# Financial Technology and Inequality

Roxana Mihet[‡]
Swiss Finance Institute at HEC Lausanne

October 28, 2025

**Abstract**

New financial information technologies were expected to democratize finance and narrow wealth gaps. I show they can do the opposite. In a noisy-rational expectations general equilibrium model of the stock market, technology works through two distinct frictions: participation and information. Lowering participation frictions broadens entry, improves risk sharing, and reduces wealth and return inequality. Lowering information costs reallocates surplus to informed traders and increases cross-sectional inequality. Broadly shared gains thus require targeting participation frictions; subsidizing information acquisition alone can exacerbate dispersion. The model's distributional predictions are consistent with recent data.

**Keywords**: Financial technology, stock market, inequality, Gini coefficient, participation, information frictions, wealth distribution.

**JEL Classification**: E21, G11, G14, L1, L15

[*]Contact: University of Lausanne, Quartier Chamberonne, Extranef 238, Lausanne, Vaud 1016, Switzerland. Email: roxana.mihet@unil.ch.

[†]The author is grateful to Anna Bayona, Jess Benhabib, Stijn Claessens, Jerome Dugast, Jon Frost, Andreas Fuster, Leonardo Gambacorta, Diego Garcia, Mark Gertler, Avi Goldfarb, Amit Goyal, Paymon Khorrami, Luc Laeven, Joseba Martinez, Guido Menzio, Emily Moschini, John Muellbauer, Cecilia Parlatore, Luigi Pistaferri, Thomas Philippon, Claudia Sahm, Thomas Sargent, Lin Shen, Hyun Shin, Johannes Stroebel, Victoria Vanasco, Laura Veldkamp, Venky Venkateswaran, Xavier Vives, Ansgar Walther and Anthony Lee Zhang, as well as participants at numerous conferences and workshops for their helpful comments and discussions. This paper is based on a chapter of the author's dissertation at NYU Stern, where it won the NYU Best Dissertation Award in the Inequality Category, the ECB Young Economist Best Paper Award, the EEA Best Paper Special Mention Award, and the WFA Cubist Strategies Prize for Outstanding Research. The author acknowledges generous funding from the Sandoz Foundation - Monique de Meuron Programme for Academic Excellence, the Internet Association Grant, and the BFI Macro-Financial Modeling Dissertation Grant. All errors are the author's responsibility.

# 1 Introduction

Recent advances in financial information technologies were expected to democratize finance and narrow wealth gaps. Fees fell, onboarding became simpler, and data access expanded.[1] Yet, a large literature in household finance documents that inequality in wealth and realized returns remains pronounced.[2] This paper explains why: the same innovations that lower the cost of entering stock markets also lower the cost of becoming informed, and these two forces have opposing distributional effects. These forces remain present even when investors can pool their resources into funds and share the cost of becoming informed.

In order to show this, I take a standard noisy rational expectations model in the spirit of Grossman and Stiglitz (1980) and Verrecchia (1982) with two familiar frictions—market participation and information—and use it as a distributional accounting framework. The novelty is to map technology acting on each friction to the two primitive components of inequality: between informed and uninformed (i.e., between-group differences), and within the informed via precision (i.e., within-group dispersion). This delivers clean, opposite-signed comparative statics in response to technology shifts for standard inequality objects (e.g., Gini coefficient, Lorenz dominance) and linked implications for realized returns, trading volumes, and price informativeness. The framework offers a single, testable lens that organizes recent facts on retail access and data-driven finance.

Technology shifts enter through three conceptually distinct costs: a fixed participation friction (i.e., onboarding, basic literacy, taxes/admin knowledge), an adoption cost of becoming informed (i.e., the set-up required to turn data into useful signals), and an information intensity cost that governs how precise signals are once informed.

[1] See, for instance, Easley, Michayluk, O'Hara and Putnins, 2021; Grennan and Michaely, 2021; Katona, Painter, Patatoukas and Zeng, 2025.

[2] E.g., Campbell, Ramadorai and Ranish, 2019; Calvet, Bach and Sodini, 2019; Fagereng, Guiso, Malacrino and Pistaferri, 2020; Smith, Zidar and Zwick, 2023.

The comparative statics results are unambiguous. Lower participation costs reduce both wealth and return inequality as the between-group gap shrinks (a larger share of the market earns roughly uninformed returns); lower information costs raise cross-sectional inequality by reallocating gains to the informed. Reducing the information adoption cost moves investors from the uninformed into the informed group, raising between-group inequality. Finally, reducing the information intensity cost largely leaves group shares unchanged, but increases precision among those already informed, which increases within-informed dispersion. Informed investors "time and size" risk more effectively—they scale positions up when signals are strong and down when signals are weak—producing more unequal outcomes even when average access improves.[3]

Measured by the Gini coefficient and related metrics, these effects are quantitatively meaningful and robust to different distributional assumptions, alternative calibrated shares of participating and/or informed investors, and magnitudes of cost cuts.[4] To quantify these effects, I adopt a *forward-looking* baseline calibrated to a high-participation environment (60% participants; 5% informed), reflective of the U.S. economy in 2023 (Gallup, 2025). Under a lognormal prior, the untargeted model-implied wealth Gini is 0.589, while under a Pareto prior it is 0.365. The Gini coefficient was 0.4885 in 2023 (U.S. Census Bureau, 2025), in between the model estimates under the two distributions. A one-at-a-time 70% cut in the participation cost lowers the Gini by about $-12\%$ under the lognormal prior ($\approx -34\%$ under Pareto prior).

[3]The cost shape matters. With a fixed information adoption component, the model features a sharp threshold between being uninformed and becoming informed, so technology that lowers adoption has a strong between-group effect. With purely marginal precision costs, the extensive margin collapses, everyone learns, and inequality movements operate mostly through within-informed dispersion (via intensity). The qualitative ranking above persists in either case.

[4]For the quantitative exercises, I adopt a lognormal benchmark for the initial wealth distribution-capturing the bulk of households, and report robustness to a Pareto upper tail emphasized in macro-inequality work, as well as to a Uniform initial distribution. The comparative statics results are identical across the three cases; heavy tails mainly amplify magnitudes because information adoption and intensity margins load disproportionately on the right tail. Robustness checks show that the results of the comparative statics are not an artifact of the initial share calibration (i.e., 60% participating investors, 5% informed), or of the magnitude of the cost cut which I vary between 20% and 80% for each specific friction.

Cutting the marginal information cost increases the Gini coefficient by about $\approx +22\%$ under the lognormal prior ($\approx +60\%$ under Pareto prior); a comparable cut in the intensity cost leaves the Gini essentially unchanged for the lognormal prior ($\approx -6\%$ under Pareto prior). The Pareto prior has larger elasticities, although the ranking is preserved, because the long right tail amplifies magnitudes as information adoption and intensity margins load on the upper tail. [5]

A parsimonious extension of the model that features delegation maps the information channel to one way in which investors may become informed in reality: through funds.[6] Delegation introduces scale economies in information production and a search friction in manager selection. Lower search costs draw mass from uninformed into delegated-informed (compressing between-group dispersion), while lower marginal research costs raise precision in the delegated sector (which can increase within-informed dispersion if self-learners do not keep pace). The effects on inequality are preserved.

The two channels in the model map cleanly to recent developments: retail participation expanded with zero-commission, mobile onboarding (participation channel), while alternative data and ML diffused unevenly across investors and intermediaries (e.g.., often via pooled funds) concentrating informational advantages (information channel). I use these facts qualitatively, but my *quantification is forward-looking*: taking the early-2020s high-participation environment as the baseline, I ask how further reductions in specific frictions reallocate surplus going forward. In this baseline, the model predicts — and descriptive evidence is consistent with — joint movements in participation, trading volume, fee compression alongside intermediation concentration, and a widening/narrowing of realized return dispersion.

This paper is, to the best of my knowledge, the first to connect classic noisy ra-

[5] This exercise is intentionally *forward-looking*: the baseline targets a high-participation steady state typical of the early-2020s, which saw a 60% rate (e.g., 2023 U.S. Gini index $\approx 0.418$; Europe $\approx 0.30$; South America $\approx 0.48$), whereas the model's Gini measures terminal wealth $W_2$.

[6] See Gârleanu and Pedersen, 2018; Sockin and Xiaolan, 2023; Bonelli and Foucault, 2024.

tional expectations models[7] to modern FinTech/AI and macro-inequality themes.[8] Peress (2004), Kacperczyk, Nosal and Stevens, 2018 and Azarmsa (2022) theoretically prove,[9] and Frost, Gambacorta and Gambacorta, 2022 and Gambacorta, Gambacorta and Mihet, 2023 empirically confirm that information acquisition by wealthier investors amplifies inequality through higher returns and through a retrenchment of less sophisticated investors into safer assets. In my framework, that force is only part of the picture because the underlying costs have been trending with technology (Easley et al., 2021; Grennan and Michaely, 2021; Evens and Armour, 2024, 2025; Katona et al., 2025). Therefore, this paper separates participation and information as distinct technological levers and shows that they have opposite implications for inequality. It derives testable distributional predictions that connect micro mechanisms to aggregate measures (e.g., the Gini coefficient) and to observable margins in stock market participation, passive-active investment, and wealth-dependent portfolio choices, echoing Roussanov (2014) insight that the composition of wealth and conditioning information shape cross-sectional returns; I shift the focus to inequality by mapping these forces into entry/adoption cutoffs and distributional statistics. Lastly, it documents robustness to a delegation extension that aligns with how information is purchased in practice and clarifies how search and scale shape the between-group and within-informed components.

**Literature review** This paper relates to several strands of literature. A first strand studies how FinTech and AI reshape household finance and distributional outcomes.

[7] See Grossman and Stiglitz, 1980; Verrecchia, 1982; Admati and Pfleiderer, 1990, among others.

[8] See Acemoglu and Restrepo, 2017; Korinek and Stiglitz, 2018; Acemoglu, 2025 on the distributional consequences of AI, and Benhabib, Bisin and Luo, 2017, 2019; Gomez, 2023; Gaillard, Wangner, Hellwig and Werquin, 2023 on tails and heterogeneous returns.

[9] Peress (2004) develops a general-equilibrium model in which heterogeneous investors choose information precision under a convex cost, implying that wealthier agents learn more and prices become more informative. Kacperczyk et al., 2018 shows the same in a static setting with multiple assets, but in that setting inequality increases because unsophisticated investors shift their portfolios towards safer assets, while Azarmsa (2022) confirms the increasing returns to information property in a dynamic setting.

These technologies may narrow gaps by lowering access costs and democratizing basic tools for participation (e.g., Duffie, Foucault, Veldkamp and Vives, 2022; Foucault, Gambacorta, Jiang and Vives, 2025). Technology shocks can also widen gaps by altering optimal participation and risk-taking: households more exposed to robot-automation rebalance away from equities and experience downward wealth mobility, highlighting an endogenous portfolio channel through which technology can amplify inequality (e.g., Gomes, Jansson and Karabulut, 2024). In parallel, market-intelligence providers now aggregate and clean an ever-growing universe of data (e.g., Grennan and Michaely, 2021; Katona et al., 2025).[10] This empirical backdrop motivates my focus on technology acting both on market entry and on information.

A second strand emphasizes that abundant data does not automatically translate into useful information or higher price informativeness (e.g., Banerjee, Davis and Gondhi, 2018; Dessaint, Foucault and Frésard, 2024); extracting value requires algorithms, organizational capacity, and expertise (e.g., Fedyk, Mihet, Rishabh and Gomes, 2023; Mihet, Rishabh and Gomes, 2025). This literature suggests a distinction between technologies that lower participation frictions and those that change the economics of becoming and being informed—precisely the two channels I formalize.

A third strand provides the theoretical backbone: classic noisy rational-expectations models of information acquisition and price formation (e.g., Grossman and Stiglitz, 1980; Verrecchia, 1982; Admati and Pfleiderer, 1990) and the portfolio-choice participation margin (e.g., Peress, 2004, 2005; Gomes and Smirnova, 2021). This participation margin is empirically relevant: higher non-tradable background risk raises the effective participation hurdle and depresses risky shares (e.g., Gomes and Michaelides, 2005; Fagereng, Gottlieb and Guiso, 2017; Gomes et al., 2024). I build on this tradition to isolate how technology shifts key frictions and to track equilibrium distributional wealth outcomes.

[10]The data used includes traditional sources such as prices, analysts' reports, and news, as well as alternative sources such as social media, satellite imagery, geolocation, or transaction records.

A fourth strand links inequality to the shape of the wealth distribution and heterogeneous returns. Heavy tails (e.g., Benhabib et al., 2017, 2019) and dispersion in realized household returns (e.g., Calvet et al., 2019; Campbell et al., 2019; Gomez, 2023; Gaillard et al., 2023) imply that forces operating on the upper tail can dominate aggregates. I therefore use a lognormal initial wealth distribution and verify robustness to a Pareto tail; the sign ranking of all the comparative statics is unchanged, while magnitudes are larger with heavier tails.

A fifth strand studies delegation and information production in asset management. Scale economies in research and search frictions in manager selection shape who becomes informed and by how much (e.g., Berk and Green (2004); Berk and van Binsbergen (2016); Gârleanu and Pedersen (2018, 2022); Greig, Ramadorai, Rossi, Utkus and Walther (2024); Bonelli and Foucault (2024)). The delegation extension nests these forces: lower search costs expand the delegated-informed group, whereas lower marginal research costs mainly raise precision within that group. Search frictions act exactly like information frictions; lowering them leads to more unequal outcomes as they raise between-group inequality.

Relative to these literatures, the paper (i) separates participation, information adoption, and information-intensity costs; (ii) maps each margin to between-group vs. within-informed components of inequality; (iii) quantifies the effects under a lognormal prior and shows robustness to a Pareto tail; and (iv) adds a delegation extension that matches how information is purchased while leaving the inequality ranking intact.

## 2 Economy with Participation and Information Frictions

In this section, I develop a *parsimonious noisy–rational-expectations economy*, in the tradition of Grossman and Stiglitz (1980) and Verrecchia (1982): prices aggregate dispersed private signals in equilibrium and investors endogenously choose whether and how to become informed. To this classic backbone I add the two margins emphasized

by Peress (2004, 2005)—*participation* and *information*—but I organize them into three conceptually distinct decisions that are central for distributional analysis: (i) *whether to enter* the market (participation), (ii) *who among participants chooses to become informed* (an extensive information margin), and (iii) *how intensively the informed acquire information* (an intensive margin). This decomposition lets the model speak to "who benefits" from reductions in participation and information frictions in a way that the baseline noisy-RE models and typical Peress-style calibrations do not.

Relative to Peress (2004, 2005), the emphasis and mapping to inequality are different. Peress uses the two margins to study participation and price informativeness; his quantitative implementations effectively place most action on entry plus an intensive information choice. In contrast, I *restore and foreground the extensive information margin among participants* and show that the three margins have *distinct, testable distributional footprints*: relaxing entry reshapes the composition of market participants (between-group effects), relaxing the extensive information margin changes who within the participant pool becomes informed (within-group selection), and relaxing the intensive margin reallocates attention/precision among those already informed (within-group reallocation). This organization delivers clean comparative statics for capital-wealth inequality and clarifies why seemingly similar "falls in information costs" can compress or widen inequality depending on which margin is actually being relaxed.

## 2.1 Model Assumptions

A continuum of investors, indexed by initial wealth $W_{0j}$ and endowed with constant relative risk-aversion (CRRA) preferences, trade a risk-free bond and a single risky asset whose payoff is observed with noise. Two frictions govern their choices: (i) a *fixed stock market entry fee* $F$, which determines the extensive margin of participation, and (ii) a *linear and convex information-acquisition cost* $C(x)$, which determines the extensive and intensive margins of private signal precision $x$ about the risky asset.

Because investors differ only in wealth, the model delivers closed-form thresholds for (i) the minimum wealth required to enter the market and (ii) the higher wealth level above which information production becomes optimal. These thresholds form the basis for comparative-static exercises in which reductions in $F$ or in the parameters of $C(\cdot)$ represent improvements in trading platforms or analytical technology. The analysis traces the resulting changes in participation, price informativeness, and the implied capital-wealth inequality, providing a transparent benchmark for the richer robustness exercises and extensions presented later in the paper.

**Agents and Preferences** A continuum of investors $j \in [0,1]$ differ solely in their initial total endowments $W_{0j} \in (0, \bar{W}_0]$ under an initial wealth distribution denoted by $V$. Without loss of generality I normalize initial stock holdings to zero and let all initial wealth be held in the bond at $t = 0$; only net trades at $t = 1$ matter for payoffs in the one-period setting. Each investor chooses a stock position $q_{1j}$ to maximize a constant-relative-risk-aversion (CRRA) utility, which is well approximated by a mean–variance objective with decreasing absolute risk aversion[11]. Hence absolute risk tolerance, $1/\rho(W_{0j})$, increases with initial wealth. The utility is given by

$$\max_{q_j} \mathbb{E}_j\big[W_{2j}\big] - \frac{\rho(W_{0j})}{2}\mathbb{V}_j\big[W_{2j}\big], \quad \text{where} \quad \rho(W_{0j}) \equiv \frac{\rho}{W_{0j}}, \;\; \rho > 0. \tag{1}$$

**Assets and Noisy Supply** The financial market consists of one risk-free asset and one risky asset. The risk-free asset is in unlimited supply, with price normalized to 1

[11]The DARA preference specification can be interpreted as a local quadratic approximation of any utility function around initial wealth, and it allows for wealth effects, similar to CRRA utility, which has strong empirical support. Concretely, the heterogeneity in absolute risk aversion implies differences in the size of investors' risky portfolios, hence different gains from purchases of information (Peress, 2004, 2005). Empirically, a large body of evidence finds absolute risk aversion declines with wealth, consistent with DARA (e.g., CRRA is a class of DARA), using portfolio, survey, and field behavior: classic household studies (Friend and Blume, 1975); Italian micro data relating willingness-to-pay for lotteries to wealth (Guiso and Paiella, 2008); administrative portfolio evidence that risky shares rise with wealth (Calvet, Campbell and Sodini, 2007); and field estimates from peer-to-peer lending show absolute risk aversion falls with a wealth proxy even within investor (Paravisini, Rappoport and Ravina, 2017). The macro-finance wealth-inequality literature routinely adopts DARA for the same reason (e.g., Benhabib et al., 2019).

and deterministic payoff $r$. The risky asset has price $p$ and a stochastic payoff $z$:

$$z = \mu_z + \epsilon, \text{ with } \epsilon \sim N(0, \sigma_z^2). \tag{2}$$

Noisy traders, who trade for reasons independent of payoffs or prices (e.g., for liquidity or hedging reasons), provide a stochastic supply $x$ of the risky asset which prevents full revelation of $z$.

$$x = \mu_x + v, \text{ with } v \sim N(0, \sigma_x^2) \tag{3}$$

**Information Technology** Investor $j$ can purchase a private signal

$$s_j = z + \delta_j, \qquad \delta_j \sim \mathcal{N}\big(0, \sigma_{s,j}^2\big),$$

where $\sigma_{s,j}^{-2} \geq 0$ is the chosen precision. Information acquisition entails the convex cost

$$C(\sigma_{s,j}^{-2}) \;=\; c_1 \sigma_{s,j}^{-2} \;+\; \frac{c_0}{2}(\sigma_{s,j}^{-2})^2 \tag{4}$$

where $c_0, c_1 > 0$.[12] This implies $C(0) = 0, C'() \geq 0$, $C''() > 0$ on $[0, +\infty)$ and $\lim_{\sigma_{s,j}^{-2} \to \infty} C'(\sigma_{s,j}^{-2}) = \infty$. These assumptions guarantee the existence of an interior solution and reflect the fact that more precise information is increasingly costlier.

Let $\mathcal{I}_j$ denote investor $j$'s information set at the trading date $t = 1$.

$$\mathcal{I}_j \;=\; \begin{cases} \mathcal{I}_j(z; p) & \text{if } \sigma_{s,j}^{-2} = 0 \quad \text{(no private signal)}, \\ \mathcal{I}_j(z; p; s_j) & \text{if } \sigma_{s,j}^{-2} > 0 \quad \text{(private signal observed)}. \end{cases}$$

Thus, every market participant observes the publicly posted price $p$ and the prior distribution of $z$; only those who have incurred $C(\sigma_{s,j}^{-2})$ additionally observe their own signal $s_j$.

[12] I need $c_1 \neq 0$, which empirically guarantees a positive marginal cost of learning (i.e., a first Bloomberg screen). Otherwise with a purely quadratic information cost, every investor ends up learning and there is no wealth threshold for information learning. With $c_1 = 0$, $W_0^{info} = 0$. This is unrealistic, although mathematically more tractable. Because I assume $c_1 > 0$, I need to find a fixed point to solve for the equilibrium.

**Participation Cost** To hold the risky asset an investor must first pay a fixed entry fee $F > 0$. If the fee is not paid, the entire portfolio is placed in the bond.

**Budget Constraint and Terminal Wealth** Let $\mathbf{1}_{\{\text{part}\}}$ be an indicator that equals one if investor $j$ pays the market-entry fee $F$ and therefore may trade the risky asset, and zero otherwise. If the investor also acquires a private signal of precision $\sigma_{s,j}^{-2}$, she incurs the information cost $C\big(\sigma_{s,j}^{-2}\big)$ defined in (4). Given the stock position $q_{1j}$ chosen at $t = 1$, terminal wealth at $t = 2$ is

$$W_{2j} \;=\; r\Big(W_{0j} - \mathbf{1}_{\{\text{part}\}}\, F - C\big(\sigma_{s,j}^{-2}\big) - p\, q_{1j}\Big) \;+\; q_{1j}\, z, \tag{5}$$

where the first bracketed term is the payoff on the bond financed by residual wealth and the second term is the payoff on the risky asset. For a non-participant $\big(\mathbf{1}_{\{\text{part}\}} = 0,\ q_{1j} = 0,\ \sigma_{s,j}^{-2} = 0\big)$, equation (5) collapses to the outside option $W_{2j} = rW_{0j}$.

**Notation.** For ease, all variables are defined in Appendix A.

## 2.2 Timing

The sequence of events unfolds over three dates.

$t = 0$ — **Planning.** Before either prices or private signals are observed, each investor decides (i) whether to incur the fixed entry fee $F$ that grants access to the stock market and (ii) how much signal precision $\sigma_{s,j}^{-2}$ to purchase. These decisions are taken under Knightian uncertainty about the realisation of the noise-trader order flow and the asset payoff.

$t = 1$ — **Trading.** The risky-asset price $p$ is set competitively, reflecting the aggregated information embedded in both noise-trader supply and the private signals of those investors who chose $\sigma_{s,j}^{-2} > 0$. Investors then observe the price and, if they

paid the entry fee, their own private signal $s_j$; conditional on this information set, $\mathcal{I}_j(z; p; s_j)$, they select their stock holding $q_{1j}$.

$t = 2$ — **Consumption.** The fundamental payoff of the risky asset is realised, portfolios pay off, and each investor consumes terminal wealth $W_{2j}$.

Because planning takes place one period *before* either the price or the signal is known, equilibrium prices remain linear in the aggregate noise, preserving the tractability of the Grossman and Stiglitz (1980) framework.

## 2.3 Equilibrium

**Equilibrium Definition** An equilibrium consists of a *linear* price for the risky asset, ($rp = a + b\,z - c\,x$), where the constants $a$, $b$, and $c$ are solved in equilibrium, such that the individual decision rules for entry ($W_0^{\text{part}}$), information acquisition ($W_0^{\text{info}}$), and portfolio choices ($q_{1j}$), and information precision choices ($\sigma_{s,j}^{-2}$) are mutually consistent. Here $x$ denotes the realised noise–trader order flow. The market maker's posterior mean of $z$ is a precision-weighted aggregator of submitted information, but $z$ itself is the fundamental payoff.

*Households.* At the planning date $t = 0$ each investor anticipates how the price will translate his private information into trading profits. Given this expectation, he (i) pays the fixed fee $F$ and enters the market if and only if initial wealth exceeds a threshold $W_0^{\text{part}}$; (ii) conditional on entry, purchases signal precision $\sigma_{s,j}^{-2}$ only when wealth passes a higher threshold $W_0^{\text{info}}$; and (iii) at the trading date $t = 1$ chooses a stock position $q_{1j}$ that maximises mean–variance utility, exploiting the information set $\mathcal{I}_j\big(z; p\big)$ or $\mathcal{I}_j\big(z; p; s_j\big)$ according to his earlier choice.

*Price formation.* A competitive market maker aggregates submitted orders and sets the price so that total demand equals the exogenous noise-trader supply $x = \mu_x + v$.

Because signals and order flow are jointly normal, the unique price that clears the market is linear in $(z, x)$ with slope $(b, -c)$, endogenously determined by investors' information choices.

*Consistency.* The wealth entry cutoff $W_0^{\text{part}}$, the wealth information cutoff $W_0^{\text{info}}$, individual demand schedules, and the price coefficients $(a, b, c)$ are pinned down jointly:

(i) each investor's strategy maximises expected utility in (1) subject to the budget constraint in (5);

(ii) the market maker sets a market clearing stock price given by (7), such as the demand of all uninformed and informed participating investors equals the noise-traders' supply;

(iii) the implied payoffs validate the thresholds $W_0^{\text{part}}$ and $W_0^{\text{info}}$.

Under the quadratic cost in (4) these conditions yield closed-form expressions for $a, b, c$, $W_0^{\text{part}}$, and $W_0^{\text{info}}$, producing a three-way partition of the population into non-participants, uninformed participants, and informed participants.[13] All the step-by-step theoretical derivations are in Appendix B.1- B.7.

**Computation.** The equilibrium is computed by solving a two–dimensional fixed point in $(s, t)$, the aggregate risk–tolerance–weighted private precision and aggregate risk tolerance, implied by market clearing and optimal information/portfolio choices. I establish existence and local uniqueness of this fixed point in Proposition 4 (Appendix B.6), and I implement a Newton step that converges locally.

[13] A two-way partition of the population into into non-participants and informed participants is also possible, provided low enough $c_0, c_1$.

**Equilibrium Solution** In equilibrium, as shown in Appendix B.1, an investor's *optimal risky asset demand* is given by

$$q_j \;=\; \frac{W_{0j}}{\rho}\,\frac{\mathbb{E}_j[z]-rp}{\mathbb{V}_j[z]}. \tag{6}$$

In equilibrium a competitive market maker posts a *linear price* that clears the market for every realization of the fundamental payoff $z$ and the noise–trader order flow $x$:

$$rp \;=\; a \;+\; b\,z \;-\; c\,x, \tag{7}$$

where the coefficients $a$, $b$, and $c$—together with the resulting price precision $h_p$—are determined exactly as in Appendix B.2.

The *optimal precision* an investor purchases, solved in Appendix B.5, is given by:

$$\sigma_{sj}^{-2}(W_0) = \frac{W_{0j}D - 2\rho c_1}{2\rho c_0}, \qquad D(s,t) \equiv \frac{\sigma_x^2 + 2st + t^2 h_0}{t^2\left(h_0 + s^2/\sigma_x^2\right)^2}, \tag{8}$$

where $W_0$ is initial wealth, and $D(s,t)$ represents the marginal gain per unit precision, solved in Appendix B.4, and is a function of $s$ (the aggregate risk-tolerance weighted private precision) and $t$ (the aggregate risk tolerance) as

$$t = \int_j \rho^{-1}(W_{0j})dV, \qquad s = \int_j \rho^{-1}(W_{0j})\sigma_{sj}^{-2}(W_0)dV. \tag{9}$$

*Wealth Cut-offs.* In equilibrium the two wealth cut-offs, derived in Appendix B.7, are

$$W_0^{\text{part}} = \frac{2\rho\,F}{D(s,t)}, \qquad W_0^{\text{info}} = \frac{2\rho\,c_1}{D(s,t)}, \tag{10}$$

where $F$ is the fixed entry fee, $c_1$ the linear information cost, $s$ the aggregate private precision, and $t$ the aggregate risk tolerance. $D(s,t)$ represents the marginal gain per unit precision, and $h_0 > 0$ the prior precision, and $\sigma_x^2 > 0$ the variance of noise trading.[14] All variables (e.g., $s$, $t$, $h_0$, $D$, et cetera) are defined in Appendix A.

[14] Only the sign properties of $D$ matter below. In the interior region $s > \sigma_x^2/(2t)$ one has $D_t < 0$ and $D_s < 0$ (Lemmas 4 and 5 in Appendix B.4.1).

*Interior region and knife-edge* The comparative statics in the following section use the interior region $s > \sigma_x^2/(2t)$, under which $D_t < 0$ and $D_s < 0$ (Appendix B.4.1 Lemmas). At the knife-edge $s = \sigma_x^2/(2t)$ these slopes flatten, but all calibrations I report lie strictly inside the interior set, so Lemmas 1–3 apply throughout.

## 2.4 How Technology Shifts Affect Trading

In this model, technology shifts enter through three conceptually distinct costs: (1) a fixed participation friction $F$ (i.e., onboarding, basic literacy, taxes/admin knowledge), (2) an adoption cost of becoming informed $c_1$ (i.e., the set-up required to turn data into useful signals), (3) and an information intensity cost, $c_0$, that governs how precise signals are once informed.[15] I now study how the two wealth thresholds, $W_0^{\text{part}}$ and $W_0^{\text{info}}$, move when either of these costs falls, one at a time. A direct change in either $F$ or $c_1$ cost shifts *only* the corresponding numerator, while general-equilibrium feedback effects run through $s$ and $t$, and implicitly through the squared Sharpe ratio $D(s,t)$, all of which need to be solved by Newton's method as a fixed point (Appendix B.8). All proofs are in Appendix B.9.

**Lemma 1** (Cut in participation cost $F$)**.** *Holding $c_1$ and $c_0$ fixed, a decrease in $F$ lowers $W_0^{\text{part}}$ and raises $W_0^{\text{info}}$; the two cutoffs move apart.*

*Proof.* See Appendix B.9.1. A lower entry cost attracts additional (mostly uninformed) investors, increasing $t$ and weakening the private incentives to acquire information (reducing $s$). The $t$ channel (with $D_t < 0$) makes prices more efficient (reduces $D$); the $s$ channel works in the opposite direction but is small locally. With $D$ lower, (10) implies $W_0^{\text{part}}$ falls directly (cheaper entry) while $W_0^{\text{info}}$ rises (costly information is justified only at higher wealth when prices already embed more information). The reallocation is from non-participants and informed to uninformed.

[15] This triple-decomposition contrasts with the common *purely quadratic* calibrations used in Peress (2005) (together with a fixed entry cost), which effectively collapse the extensive information margin (when $c_1 = 0$) and therefore mute distributional effects of information acquisition.

