Household Sentiment, Intermediary Leverage, and Financial Crises

ZIBO ZHOU
Purdue University

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Abstract

I develop a general-equilibrium macro-finance model in which rational financial intermediaries interact with sentiment-driven households in the presence of financial frictions. Household sentiment affects the ease with which intermediaries can obtain equity financing, strengthening economic resilience in downturns by lowering crash boundaries and supporting balance-sheet repair. At the same time, abundant equity funding liquidity is inherently fragile under households' speculative crowd-out, and crisis risk can build up during booms. Household sentiment also distorts consumption–investment decisions, causing equilibrium prices to rise with sentiment. The interaction between sentiment-driven price reversals and intermediary leverage generates two paradoxical stability results: the economy becomes more stable when households are more irrational (a rationality paradox) and when sentiment is more volatile (a sentiment-volatility paradox).

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1 Introduction

Household beliefs are a key force shaping both asset markets and the macroeconomy. A large empirical literature shows that households and retail investors hold systematically biased expectations about future asset returns and macroeconomic conditions (e.g., Vissing-Jorgensen (2003), Greenwood and Shleifer (2014), Kuchler and Zafar (2019)). Building on this evidence, I document new, critical patterns in U.S. data. First, I demonstrate a strong negative relationship between household sentiment and financial intermediary leverage. Second, I show that elevated household sentiment is tightly linked to boom–bust dynamics in subsequent real economic activity. These links between household sentiment, intermediaries' financial strength, and real economic activity suggest that household sentiment is not merely a psychological curiosity, but might play a central, previously underestimated role in financial stability and macroeconomic fluctuations.

Motivated by this evidence, I develop a continuous-time general equilibrium macro-finance model that embeds biased household beliefs and financial frictions. The framework builds on He and Krishnamurthy (2019) and Maxted (2024) but departs in several critical ways. Households hold irrational, procyclical expectations about capital growth, captured by a mean-reverting sentiment process. Consistent with the data, sentiment co-moves with the business cycle but drifts back toward its long-run mean in the absence of shocks. Financial intermediaries are rational, raise funds by issuing equity and (risk-free) debt, and face a state-dependent equity capital constraint. In addition, households are allowed to invest directly in physical capital, though at an additional cost that reduces returns and captures their lower efficiency relative to intermediaries. In equilibrium, they strictly prefer intermediated investment unless constraints bind. Importantly, the model is solved in full general equilibrium, without imposing special assumptions on the short rate.

In equilibrium, forced intermediary liquidations interact with tightening financial frictions to amplify distress, generating persistent fire sales and severe disruptions in real activity and growth. As intermediary capital capacity deteriorates, intermediaries are forced to liquidate capital to households, who are less efficient holders. This capital displacement reduces productive cash flows and depresses prices, activating a financial accelerator mechanism under the binding equity capital constraint.

Compared with a rational benchmark, household sentiment reshapes the distribution of crisis states. Positive sentiment shifts the crisis boundary upward, while negative sentiment compresses it. In the high-sentiment regime, speculative households crowd out financial intermediaries in the capital market once the constraint binds, forcing intermediaries to liquidate capital at depressed prices and thereby pulling up the crash boundary. Thus, although intermediaries appear strong after positive shocks, they require persistently good performance to sustain equity inflows, and even modest negative returns can trigger equity withdrawals and forced liquidations, pushing the economy into the crisis region. Crisis risk therefore builds up in frothy booms—characterized by high asset prices, compressed risk premia, and

a seemingly strong intermediary sector—consistent with extensive empirical evidence. By contrast, in the low-sentiment regime, pessimistic households are reluctant to supply equity funding, making equity issuance costly and forcing intermediaries to rely more heavily on debt. This deters the drying-up of equity funding liquidity, lowers the region in which the constraint binds, and pushes down the crash boundary. In this way, the financing channel of household sentiment—by distorting intermediaries' funding mix through changes in the perceived attractiveness of equity—enhances economic resilience in downturns while rendering abundant equity funding in booms inherently fragile.

Household sentiment also affects equilibrium capital prices through its impact on households' consumption and investment decisions. When sentiment is high (low), households reduce (increase) consumption and channel more (less) wealth into the capital market, either by holding capital directly or investing through intermediaries, thereby raising (depressing) the capital price and investment. This asset-pricing channel of household sentiment—where prices rise with sentiment—induces fluctuations in intermediary capital capacity when mean reversion in sentiment drives return fluctuations. It also generates pronounced boom–bust patterns in real investment and, hence, economic growth, consistent with the data. Moreover, because costly equity financing in low-sentiment states pushes intermediaries toward higher leverage, intermediaries earn higher returns when sentiment recovers and prices rise, helping them rebuild their balance sheets and move away from full-blown crises. The interaction between the financing and asset-pricing channels—higher leverage combined with improving prices under sentiment reversals—further reinforces economic resilience, over and above the effect of a lower crash boundary.

As households become more irrational, they are more pessimistic in downturns and cut back the supply of equity funding to intermediaries more aggressively, further raising intermediary leverage. Provided negative shocks are not extraordinarily severe, higher leverage amplifies balance-sheet repair under sentiment-driven price reversals, moving the system farther from the crisis region and reinforcing financial stability on average. I refer to this as a rationality paradox: greater household irrationality improves, rather than undermines, financial stability. Similarly, when household sentiment is more volatile, sentiment falls more sharply under negative shocks, again increasing intermediary leverage in bad times. When higher sentiment risk is priced and higher future returns are realized as sentiment recovers, highly levered intermediaries are better positioned to capture these elevated risk premia, pushing the economy toward higher intermediary capital capacity and lowering crisis likelihood on average. I refer to this as a sentiment-volatility paradox: greater volatility in biased household beliefs can stabilize the system rather than amplify crash risk.

Finally, to map the model to the data, I simulate it through the 2007–2009 financial crisis episode. The simulated intermediary capital capacity, equilibrium risk premium, and intermediary leverage are highly correlated with their empirical counterparts, and the model broadly matches the dynamics of the crisis. Moreover, the sentiment model outperforms the rational benchmark: the latter predicts

that the economy remains trapped in the crisis region for an unrealistically prolonged period. The absence of such crisis trapping in the sentiment model highlights two key features of the framework—high intermediary leverage in downturns and a lower crash boundary—which jointly enhance economic resilience and allow the system to recover more rapidly than a rational benchmark would suggest.

This paper relates to the large literature on financial frictions. Canonical rational models such as Kiyotaki and Moore (1997), He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014) and Di Tella (2017) show how binding financing constraints amplify adverse shocks through feedback loops, generating collapses in asset prices and recessions. More recent work in behavioral macro-finance—such as Maxted (2024) and Krishnamurthy and Li (2025)—seeks to explain why crises are often preceded by "frothy" conditions characterized by low credit spreads and strong economic growth, as documented by Baron and Xiong (2017), Greenwood et al. (2022), and Krishnamurthy and Muir (2025). However, both rational and behavioral approaches have focused primarily on intermediaries, rational or otherwise, while leaving households largely in the background. This paper shifts the spotlight to households, showing that household sentiment can have first-order macro-financial effects and generate results consistent with observed empirical patterns. This paper thus complements that line of work by providing a structural, behavioral macro-finance framework to analyze how household beliefs and constraints shape systemic outcomes.

A large literature studies how irrational investor behavior affects asset markets and helps explain well-documented empirical patterns (e.g., De Long et al. (1990); Hong and Stein (1999); Barberis et al. (1998, 2015); Bordalo et al. (2019); Jin and Sui (2022); Atmaz (2022); Atmaz and Zhou (2025)). This paper builds on that tradition but embeds behavioral distortions within a macro-finance environment with financial frictions to study how sentiment-driven behavior affects financial stability. Notably, household sentiment has a long-run stabilizing effect, contrasting with the conventional view that noise traders amplify volatility and cause turmoil. Recent work such as Camous and Van der Ghote (2023) and Gopalakrishna et al. (2025) shows that irrational intermediaries destabilize the economy in similar frameworks, reallocating wealth to inefficient households in the long-run. Maxted (2024) instead finds a stabilizing role for diagnostic expectations. Mine differs from these papers in that the key behavioral agents here are households—not intermediaries—highlighting a new channel through which sentiment shapes macro-financial stability.

This paper also relates to the literature on heterogeneous beliefs. Basak (2000, 2005); Dumas et al. (2009); Ehling et al. (2018); Atmaz and Basak (2018); Atmaz and Buffa (2023) show how dispersion in investor beliefs can shape asset prices, volatility, and welfare. Following Campbell and Kyle (1993) and Shleifer and Vishny (1997), I treat financial intermediaries—typically viewed as sophisticated arbitrageurs—as the rational sector, and less sophisticated households as the sentiment-driven agents. This belief heterogeneity generates rich intermediary-leverage dynamics consistent with the data and plays a central role in shaping financial stability.

This paper is most closely related to Maxted (2024) in terms of model setup, structure, and methodology, but there are several key differences. First, the mechanism driving crises differs: in Maxted (2024), crises arise solely from the binding equity-capital constraint and depend heavily on a special short-rate assumption, whereas in my model crises emerge endogenously from the reinforcing loop between fire-sales and financial frictions. Second, my framework is formulated in full general equilibrium, while Maxted (2024) is effectively partial equilibrium because of its imposed short-rate rule. Third, my model generates a negative relation between intermediary leverage and household sentiment, consistent with the data, whereas Maxted (2024) does not. Finally, under belief heterogeneity, household sentiment is the central driver of the empirical patterns in my model, while in Maxted (2024) the key driver is intermediary sentiment given the assumption of homogeneous beliefs between intermediaries and households.

The remainder of the paper is organized as follows. Section 2 documents empirical patterns of household sentiment and motivates the model setup. Section 3 introduces the general equilibrium macro-finance model. Section 4 defines equilibrium, presents the calibration and characterizes financial crises. Section 5 analyzes the behavior of equilibrium quantities and illustrates the underlying economic mechanisms. Section 6 presents three key implications of the paper and simulates the model to match the 2007–2009 financial crisis episode. Section 7 concludes. Appendices A–F provide solution algorithms and additional analysis.

2 Motivating Facts

In this section, I present two empirical facts about household sentiment. First, I document a negative relationship between intermediary leverage and household sentiment. Second, I show that household sentiment is tightly linked with boom-bust cycles in economic growth.

Specifically, household sentiment is proxied using the consumer sentiment index from surveys conducted by the University of Michigan. Intermediary leverage data come from He et al. (2017), who construct leverage ratios for publicly traded holding companies of the New York Fed's primary dealers. Real GDP growth data are obtained from the Federal Reserve Bank of St. Louis (FRED). The sample period runs from 1970 to 2018, ending in the third quarter of 2018 to align with the leverage data in He et al. (2017).

2.1 Household sentiment and intermediary leverage

Using intermediary leverage data from He et al. (2017), I construct the joint time series of (demeaned) household sentiment and intermediary leverage from 1970 to 2018, shown in Figure 1.

The figure reveals a clear negative relationship between the two series: when household sentiment

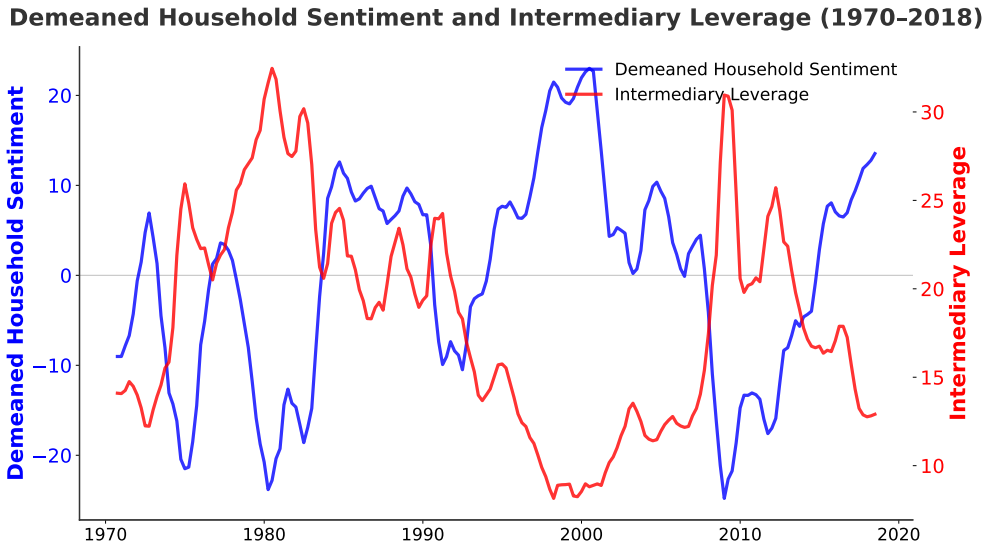


Figure 1. Household sentiment and intermediary leverage. This figure plots the time series of intermediary leverage and household sentiment from 1970 to 2018. Intermediary leverage is the leverage ratios of publicly traded holding companies of New York Fed primary dealers from He et al. (2017). To improve visualization, the series are smoothed using an 4-quarter moving average.

is low, financial intermediaries exhibit higher leverage, and when sentiment is high, leverage is lower.[1] The resulting rise in household deposits—both in the traditional banking system and in shadow banking vehicles—provides ample resources for interbank lending and short-term debt financing, for example through repurchase agreements (repos), thereby boosting the leverage of financial intermediaries, who are effectively compelled to hold more risky assets.[2] These dynamics suggest that shifts in household sentiment might be an important driver of intermediary leverage fluctuation, which turns out to play a key role in financial stability, as I will demonstrate later.

Remark 1 (Discussion on heterogeneous intermediary leverages). The literature documents heterogeneous leverage cyclicality across financial intermediaries. For instance, He et al. (2017) find that bank holding companies (BHCs) exhibit countercyclical leverage, whereas Adrian et al. (2014) show that the leverage of broker-dealers is procyclical. Although both patterns are consistent with theory—equity-constraint models such as He and Krishnamurthy (2013) versus debt-constraint models such as Brunnermeier and Pedersen (2009)—I focus on BHCs for two reasons. First, BHCs issue public

[1] OLS regression result in Table F1 confirms this strong negative relationship: the estimated coefficient is negative and statistically significant at the 1% level.

[2] Large bank holding companies (BHCs), such as JPMorgan Chase & Co., own both commercial banking and dealer divisions. Inflows of household deposits increase the leverage of the commercial banking arms, which in turn amplify system-wide leverage through interbank lending and the money multiplier mechanism. In parallel, the dealer subsidiaries of major BHCs obtain funding from money markets and internal capital transfers, which they then use to extend short-term financing to investment entities—such as hedge funds—via instruments like tri-party repo contracts. Consequently, direct household lending to BHCs tends to raise the leverage of the entire intermediary sector through these interconnected channels.

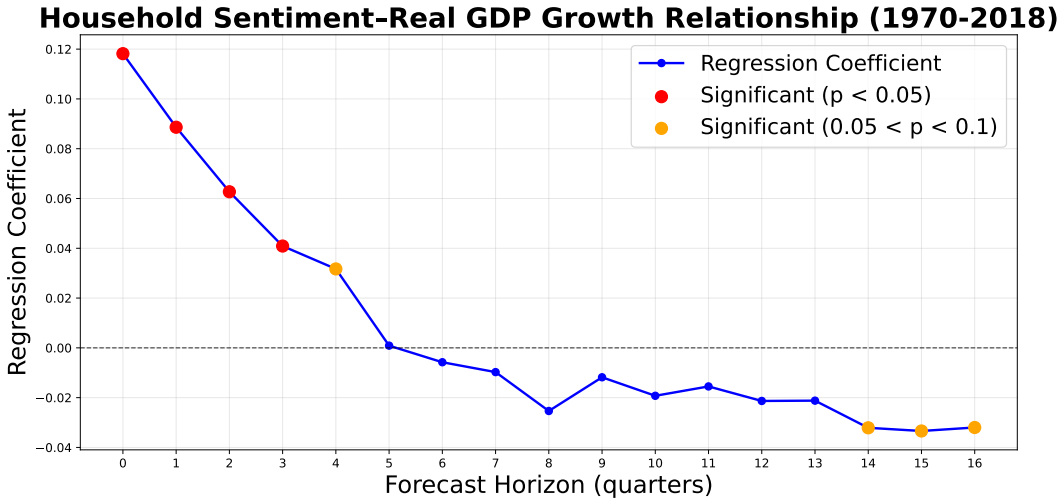


Figure 2. Predictive regressions. This figure plots the predictive regression coefficients b from the regression real GDP growth$_{t+k} = a + b \times$ sentiment$_t + \epsilon_{t+k}$, in which $k \in \{0, 1, 2, \cdots, 16\}$ denotes the forecast horizon in quarters. When $k = 0$, b corresponds to the contemporaneous regression coefficient. Red dots indicate coefficients that are statistically significant at the 5% level. The regressions are estimated over the period $1970 - 2018$.

equity and therefore align more closely with the notion of financial intermediaries in my model, as will be discussed later. Broker-dealers, by contrast, do not issue equity directly. Second, as emphasized by He et al. (2017), major BHCs own dealer subsidiaries, so their consolidated balance sheets provide a more comprehensive measure of the financial health and leverage of the intermediary sector as a whole.

2.2 Household sentiment and economic boom-bust cycles

To examine the relationship between household sentiment and real economic activity, I estimate predictive regressions—including contemporaneous ones—of real GDP growth on household sentiment across various forecast horizons over the sample period 1970–2018. Figure 2 shows that the estimated coefficients are significantly positive at short horizons but turn significantly negative in the long-run. These results indicate that household sentiment is tightly linked to the boom–bust dynamics of real activity. This empirical pattern motivates the model developed in the following sections, which explores how sentiment shapes real outcomes within a behavioral macro-finance framework.

3 Model

In this section, I develop a general-equilibrium macro-finance model to capture the evidence documented above and to examine how household sentiment influences financial stability. The framework builds on the continuous-time models of He and Krishnamurthy (2019) and Maxted (2024), but introduces sev-

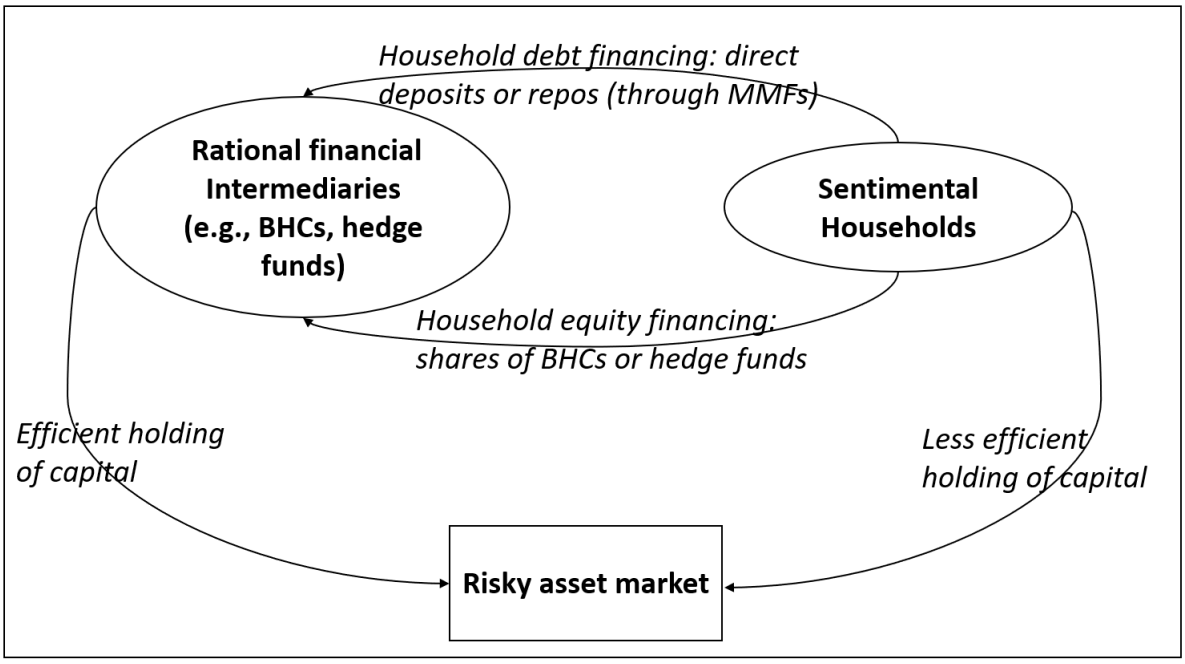


Figure 3. Scheme diagram for the model

eral key modifications. Specifically, the model has four key features. First, financial intermediaries face equity capital constraints, as in He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and He and Krishnamurthy (2019). Second, financial intermediaries are rational, while households are irrational: households hold biased, sentiment-driven beliefs about the evolution of capital. Third, households can directly invest in risky capital but face additional costs relative to investing in intermediary equity. Finally, the model is formulated in general equilibrium without imposing specific assumptions on the short rate, in contrast to He and Krishnamurthy (2019) and Maxted (2024). Figure 3 lays out the building blocks of the model.

The model proceeds in three steps. Section 3.1 describes the production side of the economy. Section 3.2 introduces intermediaries and their financing frictions. Section 3.3 turns to households, highlighting both biased beliefs and direct capital holdings.

3.1 Production economy

I consider a production economy in continuous time, with time indexed by t. The economy consists of two sectors—households and bankers—which I describe in detail below. Production follows an "AK" technology, generating per-period output Y_t as:

$$Y_t = aK_t, \tag{1}$$

in which a is a constant productivity and K_t denotes the stock of physical capital. The evolution of K_t is given by:

$$\frac{dK_t}{K_t} = \left(\Phi\left(\iota_t\right) - \delta\right) dt + \sigma d\omega_t, \tag{2}$$

where ι_t is the endogenous investment rate at time t, δ is the exogenous depreciation rate, and $\{\omega_t\}$ is a standard Brownian motion. Following Brunnermeier and Sannikov (2014), He and Krishnamurthy (2019), Maxted (2024), and Krishnamurthy and Li (2025), the term $\sigma d\omega_t$ represents a capital quality shock, which is the sole source of uncertainty in the model. Investment is subject to an adjustment cost captured by the function:

$$\Phi\left(\iota_t\right) = \frac{1}{\phi}\log\left(\phi\iota_t + 1\right). \tag{3}$$

The adjustment cost introduces technological illiquidity, making the supply of capital relatively inelastic—a key mechanism through which demand variations impact prices.

Each unit of capital costs q_t at time t in equilibrium. I postulate that q_t evolves as:

$$\frac{dq_t}{q_t} = \mu_{qt}dt + \sigma_{qt}d\omega_t, \tag{4}$$

in which μ_{qt} and σ_{qt} are determined endogenously in equilibrium. Denote the instantaneous return on holding one unit of capital as dR_t. Then we have:

$$dR_t = \frac{d\left(q_tK_t\right) + \left(a - \iota_t\right)K_t dt}{q_tK_t} = \mu_t dt + \sigma_t d\omega_t, \tag{5}$$

where

$$\mu_t = \mu_{qt} + \left(\Phi\left(\iota_t\right) - \delta\right) + \sigma\sigma_{qt} + \frac{a - \iota_t}{q_t}, \tag{6}$$

$$\sigma_t = \sigma + \sigma_{qt}. \tag{7}$$

3.2 The financial intermediary sector

There is a continuum of financial intermediaries, each managed by a single banker. I assume that bankers are not endowed with financial wealth and must raise funds from households by issuing intermediary equity and/or debt. Intermediary equity can be interpreted as publicly traded shares of bank holding companies (BHCs) or, as assets under management in hedge funds. In contrast, intermediary debt represents household lending through direct deposits or through money-market instruments such as repurchase agreements (repos). Each intermediary invests a fraction α_{It} of the total equity raised from households in the physical capital market. Therefore, if $\alpha_{It} > 1$, the financial intermediary borrows from households; the debt is assumed to be instantaneously risk-free and pays a rate r_t. Denote the

instantaneous return on intermediary equity as $d\widetilde{R}_t$. Then we can show:

$$d\widetilde{R}_t = r_t dt + \alpha_{It}\left(dR_t - r_t dt\right) = \left[r_t + \alpha_{It}\pi_t\right]dt + \alpha_{It}\sigma_t d\omega_t, \tag{8}$$

where $\pi_t = \mu_t - r_t$ is the equilibrium risk premium.

As in standard financial friction literature, intermediaries face a constraint when issuing equity to households. Specifically, following He and Krishnamurthy (2019) and Maxted (2024), the maximum equity capacity of an individual intermediary is indexed by ϵ_t, so that the intermediary can raise at most ϵ_t from households at time t. The equity capacity ϵ_t reflects the balance sheet strength or reputation of the intermediary and introduces a state-dependent financial friction, which will play a central role in financial crises. The evolution of intermediary equity capacity ϵ_t is assumed to follow:

$$\frac{d\epsilon_t}{\epsilon_t} = d\widetilde{R}_t, \tag{9}$$

implying that intermediaries that perform well in the physical capital market and earn high returns can raise more funds by issuing equity. This equity financing constraint echoes Shleifer and Vishny (1997) and Berk and Green (2004): households adjust fund flows in response to intermediary performance, withdrawing funds after poor returns and supplying capital when returns are strong.

Bankers do not consume; they make investment decisions to maximize utility derived from ϵ_t. Specifically, each banker has mean-variance preferences and rational expectations, with the objective

$$\max_{\alpha_{It},\iota_t^I} \mathrm{E}_t\left[\frac{d\epsilon_t}{\epsilon_t}\right] - \frac{\gamma^I}{2}\mathrm{Var}_t\left[\frac{d\epsilon_t}{\epsilon_t}\right] = \mathrm{E}_t\left[d\widetilde{R}_t\right] - \frac{\gamma^I}{2}\mathrm{Var}_t\left[d\widetilde{R}_t\right], \tag{10}$$

where ι_t^I denotes the intermediary's capital investment.

Since equilibrium is determined by macro variables, we now consider the aggregate financial intermediary sector. Let $\mathcal{E}_t$ denote the maximum equity that can be raised by the entire intermediary sector. Assuming each intermediary exits at rate η over period dt, we can show that $\mathcal{E}_t$ evolves as follows:

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = \frac{d\epsilon_t}{\epsilon_t} - \eta dt. \tag{11}$$

Exit ensures a stationary distribution and serves a role similar to heterogeneous impatience in Brunnermeier and Sannikov (2014).[3] Unlike He and Krishnamurthy (2019) and Maxted (2024), I do not assume

[3]Exit is introduced primarily as a technical device to prevent the state variable $\mathcal{E}_t$ from exploding in the long run. Without exit, the equity capital constraint would almost never bind asymptotically, rendering the model uninteresting. Importantly, banker exit does not destroy physical capital: it only reduces intermediaries' capacity to raise funding, and all wealth ultimately resides with households, since bankers are not endowed with any wealth of their own. An alternative interpretation of the constant exit rate is as a reduced-form way to capture the persistent depreciation of intermediaries' fundraising capacity.

additional entry of new bankers when aggregate equity capacity $\mathcal{E}_t$ is low, so no reflection barrier exists in my model.[4]

3.3 The household sector

There is a continuum of households with unit measure that collectively hold the aggregate wealth of the economy, W_t. Households are split into "debt households" and "equity households" within a representative-family framework. At the beginning of each period, each household consumes, and aggregate wealth W_t is divided such that a fraction λ is allocated to debt households and the remaining share to equity households. Each household then invests individually, with returns realized at $t+dt$. At that point, wealth is pooled to form W_{t+dt}, and the process repeats. Thus, the economy can be viewed as a continuum of representative families, each consisting of a debt household and an equity household.

