Price Distortion Created by Self-preferencing and Its Impact on Welfare*

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Abstract

We study how a platform that also sells its own product allocates visibility between itself and a competing third-party seller in a duopoly with consumer search frictions. Some consumers compare both sellers' prices, while the rest buy only from the first-listed seller. The platform chooses the probability that its own product appears first and charges the third party a commission fee. In the pricing stage, both sellers play mixed strategies, randomizing over a range of prices to balance profit margins against the likelihood of attracting consumers who compare prices. By making the ranking decision endogenous, we show that differences in cost structure alone can reverse the platform's incentives to favor its own product. Increasing the platform's visibility can either raise or lower prices across the market, depending on which seller's pricing constraint binds: greater platform visibility can lead both sellers to raise prices when the platform is constrained, or both to lower prices when the third party is constrained. The platform's optimal ranking reflects a trade-off between the gain from securing more non-searcher demand and the effect of visibility on market-wide prices. We identify the combinations of relative costs, consumer search behavior, and commission fees under which self-preferencing benefits or harms consumers. We show that regulating toward ranking neutrality can be misleading because a platform may optimally place a third-party seller at the top of search results when that seller has a substantial cost advantage, in order to preserve its overall profit margin. Notably, this outcome can arise even when the commission fee is zero.

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1 Introduction

In models of duopoly, firms compete with each other to attract customers. The outcome depends crucially on which variables firms choose to compete over. One standard model, first

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analyzed by Edgeworth, has firms competing by setting prices. The traditional assumption is that all customers purchase from the firm offering the lower price. In effect, all consumers search to find the cheapest seller. An important departure comes from Varian (1980), who introduces two types of consumers: those who search and buy from the lowest-priced seller (searchers), and those who do not search and simply buy from one seller regardless of price (non-searchers). Varian (1980) assumes that the division of the non-searchers is exogenously given, with half buying from each firm. Golding and Slutsky (1999) extend Varian (1980) to allow the division of the non-searchers to be anything, not just equal division. They continue to assume that this division is exogenously given.[1] However, in many realistic cases, the division of non-searchers is not exogenous but is instead endogenously determined by one of the firms. An example of this is when one firm owns a platform and sells a product on the platform in competition with another firm that it allows on the platform.[2] In this case, there are consumers who search and buy from the low-price seller and non-searchers who buy from one of the sellers independent of prices. Now, which firm non-searchers buy from is not exogenous but depends on which firm is listed first, something decided by the platform owner. When the platform ranks its own products more prominently than its rivals'—a practice known as self-preferencing—it can increase its share of non-searchers. In this paper, we endogenize this division: the platform chooses how to allocate non-searchers by deciding the display order of sellers. We show that the platform's optimal allocation of non-searchers need not involve ranking itself first. The platform may prefer to let the third-party seller be listed first, foregoing sales to non-searchers, when doing so leads to a more favorable pricing equilibrium.

We consider a duopoly in which one seller is also the platform owner and the other is an independent third-party seller. Both offer identical products but may face different marginal costs. The platform charges the third-party seller a per-transaction commission fee, which raises the third party's effective cost. Consumers differ in their search behavior: some compare both sellers' prices and buy from the cheaper one (searchers), while others buy only from the seller listed first in the search results (non-searchers). The platform chooses how often its own product appears first in the ranking. At one extreme, it always lists itself first (self-preferencing); at the other, it always lists the third party first; between these extremes, it can adopt an intermediate, partially preferential ranking. The timing is as follows. First, the cost and search conditions are realized and known to both firms and consumers, and the commission fee is fixed. Next, the platform selects the probability of placing its own product first in the rankings. Then both sellers set their prices simultaneously. Finally, searchers buy from the cheaper seller, while non-searchers buy from whichever seller is ranked first. Because pricing affects demand from both groups, neither firm settles on a single fixed price in equilibrium. Instead, as in Varian (1980) and Golding and Slutsky (1999), firms randomize prices over a range that balances the higher profit margin from charging more against the risk of losing searchers to the rival. The lowest price in each firm's range is determined by the point at which serving both its own non-searcher base and all searchers would earn no

[1] As in Edgeworth, firms play mixed strategies in equilibrium in Varian (1980). Varian (1980) shows that when non-searchers are evenly split, the equilibrium pricing distributions are continuous. Golding and Slutsky (1999) show that with asymmetric non-searcher shares, equilibrium pricing can involve atoms at the monopoly price.

[2] For example, Amazon does this when it sells items in competition with firms on its platform.

less than simply charging the monopoly price to its non-searchers alone.

We show that the platform's profit-maximizing ranking does not always involve placing its own product first. How the optimal ranking changes depends on which seller is more constrained in lowering prices.

When the platform is the one setting the lowest price it can profitably offer, giving it more visibility increases its base of non-searchers who will buy from it regardless of price. With this larger guaranteed demand, the platform faces less pressure to compete for searchers and can profitably raise its price without losing many customers. The rival then finds it profitable to raise its own price as well, since it no longer needs to compete as aggressively for searchers. This coordinated increase pushes up market-wide prices. In contrast, when the third-party seller is the one setting the lowest viable price, giving the platform more visibility forces the rival to lower its price to keep searchers, which in turn pushes the platform to lower its price too. In this case, greater platform visibility lowers average prices. The platform's ranking choice therefore involves a trade-off between two effects. On one hand, appearing first increases its guaranteed sales from non-searchers, which supports higher profits even without price cuts. On the other hand, changing the ranking also alters the rival's pricing incentives, which can either lead both sellers to raise their prices or both to lower them, depending on which seller's pricing constraint is binding. The platform chooses the ranking that maximizes profit by weighing the extra non-searcher sales against the profit impact of the resulting shift in market-wide prices. By allowing the ranking probability to vary continuously, we identify the combinations of relative marginal costs, the share of consumers who compare prices, and commission fees under which the platform prefers to always place itself first, to always place the rival first, or to adopt an intermediate ranking and when each choice benefits or harms consumers.

The analysis highlights that regulating ranking practices in online marketplaces can have unintended consequences. Platforms such as Amazon both operate the marketplace and sell their own products, giving them control over how prominently each seller appears in search results. This visibility can be used to favor their own products, which has drawn regulatory attention as potentially anti-competitive. Our results show that the welfare effects of self-preferencing depend critically on the market environment. In some parameter regions, increasing the platform's visibility causes both sellers to raise prices, reducing consumer surplus; in others, it induces both sellers to lower prices, benefiting consumers. The platform's optimal choice of ranking balances the sales from more non-searchers against the effect of the ranking on market-wide prices. This means that restricting self-preferencing does not guarantee improved outcomes for consumers and can even work in the platform's favor, depending on search behavior, cost asymmetries, and commission fees.

This paper extends classic models of price dispersion with search frictions, such as Varian (1980) and Golding and Slutsky (1999), by making the platform's ranking decision an endogenous choice and introducing a commission fee that can affect the two sellers differently. By endogenizing the ranking choice, we show that differences in cost structure alone can alter the platform's incentives to favor its own products, and that regulating toward search neutrality can be misleading even without considering the commission fee, often regarded as a long-term, more stable feature of the market (see Kittaka and Sato (2022)). We allow for cost differences between the platform and the third-party seller and examine how these interact with ranking and commission fees to shape pricing incentives. One advantage of

our framework is that the pricing stage features mixed strategy equilibria, which align well with the realities of digital markets where sellers often rely on algorithmic pricing tools that adjust prices dynamically in response to competitors. This approach yields explicit characterizations of when the platform prefers to list itself first, when it prefers to list the rival first, and when it chooses an interior ranking. We also show how these ranking choices affect consumer welfare, identifying the parameter combinations of relative costs, the share of consumers who compare prices, and the commission fee under which self-preferencing benefits or harms consumers. The results contribute to the literature on platform competition by demonstrating that the welfare effects of self-preferencing depend not only on whether it occurs, but also on how market structure interacts with consumer search behavior.

1.1 Literature Review

Self-preferencing by digital platforms, where a platform favors its own products over those of third-party sellers, has been widely studied in the fields of industrial organization and digital market design. A foundational survey by Kittaka, Sato, and Zennyo (2023) classifies the literature into two main strands: one focused on the effects of vertical integration between platforms and sellers, and the other on self-preferencing. They further divide self-preferencing into two forms: manipulation of search and ranking algorithms, and the use of third-party seller data. Our paper examines self-preferencing from the perspective of search and ranking algorithm manipulation and identifies how differences in seller technology influence its impact on seller's pricing strategy and consumer surplus.

A core mechanism of self-preferencing is the manipulation of search rankings. Zou and Zhou (2025) study a model of duopoly with differentiated products and show that search neutrality can weaken price competition. In their setting, consumers are of two types: one group prefers the platform's product, while the other prefers the third party product. Search neutrality may harm consumers if a large share of them initially prefers third party products. Kittaka and Sato (2022) study the effects of self-preferencing and show that its impact on welfare depends on commission rates. In particular, prohibiting self-preferencing may reduce welfare and consumer surplus when commission rates are high. In their model, products are differentiated along a Hotelling line. Consumers first search the platform and then the third-party seller, incurring an additional search cost under self-preferencing. Under search neutrality, consumers initially visit one seller at random and may visit the other by incurring a search cost. Zennyo (2022) also studies how ranking manipulation affects consumer welfare and seller profits in the long run. In this model, self-preferencing increases the likelihood that consumers participate on the platform, thereby generating an indirect network effect. When such an effect arises under certain conditions, the platform may reduce commission fees, which softens price competition and improves consumer surplus. Zou and Zhou (2025), Kittaka and Sato (2022), and Zennyo (2022) assume zero marginal production costs and that only the third-party seller pays an exogenous commission fee. As a result, the third-party seller's marginal cost is equal to the commission rate, while the platform's marginal cost remains zero. In contrast, in our paper, products are identical, but we show that search neutrality can still harm consumers when the platform has a cost advantage and many consumers face low search costs. Moreover, our model allows the platform's marginal cost and the commission rate to differ, which means the platform's marginal cost can be higher.

Thus, in our paper, a higher commission fee does not necessarily lead to higher prices. We identify conditions under which, even with a zero or low commission rate, regulating self-preferencing could still harm consumers. Additionally, rather than comparing only two cases—where the probability of being ranked first is either 1 (self-preferencing) or 1/2 (search neutrality)—we model this probability as a continuous parameter between 0 and 1. This approach allows us to identify conditions under which self-preferencing benefits or harms consumers.

Dendorfer (2024) presents another study of a duopoly model with differentiated products. They examine how platform entry affects market outcomes and show that entry can increase both consumer welfare and platform revenue by removing double marginalization. They also analyze a case where the platform offers an identical product alongside multiple third-party sellers, when the number of sellers is at least three. They find that in a pure strategy equilibrium, both the platform and the seller offering the identical product price at marginal cost, while other sellers charge above marginal cost and compete less aggressively. In their model, the platform reduces commission fees under self-preferencing, which leads the third-party seller to lower prices. However, this results in lower platform revenue and reduced consumer welfare under self-preferencing because a smaller number of consumers participate in the market. In contrast, we show that when many consumers have high search costs and the platform either has a cost disadvantage or a large cost advantage, the platform prefers self-preferencing. In this case, consumer welfare is likely lower under self-preferencing.

Another dimension of self-preferencing arises through the platform's use of proprietary third-party data. Jiang, Jerath, and Srinivasan (2011) model how platforms can strategically enter profitable product categories by monitoring third-party seller performance. While this deters entry and investment by sellers, it may also incentivize platforms to reduce commission fees to maintain a viable marketplace. Related work by Madsen and Vellodi (2025) and Hervas-Drane and Shelegia (2025) explores how different regulatory regimes affect innovation incentives. They find that unconditional bans on data usage may hinder innovation, while targeted mechanisms such as temporary data protections can preserve seller participation and promote efficient entry. In contrast to these papers, this paper examines self-preferencing through the lens of search and ranking algorithm manipulation.

Our paper relates to the literature that examines how dual role platforms affect platform profitability and consumer welfare. Hagiu and Spulber (2013) introduce the idea that platform owned retail can help jumpstart network effects by ensuring product availability. Hagiu and Wright (2015) extend this by modeling the platform's choice between reseller and marketplace models, showing that this decision depends on information asymmetries and the nature of seller participation. Anderson and Bedre Defolie (2024) show that when platforms also act as first-party sellers, endogenous third-party participation can lead to higher commissions, reduced product variety, and higher prices. Etro (2023) finds that if third-party sellers face more elastic demand, platform selling can reduce commission rates and benefit consumers. Shopova (2023) studies vertical differentiation and shows that platforms may strategically degrade the quality of their own products to induce higher prices from third-party sellers, increasing commission revenue. In contrast to these papers, we focus on a duopoly setting where the platform acts as a competing seller, and we analyze pricing competition rather than the platform's entry or business model choice.

Empirical research on self preferencing has examined algorithmic manipulation in prod-

uct suggestions and provided evidence that platforms engage in search ranking manipulation. Farronato, Fradkin, and MacKay (2023) find that Amazon branded products are ranked higher than similar alternatives in search results. N. Chen and Tsai (2024) show that Amazon systematically promotes its own products through features such as "frequently bought together," even when third party alternatives offer better value. Lam (2021) takes a structural approach by estimating a consumer search model and simulating the welfare effects of ranking manipulation. These findings are consistent with our results, which show that the platform has an incentive to engage in self-preferencing under certain conditions.

Our paper contributes to the literature on classic price dispersion and consumer search. Early theoretical work by Varian (1980) lays the foundation by modeling duopolies with heterogeneous consumer search behavior, distinguishing between price sensitive "searchers" and passive "non-searchers." Building on Varian's model, Golding and Slutsky (1999) allow for arbitrary distributions of non-searchers, though this distribution remains exogenously fixed. We extend these classical models of price competition with search frictions by endogenizing seller visibility and examining how it shapes equilibrium price distributions. In doing so, we provide a tractable framework for analyzing self-preferencing and assessing the limitations of existing regulatory approaches.

2 Model

We consider a market where two firms, firm E (a third-party seller) and firm A (a first-party seller that also operates the platform), compete by offering identical products. Let p_1, p_3 denote the prices set by firm A and firm E, respectively. The firms have marginal costs, denoted by c_3 for firm E and c_1 for firm A.

The platform charges a per transaction commission fee,[3] denoted by f, to the third-party seller for listing its product on the platform. This commission is assumed to be exogenously determined.[4] That is, the effective marginal cost for firm E becomes $c_3 + f$. We further assume that $0 \leq f < \min(r - c_1, r - c_3)$ to ensure the existence of an equilibrium where r is the consumer reservation price.[5]

We define the operating profits of each firm as follows. The profit of the third-party seller, firm E, from a single consumer is given by $\pi_3(p_3) \equiv (p_3 - c_3 - f)$, where p_3 is the price set by firm E, c_3 is its marginal cost, and f is the commission fee that firm A takes. Similarly, the profit of the platform-owning firm, firm A, from an individual consumer is given by $\pi_1(p_1) \equiv (p_1 - c_1)$, where p_1 is the price set by firm A and c_1 is its marginal cost.

[3] As of May 1, 2025, Amazon charges Individual sellers a fee of \$0.99 for each item sold.

[4] This is similar to Kittaka and Sato (2022). They note that Amazon has not changed commission fees. Our analysis focuses on a short-run case in which platforms do not adjust their commission fees. We later discuss how changes in this fee affect the equilibrium.

[5] For example, consider the case where $c_1 < c_3$ and the platform sets the fee $f \geq r - c_1$. Then:

$$\begin{aligned} \delta(1-\lambda)(r-c_1) + \lambda \cdot f &\geq \delta(1-\lambda)(r-c_1) + \lambda \cdot (r-c_1) \\ &= (\delta(1-\lambda) + \lambda)(r-c_1). \end{aligned}$$

In this case, no price below r allows the trade-off to break even: the gain in demand from searchers due to a lower price cannot sufficiently compensate for the reduction in per-unit profit. Consequently, the type of equilibrium analyzed in this paper would not exist under such a fee structure.

Consumers browse and purchase products based on the order in which they are displayed on the marketplace operated by the first-party seller. The display ranking is governed by the platform, which determines the probability that firm A appears at the top of the search results, denoted by δ.[6] Under a self-preferencing mechanism, firm A receives priority placement, corresponding to $\delta = 1$. As often referenced in the existing literature, search neutrality corresponds to the case $\delta = 0.5$, where display positions are assigned randomly with equal probability. We generalize this formulation by allowing $\delta \in [0, 1]$.

The consumer consists of a continuum of consumers with a total measure of one. Due to search costs, consumers differ in their shopping behavior. A fraction $\lambda \in (0, 1)$ of consumers has low search costs. They compare prices between the firms and choose the cheaper option. We refer to these consumers as *searchers*. The remaining fraction, $1 - \lambda$, has high search costs. They consider only the first firm displayed in the search ranking and do not engage in price comparisons. We refer to them as *non-searchers*. As a result, the firm with the lower price gains additional demand from the searchers, while the firm with the higher price only retains the baseline demand from non-searchers. We assume that each consumer buys exactly one unit at any price at or below r and no units at prices above r, similar to Salop and Stiglitz (1977) and Varian (1980).

The total demand for firm i depends on its price relative to its competitor. If firm i sets a higher price than firm j, it receives only the baseline demand. However, if firm i sets a lower price, it captures additional demand from searchers. This relationship is expressed as:

$$D_1(p_1, p_3) = \begin{cases} \delta(1-\lambda) & \text{if } p_1 > p_3, \\ \left(\delta(1-\lambda) + \frac{\lambda}{2}\right) & \text{if } p_1 = p_3, \\ (\delta(1-\lambda) + \lambda) & \text{if } p_1 < p_3, \end{cases}$$

$$D_3(p_1, p_3) = \begin{cases} (1-\delta)(1-\lambda) + \lambda) & \text{if } p_1 > p_3, \\ \left((1-\delta)(1-\lambda) + \frac{\lambda}{2}\right) & \text{if } p_1 = p_3, \\ (1-\delta)(1-\lambda) & \text{if } p_1 < p_3. \end{cases}$$

These demand functions illustrate how a firm's total demand depends on both its price relative to its competitor and the platform's display ranking. Consumers with high search costs, who make up a fraction $1-\lambda$ of the market, do not compare prices and simply purchase from the first firm they encounter. The platform determines which firm appears first through the parameter δ, so firm A receives a share $\delta(1-\lambda)$ of this baseline demand, while firm E receives the remainder. In contrast, the remaining consumers, representing a fraction λ, have low search costs and choose the firm offering the lower price. If both firms set the same price, these consumers split evenly between them. As a result, when a firm offers a lower price than its rival, it not only retains its share of the baseline demand but also attracts all of the price-sensitive consumers, the searchers. Conversely, a firm setting a higher price forgoes

[6]This setting reflects Amazon's actual practice, where the platform governs the ranking of products in search results using proprietary algorithms. The probability that Amazon's own brand appears at the top can vary, depending on factors such as conversion rates, fulfillment status, and strategic priorities. Momma et al. (2020) discuss Amazon's internal development of multi-objective relevance ranking algorithms that balance conversion rates and strategic priorities. Compiani et al. (2024) provide an empirical framework examining how platforms like Amazon rank products optimally by incorporating user behavior data and strategic considerations.

this competitive segment and relies solely on the demand from less attentive shoppers, the non-searchers.

With these demand in place, we now describe the sequence of events that determine outcomes in the market. The interaction unfolds in stages, reflecting the information structure and strategic decisions made by the firms and the platform. The timing is as follows.

1. Firms observe their marginal costs, and consumers learn their search costs, which determine whether they become searchers or non-searchers. The commission fee f is exogenously determined and publicly observable.

2. The platform sets the display probability δ.

3. The platform (as a first-party seller) and the third-party seller simultaneously choose their prices.

4. Searchers examine both listings and choose the lower priced product, while both searchers and non-searchers make their purchase decisions.

This sequence reflects a natural order of decisions in digital marketplaces. Firms first learn their costs, which influence their pricing strategies. Consumers also know their own search costs, which shape their responsiveness to price differences. The platform, in turn, sets the display ranking policy δ before prices are chosen, capturing its role as a gatekeeper that shapes market visibility independently of firms' pricing decisions. Finally, firms set prices simultaneously, recognizing how visibility and consumer behavior affect demand.

3 Pricing

Note that r is the monopoly price that maximizes profit when the firm sells exclusively to non-searchers, who make up a fraction $(1 - \lambda)$ of the market. These high-search-cost consumers purchase only from the firm that appears first in the search results and do not compare prices. Setting a price above r results in zero sales, thereby reducing profits. Therefore, a firm has no incentive to choose a price above r, as doing so would result in zero profits.

3.1 Non-existence of Pure Strategy Equilibrium

The characterization of equilibrium pricing follows a framework similar to Varian (1980) and Golding and Slutsky (1999). In the model of Golding and Slutsky (1999), they show that under certain conditions, the only possible pure strategy equilibrium occurs at the monopoly price pair. In contrast, in our framework, because the monopoly price r is identical for all firms, no pure strategy equilibrium exists analogous to Varian (1980). This result stems from the fact that searchers base their purchase decisions solely on relative prices. More precisely, a firm charging a lower price has an incentive to increase its price toward that of the higher-priced firm, since it can still attract all searchers and maintain its share of non-searchers, thereby improving its profit. Moreover, a firm always has an incentive to undercut as this allows it to capture the entire segment of searchers and increase its payoff. This creates a strict incentive to deviate, ruling out the possibility of any pure strategy equilibrium.

3.2 Mixed Strategy

Following the price dispersion framework introduced by Varian (1980), firms set prices strategically in response to heterogeneous consumer search behavior. In this environment, a mixed strategy emerges as the equilibrium outcome. Each firm i's pricing strategy is described by a cumulative distribution function $F_i(p_j)$, which gives the probability that firm i sets a price lower than its rival's price p_j.[7] This randomness prevents predictable undercutting and reflects the equilibrium behavior in markets with imperfect consumer information and strategic price competition. Accordingly, the expected demand faced by each firm depends not only on the firm's own price but also on the distribution of prices set by its competitor. Each firm retains its baseline share of non-searchers and competes probabilistically for the attention of searchers based on its relative pricing. Because firms randomize over prices in equilibrium, a given price does not guarantee winning or losing the searchers.