**Lemma 2** (Cut in marginal information cost $c_1$)**.** *Holding $F$ and $c_0$ fixed, a decrease in $c_1$ raises $W_0^{\text{part}}$ and lowers $W_0^{\text{info}}$; the two cutoffs move toward each other.*

*Proof.* See Appendix B.9.2. A lower marginal cost makes information acquisition attractive at lower wealth, so $s$ rises. Higher $s$ renders prices more informative (since $D_s < 0$, $D$ declines), which reduces the benefit of entering without information; thus $W_0^{\text{part}}$ increases. At the same time the direct effect of $c_1 \downarrow$ lowers $W_0^{\text{info}}$. The net reallocation is toward the informed group.

**Lemma 3** (Cut in quadratic information cost $c_0$)**.** *Holding $F$ and $c_1$ fixed, a decrease in the quadratic information cost $c_0$ raises both $W_0^{\text{part}}$ and $W_0^{\text{info}}$ proportionally.*

*Proof.* See Appendix B.9.3. Lower curvature makes precision cheaper for already-informed investors, so $s$ increases. With $D_s < 0$, this reduces $D$ and scales up *both* cutoffs (the adoption margin is still pinned by $c_1$; cf. $\frac{W_0^{\text{info}}}{W_0^{\text{part}}}$).

**Extensive vs. intensive margins for information acquisition** The linear term $c_1$ sets the *marginal cost at zero precision* and thus pins the extensive margin of information adoption. A lower $c_1$ reduces the wealth threshold to become informed and increases the mass of informed investors (Lemma 2). By contrast, $c_0$ governs the *curvature* of information costs. Holding participation fixed, a change in $c_0$ primarily adjusts the *intensive* precision choice of already–informed investors (and thus $s$), with only an indirect effect on cutoffs via $D(s,t)$.

## 3 Quantifying the Inequality Effect

I quantify distributional effects by running three technology counterfactuals - lower entry costs, lower information *adoption* costs, and lower information *intensity* costs - under two canonical priors for initial wealth. A lognormal captures the bulk of households around the middle of the distribution; a Pareto captures the empirically

heavy upper tail emphasized in macro-inequality work (e.g., Benhabib et al., 2019; Gomez, 2023).[16] Using both lets us separate forces that operate "in the middle" from those that load on the tail and guards against functional-form critiques.

Inequality moves through two components that the model made transparent: (i) *between-group differences* (e.g., participants vs. non-participants and, among participants, informed vs. uninformed) driven by differential access to risk-adjusted returns; and (ii) *within-group dispersion* (e.g., among the informed) driven by heterogeneity in information intensity. I summarize outcomes with the Lorenz curve and the Gini, and I report complementary top-share statistics, which follow the recent inequality literature in household finance (e.g., Badarinza, Campbell and Ramadorai, 2016; Campbell et al., 2019).

My main findings are as follow. Lower entry (participation costs) increase market access by drawing mass from the middle of the wealth distribution. The Lorenz curve shifts toward equality and the Gini coefficient falls. The compression is pronounced under a lognormal prior (large middle mass) and present, but smaller under a Pareto prior (the right tail continues to dominate). Lower information *adoption* costs (extensive information margin) reallocate investors from the uninformed to the informed group. This raises between-group gaps among participants and increases the Gini coefficient; the Lorenz curve bends away from equality at upper quantiles. Effects are modest under a lognormal prior and markedly stronger under a Pareto prior, where many investors sit near or above the adoption threshold. Lastly, lower information *intensity* costs (precision among the informed) leave group shares roughly unchanged and operates mainly through within-informed dispersion. Inequality effects are therefore modest and theoretically ambiguous: higher precision for the already-

[16]The distributions of wealth many countries, including in the US, are strikingly concentrated at the top and skewed to the right (e.g., Piketty, Saez and Zucman, 2017; Bricker, Henriques, Krimmel and Sabelhaus, 2016; Smith et al., 2023). For example, using the US administrative tax data, Smith et al., 2023 estimate that the top 0.1% and 1% wealth shares increased from 9.9% and 23.9% in 1989 to 15% and 31.5% in 2016, respectively. While the log-normal wealth distribution is illustrative, the Pareto wealth distribution fits the data better.

informed pushes toward more dispersion, while general-equilibrium improvements in price informativeness damp realized return differentials. The impact is small under a lognormal prior and more noticeable in the Pareto tail.

The main results are not driven by the participation margin: even with universal participation, lowering information costs reallocates value toward the informed and increases dispersion within participants.

The rest of the section details calibration, decomposition, and sensitivity analyses that underpin these patterns. Appendix B.10 presents all the details on the algorithm and more simulation results.

## 3.1 Quantitative Exercise

**Calibration.** I discipline the model with three baseline moments: (i) the mean of $W_2$ is normalized to 100,000 currency units[17]; (ii) the stock market participation rate is calibrated to 60% (Gallup, 2025); and (iii) the share of informed investors is calibrated to 5%.[18]

Given preferences, the return technology, and the outside option (all held fixed across experiments), the share targets in (ii)–(iii) pin down the endogenous wealth cutoffs for participation and information acquisition, $W_{\text{part}} = \frac{2\rho F}{D(s,t)}$, and $W_{\text{info}} = \frac{2\rho c_1}{D(s,t)}$ $\left( \Rightarrow \frac{W_{\text{info}}}{W_{\text{part}}} = \frac{c_1}{F} \right)$, where $(s,t)$ and $D(s,t)$ are solved through a fixed point

[17] Data regarding income for 2023 from the U.S. Census Bureau (2025) shows that the median household income was \$80,610, and the median family income was \$103,400 in 2023 CPI-U-RS adjusted dollars.

[18] Moment (i) matches the median household net worth in the U.S. Moment (ii) is intended to match broad U.S. equity participation (direct + indirect) in household surveys and the results are qualitatively identical for different levels of equity participation (see robustness in App. B.11.1); (iii) proxies the fraction that pays a nontrivial fixed cost to acquire professional-grade information (e.g., terminals, data feeds, or delegated active management). The Gini coefficient of $W_2$ is *not* separately targeted; under a lognormal prior for $W_0$ and our equilibrium mapping, the implied baseline is $G \simeq 0.589$. I verified numerically that forcing $G = 0.4885$ (Gallup, 2025) while maintaining (i)-(iii) is infeasible in this parametrization—the attainable Gini index set is narrow around 0.59. To reach substantially lower baseline inequality within my framework I would need a richer $W_0$ (e.g., a lognormal-Pareto mixture) or alternative information technology assumptions (e.g., letting $c_0$ enter the information cutoff).

algorithm. In all counterfactuals I apply a common multiplicative rescaling to market payoffs so that $\mathbb{E}[W_2]$ remains at 100,000. This mean-preserving normalization isolates distributional effects from trivial level shifts and makes inequality comparisons across scenarios meaningful.

**Counterfactuals.** I implement one-at-a-time 70% cuts in $F$, $c_1$, and $c_0$, holding fixed the return process, signal precision, and the outside option. The $F$ and $c_1$ cuts move the participation and information margins through general equilibrium: directly and also through $D(s,t)$ as shown in Section 2.4; a $c_0$ cut operates only through $D(s,t)$. I explore the Gini inequality index and the Lorenz curves across these different scenarios.[19]

**Proposition 1** (Inequality and cost cuts)**.** *The impact of a decrease in each of the three frictions in our environment—the fixed participation cost $F$, the marginal (per-unit) information cost $c_1$, and the quadratic (intensity/curvature) information cost $c_0$— on the cross-section of terminal wealth $W_2$ is given as follows:*

1. *A decrease in $F$ lowers $W_{part}$ and increases $W_{info}$, drawing in marginal households as* uninformed *participants. This compresses between-group dispersion (non-participants vs. participants), reduces the Gini coefficient, and pushes the Lorenz curve closer to the line of equality.*

2. *A decrease in $c_1$ lowers $W_{info}$ relative to $W_{part}$, raising the informed share among participants. This amplifies within-participants dispersion, increases the Gini coefficient, and pushes the Lorenz curve further from the line of equality.*

[19]The Gini index, also known as the Gini coefficient, is a statistical measure used to quantify the degree of income inequality within a population. It ranges from 0 to 1, with 0 representing perfect equality (everyone has the same income) and 1 representing perfect inequality (one person holds all the income). The index is derived from the Lorenz curve, which is a graphical representation of income or wealth inequality within a population. It plots the cumulative share of income or wealth against the cumulative share of the population, typically ordered from poorest to richest. The curve's deviation from the 45 degree line (which is the line of perfect equality where everyone has the same share) indicates the degree of inequality.

3. *A decrease in $c_0$ leaves the ratio $W_{info}/W_{part} = c_1/F$ unchanged. It acts mainly by altering $s$, $t$ and $D(s,t)$, which tends to shift both thresholds in the same direction. The net effect on the Gini coefficient and the Lorenz curve is small unless the informed premium is highly sensitive to dispersion.*

*Intuition.* The $F$ cut shifts mass from non-participants to (mostly) uninformed participants, compressing between-group dispersion per Lemma 1, reducing the spread of payoffs. This reduces inequality (Gini coefficient falls). The $c_1$ cut shifts significant mass toward informed investors, increasing within-participants dispersion per Lemma 2. The net reallocation is toward the informed group, widening the spread of payoffs and increasing inequality (Gini coefficient rises). The $c_0$ reduces both the uninformed and informed shares (the informed become better informed, however), which lowers upper-tail concentration, but raises the dispersion within the informed group per Lemma 3. The net effect on inequality is model-dependent for a cut in $c_0$: more informative prices raise the relative payoff to the informed (pushing inequality up), while higher cutoffs reduce participation at the bottom (pushing inequality up) but also reduce the mass of informed traders (pushing inequality down). The final effect on inequality is dependent on the initial wealth distribution.

**Mechanism: Sharpe ratios by group** In the model, the uninformed group's expected *squared* Sharpe equals the marginal value of information, $S_U^2(s,t) = D(s,t)$ (Appendix C). On the interior set one has $D_t < 0$ and $D_s < 0$ (Appendix B.4.1). The informed investor's optimal precision is

$$\sigma_{s,j}^{-2} = \frac{(W_j/\rho)\, D(s,t) - 2c_1}{2c_0} \quad \text{(Appendix B.5)}.$$

Hence, **(i)** $F \downarrow$ brings in mostly uninformed entrants, raises $t$ and (slightly) lowers $s$, so $D \downarrow$ and the uninformed Sharpe falls; because $\sigma_{s,j}^{-2}$ co-moves with $D$, informed Sharpe also falls. **(ii)** $c_1 \downarrow$ increases information adoption and $s$, which lowers $D$

and depresses the uninformed Sharpe, but directly raises $\sigma_{s,j}^{-2}$ via the $-2c_1$ term; in calibration the direct effect dominates, so informed Sharpe rises. **(iii)** $c_0 \downarrow$ operates on the intensive margin: $s$ increases, implying $D$ falls and the uninformed Sharpe falls; for the informed, $\sigma_{s,j}^{-2}$ rises mechanically while $D$ falls, yielding only a small (ambiguous) net change in their Sharpe ratio.[20]

## 3.2 A Lognormal Prior Distribution Spreads Mass Across the Middle: Smaller Inequality Responses

Table 1 expresses the wealth distribution, the fixed-point cutoffs, and inequality across the baseline and three one-at-a-time cost cuts (all mean-preserving) under the assumption of a lognormal initial wealth distribution.

Table 1: Wealth distribution, thresholds, and inequality across scenarios (lognormal fit), scenario color code stays fixed

| Scenario | $s$ | $t$ | $D$ ($\times 10^{-12}$) | $W_{\text{part}}$ | $W_{\text{info}}$ | Share part (%) | Share info (%) | Gini |
|---|---|---|---|---|---|---|---|---|
| Baseline | 675.18 | 2964.27 | 2.673 | 1403.40 | 9366.13 | 60.00 | 5.00 | 0.589 |
| F cut | 660.10 | 3288.21 | 2.641 | 426.09 | 9478.88 | 92.60 | 4.90 | 0.518 |
| $c_1$ cut | 787.13 | 2664.51 | 1.8 | 2084.63 | 4173.78 | 44.30 | 20.10 | 0.718 |
| $c_0$ cut | 820.92 | 2568.70 | 1.628 | 2304.94 | 15382.89 | 40.40 | 1.60 | 0.588 |

*Notes.* Each scenario applies a one-at-a-time 70% cut to the named cost. The magnitudes scale with the sensitivity of informed premia to dispersion. Variable definitions are in Appendix A.

Cutting the participation cost $F$ shifts the entry threshold far left ($W_{\text{part}}$ : 1403 $\rightarrow$ 426) while the information threshold barely moves ($W_{\text{info}}$ : 9366 $\rightarrow$ 9479), raising participation from 60.0% to 92.6% with a slight decrease in the informed share (5.0% $\rightarrow$ 4.9%); the Gini coefficient falls from 0.589 to 0.518, consistent with a compression of between-group gaps. Cutting the marginal information cost $c_1$ lowers the information threshold sharply (9366 $\rightarrow$ 4174) and raises the informed share among participants

[20] See Roussanov (2014) for evidence that the composition of aggregate wealth and conditioning information generate state-dependent risk exposures and conditional expected returns. The mechanism operates through learning $s$ and risk-bearing capacity $t$, rather than through wealth/income composition.

(unconditional informed share 5.0% → 20.1%), even as overall participation declines (60.0% → 44.3%) because $W_{\text{part}}$ moves right; the right tail thickens and the Gini coefficient rises to 0.718. By contrast, cutting the quadratic information cost $c_0$ shifts both thresholds right ($W_{\text{part}}$ : 1403 → 2305, $W_{\text{info}}$ : 9366 → 15383), reducing both participation and information adoption (40.4% and 1.6%, respectively), but leaves their ratio essentially unchanged; the opposing forces on the upper tail largely offset and the Gini coefficient is virtually unchanged (0.588).

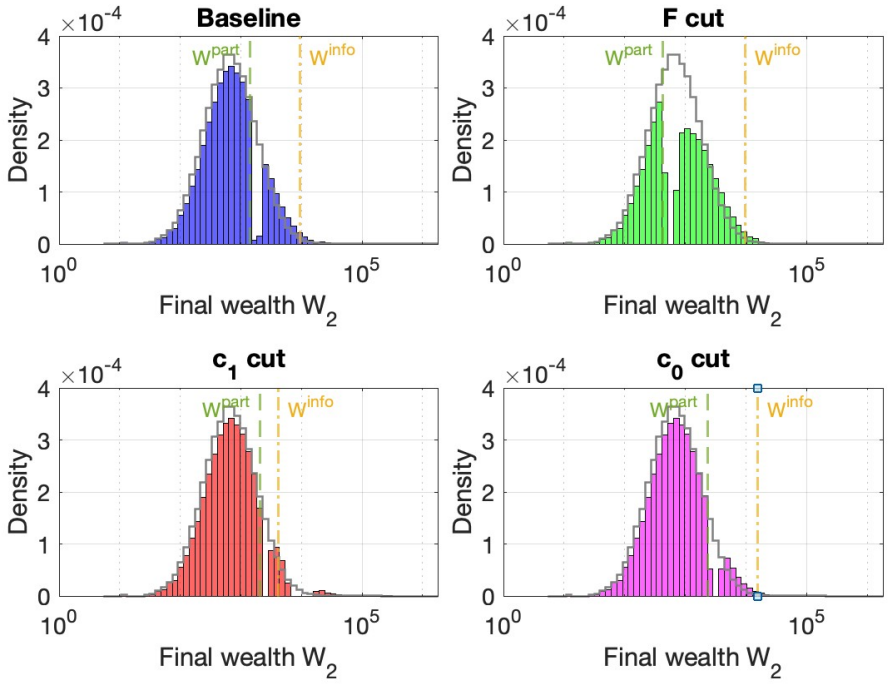
To better understand these results, in Figure 1 I plot the final wealth distributions and the endogenous cutoffs $W_0^{\text{part}}$ and $W_0^{\text{info}}$ superimposed across the different technology improvement scenarios. In the baseline, with $W_0^{\text{part}}$ to the right of many households, a sizable mass remains out of the market (i.e., 40% remains out); only a small slice above $W_0^{\text{info}}$ buys information (i.e., 5%), as in my intended calibration. The resulting $W_2$ is right-skewed, reflecting higher returns for market participants and especially for the informed.

A 70% F cut leads the $W_0^{\text{part}}$ line shift left, bringing lower-wealth households into the market. Their move from the low-return outside option to the market shifts mass from the lower tail toward the center of the $W_2$ distribution. The right tail changes little, and the distribution compresses overall. This is consistent with the lower Gini coefficient: most of the action is a reduction in *between-group gaps* generated by entry.

On the other hand, a 70% cut in the marginal information cost ($c_1$) leads the orange $W_0^{\text{info}}$ line to shift left, expanding the set of informed among existing participants while the participation margin moves as well. The payoffs to information push more mass into the upper tail; the left and middle of the distribution are largely unchanged. The $W_2$ distribution fans out with a thicker right tail, matching the higher Gini coefficient-inequality rises through *within-participants dispersion*.

Lastly, a 70% cut in the quadratic information cost ($c_0$) shifts both $W_0^{\text{part}}$ and $W_0^{\text{info}}$ to the right by roughly the same factor. In the quantitative simulation, this shows up

Figure 1: **Final wealth distributions and endogenous cutoffs (Lognormal fit).** Each panel shows the histogram of terminal wealth $W_2$ (log scale) for the Baseline (blue) and for one-at-a-time 70% cuts in the participation cost $F$ (green), the marginal information cost $c_1$ (red), and the quadratic information cost $c_0$ (magenta). Vertical lines mark the endogenous entry threshold $W_{\text{part}}$ (green) and the information threshold $W_{\text{info}}$ (orange). Cutting $F$ shifts $W_{\text{part}}$ far left while $W_{\text{info}}$ moves little, compressing the distribution toward the center; cutting $c_1$ shifts $W_{\text{info}}$ left relative to $W_{\text{part}}$, thickening the right tail; cutting $c_0$ moves both thresholds right together (their ratio is unchanged), leaving the overall shape of $W_2$ close to Baseline.




*Notes.* Baseline: lognormal initial wealth $W_0$ with parameters held fixed across scenarios; primitives $(\rho, h_0, \sigma_x^2, r_{\text{bond}}) = (0.98, 0.1, 1, 1.07)$ and baseline costs $(F, c_1, c_0) = (2, 2, 2)$. Scenarios apply a 70% cut to the named cost only; all other primitives are unchanged. For each case we solve the fixed-point conditions for $(s, t)$, compute $W_{\text{part}} = 2\rho F/D(s,t)$ and $W_{\text{info}} = 2\rho c_1/D(s,t)$, map $W_0$ into $W_2$, and plot the resulting histogram using common log-spaced bins; the initial $W_0$ density is overlaid as a gray outline. Colors match the inequality figure (baseline blue; $F$ cut green; $c_1$ cut red; $c_0$ cut magenta). Variable definitions are in Appendix A.

as lower participation and a smaller informed share, while the composition between uninformed vs. informed participants changes little on a relative basis. These shifts generate two opposing forces on the wealth distribution: (i) fewer informed investors means less mass enjoying the high information premium, which *reduces* upper-tail concentration; (ii) the fall in $c_0$ increases $s$ (technology becomes more effective), which

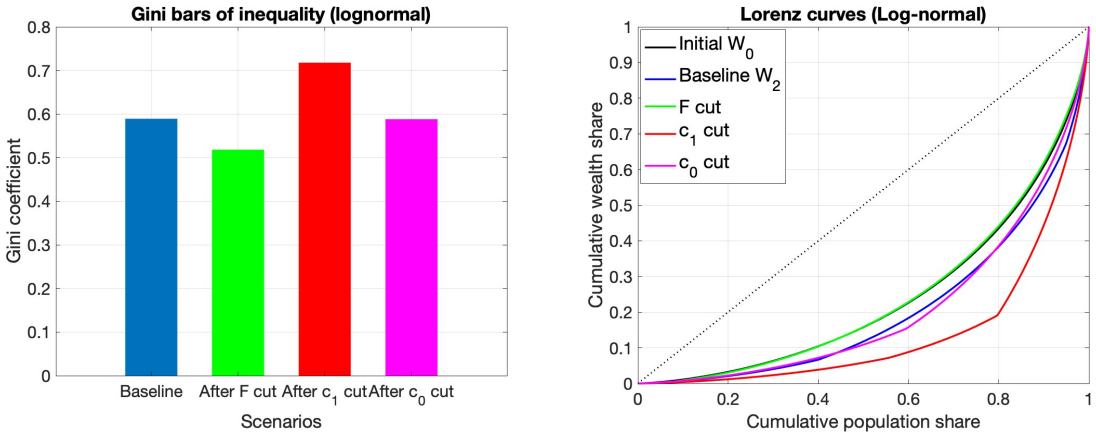
*raises* the dispersion within the informed group. These two forces nearly offset in my calibration, leaving the final wealth distribution essentially quasi-identical to the baseline.

Finally, Figure 2 summarizes how inequality (i.e., the Gini index and the Lorenz curves) responds to each cost cut. It reports the Gini coefficient (left plot) and the corresponding Lorenz curve (right plot) of $W_2$ in the baseline calibration and after a separate reduction in $F$, $c_1$, and $c_0$, holding all other parameters fixed and assuming a lognormal initial wealth distribution. In the baseline, the Gini coefficient is about 0.59. A 70% cut in the participation cost leads the Gini coefficient (in green) to fall to roughly 0.518—a decline of about -0.0710 Gini coefficient points ($\approx$ 12% relative to baseline). Lowering the fixed entry cost moves households just below the participation threshold into the market, lifting returns for the lower part of the distribution and compressing the left tail. This primarily reduces between-group inequality (non-participants vs. participants). Inequality in final wealth falls and the Lorenz curve (in green) is closer to the 45 degree line than the baseline (in blue).

On the other hand, a 70% cut in the marginal information cost ($c_1$) leads the Gini coefficient to rise to about 0.71—an increase of 0.129 ($\approx$ 21% relative). Cheaper information induces more already-participating – and typically wealthier – households to become informed, raising their expected returns. Gains accrue at the top, thickening the right tail (as seen in Figure 1) and increasing within-participants inequality. Distribution becomes wider, and inequality in final wealth rises. Thus, the Gini index (in red) rises, and the Lorenz curve (in red) shifts to the left of the baseline curve and away from the 45 degree line.

A comparable 70% cut in the quadratic information cost ($c_0$) leads the Gini coefficient at 0.588, almost unchanged from the baseline. The wealth cutoffs move to the right by roughly the same factor (the ratio $W_{\text{info}}/W_{\text{part}} = c_1/F$ is unchanged). As the two opposing forces, *reduced* upper-tail concentration and *raised* dispersion within the

Figure 2: **Inequality across scenarios.** Left: Gini coefficients of final wealth $W_2$ in the Baseline (blue) and after one-at-a-time 70% cuts in the participation cost $F$ (green), the marginal information cost $c_1$ (red), and the quadratic information cost $c_0$ (magenta). Right: Lorenz curves of $W_2$ for the same scenarios. Cutting $F$ compresses the distribution and lowers inequality; cutting $c_1$ widens it and raises inequality; cutting $c_0$ leaves inequality nearly unchanged. Colors are consistent across panels; the 45° line is shown for reference.




*Notes.* Baseline (blue): lognormal $W_0$; $\rho = 0.98$, $h_0 = 0.1$, $\sigma_x^2 = 1$, $(F, c_1, c_0) = (2, 2, 2)$, bond gross return $r_{\text{bond}} = 1.07$. Calibration targets: participation 60% and informed 5%. For each scenario we solve the fixed-point conditions for $(s, t)$, compute $W_{\text{part}}$ and $W_{\text{info}}$, map $W_0$ into $W_2$, and then compute the Gini and the Lorenz curve. Reported Ginis: baseline 0.589 (blue), $F$-cut 0.518 (green), $c_1$-cut 0.718 (red), $c_0$-cut 0.588 (magenta). Variable definitions are in Appendix A.

informed group, cancel each other the Gini coefficient (in magenta) stays essentially flat (baseline 0.589 vs. $c_0$ cut 0.588) and the corresponding Lorenz curve (in magenta) stays close to the baseline (in blue).

## 3.3 A Pareto Prior Distribution Places Mass in Upper Tail: Larger Inequality Responses

I now repeat the same exercise under the Pareto distribution and I compare the results under the lognormal and Pareto initial distributions.

Table 2 expresses the wealth distribution, the fixed-point cutoffs, and inequality across the baseline and three one-at-a-time cost cuts (all mean-preserving) under the assumption of a Pareto initial wealth distribution.

Table 2: Wealth distribution, thresholds, and inequality across scenarios (Pareto fit), scenario color code stays fixed

| Scenario | $s$ | $t$ | $D$ ($\times 10^{-8}$) | $W_{\text{part}}$ | $W_{\text{info}}$ | Share part (%) | Share info (%) | Gini |
|---|---|---|---|---|---|---|---|---|
| Baseline | 97.05 | 136.30 | 1.72 | 122.67 | 331.44 | 60.00% | 5.00% | 0.365 |
| F cut | 90.01 | 185.1852 | 1.63 | 38.84 | 348.53 | 100% | 4.40% | 0.242 |
| $c_1$ cut | 119.56 | 91.96 | 1.32 | 159.46 | 159.46 | 31.40% | 31.40% | 0.583 |
| $c_0$ cut | 147.70 | 62.22 | 1.02 | 206.89 | 559.01 | 16.20% | 1.40% | 0.345 |

*Notes.* Each scenario applies a one-at-a-time 70% cut to the named cost. No investor is counted as informed without being a participant. Reported "Share info" can never exceed "Share part". Variable definitions are in Appendix A.

Similarly, Figure 3 displays the final wealth distributions and the endogenous cutoffs $W_0^{\text{part}}$ and $W_0^{\text{info}}$ superimposed across the different technology improvement scenarios. Lastly, Figure 4 summarizes how inequality (i.e., the Gini index and the Lorenz curves) responds to each cost cut.

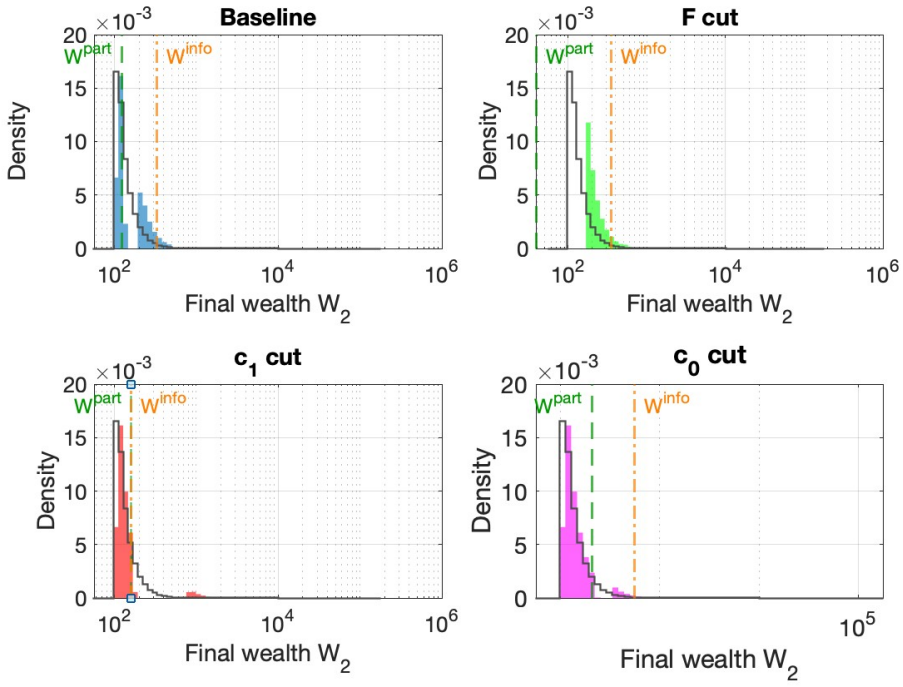
## 3.4 Lognormal vs. Pareto: What Changes and Why

Across priors, the policy ranking is robust: cutting the participation cost $F$ lowers inequality, cutting the marginal information cost $c_1$ raises it, and cutting the quadratic information cost $c_0$ has at most a modest effect. The *magnitudes*, however, differ sharply because the Pareto prior places more mass in the upper tail and has compact support at the bottom.

Under the lognormal prior (Figures 1–2), the baseline Gini coefficient is 0.589. A 70% cut in $F$ compresses between–group dispersion and lowers the Gini coefficient to 0.518 ($-0.071$, about $-12\%$); a 70% cut in $c_1$ widens within–participants dispersion and raises the Gini coefficient to 0.718 ($+0.129$, about $+22\%$); a 70% cut in $c_0$ leaves the Gini coefficient essentially unchanged at 0.588.

With a Pareto prior (Figures 3–4), the baseline Gini coefficient is lower (0.365) at our shares calibration, but policy elasticities are much larger. When $F$ falls by 70%, the participation cutoff drops below the Pareto lower bound, $W_{\min}$, so every investor

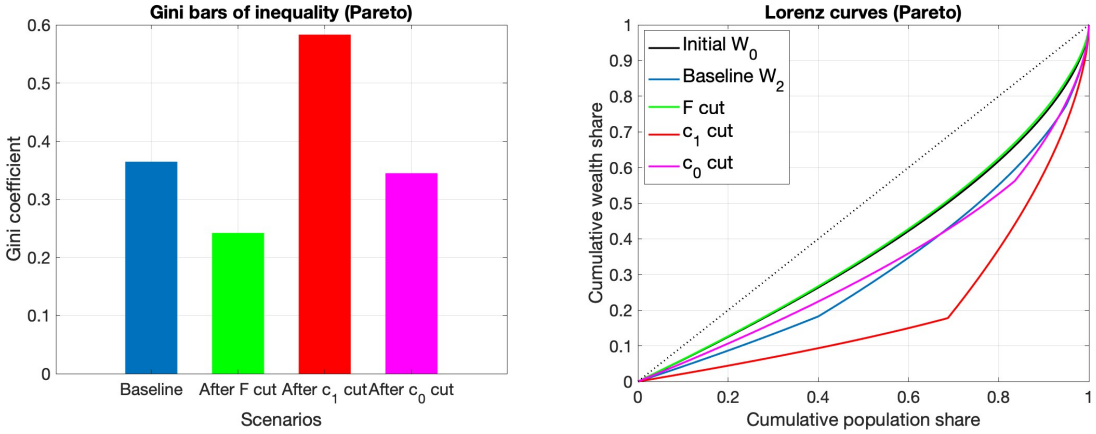
Figure 3: **Final wealth distributions and endogenous cutoffs (Pareto fit).** Each panel shows the histogram of terminal wealth $W_2$ (Pareto scale) for the baseline (blue) and for one-at-a-time 70% cuts in the participation cost $F$ (green), the marginal information cost $c_1$ (red), and the quadratic information cost $c_0$ (magenta). Vertical lines mark the endogenous entry threshold $W_{\text{part}}$ (green) and the information threshold $W_{\text{info}}$ (orange). Cutting $F$ shifts $W_{\text{part}}$ far left while $W_{\text{info}}$ moves little, compressing the distribution toward the center; cutting $c_1$ shifts $W_{\text{info}}$ left relative to $W_{\text{part}}$, thickening the right tail; cutting $c_0$ moves both thresholds right together (their ratio is unchanged), leaving the overall shape of $W_2$ close to the baseline.




