Global Value Chains and International Risk Sharing*

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Abstract

International trade in intermediate inputs via Global Value Chains activates a distinct channel of cross-border macroeconomic interdependence that operates through production costs. We study how this channel shapes the response of quantities and prices to productivity shocks, and its implications for macro risk and cross-border risk sharing. Based on the standard open-macro model, we show that, if international financial markets are incomplete, rising integration into GVCs generally improves risk sharing via cost diversification. Yet, cost diversification may not be effective for empirically reasonable input complementarity: through trade in intermediate inputs, productivity gains in one country may result in *higher* production costs, worsening the global production conditions and consumption allocation. In this case, a 'fragmentation trap' arises: a marginal reduction in interdependence may be seen as beneficial, even though deeper GVC integration would actually improve risk sharing by restoring the cost-diversification outcome. We provide quantitative evidence that the contribution of GVCs to risk sharing has been positive over the sample 2000-2014.

Key Words: Cross-Border Transmission; Incomplete Asset Markets; Intermediate Goods Trade; Global Financial Markets; Supply Chain Linkages.

JEL Codes: F32, F33, F41, F44.

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1 Introduction

As Global Value Chains (GVCs) have progressively redefined the core of international macroeconomic interdependence, geopolitical shocks and supply-chain disruptions have fuelled widespread concerns about the implied potential risks to national income and economic activity. Debates have focused on the volatility of key macroeconomic variables, and/or potential inefficiencies and market failures that distort GVCs hampering production efficiency.[1] At the root of this debate, however, lies fundamental questions concerning the global effects of the supply-side linkages creating production cost interdependence across countries and regions. How does this interdependence impinge on equilibrium relative prices (terms of trade), demand and supply at the global and cross-border level? Do GVCs magnify or reduce income risk emanating from productivity fluctuations, hampering or fostering cross-border risk sharing?

In this paper, we address these questions through the lens of open-macro theory. We analyse the general-equilibrium mechanism by which the supply-side linkages that arise from GVCs affect the international transmission of productivity shocks, uncertainty about production costs, income risk and ultimately a country's ability to minimize macroeconomic risks by sharing them across borders. Focusing on supply-side links and on the international transmission of productivity shocks allows us to highlight a 'global cost interdependence' channel linking the terms of trade and marginal costs, whose analysis is novel to the literature and complements and expands upon the contributions from Cole and Obstfeld (1991), Corsetti et al. (2008) and Corsetti et al. (Forthcoming). We characterize the equilibrium dynamics of quantities and prices, risk and risk sharing, as functions of (a) the structure of cross-border financial markets, (b) the trade elasticities determining the complementarity of domestic and imported inputs/goods, and (c) the openness of the economy to international trade.

We show analytically and quantitatively that, under incomplete international financial markets, global cost interdependence does contribute to diversifying risk when domestic and foreign inputs/goods are sufficiently substitutable. In this case, as more productive domestic firms lower supply prices, the costs of intermediate inputs and thus production costs abroad fall while foreign real income rises, activating a positive feedback that drives up consumption globally. However, if domestic and foreign inputs/goods are strong complements, i.e. if trade elasticities are sufficiently low, home productivity gains may lead to an equilibrium *appreciation* in the country's terms of trade instead, i.e. a rise in its supply price, a case which is possible at intermediate levels of GVC integration. In this case, the GVC network ends up raising production costs abroad, lowering foreign real income and demand. The possibility of such a negative feedback effect suggests that cost interdependence via GVCs does not necessarily diversify production risk. On the contrary, it may exacerbate macroeconomic risk—independently of 'disruption' shocks and/or market failures in production networks.

Our analysis draws on seminal work in international macroeconomics, showing that, under incomplete markets, terms-of-trade adjustments to productivity/supply shocks can induce ad-

[1] See, notably, Caselli, Koren, Lisicky, and Tenreyro (2020); D'Aguanno, Davies, Dogan, Freeman, Lloyd, Reinhardt, Sajedi, and Zymek (2021); Baldwin and Freeman (2022); Acemoglu and Tahbaz-Salehi (2025).

verse wealth effects on demand that magnify income risks across regions. A similar mechanism has been reconsidered by the recent literature on 'Keynesian supply shocks', which explores how supply shocks transmit through relative prices in multi-sector closed economies with uninsured households (Cesa-Bianchi and Ferrero, 2021; Guerrieri, Lorenzoni, Straub, and Werning, 2022). Contributing to these literatures, we show that the presence of GVCs can magnify inefficient international price movements—whereby productivity gains in one country *raise* production costs abroad and globally. Yet this is possible only at intermediate levels of GVC integration: through the global cost interdependence channel, a sufficiently deep integration into global value chains eventually changes the international transmission mechanism *even under strong complementarity of domestic and foreign inputs in production*, up to changing the sign of the response of the terms of trade to productivity gains and leading global production costs to actually fall. This has striking implications for the process of GVCs deepening: at intermediate levels of GVCs integration, marginal delinking improves risk sharing, while further and deeper interdependence would be globally desirable. We dub this a "fragmentation trap".

Theoretically, we ground our analysis in a canonical two-country, two-good open-economy macro model with 'roundabout' technologies that use domestic and foreign intermediate inputs in production. This class of models is widely used in academic and policy analysis, and our general specification nests a range of empirically relevant cases. In the model, households in each country consume domestic and foreign-produced final goods. In addition, production combines local labour with both domestic and foreign-produced intermediate inputs—nesting GVCs, in a stylised manner, into an otherwise standard model. Firms' output is subject to risk from country-specific productivity shocks, which brings about risks to consumption. This generates incentives for households to share risks with foreigners by trading assets through international financial markets.

In our baseline, we focus on Financial Autarky as an extreme but tractable form of market incompleteness that lends itself to closed-form solutions. We draw comparisons with the theoretical benchmark of Complete Markets where appropriate to highlight the role of international financial markets in determining how trade in intermediate inputs through GVCs affects risk. In the quantitative section of our paper, we extend our results to Bond Economies, a less extreme form of market incompleteness where international markets exist for non-contingent assets. To assess deviations from efficient risk sharing, we rely on a welfare-relevant wedge, the '*wealth gap*'.[2] This gap quantifies the misallocation of demand across countries relative to a case with Complete Markets, and reflects distortions in relative consumption and relative prices. To evaluate the total risk facing households, we examine the volatility of their consumption over the business cycle.

We conduct our analysis assessing risk and risk sharing against a wide range of trade elasticities, reflecting the range of estimates in the literature. Theoretical and empirical studies point to systemic differences in these elasticities at different horizons. In line with that, we introduce 'habits' in a model extension to differentiate between short-run and long-run elasticities. We

[2]See the definition and discussion in Corsetti, Dedola, and Leduc (2010, 2023) and Viani (2011).

also consider model extensions allowing for constant elasticity of substitution (CES) aggregators in production and endogenous labour supply.

We complement our model-based analysis with an exploratory macro-level empirical analysis of the relationship between GVC integration and the degree of international risk sharing across advanced economies. We construct model-consistent measures of both GVC integration, using data from World Input-Output Tables, and the wealth gap, combining data for real consumption expenditure, consumer prices and nominal exchange rates. Controlling for country size, final-goods trade and other factors, we show that, on average, greater GVC integration is associated with significantly less movement in the wealth gap, indicating less pronounced cross-country misalignments over the business cycle. These findings validate the conclusion from the model that GVC integration is associated with increased risk sharing.

The rest of this paper is structured as follows: Section 2 lays out the theoretical model of trade in intermediate inputs. Section 3 provides an analytically compact framework offering key insights on the implications of GVCs for the transmission of productivity shocks. Section 4 presents our main results on risk and risk sharing. Section 5 explores the quantitative relevance of the analytical results: it presents results from a calibrated version of the model with incomplete markets, considers four quantitatively relevant model extensions and assesses the model predictions with macroeconomic data. Section 6 concludes.

2 The Model

We begin by setting up a two-country real open-economy macro model with GVCs. For simplicity and analytical tractability, we assume that the two countries, Home and Foreign, are symmetric. Consumers enjoy goods produced in both countries, such that there is trade in final goods. Firms in each country produce these goods using a combination of domestically-supplied labour and both domestic and imported inputs, such that there is intermediate-goods trade, capturing, in a stylised way, a role for GVCs.

The following subsections describe the Home economy in more detail; the structure of the Foreign economy is analogous. Where necessary, the conventional $*$ denotes Foreign variables or parameters, and the subscripts H and F denote variables or parameters related to goods produced in the Home and Foreign economy, respectively.

2.1 Households

The representative Home household maximises expected lifetime utility from aggregate consumption, C_t. Their additively separable utility function is:

$$U_t = \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\frac{C_{t+\tau}^{1-\sigma} - 1}{1-\sigma} \right] \tag{1}$$

where $\mathbb{E}_t$ is the expectations operator conditional on time t information, $\beta \in (0,1)$ is the discount factor, and $\sigma > 0$ is the coefficient of relative risk aversion.

Let P_t denote the price of a unit of aggregate consumption, W_t the wage per unit of labour and L the inelastic labour supply of the household.[3] Assuming that initially there are no foreign assets/liabilities in the world economy, the intertemporal budget constraint of the representative household is: $\sum_{\tau=0}^{\infty}(P_{t+\tau}C_{t+\tau} - W_{t+\tau}L) \leq 0$. The per-period budget constraint, and hence the intertemporal optimality conditions, will depend on the financial assets available to the households, which will be determined by the nature and degree of cross-border financial linkages, as further discussed below.

Intratemporal Allocation. The aggregate consumption bundle of Home households consists of domestically-produced and imported final goods. The Home basket has the following CES form à la Armington (1969):

$$C_t \equiv \left[a_H^{\frac{1}{\phi_C}} c_{H,t}^{\frac{\phi_C - 1}{\phi_C}} + a_F^{\frac{1}{\phi_C}} c_{F,t}^{\frac{\phi_C - 1}{\phi_C}} \right]^{\frac{\phi_C}{\phi_C - 1}} \tag{2}$$

where $a_H \in (0,1)$ and $a_F = (1 - a_H)$ are the weights on H and F goods, $c_{H,t}$ and $c_{F,t}$ are the quantities consumed at time t, and $\phi_C > 0$ is the elasticity of substitution between them.[4]

Given the specification of the model, the intratemporal problem of the representative Home household at time t can be solved independently of the intertemporal problem. Minimising total expenditure, taking as given the level of aggregate consumption, C_t, and the prices of H and F goods, labelled respectively $P_{H,t}$ and $P_{F,t}$, yields the consumer price index (CPI), P_t, defined as follows:

$$P_t = \left[a_H P_{H,t}^{1-\phi_C} + a_F P_{F,t}^{1-\phi_C} \right]^{\frac{1}{1-\phi_C}} \tag{3}$$

as well as the demand schedules for H and F goods, written as functions of the terms of trade:

$$c_{H,t} = a_H \left(a_H + a_F \mathcal{T}_t^{1-\phi_C} \right)^{\frac{\phi_C}{1-\phi_C}} C_t \tag{4}$$

$$c_{F,t} = a_F \mathcal{T}_t^{-\phi_C} \left(a_H + a_F \mathcal{T}_t^{1-\phi_C} \right)^{\frac{\phi_C}{1-\phi_C}} C_t \tag{5}$$

Home's terms of trade (TOT) is defined as the price of Home's imports relative to that of domestic goods:

$$\mathcal{T}_t \equiv P_{F,t}/P_{H,t} \tag{6}$$

The Foreign consumption bundle, CPI and demand functions are analogously defined.

[3] Throughout our analytical exposition, we assume inelastic labour supply to abstract from wealth effects on labour supply that would otherwise generate counterfactually negative cross-country macroeconomic co-movement. With this assumption, in the absence of intermediate inputs, our model nests an endowment model. We assess the robustness of our quantitative model results to endogenous labour supply in Section 5.

[4] Lisack, Lloyd, and Sajedi (2022) show that the parameters of the Armington aggregator capture the sufficient statistics for first-order demand-side dynamics in a general class of open-economy macroeconomics models with homothetic preferences, including this one.

2.2 Production

In each economy, perfectly competitive final-goods firms produce a country-specific good using a 'roundabout' production function. Home firms combine domestic labour with a bundle of intermediate goods, X_t, to produce gross output, $Y_{H,t}$, according to a Cobb-Douglas technology:

$$Y_{H,t} = (A_t L)^{1-\alpha} X_t^{\alpha} \tag{7}$$

where $\alpha \in [0,1)$ is the share of intermediates in gross output, and A_t is the country-specific labour-augmenting productivity,[5] which evolves according to the following exogenous process:

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_A \tag{8}$$

The innovations, ϵ_A, are independently and identically distributed with standard deviation σ_A.