Since firms face uncertainty about their competitor's pricing behavior, they maximize expected profits by considering the probability distribution over the rival's prices. Let the support of firm A's pricing strategy F_1 be the interval (n_1, s) and the support of firm E's pricing strategy F_3 be (n_3, t). Following the approach of Golding and Slutsky (1999), each distribution function F_i can be decomposed into a continuous component, denoted $\hat{F}_i$, and a countable set of mass points located at prices P_j, for $j = 1, 2, \ldots$. Each mass point carries a probability mass $f_i^0(P_j)$, where $f_i^0(P_j) \equiv F_i(P_j) - \lim_{p \to P_j} F_i(p)$. Accordingly, each firm's expected profit when setting price p_1 and p_3 is given by:

$$
\begin{aligned}
\mathbb{E}\Pi_1(F_1, F_3) = &\int_{n_1}^{s} \left((\delta(1-\lambda) + \lambda \cdot (1 - F_3(p))) \cdot \pi_1(p) + ((1-\delta)(1-\lambda) + \lambda \cdot F_3(p)) \cdot f\right) d\hat{F}_1 \\
&+ \sum_j f_1^0(p_j) \cdot \left(\delta(1-\lambda) + \left(f_3^0(p_j)/2 + 1 - F_3(p_j)\right) \cdot \lambda\right) \cdot \pi_1(p_j) \\
&+ \sum_j f_1^0(p_j) \cdot ((1-\delta)(1-\lambda) + (f_3^0(p_j)/2 + F_3(p_j)) \cdot \lambda) \cdot f \\
\mathbb{E}\Pi_3(F_1, F_3) = &\int_{n_3}^{t} ((1-\delta)(1-\lambda) + \lambda \cdot (1 - F_1(p))) \cdot \pi_3(p) d\hat{F}_3 \\
&+ \sum_k f_3^0(p_k) \cdot \left((1-\delta)(1-\lambda) + \left(f_1^0(p_k)/2 + 1 - F_1(p_k)\right) \cdot \lambda\right) \cdot \pi_3(p_k)
\end{aligned}
$$

The continuous part integrates expected profits over the pricing distribution, while the summation terms reflect profits at discrete price points where mass points exist in the mixed strategies. The last summation term for firm A represents the revenue generated from the commission fee.

[7] This strategy aligns with the operational environment on platforms like Amazon, where third-party sellers frequently adopt automated pricing tools that adjust prices dynamically based on competitor behavior, demand conditions, and platform data. L. Chen, Mislove, and Wilson (2016) conduct a large-scale empirical study on algorithmic pricing behavior on the Amazon Marketplace. MacKay and Weinstein (2022) explain how third-party sellers use real-time data and dynamic pricing strategies. Mattes (2022) explores Amazon's pricing influence and the widespread use of automated pricing among sellers.

3.3 Equilibrium Pricing

We follow the approach of Golding and Slutsky (1999) to derive the equilibrium prices. The indifference condition ensures that, for any price p in the range $n_i \leq p \leq r$, the firm's expected profit, denoted $\mathbb{E}\Pi_i(p, F_j)$, must remain constant. The firm has no strict incentive to favor one price over another within the support of its mixed strategy. The profit trade-off between higher margins at higher prices and greater probability of attracting searchers at lower prices is exactly balanced throughout this range. As a result, firms are indifferent across all prices in their support, and randomization becomes a rational response to strategic uncertainty. To determine the supports of the equilibrium pricing distributions, define the following.

Definition 1. Define m_1 and m_3 as the solutions to the following conditions:

$$c_1 < m_1 < r \text{ and } (\delta(1-\lambda)+\lambda)\pi_1(m_1) = \delta(1-\lambda)\pi_1(r) + \lambda \cdot f,$$
$$c_3 < m_3 < r \text{ and } ((1-\delta)(1-\lambda)+\lambda)\pi_3(m_3) = (1-\delta)(1-\lambda)\pi_3(r).$$

Specifically,

$$m_1 = r - \lambda \cdot \frac{r - c_1 - f}{\delta(1-\lambda)+\lambda},$$
$$m_3 = r - \lambda \cdot \frac{r - c_3 - f}{(1-\delta)(1-\lambda)+\lambda}.$$

We define a threshold price m_i as the lowest price at which firm i can sell to all searchers (a fraction λ of the market) and to its share of non-searchers, $\delta(1-\lambda)$ for firm A and $(1-\delta)(1-\lambda)$ for firm E, while still earning the same profit it would obtain by charging the monopoly price r and serving only its share of non-searchers. These threshold prices m_1 and m_3 can be characterized formally as the unique solutions to specific profit-equality conditions. Each m_i lies strictly between the firm's marginal cost and its monopoly price, i.e., $c_i < m_i < r$, and satisfies a condition equating the firm's profit when pricing aggressively to attract searchers with its profit under monopoly pricing to non-searchers only.

Lemma 1. *The lower bound of the supports of $\hat{F}_1$ and $\hat{F}_3$ is*

$$m = \max(m_1, m_3).$$

Firms face a tension between undercutting the rival to attract searchers and maintaining sufficient markup to preserve profitability. Pricing too aggressively erodes profit margins, while pricing too high risks losing the entire segment of searchers. The threshold m_i captures a trade-off firms face in equilibrium: it is the lowest price at which the gain in demand from searchers exactly offsets the loss in per-unit profit. To balance these competing incentives, firms randomize over prices in the interval $[m_i, r)$, ensuring that no firm has a profitable deviation. Any price below m_i would lead to lower profits, as the gain in quantity sold would not sufficiently compensate for the lower markup. Thus, m_i serves as a lower bound on firm m_i's pricing incentives.

We can now define $m = \max(m_1, m_3)$. This value represents the lowest price at which either firm is willing to begin randomizing in equilibrium. If one firm's lowest price were below

that of the other, it would benefit from raising it; therefore, both firms share a common lower bound. Thus, m defines the true common lower bound of the pricing support in equilibrium and ensures both distributions overlap where firms are actively competing for demand.

Proposition 1. *The unique equilibrium price distributions, $F_1(p)$ and $F_3(p)$, are as follows.*

$$\hat{F}_1(p) = \begin{cases} \frac{\lambda+(1-\delta)(1-\lambda)}{\lambda} \cdot \frac{p-m}{p-c_3-f} & m \le p < r \\ 1 & r \le p \end{cases}$$

$$f_1^0(r) = 1 - \frac{\lambda + (1-\delta)(1-\lambda)}{\lambda} \cdot \frac{r-m}{r-c_3-f}$$

$$\hat{F}_3(p) = \begin{cases} \frac{\lambda+\delta(1-\lambda)}{\lambda} \cdot \frac{p-m}{p-c_1-f} & m \le p < r \\ 1 & r \le p \end{cases}$$

$$f_3^0(r) = 1 - \frac{\lambda + \delta(1-\lambda)}{\lambda} \cdot \frac{r-m}{r-c_1-f}$$

This proposition characterizes the unique mixed-strategy equilibrium pricing distributions for both the platform and the third-party seller. The equilibrium reflects the strategic interaction between firms in a setting where they compete for price-sensitive searchers while still retaining segments of inattentive non-searchers who do not compare prices.

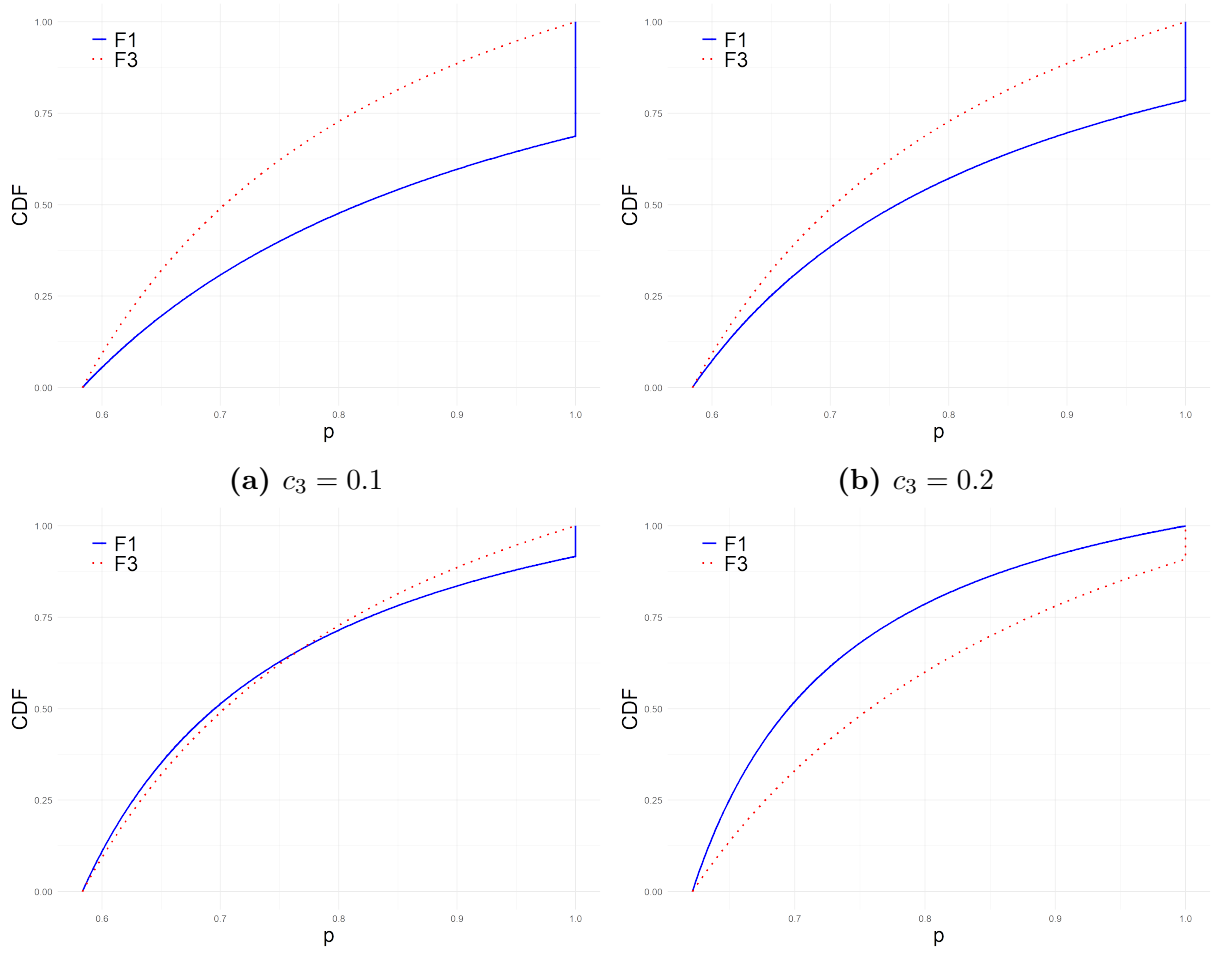
In equilibrium, each firm randomizes over a range of prices, using a distribution function that balances the trade-off between attracting more demand and maintaining higher profit margins. The lower bound of each firm's pricing support is the threshold price m, below which the firm would earn less than it would by pricing at its monopoly level. The upper bound corresponds to the monopoly price, r, beyond which the firm has no incentive to go because it would lose all demand.

Interestingly, f enters both expressions with the same (negative) sign, even though Firm E pays the fee while Firm A receives it. This symmetry arises from how f affects each firm's incentives to compete aggressively in price. For Firm E, the fee directly increases its marginal cost, reducing the profitability of more aggressive (i.e., lower) pricing. As a result, the likelihood that Firm E selects a low price can decrease as f increases.[8] For Firm A, the fee provides passive revenue whenever Firm E makes a sale. This makes higher prices more attractive for Firm A, since it can earn commission income even without capturing demand. To preserve equilibrium indifference, Firm A also becomes less aggressive in pricing as f increases. Thus, although the economic mechanisms differ, with cost inflation for Firm E and revenue substitution for Firm A, the strategic effect is the same: both firms can reduce their pricing aggressiveness as f increases. This explains why f enters both firms' pricing strategies with the same algebraic sign. To clarify this intuition, Appendix D presents a simplified discrete example that makes the logic transparent.

The typical shape of the equilibrium distribution is illustrated in Figure 1. The shape of the distribution functions ensures that each firm is indifferent across the range of prices it may choose. The probabilities are constructed so that, given the other firm's pricing strategy,

[8]This is not always the case. We analyze the effect of increasing f on equilibrium strategies more fully in Section 6.

any price in the support yields the same expected profit. The presence of point masses at r reflects the fact that firms may sometimes choose to set their monopoly price with positive probability.



(a) $c_3 = 0.1$ **(b)** $c_3 = 0.2$

(c) $c_3 = 0.3$ **(d)** $c_3 = 0.4$

Figure 1: Equilibrium under the parameter values $r = 1$, $f = 0.1$, $c_1 = 0.2$, $\lambda = 0.5$, and $\delta = 0.68$.

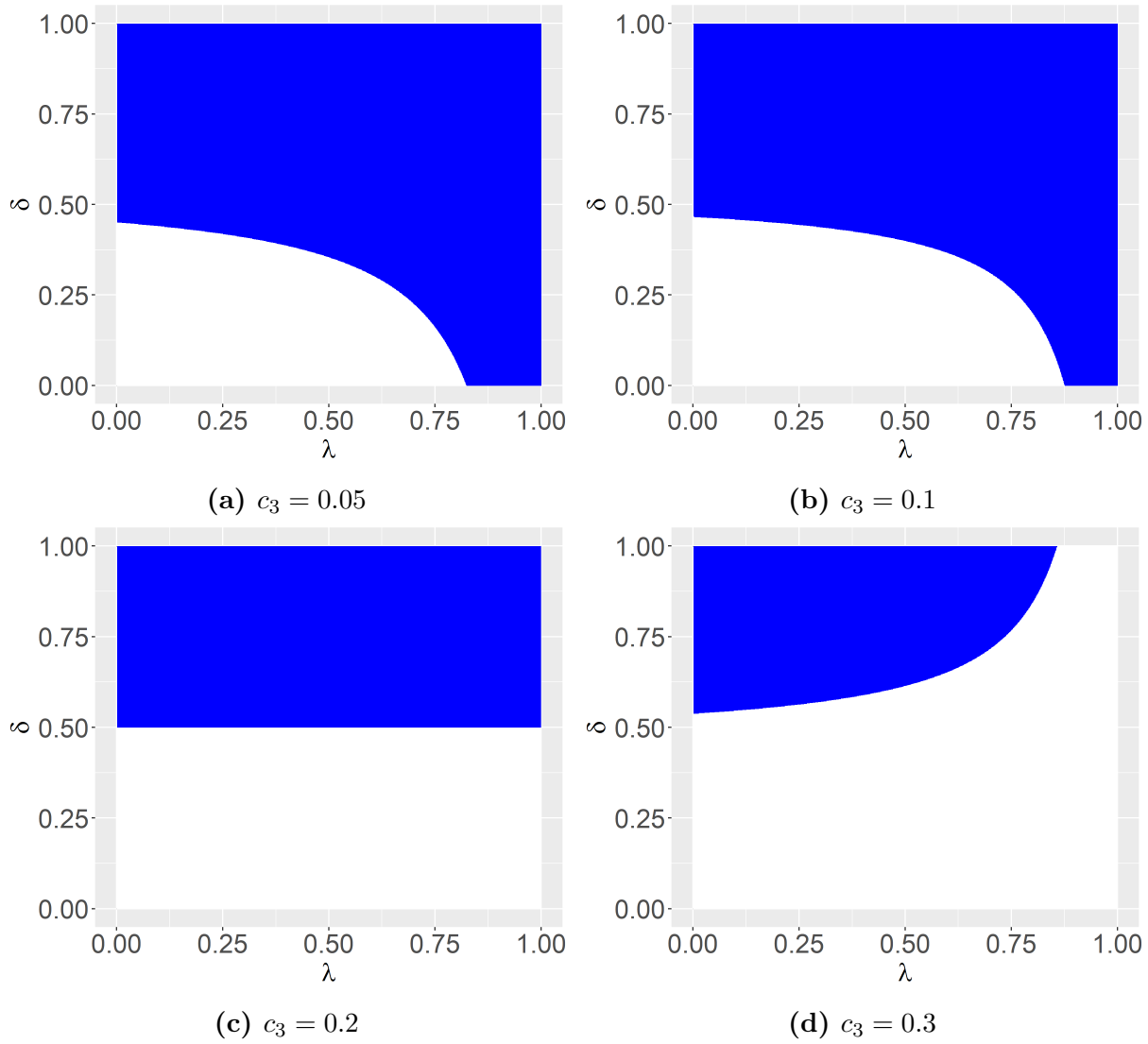
Corollary 1. *The following statements hold:*

1. $f_1^0(r) = 0$ *if and only if* $m_3 \geq m_1$.
2. $f_3^0(r) = 0$ *if and only if* $m_1 \geq m_3$.

A point mass occurs only at r, and at most one of the two distributions has a point mass at that price. Only when $m_1 = m_3$ do neither of the distributions have a point mass.

4 Optimal Display Probability in Search Rankings

Before the pricing stage, the platform determines both the display probability assigned to each firm. Self-preferencing not only shifts market share but also reshapes the equilibrium



(a) $c_3 = 0.05$ **(b)** $c_3 = 0.1$

(c) $c_3 = 0.2$ **(d)** $c_3 = 0.3$

Figure 2: The blue-shaded area represents the regions of the (λ, δ) parameter space where $m_1 > m_3$, and the white area represents where $m_1 < m_3$, under the parameter values $r = 1$, $f = 0.1$, and $c_1 = 0.2$.

price distributions. By altering the distribution of consumer demand, particularly by guaranteeing access to non-searchers, self-preferencing changes the strategic pricing incentives faced by firms. In this section, we study the optimal display probability—that is, the likelihood that a given seller appears in the top position of the search ranking.

Definition 2. Let δ^* be the value of δ that makes $m_1 = m_3$. Specifically,

$$\delta^* = \frac{(r - c_1 - f) - \lambda(r - c_3 - f)}{(1 - \lambda)(r - c_1 - f + r - c_3 - f)}.$$

Note that δ^* is the ranking frequency at which the platform's product appears at the top of the search results such that both minimum prices m_1 and m_3 are equal. The following lemma characterizes how the equilibrium pricing distribution varies with δ.

Lemma 2. *When $\delta < \delta^*$, an increase in δ leads to a first-order stochastic shift toward lower prices in the equilibrium price distributions F_1 and F_3, resulting in lower average market prices. Similarly, when $\delta > \delta^*$, a decrease in δ induces a first-order stochastic shift toward lower prices in F_1 and F_3, also leading to lower average market prices.*

To understand, consider $\delta < \delta^*$. Then, we have $m_3 > m_1$, indicating that the third-party firm's pricing is binding, while the platform retains some flexibility to lower its price further while still maintaining its monopoly-level profit. In this case, increasing δ toward δ^* directly impacts the third-party firm, forcing it to lower its price significantly in order to retain searchers. In contrast, the platform holds a larger share of non-searchers as δ increases. Thus, the platform faces two opposing forces: *Exploitative Pricing Motive*, the incentive to extract higher prices from locked-in non-searchers, and *Competitive Response Pressure*, the pressure to lower its price in response to the third-party firm's lower pricing. Since its share of non-searchers is relatively low in this case, the *Competitive Response Pressure* is relatively stronger, leading the platform to lower its price.

Now, consider $\delta > \delta^*$, or equivalently $m_1 > m_3$. In this case, the platform's pricing is binding. From $\delta = \delta^*$, increasing δ attracts more non-searchers to the platform's product. As a result, the platform is able to raise its price significantly. The third-party firm faces the *Price Coordination Incentive*, the incentive to raise its own price in response to the platform's higher pricing, which outweighs *Retention Pressure*, the need to maintain profitability by appealing to searchers. Thus, the equilibrium price distributions shift toward higher prices for both firms as δ increases or under self-preferencing ($\delta = 1$). In other words, both firms charge higher prices more frequently because firm A can consistently capture non-searchers regardless of its price, reducing its incentive to compete aggressively. As a result, firm A has less need to lower prices to attract customers, making lower prices less likely and increasing the average price level in the market.

$$\frac{d\hat{F}_1}{d\delta} = -\underbrace{\frac{1-\lambda}{\lambda}\cdot\frac{p-m}{p-c_3-f}}_{\text{Exploitative Pricing Motive}} + \underbrace{\frac{\lambda+(1-\delta)(1-\lambda)}{\lambda}\cdot\frac{1}{p-c_3-f}\cdot\left(-\frac{dm}{d\delta}\right)}_{\substack{+\text{ when } m_3 \text{ is binding: Competitive Response Pressure} \\ -\text{ when } m_1 \text{ is binding: Price Coordination Incentive}}}$$

$$\frac{d\hat{F}_3}{d\delta} = \underbrace{\frac{1-\lambda}{\lambda}\cdot\frac{p-m}{p-c_1-f}}_{\text{Retention pressure}} + \underbrace{\frac{\lambda+\delta(1-\lambda)}{\lambda}\cdot\frac{1}{p-c_1-f}\cdot\left(-\frac{dm}{d\delta}\right)}_{\substack{+\text{ when } m_3 \text{ is binding: Competitive Response Pressure} \\ -\text{ when } m_1 \text{ is binding: Price Coordination Incentive}}}$$

Definition 3. Let $\dot{\delta}$ be the value of δ that makes $LHS = RHS$ where

$$LHS = r - c_1 - f$$
$$RHS = \frac{\lambda(1+\lambda)}{((1-\delta)(1-\lambda)+\lambda)^2}\cdot(r-c_3-f).$$

Specifically,

$$\dot{\delta} = \frac{1}{1-\lambda} - \frac{1}{1-\lambda}\cdot\sqrt{\lambda(1+\lambda)\cdot\frac{r-c_3-f}{r-c_1-f}}.$$

Note that $\dot{\delta}$ is the platform's optimal ranking frequency when it sets its price equal to the third-party firm's minimum viable price. This $\dot{\delta}$ is relevant only when $m_3 \geq m_1$. In this case, increasing δ creates competitive response pressure for both firms. Since the third-party firm is the one whose pricing is binding, a small initial value of δ induces it to lower its price substantially. As a result, the platform experiences a relatively large gain from increasing δ at first. However, as δ rises, the marginal benefit of further increasing δ diminishes, while the associated loss grows, driven by a lower average price in equilibrium. The value of δ at which the marginal benefit and marginal loss shift in dominance is $\dot{\delta}$.