Debt households invest exclusively in the instantaneously risk-free debt, earning return $r_t dt$. Equity households, by contrast, can invest in both debt and intermediary equity, as in He and Krishnamurthy (2019) and Maxted (2024). A key difference in my framework is that equity households are also allowed to directly hold physical capital. This specification is natural in light of the fact that households in practice invest in both production and housing capital, and it aligns with standard modeling choices in the literature, such as Brunnermeier and Sannikov (2014) and Krishnamurthy and Li (2025).

Let w_t denote the wealth of each representative household family. The equity household allocates a fraction α^I_{Ht} of her wealth to intermediary equity and a fraction α^S_{Ht} to physical capital directly. The remaining share, $1-\alpha^I_{Ht}-\alpha^S_{Ht}$, is invested in the debt market. When this residual is negative, the equity household borrows from the debt household. In other words, equity households might compete with intermediaries for debt household lending, so that net lending from the representative family can be less than λw_t. As we will see later, equity households face a borrowing constraint (22) such that, when $b=1$, they are not allowed to borrow from debt households. But for generality, I allow this borrowing possibility for equity households.[5] When directly holding physical capital, the equity household produces output according to equation (1) and invests at rate ι^H_t. I assume that equity households are as productive as intermediaries in terms of technology a. However, their capital holding is less efficient. Specifically, for each unit invested in physical capital, the return to the equity household is

$$dR^S_t = dR_t - \varphi dt = [\mu_t - \varphi]\, dt + \sigma_t d\omega_t, \tag{12}$$

[4] In He et al. (2017) and Maxted (2024), the reflection barrier primarily affects the housing price p_t while having only a limited impact on the physical capital price q_t. Extending the model to incorporate the reflection barrier would therefore not materially alter my main results, either qualitatively or quantitatively. For simplicity, I do not impose additional boundary conditions.

[5] When equity households borrow from debt households, this simply reduces the net lending from the household sector as a whole to intermediaries. Thus, allowing equity-household borrowing is a reduced-form way to capture lower net lending from the aggregate household sector.

where $\varphi > 0$ represents the relative cost of direct capital holding. This additional cost effectively dampens the productive cash flows of capital held by households and induces fire sales during financial crises: distressed intermediaries are forced to liquidate capital assets at depressed prices into the less efficient hands of households, as we will see later.

Remark 2 (Justifications of φ). The relative cost φ can be justified in several ways. First, as emphasized in Freixas and Rochet (2008), financial intermediaries exploit economies of scale—such as diversification through risk pooling—and possess informational advantages that reduce the costs of capital allocation. Thus, φ captures the relative disadvantage of the household sector compared with intermediaries and formalizes the benefits of intermediation. Second, from a behavioral perspective (see Gennaioli et al. (2015)), households derive utility from delegating to experts, which reduces perceived uncertainty. An equivalent modeling device is to assume that direct investment lowers perceived expected returns for households. Third, φ can be viewed as an additional depreciation wedge when households hold capital, dampening future cash flows. This is analogous to the heterogeneous productivity approach in Brunnermeier and Sannikov (2014), where φ is analogous to $\frac{a^e - a^h}{q_t}$, the productivity gap between experts and households. A similar idea appears in Haddad and Muir (2021), who assume that households can access risky assets but face quadratic costs.

Household sentiment. Motivated by the large body of empirical evidence showing that retail investors hold biased expectations, I assume that households form irrational beliefs summarized by a sentiment variable X_t. Specifically, similar to Maxted (2024), households misperceive the dynamics of capital such that, in their subjective world,

$$\frac{dK_t}{K_t} = \left[\Phi\left(\iota_t\right) - \delta + \theta X_t\right] dt + \sigma d\omega_t^H, \tag{13}$$

where $\theta > 0$ captures the strength of their bias: higher θ implies stronger deviations from rational expectations. I refer to θ as *the irrationality parameter*.[6] The process $\left\{\omega_t^H\right\}$ is a Brownian motion under the household's subjective probability measure $\mathbb{P}^H$. Rather than micro-founding sentiment, I model X_t as an Ornstein–Uhlenbeck (OU) process,

$$dX_t = \kappa\left(\mu_X - X_t\right) dt + \sigma_X d\omega_t, \tag{14}$$

with constants μ_X and $\sigma_X > 0$. κ governs the mean-reversion speed such that a smaller κ would make past fundamental shocks have long-lasting impacts on sentiment. Positive σ_X ensures that sentiment is procyclical: sequences of favorable shocks push households toward greater optimism about future growth, consistent with survey evidence. Mean reversion is also empirically supported: after positive

[6] Less sophisticated households do not observe the true drift of capital accumulation, although they correctly perceive its conditional volatility. Instead, they form subjective beliefs about the expected growth rate of capital that are distorted by their sentiment, which leads them to replace the true drift with $\Phi\left(\iota_t\right) - \delta + \theta X_t$.

shocks, realized capital growth disappoints, and sentiment reverts toward its long-run mean. This OU structure is standard in extrapolative expectations models (e.g. Barberis et al. (2015) and Maxted (2024)).

Remark 3 (General sentiment dynamics). The OU specification is flexible and nests many microfoundations. For example, the diagnostic-expectations model in Maxted (2024) implies a process of the form above. Likewise, if households extrapolate capital growth as

$$X_t = \int_{-\infty}^{t} \kappa e^{-\kappa(u-t)} \frac{dK_u}{K_u},$$

then X_t follows dynamics close to the OU process but with a stochastic mean $\Phi(\iota_t) - \delta$. Since different microfoundations generate similar reduced-form dynamics, I adopt the simple OU specification for tractability.

Remark 4 (Belief heterogeneity). A key distinction from Maxted (2024) is that in my model only households are irrational, while intermediaries are rational. This choice is motivated by evidence: while some studies find extrapolative forecasts among professionals (Bordalo et al. (2019, 2020)), others show contrarian or countercyclical beliefs among asset managers and Wall Street analysts (Dahlquist and Ibert (2024)). By contrast, the evidence for extrapolative expectations among retail investors and households is overwhelming: households exhibit optimism after good times and pessimism after bad times, across individual stocks, the aggregate market, and macro variables (Vissing-Jorgensen (2003); Amromin and Sharpe (2014); Greenwood and Shleifer (2014); Da et al. (2021); Cassella and Gulen (2018); Kuchler and Zafar (2019), among others). In addition, seminal papers such as Campbell and Kyle (1993) and Shleifer and Vishny (1997) highlight that financial intermediaries—or smart-money investors—hold rational beliefs and seek to arbitrage away mispricing, thereby improving market efficiency. Reflecting this perspective, and given intermediaries' financial sophistication, it is more natural to model households—not intermediaries—as the irrational sector. One could extend the model by letting intermediaries also have biased beliefs, for example

$$\frac{dK_t}{K_t} = \left[\Phi(\iota_t) - \delta + \theta^I X_t\right] dt + \sigma d\omega_t^I,$$

where $\theta^I < \theta$. As long as households are more biased than intermediaries ($\theta > \theta^I$), the main qualitative implications should go through. What matters is belief heterogeneity. As we will see later, it is precisely belief heterogeneity that distorts the financing channel of intermediaries, altering their balance sheet dynamics and affecting financial stability.

Household optimization problem. Each representative household family consumes c_t at time t and has the stochastic differential utility with risk aversion parameter γ^H and elasticity of intertemporal

substitution (EIS) ψ. The family's objective is:

$$V_t = \max_{\left\{c_s, \alpha^I_{Hs}, \alpha^S_{Hs}, \iota^H_s\right\}_{s \geq t}} \mathrm{E}^H_t \left[\int_t^\infty f\left(c_s, V_s\right) ds \right], \tag{15}$$

subject to the following dynamic budget constraint

$$\begin{aligned} \frac{dw_t}{w_t} = & \left[r_t + \left(1-\lambda\right) \alpha^I_{Ht} \alpha_{It} \pi^H_t + \left(1-\lambda\right) \alpha^S_{Ht} \left(\pi^H_t - \varphi \right) - \frac{c_t}{w_t} \right] dt \\ & + \left(1-\lambda\right) \left(\alpha^I_{Ht} \alpha_{It} + \alpha^S_{Ht} \right) \sigma_t d\omega^H_t, \end{aligned} \tag{16}$$

where π^H_t is the household's perceived risk premium on capital:

$$\pi^H_t = \pi_t + \frac{\sigma_t}{\sigma} \theta X_t. \tag{17}$$

The aggregator function is:

$$f\left(c_t, V_t\right) = \frac{1-\gamma^H}{1-\frac{1}{\psi}} V_t \left[\left(\frac{c_t}{\left[\left(1-\gamma^H\right) V_t\right]^{\frac{1}{1-\gamma^H}}} \right)^{1-\frac{1}{\psi}} - \rho \right], \tag{18}$$

where ρ is the household's time discount rate. In addition, households face the following constraints:

1. Intermediary equity investment cap:

$$0 \leq \alpha^I_{Ht} \leq \overline{\alpha}, \tag{19}$$

   where $\overline{\alpha} \leq 1$.

2. Equity capital constraint:

$$\left(1-\lambda\right) w_t \alpha^I_{Ht} \leq \mathcal{E}_t. \tag{20}$$

3. No-short-selling constraint:

$$\alpha^S_{Ht} \geq 0. \tag{21}$$

4. Borrowing constraint:

$$\alpha^I_{Ht} + \alpha^S_{Ht} \leq b, \tag{22}$$

   where $1 \leq b \leq \frac{1}{1-\lambda}$.

First, following He and Krishnamurthy (2013) and He and Krishnamurthy (2019), I impose a cap $\overline{\alpha}$ on intermediary equity investment. This prevents households from overinvesting in intermediaries when their sentiment is high and captures the idea that they maintain some diversification and direct

asset exposure.[7] Second, because households cannot short sell physical capital, we require $\alpha^S_{Ht} \geq 0$. Third, the equity capital constraint faced by financial intermediaries effectively limits household investment in intermediary equity, which leads to (20).[8] When the constraint tightens, households withdraw equity funding from intermediaries. Finally, since equity households are allowed to borrow from debt households, I impose a leverage constraint (22). The condition that $b \leq \frac{1}{1-\lambda}$ ensures that intermediaries' balance sheets remain more concentrated and risk-exposed than household portfolios, consistent with empirical evidence on sectoral exposures.

To summarize, each representative household family chooses optimal consumption, portfolio allocations, and investment rates to maximize (15), subject to the subjective wealth dynamics (16) and the portfolio constraints (19)—(22).

4 Equilibrium and financial crises

This section defines the equilibrium and reports the calibration. It then characterizes the model's financial-crisis region, analyzes its key properties, and outlines some main mechanisms at work.

4.1 Markov equilibrium

The general equilibrium is defined as a set of prices $\{q_t, r_t\}$ and optimal policies $\left\{c_t, \alpha^I_{Ht}, \alpha^S_{Ht}, \iota^H_t, \alpha_{It}, \iota^I_t\right\}$ such that:

1. Given prices, agents choose policies that maximize their objectives under their own expectations and preferences, while respecting portfolio constraints.

2. Aggregate consumption and investment equal output:

$$C_t + \iota_t K_t = Y_t = aK_t, \tag{23}$$

   where C_t denotes aggregate household consumption.

3. Demand for risky capital from intermediaries and households equals the supply of capital:

$$\alpha_{It}\left[\alpha^I_{Ht}(1-\lambda)W_t\right] + \alpha^S_{Ht}(1-\lambda)W_t = q_t K_t. \tag{24}$$

[7] This constraint binds only in normal states when households are optimistic. Once it binds, households enter the capital market and invest directly in risky capital. Relaxing the constraint by increasing $\overline{\alpha}$ does not affect the main results, because whether households hold physical capital directly in normal times does not overturn the asset-pricing channel of household sentiment, as shown later. The cap $\overline{\alpha}$ is therefore imposed mainly for realism, to reflect that households do not overinvest in intermediary equity.

[8] The constraint can be simplified to $\alpha^I_{Ht} \leq \frac{1}{1-\lambda}\frac{e_t}{q_t}$, where e_t is defined in (26).

4. Aggregate household wealth equals the total value of capital:

$$W_t = q_t K_t. \tag{25}$$

Give the above equilibrium definition, I consider a Markov equilibrium in state variables $\mathcal{E}_t$, X_t, and K_t. Exploiting the scaling property of homothetic stochastic differential utility, and following He and Krishnamurthy (2019) and Maxted (2024), I define the intermediary's capital capacity as

$$e_t = \frac{\mathcal{E}_t}{K_t}. \tag{26}$$

The variable e_t represents the aggregate equity capacity of intermediaries scaled by the capital stock and thus measures the sector's ability to raise funds through equity issuance. It turns out that the two state variables, e_t and X_t, are sufficient to characterize the Markov equilibrium. All endogenous quantities can therefore be expressed as functions of (e_t, X_t) alone. The model is solved numerically, with full details provided in Appendix A and B.

4.2 Calibration

Table 1 reports the baseline parameter choices of the model. To discipline these choices, I solve the Kolmogorov Forward Equation (KFE) for the stationary density of the state variables and match the resulting model-implied unconditional moments to the data.[9]

Intermediation parameters. The key intermediation parameters are γ^I, λ and η. As shown in (A.2), the banker's risk aversion γ^I directly affects the equilibrium risk premium and hence the Sharpe ratio. I set $\gamma^I = 2$, which yields an unconditional mean Sharpe ratio of 0.54, well within the standard empirical range. The share of debt households, λ, influences household lending to intermediaries and thus determines intermediary leverage. I set $\lambda = 0.65$, which generates an average intermediary leverage ratio of 3.71. The intermediary exit rate, η, affects the unconditional probability of financial crises. Bordo and Meissner (2016) estimate crisis probabilities between $2\% - 4\%$. Jordà et al. (2013) report an average of 3.6%, and Sufi and Taylor (2022) find 2.5%. Setting $\eta = 0.13$ yields a model-implied probability of 3.83%.

RBC parameters. The exogenous volatility of capital, σ, affects the volatility of all endogenous quantities. I calibrate $\sigma = 6\%$, which produces an investment volatility of 7.02%, close to the U.S. data value of 6.9% (1973-2015). The depreciation rate δ is set to be 10%, as standard in the literature. For the investment adjustment cost, I adopt the same functional form as in Brunnermeier and Sannikov

[9]The numerical details of solving the stationary density via the KFE are provided in Achdou et al. (2022) and Maxted (2024). To discretize the state space and construct the transition matrix, I follow the finite-difference scheme described in Chapter 5 of Kushner and Dupuis (2001).

Table 1. Parameter values. This table reports the parameter values used in the numerical analysis. Model-implied unconditional moments are computed from the stationary density of the state variables, obtained by solving the Kolmogorov Forward Equation. "Normal periods" refer to states outside the crisis region.

Parameter		Choice	Target
Panel A: Intermediation parameters			
γ^I	Banker risk aversion	2	Mean Sharpe ratio
λ	Debt ratio	0.65	Intermediary leverage
η	Banker exit rate	0.13	Probability of crisis
Panel B: Technology parameters			
σ	Capital shock volatility	0.06	Investment volatility
δ	Capital depreciation rate	0.1	Literature
ϕ	Adjustment cost	10	Literature
a	Productivity	0.3	Mean interest rate
Panel C: Household preference parameters			
ρ	Time discount rate	0.04	Literature
γ^H	Household risk aversion	2	Free parameter
ψ	Household EIS	1.5	Literature
Panel D: Household sentiment parameters			
κ	Mean-reversion speed	0.35	Household sentiment calibration
θ	Irrationality	0.24	Free parameter
μ_X	Long-run mean of sentiment	0	Free parameter
σ_X	(conditional) Volatility of sentiment	0.06	Household sentiment calibration
Panel E: Household portfolio parameters			
$\overline{\alpha}$	Intermediary investment cap	1	Free parameter
b	Household leverage constraint	1.5	Free parameter
φ	Relative cost of direct capital holding	0.04	Free parameter
Panel F: Simulated unconditional moments			
Probability of crisis			3.83%
Mean$\left(\frac{\text{Investment}}{\text{Capital}}\right)$			17.42%
Mean$\left(\frac{\text{Consumption}}{\text{Output}}\right)$			41.94%
Mean(Sharpe Ratio)			0.54
Mean(Intermediary Leverage)			3.71
Mean(Risk-free rate) in normal periods			1.40%
Volatility(Invesment) in normal periods			7.02%
Volatility(Consumption) in normal periods			3.73%

(2016) and set $\phi = 10$, which is their baseline calibration. Finally, productivity a is calibrated to 0.3. This implies a risk-free rate of 1.40% in non-crisis periods, consistent with empirical averages.

Household preference parameters. The time discount rate is set to $\rho = 4\%$, in line with standard practice. To isolate the role of sentiment, the baseline assumes homogeneous risk aversion: $\gamma^I = \gamma^H$. The elasticity of intertemporal substitution (EIS) ψ is crucial. For the capital price q to rise with household sentiment X, we require $\psi > 1$. In this case, the substitution effect dominates the income effect, so optimism leads households to consume less and invest more, raising marginal q. Following Bansal and Yaron (2004), I set $\psi = 1.5$.[10]

Household sentiment parameters. I calibrate the sentiment process using household sentiment data from the University of Michigan Survey of Consumers under the OU specification in (14). For simplicity, I set $\mu_X = 0$. The sentiment time series is normalized to estimate the persistence parameter κ and the volatility parameter σ_X. As detailed in Appendix E.1, the calibration yields $\kappa \approx 0.35$ and $\sigma_X \approx 0.06$. The household irrationality parameter θ, which plays a central role in determining financial stability, is set to $\theta = 0.24$ in the baseline calibration, and its implications for stability are explored in Section 6.3.

Household portfolio parameters. The choice of $\overline{\alpha}$ and b would not affect the main results of the model. I set the intermediary equity cap $\overline{\alpha} = 1$, following the intermediary asset pricing literature (He and Krishnamurthy (2013), He and Krishnamurthy (2019) and Maxted (2024)). The household leverage parameter is set to $b = 1.5$, implying equity households can be at most half as levered as intermediaries. Finally, the relative cost φ governs household entry into the capital market and thus the onset of crises. I set $\varphi = 4\%$ in the baseline calibration and investigates its impact in Section 6.3.

4.3 Financial crises

As emphasized by Allen and Gale (2009) and Jordà et al. (2010), financial crises are characterized by frozen funding liquidity, spikes in risk premia and credit spreads, and—critically—sharp collapses in asset prices. Seminal models such as Brunnermeier and Pedersen (2009) and Brunnermeier and Sannikov (2014) highlight that crises arise when funding liquidity (either debt or equity financing) dries up, forcing intermediaries to liquidate assets at distressed prices, triggering a liquidity spiral and persistent fire sales. In the same spirit, I define a financial crisis in this paper as in Definition 1.

Definition 1 (**Definition of financial crises region**)**.** A financial-crisis region is defined as the set of states in which fire sales occur—that is, when stressed intermediaries are forced to liquidate capital assets to households at depressed prices.

[10] In the partial equilibrium model of He and Krishnamurthy (2019) and Maxted (2024), they also set EIS > 1. As Maxted (2024) mentions, an EIS greater than 1 is necessary to generate a positive relation between q and sentiment.

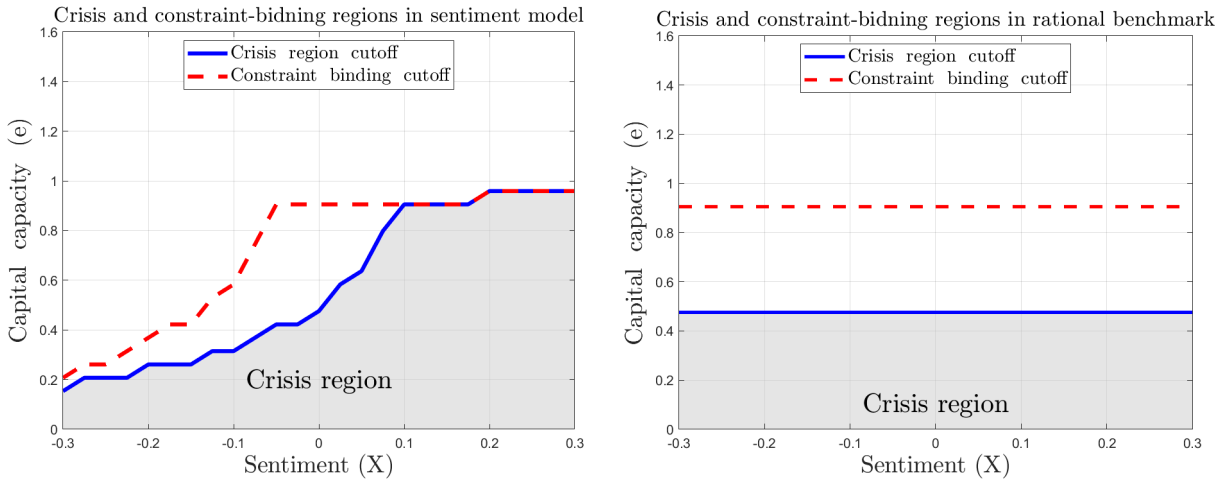


Figure 4. Crisis and constraint-binding regions. This figure plots the equity capital constraint binding region and the financial crisis region across the state space for both the main (sentiment) model and the rational benchmark. The red dashed curve marks the cutoff states where the equity capital constraint binds, while the solid blue curve marks the cutoff states for entry into the financial crisis region.

The proposition below formalizes the precise conditions under which financial crises emerge in the model.

Proposition 1 (**Financial crises in general equilibrium**). *In general equilibrium, financial crises arise if and only if households directly hold physical capital ($\alpha_{Ht}^S > 0$), the equity capital constraint binds, and the relative cost $\varphi > 0$.*

Proof. See Appendix A. □

As shown in Appendix A, if either $\alpha_{Ht}^S = 0$ or $\varphi = 0$, the economy converges to an irrational representative-household setting regarding real production. For any fixed sentiment X, the capital price is determined solely by the technology, preferences, and beliefs—independent of intermediary balance-sheet strength e. In this case, financial frictions play no role in real economic activities, even though they still distort risk sharing.[11] This is the macro-finance version of *Modigliani-Miller theorem*: the intermediary sector becomes a veil.[12] By contrast, when $\alpha_{Ht}^S > 0$ and $\varphi > 0$, inefficient household capital holdings reduce future cash flows and depress q. As the binding equity capital constraint freezes funding liquidity, constrained intermediaries are forced to liquidate capital assets at depressed prices to inefficient households. The resulting fire-sale losses further tighten the constraint, amplifying the liquidity squeeze and causing more severe asset liquidation. This mechanism—forced liquidation and capital displacement—echoes the crisis dynamics in Brunnermeier and Sannikov (2014) and leads to a

[11] Specifically, for $\alpha_{Ht}^S = 0$ or $\varphi = 0$, when the equity capital constraint binds, the risk premium π_t rises and the short rate r_t falls, but they offset each other exactly so that the expected return μ_t remains constant across e.

[12] Intuitively, as long as the expected return on capital is fixed, how we split the safe part (risk-free rate) and risky part (risk premium) does not affect the real value.

pronounced contraction in real activity.

On the left panel of Figure 4, I plot the equity-capital constraint-binding region together with the financial-crisis region in the main (sentiment) model, where households hold sentiment-driven beliefs. The right panel presents the same objects for the rational benchmark, in which households form rational expectations while all else remains identical. In both panels, the constraint-binding region fully encompasses the crisis region. This is unsurprising: once the equity-capital constraint binds—signaling frozen equity-funding liquidity—intermediaries are already on the verge of forced liquidation. In other words, constraint binding typically precedes fire sales, making the crisis region a subset of the constraint-binding region.

Relative to the rational benchmark, household sentiment distorts the distribution of the constraint-binding region. When sentiment is high, the binding regions in both models are nearly identical. However, in the low-sentiment regime, the constraint-binding region shrinks substantially in the sentiment model. To see why, recall that the equity-capital constraint takes the form $\alpha^I_{Ht} \leq \frac{1}{1-\lambda}\frac{e_t}{q_t}$. When sentiment X is high, overoptimistic households perceive a high price of risk on intermediary equity and thus provide abundant equity funding. Their aggressive equity purchases quickly hit the investment cap $\alpha^I_{Ht} = \overline{\alpha}$ before the constraint binds. In the rational benchmark, even though households are not overoptimistic, the equilibrium price of risk is still high enough that they also invest up to $\overline{\alpha}$. Because both economies start from $\alpha^I_{Ht} = \overline{\alpha}$ before the constraint binds, the constraint becomes binding at nearly the same level of capital capacity as e deteriorates.[13] In contrast, in the low-sentiment regime, pessimistic households perceive a very low price of risk on intermediary equity and thus refrain from providing equity funding. This dampens α^I_{Ht}, causing the equity-capital constraint to bind only at low levels of capital capacity e. As X becomes more negative, household equity investment declines further, lowering the binding cutoff and compressing the overall binding region. Hence, in the sentiment model, household sentiment distorts perceived risk and thereby alters households' willingness to supply equity funding, shaping the distribution of financial distress states. I refer to this mechanism as *the financing channel of household sentiment*, which also affects intermediaries' leverage dynamics and, in turn, financial stability, as I elaborate in Section 5.2.

More importantly, relative to the rational benchmark, household sentiment reshapes the distribution of the crisis region: positive sentiment lifts the crisis boundary upward, while negative sentiment compresses it. In the rational benchmark, a gap emerges between the constraint-binding region and the crisis region due to the relative cost φ. When the constraint first binds, less efficient households—facing this cost—perceive capital as overvalued and thus lend their withdrawn equity back to intermediaries

[13] When X is very high (approximately above 0.2), the equity capital constraint cutoff in the sentiment model becomes slightly higher than in the rational benchmark. This occurs because, as shown in Section 5.1, the equilibrium price q rises with X in the sentiment model. Consequently, the equity capital constraint $\frac{1}{1-\lambda}\frac{e_t}{q_t}$ crosses the investment cap $\overline{\alpha}$ at a higher level of e when q is inflated under high sentiment. However, this effect is minor, as q varies only modestly with sentiment. Moreover, it reinforces the main result that elevated household sentiment shifts the crisis region upward relative to the rational benchmark.

rather than purchasing capital directly. Intermediaries continue to lever up, which depresses the capital price q. Only when e and q decline sufficiently do households begin absorbing capital, triggering fire sales. Hence, constraint binding signals financial stress, but a true crisis arises only once capital is displaced into inefficient hands.[14]

In the low-sentiment regime of the sentiment model, the contraction of the constraint-binding region naturally leads to a smaller crisis region. Intuitively, because the economy is less exposed to equity-funding dry-ups, forced liquidations become rarer. The gap between the two regions arises for similar reasons as before: households delay entering the capital market, perceiving capital as overpriced when the constraint first binds, given both the cost φ and their pessimism distortion. In contrast, in the high-sentiment regime, households perceive capital as underpriced even after accounting for φ, and—incorrectly believing intermediaries will quickly rebuild their balance sheets—expect them to repurchase assets soon at higher prices. These misperceptions generate what I call *speculative crowd-out*: rather than lending withdrawn equity back to intermediaries, optimistic households rush into the capital market, forcing intermediaries to liquidate assets at depressed prices and triggering fire-sale dynamics. This speculative crowd-out effect causes the constraint and crisis regions to overlap, raising the crisis boundary. As a result, once the equity capital constraint binds, the economy moves immediately from financial stress into full-blown crisis. Therefore, in the high-sentiment regime, although intermediaries benefit from abundant equity funding, this liquidity is inherently fragile—it depends on sustained strong performance to support inflows. Even modest negative returns can make the constraint bind. Once it does, unlike in the rational benchmark—where the economy experiences distress without major real disruptions—the sentiment model transitions immediately into the crisis region due to speculative crowd-out, triggering forced liquidations and fire-sale dynamics.