Firms take the price of their own goods, $P_{H,t}$, the wage, W_t, and the price of the intermediate bundle, $P_{X,t}$, as given. Maximising profits, their optimal demand schedules for labour and intermediate goods are as follows:

$$W_t = (1-\alpha) P_{H,t} \frac{Y_{H,t}}{L} \tag{9}$$

$$P_{X,t} = \alpha P_{H,t} \frac{Y_{H,t}}{X_t} \tag{10}$$

Trade in Intermediate Inputs. Home firms' intermediate-inputs bundle is an aggregate of domestic inputs, $x_{H,t}$, and imported inputs, $x_{F,t}$, similar to the final consumption bundle (2):

$$X_t \equiv \left[b_H^{\frac{1}{\phi_X}} x_{H,t}^{\frac{\phi_X - 1}{\phi_X}} + b_F^{\frac{1}{\phi_X}} x_{F,t}^{\frac{\phi_X - 1}{\phi_X}} \right]^{\frac{\phi_X}{\phi_X - 1}} \tag{11}$$

where $b_H \in (0,1)$ and $b_F = (1 - b_H)$ are the weights on H and F inputs, $x_{H,t}$ and $x_{F,t}$ are the quantities of inputs used at time t, and $\phi_X > 0$ is the elasticity of substitution between them.

As with the consumption bundle, cost minimisation leads to the following definition for the intermediate-inputs price index:

$$P_{X,t} = \left[b_H P_{H,t}^{1-\phi_X} + b_F P_{F,t}^{1-\phi_X} \right]^{\frac{1}{1-\phi_X}} \tag{12}$$

and the demand functions:

$$x_{H,t} = b_H \left(b_H + b_F \mathcal{T}_t^{1-\phi_X} \right)^{\frac{\phi_X}{1-\phi_X}} X_t \tag{13}$$

$$x_{F,t} = b_F \mathcal{T}_t^{-\phi_X} \left(b_H + b_F \mathcal{T}_t^{1-\phi_X} \right)^{\frac{\phi_X}{1-\phi_X}} X_t \tag{14}$$

[5] This formulation of productivity brings out the global production-cost dependence, which is the distinctive channel of international transmission with GVCs in our model. Appendix A shows how the key results would differ under total factor productivity shocks.

2.3 Market Clearing

In equilibrium, goods markets must clear. Gross outputs of H and F goods must satisfy the world demand for both consumer goods and intermediate inputs:

$$Y_{H,t} = c_{H,t} + c^*_{H,t} + x_{H,t} + x^*_{H,t} \tag{15}$$

$$Y^*_{F,t} = c_{F,t} + c^*_{F,t} + x_{F,t} + x^*_{F,t} \tag{16}$$

Global financial markets must clear too. The expressions that ensure this depend on the exact structure of these markets, which we discuss in the next section. Nevertheless, regardless of this, the aggregate resource constraint must be satisfied in equilibrium:

$$P_{H,t}Y_{H,t} + P_{F,t}Y^*_{F,t} = P_tC_t + P^*_tC^*_t + P_{X,t}X_t + P^*_{X,t}X^*_t \tag{17}$$

2.4 Intertemporal allocation and risk sharing

For tractability, we consider two benchmark international financial market structures, contrasting Complete Markets (CM) with exogenously Incomplete Markets (IM), in the form of Financial Autarky (FA).

With CM, households can exchange a full set of state-contingent securities across borders and risk sharing will be complete. In this case, since there are no nominal or real frictions in our model, the equilibrium allocation will be first-best from the perspective of a global planner. The equilibrium risk-sharing condition follows from combining the Euler equations for each state-contingent security in each country:

$$\left(\frac{C_t}{C^*_t}\right)^{\sigma} \frac{1}{\mathcal{Q}_t} = 1 \tag{18}$$

where $\mathcal{Q}_t \equiv P^*_t/P_t$ is the real exchange rate (RER).

In contrast, IM introduce a key inefficiency, limiting the extent to which households can smooth consumption intertemporally and share risks across borders. To derive analytical insight in the next section, we focus on FA. Banning cross-border trade in financial assets, trade must be balanced in each period:

$$P_{F,t}\left(c_{F,t} + x_{F,t}\right) = P_{H,t}\left(c^*_{H,t} + x^*_{H,t}\right) \tag{19}$$

While FA is a stark assumption, we will show that the main mechanism carries over to more general IM settings, such as a Bond Economy (BE) where international trade in financial assets is limited to one non-state-contingent bond that pays a known interest rate in the next period.[6]

IM settings will give rise to deviations from the perfect risk-sharing condition, equation (18). Following Corsetti et al. (2010), we can define a 'wealth gap' measure, which acts as a summary

[6] We assess the robustness of our quantitative model findings to this alternative financial-market structure in Section 5.

statistic quantifying departures from perfect international risk sharing:

$$\mathcal{W}_t \equiv \left(\frac{C_t}{C_t^*}\right)^{\sigma} \frac{1}{\mathcal{Q}_t} \tag{20}$$

Under CM, equation (18) implies that $\mathcal{W}_t^{CM} = 1$ for all t.

Analytically, the wealth gap is the ratio of the Lagrange multipliers on the budget constraints of Foreign to Home households. CM guarantee that this gap is always closed: perfect insurance ensures that the PPP-corrected value of wealth across borders remains constant. By contrast, under IM some risk is non-traded, and the shadow value of income in the Foreign economy relative to Home fluctuates with shocks. As such, $\mathcal{W}_t > 1$ means that the marginal utility of an extra unit of income in the Foreign country exceeds that of the Home country. For given relative prices, relative consumption in the Foreign country will be inefficiently low. Equivalently, given relative consumption, the relative price of Home consumption is inefficiently high, i.e. the RER is over-appreciated. These considerations clarify that the gap comprise two wedges: one pertaining to misalignments in relative consumption across countries, and another linked to misalignments in relative prices.

For future reference, notice that the CPI definition (3) and its Foreign counterpart imply the following connection between the RER and the TOT, as defined in equation (6):

$$\mathcal{Q}_t^{1-\phi_C} = \frac{a_F + a_H \mathcal{T}_t^{1-\phi_C}}{a_H + a_F \mathcal{T}_t^{1-\phi_C}} \quad \Rightarrow \quad \widehat{\mathcal{Q}}_t = (2a_H - 1)\,\widehat{\mathcal{T}}_t \tag{21}$$

where hatted variables denote log-linear deviations from the deterministic steady state. As long as there is home bias in the consumption bundle, $a_H > 0.5$, the TOT and the RER will co-move positively.

3 Equilibrium Implications of Global Production-Cost Interdependence

The model (presented above in a stripped-down version) has been the subject of a vast literature, with and without roundabout production. Much of the analysis, however, has bypassed a key question: How does a 'global production-cost interdependence', i.e., supply-side linkages specifically created by GVCs, change the equilibrium allocation? Hereafter we propose a compact transparent framework to reconsider systematically how GVCs weigh on the equilibrium transmission of shocks.

3.1 A compact framework for analysis

In the tradition of international economics, we solve the model by looking at the relative (Home to Foreign) and the global allocation. We derive a relative supply and a relative demand schedule, taking differences between the two economies, and use these schedules to trace the equilibrium response of the TOT, consumption and economic activity to shocks.

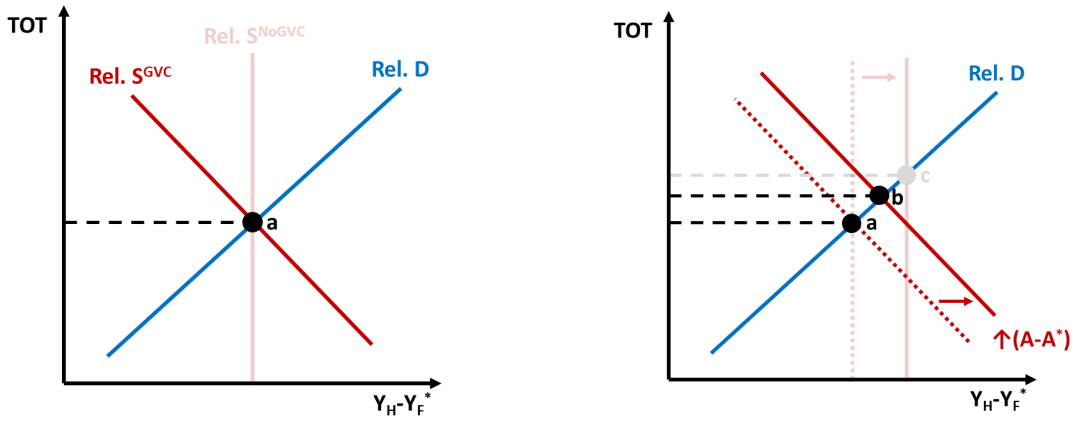


Figure 1: Relative Demand and Supply with Complete Markets or Moderate Complementarity

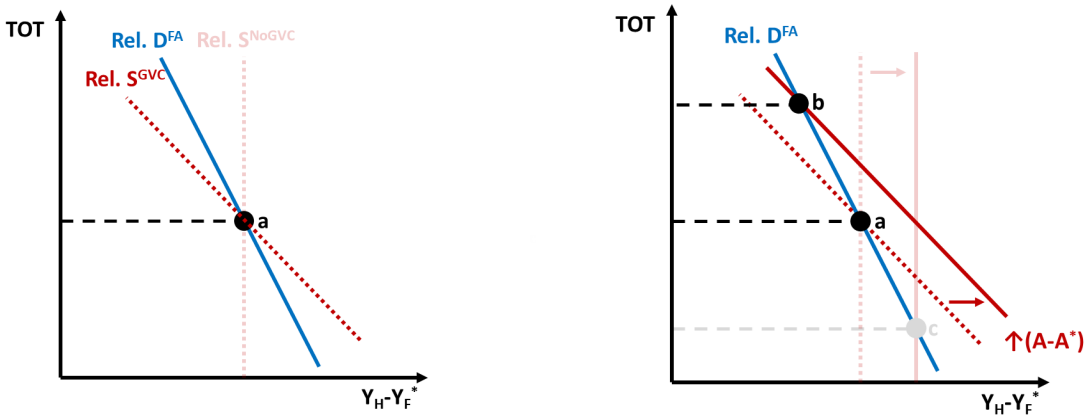


Figure 2: Incomplete Markets with Strong Complementarity but Sufficient GVCs

3.1.1 Relative Supply

The first relation is the relative supply of domestic to foreign output. To derive it, we plug the first-order condition for intermediates, equation (10), and the definition of the aggregate intermediate-input price, equation (12), into the production function, (7), and log linearise around the deterministic symmetric steady state. We obtain:

$$\widehat{Y}_{H,t} = \widehat{A}_t - b_F \frac{\alpha}{1-\alpha} \widehat{\mathcal{T}}_t \tag{22}$$

This equations clarifies that the gross output, $Y_{H,t}$, is endogenous even in an endowment economy, due to the use of intermediate inputs in production. Subtracting this two expression from its foreign analog, and we obtain the Relative Supply (RS) schedule:

$$\widehat{Y}_{R,t} \equiv \widehat{Y}_{H,t} - \widehat{Y}^*_{F,t} = \widehat{A}_{R,t} - 2 \cdot GPCI \cdot \widehat{\mathcal{T}}_t \tag{23}$$

where $\widehat{A}_{R,t} \equiv \left(\widehat{A}_t - \widehat{A}^*_t\right)$ is the relative productivity and we have defined the following indicator of global production-cost interdependence (GPCI):

$$GPCI \equiv b_F \frac{\alpha}{1-\alpha}$$

The four plots in Figures 1 and 2 show two possible RS curves in the space $\{\widehat{\mathcal{T}}, \widehat{Y}_R\}$. Relative productivity determines the position of each of these RS curves: a rise in $\widehat{A}_{R,t}$ shifts the RS horizontally to the right. The slope depends on GPCI instead. The RS curve is vertical and equal to the exogenous relative productivity $\widehat{Y}_{R,t} = \widehat{A}_{R,t}$, if either $\alpha = 0$, so that there are no intermediate inputs in production, or $b_F = 0$, so that there is no trade in intermediate inputs.

The RS curve is negatively sloped if $\alpha > 0$ and $b_F > 0$. This means that, with GVCs, Home's relative supply falls as the TOT worsens ($\mathcal{T}$ increases). Home firms decrease production if the cost of imported inputs rises relative to the price of their output. The opposite is true for foreign firms: cheaper intermediate inputs lead them to boost their production.