*Notes.* Baseline: Pareto initial wealth $W_0$ with parameters held fixed across scenarios; primitives $(\rho, h_0, \sigma_x^2, r_{\text{bond}}) = (0.98, 0.1, 1, 1.07)$ and baseline costs $(F, c_1, c_0) = (2, 2, 2)$. Scenarios apply a 70% cut to the named cost only; all other primitives are unchanged. For each case we solve the fixed-point conditions for $(s, t)$, compute $W_{\text{part}} = 2\rho F/D(s,t)$ and $W_{\text{info}} = 2\rho c_1/D(s,t)$, map $W_0$ into $W_2$, and plot the resulting histogram; the initial $W_0$ density is overlaid as a gray outline. Colors match the inequality figure (baseline blue; $F$ cut green; $c_1$ cut red; $c_0$ cut magenta). Variable definitions are in Appendix A.

enters; the Gini coefficient falls to 0.242 ($-0.123$, about $-34\%$). When $c_1$ falls by 70%, the information cutoff plummets and (economically) almost all entrants choose to be informed; the right tail thickens markedly and the Gini coefficient jumps to 0.517 ($+0.152$, about $+42\%$). A 70% cut in $c_0$ shifts both cutoffs to the right and dampens the information premium; the two forces offset and the Gini coefficient moves only

Figure 4: **Inequality across scenarios (Pareto fit).** Left: Gini coefficients of final wealth $W_2$ in the baseline (blue) and after one-at-a-time 70% cuts in the participation cost $F$ (green), the marginal information cost $c_1$ (red), and the quadratic information cost $c_0$ (magenta). Right: Lorenz curves of $W_2$ for the same scenarios. Cutting $F$ compresses the distribution and lowers inequality; cutting $c_1$ widens it and raises inequality; cutting $c_0$ leaves inequality nearly unchanged. Colors are consistent across panels; the 45° line is shown for reference.




*Notes.* Baseline (blue): Pareto $W_0$; $\rho = 0.98$, $h_0 = 0.1$, $\sigma_x^2 = 1$, $(F, c_1, c_0) = (2, 2, 2)$, bond gross return $r_{\text{bond}} = 1.07$. Calibration targets: participation 60% and informed 5%. For each scenario we solve the fixed-point conditions for $(s, t)$, compute $W_{\text{part}}$ and $W_{\text{info}}$, map $W_0$ into $W_2$, and then compute the Gini and the Lorenz curve. Reported Ginis: baseline 0.365 (blue), $F$-cut 0.242 (green), $c_1$-cut 0.517 (red), $c_0$-cut 0.345 (magenta). Variable definitions are in Appendix A.

slightly to 0.345 ($-0.020$, about $-5.5\%$).

*Conceptually*, the heavy tail of the Pareto prior amplifies any change that tilts returns toward the top (the $c_1$ cut). In contrast, the lognormal prior spreads mass more evenly across the middle, so the same policy shocks reallocate less probability into extreme tails, yielding smaller Gini coefficient responses. Finally, lowering $c_0$ preserves the ratio $W_{\text{info}}/W_{\text{part}} = c_1/F$, so it mostly translates into similar movements in both thresholds; that geometry leaves overall inequality nearly flat under the lognormal and only modestly lower under the Pareto calibration.

# 4 Robustness, Generalizations and Extensions

## 4.1 Robustness

The results are robust to different specifications (Appendix B.11). I verify robustness of the results to (i) alternative shares of participating and/or informed investors, (ii) alternative magnitudes of cost cuts, and (iii) alternative initial wealth distributions.

First, the robustness checks show that my main findings are not an artifact of the baseline share calibration, but a robust feature of the model's mapping from costs to the cross-section of final wealth (Appendix B.11.1). The heatmaps in Appendix Figure 14 plot the Gini coefficient under the lognormal and Pareto initial wealth distributions when varying the shares of participating and informed investors. The results show a clear, monotone pattern under each prior: the Gini coefficient declines with higher participation and rises with a larger informed share. Iso-Gini contours slope upward, reflecting that raising participation can offset the inequality created by more information at the top. These gradients are steeper under the Pareto prior, consistent with heavier tails making both margins more powerful.[21]

Second, the Gini coefficient preserves the ranking independently of the magnitude of the cost cut which I vary between 20% and 80% for $F$, $c_1$ and $c_0$ (Appendix B.11.2). Appendix Table 8 shows that lowering the entry cost $F$ compresses inequality, lowering the marginal information cost $c_1$ widens it, and lowering the quadratic cost $c_0$ does little, across the different magnitudes of technology improvement shocks.

Third, robustness checks show that the mechanism is not an artifact of the lognormal prior for initial wealth (Appendix B.11.3). Replacing the lognormal prior in Appendix Table 9 with heavy-tailed (Pareto) and thin-tailed (Uniform) benchmarks also preserves the ranking. What changes is the strength of the effects. With a heavy-tailed prior (Pareto), a reduction in $c_1$ shifts more mass into the right tail—amplifying

[21] To isolate distributional effects, I re-normalize the mean of $W_2$ at each grid point; combinations that violate feasibility (i.e., informed share exceeding participation) are excluded.

the Gini coefficient increase relative to lognormal—while an $F$ cut brings in many low-wealth households and produces a larger decline in the Gini coefficient. With a thin-tailed prior (Uniform), both tail effects are muted, so the rise in inequality from cheaper information is smaller and the decline from participation is still present but less pronounced. The policy implication is unchanged: interventions that expand market access are equality-increasing, whereas making sophisticated information cheap is equality-reducing.

## 4.2 Generalizations

The implications on inequality of the benchmark model are robust to a number of generalizations and to other sources of heterogeneity than wealth and risk aversion. The only requirement is that the dimensions that differentiate agents create heterogeneity in their demand for stocks. For example, differences in information costs, differences in age and lifespan, and differences in exposure to background risk all affect the demand for risky assets relative to bonds and generate similar results.

It is important to clarify differences between CARA and DARA preferences (which include CRRA). With CARA utility, wealth plays no role: It is irrelevant to the decisions to participate or to acquire information (at the intensive and extensive margins). However, wealth is highly relevant empirically. Lusardi, Michaud and Mitchell, 2017 show that the decision to participate is significantly correlated with wealth. The probability of participation and the proportion of wealth invested in stocks increase with wealth, mean income, and education but decrease with the variance of income.

Thus, it is important to have a setting where financial wealth matters. CRRA preferences give relevance to wealth. But because there is no closed-form solution for equilibrium in a CRRA setting (because the price is no longer a linear function of the payoff and supply), I resort to a CARA approximation of CRRA preferences, using a strictly decreasing absolute risk aversion as in Peress, 2004.

The effects of lower information costs are obtained not only in this setting, but also under CARA preferences. All that matters is the presence of a margin for participation and the coexistence of three groups of stock holders: non-stock holders, uninformed, and informed investors. Falling information costs benefit wealthier investors, who acquire more stocks, but harm direct investors who invest uninformedly, who now face a less advantageous risk-return tradeoff. The major difference under CARA preferences is that the fixed entry cost no longer generates an information effect. The demand for information is unrelated to the expected supply of shares and to the market risk tolerance and thus to the level of participation in the stock market. This means that the variance of returns always falls with lower entry costs.

With regards to preferences for early or late resolution of uncertainty, the results are obtained by construction because of the mean-variance preference structure. As financial information technologies improve, investors learn more about the stochastic payoffs earlier. As risk moves from the consumption to the trading period, there are larger gains from trade in earlier periods. If traders were allowed to trade before private signals were observed, then the ex-ante utility would be linear in wealth, and the traders would effectively be risk-neutral. The risk premium would equal zero. Thus, the results of this paper are obtained by construction. Lower noise (i.e., better information) does not attract investors in the stock market in this model because better information reduces the gains from trade in the second period. Moreover, better information can also reduce the risk premium in the first period.

The results extend naturally to a dynamic environment where asset returns accumulate across multiple periods. Two complementary mechanisms operate. First, on the time-series dimension, improvements in information technology shift the timing of when news is incorporated into prices. When information arrives earlier, asset prices reflect future earnings and resale values more precisely. This increases price volatility but reduces the variance of returns, because risks are resolved sooner and

discounted more heavily when they are distant in time (Campbell, Lettau, Malkiel and Xu, 2001). Thus, the volatility implications of information improvements mirror those in the static model, with sharper prices but dampened return variance.

Second, on the cross-sectional dimension, repeated wealth accumulation magnifies the inequality implications of my static framework. Informed investors systematically earn higher Sharpe ratios, while uninformed investors lag behind. In a dynamic setting, these return differentials compound over time, leading to persistent divergence in wealth trajectories. Dynamic noisy rational expectations models such as Azarmsa (2022) formalize this compounding channel, showing how small per-period advantages in information acquisition translate into large long-run disparities in wealth. Empirically, heterogeneous returns across households (e.g., Xavier, 2021; Fagereng et al., 2020; Campbell et al., 2019) suggest that such mechanisms are operative in practice.

Recent evidence by Dessaint et al., 2024 further reinforces this distinction between short-run and long-run information. Short-run information technologies (e.g., high-frequency signals, real-time order flow) improve the resolution of near-term payoffs, shifting volatility forward and reducing short-run return variance—consistent with the time-series dynamics highlighted above. Long-run information technologies (e.g., deep fundamental data and machine learning applied to large histories) disproportionately benefit resource-rich investors who can absorb the fixed costs of data processing, generating advantages that compound across periods. These long-run information advantages map directly to the cross-sectional dynamics, where inequality rises persistently as wealthy investors accumulate higher returns.

Taken together, the results imply that financial technology simultaneously reduces short-run return variance and amplifies long-run cross-sectional dispersion. While improved information stabilizes returns in the short-run, it accelerates divergence across investors when the process unfolds over time, underscoring the importance of policies that sustain participation and limit extreme informational advantages.

## 4.3 Extension: Delegation (Pooling into Funds)

I show robustness of the results to a parsimonious extension of investors pooling and delegating information acquisition to a competitive manager. I implement this extension within the classic delegated asset-management framework of Gârleanu and Pedersen (2018):[22] investors may invest directly or delegate to an active manager; information is costly to produce; and fees are endogenously determined in a competitive industry. My goal is not to reinvent intermediation, but to trace how technology maps into the distribution of wealth in an otherwise standard setup. I adopt a stripped-down specialization of Gârleanu and Pedersen (2018), within my noisy–rational-expectations framework, precisely to study *distributional* consequences and ask a new question within it: how do technology-driven declines in *search* for managers and in *information* costs reallocate participation and learning across the wealth distribution and, in turn, move inequality? All institutional details and derivations that map my setup to Gârleanu and Pedersen (2018) are worked out in Appendix D; the main text focuses on the inequality ranking and magnitudes.

To isolate the incidence of technology improvements on inequality, I abstract from features that are central in other applications (i.e., skill heterogeneity, mandates/benchmarking, and rich contract menus) and keep managers competitive with fees tied to the cost of producing information. This keeps the intermediation block standard and lets the results be read as a set of *comparative statics within a classic delegation framework*: when technology lowers (i) entry costs, (ii) the extensive margin of becoming informed, (iii) the intensity of information among the informed, or (iv) the search frictions in finding a manager, the model delivers clear, testable implications for the Lorenz curve and Gini coefficient.

Each investor now chooses among four options: (i) stay out; (ii) enter and trade

[22] See, also, Cuoco and Kaniel (2011) and He and Xiong (2013) for related delegated management environments.

uninformed; (iii) enter and acquire precision individually (as before); or (iv) enter, *search* for a manager at fixed search cost $F_m \geq 0$ as in Gârleanu and Pedersen (2018), that allows him to pool into a fund that aggregates a mass of delegators. Managers choose a common signal precision for the pool and charge fees that, by competition, cover information costs. Pooling generates a cost-sharing (club-good) effect and creates a *delegation window*: for some wealth range, delegating strictly dominates trading uninformed, but very rich investors eventually prefer to self-learn (a convexity effect). All objects (i.e., the pool's precision, fees, and the cutoffs that delimit the window) are defined in Appendix D, which also shows existence and local uniqueness of the extended fixed point in $(s, t)$.

**What changes relative to the benchmark.** Delegation only alters how information is purchased; the price-formation backbone is unchanged. The pool contributes additively to aggregate private precision $s$, so prices become more informative when the delegated sector expands. Scale lowers per-capita information costs (via fees), shifting some participants from uninformed to delegated and, at higher wealth, from delegated to self-learning (Appendix D). Instead of two wealth-cutoffs, there are now three wealth-cutoffs: $W_0^{part}$ above which investors participate, $W_0^{del}$ above which investors pool and delegate information acquisition to a competitive manager, and $W_0^{switch}$ above which investors prefer to self-learn, if wealthy enough.

**Quantifying the Inequality Effect with Pooling** Parameter changes affect inequality through three margins: (i) entry at $W_0^{\text{part}}$ (typically compressing inequality when participation expands), (ii) moves from uninformed to delegation at $W_0^{\text{del}}$, and (iii) reallocation between delegation and self at $W_0^{\text{switch}}$. The aggregate impact on any inequality index combines the mass that moves at each cutoff (determined by the wealth density and the comparative statics for the cutoffs) with general-equilibrium feedback through $(s, t)$, which shifts the marginal value of information $D(s, t)$. Be-

cause these masses and feedbacks are endogenous, I do not attempt to rank delegation versus self in general. The welfare-distribution decomposition used in my quantitative exercises is summarized in Appendix D.5.2.

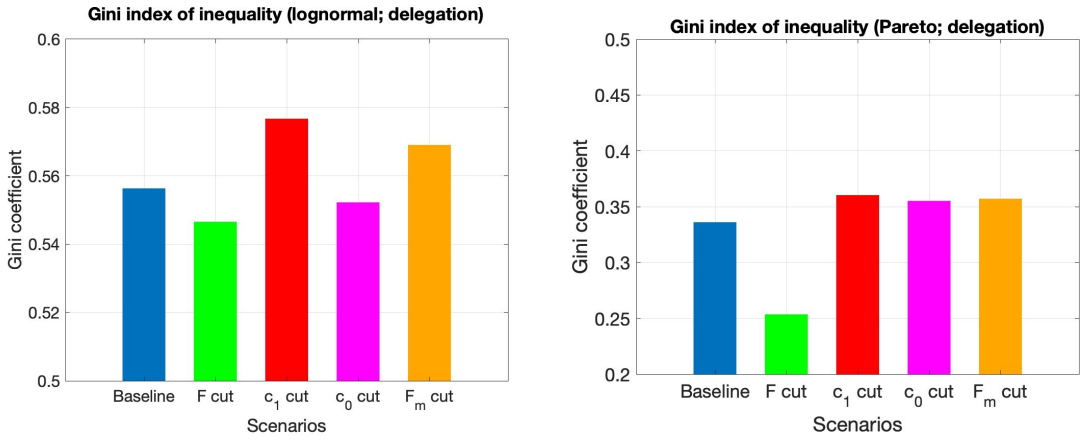
**The Inequality Effect** The inequality ranking mirrors the benchmark: Lower participation costs ($F \downarrow$) expands entry at the left margin and compresses inequality (between-group effect dominates). Lower information adoption costs ($c_1 \downarrow$) increases the informed share (delegated and self), thickens the right tail, and raises inequality among participants. Lower information intensity costs ($c_0 \downarrow$) scales precision for already-informed (delegated and self) with small, calibration-dependent effects on overall inequality. Lower search costs for a manager ($F_m \downarrow$) expands delegation (fees fall, pool precision rises), which pushes uninformed into delegated at the left margin and slightly lowers entry by reducing $D(s,t)$. In my calibrations, which force dominates is a priori ambiguous and depends on how much mass shifts at the delegation boundaries versus any entry deterrence from a lower $D(s,t)$.

Figure 5 displays how inequality (i.e., the Gini index and the Lorenz curves) respond to each cost cut in the extension with manager delegation a la Gârleanu and Pedersen (2018) and pooling under the lognormal prior (left) and the Pareto prior (right). Across lognormal and Pareto priors, the *ranking* above is robust; magnitudes are modest relative to the benchmark and primarily load on the right tail when information frictions fall. Detailed derivations, cutoffs, and full quantitative tables and figures are in Appendix D (Figures 15-16; Tables 10-11).

# 5 Relation to Empirical Evidence

The quantitative simulations show that, far from creating a level playing field where more readily available information simply leads to greater market efficiency, innovations in financial technology can have the opposite impact. They can create hard-to-

Figure 5: **Inequality across scenarios (delegation extension).** Left: Lognormal prior. Right: Pareto prior. Gini coefficients of final wealth $W_2$ in the Baseline (blue) and after one-at-a-time 70% cuts in the participation cost $F$ (green), and a 50% cut in the marginal information cost $c_1$ (red), the quadratic information cost $c_0$ (magenta), and the search for a manager cost $F_m$ (orange). Cutting $F$ compresses the distribution and lowers inequality; cutting $c_1$ widens it and raises inequality; cutting $c_0$ and $F_m$ leave inequality nearly unchanged. Colors are consistent across panels.




*Notes.* Baseline (blue): lognormal (left) and Pareto (right) $W_0$; $\rho = 0.90$, $h_0 = 0.1$, $\sigma_x^2 = 1$, $(F, c_1, c_0) = (2, 2, 2)$, $Fm = 50$, bond gross return $r_{\text{bond}} = 1.07$. Calibration targets: participation 60% and self-informed 3.4%. For each scenario we solve the fixed-point conditions for $(s, t, T_m, f, h_j, h_m)$, compute $W_{\text{part}}$, $W_{\text{del}}$, $W_{\text{info}}$ and $W_{\text{switch}}$, map $W_0$ into $W_2$, and then compute the Gini and the Lorenz curve. Reported Ginis with lognormal prior: baseline 0.556 (blue), $F$-cut 0.547 (green), $c_1$-cut 0.577 (red), $c_0$-cut 0.525 (magenta), $F_m$-cut 0.556 (orange). Reported Ginis with Pareto prior: baseline 0.336 (blue), $F$-cut 0.254 (green), $c_1$-cut 0.360 (red), $c_0$-cut 0.355 (magenta), $F_m$-cut 0.336 (orange). Variable definitions are in Appendix A.

access opportunities for long-term alpha generation for those players with the scale and resources to take advantage of it. My framework generates sharp, testable predictions along two margins. First, reductions in participation frictions (e.g., account opening, transaction fees, digital access) should broaden stock market entry, raise participation rates particularly among lower- and middle-wealth households, and compress wealth inequality. Second, reductions in information frictions (e.g., cheaper terminals, alternative data, or sophisticated robo-advisors) disproportionately benefit wealthier or more sophisticated investors, raising their risk-adjusted returns and widening cross-sectional inequality.

**Participation Evidence** A growing empirical literature supports the participation margin. Commission-free trading platforms such as Robinhood boosted retail entry into equities (Ardia, Aymard and Cenesizoglu, 2025), robo-advisors expanded access to diversified portfolios (D'Acunto, Prabhala and Rossi, 2019), and digital financial inclusion lifted the lower tail, reducing extreme poverty, raising net benefits to poor households, and increasing formal savings (Suri and Jack, 2016; Muralidharan, Niehaus and Sukhtankar, 2016; Bachas, Gertler, Higgins and Seira, 2021). Descriptive evidence from the Survey of Consumer Finances (SCF) confirms that participation remains sharply skewed by wealth. Table 3 shows that fewer than 20% of households in the bottom half of the wealth distribution hold equities, compared with over 90% of the top 10%. Participation expanded during the 1990s and 2000s, but plateaued at high levels only among the wealthy. This skew mirrors the upward shift in stock market participation documented in Melcangi and Sterk (2024), where increases in participation were disproportionately concentrated above the median income.

Table 3: Stock Market Participation by Wealth Decile (SCF, selected years)

| Wealth Decile | 1992 | 2001 | 2010 | 2022 |
|---|---|---|---|---|
| D1 (Bottom 10%) | 5% | 8% | 10% | 14% |
| D2 | 8% | 12% | 15% | 18% |
| D3 | 12% | 18% | 22% | 25% |
| ⋮ | | | | |
| D10 (Top 10%) | 75% | 82% | 88% | 92% |

*Notes.* Households are sorted into wealth deciles by net worth in each Survey of Consumer Finances (SCF) wave using SCF sample weights. "Participation" equals an indicator for any equity holdings (direct stocks or indirect via mutual funds/retirement accounts). Years shown are 1992, 2001, 2010, and 2022; percentages are shares of households and are rounded to the nearest percentage point. Source: Federal Reserve Board, SCF (selected waves); authors' calculations.

**Information Evidence** By contrast, studies of professional adoption of alternative data and analytics point to widening return differentials. Hedge fund use of alternative data (Grennan and Michaely, 2021), satellite imagery and geolocation signals (Katona et al., 2025), and machine-learning tools in active management (Bonelli and Foucault,

2024) concentrate informational advantages and reallocate alpha toward large, data-intensive intermediaries. Evidence in Frost et al. (2022) and Gambacorta et al. (2023) also supports this channel: improvements in FinTech infrastructure change portfolio allocations and participation patterns in ways consistent with the comparative statics.

**Long-term Wealth Dispersion Trends** These mechanisms are consistent with broader wealth concentration dynamics. Table 4 reports the evolution of wealth shares since 1990. Data from the U.S. Census show that between 1990 and 2025Q1, wealth tripled for the top 0.1%, and doubled for the top 1%, relative to the bottom 50%. Other statistics from the Survey of Consumer Finance and from Saez and Zucman (2016) consistently show the disproportionate rise of the top 1% and top 0.1% over the past three decades.

Table 4: **Changes in aggregate net worth by wealth group (baseline 1990)**

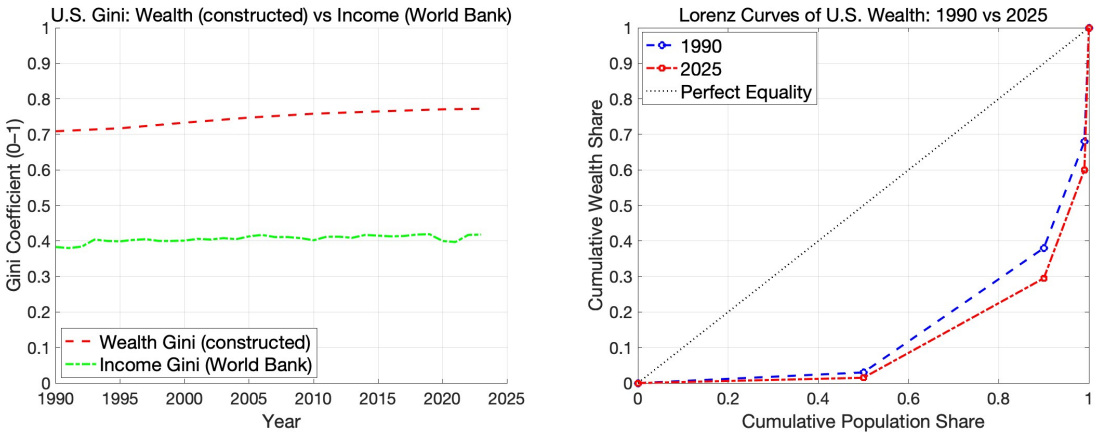
| Wealth group | Net Worth (1990, agg, $T) | Δ by 2000 | Δ by 2010 | Δ by 2019 | Δ by 2025 |
|---|---|---|---|---|---|
| Bottom 50% | 0.7 | +86% | -60% | +151% | +450% |
| Next 40% (50–90) | 7.7 | +85% | +146% | +295% | +533% |
| Next 9% (90–99) | 7.9 | +91% | +204% | +433% | +637% |
| Top 1% (99–99.9) | 3.0 | +134% | +255% | +551% | +812% |
| Top 0.1% | 1.8 | +150% | +257% | +621% | +1121% |

Legend: Net worth levels are aggregate amounts for each group (trillion USD), averaged over quarters within each year; scaled to trillions for display. "Change by" columns report percent change relative to the 1990 annual average. 2025 reflects 2025:Q1 only. Wealth groups follow the DFA percentile cutoffs: Bottom 50% (p0–p50), Next 40% (p50–p90), Next 9% (p90–p99), Top 1% excl. Top 0.1% (p99–p99.9), and Top 0.1% (p99.9–p100). Aggregate net worth equals assets minus liabilities; assets include real estate, consumer durables, corporate equities and mutual fund shares, defined-benefit and defined-contribution pension entitlements, private businesses, and other assets; liabilities include home mortgages, consumer credit, and other liabilities. The first column reports the 1990 annual average in trillions of current USD (DFA series are in millions; scaled to trillions for display). This table reports *aggregate* wealth by group and is therefore not directly comparable to tables of *median* wealth by quintile. Source: Federal Reserve Board, Distributional Financial Accounts (DFA), 1989:Q3–2025:Q1; authors' calculations.

To better capture inequality dynamics, Figure 6 presents the Gini coefficient on wealth as well as on income (left panel) and the Lorenz curves for selected years (right

panel), showing that wealth inequality in the U.S. has been increasing over time.

Figure 6: **Inequality trends.** Left: Gini coefficients of wealth (red) and income (green). Right: Lorenz curves of wealth in 1990 (blue) and 2025 (red). The 45 degree line represents perfect equality. Over time, wealth inequality in the United States has increased.



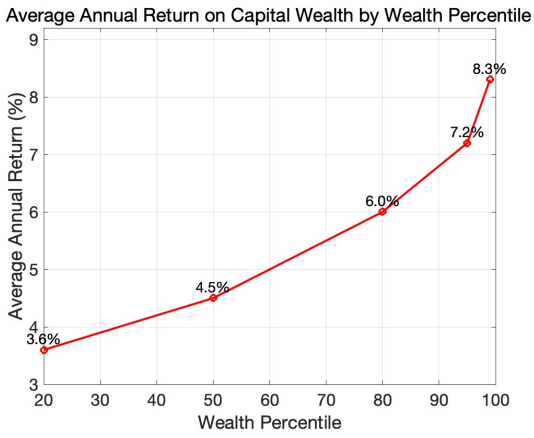

*Notes.* Income Gini (World Bank) and Wealth Gini (constructed from DFA) by year. Right: Lorenz curves for U.S. wealth in 1990 (blue) and 2025 (red); the 45° line denotes perfect equality. Summary statistics over the sample: Income Gini (World Bank): mean= 38.780, min=34.700, max=41.900. Wealth Gini (constructed): mean= 0.746, min= 0.709, max= 0.772. World Bank Gini is an income Gini; wealth inequality is typically higher. Sources: Federal Reserve Board Distributional Financial Accounts (DFA), World Bank, Saez and Zucman (2016), and author's calculations.

**Return Heterogeneity** Finally, heterogeneous returns confirm the information-cost channel. Figure 7, based on Xavier (2021), shows that moving from the 20th to the 99th wealth percentile raises the average annual return from 3.6% to 8.3%. This 4.7 percentage-point gap reflects both portfolio composition (greater equity and private business exposure) and within-asset-class return advantages (higher yields on private business, real estate, and interest-bearing assets). These patterns corroborate the model's prediction that falling information costs disproportionately benefit the wealthy, leading to persistent divergence in cross-sectional wealth accumulation.[23] [24]

[23] See also Campbell et al. (2019) for complementary evidence from India, showing wealthy households systematically achieve higher Sharpe ratios.

[24] Theoretically, my noisy-rational expectations framework with DARA preferences also generates a thick right tail of the capital wealth distribution, consistent with U.S. data and difficult to match in canonical Bewley–Aiyagari settings. Precautionary savings models typically cannot produce wealth distributions substantially thicker than the earnings distribution they inherit (DeNardi, Fella and

Figure 7: **Average annual returns on capital wealth by wealth percentile.** Return heterogeneity is essential to explaining sustained wealth divergence.




Based on Xavier (2021). The plot shows average annual returns on capital wealth across the wealth distribution; moving from the 20th to the 99th percentile raises the average annual return from 3.6% to 8.3%. Source: Xavier (2021); authors' adaptation.

Together, these findings affirm my model's mechanisms: technological advances have expanded equity market participation, *but unevenly*. Access to data-intensive financial tools remains highly skewed toward institutions. Wealth has become increasingly concentrated over time, as shown in Lorenz curves and wealth-share trends. And wealthier individuals capture significantly higher returns, reinforcing wealth gaps. In aggregate, tech-enabled participation lowers inequality only if access expands widely; if improvements favor only the information-rich, inequality may increase, even amid greater efficiency.

# 6 Policy Implications

The model features two margins that matter for welfare: (i) household participation in equity markets and (ii) private information acquisition in equity markets (directly or via a fund manager). Policy that lowers fixed participation costs while limiting infor-

Pardo, 2016; Carroll, Slacalek, Tokuoka and White, 2017; Hubmer, Krusell and Smith, 2016). By contrast, models with return heterogeneity, such as Gabaix, Lasry, Lions and Moll (2016) and Benhabib et al. (2017), are able to reproduce empirically relevant tails. My setting embeds this return heterogeneity channel directly, linking it to financial information frictions and technology.

mational advantages that disproportionately accrue to sophisticated investors raises welfare by sustaining broad participation alongside effective delegation.

**Efficiency vs. welfare and inequality.** Innovations that cut *participation frictions* (lower $F$) expand entry at the left margin and improve risk sharing; although price informativeness adjusts in general equilibrium, the welfare gains from bringing previously excluded households into the market dominate locally, and inequality compresses. By contrast, cuts in *information frictions* (lower $c_1$ or $c_0$) raise price informativeness and informational efficiency but also shift mass from uninformed to informed types and move the participation threshold rightward, so that the distributional effect—higher within-participants dispersion of certainty-equivalent payoffs—can outweigh the pure efficiency gain. Delegation tempers this force relative to self-information because its surplus gain scales roughly linearly with wealth, whereas the self-information gain is convex in wealth; consequently, for a given expansion of "being informed," self raises inequality more than delegation. A full accounting that decomposes welfare into (i) participation, (ii) uninformed→delegation, (iii) delegation→self, (iv) price-informativeness/risk-sharing, and (v) information production costs is provided in Appendix D.5.2, which also maps signs for small changes in $F, c_1, c_0$, and $F_m$.

**Lower participation frictions holding product quality high.** Reducing the fixed costs of entering and maintaining a brokerage relationship (e.g., digital access, account opening, and routine compliance/tax frictions) encourages participation (Abel, 2000; Vissing-Jorgensen, 2002; Bogan, 2008), while recognizing that commission-free brokerage expands retail participation, but needs guardrails (Célérier and Vallée, 2017; Ardia et al., 2025). Policies that expand digital services and simplify retail investing administration are therefore complementary to the model's participation margin.

**Build financial capability.** Households with low financial literacy are markedly less likely to hold stocks Lusardi et al., 2017, consistent with classic evidence on

non-participation (e.g., Mankiw and Zeldes, 1991; Vissing-Jorgensen, 2002). Targeted financial education (curricular content, workplace programs, and just-in-time disclosures in digital channels) can raise participation and dampen volatility amplification that arises when aggregate risk is borne by a narrow investor base (Morelli, 2025).