Overall, household sentiment has a profound impact on economic resilience. In the high-sentiment regime, abundant equity-funding liquidity is inherently fragile, as the economy remains close to the crash boundary under speculative crowd-out. In contrast, in the low-sentiment regime, the financing channel enhances stability by lowering the constraint-binding cutoff and shrinking the crisis region, making liquidity freezes and forced liquidations less likely.

Finally, I highlight an important difference between the crisis mechanism in this model and those in He and Krishnamurthy (2019) and Maxted (2024). In my framework, crises arise from forced liquidation under binding equity capital constraints, coupled with the presence of less efficient households who absorb capital during distress. Their inefficient ownership depresses prices and amplifies the downturn, producing fire-sale dynamics. By contrast, in He and Krishnamurthy (2019) and Maxted (2024), the

[14] It is worth reiterating that a financial crisis arises in the rational benchmark only when households inefficiently hold physical capital. Otherwise, the equilibrium price q remains flat in e under general equilibrium, for the same reason discussed earlier: the intermediary sector effectively becomes a veil—a macro-finance analogue of the Modigliani–Miller theorem. Moreover, when the constraint binds in the rational benchmark but the economy has not yet entered the crisis region, all capital remains in the efficient hands of intermediaries. Consequently, the decline in q is far milder than in the crisis region, where capital ownership shifts sharply toward households.

crisis region is defined purely as the set of states in which equity capital constraints bind—without fire sales. As I show in Appendix C, a key empirical hallmark of crises—sharp asset-price collapses—disappears in their models once they are embedded in a full general-equilibrium environment (rather than under a specific short-rate rule). In short, forced liquidation into the hands of less efficient households is essential for generating realistic crisis behavior in general equilibrium under this class of models.

5 Equilibrium characterization

In this section, I first characterize the behavior of key equilibrium quantities—such as the capital price q, the investment rate ι, and the household consumption–wealth ratio—as functions of the state variables e and X. I then show that equilibrium intermediary leverage declines in X, consistent with the negative relation documented in the data. Finally, I demonstrate that sentiment reversals generate fluctuations in intermediary capital capacity and that these fluctuations are asymmetric across sentiment regimes.

5.1 Equilibrium quantities

Figure 5 plots the equilibrium capital price q, investment rate ι, and household consumption-wealth ratio $\frac{c}{w}$ against intermediary capital capacity e and household sentiment X. The first row shows these quantities as functions of e, comparing high sentiment ($X > 0$) and low sentiment ($X < 0$).[15] The black and red dashed vertical lines indicate the respective crisis cutoffs; to the left of each line, the economy is in the financial crisis region. The second row shows the same equilibrium quantities as functions of X, under two states of intermediary capital capacity: a high state ($e = 2$) and a low state ($e = 0.5$).[16]

Amplification mechanism. As shown in the first row of Figure 5, the capital price q exhibits a classic amplification effect of financial frictions: once the economy enters the crisis region, q collapses sharply as highlighted in Section 4.3. The mechanism is straightforward. In the crises region, a negative fundamental shock erodes intermediaries' capital capacity, forcing them to fire sell capital to households, whose inefficient ownership depresses q. The resulting asset-value losses further erode intermediary equity capacity $\mathcal{E}$, tightening the equity capital constraint and triggering additional equity withdrawals. In response, intermediaries must liquidate even more capital, pushing q down further. This self-reinforcing feedback loop amplifies the initial shock and produces large real distortions.

Fire sale. Figure 6 shows that once the economy enters the crisis region, fire-sales rapidly reallocate capital to the inefficient hands of households, consistent with the Definition 1.[17] Moreover, as shown

[15] Specifically, for the high-sentiment case $X > 0$, I conisder $X = 0.133$, and for the low-sentiment case, I set $X = -0.133$; these values are approximately 1.5 standard deviations above and below the mean, respectively.

[16] For the high capital capacity state $e = 2$, the economy is never in the equity-capital-constraint region nor in the financial-crisis region. By contrast, for the low capital-capacity state $e = 0.5$, the economy remains outside the crisis region when sentiment X is low but enters the crisis region when X is high.

[17] Note that when household sentiment is high, households hold some capital assets even in normal states. Specifically,

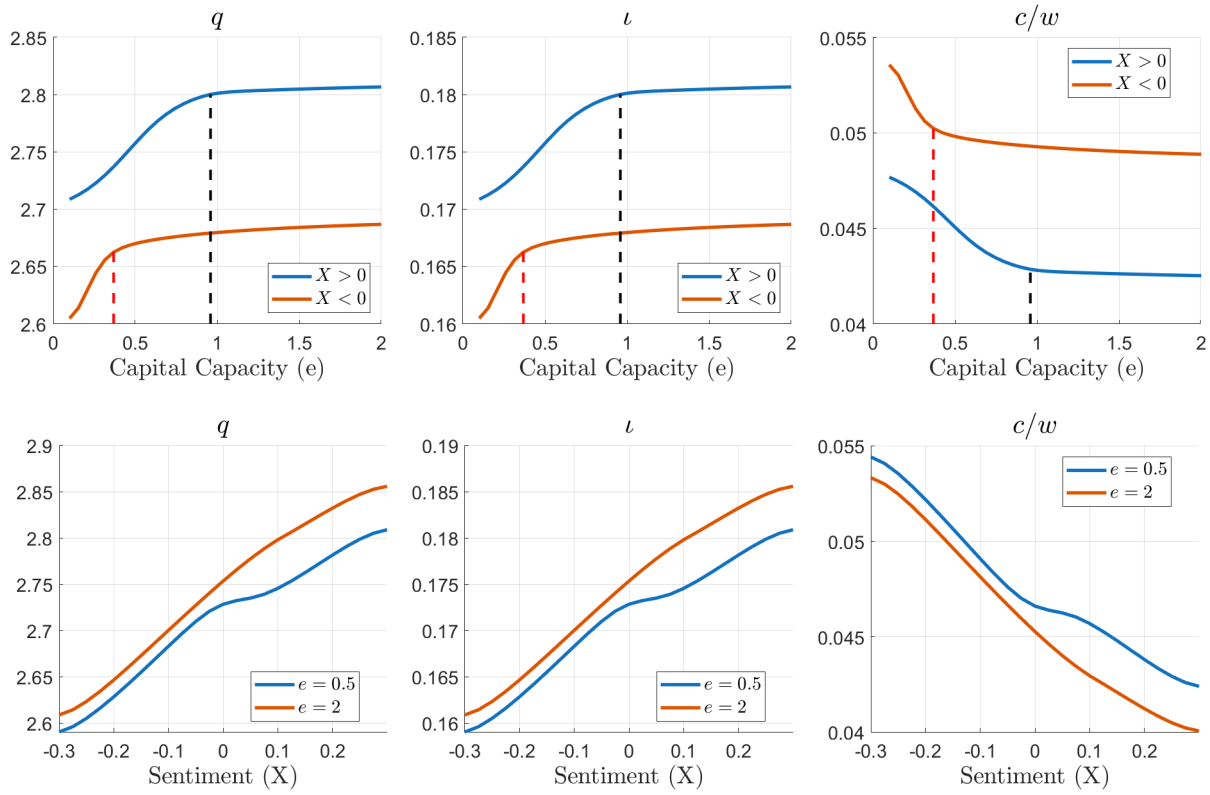


Figure 5. Equilibrium q, ι and $\frac{c}{w}$. These panels plot the equilibrium capital price q, investment rate ι, and household consumption-wealth ratio $\frac{c}{w}$ against capital capacity e and household sentiment X. The first row shows how these quantities vary with e in high sentiment ($X > 0$) and low sentiment ($X < 0$) regimes. The black (red) dashed vertical line marks the financial crisis cutoff in the high (low) sentiment regime. The second row shows their variation with X at a normal state ($e = 2$) and a distress state ($e = 0.5$).

in Figure 4, although crises appear less likely to occur in low-sentiment states, when they do arise they are more severe: the right panel of Figure 6 exhibits a steeper decline in intermediaries' capital share, indicating sharper fire sales and larger real-activity disruptions. Consistently, capital price q and investment ι collapse more dramatically in the low-sentiment regime once crises begin (Figure 5). In other words, while household sentiment—through the financing channel—reduces the frequency of crises in low-sentiment states, it also builds up vulnerability, making eventual crises far more explosive.

Anticipation effect. The figures also reveal an anticipation effect: capital prices begin to decline before the economy crosses into the crisis region. This effect, emphasized by He and Krishnamurthy

once the equity investment cap binds ($\alpha^I_{Ht} = \overline{\alpha}$), optimistic equity households borrow from debt households to purchase capital assets and increase their risk exposure until their borrowing constraint binds. This explains why the household capital share is slightly positive in the high-sentiment regime in the plot. However, this steady capital allocation between intermediaries and households does not generate sharp declines in the capital price q or investment ι. In fact, without changes in capital ownership—as occur in the crisis region—the capital price would remain essentially flat across e. Only when intermediaries are forced to liquidate their capital asset holdings to households in the fire-sale region do we observe pronounced drops in q and ι.

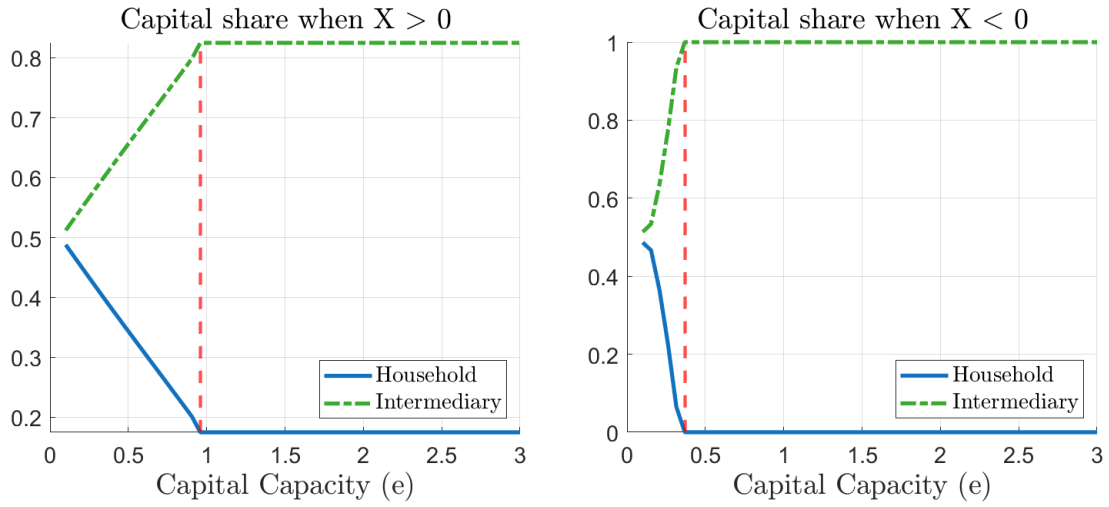


Figure 6. Physical capital shares. The left panel shows the allocation of physical capital between households and financial intermediaries in the high-sentiment regime ($X > 0$), while the right panel shows the allocation in the low-sentiment regime ($X < 0$). The red dashed vertical line marks the financial crisis cutoff for the corresponding sentiment regime.

(2019) and Maxted (2024), reflects households' forward-looking behavior. As intermediaries weaken, households perceive a rising probability of crisis. They respond by increasing consumption and diverting less wealth to capital markets directly or indirectly. This downward shift in capital demand lowers q even in pre-crisis states.

Sentiment impact. The second row of Figure 5 highlights the impact of household sentiment on the capital price q. Specifically, q increases with household optimism. The mechanism is behavioral: when sentiment X rises, households perceive stronger investment opportunities. With an elasticity of intertemporal substitution $\psi > 1$, the substitution effect dominates the income effect, so households reduce consumption and invest more. Consistent with this, the consumption–wealth ratio $\frac{c}{w}$ falls in X. As a result, more wealth flows into capital markets—either indirectly through intermediaries or directly through household capital holdings. Since the capital supply is inelastic due to adjustment costs, higher demand pushes up the price q.[18] I refer to this mechanism as *the asset pricing channel of household sentiment*. Put differently, sentiment shifts aggregate demand for capital, inflating prices when X is high and depressing them when X is low. This is the macro-finance analogue of the noise trader effect in stock markets (De Long et al. (1990); Barberis et al. (2015)), where sentiment-driven investors push equilibrium prices away from fundamentals.

Equilibrium r_t, π_t and σ_t. Figure 7 plots the equilibrium short rate r_t, the risk premium π_t, and return volatility σ_t against intermediary capital capacity e. The black dashed vertical line marks

[18] Household direct holdings of capital are not essential for this effect. Even in an economy where households cannot invest in capital directly, q still increases in X: lower consumption frees more resources to intermediaries via equity and debt, which then flows into capital markets.

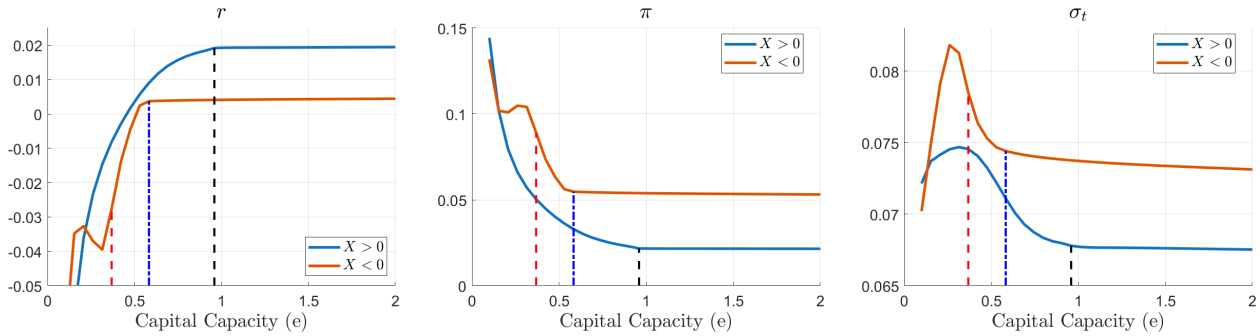


Figure 7. Equilibrium r, π and σ_t. These panels plot the equilibrium short rate r_t, equilibrium risk premium π_t and equilibrium return volatility σ_t as functions of capital capacity e_t under high sentiment ($X > 0$) and low sentiment ($X < 0$) regimes. The black (red) dashed vertical line marks the financial crisis cutoff for the high (low) sentiment regime. The blue dashed vertical line marks equity capital constraint cutoff in the low sentiment regime.

both the equity capital constraint cutoff and the crisis cutoff in the high-sentiment regime ($X > 0$). By contrast, in the low-sentiment regime, the two cutoffs differ: the blue dashed line marks the equity capital constraint, while the red dashed line marks the onset of the crisis region, consistent with the gap discussed above. We can see that, once the equity capital constraint binds, these asset pricing quantities display strong nonlinear behavior.[19] Such patterns are standard in the financial-frictions literature (He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)). Specifically, as intermediaries lever up under tighter constraints (triggered by increased equity withdrawals), they bear disproportionately larger aggregate risk. This raises risk premium π_t and volatility σ_t, while lowering the short rate r_t due to stronger precautionary savings motive. Household sentiment X further shapes these asset pricing quantities. As shown later, intermediaries operate with much higher leverage in the low-sentiment regime than in the high-sentiment regime. Consequently, the risk premium π_t and return volatility σ_t are higher, while the short rate r_t is lower, when household sentiment is pessimistic.

5.2 Intermediary leverage

As discussed in Section 4.3, household sentiment reshapes the distribution of states in which the equity capital constraint binds and thus has a profound impact on economic resilience through the financing channel. In addition, because sentiment X affects households' perceived price of risk (Appendix D), it directly influences the ease with which intermediaries can raise equity financing, with important implications for the dynamics of intermediary leverage.

When sentiment is low, pessimistic households perceive the intermediary's equity as offering a lower

[19]The non-linear asset pricing patterns arise from financial frictions that distort risk sharing, and they can appear even before the economy enters the crisis region. Specifically, in an economy without fire sales but with binding equity-capital constraints, these distortions already manifest: the surge in the risk premium π_t and the simultaneous drop in the short rate r_t offset one another, leaving expected returns unchanged while generating non-linear behaviors.

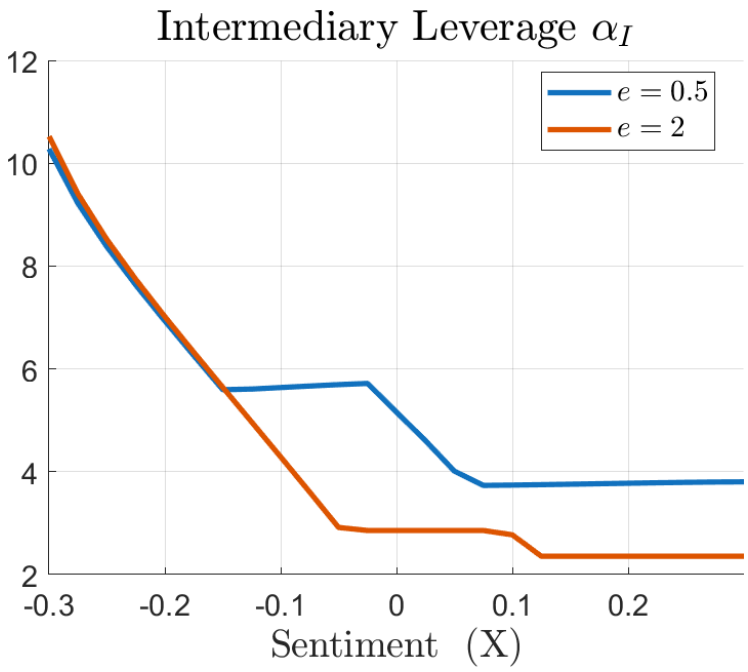


Figure 8. Equilibrium intermediary leverage. This figure plots equilibrium intermediary leverage α_I as a function of household sentiment X. Results are shown for both high capital capacity ($e = 2$) and low capital capacity ($e = 0.5$).

price of risk than they require, making them reluctant to supply equity capital. This reluctance makes equity financing prohibitively costly for intermediaries, who would need to offer a substantially higher risk premium to attract equity funding. In response, intermediaries substitute toward debt financing, allowing leverage to rise until the market price of risk adjusts to households' required compensation. As household pessimism deepens, the share of equity in the aggregate funding mix declines further, and intermediary leverage rises correspondingly. The high intermediary leverage in the low-sentiment regime, and its increase as X falls, is evident in Figure 8.

By contrast, in high-sentiment states, leverage is low for different reasons depending on the level of intermediary capital capacity. When sentiment is high and intermediary capital capacity is large ($e = 2$), optimistic households perceive intermediary equity as offering more compensation for risk than they require, making equity investment particularly attractive. They therefore invest aggressively in intermediary equity until the equity-investment cap binds ($\alpha^I_{Ht} = \overline{\alpha}$). This abundant supply of equity financing raises the equity share in the total funding mix supplied to intermediaries and results in low intermediary leverage.[20] The case of high sentiment with low intermediary capital capacity ($e = 0.5$) is more nuanced. Here the economy lies in the crisis (fire-sale) region. As sentiment rises, speculative

[20] After the equity-investment cap binds ($\alpha^I_{Ht} = \overline{\alpha}$), optimistic equity households still seek to increase their risk exposure. They borrow from debt households and invest directly in physical capital until their borrowing constraint binds. This reallocation reduces net lending to intermediaries and further lowers intermediary leverage, which is reflected in the slight decline in leverage in the high-sentiment regime when $e = 2$ (the orange curve in Figure 8).

households—who face restrictions on investment in intermediary equity—increase their direct holdings of capital assets to amplify their risk exposure. Intermediaries are consequently forced to liquidate more capital to households, shrinking their asset positions. The resulting contraction of intermediaries' balance sheets lowers their leverage, even though equity funding remains scarce, as reflected in the decline of leverage along the blue curve in Figure 8.[21]

Overall, the financing channel of household sentiment—operating through its effect on the ease with which intermediaries can raise equity—generates a strong negative relationship between household sentiment and intermediary leverage (especially in normal states), as shown in Figure 8, in line with the empirical evidence in Section 2.1. The resulting elevation of intermediary leverage in low-sentiment states has important implications for financial stability (Section 6.1) and is a key reason why the sentiment model better matches the 2007–2009 financial crisis episode (Section 6.4). Belief heterogeneity between intermediaries and households is essential for this mechanism; models with homogeneous beliefs, such as Maxted (2024), miss this feature.

5.3 Sentiment-driven balance sheet dynamics

In equilibrium, procyclical household sentiment reverts toward its stationary mean in the absence of shocks. Through the asset pricing channel—whereby the equilibrium capital price increases in X—this mean reversion affects the returns earned by intermediaries and, in turn, the strength of their balance sheets. To illustrate this mechanism, Figure 9 presents a phase diagram of the joint evolution of the state variables e_t and X_t. Each "x" marks a starting point, and the blue curves trace the ensuing five-year evolution under no fundamental shocks.

As shown in Figure 9, when sentiment is neutral ($X = 0$), the dynamics of e_t are standard in financial-frictions models: intermediary capital capacity rises when it is low (because high leverage generates high expected returns) but declines when it is high (because risk premia are compressed and exit dominates). When sentiment is high, its mean reversion lowers the capital price q, generating losses and eroding intermediary capital capacity. Conversely, when sentiment is low, its reversals raise q, deliver capital gains, and help repair intermediary balance sheets. Thus, sentiment reversals reshape the strength of the intermediary sector through the asset-pricing channel.

Maxted (2024) also generates sentiment-driven fluctuations in intermediary capital capacity and refers to this mechanism as the feedback effect of behavioral frictions. The key difference in my model is visible in the phase diagram: reversals in intermediary capital capacity associated with sentiment mean reversion are much stronger in the low-sentiment regime than in the high-sentiment regime, as seen

[21] The nearly flat intermediary leverage (for both high and low e) at very high levels of household sentiment reflects the borrowing constraint faced by equity households. Once this constraint binds, it limits the aggregate risk exposure of the household sector, leading to a stable allocation of risk between households and intermediaries and, consequently, a nearly constant level of intermediary leverage.

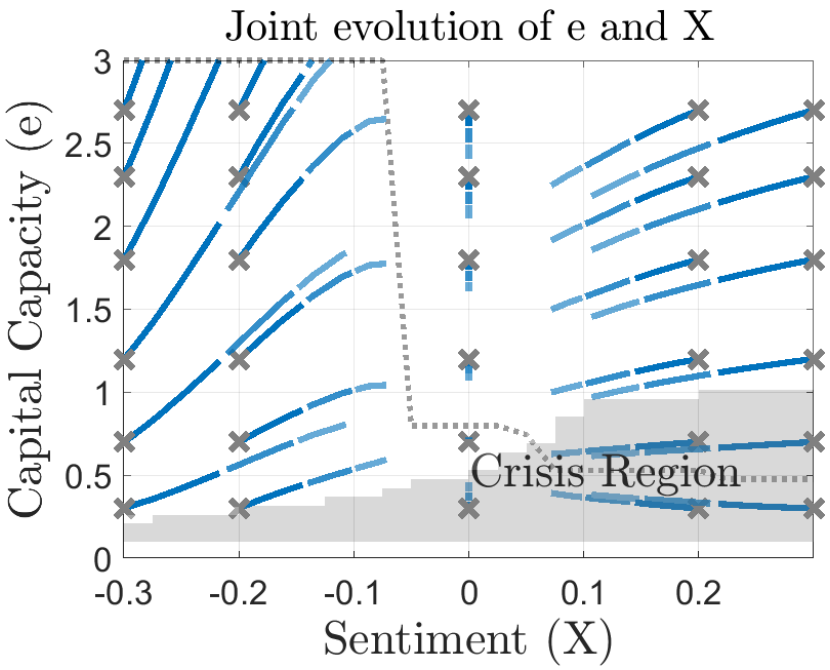


Figure 9. Sentiment-driven capital capacity fluctuations. This figure shows the three-year joint evolution of the state variables e and X in absence of fundamental shocks. Each "x" denotes a starting point, while the dotted grey line indicates the locus where the drift of e_t equals 0.

from the larger vertical movements of the transition paths at low-sentient states—an asymmetry that is absent in Maxted (2024). This feature arises from the negative relationship between intermediary leverage and household sentiment. During downturns, when sentiment is low, intermediaries operate with high leverage, so a recovery in sentiment leads to a sharp rebound in capital capacity. By contrast, in high-sentiment regimes, sentiment disappointments weaken intermediary balance sheets and can trigger crises, but this effect is mitigated by relatively low leverage. Relative to Maxted (2024), the empirically consistent negative relation between intermediary leverage and household sentiment in my model thus strengthens financial stability in both downturns and upturns: it drives the economy away from the crisis region more forcefully in bad times and attenuates the transition from sentiment reversals to full-blown crises in good times.