Here is the key take-away: the slope of the RS schedule reflects the degree of global production-cost dependence, as synthesized by the coefficient GPCI. As economies become increasingly integrated into GVCs, this coefficient rises in absolute value, and the slope of the RS curve becomes flatter. Note that in a symmetric equilibrium with relative output prices equal to 1, GPCI is equal to the share of imported intermediates in value added.

3.1.2 Relative Demand

While the expression for Relative Supply derived above does not depend on the structure of financial market and risk sharing, the expression for the Relative Demand (RD) does. This is intuitive: RD is pinned down by households' consumption and savings decisions in both countries, which depend on the menu of assets available to them. Indeed, combining the market clearing conditions with the individual demand components we arrive at the following expression

for the RD as a function of relative consumption and the TOT:

$$\widehat{Y}_{R,t} = \frac{(1-\alpha)(2a_H-1)}{1-\alpha(2b_H-1)}(\widehat{C}_t - \widehat{C}_t^*) + \frac{4\Big[(1-\alpha)\phi_C a_H a_F + \alpha\phi_X b_H b_F\Big] - 2b_F\alpha(2b_H-1)}{1-\alpha(2b_H-1)}\widehat{\mathcal{T}}_t \tag{24}$$

Both relative consumption and the TOT are endogenous and cannot be solved without specifying the assets in the households budget constraint. To proceed, we write generically:

$$\widehat{Y}_{R,t} = \mathcal{D}(\sigma, a_H, b_H, \phi_C, \phi_X)\ \widehat{\mathcal{T}}_t \tag{25}$$

and discuss how the compound parameter $\mathcal{D}(.)$ varies under different assumptions concerning financial markets.

Complete Markets Under complete international financial markets, the relationship between relative consumption and relative prices is given by the perfect risk sharing condition (18), shown below in log-linear form:

$$\sigma\left(\widehat{C}_t - \widehat{C}_t^*\right) = (2a_H - 1)\,\widehat{\mathcal{T}}_t$$

Using this to substitute out the relative consumption term, we obtain:

$$\mathcal{D}_{CM} \equiv \frac{(1-\alpha)(2a_H-1)^2\sigma^{-1} + 4\Big[(1-\alpha)\phi_C a_H a_F + \alpha\phi_X b_H b_F\Big] - 2b_F\alpha(2b_H-1)}{1-\alpha(2b_H-1)} \tag{26}$$

With perfect risk sharing, $\mathcal{D}_{CM}$ is unambiguously positive: the RD for Home output unambiguously rises with a depreciation of Home's TOT. As shown in Figure 1, the RD schedule is upward sloping in $\{\widehat{\mathcal{T}}, \widehat{Y}_R\}$ space in this case.

Financial Autarky Under Financial Autarky, the relationship between relative consumption and relative prices can be derived from the trade-balance equation:

$$\mathcal{T}_t\ (C_{F,t} + X_{F,t}) = C^*_{H,t} + X^*_{H,t} \tag{27}$$

Combining this with the individual demand functions we now have:

$$\mathcal{D}_{FA} \equiv \frac{a_F(1-\alpha)\left[1-2a_H(1-\phi_C)\right] + b_F\alpha\left[1-2b_H(1-\phi_X)\right]}{a_F(1-\alpha) + b_F\alpha} \tag{28}$$

$\mathcal{D}_{FA}$ can be positive or negative, depending on the openness to trade in intermediate and final goods, and the trade elasticities in both sectors. Hence the RD schedule can be either upward or downward sloping in $\{\widehat{\mathcal{T}}, \widehat{Y}_R\}$ space. In general, the curve becomes flatter with a larger share of imported intermediates in production, b_F, and a higher degree of GVC integration, α.

3.1.3 Equilibrium

Our analytical and graphical representation of RS and RD, together with a global equilibrium condition, provides a synthetic but insightful framework to inspect the core implications of GVCs, boiling down to the following 3 equations:

$$\begin{aligned}\widehat{Y}_{H,t} - \widehat{Y}^*_{F,t} &= \left(\widehat{A}_t - \widehat{A}^*_t\right) - 2GPCI\ \widehat{\mathcal{T}}_t \\ \widehat{Y}_{H,t} - \widehat{Y}^*_{F,t} &= \mathcal{D}_i \widehat{\mathcal{T}}_t \\ \hat{C}_t + \hat{C}^*_t &= \widehat{A}_t + \widehat{A}^*_t\end{aligned}$$

where $i = \{CM, FA\}$, and the compound parameters $\mathcal{D}_i$ are given by equations (26) and (28). GVCs impact RS directly through the global production-cost dependence channel; they impact RD indirectly via the equilibrium adjustment of relative (goods and asset) prices.

3.2 Terms-of-Trade Determination and the Transmission of Productivity Shocks

Using our framework, we combine RD and RS to obtain an expression for the TOT as a function of exogenous relative productivity under FA:

$$\widehat{\mathcal{T}}_t = \frac{1}{\mathcal{D}_{FA} + 2GPCI} \widehat{A}_{R,t} \tag{29}$$

The following proposition states a key property of the above relation, pertaining to the role of trade openness and trade elasticities in the dynamics of the TOT. For analytical transparency, but without loss of generality, the proposition is derived under the simplifying assumption that the home bias in the CES aggregator for the consumption goods and intermediate inputs is the same, i.e., $b_H = a_H$.

Proposition 1 (GVC Integration and the Terms of Trade) *Assuming $b_H = a_H$, the home terms of trade worsen in response to a relative productivity gain if the weighted-average trade elasticity defined as*

$$\Phi_\alpha = (1-\alpha)\phi_C + \alpha\phi_X \tag{30}$$

satisfies the condition

$$\Phi_\alpha < 1 - \frac{1}{2a_H} - \frac{a_F}{a_H}\frac{\alpha}{1-\alpha} \equiv \widetilde{\phi}_{TOT}(\alpha). \tag{31}$$

Proof: See Appendix D. □

This proposition shows that the TOT can either depreciate or appreciate in response to the positive Home productivity shock, depending on openness and trade elasticities, resonating with the analysis in Corsetti et al. (2008). Under IM, when goods are strong complements and demand is home-biased, the equilibrium can be dominated by the wealth effects of relative price adjustment: the incomes in the country with higher productivity must be sufficiently high to

absorb the higher supply of output, and this is only possible if the price of that output increases. Most crucially for our purposes, the proposition shows that the threshold at which the TOT response changes sign, $\widetilde{\phi}_{TOT}(\alpha)$, is a *decreasing* function of α: as intermediate inputs play a larger role in production, $\widetilde{\phi}_{TOT}(\alpha)$ declines; while Φ_α may go in either direction, depending on the relative trade elasticities.

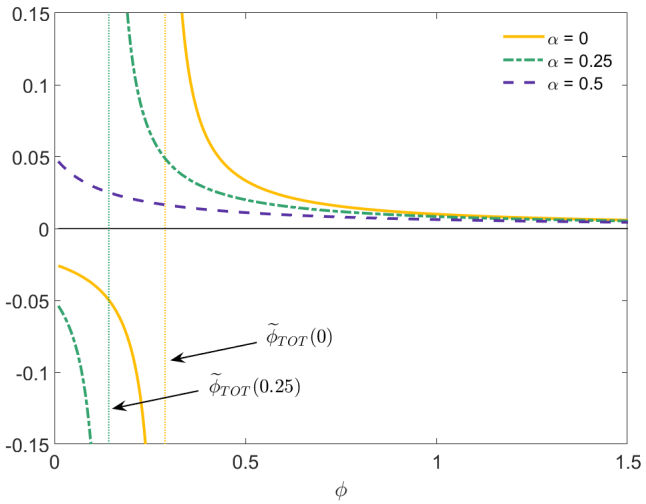
We illustrate the equilibrium effects of a positive Home productivity shock in Figures (1) and (2). Figure (1) considers the case of an economy either with CM or with IM but moderate complementarity in the use of domestic and imported intermediates, ensuring that $\Phi_\alpha > \widetilde{\phi}_{TOT}(\alpha)$, so that the RD curve is upward sloping. In an economy with no GVC, the RS schedule is vertical and output is exogenous. A positive productivity shock in the Home economy (i.e., a higher supply of Home output) would shift the equilibrium from point "a" to point "c", causing a drop in the relative price of Home goods (higher $\mathcal{T}$). In the presence of GVCs, the RS schedule is downward-sloping. Because a Home productivity gain affects production costs both at Home and abroad, the equilibrium response to the same productivity shock is a smaller adjustment in both relative prices and both output vis-à-vis the no-GVC economy, i.e., a shift from point "a" to point "b".

The impact of GVCs can be quite different in economies where domestic and imported intermediates are strongly complementary—the case in which, with $\Phi_\alpha < \widetilde{\phi}_{TOT}(\alpha)$, the RD schedule is downward sloping in $\{\widehat{\mathcal{T}}, \widehat{Y}_R\}$ space—see Figure (2). In this case, as shown by Corsetti et al. (2008), without GVCs, a positive supply shock to home productivity leads to an equilibrium *rise* in the relative price of home output (lower $\mathcal{T}$): the market equilibrium shifts from point "a" to point "c". Key to this result are strong relative wealth effects from price movements under incomplete markets: given complementarity of inputs and goods and a sufficient degree of home bias, the home relative demand must rise enough to buy the larger supply of home goods reflecting productivity gains—this is only possible if a higher supply price raises home income sufficiently.

Our novel contribution consists of highlighting the relevance of the macroeconomic interactions between relative wealth effects and global cost dependence. Starting from an equilibrium at point "c" in the figure, by continuity, terms-of-trade appreciation would also be the outcome in economies with a low degree of GVCs integration, since a small enough GPCI would not overturn the condition (31). Crucially, however, increasing global production-cost dependence eventually makes the RS schedule *flatter* than the RD curve. In other words, high GPCI eventually rules out TOT appreciation in equilibrium. As shown in the right panel of the figure, a relative productivity shock then brings the market equilibrium to point "b", where the relative price of Home goods is lower (the TOT depreciates, increasing $\mathcal{T}$).

These results are illustrated in a different way in Figure 3, where, for simplicity, we further restrict the trade elasticities for intermediate and final goods to coincide: $\phi_C = \phi_X = \phi$. The figure plots the impact response of the TOT to a positive Home productivity shock, against the trade elasticity, for different values of α. The line conditional on $\alpha = 0$ (orange line) reproduces Corsetti et al. (2008). The other lines allow for two values of α.

Figure 3: Impact Response of the Terms of Trade to a Home Productivity Shock



Note: Positive shock to A_t with $\rho_A = 0.8$, $b_H = a_H = 0.7$ and $\sigma = 2$. Picture drawn for $\phi_C = \phi_X = \phi$.

There are two takeaways from the new figure. First, an increase in GVCs integration from null to moderate, $\alpha = 0.25$ (purple line), shifts the region where the TOT appreciates to the left—the threshold $\widetilde{\phi}_{TOT}(\alpha)$ falls. A deeper degree of GVCs integration, setting $\alpha = 0.5$ (green line), eliminates this region altogether. Indeed, for a sufficiently high α, the threshold $\widetilde{\phi}_{TOT}(\alpha)$ defined above becomes non-positive.[7] Second, GVCs integration mutes the impact response of the TOT to productivity shocks when $\phi_C < \widetilde{\phi}_{TOT}(\alpha)$, but magnifies it otherwise. It is worth repeating the economic intuition. As long as $\phi_C < \widetilde{\phi}_{TOT}(\alpha)$, a higher GPCI reduces the supply price of a country when it experiences a productivity gain, since cheaper domestic intermediates translate into lower production costs abroad, hence into lower prices of foreign intermediates. These in turn lower domestic production costs and prices even more. On the contrary, when $\phi_C > \widetilde{\phi}_{TOT}(\alpha)$, through the same mechanism higher domestic supply prices in response to productivity gains, required to bring home and global demand in line with higher supply of domestic goods, imply higher production costs both domestically and abroad. As discussed below, if markets are incomplete and production risk is not shared, these relative price and demand movements have crucial implications for income risk risk sharing.