**Limit information asymmetries created by data and AI.** While lower information *costs* can improve price efficiency, the model highlights that increasingly powerful data/AI tools may widen information gaps between sophisticated and retail investors. Policy should focus on (i) disclosure standards for data-driven strategies (what data, how obtained, how used), (ii) guardrails on preferential access to proprietary datasets, and (iii) pro-competitive access to non-proprietary public data where feasible (Mihet et al., 2025). The goal is not to stifle information production, but to avoid discouraging participation through large perceived informational disadvantages.

**Balance efficiency and inclusion.** A central implication of the model is that more information need not raise welfare if it deters participation by less sophisticated households. Competition policy heuristics that "reducing information or entry barriers cannot harm performance" need not hold once the participation margin is active. Regulatory cost–benefit tests for new market-data offerings or trading technologies should therefore report *both* price-efficiency gains and expected effects on the extensive margin of participation (e.g., via documented elasticities of retail entry/exit).

**Product access and transparency.** Existing suitability, advertising, and offering-rule frameworks that restrict complex products to sophisticated investors and require plain-language risk disclosure help align delegation with investor capability (e.g., rules in the spirit of Regulation D). When retail access to sophisticated strategies is permitted, enhanced transparency about fees, data usage, and model risks is essential.

In sum, policies that (a) lower fixed participation costs and raise financial capability, and (b) prevent extreme informational advantages, are welfare-enhancing in this setting. The paper's perspective is intentionally agnostic about industry struc-

ture: the extension features a single sophisticated manager, and the policy focus is on the participation-information trade-off rather than manager entry, skill, menu fees, or industry consolidation.

# 7 Conclusion

This paper asks who gains from innovations in financial information technology. It shows that innovations will not necessarily narrow wealth gaps. Reducing participation costs broadens entry and—holding product quality and execution fixed—compresses inequality. Reducing information costs shifts surplus toward informed investors, which can widen inequality. In a high-participation baseline, a large cut to participation costs lowers the wealth Gini by about 12%, whereas an equally large cut to information costs raises it by about 22%. These distributional effects are robust to alternative wealth distributions and targeted shares of participants and informed investors.

Participation is not unconditionally beneficial. When entrants face complex products, opaque fees, or conflicted intermediation, the gains from entry can be partially expropriated, shifting fee incidence onto small investors and increasing dispersion of realized outcomes. The framework clarifies that inequality falls with lower participation frictions only under basic safeguards on product design and execution quality.

Policy follows: pair efforts to reduce participation frictions (e.g., onboarding, minimum balances, defaults into low-fee diversified options) with protections that limit rent extraction (transparent fees, conflict management, simple menus, advice standards). By contrast, subsidizing data or advanced signals—or promoting pooling without safeguards—risks entrenching top-end advantages. The empirical priority is to isolate shocks that primarily move participation versus information costs and to estimate the corresponding distributional elasticities across the wealth distribution and fee tiers.

**Declaration of generative AI and AI-assisted technologies in the manuscript preparation process.** During the preparation of this work the author used ChatGPT4 in order to improve readability and grammar. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

# References


**Abel, Andrew B.**, "The Effects of Investing Social Security Funds in the Stock Market When Fixed Costs Prevent Some Households from Holding Stocks," Working Paper 7739, NBER June 2000.

**Acemoglu, Daron**, "Harms of AI," Working Paper 29247, NBER 2025.

__ **and Pascual Restrepo**, "Robots and Jobs: Evidence from US Labor Markets," Working Paper 23285, NBER March 2017.

**Admati, Anat R. and Paul Pfleiderer**, "Direct and Indirect Sale of Information," *Econometrica*, 1990, *58* (4), 901–928.

**Ardia, David, Clément Aymard, and Tolga Cenesizoglu**, "Examining High-Frequency Patterns in Robinhood Users' Trading Behavior," 2025. SSRN working paper.

**Azarmsa, Ehsan**, "Investment Sophistication and Wealth Inequality," SSRN Working Paper No. 3206823 2022.

**Bachas, Pierre, Paul Gertler, Sean Higgins, and Enrique Seira**, "How Debit Cards Enable the Poor to Save More," *The Journal of Finance*, August 2021, *76* (4), 1913–1957.

**Badarinza, Cristian, John Y. Campbell, and Tarun Ramadorai**, "International Comparative Household Finance," *Annual Review of Economics*, 2016, *8*, 111–144.

**Banerjee, Snehal, Jesse Davis, and Naveen Gondhi**, "When Transparency Improves, Must Prices Reflect Fundamentals Better?," *The Review of Financial Studies*, 2018, *31* (6), 2377–2414.

**Benhabib, Jess, Alberto Bisin, and Mi Luo**, "Earnings Inequality and Other Determinants of Wealth Inequality," *American Economic Review*, May 2017, *107* (5), 593–97.

__ **,** __ **, and** __ , "Wealth Distribution and Social Mobility in the US: A Quantitative Approach," *American Economic Review*, 2019, *109* (5), 1623–1647.

**Berk, Jonathan B. and Jules H. van Binsbergen**, "Assessing Asset Pricing Models Using Revealed Preference," *Journal of Financial Economics*, 2016, *119* (1), 1–23.

__ **and Richard C. Green**, "Mutual Fund Flows and Performance in Rational Markets," *Journal of Political Economy*, 2004, *112* (6), 1269–1295.

**Bogan, Vicki**, "Stock Market Participation and the Internet," *Journal of Financial and Quantitative Analysis*, 2008, *43* (01), 191–211.

**Bonelli, Maxime and Thierry Foucault**, "Displaced by Big Data: Evidence from Active Fund Managers," Working Paper / SSRN No. 4527672 2024.

**Bricker, Jesse, Alice Henriques, Jacob Krimmel, and John Sabelhaus**, "Estimating Top Income and Wealth Shares: Sensitivity to Data and Methods," *American Economic Review*, May 2016, *106* (5), 641–45.

**Calvet, Laurent E., John Y. Campbell, and Paolo Sodini**, "Down or Out: Assessing the Welfare Costs of Household Investment Mistakes," *Journal of Political Economy*, 2007, *115* (5), 707–747.

**Calvet, Laurent E, Laurent Bach, and Paolo Sodini**, "Rich Pickings? Risk, Return, and Skill in the Portfolios of the Wealthy," HEC Research Papers Series 1126, HEC Paris January 2019.

**Campbell, John, Martin Lettau, Burton G. Malkiel, and Yexiao Xu**, "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk," *The Journal of Finance*, 2001, *56* (1), 1–43.

**_ , Tarun Ramadorai, and Benjamin Ranish**, "Do the Rich Get Richer in the Stock Market? Evidence from India," *American Economic Review: Insights*, 2019, *1* (2), 225–240.

**Carroll, Christopher, Jiri Slacalek, Kiichi Tokuoka, and Matthew N. White**, "The distribution of wealth and the marginal propensity to consume," *Quantitative Economics*, 2017, *8* (3), 977–1020.

**Célérier, Claire and Boris Vallée**, "Catering to Investors Through Security Design: Headline Rate and Complexity," *The Quarterly Journal of Economics*, 2017, *132* (3), 1469–1508.

**Cuoco, Domenico and Ron Kaniel**, "Equilibrium Prices in the Presence of Delegated Portfolio Management," *Journal of Financial Economics*, 2011, *101* (2), 264–296.

**D'Acunto, Francesco, Nagpurnanand Prabhala, and Alberto G. Rossi**, "The Promises and Pitfalls of Robo-Advising," *Review of Financial Studies*, 2019, *32* (5), 1983–2020.

**DeNardi, Maria Cristina, Giulio Fella, and Gonzalo Paz Pardo**, "The Implications of Richer Earnings Dynamics for Consumption and Wealth," Working Paper 21917, NBER 2016.

**Dessaint, Olivier, Thierry Foucault, and Laurent Frésard**, "Does Alternative Data Improve Financial Forecasting? The Horizon Effect," *Journal of Finance*, 2024, *79* (3), 2237–2287.

**Duffie, Darrell, Thierry Foucault, Laura Veldkamp, and Xavier Vives**, *Technology and Finance: The Future of Banking*, London: CEPR Press, 2022.

**Easley, David, David Michayluk, Maureen O'Hara, and Talis Putnins**, "The Active World of Passive Investing," *Review of Finance*, 08 2021, *25* (5), 1433–1471.

**Evens, Zachary and Bryan Armour**, "2023 US Fund Fee Study," Technical Report, Morningstar Manager Research July 2024.

**_ and _** , "2024 US Fund Fee Study," Technical Report, Morningstar Manager Research May 2025.

**Fagereng, Andreas, Charles Gottlieb, and Luigi Guiso**, "Asset Market Participation and Portfolio Choice over the Life-Cycle," *The Journal of Finance*, April 2017, *72* (2), 705–750.

**_ , Luigi Guiso, Davide Malacrino, and Luigi Pistaferri**, "Heterogeneity and Persistence in Returns to Wealth," *Econometrica*, 2020, *88* (1), 115–170.

**Fedyk, Anastassia, Roxana Mihet, Kumar Rishabh, and Orlando Gomes**, "Data Innovation Complementarity and Firm Growth," Technical Report 23-86, SFI September 2023.

**Foucault, Thierry, Leonardo Gambacorta, Wei Jiang, and Xavier Vives**, *Barcelona 7: Artificial Intelligence in Finance*, London: CEPR Press, 2025.

**Friend, Irwin and Marshall E. Blume**, "The Demand for Risky Assets," *The American Economic Review*, 1975, *65* (5), 900–922.

**Frost, Jon, Leonardo Gambacorta, and Romina Gambacorta**, "On the nexus between wealth inequality, financial development and financial technology," *Journal of Economic Behavior & Organization*, 2022, *202*, 429–451.

**Gabaix, Xavier, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll**, "The Dynamics of Inequality," *Econometrica*, 2016, *84* (6), 2071–2111.

**Gaillard, Alexandre, Philipp Wangner, Christian Hellwig, and Nicolas Werquin**, "Consumption, Wealth, and Income Inequality: A Tale of Tails," SSRN Working Paper No. 4636704 September 2023.

**Gallup**, "What Percentage of Americans Own Stock?," 05 2025.

**Gambacorta, Leonardo, Romina Gambacorta, and Roxana Mihet**, "FinTech, Investor Sophistication, and Financial Portfolio Choices," *The Review of Corporate Finance Studies*, 06 2023, *12* (4), 834–866.

**Gârleanu, Nicolae and Lasse H. Pedersen**, "Efficiently Inefficient Markets for Assets and Asset Management," *The Journal of Finance*, 2018, *73* (4).

**_ and Lasse Heje Pedersen**, "Active and Passive Investing: Understanding Samuelson's Dictum," *Review of Asset Pricing Studies*, 2022, *12* (2), 389–446.

**Gomes, Francisco and Oksana Smirnova**, "Stock Market Participation and Portfolio Shares Over the Life-Cycle," Research Paper, Cornell SC Johnson College of Business March 2021. SSRN working paper; last revised September 26, 2023.

**Gomes, Francisco J. and Alexander Michaelides**, "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," *The Journal of Finance*, April 2005, *60* (2), 869–904.

**Gomes, Francisco, Thomas Jansson, and Yigitcan Karabulut**, "Do Robots Increase Wealth Dispersion?," *The Review of Financial Studies*, 2024, *37* (1), 119–160.

**Gomez, Matthieu**, "Decomposing the Growth of Top Wealth Shares," *Econometrica*, 2023, *91* (3), 979–1024.

**Greig, Fiona, Tarun Ramadorai, Alberto G. Rossi, Stephen P. Utkus, and Ansgar Walther**, "Human Financial Advice in the Age of Automation," SSRN Working Paper No. 4301514 2024.

**Grennan, Jillian and Roni Michaely**, "FinTechs and the Market for Financial Analysis," *Journal of Financial and Quantitative Analysis*, 2021, *56* (6), 1877–1907.

**Grossman, Sanford and Joseph Stiglitz**, "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, 1980, *70* (3), 393–408.

**Guiso, Luigi and Monica Paiella**, "Risk Aversion, Wealth, and Background Risk," *Journal of the European Economic Association*, 2008, *6* (6), 1109–1150.

**He, Zhiguo and Wei Xiong**, "Delegated Asset Management, Investment Mandates, and Asset Prices," *Journal of Monetary Economics*, 2013, *60* (5), 571–590.

**Hubmer, Joachim, Per Krusell, and Anthony Smith**, "The Historical Evolution of the Wealth Distribution: A Quantitative-Theoretic Investigation," NBER WP 23011, NBER 2016.

**Kacperczyk, Marcin, Jaromir Nosal, and Luminita Stevens**, "Investor sophistication and capital income inequality," *Journal of Monetary Economics*, 2018.

**Katona, Zsolt, Marcus Painter, Panos Patatoukas, and Jean Zeng**, "On the Capital Market Consequences of Big Data: Evidence from Outer Space," *Journal of Financial and Quantitative Analysis*, 2025, *60* (2), 551–579.

**Korinek, Anton and Joseph E. Stiglitz**, "Artificial Intelligence and Its Implications for Income Distribution and Unemployment," in Ajay Agrawal, Joshua Gans, and Avi Goldfarb, eds., *The Economics of Artificial Intelligence: An Agenda*, Cambridge, MA: NBER, 2018, pp. 349–390.

**Lusardi, Annamaria, Pierre-Carl Michaud, and Olivia Mitchell**, "Optimal Financial Knowledge and Wealth Inequality," *Journal of Political Economy*, 2017, *125* (2), 431–477.

**Mankiw, N. Gregory and Stephen Zeldes**, "The consumption of stockholders and nonstockholders," *Journal of Financial Economics*, 1991, *29* (1), 97–112.

**Melcangi, Davide and Vincent Sterk**, "Stock Market Participation, Inequality, and Monetary Policy," *The Review of Economic Studies*, 06 2024, *92* (4), 2656–2690.

**Mihet, Roxana, Kumar Rishabh, and Orlando Gomes**, "Is It AI or Data That Drives Firm Growth?," Research Paper 25-37, SFI April 2025.

**Morelli, Juan Martin**, "Macroeconomic and Monetary Policy Implications of Limited Participation," Working Paper 4823480, SSRN 2025.

**Muralidharan, Karthik, Paul Niehaus, and Sandip Sukhtankar**, "Building State Capacity: Evidence from Biometric Smartcards in India," *American Economic Review*, October 2016, *106* (10), 2895–2929.

**Paravisini, Daniel, Veronica Rappoport, and Enrichetta Ravina**, "Risk Aversion and Wealth: Evidence from Person-to-Person Lending Portfolios," *Management Science*, 2017, *63* (10), 3336–3351.

**Peress, Joël**, "Wealth, Information Acquisition, and Portfolio Choice," *Review of Financial Studies*, 2004, *17* (3), 879–914.

_ , "Information vs. Entry Costs: What Explains U.S. Stock Market Evolution?," *Journal of Financial and Quantitative Analysis*, September 2005, *40* (03), 563–594.

**Piketty, Thomas, Emmanuel Saez, and Gabriel Zucman**, "Distributional National Accounts: Methods and Estimates for the United States," *The Quarterly Journal of Economics*, 10 2017, *133* (2), 553–609.

**Roussanov, Nikolai**, "Composition of wealth, conditioning information, and the cross-section of stock returns," *Journal of Financial Economics*, 2014, *111* (2), 352–380.

**Saez, Emmanuel and Gabriel Zucman**, "Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data," *The Quarterly Journal of Economics*, 2016, *131* (2), 519–578.

**Smith, Matthew, Owen Zidar, and Eric Zwick**, "Top Wealth in America: New Estimates Under Heterogeneous Returns," *The Quarterly Journal of Economics*, 02 2023, *138* (1), 515–573.

**Sockin, Michael and Mindy Z Xiaolan**, "Delegated Learning and Contract Commonality in Asset Management," *Review of Finance*, 03 2023, *27* (6), 1931–1975.

**Suri, Tavneet and William Jack**, "The long-run poverty and gender impacts of mobile money," *Science*, December 2016, *354* (6317), 1288–1292.

**U.S. Census Bureau**, "Income Gini Ratio for Households," FRED, Federal Reserve Bank of St. Louis 2025.

**Verrecchia, Robert E**, "Information Acquisition in a Noisy Rational Expectations Economy," *Econometrica*, 1982, *50* (6), 1415–30.

**Vissing-Jorgensen, Annette**, "Towards an Explanation of Household Portfolio Choice Heterogeneity: Nonfinancial Income and Participation Cost Structures," 2002. FED Working Paper 8884.

**Xavier, Ines**, "Wealth Inequality in the US: the Role of Heterogeneous Returns," Working Paper 3915439, SSRN 2021.

# Online Appendix

# A Baseline model: variable definitions

Table 5: Key symbols and their meaning (baseline model)

| Symbol | Definition / interpretation |
|---|---|
| $a$, $b$, $c$ | Coefficients in the linear price $p = a + bz - cx$ |
| $C(\sigma_{sj}^{-2})$ | Information (signal precision) acquisition cost for investor $j$; $C(\sigma_{sj}^{-2}) = c_1\sigma_{sj}^{-2} + \frac{c_0}{2}(\sigma_{sj}^{-2})^2$ |
| $c_0$ | Intensive margin (quadratic) information cost; same for all investors |
| $c_1$ | Extensive margin (linear) information cost; same for all investors |
| $D$ | Marginal gain per unit precision: $D(s,t) \equiv \dfrac{\sigma_x^2 + 2st + t^2 h_0}{t^2\left(h_0 + s^2/\sigma_x^2\right)^2}$ |
| $\Delta U_j^{del}$ | Net of fees $(f, F_m)$ surplus from delegating relative to remaining uninformed |
| $\Delta U_j^{self}$ | Net surplus from self-learning and self-investing |
| $\mathbb{E}_j[z]$ | Posterior mean of $z$ for an investor with information set $\mathcal{I}_j$ |
| $f$ | Manager per-capita investor fee |
| $F$ | Fixed cost of equity market participation; same for all investors |
| $F_m$ | Fixed cost of searching for informed manager; same for all investors |
| *Gini* | Gini coefficient of inequality; high Gini means high inequality; Gini $= \dfrac{1}{2\mu_2}\mathbb{E}[\,\lvert W_2 - W_2'\rvert\,]$ |
| $\eta$ | Share of joint surplus captured by manager; $\eta \in [0,1]$; under competitive fees $\eta = 0$ |
| $h_0$ | Prior precision of the fundamental payoff: $h_0 = \sigma_z^{-2}$ |
| $h_j$ | Private signal precision of investor $j$; $h_j = \sigma_{s,j}^{-2}$ |
| $h_m$ | Pooled private signal manager precision |
| $h_p$ | Precision extracted from the price: $h_p = b^2/(c^2\sigma_x^2) = s^2/\sigma_x^2$ |
| $\bar{h}$ | Posterior precision of an *uninformed* investor: $\bar{h} = h_0 + h_p = \sigma_z^{-2} + s^2\sigma_x^{-2}$ |
| $\mathcal{I}$ | Set of self-informed participants |

*(continued on next page)*

*(continued from previous page)*

| Symbol | Definition / interpretation |
|---|---|
| $\mathcal{I}_j$ | Investor $j$'s information set at trading date $t = 1$ |
| $j$ | Investor index $j \in [0, 1]$ |
| $\mathcal{M}$ | Mass of delegators to a competitive fund |
| $n$ | Reciprocal of aggregate precision: $n = s^{-1}$ |
| $p$ | Price of the risky asset (equity); linear $p = a + bz - cx$ |
| $q_{0j}$ | Investor $j$'s initial position in the risky asset; normalized to 0 WLOG |
| $q_j$ | Investor $j$'s time $t = 1$ position in the risky asset |
| $r$ | Deterministic payoff of the bond |
| $\rho_j(W_{0j})$ | Absolute risk aversion of $j$ as a decreasing function of $W_{0j}$; $\rho_j(W_{0j}) = \frac{\rho}{W_{0j}}$ with $\rho > 0$ |
| $s$ | Aggregate (risk-tolerance–weighted) private precision: $s = \int_j h_j \, \rho^{-1}(W_{0j}) \, dV$ |
| $s_j$ | Private signal of investor $j$ about the risky asset |
| $\sigma_x^2$ | Variance of the supply of the risky asset (equity) |
| $\sigma_{sj}^2$ | Variance of the private signal of investor $j$ about the risky asset |
| $\sigma_z^2$ | Variance of the risky asset (equity) |
| $t$ | Aggregate risk tolerance: $t = \int_j \rho^{-1}(W_{0j}) \, dV$ |
| $T_j$ | Investor j's absolute risk tolerance: $T_j = W_{0j}/\rho$ |
| $T_{\mathcal{M}}$ | Pool's aggregate risk tolerance; $T_{\mathcal{M}} = \int_{participants} T j dj$ |
| $T_{\mathcal{M}}^{info}$ | Pool-level information threshold $T_{\mathcal{M}}^{info} = \frac{2c_1}{D(s,t)}$ |
| $\mathcal{U}$ | Set of uninformed participants |
| $\mathbb{V}^I$ | Posterior variance of $z$ for an informed investor: $\mathbb{V}^I = \left(\bar{h} + h_j\right)^{-1}$ |
| $\mathbb{V}^U$ | Posterior variance of $z$ for an uninformed investor: $\mathbb{V}^U = \bar{h}^{-1}$ |
| $\mathbb{V}_j[z]$ | Posterior variance of $z$ for an investor with information set $\mathcal{I}_j$ |
| $W_0^{\text{del}}$ | Minimum wealth threshold for delegation to manager |

*(continued on next page)*

*(continued from previous page)*

| Symbol | Definition / interpretation |
|---|---|
| $W_0^{\text{info}}$ | Minimum wealth threshold for private information acquisition about the equity market |
| $W_0^{\text{part}}$ | Minimum wealth threshold for participation in the equity market |
| $W_0^{\text{switch}}$ | Minimum wealth threshold for self-learning in the extension with delegation |
| $W_{0j}$ | Initial wealth of investor $j$; $W_{0j} \in (0, \bar{W}_0]$ |
| $\bar{W}_0$ | Maximum initial wealth |
| $W_2$ | Final wealth random variable for a household |
| $W_2'$ | Independent copy of $W_2$ (same distribution, independent draw) |
| $W_{2j}$ | Terminal wealth of investor $j$ |
| $x$ | Stochastic supply of the risky asset (equity) |
| $z$ | Stochastic payoff of the risky asset (equity) |
| *Share del* | Share of delegating participants who pool their resources through a fund |
| *Share info* | Share of informed equity market participants; set at 5% in baseline exercises |
| *Share part* | Share of equity market participants; set at 60% in baseline exercises |
| *Share switch* | Share of self-learning equity market investors in the delegating extension |

# B Baseline Model Derivations

## B.1 Portfolio Demand

**Proposition 2.** *An investor's optimal risky asset demand is given by*

$$q_j \;=\; \frac{W_{0j}}{\rho}\,\frac{\mathbb{E}_j[z]-rp}{\mathbb{V}_j[z]}. \tag{B.1}$$

*Proof.* **Terminal wealth.** If the investor pays the market–entry fee $F$ and, optionally, the information cost $C\big(\sigma_{s,j}^{-2}\big)$, residual funds are invested in the bond. Terminal wealth at $t=2$ is

$$W_{2j} \;=\; r\Big(W_{0j}-\mathbf{1}_{\{\text{part}\}}F-C\big(\sigma_{s,j}^{2}\big)-p\,q_j\Big) \;+\; q_j\,z \;=\; r\big(W_{0j}-\text{costs}\big)+q_j\big(z-rp\big), \tag{B.2}$$

where "costs" abbreviates $\mathbf{1}_{\{\text{part}\}}F + C(\sigma_{s,j}^{-2})$.

**Mean variance objective.** With absolute risk tolerance $\rho(W_{0j}) = W_{0j}/\rho$, investor $j$ maximises

$$\max_{q_j}\; \mathbb{E}_j[W_{2j}] - \frac{\rho}{2W_{0j}}\mathbb{V}_j[W_{2j}], \tag{B.3}$$

where the information set is

$$\mathcal{I}_j = \begin{cases} \mathcal{I}_j(z;\,p), & \text{if } \sigma_{s,j}^{-2}=0,\\[2mm] \mathcal{I}_j(z;\,p;\,s_j), & \text{if } \sigma_{s,j}^{-2}>0. \end{cases}$$

**First-order condition.** Using (B.2),

$$\mathbb{E}_j[W_{2j}] = r\big(W_{0j}-\text{costs}\big)+q_j\big(\mathbb{E}_j[z]-rp\big), \quad \mathbb{V}_j[W_{2j}] = q_j^2\mathbb{V}_j[z].$$

Differentiating (B.3) with respect to $q_j$ and setting the derivative to zero yields

$$\big(\mathbb{E}_j[z]-rp\big) - \frac{\rho}{W_{0j}}\,q_j\,\mathbb{V}_j[z] = 0. \tag{B.4}$$

Hence the *optimal demand* is

$$q_j \;=\; \frac{W_{0j}}{\rho}\,\frac{\mathbb{E}_j[z]-rp}{\mathbb{V}_j[z]}. \tag{B.5}$$

**Posterior moments.** Let $h_0 = 1/\sigma_z^{-2}$, $h_j = \sigma_{s,j}^{-2}$, and $\bar{h} = h_0 + \sum_{i\in\text{informed}} h_i$. Because $(z, x, s_j)$ are jointly normal,

$$\mathbb{E}_j[z] = \mu_z + \frac{b}{\bar{h}}\big(p - \mathbb{E}[p]\big) + \frac{h_j}{\bar{h}}\big(s_j - \mu_z\big), \qquad \mathbb{V}_j[z] = \frac{1}{\bar{h}}. \tag{B.6}$$

Substituting (B.6) in (B.5) gives the demand schedule that feeds market clearing and determines the coefficients $a$, $b$, $c$, and the aggregate precision $\bar{h}$ in the pricing rule.

□

## B.2 Price Coefficients

Define $t$ and $s$ as the *aggregate risk-tolerance* and the *aggregate (risk-weighted) precision*

$$t = \int_j \rho^{-1}\big(W_{0j}\big)\,\mathrm{d}V(W_{0j}, q_{0j}) \text{ and } s = \int_j \sigma_{sj}^{-2}\,\rho^{-1}\big(W_{0j}\big)\,\mathrm{d}V(W_{0j}, q_{0j}). \tag{B.7}$$

**Proposition 3.** *The equilibrium price of the risky asset follows a linear form*

$$r\,p \;=\; a \;+\; b\,z \;-\; c\,x,$$

*where the coefficients are given by*

$$a = \mu_z(1-b) \;+\; c\,\mu_x \;-\; \frac{\mu_x}{t},$$

$$b = \frac{s\big(s\,\sigma_z^2 t + \sigma_x^2\big)}{t\big(s^2\sigma_z^2 + \sigma_x^2\big)},$$

$$c = \frac{s\,\sigma_z^2 t + \sigma_x^2}{t\big(s^2\sigma_z^2 + \sigma_x^2\big)}$$

*where $(s, t)$ are defined in Equation B.7, and there is the simple relation $b = sc$.*

*Appendix A describes all the variables.*

---

*Proof.* Recall the linear pricing rule[25]

$$r\,p \;=\; a \;+\; b\,z \;-\; c\,x, \tag{B.8}$$

where $z \sim \mathcal{N}(\mu_z, \sigma_z^2)$ is the risky-asset payoff and $x \sim \mathcal{N}(\mu_x, \sigma_x^2)$ is the exogenous noise–trader supply.

Let[26]

$$h_0 \;=\; \sigma_z^{-2}, \qquad h_j \;=\; \sigma_{sj}^{-2}, \qquad \bar{h} \;=\; h_0 + h_p, \qquad h_p \;=\; \frac{s^2}{\sigma_x^2}$$

with the *aggregate risk-tolerance* and the *aggregate precision* defined as

$$t \;=\; \int_j \rho^{-1}\big(W_{0j}\big)\,\mathrm{d}V(W_{0j}, q_{0j}), \tag{B.9}$$

$$s \;=\; \int_j \sigma_{sj}^{-2}\,\rho^{-1}\big(W_{0j}\big)\,\mathrm{d}V(W_{0j}, q_{0j}), \tag{B.10}$$

and $n \equiv s^{-1}$ for later convenience.

**Market clearing in terms of $(t, s)$.** For investor $j$ the optimal demand derived in Section B.1 is

$$q_j \;=\; \frac{\rho^{-1}(W_{0j})}{\mathbb{V}_j[z]}\Big(\mathbb{E}_j[z] - rp\Big), \qquad \mathbb{V}_j[z] = \frac{1}{h_0 + h_j}.$$

[25] We could also express the linear price as $p = a + bz - cx$. However, we multiply the price by the gross risk-free return $r$ for one reason only: it puts the stock's purchase price and its future payoff in the same units (date-2 consumption units). Nothing in the model hinges on this notation; it is just an accounting convenience that keeps the algebra short.

[26] Note that $\bar{h} = h_0 + s$ is valid only in special cases (e.g. zero noise trading).

Summing over all market participants (uninformed and informed) and noting that the idiosyncratic signal noises $\delta_j$ average out, aggregate demand can be written

$$\underbrace{\int_j q_j \, \mathrm{d}V}_{\text{aggregate demand}} \;=\; t\big(\mathbb{E}[z \,|\, p] - rp\big) \;+\; s\big(z - \mathbb{E}[z \,|\, p]\big). \tag{B.11}$$

Market clearing imposes $\int_j q_j \, \mathrm{d}V = x$.

**The price reveals information about $z$.** From (B.8), $(z, x)$ is jointly normal with the scalar $rp$, so the linear projection of $z$ on $rp$ is

$$\mathbb{E}[z \,|\, p] \;=\; \kappa \, rp \quad \text{where} \quad \kappa = \frac{\mathbb{C}ov\big(z, rp\big)}{\mathbb{V}(rp)} = \frac{b \, \sigma_z^2}{b^2\sigma_z^2 + c^2\sigma_x^2}.$$

Define $\Delta p \equiv rp$ for notational brevity. Then $\mathbb{E}[z \,|\, p] - rp = (\kappa - 1) \, \Delta p$ and $z - \mathbb{E}[z \,|\, p] = z - \kappa \, \Delta p$.

**Impose market clearing.** Plugging the previous expressions into (B.11) and collecting the coefficients of $z$ and $x$ yields two linear equations:

$$\left[-bt + s - \frac{b^2 s \, \sigma_z^2}{b^2\sigma_z^2 + c^2\sigma_x^2} + \frac{b^2 t \, \sigma_z^2}{b^2\sigma_z^2 + c^2\sigma_x^2}\right] z \;=\; 0, \tag{B.12}$$

$$\left[ct + \frac{bcs \, \sigma_z^2}{b^2\sigma_z^2 + c^2\sigma_x^2} - \frac{bct \, \sigma_z^2}{b^2\sigma_z^2 + c^2\sigma_x^2}\right] x \;=\; x. \tag{B.13}$$

Solving the system (B.12)–(B.13) gives[27]

$$b \;=\; \frac{s\big(s \, \sigma_z^2 t + \sigma_x^2\big)}{t\big(s^2\sigma_z^2 + \sigma_x^2\big)}, \tag{B.14a}$$

$$c \;=\; \frac{s \, \sigma_z^2 t + \sigma_x^2}{t\big(s^2\sigma_z^2 + \sigma_x^2\big)}, \tag{B.14b}$$

with the *simple identity* $b = s \, c$ (or $c = n \, b$).