6 Model implications and empirical relevance

In this section, I present three key implications of the model. First, I show that, consistent with extensive empirical evidence, crisis risk can build up in good times. Second, I demonstrate that household sentiment can generate boom–bust dynamics in the investment rate and the equilibrium risk premium. Third, I examine how household sentiment affects financial stability through comparative statics. Fi-

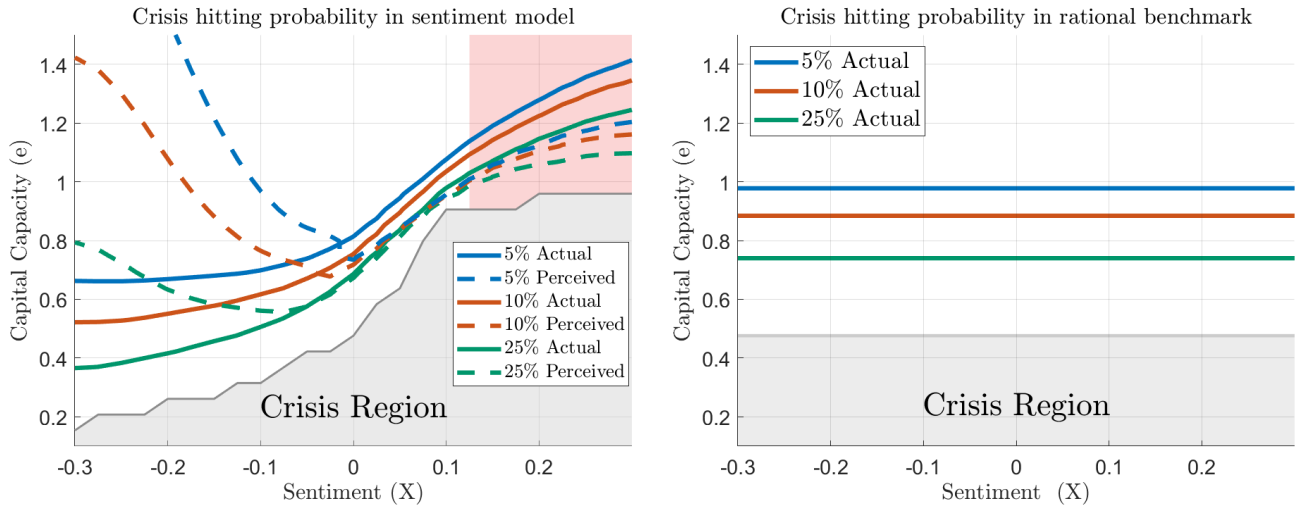


Figure 10. Crisis hitting probability. The left panel shows the probability of entering a financial crisis within the next three years in the main model, where households hold sentiment-driven beliefs. The right panel shows the corresponding probability in the rational benchmark, where households are fully rational. The blue, orange, and red contour curves indicate realized crisis probabilities of 5%, 10%, and 25%, respectively. The gray shaded region in each panel represents that model's crisis region. In the left panel, the red shaded area provides a theoretical representation of the "red zone" proposed in Greenwood et al. (2022).

nally, I simulate the model using shocks inferred from the data to assess its ability to match the 2007–2009 financial crisis episode.

6.1 Crash risk build-up in frothy booms

A large body of evidence shows that frothy financial conditions—such as rapid credit expansion, compressed credit spreads and equity risk premia, and surging asset prices—strongly predict future economic downturns and financial crises (see, e.g., Schularick and Taylor (2012); Baron and Xiong (2017); López-Salido et al. (2017); Mian et al. (2017); Greenwood et al. (2019, 2022); Kirti (2025); Krishnamurthy and Muir (2025)). Among this literature, Baron and Xiong (2017) find that the predictive power of bank credit expansion for crash risk increases with bank shareholders' optimism. Greenwood et al. (2022) propose the notion of a "red zone," where simultaneous surges in credit growth and asset prices sharply raise the likelihood of a financial crash. Moreover, López-Salido et al. (2017) and Mian et al. (2017) emphasize that credit market booms are primarily driven by credit supply shocks. However, this rapid transition from economic exuberance to collapse is difficult to reconcile with canonical financial-friction models such as He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), which require a prolonged sequence of adverse shocks to move the economy from boom to bust.

To examine whether my model can replicate these empirical patterns, I solve the Kolmogorov backward equation (KBE) to compute the probability that the economy enters the financial crisis region

within the next three years, conditional on being in the normal region today.[22] Since households' perceived state probabilities deviate from rational expectations, I also compute households' perceived crisis-hitting probability at each point in the normal region. Figure 10 reports the results. The left panel displays both the actual and perceived probabilities under the main (sentiment) model, while the right panel presents the rational benchmark in which households form rational expectations.

The left panel of Figure 10 highlights a "red zone" (red shaded area) in the model—defined as the set of states in which, relative to the rest of the state space, risk premia are at their lowest and capital prices q are at their highest—consistent with Greenwood et al. (2022). Within this red zone, the crash probability remains elevated, whereas it is essentially zero for states with the same level of capital capacity in the rational benchmark (right panel of Figure 10). In other words, under a sequence of positive shocks, crash risk in the sentiment model can build up rather than subside, while it falls to zero in the rational benchmark, underscoring the crucial role of behavioral distortions in matching these empirical patterns. Moreover, higher household optimism within this region further raises crash likelihood, consistent with the evidence in Baron and Xiong (2017).

Households perceive crash risk very differently across sentiment regimes. As shown in the left panel, pessimistic households in the low-sentiment regime overestimate crisis likelihood, leading them to cut back investment in intermediary equity—a behavior that, as discussed earlier, strengthens economic resilience. By contrast, optimistic households in the high-sentiment regime underestimate crash risk when intermediaries appear strong, supplying abundant equity financing that supports fragile funding liquidity. The influx of equity from optimistic households in the red zone finances intermediaries' demand for capital assets, effectively mirroring the credit-supply shocks that drive credit booms in the data.

The build-up of crash risk in boom periods in the sentiment model is natural given the shape of the endogenous crisis region: the crisis boundary lies relatively low in the low-sentiment regime and is shifted upward in the high-sentiment regime. As discussed in Section 4.3, under households' speculative crowd-out in high-sentiment states, the resulting abundant equity funding is inherently fragile. Even modest negative returns can then push red-zone states into the crisis region, generating an endogenous build-up of crash risk during booms. Thus, the reshaping of the crisis region by household sentiment is central to delivering the empirically consistent build-up of crash risk in frothy periods. Maxted (2024) also generates elevated crash risk in good times, but in his framework the main driver is an intermediary balance sheet recession triggered by reversals in intermediary sentiment.[23] By contrast, this paper shows that household sentiment—rather than intermediary sentiment—can reconcile this empirical regularity through an alternative mechanism.

[22] For numerical details on computing crisis-hitting probabilities via the KBE, see the appendix in Maxted (2024).

[23] As shown in Appendix F.2, the erosion of intermediary capital capacity driven by sentiment reversals through the asset-pricing channel—where mean reversion in high-sentiment states generates negative returns—also contributes to the build-up of crisis risk in the red zone. However, this mechanism plays only a minor role in the overall result.

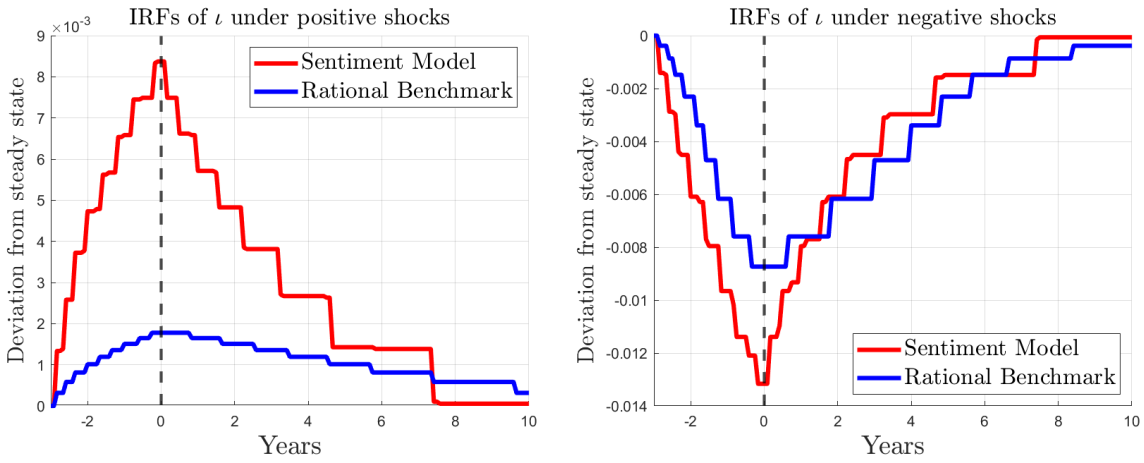


Figure 11. Investment boom-bust cycle. The left (right) panel plots the evolution of the equilibrium investment rate starting from the stochastic steady state at $t = -3$. The economy is subjected to a sequence of positive (negative) fundamental shocks until $t = 0$, after which the shocks are turned off. Results are shown for both the main model with household sentiment and the rational benchmark. In each case, the starting point at $t = -3$ corresponds to that model's own stochastic steady state.

6.2 Household-sentiment-driven boom-bust cycle

In Section 2.2, we see that periods of high household sentiment are associated with near-term economic expansions but longer-horizon slowdowns. To examine whether the model can generate this boom–bust dynamics in economic growth, I compute impulse response functions (IRFs) of investment to fundamental shocks. Starting from the stochastic steady state, I simulate the model at a monthly frequency and feed in a three-year sequence of shocks—positive or negative—that cumulate to three standard deviations.[24] Shocks are then turned off. To highlight the role of household sentiment, I also compute IRFs for the investment rate ι under the rational benchmark. The results are shown in Figure 11. As shown in the left panel of Figure 11, under a sequence of positive fundamental shocks, the model with household sentiment exhibits a pronounced boom–bust pattern in investment and thus economic growth, in contrast to the smoother adjustment in the rational benchmark. This dynamic mirrors the evidence in Section 2.2: periods of high household sentiment fuel near-term economic booms, but as sentiment subsequently reverses, they give way to longer-term slowdowns and recessions.

The mechanism is the asset pricing channel of sentiment. Beyond the strengthened intermediary capital capacity that drives higher investment in the rational benchmark, positive shocks also elevate household sentiment, inflating capital prices q and thus further boosting investment. When sentiment subsequently reverses, households consume more, wealth outflows from the capital market, and capital prices collapse—causing investment to drop sharply. Conversely, under a sequence of negative shocks, the dynamics reverse: sentiment collapses and depresses investment further in the short run, but as

[24] Appendix E.3 provides robustness checks with alternative shock magnitudes.

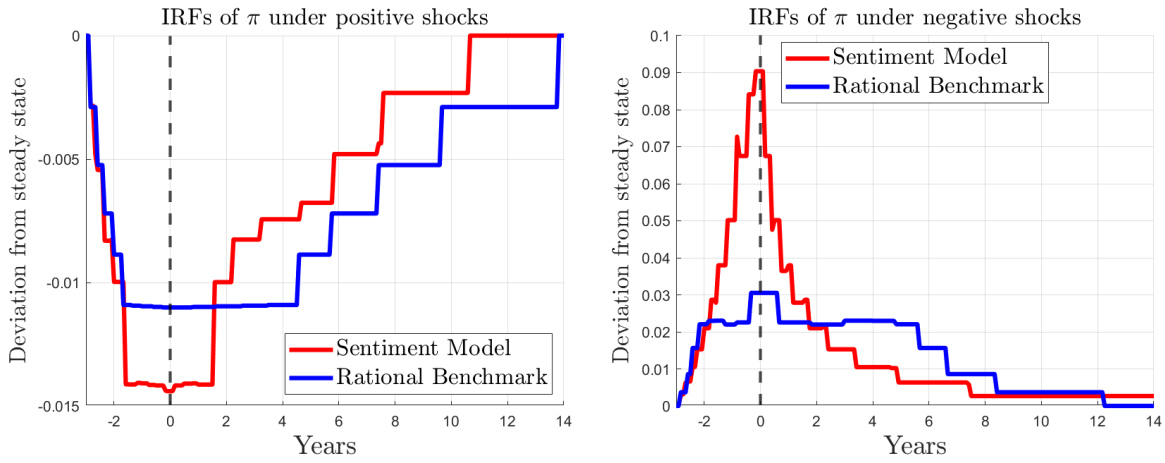


Figure 12. Risk premium boom-bust cycle. The left (right) panel plots the evolution of the equilibrium risk premium starting from the stochastic steady state at $t = -3$. The economy is subjected to a sequence of positive (negative) fundamental shocks until $t = 0$, after which the shocks are turned off. Results are shown for both the main model with household sentiment and the rational benchmark. In each case, the starting point at $t = -3$ corresponds to that model's own stochastic steady state.

sentiment rebounds, prices and investment recover strongly, generating a bust–boom cycle.

In addition to investment, I also examine how household sentiment shapes the response of the equilibrium risk premium to shocks. Using the same strategy as before, I compute IRFs of the risk premium under both positive and negative shocks, comparing the sentimental model with the rational benchmark (Figure 12).

Relative to the rational benchmark, household sentiment generates a boom–bust (bust–boom) pattern in the equilibrium risk premium following negative (positive) shocks. Consider the striking boom–bust risk-premium dynamics in the right panel of Figure 12. Under a sequence of negative shocks, pessimistic households cut back their supply of equity funding to intermediaries, which raises intermediary leverage α_{It} substantially relative to the rational benchmark. Since the equilibrium risk premium is given by $\pi_t = \gamma^I \alpha_{It} \sigma_t^2$, this heightened leverage produces a much sharper increase in the risk premium. Subsequently, when sentiment reverses, intermediary capital capacity recovers quickly because the initial risk premium and leverage are high. Leverage therefore declines more rapidly in the absence of further shocks, causing the risk premium to fall faster than in the rational benchmark. An analogous mechanism operates in the left panel under positive shocks, generating a bust–boom pattern in the risk premium.

6.3 Household-sentiment impact on financial stability

I now turn to the paper's central question: how household sentiment shapes financial stability. The strategy is straightforward: I vary key model parameters and compute the unconditional probability of

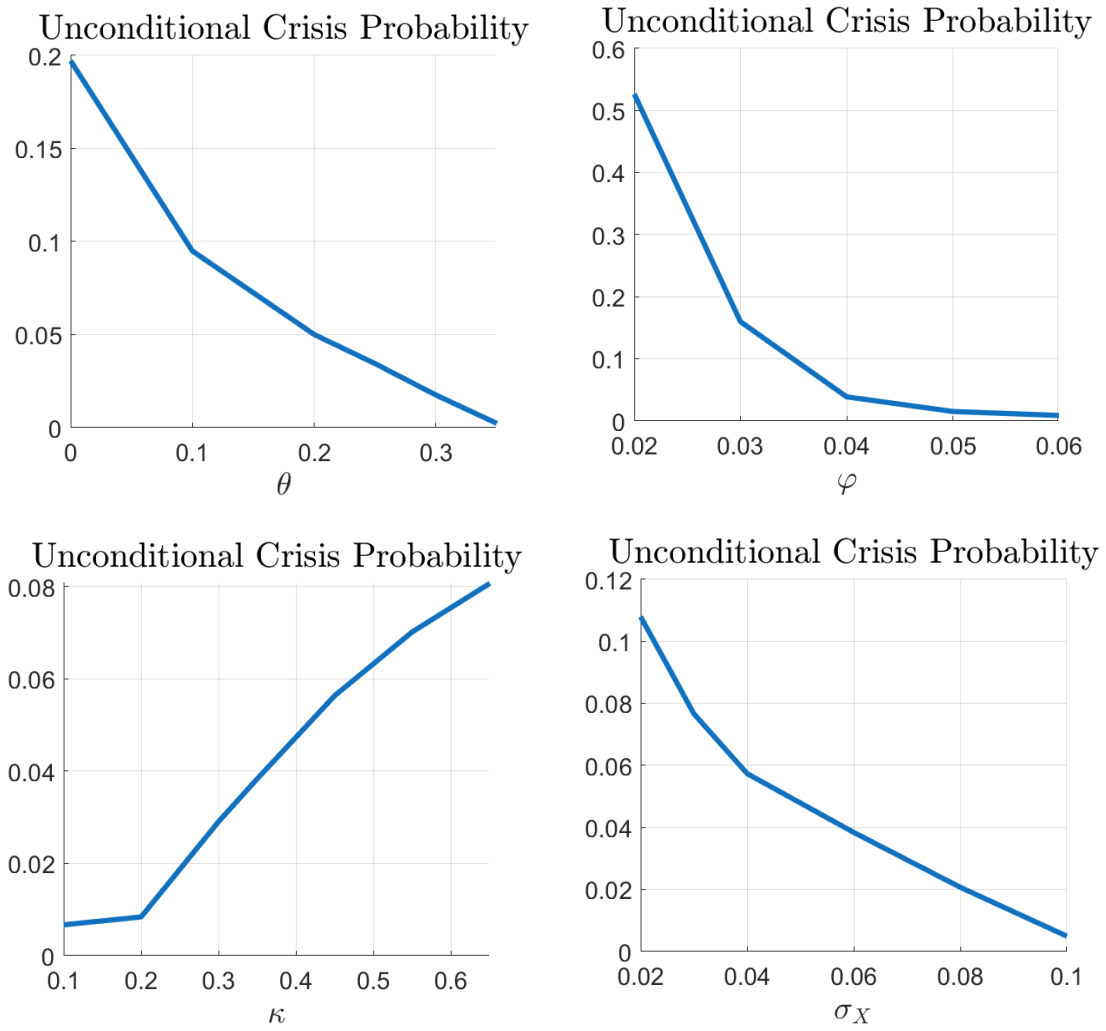


Figure 13. Comparative statics on unconditional crisis probability. These panels plot comparative statics for the unconditional probability of financial crises under alternative parameter values. The four panels report results for variations in irrationality θ, relative cost φ, sentiment persistence κ, and sentiment volatility σ_X, respectively.

financial crises. The parameters of interest are the degree of irrationality θ, the relative cost of direct investment φ, the persistence of sentiment κ, and sentiment volatility σ_X.

6.3.1 Rationality paradox

The most striking result is what I call *the rationality paradox*. As shown in the upper-left panel of Figure 13, the unconditional crisis probability actually falls as households become more irrational (higher θ). Put differently, the less rational households are, the more stable the economy appears on average.

The logic is rooted in the low-sentiment regime.[25] For a given pessimistic sentiment $X < 0$, a higher θ further lowers households' perceived price of risk on intermediary equity, $\gamma^I \alpha_{It} \sigma_t + \frac{\theta X_t}{\sigma}$ (see Appendix D), making them even more reluctant to supply equity funding. As equity financing becomes increasingly costly, intermediaries rely more on debt, resulting in even higher leverage. At first glance this appears destabilizing, but the higher leverage actually strengthens recoveries: when sentiment—and thus q—rebounds, higher leverage magnifies the resulting gains, allowing intermediaries to rebuild equity capital more quickly. Given the inherent resilience of the economy in the low-sentiment regime—manifested by the lower crash boundary—this mechanism typically pushes the economy farther away from the crisis region rather than toward it, unless shocks are extraordinarily severe. This outcome reflects the interaction of the financing and asset pricing channels of household sentiment: by amplifying leverage in bad times, higher θ creates the conditions for stronger balance-sheet repair when prices recover, thereby stabilizing the system.

As confirmed in Figure F4 in Appendix E, higher θ systematically raises intermediary leverage and risk premia—primarily through the low-sentiment regime. This delivers greater risk compensation to intermediaries on average, which accumulates into stronger balance sheets and increases the unconditional mean of capital capacity e.[26] Consistently, Figure F5 shows the ergodic distribution shifting upward as θ rises, centering at higher values of capital capacity and thus moving the economy further away from the crisis region. In addition, the reduced reliance on equity funding under larger θ causes the equity capital constraint to bind at even lower values of e in the low-sentiment regime, thereby pushing down the crash boundary further. The economy must therefore deteriorate further before entering the crisis region, which reduces the likelihood of crises.[27]

To summarize, greater household irrationality θ paradoxically safeguards the economy through the interplay between the financing and asset pricing channels. This result echoes Maxted (2024), who shows that diagnostic expectations among intermediaries stabilize the system relative to the rational benchmark.[28]

[25] As shown in Appendix F.1, the unconditional correlation between e and X is 0.76, so shocks tend to place the economy either in high-sentiment/high-capital-capacity states or in low-sentiment/low-capital-capacity states. Although crash risk can build up in the high-sentiment regime, as discussed in Section 6.1, a given sequence of positive shocks may push the economy so far into the high-e region that crash probabilities remain negligible, whereas an equally sized sequence of negative shocks only moves the economy closer to the crisis boundary. Thus, while positive shocks can endogenously raise crash risk—a feature absent in the rational benchmark—the economy is not fragile after every boom: when intermediary capital capacity becomes very high following strong positive shocks, it takes a long time for a crash to materialize.

[26] This can also been seen from the risk premium buffer term $\alpha_{It}\pi_t$ in the drift of e_t, μ_{et}, as shown in (A.6). Higher intermediary leverage and risk premia raise the drift of capital capacity e, pushing the economy away from the crisis region.

[27] This pattern is visible in Figure F5, where rising θ shifts the crisis-region cutoff curves downward in the low-sentiment regime.

[28] In fact, unconditional crisis probability is highest in the rational benchmark, corresponding to $\theta = 0$.

6.3.2 The impact of relative cost φ

In the upper-right panel of Figure 13, the unconditional crisis probability falls as the relative cost φ rises. This result is intuitive: when holding physical capital directly is more costly, households find it more attractive to provide funding liquidity, making them less inclined to force intermediaries into asset liquidations. Consistently, Figure F5 shows that the crisis region narrows considerably under higher φ, as households are less willing to absorb capital from distressed intermediaries. This finding highlights an important implication: household sentiment is more likely to trigger crashes through intermediated asset classes with low entry costs (e.g., equities) than through more complex assets where participation costs are higher (e.g., MBS). In other words, the ease with which households can bypass intermediaries shapes not only the frequency but also the locus of financial instability.

6.3.3 Sentiment volatility paradox

The second row of Figure 13 reveals another surprising result: the economy becomes more stable under higher σ_X or lower κ. Both changes raise the unconditional volatility of sentiment, given by $\frac{\sigma_X}{\sqrt{2\kappa}}$ in the OU process. In other words, when sentiment fluctuates more widely on average, the probability of financial crises actually falls. I refer to this as *the sentiment volatility paradox*.

At first glance, this result is counterintuitive: if negative shocks make sentiment fall more sharply and remain depressed for longer, the capital price q falls further and intermediary capital capacity e erodes more, seemingly making crises more likely. However, the economy is inherently resilient in the low-sentiment regime—manifested by the low crash boundary—through the financing channel. As long as negative shocks are not extraordinarily severe, the economy need not cross into the crisis region even when sentiment becomes more volatile. At the same time, greater sentiment volatility causes X to fall more under negative shocks, inducing more pessimistic households to cut back equity financing more aggressively. This raises intermediary leverage in the low-sentiment regime and accelerates the rebuilding of intermediary capital capacity when prices recover. Moreover, when higher sentiment risk is priced, intermediaries earn higher risk compensation, further cushioning their balance sheets. The resulting increase in unconditional intermediary leverage, risk premia, and return volatility shown in Figure F4 shifts the ergodic distribution away from the crisis region—as verified in Figure F5—and thus lower the unconditional probability of crises.[29] For positive shocks, higher sentiment volatility causes both sentiment and capital capacity to overshoot, pushing the economy farther away from the crisis region—even though the crash boundary is pulled upward in the high-sentiment regime. Taken together, this implies that more volatile sentiment can reduce crisis risk under both positive and negative shocks.

To summarize, both the rationality paradox and the sentiment-volatility paradox hinge on a behav-

[29] Figure F5 also shows that with higher σ_X and lower κ, the ergodic distributions are more dispersed along the X dimension.

ioral distortion of the intermediary financing mix: by making equity issuance costlier in low-sentiment states and tilting intermediaries toward leverage, the mechanism dampens the realization of otherwise severe downside risks. When negative shocks are not extraordinary, higher risk compensation allows intermediary balance sheets to rebuild more quickly and thus stabilizes the economy instead. Overall, irrational households with volatile beliefs can—within a setting with financial frictions—stabilize the economy rather than amplify crash risk, a counterintuitive result that runs against the conventional view that behavioral distortions necessarily harm economic efficiency.

6.4 Empirical relevance: matching $2007-2009$ financial crisis

In this section, I bring the model to the data and show that it can reasonably match the episode of the 2007–2009 Global Financial Crisis (GFC). Specifically, I simulate the model using shocks inferred from the household sentiment time series. Given the OU specification of sentiment dynamics in equation (14), the implied Brownian shock is $dZ_t = \frac{dX_t - \kappa(\mu_X - X_t)dt}{\sigma_X}$. Using quarterly household sentiment data from 2000Q1 to 2018Q3, I extract the corresponding series of quarterly Brownian shocks. To initialize the simulation, I map the observed intermediary capital ratio in He et al. (2017) and the household sentiment at 2000Q1 to their model counterparts, e_0 and X_0. Given these initial states and the inferred shocks, I simulate the joint evolution of e_t and X_t, and all equilibrium variables. A parallel exercise is conducted for the rational benchmark model, and more details are provided in Appendix E.2.

In the top-left panel of Figure 14, I plot the simulated intermediary capital capacity e_t, the actual (scaled) household sentiment, and the empirical intermediary capital ratio. The gray shaded area represents the crisis region implied by the sentiment model, while the red vertical band highlights the crisis episode in the simulation, defined as the period during which capital capacity crosses into this region.[30] The simulated and actual capital ratios are highly correlated ($\rho = 0.73$), and the model captures the key dynamics of the 2007–2009 crisis. Following the dot-com bust, household sentiment rises during the credit expansion from 2003 to 2006, leading to higher intermediary capital capacity as households channel more equity funding to intermediaries. Consistent with the data, the simulated risk premium and intermediary leverage (bottom panels) both fall by about half during this boom.[31] Starting in 2007, a sequence of extraordinarily negative shocks erodes intermediary capital, sharply increasing leverage and risk premia—closely mirroring the data.[32] The model predicts that the economy enters the crisis region in early 2008, remains there for about a year, and exits in 2009, broadly consistent with

[30] Specifically, given the simulated time series of household sentiment X_t, I can extract the corresponding cutoff capital capacity from the model-implied crisis region. The vertical band spans 2008Q1–2009Q2.

[31] The time-series correlation between the simulated risk premium and the Baa–Treasury yield spread in the bottom-left panel is 0.58, and the correlation between simulated intermediary leverage and the HKM intermediary leverage series is 0.70.

[32] As discussed earlier, although the economy is resilient to negative shocks in the low-sentiment regime, it cannot avoid a crisis when shocks are sufficiently severe.

