-term rate, dividend yield, and yield spread as forecasting variables, such ${}^{3}\Lambda_{e,t-p}^{10y,3m}$, or the short-term rate, dividend yield, yield spread, and consumer sentiment index, such ${}^{4}\Lambda_{e,t-p}^{10y,3m}$.

C Bayesian Information Criterion Results

Figure 6 shows four Vector of Autoregression (VAR) model order structures based on the Bayesian Information Criterion (BIC). The general approach consisted of fitting the VAR(p) model with orders $p = 0, 1, \ldots p^{max}$.

The results show that the BIC is minimized at the fifth-quarter lag for all cases. The lowest BIC occurs when the VAR investment growth rate is estimated using the equity return premium forecasted with the dividend yield. The second-lowest BIC occurs when the dividend yield and the consumer sentiment index are the forecasting variables for the estimated equity return premium. The third-lowest BIC occurs when the state variables for the return premium are the short-term rate, the dividend yield, and the yield spread. The fourth lowest is when the short-term rate, the dividend yield, the yield spread, and the consumer sentiment index are used as state variables. The results show that the marginal contribution among the state variables is not material, and the optimal lag order is consistent with that presented in Appendix **??**.

D Forecast Error Variance Decomposition Results

The Forecast Error Variance Decomposition (FEVD) measures the contribution of the shocks for a given variable affecting the forecast error variance of the model or the response of the variables that compose the VAR(p) model.

Table 5 and 6 present the results of the FEVD, including ten quarters of the forecast. Table 5 shows the change in the investment growth rate due to its own shocks and the shocks of the estimated equity return premium using four alternative state variables. The results show that when the equity return premium is estimated using the dividend yield as state variable, such ${}^{1}\Lambda_{e,t-p}^{10y,3m}$, accounts for 27.9%, 35.7% and 36.2% at the 5^{th}, 7^{th} and 10^{th} lag, respectively. The highest contribution is observed when the equity return premium is estimated using the dividend yield and the consumer sentiment index as state variables, ${}^{2}\Lambda_{e,t-p}^{10y,3m}$, accounting for 28.4%, 38.1% and 38.7% at the 5^{th}, 7^{th} and 10^{th} lag, respectively. The lowest contribution occurs when the equity return premium is estimated using the group of state variables defined by short-term rate, dividend yield, and yield spread, such ${}^{3}\Lambda_{e,t-p}^{10y,3m}$, and the short-term rate, dividend yield, yield spread, and consumer sentiment index, such ${}^{4}\Lambda_{e,t-p}^{10y,3m}$. For all the cases, the influence of each variable on the changes in the annual investment growth rates is increasing and stabilizes at the 8^{th} quarter lag. From the results presented in Table 6, the error in the forecast of the annual investment growth attributed to the equity return premium can be as low as 34.6% or as high as 38.7%, depending on the group of state variables used in tables 5 and 6 to estimate the equity return premium.

Table 6 shows the change in the estimated equity return premium due to its own shocks and the shocks in the annual investment growth rates. In this case, the results show that the shock of the change in the annual investment growth rates has a limited implication for the error of the equity return premium forecast, regardless of the state variable used to estimate the equity return premium.

Table 4: Estimated VAR model for the annual investment growth rate. This table presents the vector of autoregression results of the joint dynamics of the investment growth rate and the equity return premium estimated from the VAR(1) model. Each panel presents the same regression model, but it uses a different combination of individual or a group of state variables to estimate the equity return premium. The sample data covers the period from 1960:Q1 to 2023:Q2.

