The Pricing of Uncertain Timing

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Abstract

Are states the same as time, and does the pricing of uncertain timing resemble the pricing of uncertain outcomes? This paper studies such questions in a model-free way, assuming only the existence of a stochastic discount factor. It shows that (i) uncertain timing and uncertain outcomes are fundamentally different: when risk is purely idiosyncratic, higher order moments of uncertain timing affect valuation without constituting a risk premium. (ii) The price of an asset with uncertain timing can be characterized through the risk-free term structure, creating a tight linkage between bonds and assets with uncertain timing. (iii) Any pricing kernel must feature asymmetry for the valuation of positive and negative shocks. (iv) Systematic risk can be fully decomposed into systematic timing risk and systematic outcome risk. (v) Diversification does not remove the fundamental differences between uncertain outcomes and uncertain time. A model-free empirical test in option markets finds quotes and trades to be consistent with a pricing of uncertain timing. These results yield implications for pricing across financial markets.

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1 Introduction

Understanding sources of valuation is one of the central goals of finance. From canonical pricing results, investments that carry risk must compensate for that risk with return premia (Leahy and Whited (1996); Duffie (2001); Berk and DeMarzo (2011)). The kind of risk studied has largely been risk over outcomes. For example, the valuation of an asset that pays out in 10 days for sure, but gives \$100 with 50% probability and \$300 with 50% probability, is straightforward to compute using a pricing kernel of choice.

The literature has focused less, however, on the valuation of assets with uncertain timing. To illustrate, consider valuing an asset that pays out \$10 for sure, but does so in 100 days with 50% probability and in 300 days with 50% probability. How should such an asset be valued, and does its valuation conceptually resemble valuation of canonical uncertain outcomes?

This paper seeks to study such questions. In doing so, it speaks to the fundamental question of whether states are the same as time. Textbook approaches, and canonical asset pricing models, often use one-period-ahead results (Campbell, 2017), or derive similar approaches towards valuing outcomes and time (Cochrane, 2014). This paper finds that outcomes and time conceptually and fundamentally differ in their pricing properties, and derives model-free theoretical and empirical results for the pricing of uncertain timing.

To analyze assets with uncertain timing, I embed them in a stochastic discount factor (SDF) framework through the use of stopping times, and define idiosyncratic and systematic risk in the standard way within this framework. Using only the existence of an SDF, I derive the following results characterizing the pricing of uncertain timing, which also show how the pricing of uncertain timing is fundamentally different from the pricing of uncertain outcomes.

First, uncertainty over outcomes and over time are not the same. For any asset with purely idiosyncratic risk, a classical result in finance is that higher order moments do not affect pricing. In contrast to this result, Theorem 1 shows that higher order moments do affect price for assets with uncertain outcomes – even when risk is purely idiosyncratic.

Second, the valuation of any asset with uncertain timing which has idiosyncratic risk can be characterized entirely by the valuation of a portfolio of zero-coupon bonds

with portfolio weights identical to the asset's stopping time probabilities (Theorem 2). In particular, this implies that the effect of any SDF on uncertainty for timing is identical to its effect on the pricing of zero-coupon bonds (ZCBs). This result might seem surprising because some models specifically seek to distort uncertainty but not risk-free rates (ex. van Binsbergen et al. (2012), Wachter (2013)). Nevertheless, the theorem proves that for any SDF, any distortion must be the same for uncertain timing as for zero-coupon bonds.

This result provides a model-free way to price any asset with uncertain timing simply by observing the risk-free term structure. This risk-free term structure need not be economy-wide; for example, it can apply for any firm or household's subjective SDF. The result also closely links valuation of assets with uncertain timing to an extensive literature on bonds (Strebulaev and Whited, 2012; Krishnamurthy, 2010; van Binsbergen, 2025; Bretscher et al., 2025; Schmid et al., 2025). In particular, it implies that any result on bonds, and more generally, on risk-free term structures, can be ported for assets with uncertain timing which have purely idiosyncratic risk.

Third, another way in which outcomes and time differ is that unlike for outcomes, for which uncertainty is always valued weakly lower than certainty at the expected value, for time uncertainty can be valued higher, lower, or the same as, certainty at the expected time value (Theorem 3). Further, time features model-free asymmetry: for any SDF, if uncertainty is valued higher than certainty for positive shocks, it must be valued less than certainty for negative shocks, and vice versa (Corollary 1).

Fourth, many assets feature both uncertain outcomes and uncertain timing. For such assets, Theorem 4 provides a novel decomposition of systematic risk – the covariance of the asset's payoffs with the SDF – into systematic outcome risk and systematic timing risk. Unlike prominent decompositions in the literature (Duffie, 2001), this decomposition is complete rather than an approximation. Intuitively, systematic outcome risk is the covariance of the SDF with the asset payoffs conditional on the asset paying off, e.g. its stopping time being hit. This systematic outcome risk corresponds to how systematic risk is typically described (Caballero and Krishnamurthy, 2008; Giglio and Xiu, 2021*a*). Systematic timing risk measures whether the asset tends to pay off at times at which the SDF is higher or lower. In addition, when risk not just systematic but aggregate, it is possible to derive directionality predictions for the subset of utility-based SDFs for small shocks (Theorem 5).

Fifth and finally, diversification does not make time resemble outcomes. In par-

ticular, while uncertainty disappears with diversification, conceptual analogs to Theorems 1-3 continue to hold. Theorem 6 proves that with infinitely many independent assets with uncertain timing, payment becomes deterministic and uncertainty vanishes in the limit. However, higher orders of temporal dispersion continue to affect pricing, unlike for outcomes. Theorem 7 considers diversification which perfectly smooths payment shocks. Even in this case, higher orders of (now deterministic) dispersion continue to affect pricing; in addition, unlike diversification for assets with uncertain outcomes, the diversified portfolio can be priced higher, lower, or the same as, a certain-time deterministic asset. In addition, asymmetry continues to hold.

These model-free results, relying only on the existence of an SDF, show that outcomes and time fundamentally differ from a pricing perspective; and characterize the pricing of assets with uncertain timing.

Given that uncertain timing has not historically been focused on as a component of pricing, a natural next question is whether uncertain timing is empirically priced. To examine this in a model-free way, I use the options market as a laboratory.

Because an American option can be exercised at any time up to and including expiry, the short side of an American option is exposed to uncertain timing that the short side of an otherwise identical European option is not. While the derivatives literature has studied this difference for options with dividends, a fundamental result is that there should be no pricing difference between the American and European for call options with no dividends in their lifetime (Merton, 1973). Contrary to this predication and later advances from the options literature, a simple comparison of traded and otherwise identical American and European call options reveals a higher pricing of the American option. This higher pricing is driven primarily by a difference in asks, and is not consistent with explanations including horizon dependence (van Binsbergen et al., 2012; Giglio et al., 2023), hedging requirements (Kapadia and Pu, 2012*b*), re-investment risk (Kapadia and Willette, 2012*a*), rare disasters (Wachter, 2013), or informed trading and adversarial thinking (Hull, 2018). It is, however, consistent with a pricing of uncertain timing.

A final supplemental set of survey-elicited pricing narratives indicates that many participants think of states and times as similar. This suggests that for individuals or households, there may be an additional perception-based pricing component beyond the model-free results derived above.

Together, these results indicate that the dedicated study of uncertain timing in

finance may be promising in a number of directions. First, the model-free pricing results can be directly used in any financial setting with uncertain timing using only the assumptions that induce existence of an SDF. Uniqueness is not needed. As a result, the theorems are relevant for both complete and incomplete markets. There are many settings in finance with uncertain timing, including debt issuance, mortgage markets, venture capital, and others, where a better understanding of the timing component may be useful. Second, the option results indicate that further study of uncertain timing for financial products with embedded options, such as corporate debt which is often callable or convertible, or real options, may be fruitful. Third, the pricing narratives indicate that - on top of a first principles analysis of pricing as enabled by the theoretical results - further examination within individual and households may be interesting to study.

The paper contributes to a number of literatures. Within the pricing literature, the paper relates to model-free results on the stochastic discount factor (Harrison and Kreps, 1979; Hansen and Jagannathan, 1991) and model-free empirical tests (Giglio et al., 2016), particularly in options (van Binsbergen et al., 2022). Such results are useful because they rely on minimal assumptions and can therefore be used across settings to obtain predictions and to discipline models. In this case, the model-free results derived are for uncertain timing.

Because the paper analyzes properties of pricing, it informs a literature on the properties of different SDF choices (Wachter (2006); Papanikolaou (2011); van Binsbergen (2016); Xu (2017), among others). This literature has studied which risk should be priced and its effects (Leahy and Whited, 1996; Giglio and Xiu, 2021*a*) - a fundamental topic within pricing and finance. The current paper's results and characterization apply for all SDFs regardless of which risk properties one chooses.

The paper also relates to a neighboring literature on duration and duration risk (van Binsbergen et al., 2012; van Binsbergen, 2025). Unlike this literature which considers the pricing of uncertain outcomes over fixed time periods, the present paper focuses on the pricing of uncertain timing.

Within options markets, the paper contributes to a literature on early exercise (Kapadia and Willette, 2012*a*; Figlewski, 2022) and its consequences for pricing. The specific options studied in the model-free empirical portion of the paper have also been studied in Harvey and Whaley (1991), Fleming and Whaley (1994), among others; here, they are studied with a view towards uncertain timing.