On the other hand, when m_1 is binding, the platform is directly affected by the price coordination incentive. In this case, increasing δ benefits the platform, aside from the change in fee revenue. The effect of fee revenue on profit is symmetric in both cases, whether m_1 or m_3 is binding, and therefore does not alter this result.

$$\frac{d\mathbb{E}\Pi_1}{d\delta} = \begin{cases} \underbrace{(1-\lambda)(m_3 - c_1)}_{\substack{\text{Gain from exploitative pricing} \\ \text{(maintain same revenue} \\ \text{as monopoly pricing)}}} + \underbrace{(\delta(1-\lambda)+\lambda) \cdot \frac{dm_3}{d\delta}}_{\text{Loss from more dispersed pricing}} - \underbrace{(1-\lambda) \cdot f}_{\text{Fee revenue change}} & m_3 \geq m_1 \\ (1-\lambda) \cdot (r - c_1) - \underbrace{(1-\lambda) \cdot f}_{\text{Fee revenue change}} & m_3 \leq m_1 \end{cases}$$

The following proposition characterizes the platform's optimal choice of display probability.

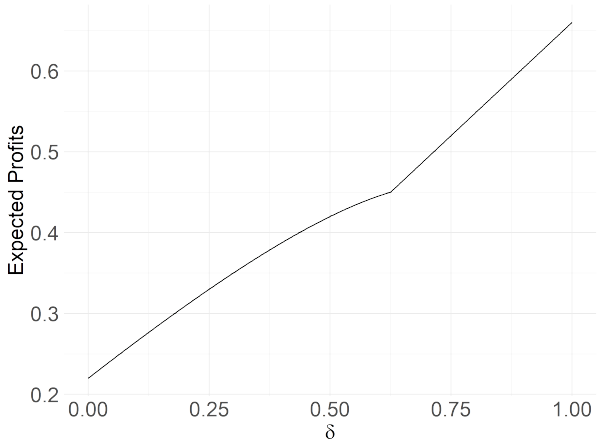
Proposition 2. *Let* $\widehat{\dot{\delta}} = \max(0, \min(1, \dot{\delta}))$. *The platform's optimal choice of* δ *is determined as follows:*

1. *If* $\delta^* \leq \dot{\delta}$, *the platform optimally sets* $\delta = 1$.
2. *If* $\dot{\delta} < \delta^*$, *the platform chooses between* $\delta = 1$ *and* $\delta = \widehat{\dot{\delta}}$ *by comparing* $\Pi_1|_{\delta=1}$ *and* $\Pi_1|_{\delta=\widehat{\dot{\delta}}}$. *In other words, the optimal* δ *is*

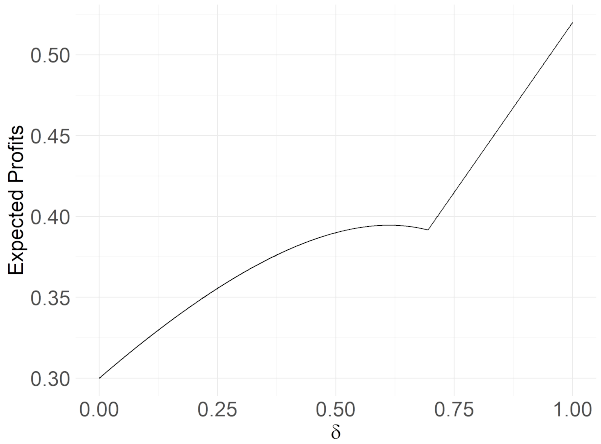
$$\arg \max_{\delta = \{\widehat{\dot{\delta}}, 1\}} \mathbb{E}\Pi_1$$

Among the values of δ where $\delta > \delta^*$, or equivalently $m_1 > m_3$, the expected profit increases with δ, making $\delta = 1$ the optimal choice. For $\delta < \delta^*$, or equivalently $m_1 < m_3$, it is possible that the expected profit does *not* strictly increase with δ. Note that when m_3 is binding, the expected profit increases with δ for $\delta < \dot{\delta}$ and decreases with δ for $\delta > \dot{\delta}$, since the RHS increases with δ while the LHS remains constant with respect to δ. Thus, the expected profit decreases with δ over the interval $(\dot{\delta}, \delta^*)$ only if $\dot{\delta} < \delta^*$, in which case the platform compares the profits at $\dot{\delta}$ and 1.

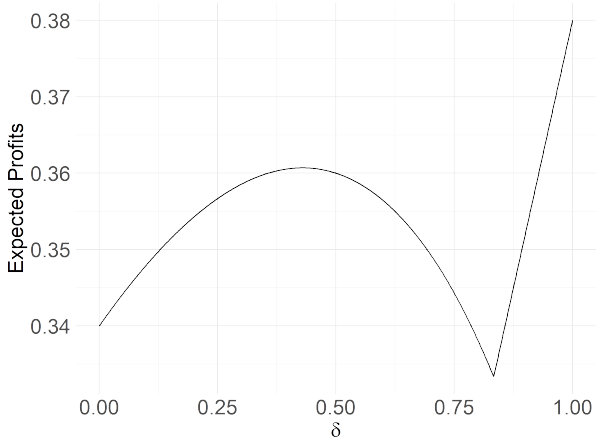
An example of the expected profit curve as a function of δ is shown in Figure 3, with additional examples provided in Appendix H. Figure 3a illustrates the case where $\delta^* < \dot{\delta}$. In this case, for small values of δ, the gain from exploitative pricing is large as the platform increases δ, while the price coordination incentive becomes relevant for large values of δ. As



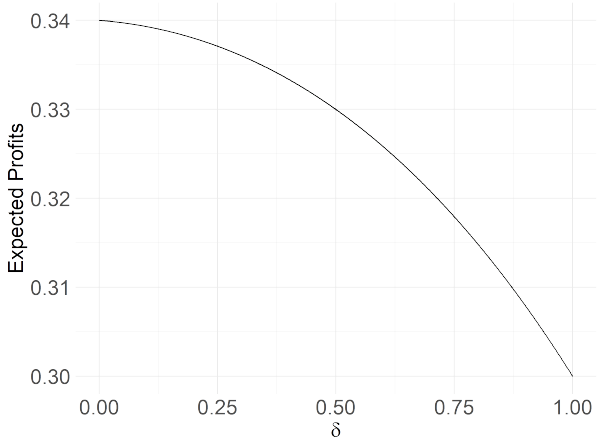
(a) $\lambda = 0.2$
$m_1 < m_3$ for small δ; $m_1 > m_3$ for big δ



(b) $\lambda = 0.4$



(c) $\lambda = 0.6$



(d) $\lambda = 0.8$
$m_1 < m_3$ for all δ

Figure 3: Expected Profit for Firm A when $r = 1$, $f = 0.1$, $c_1 = 0.2$, $c_3 = 0.4$

a result, the platform's profit increases monotonically over the entire range of δ, making it optimal for the platform to list its own product first.

Figures 3b and 3c represent the case where $\delta^* > \dot{\delta}$. Here, again, the profit increases with δ initially due to the strong gains from exploitative pricing. However, when m_3 is binding, the loss from more dispersed pricing becomes more significant, causing the platform's profit to decline. Once δ reaches and surpasses δ^*—at which point m_1 starts binding—the price coordination incentive dominates, and the platform's profit starts to rise again. In such cases, the platform compares $\Pi_1\big|\delta = 1$ with $\Pi_1\big|\delta = \dot{\delta}$ and chooses the value of δ that yields the higher profit.

Figure 3d also depicts the case where $\delta^* > \dot{\delta}$, but with $\dot{\delta} < 0 < 1 < \delta^*$. Here, the expected profit decreases throughout the range $\delta \in [0, 1]$. In this case, the platform optimally chooses a small value of δ, ultimately setting $\delta = 0$. This occurs, for example, when m_3 is binding for all values of δ, so that competitive pricing leads both firms to reduce their prices, while the loss from price dispersion remains substantial across the entire interval.

Definition 4. Let $\check{\delta}$ be the value of δ that satisfies the following:

$$\frac{(1+\lambda)\cdot\lambda}{(\delta(1-\lambda)+\lambda)^2}\cdot(r-c_1-f) = (r-c_3-f)$$

Specifically,

$$\check{\delta} = \frac{1}{1-\lambda}\cdot\sqrt{\lambda(1+\lambda)\cdot\frac{r-c_1-f}{r-c_3-f}} - \frac{\lambda}{1-\lambda}$$

$\check{\delta}$ denotes the third-party firm's profit-maximizing ranking frequency in the case where $m_1 \geq m_3$. The third-party firm's profit moves in the opposite direction of the platform's profit with respect to changes in δ. If $\check{\delta} \leq \delta^*$, the third-party firm's profit decreases as δ increases. If $\check{\delta} > \delta^*$, its profit increases only within the interval $\delta^* < \delta < \check{\delta}$ and decreases outside this range. The proof is available in the appendix J.

4.1 Discussion: When Does the Platform Engage in Self-Preferencing?

This subsection discusses the conditions under which the platform engages in self-preferencing. When the platform engages in self-preferencing, it ensures that its own product consistently appears at the top of the search results by setting $\delta = 1$. This deterministic ranking significantly influences consumer behavior, particularly for non-searchers who consider only the top-listed option. By securing the top position, the platform captures the entire segment of non-searchers, thereby reshaping the demand structure and altering the strategic pricing incentives faced by both itself and the third-party seller. The platform always prefers self-preferencing when $\delta^* \leq \dot{\delta}$, as its expected profit increases with δ over the entire interval. However, when $\delta^* > \dot{\delta}$, it is possible that the platform prefers to set $\delta = \dot{\delta}$ instead of choosing self-preferencing, depending on the relative magnitudes of the marginal gains and losses. In such cases, the trade-off between exploitative pricing and competitive response pressure determines the optimal ranking strategy. The following lemma characterizes the relationship between δ^* and $\dot{\delta}$.

Proposition 3. *The following equivalences hold:*

$$\begin{aligned}
\check{\delta} \gtreqless \delta^* \gtreqless \dot{\delta} &\Leftrightarrow \frac{r-c_1-f}{r-c_3-f} + \frac{r-c_3-f}{r-c_1-f} \gtreqless \frac{1-\lambda}{\lambda} \\
\delta^* \gtreqless 1 &\Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \frac{1}{\lambda} \\
\delta^* \gtreqless 0 &\Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \lambda \\
\dot{\delta} \gtreqless 1 &\Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \frac{1+\lambda}{\lambda} \\
\dot{\delta} \gtreqless 0 &\Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \lambda(1+\lambda) \\
\check{\delta} \gtreqless 1 &\Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \frac{1}{\lambda(1+\lambda)} \\
\check{\delta} \gtreqless 0 &\Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \frac{\lambda}{1+\lambda}
\end{aligned}$$

This proposition summarizes how the relationships among 0, 1, δ^*, $\dot{\delta}$, and $\check{\delta}$ are determined by the model parameters. The platform is most likely to engage in self-preferencing when its profit is such that moving δ toward 1 yields steady or increasing returns either because δ^* is low or because an interior profit peak is close to 1 (i.e., $\dot{\delta}$ is large), leaving a wide range of profitable δ values that favor the platform's own product. We explore the parameter configurations under which self-preferencing is likely.[9]

A low share of searchers (λ close to 0): When most consumers are non-searchers, δ^* is more likely to be small, meaning that m_1 is more likely to be binding over a wider range of δ. Intuitively, the platform has less to gain from competing for searchers and instead focuses on extracting profit from non-searchers. To do so, it increases δ so that a larger share of non-searchers are directed to its own product. Thus, self-preferencing is likely to be profitable, as the primary profit opportunities lie in the non-searcher segment.

Corollary 2. *If $\lambda \geq \frac{1}{3}$, then $\dot{\delta} \leq \delta^*$. Conversely, if $\delta^* \leq \dot{\delta}$, then $\lambda \leq \frac{1}{3}$.*

When more consumers are searchers, the searcher segment becomes increasingly relevant. If searchers constitute at least one third of the market, the competitive response pressure must always be taken into account, making $\dot{\delta}$ a potential optimal choice for the platform.

Figure 4 shows the optimal value of δ as a function of λ, given $r = 1$, $f = 0.1$, $c_1 = 0.2$, and $c_3 = 0.4$. Under this parameter combination, $\dot{\delta}$ decreases with λ. When λ is sufficiently small, $\dot{\delta}$ is close to 1 and exceeds δ^*, making $\delta = 1$ the optimal choice. However, as λ increases, there comes a point where the optimal δ switches to $\dot{\delta}$. As λ increases further and $\dot{\delta}$ becomes negative, the optimal δ eventually drops to 0.

[9]How the values of $\dot{\delta}$ and δ^* change with respect to key parameters—specifically, the derivatives of $\dot{\delta}$ and δ^* with respect to λ, c_1, c_3, and f—are presented in Appendix L.

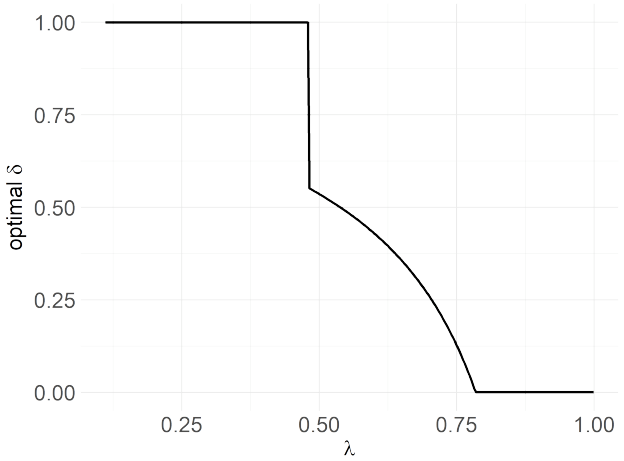


Figure 4: Optimal value of δ as a function of λ when $r = 1$, $f = 0.1$, $c_1 = 0.2$, and $c_3 = 0.4$.

High cost for the platform or low cost for the third-party firm ($c_1 \Uparrow$ or $c_3 \Downarrow$) When c_1 is large or c_3 is small, both δ^* and $\dot{\delta}$ are more likely to be small. In this case, the platform stands to gain more from favoring its own product across a wider range of δ values. Note that $\frac{d\delta^*}{dc_1} < 0$ and $\frac{d\dot{\delta}}{dc_1} < 0$, while $\frac{d\delta^*}{dc_3} > 0$ and $\frac{d\dot{\delta}}{dc_3} > 0$. Intuitively, when the platform faces higher costs or the third-party firm has lower costs, the platform is forced to compete in relatively high-price regions. As a result, it has more to gain from exploitative pricing and is therefore more likely to focus on the non-searcher segment.

However, the share of searchers must also be taken into account. In particular, when the platform has a cost advantage ($c_1 < c_3$), it is possible that $\dot{\delta}$ is relatively small while δ^* is relatively large.[10] As a result, the platform's profit may decrease over a wide range of δ values. Intuitively, this occurs because the platform, having a cost advantage, benefits significantly from the price coordination incentive. However, increasing δ also leads to greater loss from more dispersed pricing, which can outweigh the gains.

Figure 5 illustrates the relationship between the optimal value of δ and the cost difference $c_1 - c_3$, given $r = 1$, $f = 0.1$, $c_1 = 0.2$, and $\lambda = 0.7$. As c_3 decreases relative to c_1, the LHS becomes smaller compared to the RHS, so the optimal δ is initially given by $\dot{\delta}$, which decreases at first. This corresponds to the case where the platform, having a cost advantage, benefits significantly from the price coordination incentive. However, as c_3 decreases further relative to c_1, the RHS grows relative to the LHS. At some point, the loss from more dispersed pricing becomes too large, and the platform is better off choosing $\delta = 1$ rather than $\delta = 0$. Consequently, the optimal value of δ is initially given by the interior solution $\dot{\delta}$, which increases as c_3 decreases. Beyond a certain threshold, however, it becomes optimal for the platform to set $\delta = 1$.

Why would the Platform Randomize the Display Ranking Probability? We conclude this section by discussing the case in which the platform may prefer to randomize the display ranking probability. This choice is motivated by several strategic considerations. By listing the rival first, the platform may induce the third-party firm to price less aggressively,

[10] Note that $\frac{d\delta^*}{d\lambda} > 0$ if $c_1 < c_3$ and $\frac{d\delta^*}{d\lambda} < 0$ if $c_1 > c_3$.

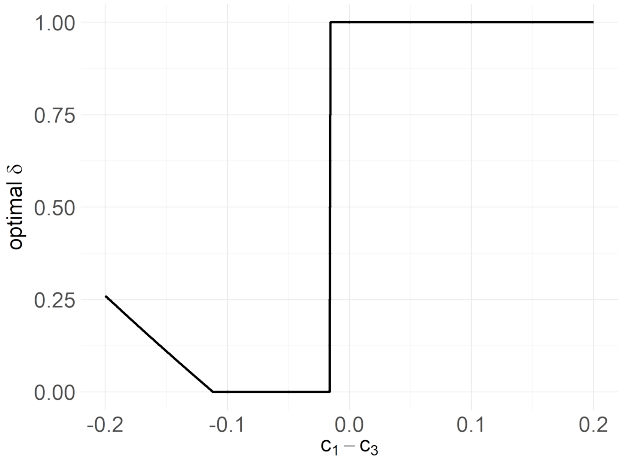


Figure 5: Optimal value of δ as a function of the cost difference, $c_1 - c_3$ when $r = 1$, $f = 0.1$, $c_1 = 0.2$, and $\lambda = 0.7$.

thereby softening competition and helping to sustain higher market prices. For example, when the platform has a cost advantage, increasing δ may subject both firms to competitive response pressure, while also amplifying the loss from more dispersed pricing. In such cases, the platform may be willing to forgo some sales to non-searchers in exchange for more favorable overall market conditions that enhance its pricing power and profit margins.

5 Consumer Welfare

The consumer's utility can be defined as $\mathbb{E}(r - p)$.

Lemma 3. *The expected utility of an individual consumer depends on their type and assigned firm as follows:*

- *Non-searchers assigned to firm A (proportion $\delta(1-\lambda)$) receive expected utility:*

$$\int_m^{r^-} \hat{F}_1 dp$$

- *Non-searchers assigned to firm E (proportion $(1-\delta)(1-\lambda)$) receive expected utility:*

$$\int_m^{r^-} \hat{F}_3 dp$$

- *Searchers (proportion λ) receive expected utility:*

$$\int_m^{r^-} \left[\hat{F}_1(z) + \hat{F}_3(z) - \hat{F}_1(z)\hat{F}_3(z)\right] dz$$

Lemma 4. *Suppose that F_{1a} first-order stochastically dominates F_{1b}, and F_{3a} first-order stochastically dominates F_{3b}; that is, $F_{1a}(z) \leq F_{1b}(z)$ and $F_{3a}(z) \leq F_{3b}(z)$ for all $z \in [0, r]$.*

Then the function $H_a(z) := F_{1a}(z)+F_{3a}(z)-F_{1a}(z)F_{3a}(z)$ *first-order stochastically dominates* $H_b(z) := F_{1b}(z) + F_{3b}(z) - F_{1b}(z)F_{3b}(z)$*; i.e.,*

$$H_a(z) \leq H_b(z) \quad \textit{for all } z \in [0, r].$$

Consumers prefer price distributions that first-order stochastically dominated by others, as these distributions reflect a higher likelihood of encountering lower prices. This translates into greater expected consumer surplus. Thus, this lemma states that if all non-searchers are better off (respectively, worse off), then searchers are also better off (respectively, worse off).

5.1 Optimal δ for Consumers

Proposition 4. *Consumer utility is maximized when* $\delta = \delta^{**}$*, where* $\delta^{**} = \max(0, \min(1, \delta^*))$*. Formally, the equilibrium price distributions of firms A and E, denoted* $F_1|_{\delta^{**}}$ *and* $F_3|_{\delta^{**}}$*, are first-order stochastically dominated by the corresponding distributions* $F_1|_\delta$ *and* $F_3|_\delta$*, induced by any other* $\delta \in [0, 1]$ *with* $\delta \neq \delta^{**}$*.*

This proposition states that, relative to any alternative δ, the price distributions at δ^{**} assign more weight to lower prices, leading to higher consumer utility. A stochastically dominated distribution places greater weight on lower price levels, implying that for any given price threshold, consumers are more likely to encounter offers below that level. Consequently, δ^{**} maximizes consumer utility, as it induces the most competitive pricing in the market.

Unless $\delta^{**} = 1$, equilibrium pricing changes under self-preferencing, and this shift in pricing behavior reduces consumer welfare, as consumers face higher average prices for the same products compared to a scenario in which firms are ranked randomly. Self-preferencing enables firm A to exploit its guaranteed visibility by charging higher prices without the competitive price pressure, thereby distorting the natural pricing mechanisms of the market. A natural question arises: if firm E can only attract searchers under self-preferencing, wouldn't it have a strong incentive to lower its prices to capture those buyers? While this intuition is reasonable, it holds only when $\delta^* \geq 1$, or equivalently, when m_3 is binding at $\delta = 1$. If m_1 is binding, the platform significantly raises its price as δ increases, and the third-party firm responds by also raising its price due to the price coordination incentive. In other words, when firm A raises its prices due to guaranteed access to non-searchers, it creates space for firm E to raise its own prices without being undercut. Rather than triggering aggressive price competition, self-preferencing softens the pricing incentives of both firms. In equilibrium, the incentive for firm E to raise prices outweighs its incentive to lower them due to weaker competition.

As a result, the market exhibits higher overall price levels, not intensified competition. Thus, when firm A engages in self-preferencing, it not only increases its own prices but also leads firm E to charge higher prices across its support if m_1 is binding. This may seem counterintuitive at first given that firm E loses access to non-searchers but it reflects the weakening of competitive pressure that arises from firm A's elevated pricing.