(a) VAR(5) Investment Growth Rate - ΔI_t^{1y}: Equity return premium $\left({}^{1}\Lambda_{e,t-p}^{10y,3m}\right)$ estimated using the dividend yield as state variable

	Coefficients on lagged variables (p)				
	1	2	3	4	5
Investment Growth Rate - ΔI_{t-p}^{1y}	0.904	-0.086	0.103	-0.471	0.307
	(15.224)**	(-1.135)	(1.384)	(-6.439)**	(5.749)**
Equity Return Premium - ${}^{1}\Lambda_{e,t-p}^{10y,3m}$	0.013	-0.03	-0.038	-0.028	-0.031
	(1.302)	(-3.009)**	(-3.675)**	(-2.642)**	(-2.915)**

(b) VAR(5) Investment Growth Rate - ΔI_t^{1y}: Equity return premium $\left({}^{2}\Lambda_{e,t-p}^{10y,3m}\right)$ estimated using the dividend yield and consumer sentiment as state variables

	Coefficients on lagged variables (p)				
	1	2	3	4	5
Investment Growth Rate - ΔI_{t-p}^{1y}	0.894	-0.084	0.102	-0.458	0.305
	(15.139)**	(-1.119)	(1.383)	(-6.295)**	(5.783)**
Equity Return Premium - ${}^{2}\Lambda_{e,t-p}^{10y,3m}$	0.016	-0.028	-0.035	-0.024	-0.032
	(1.795)	(-2.865)**	(-3.533)**	(-2.406)*	(-3.276)**

(c) VAR(5) Investment Growth Rate - ΔI_t^{1y}: Equity return premium $\left({}^{3}\Lambda_{e,t-p}^{10y,3m}\right)$ estimated using the short-term rate, dividend yield, and yield spread as state variables

	Coefficients on lagged variables (p)				
	1	2	3	4	5
Investment Growth Rate - ΔI_{t-p}^{1y}	0.916	-0.095	0.1	-0.48	0.317
	(15.362)**	(-1.238)	(1.328)	(-6.516)**	(5.902)**
Equity Return Premium - ${}^{3}\Lambda_{e,t-p}^{10y,3m}$	0.015	-0.026	-0.035	-0.025	-0.027
	(1.552)	(-2.565)**	(-3.379)**	(-2.377)*	(-2.581)**

(d) VAR(5) Investment Growth Rate - ΔI_t^{1y}: Equity return premium $\left({}^{4}\Lambda_{e,t-p}^{10y,3m}\right)$ estimated using the short-term rate, dividend yield, yield spread, and consumer sentiment as state variables

	Coefficients on lagged variables (p)				
	1	2	3	4	5
Investment Growth Rate - ΔI_{t-p}^{1y}	0.908	-0.095	0.098	-0.469	0.318
	(15.306)**	(-1.25)	(1.309)	(-6.4)**	(5.98)**
Equity Return Premium - ${}^{4}\Lambda_{e,t-p}^{10y,3m}$	0.02	-0.024	-0.032	-0.021	-0.028
	(2.16)*	(-2.453)*	(-3.195)**	(-2.117)*	(-2.861)**

(*) indicates significance at the 5% confidence level
(**) indicates significance at the 1% confidence level