While uncertain timing has not been a finance focus, prior work studies it in biology (Kacelnik and Bateson, 1996) and decision theory (Chesson and Viscusi (2003), Onay and Onculer (2007), DeJarnette et al. (2020) on expected utility, additive utility, and Epstein-Zin, respectively, with experimental results for risk aversion over time lotteries). The paper's model-free results nest preference-based approaches to pricing as special cases, while yielding new results for fundamental pricing models and financial markets.

While the paper studies time, it is conceptually distinct from other literatures on time-related concepts such as the elasticity of intertemporal substitution (Davis and Van Nieuwerburgh, 2015) and resolution period effects (Epstein et al., 2014). One way to see this is that the results hold for all SDFs, no matter their parametrization or resolution properties.

There are many settings across financial markets where uncertain timing is relevant. Such settings include, but are not limited to, real options (Strebulaev and Whited, 2012), private equity (Kaplan and Schoar, 2005; Schmid and Steri, 2023), mortgages and mortgage-backed securities (Acharya et al., 2011; Van Nieuwerburgh and Weill, 2010*b*), climate disasters (Giglio et al., 2021*b*), and venture capital (Metrick and Yasuda, 2021; Gourier et al., 2024). These varied financial markets provide considerably many avenues for future research on the empirical pricing of uncertain timing.

The paper proceeds as follows. Section 2 embeds assets with uncertain timing in an SDF framework and derives model-free pricing theorems. Of these, Section 2.1 focuses on assets with purely idiosyncratic timing; Section 2.2 on assets with systematic risk which can have both uncertain outcomes and uncertain timing; and Section 2.3 on diversification results. Section 3 performs model-free empirical tests in option markets. Section 4 discusses possible applications across financial markets. Section 5 concludes. All proofs are located in the appendix, with proof intuition discussed in the text.

2 Model-Free Pricing Results

This section derives model-free SDF results for the pricing of assets with uncertain timing. I start by defining the general SDF framework within which these assets will be defined.

Definition 1 (The Stochastic Discount Factor Framework)**.**
There exists a probability space $(\Omega, \{\mathcal{F}_t\}_{t\in T}, P)$, where Ω is a set of states, $\{\mathcal{F}_t\}$ a filtration, $T \subseteq \mathbb{R}^+$ is a set of times, and P a probability measure. In this framework, assets are adapted processes: $X = (X_1, X_2, ...)$ where each X_t is a random variable measurable with respect to $\mathcal{F}_t$. Assume at each time t the existence of a stochastic discount factor $M_t = \{M_{t\to t'}\}_{t'>t}$: the time t price of asset X under this SDF is given by $Price_t(X) = E_{P_t}[\sum_{t'} M_{t\to t'} X_{t'}]$.

Definition 2 (Assets with Pure Uncertain Outcomes)**.**
An asset with pure uncertain outcomes X is one which has non-zero payoffs at only one time $t \in T$.

Definition 3 (Stopping Time)**.**
Given the filtered probability space $(\Omega, \{\mathcal{F}_t\}_{t\in T}, P)$ from Definition 1, a stopping time is a random variable $\tau : \Omega \to T$ such that for every $t \in T$, the event $\{\omega \in \Omega : \tau(\omega) \leq t\}$ is in the filtration F_t.

Say that τ is a non-trivial stopping time if there are at least two distinct times it takes on with positive probability: $\exists t_1 \neq t_2 \in T$ such that $P(\tau = t_1), P(\tau = t_2) > 0$.

The expected time of τ is given by $E_P[\tau]$.

Definition 4 (Assets with Pure Uncertain Timing)**.**
An asset with pure uncertain timing X is one whose payoff process is entirely determined by some nontrivial stopping time τ. In particular,

$$X_t(\omega) = \begin{cases} c + a & \text{if } t = \tau(\omega) \\ c & \text{if } t \neq \tau(\omega) \end{cases}$$

for some $c, a \in \mathbb{R}$.

Example 1. *Suppose that the state space at time 0 is $\{G, B\}$, standing for good and bad. The state space at time 1 is then $\{(G,G),(G,B),(B,G),(B,B)\}$; at time 2 is $\{(G,G,G),(G,G,B),(G,B,G),(G,B,B),(B,G,G),(B,G,B),(B,B,G),(B,B,B)\}$; etc.*

An example of an asset with pure uncertain timing on this state space is one that pays out \$1 the first time the state space hits good only. This asset takes on a payoff value of 1 whenever the state is $(\underbrace{B, \ldots, B}_{\text{any number of times}}, G)$, and 0 in all other states.

An example of an asset with pure uncertain outcomes on this state space is one that pays out \$5 (in state B) or \$10 (in state G) at time 1, and 0 in all other times and states.

Definition 5 (Systematic vs. Idiosyncratic Risk)**.**
The time-t' systematic risk of an asset from the perspective of period $t \leq t'$ is given by $Cov(M_{t\to t'}, A_{t\to t'})$.

An asset has purely idiosyncratic risk from the perspective of period t if its time-t' systematic risk is equal to zero at all times $t' \geq t$.

Definition 6 (Assets with Uncertain Timing and Outcomes)**.**
An asset with uncertain timing and outcomes is characterized by a baseline payoff $c \in \mathbb{R}$, a stopping time τ, and a random shock $Y : \Omega \to \mathbb{R}$. The payoff process is

$$X_t(\omega) = c + \mathbf{1}_{\{t=\tau(\omega)\}} \cdot Y(\omega), \qquad t \in T.$$

This nests:

- *Pure uncertain timing: $Y(\omega) = a \in \mathbb{R}$ (constant shock);*
- *Pure uncertain outcomes: $\tau(\omega) = t^*$ (fixed time) and $c = 0$.*

Definition 7 (Valuation)**.**
Under Definition 1, the valuation of asset X at time t is given by $Price_t(X) = E_{P_t}[\sum_{t'} M_{t\to t'} X_{t'}]$.

Consider an asset with pure uncertain timing X which has associated stopping time τ. When $E_p[\tau] \in T$, define its certain-time equivalent,

$$X_t(\omega) = \begin{cases} c + a & \text{if } t = E_p[\tau] \\ c & \text{if } t \neq E_p[\tau] \end{cases}$$

Say that certain timing is valued higher (resp. lower) than uncertain timing if, for any asset with pure uncertain timing, its valuation is lower (resp. higher) than the valuation of its certain-time equivalent.

Definition 2 is the standard one period definition of an asset (Campbell, 2017). Definition 5 is the standard conceptualization of systematic risk and idiosyncratic risk across multiple periods (Duffie, 2001). Definitions 4 and 7 extend to the pricing

setting definitions used in the decision theory literature of time lotteries and risk aversion over them (DeJarnette et al., 2020), also earlier studied in Chesson and Viscusi (2003) and Onay and Onculer (2007). The present framework applies for any SDF; formalizes uncertainty as random variables through the use of stopping times; and allows assets to take on non-zero values away from these stopping times. Definition 6 combines uncertain outcomes and uncertain timing and will be used for the decomposition results.

All of the above definitions will play a role in analyzing the valuation of uncertain timing.

2.1 Idiosyncratic Risk

2.1.1 Timing vs. Outcomes: Higher-Order Moments

Theorem 1 shows that for idiosyncratic risk, assets with uncertain timing are fundamentally different than assets with uncertain outcomes.

In particular, a fundamental result in asset pricing is that, for an asset with idiosyncratic risk and uncertain outcomes, its higher order moments are not priced. The intuition is straightforward and illustrated in Remark 1.

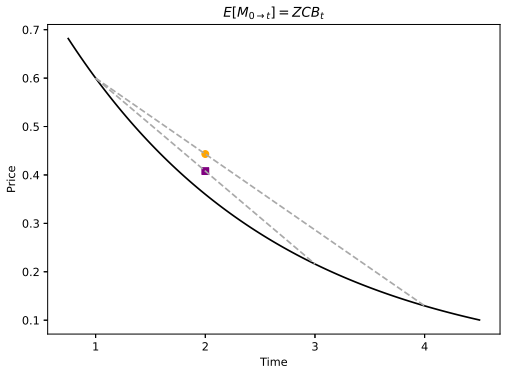
Remark 1 (Idiosyncratic Risk for Assets with Uncertain Outcomes (Duffie, 2001); fundamental result)**.** *With idiosyncratic risk, given SDF M and asset A with pure uncertain outcomes paying off at time t', $Price_t(A) = E_{P_t}[M_{t\to t'}A_{t'}] = E_{P_t}[M_{t\to t'}]E_{P_t}[A_{t'}]$, and so in particular the higher order moments of the asset such as $Var_{P_t}[A_{t'}]$ do not affect the price.*

Theorem 1, in contrast, shows that the higher order moments of the stopping time for an asset with uncertain timing do affect the price, regardless of which SDF one uses.

