Market Dynamics of Risk-On and Risk-Off Incentives and their Effect on Asset Prices

Idan Hodor *Monash University*

Fernando Zapatero *Boston University*

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Abstract

Mutual fund asset managers initially strive to outperform the index. However, as their accumulated performance improves, their focus shifts towards maintaining accumulated performance, evoking an S-shaped objective akin to prospect theory, with an inflection point equaling the benchmark rather than wealth. This paper incorporates this objective into a dynamic equilibrium asset pricing model. The resulting equilibrium, expressed in closed form, introduces two channels of predictability: (i) changes in incentives and (ii) changes in performance uncertainty. These channels generate momentum and reversal in equity returns. Our model predicts the direction of mutual fund returns in the following month with 53%–62% accuracy.

1 Introduction

Institutional investors hold a significant proportion of publicly traded stocks and account for an even larger share of daily trading volume. Consequently, their actions play a crucial role in determining stock prices, making it essential to understand the incentives driving these actions to better comprehend price dynamics. In March 2005, the U.S. Securities and Exchange Commission (SEC) implemented a rule requiring mutual funds to disclose the compensation structure of their portfolio managers in the Statement of Additional Information. These disclosures reveal that nearly all mutual fund managers' contracts include performance-based bonuses tied to the funds' accumulated performance relative to their benchmark index. Of

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even greater significance, these documents underscore the importance of maintaining consistent performance over the long term. Lastly, if the fund fails to meet expectations, the fund manager's contract can be terminated.

This paper develops a dynamic equilibrium model incorporating benchmarking incentives and highlights a critical element largely overlooked in the literature: the incentives to outperform the benchmark vary depending on how well asset managers performed so far, in light of the above contractual requirements.

Specifically, the drive to outperform the benchmark (i.e., risk-on behavior) is strong when asset managers' accumulated performance (i.e., performance since the start of the year) is below the benchmark, as they seek to earn their bonus, keep their job, and demonstrate sustained performance. However, once their accumulated performance surpasses the benchmark, these incentives diminish, and managers shift their focus toward maintaining accumulated performance (i.e., risk-off behavior) to avoid losing their bonus, their job, and incurring the reputational costs of eventual underperformance.

Our main contribution highlights asset management frictions as a potential explanation for the observed momentum and reversal patterns in asset markets, both at the asset-level time series and in long-short portfolio strategies involving a number of assets. We show that momentum and reversal naturally emerge in equilibrium within a complete information environment, where perfectly rational asset managers are incentivized to outperform the benchmark when they lag behind it, but shift their focus to maintaining accumulated performance when they are ahead.

In reality, a broad benchmark encompasses all the meaningful stocks in the equity market. These stocks are not perfectly correlated with the benchmark; some are more correlated, and some are less. Without loss of generality and to simplify the economic analysis, we focus on two assets: one that is perfectly correlated with the benchmark and one that has zero correlation with the benchmark. We refer to the first as the benchmark asset and the second as the non-benchmark asset.

Our model predicts momentum in both the benchmark and non-benchmark asset returns when asset managers outperform their benchmark, and a reversal in these asset returns when they underperform. Since these patterns persist across combinations of assets, the returns of active mutual funds exhibit the same patterns. We empirically validate these predictions. Table 1 reports the average probability of correctly predicting the direction of next month's mutual fund return, conditional on whether the fund outperformed or underperformed the benchmark in the previous month. The results indicate that momentum during outperformance and reversal during underperformance can be correctly predicted with 53% to 62% accuracy. These averages are computed across time and across all active U.S. equity funds

Table 1. Average Probability of Successful Predictions

This table presents the average probability of correctly predicting the direction of next month's mutual fund return based on the fund's performance in the previous month. If a mutual fund's return exceeded the market return by at least a given threshold (ranging from 0% to 2%) last month, we predict momentum: a positive return last month forecasts a positive return this month, while a negative return last month forecasts a negative return this month. Conversely, if the fund's return was below the market return by at least the threshold, we predict reversal: a positive return last month forecasts a negative return this month, and a negative return last month forecasts a positive return this month. The table demonstrates that mutual fund performance in the prior month can effectively predict the direction of returns in the following month. As funds become more active, and the difference between the fund's return and the market return widens, the average accuracy of predicting next month's return improves for both outperforming funds (momentum) and underperforming funds (reversal). In Panel B, we further restrict the sample to focus on funds that are active over a period of time. Specifically, we consider funds with at least 18 months of outperformance and 18 months of underperformance. Our analysis focuses on U.S. all-equity mutual funds with broad investment mandates. The predictability is robust to different specifications. A complete description of the data and methodology is provided in Section 5.1.

Funds with at least one instance of underperformance and one instance of outperformance in prior months			
	Both Detected	Momentum Detected	Reversal Detected
$\lvert\text{Portfolio} - \text{Index}\rvert > 0$	53.3%	53.8%	52.7%
$\lvert\text{Portfolio} - \text{Index}\rvert > 1.0\%$	55.8%	55.3%	55.8%
$\lvert\text{Portfolio} - \text{Index}\rvert > 2.0\%$	58.8%	56.5%	59.6%
Funds with at least 18 instances of underperformance and 18 instances of outperformance in prior months			
	Both Detected	Momentum Detected	Reversal Detected
$\lvert\text{Portfolio} - \text{Index}\rvert > 0$	53.7%	54.6%	52.7%
$\lvert\text{Portfolio} - \text{Index}\rvert > 1.0\%$	57.0%	57.4%	56.4%
$\lvert\text{Portfolio} - \text{Index}\rvert > 2.0\%$	62.1%	63.3%	60.7%

with broad U.S. mandates.

Additionally, Table 2 presents the estimated coefficients from a conditional logistic regression, including time and fund fixed effects, demonstrating that the predictive accuracy remains robust. For instance, the log-odds of momentum in the following month increase by 84% when a fund outperforms the market by at least 2%, compared to when it underperforms by the same margin. This corresponds to a 19.8% higher probability of momentum next month relative to reversal.

Given their time-varying incentives (to earn their bonus, keep their job, and demonstrate sustained performance), asset managers' payoffs are convex when they underperform the benchmark but become concave as their returns exceed the benchmark. This results in an S-shaped objective similar to prospect theory, but the inflection (or reference) point is the benchmark rather than absolute wealth. Despite this added complexity, we characterize equilibrium prices in closed form.

The introduction of an S-shaped objective and its pricing implications is novel in the context of asset management. Originally, such objective functions were developed to explain seemingly paradoxical financial behaviors observed in households. For example, Friedman

and Savage (1948) show that for an agent to simultaneously purchase insurance and lottery tickets, their utility must be concave over certain wealth levels and convex over others. Levy (1969) demonstrates that an objective function dependent on the first three moments of the wealth's distribution leads to an inverted S-shaped objective, concave below and convex above a reference point. Kahneman and Tversky (1979) introduce an S-shaped value function capturing differing risk attitudes toward gains and losses, where current wealth is the reference. More recently, Aristidou, Giga, Lee, and Zapatero (2025) demonstrate that an aspirational utility, where achieving a socially motivated goal creates a jump in utility. This jump introduces a convex segment to the utility function, making individuals risk-seekers in the presence of aspirations. The key distinction of our model is that the benchmark, not the value of assets under management, defines the reference point—significantly simplifying the equilibrium analysis.

Besides manifesting in the time-series of the benchmark and non-benchmarking asset returns, the predictability patterns also appear in investment strategies of long–short portfolios. Since Jegadeesh and Titman (1993), predictability in investment strategies is commonly examined by constructing long–short portfolios that buy past winners and sell past losers. Momentum is identified when such portfolios generate positive returns, and reversal when they yield negative returns. According to our theory, momentum should prevail in the long-short portfolio when asset managers outperform their benchmark, and reversal when they underperform it.

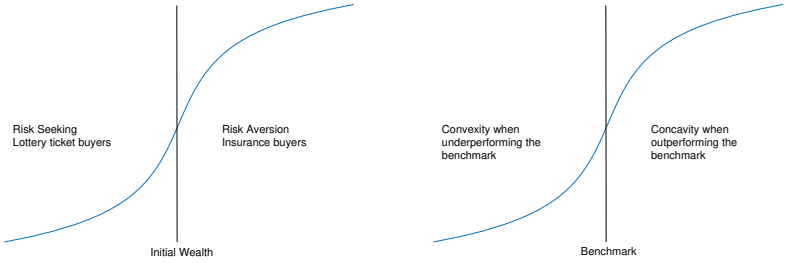


Figure 1. The figure on the left illustrates that an agent with a convex utility function over absolute wealth prefers engaging in a gamble, such as buying a lottery ticket. Conversely, when the utility function is concave, the agent is willing to pay a premium to avoid such a gamble. The figure on the right conveys a similar intuition for asset managers: they are incentivized to take on risk and "gamble" when their wealth falls below the benchmark and they are underperforming, whereas they tend to seek insurance and preserve their wealth when it exceeds the benchmark.

Pricing predictability is generally surprising because conventional wisdom suggests that if market participants expect prices to rise, they will immediately buy the asset to capture the return, causing an instantaneous price increase that eliminates the expected gain. Similarly,

if they anticipate prices will fall, they will sell the asset right away to avoid losses, triggering an immediate price drop that neutralizes the expected decline. As a result, traditional explanations for momentum and reversal typically involve either asymmetric information leading to gradual price adjustments or irrational investor behavior. In these cases, momentum and reversal arise as market participants gradually update their positions based on new information.

In contrast, the return predictability examined in this paper is fully rational and derives directly from no-arbitrage conditions, assuming all available information is instantaneously and completely reflected in prices. Predictability persists precisely because it is not optimal for asset managers to trade aggressively on it. While asset managers respond promptly to information, their trading intensity depends on their accumulated performance relative to the benchmark. These performance-dependent responses generate return predictability patterns endogenously through equilibrium dynamics.

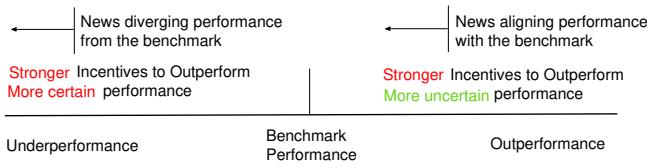


Figure 2. This figure illustrates the reactions to news during periods of underperformance and outperformance. The left side depicts scenarios where asset managers underperform, while the right side represents cases of outperformance. In both cases, lower accumulated performance intensifies incentives for managers to strive to outperform the benchmark. Conversely, news that aligns the accumulated performance of asset managers with the benchmark reduces outperformance for those ahead of the benchmark and worsens underperformance for those lagging behind. As a result, uncertainty of performance rises when accumulated performance converges toward the benchmark and declines when it diverges.

There are two equilibrium channels through which asset returns become predictable. The first channel operates through incentives: accumulated performance influences the motivation either to outperform the benchmark or to preserve existing accumulated performance. When accumulated performance falls below the benchmark, the incentive to outperform becomes stronger relative to the incentive to maintain accumulated performance. Conversely, when accumulated performance exceeds the benchmark, the incentive to preserve that level of accumulated performance dominates. Within an equilibrium, the incentives channel generates predictable price adjustments in assets across the economy—constituting a price-effect channel.

The second channel pertains to the uncertainty of (accumulated) performance: as accumulated performance diverges from the benchmark, future performance becomes more

certain, whereas convergence toward the benchmark increases uncertainty regarding future performance. Within an equilibrium, this uncertainty of performance channel induces predictable adjustments in the quantity of risk borne by assets across the economy—constituting a quantity-effect channel.

The incentives channel induces reversals in both benchmark and non-benchmark asset returns, irrespective of accumulated performance levels. In contrast, the uncertainty of performance channel generates momentum in both benchmark and non-benchmark returns when asset managers outperform the benchmark, which may shift to reversal when accumulated performance falls below the benchmark. Consequently, when asset managers underperform, both predictability channels contribute to an eventual reversal in returns. However, when asset managers outperform, the combined effect produces momentum in both benchmark and non-benchmark asset returns.

The drive to outperform the benchmark prompts asset managers to maintain a hedging position to avoid lagging behind. This hedging behavior increases demand for benchmark assets, independent of traditional risk and return considerations. Consequently, this incentive-driven demand raises the price of benchmark assets while depressing the prices of assets outside the benchmark. An unpredictable shock that worsens accumulated performance relative to the benchmark initially increases the benchmark's price and lowers the prices of non-benchmark assets due to heightened demand for the benchmark. However, this shock subsequently reduces the benchmark's expected return and increases the expected returns of the non-benchmark asset, as the market anticipates stronger future price reactions to shocks when incentives to outperform the benchmark are intensified. Thus, the incentives channel ultimately leads to asset price reversals.

Performance uncertainty increases the sensitivity of assets to news. From an economics perspective, asset managers' portfolios are not perfectly aligned with the benchmark. Therefore, when news arrives that brings the portfolio's accumulated performance closer to the benchmark, it raises performance uncertainty. This, in equilibrium, results in riskier asset markets.

When asset managers outperform, an unpredictable shock that worsens accumulated performance relative to the benchmark initially raises the benchmark's price and depresses the prices of assets outside the benchmark. This shock subsequently increases the expected return of the benchmark asset and decreases the expected return of assets outside the benchmark, as the market anticipates greater uncertainty of performance when the asset manager's accumulated performance aligns more closely with the benchmark. This uncertainty of performance channel thus drives momentum when asset managers outperform the benchmark.

In contrast, when asset managers underperform, an unpredictable shock that worsens

accumulated performance relative to the benchmark increases the expected return of the benchmark asset and decreases the expected return of assets outside the benchmark, as the market anticipates lower uncertainty of performance when the asset manager's accumulated performance diverges further from the benchmark. This uncertainty of performance channel thus drives reversal when asset managers trail the benchmark.

The remainder of the paper is organized as follows. Section 2 summarizes the related literature; Section 3 sets up the economy with the time-varying incentives to outperform and maintaining accumulated performance ; Section 4 solves for the equilibrium in closed-form; Section 5 discusses momentum and reversal in the time-series; Section 6 discusses momentum and reversal in the cross-section; and Section 7 concludes.

2 Related Literature

This paper contributes to the asset pricing literature that investigates predictable patterns in stock returns, focusing in particular on the drivers of momentum and reversal. Jegadeesh and Titman (1993) are the first to document momentum and reversal effects in the cross section of the stock market, while Chan, Jegadeesh, and Lakonishok (1996) provide a thorough investigation of the phenomenon. They show that a portfolio that buys past winners and sells past losers generates significant positive returns over a short-term window of one year or less, with the strategy's performance partially reversing over longer horizons.

There are several competing theories explaining momentum, which can be broadly categorized into a few key groups. One of the most prevalent explanations is behavioral. Within this category, Barberis, Shleifer, and Vishny (1998) examine an agent who, due to behavioral biases, forms incorrect beliefs about stock earnings. Daniel, Hirshleifer, and Subrahmanyam (1998) analyze investors who overreact to confirming evidence while underreacting to refuting evidence regarding their prior signals. Hong and Stein (1999) propose a two-agent model in which agents face different constraints on how they process information. Barberis and Shleifer (2003) study an economy with two types of investors: one who groups assets into styles and progressively reallocates funds from the worst-performing style to the best-performing style, and another who does not respond to this predictable pattern. Finally, Hillert, Jacobs, and Müller (2014) find that news coverage intensifies momentum by amplifying investors' biases.

The most prevalent rational explanation for momentum is based on information acquisition. Holden and Subrahmanyam (2002) consider a model featuring sequential information acquisition that is reflected in prices. Allen, Morris, and Shin (2006) propose a framework in which agents forecast future forecasts rather than prices directly. Shin (2006) endogenizes

firms' disclosure decisions jointly with prices. Banerjee, Kaniel, and Kremer (2009) study a heterogeneous beliefs economy where agents agree to disagree about their beliefs. Biais, Bossaerts, and Spatt (2010) analyze an overlapping generations model comprising informed and uninformed traders. Cespa and Vives (2012) compare consensus opinions and prices, demonstrating that specific patterns of information flow generate both momentum and reversal. Albuquerque and Miao (2014) show that information about future earnings, independent of current earnings, induces momentum and reversal. Ottaviani and Sørensen (2015) explore a heterogeneous beliefs economy where traders face wealth constraints. Andrei and Cujean (2017) demonstrate that short-term momentum can arise in an economy where private information arrives at an increasing rate. Finally, Cujean and Hasler (2017) investigate an economy with heterogeneous agents, some of whom update their beliefs continuously while others do so intermittently.

Another strand of the literature links momentum to firm-specific attributes such as growth options, dividend growth rate, revenues, and costs. Berk, Green, and Naik (1999), Johnson (2002), and Sagi and Seasholes (2007) explore this relationship within a partial equilibrium framework. Similarly, Liu and Zhang (2008) identify the loading on the growth rate of industrial production as a source of momentum, while Liu and Zhang (2014) examine momentum in a partial equilibrium, production-based setting.

More recent studies link the asset management industry to momentum and reversal, making this strand of literature particularly relevant to this paper. Dasgupta, Prat, and Verardo (2011) assume that investors observe asset managers' actions and form beliefs about their ability to perform. Asset managers, internalizing the reputational costs of underperformance, engage in herding behavior that positively predicts short-term returns. Vayanos and Woolley (2013) suggest that the slow-moving flow of capital between active and passive funds, driven by active funds' performance, might be a key driver of momentum and reversal.

Our paper is the first to demonstrate that a predictable pattern in asset managers' risk preferences generates momentum and reversal. We show that these predictable changes in risk preferences stem from the time-varying nature of their incentive contracts. Specifically, asset managers who underperform face stronger incentives to outperform the benchmark. However, as performance improves, the incentives to further exceed the benchmark diminish, and instead, asset managers shift their focus toward maintaining their performance to avoid losing their bonus.

Furthermore, our article contributes to the growing literature examining the impact of professional asset management on asset prices. In particular, our research aligns with a strand of this literature that explores the asset pricing implications of benchmarking incentives.

The paper most closely related to ours is Sotes-Paladino and Zapatero (2019). Their

model introduces a kink in the asset manager's performance-based fees, which alters the sensitivity of the manager's objective to wealth relative to the benchmark. Similar to our approach, the kink represents a shift in performance sensitivity between the end-of-period underperformance and outperformance regions. While this kink introduces convexity to the asset manager's objective, the slope in the outperformance region can be concave, a feature consistent with our setup. They find that asset managers overinvest in assets outside the benchmark and underinvest in the benchmark, which aligns with our predictions for periods when asset managers are likely to outperform (the risk-off region). The primary distinction between their work and ours is that they analyze asset manager behavior given prices in a partial equilibrium framework, whereas our focus is on the pricing implications within an equilibrium setting.

By and large, the asset management literature assumes that the asset manager's objective is always convex and does not become concave when outperformance is likely, as part of maintaining performance. The idea of embedding the benchmark into the asset manager's objective was initiated by Brennan (1993) and Gómez and Zapatero (2003). Cuoco and Kaniel (2011) were the first to analyze the equilibrium implications of benchmarking incentives through performance-based fees. Subsequently, Basak and Pavlova (2013) and Basak and Pavlova (2016) introduced a reduced-form approach to incorporate the benchmark, allowing for greater tractability. More recently, Buffa and Hodor (2023) extended this framework to study the pricing implications of heterogeneous benchmarking incentives, and Hodor and Zapatero (2023) examined the pricing effects of heterogeneous investment horizons.

Unlike previous literature, we investigate the pricing implications of time-varying benchmarking incentives for the asset manager. While the asset manager has strong incentives to outperform the benchmark when underperformance is likely (consistent with existing literature), these incentives weaken and eventually reverse when outperformance becomes more likely, as the focus shifts from outperforming the benchmark to preserving performance.

Lastly, our paper relates to the early work of Friedman and Savage (1948), Levy (1969), Kahneman and Tversky (1979), and Kahneman and Tversky (1991) who examine a scenario in which the curvature of the objective function changes from convex below a reference wealth level to concave above a reference wealth level. We incorporate the key insights of these papers into an asset management framework with relative wealth concerns. Specifically, when asset managers expect their wealth to fall below the benchmark index, they behave like poor individuals and exhibit risk-on incentives—akin to purchasing lottery tickets—with convex preferences that increase their marginal utility of wealth the further their wealth declines relative to the index. Conversely, when their wealth is expected to exceed the

index, asset managers behave like wealthy individuals and face risk-off incentives—similar to purchasing insurance—with concave preferences that reduce their marginal utility of wealth as their wealth increases above the index. More broadly, this paper fits into the reference-dependence literature, which originated with Rabin (2000) and Kőszegi and Rabin (2006) and was expanded more recently by DellaVigna and Pope (2017), Pagel (2017), and Khaw, Li, and Woodford (2020).

3 The Economy

This section lays out a simple and tractable model to study the joint equilibrium effect of risk-on and risk-off incentives of asset managers. We consider a simple and tractable standard pure-exchange finite horizon economy in which time t is continuous and goes from zero to T. The economy is populated by two types of investors: passive investors, $\mathcal{P}$, and asset managers, $\mathcal{A}$.

A broad benchmark encompasses all the meaningful stocks in the equity market. These stocks are not perfectly correlated with the benchmark; some are more correlated, and some are less. To simplify the economic analysis, we focus on two assets: one that is perfectly correlated with the benchmark and one that has zero correlation with the benchmark. We refer to the first as the benchmark (or index) asset and the second as the non-benchmark (or non-index) asset. While the economy could be generalized and accommodate multiple assets and different specifications of the index against which the asset managers' accumulated performance is evaluated, the main results do not qualitatively change. Accordingly, uncertainty is driven by a (3)-dimensional Brownian motion $\mathbf{Z} = (Z_1, Z_2, Z_3)'$.