This result highlights a central welfare concern in platform markets: the strategic use of ranking power weakens competitive pressure across the board. Although it may seem intuitive that firm E, relying on price-sensitive consumers, would respond by pricing more

aggressively, the opposite occurs in equilibrium. Because firm A captures all non-searchers through self-preferencing, it effectively reduces the intensity of price competition in the market. This, in turn, gives both firms room to raise prices without fear of being undercut.

As a result, self-preferencing does not enhance competition—it weakens it. Rather than delivering lower prices or increased consumer surplus, it could result in higher market-wide prices and a corresponding reduction in overall consumer welfare. The equilibrium outcome is characterized by elevated price levels, reducing consumer surplus relative to more neutral or randomized ranking regimes.

Corollary 3. *The following holds:*

1. $\delta^* > 0.5$ *if* $c_1 < c_3$,
2. $\delta^* < 0.5$ *if* $c_1 > c_3$, *and*
3. $\delta^* = 0.5$ *if* $c_1 = c_3$.

Corollary 3 characterizes how the consumer-optimal ranking, δ^*, depends on the relative marginal costs of the platform and the third-party firm. When both firms have equal marginal costs ($c_1 = c_3$), the consumer-optimal ranking is symmetric, with $\delta^* = 0.5$, regardless of the fraction of searchers or the commission fee. In this case, price competition alone drives consumer welfare, and neutrality in ranking maximizes utility. Any preferential treatment in the display order distorts the alignment between visibility and pricing efficiency, thereby reducing consumer surplus.

When one firm has a cost advantage, consumer welfare is maximized by giving that firm more than equal visibility. For example, if the platform has a lower cost ($c_1 < c_3$), then $\delta^* > 0.5$, meaning it is optimal for consumers to give the platform's product greater visibility in the ranking. This ensures that more consumers are directed toward the more efficient seller, resulting in lower average prices and higher consumer welfare.

5.2 Discussion: Consumer Welfare vs. Platform Profit

Unless $\delta^* = \dot{\delta}$, consumer and platform interests are generally misaligned. Since the platform can endogenously set the display probability δ, it effectively uses product ranking as a strategic lever to influence the competitive dynamics between itself and the third-party seller. By adjusting δ, the platform can strategically tilt the balance between competition and market power, influencing both the resulting price distribution and its own profitability. The platform has a natural incentive to place its own product at the top of search results by setting $\delta = 1$, as this position grants access to non-searchers and enables the platform to charge prices closer to the monopoly level, capitalizing on the relatively inelastic demand from those consumers. When $\delta^* \leq \dot{\delta}$, the platform optimally sets $\delta = 1$. Alternatively, it may benefit from setting $\delta = \dot{\delta}$, which allows it to sustain higher prices by softening competition. Specifically, when $\dot{\delta} < \delta^*$, the platform chooses either $\delta = \dot{\delta}$ or $\delta = 1$, depending on which yields higher profit. In contrast, consumers benefit most when $\delta = \delta^*$ as it induces the most competitive pricing.

Does self-preferencing harm consumers? When $\delta = \delta^{**}$, pricing is at its most competitive. If $\delta^* \geq 1$, self-preferencing actually benefits consumers, since the most competitive pricing occurs at $\delta^{**} = 1$. This situation arises when $\frac{r-c_1-f}{r-c_3-f} \geq \frac{1}{\lambda}$. Thus, this outcome is only possible when the platform has a cost advantage, i.e., $c_1 < c_3$. The cost-disadvantaged firm is more likely to be the one with binding pricing constraints. In particular, if the third-party firm has a cost disadvantage, it is more likely to be binding in pricing, making both firms susceptible to competitive response pressure as δ increases. Additionally, a high share of searchers in the market, i.e., a large λ, amplifies this effect. As more consumers actively compare prices, both firms are more likely to respond competitively to changes in δ, further intensifying price competition.

On the other hand, if $\delta^* < 1$, self-preferencing always harms consumers. Specifically, for $\delta^* \leq \delta \leq 1$, since m_1 is binding, both firms face price coordination incentives. In this range, increasing δ strengthens these incentives, causing both firms to raise their prices. A more interesting question, then, is whether it is credible for the platform to engage in self-preferencing in this case. If $\delta^* < \dot{\delta}$, self-preferencing is always credible. Even when $\delta^* > \dot{\delta}$, as discussed earlier, a low share of searchers, high platform costs, or low costs for the third-party firm can make self-preferencing a credible strategy for the platform.

Does regulating $\delta < 1$ improve consumer welfare or disadvantage the platform? While consumers benefit from more competitive pricing, the platform's objective is to maximize its own profit, which may not coincide with minimizing prices. If $1 \leq \dot{\delta}$ and $1 \leq \delta^*$, a regulation that forces the platform to lower δ can harm both consumers and the platform. This occurs when $\frac{r-c_1-f}{r-c_3-f} \geq \frac{1}{\lambda} + 1$. Such a condition arises when the platform has a cost advantage ($c_1 < c_3$) and the majority of consumers are searchers. In this case, both firms are likely to face competitive response pressure, and the cost-advantaged platform has little incentive to raise δ, as there are relatively few non-searchers to target. As a result, the natural market dynamics already foster competitive pricing. Imposing a restriction that lowers δ could therefore reduce efficiency and harm both consumers and the platform.

When $\delta^* \leq \dot{\delta}$, then forcing the platform to lower δ below 1 would reduce its profit while improving consumer welfare. However, δ is pushed too far below δ^*, both consumer utility and platform profit may decline. When $\dot{\delta} < \delta^*$, the trade-off becomes more nuanced. Initially, reducing δ from 1 toward δ^* improves consumer welfare but lowers platform profit. Yet, if δ is forced below δ^*, the platform may actually benefit by continuing to lower δ toward $\dot{\delta}$. In this case, the platform prefers to feature the third-party seller more frequently in the top position, which softens price competition and supports higher equilibrium prices. Although this reduces consumer welfare due to higher prices, it can increase the platform's profit by preserving margins and segmenting the market more effectively.

In particular, when $\dot{\delta} < 1 \leq \delta^*$, a regulation that forces the platform to lower δ can unintentionally favor the platform while harming consumers. Note that this occurs when $\frac{1}{\lambda} \leq \frac{r-c_1-f}{r-c_3-f} \leq \frac{1}{\lambda} + 1$. This type of misalignment is most likely to arise in environments where the platform has a cost advantage ($c_1 < c_3$), the cost difference is not too large, and the shares of searchers and non-searchers are relatively balanced. In such cases, since the platform has a cost advantage and m_3 is binding, both firms face competitive response pressure. The platform also suffers some loss from more dispersed pricing as δ increases,

but the gain from exploitative pricing is limited because there are not many non-searchers. However, if a regulation forces δ below 1, the competitive response pressure weakens, allowing the platform to raise its price without a corresponding reaction from the third-party firm. Consequently, such a regulation may increase the platform's profit while reducing consumer welfare, producing an unintended policy outcome.

This observation highlights an important regulatory implication: without accounting for the broader market environment, blocking self-preferencing could unintentionally benefit the platform while harming consumers. As a result, policymakers aiming to promote fair and competitive digital markets should look beyond the surface-level question of which firm is ranked first. Instead, regulatory focus should extend to how firm-level cost efficiency, technological capabilities, and the ranking mechanism collectively shape the resulting endogenous price distributions. In addition, attention should be given to whether those distributions support or undermine the broader objective of ensuring consumer access to lower prices and fair competition.

6 Commission Fee

So far, we have assumed that the commission fee is fixed.[11] However, in the long run, the platform may adjust the fee. In this section, we analyze how such changes affect equilibrium pricing and consumer welfare.

6.1 Effects of an Increasing Commission Fee on Equilibrium Pricing and Consumer Welfare

Proposition 5. *The signs of $\frac{d\hat{F}_1}{df}$ and $\frac{d\hat{F}_3}{df}$ are characterized as follows:*

1. $\frac{d\hat{F}_1}{df} < 0$ *if* $c_1 > c_3$;
2. $\frac{d\hat{F}_3}{df} < 0$ *if* $c_1 < c_3$;

In particular:

3. $\frac{d\hat{F}_1}{df} < 0$ *and* $\frac{d\hat{F}_3}{df} < 0$ *if* $\delta^* \geq 1$ *and* $c_3 > c_1$, *or if* $\delta^* \leq 0$ *and* $c_1 < c_3$;
4. $\frac{d\hat{F}_1}{df} < 0$ *and* $\frac{d\hat{F}_3}{df} > 0$ *if* $\frac{\lambda}{(1-\delta)(1-\lambda)} \cdot (c_1 - c_3) > r - c_1 - f$;
5. $\frac{d\hat{F}_1}{df} > 0$ *and* $\frac{d\hat{F}_3}{df} < 0$ *if* $\frac{\lambda}{\delta(1-\lambda)} \cdot (c_3 - c_1) > r - c_3 - f$.

Proposition 5 analyzes how changes in the platform's commission fee f affect the equilibrium pricing distributions of the platform and the third-party firm, and thereby influence consumer welfare. The basic result is that a higher commission fee leads to higher prices for

[11] We do not model consumer or firm entry and exit, as in Kittaka and Sato (2022). As a result, holding other parameters fixed, the platform's profit increases strictly with the commission fee.

the non-searchers of the higher-cost firm. Note that, the commission fee enters the equilibrium pricing distributions for both firms with the same sign. For the third-party firm, the fee acts as a direct marginal cost, reducing its incentive to lower prices. For the platform, the fee represents an alternative revenue source, which similarly reduces its incentive to compete on price since it can earn more by collecting the fee. When the firm with the higher marginal cost faces the fee increase, it places even more weight on higher pricing. When cost asymmetries are large (when $\delta^* \geq 1$ or $\delta^* \leq 0$), even the lower-cost firm may respond to the commission fee by raising its price, leading to a decline in consumer welfare for all consumers, including searchers.

However, the effect becomes ambiguous under certain conditions. When most consumers are searchers (λ is close to 1), and the platform sets δ in a way that gives sufficient visibility to the cost-disadvantaged firm, the utility of the cost-advantaged firm's non-searching consumers can actually increase in response to a higher commission fee. This occurs, for example, when the third-party firm has the cost advantage ($c_3 < c_1$), δ is close to 1 (favoring the platform itself in search rankings), and λ is high; or conversely, when the platform has the cost advantage ($c_1 < c_3$), δ is close to 0 (favoring the third-party firm), and λ is high. In these cases, competitive pressure may lead the cost-advantaged firm to lower its price, improving utility for its non-searching consumers despite the fee increase.

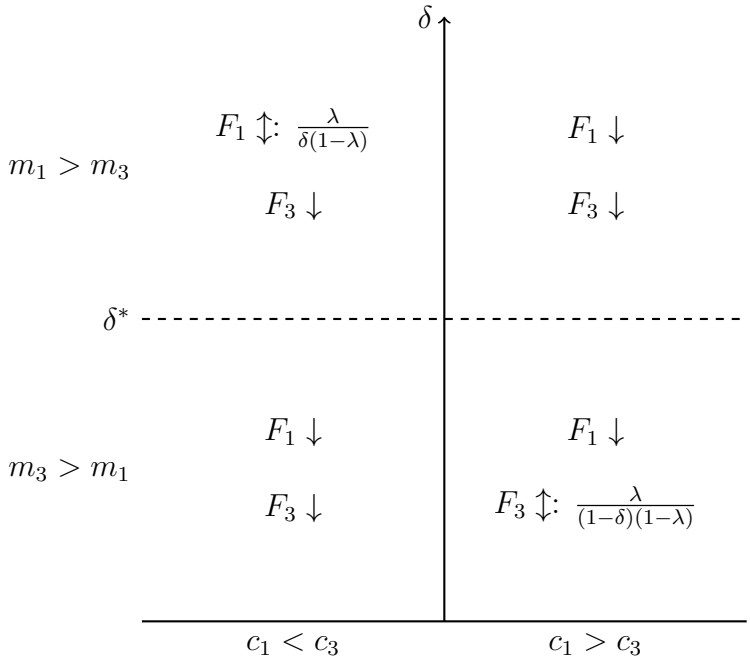


Figure 6: Description of Proposition 5

Proposition 6. *The following holds:*

1. $\frac{d\delta^*}{df} > 0$ *and* $\frac{d\dot{\delta}}{df} > 0$ *if* $c_1 < c_3$*; and*
2. $\frac{d\delta^*}{df} < 0$ *and* $\frac{d\dot{\delta}}{df} < 0$ *if* $c_1 > c_3$

Proposition 6 characterizes how the consumer-optimal ranking, δ^*, and the platform's local optimal ranking frequency, $\dot{\delta}$, change depending on the commission fee f. Recall that δ^* represents the value of δ that maximizes expected consumer utility. Note that $\delta^* = 0.5$ if $c_1 = c_3$ as discussed in Corollary 3. In this case, any preferential treatment in the display order distorts the alignment between visibility and pricing efficiency, thereby reducing consumer surplus regardless of the commission fee.

When one firm has a cost advantage, the ranking that maximizes consumer utility shifts further in favor of that firm as the commission fee increases. For example, if the platform has lower cost ($c_1 < c_3$), then $\delta^* > 0.5$, and an increase in the commission fee f leads to a further increase in δ^*, implying that giving the platform greater visibility becomes more aligned with consumer interest. Conversely, if the third-party firm has lower cost ($c_1 > c_3$), then $\delta^* < 0.5$, and a higher f leads to a further decrease in δ^*, indicating that visibility should shift more toward the third-party product to maintain consumer-optimal outcomes.

Corollary 4. *An increase in f always reduces the utility of all consumer types under either of the following conditions:*

1. *$c_1 < c_3$, $\delta^* > 0.5$, and the platform selects $\delta < \delta^*$; or*
2. *$c_1 > c_3$, $\delta^* < 0.5$, and the platform selects $\delta > \delta^*$.*

6.2 Marginal Profit Gain from an Increase in the Commission Fee

Proposition 7. *Suppose the platform sets $\delta = \delta_0$. Then the marginal profit gain from a one-unit increase in the commission fee, $\frac{d\mathbb{E}\Pi_1}{df}$, is characterized as follows:*

Case 1: *If $c_1 > c_3$, then*

$$\frac{d\mathbb{E}\Pi_1}{df} \begin{cases} = A\,(< B) & \text{if } \delta_0 < \delta^* < 0.5, \\ = B\,(> A) & \text{if } \delta^* < \delta_0 < 0.5, \\ = B\,(< A) & \text{if } \delta^* < 0.5 < \delta_0. \end{cases}$$

Case 2: *If $c_3 > c_1$, then*

$$\frac{d\mathbb{E}\Pi_1}{df} \begin{cases} = A\,(< B) & \text{if } \delta_0 < 0.5 < \delta^*, \\ = A\,(> B) & \text{if } 0.5 < \delta_0 < \delta^*, \\ = B\,(< A) & \text{if } 0.5 < \delta^* < \delta_0. \end{cases}$$

where $A = \frac{(1-\delta_0)^2(1-\lambda)^2+\lambda}{(1-\delta_0)(1-\lambda)+\lambda}$ and $B = (1-\delta_0)(1-\lambda)+\lambda$. Note: A change in the slope may occur if f continues to increase, potentially reversing the inequality when δ_0 crosses δ^*.

Proposition 7 examines how the platform's profit changes when it slightly increases the commission fee it charges to third-party sellers, assuming that the platform keeps its product's position in the search ranking fixed at a given level, denoted by δ_0. The question is how the marginal profit gain from increasing the fee depends on the relationship between δ_0, the cost structure of the platform and third-party firm, and δ^*, which is the ranking that would maximize consumer welfare.

The proposition distinguishes two scenarios based on which firm has the lower marginal cost. In the first case, where the platform's cost is higher than the third-party's, the profit gain from increasing the fee depends on whether δ_0 is above or below both 0.5 and δ^*. Specifically, when the ranking δ_0 is greater than 0.5 and also greater than δ^*, the marginal profit gain is relatively low. However, if δ_0 lies between δ^* and 0.5, then the profit gain is higher, and when δ_0 is less than both 0.5 and δ^*, the profit gain becomes smaller again. In the second case, where the third-party firm has the higher cost, a similar pattern emerges with profit gains switching depending on whether the platform's ranking advantage δ_0 lies above or below δ^* and 0.5.

An interesting feature is that if the commission fee continues to increase, it can endogenously shift the value of δ^*, potentially reversing the relationship between δ_0 and δ^*. For example, when the platform's cost is higher than the third-party's ($c_1 > c_3$), a low value of δ_0 may initially not align with what maximizes consumer utility. However, as the fee f increases, the resulting changes in prices may shift δ^* in such a way that it becomes equal to the originally chosen δ_0. In that case, the ranking choice may appear to match consumer interests. However, this does not imply that consumer welfare has improved. As shown in Proposition 5, increasing f reduces the utility of non-searchers for the platform, while the utility from the third-party product depends on the size of the term $\frac{\lambda}{(1-\delta)(1-\lambda)}$. A similar logic applies when $c_1 < c_3$ and δ_0 is set too high as continued increases in f may push δ^* upward, making it equal to δ_0, even though the overall effect on consumer utility may still be negative.

Another regulatory concern arises when the third-party firm has a higher marginal cost than the platform ($c_3 > c_1$), and the platform sets its product's display probability δ_0 just below the consumer-optimal level δ^*, within the range $0.5 < \delta_0 < \delta^*$, either by its own strategic choice or due to regulatory intervention. In this case, the platform has a strong incentive to raise the commission fee, as the marginal profit from doing so is relatively high, and the resulting shift may still align with consumer interests. However, when the platform has a cost disadvantage ($c_1 > c_3$) and chooses δ_0 such that $\delta^* < \delta_0 < 0.5$, it may similarly have a strong incentive to raise the fee even though doing so would be harmful to consumers, despite δ_0 being below 0.5. This underscores the need for regulators to monitor not only ranking algorithms but also the interaction between rankings and fee structures, especially in cases where small deviations from optimal ranking can generate large profit incentives without triggering obvious consumer harm.

7 Conclusion

Online platforms such as Amazon have become central to modern commerce, not only facilitating transactions between buyers and sellers but also participating directly as sellers of their own private-label or in-house products alongside independent third-party offerings. This dual role as both market-maker and market participant gives platforms considerable control over the competitive environment, raising questions about fairness and market power. One widely scrutinized practice is self-preferencing, in which the platform favors its own products in search results, recommendations, or labeling. Examples include the use of tags such as "Amazon's Choice" or ranking algorithms that place Amazon-branded products more

prominently than those of competitors.

This paper examines how a platform that also sells its own product chooses to allocate visibility between itself and a competing third-party seller when consumers differ in their search behavior. The analysis builds on classic models of price dispersion with search frictions by making the ranking decision endogenous and allowing for cost asymmetries and commission fees. We show that changing the platform's visibility affects market prices through two distinct forces: the size of its locked-in non-searcher base and the rival's pricing incentives. When greater visibility increases the platform's guaranteed demand, it softens its competition for searchers and raises its price; the rival then follows suit, leading to higher market prices. Conversely, when the rival is the one constrained by the lowest price it can profitably set, giving the rival more visibility prompts both sellers to lower prices. The platform's optimal ranking choice balances the gain from securing more non-searchers against the impact on market-wide prices.

From a policy perspective, self-preferencing has attracted growing regulatory attention as potentially anti-competitive, yet its economic implications remain incompletely understood. These results imply that regulating platforms toward ranking neutrality, a policy often proposed to address self-preferencing, can have unintended consequences. Depending on relative costs, the share of consumers who compare prices, and the commission fee, such regulation can either improve or reduce consumer welfare, and in some cases can even increase the platform's profits at consumers' expense. Our analysis shows that regulating ranking practices with the aim of promoting neutrality can be misleading, as such interventions may in some circumstances benefit the platform while harming consumers.

While our model captures key strategic interactions in a simple duopoly setting, it abstracts from environments in which additional sellers can enter the market. Allowing for entry would introduce new competitive dynamics, as the platform's ranking choice would then determine not only the distribution of non-searchers between two incumbents but also which entrants gain visibility and survive. This could either intensify price competition if multiple low-cost sellers gain prominence or soften it if the platform favors a small set of sellers, including itself, to maintain higher margins. Extending the analysis to such multi-seller settings, possibly combined with product differentiation or dynamic pricing adjustments beyond the mixed-strategy equilibrium, would be another direction for future research.

Overall, the findings highlight that the welfare effects of self-preferencing depend not only on whether it occurs, but also on how market structure and consumer search behavior interact—underscoring the need for careful, evidence-based policy design in online marketplace regulation.

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Appendix A Search Result in Amazon

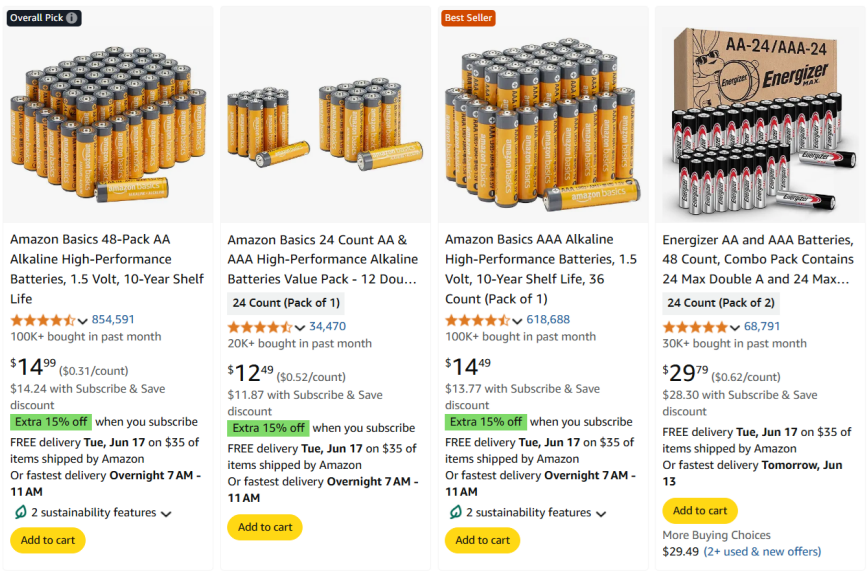


Figure 7: Amazon search results for "batteries" on June 12, 2025.