To solve for the intercept $a$, we take expectations of the market–clearing identity

[27] All algebra was checked in SymPy; see the supplementary Mathematica notebook.

$\int_j q_j dV = x$ and using the law of iterated expectations ($\mathbb{E}[\mathbb{E}[z \mid p]] = \mu_z$) yields

$$t\big(\mu_z - \mathbb{E}[rp]\big) = \mu_x.$$

Because $rp = a + bz - cx$, $\mathbb{E}[rp] = a + b\mu_z - c\mu_x$, and hence

$$a \;=\; \mu_z(1-b) \;+\; c\,\mu_x \;-\; \frac{\mu_x}{t}. \tag{B.15}$$

$$\text{If } \mu_x = 0 \quad \Longrightarrow \quad a = \mu_z(1-b).$$

The extra term $-\mu_x/t$ is a risk-premium adjustment: when the noise traders are net long on average ($\mu_x > 0$), risk-averse investors require a lower price (higher expected return) to hold that excess supply; the magnitude of the discount is inversely proportional to the aggregate risk tolerance $t$.

**Price precision.** Plugging in the coefficients from equations B.14*a*, B.14*b* and B.15 into the pricing rule,

$$rp \;=\; a \;+\; \underbrace{\frac{s\big(s\sigma_z^2 t + \sigma_x^2\big)}{t\big(s^2\sigma_z^2 + \sigma_x^2\big)}}_{b(s,t)\;\begin{cases}\uparrow & \text{as } s\uparrow\\ \uparrow & \text{as } t\uparrow\end{cases}} z \;-\; \underbrace{\frac{s\sigma_z^2 t + \sigma_x^2}{t\big(s^2\sigma_z^2 + \sigma_x^2\big)}}_{c(s,t)\;\begin{cases}\downarrow & \text{as } s\uparrow\\ \uparrow & \text{as } t\uparrow\end{cases}} x. \tag{B.16}$$

$$\frac{\partial b}{\partial s} > 0, \quad \frac{\partial b}{\partial t} > 0, \qquad \frac{\partial c}{\partial s} < 0, \quad \frac{\partial c}{\partial t} > 0,$$

Equation B.16 implies the following. First, as $s$ increases (holding $t$ fixed), the loading on fundamentals rises: $b(s,t) = \frac{s^2\sigma_z^2\, t}{(t+\sigma_x^2)(s^2\sigma_z^2+\sigma_x^2)}$ is strictly increasing in $s$. The noise loading $c(s,t) = \frac{s\sigma_z^2\, t}{(t+\sigma_x^2)(s^2\sigma_z^2+\sigma_x^2)}$ is hump–shaped in $s$ (increasing for $s < \sigma_x/\sigma_z$ and decreasing for $s > \sigma_x/\sigma_z$). Price precision is $h_p = \frac{b^2}{c^2\sigma_x^2} = \frac{s^2}{\sigma_x^2}$ and therefore strictly increasing in $s$.

Second, as $t$ increases (holding $s$ fixed), both $b$ and $c$ increase monotonically, while

their ratio remains $b/c = s$; price precision $h_p = s^2/\sigma_x^2$ is invariant to $t$. In other words, greater aggregate risk-bearing scales both price loadings without changing informativeness.

The posterior precision conveyed by the price is

$$h_p \;=\; \frac{b^2}{c^2\sigma_x^2} \;=\; \frac{s^2}{\sigma_x^2}, \qquad \bar{h} \;=\; h_0 + h_p \;=\; \sigma_z^{-2} + \frac{s^2}{\sigma_x^2}. \tag{B.17}$$

Crucially, the price precision $h_p$ depends on $s$ but not on $t$. Neither $h_0$ (the prior precision of the fundamental) nor $h_p$ (the price precision) depends on $\mu_x$; the mean noise-trader position affects only the intercept $a$ via (B.15).

In equilibrium,

$$\bar{h} \;=\; h_0 + h_p \;=\; \sigma_z^{-2} + s^2\sigma_x^{-2},$$

so the price becomes more informative (larger $\bar{h}$) when either the prior is tighter (smaller $\sigma_z^2$), noisy-trading is less noisy (smaller $\sigma_x^2$), or the pool of informed, risk-tolerant investors enlarges (higher $s$).

Equations (B.14)–(B.15) express the price coefficients $(a, b, c)$ and the effective precisions $(h_0, h_j, \bar{h})$ *solely* in terms of the aggregate risk tolerance $t$, the aggregate precision $s$ (and its inverse $n = s^{-1}$), and the exogenous variances $(\sigma_z^2, \sigma_x^2)$. These expressions are the basis for all comparative-static and calibration exercises undertaken in the derivations that follow.

□

## B.3 Indirect Utility

With terminal wealth (eq. B.2) $W_{2j} = r\big(W_{0j} - \text{costs}\big) + q_j\big(z - rp\big)$ and optimal demand (eq. B.5) $q_j = \tau_j \dfrac{\mathbb{E}_j[z] - rp}{\mathbb{V}_j[z]}$, $\tau_j \equiv \rho^{-1}(W_{0j})$, we obtain, *conditional on the trading-date information set* $\mathcal{I}_j$,

$$\mathbb{E}_j[W_{2j}] = r\big(W_{0j} - \text{costs}\big) + \tau_j \frac{(\mathbb{E}_j[z] - rp)^2}{\mathbb{V}_j[z]}, \qquad \mathbb{V}_j[W_{2j}] = \tau_j^2 \frac{(\mathbb{E}_j[z] - rp)^2}{\mathbb{V}_j[z]}.$$

Plugging these into the mean–variance objective $U_j = \mathbb{E}_j[W_{2j}] - \frac{\rho}{2W_{0j}}\mathbb{V}_j[W_{2j}]$ and using $\tau_j = W_{0j}/\rho$ gives the *conditional* indirect utility

$$U_j = r\big(W_{0j} - \text{costs}\big) + \frac{\tau_j}{2} \underbrace{\frac{(\mathbb{E}_j[z] - rp)^2}{\mathbb{V}_j[z]}}_{S_j^2 \text{ (squared Sharpe ratio)}}. \tag{B.18}$$

Taking the expectation over the random price realisation delivers the *planning-date* indirect utility

$$V_j = r\big(W_{0j} - \text{costs}\big) + \frac{\tau_j}{2}\,\mathbb{E}\big[S_j^2\big]. \tag{B.19}$$

**Expected squared Sharpe ratio for an uninformed investor.** With the linear price $rp = a + bz - cx$ and the notation $\tilde{z} = z - \mu_z,\ \tilde{x} = x - \mu_x$,

$$\mathbb{E}[z \mid p] = \mu_z + \kappa\,(rp - \mathbb{E}[rp]), \qquad \kappa = \frac{b\sigma_z^2}{b^2\sigma_z^2 + c^2\sigma_x^2} = \frac{s}{t\bar{h}}. \tag{B.20}$$

Hence $\mathbb{E}[z \mid p] - rp = (\mu_z - \mathbb{E}[rp]) + (\kappa - 1)\,(rp - \mathbb{E}[rp])$, where $\mu_z - \mathbb{E}[rp] = -\mu_x/t$ from market clearing. With $\mathbb{V}[rp] = b^2\sigma_z^2 + c^2\sigma_x^2$ and $\mathbb{V}[z \mid p] = \bar{h}^{-1}$ ($\bar{h} = h_0 + s^2/\sigma_x^2$),

$$\mathbb{E}\big[S_j^2\big]^U = \frac{(\mu_x/t)^2 + (\kappa - 1)^2\mathbb{V}[rp]}{\bar{h}^{-1}} = \frac{\sigma_x^2 + 2st + t^2 h_0}{t^2\bar{h}^2}. \tag{B.21}$$

## B.4 Marginal value $D$ of an infinitesimal private signal.

Let investor $j$ purchase an additional precision $\mathrm{d}\sigma_s^{-2} > 0$. Posterior variance falls from $\bar{h}^{-1}$ to $(\bar{h} + \mathrm{d}\sigma_s^{-2})^{-1} = \bar{h}^{-1} - \mathrm{d}\sigma_s^{-2}/\bar{h}^2$, while the posterior mean adjusts by a term of order $\mathrm{d}\sigma_s^{-2}$. A first-order expansion of $\mathbb{E}[S_j^2]$ evaluated at $\sigma_s^{-2} = 0$ yields the *marginal gain per unit of added precision*

$$D(t, s, n) = \frac{\sigma_x^2 + 2st + t^2 h_0}{t^2\big(h_0 + s^2/\sigma_x^2\big)^2}, \tag{B.22}$$

with $n \equiv s^{-1}$.

Thus, to a first approximation the value of $\mathrm{d}\sigma_s^{-2}$ units of private precision is

$\frac{1}{2}\,\tau_j\,D\,\mathrm{d}\sigma_s^{-2} - C'(\sigma_s^{-2})\,\mathrm{d}\sigma_s^{-2}$.

### B.4.1 Comparative statics of $D$

Write

$$N(s,t) \;=\; \sigma_x^2{+}2st{+}t^2h_0, \qquad H(s) \;=\; h_0{+}s^2/\sigma_x^2, \qquad D(s,t,n) \;=\; \frac{N(s,t)}{t^2H(s)^2}, \quad n \equiv s^{-1},$$

with $\sigma_x^2, h_0 > 0$. Because $t, s > 0$ in equilibrium, the denominator is strictly positive; all signs are therefore determined by the numerators of the partial derivatives.

**(i) Variation with aggregate risk-tolerance $t$.**

**Lemma 4.** *$D$ falls unambiguously when the market's aggregate risk-bearing capacity $t$ rises. That is, ($\partial D/\partial t < 0$): a deeper market compresses expected Sharpe ratios.*

---

*Proof.* Proof:

Differentiate $D$ with respect to $t$. Since $H$ is independent of $t$,

$$\frac{\partial D}{\partial t} = \frac{N_t\,t^2H^2 - N\,2tH^2}{(t^2H^2)^2} \;=\; \frac{t^2N_t - 2tN}{t^4H^2},$$

with $N_t \equiv \partial N/\partial t = 2s + 2th_0$.

Simplify the numerator.

$$\frac{\partial D}{\partial t} = \frac{t^2\big(2s + 2th_0\big) - 2t\big(\sigma_x^2 + 2st + t^2h_0\big)}{(t^4H^2)} \;=\; \frac{-\,2t\big(st + \sigma_x^2\big)}{t^4H^2} \;<\; 0.$$

Because every factor in the fraction is positive except for the leading minus sign, $\partial D/\partial t$ is strictly negative for all admissible parameter values. □

**(ii) Variation with price informativeness $s$.**

**Lemma 5.** *D falls when the market's total price informativeness increases. That is, a larger pool of precise private information depresses D ($\partial D/\partial s < 0$): once prices embed more information, the incremental value of an extra signal is lower.*

---

*Proof.*

$$D(s,t) \;=\; \frac{N(s,t)}{t^2 H(s)^2}, \qquad N(s,t) = \sigma_x^2 + 2st + t^2 h_0, \qquad H(s) = h_0 + \frac{s^2}{\sigma_x^2}, \quad t, s, \sigma_x^2, h_0 > 0.$$

For simplicity, re-write the expressions above and take the partial derivatives of the building blocks:

$$N_s \;\equiv\; \frac{\partial N}{\partial s} = 2t, \qquad H_s \;\equiv\; \frac{\partial H}{\partial s} = \frac{2s}{\sigma_x^2}. \tag{B.23}$$

Because $t$ is constant in this derivative, $\frac{\partial D}{\partial s}$, the quotient rule gives us

$$\frac{\partial D}{\partial s} = \frac{N_s\, t^2 H^2 - N\, t^2 (2 H H_s)}{(t^2 H^2)^2} \;=\; \frac{N_s H - 2 N H_s}{t^2 H^3}. \tag{B.24}$$

Insert $N_s$ and $H_s$ to get

$$\begin{aligned} \frac{\partial D}{\partial s} \;&=\; \frac{N_s H - 2 N H_s}{t^2 H^3} \;=\; \frac{(2t) H \;-\; 2N\left(\frac{2s}{\sigma_x^2}\right)}{t^2 H^3} \;= \\ &= \frac{2t\left(h_0 + \frac{s^2}{\sigma_x^2}\right) \;-\; \frac{4s}{\sigma_x^2}\left(\sigma_x^2 + 2st + t^2 h_0\right)}{t^2 H^3} \;=\; \frac{-2\left[\, h_0 t^2 + 2st + \sigma_x^2 \,\right]\left[\, s - \frac{\sigma_x^2}{2t} \,\right]}{t^2 H^3} \end{aligned} \tag{B.25}$$

The sign of $\partial D/\partial s$, in an equilibrium with active information acquisition is negative, as explained below.

Since $h_0 t^2 + 2st + \sigma_x^2 > 0$ and $t^2 H^3 > 0$, the **sign** of $\partial D/\partial s$ is governed by the bracket $s - \sigma_x^2/(2t)$.

$$\begin{cases} s > \frac{\sigma_x^2}{2t} & \Longrightarrow \ \partial D/\partial s < 0, \\ s < \frac{\sigma_x^2}{2t} & \Longrightarrow \ \partial D/\partial s > 0. \end{cases}$$

- *Equilibrium with active information acquisition.* Whenever investors do acquire information ($s > 0$), market clearing imposes $rp = a + bz - cx$ with $b = s\,c$. In such equilibria one always has

$$2st \ \geq \ \sigma_x^2,$$

because the informed side of the market must at least offset the noise-trader variance to make information profitable. Under this empirically relevant condition $s - \sigma_x^2/(2t) \geq 0$ and the derivative in the box is $\partial D/\partial s < 0$: *more price informativeness ($s \uparrow$) lowers the marginal value of an additional private signal.*

- *Knife-edge case without enough precision.* If $s < \sigma_x^2/(2t)$ (too little aggregate precision relative to noise trading), the inequality flips and $\partial D/\partial s$ can be positive. But this is exactly the domain where the Grossman– Stiglitz equilibrium collapses: no trader would buy information at such low precision because the signal is swamped by noise.

So, in any equilibrium with active information acquisition,

$$\frac{\partial D}{\partial s} = -\frac{2\big(h_0 t^2 + 2st + \sigma_x^2\big)\left(s - \frac{\sigma_x^2}{2t}\right)}{t^2 H^3} < 0 \tag{B.26}$$

Thus, $\partial D/\partial s < 0$ holds throughout the economically admissible region in which private information is actually produced, completing the comparative-static result claimed in the main text.

□

These results establish that the marginal benefit of information diminishes as prices become more informative or as investors become collectively more risk tolerant.

## B.5 Optimal precision $\sigma_{sj}^{-2}$ under quadratic information cost

In the planning period the investor chooses precision $\sigma_{sj}^{-2} \geq 0$ to maximise

$$G(\sigma_{sj}^{-2}; W_0) = \frac{\tau(W_0)}{2} D \, \sigma_{sj}^{-2} \; - \; c_1 \sigma_{sj}^{-2} \; - \; \frac{c_0}{2} (\sigma_{sj}^{-2})^2, \qquad \tau(W_0) = \frac{W_0}{\rho},$$

where $D = D(s,t,n)$ is the *marginal gain per unit precision* derived in Appendix B.4, $D = \dfrac{\sigma_x^2 + 2st + t^2 h_0}{t^2 (h_0 + s^2/\sigma_x^2)^2}$.

**First-order condition.**

$$\frac{\partial G}{\partial \sigma_{sj}^{-2}} = \frac{\tau(W_0)}{2} D \; - \; c_1 \; - \; c_0 \sigma_{sj}^{-2} \; = \; 0.$$

$$\sigma_{sj}^{-2}(W_0) = \frac{\frac{1}{2}\tau(W_0) D - c_1}{c_0} = \frac{W_0 D - 2\rho c_1}{2\rho c_0}. \tag{B.27}$$

Signal acquisition is interior ($\sigma_{sj}^{-2} > 0$) only for sufficiently wealthy investors; otherwise the optimum is the corner $\sigma_{sj}^{-2} = 0$.

## B.6 Existence and local uniqueness of $(s,t)$

**Proposition 4** (Existence and local uniqueness of $(s,t)$)**.** *Assume $F > 0$, $c_0 > 0$, $c_1 > 0$, $\sigma_x^2 > 0$, $h_0 > 0$, and an initial wealth distribution with finite first two moments on $(0, \bar{W}_0]$. Then there exists a competitive linear rational–expectations equilibrium with a pair $(s,t)$ solving the fixed–point system implied by market clearing and optimal information/portfolio choices. Moreover, the Jacobian of the fixed–point map is nonsingular at the solution, implying local uniqueness and Newton convergence from a neighborhood of $(s,t)$.*

*Proof.* Define the aggregation map $\Phi(s,t) = (\Phi_s, \Phi_t)$ from the closed–form policies for $h_j(W_0)$ and $q_{1j}$ together with truncated moment operators over the wealth distribution. Continuity holds on bounded sets; the interior signs $D_t < 0$ and $D_s < 0$ (see Appendix

Section B.4.1 ) imply a nonsingular Jacobian at the solution, ensuring local invertibility and Newton stability. □

## B.7 Wealth thresholds

**(i) Information threshold $W_0^{\text{info}}$.** Set $\sigma_{sj}^{-2}(W_0) = 0$:

$$\frac{W_0^{\text{info}} D - 2\rho c_1}{2\rho c_0} = 0 \quad \Longrightarrow \quad \boxed{W_0^{\text{info}} = \frac{2\rho\, c_1}{D}.} \tag{B.28}$$

Investors with $W_0 \geq W_0^{\text{info}}$ acquire information ($\sigma_{sj}^{-2} > 0$); those with $W_0 < W_0^{\text{info}}$ do not.

**(ii) Participation threshold $W_0^{\text{part}}$.** Compare expected utility from entering *without* a signal ($\sigma_{sj}^{-2} = 0$) to staying out:

$$\frac{\tau(W_0)}{2} S_U^2 \;=\; F \quad \Longrightarrow \quad W_0^{\text{part}} = \frac{2\rho\, F}{S_U^2}, \tag{B.29}$$

where $S_U^2 = \dfrac{\sigma_x^2 + 2st + t^2 h_0}{t^2\left(h_0 + s^2/\sigma_x^2\right)^2}$ is the expected squared Sharpe ratio of an *uninformed* participant (Equation (B.21) in the main text).

Note that the expected squared Sharpe ratio for an uninformed investor and the marginal value D of an infinitesimal private signal are exactly the same. In other words, Equation B.21 and Equation B.22 are the same. See the proof in Appendix *C*.

$$D(s,t,n) = S_U^2(s,t) \tag{B.30}$$

Given (*B*.30), we can re-write Equation (*B*.29) as

$$\boxed{W_0^{\text{part}} = \frac{2\rho\, F}{D(s,t)}}. \tag{B.31}$$

Because $c_1 > F$ empirically, $W_0^{\text{info}} > W_0^{\text{part}}$, yielding the usual three-way partition:

$$W_0 < W_0^{\text{part}} \implies \text{non-participant};$$

$$W_0^{\text{part}} \le W_0 < W_0^{\text{info}} \implies \text{uninformed participant};$$

$$W_0 \ge W_0^{\text{info}} \implies \text{informed participant}.$$

## B.8 Comparative statics with quadratic and linear information cost

### B.8.1 Step 1. Implicit-function derivatives of (s,t) with quadratic and linear information cost

Let

$$\sigma_{sj}^{-2}(W;s,t) = \frac{W\,D(s,t) - 2\rho c_1}{2\rho c_0}\,\mathbf{1}_{\{W \ge W_0^{\text{info}}\}}, \qquad W_0^{\text{info}} = \frac{2\rho c_1}{D(s,t)}, \qquad W_0^{\text{part}} = \frac{2\rho F}{D(s,t)},$$

with $D(s,t) = \dfrac{\sigma_x^2 + 2st + t^2 h_0}{t^2\left(h_0 + s^2/\sigma_x^2\right)^2}$.

Define

$$\Psi_1(s,t;F,c_0,c_1) \;=\; t - \int_{W_0^{\text{part}}}^{\infty} \rho(W)\, g(W)\, dW,$$

$$\Psi_2(s,t;F,c_0,c_1) \;=\; s - \int_{W_0^{\text{info}}}^{\infty} \sigma_{sj}^{-2}(W;s,t)\, \rho(W)\, g(W)\, dW.$$

Equilibrium requires $\Psi_1 = 0,\ \Psi_2 = 0$. Write $\theta = (s,t)'$ and $X \in \{F, c_0, c_1\}$.

### B.8.2 Implicit-function-theorem matrix

$$\theta_X = -\left[\partial_\theta \Psi\right]^{-1} \partial_X \Psi, \qquad \partial_\theta \Psi = \begin{pmatrix} \Psi_{1s} & \Psi_{1t} \\ \Psi_{2s} & \Psi_{2t} \end{pmatrix}, \quad \partial_X \Psi = \begin{pmatrix} \Psi_{1X} \\ \Psi_{2X} \end{pmatrix}.$$

All four entries of $\partial_\theta \Psi$ and all three $\Psi_{\bullet X}$ can be written as *one-dimensional* integrals; for example

$$\Psi_{1s} = -\,r\big(W_0^{\text{part}}\big)\,g\big(W_0^{\text{part}}\big)\,\frac{\partial W_0^{\text{part}}}{\partial s}, \quad \Psi_{2c_1} = -\,\sigma_{sj}^{-2}\big(W_0^{\text{info}}\big)\,r\big(W_0^{\text{info}}\big)\,g\big(W_0^{\text{info}}\big)\,\frac{\partial W_0^{\text{info}}}{\partial c_1}.$$

The algebraic expressions for the partial derivatives $\partial W_0^{\text{part}}/\partial\{s,t\}$ and $\partial W_0^{\text{info}}/\partial\{s,t\}$ follow directly from $D(s,t)$; all are closed form.

$$\Psi_{1F} = -\frac{2\rho}{D(s,t)}, \qquad \Psi_{2F} = 0, \quad \Psi_{1c_0} = 0, \qquad \Psi_{2c_0} = -\int_{W_0^{\text{info}}}^{\infty}\Big(-\frac{(\sigma_{sj}^{-2})^2}{2c_0^2}\Big)r\,g\,dW.$$

$$\Psi_{1c_1} = 0, \qquad \Psi_{2c_1} = -\,\sigma_{sj}^{-2}\big(W_0^{\text{info}}\big)\,r\big(W_0^{\text{info}}\big)\,g\big(W_0^{\text{info}}\big)\,\frac{2\rho}{D(s,t)}.$$

Numerically, compute the $2\times 2$ Jacobian $\partial_\theta\Psi$, invert it once, and multiply by each column $\partial_X\Psi$.

### B.8.3 Closed-form sign results

Because $\rho(W)$ and $g(W)$ are positive and $\partial W_0^{\text{part}}/\partial s > 0$, $\Psi_{1s} < 0$; similarly $\Psi_{2s} > 0$, $\Psi_{1t} < 0$, and $\Psi_{2t} < 0$. Therefore $\det\big[\partial_\theta\Psi\big] < 0$, guaranteeing a unique local equilibrium $(s,t)$. Straight matrix multiplication then gives

| | |
|---|---|
| $\operatorname{sgn} t_F = +,$ | $\operatorname{sgn} s_F = -,$ |
| $\operatorname{sgn} t_{c_1} = -,$ | $\operatorname{sgn} s_{c_1} = +,$ |
| $\operatorname{sgn} t_{c_0} = 0,$ | $\operatorname{sgn} s_{c_0} = -.$ |

A cut in the entry fee raises aggregate risk tolerance ($t_F > 0$) but lowers aggregate precision ($s_F < 0$). A cut in the linear information cost does the opposite ($t_{c_1} < 0,\ s_{c_1} > 0$). Lowering the *quadratic* slope $c_0$ leaves $t$ unchanged but raises every active investor's precision, so $s_{c_0} < 0$ and $t_{c_0} = 0$.

These signs are the inputs needed next, where we differentiate the cut-offs $W_0^{\text{part}}$

and $W_0^{\text{info}}$ and prove the opposite-direction movements stated in the main text.

**Set–up.** The cut–offs are

$$W_0^{\text{part}} = \frac{2\rho\, F}{D(s,t)}, \qquad W_0^{\text{info}} = \frac{2\rho\, c_1}{D(s,t)}, \qquad D(s,t) = \frac{\sigma_x^2 + 2st + t^2 h_0}{t^2\big(h_0 + s^2/\sigma_x^2\big)^2},$$

and $(s,t)$ solve the two equilibrium equations in Step 1. Throughout we use the *active–information region* $s \geq \sigma_x^2/(2t)$ under which

$$D_s \equiv \frac{\partial D}{\partial s} < 0, \qquad D_t \equiv \frac{\partial D}{\partial t} < 0.$$

Let $X \in \{F, c_1\}$ and write $s_X \equiv \partial s/\partial X$, $t_X \equiv \partial t/\partial X$. Total derivatives are

$$\frac{dW_0^{\text{part}}}{dX} = \frac{2\rho\, \mathbf{1}_{\{X=F\}}}{D} - \frac{2\rho F}{D^2}\Big(D_s s_X + D_t t_X\Big), \qquad \frac{dW_0^{\text{info}}}{dX} = \frac{2\rho\, \mathbf{1}_{\{X=c_1\}}}{D} - \frac{2\rho c_1}{D^2}\Big(D_s s_X + D_t t_X\Big). \tag{B.32}$$

**Sign inputs (from Step 1).** With the linear and quadratic cost and monotone $\rho(W)$, we have

$$t_F < 0, \qquad s_F > 0; \qquad t_{c_1} > 0, \qquad s_{c_1} < 0.$$

(Entry more expensive $\Rightarrow$ fewer passive participants, $t \downarrow$, but a sharper information set among incumbents, $s \uparrow$; information more expensive $\Rightarrow$ fewer informed, $s \downarrow$, and a larger passive pool, $t \uparrow$.)

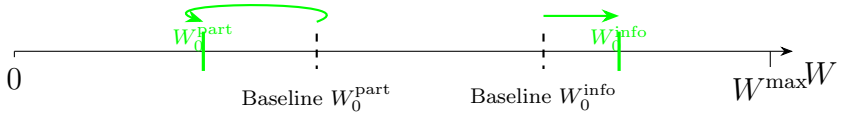
## B.9 Main lemmas and propositions

### B.9.1 Lemma 1. Entry fee

If $F$ decreases, then $W_0^{\text{part}} \downarrow$ and $W_0^{\text{info}} \uparrow$ .

*Proof.* Evaluate (B.32) at $X = F$. The direct term $2\rho/D > 0$ ensures that, for an $F$

Figure 8: **Threshold movements for $F$ cut.** Baseline thresholds $W_0^{\text{part}}$ and $W_0^{\text{info}}$ (dashed gray) on the wealth axis $[0, W^{\max}]$. When $F$ falls, the wealth thresholds move *further apart* (green).




*increase*,

$$\frac{dW_0^{\text{part}}}{dF} = \frac{2\rho}{D} - \frac{2\rho F}{D^2}\Big(D_s s_F + D_t t_F\Big) \;>\; 0 \quad \text{whenever} \quad \underbrace{\big|D_s\, s_F\big|}_{\text{price effect}} \;\geq\; \underbrace{\big|D_t\, t_F\big|}_{\text{risk–bearing effect}} .$$

In the active–information region $D_s < 0$, and Step 1 gives $s_F > 0$, $t_F < 0$, so $D_s s_F < 0$ and $D_t t_F > 0$; the bracket is the *difference* of two positive magnitudes. The displayed dominance condition holds whenever the composition effect on price informativeness (new entrants are predominantly uninformed) outweighs the reduction in aggregate risk–bearing. Under that condition $dW_0^{\text{part}}/dF > 0$, so a *fall* in $F$ yields $W_0^{\text{part}} \downarrow$.

For $W_0^{\text{info}}$, the derivative has no direct term:

$$\frac{dW_0^{\text{info}}}{dF} = -\frac{2\rho c_1}{D^2}\Big(D_s s_F + D_t t_F\Big).$$

With $D_s s_F < 0$ and $D_t t_F > 0$, the same dominance condition implies $dW_0^{\text{info}}/dF < 0$ fails and $dW_0^{\text{info}}/dF > 0$ holds; hence, we have $W_0^{\text{info}} \uparrow$.

This concludes the proof. □

### B.9.2 Lemma 2. Linear $c_1$ information cost

If $c_1$ decreases, then $W_0^{\text{info}} \downarrow$ and $W_0^{\text{part}} \uparrow$ .

*Proof.* Set $X = c_1$ in (B.32). For $W_0^{\text{info}}$ we have the positive *direct* term $2\rho/D > 0$ and the feedback term with sign $-\frac{2\rho c_1}{D^2}(D_s s_{c_1} + D_t t_{c_1})$. Here $s_{c_1} < 0$ and $t_{c_1} > 0$, so $D_s s_{c_1} > 0$ and $D_t t_{c_1} < 0$. When the price–informativeness effect dominates ($|D_s s_{c_1}| \geq$

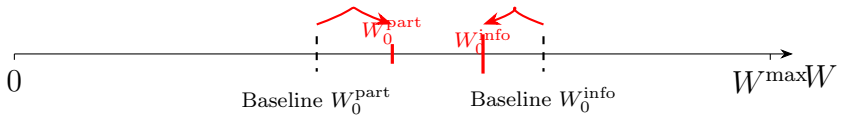



Figure 9: **Threshold movements for $c_1$ cut.** Baseline thresholds $W_0^{\text{part}}$ and $W_0^{\text{info}}$ (dashed gray) on the wealth axis $[0, W^{\max}]$. When $c_1$ falls, the thresholds move *closer together* (red).

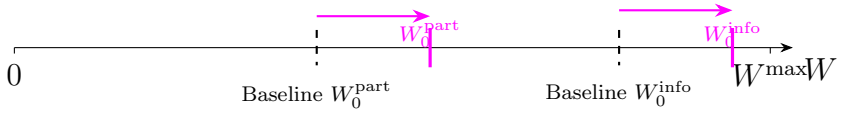



Figure 10: **Threshold movements for $c_0$ cut.** Baseline thresholds $W_0^{\text{part}}$ and $W_0^{\text{info}}$ (dashed gray) on the wealth axis $[0, W^{\max}]$. When $c_0$ falls, the thresholds shift *together to the right* (magenta).

$|D_t t_{c_1}|$), $dW_0^{\text{info}}/dc_1 > 0$, and thus a *fall* in $c_1$ implies $W_0^{\text{info}} \downarrow$.