3.1 Investment Opportunities

Our economy has 3 risky assets and a single riskless bond. We set the bond as numeraire and normalize its interest rate to zero. The risky asset is denoted by $S_{kt}, k = 1, 2, 3$, and represents a claim on the dividends D_{kT} arriving at time T. We assume each risky asset is in unit supply and posit that it follows

$$dS_{kt} = S_{kt}\left(\mu_{kt}^S dt + \boldsymbol{\sigma}_{kt}^{S\prime} d\boldsymbol{Z}_t\right). \tag{1}$$

Prices (S_{kt}), the (instantaneous) vector of expected returns $\boldsymbol{\mu}_t^S \equiv (\mu_{1t}^S, \mu_{2t}^S, \mu_{3t}^S)'$, and the (instantaneous) volatility matrix $\boldsymbol{\Sigma}_t^S \equiv \left(\boldsymbol{\sigma}_{1t}^S, \boldsymbol{\sigma}_{2t}^S, \boldsymbol{\sigma}_{3t}^S\right)'$ are endogenous and determined in equilibrium. The bold symbols represent vectors and the $\prime$ represents the transpose throughout the analysis.

The dividends (D_{kT}) are determined by the dynamics of the processes

$$dD_{kt} = D_{kt}\left(\mu_k dt + \boldsymbol{\sigma}_k^{D\prime} d\boldsymbol{Z}_t\right), \quad D_{k0} > 0, \tag{2}$$

which we refer to as dividend news. The parameter μ_k and the vector $\boldsymbol{\sigma}_k^D$ are constants for the first two dividends. We follow Menzly, Santos, and Veronesi (2004), Basak and Pavlova (2013), and Buffa and Hodor (2023) and leave the remaining dividend news process unspecified, and, instead, we specify an aggregate news process as discussed below. The procedure substantially improves the tractability of the model and provides closed-form precise equilibrium characterization.

We assume that stocks' fundamentals are independent, implying that the vector $\boldsymbol{\sigma}_k^D$ (2) has a positive kth element equaling $\sigma_k > 0$, while the remaining entries are zero. This assumption provides clear interpretation to the type of shocks that affect prices. We assume that the distributional properties of news are the same, $\sigma_1 = \sigma_2 = \sigma_3$ and $\mu_1 = \mu_2 = \mu_3$.

3.2 The Index

We assume that the first asset is the index.

$$S_t^I \equiv S_{1t}. \tag{3}$$

The asset manager's accumulated performance is evaluated against the performance of this index and the passive investor invests its entire wealth in it.

For consistency of the analysis, we refer to I_t as the *index news*, which equals the news about asset 1,

$$I_t \equiv D_{1t}, \quad \boldsymbol{\sigma}^I \equiv \boldsymbol{\sigma}_1^D. \tag{4}$$

3.3 Investors

The passive investors ($\mathcal{P}$) are endowed with $(1-\lambda)$ units of the index at time 0

$$W_0^{\mathcal{P}} = (1-\lambda)\, S_0^I, \tag{5}$$

and remain passive until time T. The asset managers ($\mathcal{A}$) are endowed with the remaining (λ) units of the index and holds the entire supply of the other assets,

$$W_0^{\mathcal{A}} = \lambda S_0^I + S_{20} + S_{30} \equiv S_0^{\mathcal{A}}. \tag{6}$$

The asset managers' initial endowment identifies the active section of the asset market, which is a claim on the total asset market dividends minus the passive index holdings' dividends,

$$V_T \equiv I_{1T} + D_{2T} + D_{3T} - (1 - \lambda) I_{1T}, \tag{7}$$

where we assume that this terminal value is determined by the process

$$dV_t = V_t \left(\eta dt + \boldsymbol{\nu}' d\boldsymbol{Z}_t \right), \quad V_0 > 0. \tag{8}$$

The parameter η is positive and the vector $\boldsymbol{\nu} \equiv (\nu_1, \nu_2, \nu_3)'$ has positive entries. We assume that $(\nu_1, \nu_2, \nu_3) = (\lambda\sigma_1/3, \sigma_2/3, \sigma_3/3)$ to mimic the aggregate active dividends and following the standard in the literature. We refer to V_t as the *market news*.

3.4 Risk-On and Risk-Off Incentives

Empirical evidence indicates that individuals sometimes choose to pay an insurance premium to avoid a fair gamble, while at other times they prefer to take the gamble. At first glance, these behaviors appear inconsistent with the expected utility maximization theory, which would require a rational agent to pay for insurance in some situations and buy a lottery ticket in others.

Friedman and Savage (1948) addressed this puzzle by introducing utility characteristics that explain these two behaviors within a rational, expected utility-maximizing framework. They demonstrated that rationalizing these behaviors means that, at certain wealth levels, an agent's marginal utility increases with wealth, while at other wealth levels, the marginal utility decreases.

The empirical evidence on asset managers and their benchmarking incentives aligns with the behaviors of individual agents as described by Friedman and Savage (1948), Kahneman and Tversky (1979), Kahneman and Tversky (1991), Rabin (2000), and Aristidou et al. (2025).

Specifically, when wealth is expected to fall below the index, asset managers act like individuals willing to take a fair gamble, facing risk-on incentives. These convex incentives increase their marginal utility of wealth as it falls further below the index. Conversely, when wealth is expected to exceed the index, asset managers behave like individuals willing to purchase insurance to avoid a fair gamble, facing risk-off incentives. These concave incentives reduce their marginal utility of wealth as it rises above the index.

Crucially, the arguments and insights of Friedman and Savage (1948) apply to asset managers' risk-on and risk-off incentives because the key factor for prices is the marginal

utility of wealth. Changes in the index impact prices solely through their effect on the marginal utility of wealth.

Therefore, Friedman and Savage (1948)'s arguments in this context suggest that asset managers' marginal utility of wealth *increases* with the index level when their wealth is below the index, ($U_{WS^I} > 0$), indicating a stronger desire to increase wealth when they are underperforming. Conversely, when wealth is above the index, their marginal utility of wealth *decreases*, ($U_{WS^I} < 0$), indicating a weaker desire to increase wealth when they are outperforming the benchmark. Any changes in the index level that do not affect the marginal utility of wealth have no implications for prices. Figure 3 illustrates this argument.

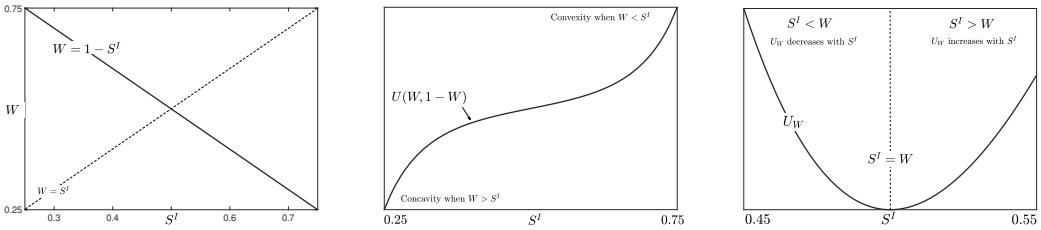


Figure 3. The left figure plots the $S^I - W$ plane. At the top left triangle, the asset managers have concave incentives because $W > S^I$, while at the bottom right triangle, the asset managers have convex incentives because $W < S^I$, where the solid linear line represents $W = -S^I + 1$. As we trace that line from the top left corner to the bottom right corner, incentives shift from concave to convex incentives. Indeed, the middle figure shows that the cross derivative U_{WS^I} is concave initially and becomes convex as the line crosses the dashed line. The cross derivative (U_{WS^I}) in the direction of the solid line (in the left figure) can be captured by the rate of change in $U_W(W, S^I)$ in the direction $(0, 1)$, leading to a cross derivative: $U_{WS^I}(W, S^I)$. The right figure shows what the cross derivative captures. When $W < S^I$ (right side of the figure), the marginal utility of wealth increases when S^I increases, indicating the asset manager desire to outperform. In contrast, when $W > S^I$ (left side of the figure), the marginal utility of wealth decreases when S^I increases, indicating the asset manager desire to maintain accumulated performance. The function U represents the asset managers' objective (11) with $\alpha_1 = 0.5$ and $\alpha_2 = 4.5$.

While many potential objective functions may satisfy the requirements for risk-on and risk-off incentives. The goal is to introduce an objective that switches between risk-on and risk-off incentives based on the state of asset managers' wealth relative to the index without losing tractability.

To do so, we start with a variation of Basak and Pavlova (2013)s' asset manager objective function, and introduce the *risk-on objective*

$$-E\left[\frac{\left(S_T^I\right)^{1-\gamma_1}\left(W_T^A\right)^{1-\gamma_2}}{\left(\gamma_2-1\right)\left(1-\gamma_1\right)}\right], \quad \gamma_2 > 1 > \gamma_1 > 0. \tag{9}$$

Similar to the original objective function of Basak and Pavlova (2013), the risk-on objective (9) is always convex, $U_{WS^I} = \left(S_T^I\right)^{-\gamma_1}\left(W_T^{\mathcal{A}}\right)^{-\gamma_2} > 0$. The parameter of risk aversion in this case is γ_2.

If we alternate the wealth and the index in the risk-on objective (9) and remove the minus sign, we attain a *risk-off objective*,

$$E\left[\frac{\left(W_T^{\mathcal{A}}\right)^{1-\gamma_1}\left(S_T^I\right)^{1-\gamma_2}}{(\gamma_2-1)(1-\gamma_1)}\right], \quad \gamma_2 > 1 > \gamma_1 > 0, \tag{10}$$

which is always concave, $U_{WS^I} = -\left(S_T^I\right)^{-\gamma_2}\left(W_T^{\mathcal{A}}\right)^{-\gamma_1} < 0$. The parameter of risk aversion in this case is γ_1.

Notice that the parameter of risk aversion γ_2 applies when the manager is wealthier than when γ_1 applies, and $\gamma_2 > \gamma_1$. This means that the wealthier the asset manager, the lower their risk aversion, which is consistent with empirical evidence suggesting that absolute risk aversion decreases with wealth.

Perhaps more importantly, by combining the risk-on (9) and risk-off (10) incentives we attain an objective function that switches between risk-on and risk-off depending on whether the wealth is below or above the index.

$$U\left(W^{\mathcal{A}}, S^I\right) \equiv E\left[\mathbf{1}_{\text{Off}}\frac{\left(S_T^I\right)^{1-\gamma_2}\left(W_T^{\mathcal{A}}\right)^{1-\gamma_1}}{(\gamma_2-1)(1-\gamma_1)} - \mathbf{1}_{\text{On}}\frac{\left(S_T^I\right)^{1-\gamma_1}\left(W_T^{\mathcal{A}}\right)^{1-\gamma_2}}{(\gamma_2-1)(1-\gamma_1)}\right], \quad \gamma_2 > 1 > \gamma_1 > 0. \tag{11}$$

The indicators $\mathbf{1}_{\text{Off}}, \mathbf{1}_{\text{On}}$ in (11) allow to separate the asset pricing implications of each incentive individually and their joint effect. A fully risk-on asset manager has only the second component in (11), $\mathbf{1}_{\text{Off}} = 0, \mathbf{1}_{\text{On}} = 1$, while a fully risk-off asset manager has only the second component in (11), $\mathbf{1}_{\text{Off}} = 1, \mathbf{1}_{\text{On}} = 0$. When taken together, $\mathbf{1}_{\text{Off}} = 1, \mathbf{1}_{\text{On}} = 1$, the risk-off component dominates when $S^I < W^{\mathcal{A}}$, while the risk-on component dominates when $S^I > W^{\mathcal{A}}$, and the asset manager has both risk-on and risk-off incentives, in line with Friedman and Savage (1948) main insights. The parameter of risk aversion in this case is endogenous and time-varying.

Besides the risk-on and risk-off incentives, the asset manager objective function (11) satisfies other useful characteristics: it increases and is concave in wealth ($U_W > 0, U_{WW} < 0$), and satisfies the inada conditions stating that the marginal utility of wealth (U_W) takes the value of ∞ as wealth approaches zero, and the value 0 as wealth approaches infinity. These conditions are at the core of expected utility maximization.

There are two variations with respect to the original Basak and Pavlova (2013) objective function. First, we introduce a risk aversion parameter larger than one (γ_2) instead of the

typical log utility of wealth, and second, the objective is a power function of the index (γ_1) instead of being linear. The are two reasons for these variations.

The symmetrical structure of the asset manager's objective function (11) is the primary characteristic, inducing the shift between risk-on and risk-off incentives. If we were to follow Basak and Pavlova (2013) log utility of wealth for the risk-on incentives, the symmetrical structure would require a log of the index for the risk-off incentives. This outcome would, in turn, violate the requirement for the increasing-in-wealth objective function ($U_W > 0$) because the index log becomes negative when the index level falls below one, and U_W would eventually turn negative. Similarly, if we were to follow Basak and Pavlova (2013)s' linearity in the index for risk-on incentives, the symmetrical structure would require linearity in wealth for the risk-off incentives. This outcome would, in turn, violate the requirement for a decreasing marginal utility in wealth, $U_{WW} < 0$. Due to these two reasons, we introduce the two variations to the original asset manager objective function, as laid out by Basak and Pavlova (2013).

Starting with their initial endowments, the asset managers dynamically choose a portfolio $\boldsymbol{\pi}_t^{\mathcal{A}}$, which represents the fraction of wealth invested in each of the risky assets. The wealth process of the asset managers, therefore, follows the dynamics

$$\frac{dW_t^{\mathcal{A}}}{W_t^{\mathcal{A}}} = \boldsymbol{\pi}_t^{\mathcal{A}\prime}\left(\boldsymbol{\mu}_{St}dt + \boldsymbol{\Sigma}_t^S d\boldsymbol{Z}_t\right), \quad t \leq T. \tag{12}$$

4 The Equilibrium

We define the equilibrium in a standard way: equilibrium prices and portfolio holdings are such that (i) the asset managers choose their optimal portfolio for given prices, and (ii) stocks, bond and consumption-good markets clear.

Asset managers' risk-on and risk-off objectives are of a constant relative risk aversion type individually. However, when taken together, the relative risk aversion of the asset managers' objective (11) is a weighted average of the two risk aversion coefficients (γ_1, γ_2), with time-varying weights that depend on the likelihood of the asset managers to outperform or underperform. We denote the relative risk aversion by $RA(W, S^I)$. By applying the definition of relative risk aversion to the asset managers' objective function, we find that:

$$RA(W,S^I) = \gamma_1 \underbrace{\left(1 - \frac{1}{1+\frac{(1-\gamma_1)}{(\gamma_2-1)}\,(W/S^I)^{(\gamma_2-\gamma_1)}}\right)}_{\equiv 1-w} + \gamma_2 \underbrace{\left(\frac{1}{1+\frac{(1-\gamma_1)}{(\gamma_2-1)}\,(W/S^I)^{(\gamma_2-\gamma_1)}}\right)}_{\equiv w}, \tag{13}$$

where $w \in (0,1)$ since $0 < \gamma_1 < 1 < \gamma_2$. The formulation shows that risk aversion,

$RA(W, S^I)$, is endogenous, time-varying, and dependent on the ratio of the asset managers' wealth to the index. When asset managers' wealth is low relative to the index, they are in the risk-on region since $W < S^I$. In this case, the risk aversion aligns more closely to the risk-aversion coefficient in the risk-on region (γ_2) since the weight on the risk-off risk-aversion (γ_1) declines.

As the accumulated performance of asset managers improve, their total wealth increases relative to the index. Consequently, the weight on the risk-off risk-aversion parameter rises while the weight on the risk-on risk-aversion parameter falls, resulting in an overall objective function that is decreasing in relative risk aversion: the risk aversion decreases as asset managers become wealthier.

Since market clearing requires asset managers to hold the entire active supply, the index price must equal its dividends at maturity, and in equilibrium, $RA(W, S^I) = RA(V, I)$. Interestingly, the state variable (V/I) that determines the variations in risk aversion also dictates how asset managers hedge against shifts between the risk-off and the risk-on regions, ultimately determining the equilibrium quantities.

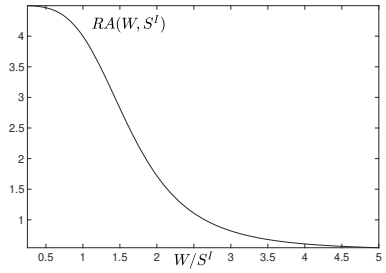


Figure 4. This figure plots the asset managers' risk aversion as a function of W/S^I. It converges to γ_2, the risk aversion coefficient of the risk-on region when $W/S^I \downarrow 0$, and to γ_1, the risk aversion coefficient of the risk-off region when $W/S^I \uparrow \infty$, resulting in a decreasing relative risk aversion utility.

Before we start the equilibrium analysis we first define the asset managers' *risk-on hedging demand*, $\mathcal{H}_{it}$, which is given by

$$\mathcal{H}_{it} \equiv \frac{1}{1 + \frac{1-\gamma_1}{\gamma_2-1} \left(V_t/I_t\right)^{\gamma_2-\gamma_1} \mathcal{E}_{it}}, \quad i = 1, 2, 3, 4, \tag{14}$$

where i refers to four different determinstic functions of time, $\mathcal{E}_{1t}$, $\mathcal{E}_{2t}$, $\mathcal{E}_{3t}$, $\mathcal{E}_{4t}$, given in (A.3), (A.6), (A.9), (A.12), respectively. We refer to $\mathcal{H}_{it}$ as the risk-on hedging demand because the closer it is to one, the more correlated the equilibrium quantities of the joint incentives case are with those of the purely risk-on incentives case. The deterministic functions $\mathcal{E}_{it}$ determines the speeds at which the hedging demand changes with the market-index ratio,

$\partial \mathcal{H}_{it}/\partial(V/I)$. Thus, we refer to $\mathcal{H}_{it}$ for $i = 1, 2, 3, 4$ as the risk-on hedging demand. The market-index ratio governs both the risk-on hedging demand and the risk aversion parameter, and it serves as the key state variable that determines all equilibrium quantities.

We begin by analyzing the partial equilibrium effect of how shocks impact the wealth of asset managers, given prices. This approach allows us to identify the different shock propagation channels for risk-on and risk-off incentives, both individually and jointly. To facilitate this, let us denote the (equilibrium) discount factor by ξ_t and its dynamics by

$$d\xi_t = -\xi_t \left(\boldsymbol{\theta}_t' d\boldsymbol{Z}_t \right). \tag{15}$$

By definition of the discount factor, the vector of market prices of risk process, $\boldsymbol{\theta}_t$, can be represented as

$$\boldsymbol{\theta}_t = \left(\boldsymbol{\Sigma}_t^S \right)^{-1} \boldsymbol{\mu}_t^S, \tag{16}$$

which in a single asset economy simplifies to the ratio of an asset's (excess) expected return and its standard deviation.

Lemma 1 (**Risk Exposure**). *The asset managers' risk exposure due to risk-on incentives is given by*

$$\boldsymbol{\Sigma}_t^{S\prime} \boldsymbol{\pi}_t^{\mathcal{A}} = \boldsymbol{\theta}_t - (\gamma_2 - 1)\, \boldsymbol{\nu} + (1 - \gamma_1)\, \boldsymbol{\sigma}^I, \tag{17}$$

while the risk exposure due to risk-off incentives is given by

$$\boldsymbol{\Sigma}_t^{S\prime} \boldsymbol{\pi}_t^{\mathcal{A}} = \boldsymbol{\theta}_t + (1 - \gamma_1)\, \boldsymbol{\nu} - (\gamma_2 - 1)\, \boldsymbol{\sigma}^I. \tag{18}$$

In the presence of both risk-on and risk-off incentives the risk exposure is given by

$$\begin{aligned} &\boldsymbol{\Sigma}^{S\prime} \boldsymbol{\pi}_t^{\mathcal{A}} \\ &= \boldsymbol{\theta}_t + \left[(1 - \gamma_1)\,(1 - \mathcal{H}_{2t}) - (\gamma_2 - 1)\, \mathcal{H}_{2t} \right] \boldsymbol{\nu} + \left[(1 - \gamma_1)\, \mathcal{H}_{2t} - (\gamma_2 - 1)\,(1 - \mathcal{H}_{2t}) \right] \boldsymbol{\sigma}^I. \end{aligned} \tag{19}$$

The function $\mathcal{H}_{2t}$ is defined in (14).

There are three shock propagation channels. The first is the myopic mean-variance channel, where asset managers seek a risk exposure proportional to the assets' risk-return tradeoff, correlating their wealth with the market price of risk, $\boldsymbol{\theta}_t$. Lemma 1 demonstrates that this channel remains unaffected regardless of the type of incentives. However, the exact exposure will be determined in equilibrium once the market prices of risk are identified.

The second channel is a risk-aversion adjustment channel, and Lemma 1 shows that it

has different implications for risk-on and risk-off incentives. In the pure risk-on incentives case (17), asset managers are risk averse with respect to the market and desire to hedge against adverse market moves (buy insurance on the market). They do so by aspiring to higher wealth following adverse market movements, as the risk aversion coefficient exceeds one, $\gamma_2 > 1$. The logic reverses for risk-off incentives. In pure risk-off incentives (18), asset managers are risk takers with respect to the market (gamble on the market) and correlate their wealth with the market, since the risk aversion coefficient is less than one, $\gamma_1 < 1$.

The third channel is the benchmark consideration. Similar to the risk aversion adjustment channel, it has opposing implications for risk-on and risk-off incentives. When asset managers trail the index and aim to outperform it (17), they act as risk-takers with respect to the index, seeking higher returns when the benchmark is high compared to when it is low — an effort to surpass the benchmark, as evident in their optimal positive risk exposure, $(1-\gamma_1)\,\sigma^I$. In particular, when the benchmark is high and the asset managers trail it, their marginal utility of wealth is convex with respect to the benchmark, $U_{WS^I} > 0$. This convexity induces asset managers to take larger risk when they are further below the benchmark, similar to the findings in the asset management literature.

Conversely, when asset managers outperform the index and aim to maintain their lead (18), they are risk-averse relative to the benchmark, seeking higher returns when the benchmark is low compared to when it is high — an effort to maintain their lead over the benchmark, as evident in their optimal negative risk exposure, $-(\gamma_2-1)\,\sigma^I$. Specifically, when the benchmark is low and asset managers lead, their marginal utility of wealth is concave with respect to the benchmark, $U_{WS^I} < 0$. This concavity induces asset managers to protect their wealth more when they are outperforming the benchmark by a larger margin.