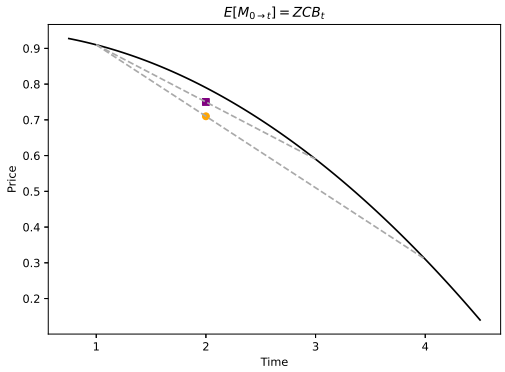
Theorem 1 (Higher Order Moments for Uncertain Timing)**.** *Consider an asset with pure uncertain timing, with associated stopping time τ which has purely idiosyncratic risk. The higher order moments of τ affect the valuation of the asset.*

The intuition is as follows. When an asset with uncertain outcomes has purely idiosyncratic risk, its price as discussed in Remark 1 becomes an expectation of the SDF times an expectation of the asset payoff. When the asset has uncertain timing,

FIGURE 1: Pricing Assets with Uncertain Timing with Purely Idiosyncratic Risk



(A) Sample SDF 1 - Receiving \$1 at Time t



(B) Sample SDF 2 - Receiving \$1 at Time t

Note: These figures plot the price of two assets for two sample SDFs. In each figure, the curve corresponding to the price of zero-coupon bonds arising from the SDF is shown. In Figures 1a, this curve is convex, and in Figure 1b, it is concave. In each figure, the price of two assets is shown. The asset whose price is depicted by a purple square pays out \$1 at time 1 with 50% probability and at time 3 with 50% probability. The asset whose price is depicted by an orange circle pays out \$1 at time 1 with 66.6% probability and at time 4 with 33.3% probability. Both assets are assumed to have purely idiosyncratic risk. For Theorem 1, observe that both assets have an expected stopping time of 2 (thus lining up on the x axis), yet their prices differ regardless of the SDF used. For Theorem 2, observe that $E[M_{0\to t}]$ is exactly the price of a zero coupon bond maturing in t periods; then re-interpret the probabilities on the line as a portfolio.

the price is similarly an expectation of the SDF, but that expectation is taken over time. If the SDF has any curvature - as shown for example in Figure 1 - this expectation embeds nonlinearity in it. Because of the nonlinearity, the higher order moments of the stopping time affect valuation.

To illustrate, Figure 1 plots the valuation of two assets with purely idiosyncratic risk. One pays out \$1 at Time 1 with 1/2 probability and at Time 3 with 1/2 probability. The other pays out \$1 at Time 1 with 2/3 probability and at Time 4 with 1/3 probability. Both have an expected value in the stopping time of 2. But they have different higher order moments - for example, the variance of stopping time of the former asset is 1, and of the latter asset 2. Because of this difference in the higher order moments of the stopping time, they are priced differently for any SDF, whenever the SDF has curvature in time. Figure 1 shows the different pricing for an SDFs with different types of curvature, illustrating that the valuation is different regardless of the SDF used, except in the special case where the SDF is linear.

2.1.2 Model-Free Characterization

Theorem 1 indicates that higher-order moments matter for pricing an asset with uncertain timing. Is it possible to say anything more about the price of such assets, in a model-free way? Or is the pricing of these higher order moments sufficiently complex to warrant use of statistical techniques (Erickson et al., 2017)?

I show in Theorem 2 that it is possible to completely characterize the price of an asset with uncertain timing using only the assumption of existence of an SDF, in a straightforward way. The theorem proves that the price of an asset with uncertain timing is exactly equal to the price of a portfolio of zero-coupon bonds with portfolio weights equal to the asset's stopping time probabilities.

The theorem shows a tight linkage between bond pricing results and the pricing of assets with uncertain timing. For example, by leveraging this result, known results for the pricing of bonds can be directly applied to price assets with uncertain timing which have purely idiosyncratic risk.

Theorem 2 (Zero-Coupon Bond Equivalent)**.** *Consider an asset X with pure uncertain timing, associated stopping time τ, parameters (c, a), and purely idiosyncratic risk from time t. Define:*

- $ZCB_{t'}$ *= a zero coupon bond paying* \$1 *at time* t' *and* \$0 *otherwise*
- $Portfolio_\tau = \sum_{t' \geq t} P_t(\tau = t') \cdot ZCB_{t'}$

Then:

$$Price_t(X) = c \cdot \sum_{t' \geq t} Price_t(ZCB_{t'}) + a \cdot Price_t(Portfolio_\tau)$$

The intuition behind Theorem 2 is as follows. When writing out the price of an asset with uncertain timing which has purely idiosyncratic risk, the covariance between the SDF and the asset vanishes by virtue of the risk being idiosyncratic. What is left is a weighted sum over the expected value of the SDF at each point in time, where the weightage on the time is exactly the stopping time probability.

Now notice that probabilities sum to one. What else sums to one? Portfolio weights. Using a straightforward relabeling, one obtains that the pricing equation is precisely equal to that of a portfolio of suitably chosen zero-coupon bonds. The visual intuition is depicted in Figure 1.

2.1.3 Model-Free Directionality and Asymmetry

Theorem 3 derives another model-free way in which time fundamentally differs from outcomes. For assets with uncertain outcomes which have pure idiosyncratic risk, the valuation is equivalent to that of the certain-outcome equivalent, from Remark 1.

For assets with uncertain timing, however, whether or not the valuation of the asset is higher or lower than the certain-time equivalent is characterized by the risk-free term structure. This means that, unlike for outcomes, the asset with pure uncertain timing can have higher, lower, or equal valuation as its certain-time equivalent.

Theorem 3 shows this difference between states and time formally. The intuition is that when the yield curve is strongly convex, the value of possibly receiving a positive payment at some point later in the future is much less than the value of receiving it at the expected stopping time. The intuition flips for negative payments, since the negative effect of cash flowing out is much less at a possible later time than at the expected stopping time. The sign intuition is similar to Onay and Onculer (2007), who find opposite behavior for gains and losses with additive utility, but here applies for any stochastic discount factor, proven through the use of zero-coupon bonds. The proof intuition is similar to that used for convexity in bonds (Berk and DeMarzo, 2011), but here applies for a dimension of uncertainty.

Definition 8 (Term Structure)**.**
Given the discount function $m(T) := Price_t(ZCB_T)$ *representing the time-t price of a zero coupon bond with maturity* T*, the yield to maturity (or spot rate)* $y(T)$ *is defined by:* $m(T) = e^{-T \cdot y(T)}$*. Equivalently,* $y(T) = -\frac{\ln m(T)}{T}$*.*

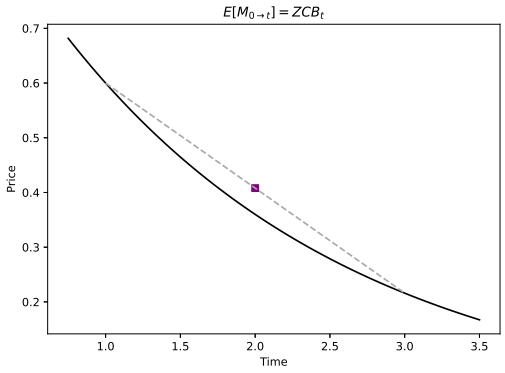
Theorem 3 (Model-Free Directionality: Characterized by the Risk-Free Term Structure)**.** *Consider any asset with uncertain timing with associated stopping time* τ *and payoffs* $(c, c+a)$ *which has purely idiosyncratic risk. Let* $y(T)$ *denote the yield to maturity at time* T*.*

Then uncertain timing has a higher (resp. lower) valuation than certain timing for $a > 0$ *(resp.* $a < 0$*) if and only if*

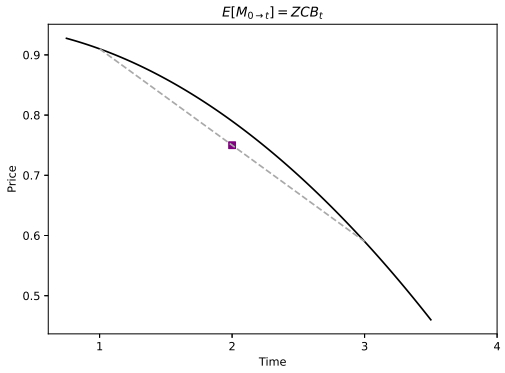
$$(y(T) + T \cdot y'(T))^2 > 2y'(T) + T \cdot y''(T) \quad \textit{for all } T \in \mathcal{T}$$

The proof intuition is depicted in Figure 2. The directionality - whether the asset price lies above or below the black curve at the expected stopping time - depends on

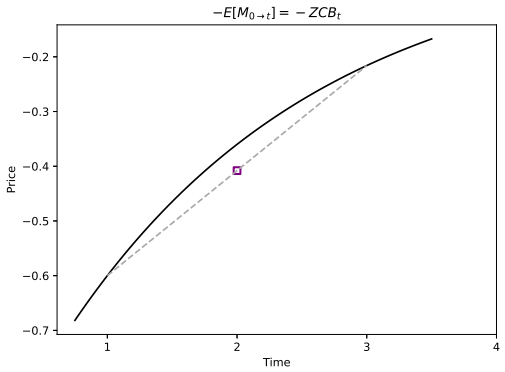
FIGURE 2: Certainty vs. Uncertainty and Asymmetry



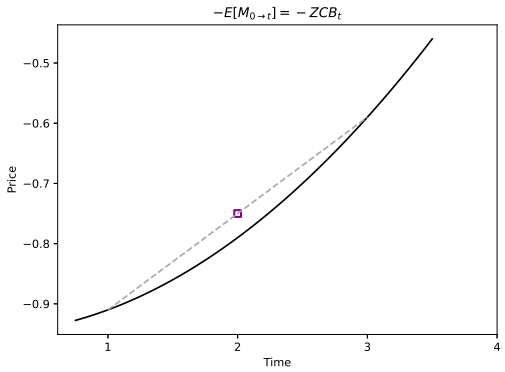
(A) Sample SDF 1 - Receiving $1 at Time t



(B) Sample SDF 2 - Receiving $1 at Time t



(C) Sample SDF 1 - Receiving (-$1) at Time t



(D) Sample SDF 2 - Receiving (-$1) at Time t

Note: These figures plot the price of gaining (subfigures 2a and 2b) and losing (subfigures 2c and 2d) $1 of purely idiosyncratic risk, for two sample SDFs (the same sample SDFs as in Figure 1). Observe that, for losing $1, the expected SDF curve flips over the x-axis (subfigures 2a and 2c show the flip for sample SDF 1; and subfigures 2b and 2d for sample SDF 2). The top two figures also depict, as a filled purple square, the price of an asset with uncertain timing with purely idiosyncratic risk which pays $1 in period 1 with 50% probability and in period 3 with 50% probability. The bottom two figures depict, as a hollow purple square, for each sample SDF, the price of an asset with uncertain timing with purely idiosyncratic risk which pays (-$1) in period 1 with 50% probability and in period 3 with 50% probability. For Theorem 3, observe that whether certainty is preferred to uncertainty is visually whether the purple square is below or above the black line; and that depending on the SDF, either direction can occur. For Corollary 1, observe that whenever the filled purple square is above the black line, the hollow purple square is below the black line, and vice versa; e.g., compare subfigures 2a and 2c. Similarly compare subfigures 2b and 2d. Because of the flipping over the x-axis, for *any* SDF, a higher valuation for certainty for positive shocks must be coupled with a higher valuation of uncertainty for negative shocks, and vice versa.

the curvature of the the price of zero-coupon bonds. This pricing curve in turn can be characterized by the yield curve.