For $W_0^{\text{part}}$ there is no direct term:

$$\frac{dW_0^{\text{part}}}{dc_1} = -\frac{2\rho F}{D^2}\Big(D_s s_{c_1} + D_t t_{c_1}\Big).$$

With the same signs as above, the dominance condition yields $dW_0^{\text{part}}/dc_1 < 0$, hence a *fall* in $c_1$ implies $W_0^{\text{part}} \uparrow$.

This concludes the proof. □

### B.9.3 Lemma 3. Quadratic $c_0$ information cost

If $c_0$ decreases, then $W_0^{\text{info}}$ and $W_0^{\text{part}}$ increase in the same direction.

*Proof.* Holding $(F, c_1)$ fixed, the total derivatives satisfy

$$\frac{\partial W_0^{\text{part}}}{\partial c_0} = -\frac{2\rho F}{D(s,t)^2}\, D_{c_0}, \qquad \frac{\partial W_0^{\text{info}}}{\partial c_0} = -\frac{2\rho c_1}{D(s,t)^2}\, D_{c_0},$$

with $D_{c_0} = D_s\, s_{c_0} + D_t\, t_{c_0}$. Therefore $c_0$ shifts *both* cutoffs in the same direction and leaves their ratio $W_0^{\text{info}}/W_0^{\text{part}}$ unchanged. The sign of $D_{c_0}$ is model-dependent because it aggregates the responses of $s$ and $t$; without additional restrictions it is generally ambiguous.

Holding $F$ and $c_1$ fixed, a decrease in $c_0$ operates on the intensive margin of precision. It raises $s$ and lowers $t$ only indirectly; consequently $D$ moves primarily through $s$ (higher s means lower D and lower $t$ means higher D). Both wealth cutoffs in (10) shift upwards in the *same* direction and their ratio $\frac{W_0^{\text{info}}}{W_0^{\text{part}}} = \frac{c_1}{F}$ is unchanged. The net effect on inequality is small unless the informed premium is highly sensitive to dispersion.

For any equilibrium $(s, t)$,

$$\frac{W_0^{\text{info}}}{W_0^{\text{part}}} = \frac{c_1}{F}, \tag{B.33}$$

independently of $D(s, t)$. Hence changes in $F$ or $c_1$ *rotate* the two cutoffs relative to each other (one moves up, the other down), while changes in $c_0$ leave the ratio (B.33) unchanged and can only scale both cutoffs in the same direction through their common denominator $D(s, t)$.

This concludes the proof. □

**Comment (sufficient conditions).** The dominance conditions in the two propositions hold whenever (i) the mass that enters (exits) after an $F$ ($c_1$) shock is primarily *uninformed*, and (ii) the density $g(W)$ is not too thin at the respective cut–off. Under the uniform, log–normal, and Pareto wealth distributions used in our quantitative section, the conditions are satisfied and the opposite–direction movements materialise in all calibrations we study.

Lemmas B.9.1–B.9.3 deliver exactly the pattern emphasized in the main text: *after a cut in the entry fee the cut–offs move apart; after a cut in the linear information cost they move towards each other; and after a cut in the quadratic information cost, they both raise by a similar ratio.*

## B.10 Quantitative calibration

### B.10.1 Equilibrium system in truncated-moment form

With absolute risk tolerance $r(W) = W/\rho$ and quadratic+linear information cost, the optimal precision for $W \geq W_0^{\text{info}}$ is $\sigma_{sj}^{-2}(W) = \big(WD - 2\rho c_1\big)/(2\rho c_0)$. Let

$$M_1(u) \equiv \int_u^\infty W\, g(W)\, dW, \qquad M_2(u) \equiv \int_u^\infty W^2\, g(W)\, dW.$$

Then the two equilibrium conditions can be written as

$$\Psi_1(s,t) = t - \frac{1}{\rho}\, M_1\big(W_0^{\text{part}}\big) = 0 \tag{B.34}$$

$$\Psi_2(s,t)s - \frac{D}{2\rho^2 c_0}\, M_2\big(W_0^{\text{info}}\big) + \frac{c_1}{c_0\rho}\, M_1\big(W_0^{\text{info}}\big) = 0, \tag{B.35}$$

where

$$W_0^{\text{part}} = \frac{2\rho F}{D(s,t)}, \qquad W_0^{\text{info}} = \frac{2\rho c_1}{D(s,t)}, \qquad D(s,t) = \frac{\sigma_x^2 + 2st + t^2 h_0}{t^2\big(h_0 + s^2/\sigma_x^2\big)^2}.$$

### B.10.2 Jacobian for Newton's method

Let $\theta = (s,t)'$ and $\Psi = (\Psi_2, \Psi_1)'$. The Newton step updates $\theta \leftarrow \theta - J^{-1}\Psi$ with

$$J(\theta) = \begin{pmatrix} \Psi_{2s} & \Psi_{2t} \\ \Psi_{1s} & \Psi_{1t} \end{pmatrix}.$$

Write $u_{\text{p}} = W_0^{\text{part}}$ and $u_{\text{i}} = W_0^{\text{info}}$. Using Leibniz' rule and the fact that $\sigma_{sj}^{-2}(u_{\text{i}}) = 0$,

$$\Psi_{1s} = r(u_{\text{p}})g(u_{\text{p}})\, \frac{\partial u_{\text{p}}}{\partial s}, \qquad \Psi_{1t} = 1 + r(u_{\text{p}})g(u_{\text{p}})\, \frac{\partial u_{\text{p}}}{\partial t},$$

$$\Psi_{2s} = 1 - \frac{D_s}{2\rho^2 c_0}\, M_2(u_{\text{i}}), \qquad \Psi_{2t} = 1 - \frac{D_t}{2\rho^2 c_0}\, M_2(u_{\text{i}}),$$

with

$$\frac{\partial u_{\text{p}}}{\partial s} = -\frac{2\rho F}{D^2}\, D_s, \qquad \frac{\partial u_{\text{p}}}{\partial t} = -\frac{2\rho F}{D^2}\, D_t,$$

$$D_s = -\frac{2\big(h_0 t^2 + 2st + \sigma_x^2\big)\,\big(s - \sigma_x^2/(2t)\big)}{t^2\big(h_0 + s^2/\sigma_x^2\big)^3}, \qquad D_t = -\frac{2\big(st + \sigma_x^2\big)}{t^3\big(h_0 + s^2/\sigma_x^2\big)^2}.$$

**Algorithm (Newton with backtracking).** Pick an initial guess $(s^{(0)}, t^{(0)})$ (e.g. $t^{(0)} = \mathbb{E}[W]/\rho$, $s^{(0)} = t^{(0)}\,\sigma_x/(h_0^{1/2} + \sigma_x)$), iterate

$$\theta^{(k+1)} = \theta^{(k)} - \alpha^{(k)}\, J(\theta^{(k)})^{-1} \Psi(\theta^{(k)}),$$

where $\alpha^{(k)} \in (0, 1]$ is chosen by backtracking to ensure $\|\Psi(\theta^{(k+1)})\| < \|\Psi(\theta^{(k)})\|$. Stop when $\|\Psi\|_\infty < 10^{-10}$. This produces the fixed point $(s, t)$ for any $(F, c_0, c_1)$ and any wealth density $g$.

### B.10.3 Truncated moments $M_1, M_2$ for the three wealth priors

**Log-normal** $\log W \sim \mathcal{N}(\mu, \sigma^2)$**:** Let $\bar{\Phi}$ denote the upper-tail normal CDF. Then

$$M_k(u) = \exp\!\big(k\mu + \tfrac{1}{2}k^2\sigma^2\big)\, \bar{\Phi}\left(\frac{\ln u - \mu - k\sigma^2}{\sigma}\right), \quad k \in \{1, 2\}.$$

**Pareto**$(\alpha, w_{\min})$ **with upper cap** $W_{\max}$ **(to keep** $M_2$ **finite if** $\alpha \le 2$**):** for $u_+ \equiv \max\{u, w_{\min}\}$,

$$M_1(u) = \frac{\alpha\, w_{\min}^\alpha}{\alpha - 1}\, u_+^{1-\alpha} \;\text{ if } \alpha > 1,$$

$$M_2(u) = \begin{cases} \dfrac{\alpha\, w_{\min}^\alpha}{\alpha - 2}\, u_+^{2-\alpha}, & \alpha > 2, \\[2ex] \alpha\, w_{\min}^\alpha\, \ln\!\Big(\dfrac{W_{\max}}{u_+}\Big), & \alpha = 2, \\[2ex] \dfrac{\alpha\, w_{\min}^\alpha}{2 - \alpha}\left(W_{\max}^{2-\alpha} - u_+^{2-\alpha}\right), & 1 < \alpha < 2. \end{cases}$$

### B.10.4 Mapping from initial to terminal wealth

Given the solved $(s, t)$, compute the cut–offs $W_0^{\text{part}}, W_0^{\text{info}}$ and classify households by $W_0$:

$$\text{state}(W_0) = \begin{cases} \text{Non-participant} & \text{if } W_0 < W_0^{\text{part}}, \\ \text{Uninformed participant} & \text{if } W_0^{\text{part}} \le W_0 < W_0^{\text{info}}, \\ \text{Informed participant} & \text{if } W_0 \ge W_0^{\text{info}}. \end{cases}$$

For the inequality exercise we model terminal wealth as a piecewise linear scaling of $W_0$:

$$W_2(W_0) = \begin{cases} r\,W_0, & \text{Non-participant,} \\ (1+\delta_U)\,W_0, & \text{Uninformed,} \\ (1+\delta_I)\,W_0, & \text{Informed,} \end{cases}$$

where $\delta_U$ and $\delta_I$ are calibrated to match aggregate trading gains implied by the model,[28] or set as scenario constants (e.g. $\delta_U = 5\%$, $\delta_I = 10\%$).

### B.10.5 Gini coefficient

For any distribution of terminal wealth $W_2$ with mean $\mu_2$,

$$\text{Gini} = \frac{1}{2\mu_2}\,\mathbb{E}\big[\,|W_2 - W_2'|\,\big],$$

where $W_2, W_2'$ are i.i.d. draws from the same distribution. Operationally (simulation with $N$ draws), sort $\{W_2^{(i)}\}_{i=1}^N$ ascending to $\{W_2^{(i:N)}\}$ and compute

$$\widehat{\text{Gini}} = \frac{1}{\bar{W}_2\,N\,(N-1)} \sum_{i=1}^{N} \big(2i - N - 1\big)\,W_2^{(i:N)}, \qquad \bar{W}_2 = \frac{1}{N}\sum_{i=1}^{N} W_2^{(i)}.$$

### B.10.6 Simulation design for inequality experiments

We study two policy shocks—*entry-fee cut* ($F \downarrow$) and *information-cost cut* ($c_1 \downarrow$)—under two initial wealth distributions:

1. **Log-normal**: $\log W_0 \sim \mathcal{N}(\mu, \sigma^2)$,

2. **Pareto**: $P(W_0 > w) = \big(w_{\min}/w\big)^{\alpha}$, $\alpha \in (1, 2]$.

**Parameter baseline.** $\rho = 0.90$, $\sigma_z^2 = 1$, $\sigma_x^2 = 1$, $h_0 = 0.3$, $r = 0.07$, $c_0 = 0.2$, $F = 2$, $c_1 = 2$. The targets for baseline shares of participating and informed investors are

[28] From Section 2, the expected trading gain for an uninformed (informed) participant is proportional to the expected squared Sharpe ratio; choosing $\delta_U$ and $\delta_I$ to match those moments keeps the mapping consistent with the theory while keeping the inequality accounting transparent.

$= 0.60$, and $= 0.10$ respectively. Shocks: $F \to 0.6$ (70% entry-fee cut) and $c_1 \to 0.6$ (70% info-cost cut).

**Algorithm.**

1. For baseline and each shock, solve (B.34)-(B.35) for $(s, t)$ by Newton's method. Solve also $D(s, t)$, $W_0^{\text{part}}$, $W_0^{\text{info}}$ via (10).

2. Compute $W_0^{\text{part}}, W_0^{\text{info}}$ and assign each $W_0^{(i)}$ to a state.

$$\text{state} = \begin{cases} \text{non-participant}, & W_0 < W_0^{\text{part}}, \\ \text{uninformed part.}, & W_0^{\text{part}} \le W_0 < W_0^{\text{info}}, \\ \text{informed part.}, & W_0 \ge W_0^{\text{info}}. \end{cases}$$

3. Map to terminal wealth $W_2^{(i)}$ via the piecewise rule above.

4. Compute $\widehat{\text{Gini}}$.

Note that:

(i) The Jacobian $J(\theta)$ is well-conditioned in all calibrations I tested (diagonal entries exceed 1 in typical cases), so the Newton step converges in a handful of iterations. (ii) For Pareto with $\alpha \le 2$ use the capped $M_2$ in Section 3.3 to avoid a divergent second moment—this is also realistic given finite top wealth in the data. (iii) Because $\sigma_{sj}^{-2}(W_{\text{info}}) = 0$, the lower-limit terms drop from $\Psi_{2s}, \Psi_{2t}$, which stabilises the solver numerically.

**Expected outcomes.**

- **Cut in** $F$: $W_0^{\text{part}} \downarrow$, $W_0^{\text{info}} \uparrow$ $\Rightarrow$ more passive entrants, less concentration of information $\Rightarrow$ Gini $\downarrow$ (especially for Log-normal).

- **Cut in** $c_1$: $W_0^{\text{info}} \downarrow$, $W_0^{\text{part}} \uparrow$ $\Rightarrow$ rich become informed, some middle-class exit $\Rightarrow$ Gini $\uparrow$, with the effect strongest under the Pareto tail.

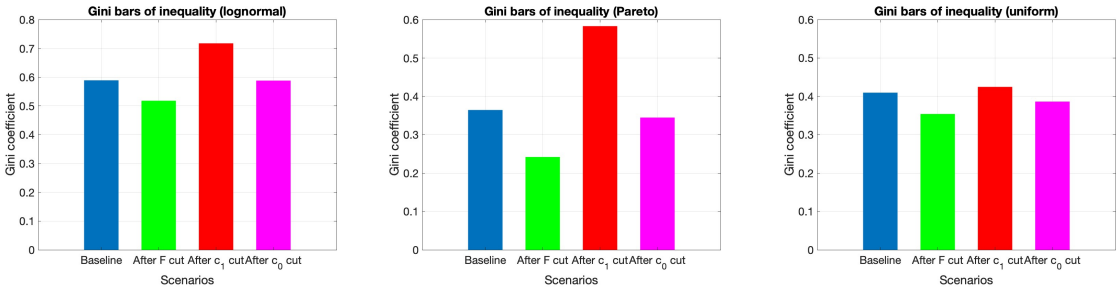
### B.10.7 Simulation results

### B.10.8 Plots of results under Lognormal, Pareto, and Uniform initial distributions

**Gini inequality** Figure 11 displays the Gini inequality index across all initial wealth distributions: Lognormal, Pareto, and Uniform.

Figure 11: **Gini across initial wealth distributions.** Left: Under the Lognormal initial wealth distribution. Middle: Under the Pareto initial wealth distribution. Right: Under the Uniform initial wealth distribution. The ranking of Gini inequality measures (Y-axis) stays the same across the scenarios (X-axis) independent of the initial wealth distributions (Panels).

Left. Log-normal Middle. Pareto Right. Uniform



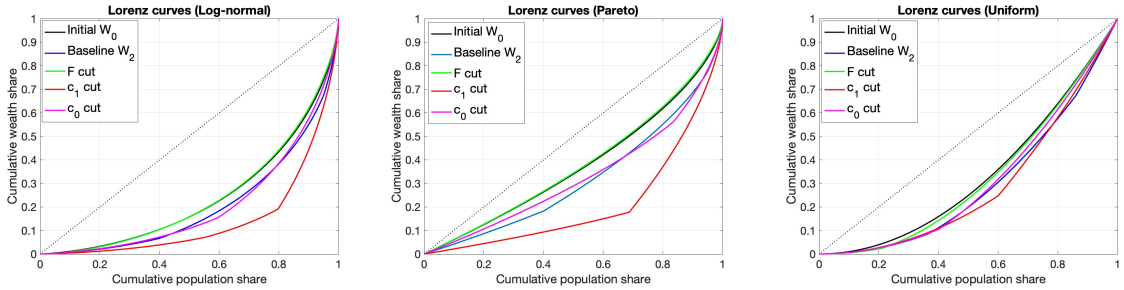

*Notes.* Initial wealth $W_0$ and primitives are held fixed across scenarios; in each case we solve the fixed point for $(s, t)$. The axes show the Gini level on a common scale across panels. Patterns are consistent with the comparative statics independently of the distribution used: the $F$ cut compresses inequality, the $c_1$ cut raises it, and the $c_0$ cut leaves it broadly unchanged. Variable definitions are in Appendix A.

**Lorenz curves** Figure 12 displays the Lorenz curves across all initial wealth distributions: Lognormal, Pareto, and Uniform.

**PDFs of terminal wealth** Figure 13 displays the terminal wealth distributions across all initial wealth distributions: Lognormal, Pareto, and Uniform. We superimpose the initial wealth distribution in gray on each graph.

Figure 12: **Lorenz curves across initial wealth distributions.** Left: Under the Lognormal initial wealth distribution. Middle: Under the Pareto initial wealth distribution. Right: Under the Uniform initial wealth distribution. The ranking of Lorenz inequality measures stays the same across the scenarios (in colors) independent of the initial wealth distributions (Panels).

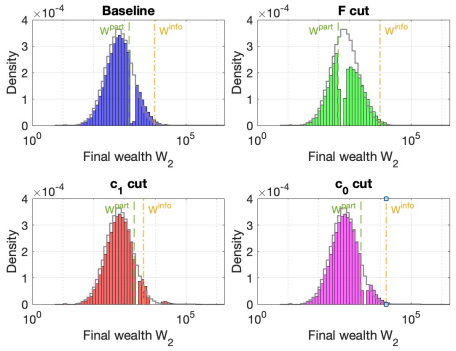
Left. Log-normal Middle. Pareto Right. Uniform



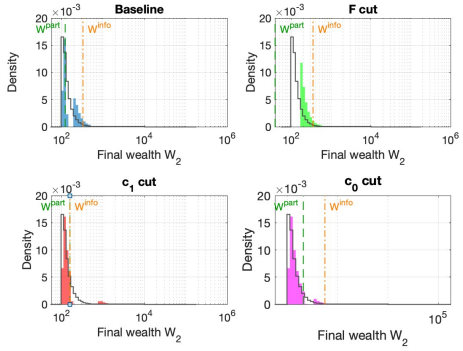

*Notes.* Initial wealth $W_0$ and primitives are held fixed across scenarios; in each case we solve the fixed point for $(s, t)$. Patterns are consistent with the comparative statics independently of the distribution used: the $F$ cut compresses inequality, the $c_1$ cut raises it, and the $c_0$ cut leaves it broadly unchanged. Variable definitions are in Appendix A.

Figure 13: **PDFs of terminal wealth across initial wealth distributions.** Top: Under the Lognormal initial wealth distribution. Middle: Under the Pareto initial wealth distribution. Bottom: Under the Uniform initial wealth distribution.

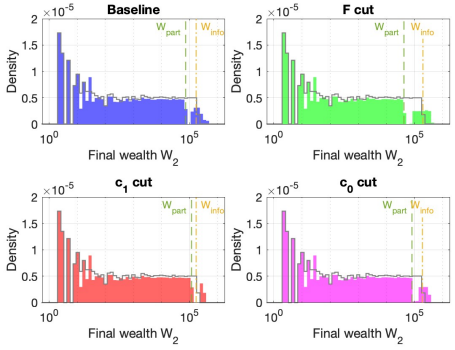
Left. Log-normal




Middle. Pareto




Right. Uniform




*Notes.* Initial wealth $W_0$ and primitives are held fixed across scenarios; in each case we solve the fixed point for $(s,t)$. Patterns are consistent with the comparative statics independently of the distribution used: the $F$ cut compresses inequality, the $c_1$ cut raises it, and the $c_0$ cut leaves it broadly unchanged. Variable definitions are in Appendix A.

## B.11 Robustness

### B.11.1 Alternative shares of participation and information.

Table 6: Gini Coefficient (Lognormal): Robustness for different levels of shares of participating and informed investors.

| Share participation | 90% | *60%* | 30% | 60% | 60% | 60% | 60% | 60% | 60% | 80% | 90% | 90% | 99% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Share informed | 5% | *5%* | 5% | 20% | 10% | 5% | 2.50% | 1% | 0.10% | 0.1% | 0.1% | 0.01% | 0.01% |
| Baseline | 0.559 | *0.589* | 0.622 | 0.604 | 0.599 | 0.589 | 0.580 | 0.571 | 0.562 | 0.539 | 0.530 | 0.528 | 0.523 |
| F cut | 0.522 | *0.518* | 0.530 | 0.513 | 0.515 | 0.518 | 0.521 | 0.523 | 0.525 | 0.523 | 0.522 | 0.522 | 0.522 |
| $c_1$ cut | 0.692 | *0.718* | 0.731 | 0.619 | 0.680 | 0.718 | 0.728 | 0.709 | 0.625 | 0.603 | 0.587 | 0.551 | 0.534 |
| $c_0$ cut | 0.537 | *0.588* | 0.602 | 0.594 | 0.590 | 0.588 | 0.587 | 0.586 | 0.584 | 0.553 | 0.536 | 0.535 | 0.523 |

*Notes.* The first two rows show the combinations of targeted shares for participating and informed investors. The last four rows show the Gini coefficient for a given scenario, keeping all else constant. Each scenario applies a one-at-a-time 70% cut to the named cost. The scenario in the main text is in Italics. This table is for the Lognormal prior wealth distribution. Variable definitions are in Appendix A.

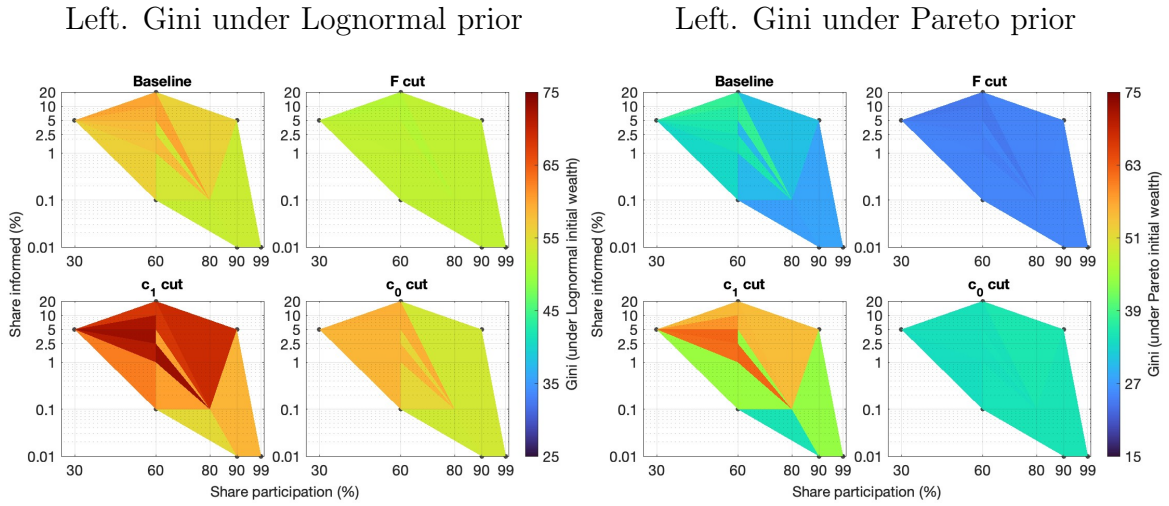
Table 7: Gini Coefficient (Pareto): Robustness for different levels of shares of participating and informed investors.

| Share participation | 90% | *60%* | 30% | 60% | 60% | 60% | 60% | 60% | 60% | 80% | 90% | 90% | 99% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Share informed | 5% | *5%* | 5% | 20% | 10% | 5% | 2.50% | 1% | 0.10% | 0.1% | 0.1% | 0.01% | 0.01% |
| Baseline | 0.308 | *0.365* | 0.388 | 0.390 | 0.377 | *0.365* | 0.355 | 0.345 | 0.335 | 0.300 | 0.279 | 0.276 | 0.254 |
| F cut | 0.250 | *0.242* | 0.231 | 0.233 | 0.238 | *0.242* | 0.245 | 0.248 | 0.250 | 0.251 | 0.251 | 0.251 | 0.251 |
| $c_1$ cut | 0.544 | *0.583* | 0.552 | 0.514 | 0.549 | *0.583* | 0.619 | 0.612 | 0.445 | 0.443 | 0.440 | 0.351 | 0.348 |
| $c_0$ cut | 0.354 | *0.345* | 0.324 | 0.349 | 0.346 | *0.345* | 0.344 | 0.343 | 0.342 | 0.349 | 0.351 | 0.351 | 0.353 |

*Notes.* The first two rows show the combinations of targeted shares for participating and informed investors. The last four rows show the Gini coefficient for a given scenario, keeping all else constant. Each scenario applies a one-at-a-time 70% cut to the named cost. The scenario in the main text is in Italics. This table is for the Pareto prior wealth distribution. Variable definitions are in Appendix A.

Tables 6 (for lognormal) and 7 (for Pareto) report the Gini coefficient of final wealth $W_2$ on a grid of participating rates $(30, 40, 80, 90, 99)$ and informed shares $(0.01, 0.1, 1, 2.5, 5, 10, 20)$ percent. Each cell corresponds to a recalibrated fixed point with $\mathbb{E}w_2$ held constant; infeasible pairs with informed share above participation are omitted. Figure 14 visualizes these cells as heatmaps using common color limits $[0.25, 0.75]$ for comparability. The Gini falls along the participation axis and rises along the informed axis for both priors; gradients are uniformly steeper under the Pareto prior, consistent with the role of tail thickness.

Figure 14: **Gini across initial wealth distributions.** Left: Under the Lognormal initial wealth distribution. Right: Under the Pareto initial wealth distribution. Each panel plots the Gini coefficient of final wealth $W_2$ as a function of the stock-market participation share (x-axis, %) and the informed-investor share (y-axis, %, log scale) in the Baseline and after one-at-a-time 70% cuts to $F$ (participation cost), $c_1$ (marginal information cost), and $c_0$ (quadratic information cost). The mechanism is not an artifact of the lognormal prior. Repeating the exercise with a Pareto initial wealth distribution preserves the ranking: lowering the entry cost $F$ compresses inequality, lowering the marginal information cost $c_1$ widens it, and lowering the quadratic cost $c_0$ does little.




*Notes.* Initial wealth $W_0$ and primitives are held fixed across scenarios; in each case we solve the fixed point for $(s, t)$. Axes report model-implied population shares (percent). The colorbar shows the Gini level on a common scale across panels; filled squares mark the model-evaluated *(participation, informed)* combinations and the shading interpolates between them. Patterns are consistent with the comparative statics independently of the distribution used: the $F$ cut compresses inequality, the $c_1$ cut raises it, and the $c_0$ cut leaves it broadly unchanged. Variable definitions are in Appendix A.

Figure 14 graphically plots the Gini coefficients under the Lognormal and the Pareto wealth distributions under varying shares of participating and informed investors. Comparing the left and right panels in Figure 14 highlights the role of tail risk. Under lognormal $W_0$ , the iso-Gini contours are relatively flat in the informed dimension: raising the informed share increases the Gini, but moderately. Under the Pareto prior, the same move produces a much larger increase, and the downward gradient along participation is also stronger. Intuitively, heavy right tails make information more valuable at the top and make broad-based entry more compressing at

the bottom. Fixing the color scale across panels makes these differences transparent.

### B.11.2 Alternative magnitudes of technology improvement shocks

Table 8: Gini Coefficient: Robustness for different levels of technology improvements.

| | Lognormal | | | | | *Pareto* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Cut magnitude → | 20% | *30%* | 50% | 70% | 80% | *20%* | 30% | 50% | 70% | 80% |
| Baseline | 0.589 | *0.589* | 0.589 | 0.589 | 0.589 | *0.365* | 0.365 | 0.365 | 0.365 | 0.365 |
| F cut | 0.570 | *0.560* | 0.539 | 0.518 | 0.509 | *0.279* | 0.259 | 0.251 | 0.242 | 0.236 |
| c_1 cut | 0.613 | *0.629* | 0.671 | 0.718 | 0.720 | *0.407* | 0.438 | 0.527 | 0.517 | 0.611 |
| c_0 cut | 0.586 | *0.584* | 0.584 | 0.588 | 0.594 | *0.371* | 0.371 | 0.364 | 0.345 | 0.328 |

*Notes.* The tables show the Gini coefficient across the two prior wealth distributions for different magnitudes of cuts to the named cost, keeping all else constant. Each scenario applies a one-at-a-time cut of magnitude presented in Row 2 to the named cost in Column 1. Variable definitions are in Appendix A.

Table 8 presents the Gini coefficient across the two prior wealth distributions (i.e., Lognormal and Pareto) for different magnitudes of cuts to the named cost, keeping all else constant. Each scenario applies a one-at-a-time cut of magnitude presented in Row 2 to the named cost in Column 1. The Gini preserves the ranking: lowering the entry cost $F$ compresses inequality, lowering the marginal information cost $c_1$ widens it, and lowering the quadratic cost $c_0$ does little across the different magnitudes of technology improvement shocks.