The joint effect is a time-varying weighted average of the incentives to outperform the benchmark and maintain accumulated performance. Without factoring in equilibrium pricing effects, when asset managers are far below the benchmark ($\mathcal{H}_{2t} \to 1$), they act as risk-takers and aim to achieve higher returns by gaining significant exposure to the benchmark (for example, through leverage). As asset managers' accumulated performance improves, they more closely balance risk-on and risk-off portfolio strategies ($0 < \mathcal{H}_{2t} < 1$), which reduces their optimal benchmark exposure (for example, fully replicating the benchmark). Finally, when the accumulated performance of asset managers continues to improve ($\mathcal{H}_{2t} \to 0$), they become risk-averse relative to the benchmark and seek to take large risks negatively correlated with the benchmark (for example, by taking a short benchmark position).

The incentives of asset managers to leverage, replicate, or short the benchmark induce demand pressures that are counteracted in equilibrium by changes in the risk-return trade-offs of assets, which are captured by the market price of risk. We proceed by deriving

closed-form expressions for the equilibrium quantities, beginning with the market price of risk.

Proposition 1 (**Market Price of Risk**). *The market price of risk in the presence of risk-on incentives is given by*

$$\boldsymbol{\theta}_t = -\left(1-\gamma_1\right)\boldsymbol{\sigma}^I + \gamma_2\boldsymbol{\nu}, \tag{20}$$

while in the presence of risk-off incentives, it is given by

$$\boldsymbol{\theta}_t = \left(\gamma_2-1\right)\boldsymbol{\sigma}^I + \gamma_1\boldsymbol{\nu}. \tag{21}$$

In the presence of both risk-on and risk-off incentives the market prices of risk are given by

$$\boldsymbol{\theta}_t = \left[\gamma_1\left(1-\mathcal{H}_{1t}\right) + \gamma_2\left(\mathcal{H}_{1t}\right)\right]\boldsymbol{\nu} + \left[-\left(1-\gamma_1\right)\mathcal{H}_{1t} + \left(\gamma_2-1\right)\left(1-\mathcal{H}_{1t}\right)\right]\boldsymbol{\sigma}^I. \tag{22}$$

The function $\mathcal{H}_{1t}$ is defined in (14).

In the risk on incentives case, (20), asset managers trail the benchmark. In this case, the market price of risk aligns with findings documented in the literature over the past decade: equilibrium counteracts the heightened demand for assets correlated with the benchmark by decreasing the market price of risk associated with those assets, making them less attractive from a risk-return trade-off perspective, $-\left(1-\gamma_1\right)\boldsymbol{\sigma}^I$.

The logic reverses for pure risk-off incentives (21). In this case, when asset managers' wealth exceeds the benchmark, they maintain their lead over the benchmark by taking large risky positions negatively correlated with the benchmark. Equilibrium counteracts the heightened demand for assets negatively correlated with the benchmark by increasing the market price of risk associated with those assets, making them less attractive instruments from a risk-return trade-off perspective, $\left(\gamma_2-1\right)\boldsymbol{\sigma}^I$.

The joint effect is a time-varying weighted average of the market price of risk during underperformance and that during outperformance, with $\mathcal{H}_{1t}$ determining the weights. When asset managers underperform by a large margin ($\mathcal{H}_{1t} \to 1$), the market price of risk in underperformance dominates, leading to a lower market price of risk for assets correlated with the benchmark. As accumulated performance improves and aligns more closely with the benchmark, asset managers balance the risk-on and risk-off market prices of risk more closely ($0 < \mathcal{H}_{1t} < 1$). The reduced demand for the benchmark leads to a recovery in the market price of risk correlated with the benchmark. Finally, when asset managers outperform and significantly exceed the benchmark ($\mathcal{H}_{1t} \to 0$), the market price of risk associated with

outperformance dominates, leading to an increase in the market price of risk for assets correlated with the benchmark.

The market price of risk associated with the market exposure is also time-varying. In the underperformance region, asset managers are risk-averse relative to the market, seeking to negatively correlate their wealth with the market. Equilibrium prices respond by increasing the market price of risk for assets correlated with the market, $\gamma_2 \boldsymbol{\nu}$. As accumulated performance improves, the demand for assets negatively correlated with the market decreases, and equilibrium responds by reducing the market price of risk associated with the market, $\gamma_1 \boldsymbol{\nu}$. When accumulated performance aligns more closely with the benchmark, equilibrium balances the risk-on and risk-off market prices of risk, with the weights determined by $\mathcal{H}_{1t}$.

We proceed by characterizing the risk exposures.

Proposition 2 (**Volatility**). *The return volatilities in the presence of either risk-on incentives or risk-off incentives, separately, are given by*

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\sigma}_{1t}^D, \qquad \boldsymbol{\sigma}_{2t}^S = \boldsymbol{\sigma}_{2t}^D. \tag{23}$$

The return risk exposures in the presence of joint risk-on and risk-off incentives are given by

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\sigma}_1^D + (\gamma_2 - \gamma_1)\left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right)\left(\mathcal{H}_{3t} - \mathcal{H}_{1t}\right), \tag{24}$$

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\sigma}_2^D - (\gamma_2 - \gamma_1)\left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right)\left(\mathcal{H}_{1t} - \mathcal{H}_{4t}\right), \tag{25}$$

where $\mathcal{H}_{1t}$, $\mathcal{H}_{3t}$, $\mathcal{H}_{4t}$ are given in (14), and $\mathcal{H}_{3t} - \mathcal{H}_{1t} > 0$, $\mathcal{H}_{1t} - \mathcal{H}_{4t} > 0$.

The index asset sensitivity to news is the largest when $\mathcal{H}_{1t} + \mathcal{H}_{3t} = 1$, and for the non-index asset is when $\mathcal{H}_{1t} + \mathcal{H}_{4t} = 1$.

Positive index news increases the index price, while positive non-index news decreases the index price:

$$\boldsymbol{\sigma}_{1t}^S(1) > 0, \ \boldsymbol{\sigma}_{1t}^S(2) < 0, \ \boldsymbol{\sigma}_{1t}^S(3) < 0. \tag{26}$$

In contrast, positive index news decreases the non-index price, while positive non-index news increases the non-index price:

$$\boldsymbol{\sigma}_{2t}^S(1) < 0, \ \boldsymbol{\sigma}_{2t}^S(2) > 0, \ \boldsymbol{\sigma}_{2t}^S(3) > 0. \tag{27}$$

In a traditional endowment economy without benchmark incentives, prices only reflect news about the assets' fundamentals. In such environments, optimal portfolios remain unchanged by the arrival of news, and, therefore, equilibrium does not alter the exposure of prices to future news, leading to the immediate reflection of news in prices.

Proposition 2 reveals that pure incentives cause asset managers to respond to news differently than market participants in a traditional setup. However, similar to a traditional endowment economy, optimal portfolios (and risk exposures) remain unchanged by the arrival of news. Therefore, equilibrium does not alter the exposure of prices to future news to counteract the desire to change optimal portfolios.

In contrast, in previous pure risk-on models, such as those by Basak and Pavlova (2013), and Buffa and Hodor (2023), asset managers desire to change their portfolios with the arrival of news, since the incentives to outperform the benchmark increases as accumulated performance deteroriates. As a result, equilibrium changes how the index price will react to future news. This behavior occurs in the joint incentive case, where the incentive to outperform becomes stronger as accumulated performance deteriorates.

In particular, in the joint incentives case, equilibrium reacts to changes in the incentives to outperform the benchmark relative to the incentives to maintain accumulated performance. As the accumulated performance of asset managers converges to the benchmark ($\mathcal{H}_{1t} + \mathcal{H}_{3t}$ and $\mathcal{H}_{1t} + \mathcal{H}_{4t}$ get closer to one), assets become riskier due to the higher uncertainty of performance.

Furthermore, when positive index news arrives, the index price increases, making asset managers more likely to fall behind the benchmark and underperform. Anticipating this, asset managers aim to buy more of the benchmark as positive news arrives, increasing the demand for the index. Consequently, equilibrium causes the index price to overreact to index news. In contrast, the non-index asset becomes less attractive to asset managers because it does not fulfill the role of risk-on incentives. Consequently, positive index news translates into a negative price impact on the non-index asset.

Lastly, when positive non-index news arrives, the non-index asset price increases, boosting the overall market relative to the index. In this scenario, asset managers are more likely to outperform. Anticipating this, asset managers buy less of (or sell) the benchmark as positive non-index news arrives, decreasing the demand for the index. Consequently, equilibrium causes the index price to underreact to positive non-index news.

We proceed with analyzing the predictability of asset returns.

5 Time-Series Predictability

This section examines how an asset's past returns can predict its current return. Positive autocorrelation is known as time-series *momentum*, while negative autocorrelation is referred to as time-series *reversal*. Predictability arises from predictable changes to both quantities of risk and market prices of risk. We separate these two predictability channels throughout

the analysis.

We start by providing preliminary yet compelling evidence indicating that mutual fund returns are more likely to exhibit momentum when they outperform their benchmark, and reversal when they underperform it. Next, to deepen our understanding of the equilibrium mechanism, we analyze how the market prices of risk and the quantities of risk react to news.

5.1 Further Preliminary Evidence

This literature often documents positive return autocorrelation (time-series momentum) at high frequencies, such as daily, weekly and monthly, which reverts to negative autocorrelation (time-series reversal) at lower frequencies, such as annually and over several years. See for example, Fama and French (1988), Lo and MacKinlay (1988), James M. Poterba (1988), and Cutler, Poterba, and Summers (1991). More recently, Moskowitz, Ooi, and Pedersen (2012) documented time series momentum in equity index returns. However, Huang, Li, Wang, and Zhou (2020) question the significance of these findings, claiming that the evidence for time series momentum in the stock level and index level is weak.

Our results indicate that time-series momentum is strongly present in mutual fund returns and stock level returns—but only when the fund outperforms the market. When it underperforms the market, strong return reversal is observed. This evidence supports the main findings of our theory.

In this section, we focus on U.S. all equity mutual funds with a broad U.S. category and empirically document that their performance relative to the market during this month has significant predictive power for the direction of the fund's return in the following month. In particular, when the fund's monthly return underperforms the market return, the return next month is significantly more likely to exhibit reversal: a positive return this month predicts a negative return next month, while a negative return this month predicts a positive return next month. In contrast, when the fund's monthly return outperforms the market return, the next monthly return is significantly more likely to exhibit momentum: a positive return this month predicts a positive return next month, while a negative return this month predicts a negative return next month.

Table 2 presents the estimated coefficient of a conditional logistic regression: Panel A includes fund fixed effects, and Panel B includes both fund and year fixed effects. Each column represents a different threshold for outperformance and underperformance: in the first column, the threshold is 0%; in the second, it is 1%; and in the third, it is 2%. In the second and third columns, performance may fall between the thresholds for underperformance and outperformance. In such cases, we exclude the month from the dataset. In the first column,

there are no exclusions, and the dataset contains 295,583 fund-month observations. The restrictions in columns two and three exclude fund-month observations where the funds are less active and generate performance within the threshold (1% or 2%) relative to the market. Indeed, the estimated coefficients in columns two and three are economically larger than in column one.

Each estimated coeficient presents the change in the log odds from predicting reversal (predicted direction=0) to predicting momentum (predicted direction=1). In panel A, in column three, the performance threshold is 2% and the coeficient is 0.84. This means that the log odds of momentum next month increase by 84% when a fund outperforms the market by at least 2% relative to when that fund underperforms the market by at least 2%. In other words, the probability of momentum next month is 19.8% higher than reversal if the fund outperforms the market by at least 2% this month. In the lower section of Panel A, we control for year fixed effects. The results indicate that the log odds increase by 80%. The year fixed effects indicate that the effect we identify appears both cross-sectionally, across underperforming and outperforming funds within a given year, and within funds, between months of outperformance and underperformance.

In Panel B, we further restrict the sample to focus on funds that are more active over time. Specifically, we consider funds with at least 18 months of outperformance and 18 months of underperformance over their lifetime. When the performance threshold is 2% (column 3), the log odds ratio increases by 97% when a fund outperforms the market by at least 2%, compared to when it underperforms the market by at least 2%. This implies that the probability of momentum in the following month is 22.5% higher than that of reversal when the fund outperforms the benchmark by at least 2% this month. In the second row of Panel B, we control for year fixed effects, and the results indicate that this pattern persists cross-sectionally, across underperforming and outperforming funds within a given year.

5.1.1 Data. We obtain monthly mutual fund return data from the CRSP Mutual Fund database, which contains 9,580,005 fund-month observations. We merge this dataset with the CRSP Mutual Fund Summary data based on CRSP fund number. This data has 675,790 fund-year observations. The merge leads to 7,844,130 fund-month observations. We removed index funds (resulting in 7,338,637), ETF and ETN funds (resulting in 7,191,153), and Variable Annuity funds (resulting in 6,493,363). We identify all-equity mutual funds by the 'Morningstar Category Group' equaling 'Equity', and the broad market mandate by the 'U.S. Category Group' equaling 'U.S. Equity'.[1] We merge these two datasets based on the

[1]This identification is robust to several other specifications, such as requiring the prospectus benchmark to be a broad benchmark.

mutual fund CUSIP code, and exclude all funds that do not match. Our analysis starts at 1/1/2010 and end at 31/12/2023. The market's monthly return is proxied by SPY, the SPDR ETF that tracks the performance of the S&P 500 Index, with data downloaded from CRSP. The merge between SPY, Morningstar, based on the above characteristics leads to 295,583 fund-month observations and 2,326 unique mutual funds.

Table 2. Conditional Logistic Regression: Direction of Next Month Return

This table presents the estimated coeficients of a conditional logistic panel regression. The dependent variable is the direction of the monthly return, and the independent variable is the predicted return direction based on whether the mutual fund outperformed or underperformed the market in the previous month. Both the dependent and independent variables take the value 0 for a negative return and 1 for a positive return. If a mutual fund's return was above the market return (plus the threshold, ranging from 0% to 2%) last month, we predict momentum in returns: positive return last month predict a positive return this month; negative return last month predict a negative return this month. Conversely, if a mutual fund's return was below the market return (minus the threshold) last month, we predict reversal in returns: positive return last month predict a negative return this month; negative return last month predict a positive return this month. Our analysis focuses on U.S. all-equity mutual funds with a broad category.

	$\lvert\text{Portfolio} - \text{Index}\rvert > 0$	$\lvert\text{Portfolio} - \text{Index}\rvert > 0.01$	$\lvert\text{Portfolio} - \text{Index}\rvert > 0.02$
Panel A: Funds with at Least One Instance of Outperformance and One Instance of Underperformance in Prior Month			
Predicted direction $(t-1)$	0.30 (39.89)	0.56 (53.38)	0.84 (56.67)
Fund Fixed Effects	Yes	Yes	Yes
Year Fixed Effects	No	No	No
Concordance	54.0%	58.0%	61.0%
Unique Funds	2,326	2,310	2,153
Monthly Time Period	11–167	2–139	2–104
N	295,583	152,536	77,381
Predicted direction $(t-1)$	0.32 (41.59)	0.56 (52.07)	0.80 (52.41)
Fund Fixed Effects	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes
Concordance	64.0%	64.0%	66.0%
Unique Funds	2,326	2,310	2,153
Monthly Time Period	11–167	2–139	2–104
N	295,583	152,536	77,381
Panel B: Funds with at Least 18 Instances of Outperformance and 18 Instances of Underperformance in Prior Month			
Predicted direction $(t-1)$	0.30 (40.51)	0.60 (54.2)	0.97 (50.22)
Fund Fixed Effects	Yes	Yes	Yes
Year Fixed Effects	No	No	No
Concordance	54.0%	58.0%	62.0%
Unique Funds	2,146	1,693	740
Monthly Time Period	40–167	38–139	38–104
N	290,852	136,303	45,818
Predicted direction $(t-1)$	0.33 (42.18)	0.60 (52.78)	0.93 (46.35)
Fund Fixed Effects	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes
Concordance	64.0%	65.0%	67.0%
Unique Funds	2,146	1,693	740
Monthly Time Period	40–167	38–139	38–104
N	290,852	136,303	45,818

5.2 Equilibrium Response to Frictions

Prices not only react to information about the fundamentals but also react to news about management frictions.

To illustrate the intuitions clearly, we rewrite the vector of market prices of risk, (22), as

a sum of the *index news exposure* ($\boldsymbol{\sigma}^I$) and the *market news exposure* ($\boldsymbol{\nu}$),

$$\boldsymbol{\theta}_t = \kappa_t^I \boldsymbol{\sigma}^I + \kappa_t^\nu \boldsymbol{\nu}, \tag{28}$$

where

$$\kappa_t^I \equiv (\gamma_2 - 1) - (\gamma_2 - \gamma_1)(\mathcal{H}_{1t}), \qquad \kappa_t^\nu \equiv \gamma_1 + (\gamma_2 - \gamma_1)(\mathcal{H}_{1t}). \tag{29}$$

When asset managers are purely risk-on or purely risk-off, the market prices of risk do not reflect news about the management frictions because the risk-on hedging demand ($\mathcal{H}_{1t}$) is a constant that equals one or zero, respectively, and never changes. As a result, prices fully incorporate fundamental information. However, when asset managers balance both strategies, market prices of risk reflect not only fundamentals but also news about asset management frictions. Note that from a purely fundamental perspective, the price reaction to management friction might appear as incorrect pricing, or an overreaction.

As asset managers' accumulated performance deteriorates, risk-on incentives become more pronounced than risk-off incentives, leading asset managers to invest more in the benchmark and less in assets outside the benchmark. To restore equilibrium, market prices of risk associated with the benchmark are depressed to make the index assets less attractive and more expensive. Conversely, the market prices of risk associated with assets outside the benchmark increase to make these assets more attractive and less expensive.

Similar to the market price of risk, we rewrite the vector of return exposures (quantities of risk) as a sum of index news exposure ($\boldsymbol{\sigma}^I$) and market news exposure ($\boldsymbol{\nu}$),

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\sigma}_1^D + (\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t})\boldsymbol{\sigma}^I - (\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t})\boldsymbol{\nu}, \tag{30}$$

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\sigma}_2^D - (\gamma_2 - \gamma_1)(\mathcal{H}_{1t} - \mathcal{H}_{4t})\boldsymbol{\sigma}^I + (\gamma_2 - \gamma_1)(\mathcal{H}_{1t} - \mathcal{H}_{4t})\boldsymbol{\nu}. \tag{31}$$

The representations reveal that when asset managers are purely risk-on or purely risk-off, asset prices react only to news about fundamentals since the risk-on hedging demands are constant, either equaling one or zero, respectively. In contrast, when asset managers weigh both strategies, assets incorporate news about management frictions in addition to the news about fundamentals.

The difference between the risk-on hedging demands attain its maximum when the sum of the risk-on hedging damands equals one, $\mathcal{H}_{it} + \mathcal{H}_{jt} = 1$, $i, j = 1, 2, 3, 4$. The maximum values for different i and j are reached close to each other and identify the state at which asset managers have an equal likelihood of outperforming or underperforming the benchmark.

This likelihood influences prices. The arrival of news can cause the asset managers' accumulated performance to either align more closely with the benchmark or deviate away

from the benchmark. The uncertainty about accumulated performance is strongest when asset managers' accumulated performance closely matches the benchmark, compared to when their performances diverge. Consequently, the price sensitivity to news is the largest in this scenario.

When asset managers' accumulated performance diverges towards underperformance, their portfolios weigh more towards the risk-on strategy relative to the risk-off strategy. In this case, it becomes more likely that asset managers will underperform. Equilibrium reflects this higher certainty of underperformance by reducing the risks in the asset markets. Conversely, when asset managers' accumulated performance diverges towards outperformance, their portfolios weigh more towards the risk-off strategy relative to the risk-on strategy. Equilibrium reflects this higher certainty of outperformance by reducing the risks in the asset markets.

The equilibrium response to news about management frictions, through both the market prices of risk and quantities of risk, reveals two predictability channels.

5.3 Time-Series Predictability

So far, we have shown that market prices of risk and quantities of risk react to news about asset management frictions.

Before we delve into why these responses are predictable, let us first establish how we identify predictability. Let's begin by defining the expected return dynamics for both the index and non-index assets as follows

$$d\mu_{1t}^{S} = \alpha_{1t}^{\mu} dt + \boldsymbol{b}_{1t}^{\mu'} d\boldsymbol{Z}_t, \qquad d\mu_{2t}^{S} = \alpha_{2t}^{\mu} dt + \boldsymbol{b}_{2t}^{\mu'} d\boldsymbol{Z}_t. \tag{32}$$

The covariance between the expected returns in time t and $t+\Delta$ (for $\Delta > 0$) is given by

$$\begin{aligned} \mathbb{C}\mathrm{ov}_t\left(\frac{dS_{kt}}{S_{kt}}, \frac{dS_{kt+\Delta}}{S_{kt+\Delta}}\right) =& \mathbb{C}\mathrm{ov}_t\left(\left(\mu_{kt}^{S} dt + \boldsymbol{\sigma}_{kt}^{S\prime} d\boldsymbol{Z}_t\right), \left(\mu_{kt+\Delta}^{S} d\left(t+\Delta\right) + \boldsymbol{\sigma}_{kt+\Delta}^{S\prime} d\boldsymbol{Z}_{t+\Delta}\right)\right) \\ =& \mathbb{C}\mathrm{ov}_t\left(\boldsymbol{\sigma}_{kt}^{S\prime} d\boldsymbol{Z}_t, \mu_{kt+\Delta}^{S} d\left(t+\Delta\right)\right) = \boldsymbol{\sigma}_{kt}^{S\prime} \mathbb{E}_t\left[\left(d\boldsymbol{Z}_t\right) \mu_{kt+\Delta}^{S}\right] d\left(t+\Delta\right). \end{aligned} \tag{33}$$

This covariance indicates that the predictability of asset k depends on how today's news ($d\boldsymbol{Z}_t$) influences the asset's expected return in $t+\Delta$. It reflects the response of the expected return $\mu_{kt+\Delta}^{S}$ to the news $d\boldsymbol{Z}_t$.