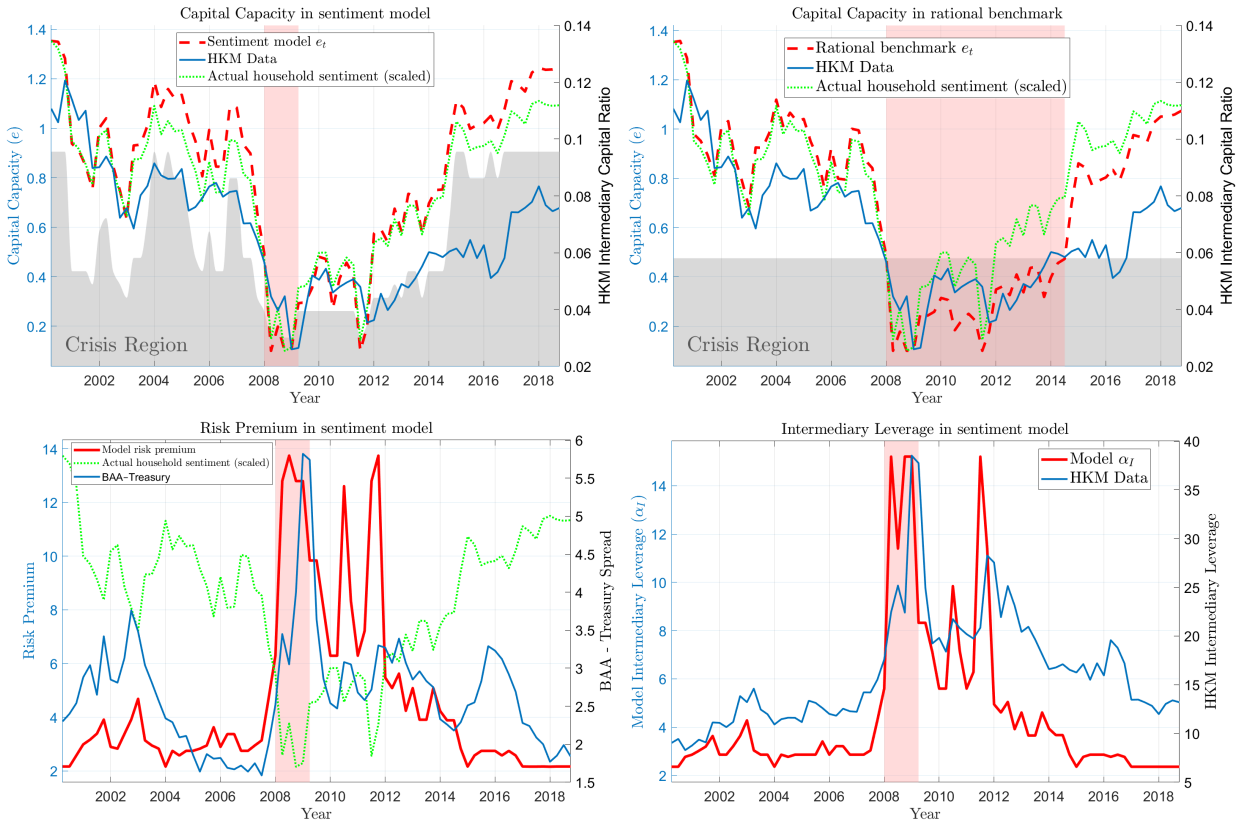


Figure 14. Simulating $2007-2009$ GFC. These panels present simulated results over the period from 2000Q1 to 2018Q3. The upper-left panel plots the simulated intermediary capital capacity e_t from the sentiment model together with the intermediary capital ratio from He et al. (2017); the shaded area denotes the model's crisis region. The upper-right panel shows the corresponding results for the rational benchmark. The bottom-left panel compares the simulated risk premium in the sentiment model with the empirical yield spread between Moody's Baa-rated corporate bonds and 10-year Treasury yields. The bottom-right panel reports the simulated intermediary leverage alongside the empirical leverage series from He et al. (2017). The red vertical bands highlight the crisis episode as implied by both the sentiment model and the rational benchmark.

the timing of the GFC.[33] Afterward, the recovery phase features a rebuilding of intermediary capital and a normalization of leverage and risk premia, again in line with the data.

As shown in the top-right panel of Figure 14, the rational benchmark also predicts that the economy enters the crisis region in 2008 given the extraordinarily negative shocks. However, the crisis lasts unrealistically long—the economy does not exit the crisis region until after 2014. This prolonged "crisis

[33]One limitation of the simulation is the brief entry into the crisis region in 2011, lasting roughly one quarter. Consistently, the bottom panels show a spike in the simulated risk premium and intermediary leverage. This episode is mainly driven by the sharp drop in household sentiment at that time, likely related to the 2011 U.S. debt-ceiling standoff, the sovereign downgrade, and heightened global uncertainty (including the Eurozone debt crisis). In reality, post-GFC policy interventions provided substantial support to the financial system even under severe shocks—forces that are absent from the model. Thus, the model's predicted short-lived crisis in 2011 is best interpreted as a period of heightened fragility and panic, rather than a full-blown financial crisis.

trap" reflects two key features that are absent in the rational model but present in the sentiment model: an inherently more resilient economy during downturns and stronger intermediary recovery driven by higher leverage. As highlighted earlier, in the sentiment model the economy is more resilient in the low-sentiment regime, as reflected by a lower crash boundary. By contrast, the crash boundary in the rational benchmark does not vary. This difference is evident in the figure: during 2009–2012, the crisis cutoff in the sentiment model is roughly half that in the rational benchmark, preventing the economy from remaining in the crisis region for an extended period. Moreover, under low sentiment, intermediaries operate with higher leverage and earn higher risk premia, which accelerates balance-sheet repair. Consequently, the sentiment model delivers a faster exit from crises and a stronger post-crisis recovery than the rational benchmark. This superior performance of the sentiment model underscores a key message of the paper—household sentiment shapes economic resilience by altering the endogenous crisis boundary and intermediary leverage dynamics.

Overall, the sentiment model provides a close match to the 2007–2009 GFC. It highlights the frothy economic conditions preceding the crisis—high capital capacity and low risk premia—and its aftermath, where the sentiment-driven resilience enables faster recovery than the rational benchmark.

7 Conclusion

In this paper, I study how household sentiment shapes macro-financial dynamics in a continuous-time general-equilibrium model featuring rational financial intermediaries and sentiment-driven households in the presence of financial frictions. The model delivers results consistent with the empirical evidence, yields sharp implications for financial stability, and can broadly match the dynamics of the 2007–2009 financial crisis episode.

The main results are as follows. First, through a financing channel, household sentiment affects the ease with which intermediaries can issue equity. When sentiment is low and equity financing is costly, intermediaries increase their reliance on debt and raise leverage, generating a negative relationship between intermediary leverage and household sentiment, as observed in the data. Second, through an asset-pricing channel, household sentiment distorts consumption–investment decisions, causing equilibrium capital prices to rise with sentiment. When sentiment mean-reverts, these price reversals induce fluctuations in intermediary capital capacity and generate pronounced boom–bust patterns in investment and, hence, in real economic activity. Third, the economy is inherently more resilient during downturns. In the low-sentiment regime, the crash boundary is pushed down, and higher intermediary leverage—combined with subsequent price recoveries under sentiment reversals—helps rebuild intermediary balance sheets and drives the system away from the crisis region. By contrast, in booms, abundant equity funding liquidity is inherently fragile: households' speculative crowd-out raises crash risk and allows crises to build up in "frothy" conditions. Finally, and in contrast to conventional intuition, more

irrational and more volatile household beliefs can strengthen resilience on average rather than trigger instability.

Beyond its specific contributions, this paper speaks to a broader research agenda. First, it underscores the importance of shifting attention from intermediaries to households as central players in financial stability. Second, it suggests new ways to interpret historical crises, where periods of optimism may have sown the seeds of vulnerability not only through credit booms but also through household portfolio and consumption choices. Finally, it opens avenues for policy: if household sentiment shapes systemic risk, then monitoring and understanding retail investor behavior may be as important as tracking intermediary balance sheets.

Appendix

The appendix is organized as follows. Appendix A details the model's solution algorithms and sketches the proof of Proposition 1. Appendix B presents the solution algorithm for the household portfolio problem. Appendix C explains why the pricing effects in He and Krishnamurthy (2019) and Maxted (2024) vanish in general equilibrium. Appendix D analyzes how household sentiment shapes intermediaries' financing. Appendix E contains additional material referenced in the main text and supplementary empirical evidence for Section 2.

Appendix A: Equilibrium solution

In this appendix, I describe the solution method for the macro-finance model with belief heterogeneity and financial frictions.

Intermediary's portfolio choice

From the banker's mean–variance objective (10), we can easily show that

$$\alpha_{It} = \frac{\pi_t}{\gamma^I \sigma_t^2}, \tag{A.1}$$

implying the equilibrium risk premium

$$\pi_t = \gamma^I \alpha_{It} \sigma_t^2. \tag{A.2}$$

The intermediary's optimal investment rate solves

$$\max_{\iota_t^I} \mu_t = \mu_{qt} + \Phi\left(\iota_t^I\right) - \delta + \sigma\sigma_{qt} + \frac{a - \iota_t^I}{q_t},$$

which yields

$$\iota_t^I = \frac{q_t - 1}{\phi}. \tag{A.3}$$

State variables dynamics

As discussed in Section 4.1, equilibrium is fully characterized by the two state variables e_t and X_t. Their dynamics are

$$\frac{de_t}{e_t} = \mu_{et}dt + \sigma_{et}d\omega_t, \tag{A.4}$$

$$dX_t = \mu_{Xt}dt + \sigma_X d\omega_t, \tag{A.5}$$

where

$$\mu_{et} = r_t + \alpha_{It}\pi_t - \eta - (\Phi(\iota_t) - \delta) + \sigma^2 - \alpha_{It}\sigma_t\sigma, \tag{A.6}$$

$$\sigma_{et} = \alpha_{It}\sigma_t - \sigma, \tag{A.7}$$

and

$$\mu_{Xt} = \kappa(\mu_X - X_t), \tag{A.8}$$

Endogenous risk

By Ito's lemma, we have

$$\sigma_{qt} = \frac{q_e e_t \sigma_{et} + q_X \sigma_X}{q_t}.$$

Combining this with (7) and (A.7), the equilibrium return volatility is

$$\sigma_t = \frac{\left[1 - \frac{q_e e_t}{q_t}\right]\sigma + \frac{q_X}{q_t}\sigma_X}{1 - \frac{q_e e_t \alpha_{It}}{q_t}}. \tag{A.9}$$

Household's optimization problem

The household's optimization problem (15) yields the HJB equation

$$0 = \max_{c_t, \alpha^I_{Ht}, \alpha^S_{Ht}, \iota^H_t} f(C_t, V_t) + \frac{\mathrm{E}^H_t[dV_t]}{dt}, \tag{A.10}$$

subject to the portfolio constraints (19)—(22). The household's optimal investment rate solves

$$\max_{\iota^H_t} \mu^H_t = \mu_{qt} + \Phi\left(\iota^I_t\right) - \delta + \sigma\sigma_{qt} + \frac{a - \iota^I_t}{q_t} + \frac{\sigma_t}{\sigma}\theta X_t,$$

from which we can obtain

$$\iota^H_t = \frac{q_t - 1}{\phi}. \tag{A.11}$$

Therefore, intermediaries and households make identical capital investment decisions and I denote the common investment rate by ι_t:

$$\iota_t = \iota^I_t = \iota^H_t = \frac{q_t - 1}{\phi}. \tag{A.12}$$

We conjecture that the household's value function

$$V_t = \frac{w_t^{1-\gamma^H}}{1-\gamma^H} J_t^{\frac{\gamma^H-1}{1-\psi}}, \tag{A.13}$$

with

$$\frac{dJ_t}{J_t} = \mu_{Jt}dt + \sigma_{Jt}d\omega_t = \mu_{Jt}^H dt + \sigma_{Jt}d\omega_t^H, \tag{A.14}$$

and

$$\mu_{Jt}^H = \mu_{Jt} + \frac{\sigma_{Jt}}{\sigma}\theta X_t. \tag{A.15}$$

Combing the HJB equation (A.10) with the conjecture (A.13), we can show that

$$\begin{aligned} 0 = {} & J_t^{-1} - \psi\rho + (\psi-1)\left[r_t + (1-\lambda)\,\alpha_{Ht}^I\alpha_{It}\left(\pi_t + \frac{\sigma_t}{\sigma}\theta X_t\right) + (1-\lambda)\,\alpha_{Ht}^S\left(\pi_t + \frac{\sigma_t}{\sigma}\theta X_t - \varphi\right)\right] \\ & - \frac{1}{2}\gamma^H(\psi-1)(1-\lambda)^2\left(\alpha_{Ht}^I\alpha_{It} + \alpha_{Ht}^S\right)^2\sigma_t^2 + \frac{\sigma_{Jt}}{\sigma}\theta X_t + \frac{1}{2}\left(\frac{\gamma^H-1}{1-\psi} - 1\right)\sigma_{Jt}^2 \\ & + \left(1-\gamma^H\right)(1-\lambda)\left(\alpha_{Ht}^I\alpha_{It} + \alpha_{Ht}^S\right)\sigma_t\sigma_{Jt} + \mu_{Jt}. \end{aligned} \tag{A.16}$$

From the first order condition with respect to c_t in (A.10), it follows that

$$J_t = \frac{w_t}{c_t}, \tag{A.17}$$

so the unknown function J in (A.13) is the household's wealth-consumption ratio. Furthermore, using the consumption goods market clearing condition (23), we obtain

$$\frac{q_t}{J_t} = a - \iota_t = a - \frac{q_t - 1}{\phi}, \tag{A.18}$$

from which we can solve for σ_{Jt} as

$$\sigma_{Jt} = \left(1 + \frac{q}{\phi\,(a-\iota_t)}\right)\sigma_{qt}. \tag{A.19}$$

Solving for the household's portfolio choices is more involved. Specifically, we must determine α_{It}, α_{Ht}^I, α_{Ht}^S, σ_t and π_t simultaneously for each state (e_t, X_t). Because the derivations are lengthy, I describe the full solution algorithm in Appendix B.

Short rate r_t and capital price drift μ_{qt}

We can solve for short rate r_t using the consumption goods market clearing condition. Specifically, from (23), we have

$$dC_t = d\left[\left(a - \iota_t\right) K_t\right].$$

By matching the drift terms of this equation and using (A.16), we obtain

$$\begin{aligned} r_t = \rho + \frac{J_t}{\psi q_t}\left[\left(a-\iota_t\right)\left(\Phi\left(\iota_t\right)-\delta\right) - \frac{q_t}{\phi}\left(\mu_{qt}+\sigma\sigma_{qt}\right)\right] - \frac{1}{\psi}\{\psi\left(1-\lambda\right)\left[\alpha_{Ht}^{I}\alpha_{It}\pi_t + \alpha_{Ht}^{S}\left(\pi_t-\varphi\right)\right] \\ + \frac{1}{2}\frac{\gamma^H-\psi}{1-\psi}\sigma_{Jt}^2 - \gamma^H\sigma_t\sigma_{Jt} - \frac{1}{2}\gamma^H\left(\psi-1\right)\sigma_t^2 + \left[\left(\psi-1\right)\frac{\sigma_t}{\sigma} + \frac{\sigma_{Jt}}{\sigma}\right]\theta X_t\}. \end{aligned} \tag{A.20}$$

From (6), we can obtain the following identity:

$$\pi_t + r_t = \mu_t = \mu_{qt} + \left(\Phi\left(\iota_t\right)-\delta\right) + \sigma\sigma_{qt} + \frac{a-\iota_t}{q_t}. \tag{A.21}$$

Combining (A.20) and (A.21), we can solve for μ_{qt} as

$$\mu_{qt} = \frac{\begin{array}{c}\pi_t + \rho + \frac{J_t}{\psi q_t}\left[\left(a-\iota_t\right)\left(\Phi\left(\iota_t\right)-\delta\right) - \frac{q_t}{\phi}\sigma\sigma_{qt}\right] - \frac{1}{\psi}\{\psi\left(1-\lambda\right)\left[\alpha_{Ht}^{I}\alpha_{It}\pi_t + \alpha_{Ht}^{S}\left(\pi_t-\varphi\right)\right] \\ + \frac{1}{2}\frac{\gamma^H-\psi}{1-\psi}\sigma_{Jt}^2 - \gamma^H\sigma_t\sigma_{Jt} - \frac{1}{2}\gamma^H\left(\psi-1\right)\sigma_t^2 + \left[\left(\psi-1\right)\frac{\sigma_t}{\sigma} + \frac{\sigma_{Jt}}{\sigma}\right]\theta X_t\} - \left[\Phi\left(\iota_t\right) - \delta + \sigma\sigma_{qt} + \frac{a-\iota_t}{q_t}\right]\end{array}}{1 + \frac{J_t}{\psi\phi}}. \tag{A.22}$$

Thus, we can solve for r_t and, in turn, obtain μ_{et} from (A.20) and (A.6).

Close the model

We solve the model numerically by conjecturing a capital price function $q\left(e_t, X_t\right)$ over the state space $\left(e_t, X_t\right)$. Conditional on this conjecture, we compute all endogenous quantities as described above. The model is then closed by updating $q\left(e_t, X_t\right)$ using the PDE

$$0 = e_t\mu_{et}q_t + \mu_{Xt}q_X + \frac{1}{2}\left(e_t\sigma_{et}\right)^2 q_{ee} + \frac{1}{2}\sigma_{Xt}^2 q_{XX} + e_t\sigma_{et}\sigma_{Xt}q_{eX} - q_t\mu_{qt}, \tag{A.23}$$

which follows from Itô's lemma. We iteratively update q until convergence. The PDE (A.23) is solved using a Newton-based iterative scheme known as pseudo-transient continuation.[34]

[34] Gomez (2025) also applies this method, and Kelley and Keyes (1998) illustrate how to construct the numerical scheme.

Proof of Proposition 1

Here I provide a sketch of proof and the intuition is straightforward. Recall that I define a financial crisis as a state in which the capital price q collapses while the risk premium spikes. Therefore, pure financial-market turmoil without a contraction in real cash flows does not qualify. For either $\alpha_{Ht}^S = 0$ or $\varphi = 0$, we can show that the economy on the real side is equivalent to one with a (behaviorally) representative household that holds the entire capital stock. The representative household faces the following optimization problem:

$$V_t = \max_{\{c_s, \alpha_s, \iota_s\}_{s \geq t}} \mathrm{E}_t^H \left[\int_t^\infty f\left(c_s, V_s\right) ds \right],$$

subject to the following dynamic budget constraint

$$\frac{dw_t}{w_t} = \left[r_t + \alpha_t \pi_t^H - \frac{c_t}{w_t} \right] dt + \alpha_t \sigma_t d\omega_t^H,$$

and does not face any portfolio constraints such that $\alpha_t = 1$ in equilibrium. In this equivalent representative-household economy, sentiment risk is priced; however, the financial friction variable e_t no longer appears and is irrelevant for real outcomes. For any fixed X, the triplet $(q_t,\, \mu_t,\, \frac{c_t}{w_t}\,)$ is pinned down by technology, preferences, and beliefs and does not vary with e. Hence there is no pricing impact from the financial friction and no crisis as defined.

When households directly hold (some) capital and face a relative cost $\varphi > 0$, we can show that the real side of the economy is again equivalent to a representative household problem:

$$V_t = \max_{\{c_s, \alpha_s, \iota_s\}_{s \geq t}} \mathrm{E}_t^H \left[\int_t^\infty f\left(c_s, V_s\right) ds \right],$$

subject to the following dynamic budget constraint

$$\frac{dw_t}{w_t} = \left[r_t + \alpha_t \pi_t^H \underbrace{- \left(1 - \lambda\right) \alpha_{Ht}^S \varphi}_{\text{Capital misallocation}} - \frac{c_t}{w_t} \right] dt + \alpha_t \sigma_t d\omega_t^H,$$

and does not face any portfolio constraint such that $\alpha_t = 1$ in equilibrium. Crucially, the financial friction variable e now affects real outcomes indirectly through the capital misallocation wedge $\left(1 - \lambda\right) \alpha_{Ht}^S \varphi$. A tighter intermediary equity-capital constraint—reflected by a lower e—raises the household's direct investment share α_{Ht}^S. As a result, in this representative household economy, real production is further dampened, and future cash flows decline, as reflected in the lower growth rate of wealth. Endogenously, q falls.

It is also important to note that the wedge $\left(1 - \lambda\right) \alpha_{Ht}^S \varphi$ would have been a constant over e if

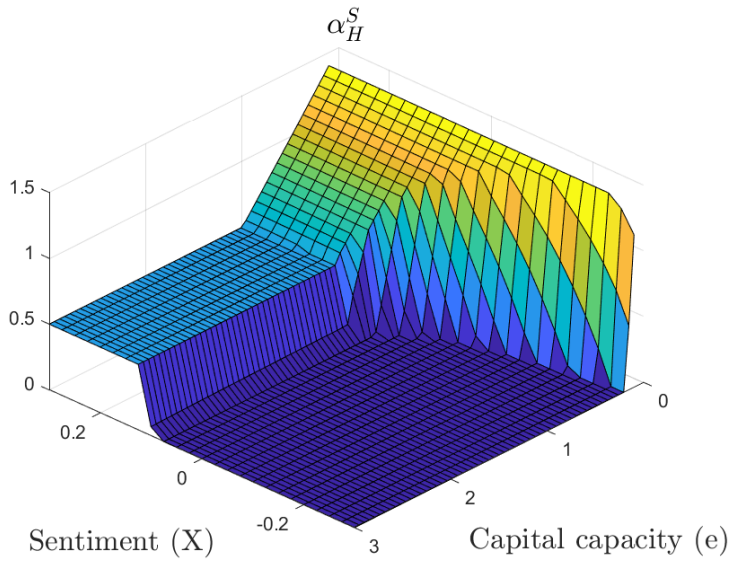


Figure 15. Equilibrium α_{Ht}^S. This figure plots the equilibrium fraction of household wealth invested in the physical capital.

there was no state-dependent equity capital constraint, leading the financial friction to play no role. To demonstrate this, I plot the equilibrium α_{Ht}^S in the actual economy in Figure 15. We can easily see that the binding equity capital constraint leads to the variation of α_{Ht}^S over e such that a decline in e dampens the real production. Otherwise, without state dependence, α_{Ht}^S would be flat in e, so only sentiment X would move q through demand, with no pricing role for the friction e. Thus, the state-dependent constraint is what generates crisis states—spiking risk premia and collapsing capital prices—via the financial accelerator.

Appendix B: Portfolio choices solution algorithm

In this appendix, I provide a detailed solution algorithm for investors' portfolio choices α_{It}, α_{Ht}^I and α_{Ht}^S along with two endogenous asset pricing quantities σ_t and π_t for each state (e_t, X_t). Before presenting the algorithm itself, however, I first develop some preliminary analysis that helps build intuition and clarifies the steps of the procedure.

Portfolio choices analysis

From the above HJB equation (A.10), we can extract each household's portfolio choice problem. Specifically, the household chooses α_{Ht}^I and α_{Ht}^S to

$$\begin{aligned}&\max_{\alpha_{Ht}^I,\alpha_{Ht}^S}\ (1-\lambda)\,\alpha_{Ht}^I\alpha_{It}\pi_t^H+(1-\lambda)\,\alpha_{Ht}^S\left(\pi_t^H-\varphi\right)\\&-\frac{1}{2}\gamma^H\,(1-\lambda)^2\left(\alpha_{Ht}^I\alpha_{It}+\alpha_{Ht}^S\right)^2\sigma_t^2+\frac{\gamma^H-1}{1-\psi}\,(1-\lambda)\left(\alpha_{Ht}^I\alpha_{It}+\alpha_{Ht}^S\right)\sigma_t\sigma_{Jt},\end{aligned}\tag{B.1}$$

subject to the following constraints:

$$0\le\alpha_{Ht}^I\le\overline{\alpha},\tag{B.2}$$

$$\alpha_{Ht}^I\le\frac{1}{1-\lambda}\frac{e_t}{q_t},\tag{B.3}$$

$$\alpha_{Ht}^S\ge 0,\tag{B.4}$$

$$\alpha_{Ht}^I+\alpha_{Ht}^S\le b.\tag{B.5}$$

Let the Lagrange multipliers associated with the constraints $0\le\alpha_{Ht}^I$, $\alpha_{Ht}^I\le\overline{\alpha}$, $\alpha_{Ht}^I\le\frac{1}{1-\lambda}\frac{e_t}{q_t}$, $\alpha_{Ht}^S\ge 0$ and $\alpha_{Ht}^I+\alpha_{Ht}^S\le b$ be denoted by $\lambda_t^{I,1}$, $\lambda_t^{I,2}$, $\lambda_t^{I,3}$, λ_t^S and λ_t^{IS}, respectively. The unconstrained portfolio problem for each household can then be written as:

$$\begin{aligned}&\max_{\alpha_{Ht}^I,\alpha_{Ht}^S}\ (1-\lambda)\,\alpha_{Ht}^I\alpha_{It}\pi_t^H+(1-\lambda)\,\alpha_{Ht}^S\left(\pi_t^H-\varphi\right)\\&-\frac{1}{2}\gamma^H\,(1-\lambda)^2\left(\alpha_{Ht}^I\alpha_{It}+\alpha_{Ht}^S\right)^2\sigma_t^2+\frac{\gamma^H-1}{1-\psi}\,(1-\lambda)\left(\alpha_{Ht}^I\alpha_{It}+\alpha_{Ht}^S\right)\sigma_t\sigma_{Jt}\\&+\lambda_t^{I,1}\alpha_{Ht}^I+\lambda_t^{I,2}\left(\overline{\alpha}-\alpha_{Ht}^I\right)+\lambda_t^{I,3}\left(\frac{1}{1-\lambda}\frac{e_t}{q_t}-\alpha_{Ht}^I\right)+\lambda_t^S\alpha_{Ht}^S+\lambda_t^{IS}\left(b-\left[\alpha_{Ht}^I+\alpha_{Ht}^S\right]\right).\end{aligned}\tag{B.6}$$

Therefore, we can obtain the first order condition with respect to α_{Ht}^I as

$$\begin{aligned}0=&(1-\lambda)\,\pi_t^H-\gamma^H\,(1-\lambda)^2\,\sigma_t^2\left(\alpha_{Ht}^I\alpha_{It}+\alpha_{Ht}^S\right)\\&+\frac{\gamma^H-1}{1-\psi^H}\,(1-\lambda)\,\sigma_t\sigma_{Jt}+\frac{\lambda_t^{I,1}}{\alpha_{It}}-\frac{\lambda_t^{I,2}}{\alpha_{It}}-\frac{\lambda_t^{I,3}}{\alpha_{It}}-\frac{\lambda_t^{IS}}{\alpha_{It}},\end{aligned}\tag{B.7}$$

and the first order condition with respect to α_{Ht}^S is

$$\begin{aligned}0=&(1-\lambda)\left(\pi_t^H-\varphi\right)-\gamma^H\,(1-\lambda)^2\left(\alpha_{Ht}^I\alpha_{It}+\alpha_{Ht}^S\right)\sigma_t^2\\&+\frac{\gamma^H-1}{1-\psi^H}\,(1-\lambda)\,\sigma_t\sigma_{Jt}+\lambda_t^S-\lambda_t^{IS}.\end{aligned}\tag{B.8}$$

Given the two optimality conditions (B.7) and (B.8), we can derive several useful results, stated as lemmas below. These lemmas characterize equilibrium portfolio allocations and will serve as building blocks for the subsequent solution algorithm.

Lemma 1 (**Equilibrium** α_{It})**.** $\alpha_{It} \geq 1$ *in equilibrium.*

Proof of Lemma 1. Given the capital market clearing condition (24) and the identity (25), we obtain

$$\alpha_{It}\alpha_{Ht}^{I} + \alpha_{Ht}^{S} = \frac{1}{1-\lambda}. \tag{B.9}$$

Rearranging gives

$$(\alpha_{It} - 1)\,\alpha_{Ht}^{I} = \frac{1}{1-\lambda} - \left(\alpha_{Ht}^{I} + \alpha_{Ht}^{S}\right).$$

By the leverage constraint (B.5), we know that $\alpha_{Ht}^{I} + \alpha_{Ht}^{S} \leq b$. Hence,

$$(\alpha_{It} - 1)\,\alpha_{Ht}^{I} \geq \frac{1}{1-\lambda} - b,$$

As shown later, $\alpha_{Ht}^{I} > 0$ in equilibrium. Therefore, under the assumption $b \leq \frac{1}{1-\lambda}$, it follows that $\alpha_{It} \geq 1$. □

Lemma 2 (**Equilibrium** α_{Ht}^{I})**.** $\alpha_{Ht}^{I} > 0$ *in equilibrium.*

Proof of Lemma 2. It is not trivial to show that α_{Ht}^{I} is strictly positive in equilibrium, implying that the constraint $\alpha_{Ht}^{I} \geq 0$ never binds and is therefore redundant. Intuitively, when sentiment is very low, households might have an incentive to short sell intermediary equity, so we must rule out this possibility formally.