Corollary 1 (Asymmetry)**.** *Any SDF that has a higher valuation for uncertain timing than certain-time equivalents for positive shocks must have a higher valuation for certain-time equivalents than uncertain timing for negative shocks. Any SDF that has a higher valuation for certain-time equivalents than uncertain timing for positive shocks must have a higher valuation for uncertain timing than certain-time equivalents for negative shocks.*

The intuition is shown in Figure 2. For example, compare Figures 2a and 2c. These both depict the same SDF. In the former, the payoff is positive - think of receiving \$1 in t periods for each zero-coupon bond. Similarly the asset payoffs are positive: for example the purple square depicts receiving \$1 in period 1 with 50% probability and in period 3 with 50% probability. When considering an asset with uncertain timing which has negative payoffs, e.g. losing \$1 in period 1 with 50% probability and losing it in period 3 with 50% probability, the relevant zero coupon-bond comparison (Theorem 2) also becomes negative: losing \$1 at each possible maturity t. As a result, the SDF curve gets flipped over the x-axis. Because of this flipping, if the curve was convex when positive, it becomes concave when negative, and vice versa. For example, in Figure 2a the SDF was convex when positive, and as a result the purple square (the valuation of the asset with uncertain timing paying off \$1) was above the black curve; when that flipped over the x-axes in Figure 2c, the purple hollow square (the valuation of the asset with uncertain timing paying off -\$1) goes below the black curve. Similarly for Figures 2b and 2d, where concavity in the positive axis indicates certainty is valued higher than uncertainty for positive shocks; and the corresponding convexity when flipped over the axis indicates the opposite, e.g uncertainty is valued higher than certainty for negative shocks.

Remark 2 (Yield Curve Translation)**.** *Uncertain timing is always valued higher (resp. lower) than certain timing for positive (resp. negative) shocks when:*

- *The yield curve is flat and nonzero:* $y' = 0, y'' = 0, y \neq 0$.
- *The yield curve is downward sloping and weakly convex:* $y' \leq 0, y'' \leq 0$.
- *The yield curve does not rise too steeply:* $2y'(t) \leq Ty''(t)$.

2.2 Systematic Risk

Many real-world assets have both uncertain timing and uncertain outcomes. For such assets, this section introduces a novel decomposition of systematic risk into systematic outcome risk and systematic timing risk.

These two components can be intuitively described as follows. Systematic outcome risk is what is typically thought of when discussing systematic risk (Berk and van Binsbergen, 2016; Campbell, 2017): conditional on the asset's stopping time being hit at time t, the covariance of the level of the SDF with the level of the asset payout. Systematic timing risk measures whether the asset is more likely to pay out in states of the world where the SDF is higher or lower. Theorem 4 and Corollary 2 derive the decomposition and discuss the intuition.

Directionality results for systematic risk will depend on the form of the SDF, by definition of the covariance term (Definition 5). However, it is possible to derive directionality results for a wide class of utility-based SDFs when the asset represents aggregate risk (Theorem 5).

2.2.1 Systematic Risk Decomposition

This section derives the systematic risk decomposition into systematic outcome risk and systematic timing risk, and provides a numerical example for the decomposition.

Systematic outcome risk is standard: conditional on the asset paying off at time t, it is the covariance of the asset payoff with the SDF level at time t. To weigh it in the overall decomposition, it is multiplied by the probability that the asset does pay off at time t.

Systematic timing risk is computed at each date t by first computing the number of times at which the asset's stopping time takes on a non-zero probability; call this number N. Then create N mega-states: one for each time that the stopping time can take on with non-zero probability. Compute the conditional SDF in each of these mega-states. Compute the conditional asset payoff in each of these mega-states, then set the conditional amount to zero in all states except the one in which the asset pays out at time t. The probability-weighted covariance of these two series – the conditional SDFs against the conditional shock payoffs – is systematic timing risk. Intuitively, it measures whether, in the cases where the asset actually pays at t, the expected conditional discount factor at t tends to be higher or lower than its average

across all possible paying dates.

The proof intuition is a direct application of the law of total covariance. One appealing property of this decomposition is that, unlike common decompositions used in the literature, which depend on a first-order Taylor approximation (Duffie, 2001), the decomposition shown here is complete.

Theorem 4 (Time-t Systematic Risk Decomposition)**.** *Consider an asset with uncertain timing and outcomes as in Definition 6. At each time t, the systematic risk decomposes as:*

$$Cov(M_{0\to t}, X_t) = \underbrace{P(\tau = t) \cdot Cov(M_{0\to t}, Y \mid \tau = t)}_{\textit{Systematic Outcome Risk at } t} + \underbrace{Cov\left(\mathbb{E}[M_{0\to t} \mid \tau], \mathbf{1}_{\{t=\tau\}} \cdot \mathbb{E}[Y \mid \tau]\right)}_{\textit{Systematic Timing Risk at } t}$$

Corollary 2 (Overall Systematic Risk Decomposition)**.** *Let the asset pay the shock Y at the (random) date τ. Then*

$$\begin{aligned}\mathrm{Cov}\left(\sum_t \mathbf{1}_{\{\tau=t\}} M_{0\to t}, Y\right) = &\underbrace{\sum_t \mathbb{P}(\tau = t) \cdot \mathrm{Cov}(M_{0\to t}, Y \mid \tau = t)}_{\textit{Total systematic outcome risk}} \\ &+ \underbrace{\mathrm{Cov}\left(\mathbb{E}\left[\sum_t \mathbf{1}_{\{\tau=t\}} M_{0\to t} \,\middle|\, \tau\right], \mathbb{E}[Y \mid \tau]\right)}_{\textit{Total systematic timing risk}},\end{aligned}$$

where the first term exactly corresponds to the sum of similar terms in Theorem 4 and the second term is a probability weighted covariance over two random variables, where the probability weights are given by the stopping time τ's probabilities.

Remark 3 (Special Cases of the Systematic Risk Decomposition)**.** *Two special cases of the systematic risk decomposition are:*

1. *Pure systematic timing risk. If $Y = a$ (constant), then for every t, $\mathrm{Cov}(M_{0\to t}, Y \mid \tau = t) = \mathrm{Cov}(M_{0\to t}, a \mid \tau = t) = 0$, so the outcome component vanishes and all systematic risk is timing risk.*

2. *Pure systematic outcome risk. If $\mathbb{P}(\tau = t^*) = 1$ (deterministic timing), then the timing term $\mathrm{Cov}\big(\mathbb{E}[M_{0\to t} \mid \tau],\, \mathbf{1}_{\{\tau=t\}}\mathbb{E}[Y \mid \tau]\big)$ is zero, and only $\mathrm{Cov}(M_{0\to t^*}, Y \mid \tau = t^*) = \mathrm{Cov}(M_{0\to t^*}, Y)$ remains.*

Example 2 (Numerical Example: Systematic Risk Decomposition)**.** *Consider a probability space with state space* $\Omega = \{GG, GB, BG, BB\}$*, times* $T = \{1, 2\}$*, and probability measure* P *given in the table below. Define an asset with uncertain timing and outcomes (Definition 6) with baseline* $c = 2$*, stopping time* τ*, and shock* Y *as follows:*

State ω	$P(\omega)$	$\tau(\omega)$	$Y(\omega)$	c	*Asset Payoff at Time 1*	*Asset Payoff at Time 2*	$M_1(\omega)$	$M_2(\omega)$
GG	0.30	1	13.0	2.0	15.0	2.0	0.90	0.85
GB	0.30	1	5.0	2.0	7.0	2.0	0.94	0.91
BG	0.25	2	2.0	2.0	2.0	4.0	0.96	0.93
BB	0.15	2	0.0	2.0	2.0	2.0	0.97	0.95

where $M_t(\omega) := M_{0\to t}(\omega)$ *denotes the stochastic discount factor from time 0 to time* t *in state* ω*.*

The asset's payoff process is:

$$X_t(\omega) = c + \mathbf{1}_{\{t=\tau(\omega)\}} \cdot Y(\omega) \quad \textit{for } t \in \{1, 2\}.$$

We compute the time 1 systematic risk decomposition.