To obtain a closed-form characterization, we examine predictability as Δ approaches 0. Plugging the dynamics of μ_{kt}^{S} and taking the covariance leads to

$$\mathbb{C}\mathrm{ov}_t\left(\frac{dS_{kt}}{S_{kt}}, \frac{dS_{kt+\Delta\downarrow 0}}{S_{kt+\Delta\downarrow 0}}\right) = \boldsymbol{\sigma}_{kt}^{S\prime} \boldsymbol{b}_{kt}^{\mu} dt, \tag{34}$$

where $\boldsymbol{b}_{kt}^{\mu}$ is the exposure of asset k expected return to news, (32). The following definition characterizes time-series momentum and reversal.

Definition 1 (**Time-Series Momentum and Reversal**). *Asset k exhibits* momentum *if both the asset return and its expected return move in the same direction,*

$$\boldsymbol{\sigma}_{kt}^{S\prime}\boldsymbol{b}_{kt}^{\mu} > 0, \tag{35}$$

and exhibits reversal *if the asset return and the expected return move in opposite directions,*

$$\boldsymbol{\sigma}_{kt}^{S\prime}\boldsymbol{b}_{kt}^{\mu} < 0, \tag{36}$$

where $k = 1, 2$ and identifies the index and non-index assets, respectively.

The definition is straightforward. We identify momentum when returns rise (fall) and the equilibrium expectation is that they will continue to rise (fall). Conversely, we identify reversal when returns rise (fall) but the equilibrium expectation is that they will fall (rise) in the future. The direction of predictability in equilibrium depends on the average effect across all shocks, which makes the analysis particularly challenging.

There are two potential predictable channels that emerge from the no arbitrage condition, $\boldsymbol{\theta}_t'\boldsymbol{\sigma}_{it}^{S} = \mu_{it}^{S}$. The first channel is a *pricing effect*, which is driven by the predictable change in the market prices of risk, $\boldsymbol{\theta}_t$. The second channel is a *quantity-of-risk effect*, which is driven by the predictable change in the asset's risk exposures, $\boldsymbol{\sigma}_{it}^{S}$.

To disentangle these two channels and examine how shocks in each channel propagate to expected returns, we adopt the approach of Buffa and Hodor (2023). This approach introduces the concept of elasticity between two quantities within a dynamic system, akin to an impulse-response function.[2] Accordingly, we define the shock elasticity at time t of an equilibrium quantity X at time t as the time t expectation of the Malliavin derivative $\mathcal{D}_t$ of X_t, normalized by the absolute value of the time t expectation of X_t,

$$\varepsilon_t(X, \ell) \equiv \frac{\mathbb{E}_t[\mathcal{D}_t^{\ell} X_t]}{|\mathbb{E}_t[X_t]|} = \frac{\mathcal{D}_t^{\ell} X_t}{|X_t|} = \frac{\sigma_t^X(\ell)}{|X_t|}, \tag{37}$$

where $\sigma_t^X(\ell)$ is entry ℓ of the diffusion term of dX_t. The shock elasticity $\varepsilon_t(X, \ell)$ is well-defined for $X_t \neq 0$. News in the economy can be separated into two mutually exclusive types: index news and non-index news, which implies that there are two shock elasticities,

[2]This methodology was originally presented by Borovička, Hansen, Hendricks, and Scheinkman (2011) and Borovička, Hansen, and Scheinkman (2014), which, in continuous time, builds on Malliavin calculus. Please refer to Ocone and Karatzas (1991), Øksendal (1997), Detemple, Garcia, and Rindisbacher (2003) for further discussion on Malliavin calculus with Markovian processes in finance.

one for each type of news. By applying the chain rule, we find that

$$\varepsilon_t(\mu_{it}^S, \ell) = \varepsilon_t(\boldsymbol{\sigma}_{it}^{S\prime}\boldsymbol{\theta}_t, \ell) = \frac{\boldsymbol{\sigma}_{it}^{S\prime}}{|\mu_{it}^S|}\left(\underbrace{\left(\mathcal{D}_t^\ell \boldsymbol{\theta}_t\right)}_{\substack{\text{benchmarking}\\ \text{incentives}}}\right) + \frac{\boldsymbol{\theta}_t'}{|\mu_{it}^S|}\left(\underbrace{\left(\mathcal{D}_t^\ell \boldsymbol{\sigma}_{it}^S\right)}_{\substack{\text{uncertainty of}\\ \text{performance}}}\right), \tag{38}$$

where the vector of Malliavin derivatives is the vector of derivatives for every entry. For example, $\mathcal{D}_t^\ell \boldsymbol{\theta}_t$ is the Malliavin derivative of each entry of the market price of risk vector with respect to news ℓ, which could be either index news or non-index news.

Let $\boldsymbol{\varepsilon}_t(\mu_{it}^S)$ be the vector of shock-elasticities. The definition of momentum implies a positive cross product between the shock elasticities of the expected return and the shock elasticities of the asset. Conversely, the definition of reversal is indicated by a negative cross product,

$$\text{Momentum} \Rightarrow \boldsymbol{\varepsilon}_t(\mu_{it}^S)'\boldsymbol{\varepsilon}_t(S_{it}) > 0, \quad \text{Reversal} \Rightarrow \boldsymbol{\varepsilon}_t(\mu_{it}^S)'\boldsymbol{\varepsilon}_t(S_{it}) < 0. \tag{39}$$

5.4 Incentives Channel

This channel causes a reversal in asset returns. Suppose news arrives that increases the likelihood of underperformance, whether it is positive index news or negative non-index news. Asset managers respond by hedging against further declines in accumulated performance, as their portfolio is not fully correlated with the benchmark. Consequently, asset managers seek to increase their allocation to the benchmark at the expense of other assets. To maintain equilibrium, the benchmark asset must become less attractive (and the non-index asset more attractive) to offset the asset managers' demand. This reduces the market prices of risk correlated with the benchmark and increasing the market prices of risk negatively correlated with the benchmark. As a result, the expected return of the index asset falls, while the expected return of the non-index asset rises.

Let's examine this channel more closely, which is derived by the Malliavin derivative of the market prices of risk with respect to a particular shock ℓ, which is given by

$$\mathcal{D}_t^\ell \boldsymbol{\theta}_t = -\left(\gamma_2 - \gamma_1\right)^2 \mathcal{H}_{1t}\left(1 - \mathcal{H}_{1t}\right)\left(\boldsymbol{\sigma}^I(\ell) - \boldsymbol{\nu}(\ell)\right)\left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right). \tag{40}$$

Since $\boldsymbol{\theta}_t$ is a vector, the Malliavin derivative with respect to shock ℓ of this vector results in a vector of Malliavin derivatives for each entry. Let $\mathcal{D}_t \boldsymbol{\theta}_t$ be the vector Malliavin derivatives with respect for all shocks, $\ell = 1, 2, 3$. The resulting Malliavin derivative has a matrix form

identified by

$$\mathcal{D}_t\boldsymbol{\theta}_t = -\left(\gamma_2-\gamma_1\right)^2\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right)\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right)'. \tag{41}$$

The signs of the each of the entries in this matrix are given by

$$\operatorname{sign}\left(\mathcal{D}_t\boldsymbol{\theta}_t\right) = \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix}, \tag{42}$$

and they do not change with the manager's accumulated performance relative to the benchmark. Each column represents the market price of risk with respect to particular news. Each row represents how this market price of risk change with arrival of particular news. For example, the entry $(2,1)$ represent how the market price of risk that is associated with index news ($\ell=1$) changes with the arrival of non-index news ($\ell=2$). The $+$ sign implies that the market price of this risk increases.

The changes in the expected returns of the index and non-index assets due to the incentives channel are given by

$$\operatorname{sign}\left(\boldsymbol{\varepsilon}_t(\mu^S_{1t})'\right) = \operatorname{sign}\left(\boldsymbol{\sigma}^{S\prime}_{1t}\left(\mathcal{D}_t\boldsymbol{\theta}_t\right)\right) = \begin{pmatrix} + & - & - \end{pmatrix} \times \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix} = \begin{pmatrix} - & + & + \end{pmatrix}, \tag{43}$$

$$\operatorname{sign}\left(\boldsymbol{\varepsilon}_t(\mu^S_{2t})'\right) = \operatorname{sign}\left(\boldsymbol{\sigma}^{S\prime}_{2t}\left(\mathcal{D}_t\boldsymbol{\theta}_t\right)\right) = \begin{pmatrix} - & + & + \end{pmatrix} \times \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix} = \begin{pmatrix} + & - & - \end{pmatrix}. \tag{44}$$

This cross multiplication reveals that for any enrty whereby $\mathcal{D}_t\boldsymbol{\theta}_t$ is negative, $\boldsymbol{\sigma}^S_{1t}$ is positive, and vice versa. That is, index news ($\ell=1$) decreases the expected return of the index, while non-index news ($\ell=2$) increases it. The opposite occurs for the non-index asset.

To determine whether the *incentives* channel generates price momentum or reversal, we identify the sign of the index and non-index assets shock elasticities. These signs are given by

$$\operatorname{sign}\left(\boldsymbol{\varepsilon}_t(S_{1t})'\right) = \begin{pmatrix} + & - & - \end{pmatrix}, \qquad \operatorname{sign}\left(\boldsymbol{\varepsilon}_t(S_{2t})'\right) = \begin{pmatrix} - & + & + \end{pmatrix}, \tag{45}$$

which reveals that positive index news boosts the index return, while positive non-index news reduces it. Conversely, positive index news reduces the non-index asset return, whereas positive non-index news increases it.

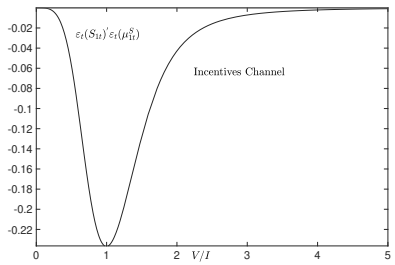


Figure 5. The figure shows that the overall effect coming from the *incentives* channel produces a reversal in asset returns. The parameters are $\gamma_2 = 4.5$, $\gamma_1 = 0.5$, $\mu_1 = \mu_2 = 0.05$, $\sigma_1 = \sigma_2 = 0.25$, $\lambda = 0.5$, $V_t = 5$, $V_0 = 2.5$, $I_0 = 1$, $T = 5$, $t = 2$.

By cross multiplying the signs of the two shock elasticities,

$$\text{sign}\left(\boldsymbol{\varepsilon}_t(\mu^S_{1t})'\right)\text{sign}\left(\boldsymbol{\varepsilon}_t(S_{1t})\right) = \begin{pmatrix} - & + & + \end{pmatrix} \times \begin{pmatrix} + \\ - \\ - \end{pmatrix} = \begin{pmatrix} - \end{pmatrix} \Rightarrow \text{ Reversal,} \tag{46}$$

$$\text{sign}\left(\boldsymbol{\varepsilon}_t(\mu^S_{2t})'\right)\text{sign}\left(\boldsymbol{\varepsilon}_t(S_{2t})\right) = \begin{pmatrix} + & - & - \end{pmatrix} \times \begin{pmatrix} - \\ + \\ + \end{pmatrix} = \begin{pmatrix} - \end{pmatrix} \Rightarrow \text{ Reversal.} \tag{47}$$

The analysis reveals that, in sum, the *incentives* channel induce an opposite reaction on the expected return and asset return shock elasticities: whenever one shock elasticity is positive the other is negative, and vice versa. Therefore, this channel causes asset price *reversals* during periods of both underperformance and outperformance.

5.5 Uncertainty of Performance Channel

This mechanism leads to momentum in asset returns when asset managers outperform the benchmark, resulting in a reversal when they underperform the benchmark, and resumes momentum with severe underperformance relative to the benchmark.

Higher uncertainty of performance implies that positive asset sensitivities become more positive and negative asset sensitivities become more negative. However, since market prices of risk are always positive and do not change signs, the sign of the expected return is determined by the average effect.

The average effect is dominated by the index news sensitivity. When accumulated performance converges to the benchmark, it becomes less certain whether asset managers will outperform or underperform the benchmark. This uncertainty increases the assets response to news because asset managers balance risk-on and risk-off strategies more closely. Equi-

librium market clearing compensates asset managers for the heightened uncertainty by increasing expected returns.

When asset managers outperform the benchmark, news that brings their accumulated performance more closely in line with the benchmark boosts the benchmark asset return and due to the heightened uncertainty also its expected return. Therefore, the benchmark asset returns exhibit momentum when asset managers outperform the benchmark. In contrast, when asset managers underperform the benchmark, news that aligns their accumulated performance more closely with the benchmark decreases the benchmark asset return, but due to the heightened uncertainty its expected return increases. As a result, the benchmark asset returns exhibit reversal when asset managers underperform the benchmark.

In cases of severe underperformance, the average effect is determined by non-index news sensitivity. The demand for the benchmark asset surges to such an extent that, to maintain equilibrium market-clearing, the market price of risk associated with index news drops significantly. As a result, the pricing effect of uncertainty of performance is determined by the negative non-index sensitivity, which reduces the expected return of the benchmark asset. Overall, when asset managers severely underperform, the benchmark asset returns exhibit momentum.

This channel is driven by the predictable change in assets' risk exposures, (38), which is given by the derivatives

$$\mathcal{D}_t^\ell \boldsymbol{\sigma}_{1t}^S = (\gamma_2 - \gamma_1)^2 (\mathcal{H}_{3t} - \mathcal{H}_{1t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) \left(\boldsymbol{\sigma}^I(\ell) - \boldsymbol{\nu}(\ell)\right) \left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right)', \tag{48}$$

$$\mathcal{D}_t^\ell \boldsymbol{\sigma}_{2t}^S = -(\gamma_2 - \gamma_1)^2 (\mathcal{H}_{1t} - \mathcal{H}_{4t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) \left(\boldsymbol{\sigma}^I(\ell) - \boldsymbol{\nu}(\ell)\right) \left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right)'. \tag{49}$$

Since the return exposures, $\boldsymbol{\sigma}_{1t}^S$ and $\boldsymbol{\sigma}_{2t}^S$, are vectors, the Malliavin derivative with respect to shock ℓ of these vectors result in a vector of Malliavin derivatives for each entry. Let $\mathcal{D}_t \boldsymbol{\sigma}_{it}^S$, $i = 1, 2$, be the vector Malliavin derivatives with respect to all shocks, $\ell = 1, 2, 3$. The resulting Malliavin derivatives have a matrix form identified by

$$\mathcal{D}_t \boldsymbol{\sigma}_{1t}^S = (\gamma_2 - \gamma_1)^2 (\mathcal{H}_{3t} - \mathcal{H}_{1t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) \left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right) \left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right)', \tag{50}$$

$$\mathcal{D}_t \boldsymbol{\sigma}_{2t}^S = -(\gamma_2 - \gamma_1)^2 (\mathcal{H}_{1t} - \mathcal{H}_{4t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) \left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right) \left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right)'. \tag{51}$$

Different from the incentives channel, the signs of each of the entries in these matrices (representing the uncertainty of performance) change with the manager's accumulated performance relative to the benchmark. This is because in outperformance, the risk-on hedging demands ($\mathcal{H}_{1t}$, $\mathcal{H}_{3t}$, and $\mathcal{H}_{4t}$) are close to zero, whereas in underperformance, they are close to one, which flip the signs of all the entries in these matrices. Therefore, the signs of these

matrices are given by

$$\text{Outperformance} \Rightarrow 1 > \mathcal{H}_{1t} + \mathcal{H}_{3t} \Rightarrow \text{sign}\left(\mathcal{D}_t \boldsymbol{\sigma}_{1t}^S\right) = \begin{pmatrix} + & - & - \\ - & + & + \\ - & + & + \end{pmatrix}, \tag{52}$$

$$\text{Underperformance} \Rightarrow 1 < \mathcal{H}_{1t} + \mathcal{H}_{3t} \Rightarrow \text{sign}\left(\mathcal{D}_t \boldsymbol{\sigma}_{1t}^S\right) = \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix}. \tag{53}$$

Each column represents the return exposure with respect to particular news. Each row represents how this return exposure changes with arrival of particular news. For example, the entry $(2, 1)$ represent how the benchmark asset return exposure to index news ($\ell = 1$) changes with the arrival of non-index news ($\ell = 2$). The "$-$" sign in outperformance implies that the return exposure shrinks when asset managers outperform the benchmark, but it flips when they underperform the benchmark.

Similarly, the signs flip for the non-benchmark asset,

$$\text{Outperformance} \Rightarrow 1 > \mathcal{H}_{1t} + \mathcal{H}_{4t} \Rightarrow \text{sign}\left(\mathcal{D}_t \boldsymbol{\sigma}_{2t}^S\right) = \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix}, \tag{54}$$

$$\text{Underperformance} \Rightarrow 1 < \mathcal{H}_{1t} + \mathcal{H}_{4t} \Rightarrow \text{sign}\left(\mathcal{D}_t \boldsymbol{\sigma}_{2t}^S\right) = \begin{pmatrix} + & - & - \\ - & + & + \\ - & + & + \end{pmatrix}. \tag{55}$$

Overall, the signs in these matrices, (52), (53), (54), (55), determine whether the arrival of news increases or decreases the assets' senesitivites. However, the changes in the expected returns of the index and non-index assets due to the uncertainty of performance channel cannot immediately be determined because the market prices of risk are positive and do not change signs,

$$\text{sign}\left(\boldsymbol{\theta}_t'\right) = \begin{pmatrix} + & + & + \end{pmatrix}. \tag{56}$$

To determine whether this channel induces momentum or reversal we apply (39) and find

that

$$\begin{aligned}\operatorname{sign}\left(\boldsymbol{\varepsilon}_t(\mu_{1t}^S)'\right)\operatorname{sign}\left(\boldsymbol{\varepsilon}_t(S_{1t})\right) &= \operatorname{sign}\left(\boldsymbol{\theta}_t'\left(\mathcal{D}_t\boldsymbol{\sigma}_{1t}^S\right)\right)\operatorname{sign}\left(\boldsymbol{\varepsilon}_t(S_{1t})\right)\\ &= \begin{pmatrix}+ & + & +\end{pmatrix}\times\underbrace{\begin{pmatrix}+ & - & -\\ - & + & +\\ - & + & +\end{pmatrix}}_{\text{outperformance}}\begin{pmatrix}+\\ -\\ -\end{pmatrix} = \begin{pmatrix}+ & + & +\end{pmatrix}\times\begin{pmatrix}+\\ -\\ -\end{pmatrix} = \pm, \qquad (57)\\ &= \begin{pmatrix}+ & + & +\end{pmatrix}\times\underbrace{\begin{pmatrix}- & + & +\\ + & - & -\\ + & - & -\end{pmatrix}}_{\text{underperformance}}\begin{pmatrix}+\\ -\\ -\end{pmatrix} = \begin{pmatrix}+ & + & +\end{pmatrix}\times\begin{pmatrix}-\\ +\\ +\end{pmatrix} = \mp. \qquad (58)\end{aligned}$$

By pluggging $\boldsymbol{\theta}_t'$, $\left(\mathcal{D}_t\boldsymbol{\sigma}_{1t}^S\right)$, and $\boldsymbol{\varepsilon}_t(S_{1t})$, respectively given in (28), (50), and (30), we find that the predictability pattern due to the uncertainty of performance channel is characterized by

$$\begin{aligned}\boldsymbol{\theta}_t'\left(\mathcal{D}_t\boldsymbol{\sigma}_{1t}^S\right)\boldsymbol{\varepsilon}_t(S_{1t}) = &\left(\gamma_2-\gamma_1\right)^3\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right\|^2\left(\bar{\mathcal{H}}_1-\mathcal{H}_{1t}\right)\times\\ &\left[\left(\left\|\boldsymbol{\sigma}^I\right\|^2-\boldsymbol{\sigma}^{I\prime}\boldsymbol{\nu}\right)+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)\left\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right\|^2\right], \qquad (59)\end{aligned}$$

where

$$\bar{\mathcal{H}}_1 \equiv \frac{\left(\gamma_2-1\right)\boldsymbol{\sigma}^{I\prime}\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right)-\gamma_1\boldsymbol{\nu}'\left(\boldsymbol{\nu}-\boldsymbol{\sigma}^I\right)}{\left(\gamma_2-\gamma_1\right)\left\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right\|^2} > 0. \qquad (60)$$

The signs of the shock elasticities, (57) and (58), show that index news and non-index news contribute to opposite predictability patterns: whenever the index news implies positive predictability, the non-index news implies negative predictability, and vice versa. The sum of these two contributions is determined by whether the hedging demand $\mathcal{H}_{1t}$ is above or below the parameter $\bar{\mathcal{H}}_1$, given in (60).

The predictability pattern is determined by the relative magnitudes of the market prices of risk for index and non-index news, as follows

$$\bar{\mathcal{H}}_1-\mathcal{H}_{1t} = \frac{\kappa_t^I\left(\left\|\boldsymbol{\sigma}^I\right\|^2-\boldsymbol{\sigma}^{I\prime}\boldsymbol{\nu}\right)-\kappa_t^\nu\left(\left\|\boldsymbol{\nu}\right\|^2-\boldsymbol{\sigma}^{I\prime}\boldsymbol{\nu}\right)}{\left(\gamma_2-\gamma_1\right)\left\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right\|^2}. \qquad (61)$$

When $\mathcal{H}_{1t} < \bar{\mathcal{H}}_1$, the numerator in (61) is positive. In this case, the overall predictability pattern is determined by the predictability pattern of the index news (the first entries in the last cross products in (57) and (58)).

In outperformance, the hedging demands $\mathcal{H}_{1t}$ and $\mathcal{H}_{3t}$ are small, which implies that the benchmark asset exhibits momentum because $1-\mathcal{H}_{1t}-\mathcal{H}_{3t} > 0$ and the predictability pattern is determined by index news ($\bar{\mathcal{H}}_1 > \mathcal{H}_{1t}$), implying that " $+$ " in (57) determines the predictability. Therefore, the benchmark asset return exhibits momentum when asset

managers outperform the benchmark.

In contrast, in underperformance, the hedging demands become larger, leading $1-\mathcal{H}_{1t}-\mathcal{H}_{3t}<0$. As long as the underperformance is not too severe, the predictability pattern is determined by the index news ($\bar{\mathcal{H}}_1>\mathcal{H}_{1t}$), implying that " $-$ " in (58) determines the predictability. Therefore, the benchmark asset return exhibits reversal when asset managers underperform the benchmark.