(a) VAR Model Order Structure Using Different State Variables

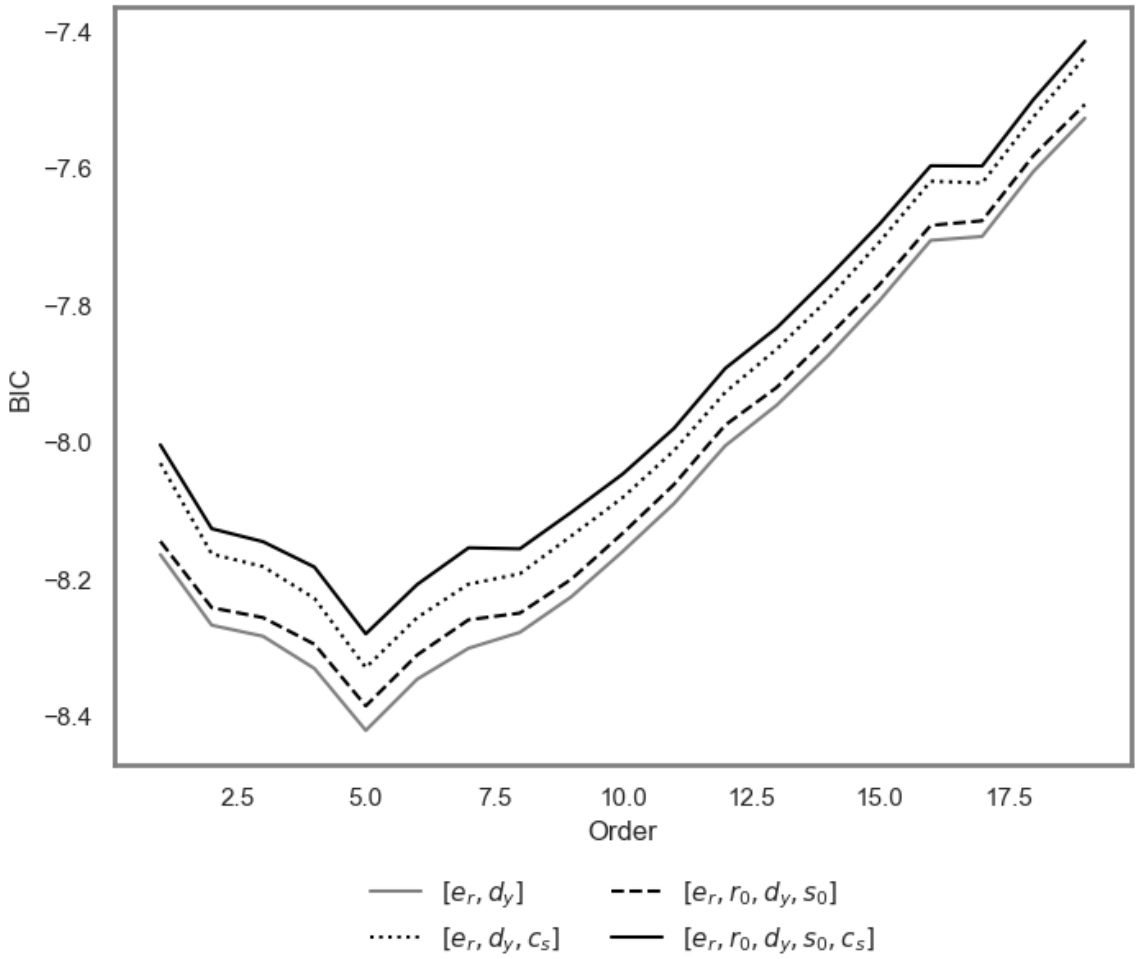


Figure 6: VAR Model Order Structure Results of Bayesian Information Criterion (BIC). Figure 6 shows the Vector of Autoregression (VAR) model order structure based on the Bayesian Information Criterion (BIC). The data-generating process considered the VAR model with the annual Investment Growth Rate and the 10-year and 3-month equity return premium using the equity return (e_r) and an individual or a group of state variables such as the short-term rate (r_0), dividend yield (d_y), yield spread (s_0), and consumer sentiment index (c_s) as state variables. The sample period is from 1960:Q1 to 2023:Q2. The general approach consisted of fitting the VAR(p) model with orders $p = 0, 1, \dots p^{max}$ and choosing the value of p (lag), which minimizes the model selection criteria based on the BIC. The BIC is an information criterion that identifies the best-fitting VAR model order by maximizing the likelihood of the data and penalizing the number of parameters of the model, preventing the model from being overfitted to the data.

Table 5: Forecast Error Variance Decomposition (FEVD) - Annual Investment Growth. Table 5 presents the FEVD results showing the proportion of the forecast error variance of the annual investment growth rate that is explained by its own shocks and the equity return premium. The analysis of the FEVD uses a forecast horizon of ten quarters, based on data from 1960:Q1 to 2023:Q2. The FEVD results includes four different categories that uses the shocks of the equity return premium when is estimated using different state variables, such as the short-term rate (r_0), the dividend yield (d_y), the yield spread (s_0), and the consumer sentiment index (c_s).