For systematic outcome risk, we calculate the covariance of the SDF with the shock conditional on the asset paying at time 1, times the probability of paying at 1. Conditional on $\tau = 1$*:*

$$P(GG \mid \tau = 1) = \frac{0.30}{0.60} = 0.5, \qquad P(GB \mid \tau = 1) = \frac{0.30}{0.60} = 0.5.$$

The conditional means are:

$$\begin{aligned} \mathbb{E}[M_1 \mid \tau = 1] &= 0.90 \cdot 0.5 + 0.94 \cdot 0.5 = 0.92, \\ \mathbb{E}[Y \mid \tau = 1] &= 13.0 \cdot 0.5 + 5.0 \cdot 0.5 = 9.0. \end{aligned}$$

The conditional covariance is:

$$\begin{aligned} \mathrm{Cov}(M_1, Y \mid \tau = 1) &= \mathbb{E}[M_1 Y \mid \tau = 1] - \mathbb{E}[M_1 \mid \tau = 1] \cdot \mathbb{E}[Y \mid \tau = 1] \\ &= (0.90 \cdot 13.0 \cdot 0.5 + 0.94 \cdot 5.0 \cdot 0.5) - 0.92 \cdot 9.0 \\ &= (5.85 + 2.35) - 8.28 = -0.08. \end{aligned}$$

Therefore, systematic outcome risk at time 1 is $P(\tau = 1) \cdot \mathrm{Cov}(M_1, Y \mid \tau = 1) = 0.6 \cdot (-0.08) = -0.048$*..*

We now compute systematic timing risk at time 1. Since there are two times at which the asset pays off with positive probability, create two mega-states: one where the asset pays at time 1 and one where it pays out at time 2. Then:

$$\begin{aligned}
\mathbb{E}[M_1 \mid \tau = 1] &= 0.92, \\
\mathbb{E}[M_1 \mid \tau = 2] &= \frac{0.96 \cdot 0.25 + 0.97 \cdot 0.15}{0.40} = 0.96375, \\
\mathbb{E}[Y \mid \tau = 1] &= 9.0, \\
\mathbb{E}[Y \mid \tau = 2] &= \frac{2.0 \cdot 0.25 + 0.0 \cdot 0.15}{0.40} = 1.25.
\end{aligned}$$

Computing the covariance and setting $\mathbb{E}[Y \mid \tau = 2]$ *to 0 using the indicator* $\mathbf{1}_{\{\tau=1\}}$*:*

$$\begin{aligned}
\mathbb{E}[\mathbb{E}[M_1 \mid \tau]] &= 0.92 \cdot 0.6 + 0.96375 \cdot 0.4 = 0.9375, \\
\mathbb{E}\big[\mathbf{1}_{\{\tau=1\}} \cdot \mathbb{E}[Y \mid \tau]\big] &= 9.0 \cdot 0.6 + 0.0 \cdot 0.4 = 5.4, \\
\mathbb{E}\big[\mathbb{E}[M_1 \mid \tau] \cdot \mathbf{1}_{\{\tau=1\}} \cdot \mathbb{E}[Y \mid \tau]\big] &= 0.92 \cdot 9.0 \cdot 0.6 + 0.96375 \cdot 0.0 \cdot 0.4 = 4.968.
\end{aligned}$$

Therefore, systematic timing risk at time 1:

$$\operatorname{Cov}\big(\mathbb{E}[M_1 \mid \tau], \mathbf{1}_{\{\tau=1\}} \cdot \mathbb{E}[Y \mid \tau]\big) = 4.968 - 0.9375 \cdot 5.4 = -0.0945.$$

The total systematic risk at time 1 is the sum of the two components:

$$\operatorname{Cov}(M_1, X_1) = -0.048 + (-0.0945) = -0.1425.$$

2.2.2 Directionality and Aggregate Risk: Utility-Based SDFs

The systematic risk results above did not make any mention of directionality. When the asset represents aggregate risk, e.g. it is the only source of uncertainty in the economy, it is possible to obtain directionality results for assets with uncertain timing (c, a) where the shock amount a is small and the baseline payoff $c > 0$, for a wide class of utility functions.

Theorem 5 derives directionality results for any SDF arising from the large class of Kreps-Porteus preferences, which are characterized by the arguably attractive properties of dynamic consistency, history independence, and no probability distortion (Kreps and Porteus, 1978). As a result, this class encompasses many commonly used

utility functions. For small shocks, Theorem 5 therefore generalizes to a larger class the directionality results in the decision theory literature on expected utility (Chesson and Viscusi, 2003), additive utility (Onay and Onculer, 2007), and Epstein-Zin (DeJarnette et al., 2020).[1] In particular, one implication is that even moving outside these standard functions, strong directional predictions would remain.

The theorem uses utility levels. While the general link between utility and prices is textbook (Duffie, 2001), for precision, Lemma 1 provides an intermediate linkage between price for aggregate risk and utility level for assets with uncertain timing.

Lemma 1. *Suppose that the SDF derives from some differentiable utility function U, and asset A with uncertain timing and payoffs $(c, c+a)$ represents aggregate risk. Then, for a sufficiently small, asset A is valued more than its certain-time equivalent if and only if the utility of asset A is higher than that of its certain-time equivalent.*

Theorem 5 (Aggregate Risk Directionality: Utility Class)**.** *Suppose the representative agent's SDF derives from a Kreps-Porteus representation with convex time discounting. Then for any asset with uncertain timing and payoffs $(c, c+a)$, where the representation is smooth at c and when the asset represents aggregate risk, uncertain timing is valued higher (resp. lower) than certain timing when $a > 0$ (resp. $a < 0$) for sufficiently small $|a|$.*

The proof uses a Taylor expansion, which makes smoothness at c (e.g. derivatives being well-defined) key. It is worth noting that commonly used utility aggregators do not have a well-defined derivative at zero, so some positive aggregate consumption c, leveraging non-zero consumption away from stopping times in Definition 4, is needed for this proof approach.

2.3 Diversification

For assets with uncertain outcomes, it is often said that diversification is the only free lunch in finance (Van Nieuwerburgh and Veldkamp, 2010*a*; Campbell, 2017) because pooling assets with uncorrelated risks improves the risk profile while not affecting returns. This is one of the fundamental results in finance, and has been applied widely across markets (ex. Barber and Odean (2008, 2013)).

[1] See also Dillenberger et al. (2024) for other time-related properties.

A natural question is whether the price impact of an asset with uncertain timing can be neutralized by holding a pool of such assets.

Theorems 6 and 7 seek to answer this question. Theorem 6 considers independent stopping times and shows that in the limit, flows become predictable, removing uncertainty. But pooling does not eliminate the impact of higher-order moments, even when each asset has purely idiosyncratic risk. The higher order impact is no longer due to risk – because flows become predictable – but rather due to the predictable flow's dispersion.

A natural next question is whether the time vs. outcome gap vanishes when the portfolio is chosen to smooth any shocks, for example as with perfectly negatively correlated assets. Theorem 7 shows that the difference in pricing time vs. pricing outcomes remains even with perfectly deterministic and smoothed payments through diversification. In particular, the higher-order moment and asymmetry results that applied for individual assets continue to hold even after a synthetic risk-free asset is created.

2.3.1 Independent Stopping Times

Theorem 6 treats the case with infinitely many independent stopping times and shows that while payments are now deterministic, the conceptual differences highlighted earlier in the paper for individual assets continue to hold.

Theorem 6 (Diversification of Idiosyncratic Uncertain Timing: Independent Stopping Times)**.** *Consider a set of N assets $A_1, .., A_n$ with uncertain timing which have payoffs $(c_i, a_i)_{i\in\{1,2,..,n\}}$ and associated stopping times $\tau_1, .., \tau_n$. Each asset A_i has purely idiosyncratic risk. The stopping times are assumed to be independent of each other. In addition, the limits $\bar{c} := \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} c_i$ and $\bar{a}(t') := \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} a_i\, P_t(\tau_i = t')$ are assumed to be well-defined.*

Define the equally weighted portfolio $P^{(n)}$ as

$$P_{t'}^{(n)} := \frac{1}{n}\sum_{i=1}^{n} A_{t'}^{(i)} = \underbrace{\frac{1}{n}\sum_{i=1}^{n} c_i}_{\textit{baseline}} + \underbrace{\frac{1}{n}\sum_{i=1}^{n} a_i\, \mathbf{1}_{\{\tau_i = t'\}}}_{\textit{timing shock at } t'}.$$

Then:

(a) *The payoff stream becomes deterministic in the limit.*

(b) *For any SDF* $\{M_{0\to t}\}_{t\in T}$*, the price of the portfolio is equivalent to the price of an appropriately selected portfolio of zero-coupon bonds.*

(c) *The higher order moments of the dispersion across time, although now deterministic, affect valuation.*

2.3.2 Perfect Smoothing

We now study what happens with perfectly negatively correlated stopping times, so that payments are deterministic and smooth. For example, suppose that asset A_1 pays out \$1 at time 1 and at time 3 with 50% probability each, and asset A_2 does the same but is perfectly negatively correlated with asset A_1. Then holding a portfolio of $1/2$ A_1 and $1/2$ A_2 gives a payoff of \$0.5 at time 1 and \$0.5 at time 3 deterministically – a synthetic risk-free asset. Does this perfect negative correlation change the time vs. outcomes results of the earlier theorems?