In severe underperformance, the benchmark asset becomes so desired that its market prices of risk κ_t^I drops so low that the numerator in (61) becomes negative. In this case, the predictability pattern is determined by the non-index news ($\bar{\mathcal{H}}_1<\mathcal{H}_{1t}$), implying that " $+$ " in (58) determines the predictability. Therefore, the benchmark asset return exhibits momentum when asset managers severly underperform the benchmark.

Like the benchmark asset, we determine whether the uncertainty of performance channel drives momentum or reversal in the non-index asset using (39) as follows.

$$\begin{aligned}
&\operatorname{sign}\left(\boldsymbol{\varepsilon}_t(\mu_{2t}^S)'\right)\operatorname{sign}\left(\boldsymbol{\varepsilon}_t(S_{2t})\right)=\operatorname{sign}\left(\boldsymbol{\theta}_t'\left(\mathcal{D}_t\boldsymbol{\sigma}_{2t}^S\right)\right)\operatorname{sign}\left(\boldsymbol{\varepsilon}_t(S_{2t})\right)\\
&\quad=\begin{pmatrix}+&+&+\end{pmatrix}\times\underbrace{\begin{pmatrix}-&+&+\\+&-&-\\+&-&-\end{pmatrix}}_{\text{outperformance}}\begin{pmatrix}-\\+\\+\end{pmatrix}=\begin{pmatrix}+&+&+\end{pmatrix}\times\begin{pmatrix}+\\-\\-\end{pmatrix}=\pm, \qquad (62)\\
&\quad=\begin{pmatrix}+&+&+\end{pmatrix}\times\underbrace{\begin{pmatrix}+&-&-\\-&+&+\\-&+&+\end{pmatrix}}_{\text{underperformance}}\begin{pmatrix}-\\+\\+\end{pmatrix}=\begin{pmatrix}+&+&+\end{pmatrix}\times\begin{pmatrix}-\\+\\+\end{pmatrix}=\mp. \qquad (63)
\end{aligned}$$

By pluggging $\boldsymbol{\theta}_t'$, $\left(\mathcal{D}_t\boldsymbol{\sigma}_{2t}^S\right)$, and $\boldsymbol{\varepsilon}_t(S_{2t})$, respectively given in (28), (51), and (31), we find that the non-benchmark asset has similar predictability pattern to the benchmark asset due to the uncertainty of performance, which is characterized by

$$\begin{aligned}
\boldsymbol{\theta}_t'\left(\mathcal{D}_t\boldsymbol{\sigma}_{2t}^S\right)\boldsymbol{\varepsilon}_t(S_{2t})=&\left(\gamma_2-\gamma_1\right)^3\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left(1-\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right\|^2\left(\bar{\mathcal{H}}_1-\mathcal{H}_{1t}\right)\times\\
&\left[\left(\left\|\boldsymbol{\sigma}^I\right\|^2-\boldsymbol{\sigma}^{I\prime}\boldsymbol{\nu}\right)+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right\|^2\right]. \qquad (64)
\end{aligned}$$

where $\bar{\mathcal{H}}_1$ is given in (60).

The characterization reveals that overall, the predictability pattern in the benchmark asset are similar to those in the non-benchmark asset. The reason is that outperformance in the non benchmark asset $1>\mathcal{H}_{1t}+\mathcal{H}_{4t}$ aligns closely with the outperformance in the benchmark asset $1>\mathcal{H}_{1t}+\mathcal{H}_{3t}$.

In outperformance, the hedging demands $\mathcal{H}_{1t}$ and $\mathcal{H}_{4t}$ are small, which implies that the benchmark asset exhibits momentum because $1-\mathcal{H}_{1t}-\mathcal{H}_{4t}>0$ and the predictability

pattern is determined by index news ($\bar{\mathcal{H}}_1 > \mathcal{H}_{1t}$), implying that " $+$ " in (62) determines the predictability. Therefore, the non-benchmark asset return exhibits momentum when asset managers outperform the benchmark.

In contrast, in underperformance, the hedging demands become larger, leading $1 - \mathcal{H}_{1t} - \mathcal{H}_{4t} < 0$. As long as the underperformance is not too severe, the predictability pattern is determined by the index news ($\bar{\mathcal{H}}_1 > \mathcal{H}_{1t}$), implying that " $-$ " in (63) determines the predictability. Therefore, the benchmark asset return exhibits reversal when asset managers underperform the benchmark.

In severe underperformance, the benchmark asset becomes so desired that its market prices of risk κ_t^I drops so low that the numerator in (61) becomes negative. In this case, the predictability pattern becomes determines by the non-index news ($\bar{\mathcal{H}}_1 < \mathcal{H}_{1t}$), implying that " $+$ " in (63) determines the predictability. Therefore, the benchmark asset return exhibits momentum when asset managers severly underperform the benchmark.

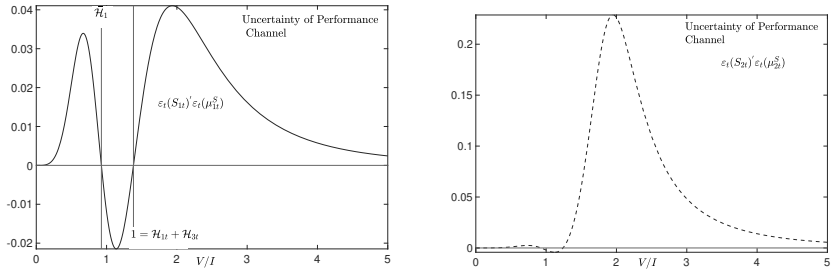


Figure 6. The figures show the predictability arising from the *unceratinty of performance* channel. When V/I is above the vertical line $\bar{\mathcal{H}}_1$, the overall predictability patterns is determined by the predictability in index news. In this case, the *unceratinty of performance* generates momentum when V/I is above the vertical line $1 = \mathcal{H}_{1t} + \mathcal{H}_{3t}$ (indicating the outperformance region), and reversal when it is below (indicating the underperformance region). However, when V/I is below the vertical line $\bar{\mathcal{H}}_{1t}$, the predictability patterns is determined by the predictability in the non-index news. In this case, the *unceratinty of performance* generates momentum in asset returns. The parameters are as in Figure 5.

To summarize, when the asset manager outperforms the benchmark or severely underperforms the benchmark, the *uncertainty of performance* induces the same reaction in the expected return and the asset return, leading to momentum regions. In mild underperformance, this channel induces an opposite reaction in the expected return and the asset return, leading to a reversal region.

$$\text{Outperformance:} \quad \Rightarrow \quad \boldsymbol{\varepsilon}_t(\mu_{it}^S)'\boldsymbol{\varepsilon}_t(S_{it}) > 0, \quad \Rightarrow \text{Momentum}, \tag{65}$$

$$\text{Underperformance: } (\mathcal{H}_{1t} \leq \bar{\mathcal{H}}_1) \quad \Rightarrow \quad \boldsymbol{\varepsilon}_t(\mu_{it}^S)'\boldsymbol{\varepsilon}_t(S_{it}) < 0, \quad \Rightarrow \text{Reversal}, \tag{66}$$

$$\text{Underperformance: } (\mathcal{H}_{1t} > \bar{\mathcal{H}}_1) \quad \Rightarrow \quad \boldsymbol{\varepsilon}_t(\mu_{it}^S)'\boldsymbol{\varepsilon}_t(S_{it}) > 0 \quad \Rightarrow \text{Momentum}. \tag{67}$$

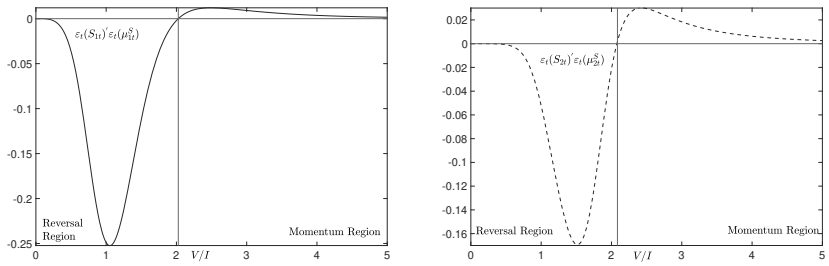


Figure 7. These figures show that momentum, which arises from the uncertainty-of-performance channel, dominates when asset managers outperform. Conversely, reversal, which stems from the benchmarking-incentives channel, dominates when asset managers underperform. The parameters are as in Figure 5.

5.6 The Joint Effect of the Two Predictable Channels

The two predictable channels, *incentives* and *uncertainty of performance*, work in opposite directions when asset managers outperform the benchmark, $\mathcal{H}_{1t} + \mathcal{H}_{3t} < 1$, but also when they severe underperform the benchmark, $\mathcal{H}_{1t} > \bar{\mathcal{H}}_1$.

In this case, the sum of the two predictable channels lead to an asymmetric price reaction in these two regions. When asset managers underperform the benchmark, the benchmarking incentives channel dominates and lead to price reversals. However, when asset managers outperform the benchmark, the uncertainty of performance channel dominates and leads to price momentum.

In the following proposition, we identify a parametric restriction on the risk-on risk aversion parameter, γ_2. This ensures that the *benchmarking incentives* is at least as significant as the *uncertainty of performance*. Specifically, when the risk-on risk-aversion parameter falls within the specified range, $\underline{\gamma}_2 \leq \gamma_2 \leq \bar{\gamma}_2$, prices show overall momentum during outperformance periods and reversal in underperformance periods.

The same two mechanisms apply similarly to the non-index asset, though with a slightly different parametric restriction on the risk-on risk-aversion parameter. When asset managers underperform, the reversal channel is more dominant. Conversely, when outperformance is more likely, the momentum channel is stronger under the parametric restriction $\underline{\underline{\gamma}}_2 \leq \gamma_2 \leq \bar{\bar{\gamma}}_2$. The following proposition summarizes our main findings.

Proposition 3 (**Time-Series Momentum and Reversal**). *The index asset exhibits momentum when asset managers outperform and reversal if they underperform, if and only if*

$$\underline{\gamma}_2 < \gamma_2 < \bar{\gamma}_2. \tag{68}$$

Similarly, the non-index asset exhibits momentum when asset managers outperform and reversal if they underperform, if and only if

$$\underline{\underline{\gamma}}_2 < \gamma_2 < \bar{\bar{\gamma}}_2. \tag{69}$$

The risk-on risk-aversion parameter thresholds $\underline{\gamma}_2$, $\bar{\gamma}_2$, $\underline{\underline{\gamma}}_2$, *and* $\bar{\bar{\gamma}}_2$ *depend on the risk-off risk aversion parameter,* γ_1, *and the active management share* λ. *For a given* γ_1, *there always exist an active management share* λ *such that (68) holds. The same is true for the non-index asset under the parametric restriction in (A.86). The thresholds are given in (A.66), (A.68), (A.71), and (A.72), respectively.*

6 Predictability in Long-short Portfolio Strategies

Since the work of Jegadeesh and Titman (1993), the analysis of predictability in asset returns has involved constructing long-short portfolios by purchasing assets with high returns and selling those with low returns. According to this framework, momentum is identified when the long-short portfolio yields a positive return, whereas reversal is identified when the portfolio yields a negative return.

In this section, we show that the predictability patterns obsereved in the time-series of each asset separately also exist in the long-short portfolio. Our theory implies instantaneous momentum in the long-short portfolio when asset managers outperform their benchmark index and instantaneous reversal when they underperform it.

It is neither obvious nor immediate that time-series predictability would manifest in the long-short portfolio. This phenomenon occurs because index and non-index asset sensitivities are opposite to one another. Positive news for one implies negative news for the other. The long-short portfolio buys the asset that has risen and sells the asset that has fallen, as the former has a positive return and the latter a negative return. When asset managers outperform the benchmark, momentum causes the rising asset to continue rising and the falling asset to continue falling. Consequently, this strategy generates a positive return, aligning with traditional long-short portfolio momentum.

Conversely, when asset managers underperform the benchmark, a reversal causes the rising asset to fall in the future and the falling asset to rise in the future. As a result, this strategy generates a negative return, aligning with traditional long-short portfolio reversal. The following proposition summarizes our key findings.

Proposition 4 (**Momentum and Reversal in the Cross-Section**). *The shock sensitivity of the index asset is positive if and only if the shock sensitivity of the non-index asset is*

negative,

$$\varepsilon_t(S_{1t}, \ell) > 0 \iff \varepsilon_t(S_{2t}, \ell) < 0, \quad \ell = 1, 2, 3. \tag{70}$$

Suppose that the risk-on risk aversion parameter satisfies the conditions (68) and (69), which guarentee time-series momentum in outperformance and time-series reversal in underperformance.

A long-short portfolio that takes a long position on the asset with positive shock sensitivity and a short position on the asset with negative shock sensitivity generates momentum when asset managers outperform the benchmark and reversal when they underperform the benchmark.

6.1 Prices and Expected Returns

To clarify the mechanism, this section characterizes the index and non-index prices and their expected returns. This analysis provides further intuitions for why the long-short portfolio generates momentum in the outperformance region and reversal in the underperformance region. We start by characterizing the index and non-index prices.

Proposition 5 (**Stock Prices**). *The index and non-index price-dividend ratios in the presence of risk-on incentives are given by*

$$S_t^I / I_t = \frac{\mathcal{E}_{3t}^{On}}{\mathcal{E}_{1t}^{On}}, \qquad S_{2t}/D_{2t} = \frac{\mathcal{E}_{4t}^{On}}{\mathcal{E}_{1t}^{On}}, \tag{71}$$

while in the presence of risk-off incentives, they are given by

$$S_t^I / I_t = \frac{\mathcal{E}_{3t}^{Off}}{\mathcal{E}_{1t}^{Off}}, \qquad S_{2t}/D_{2t} = \frac{\mathcal{E}_{4t}^{Off}}{\mathcal{E}_{1t}^{Off}}. \tag{72}$$

The index and non-index prices in the presence of both risk-on and risk-off incentives are given by

$$S_t^I / I_t = \frac{\mathcal{E}_{3t}^{On}}{\mathcal{E}_{1t}^{On}} \left[\frac{\mathcal{H}_{1t}}{\mathcal{H}_{3t}}\right], \qquad S_{2t}/D_{2t} = \frac{\mathcal{E}_{4t}^{On}}{\mathcal{E}_{1t}^{On}} \left[\frac{\mathcal{H}_{1t}}{\mathcal{H}_{4t}}\right]. \tag{73}$$

The comparison between the index and non-index price-dividend ratios reveals that the index price-dividend ratio

- *is higher than the non-index one with pure risk-on incentives.*
- *is lower than the non-index one with pure risk-off incentives, if the difference between risk-on and risk-off risk aversion parameters is greater than one:* $\gamma_2 - \gamma_1 \geq 1$.

In the joint incentives case, if $\gamma_2 - \gamma_1 \geq 1$, the index price-dividend ratio is higher than the non-index one if and only if the ratio of the market news to the index news is below a deterministic threshold,

$$\frac{V_t}{I_t} \leq \left[\frac{\left(\mathcal{E}_{3t}^{On} - \mathcal{E}_{4t}^{On}\right)}{\left(\mathcal{E}_{4t}^{Off} - \mathcal{E}_{3t}^{Off}\right)} \frac{(\gamma_2 - 1)}{(1 - \gamma_1)} \right]^{\frac{1}{\gamma_2 - \gamma_1}}. \tag{74}$$

Further, in the joint incentives case, the index price-dividend ratio S_t^I/I_t

- *decreases following news about the market: $\partial\left(S_t^I/I_t\right)/\partial V_t < 0$,*
- *increases following news about the index: $\partial\left(S_t^I/I_t\right)/\partial I_t > 0$.*

The results flip for the non-index price-dividend ratio, S_{2t}/D_{2t}; it

- *increases following news about the market: $\partial\left(S_{2t}/D_{2t}\right)/\partial V_t > 0$,*
- *decreases following news about the index: $\partial\left(S_{2t}/D_{2t}\right)/\partial I_t < 0$,*

where $\mathcal{H}_{1t}$, $\mathcal{H}_{3t}$, and $\mathcal{H}_{4t}$ are given in (14), and $\mathcal{E}_{1t}^{Off}$, $\mathcal{E}_{1t}^{On}$, $\mathcal{E}_{3t}^{Off}$, $\mathcal{E}_{3t}^{On}$, $\mathcal{E}_{4t}^{Off}$, $\mathcal{E}_{4t}^{On}$ are given in (A.1), (A.2), (A.7), (A.8), (A.10), and (A.11), respectively.

In pure risk-on incentives, asset managers desire to post higher returns when the benchmark is high rather than when it is low. These risk-on incentives induce asset managers to buy the benchmark to hedge against unexpected increases in the benchmark, which, in turn, creates extra demand for the benchmark. Equilibrium is restored by raising the prices of assets within the benchmark and lowering the prices of assets outside the benchmark, thereby diminishing the benchmark's attractiveness relative to non-index assets.

In pure risk-off incentives, asset managers desire to maintain their accumulated performance. In this case, asset managers are risk-averse and protect their portfolio against scenarios where the benchmark declines. To achieve that goal, asset managers substantially reduce their benchmark position, which increases the available supply for the benchmark. Equilibrium is restored by decreasing the prices of benchmark assets and increasing the prices of non-benchmark assets, which in turn, enhances the attractiveness of the benchmark relative to the market.

In the joint incentives case, asset managers weigh the risk-on and risk-off strategies, depending on the likelihood that they will outperform the benchmark. Since their portfolio is not fully correlated with the benchmark, the likelihood of outperformance is influenced by the ratio of market news to index news, V/I. When V is large relative to I, the asset managers are likely to outperform, whereas when V is small relative to I, the asset managers are likely to underperform.

Thus far, we have demonstrated a negative correlation between index and non-index prices, an increase in the index price leads to a decrease in the non-index price, and vice versa. Next, we discuss the implications for expected returns.

By applying the covariaion between between the discount factor exposure (28) and the assets exposure, (24) and (25), we find that the expected return (per unit of time) of the index is given by

$$\begin{aligned}\mu_{1t}^{S} = &\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)\times \\ &\left\{\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\boldsymbol{\sigma}^{I\prime}\left(\boldsymbol{\sigma}^{I}-\boldsymbol{\nu}\right)-\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\boldsymbol{\nu}'\left(\boldsymbol{\nu}-\boldsymbol{\sigma}^{I}\right)\right\} \\ &+\left\{\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\boldsymbol{\sigma}^{I\prime}\left(\boldsymbol{\sigma}_1^{D}\right)+\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\boldsymbol{\nu}'\left(\boldsymbol{\sigma}_1^{D}\right)\right\},\end{aligned} \tag{75}$$

and the expected return (per unit of time) of the non-index asset is given by

$$\begin{aligned}\mu_{2t}^{S} = &\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\times \\ &\left\{-\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\boldsymbol{\sigma}^{I\prime}\left(\boldsymbol{\sigma}^{I}-\boldsymbol{\nu}\right)+\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\boldsymbol{\nu}'\left(\boldsymbol{\nu}-\boldsymbol{\sigma}^{I}\right)\right\} \\ &+\left\{\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\boldsymbol{\sigma}^{I\prime}\left(\boldsymbol{\sigma}_2^{D}\right)+\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\boldsymbol{\nu}'\left(\boldsymbol{\sigma}_2^{D}\right)\right\}.\end{aligned} \tag{76}$$

There are two curly brackets in each representation. The bottom curly bracket in each representation identifies the contribution to the expected return due to the fundamental risk exposures $\left(\boldsymbol{\sigma}_1^{D}\right)$ and $\left(\boldsymbol{\sigma}_2^{D}\right)$. The top curly barckets identify the contribution to the expected return due to the changes in the market prices of risk, which reflects the *incentives* channel. As asset managers' accumulated performance declines, equilibrium lowers the market prices of risk that correlate with the benchmark and raises the market prices of risk orthogonal to the benchmark. Consequently, this channel reduces the expected return of benchmark assets and increases the expected return of assets outside the benchmark.

The leading factor multiplying the first curly bracket in each representation highlights the assets' response to the *uncertainty of performance*. As asset managers' accumulated performance aligns more closely with the benchmark, assets become increasingly sensitive to news in equilibrium, which increases the expected returns of both assets.

Overall, the expected returns are influenced by the two channels and the contribution of the fundamental effect. Figure 8 illustrates the effects of the two channels, and Figure 9 illustrates the combined effect of the two channels and the fundamental effect. The fact that the expected returns of the index and non-index assets work in opposite direction is the key driver of the cross-sectional predictability: if the arrival of news increases the expected return of the index, it also decreases the expected return of assets outside the index. Figure 9 also illustrates the predictability patterns in the cross-section.

The following proposition summarizes our key findings.

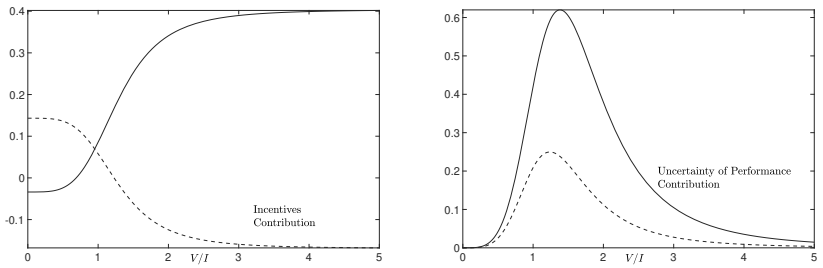


Figure 8. The solid line in the left figure plots the top curly brackets for the index asset (75), while the dashed line represents the curly barckets for the non-index asset (76). The figure shows that as accumulated performance deteriorates, the benchmarking incentives (changes to the market prices of risk) lead the benchmark asset's expected return to decline and the non-index asset's expected return to rise. The solid line in the right figure plots the leading factor multiplying the first curly bracket for the index asset (75), while the dashed line represents the leading factor for the non-index asset (76). First, the figure shows that the expected returns of both assets rise due to the uncertainty of performance. Second, it shows that as accumulated performance deteriorates, the expected return of the benchmark asset falls, while that of the non-index asset rises. The parameters are as in Figure 5.