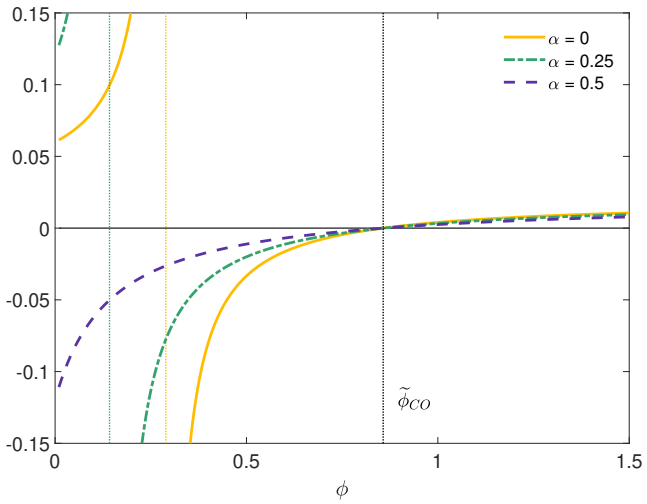
4 GVCs and Risk Sharing

In light of the transmission mechanism explained above, we now turn to the core question in our paper, concerning the effects of global production-cost dependence on the cross-border sharing of production risk.

To this end, we rely on the wealth gap in equation (20) as the welfare-relevant index of the distance from perfect risk sharing. This gap is reproduced below, for convenience, in log-

[7] Note that $\widetilde{\phi}_{TOT}(\widetilde{\alpha}) = 0$ at a critical degree of GVC integration $\widetilde{\alpha} = 1 - 2a_F$.

Figure 4: Impact Response of Risk-sharing Wedge to Home Productivity Shock



Note: Positive shock to A_t with $\rho_A = 0.8$, $b_H = a_H = 0.7$ and $\sigma = 2$. Picture drawn for $\phi_C = \phi_X = \phi$.

linearised form:

$$\widehat{\mathcal{W}}_t = \sigma\left(\widehat{C}_t - \widehat{C}_t^*\right) - \widehat{\mathcal{Q}}_t = \sigma\left(\widehat{C}_t - \widehat{C}_t^*\right) - (2a_H - 1)\,\widehat{\mathcal{T}}_t \tag{32}$$

Figure 4 plots the wealth gap in response to productivity shocks against the weighted average trade elasticity (Φ_α), under the same setting as Figure 3, i.e. an economy under FA and equating home bias coefficients as well as the elasticities $\phi_C = \phi_X = \phi$ for simplicity.

The wealth gap response is clearly different for low and high elasticities. Above the threshold $\widetilde{\phi}_{CO}$, to be defined below, the gap is very small and it worsens (at the margin) with a rising GVCs integration. For elasticities below $\widetilde{\phi}_{CO}$, instead, the gap can be significantly large and can switch sign as goods and intermediates become strong complements (ϕ shrinks towards zero). This suggests that, as we will show next, depending on the elasticities and the initial GPCI, deepening GVCs can have opposing effects on risk sharing.

To show this most clearly, we start from a key property of the model, highlighted by the figure. Independently of α, at $\phi = \widetilde{\phi}_{CO}$, the wealth gap is zero—there is no gain in sharing production risk, at any degree of GVCs integration. This is a generalization of the well-known result of Cole and Obstfeld (1991) to an economy with GPCI.

Lemma 1 (Cole-Obstfeld with GVCs) *Assuming $b_H = a_H$, perfect international risk sharing can be attained under Financial Autarky when the weighted-average trade elasticity, Φ_α, satisfies*

$$\Phi_\alpha = \frac{1}{2a_H} + \frac{2a_H - 1}{\sigma 2a_H} \equiv \widetilde{\phi}_{CO}$$

Proof: See Appendix G. □

At $\phi = \widetilde{\phi}_{CO}$, in equilibrium, the TOT responds to productivity shocks proportionally to relative output, so as to keep the *income* of home and foreign residents constant. That way,

output variability does not translate into income risk. Trade in assets is irrelevant and there are no gains from moving from financial autarky to either a bond economy or to complete markets, provided productivity is the only source of uncertainty. Relative to Cole and Obstfeld (1991), the above lemma underscores that the TOT movements providing insurance are endogenous to, and therefore vary with, GVCs.

Going back to Figure 4, for elasticities above $\widetilde{\phi}_{CO}$, goods and intermediates are relatively more substitutable, and the TOT responds less than relative output: the income of the residents in a country hit by a positive (negative) productivity shock rises (falls), generating income risk. Since a higher GPCI tends to contain the response of the TOT in this case, GVCs integration increases the income risk at the margin, worsening the wealth gap under incomplete markets. Yet a relatively high substitutability of domestic and foreign goods and intermediates ensures that the movements of the TOT are small, and the wealth gap remains overall negligible.

The picture changes significantly for elasticities below $\widetilde{\phi}_{CO}$. For a given α, with low elasticities the TOT response becomes larger than the change in relative net output. If $\widetilde{\phi}_{TOT} < \phi < \widetilde{\phi}_{CO}$ the supply price of the country experiencing a productivity gain falls so much that the residents become poorer, despite producing more. The opposite is true for $\phi < \widetilde{\phi}_{TOT}$, where the income of such a country is boosted by the equilibrium appreciation of their supply price. In either case, the wealth effects from the TOT adjustment amplify income risk, translating into large risk-sharing wedges.

Key to our analysis, however, is that, for a given elasticity ϕ, deepening GVCs integration changes the threshold $\widetilde{\phi}_{TOT}$, hence it may bring about dramatic changes in the international transmission mechanism. We synthesize our main result in the proposition and corollary below, both derived under the simplifying assumptions that $b_H = a_H$ and $\phi_X = \phi_C = \phi$.

Proposition 2 (GVC Integration and Risk Sharing) *Positing $b_H = a_H$ and $\phi_X = \phi_C = \phi$, provided $\phi < \widetilde{\phi}_{CO}$, the marginal impact of increased GVC integration is to reduce risk sharing whenever $\phi < \widetilde{\phi}_{TOT}(\alpha)$, and improve risk sharing whenever $\phi > \widetilde{\phi}_{TOT}(\alpha)$.*

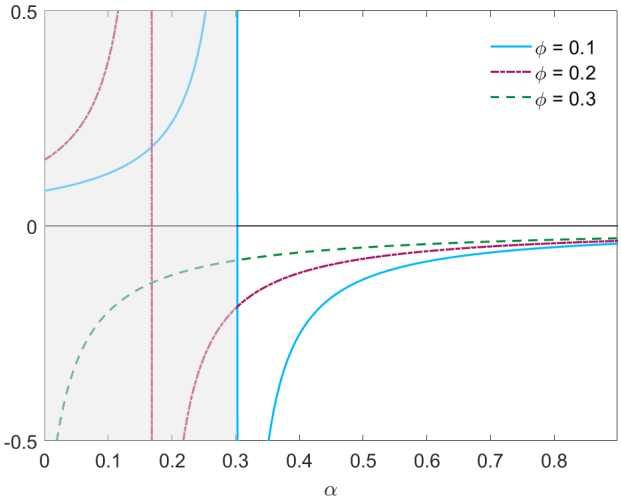
Proof: See Appendix H. □

Corollary (Fragmentation Traps) *When $b_H = a_H$ and $\phi_X = \phi_C = \phi < \widetilde{\phi}_{TOT}$, starting from a low degree of GVCs integration, raising GPCI marginally worsens risk sharing, while sufficiently deeper integration improves it.*

Proof: See Appendix I. □

The corollary is best illustrated by using Figure 5, which plots the risk-sharing wedge against α for three different values of the trade elasticity. Note that the figure is drawn for relatively low elasticities, in the region of high complementarity of intermediate inputs (and goods). In the graph, a negative response of the risk sharing wedge corresponds to the case of a trade elasticity ϕ above $\widetilde{\phi}_{TOT}$—so that the Home terms of trade depreciates in response to Home productivity gains. With a negative wedge, deeper integration into GVCs (a higher α) reduces

Figure 5: Impact Response of the Risk-sharing Wedge to Home Productivity Shock against GVC integration



Note: Positive shock to A_t with $\rho_A = 0.8$, $b_H = a_H = 0.7$, $\sigma = 2$. Pictures drawn for $\phi_C = \phi_X = \phi$. Here, $\widetilde{\phi}_{TOT} = 0.29$.

the size of the wedge by moderating the excessive depreciation of the terms of trade. A positive risk sharing wedge instead corresponds to the case when ϕ is below $\widetilde{\phi}_{TOT}$, so that the terms of trade *appreciate*. With a positive wedge, deeper integration worsens, instead of ameliorating, risk sharing. But raising α's also moves the threshold $\widetilde{\phi}_{TOT}$ to the left, implying that GVCs integration at some point can change the sign of the transmission mechanism, from positive to negative. Because of this, the "fragmentation trap" stated in the corollary can arise.

Looking at the figure, focus on the line drawn for $\phi = 0.1$ and start at a very low level of integration, $\alpha = 0.1$. From this point, a marginal reduction in GPCI (an even lower α) reduces the risk-sharing wedge. Since the threshold $\widetilde{\phi}_{TOT}$ does not move enough to fall below the trade elasticity, a marginal reduction in exposure to global cost shocks reduces the volatility of the terms of trade. Strengthening interconnections further, however, changes the transmission mechanism—the TOT response to productivity gains switches sign. In this case, as shown in the figure, GVC deepening (for higher values of α) first widens the risk-sharing wedge, but ultimately improves risk sharing.

We label this result *Fragmentation Trap* because, starting from an intermediate GPCI (marked by the shaded area in the figure), non-monotonicity in the effects of GVC integration may create incentives to delink domestic production to contain production cost risk, while this risk could be better shared via a much stricter integration. The position and size of the parameter region where the fragmentation trap arises is a quantitative question.

We should note that, through risk sharing, GVCs impinge on the allocation of risk across countries. As a standard measure of macro risk, we rely on the volatility of aggregate consumption in each country, which under CRRA preferences is proportional to the volatility of

the stochastic discount factor. In the model, this is a function of the variance of the productivity shocks, the degree of global production-cost dependence, and the structure of international financial markets, as shown in Appendix B:

$$\text{Var}(\widehat{C}) = \frac{1}{2}\sigma_A \left[1 + \left(\frac{\mathcal{G}_i}{\mathcal{D}_i + 2GPCI}\right)^2\right]$$

As shown in that Appendix, this formula implies that fragmentation traps arise also total risk: for intermediate levels of integration, total risk falls with marginal de-linking from GVCs, while it could actually fall much more with non-marginal deeper integration. While the range of GVC integration and trade elasticity values at which the trap arises in risk-sharing space does not coincide exactly with that for the trap in the domain of total risk, the two are related and the overall conclusion stands.[8] By the international transmission mechanism embedded in the model, in general, deeper integration of foreign intermediate inputs that are complementary to domestic ones in production tend to reduce exposure to national production shocks.

In light of this result, one may note that, to the extent that national security concerns and geopolitical risk motivate delinking, reshoring and source-diversification strategies, the global economy could move to a configuration featuring lower GPCI and higher trade elasticities. This repositioning of the economy may actually raise the likelihood of a fragmentation trap scenario, potentially reinforcing the incentives to delink further.

5 Quantitative and Empirical Evidence

In this section, we take a first pass to assessing the quantitative and empirical relevance of our analysis. Relaxing the parameter restrictions imposed so far for analytical transparency, we explore the implications of GVC integration for risk sharing calibrating the key parameters, a_H, b_H and α, to the data. We also extend our baseline model in several directions, allowing for differences in the trade elasticity in the short- and the long-run, trade in non-contingent bonds, CES aggregators in production, and endogenous labour supply. We complement the model-based analysis with reduced-form macroeconomic evidence relying on empirical measures of the wealth gap for a panel of advanced economies.

5.1 Calibrated Model

The simplifying assumptions $a_H = b_H$ and $\phi_C = \phi_X$ we used in the previous section allowed us to highlight the supply-side impact of GVCs removing demand-side differences between trade in final goods and intermediate inputs. This meant that we could capture the degree of GVC integration with α, the share of intermediate inputs in production. Allowing for evidence-based differences in a_H and b_H, the degree of GVC integration will is instead captured by a

[8] As shown in equation (20), risk sharing is a function of relative consumption and RER movements. By contrast, total risk only reflects consumption movements.

combination of α and b_H.[9] In order to explore the implications of the model in its more general form, we turn to data to pin down the values of these parameters.

Data. We use the World Input-Output Tables (WIOT) data, which is available for 28 advanced economies, at annual frequency over the period 2000-2014.[10] This data allows us to measure the share of intermediates in gross output and the imported share of both intermediate inputs and final goods.