It turns out that time and outcomes continue to differ. Whereas for outcomes, a synthetic risk-free asset created by combining two assets with identical payoffs and perfect negative correlation, is valued identically to the deterministic asset that pays the expected payoff, in the time dimension this is no longer the case. That is, the valuation of synthetic risk-free assets can differ from the valuation of a true risk-free asset paying at the expected stopping time.

Similarly, as in Theorem 6, the higher order moments of the temporal dispersion continue to affect valuation for time although higher order moments do not matter for diversified outcomes.

Theorem 7 proves the general case. Its statement is more general than perfect negative correlation: it says, instead, suppose there is a way to combine assets with uncertain timing to achieve identical and deterministically predictable payoffs at each date. Perfect negative correlation is one way to achieve this; there may be others. Even so, the above more nuanced time vs. outcomes differences continue to hold.

Theorem 7 (Diversification of Assets with Uncertain Timing: Smoothing Shocks)**.** *Let* $A_1, \ldots, A_n$ *be assets with uncertain timing with purely idiosyncratic risk, where asset* i *pays* c_i *at all times plus an additional* a_i *at a random stopping time* τ_i*. Assume further that each* τ_i *has the same marginal distribution.*

Suppose there exists a portfolio with weights $p = (p_1, \ldots, p_n)$ *such that the portfolio deterministically pays amount* c *at all times plus an additional* s *at each shock time*

in S.

Then:

(a) The price of the portfolio is equivalent to the price of a portfolio of suitably chosen zero-coupon bonds.

(b) The temporal dispersion of the shock times affects valuation.

(c) The portfolio value can be higher, lower, or equal to its certain time equivalent, e.g. a payment of the same total amount at time $\mathbb{E}[\tau_1]$ (plus the baseline c at all times).

(d) If the portfolio value exceeds (resp. is below) the certain-time equivalent when the shock amount $s > 0$, it must be below (resp. exceed) it when $s < 0$.

Remark 4 (Contrast with Outcomes)**.** *Combining perfectly negatively correlated assets which have uncertain outcomes and pay at the same date t creates a risk-free asset. In particular, if A_1 and A_2 are assets with uncertain outcomes, and they have identical cumulative distribution functions but are perfectly negatively correlated, holding a portfolio with equal portfolio weights on each of them creates a risk-free asset whose valuation is identical to the asset that deterministically pays $E[A_1]$ at time t.*

By contrast, Theorem 7 shows that even when aggregate timing is made deterministic, for example via perfect negative correlation, the present value generally differs from paying the same total amount once at the average time. The wedge is a curvature effect of the SDF, not a risk premium.

In addition, the asymmetry implication of Corollary 1 remains, even though there is no uncertainty: if the wedge is positive (resp. negative) for positive shocks ($s > 0$), the wedge must be negative (resp. positive) for negative shocks ($s < 0$).

3 Model-Free Financial Market Tests

Section 2 derived model-free theoretical results for the pricing of uncertain timing.

A natural question is whether uncertain timing is actually priced in financial markets. This is a challenging question to answer because many financial settings which have uncertain timing also have uncertain outcomes. And any structural approach – any choice of SDF – embeds predictions in it, as shown in Section 2.

To circumvent these difficulties, I use option markets as a laboratory in which to conduct model-free tests for whether uncertain timing is priced. These tests are even more model-free than the theoretical results, and do not even assume existence of an SDF.

The tests will therefore necessarily be limited in their ability to pinpoint parameters: their model-free nature means that this or structurally pointing towards sources of pricing will not be possible. Nevertheless, this simple model-free comparison is consistent with a pricing of uncertain timing over and above predictions from the options literature. For example, they find a departure from the classic derivatives theorem of Merton (1973).

I start by describing the data and its linkage to the theory before describing the null hypotheses from the derivatives literature, and how the empirical results are not consistent with any of these but are consistent with a pricing of uncertain timing.

3.1 Empirical Strategy

There are two types of commonly traded options: American and European. European options may be exercised only on the designated expiration date. American options can be exercised any time up to and including expiry. As a result, the short side of an American is exposed to uncertain timing that the short side of a European is not.

Derivatives theory links the optimal exercise of an American option to dividend considerations (Duffie, 2001). When call options contain no dividends in their lifetime, it is never theoretically optimal to exercise an American option early (Merton, 1973). Nevertheless, empirically early exercise does occur (Finucane, 1997; Poteshman and Serbin, 2003; Jensen and Pedersen, 2016). As a result, the short side of an American is exposed to uncertain timing through this empirical early exercise (which is not theoretically optimal). One can ask whether this uncertain timing is priced above and beyond standard considerations from the derivatives literature.

To do so, I compare the pricing of otherwise identical American and European option contracts. The Chicago Board Option Exchange (CBOE) offers both American and European options on the S&P 100, which have the identical contract size, trading fees, settlement times, and other contract specifications.[2] I consider the difference in

[2]There also exist both American and European options on the S&P 500; in particular, American-style contracts on SPDR and European-style contracts on the S&P 500 index. However, these differ in at least three ways that OEX and XEO do not. First, the most liquid of the S&P 500 options,

pricing between these otherwise identical American and European options in both quotes and trades. In quotes, to focus on the equilibrium short side pricing, the main object of consideration is the ask difference. In trades, it is the traded price difference.

This exercise contributes to a literature on the pricing of American options relative to European ones. Prior papers in this literature focuses on the early exercise premium (Sung, 1995; Zivney, 1991; Harvey and Whaley, 1991; Li et al., 2024), typically using midpoints; I focus here on uncertain timing, towards which it is helpful to separate out bid and ask quotes.[3]

The linkage to the theory is conceptually a dominance relationship. From the perspective of the short side of an option, money can arrive or depart only at expiry for a European option but at any time up to and including expiry for an American option. As a result, an example that would conceptually correspond to the options comparison is \$x in, say, 7 days for sure ('Sure Later Timing'), against \$x in 1 day with some probability; 2 days with some probability; and so on, up to 7 days ('Uncertain Sooner Timing'). The second lottery time-dominates the first one, in that all payments are received or made weakly sooner. While the sign of x would be informative for directionality results, it is not observed in the data; nor is empirical early exercise. As a result, it is not possible to directly estimate the cost to the short side of early exercise; the most that can be said is whether quotes are trades are consistent with a pricing of uncertain timing.

SPY and SPX, have different contract sizes. In contrast, OEX and XEO have the same contract size. Second, all S&P 500 index options have different CBOE trading fees than SPY, meaning that even after matching on contract size, one would not be able to differentiate fees from pricing premia (CBOE, 2024*a*). This is because it is not possible to see participant identity in the data, and the fees differ by participant type. In contrast, OEX and XEO have the same CBOE trading fees for all participants. Third, SPY versus SPX (and other S&P 500 index options) settle 15 minutes apart; that difference in settlement time can contribute to what is known as a wildcard premium (Fleming and Whaley, 1994), further affecting pricing. In contrast, OEX and XEO have the same settlement time.

[3]To my knowledge, there is one prior paper with a reduced-form comparison of American to European options that considers bids versus asks; it finds that most of the premium shows up on the ask side (Dueker and Miller, 2002). The paper hypothesizes that it may be because the options in question were about to be discontinued leading to wider bid-ask spreads. Uncertain timing provides a more general way to rationalize why differences in American versus European options may show up in ask quotes - the short side may requires compensation to bear uncertain timing. Further, the options that I consider here are continuously traded and not discontinued at any point in the sample.

3.1.1 Data

I use minute-by-minute quote data from CBOE, which offers both OEX and XEO.[4] I compare quotes within the same minute. CBOE treats both OEX and XEO identically for the purpose of fees, settlement dates and times, contract size, expiration dates, tick size, and other institutional features (CBOE, 2024*c*,*b*). The minute-by-minute quote data is available starting in 2012. Starting in 2020, the volume of contracts traded across these asset classes drops below 100,000; for example, in 2024 it was only on the order of 3,000 contracts. The sample I use is therefore Jan 2012 - Dec 2019.

Data on dividends paid by companies in the S&P 100 comes from Compustat, whose series starts in 2002; data on end-of-day levels of the S&P 100 also comes from Compustat. Data on Federal Open Market Committee meeting dates comes from the Federal Reserve Board. For robustness, Appendix B.3 also collects dividend data by hand and shows that it maps well to Compustat.

Following standard data cleaning procedure for options data (Muravyev, 2016), I drop any quotes with a bid less than \$0.05 or where the bid-ask spread is negative. Following Constantinides et al. (2013) and Ni et al. (2008), to focus on credible quotes, I drop options with a moneyness (defined as (underlying-strike)/strike) of below -0.2 or above 0.2. As is the case in other option markets, these tend to be less traded than those within the 0.2 moneyness band.

Because both OEX and XEO options settle to the closing price on the day of exercise, I also drop quotes where the option expires on the same day as the quote.

While most contract specifications are held fixed by CBOE, there are two features that differentiate the American and European options that should be controlled for. The first and most important is dividends. When a stock is known to start paying a dividend on a particular date, the stock price adjusts downwards by the dividend amount on that date (Hull, 2018). The S&P 100 is a weighted average of stock prices. I therefore restrict to options in whose lifetime, including at initiation and expiry, there are no dividend considerations for any of the stocks in the S&P 100. This restriction substantially reduces the sample, but makes for a clean sample by

[4]Past research has shown that minute-by-minute data is superior to end-of-day quotes (such as those offered by OptionMetrics), which are subject to end-of-day volatility (Andersen et al., 2017) and end-of-day quotes occurring after the settlement value of the underlying is determined (van Binsbergen et al., 2012). For completeness, I re-do analysis in OptionMetrics data, in Online Appendix B.

removing the standard explanation for American option premia.[5]

The second feature that should be controlled for is liquidity: in this case, the American option is the more traded product, perhaps because it was introduced before the European version (Harvey and Whaley, 1991). Following Amihud (2002) and Bryzgalova et al. (2022), I use the bid-ask spread, (ask - bid)/midpoint, as the illiquidity measure. I use a 5% threshold in the main text, e.g. restrict attention to minutes where the difference in bid-ask spreads between the OEX and XEO quotes is less than 5%. For robustness, Table B.1 in Appendix B.2 re-performs the analysis for other thresholds and also without liquidity corrections. Regardless of the level of liquidity correction (or its removal), the difference in asks is positive and larger than the difference in bids, contrary to all null hypotheses from the derivatives literature in Section 3.1.2.