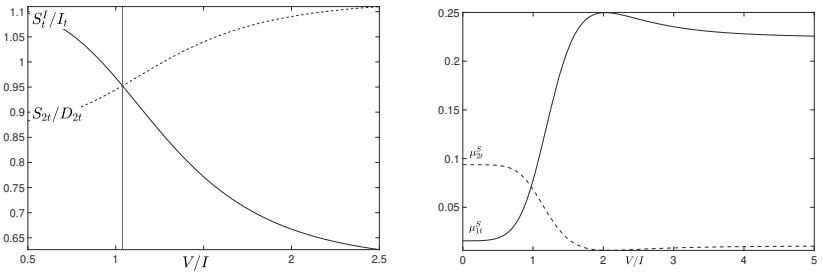


Figure 9. The left figure plots the index and non-index price-dividend ratios in the joint incentives case as a function of (V/I), the state variable governing the likelihood of accumulated performance. The vertical line is given in (74) and captures the location at which the price-dividend ratios is at the threshold. The figure shows that when the arrival of news causes one asset to rise, it also causes the other asset to fall. The right figure plots the expected returns of index and non-index assets as a function of (V/I). The long-short portfolio buys the asset that has risen (positive slope in the left figure) and sells the asset that has fallen (negative slope in the left figure). When asset managers outperform the benchmark, momentum causes the rising asset to continue rising (positive slope in the right figure) and the falling asset to continue falling (negative slope in the right figure). Consequently, this strategy results in a positive return, aligning with traditional cross-sectional momentum. Conversely, when asset managers underperform the benchmark, reversal causes the rising asset to fall (negative slope in the right figure) in the future and the falling asset to rise (positive slope in the right figure). As a result, this strategy leads to a negative return, aligning with traditional cross-sectional reversal. The parameters are as in Figure 5.

Proposition 6 (**Expected Return**). *The expected returns for the index and non-index assets are given by (75) and (76), respectively. The expected return of the index in the case*

of extreme outperformance is strictly higher than in the case of extreme underperformance ($\mu^S_{1t}\,|_{\mathcal{H}_{it}\downarrow 0} - \mu^S_{1t}\,|_{\mathcal{H}_{it}\uparrow 1} > 0$). *In contrast, the expected return of the non-index asset in extreme outperformance is strictly lower than in the case of extreme underperformance* ($\mu^S_{2t}\,|_{\mathcal{H}_{it}\downarrow 0} - \mu^S_{2t}\,|_{\mathcal{H}_{it}\uparrow 1} < 0$). *Furthermore, the index expected return*

- *decreases in outperformance if and only if* $\underline{\gamma}_2 \leq \gamma_2$,
- *increases in underperformance if and only if* $\bar{\gamma}_2 \geq \gamma_2$.

Conversly, the non-index expected return

- *increases in outperformance if and only if* $\underline{\underline{\gamma}}_2 \leq \gamma_2$,
- *decreases in underperformance if and only if* $\bar{\bar{\gamma}}_2 \geq \gamma_2$,

where $\underline{\gamma}_2$, $\bar{\gamma}_2$, $\underline{\underline{\gamma}}_2$, *and* $\bar{\bar{\gamma}}_2$ *are given in (A.66), (A.68), (A.71), and (A.72), respectively.*

7 Conclusion

Mutual fund managers aim to balance incentives to outperform the index with incentives to maintain accumulated performance levels when ahead of the index. This balance leads asset managers to optimally adjust their risk-on and risk-off portfolio positions based on their accumulated performance. When managers are likely to underperform the index, they increase their allocation to risk-on assets to improve accumulated performance relative to the benchmark. As their accumulated performance improves, they shift toward a risk-off portfolio, reducing their exposure to risk-on assets.

This paper incorporates this time-varying incentives into a dynamic, continuous-time asset pricing model and derives closed-form solutions for equilibrium quantities.

Asset managers' efforts to balance risk-on and risk-off investment strategies provide a potential explanation for the observed momentum and reversal patterns in asset markets, both over time in individual assets. We observe time-series momentum when asset managers outperform the benchmark, and time-series reversal when they underperform.

We demonstrate that these predictability patterns also appear in long-short portfolios: a strategy that buys assets with higher returns and sells those with lower returns generates positive returns (momentum) when asset managers outperform the benchmark, and negative returns (reversal) when they underperform.

Return predictability arises from two primary channels. The first is driven by predictable changes in incentive contracts: as new information becomes available, asset managers adjust the optimal balance between risk-on and risk-off portfolios, which affects returns through

market clearing. The second channel relates to performance uncertainty. When the probabilities of outperformance and underperformance are balanced, uncertainty about asset managers' accumulated performance is heightened, amplifying asset sensitivity to news in equilibrium.

References

Albuquerque, R., and J. Miao. 2014. Advance Information and Asset Prices. *Journal of Economic Theory* 149:236–275.

Allen, F., S. Morris, and H. S. Shin. 2006. Beauty Contests and Iterated Expectations in Asset Markets. *Review of financial studies* 19:719–752.

Andrei, D., and J. Cujean. 2017. Information Percolation, Momentum and Reversal. *Journal of Financial Economics* 123:617–645.

Aristidou, A., A. Giga, S. Lee, and F. Zapatero. 2025. Aspirational Utility and Investment Behavior. *Journal of Financial Economics* 163:103970.

Banerjee, S., R. Kaniel, and I. Kremer. 2009. Price Drift as an Outcome of Differences in Higher-Order Beliefs. *The Review of Financial Studies* 22:3707–3734.

Barberis, N., and A. Shleifer. 2003. Style Investing. *Journal of Financial Economics* 68:161–199.

Barberis, N., A. Shleifer, and R. Vishny. 1998. A Model of Investor Sentiment. *Journal of Financial Economics* 49:307–343.

Basak, S., and A. Pavlova. 2013. Asset Prices and Institutional Investors. *American Economic Review* 103:1728–1758.

Basak, S., and A. Pavlova. 2016. A Model of Financialization of Commodities. *Journal of Finance* 71:1511–1556.

Berk, J. B., R. C. Green, and V. Naik. 1999. Optimal Investment, Growth Options, and Security Returns. *Journal of Finance* 54:1553–1607.

Biais, B., P. Bossaerts, and C. Spatt. 2010. Equilibrium Asset Pricing and Portfolio Choice Under Asymmetric Information. *The Review of Financial Studies* 23:1503–1543.

Borovička, J., L. P. Hansen, M. Hendricks, and J. A. Scheinkman. 2011. Risk-Price Dynamics. *Journal of Financial Econometrics* 9:3–65.

Borovička, J., L. P. Hansen, and J. A. Scheinkman. 2014. Shock Elasticities and Impulse Responses. *Mathematics and Financial Economics* 8:333–354.

Brennan, M. J. 1993. Agency and Asset Pricing. *Working Paper* .

Buffa, A. M., and I. Hodor. 2023. Institutional Investors, Heterogeneous Benchmarks and the Comovement of Asset Prices. *Journal of Financial Economics* 147:352–381.

Cespa, G., and X. Vives. 2012. Dynamic Trading and Asset Prices: Keynes vs. Hayek. *The Review of Economic Studies* 79:539–580.

Chan, L. K. C., N. Jegadeesh, and J. Lakonishok. 1996. Momentum Strategies. *Journal of Finance* 51:1681–1713.

Cujean, J., and M. Hasler. 2017. Why Does Return Predictability Concentrate in Bad Times? *Journal of Finance* 72:2717–2758.

Cuoco, D., and R. Kaniel. 2011. Equilibrium Prices in the Presence of Delegated Portfolio Management. *Journal of Financial Economics* 101:264–296.

Cutler, D. M., J. M. Poterba, and L. H. Summers. 1991. Speculative Dynamics. *Review of Economic Studies* 58:529–546.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam. 1998. Investor Psychology and Security Market Under- and Overreactions. *Journal of Finance* 53:1839–1885.

Dasgupta, A., A. Prat, and M. Verardo. 2011. The Price Impact of Institutional Herding. *The Review of Financial Studies* 24:892–925.

DellaVigna, S., and D. Pope. 2017. What Motivates Effort? Evidence and Expert Forecasts. *Review of Economic Studies* 85:1029–1069.

Detemple, J. B., R. Garcia, and M. Rindisbacher. 2003. A Monte Carlo Method for Optimal Portfolios. *Journal of Finance* 58:401–446.

Fama, E. F., and K. R. French. 1988. Permanent and Temporary Components of Stock Prices. *Journal of Political Economy* 96:246–273.

Friedman, M., and L. J. Savage. 1948. The Utility Analysis of Choices Involving Risk. *Journal of Political Economy* 56:279–304.

Gómez, J.-P., and F. Zapatero. 2003. Asset Pricing Implications of Benchmarking: A Two-Factor CAPM. *European Journal of Finance* 9:343–357.

Hillert, A., H. Jacobs, and S. Müller. 2014. Media Makes Momentum. *The Review of Financial Studies* 27:3467–3501.

Hodor, I., and F. Zapatero. 2023. Asset Pricing Implications of Heterogeneous Investment Horizons. Working paper, SSRN.

Holden, C., and A. Subrahmanyam. 2002. News Events, Information Acquisition, and Serial Correlation. *Journal of Business* 75:1–32.

Hong, H., and J. C. Stein. 1999. A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets. *Journal of Finance* 54:2143–2184.

Huang, D., J. Li, L. Wang, and G. Zhou. 2020. Time Series Momentum: Is it there? *Journal of Financial Economics* 135:774–794.

James M. Poterba, L. H. S. 1988. Mean Reversion in Stock Prices: Evidence and Implications. *Journal of Financial Economics* 22:27–59.

Jegadeesh, N., and S. Titman. 1993. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance* 48:65–91.

Johnson, T. C. 2002. Rational Momentum Effects. *Journal of Finance* 57:585–608.

Kahneman, D., and A. Tversky. 1979. Prospect Theory: An Analysis of Decision under Risk. *Econometrica* 47:263–291.

Kahneman, D., and A. Tversky. 1991. Loss Aversion in Riskless Choice: A Reference-Dependent Model. *Quarterly Journal of Economics* 106:1039–1061.

Khaw, M. W., Z. Li, and M. Woodford. 2020. Cognitive Imprecision and Small-Stakes Risk Aversion. *Review of Economic Studies* 88:1979–2013.

Kőszegi, B., and M. Rabin. 2006. A Model of Reference-Dependent Preferences. *Quarterly Journal of Economics* 121:1133–1165.

Levy, H. 1969. A Utility Function Depending on the First Three Moments. *Journal of Finance* 24:715–719.

Liu, L. X., and L. Zhang. 2008. Momentum Profits, Factor Pricing, and Macroeconomic Risk. *The Review of Financial Studies* 21:2417–2448.

Liu, L. X., and L. Zhang. 2014. A Neoclassical Interpretation of Momentum. *Journal of Monetary Economics* 67:109–128.

Lo, A. W., and A. C. MacKinlay. 1988. Stock Market Prices do not Follow Random Walks: Evidence from a Simple Specification Test. *Review of Financial Studies* 1:41–66.

Menzly, L., T. Santos, and P. Veronesi. 2004. Understanding Predictability. *Journal of Political Economy* 112:1–47.

Moskowitz, T. J., Y. H. Ooi, and L. H. Pedersen. 2012. Time Series Momentum. *Journal of Financial Economics* 104:228–250.

Ocone, D. L., and I. Karatzas. 1991. A Generalized Clark Representation Formula, with Application to Optimal Portfolios. *Stochastics and Stochastic Reports* 34:187–220.

Øksendal, B. 1997. An Introduction to Malliavin Calculus with Applications to Economics. Tech. rep., Unpublished Manuscript.

Ottaviani, M., and P. N. Sørensen. 2015. Price Reaction to Information with Heterogeneous Beliefs and Wealth Effects: Underreaction, Momentum, and Reversal. *American Economic Review* 105:1–34.

Pagel, M. 2017. Expectations-Based Reference-Dependent Life-Cycle Consumption. *Review of Economic Studies* 84:885–934.

Rabin, M. 2000. Risk Aversion and Expected-utility Theory: A Calibration Theorem. *Econometrica* 68:1281 – 1292.

Sagi, J. S., and M. S. Seasholes. 2007. Firm-specific Attributes and the Cross-section of Momentum. *Journal of Financial Economics* 84:389–434.

Shin, H. S. 2006. Disclosure Risk and Price Drift. *Journal of Accounting Research* 44:351–379.

Sotes-Paladino, J., and F. Zapatero. 2019. Riding the Bubble with Convex Incentives. *Review of Financial Studies* 32:1416–1456.

Vayanos, D., and P. Woolley. 2013. An Institutional Theory of Momentum and Reversal. *The Review of Financial Studies* 26:1087–1145.

A Internet Appendix – Proofs

Before we begin with the formal proofs, we solve the following expected values.

$$\mathcal{E}_{1t}^{\text{Off}} = (\mathbf{1}_{\text{Off}})\,\mathcal{E}_{1t}\,(1-\gamma_2,-\gamma_1) \equiv \frac{E_t\left[(I_T)^{1-\gamma_2}\,(V_T)^{-\gamma_1}\right]}{(I_t)^{1-\gamma_2}\,(V_t)^{-\gamma_1}} \tag{A.1}$$
$$= e^{\left((-\gamma_1)(\eta)-(\gamma_2-1)(\mu_1)+(\sigma_1\nu_1)(\gamma_1)(\gamma_2-1)+\frac{1}{2}(\sigma_1)^2(\gamma_2)(\gamma_2-1)+\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_1)(1+\gamma_1)\right)(T-t)},$$

$$\mathcal{E}_{1t}^{\text{On}} = (\mathbf{1}_{\text{On}})\,\mathcal{E}_{1t}\,(1-\gamma_1,-\gamma_2) \equiv \frac{E_t\left[(I_T)^{1-\gamma_1}\,(V_T)^{-\gamma_2}\right]}{(I_t)^{1-\gamma_1}\,(V_t)^{-\gamma_2}} \tag{A.2}$$
$$= e^{\left((-\gamma_2)(\eta)+(1-\gamma_1)(\mu_1)-(\sigma_1\nu_1)(\gamma_2)(1-\gamma_1)-\frac{1}{2}(\sigma_1)^2(\gamma_1)(1-\gamma_1)+\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_2)(1+\gamma_2)\right)(T-t)},$$

$$\frac{\mathcal{E}_{1t}^{\text{Off}}}{\mathcal{E}_{1t}^{\text{On}}} \equiv \mathcal{E}_{1t} = e^{\left((\gamma_2-\gamma_1)\left(\eta-\mu_1+\sigma_1\nu_1+\frac{1}{2}(\sigma_1)^2(\gamma_1+\gamma_2-1)-\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_1+\gamma_2+1)\right)\right)(T-t)}, \tag{A.3}$$

$$\mathcal{E}_{2t}^{\text{Off}} = (\mathbf{1}_{\text{Off}})\,\mathcal{E}_{2t}\,(1-\gamma_2,1-\gamma_1) \equiv \frac{E_t\left[(I_T)^{1-\gamma_2}\,(V_T)^{1-\gamma_1}\right]}{(I_t)^{1-\gamma_2}\,(V_t)^{1-\gamma_1}} \tag{A.4}$$
$$= e^{\left((1-\gamma_1)(\eta)-(\gamma_2-1)(\mu_1)-(\sigma_1\nu_1)(1-\gamma_1)(\gamma_2-1)+\frac{1}{2}(\sigma_1)^2(\gamma_2)(\gamma_2-1)+\frac{1}{2}\|\boldsymbol{\nu}\|^2(-\gamma_1)(1-\gamma_1)\right)(T-t)},$$

$$\mathcal{E}_{2t}^{\text{On}} = (\mathbf{1}_{\text{On}})\,\mathcal{E}_{2t}\,(1-\gamma_1,1-\gamma_2) \equiv \frac{E_t\left[(I_T)^{1-\gamma_1}\,(V_T)^{1-\gamma_2}\right]}{(I_t)^{1-\gamma_1}\,(V_t)^{1-\gamma_2}} \tag{A.5}$$
$$= e^{\left((1-\gamma_2)(\eta)+(1-\gamma_1)(\mu_1)-(\sigma_1\nu_1)(\gamma_2-1)(1-\gamma_1)-\frac{1}{2}(\sigma_1)^2(\gamma_1)(1-\gamma_1)+\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_2)(\gamma_2-1)\right)(T-t)},$$

$$\frac{\mathcal{E}_{2t}^{\text{Off}}}{\mathcal{E}_{2t}^{\text{On}}} \equiv \mathcal{E}_{2t} = e^{\left((\gamma_2-\gamma_1)\left(\eta-\mu_1+\frac{1}{2}\left[(\sigma_1)^2-\|\boldsymbol{\nu}\|^2\right](\gamma_1+\gamma_2-1)\right)\right)(T-t)}, \tag{A.6}$$

$$\mathcal{E}_{3t}^{\text{Off}} = (\mathbf{1}_{\text{Off}})\,\mathcal{E}_{3t}\,(2-\gamma_2,-\gamma_1) \equiv \frac{E_t\left[(I_T)^{2-\gamma_2}\,(V_T)^{-\gamma_1}\right]}{(I_t)^{2-\gamma_2}\,(V_t)^{-\gamma_1}} \tag{A.7}$$
$$= e^{\left(-\gamma_1\eta+(2-\gamma_2)(\mu_1)-(\sigma_1\nu_1)\gamma_1(2-\gamma_2)+\frac{1}{2}(\sigma_1)^2(\gamma_2-1)(\gamma_2-2)+\frac{1}{2}\|\boldsymbol{\nu}\|^2\gamma_1(1+\gamma_1)\right)(T-t)},$$

$$\mathcal{E}_{3t}^{\text{On}} = (\mathbf{1}_{\text{On}})\,\mathcal{E}_{3t}\,(2-\gamma_1,-\gamma_2) \equiv \frac{E_t\left[(I_T)^{2-\gamma_1}\,(V_T)^{-\gamma_2}\right]}{(I_t)^{2-\gamma_1}\,(V_t)^{-\gamma_2}} \tag{A.8}$$
$$= e^{\left(-\gamma_2\eta+(2-\gamma_1)(\mu_1)-(\sigma_1\nu_1)\gamma_2(2-\gamma_1)+\frac{1}{2}(\sigma_1)^2(\gamma_1-1)(\gamma_1-2)+\frac{1}{2}\|\boldsymbol{\nu}\|^2\gamma_2(1+\gamma_2)\right)(T-t)},$$

$$\frac{\mathcal{E}_{3t}^{\text{Off}}}{\mathcal{E}_{3t}^{\text{On}}} \equiv \mathcal{E}_{3t} = e^{(\gamma_2-\gamma_1)\left(\eta-\mu_1+2(\sigma_1\nu_1)+\frac{1}{2}(\sigma_1)^2(\gamma_2+\gamma_1-3)-\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_2+\gamma_1+1)\right)(T-t)}, \tag{A.9}$$

$$\begin{aligned}\mathcal{E}_{4t}^{\text{Off}} &= (\mathbf{1}_{\text{Off}})\,\mathcal{E}_{4t}\,(1-\gamma_2,-\gamma_1) \equiv \frac{E_t\left[(I_T)^{1-\gamma_2}\,(V_T)^{-\gamma_1}\,D_{2T}\right]}{(I_t)^{1-\gamma_2}\,(V_t)^{-\gamma_1}\,D_{2t}} \\ &= e^{\left[-(\gamma_2-1)\mu_1-\gamma_1\eta+\mu_2+(\sigma_1\nu_1)(\gamma_1)(\gamma_2-1)-(\sigma_2\nu_2)(\gamma_1)+\frac{1}{2}(\sigma_1)^2(\gamma_2-1)(\gamma_2)+\frac{1}{2}\|\boldsymbol{\nu}\|^2\gamma_1(1+\gamma_1)\right](T-t)},\end{aligned} \tag{A.10}$$

$$\begin{aligned}\mathcal{E}_{4t}^{\text{On}} &= (\mathbf{1}_{\text{On}})\,\mathcal{E}_{4t}\,(1-\gamma_1,-\gamma_2) \equiv \frac{E_t\left[(I_T)^{1-\gamma_1}\,(V_T)^{-\gamma_2}\,D_{2T}\right]}{(I_t)^{1-\gamma_1}\,(V_t)^{-\gamma_2}\,D_{2t}} \\ &= e^{\left[-(\gamma_1-1)\mu_1-\gamma_2\eta+\mu_2+(\sigma_1\nu_1)(\gamma_2)(\gamma_1-1)-(\sigma_2\nu_2)(\gamma_2)+\frac{1}{2}(\sigma_1)^2(\gamma_1-1)(\gamma_1)+\frac{1}{2}\|\boldsymbol{\nu}\|^2\gamma_2(1+\gamma_2)\right](T-t)},\end{aligned} \tag{A.11}$$

$$\frac{\mathcal{E}_{4t}^{\text{Off}}}{\mathcal{E}_{4t}^{\text{On}}} \equiv \mathcal{E}_{4t} = e^{(\gamma_2-\gamma_1)\left[-\mu_1+\eta+(\sigma_1\nu_1)+(\sigma_2\nu_2)+\frac{1}{2}(\sigma_1)^2(\gamma_2+\gamma_1-1)-\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_2+\gamma_1+1)\right](T-t)}. \tag{A.12}$$

We reference to these expected values throughout the proofs. Note that $\mathcal{E}_{it}$, $i = 1,2,3,4$ is not well defined in the risk-off incentive case, since $\mathcal{E}_{it}^{\text{On}} = 0$. In pure risk-on incentives $\mathcal{E}_{it} = 0$, $i = 1,2,3,4$ since $\mathcal{E}_{it}^{\text{Off}} = 0$. By plugging the definition of $\mathcal{E}_{it}$ into $\mathcal{H}_{it}$ we find that

$$\mathcal{H}_{it} \equiv \frac{1}{1+\frac{1-\gamma_1}{\gamma_2-1}\,(V_t/I_t)^{\gamma_2-\gamma_1}\,\mathcal{E}_{it}} = \frac{\mathcal{E}_{it}^{\text{On}}}{\mathcal{E}_{it}^{\text{On}}+\frac{1-\gamma_1}{\gamma_2-1}\,(V_t/I_t)^{\gamma_2-\gamma_1}\,\mathcal{E}_{it}^{\text{Off}}}, \tag{A.13}$$

implying that $\mathcal{H}_{it} = 0$ in the pure risk-off incentives and $\mathcal{H}_{it} = 1$ in the pure risk-on incentives. Notice that

$$\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu} = \frac{\sigma_1^2}{N}\,(N-\lambda) > 0, \tag{A.14}$$

$$\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I = \frac{\sigma_1^2}{N^2}\,(1-\lambda)\,((N-1)-\lambda)\,, \tag{A.15}$$

implying that $\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I$ approached zero as $\lambda \to 1$, and that

$$\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu} > \|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I. \tag{A.16}$$