In the model, we can combine the firms' optimal demand for intermediate inputs, equation (10), and the optimal demand for imported intermediate inputs, equation (14), to write imported intermediate goods as a share of gross output:

$$\frac{P_{F,t}\, x_{F,t}}{P_{H,t}\, Y_{H,t}} = \alpha b_F \left(\frac{P_{F,t}}{P_{X,t}}\right)^{1-\phi_X}. \tag{33}$$

If, on average over a sufficiently long period, relative prices are equal to one, then the mean of this measure will equal αb_F, capturing structural integration into GVCs. In light of this, we calculate shares taking averages over time for each country, essentially smoothing out the effect of relative prices.

As is well documented in the literature, import shares tend to reflect the size of economies, with smaller countries being more dependent on trade and therefore having higher import shares. Since our model features symmetric country sizes, we need to correct for this. We do so by adjusting import shares by country size following De Paoli (2009). Namely, we divide import shares by $(1-n)$, where n is the relative size of the country, defined as their GDP relative to total GDP of the group of 28 countries, on average over the period 2000-14.[11] The adjusted measures of import shares for intermediate inputs and final goods translate into our measures of b_F and a_F respectively.

Three calibrations. We will compare the model under the three calibrations of GVCs integration presented in Table 1. The first is an 'Average' calibration, corresponding to the unweighted average of a_F, b_F and α across countries, shown in the first row of the table. On average, home bias is signficant in both final consumption and intermediate inputs, with import shares as high as 18% and 26%, respectively, and intermediate inputs accounting for 51% of gross output.

The other two calibrations are labelled 'Most Integrated' and 'Least Integrated', based on the parameter values of the country with, respectively, the highest and the lowest GVC integration as measured by αb_F.[12] In the Most Integrated calibration, the import shares in final goods and

[9] Baldwin, Freeman, and Theodorakopoulos (2022) explore various ways of measuring GVC integration, and discuss the differences between considering imported inputs as a share of total inputs (b_F in our model), or a share of gross output ($\alpha\, b_F$ in our model).

[10] The full set of countries are: AUS, AUT, BEL, CAN, CHE, CZE, DEU, DNK, ESP, EST, FIN, FRA, GBR, GRC, IRL, ITA, JPN, KOR, LTU, LUX, LVA, NLD, NOR, PRT, SVK, SVN, SWE, USA.

[11] GDP data is taken from the OECD.

[12] These calibrations correspond to Luxembourg and Japan respectively.

Table 1: Calibrated Parameter Values

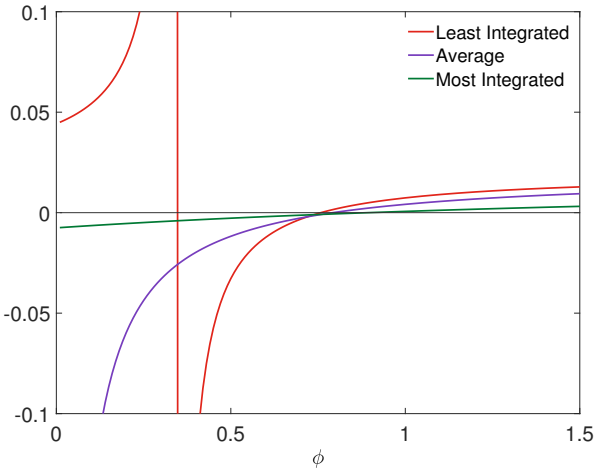
	a_F	b_F	α
Average	18%	26%	51%
Most Integrated	40%	58%	65%
Least Integrated	6%	11%	47%

Note: Average refers to the unweighted mean over the 28 countries. 'Most Integrated' and 'Least Integrated' refer to the highest and lowest αb_F, respectively.

intermediate inputs are 40% and 58% respectively, while the share of intermediate inputs in production is 65%. Conversely, in the Least Integrated calibration there is significantly more home bias, with import shares as low as 6% and 11%, yet the share of inputs in production is still relatively high, at 47%.[13]

While we solve our model using the three calibrations for the parameters α, b_H and a_H shown in the table, we allow for different values for the trade elasticity keeping the restriction $\phi_X = \phi_C = \phi$. As is well known, ϕ_C and ϕ_X, are famously difficult to estimate and studies have come up with a wide range of estimates (Feenstra, Luck, Obstfeld, and Russ, 2018; Freeman, Larch, Theodorakopoulos, and Yotov, 2021). For transparency of interpretation, we find it preferable to explore this range moving a single parameter.

Figure 6: Risk-sharing Wedge Responses to Home Productivity Shock



Note: Average, Most Integrated and Least Integrated are defined as in Table 1.

Results. Figure 6 shows the response of the risk-sharing wedge to a Home productivity shock under our three calibrations. The pattern is close to Figure 4.[14] For the Least Integrated

[13] 65% is the highest α in our sample, while the lowest α is actually 42%. The correlation of α with the two import shares is around 0.55-0.60, highlighting that, while this parameter is related to GVC integration, it also captures other structural factors such as the sectoral composition. Using $\alpha = 42\%$ in the Least Integrated calibration does not significantly change the results.

[14] Once the restriction $a_H = b_H$ is relaxed, the three lines in the figure do not need to cross zero at the same point.

calibration, the impact response of the wealth gap is positive to the left of the graph, and negative to the right. The change in sign is determined by the elasticity threshold ϕ_{CO}, defined in Lemma 1 under parametric restrictions, and here computed allowing for differences in home bias across consumption and production. In the Average calibration, this elasticity threshold essentially drops to zero, and the wealth gap is negative for all elasticities. The same applies to the Most Integrated calibration. In this case, however, the wealth gap remains remarkably small for all ϕ's. The high level of GVCs integration appears to be effective in containing deviations from perfect risk sharing.

Most crucially, however, our results suggest that, for low trade elasticity values, the wealth-gap ranking between the Average and Least Integrated calibrations reverses. That is, the Least Integrated calibration exhibits better risk sharing than the Average calibration in this region, although the Most Integrated has better risk sharing than both. Our empirically-backed quantitative assessment thus replicates the 'fragmentation traps' result from Section 3.

5.2 Model Extensions

We verify our results in four quantitatively relevant model extensions.

Time-Varying Trade Elasticities. In line with the literature, our analysis highlights the importance of the trade elasticity for the transmission of shocks across borders. It stands to reason that there could be differences in this elasticity, i.e. it can be expected to be lower in the short run than in the long run—see Boehm, Levchenko, and Pandalai-Nayar (2023) for recent evidence. The difference reflects the time and cost required to adjust trade patterns. With GVCs, it also captures the idea that firms may need time to modify their production process in response to relative price movements in their inputs.

One way to model different short- and long-run trade elasticities in our framework is to assume external habits in input demand, à la Alessandria, Bai, and Woo (2023). Specifically, we amend the intermediate-input aggregator as follows:

$$X_t \equiv \left[b_H^{\frac{1}{\phi_X}} \left(\frac{x_{H,t}}{(x_{H,t-1}/X_{t-1})^{\frac{\theta}{1-\phi_X}}} \right)^{\frac{\phi_X-1}{\phi_X}} + b_F^{\frac{1}{\phi_X}} \left(\frac{x_{F,t}}{(x_{F,t-1}/X_{t-1})^{\frac{\theta}{1-\phi_X}}} \right)^{\frac{\phi_X-1}{\phi_X}} \right]^{\frac{\phi_X}{\phi_X-1}},$$

where the parameter $\theta \in [0, 1)$ controls the importance of habits. When $\theta = 0$, we return to the baseline model with no habit formation.

The specification of habits as *external* means that firms take the additional lagged terms in the above expression as given, i.e. do not internalise that their current decisions will impact those terms in the next period. This implies that the short-run trade elasticity remains ϕ_X. The long-run elasticity, instead, is given by $\phi_X/(1-\theta)$. Thus, the higher the incidence of habit formation, the larger the discrepancy between short-run and long-run elasticities, with the latter

always being larger.[15]

The extended model has two crucial properties. First, for a given short-run trade elasticity, ϕ_X, the *impact* response of all variables to productivity shocks is independent of the degree of habit formation, θ. Thus the response of the risk-sharing wedge, as plotted in Figure 6, is unchanged, regardless of the long-run trade elasticity. Second, habit formation affects the dynamics of the model over time.

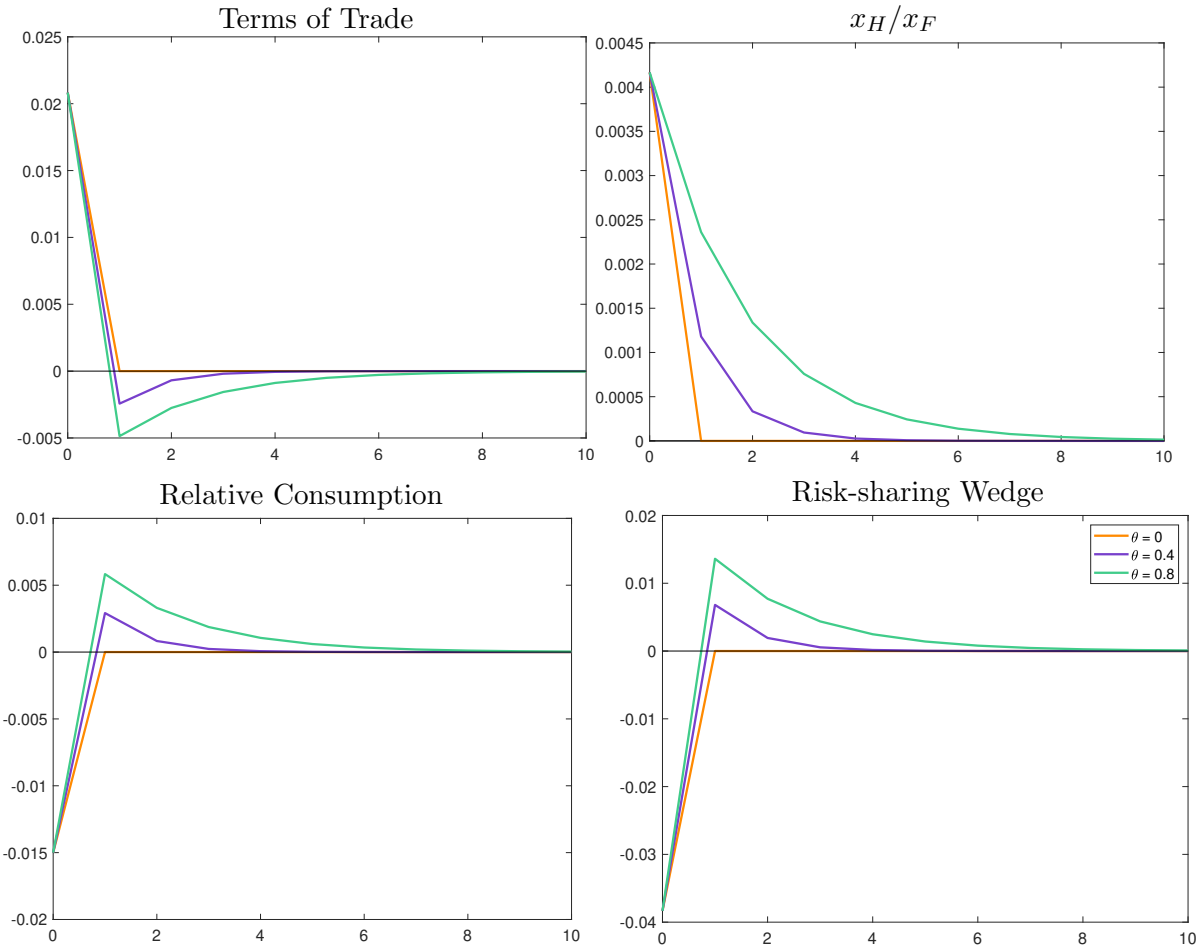
To illustrate these properties most clearly, in Figure 7 we plot the *impulse* response functions following a one-off productivity shock. In our baseline without habits, $\theta = 0$, all responses return to zero in the second period, as the shock disappears. The TOT depreciation on impact raises the relative demand for Home inputs in accordance with the rather low trade elasticity 0.2, lowering relative consumption and widening the risk-sharing wedge, but the shock has no lasting consequences. With habits, while the impact responses are unchanged, the impulse responses from period 1 on exhibit endogenous persistence in the use of intermediates. By force of habit, the rise in the relative demand for Home inputs in period 0 induces higher demand for the same inputs in period 1, acting like a positive demand shock that induces an appreciation of Home's TOT. The wealth effects associated with the appreciation actually raise Home consumption relative to Foreign consumption, turning the wealth gap from negative to positive. The higher θ is, the larger and more persistent this dynamic effect of shocks is, leading to longer lasting deviations from perfect risk sharing.