Figure 7 shows that Amazon's private-label product appears in the top-left position of the search results.

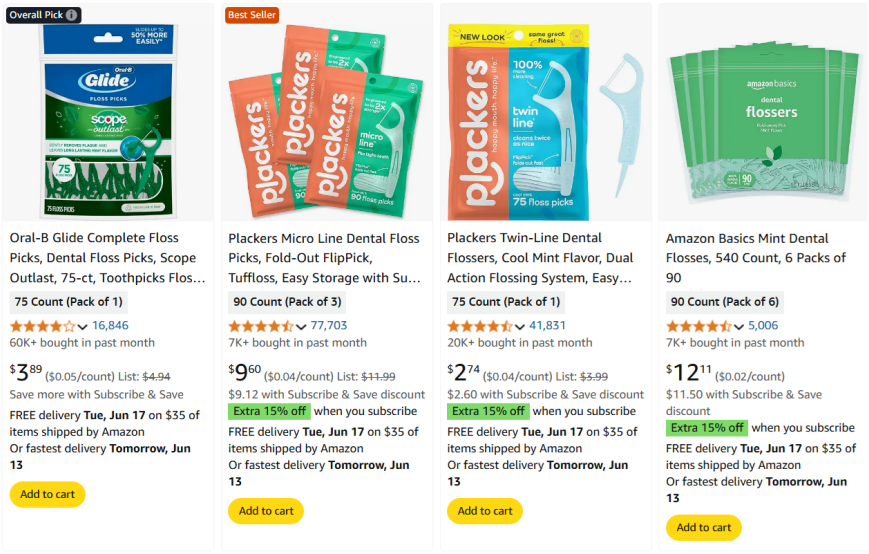


Figure 8: Amazon search results for "flossers" on June 12, 2025.

In contrast, Figure 8 shows a case in which Amazon does not place its own product at the top of the results. The Amazon-branded flosser appears on the right-hand side, following several third-party sellers' products displayed on the left. In this example, Amazon's product has a unit price of \$0.02, compared with unit prices of \$0.05 and \$0.04 for third-party sellers.

Appendix B Details about Definition 1

For firm 1 (the platform), m_1 solves:

$$c_1 < m_1 < r \text{ and } (\delta(1-\lambda)+\lambda)\pi_1(m_1) = \delta(1-\lambda)\pi_1(r) + \lambda \cdot f$$

Thus,

$$\begin{aligned}
(\delta(1-\lambda)+\lambda)\pi_1(m_1) &= \delta(1-\lambda)\pi_1(r) + \lambda \cdot f \\
(\delta(1-\lambda)+\lambda)(m_1 - c_1) &= \delta(1-\lambda)(r-c_1) + \lambda \cdot f \\
(\delta(1-\lambda)+\lambda) \cdot m_1 &= (\delta(1-\lambda)+\lambda) \cdot c_1 + \delta(1-\lambda) \cdot r - \delta(1-\lambda) \cdot c_1 + \lambda \cdot f \\
&= \delta(1-\lambda) \cdot r + \lambda \cdot (c_1 + f) \\
&= (\delta(1-\lambda)+\lambda) \cdot r - \lambda \cdot (r - c_1 - f) \\
m_1 &= r - \lambda \cdot \frac{r - c_1 - f}{\delta(1-\lambda)+\lambda}.
\end{aligned}$$

Similarly, for firm 3 (the third-party seller), m_3 solves:

$$c_3 < m_3 < r \text{ and } ((1-\delta)(1-\lambda)+\lambda)\pi_3(m_3) = (1-\delta)(1-\lambda)\pi_3(p_3^M).$$

Thus,

$$\begin{aligned}
((1-\delta)(1-\lambda)+\lambda)(m_3 - c_3 - f) &= (1-\delta)(1-\lambda)(r - c_3 - f) \\
((1-\delta)(1-\lambda)+\lambda) \cdot m_3 &= ((1-\delta)(1-\lambda)+\lambda) \cdot (c_3 + f) + (1-\delta)(1-\lambda) \cdot r \\
&\quad - (1-\delta)(1-\lambda) \cdot (c_3 + f) \\
((1-\delta)(1-\lambda)+\lambda) \cdot m_3 &= \lambda \cdot (c_3 + f) + (1-\delta)(1-\lambda) \cdot r + \lambda \cdot r - \lambda \cdot r \\
((1-\delta)(1-\lambda)+\lambda) \cdot m_3 &= ((1-\delta)(1-\lambda)+\lambda) \cdot r - \lambda \cdot (r - c_3 - f) \\
m_3 &= r - \lambda \cdot \frac{r - c_3 - f}{(1-\delta)(1-\lambda)+\lambda} \\
&= r - \lambda \cdot \frac{r - c_3 - f}{(1-\lambda) - \delta(1-\lambda)+\lambda} \\
&= r - \lambda \cdot \frac{r - c_3 - f}{1 - \delta(1-\lambda)} \\
&= r - \frac{\lambda}{1-\lambda} \cdot \frac{r - c_3 - f}{\frac{1}{1-\lambda} - \delta}.
\end{aligned}$$

Appendix C Proof of Proposition 1

For any fixed (δ, λ, f), the pricing subgame is a two–player simultaneous-move game in which firms A and E choose prices from compact intervals $[c_1, r]$ and $[c_3 + f, r]$, respectively. Because searchers buy from the lowest-priced firm and tie–breaking is deterministic, each firm's demand exhibits a discontinuity at price equalities. Hence payoffs are not continuous, and standard fixed-point theorems do not apply. Equilibrium must therefore be established constructively, following the method of Varian 1980, Golding and Slutsky 1999, and Spiegler 2006.

Denote by F_1 and F_3 the cumulative distributions over prices chosen by firms A and E. Let m_1 and m_3 denote the lowest prices in the supports of F_1 and F_3, and let p_1^M and p_3^M denote the respective monopoly prices. We show that the pricing game admits a mixed-strategy Nash equilibrium with supports $[m, r]$ for both firms and with the CDFs stated in the proposition.

Step 1. No firm places an atom strictly below r. Suppose firm i places an atom on some $p < r$. If no mass is placed by the opponent in a neighborhood above p, a profitable deviation exists by shifting the atom slightly upward: searcher demand remains unchanged while revenue strictly increases. If instead the opponent places positive weight arbitrarily close above p, then the opponent can profitably shift that mass slightly downward. Hence no atom can occur below r, and both strategies are continuous on $[m_i, r)$.

Step 2. The supremum of both supports is r. If a firm's supreme price $p_H < r$, then by Step 1 the opponent assigns no mass to $(p_H, r]$. At p_H the firm strictly earns less than at r, because pricing at r retains the same non-searcher demand while extracting a strictly larger markup. This contradicts optimality. Hence the upper endpoint of both supports must be r.

Step 3. Indifference within the support and consistency across firms. Let $\Pi_i(p)$ denote firm i's expected profit conditional on choosing the pure price p. In any mixed–strategy equilibrium, every price in the support of F_i must yield the same expected payoff; otherwise firm i would strictly prefer one price in its support to another and would eliminate the inferior price from its strategy. Hence, for each firm $i \in \{1, 3\}$, there exists a constant K_i such that

$$\Pi_i(p) = K_i \qquad \text{for all } p \in \text{supp}(F_i).$$

This establishes the indifference condition that characterizes the equilibrium mixed strategies.

Step 4. Determination of the common lower bound m. Since $F_i(m) = 0$, expected profits at the lower bound must equal the constant equilibrium payoff K_i for each firm. For firm E, choosing $p = m$ yields

$$K_3 \;=\; ((1-\delta)(1-\lambda) + \lambda)\; \pi_3(m),$$

because at $p = m$ all searchers purchase from E.

Similarly, for firm A we obtain

$$K_1 \;=\; (\delta(1-\lambda)+\lambda)\;\pi_1(m) + (1-\delta)(1-\lambda)\,f,$$

because non–searchers purchase from A with probability δ.

These boundary conditions uniquely determine the common lower bound m as the price at which each firm obtains its equilibrium payoff when facing no probability of being undercut. The value of m then enters the indifference conditions that characterize the equilibrium distributions.

Step 5. Strict monotonicity of both CDFs on $[m, r)$. The probability that firm j is undercut when it chooses price p_j is $\Pr(p_i < p_j) = F_i(p_j)$. If F_i is flat on an interval (p, p'), then $F_i(p) = F_i(p')$, which means firm i assigns zero probability to all prices between p and p'. Consequently, firm j faces exactly the same probability of being undercut at p as at p'. If both p and p' belonged to the support of F_j, then both prices would have to yield the same expected profit to firm j (the indifference condition for mixed strategies). Hence we may treat p and p' as candidate best responses. However, since firm j faces the same probability of being undercut at both prices but earns a strictly higher markup at p', it follows that $\Pi_j(p') > \Pi_j(p)$, contradicting the requirement that both p and p' be optimal. Therefore no such flat interval can occur in equilibrium, and F_i must be strictly increasing on $[m, r)$.

Step 6. Solving the indifference conditions yields the unique CDFs. Since firms are using a mixed strategy, the expected profit at any price p must be equal to the expected profit at the lowest competitive price m_i. This ensures that firm E is indifferent across all prices in the range $m \le p < r$.

Denote this constant by $\mathbb{E}\Pi_3(p) = ((1-\delta)(1-\lambda) + \lambda(1-F_1))\,\pi_3(p) = K_3$. Since $F_1(m) = 0$, we obtain $K_3 = ((1-\delta)(1-\lambda)+\lambda)\cdot\pi_3(m)$. Firm E's expected profit at price p is equal to a weighted fraction of its monopoly profit at m_3. This fraction depends on λ, the proportion of consumers with low search costs. Rearranging this condition leads to the cumulative price distribution function for firm A:

$$\begin{aligned}
F_1(p) &= \frac{\lambda + (1-\delta)(1-\lambda)}{\lambda} - \frac{1}{\lambda}\cdot\frac{K_3}{\pi_3(p)} \\
&= \frac{\lambda + (1-\delta)(1-\lambda)}{\lambda} - \frac{1}{\lambda}\cdot\frac{((1-\delta)(1-\lambda)+\lambda)\cdot\pi_3(m)}{\pi_3(p)} \\
&= \frac{\lambda + (1-\delta)(1-\lambda)}{\lambda}\cdot\left(1 - \frac{\pi_3(m)}{\pi_3(p)}\right).
\end{aligned}$$

This function describes the probability that firm A sets a price lower than p. The profit earned from the commission for firm A is $((1-\delta)(1-\lambda)+\lambda\cdot F_3(p))\cdot f$. Denote $\mathbb{E}\Pi_1(p) = (\delta(1-\lambda)+\lambda(1-F_3(p)))\,\pi_1(p) + ((1-\delta)(1-\lambda)+\lambda\cdot F_3(p))\cdot f = K_1$. Since $F_3(m) = 0$, we obtain $K_1 = (\delta(1-\lambda)+\lambda)\,\pi_1(m) + (1-\delta)(1-\lambda)\cdot f$. Thus,

$$\begin{aligned}
(f - \pi_1(p))\cdot F_3(p) &= \frac{\delta(1-\lambda)+\lambda}{\lambda}\cdot(\pi_1(m) - \pi_1(p)) \\
F_3(p) &= \frac{\delta(1-\lambda)+\lambda}{\lambda}\cdot\frac{\pi_1(p) - \pi_1(m)}{\pi_1(p) - f}.
\end{aligned}$$

Both F_1 and F_3 are continuous, strictly increasing on $[m, r)$, and satisfy $F_i(m) = 0$ and $F_i(r) = 1$. They are therefore valid cumulative distributions.

Step 7. Prices outside the support are strictly suboptimal. For any $p < m$, either the markup is too small or the firm loses too many non–searcher customers, implying $\Pi_i(p) < K_i$. For any $p > r$, demand is zero by assumption, so profit is zero $< K_i$. Thus no profitable deviation exists outside the support.

The strategies (F_1, F_3) therefore constitute a mixed-strategy Nash equilibrium in the pricing subgame. □

Appendix D Why the Fee Appears with the Same Sign in Both Strategies

In this section, we provide a simple example to illustrate why the fee appears with the same sign in both firms' equilibrium strategies.

Consider a two-firm pricing game in which each firm chooses either a low price (L) or a high price (H). All consumers are searchers, so that they purchase from the firm offering the lower price. If both firms choose the same price, they split the market evenly. Let p_L and p_H denote the low and high price options, respectively, with $p_H > p_L$. Each firm $i \in \{1, 3\}$ faces marginal cost c_i, where $c_i < p_L$. If Firm 3 sells any quantity, it must pay a per-unit commission fee f to Firm 1. The payoff matrix, with entries denoted as $[\pi_1, \pi_3]$, is as follows:

	Firm 3: L	**Firm 3: H**
Firm 1: L	$\left[\frac{1}{2}(p_L - c_1) + \frac{f}{2},\ \frac{1}{2}(p_L - c_3 - f)\right]$	$[p_L - c_1,\ 0]$
Firm 1: H	$[f,\ p_L - c_3 - f]$	$\left[\frac{1}{2}(p_H - c_1) + \frac{f}{2},\ \frac{1}{2}(p_H - c_3 - f)\right]$

Table 1: Payoff matrix for Firm 1 and Firm 3 under pricing competition with commission.

Let x^* be the probability that Firm 1 plays L, and y^* be the probability that Firm 3 plays L. The unique mixed strategy Nash equilibrium is given by:

$$x^* = 1 - \frac{(p_L - c_3 - f)}{p_H - p_L}, \qquad y^* = 1 - \frac{(p_L - c_1 - f)}{p_H - p_L}$$

These probabilities ensure that each firm is indifferent between its two pricing strategies, given the mixing behavior of the other.

At first glance, it may seem surprising that f enters both expressions with the same (negative) sign, given that Firm 3 *pays* the fee while Firm 1 *receives* it. However, this symmetry arises from how f affects each firm's incentives to compete aggressively by choosing the low price. For Firm 3, the fee directly increases its marginal cost. The profit from pricing low is reduced by f, making aggressive pricing less attractive. Hence, its probability y^* of pricing low decreases as f increases. For Firm 1, the fee provides revenue when Firm 3 wins the market. This passive income makes pricing high more attractive. To maintain indifference, Firm 1 must also price low less often. Thus, x^* also decreases in f.

While the economic effects of the fee differ—raising costs for one firm and creating passive gains for the other—the strategic consequence is the same: both firms become less aggressive as f increases. This is why f enters both expressions in the same algebraic direction.

Appendix E Proof of Lemma 2

Consider the case $\delta < \delta^*$. Then, $m_3 > m_1$ and $f_1^0 = 0$. Note that

$$m_3 = r - \lambda \cdot \frac{r - c_3 - f}{(1-\delta)(1-\lambda) + \lambda}$$
$$\frac{dm_3}{d\delta} = -\frac{\lambda(1-\lambda)}{((1-\delta)(1-\lambda)+\lambda)^2} \cdot (r - c_3 - f) < 0$$

$$\begin{aligned}
\frac{d\hat{F}_1}{d\delta} &= -\frac{1-\lambda}{\lambda} \cdot \frac{p - m_3}{p - c_3 - f} - \frac{\lambda + (1-\delta)(1-\lambda)}{\lambda} \cdot \frac{1}{p - c_3 - f} \cdot \frac{dm_3}{d\delta} \\
&= -\frac{1}{\lambda} \cdot \frac{1}{p - c_3 - f} \cdot \left((1-\lambda) \cdot (p - m_3) + (\lambda + (1-\delta)(1-\lambda)) \cdot \frac{dm_3}{d\delta} \right) \\
&= -\frac{1}{\lambda} \cdot \frac{1}{p - c_3 - f} \cdot \left((1-\lambda) \cdot (p - m_3) - \frac{\lambda(1-\lambda)}{(1-\delta)(1-\lambda)+\lambda} \cdot (r - c_3 - f) \right) \\
&= -\frac{1-\lambda}{\lambda} \cdot \frac{1}{p - c_3 - f} \cdot \left(p - \left(r - \lambda \cdot \frac{r - c_3 - f}{(1-\delta)(1-\lambda)+\lambda} \right) - \frac{\lambda}{(1-\delta)(1-\lambda)+\lambda} \cdot (r - c_3 - f) \right) \\
&= -\frac{1-\lambda}{\lambda} \cdot \frac{1}{p - c_3 - f} \cdot (p - r) \\
&= \frac{1-\lambda}{\lambda} \cdot \frac{1}{p - c_3 - f} \cdot (r - p) \\
&> 0
\end{aligned}$$

$$\begin{aligned}
\frac{d\hat{F}_3}{d\delta} &= \frac{1-\lambda}{\lambda} \cdot \frac{p - m_3}{p - c_1 - f} - \frac{\lambda + \delta(1-\lambda)}{\lambda} \cdot \frac{1}{p - c_1 - f} \cdot \frac{dm_3}{d\delta} \\
&= \frac{1}{\lambda} \cdot \frac{1}{p - c_1 - f} \cdot \left((1-\lambda)(p - m_3) - (\lambda + \delta(1-\lambda)) \cdot \frac{dm_3}{d\delta} \right) \\
&= \frac{1}{\lambda} \cdot \frac{1}{p - c_1 - f} \cdot \left((1-\lambda)(p - m_3) + (\lambda + \delta(1-\lambda)) \cdot \frac{\lambda(1-\lambda)}{((1-\delta)(1-\lambda)+\lambda)^2} \cdot (r - c_3 - f) \right) \\
&= \frac{1-\lambda}{\lambda} \cdot \frac{1}{p - c_1 - f} \cdot \left((p - m_3) + \frac{\lambda + \delta(1-\lambda)}{(1-\delta)(1-\lambda)+\lambda} \cdot \frac{\lambda}{(1-\delta)(1-\lambda)+\lambda} \cdot (r - c_3 - f) \right) \\
&= \frac{1-\lambda}{\lambda} \cdot \frac{1}{p - c_1 - f} \cdot \left((p - m_3) + \frac{\lambda + \delta(1-\lambda)}{(1-\delta)(1-\lambda)+\lambda} \cdot (r - m_3) \right) \\
&= \frac{1-\lambda}{\lambda} \cdot \frac{1}{p - c_1 - f} \cdot \left((r - p) \cdot \frac{\lambda + \delta(1-\lambda)}{(1-\delta)(1-\lambda)+\lambda} + \frac{1+\lambda}{(1-\delta)(1-\lambda)+\lambda} \cdot (p - m_3) \right) \\
&> 0
\end{aligned}$$

$$\begin{aligned}
\frac{df_3^0}{d\delta} &= -\frac{1-\lambda}{\lambda}\cdot\frac{r-m_3}{r-c_1-f} + \frac{\lambda+\delta(1-\lambda)}{\lambda}\cdot\frac{1}{r-c_1-f}\cdot\frac{dm_3}{d\delta} \\
&= -\frac{1}{\lambda}\cdot\frac{1}{r-c_1-f}\cdot\left((1-\lambda)(r-m_3) - (\lambda+\delta(1-\lambda))\cdot\frac{dm_3}{d\delta}\right) \\
&= -\frac{1-\lambda}{\lambda}\cdot\frac{1}{r-c_1-f}\cdot\left((r-r)\cdot\frac{\lambda+\delta(1-\lambda)}{(1-\delta)(1-\lambda)+\lambda} + \frac{1+\lambda}{(1-\delta)(1-\lambda)+\lambda}\cdot(p-m_3)\right) \\
&= -\frac{1-\lambda}{\lambda}\cdot\frac{1}{r-c_1-f}\cdot\frac{1+\lambda}{(1-\delta)(1-\lambda)+\lambda}\cdot(p-m_3) \\
&< 0
\end{aligned}$$

Thus, an increase in δ induces price distributions F_1 and F_3 that are further first-order stochastically dominated.