To establish $\alpha_{Ht}^{I} > 0$, it suffices to show that the alternative case $\alpha_{Ht}^{I} = 0$ cannot hold in equilibrium. Suppose $\alpha_{Ht}^{I} = 0$. Then $\lambda_t^{I,1} \geq 0$ while $\lambda_t^{I,2} = \lambda_t^{I,3} = 0$. Under this assumption, the first-order condition (B.7) reduces to

$$\begin{aligned} 0 = {} & (1-\lambda)\,\pi_t^{H} - \gamma^{H}\,(1-\lambda)^2\,\sigma_t^2\left(\alpha_{Ht}^{I}\alpha_{It} + \alpha_{Ht}^{S}\right) \\ & + \frac{\gamma^{H}-1}{1-\psi^{H}}\,(1-\lambda)\,\sigma_t\sigma_{Jt} + \frac{\lambda_t^{I,1}}{\alpha_{It}} - \frac{\lambda_t^{IS}}{\alpha_{It}}. \end{aligned} \tag{B.10}$$

Combining (B.10) with (B.8) gives

$$0 = (1-\lambda)\,\varphi + \frac{\lambda_t^{I,1}}{\alpha_{It}} - \lambda_t^{S} + \left(1 - \frac{1}{\alpha_{It}}\right)\lambda_t^{IS},$$

which can be rearranged as

$$\lambda_t^S = (1-\lambda)\,\varphi + \frac{\lambda_t^{I,1}}{\alpha_{It}} + \left(1 - \frac{1}{\alpha_{It}}\right)\lambda_t^{IS} > 0$$

Since Lemma 1 established that $\alpha_{It} \geq 1$, it follows that $\lambda_t^S > 0$. By complementary slackness, this implies $\alpha_{Ht}^S = 0$ when $\alpha_{Ht}^I = 0$. Intuitively, if households wish to short sell intermediary equity, they would also want to short sell the relatively more costly physical capital. But if both α_{Ht}^I and α_{Ht}^S are 0, then the capital market cannot clear, contracting equilibrium. Therefore, the case $\alpha_{Ht}^I = 0$ is not feasible, and we conclude $\alpha_{Ht}^I > 0$ in equilibrium. □

Lemma 3 (**Unconstrained** α_{Ht}^I)**.** *If the multipliers* $\lambda_t^{I,2} = \lambda_t^{I,3} = 0$*, so that* $\alpha_{Ht}^I < \min\left\{\overline{\alpha}, \frac{1}{1-\lambda}\frac{e_t}{q_t}\right\}$*, then the equilibrium allocation satisfies* $\alpha_{Ht}^S = 0$*, and the interior solution for* α_{Ht}^I *is given by*

$$\widetilde{\alpha_{Ht}^I} = \frac{1}{(1-\lambda)\,\alpha_{It}}\frac{\pi_t^H}{\gamma^H \sigma_t^2} + \frac{1}{(1-\lambda)\,\alpha_{It}}\frac{\gamma^H - 1}{1-\psi}\frac{\sigma_{Jt}}{\gamma^H \sigma_t}. \tag{B.11}$$

Proof of Lemma 3. In the case that $\lambda_t^{I,2} = \lambda_t^{I,3} = 0$, we can rewrite (B.7) as

$$\begin{aligned} 0 = {} & (1-\lambda)\,\pi_t^H - \gamma^H (1-\lambda)^2 \sigma_t^2 \left(\alpha_{Ht}^I \alpha_{It} + \alpha_{Ht}^S\right) \\ & + \frac{\gamma^H - 1}{1-\psi^H}(1-\lambda)\,\sigma_t \sigma_{Jt} - \frac{\lambda_t^{IS}}{\alpha_{It}}. \end{aligned} \tag{B.12}$$

Combining (B.12) with (B.8), we obtain

$$0 = (1-\lambda)\,\varphi - \lambda_t^S + \left(1 - \frac{1}{\alpha_{It}}\right)\lambda_t^{IS},$$

or equivalently,

$$\lambda_t^S = (1-\lambda)\,\varphi + \left(1 - \frac{1}{\alpha_{It}}\right)\lambda_t^{IS} > 0,$$

which implies that $\alpha_{Ht}^S = 0$. Intuitively, households have no incentive to hold the more costly physical capital when they are still permitted to purchase intermediary equity. Moreover, in this case the leverage constraint cannot bind, since

$$\alpha_{Ht}^I + \alpha_{Ht}^S = \alpha_{Ht}^I \leq \overline{\alpha},$$

where $\overline{\alpha} < b$ under assumption. Therefore, $\lambda_t^{IS} = 0$. Substituting this back into (B.12), we obtain

$$0 = (1-\lambda)\,\pi_t^H - \gamma^H (1-\lambda)^2 \sigma_t^2 \widetilde{\alpha_{Ht}^I} \alpha_{It} + \frac{\gamma^H - 1}{1-\psi^H}(1-\lambda)\,\sigma_t \sigma_{Jt},$$

from which we can solve for the unconstrained solution $\widetilde{\alpha_{Ht}^I}$ as stated in (B.11). □

Lemma 4 (**Equilibrium** α_{Ht}^S)**.** *Households hold capital only if* $\alpha_{Ht}^I = \min\left\{\overline{\alpha}, \frac{1}{1-\lambda}\frac{e_t}{q_t}\right\}$*. In this case,* α_{Ht}^S *can take one of three possible values:* $\alpha_{Ht}^S \in \left\{0, \widetilde{\alpha_{Ht}^S}, b - \min\left\{\overline{\alpha}, \frac{1}{1-\lambda}\frac{e_t}{q_t}\right\}\right\}$*, where*

$$\widetilde{\alpha_{Ht}^S} = \frac{1}{1-\lambda}\frac{\pi_t^H - \varphi}{\gamma^H \sigma_t^2} + \frac{1}{1-\lambda}\frac{\gamma^H - 1}{1-\psi}\frac{\sigma_{Jt}}{\gamma^H \sigma_t} - \min\left\{\overline{\alpha}, \frac{1}{1-\lambda}\frac{e_t}{q_t}\right\}\alpha_{It}. \tag{B.13}$$

Proof of Lemma 4. It is easy to see that $\alpha_{Ht}^S > 0$ only if α_{Ht}^I is constrained by either the equity capital requirement or the investment cap $\alpha_{Ht}^I \leq \overline{\alpha}$. If neither of these constraints bind (so that $\lambda_t^{I,2} = \lambda_t^{I,3} = 0$), then by Lemma 3, we must have $\alpha_{Ht}^S = 0$. Now suppose instead that α_{Ht}^I binds, i.e., $\alpha_{Ht}^I = \min\left\{\overline{\alpha}, \frac{1}{1-\lambda}\frac{e_t}{q_t}\right\}$. In this case, one of the following two equations must hold:

$$\lambda_t^S + \frac{\lambda_t^{I,2}}{\alpha_{It}} = (1-\lambda)\varphi + \left(1 - \frac{1}{\alpha_{It}}\right)\lambda_t^{IS}, \tag{B.14}$$

$$\lambda_t^S + \frac{\lambda_t^{I,3}}{\alpha_{It}} = (1-\lambda)\varphi + \left(1 - \frac{1}{\alpha_{It}}\right)\lambda_t^{IS}. \tag{B.15}$$

(B.14) applies when $\alpha_{Ht}^I = \overline{\alpha}$, while (B.15) applies when $\alpha_{Ht}^I = \frac{1}{1-\lambda}\frac{e_t}{q_t}$. In either case, the complementary slackness conditions imply that λ_t^S can be 0 or positive, and λ_t^{IS} can also be 0 or positive. This implies that α_{Ht}^S can take three possible values: $\alpha_{Ht}^S = 0$ when $\lambda_t^S > 0$, $\alpha_{Ht}^S = \widetilde{\alpha_{Ht}^S}$ when $\lambda_t^S = \lambda_t^{IS} = 0$, or $\alpha_{Ht}^S = b - \min\left\{\overline{\alpha}, \frac{1}{1-\lambda}\frac{e_t}{q_t}\right\}$ when $\lambda_t^{IS} > 0$. Finally, in the interior case with $\lambda_t^S = \lambda_t^{IS} = 0$, the first-order condition (B.8) simplifies to

$$0 = (1-\lambda)\left(\pi_t^H - \varphi\right) - \gamma^H (1-\lambda)^2 \left(\alpha_{Ht}^I \alpha_{It} + \widetilde{\alpha_{Ht}^S}\right)\sigma_t^2 + \frac{\gamma^H - 1}{1-\psi^H}(1-\lambda)\sigma_t \sigma_{Jt},$$

where $\alpha_{Ht}^I = \min\left\{\overline{\alpha}, \frac{1}{1-\lambda}\frac{e_t}{q_t}\right\}$. Therefore, we can solve for the unconstrained $\widetilde{\alpha_{Ht}^S}$ as given in (B.13). □

Solution algorithm for α_{It}**,** α_{Ht}^I **and** α_{Ht}^S

For each state (e_t, X_t), the equilibrium allocations α_{It}, α_{Ht}^I and α_{Ht}^S can be solved jointly by following these steps:

- **Step 1: solve for** e^* **given** X_t

  Given the two constraints on α_{Ht}^I: $\alpha_{Ht}^I \leq \overline{\alpha}$ and $\alpha_{Ht}^I \leq \frac{1}{1-\lambda}\frac{e_t}{q_t}$, it is clear that they cannot bind simultaneously. At any state (e_t, X_t), only one of these constraints may be binding. For any given X_t, define the cutoff capital capacity e^* as the solution to

  $$e^* = (1-\lambda)\overline{\alpha} q\left(e^*, X\right). \tag{B.16}$$

Once we solved e^* from (B.16), we know that for $e_t \leq e^*$, only the equity capital constraint $\alpha^I_{Ht} \leq \frac{1}{1-\lambda}\frac{e_t}{q_t}$ is possible to bind; While for $e_t > e^*$, only the investment cap $\alpha^I_{Ht} \leq \overline{\alpha}$ may bind.

- **Step 2: solve portfolios using guess-and-verify method**

  For a given X and the corresponding cutoff e^* solved from (B.16), we determine α_{It}, α^I_{Ht} and α^S_{Ht} in two steps: first for the region $e_t \leq e^*$, and then for the region $e_t > e^*$.

  ▷ $e_t \leq e^*$:

  For $e_t \leq e^*$, only the equity capital constraint $\alpha^I_{Ht} \leq \frac{1}{1-\lambda}\frac{e_t}{q_t}$ is possible to bind. We begin with the conjecture that this constraint binds together with the leverage constraint. That is, $\alpha^I_{Ht} = \frac{1}{1-\lambda}\frac{e_t}{q_t}$ and $\alpha^S_{Ht} = b - \frac{1}{1-\lambda}\frac{e_t}{q_t}$.[35] Given this conjecture, we can obtain the corresponding α_{It} as follows:

$$\alpha_{It} = \frac{\frac{1}{1-\lambda} - \left(b - \frac{1}{1-\lambda}\frac{e}{q}\right)}{\frac{1}{1-\lambda}\frac{e}{q}},$$

  from which we can also solve for associated σ_t, π_t and σ_{Jt} using (A.9), (A.2) and (A.19), respectively. Given all the solutions under this conjecture, we can now compute the corresponding unconstrained values of household portfolio shares. Specifically, the interior solutions for α^I_{Ht} and α^S_{Ht} are given by

$$\widetilde{\alpha^I_{Ht}} = \frac{1}{(1-\lambda)\,\alpha_{It}}\frac{\pi^H_t}{\gamma^H\sigma^2_t} + \frac{1}{(1-\lambda)\,\alpha_{It}}\frac{\gamma^H-1}{1-\psi}\frac{\sigma_{Jt}}{\gamma^H\sigma_t}, \tag{B.17}$$

$$\widetilde{\alpha^S_{Ht}} = \frac{1}{1-\lambda}\frac{\pi^H_t-\varphi}{\gamma^H\sigma^2_t} + \frac{1}{1-\lambda}\frac{\gamma^H-1}{1-\psi}\frac{\sigma_{Jt}}{\gamma^H\sigma_t} - \frac{1}{1-\lambda}\frac{e_t}{q_t}\alpha_{It}. \tag{B.18}$$

  We then check whether $\widetilde{\alpha^I_{Ht}} \geq \frac{1}{1-\lambda}\frac{e_t}{q_t}$ and $\widetilde{\alpha^S_{Ht}} \geq b - \frac{1}{1-\lambda}\frac{e_t}{q_t}$ both hold. If conditions hold, the conjecture is internally consistent, and we retain all solutions under this case. If either condition fails, the conjecture cannot hold, and we proceed to the next guess. In the second conjecture, we set $\alpha^I_{Ht} = \frac{1}{1-\lambda}\frac{e_t}{q_t}$ and $\alpha^S_{Ht} = \widetilde{\alpha^S_{Ht}}$. In this case, the intermediary share α_{It} must satisfy

$$\frac{\pi^H_t-\varphi}{\gamma^H\sigma^2_t} + \frac{\gamma^H-1}{1-\psi}\frac{\sigma_{Jt}}{\gamma^H\sigma_t} = 1. \tag{B.19}$$

  Solving (B.19) yields α_{It}, from which all other relevant quantities can be computed. We then obtain the unconstrained values of α^I_{Ht} and α^S_{Ht} using (B.17) and (B.18), respectively. If $\widetilde{\alpha^I_{Ht}} \geq \frac{1}{1-\lambda}\frac{e}{q}$ and $0 < \widetilde{\alpha^S_{Ht}} < b - \frac{1}{1-\lambda}\frac{e}{q}$ both hold, we accept the solution under this conjecture. If either condition fails, the conjecture cannot be true, and we proceed to the

[35] Since the numerical scheme starts from the smallest values of e for each X, it is natural to observe that both the equity capital constraint and the leverage constraint bind. The reasoning is twofold: when capital capacity e is low, the equity capital constraint is tight, and at the same time, the high risk premium in the capital market induces households to borrow aggressively in order to hold physical capital.

next case. In the third conjecture, we set $\alpha^I_{Ht} = \frac{1}{1-\lambda}\frac{e}{q}$ and $\alpha^S_{Ht} = 0$ so that the no–short-selling constraint on α^S_{Ht} binds. Under this conjecture, the intermediary portfolio share is given by

$$\alpha_{It} = \frac{q}{e}.$$

With α_{It} determined, we can then solve for all other quantities and compute the unconstrained values of α^I_{Ht} and α^S_{Ht} using (B.17) and (B.18), respectively. If both $\widetilde{\alpha^I_{Ht}} \geq \frac{1}{1-\lambda}\frac{e}{q}$ and $\widetilde{\alpha^S_{Ht}} \leq 0$ hold, we accept the solution under this conjecture. If either condition fails, the conjecture is rejected and we proceed to the final case. In the last conjecture, we set $\alpha^I_{Ht} = \widetilde{\alpha^I_{Ht}}$ and thus we know that $\alpha^S_{Ht} = 0$ according to Lemma 3. In this case, the intermediary portfolio α_{It} must satisfy

$$\frac{\pi^H_t}{\gamma^H \sigma_t^2} + \frac{\gamma^H - 1}{1-\psi^H}\frac{\sigma_{Jt}}{\gamma^H \sigma_t} = 1. \tag{B.20}$$

Once α_{It} is solved from this equation, we can obtain $\alpha^I_{Ht} = \frac{1}{(1-\lambda)\alpha_{It}}$ from the capital market clearing condition (B.9).

▷ $e_t > e^*$:

For the regime $e_t > e^*$, the solution algorithm proceeds in the same way as above. First, we conjecture that $\alpha^I_{Ht} = \overline{\alpha}$ and $\alpha^S_{Ht} = b - \overline{\alpha}$ so that both the equity cap and the leverage constraint bind simultaneously. The intermediary portfolio α_{It} then is given by

$$\alpha_{It} = \frac{\frac{1}{1-\lambda} - (b - \overline{\alpha})}{\overline{\alpha}},$$

With α_{It} determined, we solve for σ_t, π_t and σ_{Jt} using (A.9), (A.2) and (A.19), respectively. Next, the unconstrained household allocations of α^I_{Ht} and α^S_{Ht} are given by

$$\widetilde{\alpha^I_{Ht}} = \frac{1}{(1-\lambda)\,\alpha_{It}}\frac{\pi^H_t}{\gamma^H \sigma_t^2} + \frac{1}{(1-\lambda)\,\alpha_{It}}\frac{\gamma^H - 1}{1-\psi}\frac{\sigma_{Jt}}{\gamma^H \sigma_t}, \tag{B.21}$$

$$\widetilde{\alpha^S_{Ht}} = \frac{1}{1-\lambda}\frac{\pi^H_t - \varphi}{\gamma^H \sigma_t^2} + \frac{1}{1-\lambda}\frac{\gamma^H - 1}{1-\psi}\frac{\sigma_{Jt}}{\gamma^H \sigma_t} - \overline{\alpha}\alpha_{It}. \tag{B.22}$$

If $\widetilde{\alpha^I_{Ht}} \geq \overline{\alpha}$ and $\widetilde{\alpha^S_{Ht}} \geq b - \overline{\alpha}$, the conjecture is consistent and we keep the solutions. Otherwise, this conjecture fails and we move to the next case. In the second conjecture, we assume that $\alpha^I_{Ht} = \overline{\alpha}$ and $\alpha^S_{Ht} = \widetilde{\alpha^S_{Ht}}$. Under this conjecture, the intermediary portfolio α_{It} must satisfy (B.19). Once α_{It} is obtained, we can compute all other quantities and then solve for the unconstrained values of α^I_{Ht} and unconstrained α^S_{Ht} using (B.21) and (B.22), respectively. If $\widetilde{\alpha^I_{Ht}} \geq \overline{\alpha}$ and $0 < \widetilde{\alpha^S_{Ht}} < b - \overline{\alpha}$, the conjecture is consistent and we retain all solutions. If either condition fails, the conjecture is rejected and we proceed to the next case. In the third

conjecture, we assume that $\alpha_{Ht}^I = \overline{\alpha}$ and $\alpha_{Ht}^S = 0$ so that the no–short-selling constraint on α_{Ht}^S binds. Under this conjecture, the intermediary portfolio is

$$\alpha_{It} = \frac{1}{(1-\lambda)\,\overline{\alpha}}.$$

We can then compute all other relevant quantities and obtain the unconstrained values $\widetilde{\alpha_{Ht}^I}$ and $\widetilde{\alpha_{Ht}^S}$. If both $\widetilde{\alpha_{Ht}^I} \geq \overline{\alpha}$ and $\widetilde{\alpha_{Ht}^S} \leq 0$ hold, the conjecture is consistent and we keep the solutions. If either condition fails, the conjecture is rejected and we proceed to the final case. In the last case, we set $\alpha_{Ht}^I = \widetilde{\alpha_{Ht}^I}$ and thus $\alpha_{Ht}^S = 0$ according to Lemma 3. In this situation, the intermediary portfolio α_{It} is determined by (B.20). Once α_{It} is solved, the household's position α_{Ht}^I follows directly from the capital market clearing condition (B.9): $\alpha_{Ht}^I = \frac{1}{(1-\lambda)\alpha_{It}}$.

Appendix C: Comments on HK-2019 and Maxted-2024

In this part of the Appendix, I solve the models of He and Krishnamurthy (2019) (HK-2019 hereafter) and Maxted (2024) (Maxted-2024 hereafter) in a general equilibrium setting and show that the pricing impact of financial frictions disappears in the crisis region. Put differently, once embedded in general equilibrium, their frameworks lose the feature of collapsing asset prices during crises.

This result underscores an important limitation: the amplification mechanisms highlighted in their original setups do not generate sharp price declines. This motivates the search for alternative and realistic channels that can deliver collapsing asset prices in crisis states—such as the fire sales from financial intermediaries to households emphasized in this paper and in Brunnermeier and Sannikov (2014).

C.1 HK-2019 in general equilibrium

Relative to HK-2019, I adopt all of their assumptions except for three main changes:

1. Single risky asset: There is only productive capital as the risky asset. I eliminate housing capital for simplicity.[36]

2. Endogenous risk-free rate: The instantaneous risk-free rate is determined in general equilibrium.

[36] One can show that this result—that financial frictions have no pricing impact in general equilibrium—continues to hold even when housing capital is introduced alongside productive capital.

3. No reflection barrier: I do not assume a reflection barrier when e becomes too low. [37]

In this setup, the state variable e_t evolves according to

$$\frac{de_t}{e_t} = \mu_{et}dt + \sigma_{et}d\omega_t,$$

where

$$\begin{aligned}\mu_{et} &= r_t + \alpha_t\pi_t - \eta - (\iota_t - \delta) + \sigma^2 - \alpha_t\sigma_t\sigma,\\ \sigma_{et} &= \alpha_t\sigma_t - \sigma.\end{aligned}$$

The equilibrium risk premium $\pi_t = \gamma^I \alpha_t \sigma_t^2$ where $\alpha_t = \frac{1}{\min\left\{\frac{e_t}{q_t},(1-\lambda)\right\}}$. Moreover, asset return volatility satisfies $\sigma_t = \frac{1-\frac{q_e e_t}{q_t}}{1-\frac{q_e e_t}{q_t}\alpha_t}\sigma$.

In HK-2019, households have CRRA preferences, and their optimization problem is

$$V_t \coloneqq \max_{c_t,\alpha_{Ht}} \mathrm{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{c_s^{1-\gamma^H}}{1-\gamma^H}ds\right],$$

subject to the dynamic budget constraint

$$\frac{dw_t}{w_t} = \left[r_t + (1-\lambda)\,\alpha_{Ht}\alpha_t\pi_t - \frac{c_t}{w_t}\right]dt + (1-\lambda)\,\alpha_{Ht}\alpha_t\sigma_t d\omega_t,$$

and portfolio constraints

$$\begin{aligned}E_t &= \alpha_{Ht}\,(1-\lambda)\,w_t \le \mathcal{E}_t,\\ 0 &\le \alpha_{Ht} \le 1,\end{aligned}$$

where α_{Ht} denotes the fraction of household wealth invested in intermediary equity. The Hamilton–Jacobi–Bellman equation is

$$\rho V_t = \max_{c_t,\alpha_{Ht}} \frac{\mathrm{E}_t\left[dV_t\right]}{dt} + \frac{c_t^{1-\gamma^H}}{1-\gamma^H}.$$

Conjecturing a value function of the form

$$V_t = \frac{w_t^{1-\gamma^H}}{1-\gamma^H}J_t^{\gamma^H},$$

we obtain the following ODE:

[37] Financial crises and sharp declines in asset prices should not be artifacts of arbitrary reflection barriers. By eliminating this assumption, I allow the dynamics of the model to fully reflect economic forces.

$$0 = \frac{1-\gamma^H}{\gamma^H}\left[r_t + (1-\lambda)\,\alpha_{Ht}\alpha_t\pi_t\right] - \frac{1}{2}\left(1-\gamma^H\right)(1-\lambda)^2\,\alpha_{Ht}^2\alpha_t^2\sigma_t^2$$
$$+ \frac{1}{2}\left(\gamma^H-1\right)\sigma_{Jt}^2 + \left(1-\gamma^H\right)(1-\lambda)\,\alpha_{Ht}\alpha_t\sigma_t\sigma_{Jt} + J_t^{-1} - \frac{\rho}{\gamma^H} + \mu_{Jt}, \tag{C.1}$$

We can further show that $J_t = \frac{w_t}{c_t}$ and $\sigma_{Jt} = \left(1 + \frac{q_t^2}{\phi[a-\Phi(\iota_t)]}\right)\sigma_{qt}$. From the consumption goods market clearing condition, we solve for the general equilibrium short rate

$$r_t = \rho + \gamma^H\frac{J_t}{q_t}\left[\left(\frac{q_t-1}{\phi}\right)(a-\Phi(\iota_t)) - \frac{q_t^2}{\phi}\left[\mu_{qt} + \frac{1}{2}\sigma_{qt}^2 + \sigma\sigma_{qt}\right]\right]$$
$$- \left\{(1-\lambda)\,\alpha_{Ht}\alpha_t\pi_t + \frac{\gamma^H\left(1+\gamma^H\right)}{2}\sigma_{Jt}^2 - \left(\gamma^H\right)^2(1-\lambda)\,\alpha_{Ht}\alpha_t\sigma_t\sigma_{Jt} - \frac{\gamma^H\left(1-\gamma^H\right)}{2}(1-\lambda)^2\,\alpha_{Ht}^2\alpha_t^2\sigma_t^2\right\}. \tag{C.2}$$

By definition,

$$\pi_t + r_t = \mu_t = \mu_{qt} + \iota_t - \delta + \sigma\sigma_{qt} + \frac{a-\Phi(\iota_t)}{q_t}.$$

Combining the above two equations yields

$$\mu_{qt} = \frac{\begin{array}{c}\pi_t + \rho + \gamma^H\frac{J_t}{q_t}\left[\left(\frac{q_t-1}{\phi}\right)(a-\Phi(\iota_t)) - \frac{q_t^2}{\phi}\left[\frac{1}{2}\sigma_{qt}^2 + \sigma\sigma_{qt}\right]\right] \\ - \left\{(1-\lambda)\,\alpha_{Ht}\alpha_t\pi_t + \frac{\gamma_H\left(1+\gamma^H\right)}{2}\sigma_{Jt}^2 - \left(\gamma^H\right)^2(1-\lambda)\,\alpha_{Ht}\alpha_t\sigma_t\sigma_{Jt} - \frac{\gamma^H\left(1-\gamma^H\right)}{2}(1-\lambda)^2\,\alpha_{Ht}^2\alpha_t^2\sigma_t^2\right\} \\ - \left[\iota_t - \delta + \sigma\sigma_{qt} + \frac{a-\Phi(\iota_t)}{q_t}\right]\end{array}}{1 + \frac{\gamma^H J_t q_t}{\phi}}, \tag{C.3}$$

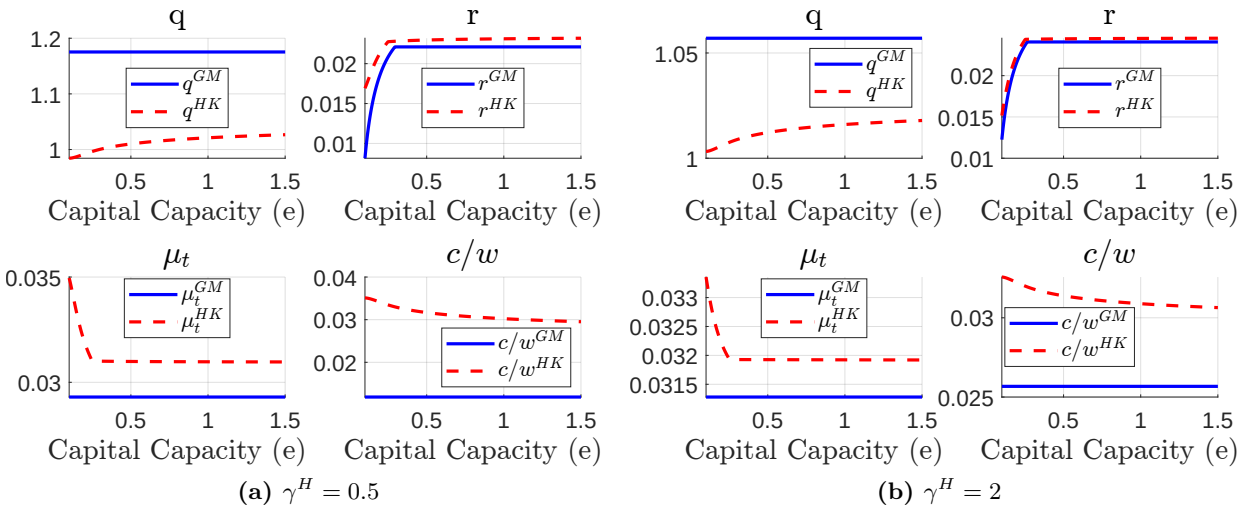
from which we can further solve for r_t and μ_{et}. The model is then closed with the following ODE

$$0 = e_t\mu_{et}q_e + \frac{1}{2}\left(e_t\sigma_{et}\right)^2 q_{ee} - q_t\mu_{qt}.$$

Remark 5 (Short rate assumption in HK-2019). In HK-2019, the short rate is defined as

$$r_t^{HK} = \rho + \gamma^H\frac{\mathrm{E}_t\left[\frac{dc_t}{c_t}\right]}{dt} - \frac{\gamma^H\left(1+\gamma^H\right)}{2}\frac{\mathrm{Var}_t\left[\frac{dc_t}{c_t}\right]}{dt}. \tag{C.4}$$

This coincides with the general equilibrium short rate in (C.2) only when portfolio constraints are slack. Formally, r_t^{HK} is the minus-drift term of the household SDF, $\xi_t = e^{-\rho t}c_t^{-\gamma^H}$, and therefore represents the "general equilibrium" short rate in a fictitious complete market. However, in the actual incomplete



(a) $\gamma^H = 0.5$ **(b)** $\gamma^H = 2$

Figure C1. Endogenous quantities in GE and PE. This figure compares endogenous quantities in a general equilibrium model and the partial-equilibrium framework of HK-2019, which imposes an exogenous short-rate rule for r_t. The left (right) panel reports results for EIS = 2 (EIS = 0.5) for both models.

market with financial frictions, the drift of the household SDF corresponds to a shadow short rate, not the true equilibrium short rate. When constraints bind, the shadow short rate r_t^{HK} deviates from the actual general equilibrium short rate in (C.2). Alternatively, denoting the risk-free bond price by B_t, the standard Euler equation $\mathrm{E}_t\left[\frac{\xi_s}{\xi_t}\frac{B_s}{B_t}\right] = 1$ no longer holds in the constrained economy. When the intermediary's capital constraint binds, it becomes $\mathrm{E}_t\left[\frac{\xi_s}{\xi_t}\frac{B_s}{B_t}\right] < 1$.[38] Therefore, the equilibrium solved in HK-2019 can be viewed as a partial equilibrium, since it relies on the shadow short rate r_t^{HK} rather than the true general equilibrium short rate (C.2).