While the predictions of the habit-augmented model are stark, they underscore two crucial empirical issues. The first is the implications of strong input complementarity in GVCs (whether due to technology or to informational frictions in contracts) for the transmission of shocks. These may constrain firms' production for some months, in line with evidence from Carvalho et al. (2021). In our framework the short run is insulated from the long run, but in other model specifications this may not be true. Yet the dynamics of the TOT and other macroeconomic variables over time may still be heavily affected by complementarity. The second is the extent to which adjustment costs may create endogenous fluctuations in demand, wealth and relative prices that impinge on the dynamics of risk sharing over time.

Figure 8 turns to a very persistent/nearly permanent productivity shock. Without time-varying trade elasticities, the impulse-response profiles of all key variables fully reflect their responses on impact: once an exogenous shock raises the relative productivity of Home firms in a persistent way, the TOT depreciate persistently and Home's consumption falls persistently below Foreign consumption, opening a lasting risk-sharing wedge. With time-varying elasticities, producers can further adjust the mix of inputs over time, and that progressively mitigates the movement in relative output. Thanks to that, relative consumption and the TOT fall back over time, closing the risk-sharing wedge. Therefore, under persistent productivity shocks GVCs help improve risk sharing even in economies with rather low trade elasticities like the one considered here.

[15] Full derivations in Appendix J.

Figure 7: Impulse Responses to a Transitory Shock with Time-Varying Trade Elasticities

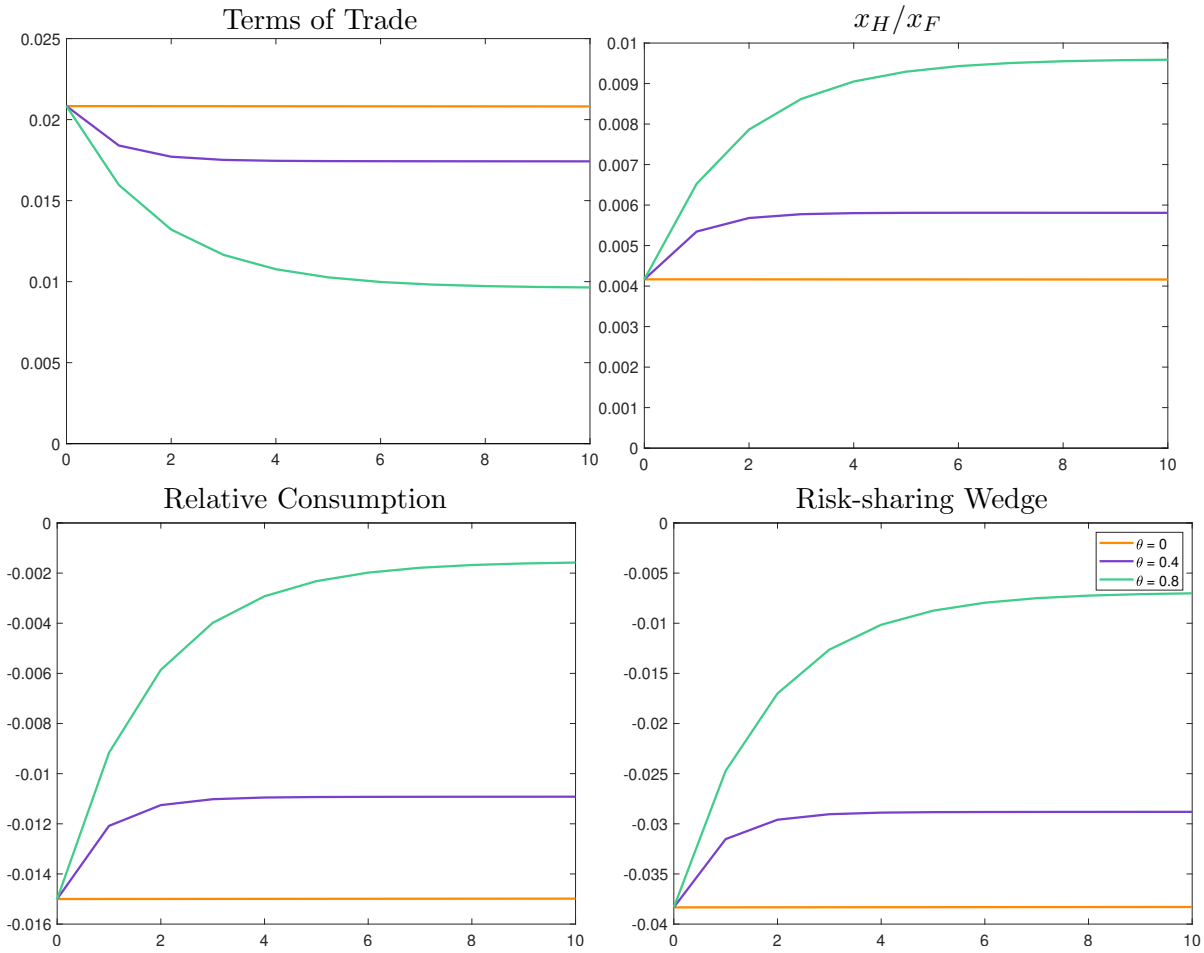


Note: Impulse responses to a one-off positive shock to A_t, with $\phi = 0.2$ and $\alpha = 0.5$.

Bond Economy. Our benchmark results are derived assuming an extreme form of financial market incompleteness: FA. A key question is the extent to which our quantitative findings are robust to modelling a bond economy, in which households have access to non-state-contingent one-period international bonds, in zero net supply across countries.[16] The impact response of the risk-sharing wedge to a productivity shock in this economy is shown in Panel A of Figure 9, again for the three calibrations of GVC integration introduced above. The overall pattern across the different degrees of integration is qualitatively unchanged. Yet, the scale of the y-axis is one tenth of the baseline figure. Although financial markets are not complete, international borrowing and lending allow households to self-insure smoothing consumption across periods. This reduces the gap relative to CM economies.

[16] Following Bodenstein (2011), we additionally include Uzawa-style discounting in this model variant to ensure that (i) bond positions are stationary and (ii) the equilibrium is unique.

Figure 8: Impulse Responses to a Persistent Shock with Time-Varying Trade Elasticities



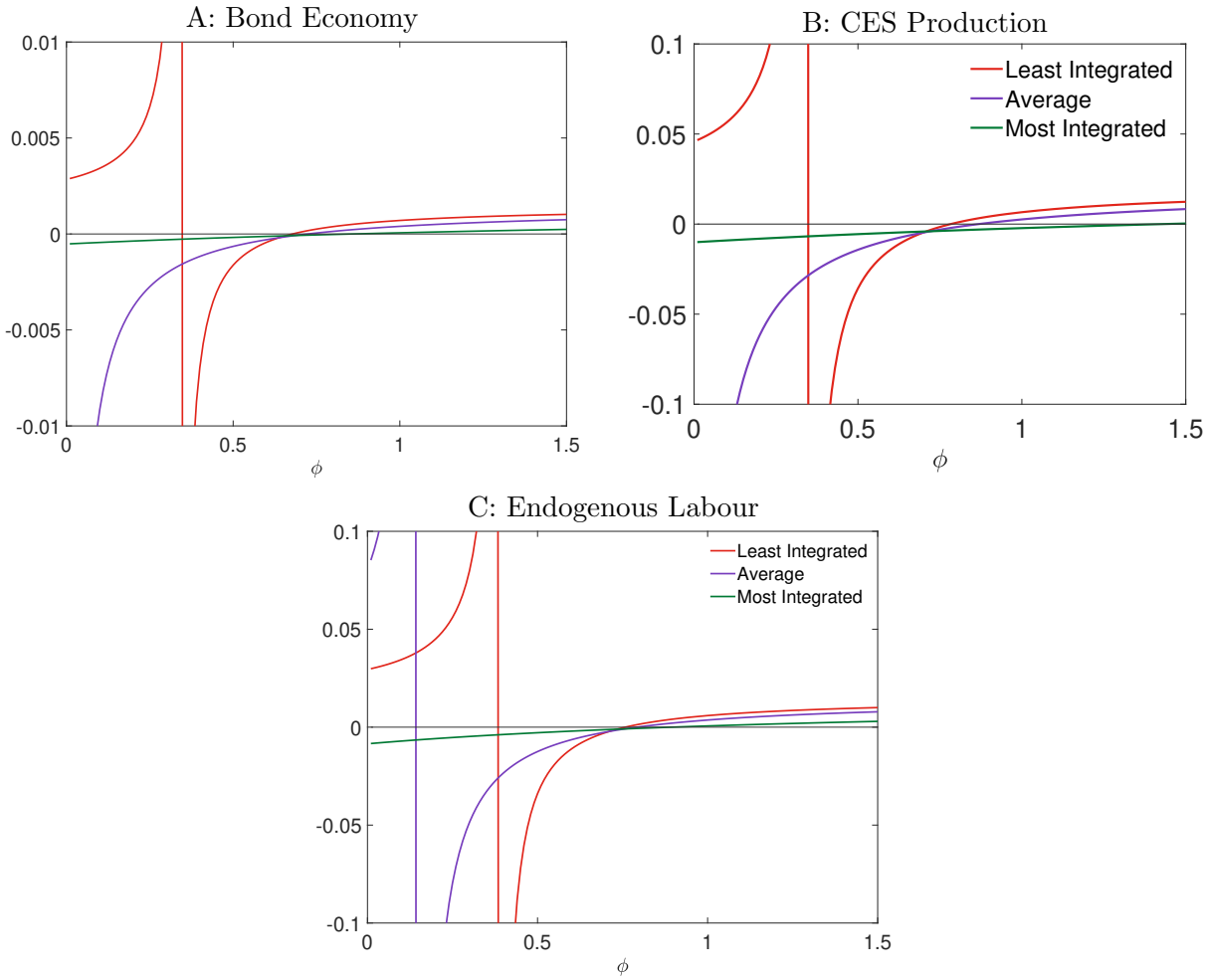
Note: Impulse responses to a persistent positive shock to A_t, $\rho = 0.9999$, with $\phi = 0.2$ and $\alpha = 0.5$.

CES Production. In another exercise, we replace the Cobb-Douglas specification of the production function (7) with a CES production function. We assume that the elasticity of substitution between labour and intermediate inputs is 0.85, consistent with a range of estimates that suggest a value weakly below 1.[17] As seen in Panel B of Figure 9, the impact response of the wealth gap to a productivity shock looks very similar to the baseline. The notable difference is that the Most Integrated calibration has a negative wealth-gap response for a wider range of elasticities.

Endogenous Labour Supply. In our baseline, we abstract from wealth effects via labour supply by assuming that this is exogenously given. These effects can clearly impinge on the cross-country co-movements in IM models. To verify the robustness of our quantitative findings to endogenous labour supply, we extend the model assuming that households have period-by-

[17] This is the central estimate in Atalay (2017), in the middle of the range of estimates in Oberfield and Raval (2021), and within the confidence bands of the estimates of Peter and Ruane (2024).

Figure 9: Robustness of Risk-sharing Wedge Responses



Note: Average, Most Integrated and Least Integrated are defined as in Table 1.

period labour disutility equal to $L_t^{1+1/\psi}/(1+1/\psi)$—where ψ is the inverse Frisch elasticity of labour supply. We calibrate this parameter to a standard value, $\psi = 0.5$. As shown in Panel C of Figure 9, while the overall patterns are overall unchanged, the responses are amplified relative to our baseline. Most crucially, the elasticity threshold is now strictly larger than zero also for the Average calibration.

5.3 Macro-Level Evidence

To explore the evidence on the mechanism highlighted in the model, we turn to an empirical exercise. We construct empirical measures of the wealth gap and GVC integration for the same sample of countries and time period we use in our model calibration.[18]

[18] Relative to the previous sub-section, we use the same countries with the exception of CZE, EST, LTU, LVA, SVK and SVN, which we drop owing to limited data availability for the wealth gap. This leaves us with 22 countries.

GVC Integration. Our measure is the share of imported intermediate inputs in gross output, as in equation (33). As discussed above, this measure provides a good estimate of αb_F if averaged over a sufficiently long period, so to reduce the distortions from relative price fluctuations. Now, in our empirical exercise, we need a GVC integration measure at an annual frequency. To improve its reliability, we take three-year moving averages.