Now, consider the case $\delta > \delta^*$. Then, $m_1 > m_3$ and $f_3^0 = 0$. Note that

$$\begin{aligned}
m_1 &= r - \lambda\cdot\frac{r-c_1-f}{\delta(1-\lambda)+\lambda} \\
\frac{dm_1}{d\delta} &= \frac{\lambda(1-\lambda)}{(\delta(1-\lambda)+\lambda)^2}\cdot(r-c_1-f) > 0
\end{aligned}$$

$$\begin{aligned}
\frac{d\hat{F}_1}{d\delta} &= -\frac{1-\lambda}{\lambda}\cdot\frac{p-m_1}{p-c_3-f} - \frac{\lambda+(1-\delta)(1-\lambda)}{\lambda}\cdot\frac{1}{p-c_3-f}\cdot\frac{dm_1}{d\delta} \\
&= -\frac{1}{\lambda}\cdot\frac{1}{p-c_3-f}\cdot\left((1-\lambda)\cdot(p-m_1) + (\lambda+(1-\delta)(1-\lambda))\cdot\frac{dm_1}{d\delta}\right) \\
&= -\frac{1-\lambda}{\lambda}\cdot\frac{1}{p-c_3-f}\cdot\left((p-m_1) + \frac{\lambda+(1-\delta)(1-\lambda)}{\delta(1-\lambda)+\lambda}\cdot\frac{\lambda}{\delta(1-\lambda)+\lambda}\cdot(r-c_1-f)\right) \\
&= -\frac{1-\lambda}{\lambda}\cdot\frac{1}{p-c_3-f}\cdot\left((p-m_1) + \frac{\lambda+(1-\delta)(1-\lambda)}{\delta(1-\lambda)+\lambda}\cdot(r-m_1)\right) \\
&= -\frac{1-\lambda}{\lambda}\cdot\frac{1}{p-c_3-f}\cdot\left((r-p)\cdot\frac{\lambda+(1-\delta)(1-\lambda)}{\delta(1-\lambda)+\lambda} + \frac{1+\lambda}{\delta(1-\lambda)+\lambda}\cdot(p-m_1)\right) \\
&< 0
\end{aligned}$$

$$\begin{aligned}
\frac{df_1^0}{d\delta} &= -\frac{1-\lambda}{\lambda}\cdot\frac{m_1-c_3-f}{r-c_3-f} + \frac{\lambda+(1-\delta)(1-\lambda)}{\lambda}\cdot\frac{1}{r-c_3-f}\cdot\frac{dm_1}{d\delta} + \frac{1-\lambda}{\lambda} \\
&= \frac{1}{\lambda}\cdot\frac{1}{r-c_3-f}\cdot\left((1-\lambda)(r-m_1) + (\lambda+(1-\delta)(1-\lambda))\cdot\frac{dm_1}{d\delta}\right) \\
&= \frac{1-\lambda}{\lambda}\cdot\frac{1}{r-c_3-f}\cdot\left((r-r)\cdot\frac{\lambda+(1-\delta)(1-\lambda)}{\delta(1-\lambda)+\lambda} + \frac{1+\lambda}{\delta(1-\lambda)+\lambda}\cdot(p-m_1)\right) \\
&= \frac{1-\lambda}{\lambda}\cdot\frac{1}{r-c_3-f}\cdot\frac{1+\lambda}{\delta(1-\lambda)+\lambda}\cdot(p-m_1) \\
&> 0
\end{aligned}$$

$$
\begin{aligned}
\frac{d\hat{F}_3}{d\delta} &= \frac{1-\lambda}{\lambda}\cdot\frac{p-m_1}{p-c_1-f} - \frac{\lambda+\delta(1-\lambda)}{\lambda}\cdot\frac{1}{p-c_1-f}\cdot\frac{dm_1}{d\delta} \\
&= \frac{1}{\lambda}\cdot\frac{1}{p-c_1-f}\cdot\left((1-\lambda)(p-m_1) - (\lambda+\delta(1-\lambda))\cdot\frac{dm_1}{d\delta}\right) \\
&= \frac{1}{\lambda}\cdot\frac{1}{p-c_1-f}\cdot\left((1-\lambda)(p-m_1) - (\lambda+\delta(1-\lambda))\cdot\frac{\lambda(1-\lambda)}{(\delta(1-\lambda)+\lambda)^2}\cdot(r-c_1-f)\right) \\
&= \frac{1-\lambda}{\lambda}\cdot\frac{1}{p-c_1-f}\cdot\left((p-m_1) - \frac{\lambda+\delta(1-\lambda)}{\delta(1-\lambda)+\lambda}\cdot\frac{\lambda}{\delta(1-\lambda)+\lambda}\cdot(r-c_1-f)\right) \\
&= \frac{1-\lambda}{\lambda}\cdot\frac{1}{p-c_1-f}\cdot((p-m_1)-(r-m_1)) \\
&= -\frac{1-\lambda}{\lambda}\cdot\frac{1}{p-c_1-f}\cdot(r-p) \\
&< 0
\end{aligned}
$$

Thus, when $\delta > \delta^*$, a decrease in δ induces distributions F_1 and F_3 that are further first-order stochastically dominated.

$$
\frac{d\hat{F}_1}{d\delta}\begin{cases} < 0 & \text{if } \delta > \delta^* \\ > 0 & \text{if } \delta < \delta^* \end{cases}
$$
$$
\frac{d\hat{F}_3}{d\delta}\begin{cases} < 0 & \text{if } \delta > \delta^* \\ > 0 & \text{if } \delta < \delta^* \end{cases}
$$

Appendix F Proof of Proposition 2

$$
\mathbb{E}\Pi_1 = \begin{cases} \delta(1-\lambda)\cdot(r-c_1) + ((1-\delta)(1-\lambda)+\lambda)\cdot f & m = m_1 \\ (\delta(1-\lambda)+\lambda)\cdot(m_3-c_1) + (1-\delta)(1-\lambda)\cdot f & m = m_3 \end{cases}
$$

$$
\frac{d\mathbb{E}\Pi_1}{d\delta} = \begin{cases} (1-\lambda)\cdot(r-c_1-f) & m = m_1 \\ (1-\lambda)\cdot(r-c_1-f) & c_1 \geq c_3 \text{ and } m = r \\ (1-\lambda)\cdot(r-c_1-f) & c_1 \leq c_3 \text{ and } m = r \\ (1-\lambda)\cdot(m_3-c_1) + (\delta(1-\lambda)+\lambda)\cdot\frac{dm_3}{d\delta} - (1-\lambda)\cdot f & m = m_3 \end{cases}
$$

Since

$$
m_3 = r - \lambda\cdot\frac{r-c_3-f}{(1-\delta)(1-\lambda)+\lambda}
$$

Thus,

$$\frac{dm_3}{d\delta} = -\frac{\lambda(1-\lambda)}{((1-\delta)(1-\lambda)+\lambda)^2} \cdot (r - c_3 - f)$$

When $m = m_3$,

$$\begin{aligned}
\frac{d\mathbb{E}\Pi_1}{d\delta} \geq 0 &\Leftrightarrow (1-\lambda)\cdot(m_3 - c_1) + (\delta(1-\lambda)+\lambda)\cdot\frac{dm_3}{d\delta} - (1-\lambda)\cdot f \geq 0 \\
&\Leftrightarrow (m_3 - c_1) - \frac{\lambda(\delta(1-\lambda)+\lambda)}{((1-\delta)(1-\lambda)+\lambda)^2}\cdot(r - c_3 - f) - f \geq 0 \\
&\Leftrightarrow r - \lambda\cdot\frac{r - c_3 - f}{(1-\delta)(1-\lambda)+\lambda} - c_1 - f \geq \frac{\lambda(\delta(1-\lambda)+\lambda)}{((1-\delta)(1-\lambda)+\lambda)^2}\cdot(r - c_3 - f) \\
&\Leftrightarrow (r - c_1 - f)\cdot((1-\delta)(1-\lambda)+\lambda) \geq \lambda\cdot\left(1 + \frac{(\delta(1-\lambda)+\lambda)}{(1-\delta)(1-\lambda)+\lambda}\right)\cdot(r - c_3 - f) \\
&\Leftrightarrow (r - c_1 - f) \geq \frac{\lambda(1+\lambda)}{((1-\delta)(1-\lambda)+\lambda)^2}\cdot(r - c_3 - f)
\end{aligned}$$

Thus, the optimal value of δ is determined as follows:

1. $\delta = 1$ if $\delta^*(1-\delta^*) \geq \left(\frac{\lambda}{1-\lambda}\right)^2$ or $\delta^* < \dot{\delta}$.
2. If $LHS < RHS|_{\delta=0}$, the platform chooses between $\delta = 1$ or $\delta = 0$ by comparing $\Pi_1|_{\delta=1}$ and $\Pi_1|_{\delta=0}$.
3. Otherwise, i.e., if $RHS|_{\delta=0} < LHS < RHS|_{\delta=\delta^*}$ and $\delta^* > \dot{\delta}$, the platform chooses between $\delta = 1$ or $\delta = \dot{\delta}$ by comparing $\Pi_1|_{\delta=1}$ and $\Pi_1|_{\delta=\dot{\delta}}$.

Appendix G m_1 and m_3 Curve with Respect to δ

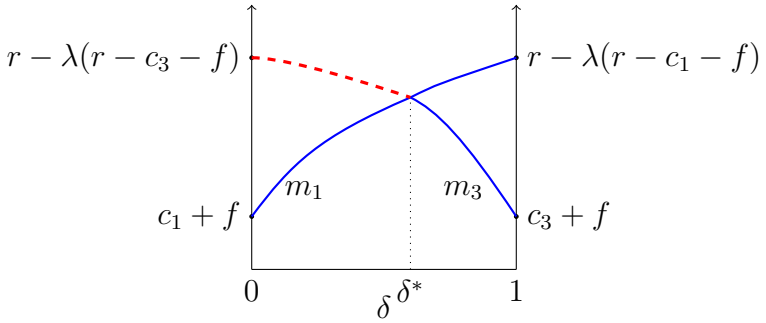
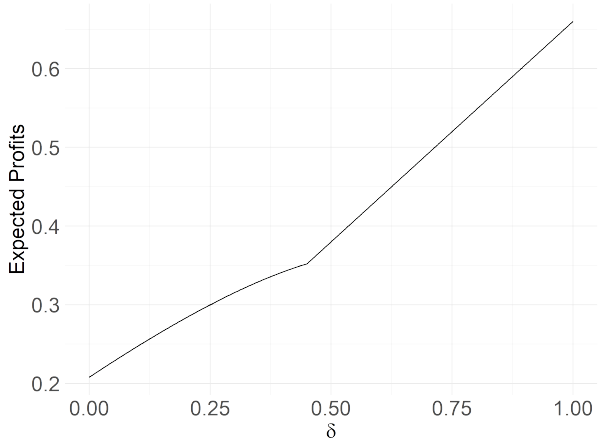
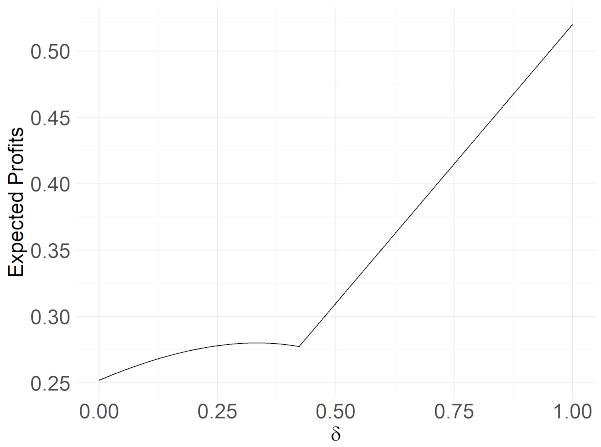


Figure 9: m_1 and m_3 as functions of δ.

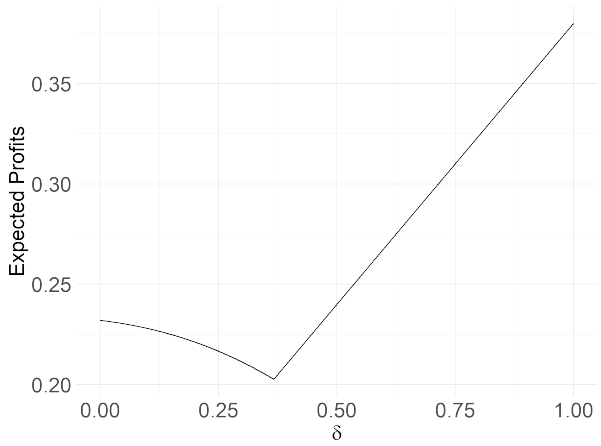
Appendix H Expected Profit Curve with Respect to δ



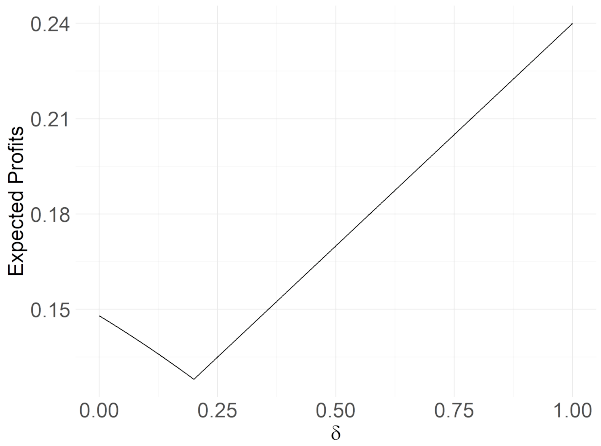
(a) $\lambda = 0.2$
$m_1 < m_3$ for small δ; $m_1 > m_3$ for big δ



(b) $\lambda = 0.4$

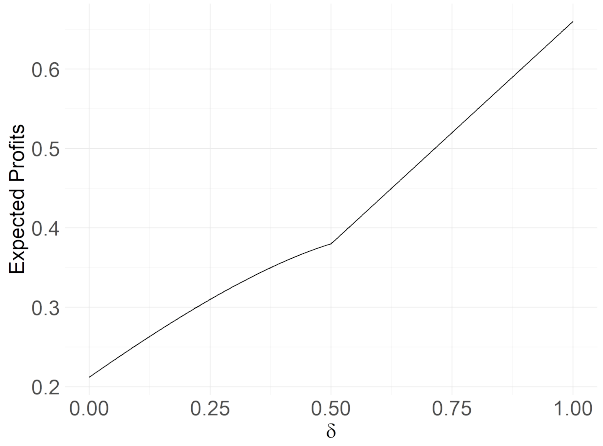


(c) $\lambda = 0.6$

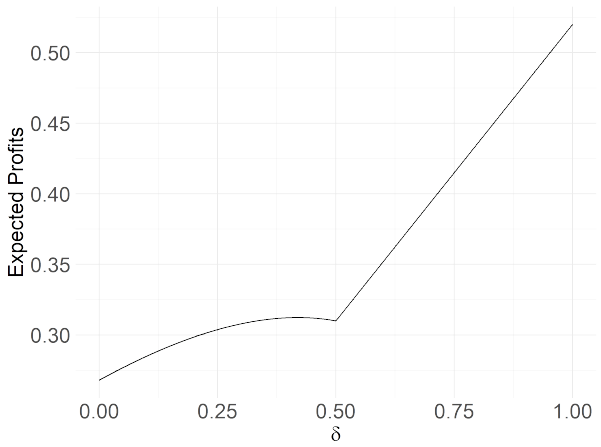


(d) $\lambda = 0.8$

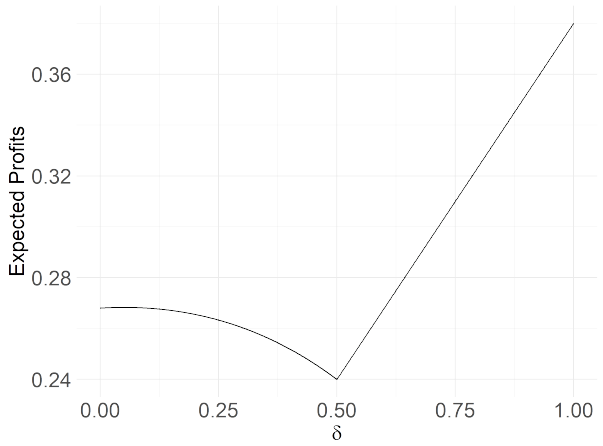
Figure 10: Expected Profit for Firm A when $r = 1$, $f = 0.1$, $c_1 = 0.2$, $c_3 = 0.1$



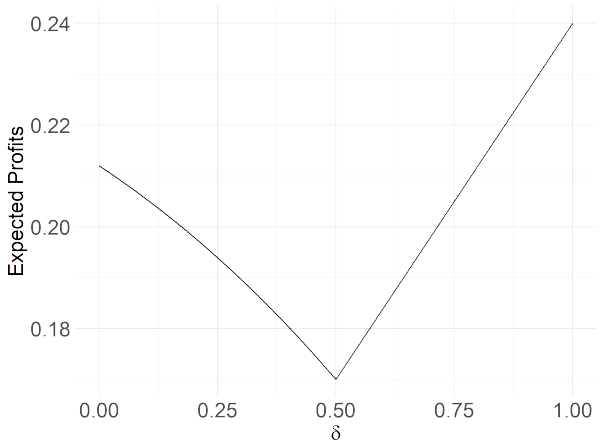
(a) $\lambda = 0.2$
$m_1 < m_3$ for small δ; $m_1 > m_3$ for big δ



(b) $\lambda = 0.4$

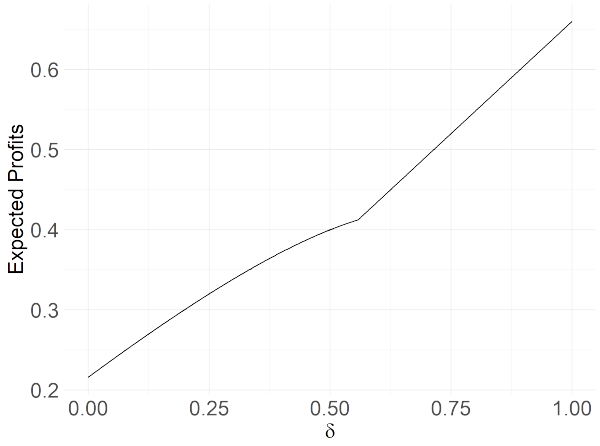


(c) $\lambda = 0.6$

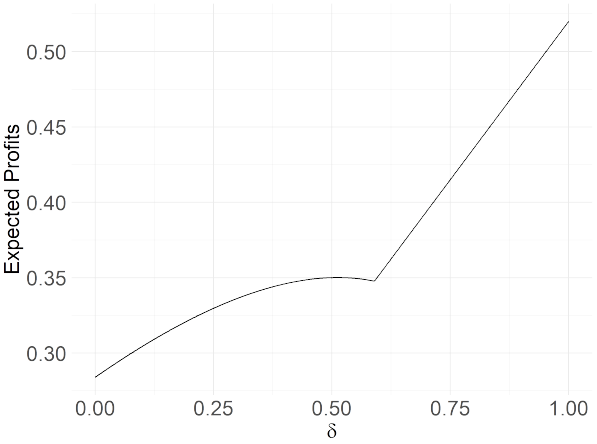


(d) $\lambda = 0.8$

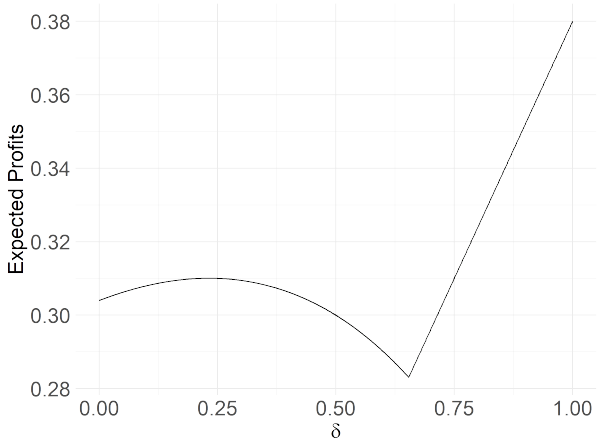
Figure 11: Expected Profit for Firm A when $r = 1$, $f = 0.1$, $c_1 = 0.2$, $c_3 = 0.2$



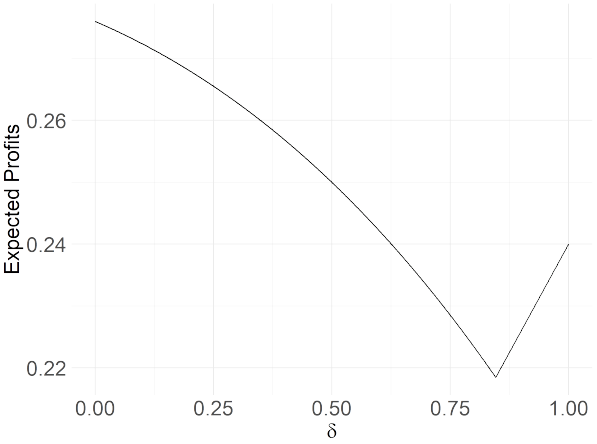
(a) $\lambda = 0.2$
$m_1 < m_3$ for small δ; $m_1 > m_3$ for big δ



(b) $\lambda = 0.4$

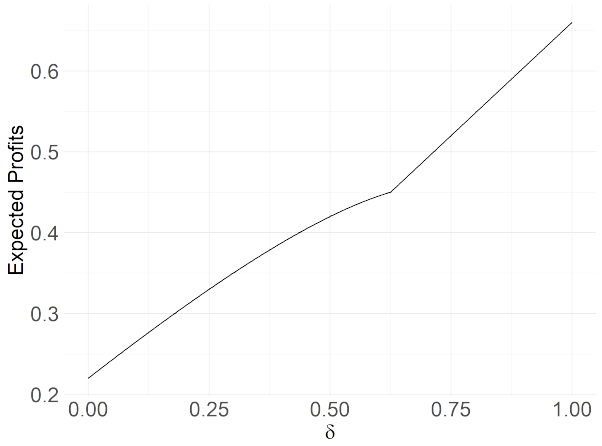


(c) $\lambda = 0.6$

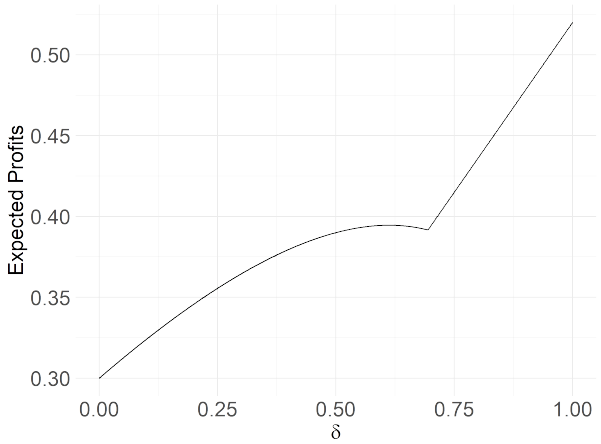


(d) $\lambda = 0.8$

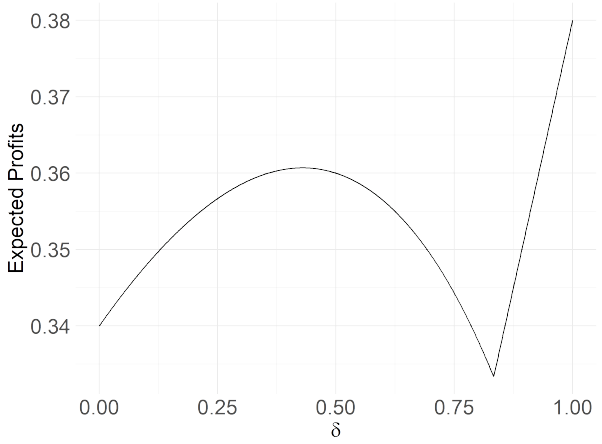
Figure 12: Expected Profit for Firm A when $r = 1$, $f = 0.1$, $c_1 = 0.2$, $c_3 = 0.3$