The notional value of all options in the final cleaned dataset is $3.65 billion.

3.1.2 Null Hypotheses

There are three null hypotheses from the derivatives literature that inform the prior on what the pricing results might look like.

The first and most fundamental is Merton's theorem (Merton, 1973). This says that for call options without dividends in their lifetime, the value of an American and European option should be the same. In particular, this also implies that the ask difference should be the same: $Ask_{American} - Ask_{European} = 0$.

The second and third null hypotheses arise from later advances in the derivatives literature.

Because the long side of an option may value the flexibility of early exercise, the long side may be willing to pay more for an American than otherwise identical European (Poteshman and Serbin, 2003). Proxying for the equilibrium willingness to be short with the ask and equilibrium willingness to be long with the bid, this second null hypothesis says that: $(Ask_{American} - Ask_{European}) - (Bid_{American} - Bid_{European}) = 0$.

That is, there may be a pricing difference arising from the long side, but after that

[5]Li et al. (2024) also consider the midpoints of options I study here, and find that at longer horizons, OEX is more expensive than XEO 86% of the time, consistent with an aversion-to-timing-uncertainty interpretation. However, they (1) do not correct for dividend concerns and (2) do not adjust for liquidity differences, and (3) do not separate the early exercise premium on the long side from the pricing of holding uncertain timing on the short side.

is accounted for, there should be no additional pricing difference for the short side of the American option.

Third, the short side may anticipate that the long side may suboptimally exercise early. Early exercise for a call option with no dividends in the lifetime of the option indicates that the long side is not taking full advantage of the option value (Duffie, 2001). This kind of adversarial thinking in advance should therefore indicate that the short side is willing to take less compensation for the option at the onset: $(Ask_{American} - Ask_{European}) - (Bid_{American} - Bid_{European}) \leq 0$.

3.1.3 Summary Statistics

Table 1 provides summary statistics for moneyness, days to expiry, and bid-ask spreads for all options in the analysis dataset post cleaning (Section 3.1.1). The bid-ask spreads are similar because of the imposed liquidity controls. Because companies within the S&P 100 pay out dividends at different times (Harvey and Whaley, 1991), in this data, the longest lived contracts with strictly zero dividend considerations have a lifetime from quote date to expiry of seven days. Appendix B.1 provides further descriptive statistics for these options.

TABLE 1: Summary Statistics for OEX and XEO Options

	Both	**OEX**	**XEO**
Bid-Ask Spread (%):			
Mean		6.27%	6.16%
Median		2.90%	2.69%
Standard Deviation		11.22%	11.30%
Moneyness:			
Mean	0.09		
Median	0.08		
Standard Deviation	0.06		
N. Contracts by Days to Expiry:			
1	743,713		
2	240,515		
3	112,668		
4	68,476		
5	11,036		
7	26,577		

Note: This table lists summary statistics for the main sample of OEX and XEO options, cleaned according to the procedure in Section 3.1.1. The bid-ask spread is calculated as $100 * (ask - bid)/midpoint$. Days to expiry is the number of days between the quote date and the expiration date of the option. Moneyness is calculated as $(underlying - strike)/strike$.

3.2 Results

3.2.1 Quotes

Table 2, Column 1 provides the baseline results. Unlike any of the null hypotheses (Section 3.1.2), the mean ask difference (OEX - XEO) is positive and statistically significant ($p < 0.01$) at \$0.33; and the ask difference less the bid difference is a positive and statistically significant ($p < 0.01$) at \$0.11 on average. These correspond to 68 basis points and 11 basis points as a percentage of the midpoint option price, respectively.

Figure B.3 in Appendix B.1 plots the average ask, bid, and midpoint difference, aggregated by week, over time. This figure shows that the ask difference is usually higher than the bid difference across all time periods considered. This indicates that the averages reported in Table 2 are not the result of a few positive outliers, but rather arise from a largely consistently positive ask difference less bid difference.

While these results are not consistent with the null hypotheses in Section 3.1.2, they are consistent with a pricing of uncertain timing. The uncertain timing here arises from the long side possibly exercising early. I take no position on why the long side might exercise early, and do not observe empirical early exercise in the data. To check for one crude possible proxy for likelihood of early exercise and whether it correlates with additional compensation required by (the proxy for) the short side, I consider moneyness; the more in-the-money an option is, the more tempted the long side may be to exercise. Figure 3 groups the ask difference − bid difference by moneyness bands, and finds that this difference-in-difference is approximately monotonic in moneyness.

Towards initial robustness checks, Column 2 of Table 2 restricts to in-the-money options; Column 3 restricts to dates in which there are no FOMC meetings; and Column 4 restricts to options with strictly positive open interest. The same results qualitatively hold throughout.

The robustness section, next, finds that these results continue to hold in various additional subsamples of the data.

3.2.2 Trades

As in Section 3.1.1, I consider only options in whose lifetime there are no dividends. I then match each XEO trade with an OEX trade with the same strike price and

TABLE 2: Pricing Differences between Dividend-Free American (OEX) and European (XEO) Options

	Baseline	In the Money	No Fed Meetings	Positive Open Interest
Ask Difference - Bid Difference	\$0.11***	\$0.11***	\$0.11***	\$0.05***
	(0.04)	(0.04)	(0.04)	(0.02)
Ask Difference	\$0.33***	\$0.34***	\$0.33***	\$0.22***
	(0.03)	(0.03)	(0.03)	(0.02)
Midpoint Difference	\$0.28***	\$0.28***	\$0.28***	\$0.19***
	(0.02)	(0.02)	(0.02)	(0.02)
Bid Difference	\$0.22***	\$0.23***	\$0.22***	\$0.16***
	(0.03)	(0.03)	(0.03)	(0.03)
Ask Difference Quartiles:				
25%	\$0.20	\$0.20	\$0.20	\$0.00
50%	\$0.30	\$0.30	\$0.30	\$0.20
75%	\$0.50	\$0.50	\$0.50	\$0.40
Midpoint Difference Quartiles:				
25%	\$0.15	\$0.15	\$0.15	\$0.025
50%	\$0.25	\$0.25	\$0.25	\$0.15
75%	\$0.40	\$0.40	\$0.40	\$0.30
Bid Difference Quartiles:				
25%	\$0.10	\$0.10	\$0.10	\$0.00
50%	\$0.20	\$0.20	\$0.20	\$0.10
75%	\$0.40	\$0.40	\$0.40	\$0.30
Number of Observations	1,202,985	1,169,096	1,202,985	205,223
Number of Unique Days	101	101	101	98

*Standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01.*

Note: This table shows the difference in quoted prices (OEX (American) - XEO (European)) between otherwise identical OEX and XEO options in dollar terms. Standard errors are clustered by date and by contract, where a contract is a (strike price, expiration date) pair. The baseline column performs this analysis on the data cleaned as described in Section 3.1.1. The in-the-money column restricts the cleaned data to only in-the-money options. The no-fed-in-lifetime column restricts the cleaned data to only options in whose lifetime there is no Fed meeting. The positive open interest column restricts the cleaned data to only option quotes and minutes for which both OEX and XEO have strictly positive open interest.

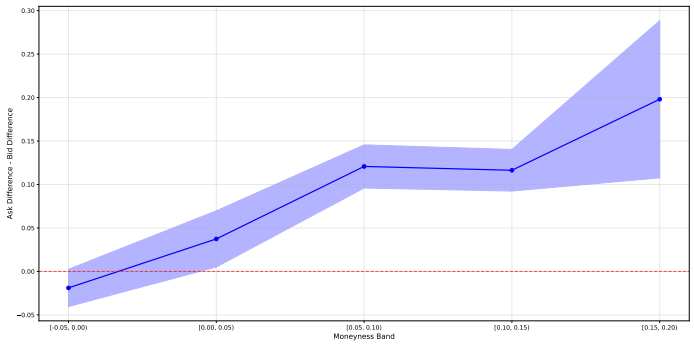


FIGURE 3: Ask Difference - Bid Difference by Moneyness

Note: This figure shows average and 95% confidence intervals for (Ask(OEX) - Ask(XEO)) - (Bid(OEX) - Bid(XEO)), with standard errors clustered at the date and contract level. Moneyness is defined as $(underlying - strike)/strike$. Moneyness bands are taken within 0.05 increments. For all options with a given moneyness band, the ask difference less bid difference is regressed on a constant, with standard errors clustered at the date and contract level. The resulting intercept estimate and its confidence intervals are shown.

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expiry, where both trades occur within 10 minutes of each other. If there are multiple such OEX trades, I take the one closest in time to the XEO trade. I then compare the traded price for the OEX and XEO trades. This method of cleaning yields 1189 trades in the sample across 62 days.