Wealth Gap. As shown in our model section, this gap combines country-level data for real final consumption expenditure with the consumer price indices (CPI) and the nominal exchange rate, with a weight given by risk aversion. We use consumer and price data from the OECD, and the nominal exchange rate data (measuring currency per US dollar) from the BIS. For each country, we construct *relative* consumption by defining 'foreign' consumption as a PPP-weighted average of the remaining countries in the sample—with weights from the IMF World Economic Outlook data. Real exchange rates are given by the ratio of CPI across countries, converted into US dollar terms using the nominal exchange rate data, where, again, 'foreign' prices are PPP-weighted averages of the remaining countries in each period.

The empirical measures of the wealth gap at country level are then derived applying the definition in equation (32), under alternative parameterisations of σ. Formally, we follow the approach of Corsetti, Dedola, and Viani (2012a,b) and first construct a measure of the cyclical component of the wealth gap at quarterly frequency. To do this, we use data for 1990Q1-2019Q4 and use the Hodrick-Prescott filter to detrend each series.[19] We then calculate the cyclical wealth gap by combining the cyclical components of each variable as in equation (32). Finally, to match the annual periodicity of our GVC integration measure, we isolate end-of-year values of the wealth gap for the period 2000-2014 for our regression analysis.

Regression Analysis. Table 2 presents results from a cross-country panel regression. We regress the absolute values of the wealth gap in logarithms on countries' degree of GVC integration—measured by the three-year moving average of intermediate import share—including modularly a number of controls.

Consistent with the theoretical predictions outlined in Section 5.1, the results in the table show that GVC integration and deviations from full risk sharing are negatively related. Since the regression equations take a log-linear form, the estimated coefficients can be interpreted as semi-elasticities. A 1 percentage point increase in a country's share of imported intermediate goods is associated with a $1-2$ percent decrease in its absolute wealth gap.

Columns (1) and (5) of the table present the bivariate association between GVC integration and the absolute log wealth gap—measured setting $\sigma = 1, 2$, respectively. Columns (2)-(3)-(4) and (6)-(7)-(8) build on this by reporting coefficient estimates conditional on additional controls. In columns (2) and (6), we include the three-year moving average of countries' import share of consumption to control for openness in final goods trade, and relative GDP to control

[19] Our results are robust to using data for 2000Q1-2014Q4 to construct the wealth gap measure. We prefer the longer sample for the purposes of detrending however.

Table 2: Association Between GVC Integration and Absolute Log Wealth Gap

	Dep. Var.: Absolute Log Wealth Gap $\lvert \ln \mathcal{W}_{i,t} \rvert$							
	$\sigma = 1$				$\sigma = 2$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Int. Import Share MA	-0.03***	-0.10**	-0.11***	-0.10***	-0.02**	-0.10	-0.12*	-0.16***
	(0.010)	(0.041)	(0.035)	(0.034)	(0.010)	(0.065)	(0.061)	(0.057)
Cons. Import Share MA		-0.20*	-0.31**	-0.24		-0.12	-0.24	-0.25
		(0.119)	(0.153)	(0.214)		(0.159)	(0.161)	(0.220)
Relative GDP		-0.04*	-0.05	-0.06**		-0.06***	-0.07**	-0.08***
		(0.026)	(0.031)	(0.028)		(0.024)	(0.030)	(0.026)
VA Share MA			-0.14	-0.07			-0.15	-0.08
			(0.104)	(0.098)			(0.105)	(0.122)
Observations	286	286	286	286	286	286	286	286
Country FE	No	Yes	Yes	Yes	No	Yes	Yes	Yes
Year FE	No	No	No	Yes	No	No	No	Yes
# Countries, N	22	22	22	22	22	22	22	22
Within R-squared	0.015	0.036	0.041	0.155	0.008	0.032	0.037	0.117

Note: Country-level GVC linkages measured using the three-year moving average of gross output share of imported intermediate inputs for the period 2000-2014 using the data from WIOT. Country-level wealth gap measured using HP-filtered relative consumption and CPI data from OECD and nominal exchange rate data from BIS, with foreign quantities PPP-weighted for the period 1980Q1-2019Q4. Regression estimated for 2000-2014 using end-of-year data for absolute wealth gap. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

for country size and country-specific changes in income.[20] In addition, we include country fixed effects to account for country-specific features. To ensure that our result on the first row is specifically related to the share of imported inputs in production, in columns (3) and (7), we add the three-year moving average of countries' value-added share. Finally, in columns (4) and (8), we include year fixed effects to account for global shocks. In all these cases, the association between deviations from full risk sharing and GVC integration remain robustly negative and significant.

Our explorative empirical analysis appears to lend support that in the sample higher GVC integration is associated with smaller deviations from perfect risk sharing.

6 Conclusion

In this paper, we offer a macroeconomic assessment of the notion that, barring risks of disruption of global value chains reflecting geopolitical or natural events, a high degree of integration into GVCs tends to foster efficient sharing of production risk via cost diversification. While offering theoretical and empirical support to this notion, we also show that, for empirically reasonable level of integration, intermediate-goods-trade linkages can actually hamper a country ability to share risks, even if GVCs operate frictionlessly. Under incomplete international financial markets, this occurs because, in response to productivity gains in one country, GVCs may actually *raise* production costs abroad instead of lowering them, worsening global production conditions.

This result is derived in a standard two-country, two-good open-economy macro model with

[20] Relative GDP for each country is defined in the same way as relative consumption.

roundabout production that is widely used in academic and policy analyses, with a general specification that nests a range of empirically-relevant cases. Under educated simplifying assumptions, we have derived transparent and insightful analytical expressions, shedding light on the economics of the international transmission of shocks varying the degree of GVCs integration.

Our analysis contributes a macro-global perspective to the contemporary policy debates around the benefits and risks associated with GVC integration. On one hand, the results from our calibrated theoretical model and exploratory macro-level empirical analysis indicate that the empirically relevant state of the global economy is one in which GVC integration serve to reduce misalignments in relative demand and prices, effectively enhancing the insurance of macro risks. On the other hand, the widely used policy model we adopt in our analysis also suggests that GVCs deepening is not necessarily conducive to production risk sharing—a result that is rooted in a general equilibrium feedback from international prices to relative wealth and demand across countries.

Looking ahead, to the extent that national security concerns and geopolitical risk motivate delinking, re-shoring and source-diversification strategies, the global economy could move to a configuration featuring lower GPCI and higher trade elasticities than reflected in our average calibration. This repositioning of the economy may actually raise the likelihood of the fragmentation trap scenario we discuss in this text, potentially reinforcing the incentives to delink further.

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Appendices

A Total Factor Productivity Shocks

In this appendix we compare the model with total factor productivity (TFP) against the one with labour-augmenting productivity. We can re-specify the production function:

$$Y_{H,t} = A_t L^{1-\alpha} X_t^{\alpha}$$

where A_t is now the exogenous country-specific TFP.

The first-order conditions of the firm with respect to labour and intermediates remain unchanged. Plugging these in to this production function, we derive the expression for gross output as a function of the TOT:

$$Y_{H,t} = A_t^{1/(1-\alpha)} L(\alpha)^{\frac{\alpha}{1-\alpha}} \left[b_H + b_F \mathcal{T}_t^{1-\phi_X} \right]^{-\frac{1}{1-\phi_X}\frac{\alpha}{1-\alpha}}$$

and the Foreign equivalent:

$$Y_{F,t}^* = A_t^{*\,1/(1-\alpha)} L^*(\alpha)^{\frac{\alpha}{1-\alpha}} \left[b_H + b_F \mathcal{T}_t^{-(1-\phi_X)} \right]^{-\frac{1}{1-\phi_X}\frac{\alpha}{1-\alpha}}$$

Bringing these together, the log-linearised expression for relative supply is:

$$\widehat{Y}_{H,t} - \widehat{Y}_{F,t} = \frac{1}{1-\alpha}\left(\widehat{A}_t - \widehat{A}_t^*\right) - 2b_F \frac{\alpha}{1-\alpha}\widehat{\mathcal{T}}_t. \tag{A.1}$$

The key difference with labour-augmenting technology shocks is that here GVC integration also affects the direct impact of relative productivity shocks on relative output, whereas in equation (23) it only affects the size of global production-cost dependence. Because α multiplies the relative productivity term in (A.1), the presence of intermediate inputs in the production function amplifies the effect of a given relative TFP shock on gross output. This effects comes out particularly clearly at high values of the trade elasticity, ϕ, where the equilibrium response of the TOT, and hence global production-cost interdependence, is small, and therefore this direct impact dominates. In contrast, labour-augmenting productivity shocks allow us to isolate production-cost interdependence from GVCs, abstracting from this direct impact.

B Total Risk

Here we derive and analyse the expressions for the total volatility of consumption in Section 4.

Note, under symmetry:

$$\mathrm{Var}(\widehat{C} + \widehat{C}^*) = 2\mathrm{Var}(\widehat{C}) + 2\mathrm{Cov}(\widehat{C}, \widehat{C}^*)$$
$$\mathrm{Var}(\widehat{C} - \widehat{C}^*) = 2\mathrm{Var}(\widehat{C}) - 2\mathrm{Cov}(\widehat{C}, \widehat{C}^*)$$

So that:

$$\text{Var}(\widehat{C}) = \frac{1}{4}\left[\text{Var}(\widehat{C}+\widehat{C}^*) + \text{Var}(\widehat{C}-\widehat{C}^*)\right] \tag{B.1}$$

Aggregate resource constraint:

$$\widehat{Y}_{H,t} + \widehat{Y}^*_{F,t} = \widehat{C}_t + \widehat{C}^*_t$$

From the supply function (22) and its Foreign counterpart:

$$\widehat{Y}_{H,t} + \widehat{Y}^*_{F,t} = \widehat{A}_t + \widehat{A}^*_t$$

Hence

$$\widehat{C}_t + \widehat{C}^*_t = \widehat{A}_t + \widehat{A}^*_t \tag{B.2}$$

Recall the expressions for relative consumption as a function of the terms of trade:

$$\begin{aligned} \text{CM:} \quad & \widehat{C}_t - \widehat{C}^*_t = \sigma^{-1}\left(2a_H - 1\right)\widehat{\mathcal{T}}_t \\ \text{FA:} \quad & \widehat{C}_t - \widehat{C}^*_t = \left(\mathcal{D}_{FA} - 2a_F\right)\widehat{\mathcal{T}}_t \end{aligned}$$

and the equilibrium terms of trade as a function of the shocks:

$$\widehat{\mathcal{T}}_t = \frac{1}{\mathcal{D}_i + 2\frac{\alpha}{1-\alpha}b_F}\left(\widehat{A}_t - \widehat{A}^*_t\right)$$

Hence write relative consumption as a function of the shocks:

$$\widehat{C}_t - \widehat{C}^*_t = \frac{\mathcal{G}_i}{\mathcal{D}_i + 2\frac{\alpha}{1-\alpha}b_F}\left(\widehat{A}_t - \widehat{A}^*_t\right) \tag{B.3}$$

where

$$\mathcal{G}_{CM} \equiv \sigma^{-1}\left(2a_H - 1\right), \qquad \mathcal{G}_{FA} \equiv \mathcal{D}_{FA} - 2a_F$$

Using (B.2) and (B.3):

$$\begin{aligned} \text{Var}(\widehat{C}+\widehat{C}^*) &= \text{Var}(\widehat{A}+\widehat{A}^*) = 2\sigma_A \\ \text{Var}(\widehat{C}-\widehat{C}^*) &= 2\sigma_A\left(\frac{\mathcal{G}_i}{\mathcal{D}_i + 2\frac{\alpha}{1-\alpha}b_F}\right)^2 \end{aligned}$$

where we have used the fact that the shocks to A and A^* are independent of each other and have the same variance. Plugging these into (B.1):

$$\text{Var}(\widehat{C}) = \frac{1}{2}\sigma_A\left[1 + \left(\frac{\mathcal{G}_i}{\mathcal{D}_i + 2\frac{\alpha}{1-\alpha}b_F}\right)^2\right]$$

Impact of GVC Integration Let's assume for a moment that $\mathcal{D}_i$ and $\mathcal{G}_i$ are independent of GVCs.[21] In this case, we can summarise the impact of GVC integration as a rise in the GPCI term $\frac{\alpha}{1-\alpha}b_F$ in the denominator of the fraction above. Under this simplification, we can consider the impact of GVC integration on the volatility of aggregate consumption:

$$\frac{\partial \text{Var}(\widehat{C})}{\partial GPCI} = \frac{\partial}{\partial GPCI}\left[\left(\frac{\mathcal{G}_i}{\mathcal{D}_i + 2GPCI}\right)^2\right] = -2\frac{\mathcal{G}_i^2}{(\mathcal{D}_i + 2GPCI)^3}$$

The sign of this derivative depends inversely on the sign of $(\mathcal{D}_i + 2GPCI)$, which is precisely the term that determines whether the terms of trade appreciates or depreciates in response to a relative productivity shock.