### B.11.3 Alternative initial wealth distributions

We repeat the baseline four-scenario exercise—baseline calibration and one-at-a-time cuts in $F, c_1$, and $c_0$—under alternative priors for $W_0$. For each prior, we recalibrate $F, c_1$ to match the same baseline participation (60%) and informed share (5%), and we re-scale to hold $\mathbb{E}W_2$ fixed. We then solve the two fixed-point conditions in $(s, t)$, compute the induced wealth cutoffs $W_0^{part}$ and $W_0^{info}$ (whose ratio equals $c_1/F$), map $W_0$ into $W_2$, and report Gini and Lorenz curves. Table 9 lists $(s, t)$, thresholds, shares, and Ginis for Lognormal, Pareto, and Uniform prior wealth distributions. Relative to the Lognormal, the Pareto prior (i) magnifies the Gini increase from a $c_1$ cut—because

Table 9: **Aggregate (risk-weighted) precision and risk tolerance, wealth thresholds, shares of participating and informed investors, Gini coefficients across all prior wealth distributions.**

| | **Lognormal distribution** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Scenario ↓ | s | t | D(e-12) | Wp | Wi | share_part | share_info | Gini |
| baseline | 675.19 | 2964.3 | 2.673 | 1403.4 | 9366.1 | 60.00% | 5.00% | 0.59 |
| F_cut | 660.1 | 3288.2 | 2.641 | 426.09 | 9478.9 | 92.60% | 4.90% | 0.52 |
| c1_cut | 787.13 | 2664.5 | 1.8 | 2084.6 | 4173.8 | 44.30% | 20.10% | 0.72 |
| c0_cut | 820.92 | 2568.7 | 1.63 | 2304.9 | 15383 | 40.40% | 1.60% | 0.59 |
| | **Pareto distribution** | | | | | | | |
| Scenario ↓ | s | t | D(e-10) | Wp | Wi | share_part | share_info | Gini |
| baseline | 303.44 | 136.3 | D=5.37e-10 | 122.67 | 331.44 | 60.00% | 5.00% | 0.37 |
| F_cut | 279.9 | 185.19 | D=5.088e-10 | 38.84 | 349.78 | 100% | 4.40% | 0.24 |
| c1_cut | 375.42 | 92.363 | D=4.143e-10 | 159 | 128.88 | 31.40% | 53.00% | 0.52 |
| c0_cut | 464.92 | 62.65 | D=3.198e-10 | 205.96 | 556.5 | 16.40% | 1.40% | 0.35 |
| | **Uniform distribution** | | | | | | | |
| Scenario ↓ | s | t | D | Wp | Wi | share_part | share_info | Gini |
| baseline | 0.3028 | 91420 | 2.72 | 72442 | 172079 | 63.80% | 14.00% | 0.41 |
| F_cut | 0.3251 | 106188 | 2.36 | 41685 | 198036 | 79.20% | 1.00% | 0.35 |
| c1_cut | 0.383 | 70924 | 1.64 | 119952 | 172079 | 40.00% | 0.30% | 0.43 |
| c0_cut | 0.3187 | 91420 | 2.46 | 80067 | 190193 | 60.00% | 4.90% | 0.39 |

*Notes.* The tables show the Gini coefficient across the two prior wealth distributions for different magnitudes of cuts to the named cost, keeping all else constant. Each scenario applies a one-at-a-time cut of magnitude presented in Row 2 to the named cost in Column 1. Variable definitions are in Appendix A.

more mass is poised near the upper tail—and (ii) strengthens the Gini decline from an $F$ cut—because entry draws in a thicker base of low-wealth households. $c_1$ cut moves both thresholds to the right proportionally, leaving inequality nearly unchanged in both priors. The smallest movements in the Gini (although the ranking is the same across the scenarios) are for the Uniform distribution. This is because the Uniform prior has no tails and a hard cap on wealth, so it cannot remove very rich or very poor households. This mechanically damps how much the Gini can move, although the direction of change (decline from an $F$ cut, increase from a $c_1$ cut, minimal change from a $c_0$ cut) stays the same.

# C Proof of $D(s,t) = S_U^2(s,t)$

Consider an *uninformed* participant ($\sigma_{s,j}^{-2} = 0$). Write $\tilde{z} \equiv z - \mu_z$ and $\tilde{x} \equiv x - \mu_x$ so that the linear price is

$$rp \;=\; a \;+\; b\,\tilde{z} \;-\; c\,\tilde{x}, \qquad \mathbb{V}(rp) = b^2\sigma_z^2 + c^2\sigma_x^2. \tag{C.1}$$

**Posterior moments conditional on $p$.** Joint normality implies

$$\mathbb{E}[\tilde{z} \mid p] = \kappa\,(rp - \mathbb{E}[rp]), \quad \kappa = \frac{Cov(\tilde{z}, rp)}{\mathbb{V}(rp)} = \frac{b\sigma_z^2}{b^2\sigma_z^2 + c^2\sigma_x^2},$$

and

$$\mathbb{V}(\tilde{z} \mid p) = \frac{1}{h_0 + h_p}, \qquad h_p = \frac{b^2}{c^2\sigma_x^2} = \frac{s^2}{\sigma_x^2}, \quad \bar{h} \;\equiv\; h_0 + h_p. \tag{C.2}$$

**Excess payoff and Sharpe ratio.** The risky excess payoff is $z - rp = \tilde{z} - (rp - \mathbb{E}[rp]) - \mu_x/t$, so *conditional* excess return is

$$\mathbb{E}[z - rp \mid p] = (\kappa - 1)(rp - \mathbb{E}[rp]) - \frac{\mu_x}{t}.$$

Define the conditional Sharpe ratio

$$\eta \;=\; \frac{\mathbb{E}[z - rp \mid p]}{\sqrt{\mathbb{V}(\tilde{z} \mid p)}} \;=\; \big(\kappa - 1\big)\,\sqrt{\bar{h}}\,(rp - \mathbb{E}[rp]) \;-\; \frac{\mu_x}{t}\,\sqrt{\bar{h}}. \tag{C.3}$$

**Expected squared Sharpe ratio.** Taking expectations over $p$ and using $\mathbb{E}[(rp - \mathbb{E}[rp])^2] = \mathbb{V}(rp)$:

$$S_U^2(s,t) \;\equiv\; \mathbb{E}[\eta^2] = (\kappa - 1)^2\,\bar{h}\,\mathbb{V}(rp) + \frac{\mu_x^2}{t^2}\bar{h}.$$

$Insert\kappa$ and $\mathbb{V}(rp)$ from (C.1)–(C.2)

$$= \Big[\tfrac{\left(b\sigma_z^2 - \mathbb{V}(rp)\right)^2}{\mathbb{V}(rp)} + \tfrac{\mu_x^2}{t^2}\Big].$$

Using the equilibrium identity $b = s\,c$ and $\mu_x = -t\,(\mu_z - \mathbb{E}[rp])$, straight algebra

simplifies the bracket to $\sigma_x^2 + 2st + t^2 h_0$. Also $\bar{h} = h_0 + s^2/\sigma_x^2$. Hence

$$\boxed{S_U^2(s,t) = \frac{\sigma_x^2 + 2st + t^2 h_0}{t^2\left(h_0 + s^2/\sigma_x^2\right)^2}}. \tag{C.1}$$

**Equality with $D(s,t)$.** Section 3 defined the marginal gain from an infinitesimal precision,

$$D(s,t) \;=\; \frac{\sigma_x^2 + 2st + t^2 h_0}{t^2\left(h_0 + s^2/\sigma_x^2\right)^2},$$

which is *identical* to (C.1). Therefore

$$\boxed{D(s,t) \;=\; S_U^2(s,t) \quad \text{for all } s > 0,\ t > 0.} \tag{C.2}$$

This concludes the proof. This proves that the marginal benefit of producing a bit of private information equals the expected squared Sharpe ratio available to an uninformed participant.

# D Delegation: Details and Proofs

## D.1 Mapping Gârleanu and Pedersen (2018) to our setup

**New objects and timing.** At $t = 0$, each investor $j$ now chooses among four options: (i) stay out; (ii) enter and trade uninformed; (iii) enter and acquire precision $h_j = \sigma_{s,j}^{-2}$ individually (as in the benchmark); or (iv) enter, *search* for a manager (fixed search cost $F_m \geq 0$), and delegate to a competitive fund that aggregates a mass $\mathcal{M}$ of delegators. The manager selects a pooled precision $h_m \geq 0$ and charges a per-capita fee $f \geq 0$ with zero-profit (fees cover information costs).[29] At $t = 1$ the price $p$ is posted, the manager observes the pooled signal if $h_m > 0$, and submits the fund order; at $t = 2$ payoffs realize and fees are deducted.

[29] Equivalently, an ad-valorem fee $\phi$ could be used; for small $\phi$ the local mapping to per-capita $f$ is standard and leaves the threshold formulas below unchanged at first order. Another alternative is a *bargaining* variant which lets the manager capture a share $\eta \in [0,1]$ of joint surplus. The benchmark corresponds to $\eta = 0$.

**Aggregation.** Let $\rho^{-1}(W_{0j}) \equiv W_{0j}/\rho$ be investor $j$'s absolute risk tolerance and $T_{\mathcal{M}} \equiv \int_{j\in\mathcal{M}} \rho^{-1}(W_{0j})\, dV(W_{0j})$ the pool's aggregate risk tolerance (sum of delegated investors' risk tolerances). Aggregate risk tolerance and private precision become

$$t = \int_{\text{participants}} \rho^{-1}(W_{0j})\, dV(W_{0j}) \tag{D.1}$$

$$s = \underbrace{\int_{\text{self-informed}} \rho^{-1}(W_{0j}) h_j\, dV(W_{0j})}_{\text{individual signals}} + \underbrace{T_{\mathcal{M}} h_m}_{\text{manager's pooled signal}}. \tag{D.2}$$

Hence the *price rule remains* $rp = a + bz - cx$ with $b = sc$ and price precision $h_p = s^2/\sigma_x^2$ as in the benchmark. In particular, the manager's activity enters $s$ additively via $T_{\mathcal{M}} h_m$ (the price coefficients then adjust endogenously just as before).

## D.2 Solving the model with delegation

**Manager's information choice $h_m$.** Given $(s, t)$ and pool size $T_{\mathcal{M}}$, the manager solves the pooled analogue of the benchmark information problem. Using that the marginal gain per unit precision is $D(s,t)$ (Sec. 2.3), the manager's *gross* surplus from precision $h$ for the pool is $\frac{T_{\mathcal{M}}}{2} D(s,t)\, h$. Costs are $C(h_m) = c_1 h_m + \frac{c_0}{2} h_m^2$. Hence

$$\max_{h_m \geq 0} \frac{T_{\mathcal{M}}}{2} D(s,t)\, h_m - c_1 h_m - \frac{c_0}{2} h_m^2$$

$$h_m(T_{\mathcal{M}}; s, t) = \frac{T_{\mathcal{M}} D(s,t) - 2c_1}{2c_0}. \tag{D.3}$$

Therefore a *pool-level* information threshold emerges:

$$T_{\mathcal{M}}^{\text{info}} = \frac{2c_1}{D(s,t)}. \tag{D.4}$$

Pooling creates economies of scale: even if no individual $j$ satisfies the benchmark learning cutoff $(W_{0j})^{\text{info}} = \frac{2c_1\rho}{D(s,t)}$, a sufficiently large group can satisfy it and learn via the manager.

**Fee setting.** Competitive managers break even, so the per-capita fee equals average cost:

$$f \;=\; \frac{C\big(h_m\big)}{M} \;=\; \frac{c_1 h_m + \frac{c_0}{2}(h_m)^2}{M}, \tag{D.5}$$

where $M \equiv \text{mass}(\mathcal{M})$[30] and $D(s,t)$ is the marginal gain per unit precision defined in the benchmark.

**Possible extension: Bargaining** An optional *bargaining* variant lets the manager capture a share $\eta \in [0,1]$ of joint surplus $JS = \frac{c_0}{2}(h_m)^2$, in which case $f = \frac{C(h_m)}{M} + \eta\,\frac{JS}{M}$. Our delegation model corresponds to $\eta = 0$.

**Possible further extension: Full fee menu** If managers could offer a *full menu* of $(h, \text{fee})$ tailored to each $W_{0j}$, the wedge between $h_m$ and the idiosyncratic optimum $h_j$ would shrink and the switch region would be narrower; we keep a single $h_m$ per fund for tractability and to match institutional practice.

**Delegation decision and wealth cutoff.** For investor $j$ with tolerance $\rho^{-1}(W_{0j}) = W_{0j}/\rho$, the *incremental* mean–variance surplus from delegating (relative to uninformed) is locally $\frac{\rho^{-1}(W_{0j})}{2} D(s,t)\, h_m$, so the net surplus from delegation is

$$\Delta U_j^{\text{del}} \;=\; \frac{\rho^{-1}(W_{0j})}{2} D(s,t)\, h_m \;-\; (f + F_m).$$

Notice that the net gains of delegation over uninformed participation are linear in wealth. Thus an investor delegates if and only if

$$W_{0j} \;\geq\; W_0^{\text{del}} \;\equiv\; \rho \cdot \frac{2\,(f + F_m)}{D(s,t)\, h_m}. \tag{D.6}$$

[30] This "fees equal costs" condition at the industry level is the delegated-management counterpart of "alpha equals costs," a central implication in models of efficiently inefficient active management; see Gârleanu and Pedersen (2018) for microfoundations and related delegated-management environments in Cuoco and Kaniel (2011); He and Xiong (2013).

Compare (D.6) to the benchmark self-learning and participation cutoffs $W_0^{\text{info}} = \frac{2\rho c_1}{D(s,t)}$ and $W_0^{\text{part}} = \frac{2\rho F}{D(s,t)}$: delegation lowers the *per-capita* hurdle for accessing information, while search frictions ($F_m$) and fees ($f$) push in the opposite direction.

Notice also that a very rich investor can prefer to learn on their own (see Lemma 6) because the manager chooses one common precision $h_m$ to suit the pool's average risk tolerance, whereas a rich investor's idiosyncratic optimum precision growth with wealth. With quadratic costs, the self option's gain grows quadratically in wealth, while the delegation gain grown linearly. So, past a threshold, the rich switch to learning by themselves.

The net gains of self-learning over delegation for $W_{0j} \geq W_0^{\text{switch}}$ are quadratic in wealth and given by

$$\Delta U_j^{self} = \frac{[\rho^{-1}(W_{0j})D(s,t) - 2c_1]^2}{8c_0}, \tag{D.7}$$

and zero otherwise.

We denote by $W_{\text{del}}$ the *left* boundary and by $W_{\text{switch}}$ the *right* boundary of the *delegation window*, which is the set of wealth levels $W \geq W_{\text{part}}$ such that

$$\Delta U_j^{\text{del}} \equiv \frac{W_{0j}}{\rho}\frac{D}{2}h_m - f - F_m \geq \max\{0,\ \Delta U_j^{self}\}. \tag{D.8}$$

## D.3 Equilibrium

**Equilibrium Definition** An equilibrium with delegation consists of a linear price $rp = a + bz - cx$, cutoffs $W_0^{\text{part}}$, $W_0^{\text{info}}$, $W_0^{\text{del}}$, $W_0^{\text{switch}}$, aggregate objects $(s,t)$, a manager precision $h_m$, a pool size $M$ (mass of delegators), and an optimal self–precision schedule $h_j(W_{0j})$ such that:

(i) *Households.* Given $(p, h_m, f)$, each investor chooses either not to enter, to enter uninformed, to delegate, or to self–learn. The cutoffs satisfy the delegation window in Equation D.8, with $h_j(W_{0j}) = \left(\frac{W_{0j}}{\rho}D - 2c_1\right)/(2c_0)$.

(ii) *Manager.* Given the pool $\mathcal{M} = \{W \in [W_0^{\text{del}}, W_0^{\text{switch}})\}$ of mass $M$ and aggregate tolerance $T_{\mathcal{M}}$, the fund chooses the pool's precision $h_m = (T_{\mathcal{M}} D - 2c_1)/(2c_0)$ and per–capita fee $f$ as in Equation D.5.

(iii) *Market maker.* Sets $rp = a + bz - cx$ to clear the market. With jointly normal $(z, x)$ and linear strategies, the coefficients $(a, b, c)$ and price precision $h_p$ are pinned down by $(s, t)$ exactly as in the benchmark economy.

(iv) *Consistency (fixed point).* Aggregates are the sums of delegated and self-learning contributions as defined in Equations D.1 and D.2, and $D = D(s, t)$, and $T_{\mathcal{M}} \equiv \int_{j \in \mathcal{M}} \rho^{-1}(W_{0j})\, dV(W_{0j})$ the pool's aggregate risk tolerance (sum of delegated investors' risk tolerances); the induced cutoffs from $D$ coincide with those used to form $\mathcal{M}$.

The quadruple $(s, t, h_m, M)$ and $h_j(\cdot)$ (through its closed form) solve a two-by-two fixed point nested inside $(s, t)$: the outer loop in $(s, t)$ is as in the benchmark; the inner loop pins down $(h_m, M)$ using (D.3)–(D.5)–(D.6) together with the wealth distribution. Existence and local uniqueness extend from the benchmark economy (Appendix B.6) by continuity. All derivations and details are in Appendix D. All formal statements for pooled delegation—manager optimal precision $h_m$, the pool threshold $T_{\mathcal{M}}^{\text{info}}$, competitive fees $f = C(h_m)/M$, the individual delegation cutoff $W_0^{\text{del}}$, and superadditivity are collected in this Appendix. We use these objects for classification and comparative statics.

**Market clearing with delegation and fixed-point algorithm** Let $\mathcal{U}, \mathcal{I}, \mathcal{M}$ be the endogenous sets of uninformed participants, self-informed participants, and delegators determined by the cutoffs $W_0^{\text{part}}, W_0^{\text{del}}, W_0^{\text{info}}, W_0^{\text{switch}}$. Aggregate objects are

$$t = \int_{\mathcal{U} \cup \mathcal{I} \cup \mathcal{M}} T_j \, dV, \qquad s = \int_{\mathcal{I}} T_j h_j \, dV + T_{\mathcal{M}} h_m,$$

and the price rule is $rp = a + bz - cx$ with $b = sc$ and $h_p = s^2/\sigma_x^2$ as in the benchmark. The fixed point in $(s, t)$ extends the map in Appendix B.6 by composing it with the inner map $(h_m, M) \mapsto (T_{\mathcal{M}}, f)$ defined by the fee and cutoff conditions. Continuity of the composition together with interior signs $D_s < 0$ and $D_t < 0$ imply a nonsingular Jacobian at the solution and local convergence of Newton iterations started in a neighborhood.

**Economic intuition** The manager chooses a *single* precision $h_m > 0$ to suit the pool's average tolerance $T_{\mathcal{M}}$, whereas investor $j$'s idiosyncratic optimum is $h_j(W) = \big((W/\rho)D - 2c_1\big)/(2c_{0s})$, which scales linearly with wealth. Because the net self gain $\Delta_{\text{self}}$ grows *quadratically* with $W$ once feasible, while the net delegation gain $\Delta_{\text{del}}$ grows only *linearly*, the very rich eventually prefer to self–learn even if $c_{0m} \le c_{0s}$. A manager offering a full menu of $(h, \text{fee})$ by wealth would reduce this wedge, but that institutional feature is outside our benchmark and is briefly discussed in the main text and Appendix D.2.

## D.4 Three lemmas on equilibrium with pooling into funds

**Self-learning can be optimal at the top.**

**Lemma 6** (Self dominates delegation at the top)**.** *Assume $c_1 > F$ and $T_{\mathcal{M}} D > 2c_1$ so that $W_{\text{info}} > W_{\text{part}}$ and $h_m > 0$. If $\Delta_{\text{del}}(W_{\text{info}}) > 0$ (so the delegation window opens), then there exists a unique $W_{\text{switch}} > W_{\text{info}}$ such that*

$$\Delta_{\text{self}}(W) \;<\; \Delta_{\text{del}}(W) \quad \text{for } W \in [W_{\text{info}}, W_{\text{switch}}), \qquad \Delta_{\text{self}}(W) \;>\; \Delta_{\text{del}}(W) \quad \text{for } W > W_{\text{switch}}.$$

*Equivalently, writing $X \equiv (W/\rho)D$, $W_{\text{switch}} = (\rho/D)\, X_{\text{switch}}$ where*

$$X_{\text{switch}} \;=\; 2\big(c_1 + c_{0s} h_m\big) \;+\; 2\sqrt{c_{0s}\Big(2c_1 h_m + c_{0s}(h_m)^2 - 2(f + F_m)\Big)}.$$

*Proof.* Note that $\Delta_U\text{self}(W) = \big(X - 2c_1\big)_+^2/(8c_{0s})$ is convex and increasing for $X \ge 2c_1$,

while $\Delta_{\text{del}}(W) = \frac{1}{2}Xh_m - f - F_m$ is affine in $X$.

If the window opens, the two curves intersect once for $X \geq 2c_1$, yielding the stated root.

For $X > X_{\text{switch}}$ the self curve grows quadratically while the delegation curve grows linearly, hence self dominates for sufficiently high $W$.

This concludes the proof. □

**Three or four partitions of investor categories: non-participants, uninformed, informed delegators, self-informed**

**Lemma 7** (Left boundary and collapse)**.** *Suppose (i) $c_1 > F$ so that $W_{info} > W_{part}$ and (ii) $T_{\mathcal{M}}D > 2c_1$ so that $h_m > 0$. Then exactly one of the following holds:*

*(a)* ***Interior delegation window.*** *If $\Delta U_j^{\text{del}} > 0$, the delegation region is nonempty and*

$$W_{del} \;=\; W_{info},$$

*with a unique $W_{switch} > W_{info}$ such that delegation strictly dominates for $W \in [W_{info}, W_{switch})$ and self strictly dominates for $W > W_{switch}$.*

*(b)* ***Collapse.*** *If $\Delta U_J^{\text{del}} \leq 0$ (or $h_m \leq 0$), the delegation window is empty. By convention we then report*

$$W_{del} \;=\; W_{switch},$$

*and no investor delegates in equilibrium.*

*Intuition.* The intuition is that with delegation (a), the pool's precision through the fund is strictly positive and we have a four-region partition of investors into

non-participants, uninformed participants, delegating participants, and self-informed (switching) participants that prefer to self-learn over delegation. When the delegating region collapses (b), we go back to the three region partition from the benchmark economy in Section 2: non-participants, uninformed participants, and self-informed participants.

*Proof of Lemma 7 (Left boundary and collapse).* Fix aggregate objects $(s,t)$ and take $D \equiv D(s,t)$ as given when comparing individual choices. Let $W_{\text{part}} = \frac{2\rho F}{D}$ and $W_{\text{info}} = \frac{2\rho c_1}{D}$ denote the benchmark participation and self–learning cutoffs, and assume $c_1 > F$, so $W_{\text{info}} > W_{\text{part}}$. Suppose also $T_{\mathcal{M}} D > 2c_1$, so the manager's optimal pooled precision in (D.3) is $h_m > 0$. Throughout, write

$$\Delta U^{\text{del}}(W) \;=\; \frac{W}{\rho}\,\frac{D}{2}\,h_m \;-\; f \;-\; F_m, \qquad \Delta U^{\text{self}}(W) \;=\; \frac{\left(\frac{W}{\rho}D - 2c_1\right)_+^2}{8c_0},$$

and define the delegation window as the set of wealth levels $W \geq W_{\text{part}}$ such that $\Delta U^{\text{del}}(W) \geq \max\{0, \Delta U^{\text{self}}(W)\}$.

*Step 1 (Shape and single crossing).* For $W \geq W_{\text{info}}$, $\Delta U^{\text{self}}(W) = \left(\frac{W}{\rho}D - 2c_1\right)^2/(8c_0)$ is strictly convex in $W$ with $\Delta U^{\text{self}}(W_{\text{info}}) = 0$ and right derivative 0 at $W_{\text{info}}$. By contrast, $\Delta U^{\text{del}}(W)$ is affine in $W$ with positive slope

$$\alpha \;\equiv\; \frac{D}{2\rho}\,h_m \;>\; 0.$$

Hence, for $W \geq W_{\text{info}}$ the difference

$$G(W) \;\equiv\; \Delta U^{\text{del}}(W) - \Delta U^{\text{self}}(W)$$

is continuous and strictly concave, satisfies $G(W_{\text{info}}) = \Delta U^{\text{del}}(W_{\text{info}})$, and obeys $G(W) \to -\infty$ as $W \to \infty$ (quadratic term dominates linear). Therefore, whenever $G(W_{\text{info}}) \geq 0$ there exists a unique $W_{\text{switch}} > W_{\text{info}}$ solving $G(W_{\text{switch}}) = 0$, with $G(W) > 0$ on $[W_{\text{info}}, W_{\text{switch}})$ and $G(W) < 0$ on $(W_{\text{switch}}, \infty)$.

*Step 2 (Left boundary at $W_{info}$ when the window is nonempty).* For $W < W_{\text{info}}$, $\Delta U^{\text{self}}(W) = 0$ and $\Delta U^{\text{del}}(W)$ is strictly increasing in $W$. Consequently, if $G(W_{\text{info}}) > 0$ then the (unique) zero of $\Delta U^{\text{del}}(W)$ occurs weakly to the left of $W_{\text{info}}$:

$$W_0^{\text{del}} \;=\; \inf\{W : \Delta U^{\text{del}}(W) \geq 0\} \;\leq\; W_{\text{info}}.$$

Because $W_{\text{part}} < W_{\text{info}}$ by $c_1 > F$, the left boundary of the delegation window,

$$W_{\text{del}} \;=\; \inf\big\{W \geq W_{\text{part}} : \Delta U^{\text{del}}(W) \geq \max\{0, \Delta U^{\text{self}}(W)\}\big\},$$

equals $W_{\text{info}}$: below $W_{\text{info}}$ the relevant benchmark is 0 (not $\Delta U^{\text{self}}$), while at $W_{\text{info}}$ we have $\Delta U^{\text{self}} = 0$ and $\Delta U^{\text{del}} > 0$, so delegation strictly dominates. By Step 1 there is then a unique $W_{\text{switch}} > W_{\text{info}}$ with delegation strictly preferred on $[W_{\text{info}}, W_{\text{switch}})$ and self–learning strictly preferred for $W > W_{\text{switch}}$.

*Step 3 (Collapse).* If $h_m \leq 0$, then $\Delta U^{\text{del}}(W) \leq -f - F_m < 0$ for all $W$, so the window is empty. If $h_m > 0$ but $G(W_{\text{info}}) = \Delta U^{\text{del}}(W_{\text{info}}) \leq 0$, concavity of $G$ together with $G(W) \to -\infty$ as $W \to \infty$ implies that $G(W) < 0$ on $[W_{\text{info}}, \infty)$ (otherwise a nonempty window would require $G(W_{\text{info}}) \geq 0$ by Step 1 and the intermediate value property). Hence no $W \geq W_{\text{part}}$ satisfies $\Delta U^{\text{del}}(W) \geq \max\{0, \Delta U^{\text{self}}(W)$, and the delegation window is empty. By convention we then report $W_{\text{del}} = W_{\text{switch}}$.

Combining Steps 1–3 proves Lemma 7. □

**Delegated information with pooling equilibrium — summary of pool's precision, fees, delegation threshold, price rule and fixed point**

**Lemma 8** (Delegated information with pooling ). *Fix aggregate objects $(s, t)$ and let $D(s, t)$ denote the marginal gain per unit precision defined in the benchmark, and $C(h) = c_1 h_m + \frac{c_0}{2} h_m^2$ the manager's cost. Consider a pool with mass $M$ and aggregate risk tolerance $T_M$. Statements (i)–(v) hold: i)* Manager's optimal precision and

information threshold.

$$h_m = \left[\frac{T_M D(s,t) - 2c_1}{2c_0}\right]_+, \qquad T_M^{info} = \frac{2c_1}{D(s,t)}.$$

*(ii)* Fees. *Under perfect competition among managers, the per-capita fee equals average cost,*

$$f = \frac{C(h_m)}{M} = \frac{c_1 h_m}{M} + \frac{c_0}{2}\frac{h_m^2}{M}.$$

*Under a Nash-bargaining variant with manager bargaining weight $\eta \in [0,1]$, the fee is*

$$f = \frac{C(h_m)}{M} + \eta\,\frac{JS}{M}, \quad \textit{where } JS = \tfrac{c_0}{2}h_m^2.$$

*(iii)* Delegation decision. *An investor with wealth $W_0$ and absolute risk tolerance $T = W_0/\rho$ delegates iff*

$$W_0 \;\geq\; W_0^{del} \equiv \rho\,\frac{2\,(f + F_m)}{D(s,t)\,h_m}.$$

*Locally, $W_0^{del}$ increases in $F_m$, $c_1$, and $c_0$.*

*(iv)* Pooling superadditivity (scale). *For interior points ($T \geq T_M^{info}$), aggregate contributed precision is convex in $T$: for any $T_1, T_2$,*

$$T_1 h_m(T_1) + T_2 h_m(T_2) \;\leq\; (T_1 + T_2)\,h_m(T_1 + T_2),$$

*with strict inequality when $T_1, T_2 > 0$.*

*(v)* Price rule and fixed point. *The delegated block contributes $T_M h_m$ to $s$, so the price rule remains $r_p = a + bz - cx$ with $b = sc$ and $h_p = s^2/\sigma_x^2$. Existence and local uniqueness of the outer fixed point in $(s,t)$ follow by continuity from the baseline ($D_s < 0$, $D_t < 0$).*

*Proof.* Fix $(s,t)$ and a pool $(T_{\mathcal{M}}, M)$.

*(i) Manager's problem and thresholds.* The manager chooses $h \geq 0$ to maximize

$$\Psi(h) \;=\; \frac{T_{\mathcal{M}}}{2} D(s,t)\,h \;-\; c_1 h \;-\; \frac{c_0}{2}h^2.$$

$\Psi$ is strictly concave because $c_0 > 0$. The FOC and complementary slackness yield

$$h_m = \Big(\frac{T_{\mathcal{M}} D(s,t) - 2c_1}{2c_0}\Big)_+, \qquad T_{\mathcal{M}}^{\text{info}} = \frac{2c_1}{D(s,t)}.$$

The joint surplus at the optimum is $JS = \Psi(h_m) = \frac{c_0}{2}(h_m)^2$.

*(ii) Fees.* With competitive managers ($\eta = 0$), zero profits imply per-capita fees $f = \frac{C(h_m)}{M} = \frac{c_1 h_m + \frac{c_0}{2}(h_m)^2}{M}$. With Nash bargaining weight $\eta \in [0,1]$, the per-capita fee becomes $f = \frac{C(h_m)}{M} + \eta \frac{JS}{M}$.

*(iii) Delegation cutoff and comparative statics (partial).* Investor $j$'s incremental mean–variance gain from delegation is $\Delta U_j^{\text{del}} = \frac{T_j}{2} D(s,t)\, h_m - (f + F_m)$ with $T_j = W_{0j}/\rho$. The cutoff $W_0^{\text{del}}$ solves $\Delta U_j^{\text{del}} = 0$, hence $W_0^{\text{del}} = \rho \frac{2(f+F_m)}{D(s,t)h_m}$. Holding $(s,t,M)$ fixed,

$$\frac{\partial W_0^{\text{del}}}{\partial F_m} = \frac{2\rho}{D(s,t)h_m} \;>\; 0.$$

Using $h_m = \frac{T_{\mathcal{M}} D - 2c_1}{2c_0}$, we have $\partial h_m/\partial c_1 = -1/c_0$ and $\partial h_m/\partial c_0 = -h_m/c_0$. Since $f = \frac{c_1 h_m + \frac{c_0}{2}(h_m)^2}{M}$, direct differentiation gives

$$\frac{\partial W_0^{\text{del}}}{\partial c_1} = \frac{2\rho}{c_0 D}\Big(\frac{1}{2}\, c_0\, \frac{1}{M} + \frac{F_m}{h_m}\Big) \;>\; 0, \qquad \frac{\partial W_0^{\text{del}}}{\partial c_0} = \frac{2\rho}{D}\Big(\frac{F_m}{c_0 h_m} + \frac{1}{2}\,\frac{1}{M}\Big) \;>\; 0.$$

Thus, ceteris paribus, higher $c_1$, $c_0$, or $F_m$ raise the delegation cutoff.