Forecast Error Variance Decomposition (FEVD) - Annual Investment Growth

Quarter	ΔI_t^{1y}	${}^{1}\Lambda_{e,t}^{10y,3m}$	Step	ΔI_t^{1y}	${}^{2}\Lambda_{e,t}^{10y,3m}$	Step	ΔI_t^{1y}	${}^{3}\Lambda_{e,t}^{10y,3m}$	Step	ΔI_t^{1y}	${}^{4}\Lambda_{e,t}^{10y,3m}$
0	1.000	0.000	0	1.000	0.000	0	1.000	0.000	0	1.000	0.000
1	0.996	0.004	1	0.993	0.007	1	0.995	0.005	1	0.989	0.011
2	0.992	0.008	2	0.993	0.007	2	0.995	0.005	2	0.992	0.008
3	0.940	0.060	3	0.946	0.054	3	0.958	0.042	3	0.964	0.036
4	0.845	0.155	4	0.852	0.148	4	0.881	0.119	4	0.893	0.107
5	0.721	0.279	5	0.716	0.284	5	0.767	0.233	5	0.771	0.229
7	0.643	0.357	7	0.619	0.381	7	0.682	0.318	7	0.665	0.335
10	0.638	0.362	10	0.613	0.387	10	0.674	0.326	10	0.654	0.346

Notes:

- ${}^{1}\Lambda_{e,t}^{10y,3m}$: Equity return premium estimated using dividend yield as a state variable.
- ${}^{2}\Lambda_{e,t}^{10y,3m}$: Equity return premium estimated using dividend yield and consumer sentiment index as state variables.
- ${}^{3}\Lambda_{e,t}^{10y,3m}$: Equity return premium estimated using short-term rate, dividend yield, and yield spread as state variables.
- ${}^{4}\Lambda_{e,t}^{10y,3m}$: Equity return premium estimated using short-term rate, dividend yield, yield spread, and consumer sentiment index as state variables.

Table 6: Forecast Error Variance Decomposition (FEVD) - Equity Return Premium. Table 6 presents the FEVD results showing the proportion of the forecast error variance of the equity return premium that is explained by its own shocks and the investment growth rate. The analysis of the FEVD uses a forecast horizon of ten quarters, based on data from 1960:Q1 to 2023:Q2. The FEVD results includes four different categories that uses the shocks of the equity return premium when is estimated using different state variables, such as the short-term rate (r_0), the dividend yield (d_y), the yield spread (s_0), and the consumer sentiment index (c_s).

Forecast Error Variance Decomposition (FEVD) - Estimated Equity Return Premium

Quarter	ΔI_t^{1y}	${}^{1}\Lambda_{e,t}^{10y,3m}$	Step	ΔI_t^{1y}	${}^{2}\Lambda_{e,t}^{10y,3m}$	Step	ΔI_t^{1y}	${}^{3}\Lambda_{e,t}^{10y,3m}$	Step	ΔI_t^{1y}	${}^{4}\Lambda_{e,t}^{10y,3m}$
0	0.003	0.997	0	0.001	0.999	0	0.006	0.994	0	0.004	0.996
1	0.015	0.985	1	0.015	0.985	1	0.025	0.975	1	0.025	0.975
2	0.031	0.969	2	0.035	0.965	2	0.044	0.956	2	0.050	0.950
3	0.036	0.964	3	0.040	0.960	3	0.050	0.950	3	0.056	0.944
4	0.036	0.964	4	0.040	0.960	4	0.051	0.949	4	0.056	0.944
5	0.037	0.963	5	0.041	0.959	5	0.051	0.949	5	0.057	0.943
7	0.039	0.961	7	0.043	0.957	7	0.053	0.947	7	0.057	0.943
10	0.040	0.960	10	0.043	0.957	10	0.053	0.947	10	0.058	0.942

Notes:

- ${}^{1}\Lambda_{e,t}^{10y,3m}$: Equity return premium estimated using dividend yield as a state variable.
- ${}^{2}\Lambda_{e,t}^{10y,3m}$: Equity return premium estimated using dividend yield and consumer sentiment index as state variables.
- ${}^{3}\Lambda_{e,t}^{10y,3m}$: Equity return premium estimated using short-term rate, dividend yield, and yield spread as state variables.
- ${}^{4}\Lambda_{e,t}^{10y,3m}$: Equity return premium estimated using short-term rate, dividend yield, yield spread, and consumer sentiment index as state variables.

* * *