C Terms-of-Trade Determination

Inserting the individual demand functions into the trade-balance condition, equation (19), we can derive an expression for relative demand as a function of the TOT:

$$\widehat{Y}_{H,t} - \widehat{Y}_{F,t} = \mathcal{D}_i \widehat{\mathcal{T}}_t, \tag{C.1}$$

with the composite parameter $\mathcal{D}_i$ defined as in equation (28). In equilibrium, RD equals RS, meaning that we can equate equations (C.1) and (23):

$$\mathcal{D}_i \widehat{\mathcal{T}}_t = \left(\widehat{A}_t - \widehat{A}_t^*\right) - 2b_F \frac{\alpha}{1-\alpha}\widehat{\mathcal{T}}_t.$$

Rearranging this yields the equilibrium TOT equation:

$$\widehat{\mathcal{T}}_t = \frac{1}{\mathcal{D}_\alpha + 2b_F \frac{\alpha}{1-\alpha}}\left(\widehat{A}_t - \widehat{A}_t^*\right), \tag{C.2}$$

See the Supplemental Appendix for full derivations.

D Proof of Proposition 1 (GVC Integration and the Terms of Trade)

Inserting $b_H = a_H$ into equation (C.2), the equilibrium expression for the TOT becomes:

$$\widehat{\mathcal{T}}_t = \frac{1}{1 - 2a_H(1-\Phi_\alpha) + 2a_F \frac{\alpha}{1-\alpha}}\left(\widehat{A}_t - \widehat{A}_t^*\right), \tag{D.1}$$

so that:

$$\text{sign}\left(\frac{\partial \widehat{\mathcal{T}}_t}{\partial \widehat{A}_{R,t}}\right) = \text{sign}\left(1 - 2a_H(1-\Phi_\alpha) + 2a_F\frac{\alpha}{1-\alpha}\right).$$

[21] We know that we can impose this under FA by setting the CES parameters for consumption goods and intermediate inputs to be equal. We haven't found a similar condition for CM, however $\mathcal{D}_{CM}$ does appear to generically be positive, which is sufficient for most of the discussion below.

Since the expression on the right-hand side is a continuous and monotonic function of Φ_α, we can define the threshold value as the value of the elasticity that sets that expression to zero:

$$1 - 2a_H(1 - \widetilde{\phi}_{TOT}(\alpha)) + 2a_F\frac{\alpha}{1-\alpha} = 0 \quad \Rightarrow \quad \widetilde{\phi}_{TOT}(\alpha) = 1 - \frac{1}{2a_H} - \frac{a_F}{a_H}\frac{\alpha}{1-\alpha}$$

E Relative-Consumption Determination

Here we show that relative consumption is pinned down by the TOT in the FA equilibrium.

We first use the trade-balance and market-clearing conditions to derive the following relationships between aggregate gross output and aggregate consumption in each country:

$$\widehat{C}_t = \widehat{Y}_{H,t} + \widehat{P}_{H,t} - \widehat{P}_t, \qquad \widehat{C}^*_t = \widehat{Y}_{F,t} + \widehat{P}_{F,t} - \widehat{P}^*_t.$$

Taking the difference between $\widehat{C}_t$ and $\widehat{C}^*_t$, plugging in relative demand from equation (C.1), and using the relationship between the RER and the TOT in equation (21), we arrive at the equilibrium relative consumption equation:

$$\widehat{C}_t - \widehat{C}^*_t = (\mathcal{D}_{FA} - 2a_F)\,\widehat{\mathcal{T}}_t \tag{E.1}$$

with $\mathcal{D}_{FA}$ defined as in equation (28). See the Supplemental Appendix for full derivations.

F GVC Integration and Relative Consumption

Inserting $b_H = a_H$ into the relative consumption equation (E.1), this simplifies to:

$$\widehat{C}_t - \widehat{C}^*_t = (2a_H\Phi_\alpha - 1)\,\widehat{\mathcal{T}}_t \tag{F.1}$$

so that:

$$\text{sign}\left(\frac{\partial \widehat{C}_{R,t}}{\partial \widehat{A}_{R,t}}\right) = \text{sign}\,(2a_H\Phi_\alpha - 1)$$

where $\widehat{C}_{R,t} \equiv (\widehat{C}_t - \widehat{C}^*_t)$. As before, because the right-hand side is a continuous and monotonic function of Φ_α, we can define the threshold as the value that sets the expression to zero:

$$2a_H\widetilde{\phi}_{CORR} - 1 \quad \Rightarrow \quad \widetilde{\phi}_{CORR} = \frac{1}{2a_H}$$

G Proof of Lemma 1 (Cole-Obstfeld with GVCs)

From equation (32), take the derivative of $\widehat{\mathcal{W}}_t$ with respect to the relative productivity shock:

$$\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}} = \sigma\frac{\partial \widehat{C}_{R,t}}{\partial \widehat{\mathcal{T}}_t}\frac{\partial \widehat{\mathcal{T}}_t}{\partial \widehat{A}_{R,t}} - (2a_H - 1)\frac{\partial \widehat{\mathcal{T}}_t}{\partial \widehat{A}_{R,t}} = \left(\sigma\frac{\partial \widehat{C}_{R,t}}{\partial \widehat{\mathcal{T}}_t} - (2a_H - 1)\right)\frac{\partial \widehat{\mathcal{T}}_t}{\partial \widehat{A}_{R,t}} \tag{G.1}$$

The two derivatives in the final expression can be read from equations (D.1) and (F.1), with the parameter restriction $b_H = a_H$:

$$\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}} = \frac{\sigma(2a_H\Phi_\alpha - 1) - (2a_H - 1)}{1 - 2a_H(1-\Phi_\alpha) + 2a_F\frac{\alpha}{1-\alpha}} \tag{G.2}$$

The response of the wealth gap is zero when the numerator is equal to zero. This occurs when:

$$\Phi_\alpha = \widetilde{\phi}_{CO} \equiv \frac{1}{2a_H} + \frac{2a_H - 1}{\sigma 2a_H}$$

See the Supplemental Appendix for full derivations.

H Proof of Proposition 2 (GVC Integration and Risk Sharing)

To evaluate the marginal impact of GVC integration on risk sharing, we study how the absolute value of the derivative in equation (G.2) varies with α. Recall that for any function $f(x)$:

$$\frac{\partial |f(x)|}{\partial x} = \text{sign}(f(x))\ \frac{\partial f(x)}{\partial x} = \frac{|f(x)|}{f(x)}\ \frac{\partial f(x)}{\partial x} \tag{H.1}$$

where $\text{sign}(x) = |x|/x$ is the sign function, which is equal to 1 when its argument is positive and -1 when it is negative.[22]

Imposing the parameter restriction $\phi_C = \phi_X = \phi$ on equation (G.2), and taking the derivative with respect to α, we can show that:[23]

$$\frac{\partial}{\partial \alpha}\left(\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}}\right) = -2a_F \frac{\sigma(2a_H\phi - 1) - (2a_H - 1)}{\left[(1-\alpha)\left(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right)\right]^2}.$$

Plugging this into (H.1) and simplifying:

$$\begin{aligned}\frac{\partial}{\partial \alpha}\left(\left|\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}}\right|\right) &= \left|\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}}\right| \left(\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}}\right)^{-1} \frac{\partial}{\partial \alpha}\left(\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}}\right) \\ &= -\frac{2a_F}{(1-\alpha)^2}\left|\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}}\right| \left(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right)^{-1},\end{aligned}$$

hence:

$$\text{sign}\left(\frac{\partial}{\partial \alpha}\left(\left|\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}}\right|\right)\right) = -\text{sign}\left(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right).$$

As in Proposition 1, we know that this sign depends on the trade elasticity relative to the threshold $\widetilde{\phi}_{TOT}$. Specifically, when $\phi < \widetilde{\phi}_{TOT}$, the magnitude of the response of the wealth

[22] Intuitively, the derivative of the absolute value is equal to the derivative of the function wherever the function is positive, and minus the derivative wherever the function is negative.

[23] The restriction that $\phi_X = \phi_C$ is imposed in order to avoid including the derivative of Φ_α with respect to α. In other words, we can interpret Proposition 2 without this restriction, with the additional assumption that we abstract from the impact of increased GVC integration on the weighted-average elasticity.

gap is increasing in α, such that more GVC integration reduces risk sharing. Conversely, when $\phi > \widetilde{\phi}_{TOT}$, the magnitude of the response of the wealth gap is decreasing in α, such that more GVC integration improves risk sharing. See the Supplemental Appendix for full derivations.

I Proof of Corollary (Fragmentation Traps)

Recall that the threshold $\widetilde{\phi}_{TOT}(\alpha)$ is a decreasing function of α, with $\widetilde{\phi}_{TOT}(\alpha) \in [0, \widetilde{\phi}_{TOT}(0)]$ for $\alpha \in [0, 1)$. $\widetilde{\phi}_{TOT}(0)$ is the threshold in the no-GVC benchmark of Corsetti et al. (2008).

A given elasticity, $\phi < \widetilde{\phi}_{TOT}(0)$, therefore, will satisfy $\phi < \widetilde{\phi}_{TOT}(\alpha)$ for low α, and $\phi > \widetilde{\phi}_{TOT}(\alpha)$ for high α. Applying Proposition 2, this means that at such elasticities, more GVC integration reduces risk sharing at the margin for low α, and improves it for high α.

J Time-Varying Trade Elasticities

Consider the preferences over domestic and foreign intermediate inputs featuring external habits:

$$X_t \equiv \left[b_H^{\frac{1}{\phi_X}} \left(\frac{x_{H,t}}{(x_{H,t-1}/X_{t-1})^{\frac{\theta}{1-\phi_X}}} \right)^{\frac{\phi_X - 1}{\phi_X}} + b_F^{\frac{1}{\phi_X}} \left(\frac{x_{F,t}}{(x_{F,t-1}/X_{t-1})^{\frac{\theta}{1-\phi_X}}} \right)^{\frac{\phi_X - 1}{\phi_X}} \right]^{\frac{\phi_X}{\phi_X - 1}},$$

where the parameter $\theta \in [0, 1)$ controls the importance of habits. When $\theta = 0$, we return to the baseline model with no habits. It is convenient to re-arrange this expression to:

$$X_t \equiv \left[\kappa_H^{\frac{1}{\phi_X}} x_{H,t}^{\frac{\phi_X - 1}{\phi_X}} + \kappa_F^{\frac{1}{\phi_X}} x_{F,t}^{\frac{\phi_X - 1}{\phi_X}} \right]^{\frac{\phi_X}{\phi_X - 1}},$$

with

$$\kappa_H \equiv b_H \left(\frac{x_{H,t-1}}{X_{t-1}} \right)^{\theta} \quad \text{and} \quad \kappa_F \equiv b_F \left(\frac{x_{F,t-1}}{X_{t-1}} \right)^{\theta}.$$

In this way, the expressions derived previously all apply, with κ_H and κ_F replacing b_H and b_F. Specifically, the intermediate-inputs price index becomes:

$$P_{X,t} = \left[\kappa_H P_{H,t}^{1-\phi_X} + \kappa_F P_{F,t}^{1-\phi_X} \right]^{\frac{1}{1-\phi_X}}$$

and the relative-demand function can be written as:

$$\frac{x_{H,t}}{x_{F,t}} = \frac{\kappa_H}{\kappa_F} \mathcal{T}_t^{\phi_X}. \tag{J.1}$$

From equation (J.1), it is evident that the short-run trade elasticity is given by:

$$\eta_{SR} \equiv \frac{d \ln(x_{H,t}/x_{F,t})}{d \ln(\mathcal{T}_t)} = \phi_X.$$

In the long run, equation (J.1) reduces to:

$$\frac{x_H}{x_F} = \frac{b_H}{b_F}\left(\frac{x_H}{x_F}\right)^{\theta}\mathcal{T}^{\phi_X} = \left(\frac{b_H}{b_F}\right)^{\frac{1}{1-\theta}}\mathcal{T}^{\frac{\phi_X}{1-\theta}},$$

which implies the following long-run elasticity of substitution:

$$\eta_{LR} \equiv \frac{d\ln(x_H/x_F)}{d\ln(\mathcal{T})} = \frac{\phi_X}{1-\theta}.$$