By using the inequalities (A.14) and (A.15), and the that positive news have a positive

impact on fundamentals, $\sigma_2 > 0$ and $\nu_2 > 0$, we find that

$$\mathcal{H}_{3t} > \mathcal{H}_{1t} \iff \mathcal{E}_{1t} > \mathcal{E}_{3t} \iff e^{(\gamma_2-\gamma_1)\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)(T-t)} > 1, \tag{A.17}$$

$$\mathcal{H}_{1t} > \mathcal{H}_{4t} \iff \mathcal{E}_{4t} > \mathcal{E}_{1t} \iff e^{(\gamma_2-\gamma_1)(\sigma_2\nu_2)(T-t)} > 1, \tag{A.18}$$

$$\mathcal{H}_{1t} > \mathcal{H}_{2t} \iff \mathcal{E}_{2t} > \mathcal{E}_{1t} \iff e^{(\gamma_2-\gamma_1)\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)(T-t)} > 1, \tag{A.19}$$

$$\mathcal{H}_{3t} > \mathcal{H}_{2t} \iff \mathcal{E}_{2t} > \mathcal{E}_{3t} \iff e^{(\gamma_2-\gamma_1)\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)(T-t)} > 1. \tag{A.20}$$

The definition of the risk-on hedging demand (14) and the ensuing analysis implies that in extreme underperformance the ratio of market news to index news approaches zero, $V/I \downarrow 0$, while in extreme outperformance the ratio approaches infinity, $V/I \downarrow \infty$, implying that

$$\lim_{V/I\to 0} \mathcal{H}_{it} - \mathcal{H}_{jt} \to 0, \qquad \lim_{V/I\to \infty} \mathcal{H}_{it} - \mathcal{H}_{jt} \to 0. \tag{A.21}$$

The derivative of the risk-on hedging demand with respect to V/I equals

$$\frac{d\mathcal{H}_{it}}{d\frac{V}{I}} = -\left(\gamma_2 - \gamma_1\right)\mathcal{H}_{it}\left(1-\mathcal{H}_{it}\right)\left(V_t/I_t\right)^{-1} < 0, \tag{A.22}$$

and the derivative of $\mathcal{H}_{it} - \mathcal{H}_{jt}$ with respect to V/I is given by

$$\frac{d\mathcal{H}_{it}}{d\frac{V}{I}} - \frac{d\mathcal{H}_{jt}}{d\frac{V}{I}} = -\left(\gamma_2 - \gamma_1\right)\left(V_t/I_t\right)^{-1}\left(\mathcal{H}_{it} - \mathcal{H}_{jt}\right)\left(1-\mathcal{H}_{it}-\mathcal{H}_{jt}\right), \tag{A.23}$$

which approaches zero at extreme outperformance and underperformance due to (A.21). Next, we find that $\mathcal{H}_{3t} - \mathcal{H}_{1t}$ converges to zero as fast as $\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)$ converges to zero, both when $\mathcal{H}_{it} \to 0$ and $\mathcal{H}_{it} \to 1$, and the limits are given by

$$\lim_{\mathcal{H}_{it}\to 0} = \lim_{V/I\to\infty} \frac{\mathcal{H}_{jt} - \mathcal{H}_{1t}}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)} = \frac{\mathcal{E}_{1t} - \mathcal{E}_{jt}}{\mathcal{E}_{jt}} > 0, \quad j = 3,4, \tag{A.24}$$

$$\lim_{\mathcal{H}_{it}\to 1} = \lim_{V/I\to 0} \frac{\mathcal{H}_{jt} - \mathcal{H}_{1t}}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)} = \frac{\mathcal{E}_{1t} - \mathcal{E}_{jt}}{\mathcal{E}_{1t}} > 0. \tag{A.25}$$

Lastly,

$$\lim_{V/I\to 0} \frac{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}{V/I} = \lim_{V/I\to\infty} \frac{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}{V/I} = 0, \tag{A.26}$$

$$\lim_{V/I\to 0} \frac{\mathcal{H}_{jt} - \mathcal{H}_{1t}}{V/I} = \lim_{V/I\to\infty} \frac{\mathcal{H}_{jt} - \mathcal{H}_{1t}}{V/I} = 0, \quad j = 3,4. \tag{A.27}$$

Proof of Proposition 7 (Discount Factor). The security market is dynamically com-

plete. As such, there exists a unique state price density process, ξ, and the no arbitrage relations

$$\xi_t S_{kt} = \mathbb{E}_t \left[\xi_T D_{kT} \right], \quad t \in [0, T], k = 1, 2, 3, \tag{A.28}$$

is always satisfied. In our setup, we set $r = 0$ for $t \leq T$, and thus, the state price density evolves according to

$$d\xi_t = -\xi_t \left(\boldsymbol{\theta}_t' d\boldsymbol{Z}_t \right), \quad t \leq T. \tag{A.29}$$

The vector process $\boldsymbol{\theta}_t$ is the cash-flow news market risk prices. Restating the dynamic budget constraint as

$$\xi_t W_t^{\mathcal{A}} = \mathbb{E}_t \left[\xi_T W_T^{\mathcal{A}} \right], \quad t \in [0, T], \tag{A.30}$$

and maximizing the asset manager's objective function (11), subject to (A.30) at time $t = 0$, we obtain the first order condition

$$y^{\mathcal{A}} \xi_T = \mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} \left(W_T^{\mathcal{A}} \right)^{-\gamma_1}}{(\gamma_2 - 1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} \left(W_T^{\mathcal{A}} \right)^{-\gamma_2}}{(1 - \gamma_1)}, \tag{A.31}$$

where $y^{\mathcal{A}}$ is the Lagrange multiplier, and due to no-arbitrage condition at time T, $S_T^I = I_T$.

To clear markets, the asset managers must hold the entire active supply S_t^A, which at time T equals V_T, leading to

$$y^{\mathcal{A}} \xi_T = \mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1}}{(\gamma_2 - 1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2}}{(1 - \gamma_1)}. \tag{A.32}$$

Since ξ_t is a martingale, we have

$$\xi_{0,t} = \frac{\mathbb{E}_t \left[\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1}}{(\gamma_2 - 1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2}}{(1-\gamma_1)} \right]}{\mathbb{E} \left[\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1}}{(\gamma_2 - 1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2}}{(1-\gamma_1)} \right]}. \tag{A.33}$$

By plugging (A.1) and (A.2) into (A.33) we find that

$$\xi_{0,t} = \frac{\frac{\mathbf{1}_{\text{Off}}}{\gamma_2 - 1} (I_t)^{1-\gamma_2} (V_t)^{-\gamma_1} \mathcal{E}_{1t}^{\text{Off}} + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1} (I_t)^{1-\gamma_1} (V_t)^{-\gamma_2} \mathcal{E}_{1t}^{\text{On}}}{\frac{\mathbf{1}_{\text{Off}}}{\gamma_2 - 1} (I_0)^{1-\gamma_2} (V_0)^{-\gamma_1} \mathcal{E}_{10}^{\text{Off}} + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1} (I_0)^{1-\gamma_1} (V_0)^{-\gamma_2} \mathcal{E}_{10}^{\text{On}}}. \tag{A.34}$$

To find the Lagrange multiplier, we plug the equilibrium discount factor (A.32) into the dynamic budget constraint (A.30) evaluated at time $t = 0$, apply the market clearing condition

so that the asset managers must hold the entire active supply and find that

$$\xi_0 S_0^A = \mathbb{E}\left[\frac{1}{y^{\mathcal{A}}}\left(\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{1-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{1-\gamma_2}}{(1-\gamma_1)}\right)\right], \tag{A.35}$$

which eventually leads to

$$\begin{aligned} y^{\mathcal{A}} &= \frac{1}{\xi_0 S_0^A}\mathbb{E}\left[\left(\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{1-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{1-\gamma_2}}{(1-\gamma_1)}\right)\right] \\ &= \frac{1}{\xi_0 S_0^A}\left[\frac{\mathbf{1}_{\text{Off}}}{\gamma_2-1}(I_0)^{1-\gamma_2}(V_0)^{1-\gamma_1}\mathcal{E}_{20}(1-\gamma_2,1-\gamma_1) + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1}(I_0)^{1-\gamma_1}(V_0)^{1-\gamma_2}\mathcal{E}_{20}(1-\gamma_1,1-\gamma_2)\right], \end{aligned} \tag{A.36}$$

where the second equality is due to (A.4) and (A.5) evaluated at time $t = 0$.

We conclude the proof with the discount factor derivatives. It is straightforward to show that $\xi_{0,t}$ is decreasing in V_t and increasing in I_t in the risk-on incentives case, (B.1), and $\xi_{0,t}$ is decreasing in V_t and I_t in the risk-off incentives case, (B.2). The derivative of the discount factor with respect to I_t in the joint incentives case equals

$$\frac{\partial \xi_{0,t}}{\partial I_t} = (1-\gamma_1)\frac{1}{I_t}\xi_{0,t} - \xi_{0,t}\frac{1}{\mathcal{H}_{1t}}\frac{\partial \mathcal{H}_{1t}}{\partial I_t}, \tag{A.37}$$

where the derivative of the risk-on hedging demand with respect to I_t equals

$$\frac{\partial \mathcal{H}_{1t}}{\partial I_t} = (\mathcal{H}_{1t})^2\left(\frac{1-\gamma_1}{\gamma_2-1}(\gamma_2-\gamma_1)(I_t)^{\gamma_1-\gamma_2-1}(V_t)^{\gamma_2-\gamma_1}\mathcal{E}_{1t}\right) > 0. \tag{A.38}$$

By plugging the hedging demand derivative (A.38) into the discount factor derivative (A.37), and by removing strictly positive terms and rearranging, we find that $\partial \xi_{0,t}/\partial I_t < 0$ if and only if

$$(1-\gamma_1) - (\gamma_2-\gamma_1)(-\mathcal{H}_{1t}+1) < 0, \tag{A.39}$$

which leads to our desired result in (B.4). Similarly, the derivative of the discount factor with respect to V_t in the joint incentives case equals

$$\frac{\partial \xi_{0,t}}{\partial V_t} = (-\gamma_2)\frac{1}{V_t}\xi_t - \xi_t\frac{1}{\mathcal{H}_{1t}}\frac{\partial \mathcal{H}_{1t}}{\partial V_t}, \tag{A.40}$$

where the derivative of the risk-on hedging demand with respect to V_t equals

$$\frac{\mathcal{H}_{it}}{\partial V_t} = -(\mathcal{H}_{it})^2\left(\frac{1-\gamma_1}{\gamma_2-1}(\gamma_2-\gamma_1)(I_t)^{\gamma_1-\gamma_2}(V_t)^{\gamma_2-\gamma_1-1}\mathcal{E}_{1t}\right) < 0, \quad i = 1,2,3,4. \tag{A.41}$$

By plugging the hedging demand derivative (A.41) into the discount factor derivative (A.40) and rearranging, we find that $\partial \xi_{0,t}/\partial V_t < 0$ if and only if

$$(\gamma_2 - \gamma_1)\,\mathcal{H}_{1t} + \gamma_1 > 0, \tag{A.42}$$

which is always true because $\gamma_2 > \gamma_1$ and $\mathcal{H}_{1t} > 0$.

□

Proof of Lemma 1 (Risk Exposures). We utilize the no-arbitrage condition (A.30) at time t, the market clearing condition, and write

$$\xi_t W_t^{\mathcal{A}} = \mathbb{E}_t\left[\xi_T V_T\right] = \frac{1}{y^{\mathcal{A}}}\mathbb{E}_t\left[\left(\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}\,(V_T)^{1-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}\,(V_T)^{1-\gamma_2}}{(1-\gamma_1)}\right)\right]. \tag{A.43}$$

By plugging (A.4) and (A.5) we obtain

$$\xi_t W_t^{\mathcal{A}} = \frac{1}{y^{\mathcal{A}}}\left[\frac{\mathbf{1}_{\text{Off}}}{\gamma_2 - 1}\,(I_t)^{1-\gamma_2}\,(V_t)^{1-\gamma_1}\,\mathcal{E}_{2t}^{\text{Off}} + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1}\,(I_t)^{1-\gamma_1}\,(V_t)^{1-\gamma_2}\,\mathcal{E}_{2t}^{\text{On}}\right] \tag{A.44}$$

Applying Itô's Lemma on both sides of the above equation leads to the desired result. □

Proof of Proposition 1 (Market Price of Risk). Applying Itô's Lemma on both sides of (A.34) leads to the desired result.

□

Proof of Proposition 5 (Stock Prices). We start from the no arbitrage condition (A.28), plug $\xi_{0,T}$ (A.34), and find that the deflated index price satisfies

$$S_t^I \xi_{0,t} = \frac{\mathbb{E}_t\left[\left(\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2}}{(1-\gamma_1)}\right) I_T\right]}{\mathbb{E}\left[\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2}}{(1-\gamma_1)}\right]}. \tag{A.45}$$

By plugging the expected values from (A.7) and (A.8), $\xi_{0,t}$ (A.34), and rearranging we obtain the desired result. Next, we characterize the non-index price. Similar to the index price, we find that the deflated non-index price satisfies

$$S_{2t} \xi_{0,t} = \frac{\mathbb{E}_t\left[\left(\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1} D_{2T}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2} D_{2T}}{(1-\gamma_1)}\right)\right]}{\mathbb{E}\left[\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2}}{(1-\gamma_1)}\right]}. \tag{A.46}$$

By plugging the expected values from (A.10) and (A.11) and $\xi_{0,t}$ (A.34), and rearranging we obtain the desired result. We conclude the proof by taking the derivative of the index and

non-index price dividend ratios, and find that

$$\partial\left(S_t^I/I_t\right)/\partial V_t = \frac{\mathcal{E}_{3t}^{\mathrm{On}}}{\mathcal{E}_{1t}^{\mathrm{On}}\left(\mathcal{H}_{3t}\right)^2}\left[\left(\partial\mathcal{H}_{1t}/\partial V_t\right)\mathcal{H}_{3t} - \left(\partial\mathcal{H}_{3t}/\partial V_t\right)\mathcal{H}_{1t}\right] \tag{A.47}$$

$$\partial\left(S_{2t}/D_{2t}\right)/\partial V_t = \frac{\mathcal{E}_{4t}^{\mathrm{On}}}{\mathcal{E}_{1t}^{\mathrm{On}}\left(\mathcal{H}_{4t}\right)^2}\left[\left(\partial\mathcal{H}_{1t}/\partial V_t\right)\mathcal{H}_{4t} - \left(\partial\mathcal{H}_{4t}/\partial V_t\right)\mathcal{H}_{1t}\right]. \tag{A.48}$$

By plugging the derivative of $\mathcal{H}_{it}$, $i = 1, 3, 4$, (14) with respect to V_t, into the price-dividend ratios (A.47) and (A.48), and by removing strictly positive terms and rearranging, we find that

$$\partial\left(S_t^I/I_t\right)/\partial V_t < 0 \iff \mathcal{E}_{3t} - \mathcal{E}_{1t} < 0, \tag{A.49}$$

$$\partial\left(S_{2t}/D_{2t}\right)/\partial V_t > 0 \iff \mathcal{E}_{4t} - \mathcal{E}_{1t} > 0. \tag{A.50}$$

The right hand side inequalities are true due to (A.17) and (A.18), respectively, implying that the left hand side inequality are also true, which is our desire result. In similar fashion, we obtain

$$\partial\left(S_t^I/I_t\right)/\partial I_t > 0 \iff \mathcal{E}_{3t} - \mathcal{E}_{1t} < 0, \tag{A.51}$$

$$\partial\left(S_{2t}/D_{2t}\right)/\partial I_t < 0 \iff \mathcal{E}_{4t} - \mathcal{E}_{1t} > 0. \tag{A.52}$$

The right hand side is always satisfied due to (A.17) and (A.18), respectively, implying that the left hand side inequality are also true, which is our desire result.

We continue with the comparison of the index price-dividend ratio and the non-index price-dividend ratio. Dividing the index price-dividend ratio by the non-index price-dividend ratio (71) in the pure risk-on incentives, we obtain

$$\begin{aligned}\frac{S_t^I/I_t}{S_{2t}/D_{2t}} = \frac{\mathcal{E}_{3t}^{\mathrm{On}}}{\mathcal{E}_{4t}^{\mathrm{On}}} &= e^{\left(\mu_1-\mu_2-(\sigma_1\nu_1)\gamma_2+(\sigma_2\nu_2)\gamma_2+(\sigma_1)^2(1-\gamma_1)\right)(T-t)} \\ &= e^{\left(-\left(\lambda\sigma^2/3\right)\gamma_2+\left(\sigma^2/3\right)\gamma_2+(\sigma)^2(1-\gamma_1)\right)(T-t)} > 1.\end{aligned} \tag{A.53}$$

We obtain the second equality by plugging the definitions of $\mathcal{E}_{3t}^{\mathrm{On}}$ (A.8) and $\mathcal{E}_{4t}^{\mathrm{On}}$ (A.11) and cancelling similar terms, and the last equality by using the assumptions stating that $\mu_1 = \mu_2$, $\sigma_1 = \sigma_2$, and $\nu_1 = \lambda\sigma/3$, $\nu_2 = \sigma/3$. In similar fashion, dividing the index price-dividend ratio by the non-index price-dividend ratio (72) in the pure risk-off incentives, we obtain

$$\begin{aligned}\frac{S_t^I/I_t}{S_{2t}/D_{2t}} = \frac{\mathcal{E}_{3t}^{\mathrm{Off}}}{\mathcal{E}_{4t}^{\mathrm{Off}}} &= e^{\left(\mu_1-\mu_2-(\sigma_1\nu_1)\gamma_1+(\sigma_2\nu_2)\gamma_1-(\sigma_1)^2(\gamma_2-1)\right)(T-t)} \\ &= e^{\left(-\left(\lambda\sigma^2/3\right)\gamma_1+\left(\sigma^2/3\right)\gamma_1-(\sigma)^2(\gamma_2-1)\right)(T-t)} < 1.\end{aligned} \tag{A.54}$$

We obtain the second equality by plugging the definitions of $\mathcal{E}_{3t}^{\text{Off}}$ (A.7) and $\mathcal{E}_{4t}^{\text{Off}}$ (A.10) and cancelling similar terms, and the last equality by using the assumptions stating that $\mu_1 = \mu_2$, $\sigma_1 = \sigma_2$, and $\nu_1 = \lambda\sigma/3$, $\nu_2 = \sigma/3$. The inequality is true because the exponent simplifies to $\sigma^2\left(\frac{\gamma_1}{3}(1-\lambda)+1-\gamma_2\right) < 0$, which is negative because we assume that $\gamma_2 - \gamma_1 > 1$.

Lastly, dividing the index price-dividend ratio by the non-index price-dividend ratio (73) in the joint incentives case, we obtain

$$\frac{S_t^I/I_t}{S_{2t}/D_{2t}} = \frac{\frac{\mathcal{E}_{3t}^{\text{On}}}{\mathcal{H}_{3t}}}{\frac{\mathcal{E}_{4t}^{\text{On}}}{\mathcal{H}_{4t}}} = \frac{\left(\mathcal{E}_{3t}^{\text{On}} + \frac{1-\gamma_1}{\gamma_2-1}\left(V_t/I_t\right)^{\gamma_2-\gamma_1}\mathcal{E}_{3t}^{\text{Off}}\right)}{\left(\mathcal{E}_{4t}^{\text{On}} + \frac{1-\gamma_1}{\gamma_2-1}\left(V_t/I_t\right)^{\gamma_2-\gamma_1}\mathcal{E}_{4t}^{\text{Off}}\right)}, \tag{A.55}$$

where the last equality is obtained by plugging the definitions of $\mathcal{E}_{3t}^{\text{On}}$ (A.8) and $\mathcal{E}_{4t}^{\text{On}}$ (A.11) and the definitions of $\mathcal{H}_{3t}$ and $\mathcal{H}_{4t}$, (14), and cancelling similar terms. Thus, we find that $\frac{S_t^I/I_t}{S_{2t}/D_{2t}} > 1$ if and only if V/I is below the deterministic threshold in (74). The threshold is well defined because $\mathcal{E}_{3t}^{\text{On}} > \mathcal{E}_{4t}^{\text{On}}$ and $\mathcal{E}_{4t}^{\text{Off}} > \mathcal{E}_{3t}^{\text{Off}}$ since $\gamma_2 - \gamma_1 > 1$. □

Proof of Proposition 2 (Risk Exposures). We start with the joint incentives case. By applying Itô's Lemma on both sides of the deflated index price (A.45), given (A.7) and (A.8), we obtain

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\theta}_t + (2-\gamma_2)\,\boldsymbol{\sigma}^I - \gamma_1\boldsymbol{\nu} + (\gamma_2-\gamma_1)\left(\mathcal{H}_{3t}\right)\left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right). \tag{A.56}$$

And, plugging θ_t (22) and rearranging leads to the desired result.