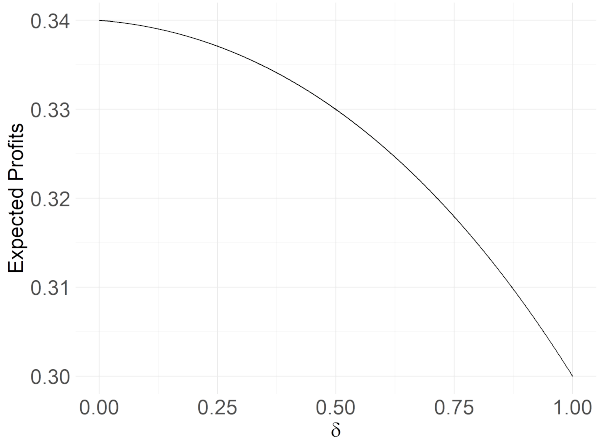
(a) $\lambda = 0.2$
$m_1 < m_3$ for small δ; $m_1 > m_3$ for big δ



(b) $\lambda = 0.4$



(c) $\lambda = 0.6$



(d) $\lambda = 0.8$
$m_1 < m_3$ for all δ

Figure 13: Expected Profit for Firm A when $r = 1$, $f = 0.1$, $c_1 = 0.2$, $c_3 = 0.4$

Appendix I Proof of Proposition 3

Note that

$$\delta^* = \frac{(r - c_1 - f) - \lambda(r - c_3 - f)}{(1-\lambda)(r - c_1 - f + r - c_3 - f)}$$

$$\dot{\delta} = \frac{1}{1-\lambda} - \frac{1}{1-\lambda} \cdot \sqrt{\lambda(1+\lambda) \cdot \frac{r - c_3 - f}{r - c_1 - f}}$$

$$\begin{aligned}
\delta^* \geq \dot{\delta} &\Leftrightarrow \frac{r_1 - \lambda r_3}{(1-\lambda)(r_1 + r_3)} \geq \frac{1}{1-\lambda}\left(1 - \sqrt{\lambda(1+\lambda) \cdot \frac{r_3}{r_1}}\right) \\
&\Leftrightarrow \sqrt{\lambda(1+\lambda) \cdot \frac{r_3}{r_1}} \geq 1 - \frac{r_1 - \lambda r_3}{r_1 + r_3} = (1+\lambda) \cdot \frac{r_3}{r_1 + r_3} \\
&\Leftrightarrow \lambda \cdot \frac{1}{r_1} \geq (1+\lambda) \cdot \frac{r_3}{(r_1 + r_3)^2} \\
&\Leftrightarrow \frac{\lambda}{1+\lambda} \geq \frac{r_1 \cdot r_3}{(r_1 + r_3)^2} \\
&\Leftrightarrow \frac{1+\lambda}{\lambda} \leq \frac{(r_1 + r_3)^2}{r_1 \cdot r_3} = \frac{r_1^2 + 2r_1 r_3 + r_3^2}{r_1 \cdot r_3} \\
&\Leftrightarrow \frac{1}{\lambda} - 1 \leq \frac{r_1}{r_3} + \frac{r_3}{r_1}
\end{aligned}$$

$$\begin{aligned}
\delta^* \gtreqless 1 &\Leftrightarrow \frac{r_1 - \lambda r_3}{(1-\lambda)(r_1 + r_3)} \gtreqless 1 \\
&\Leftrightarrow r_1 - \lambda r_3 \gtreqless (1-\lambda)(r_1 + r_3) \\
&\Leftrightarrow \lambda \gtreqless \frac{r_3}{r_1} \\
&\Leftrightarrow \frac{r_1}{r_3} \gtreqless \frac{1}{\lambda}
\end{aligned}$$

$$\begin{aligned}
\delta^* \gtreqless 0 &\Leftrightarrow \frac{r_1 - \lambda r_3}{(1-\lambda)(r_1 + r_3)} \gtreqless 0 \\
&\Leftrightarrow r_1 - \lambda r_3 \gtreqless 0 \\
&\Leftrightarrow \frac{r_1}{r_3} \gtreqless \lambda
\end{aligned}$$

$$\begin{aligned}
\dot{\delta} \gtreqless 1 &\Leftrightarrow \frac{1}{1-\lambda}\left(1-\sqrt{\lambda(1+\lambda)\cdot\frac{r_3}{r_1}}\right) \gtreqless 1 \\
&\Leftrightarrow 1-\sqrt{\lambda(1+\lambda)\cdot\frac{r_3}{r_1}} \gtreqless 1-\lambda \\
&\Leftrightarrow \frac{\lambda}{1+\lambda} \gtreqless \frac{r_3}{r_1} \\
&\Leftrightarrow \frac{r_1}{r_3} \gtreqless \frac{1}{\lambda}+1
\end{aligned}$$

$$\begin{aligned}
\dot{\delta} \gtreqless 0 &\Leftrightarrow \frac{1}{1-\lambda}\left(1-\sqrt{\lambda(1+\lambda)\cdot\frac{r_3}{r_1}}\right) \gtreqless 0 \\
&\Leftrightarrow 1-\sqrt{\lambda(1+\lambda)\cdot\frac{r_3}{r_1}} \gtreqless 0 \\
&\Leftrightarrow \frac{1}{\lambda(1+\lambda)} \gtreqless \frac{r_3}{r_1} \\
&\Leftrightarrow \frac{r_1}{r_3} \gtreqless \lambda(1+\lambda)
\end{aligned}$$

$$\begin{aligned}
\delta^* \gtreqless \check{\delta} &\Leftrightarrow \frac{(r-c_1-f)-\lambda(r-c_3-f)}{(1-\lambda)(r-c_1-f+r-c_3-f)} \gtreqless \frac{1}{1-\lambda}\cdot\sqrt{\lambda(1+\lambda)\cdot\frac{r-c_1-f}{r-c_3-f}}-\frac{\lambda}{1-\lambda} \\
&\Leftrightarrow \frac{(r-c_1-f)-\lambda(r-c_3-f)}{(r-c_1-f+r-c_3-f)}+\lambda \gtreqless \sqrt{\lambda(1+\lambda)\cdot\frac{r-c_1-f}{r-c_3-f}} \\
&\Leftrightarrow (1+\lambda)\cdot\frac{(r-c_1-f)^2}{(r-c_1-f+r-c_3-f)^2} \gtreqless \lambda\cdot\frac{r-c_1-f}{r-c_3-f} \\
&\Leftrightarrow \frac{1+\lambda}{\lambda} \gtreqless \frac{(r-c_1-f+r-c_3-f)^2}{(r-c_1-f)(r-c_3-f)} = \frac{r-c_1-f}{r-c_3-f}+2+\frac{r-c_3-f}{r-c_1-f} \\
&\Leftrightarrow \frac{1-\lambda}{\lambda} \gtreqless \frac{r-c_1-f}{r-c_3-f}+\frac{r-c_3-f}{r-c_1-f}
\end{aligned}$$

$$\begin{aligned}
\dot{\delta} \gtreqless \check{\delta} &\Leftrightarrow \frac{1}{1-\lambda}-\frac{1}{1-\lambda}\cdot\sqrt{\lambda(1+\lambda)\cdot\frac{r-c_3-f}{r-c_1-f}} \gtreqless \frac{1}{1-\lambda}\cdot\sqrt{\lambda(1+\lambda)\cdot\frac{r-c_1-f}{r-c_3-f}}-\frac{\lambda}{1-\lambda} \\
&\Leftrightarrow 1-\sqrt{\lambda(1+\lambda)\cdot\frac{r-c_3-f}{r-c_1-f}} \gtreqless \sqrt{\lambda(1+\lambda)\cdot\frac{r-c_1-f}{r-c_3-f}}-\lambda \\
&\Leftrightarrow \sqrt{\frac{1+\lambda}{\lambda}} \gtreqless \sqrt{\frac{r-c_1-f}{r-c_3-f}}+\sqrt{\frac{r-c_3-f}{r-c_1-f}}
\end{aligned}$$

$$\check{\delta} \gtreqless 0 \Leftrightarrow \frac{1}{1-\lambda} \cdot \sqrt{\lambda(1+\lambda) \cdot \frac{r-c_1-f}{r-c_3-f}} - \frac{\lambda}{1-\lambda} \gtreqless 0$$
$$\Leftrightarrow \sqrt{\lambda(1+\lambda) \cdot \frac{r-c_1-f}{r-c_3-f}} \gtreqless \lambda$$
$$\Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \frac{\lambda}{1+\lambda}$$

$$\check{\delta} \gtreqless 1 \Leftrightarrow \frac{1}{1-\lambda} \cdot \sqrt{\lambda(1+\lambda) \cdot \frac{r-c_1-f}{r-c_3-f}} - \frac{\lambda}{1-\lambda} \gtreqless 1$$
$$\Leftrightarrow \sqrt{\lambda(1+\lambda) \cdot \frac{r-c_1-f}{r-c_3-f}} \gtreqless 1$$
$$\Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \frac{1}{\lambda(1+\lambda)}$$

Thus, the following equivalences hold:

$$\check{\delta} \gtreqless \delta^* \gtreqless \dot{\delta} \Leftrightarrow \frac{r-c_1-f}{r-c_3-f} + \frac{r-c_3-f}{r-c_1-f} \gtreqless \frac{1-\lambda}{\lambda}$$
$$\delta^* \gtreqless 1 \Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \frac{1}{\lambda}$$
$$\delta^* \gtreqless 0 \Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \lambda$$
$$\dot{\delta} \gtreqless 1 \Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \frac{1+\lambda}{\lambda}$$
$$\dot{\delta} \gtreqless 0 \Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \lambda(1+\lambda)$$
$$\check{\delta} \gtreqless 1 \Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \frac{1}{\lambda(1+\lambda)}$$
$$\check{\delta} \gtreqless 0 \Leftrightarrow \frac{r-c_1-f}{r-c_3-f} \gtreqless \frac{\lambda}{1+\lambda}$$

Appendix J The Third-party Firm's Profit

$$\mathbb{E}\Pi_3 = \begin{cases} ((1-\delta)(1-\lambda)+\lambda) \cdot (m_1 - c_3 - f) & \text{if } m = m_1 \\ (1-\delta)(1-\lambda) \cdot (r - c_3 - f) & \text{if } m = m_3 \end{cases}$$

If $m_1 \geq m_3$,

$$\begin{aligned}
\frac{d\mathbb{E}\Pi_3}{d\delta} &= -(1-\lambda)\cdot(m_1 - c_3 - f) + ((1-\delta)(1-\lambda)+\lambda)\cdot\frac{dm_1}{d\delta} \\
&= -(1-\lambda)\cdot(m_1 - c_3 - f) + ((1-\delta)(1-\lambda)+\lambda)\cdot\lambda(r - c_1 - f)\cdot\frac{1-\lambda}{(\delta(1-\lambda)+\lambda)^2} \\
&= -(1-\lambda)\cdot(r - c_3 - f) + \frac{\lambda\cdot(1-\lambda)}{\delta(1-\lambda)+\lambda}\cdot(r - c_1 - f) + \frac{((1-\delta)(1-\lambda)+\lambda)\cdot\lambda\cdot(1-\lambda)}{(\delta(1-\lambda)+\lambda)^2}\cdot(r - c_1 - \\
&= -(1-\lambda)\cdot(r - c_3 - f) + \frac{(1+\lambda)\cdot\lambda\cdot(1-\lambda)}{(\delta(1-\lambda)+\lambda)^2}\cdot(r - c_1 - f) \\
&= (1-\lambda)\cdot\left(\frac{(1+\lambda)\cdot\lambda}{(\delta(1-\lambda)+\lambda)^2}\cdot(r - c_1 - f) - (r - c_3 - f)\right)
\end{aligned}$$

Thus,

$$\frac{d\mathbb{E}\Pi_3}{d\delta} = \begin{cases} (1-\lambda)\cdot\left(\frac{(1+\lambda)\cdot\lambda}{(\delta(1-\lambda)+\lambda)^2}\cdot(r - c_1 - f) - (r - c_3 - f)\right) & \text{if } m_1 \geq m_3 \\ -(1-\lambda)\cdot(r - c_3 - f) & \text{if } m_3 \geq m_1 \end{cases}$$

Let $\check{\delta}$ be the value of δ that satisfies the following:

$$\frac{(1+\lambda)\cdot\lambda}{(\delta(1-\lambda)+\lambda)^2}\cdot(r - c_1 - f) = (r - c_3 - f)$$

Specifically,

$$\check{\delta} = \frac{1}{1-\lambda}\cdot\sqrt{\lambda(1+\lambda)\cdot\frac{r - c_1 - f}{r - c_3 - f}} - \frac{\lambda}{1-\lambda}$$

Then for $m_1 \geq m_3$,

$$\frac{d\mathbb{E}\Pi_3}{d\delta} \gtreqless 0 \Leftrightarrow \check{\delta} \gtreqless \delta$$

Thus,

1. If $\check{\delta} \leq \delta^*$, $\frac{d\mathbb{E}\Pi_3}{d\delta} < 0$
2. If $\delta^* < \check{\delta}$, $\frac{d\mathbb{E}\Pi_3}{d\delta} > 0$ for $\delta^* < \delta < \check{\delta}$ and $\frac{d\mathbb{E}\Pi_3}{d\delta} < 0$ otherwise.

Appendix K The derivatives of m_1 and m_3 with respect to δ

$$
\begin{aligned}
m_1 &= r - \frac{\lambda}{1-\lambda} \cdot \frac{r - c_1 - f}{\delta + \frac{\lambda}{1-\lambda}} \\
\frac{dm_1}{d\delta} &= \frac{\lambda}{1-\lambda} \cdot (r - c_1 - f) \cdot \frac{1}{\left(\delta + \frac{1}{1-\lambda}\right)^2} > 0 \\
\frac{d^2 m_1}{d\delta^2} &= -\frac{\lambda}{1-\lambda} \cdot (r - c_1 - f) \cdot \frac{2}{\left(\delta + \frac{1}{1-\lambda}\right)^3} < 0 \\
m_3 &= r - \frac{\lambda}{1-\lambda} \cdot \frac{r - c_3 - f}{\frac{1}{1-\lambda} - \delta} \\
\frac{dm_3}{d\delta} &= -\frac{\lambda}{1-\lambda} \cdot (r - c_3 - f) \cdot \frac{1}{\left(\frac{1}{1-\lambda} - \delta\right)^2} < 0 \\
\frac{d^2 m_3}{d\delta^2} &= -\frac{\lambda}{1-\lambda} \cdot (r - c_3 - f) \cdot \frac{2}{\left(\frac{1}{1-\lambda} - \delta\right)^3} < 0
\end{aligned}
$$

Appendix L The derivatives of $\dot{\delta}$ and δ^* with respect to λ, c_1, c_3, and f

$$
\begin{aligned}
(r - c_1 - f) \cdot (1 - \dot{\delta}(1-\lambda))^2 &= \lambda(1+\lambda) \cdot (r - c_3 - f) \\
\dot{\delta} &= \frac{1}{1-\lambda} - \frac{1}{1-\lambda} \cdot \sqrt{\lambda(1+\lambda) \cdot \frac{r - c_3 - f}{r - c_1 - f}}, \\
\delta^* \cdot (1-\lambda)(2r - 2f - c_1 - c_3) &= (1-\lambda)(r-f) - c_1 + \lambda c_3 \\
\delta^* &= \frac{(1-\lambda)(r-f) - c_1 + \lambda c_3}{(1-\lambda)(2r - 2f - c_1 - c_3)}.
\end{aligned}
$$

By the Implicit Function Theorem,

$$\begin{aligned}
\frac{d\delta^*}{d\lambda} &= \frac{c_3 - c_1}{(1-\lambda)^2(r - c_1 - f + r - c_3 - f)} \\
\frac{d\delta^*}{dc_1} &= -\frac{1 - \delta^*(1-\lambda)}{(1-\lambda)(2r - 2f - c_1 - c_3)} < 0 \\
\frac{d\delta^*}{dc_3} &= -\frac{-\delta^*(1-\lambda) - \lambda}{(1-\lambda)(2r - 2f - c_1 - c_3)} = \frac{\delta^*(1-\lambda) + \lambda}{(1-\lambda)(2r - 2f - c_1 - c_3)} > 0 \\
\frac{d\delta^*}{df} &= \frac{2\delta^* - 1}{(r - c_1 - f) + (r - c_3 - f)} \\
\frac{d\dot{\delta}}{d\lambda} &= \frac{(r - c_1 - f) \cdot 2(1 - \dot{\delta}(1-\lambda)) \cdot \dot{\delta} - (1 + 2\lambda)(r - c_3 - f)}{2(r - c_1 - f)(1 - \dot{\delta}(1-\lambda))(1-\lambda)} \\
&= \frac{2 \cdot \dot{\delta} \cdot \lambda(1+\lambda) - (1+2\lambda)(1 - \dot{\delta}(1-\lambda))}{2(r - c_1 - f)(1 - \dot{\delta}(1-\lambda))^2(1-\lambda)} \cdot (r - c_3 - f) \\
&= \frac{\dot{\delta}(1+3\lambda) - (1+2\lambda)}{2(r - c_1 - f)(1 - \dot{\delta}(1-\lambda))^2(1-\lambda)} \cdot (r - c_3 - f) \\
\frac{d\dot{\delta}}{dc_1} &= -\frac{(1 - \dot{\delta}(1-\lambda))^2}{2(r - c_1 - f)(1 - \dot{\delta}(1-\lambda))(1-\lambda)} < 0 \\
\frac{d\dot{\delta}}{dc_3} &= \frac{\lambda(1+\lambda)}{2(r - c_1 - f)(1 - \dot{\delta}(1-\lambda))(1-\lambda)} > 0 \\
\frac{d\dot{\delta}}{df} &= \frac{\lambda(1+\lambda) - (1 - \dot{\delta}(1-\lambda))^2}{2(r - c_1 - f)(1 - \dot{\delta}(1-\lambda))(1-\lambda)} \\
&= \lambda(1+\lambda) \cdot \frac{c_3 - c_1}{2(r - c_1 - f)^2(1 - \dot{\delta}(1-\lambda))(1-\lambda)}
\end{aligned}$$

$$\frac{d\dot{\delta}}{d\lambda} < 0 \Leftrightarrow \frac{4\lambda(1+\lambda)}{1+3\lambda} < \frac{r - c_3 - f}{r - c_1 - f}$$

Appendix M Proof of Corollary 2

$$\begin{aligned}
\dot{\delta} &= \frac{1}{1-\lambda} - \frac{1}{1-\lambda} \cdot \sqrt{\lambda(1+\lambda) \cdot \frac{r - c_3 - f}{r - c_1 - f}}, \\
\delta^* &= \frac{(1-\lambda)(r-f) - c_1 + \lambda c_3}{(1-\lambda)(2r - 2f - c_1 - c_3)}.
\end{aligned}$$

$$\dot{\delta} \geq \delta^* \Leftrightarrow \frac{1}{1-\lambda} - \frac{1}{1-\lambda} \cdot \sqrt{\lambda(1+\lambda) \cdot \frac{r-c_3-f}{r-c_1-f}} \geq \frac{(1-\lambda)(r-f)-c_1+\lambda c_3}{(1-\lambda)(2r-2f-c_1-c_3)}$$

$$\Leftrightarrow 1 - \sqrt{\lambda(1+\lambda) \cdot \frac{r-c_3-f}{r-c_1-f}} \geq \frac{(r-c_1-f)-\lambda(r-c_3-f)}{(r-c_1-f)+(r-c_3-f)}$$

$$\Leftrightarrow \frac{(1+\lambda)(r-c_3-f)}{(r-c_1-f)+(r-c_3-f)} \geq \sqrt{\lambda(1+\lambda) \cdot \frac{r-c_3-f}{r-c_1-f}}$$

$$\Leftrightarrow \frac{(1+\lambda)(r-c_3-f)}{((r-c_1-f)+(r-c_3-f))^2} \geq \frac{\lambda}{r-c_1-f}$$

$$\Leftrightarrow \frac{1+\lambda}{\lambda} \geq \frac{((r-c_1-f)+(r-c_3-f))^2}{(r-c_1-f)(r-c_3-f)}$$

$$\Leftrightarrow \frac{1-\lambda}{\lambda} \geq \frac{(r-c_1-f)^2+(r-c_3-f)^2}{(r-c_1-f)(r-c_3-f)}$$

$$\Leftrightarrow \frac{1-\lambda}{\lambda} \geq \frac{(r-c_1-f)}{(r-c_3-f)} + \frac{(r-c_3-f)}{(r-c_1-f)}$$

Thus, if $\lambda \geq \frac{1}{3}$,

$$\frac{1-\lambda}{\lambda} \leq 2 \leq \frac{r-c_3-f}{r-c_1-f} + \frac{r-c_1-f}{r-c_3-f} \Rightarrow \dot{\delta} \leq \delta^*$$

Appendix N Proof of Lemma 3

First, consider the non-searchers.

$$\begin{aligned}
\mathbb{E}(r-p) &= r - \mathbb{E}(p) \\
&= r - \left(\int_m^{r-} p d\hat{F} + r \cdot f^0(r) \right) \\
&= \left(1 - f^0(r)\right) \cdot r - \int_m^{r-} p d\hat{F} \\
&= \left(1 - f^0(r)\right) \cdot r - \left(p\hat{F} \Big|_m^{r-} - \int_m^{r-} \hat{F} dp \right) \\
&= \left(1 - f^0(r)\right) \cdot r - (r) \cdot \left(1 - f^0(r)\right) + \int_m^{r-} \hat{F} dp \\
&= \int_m^{r-} \hat{F} dp
\end{aligned}$$

Next, consider the searchers. Let's define: $Z := \min(p_1, p_3)$. Then, the CDF of Z is:

$$\begin{aligned} H(z) := \mathbb{P}(Z \leq z) &= 1 - \mathbb{P}(p_1 > z, p_3 > z) \\ &= 1 - (1 - F_1(z))(1 - F_3(z)) \\ &= F_1(z) + F_3(z) - F_1(z)F_3(z) \qquad\qquad \forall z \in [m, r) \\ \mathbb{P}(Z = r) &= f_1^0 f_3^0 \end{aligned}$$