General vs. Partial Equilibrium

To illustrate the difference between the general equilibrium (as in my model) and the partial equilibrium approach of HK-2019, I solve the model numerically using a finite difference method under the following parameter values: $\gamma^I = 2$, $\gamma^H \in \{0.5, 2\}$, $\lambda = 0.75$, $\eta = 0.15$, $\sigma = 0.03$, $\delta = 0.1$, $\phi = 10$, $a = 0.133$, $\rho = 0.02$.[39]

[38] Specifically, in this simplified version of HK-2019, two constraints can affect the shadow short rate r_t^{HK}: the pure borrowing constraint $\alpha_{Ht} \leq 1$ and and the equity capital constraint $\alpha_{Ht} \leq \frac{1}{1-\lambda}\frac{e_t}{q_t}$. Chabakauri (2013, 2015) provide detailed analyses of how fictitious complete markets can be constructed from constrained economies using the duality approach introduced by Cvitanić and Karatzas (1992). Chapter 14 of Dumas and Luciano (2017) also offers a detailed explanation of how to solve models in incomplete markets.

[39] To address concerns about the role of the elasticity of intertemporal substitution (EIS), I consider two different values of γ^H. In addition, to better visualize the difference between the two equilibria, I set the investment adjustment cost to $\phi = 10$, which generates more variation in the solutions. The qualitative results remain unchanged under the baseline calibration $\phi = 3$ used in HK-2019.

As shown in Figure (C1), unlike the partial-equilibrium framework of HK-2019, the binding financial friction has no pricing impact in general equilibrium: asset prices do not collapse when the constraint binds. The mechanism is straightforward: without a specific short-rate rule, the endogenous short rate r_t falls sharply as the equity-capital constraint tightens. This decline offsets the simultaneous rise in the risk premium π_t. Since the total expected return $\mu_t = r_t + \pi_t$ remains essentially unchanged, the capital price q_t is stabilized rather than collapsing. This outcome reflects the macro-finance analogue of the Modigliani–Miller theorem, as discussed in Section 4.3.

C.2 Maxted-2024 in general equilibrium

I now show that once Maxted (2024) is embedded in general equilibrium, the pricing impact of the financial friction vanishes: capital prices do not collapse when the equity-capital constraint binds. To keep the mapping transparent, I retain Maxted's assumptions and notation while make the following changes:

1. The economy contains only productive capital as the risky asset, with no housing capital;
2. The instantaneous risk-free rate is determined endogenously in general equilibrium;
3. No reflection barrier is imposed when e is low.

In this setup, with diagnostic expectations, the state variable e_t evolves as

$$\frac{d\widehat{e_t}}{e_t} = \widehat{\mu_{et}}dt + \widehat{\sigma_{et}}d\omega_t,$$

where

$$\begin{aligned}\widehat{\mu_{et}} &= r_t + \alpha_t\widehat{\pi}_t - \eta - [\iota_t - \delta + \theta\mathcal{I}_t] + \sigma^2 - \alpha_t\sigma\widehat{\sigma}_t,\\ \widehat{\sigma_{et}} &= \alpha_t\widehat{\sigma}_t - \sigma.\end{aligned}$$

The sentiment variable $\mathcal{I}_t$ evolves as

$$\frac{d\widehat{\mathcal{I}_t}}{\mathcal{I}_t} = -\kappa dt.$$

The perceived risk premium under diagnostic expectation is $\widehat{\pi}_t = \gamma^I\alpha_t{\widehat{\sigma}_t}^2$ where $\alpha_t = \frac{1}{\min\left\{\frac{e}{q}, 1-\lambda\right\}}$. In addition, we can show that $\widehat{\sigma}_t = \sigma + \widehat{\sigma_{qt}} = \frac{1-\frac{q_e e_t}{q_t}}{1-\frac{q_e e_t \alpha_t}{q_t}}\sigma$.

In Maxted-2024, households have CRRA preferences and solve

$$V_t \coloneqq \max_{c_t, \alpha_{Ht}} \widehat{\mathrm{E}_t}\left[\int_t^\infty e^{-\rho(s-t)}\frac{c_s^{1-\gamma^H}}{1-\gamma^H}ds\right],$$

subject to the dynamic budget constraint

$$\frac{dw_t}{w_t} = \left[r_t + (1-\lambda)\,\alpha_{Ht}\alpha_t\widehat{\pi}_t - \frac{c_t}{w_t}\right]dt + (1-\lambda)\,\alpha_{Ht}\alpha_t\widehat{\sigma}_t d\omega_t,$$

and portfolio constraints

$$\begin{aligned} &0 \leq \alpha_{Ht} \leq 1, \\ &(1-\lambda)\,\alpha_{Ht} w_t \leq \mathcal{E}_t, \end{aligned}$$

where α_{Ht} denotes the share of household wealth invested in intermediary equity.

Conjecturing $V_t = \frac{w_t^{1-\gamma^H}}{1-\gamma^H} J_t^{\gamma^H}$, we can obtain the following equation

$$\begin{aligned} 0 = &\frac{1}{J_t} - \frac{\rho}{\gamma^H} + \frac{1-\gamma^H}{\gamma^H}\left[r_t + (1-\lambda)\,\alpha_{Ht}\alpha_t\widehat{\pi}_t\right] \\ &- \frac{1}{2}\left(1-\gamma^H\right)(1-\lambda)^2\,\alpha_{Ht}^2\alpha_t^2\widehat{\sigma}_t{}^2 + \frac{1}{2}\left(\gamma^H - 1\right)\widehat{\sigma_{Jt}}{}^2 \\ &+ \left(1-\gamma^H\right)(1-\lambda)\,\alpha_{Ht}\alpha_t\widehat{\sigma}_t\widehat{\sigma_{Jt}} + \widehat{\mu_{Jt}}, \end{aligned} \tag{C.5}$$

and it follows that $J_t = \frac{w_t}{c_t}$ and $\widehat{\sigma_{Jt}} = \left(1 + \frac{q_t^2}{\phi(a-\Phi(\iota_t))}\right)\widehat{\sigma_{qt}}$ in equilibrium.

We can solve for r_t using the consumption-goods market clearing condition as

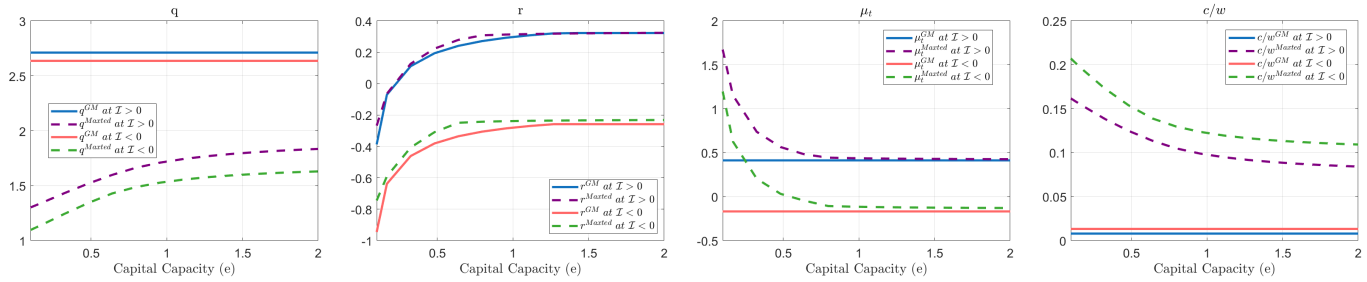
$$\begin{aligned} r_t = &\rho + \gamma^H\frac{J_t}{q_t}\left[(\iota_t - \delta + \theta\mathcal{I}_t)\,(a - \Phi(\iota_t)) - \frac{q_t^2}{\phi}\left(\widehat{\mu_{qt}} + \frac{1}{2}\widehat{\sigma_{qt}}{}^2 + \sigma\widehat{\sigma_{qt}}\right)\right] \\ &- \gamma^H\left[\frac{(1-\lambda)\,\alpha_{Ht}\alpha_t\widehat{\pi}_t}{\gamma^H} + \frac{\left(1+\gamma^H\right)\widehat{\sigma_{Jt}}{}^2}{2} - \gamma^H(1-\lambda)\,\alpha_{Ht}\alpha_t\widehat{\sigma}_t\widehat{\sigma_{Jt}} - \frac{\left(1-\gamma^H\right)(1-\lambda)^2\,\alpha_{Ht}^2\alpha_t^2\widehat{\sigma}_t{}^2}{2}\right]. \end{aligned} \tag{C.6}$$

By definition, the expected return under diagnostic expectation satisfies

$$\widehat{\pi}_t + r_t = \widehat{\mu_t} = \widehat{\mu_{qt}} + [\iota_t - \delta + \theta\mathcal{I}_t] + \sigma\widehat{\sigma_{qt}} + \frac{a - \Phi(\iota_t)}{q_t}.$$

Combining the above two equations yields

$$\begin{aligned} &\widehat{\pi}_t + \rho + \gamma^H\tfrac{J_t}{q_t}\left[(\iota_t - \delta + \theta\mathcal{I}_t)\,(a - \Phi(\iota_t)) - \tfrac{q_t^2}{\phi}\left(\tfrac{1}{2}\widehat{\sigma_{qt}}{}^2 + \sigma\widehat{\sigma_{qt}}\right)\right] \\ &-\gamma^H\left[\tfrac{(1-\lambda)\alpha_{Ht}\alpha_t\widehat{\pi}_t}{\gamma^H} + \tfrac{\left(1+\gamma^H\right)\widehat{\sigma_{Jt}}{}^2}{2} - \gamma^H(1-\lambda)\,\alpha_{Ht}\alpha_t\widehat{\sigma}_t\widehat{\sigma_{Jt}} - \tfrac{\left(1-\gamma^H\right)(1-\lambda)^2\alpha_{Ht}^2\alpha_t^2\widehat{\sigma}_t{}^2}{2}\right] \\ \widehat{\mu_{qt}} = &\frac{-\left[\iota_t - \delta + \theta\mathcal{I}_t + \sigma\widehat{\sigma_{qt}} + \frac{a-\Phi(\iota_t)}{q_t}\right]}{1 + \frac{\gamma^H J_t q_t}{\phi}}, \end{aligned} \tag{C.7}$$



(a) q comparisons **(b)** r comparisons **(c)** μ_t comparisons **(d)** $\frac{c}{w}$ comparisons

Figure C2. Endogenous quantities in GE vs. PE for EIS > 1. This figure compares endogenous quantities in the general-equilibrium model and the partial-equilibrium benchmark of Maxted-2024 under high- and low-sentiment regimes, for EIS > 1.

from which we can further solve for r_t and $\widehat{\mu_{et}}$. The model is then closed with the PDE

$$0 = \left(e_t \widehat{\mu_{et}}\right) q_e + \frac{1}{2}\left(e_t \widehat{\sigma_{et}}\right)^2 q_{ee} - \kappa \mathcal{I}_t q_{\mathcal{I}} - q_t \widehat{\mu_{qt}}. \tag{C.8}$$

Remark 6 (Short rate assumption in Maxted-2024). Same as in HK-2019, Maxted-2024 assumes that the equilibrium short rate r_t takes the form

$$r_t^{Maxted} = \rho + \gamma^H \frac{\widehat{\mathrm{E}_t}\left[\frac{d\widehat{c_t}}{c_t}\right]}{dt} - \frac{\gamma^H\left(1+\gamma^H\right)}{2}\frac{\widehat{\mathrm{Var}_t}\left[\frac{d\widehat{c_t}}{c_t}\right]}{dt}. \tag{C.9}$$

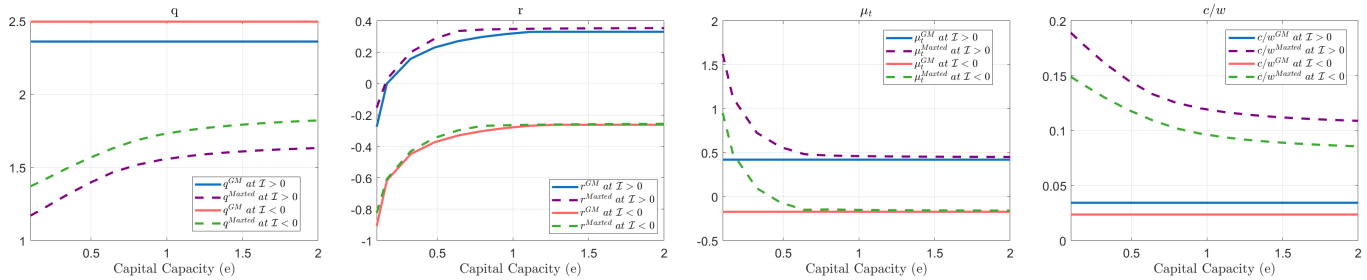
This expression coincides with the general equilibrium short rate in (C.6) when constraints are slack. Formally, r_t^{Maxted} is implied by the minus-drift term of the household's SDF, $\xi_t = e^{-\rho t} c_t^{-\gamma^H}$. However, as discussed in Remark (5), the short rate derived from the SDF is in fact a shadow short rate. When portfolio constraints bind, this shadow rate no longer coincides with the actual general equilibrium short rate (C.6). Therefore, the equilibrium solved in Maxted-2024 can be viewed as a partial equilibrium.

General vs. Partial Equilibrium

To show the difference between the general equilibrium and the partial equilibrium setup in Maxted-2024, I solve the model numerically using a finite difference method under the following parameter values: $\gamma^I = 2$, $\gamma^H \in \{0.9, 1.1\}$, $\lambda = 0.5$, $\eta = 0.126$, $\sigma = 0.15$, $\delta = 0.1$, $\phi = 15$, $a = 1/3$, $\rho = 0.02$, $\kappa = 0.116$, $\theta = 0.2$.[40]

As we can see from both Figure C2 and Figure C3, unlike in the partial equilibrium of Maxted-2024, capital price q does not collapse in the constraint binding region in general equilibrium. The mechanism

[40] I also consider two different values of the elasticity of intertemporal substitution (EIS) to demonstrate how sentiment affects the capital price q. In addition, I deviate slightly from the parameter values in Maxted-2024 to generate clearer visualizations.



(a) q comparisons **(b)** r comparisons **(c)** μ_t comparisons **(d)** $\frac{c}{w}$ comparisons

Figure C3. Endogenous quantities in GE vs. PE for EIS < 1. This figure compares endogenous quantities in the general-equilibrium model and the partial-equilibrium benchmark of Maxted-2024 under high- and low-sentiment regimes, for EIS < 1.

mirrors the HK case: when the equity-capital constraint binds, the endogenous short rate r falls enough to offset the rise in the risk premium π_t, leaving μ_t and thus q unchanged. Moreover, as shown in Figure C2 (C3), capital price q increases (decreases) in sentiment $\mathcal{I}$ when EIS > 1 (EIS < 1) as expected.

Appendix D: The impact of household sentiment on financing

In this part of appendix, I explore the impact of household sentiment on the financing for financial intermediaries through a rigorous analysis. To simplify things, I assume that $\gamma^H = \frac{1}{\psi}$ such that the household's recursive utility reduces to a standard CRRA utility. It turns out the reduced form delivers the same insights on the financing channel of household sentiment.

Given the dynamics of instantaneous return on intermediary equity (8), we know that the price of risk delivered by the intermediary equity is $\frac{\pi_t}{\sigma_t} = \gamma^I \alpha_{It} \sigma_t$. Under the CRRA utility, we know that the household's optimization problem is as follows

$$V_t = \max_{\{\alpha^I_{Hs}, \alpha^S_{Hs}, c_s, \iota^H_s\}} \mathrm{E}^H_t \left[\int_t^\infty e^{-\rho(s-t)} \frac{c_s^{1-\gamma^h}}{1-\gamma^h} ds \right],$$

subject to the following dynamic budget constraint

$$\begin{aligned} \frac{dw_t}{w_t} = & \left[r_t + (1-\lambda)\, \alpha^I_{Ht} \alpha_{It} \pi^H_t + (1-\lambda)\, \alpha^S_{Ht} \left(\pi^H_t - \varphi \right) - \frac{c_t}{w_t} \right] dt \\ & + (1-\lambda) \left(\alpha^I_{Ht} \alpha_{It} + \alpha^S_{Ht} \right) \sigma_t d\omega^H_t, \end{aligned}$$

and the following portfolio constraints:

$$\begin{aligned}
&\alpha_{Ht}^{I} \leq \overline{\alpha}, \\
&\alpha_{Ht}^{I} \leq \frac{1}{1-\lambda}\frac{e_t}{q_t}, \\
&\alpha_{Ht}^{S} \geq 0, \\
&\alpha_{Ht}^{I} + \alpha_{Ht}^{S} \leq b.
\end{aligned}$$

Therefore, we can obtain the following HJB equation

$$\rho V_t = \max_{c_t, \alpha_{Ht}^{I}, \alpha_{Ht}^{S}, \iota_t^H} \frac{c_t^{1-\gamma^H}}{1-\gamma^H} + \frac{\mathrm{E}_t^H\left[dV_t\right]}{dt}, \tag{D.1}$$

subject to the portfolio constraints. Let me now conjecture that

$$V_t = e^{\rho t}\xi_{ht} w_t,$$

and I postulate the dynamics of ξ_{ht} as follows:

$$\frac{d\xi_{ht}}{\xi_{ht}} = -\mu_{\xi_h t}dt - \varsigma_t^h d\omega_t^H. \tag{D.2}$$

As a result, given (D.1), we can show that

$$\begin{aligned}
0 = &\max_{c_t, \alpha_{Ht}^{I}, \alpha_{Ht}^{S}, \iota_t^H} \frac{c_t^{1-\gamma^h}}{1-\gamma^h} - V_t\mu_{\xi_h t} + V_t\left[r_t + \left(1-\lambda\right)\alpha_{Ht}^{I}\alpha_{It}\pi_t^H + \left(1-\lambda\right)\alpha_{Ht}^{S}\left(\pi_t^H - \varphi\right) - \frac{c_t}{w_t}\right] \\
&- V_t\varsigma_t^h\left(1-\lambda\right)\left(\alpha_{Ht}^{I}\alpha_{It} + \alpha_{Ht}^{S}\right)\sigma_t,
\end{aligned}$$

subject to the portfolio constraints. We can then introduce Lagrangian multipliers $V_t\lambda_t^{I,1}$, $V_t\lambda_t^{I,2}$, $V_t\lambda_t^{S}$ and $V_t\lambda_t^{IS}$ to transform the above constrained optimization problem into the following unconstrained optimization problem:

$$\begin{aligned}
0 = &\max_{c_t, \alpha_{Ht}^{I}, \alpha_{Ht}^{S}, \iota_t^H} \frac{c_t^{1-\gamma^h}}{1-\gamma^h} - V_t\mu_{\xi_h t} + V_t\left[r_t + \left(1-\lambda\right)\alpha_{Ht}^{I}\alpha_{It}\pi_t^H + \left(1-\lambda\right)\alpha_{Ht}^{S}\left(\pi_t^H - \varphi\right) - \frac{c_t}{w_t}\right] \\
&- V_t\varsigma_t^h\left(1-\lambda\right)\left(\alpha_{Ht}^{I}\alpha_{It} + \alpha_{Ht}^{S}\right)\sigma_t + V_t\lambda_t^{I,1}\left(\overline{\alpha} - \alpha_{Ht}^{I}\right) \\
&+ V_t\lambda_t^{I,2}\left(\frac{1}{1-\lambda}\frac{e_t}{q_t} - \alpha_{Ht}^{I}\right) + V_t\lambda_t^{S}\alpha_{Ht}^{S} + V_t\lambda_t^{IS}\left(b - \left[\alpha_{Ht}^{I} + \alpha_{Ht}^{S}\right]\right).
\end{aligned}$$

From the first order condition with respect to c_t, we have

$$\xi_{ht} = e^{-\rho t}c_t^{-\gamma^h},$$

such that we know that the (discounted) marginal value of wealth ξ_{ht} is the SDF of the household, which is a standard result for the time-additive CRRA utility. Therefore, we know that ς_t^h in (D.2) is the household's required price of risk.

From the first order conditions with respect to α_{Ht}^I and α_{Ht}^S, we can obtain the follows:

$$\frac{\pi_t}{\sigma_t} + \frac{\theta X_t}{\sigma} = \varsigma_t^h + \frac{\lambda_t^{I,1}}{(1-\lambda)\,\alpha_{It}\sigma_t} + \frac{\lambda_t^{I,2}}{(1-\lambda)\,\alpha_{It}\sigma_t} + \frac{\lambda_t^{IS}}{(1-\lambda)\,\alpha_{It}\sigma_t}, \tag{D.3}$$

$$\frac{\pi_t - \varphi}{\sigma_t} + \frac{\theta X_t}{\sigma} = \varsigma_t^h - \frac{\lambda_t^{S}}{(1-\lambda)\,\sigma_t} + \frac{\lambda_t^{IS}}{(1-\lambda)\,\sigma_t}. \tag{D.4}$$

We can further rewrite the above two identities as follows

$$\underbrace{\gamma^I \alpha_{It}\sigma_t + \overbrace{\frac{\theta X_t}{\sigma}}^{\text{Belief distortion}}}_{\substack{\text{Household perceived price of risk} \\ \text{of intermediary equity}}} = \underbrace{\varsigma_t^h}_{\text{Household required price of risk}} + \breve{\lambda}_t^{I,1} + \breve{\lambda}_t^{I,2} + \breve{\lambda}_t^{IS}, \tag{D.5}$$

$$\underbrace{\gamma^I \alpha_{It}\sigma_t \overbrace{-\frac{\varphi}{\sigma_t}}^{\text{Capital displacement}} + \overbrace{\frac{\theta X_t}{\sigma}}^{\text{Belief distortion}}}_{\substack{\text{Household perceived price of risk} \\ \text{of capital asset}}} = \underbrace{\varsigma_t^h}_{\text{Household required price of risk}} - \mathring{\lambda}_t^{S} + \mathring{\lambda}_t^{IS}. \tag{D.6}$$

in which $\breve{\lambda}_t^{I,1} \geq 0$, $\breve{\lambda}_t^{I,2} \geq 0$, $\breve{\lambda}_t^{IS} \geq 0$, $\mathring{\lambda}_t^{S} \geq 0$ and $\mathring{\lambda}_t^{IS} \geq 0$ are all scaled Lagrangian multipliers.

Let's first look at the first identity (D.5), which determines the financing between financial intermediaries and households. Suppose we are now at the high sentiment regime such that $X_t > 0$. In this case, (D.5) implies that $\gamma^I \alpha_{It}\sigma_t + \frac{\theta X_t}{\sigma} > \varsigma_t^h$, where $\gamma^I \alpha_{It}\sigma_t$ is the actual price of risk delivered by intermediary equity. The additional term $\frac{\theta X_t}{\sigma}$ reflects belief distortion under optimism, meaning that households perceive a higher price of risk from holding intermediary equity than they require. As a result, intermediary equity becomes particularly attractive to optimistic households, who then invest aggressively in it. Consequently, equity financing for intermediaries becomes excessively easy in high-sentiment states. This causes the intermediary equity constraint to bind more readily when sentiment is elevated, generating a strictly positive Lagrange multiplier and restoring the equality in the condition. Conversely, in the low sentiment regime such that $X < 0$, we typically have $\gamma^I \alpha_{It}\sigma_t + \frac{\theta X_t}{\sigma} < \varsigma_t^h$. In this situation, pessimistic households perceive a lower price of risk from intermediary equity than they require, making equity investment unattractive. Consequently, households become unwilling to supply equity capital to intermediaries, rendering equity financing prohibitively costly. Financial intermediaries are therefore forced to borrow aggressively in order to fund investment, which pushes up their leverage

α_{It} until $\gamma^I \alpha_{It}\sigma_t$ rises sufficiently to meet households' required compensation, leading to unconstrained risk sharing in normal periods and we have $\breve{\lambda}_t^{I,1} = \breve{\lambda}_t^{I,2} = \breve{\lambda}_t^{IS} = 0$.