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\sigma}_1^D + (\gamma_2-\gamma_1)\left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right)\left(\mathcal{H}_{3t} - \mathcal{H}_{1t}\right). \tag{A.57}$$

Note that $\mathcal{H}_{3t} > \mathcal{H}_{1t}$ since $\mathcal{E}_{1t} > \mathcal{E}_{3t}$, as (A.17) shows. Therefore, the first entry is always positive, $\boldsymbol{\sigma}_{1t}^S(1) > 0$ since $\nu_1 = \lambda\sigma/3 < \sigma$, while the second and third entries are always negative because $\boldsymbol{\sigma}^I \equiv \boldsymbol{\sigma}_1^D = 0$ for these two entries. In similar fashion, by applying Itô's Lemma on both sides of the deflated non-index price (A.46), given (A.7) and (A.8), we obtain

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\theta}_t + \boldsymbol{\sigma}_2^D - (\gamma_2-1)\,\boldsymbol{\sigma}^I - \gamma_1\boldsymbol{\nu} + (\gamma_2-\gamma_1)\left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right)\left(\mathcal{H}_{4t}\right). \tag{A.58}$$

And, plugging θ_t (22) and rearranging leads to the desired result. Note that $\mathcal{H}_{4t} < \mathcal{H}_{1t}$ since $\mathcal{E}_{1t} < \mathcal{E}_{4t}$, as (A.18) shows. Therefore, the first entry is always negative, $\boldsymbol{\sigma}_{2t}^S(1) < 0$, because $\nu_1 = \lambda\sigma/3 < \sigma$ and $\boldsymbol{\sigma}_2^D(1) = 0$, while the second and third entries are always positive because

$\boldsymbol{\sigma}^I = 0$ and $\boldsymbol{\nu} > 0$ for the second and third entries, and $\boldsymbol{\sigma}_2^D(2) = \sigma > 0$.

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\sigma}_2^D + (\gamma_2 - \gamma_1)\left(\boldsymbol{\nu} - \boldsymbol{\sigma}^I\right)(\mathcal{H}_{1t} - \mathcal{H}_{4t}). \tag{A.59}$$

The pure incentives cases are two knife-edge cases of the joint incentives case. In the pure risk-off incentives ($\mathbf{1}_{\text{Off}} = 1, \mathbf{1}_{\text{On}} = 0$), the risk-on hedging demand equals zero, $\mathcal{H}_{it} = 0$, which leads to the simplifications in (23). Similarly, in the pure risk-on incentives ($\mathbf{1}_{\text{Off}} = 0, \mathbf{1}_{\text{On}} = 1$), the risk-on hedging demand equals one, $\mathcal{H}_{it} = 1$, which leads to the same simplifications. □

Proof of Proposition 6 (Expected Return). The index and non-index expected returns are given by (75) and (76), respectively, and obtained by taking the inner product between the market price of risk (28) and the return exposures (30) and (31). The six channels are identified by the six combinations of inner products between the two components of the market price of risk (σ^I, ν) and the three components of the asset exposures (30) and (31) (σ_i^D, σ^I, ν). By multiplying the characterizations for θ_t and $\boldsymbol{\sigma}_{1t}^S$, we obtain our desired result in (75) and (76)

Next, we show that the expected return in extreme undeperformance is strictly lower than the expected return in extreme outperformance. First, notice that the term,

$$(\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t}), \tag{A.60}$$

in the first four channels is strictly positive (A.17) and approaches zero in extreme outperformance and underperformance (A.21). Therefore, in extreme outperformance the index asset expected return becomes

$$\mu_{1t}^S \mid_{\mathcal{H}_{it}\downarrow 0} = (\gamma_2 - 1)\,\boldsymbol{\sigma}^{I'}\boldsymbol{\sigma}_1^D + \gamma_1 \boldsymbol{\nu}'\boldsymbol{\sigma}_1^D, \tag{A.61}$$

while in extreme underperformance it becomes

$$\mu_{1t}^S \mid_{\mathcal{H}_{it}\uparrow 1} = -(1 - \gamma_1)\,\boldsymbol{\sigma}^{I'}\boldsymbol{\sigma}_1^D + \gamma_2 \boldsymbol{\nu}'\boldsymbol{\sigma}_1^D.$$

The difference between extreme outperformance and underperformance is strictly positive,

$$\mu_{1t}^S \mid_{\mathcal{H}_{it}\downarrow 0} - \mu_{1t}^S \mid_{\mathcal{H}_{it}\uparrow 1} = (\gamma_2 - \gamma_1)\left(\boldsymbol{\sigma}^{I'}\boldsymbol{\sigma}_1^D - \boldsymbol{\nu}'\boldsymbol{\sigma}_1^D\right) > 0, \tag{A.62}$$

because $\gamma_2 - \gamma_1 > 0$, and (A.14). In similar fashion, the difference between extreme outperformance and underperformance for the non-index asset is

$$\mu_{2t}^{S}\mid_{\mathcal{H}_{it}\downarrow 0} -\mu_{2t}^{S}\mid_{\mathcal{H}_{it}\uparrow 1} = -\left(\gamma_2-\gamma_1\right)\boldsymbol{\nu}^{'}\boldsymbol{\sigma}_2^{D} < 0. \tag{A.63}$$

Next, we establish that the index expected return decreases in outperformance and increases in underperformance. We achieve it by showing the derivative of the expected return with respect to V/I is negative as $V/I \to \infty$ ($\mathcal{H}_{it} \to 0$) and positive as $V/I \to 0$ ($\mathcal{H}_{it} \to \infty$). We derive the derivative, $d\mu_{1t}^{S}/d(V/I)$, by plugging the derivatives of $\mathcal{H}_{it}$ and $\mathcal{H}_{it}-\mathcal{H}_{jt}$, given in (A.22) and (A.23), respectively, and grouping similar terms, which leads to

$$\begin{aligned} d\mu_{1t}^{S}/d(V/I) = &\left(V_t/I_t\right)^{-1}\left(\gamma_2-\gamma_1\right)^2\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)\times\Bigg\{ \\ &-\frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\sigma}^{I}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right) \\ &+\frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}^{'}\boldsymbol{\sigma}^{I}\right) \\ &+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)\|\boldsymbol{\sigma}^{I}-\boldsymbol{\nu}\|^2+\left(\|\boldsymbol{\sigma}^{I}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\Bigg\}. \end{aligned} \tag{A.64}$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I \to 0$ and $V/I \to \infty$ due to (A.26) and (A.27). Therefore, to show that the derivative is negative as $V/I \to \infty$, we establish that the function inside the curly brackets is negative as $V/I \to \infty$. Accordingly, the limit $V/I \to \infty$ of the function inside the curly brackets equals

$$-\frac{\mathcal{E}_{1t}-\mathcal{E}_{3t}}{\mathcal{E}_{3t}}\left(\gamma_2-1\right)\left(\|\boldsymbol{\sigma}^{I}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)+\frac{\mathcal{E}_{1t}-\mathcal{E}_{3t}}{\mathcal{E}_{3t}}\gamma_1\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}^{'}\boldsymbol{\sigma}^{I}\right)+\left(\|\boldsymbol{\sigma}^{I}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right) \tag{A.65}$$

because the limit of $\frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}$ is a strictly positive constant (A.25). Simplifying the term, we find that the derivative is negative if and only if

$$\underline{\gamma}_2 \equiv \frac{\mathcal{E}_{1t}}{\mathcal{E}_{1t}-\mathcal{E}_{3t}}+\gamma_1\frac{\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}^{'}\boldsymbol{\sigma}^{I}\right)}{\left(\|\boldsymbol{\sigma}^{I}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)} < \gamma_2. \tag{A.66}$$

Similarly, the limit $V/I \to 0$ of the function inside the curly brackets equals

$$-\frac{\mathcal{E}_{1t}-\mathcal{E}_{3t}}{\mathcal{E}_{1t}}\left(1-\gamma_1\right)\left(\|\boldsymbol{\sigma}^{I}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)-\frac{\mathcal{E}_{1t}-\mathcal{E}_{3t}}{\mathcal{E}_{1t}}\gamma_2\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}^{'}\boldsymbol{\sigma}^{I}\right)+\left(\|\boldsymbol{\sigma}^{I}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right). \tag{A.67}$$

Simplifying the term, we find that the derivative is positive if and only if

$$\gamma_2 < \frac{\left(\|\boldsymbol{\sigma}^{I}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}{\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}^{'}\boldsymbol{\sigma}^{I}\right)}\left(\frac{\mathcal{E}_{3t}}{\mathcal{E}_{1t}-\mathcal{E}_{3t}}+\gamma_1\right) \equiv \bar{\gamma}_2. \tag{A.68}$$

Similar to $d\mu_{1t}^{S}/d(V/I)$, we plug the derivatives of $\mathcal{H}_{it}$ and $\mathcal{H}_{it}-\mathcal{H}_{jt}$, given in (A.22) and

(A.23), respectively, and group similar terms, leading to

$$
\begin{aligned}
d\mu_{2t}^{S}/d(V/I) = {} & (V_t/I_t)^{-1}(\gamma_2-\gamma_1)^2\,\mathcal{H}_{1t}(1-\mathcal{H}_{1t})\times\Bigg\{ \\
& \frac{(\mathcal{H}_{1t}-\mathcal{H}_{4t})}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}(1-\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[(\gamma_2-1)-(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right) \\
& -\frac{(\mathcal{H}_{1t}-\mathcal{H}_{4t})}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}(1-\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[\gamma_1+(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right) \\
& -(\gamma_2-\gamma_1)(\mathcal{H}_{1t}-\mathcal{H}_{4t})\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D\Bigg\}. \qquad \text{(A.69)}
\end{aligned}
$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I \to 0$ and $V/I \to \infty$ due to (A.26) and (A.27). Therefore, to show that the derivative is positive as $V/I \to \infty$, we establish that the function inside the curly brackets is positive as $V/I \to \infty$. Accordingly, the limit $V/I \to \infty$ of the function inside the curly brackets equals

$$
\frac{\mathcal{E}_{4t}-\mathcal{E}_{1t}}{\mathcal{E}_{4t}}(\gamma_2-1)\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)-\frac{\mathcal{E}_{4t}-\mathcal{E}_{1t}}{\mathcal{E}_{4t}}\gamma_1\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)-\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D, \qquad \text{(A.70)}
$$

which is positive if and only if

$$
\gamma_2 > 1+\frac{1}{(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu})}\left[\gamma_1\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)+\frac{\mathcal{E}_{4t}}{\mathcal{E}_{4t}-\mathcal{E}_{1t}}\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D\right] \equiv \underset{=2}{\gamma}. \qquad \text{(A.71)}
$$

Similarly, the limit $V/I \to 0$ of the function inside the curly brackets is negative if and only if

$$
\gamma_2 < \frac{1}{(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I)}\left[\frac{\mathcal{E}_{1t}}{\mathcal{E}_{4t}-\mathcal{E}_{1t}}\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D-(1-\gamma_1)\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\right] \equiv \bar{\bar{\gamma}}_2 \qquad \text{(A.72)}
$$

□

Proof of Proposition 3 (Momentum and Reversal). By taking Itó's Lemma of the index asset expected return (μ_{1t}^S), given in (75), and matching the $d\boldsymbol{Z}_t$ terms with (32) we find that

$$
\begin{aligned}
\boldsymbol{b}_{1t}^{\mu} = {} & (\gamma_2-\gamma_1)^2\left[(\mathcal{H}_{3t}-\mathcal{H}_{1t})(1-\mathcal{H}_{1t}-\mathcal{H}_{3t})\right]\left[(\gamma_2-1)-(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)(\boldsymbol{\sigma}^I-\boldsymbol{\nu}) \\
& -(\gamma_2-\gamma_1)^2\left[(\mathcal{H}_{3t}-\mathcal{H}_{1t})(1-\mathcal{H}_{1t}-\mathcal{H}_{3t})\right]\left[\gamma_1+(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)(\boldsymbol{\sigma}^I-\boldsymbol{\nu}) \\
& -(\gamma_2-\gamma_1)^3(\mathcal{H}_{3t}-\mathcal{H}_{1t})\left[\mathcal{H}_{1t}(1-\mathcal{H}_{1t})\right]\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2(\boldsymbol{\sigma}^I-\boldsymbol{\nu}) \\
& -\left[(\gamma_2-\gamma_1)^2\mathcal{H}_{1t}(1-\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)(\boldsymbol{\sigma}^I-\boldsymbol{\nu}). \qquad \text{(A.73)}
\end{aligned}
$$

By taking the inner product of $\boldsymbol{b}_{1t}^{\mu}$ and $\boldsymbol{\sigma}_{1t}^{S}$, we find that

$$
\begin{aligned}
\boldsymbol{\sigma}_{1t}^{S'}\boldsymbol{b}_{1t}^{\mu} = & \left(\gamma_2-\gamma_1\right)^2\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)\times\Bigg\{ \\
& \left(\gamma_2-\gamma_1\right)\frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)^2}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
& -\left(\gamma_2-\gamma_1\right)\frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)^2}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
& -\left(\gamma_2-\gamma_1\right)^2\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)^2\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
& -\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
& \frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)^2 \\
& -\frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right) \\
& -\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right) \\
& -\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)^2\Bigg\}. \qquad \text{(A.74)}
\end{aligned}
$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I \to 0$ and $V/I \to \infty$ due to (A.26) and (A.27). Therefore, to show that the momentum condition is satisfied as $V/I \to \infty$, we establish that the function inside the curly brackets is positive as $V/I \to \infty$ ($\mathcal{H}_{it} \to 0$). Accordingly, the limit $V/I \to \infty$ ($\mathcal{H}_{it} \to 0$) of the function inside the curly brackets is positive if and only if

$$\gamma_2 > \underline{\gamma}_2. \qquad \text{(A.75)}$$

Similarly, the reversal condition is satisfied as $V/I \to 0$ ($\mathcal{H}_{it} \to 1$), if and only if

$$\gamma_2 < \bar{\gamma}_2. \qquad \text{(A.76)}$$

We conclude by showing that there always exist an active share λ such that

$$\underline{\gamma}_2 < \bar{\gamma}_2. \qquad \text{(A.77)}$$

First, comparing the factors multiplying γ_1 in (A.66) and (A.68), it is clear that

$$\gamma_1\frac{\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}{\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)} > \gamma_1\frac{\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)}{\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)},$$

due to (A.16). Second, comparing the remaining factors in (A.66) and (A.68), it is clear that

$$\frac{\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}{\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)}\frac{\mathcal{E}_{3t}}{\mathcal{E}_{1t}-\mathcal{E}_{3t}}>\frac{\mathcal{E}_{1t}}{\mathcal{E}_{1t}-\mathcal{E}_{3t}} \tag{A.78}$$

because the denominator shrinks to zero as the active increases, (A.15). Therefore, there is always exist a γ_2 such that index asset exhibits momentum and reversal.

We continue with the non-index asset. By taking Itó's Lemma of the non-index asset expected return (μ_{2t}^S), given in (76), and matching the $d\boldsymbol{Z}_t$ terms with (32) we find that

$$\begin{aligned}\boldsymbol{b}_{2t}^{\mu}=&-\left(\gamma_2-\gamma_1\right)^2\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left(1-\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right)\\&+\left(\gamma_2-\gamma_1\right)^2\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left(1-\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right)\\&+\left(\gamma_2-\gamma_1\right)^3\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right)\\&+\left(\gamma_2-\gamma_1\right)^2\left(\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)\right)\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right).\end{aligned} \tag{A.79}$$

By taking the inner product of $\boldsymbol{b}_{2t}^{\mu}$ and $\boldsymbol{\sigma}_{2t}^S$, we find that

$$\begin{aligned}\boldsymbol{\sigma}_{2t}^{S'}\boldsymbol{b}_{2t}^{\mu}=&\left(\gamma_2-\gamma_1\right)^2\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)\times\Bigg\{\\&\left(\gamma_2-\gamma_1\right)\frac{\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)^2}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\\&-\left(\gamma_2-\gamma_1\right)\frac{\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)^2}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\\&-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\\&-\left(\gamma_2-\gamma_1\right)^2\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)^2\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\\&-\frac{\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)\boldsymbol{\sigma}_2^{D\prime}\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right)\\&+\frac{\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\boldsymbol{\sigma}_2^{D\prime}\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right)\\&+\left(\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D\right)\boldsymbol{\sigma}_2^{D\prime}\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right)+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}-\mathcal{H}_{4t}\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\boldsymbol{\sigma}_2^{D\prime}\left(\boldsymbol{\sigma}^I-\boldsymbol{\nu}\right)\Bigg\}.\end{aligned} \tag{A.80}$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I\to 0$ and $V/I\to\infty$ due to (A.26) and (A.27). Therefore, to show that the reversal condition is satisfied as $V/I\to\infty$, we establish that the function inside the curly brackets is negative as $V/I\to\infty$ ($\mathcal{H}_{it}\to 0$). Accordingly, the limit $V/I\to\infty$ ($\mathcal{H}_{it}\to 0$) of the function inside the curly brackets is positive if and only if

$$\gamma_2>\underline{\underline{\gamma}}_2. \tag{A.81}$$

Similarly, the reversal condition is satisfied as $V/I\to 0$ ($\mathcal{H}_{it}\to 1$), if and only if

$$\gamma_2<\bar{\bar{\gamma}}_2 \tag{A.82}$$

We conclude the proof by showing that there always exist an active share λ such that

$$\underline{\underline{\gamma}}_2 < \bar{\bar{\gamma}}_2. \tag{A.83}$$

First, comparing the factors multiplying γ_1 in (A.71) and (A.72), it is clear that

$$\gamma_1 \frac{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}{\left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)} > \gamma_1 \frac{\left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)}{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}, \tag{A.84}$$

due to (A.16). Second, comparing the remaining factors in (A.71) and (A.72), it is clear that

$$1 + \frac{1}{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}\left[\frac{\mathcal{E}_{4t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}}\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D\right] < \frac{1}{\left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)}\left[\frac{\mathcal{E}_{1t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}}\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D - \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\right] \tag{A.85}$$

is true if

$$\frac{\mathcal{E}_{1t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}} \geq \frac{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}{\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D}. \tag{A.86}$$

The denominator in the right hand side of (A.85) shrinks to zero as the active increases due to (A.15). Therefore, there is always exist a γ_2 such that the non-index asset exhibits momentum in outperformance and reversal in underperformance. □

B Internet Appendix – Further Discussion

Proposition 7 (**Discount Factor**). *The discount factor in the presence of risk-on incentives is given by*

$$\xi_{0,t} = (I_t/I_0)^{1-\gamma_1} (V_t/V_0)^{-\gamma_2} \mathcal{E}_{1t}^{On}/\mathcal{E}_{10}^{On}, \tag{B.1}$$

while it is given by

$$\xi_{0,t} = (I_t/I_0)^{-(\gamma_2-1)} (V_t/V_0)^{-\gamma_1} \mathcal{E}_{1t}^{Off}/\mathcal{E}_{10}^{Off} \tag{B.2}$$

in presence of risk-off incentives. In the presence of both risk-on and risk-off incentives, the discount factor is given by

$$\xi_{0,t} = (I_t/I_0)^{1-\gamma_1} (V_t/V_0)^{-\gamma_2} \mathcal{E}_{1t}^{On}/\mathcal{E}_{10}^{On} \frac{1}{\mathcal{H}_{1t}/\mathcal{H}_{10}}, \tag{B.3}$$

where $\mathcal{E}_{1t}^{Off}$ and $\mathcal{E}_{1t}^{On}$ are determinstic functions of time given in (A.1) and (A.2), respectively, and $\mathcal{H}_{1t}$ is given in (14). The discount factor exhibits the following characteristics:

- *It decreases following market news: $\partial\xi_t/\partial V_t < 0$.*
- *It decreases following index news in the risk-off incentives case: $\partial\xi_t/\partial I_t < 0$.*
- *It increases following index news in the risk-on incentives case: $\partial\xi_t/\partial I_t > 0$.*
- *It has a U-shape with index news in the joint incentives case. It decreases with the index news ($\partial\xi_t/\partial I_t < 0$) if and only if the risk-on hedging demand is below a threshold,*

$$\mathcal{H}_{1t} < \frac{\gamma_2 - 1}{\gamma_2 - \gamma_1} < 1, \tag{B.4}$$

and increases with the index news ($\partial\xi_t/\partial I_t > 0$) when $\mathcal{H}_{1t}$ passes the threshold.

Regardless of the asset managers' incentives, Proposition 7 reveals that the state price density is inversely related to the market news, V_t — a feature that is similar to a traditional asset pricing model: an asset that pays off in bad states get a high value.

The risk-on incentives imply that assets that pay off when the index is high get a high value because asset managers with risk-on incentives are concerned about underperforming the index and are willing to pay high prices for assets that pay in these states. This feature is typical to an asset pricing model with benchmarking concerns, such as Basak and Pavlova (2013), Buffa and Hodor (2023), Hodor and Zapatero (2023), among others.

The logic flips with risk-off incentives: assets that pay off when the index is high get a low value. With risk-off incentives, asset managers are outperforming the index and do not desire to hold assets that pay off when the benchmark is high. Instead of hedging unexpected increases in the benchmark, outperforming asset managers hedge unexpected falls by taking a short position on the benchmark and investing the proceeds in other assets — a feature unique to risk-off incentives.

When asset managers face both risk-on and risk-off incentives, the joint incentives discount factor fluctuates between risk-on and risk-off pricing environments. When the benchmark news (I) is low relative to the market news (V), the risk-on hedging demand ($\mathcal{H}_{1t}$) is small, and the asset prices correlate with the purely risk-off prices. In these states, asset managers are more likely to outperform, and prices adjust to accommodate their risk-off portfolios. As the benchmark news (I) increases, the risk-on hedging demand ($\mathcal{H}_{1t}$) becomes more pronounced, and prices adjust to accommodate the heightened likelihood that asset managers will underperform. Eventually, the risk-on hedging demand ($\mathcal{H}_{1t}$) cross the threshold in (B.4) and becomes so pronounced that prices correlate with the purely risk-on prices. Figure 10 illustrates this logic.

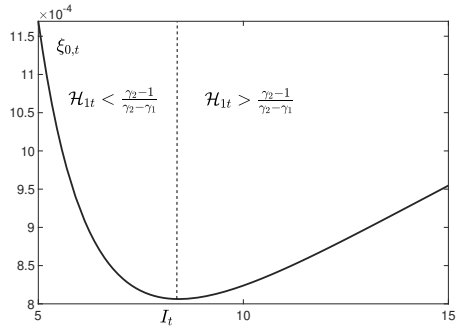


Figure 10. This figure plots the discount factor as a function of index news in the joint incentives case. The discount factor sets prices similar to the risk-off environment when the risk-on hedging demand is below a threshold. In contrast, the discount factor sets prices similar to the risk-on environment when the risk-on hedging demand crosses that threshold. The parameters are as in Figure 5.