Note that

$$\begin{aligned} H(r^-) &= F_1(r^-) + F_3(r^-) - F_1(r^-)F_3(r^-) \\ &= (1 - f_1^0) + (1 - f_3^0) - (1 - f_1^0)(1 - f_3^0) \\ &= 1 - f_1^0 f_3^0 \end{aligned}$$

$$\begin{aligned} \mathbb{E}[\min(p_1, p_3)] := \mathbb{E}[Z] &= \int_m^{r^-} z\, dH(z) + r \cdot \mathbb{P}(Z = r) \\ &= \int_m^{r^-} z\, dH(z) + r \cdot f_1^0 f_3^0 \\ &= zH(z)\big|_m^{r^-} - \int_m^{r^-} H(z)\, dz + r \cdot f_1^0 f_3^0 \\ &= r(1 - f_1^0 f_3^0) - \int_m^{r^-} H(z)\, dz + r \cdot f_1^0 f_3^0 \\ &= r - \int_m^{r^-} H(z)\, dz \\ &= r - \int_m^{r^-} [F_1(z) + F_3(z) - F_1(z)F_3(z)]\, dz \end{aligned}$$

Appendix O Proof of Lemma 4

Suppose F_1 is first-order stochastically dominated by F, i.e., $F_1(z) \geq F(z)$ for all $z \in [0, r]$. Suppose also that G_1 is first-order stochastically dominated by G, i.e., $G_1(z) \geq G(z)$ for all $z \in [0, r]$. Let $H(z)$ and $H_1(z)$ be the following:

$$\begin{aligned} H(z) &:= F(z) + G(z) - F(z)G(z) \\ H_1(z) &:= F_1(z) + G_1(z) - F_1(z)G_1(z) \end{aligned}$$

Note that

$$\begin{aligned} F_1G_1 - FG &= F_1G_1 - F_1G + GF_1 - FG_1 + FG_1 + FG - FG - GF \\ &= (F_1 - F)(G_1 - G) + F(G_1 - G) + G(F_1 - F) \end{aligned}$$

Thus,

$$\begin{aligned}
H_1(z) - H(z) &= (F_1 + G_1 - F_1G_1) - (F + G - FG) \\
&= (F_1 - F) + (G_1 - G) - (F_1G_1 - FG) \\
&= (F_1 - F) + (G_1 - G) - [(F_1 - F)(G_1 - G) + F(G_1 - G) + G(F_1 - F)] \\
&= (F_1 - F)(1 - G) + (G_1 - G)(1 - F) - (F_1 - F)(G_1 - G) \\
&= (F_1 - F)(1 - G - G_1 + G) + (G_1 - G)(1 - F) \\
&= (F_1 - F)(1 - G_1) + (G_1 - G)(1 - F) \\
&\geq 0
\end{aligned}$$

That is:

$$F_1 + G_1 - F_1G_1 \geq F + G - FG \quad \text{for all } z \in [0, r]$$

Appendix P Proof of Corollary 3

$$\delta^* = \frac{(r - c_1 - f) - (r - c_3 - f) \cdot \lambda}{(1 - \lambda) \cdot (r - c_1 - f + r - c_3 - f)}$$

Thus,

$$\begin{aligned}
\delta^* \gtreqless 0.5 &\Leftrightarrow \frac{(r - c_1 - f) - (r - c_3 - f) \cdot \lambda}{(1 - \lambda) \cdot (r - c_1 - f + r - c_3 - f)} \gtreqless 0.5 \\
&\Leftrightarrow 2(r - c_1 - f) - 2(r - c_3 - f) \cdot \lambda \gtreqless (1 - \lambda) \cdot (r - c_1 - f + r - c_3 - f) \\
&\Leftrightarrow 2(r - c_1 - f) - (r - c_1 - f + r - c_3 - f) \gtreqless 2(r - c_3 - f) \cdot \lambda - \lambda \cdot (r - c_1 - f + r - c_3 - f) \\
&\Leftrightarrow c_3 - c_1 \gtreqless \lambda \cdot (c_1 - c_3) \\
&\Leftrightarrow (1 + \lambda)(c_3 - c_1) \gtreqless 0 \\
&\Leftrightarrow c_3 \gtreqless c_1
\end{aligned}$$

Appendix Q Proof of Proposition 5

Note that

$$\begin{aligned}
\frac{d\hat{F}_1}{df} &= \frac{\lambda + (1 - \delta)(1 - \lambda)}{\lambda} \cdot \frac{p - m - \frac{dm}{df} \cdot (p - c_3 - f)}{(p - c_3 - f)^2} \\
\frac{d\hat{F}_3}{df} &= \frac{\lambda + \delta(1 - \lambda)}{\lambda} \cdot \frac{p - m - \frac{dm}{df} \cdot (p - c_1 - f)}{(p - c_1 - f)^2}
\end{aligned}$$

Note also that

$$\begin{aligned}
\frac{dm_1}{df} &= \frac{\lambda}{\delta(1-\lambda)+\lambda} \\
\frac{dm_3}{df} &= \frac{\lambda}{(1-\delta)(1-\lambda)+\lambda} \\
p - m_3 - \frac{dm_3}{df}\cdot(p - c_3 - f) &= -(r-p)\cdot\frac{(1-\delta)(1-\lambda)}{(1-\delta)(1-\lambda)+\lambda} \\
p - m_3 - \frac{dm_3}{df}\cdot(p - c_1 - f) &= -(r-p)\cdot\frac{(1-\delta)(1-\lambda)}{(1-\delta)(1-\lambda)+\lambda} + \lambda\cdot\frac{-c_3+c_1}{(1-\delta)(1-\lambda)+\lambda} \\
&= -\frac{(r-p)(1-\delta)(1-\lambda)+\lambda\cdot(c_3-c_1)}{(1-\delta)(1-\lambda)+\lambda} \\
p - m_1 - \frac{dm_1}{df}\cdot(p - c_1 - f) &= -(r-p)\cdot\frac{\delta(1-\lambda)}{\delta(1-\lambda)+\lambda} \\
p - m_1 - \frac{dm_1}{df}\cdot(p - c_3 - f) &= -(r-p)\cdot\frac{\delta(1-\lambda)}{\delta(1-\lambda)+\lambda} + \lambda\cdot\frac{-c_1+c_3}{\delta(1-\lambda)+\lambda} \\
&= -\frac{(r-p)\cdot\delta(1-\lambda)+\lambda\cdot(c_1-c_3)}{\delta(1-\lambda)+\lambda}
\end{aligned}$$

Consider the case: $m_3 > m_1$,

$$\begin{aligned}
\frac{d\hat{F}_1}{df} &= -\frac{\lambda+(1-\delta)(1-\lambda)}{\lambda}\cdot\frac{1}{(p-c_3-f)^2}\cdot\frac{(r-p)(1-\delta)(1-\lambda)}{(1-\delta)(1-\lambda)+\lambda} < 0 \\
\frac{d\hat{F}_3}{df} &= -\frac{\lambda+\delta(1-\lambda)}{\lambda}\cdot\frac{1}{(p-c_1-f)^2}\cdot\frac{(r-p)(1-\delta)(1-\lambda)+\lambda\cdot(c_3-c_1)}{(1-\delta)(1-\lambda)+\lambda}
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{d\hat{F}_3}{df} > 0 &\Leftrightarrow (r-p)(1-\delta)(1-\lambda)+\lambda\cdot(c_3-c_1) < 0 \\
&\Leftrightarrow r + \frac{\lambda}{(1-\delta)(1-\lambda)}\cdot(c_3-c_1) < p
\end{aligned}$$

Thus, if $r + \frac{\lambda}{(1-\delta)(1-\lambda)}\cdot(c_3-c_1) < m_3$, then for all p, $\frac{d\hat{F}_3}{df} > 0$.

$$\begin{aligned}
r + \frac{\lambda}{(1-\delta)(1-\lambda)}\cdot(c_3-c_1) < m_3 &\Leftrightarrow r + \frac{\lambda}{(1-\delta)(1-\lambda)}\cdot(c_3-c_1) < r - \lambda\cdot\frac{r-c_3-f}{(1-\delta)(1-\lambda)+\lambda} \\
&\Leftrightarrow \frac{c_3-c_1}{(1-\delta)(1-\lambda)} < -\frac{r-c_3-f}{(1-\delta)(1-\lambda)+\lambda} \\
&\Leftrightarrow (c_1-c_3)\cdot((1-\delta)(1-\lambda)+\lambda) > (1-\delta)(1-\lambda)\cdot(r-c_3-f) \\
&\Leftrightarrow \frac{\lambda}{(1-\delta)(1-\lambda)}\cdot(c_1-c_3) > r-c_1-f
\end{aligned}$$

Therefore, if $c_1 > c_3$ and $\frac{\lambda}{(1-\delta)(1-\lambda)}$ is sufficiently big, $\frac{d\hat{F}_3}{df} > 0$.
Consider another case: $m_3 < m_1$,

$$\frac{d\hat{F}_1}{df} = -\frac{\lambda + (1-\delta)(1-\lambda)}{\lambda} \cdot \frac{1}{(p - c_3 - f)^2} \cdot \frac{(r-p) \cdot \delta(1-\lambda) + \lambda \cdot (c_1 - c_3)}{\delta(1-\lambda) + \lambda}$$
$$\frac{d\hat{F}_3}{df} = -\frac{\lambda + \delta(1-\lambda)}{\lambda} \cdot \frac{1}{(p - c_1 - f)^2} \cdot \frac{(r-p) \cdot \delta(1-\lambda)}{\delta(1-\lambda) + \lambda} < 0$$

Thus,

$$\begin{aligned} \frac{d\hat{F}_1}{df} > 0 &\Leftrightarrow (r-p) \cdot \delta(1-\lambda) + \lambda \cdot (c_1 - c_3) < 0 \\ &\Leftrightarrow r + \frac{\lambda}{\delta(1-\lambda)} \cdot (c_1 - c_3) < p \end{aligned}$$

Thus, if $r + \frac{\lambda}{\delta(1-\lambda)} \cdot (c_1 - c_3) < m_1$, then for all p, $\frac{d\hat{F}_1}{df} > 0$.

$$\begin{aligned} r + \frac{\lambda}{\delta(1-\lambda)} \cdot (c_1 - c_3) < m_1 &\Leftrightarrow r + \frac{\lambda}{\delta(1-\lambda)} \cdot (c_1 - c_3) < r - \lambda \cdot \frac{r - c_1 - f}{\delta(1-\lambda) + \lambda} \\ &\Leftrightarrow \frac{(c_3 - c_1)}{\delta(1-\lambda)} > \frac{r - c_1 - f}{\delta(1-\lambda) + \lambda} \\ &\Leftrightarrow c_3 \cdot \left(1 + \frac{\lambda}{\delta(1-\lambda)}\right) - c_1 \cdot \frac{\lambda}{\delta(1-\lambda)} > r - f \\ &\Leftrightarrow \frac{\lambda}{\delta(1-\lambda)} \cdot (c_3 - c_1) > r - c_3 - f \end{aligned}$$

Therefore, if $c_3 > c_1$ and $\frac{\lambda}{\delta(1-\lambda)}$ is sufficiently big, $\frac{d\hat{F}_1}{df} > 0$.

Appendix R Proof of Lemma 6

Note that

$$\begin{aligned} r - \lambda \cdot \frac{r - c_1 - f}{\delta^*(1-\lambda) + \lambda} &= r - \lambda \cdot \frac{r - c_3 - f}{(1-\delta^*)(1-\lambda) + \lambda} \\ \frac{r - c_1 - f}{\delta^*(1-\lambda) + \lambda} &= \frac{r - c_3 - f}{(1-\delta^*)(1-\lambda) + \lambda} \\ (r - c_1 - f) \cdot ((1-\delta^*)(1-\lambda) + \lambda) &= (r - c_3 - f) \cdot (\delta^*(1-\lambda) + \lambda) \end{aligned}$$

Then,

$$\begin{aligned}\frac{d\delta^*}{df} &= -\frac{(\delta^*(1-\lambda)+\lambda)-((1-\delta^*)(1-\lambda)+\lambda)}{-(r-c_1-f)(1-\lambda)-(r-c_3-f)(1-\lambda)} \\ &= -\frac{\delta^*(1-\lambda)-(1-\delta^*)(1-\lambda)}{-(r-c_1-f)(1-\lambda)-(r-c_3-f)(1-\lambda)} \\ &= -\frac{\delta^*-(1-\delta^*)}{-(r-c_1-f)-(r-c_3-f)} \\ &= \frac{2\delta^*-1}{(r-c_1-f)+(r-c_3-f)}\end{aligned}$$

Thus, $\frac{d\delta^*}{df} > 0$ if $\delta^* > 0.5$ and $\frac{d\delta^*}{df} < 0$ if $\delta^* < 0.5$.
When $f = 0$, δ^* satisfies the following:

$$\begin{aligned}(r-c_1)\cdot((1-\delta^*)(1-\lambda)+\lambda) &= (r-c_3)\cdot(\delta^*(1-\lambda)+\lambda) \\ \delta^* &= \frac{(r-c_1)-(r-c_3)\cdot\lambda}{(1-\lambda)\cdot(2r-c_1-c_3)}\end{aligned}$$

Thus,

$$\begin{aligned}\delta^* > 0.5 &\Leftrightarrow \frac{(1-\lambda)\cdot r-(c_1-c_3\cdot\lambda)}{(1-\lambda)\cdot(2r-c_1-c_3)} > 0.5 \\ &\Leftrightarrow (1-\lambda)\cdot 2r-2(c_1-c_3\cdot\lambda) > (1-\lambda)\cdot(2r-c_1-c_3) \\ &\Leftrightarrow -2(c_1-c_3\cdot\lambda) > -(1-\lambda)\cdot(c_1+c_3) \\ &\Leftrightarrow 2c_1-2c_3\cdot\lambda < (1-\lambda)\cdot c_1+(1-\lambda)\cdot c_3 \\ &\Leftrightarrow 2c_1-(1-\lambda)\cdot c_1 < 2c_3\cdot\lambda+(1-\lambda)\cdot c_3 \\ &\Leftrightarrow (1+\lambda)\cdot c_1 < (1+\lambda)\cdot c_3 \\ &\Leftrightarrow c_1 < c_3\end{aligned}$$

Also,

$$\frac{d\dot{\delta}}{df} = (c_3-c_1)\cdot\frac{\lambda(1+\lambda)}{2(r-c_1-f)^2(1-\dot{\delta}(1-\lambda))(1-\lambda)}$$

Thus, $\frac{d\delta^*}{df} > 0$ and $\frac{d\dot{\delta}}{df} > 0$ if $c_1 < c_3$ and $\frac{d\delta^*}{df} < 0$ and $\frac{d\dot{\delta}}{df} < 0$ if $c_1 > c_3$.

Appendix S Proof of Proposition 7

Note that

$$\frac{dm_3}{df} = \frac{\lambda}{(1-\delta)(1-\lambda)+\lambda}$$

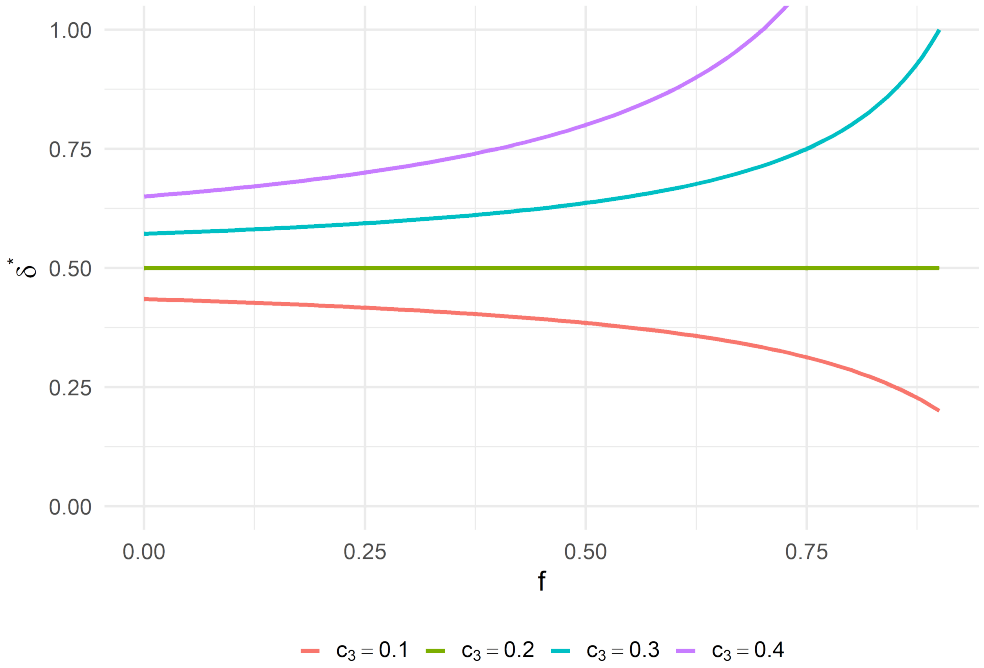


Figure 14: δ^* over f depending on c_3 when $r = 1.3$, $c_1 = 0.2$, and $\lambda = 0.5$.

Thus,

$$\begin{aligned}
\frac{d\mathbb{E}\Pi_1}{df} &= \begin{cases} (\delta(1-\lambda)+\lambda)\cdot\frac{dm_3}{df} + (1-\delta)(1-\lambda) & m_3 \geq m_1 \\ (1-\delta)(1-\lambda)+\lambda & m_1 \geq m_3 \end{cases} \\
&= \begin{cases} (\delta(1-\lambda)+\lambda)\cdot\frac{\lambda}{(1-\delta)(1-\lambda)+\lambda} + (1-\delta)(1-\lambda) & m_3 \geq m_1 \\ (1-\delta)(1-\lambda)+\lambda & m_1 \geq m_3 \end{cases} \\
&= \begin{cases} \frac{(1-\delta)^2(1-\lambda)^2+\lambda}{(1-\delta)(1-\lambda)+\lambda} & m_3 \geq m_1 \\ (1-\delta)(1-\lambda)+\lambda & m_1 \geq m_3 \end{cases}
\end{aligned}$$

Note that $\frac{d\mathbb{E}\Pi_1}{df} > 0$ always holds when the display probability is fixed, although the slope varies depending on the relative sizes of m_1 and m_3.

$$\begin{aligned}
\frac{(1-\delta)^2(1-\lambda)^2+\lambda}{(1-\delta)(1-\lambda)+\lambda} > (1-\delta)(1-\lambda)+\lambda &\Leftrightarrow (1-\delta)^2(1-\lambda)^2+\lambda > ((1-\delta)(1-\lambda)+\lambda)^2 \\
&\Leftrightarrow \lambda > 2\lambda(1-\delta)(1-\lambda)+\lambda^2 \\
&\Leftrightarrow 1-\lambda > 2(1-\delta)(1-\lambda) \\
&\Leftrightarrow \delta > 0.5
\end{aligned}$$

Thus, the proposition holds.

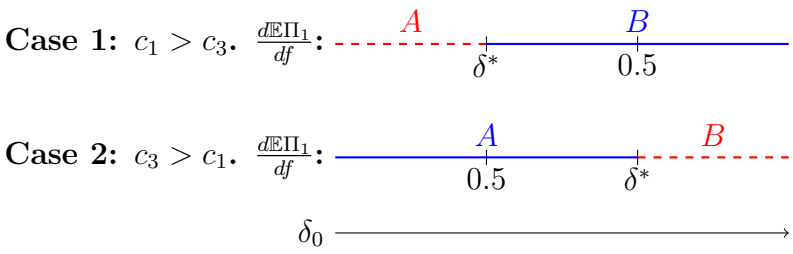


Figure 15: Description of Proposition 7
Note: As the commission fee f increase, the dashed segment shortens due to the corresponding shift in δ^*.

Appendix T Note on the commission fee

$$\dot{\delta} = \frac{1}{1-\lambda} - \frac{1}{1-\lambda} \cdot \sqrt{\lambda(1+\lambda) \cdot \frac{r - c_3 - f}{r - c_1 - f}},$$
$$\delta^* = \frac{(1-\lambda)(r-f) - c_1 + \lambda c_3}{(1-\lambda)(2r - 2f - c_1 - c_3)}.$$

$$\begin{aligned}\frac{d\dot{\delta}}{df} &= (c_3 - c_1) \cdot \frac{\lambda(1+\lambda)}{2(r - c_1 - f)^2(1 - \dot{\delta}(1-\lambda))(1-\lambda)} \\ &= \frac{\lambda(1+\lambda)}{2(1-\lambda)} \cdot \frac{(c_3 - c_1)}{r_1 \cdot r_1 \cdot (1 - \dot{\delta}(1-\lambda))}\end{aligned}$$

$$\begin{aligned}\frac{d\delta^*}{df} &= \frac{2\delta^* - 1}{(r - c_1 - f) + (r - c_3 - f)} \\ &= \frac{2\delta^* - 1}{r_1 + r_3}\end{aligned}$$

$$\begin{aligned}\frac{d\dot{\delta}}{df} &\geq \frac{d\delta^*}{df} \\ \Leftrightarrow \frac{\lambda(1+\lambda)}{2(1-\lambda)} \cdot \frac{(c_3 - c_1)}{r_1 \cdot r_1 \cdot (1 - \dot{\delta}(1-\lambda))} &\geq \frac{2\delta^* - 1}{r_1 + r_3} \\ \Leftrightarrow \frac{\lambda(1+\lambda)}{2(1-\lambda)} \cdot (c_3 - c_1) \cdot \frac{r_1 + r_3}{r_1 \cdot r_1} &\geq (2\delta^* - 1) \cdot (1 - \dot{\delta}(1-\lambda)) \\ \Leftrightarrow \frac{\lambda(1+\lambda)}{2(1-\lambda)} \cdot (c_3 - c_1) \cdot \frac{r_1 + r_3}{r_1 \cdot r_3} &\geq (2\delta^* - 1) \cdot (1 - \dot{\delta}(1-\lambda)) \cdot \frac{r_1}{r_3} \\ &= (2\delta^* - 1) \cdot (1 - \dot{\delta}(1-\lambda)) \cdot \frac{\delta^*(1-\lambda) + \lambda}{1 - \delta^*(1-\lambda)} \\ &= (2\delta^* - 1) \cdot (\delta^*(1-\lambda) + \lambda) \cdot \frac{(1 - \dot{\delta}(1-\lambda))}{1 - \delta^*(1-\lambda)}\end{aligned}$$