In addition, as we can see from the second identity (D.6), in the high sentiment regime $X > 0$, we typically have $\gamma^I \alpha_{It}\sigma_t - \frac{\varphi}{\sigma_t} + \frac{\theta X_t}{\sigma} > \varsigma_t^h$. Therefore, equity households find it attractive to enter the physical capital market and thus they borrow from debt households until the borrowing constraint binds or $\mathring{\lambda}_t^{IS} > 0$, restoring the equality. Consequently, the net lending from the household sector to financial intermediaries is reduced and thus generates lower intermediary leverage α_{It} in the high sentiment regime. On the other hand, when household sentiment X is low, $\gamma^I \alpha_{It}\sigma_t - \frac{\varphi}{\sigma_t} + \frac{\theta X_t}{\sigma} < \varsigma_t^h$ always holds such that households have incentives to short sell physical capital, leading the no-short-sell constraint $\alpha_{Ht}^S \geq 0$ to bind such that we have $\mathring{\lambda}_t^S > 0$ and thus the equality holds.

To summarize, in the high household sentiment regime, optimistic households perceive intermediary equity as highly attractive and aggressively supply equity capital until the equity investment constraints bind. Once the constraints bind, optimistic households enter the physical capital market, reducing the net lending to financial intermediaries and thus lowering intermediary leverage. By contrast, in low-sentiment states, pessimistic households are unwilling to supply equity to intermediaries, making equity financing expensive. Intermediaries therefore rely more heavily on debt financing, which causes leverage to rise.

Appendix E: Sentiment calibration and 2007-2009 crisis simulation

In this section, I first detail the calibration of the sentiment parameters in the model to the household sentiment index from the University of Michigan Consumer Survey. I then provide additional information on the simulation of the 2007–2009 Global Financial Crisis episode discussed in Section 6.4.

E.1 Household sentiment calibration

Given the assumption that household sentiment X_t follows a standard OU process as in (14), and setting the long-run mean $\mu_X = 0$, only two parameters require calibration: the persistence parameter κ and the diffusion parameter σ_X. I calibrate these parameters using the University of Michigan Consumer Sentiment Index over the sample period 1970Q1–2018Q3, consistent with the empirical analysis in Section 2.

Given the continuous time OU process (14) and setting $\mu_X = 0$, we can write

$$d\left(e^{\kappa t} X_t\right) = \sigma_X e^{\kappa t} d\omega_t.$$

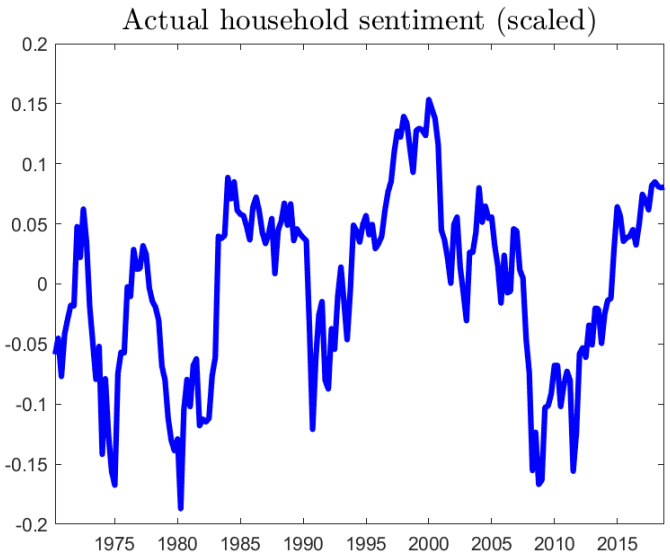


Figure E1. Normalized household sentiment time series. This figure plots the normalized household sentiment time series over the sample period 1970Q1–2018Q3.

Integrating both sides yields

$$X_{t+\Delta} = e^{-\kappa\Delta} X_t + \sigma_X \int_t^{t+\Delta} e^{-\kappa(t+\Delta-s)} d\omega_s, \tag{E.1}$$

where Δ denotes the discrete-time interval. Since I use quarterly sentiment index time series, we set $\Delta = \frac{1}{4}$. From this discrete-time representation, we obtain the predictive regression

$$X_{t+\Delta} = \mathfrak{b} X_t + \mathfrak{u}_{t+\Delta}, \tag{E.2}$$

where $\mathfrak{b} = e^{-\kappa\Delta}$ and $\mathfrak{u}_{t+\Delta}$ is conditionally normally distributed with mean 0 and variance $\frac{\sigma_X^2}{2\kappa}\left(1 - e^{-2\kappa\Delta}\right)$. The goal is to estimate the parameters κ and σ_X by fitting this discrete-time regression equation to the household sentiment data. The estimation proceeds as follows:

1. I begin with the raw household sentiment index and restrict the sample to 1970Q1–2018Q3. I then demean the truncated series to align with the model assumption that $\mu_X = 0$. Let the demeaned time series be denoted by Z_t.

2. Next, I rescale Z_t so that the empirical sentiment index fits within the model's range $X \in [-0.3, 0.3]$. Since an OU process can, in principle, fluctuate without bound, I define the upper bound 0.3 as four standard deviations above the mean of Z_t, and the lower bound -0.3 as

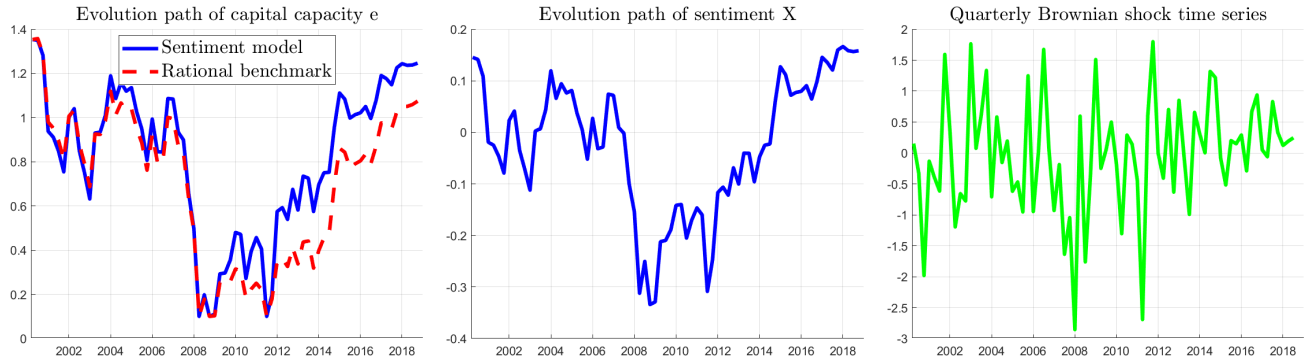


Figure E2. Evolution paths of e_t and X_t and Brownian shocks time series. The left panel plots the simulated evolution of intermediary capital capacity e_t in the sentiment model (blue solid line) and in the rational benchmark (red dashed line). The middle panel displays the simulated path of household sentiment X_t. The right panel shows the inferred quarterly Brownian shock time series.

four standard deviations below it.[41] The scaled series is given by

$$S_t = \frac{0.3 \times Z_t}{4 \times \text{std}\,(Z_t)},$$

where $\text{std}\,(Z_t)$ denotes the standard deviation of the demeaned series. The scaled series S_t serves as the empirical counterpart of model sentiment X_t.

3. Using the scaled series S_t, I estimate the predictive regression

$$S_{t+1} = bS_t + u_{t+1} \tag{E.3}$$

obtaining the coefficient estimate $\hat{b}$ and the standard deviation of residuals $\hat{\sigma_u}$.

4. Given that the data are quarterly ($\Delta = \frac{1}{4}$), I convert the regression estimates to their annualized counterparts as follows:

$$\kappa = -\frac{\log\left(\hat{b}\right)}{\Delta} = -4\log\left(\hat{b}\right), \tag{E.4}$$

$$\sigma_X = \frac{\hat{\sigma_u}}{\sqrt{2\Delta}} = 2\hat{\sigma_u}. \tag{E.5}$$

From the above procedure, I obtain the regression estimates $\hat{b} = 0.9133$ and $\hat{\sigma_u} = 0.0307$, leading to (approximately) $\kappa = 0.35$ and $\sigma_X = 0.06$ in the model's baseline calibration. The normalized household sentiment time series constructed from these estimates is shown in Figure E1.

[41] This scaling choice does not affect the persistence parameter κ, but it does influence the estimated diffusion parameter σ_X. The resulting calibration yields σ_X roughly equal to the model's fundamental volatility σ, ensuring numerical stability and realistic unconditional macroeconomic moments.

E.2 Simulation of $2007-2009$ financial crisis episode

As discussed in Section (6.4), we need to infer Brownian shocks from the household sentiment time series and determine the initial states for simulation. Under the OU specification, the Brownian shock satisfies $dZ_t = \frac{dX_t - \kappa(\mu_X - X_t)dt}{\sigma_X}$. Given $\mu_X = 0$ and $dt = \Delta$ in discrete time, this implies

$$\Delta Z_t \approx \frac{X_{t+\Delta} - X_t + \kappa \Delta X_t}{\sigma_X} \approx \frac{X_{t+\Delta} - e^{-\kappa\Delta} X_t}{\sigma_X} = \frac{\mathfrak{u}_{t+\Delta}}{\sigma_X}.$$

Therefore, from the predictive regression (E.3), the quarterly Brownian shocks can be backed out as

$$\xi_{t+1} = \frac{S_{t+1} - \hat{b} S_t}{\hat{\sigma_X}}. \tag{E.6}$$

The initial sentiment state X_0 is is obtained directly from the normalized household sentiment index in 2000Q1, which equals $X_0 = 0.1455$, corresponding to the optimism at the peak of the dot-com boom. To determine the initial intermediary capital capacity e_0, I use the empirical intermediary capital ratio from He et al. (2017) at 2000Q1. I convert this value into a Z-score relative to its empirical distribution and then map it to the model's distribution of e_t, yielding $e_0 = 1.3535$.

With these initial states $(e_0, X_0) = (1.3535, 0.1455)$ and the sequence of inferred Brownian shocks ξ_t, I simulate the joint evolution of e_t and X_t and obtain all corresponding equilibrium quantities. For the rational benchmark, I initialize with the same e_0 and feed in the same sequence of shocks to generate the evolution of e_t and its corresponding equilibrium quantities. Figure E2 displays the resulting paths of e_t, X_t and the inferred quarterly Brownian shocks.

Appendix F: Additional details

In this part of appendix, I provide additional details that complement various parts of the main text and present the regression results underlying Section 2.

F.1 Ergodic distribution in the baseline calibration

Figure F1 displays the ergodic distribution of the state variables e_t and X_t under the baseline calibration. The distribution implies a high unconditional correlation of 0.76 between e and X, which naturally arises from their shared exposure to the single Brownian shock driving the economy.

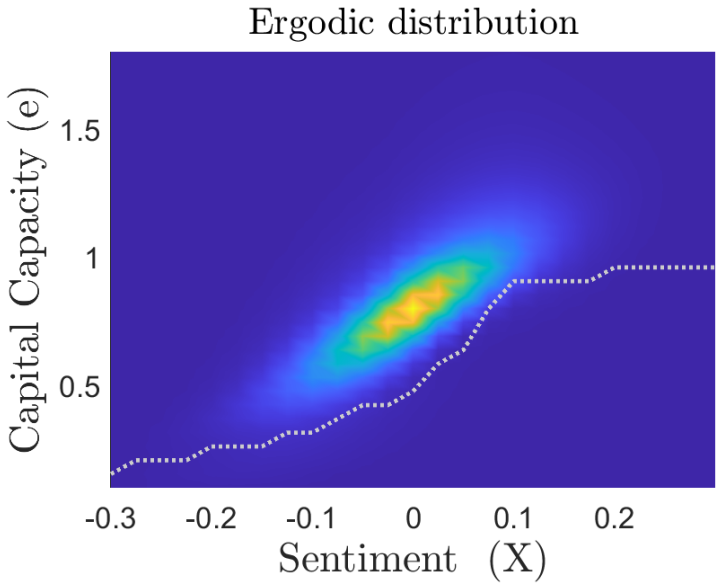


Figure F1. Ergodic distribution in the baseline calibration. This figure plots the ergodic distribution for the state variables e and X under the baseline calibration. The distribution is obtained by solving a Kolmogrov forward equation. The grey dotted line represents the cutoff curve of the crisis region.

F.2 The contribution of asset pricing channel to crisis predictability

As discussed in Footnote 23 in Section 6.1, the asset pricing channel of household sentiment also contributes to the crisis risk build-up in the high sentiment regime as shown in Figure 10. To illustrate this point, I construct a hypothetical economy in which the cutoff states that define the crisis region are held fixed across different sentiment levels. This allows me to isolate the effect of the shape of the crisis region on the crisis-hitting probabilities. Furthermore, when applying the KBE to compute these probabilities, I turn off the stochastic channels by setting the diffusion terms to zero. In this way, I can focus on the pure effect of price reversals through their impact on the deterministic evolution of the state variables. Under this implementation, I obtain the three-year crisis-hitting probabilities across different hypothetical economies, as reported in Figure F2.

As show in Figure F2, across different hypothetical economies with varying crisis-region depths, the crisis-hitting probabilities are elevated in the high-sentiment regime but essentially zero in the low-sentiment regime. This pattern underscores the role of the asset-pricing channel of household sentiment. The intuition, consistent with Figure 9, is that in the high-sentiment regime, price reversals driven by sentiment reversals reduce the returns earned by financial intermediaries, thereby depressing e and raising crisis-hitting probabilities. Conversely, in the low-sentiment regime, the same mechanism improves intermediary returns, strengthens e, and lowers the likelihood of hitting the crisis region.

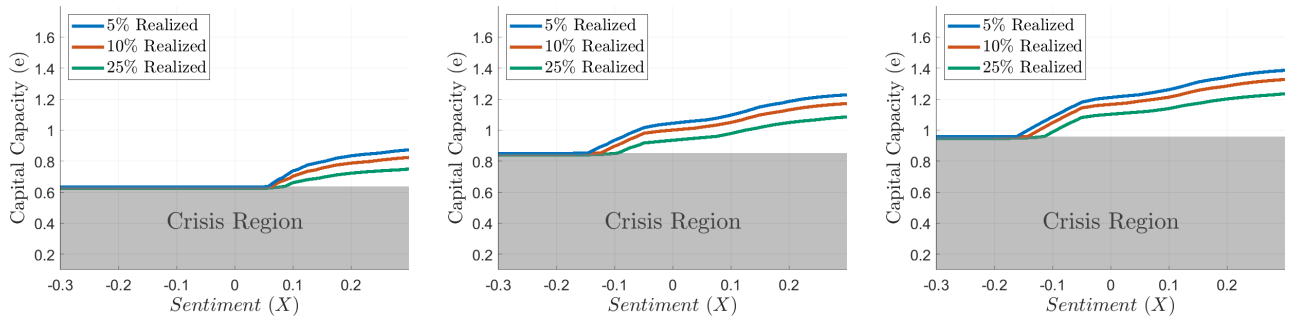
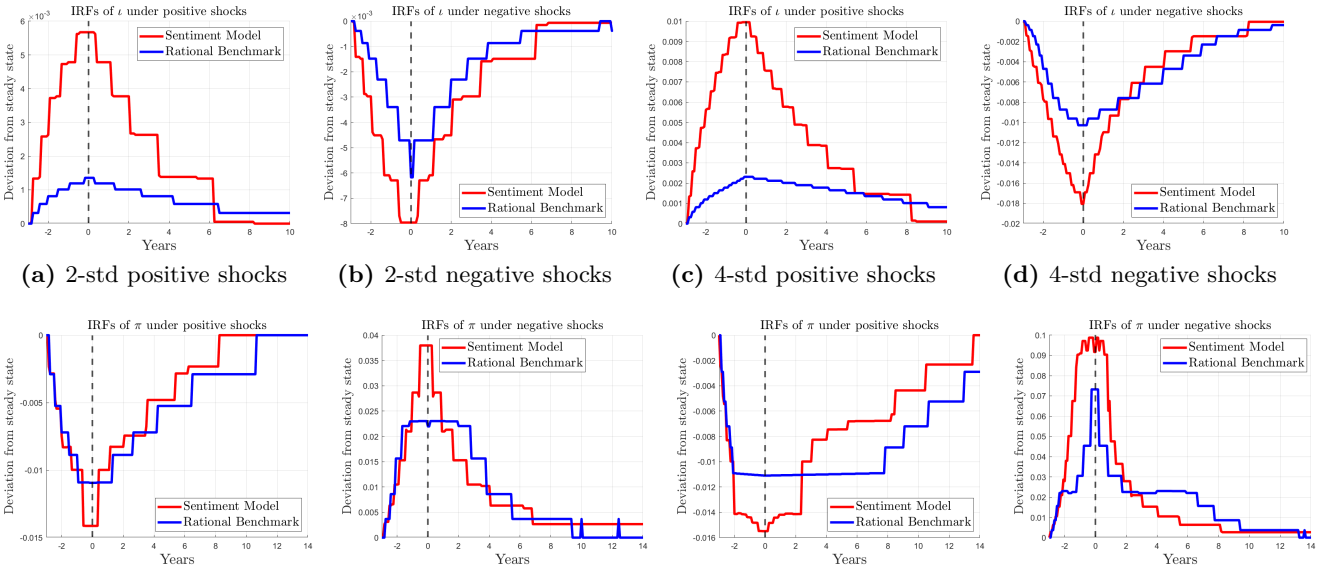


Figure F2. Crisis hitting probability in next 3 years in hypothetical economies. These panels plot the three-year crisis-hitting probability in hypothetical economies where the crisis-region cutoffs are held fixed across sentiment states. For this exercise, the KBE is solved with diffusion terms set to zero.



(a) 2-std positive shocks **(b)** 2-std negative shocks **(c)** 4-std positive shocks **(d)** 4-std negative shocks

(e) 2-std positive shocks **(f)** 2-std negative shocks **(g)** 4-std positive shocks **(h)** 4-std negative shocks

Figure F3. Boom-bust cycles under shocks with different magnitudes. This figure plots impulse response functions (IRFs) for the investment rate ι_t and the equilibrium risk premium π_t following cumulative shocks of 2 standard deviations and 4 standard deviations, shown for both positive and negative shock sequences.

F.3 Robustness check for the boom-bust cycles

For the robustness check of the boom–bust cycles in the investment rate and the equilibrium risk premium driven by household sentiment, as discussed in Section 6.2, I feed in a three-year sequence of either positive or negative shocks that cumulate to two or four standard deviations over the three-year horizon. The resulting dynamics are shown in Figure F3. We can see that the boom–bust cycles in the investment rate and the equilibrium risk premium persist even under alternative shock magnitudes.

F.4 Additional figures for financial stability analysis

As discussed in Section 6.3, the effects of θ, φ, κ and σ_X on financial stability are best understood by comparing ergodic distributions across different parameter values. These distributions are reported in Figure F5. In addition, F4 plots the unconditional means of intermediary leverage α_{It}, equilibrium return volatility σ_t, and the equilibrium risk premium π_t.

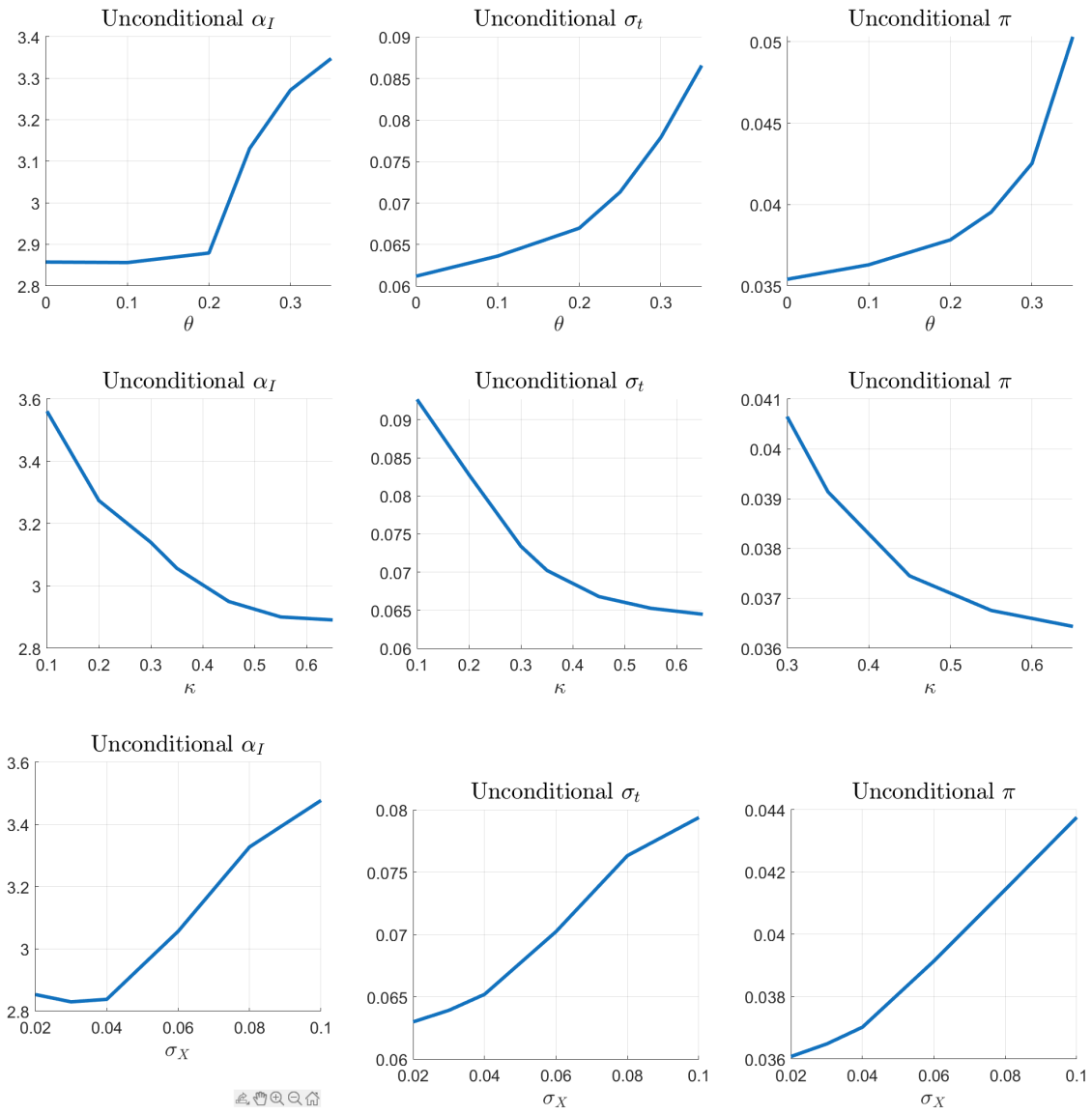
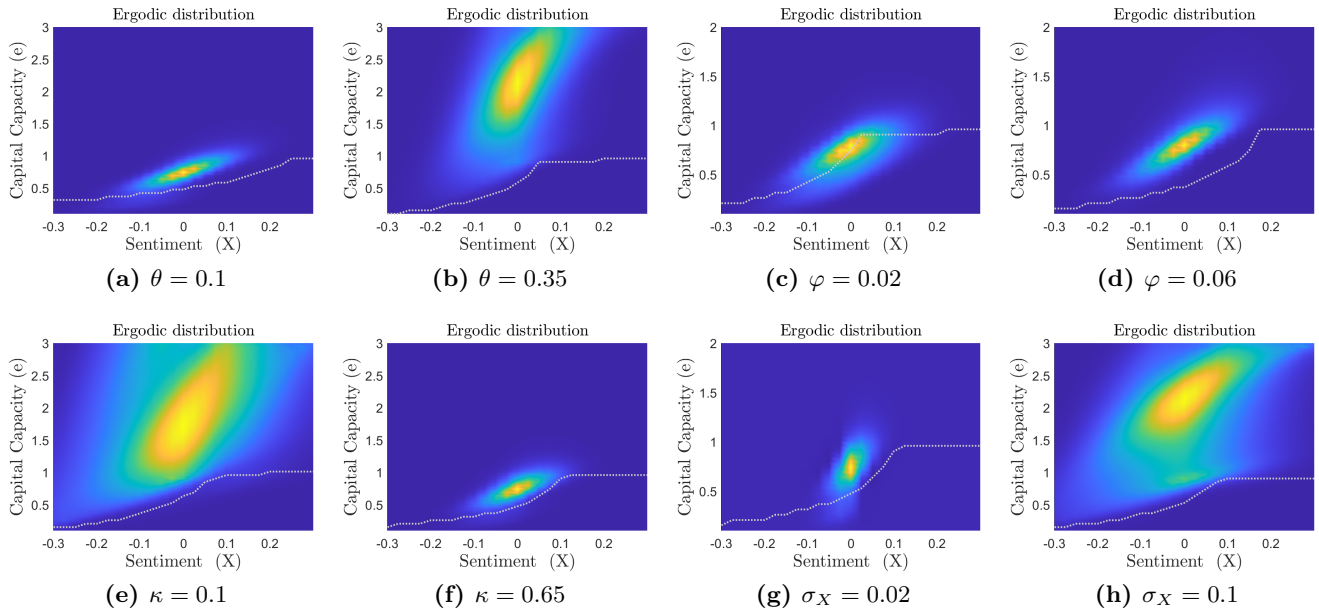


Figure F4. Unconditional mean of α_I, π and σ_t. This figure plots unconditional intermediary leverage α_I, unconditional mean of σ_t and unconditional mean of π_t under alternative values of θ, φ, κ and σ_X.



(a) $\theta = 0.1$ **(b)** $\theta = 0.35$ **(c)** $\varphi = 0.02$ **(d)** $\varphi = 0.06$

(e) $\kappa = 0.1$ **(f)** $\kappa = 0.65$ **(g)** $\sigma_X = 0.02$ **(h)** $\sigma_X = 0.1$

Figure F5. Ergodic distributions under various θ, φ, κ and σ_X. These panels plot the ergodic distributions of the state variables e and X under alternative values of θ, φ, κ and σ_X. The grey dotted line represents the cutoff curve of the crisis region.

F.5 Additional empirical details

Table F1 reports the OLS regression results corresponding to Figure 1.

	Intermediary leverage
OLS regression coefficient	-0.3912^{***} (-13.417)

Table F1. OLS regression for intermediary leverage. The table reports the regression result of intermediary leverage on household sentiment. The regression is of the form $y_t = a + \beta x_t + \epsilon_t$, where x_t is the household sentiment index at time t and y_t is the intermediary leverage ratio at time t. Reported coefficients are for β. $***$ denote significance at 1% levels. t-statistic is reported in parentheses. The sample covers the period $1980 - 2018$.

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