Following the matching procedure above finds that American options trade on average 3.75 cents higher than European options ($p < 0.05$, with standard errors clustered by date); as a percentage of the executed option price, this difference is on average 5.5%. The larger difference (as a percentage) for trades than for quotes comes from the average traded price being much lower than the average quoted price, as is typical for short-dated options where trading volume tends to concentrate in cheaper contracts (Andersen et al., 2017).

3.2.3 Assessing Possible Explanations

A pricing of uncertain timing is only one story that is consistent with the baseline results from quotes and trades; there may be many others. In this section, I consider alternative explanations from the literature and discuss their ability to capture the results.

Wildcard Premium Differences. Fleming and Whaley (1994) document that some options have wildcard premiums, meaning that the settlement price of the contract is established before the last time at which the option holder can decide to

exercise. This would affect a comparison of options on the S&P 500, because PM-settled SPX (European) options are exercised at the close of the trading day, but holders of SPY (American) have the option to exercise after the closing price of the underlying - so one is subject to a wildcard premium while the other is not. This is one of the reasons I chose not to use the S&P 500 for the reduced form comparison. A benefit of the options on the S&P 100 considered here is that both stop trading at the same time and have identical settlement specifications, and therefore there is no wildcard premium difference between them.

Re-Investment Risk. If interest rates are expected to changing during the life-time of the option in a highly volatile way, this could be a reason to avoid holding uncertain timing. Here, however, most options are short dated, making interest rate considerations unlikely to play an important role. As a further explicit check, Column 3 of Table 2 restricts attention to options in whose lifetime there are no FOMC meetings and finds identical results to the baseline.

Informed Trading. The next set of explanations explores the idea that the short side of the market may be strategically reacting to actions from the long side of the market. There are several reasons to believe that the trades corresponding to the main sample are uninformed. First, past research shows that trading in index options is generally uninformed, with informed investors typically locating in single-name stock options, rather than index options (Pan and Poteshman, 2006; Chan et al., 2002). Second Muravyev (2016) finds that small option trades tend to be uninformed in that they do not predict market movements. Appendix B.5 checks the proportion of small trades (few contracts traded, small monetary amount transacted, or both) and shows that these small trades constitute over over 50% of trades corresponding to the main sample. This proportion of small trades is similar across the American and European classes.

Hedging Considerations. One reason American options may be harder to hold for the short side than European ones is that, when there is uncertainty over the day of exercise, hedging becomes more complicated, and the short side of the American may have a practical or psychological aversion to dealing with this complexity (Hull, 2018). Appendix B.4 re-performs the analysis for only options whose hedging requirements

are not expected to change over their lifetime, e.g. whose gamma is close to zero. Though the sample becomes an order of magnitude smaller, the qualitative result: a pricing difference in asks, continues to hold.

Inventory Pressure It is possible that the long side moves differently than the short side. For example, under some models (Ho and Stoll, 1981), if a market maker has accumulated excessive long positions due to rising bids, they might lower ask prices to encourage sales and rebalance inventory. To allow for this flexibility, I run the regression: $(\text{ask}_{\text{American}} - \text{ask}_{\text{European}})_{m,o} = \beta_0 + \beta_1 (\text{bid}_{\text{American}} - \text{bid}_{\text{European}})_{m,o} + \epsilon_{m,o}$, where m denotes the minute and o the option. The regression yields a statistically significant $\beta_0 = 0.37$ ($p < 0.01$, clustering standard errors by date), consistent with compensation for being short the American over and above flexible pricing incorporation of the long side.

Rare Disasters. Seo and Wachter (2019) find that index put options are routinely more expensive to purchase than predicted by Black-Scholes, and finds that a model including a small probability of a rare disaster provides a better approximation of prices. My findings do not contradict this, but neither can they be fully explained by rare disasters. First, the analysis is on call options which, all else equal, should be less appealing in the face of rare disasters, while the analysis of Seo and Wachter (2019) focuses on put options, which provide insurance should a rare disaster occur. Second, the empirical test is reduced form and within-index options; it does not rely on Black-Scholes assumptions in any way.

Duration-Based Explanations van Binsbergen et al. (2012) and Giglio et al. (2023), among others, find that risk premia differ by the payoff horizon of the asset. Here, the maturity dates of the American and European are matched. In addition, so long as quotes move with early exercise value and incorporate a time value of money, it should never be optimal to exercise early per Merton (1973), no matter the underlying price process or pricing kernel.

3.3 Supplemental Pricing Narratives

To supplement the real world pricing results of Section 3, this section elicits survey-based pricing narratives. Respondents were asked to price uncertain timing and

uncertain outcomes, and asked their procedure for pricing after pricing uncertain timing. Detailed survey procedures are provided in Online Appendix A.

The pricing narratives are shown in Table 3. Many respondents (51% in an Large Language Model classification) do not differentiate between states and time; among those that do differentiate, a further 17% report valuing certainty higher in both domains. This indicates that there may be an intuitive similarity between the two domains, although the model-free pricing results of Section 2, for idiosyncratic risk, systematic risk, and diversification, indicate that these two types of uncertainty are treated differently in the theory regardless of pricing kernel used.

TABLE 3: Pricing Narratives

Panel A: Sample Narratives	**Panel B: Classification**	
(1) I simply just went for less risks.	GENERAL PRICING RULE	51%
(2) I tried to play it relatively safe.	MONEY CERTAINTY PRICING, TIME UNCERTAINTY TOLERANT	17%
(3) I chose the safe option with no probability and was a guarantee up until $4 when I changed to the probability option.	MONEY CERTAINTY PRICING, TIME CERTAINTY PRICING	17%
(4) For lotteries that include a 50% chance of receiving nothing, I elected to receive everything over a minimal amount. For lotteries that paid out and varied only in time, I elected to receive the deferred payment for anything less than the immediate award.	MONEY UNCERTAINTY TOLERANT, TIME UNCERTAINTY TOLERANT	11%
	MONEY UNCERTAINTY TOLERANT, TIME CERTAINTY PRICING	3%
(5) I chose to not look at the timeframe of when I would get the bonus because I knew that I would get it eventually. Instead, I only focused on the dollar amount that would be given to me. In the case of a lottery, I calculate the dollar amount by taking the expected payoff from the lottery.		

4 Potential Applications Across Financial Markets

Future work can explore the theoretical or empirical pricing of uncertain timing across a variety of financial markets.

Bond Spreads. Callable and convertible bonds are subject to uncertain timing. Pricing models for these bonds typically conceptualize them as a standard bond plus an option (Berk and DeMarzo, 2011). Just as, in Section 3, options prices differ from predictions of classical theorems, consistent with a pricing of uncertain timing (Section 3), corporate bond spreads may differ as well.

Corporate Decisions. Companies often have uncertainty not only over the magnitude of cash flows on an uncertain project, but also when they will occur. One puzzle in the corporate literature is that companies appear to use hurdle rates that are higher than their weighted average cost of capital (Graham, 2022). It may be the case that one reason for these high hurdle rates is that the cost of capital estimate does not account for the uncertain timing of return inherent in some projects.

Mortgage-Backed Securities. In mortgages, prepayment risk is at heart uncertainty over the time at which the mortgage will be paid back. In practice, prepayment is often predicted using interest rate factors, but research shows that much of the pricing of MBS reflects something beyond interest rates (Boyarchenko et al., 2019). It may be interesting to explore idiosyncratic risk at the aggregate level in MBS and the extent to which this is priced.

Distressed Debt. When a borrower has trouble paying debt back under the initially specified timeline, lenders sometimes extend the timeline of payments and pretend that doing so pays back the debt in full in a practice called 'extend and pretend' (Bloomberg, 2023). The results in this paper suggest that moving from a certain to uncertain time of repayment affects valuation beyond the total amount of debt paid back.

Private Equity. The time of capital call for private equity is uncertain, as studied through an additive utility model in Gourier et al. (2024); however, this approach embeds strong directional assumptions. It may be interesting to study valuation of uncertain timing in private equity in a more general and model-free way.

Policy Investment Decisions. A number of policy decisions involve uncertainty both over the outcome and over the timeline of implementation. For example, in-

frastructure projects include both types of uncertainty. A pricing of uncertain timing may therefore be relevant in policy contexts.

Optimal Information Structures. Prior work finds that Poisson signals are optimal under expected utility and positive payoffs (Zhong, 2022). The results in the current paper suggest that Poisson may no longer be optimal for negative payoffs and for more general valuation functions.

Convenience Yields. In times of fiscal shutdown or economic crisis, both the likelihood of default and the probability that payments might be delayed, may shift. Exploration of whether convenience yields (Krishnamurthy and Vissing-Jorgensen, 2012) are affected in such instances due not only to outcome uncertainty but also due to uncertain timing may be interesting to study.

5 Conclusion

This paper studied the pricing of uncertain timing in a model-free way, relying only on the existence of a stochastic discount factor. The paper shows theoretically that time and outcomes are fundamentally different, both for individual assets and when diversified. For idiosyncratic risk, the paper characterized pricing and introduced an equivalence results for zero-coupon bonds; it also showed that uncertain time is affected by higher order moments and asymmetry, while uncertain outcomes are not. For systematic risk, the paper introduced a novel decomposition. For diversification, the paper showed that conceptual analogs of the time vs. outcomes difference persist even with deterministic cash flows. Empirically, the paper performed model-free tests in option markets that indicate quotes and trades are consistent with a pricing of uncertain timing beyond canonical results from derivatives.

The paper informs foundational pricing models by showing that time and outcomes fundamentally differ; and the results suggest that the study of uncertain timing can yield useful insights in financial markets.

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