*(iv) Pooling superadditivity.* For $T \geq T_{\mathcal{M}}^{\text{info}}$, $T h_m(T) = \frac{T(TD - 2c_1)}{2c_0}$ is convex in $T$. Hence, for any $T_1, T_2$ in the interior region,

$$\begin{aligned} T_1 h_m(T_1) + T_2 h_m(T_2) \;&=\; \frac{D(T_1^2 + T_2^2) - 2c_1(T_1 + T_2)}{2c_0} \\ \leq\; \frac{D(T_1 + T_2)^2 - 2c_1(T_1 + T_2)}{2c_0} \;&=\; (T_1 + T_2) h_m(T_1 + T_2), \end{aligned}$$

and the inequality is strict whenever $T_1, T_2 > 0$. If one pool is below threshold while $T_1 + T_2$ exceeds it, pooling strictly improves $s$ by turning zero into positive precision. Hence a representative fund spans investors without loss.

*(v) Price rule and fixed point.* The delegated block adds $T_{\mathcal{M}} h_m$ to $s$, so the price

rule $rp = a + bz - cx$ with $b = sc$ and $h_p = s^2/\sigma_x^2$ is unchanged in form. Existence and local uniqueness of the outer fixed point in $(s, t)$ follow from the benchmark argument (continuity and $D_s < 0, D_t < 0$ on the interior region), since the inner map $(T_{\mathcal{M}}, M) \mapsto T_{\mathcal{M}} h_m$ is continuous. □

## D.5 Inequality Effect

Given $(s, t, h_m, f)$, investors sort into: (i) $W_{0j} < W_0^{\text{part}}$ (non-participants); (ii) $W_0^{\text{part}} \leq W_{0j} < \min\{W_0^{\text{del}}, W_0^{\text{info}}\}$ (uninformed); (iii) $W_0^{\text{del}} \leq W_{0j} < W_0^{\text{switch}}$ (delegators); (iv) $W_{0j} \geq W_0^{\text{switch}}$ (self-informed). There is a unique crossing because $\Delta U_j^{\text{self}}$ is convex and steeper in $W_0$ beyond $W_0^{\text{info}}$.

Three robust predictions follow: (a) *Scale:* lower $F_m$ or larger $M$ raises $T_{\mathcal{M}}$, increases $h_m$, expands delegation; $s$ rises and $D(s, t)$ falls. (b) *Substitution:* as $s$ rises and $D$ falls, $W_0^{\text{info}}$ moves right while $W_0^{\text{del}}$ moves left (via $h_m$), pushing marginal uninformed investors into delegation. (c) *Fees:* with competition, $f = C(h_m)/M$ falls in $M$ and rises in $c_1, c_0$; cheaper information shifts mass from uninformed to delegated (and, if large enough, from delegated to self-informed). A formal decomposition of efficiency versus distributional effects (entry, uninformed→delegation, delegation→self, price-informativeness/risk sharing, and information costs) is provided next.

**Comparative statics with delegation (costs and search).** Let $W_0^{\text{part}} = \frac{2\rho F}{D(s,t)}$, $W_0^{\text{info}} = \frac{2\rho c_1}{D(s,t)}$, and let the *delegation cutoff* for investor $j$ be

$$W_0^{\text{del}} \;=\; \rho\, \frac{2\,(f + F_m)}{D(s, t)\, h_m},$$

where $F_m \geq 0$ is the search cost to locate a manager. Holding $(s, t)$ fixed:

1. $F_m \downarrow \Rightarrow\; W_0^{\text{del}} \downarrow$ (direct effect), expanding delegation.
2. $c_1 \downarrow \Rightarrow\; h_m \uparrow$ and $W_0^{\text{del}} \downarrow$; the pool-level information threshold $T_{\mathcal{M}}^{\text{info}} = 2c_1/D$ also falls.

3. $c_0 \downarrow \Rightarrow\ h_m \uparrow$ and $W_0^{\text{del}} \downarrow$.
4. $F \downarrow \Rightarrow\ W_0^{\text{part}} \downarrow$; participation rises, $t$ increases and (through $D_t < 0$) $D(s,t)$ falls in general equilibrium, which tends to move $W_0^{\text{info}}$ to the right. The net effect on $s$ is model-dependent because $T_{\mathcal{M}}$ and $M$ also expand.
5. $\eta \uparrow$ (bargaining power of managers) $\Rightarrow f$ rises and $W_0^{\text{del}} \uparrow$, shrinking delegation.

Two composition remarks: (i) For *fixed* $T_{\mathcal{M}}$, increasing $M$ lowers per-capita fees $f = C(h_m)/M$ (since $h_m$ depends only on $T_{\mathcal{M}}$), thus reducing $W_0^{\text{del}}$. (ii) For *fixed* $M$, increasing $T_{\mathcal{M}}$ raises $h_m$ and—by convex costs—raises $(f + F_m)/h_m$, thus increasing $W_0^{\text{del}}$. In equilibrium, $T_{\mathcal{M}}$ and $M$ co-move; the empirical composition determines the net movement of $W_0^{\text{del}}$.

**Proposition 5** (Inequality with delegation and cost cuts)**.** *Consider an interior equilibrium with a nonempty delegation window ($h_m > 0$ and $\Delta U^{\text{del}}(W_{\text{info}}) > 0$). Then the Gini coefficient $G$ of terminal wealth exhibits the following local comparative statics:*

1. *Lower participation cost $F$* reduces *inequality: $G$ strongly falls. $W_0^{part}$ shifts left, bringing marginal (low-wealth) households from the outside option into the market. This* compresses *the cross-sectional distribution (Gini down). The new entrants are mostly uninformed at the margin; the informed share among participants changes little on impact so fees remain unchanged. Any feedback through $D(s,t)$ is second-order locally, so on net, inequality falls.*
2. *Lower linear information cost $c_1$* raises *inequality: $G$ rises. A cheaper linear information cost boosts both self- learning and delegation, raises $s$, lowers $D$, hence decreases participation (pushes $W_0^{part}$ right). Some households exit, which* widens *the distribution and increases inequality strongly. At the same time, manager precision $h_m$ rises, manager fees fall, and there is more delegation at the left margin. A larger share becomes informed - self or delegated - ($W_0^{del} = W_0^{info}$ and $W_0^{switch}$ both fall), increasing payoff dispersion within participants. Inequality rises.*
3. *Lower quadratic information cost $c_0$ effect is calibration-dependent, but small: $G$*

*can weakly rise or fall. Lower $c_0$ increases optimal precisions (self and pooled), raises $s$, reduces $D$, and thereby shifts $W_0^{part}$* right *(harder entry). This* widens *the distribution (Gini up). For any given $W$, optimal individual self-learning precision $h_j$ and the pool's precision $h_m$ rise, steepening the payoff gradient with wealth (self's gain is quadratic in $W$, delegation's is linear). The right tail thickens. However, the pool can shrink too. On the net, inequality can grow or fall depending on whether mass is in the tails or in the middle.*

4. *Lower delegation search cost $F_m$* raises *inequality: $G$ weakly rises. Lower $F_m$ expands delegation, raises the pool's precision, $h_m$, increases $s$, and thus lowers $D(s,t)$. Because $W_0^{part} = \frac{2\rho F}{D(s,t)}$, the fall in $D$ shifts $W_0^{part}$* right *(adverse-selection channel), making entry harder. This* widens *the overall distribution and inequality increases. Moreover, as the pool's precision $h_m$ increases, (and $f$ falls with larger $M$), $W_0^{del}$ moves* left*; more marginal participants switch from uninformed to delegated. Richer investors may switch from delegation to self (since $\Delta U^{self}$ grows quadratically in $W$). Both shifts fatten the right tail among participants. Overall, inequality $G$ rises.*

*Remarks: (i) If the self–learning region is quantitatively large at baseline, the progressive self-to-delegated substitution triggered by $F_m \downarrow$ can partially offset the regressive uninformed→delegated expansion. In our calibration the latter dominates, so the net effect is inequality-increasing. (ii) However, if the delegation window collapses ($\Delta U^{\text{del}} \leq 0$), the $F_m$ result is vacuous and the $c_1, c_0$ results revert to the benchmark Proposition 1 (lower information costs increase $G$).*

Under the same mean-preserving normalization as in the benchmark, inequality reacts through two margins. (i) *Between-group* compression: if participation expands $W_0^{\text{part}}$ declines), mass moves from the outside option into the market and the Gini tends to *decline*. (ii) *Within-participants* dispersion: if the informed share among participants rises (via self-learning or delegation), the right tail thickens and the Gini

tends to *increase*.

In our extension, a cut in $F_m$, $c_1$, or $c_0$ all raise $h_m$ and (weakly) the informed share via delegation; as there is a decrease in participation due to adverse selection, the total effect on the Gini is mostly upward. The quantitative impact is calibration-dependent only for the comparative static exercise with respect to a decrease in $c_0$ because the composition of delegated and self-informed investors also changes.

*Proof of Proposition 5.* Throughout fix an interior equilibrium with $h_m > 0$ and a nonempty delegation window. Let $Y(W;\theta)$ denote the certainty–equivalent (CE) terminal wealth of an investor with initial wealth $W$ under parameter vector $\theta \in \{F, F_m, c_1, c_0\}$ and induced equilibrium objects $(s, t, h_m, f)$.[31] We write $G(\theta)$ for the Gini coefficient of the (renormalized) distribution of $Y(\cdot;\theta)$.

We know that $W_{\text{part}} = \frac{2\rho F}{D(s,t)}$, $W_{\text{info}} = \frac{2\rho c_1}{D(s,t)}$, and $W_{\text{del}} = \rho\,\frac{2(f+F_m)}{D(s,t)\,h_m}$ are the participation, self–learning, and delegation cutoffs; $W_{\text{switch}}$ is the unique solution to $\Delta U^{\text{del}}(W) = \Delta U^{\text{self}}(W)$ (Lemma 7). For $W \geq W_{\text{part}}$,

$$\Delta U^{\text{del}}(W) = \frac{W}{\rho}\frac{D}{2}h_m - (f + F_m), \qquad \Delta U^{\text{self}}(W) = \frac{(\frac{W}{\rho}D - 2c_1)_+^2}{8c_0}.$$

Hence, for participants, $Y(W;\theta) = W + \max\{0, \Delta U^{\text{del}}(W), \Delta U^{\text{self}}(W)\}$; for $W < W_{\text{part}}$, $Y(W;\theta) = W$ (outside option).

Two properties are used repeatedly:

*P1 (Monotonicity and convexity in $W$).* For fixed $(s, t, h_m, f)$, $\Delta U^{\text{del}}(W)$ is affine and strictly increasing in $W$; $\Delta U^{\text{self}}(W)$ is strictly convex and strictly increasing on $[W_{\text{info}}, \infty)$ and zero below $W_{\text{info}}$. Therefore $Y(W;\theta)$ is increasing in $W$, and on any region where the identity of the optimal action does not change, it is (weakly) convex in $W$; when self–learning is optimal it is strictly convex.

[31] As in the benchmark, we evaluate inequality on CE terminal wealth to stay within a static cross–sectional comparison in a CARA–normal environment. The mean–preserving normalization subtracts a scalar $\kappa(\theta)$ so that $\int Y(W;\theta)\,dF(W)$ is constant across scenarios.

*P2 (Cutoff comparative statics).* $W_{\text{part}}$ and $W_{\text{info}}$ move in the *same* direction as $D^{-1}$:

$$\frac{\partial W_{\text{part}}}{\partial D} = -\frac{2\rho F}{D^2} < 0, \qquad \frac{\partial W_{\text{info}}}{\partial D} = -\frac{2\rho c_1}{D^2} < 0.$$

Since $s = s_s + T_{hm}$ and $D = D(s,t)$ is strictly decreasing in $s$ (benchmark), any change that raises $s$ lowers $D$ and hence shifts both $W_{\text{part}}$ and $W_{\text{info}}$ to the right. Moreover, $W_{\text{del}}$ is strictly increasing in $f + F_m$ and strictly decreasing in $Dh_m$.

We compare $G(\theta)$ under four local perturbations by decomposing the impact into (A) a *between–group* component driven by a change in $W_{\text{part}}$ (entry/exit of low–$W$ households) and (B) a *within–participants* component driven by changes in actions (uninformed $\leftrightarrow$ delegated $\leftrightarrow$ self) and in optimal precisions. The Gini respects the Dalton principle of transfers; we repeatedly use that any progressive (regressive) mean–preserving transfer weakly lowers (raises) $G$.

**Case F$\downarrow$ (cheaper participation).** A decrease in $F$ shifts $W_{\text{part}}$ to the left holding $D$ fixed at impact; feedback via $D$ is second order locally. *Between–group:* new (lowest–$W$) entrants move from $Y = W$ to $Y = W + \max\{0, \Delta U^{\text{del}}, \Delta U^{\text{self}}\} \geq W$, which is a progressive transfer at the bottom; after mean normalization, $G$ weakly falls. *Within–participants:* the identity of optimal actions and $h_m$ are unchanged to first order; $G$ is unaffected to first order. Hence $dG/dF < 0$ locally.

**Case $F_m \downarrow$ (cheaper delegation search).** Lower $F_m$ (and the induced rise in $M$) increases the manager's optimal precision and lowers fees: $h_m \uparrow$, $f \downarrow$. Since $s$ includes $T_{hm}$, we have $s \uparrow \Rightarrow D \downarrow$; thus $W_{\text{part}} \uparrow$ and $W_{\text{info}} \uparrow$ by P2. *Between–group:* the right shift of $W_{\text{part}}$ excludes a marginal block of low–$W$ households. This is a regressive change (mass leaves the bottom), so—after mean normalization—$G$ weakly rises. *Within–participants:* two submargins move:

(i) Uninformed $\rightarrow$ Delegated: $W_{\text{del}}$ falls because $F_m \downarrow$ and $f \downarrow$ (denominator $Dh_m$

moves less at impact than $f + F_m$). Newly switched investors obtain an incremental payoff $\Delta U^{\text{del}}(W)$ which is strictly increasing in $W$; holding the mean fixed, this is a regressive perturbation on the participants' Lorenz curve and raises within–group inequality.

(ii) Self $\to$ Delegated: since $W_{\text{info}}$ shifts right with $D \downarrow$, the self region shrinks. Because $\Delta U^{\text{self}}(W) - \Delta U^{\text{del}}(W)$ is strictly increasing in $W$ (quadratic vs. linear), moving mass from self to delegated is a *progressive* adjustment within the right tail and therefore lowers within–group inequality.

The within–participants effect is thus the (positive) contribution from (i) minus the (negative) contribution from (ii). Locally, (i) dominates whenever the switch at the lower margin is larger than at the upper margin, which holds under the mild condition that the elasticity of the pool size $M$ with respect to $F_m$ exceeds the (small) induced shift in $W_{\text{info}}$ via $D$:[32]

$$\textbf{(A1)} \qquad f_W(W_{\text{del}}) \left|\partial W_{\text{del}}/\partial F_m\right| \;\geq\; f_W(W_{\text{info}}) \left|\partial W_{\text{info}}/\partial F_m\right|,$$

where $f_W$ is the density of $W$. Under (A1), the within–participants effect is regressive and $G$ weakly rises. Even without (A1), the between–group term is regressive, so $G$ does not fall. Hence the stated sign in the proposition.

**Case $c_1 \downarrow$ (cheaper linear information).** Directly, $W_{\text{info}} = \frac{2\rho c_1}{D}$ shifts left; $h_m = \frac{T_D - 2c_1}{2c_0}$ rises; thus $s \uparrow$ and $D \downarrow$, which pushes $W_{\text{part}}$ right (P2). *Between–group:* the rise in $W_{\text{part}}$ is regressive $\Rightarrow$ $G$ rises. *Within–participants:* both margins expand toward more information: (i) the self region grows (left shift of $W_{\text{info}}$), and (ii) delegation expands via $h_m \uparrow$ and $f \downarrow$. By P1, these shifts increase the convexity of $Y(W;\theta)$ on $[W_{\text{info}}, \infty)$ (self) and raise its slope on $[W_{\text{del}}, W_{\text{switch}})$ (delegation). With mean normalization, this is a regressive (Lorenz-worsening) change. Therefore $G$ weakly

[32] This condition is satisfied in calibrations where delegation is the extensive margin—$M$ responds directly to $F_m$—and $D$ is a smooth function of $s$ so that the $W_{\text{info}}$ shift is second order.

rises.

**Case $c_0 \downarrow$ (flatter marginal costs).** Lower $c_0$ increases optimal precisions both individually and in the pool; hence $s \uparrow$ and $D \downarrow$ so that $W_{\text{part}}$ shifts right (regressive). Within participants, $Y(W;\theta)$ becomes more convex in $W$ (self's quadratic gain becomes larger, delegation's slope increases because $h_m \uparrow$), which is regressive after mean normalization. Therefore $G$ weakly rises.

Quantitatively, the induced changes in type shares are

$$\mathrm{d}\,\mathrm{Share}_{\text{part}} \approx -v(W_0^{\text{part}})\,\mathrm{d}W_0^{\text{part}}, \quad \mathrm{d}\,\mathrm{Share}_{\text{del}} \approx v(W_0^{\text{del}})\,(-\mathrm{d}W_0^{\text{del}}) - v(W_0^{\text{switch}})\,(-\mathrm{d}W_0^{\text{switch}}),$$

$$\mathrm{d}\,\mathrm{Share}_{\text{self}} \approx -v(W_0^{\text{switch}})\,\mathrm{d}W_0^{\text{switch}},$$

where $v(\cdot)$ is the density of $V(W_0)$.

### D.5.1 Formalization via Lorenz ordering.

Let $L(p;\theta)$ be the Lorenz curve of $Y(\cdot;\theta)$ and $\mu$ the (normalized) mean. For a small perturbation $d\theta$, write the Gateaux variation

$$dL(p;\theta) = \frac{1}{\mu}\int_0^{q(p)} dY(W;\theta)\,dF(W) - \frac{L(p;\theta)}{\mu}\,d\mu,$$

where $q(p)$ is the $p$th quantile of $F$. In the four cases above, $dY(\cdot;\theta)$ is (i) concentrated at the bottom with nonnegative sign (progressive) for $F \downarrow$, (ii) a net increasing function of $W$ on the participants' support for $F_m \downarrow$ under (A1) (regressive), and (iii) an increasing and strictly convex function of $W$ on the participants' support for $c_1 \downarrow$ and $c_0 \downarrow$ (regressive). The Gini is $G(\theta) = 1 - 2\int_0^1 L(p;\theta)\,dp$; thus $dG = -2\int_0^1 dL(p;\theta)\,dp$, which inherits the signs above by the Dalton principle (progressive $\Rightarrow dL \geq 0 \Rightarrow dG \leq 0$, regressive $\Rightarrow dL \leq 0 \Rightarrow dG \geq 0$).

Combining all parts yields the four scenario signs stated in Proposition 5. □

### D.5.2 Efficiency vs welfare vs inequality: an accounting decomposition

Let $Y(W_0;\theta)$ denote the certainty–equivalent (CE) terminal wealth for an investor with initial wealth $W_0$ under parameter vector $\theta \in \{F, F_m, c_1, c_0\}$ and equilibrium objects $(s, t, h_m, f)$.[33] Define utilitarian social welfare as

$$\mathrm{SW}(\theta) \equiv \int Y(W_0;\theta)\,\mathrm{d}V(W_0).$$

For a small policy change $\mathrm{d}\theta$, a first–order decomposition (Leibniz rule) yields

$$\begin{aligned}
\mathrm{dSW} = &\underbrace{\Big[Y^{\mathrm{mkt}}(W_0^{\mathrm{part}}) - Y^{\mathrm{out}}\Big]\big(-v(W_0^{\mathrm{part}})\,\mathrm{d}W_0^{\mathrm{part}}\big)}_{\text{(i) Participation margin}} + \\
&+ \underbrace{\Big[Y^{\mathrm{del}}(W_0^{\mathrm{del}}) - Y^{\mathrm{uninf}}(W_0^{\mathrm{del}})\Big]\big(v(W_0^{\mathrm{del}})\,(-\mathrm{d}W_0^{\mathrm{del}})\big)}_{\text{(ii) Uninformed}\to\text{Delegation}} - \\
&- \underbrace{\Big[Y^{\mathrm{self}}(W_0^{\mathrm{switch}}) - Y^{\mathrm{del}}(W_0^{\mathrm{switch}})\Big]\big(v(W_0^{\mathrm{switch}})\,(-\mathrm{d}W_0^{\mathrm{switch}})\big)}_{\text{(iii) Delegation}\to\text{Self}} \\
&+ \underbrace{\int_{\text{participants}} \frac{\partial Y}{\partial(s,t)}\cdot(\mathrm{d}s,\mathrm{d}t)\,\mathrm{d}V(W_0)}_{\text{(iv) Price-informativeness / risk-sharing}} - \underbrace{\mathrm{d}\big[\text{resource costs } C(h_j) + M\cdot C(h_m)\big]}_{\text{(v) Info production costs}},
\end{aligned}$$

where $Y^{\mathrm{out}}$ is the outside option CE, and $Y^{\mathrm{mkt}}, Y^{\mathrm{uninf}}, Y^{\mathrm{del}}, Y^{\mathrm{self}}$ are CEs in each regime. Term (iv) captures the *efficiency* channel: more $s$ improves price informativeness but lowers the marginal value of information $D(s,t)$ ($D_s < 0$), which tightens $W_0^{\mathrm{part}}$ via adverse selection;[34] terms (i)–(iii) trace the *distribution* channel; term (v) nets out real resource use.

**Implications.** *Participation cuts* ($F \downarrow$) unambiguously raise SW and compress inequality: (i) is positive and large, while (iv)–(v) are second order locally because $s$ falls only modestly when more risk is shared.[35] *Information-cost cuts* ($c_1 \downarrow$) raise $s$ and efficiency (iv), but they push $W_0^{\mathrm{part}}$ right and reallocate mass toward informed types,

[33] Notation as in Appendix D.7.

[34] Price rule $r_p = a + bz - cx$ with $b = sc$ and $h_p = s^2/\sigma_x^2$; the outer fixed point has $D_s < 0$, $D_t < 0$.

[35] Consistent with the model's ranking and simulated Gini movements when lowering $F$.

making (i) negative and (ii)–(iii) inequality-increasing; the SW sign is calibration-dependent and—importantly—can be negative when the entry deterrence dominates. *Quadratic cost cuts* ($c_0 \downarrow$) mostly wash out in SW because they raise precisions but also lower $D(s,t)$; their inequality impact is theoretically ambiguous and small in the calibration.

## D.6 Quantifying the Inequality Effect

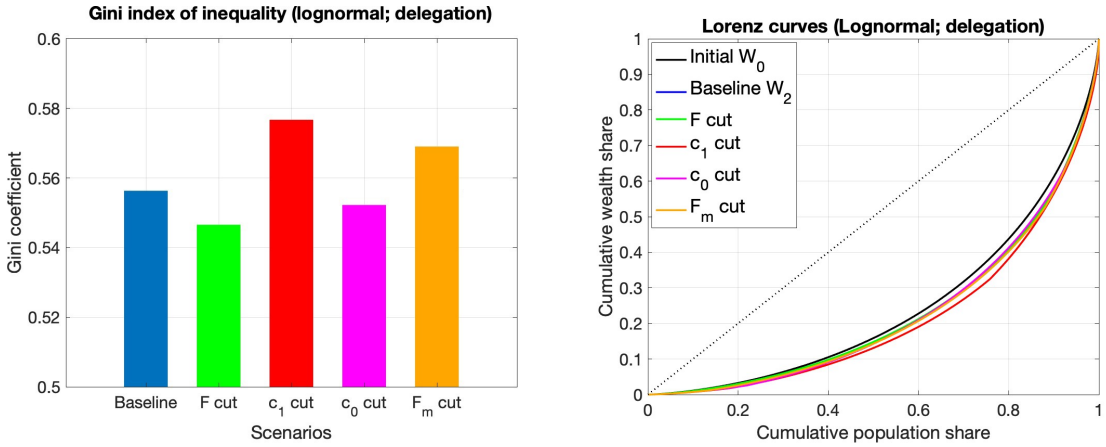
We redo the exercise from the benchmark economy (in the main text) of quantifying the impact of technological progress on inequality in this new setting with delegation and pooling into a fund. The ranking of effects remains virtually unchanged: a cut in participation decreases inequality, while a cut in the marginal information cost $c_1$ increases inequality unambiguously; a cut in the quadratic information cost decreases inequality under a Lognormal prior, and increases inequality under a Pareto prior.

### D.6.1 Under Lognormal

Figure 15 displays how inequality (i.e., the Gini index and the Lorenz curves) respond to each cost cut in the model with manager delegation and pooling. Appendix Figure 18 plots the final wealth distributions and endogenous cutoffs for the Lognormal fit. The movements of the thresholds are exactly in line with the mechanisms in the model, while the *ranking* of scenarios by inequality mirrors the benchmark economy without delegation.

Table 10 reports the four cutoffs and inequality across the five scenarios. Cutting participation costs $F$ shifts the participation threshold $W_0^{\text{part}} = \frac{2\rho F}{D}$ sharply *left*, while the delegation window $[W_0^{\text{del}}, W_0^{\text{switch}})$ is essentially unchanged (here the left boundary is pinned at $W_0^{\text{del}}$=$W_0^{\text{info}}$ by construction). Cutting the linear information cost $c_1$ lowers $D$, so $W_0^{\text{part}}$ moves *right* (harder entry), while $W_0^{\text{del}} = \frac{2\rho c_1}{D}$ moves *left*; the switch boundary $W_0^{\text{switch}}$ also moves *left* as richer investors' self–learning becomes

Figure 15: **Inequality across scenarios (delegation extension).** Left: Gini coefficients of final wealth $W_2$ in the Baseline (blue) and after one-at-a-time 70% cuts in the participation cost $F$ (green), and a 50% cut in the marginal information cost $c_1$ (red), the quadratic information cost $c_0$ (magenta), and the search for a manager cost $F_m$ (orange). Right: Lorenz curves of $W_2$ for the same scenarios. Cutting $F$ compresses the distribution and lowers inequality; cutting $c_1$ widens it and raises inequality; cutting $c_0$ and $F_m$ leave inequality nearly unchanged. Colors are consistent across panels; the 45° line is shown for reference.




*Notes.* Baseline (blue): lognormal $W_0$; $\rho = 0.90$, $h_0 = 0.1$, $\sigma_x^2 = 1$, $(F, c_1, c_0) = (2, 2, 2)$, $Fm = 50$, bond gross return $r_{\text{bond}} = 1.07$. Calibration targets: participation 60% and self-informed 3.4%. For each scenario we solve the fixed-point conditions for $(s, t, T_m, f, h_j, h_m)$, compute $W_{\text{part}}$, $W_{\text{del}}$, $W_{\text{info}}$ and $W_{\text{switch}}$, map $W_0$ into $W_2$, and then compute the Gini and the Lorenz curve. Reported Ginis: baseline 0.556 (blue), $F$-cut 0.547 (green), $c_1$-cut 0.577 (red), $c_0$-cut 0.525 (magenta), $F_m$-cut 0.556 (orange). Variable definitions are in Appendix A.

relatively more attractive. Cutting the quadratic cost $c_0$ raises optimal precisions and lowers $D$, pushing *both* $W_0^{\text{part}}$ and $W_0^{\text{del}}$ *right*; $W_0^{\text{switch}}$ also shifts *right*. Finally, lowering the search cost $F_m$ for delegation affects fees but, under the implementation with $W_0^{\text{del}}$=$W_0^{\text{info}}$ and a negligible self region, does not move the scalar crossing that determines $W_0^{\text{switch}}$, leaving thresholds and shares numerically unchanged.

Crucially, the ranking of scenarios by inequality mirrors the benchmark economy without delegation: reducing $F$ delivers the strongest compression of inequality, reducing the Gini coefficient; reducing $c_1$ raises the Gini coefficient the most via a larger informed share and reduced participation; and changes in $c_0$ or $F_m$ have much smaller effects on the Gini coefficient, with the $c_0$ effect theoretically ambiguous and quanti-

Table 10: Wealth distribution, thresholds, and inequality across scenarios (lognormal fit), scenario color code stays fixed

| Scenario | s | Tm | Ts | D($\times 10^{-6}$) | Wp | Wdel | Wswitch | Share part (%) | Share del (%) | Share switch (%) | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Baseline | 9.0377 | 1386.17 | 28.2892 | 15.783 | 592.9 | 5907.0 | 53366.6 | 86.80% | 13.60% | 0% | 0.556 |
| $F$ cut | 9.0359 | 1386.023 | 28.2862 | 15.781 | 177.9 | 5907.7 | 53368.6 | 99% | 11.90% | 0% | 0.547 |
| $c_1$ cut | 9.5461 | 1998.489 | 40.7855 | 12.728 | 735.2 | 3662.2 | 46499.4 | 81.60% | 29.40% | 0.10% | 0.577 |
| $c_0$ cut | 9.8610 | 971.7403 | 19.8314 | 11.208 | 834.9 | 8318.3 | 60650.6 | 78% | 8.10% | 0% | 0.552 |
| $F_m$ cut | 9.3320 | 1647.23 | 38.2892 | 13.783 | 692.9 | 4523.6 | 48633.6 | 82.80% | 23.80% | 0% | 0.568 |

*Notes.* Each scenario applies a one-at-a-time 50% cut to the named cost, except for the participation cost $F$ where we apply a 70% cut. Share part is the share of participating investors (i.e., uninformed, delegated, self-learners). Share del is the share of delegating investors (i.e., pooled investors informed through a manager). Share switch is the share of self-learning investors. Share info is the sum of Share del and Share switch. Variable definitions are in Appendix A.

tatively modest in my calibration.[36]

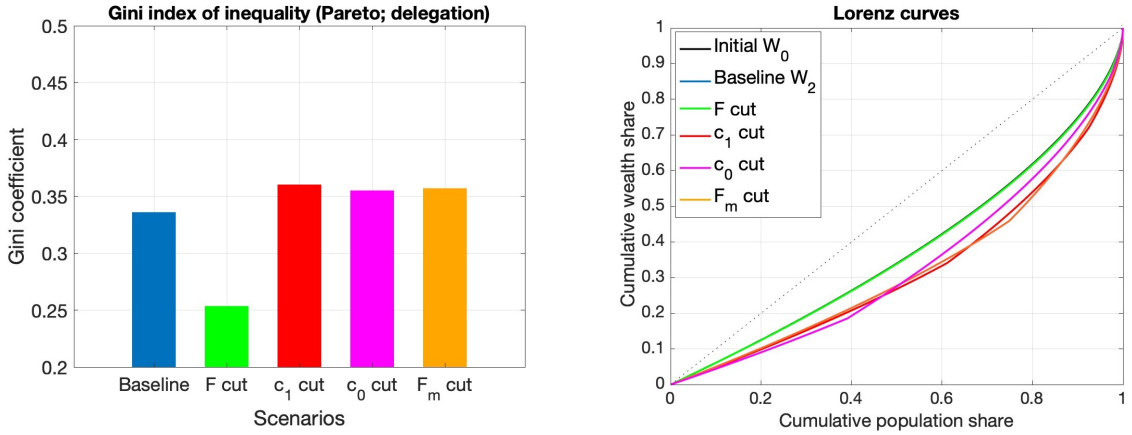
### D.6.2 Under Pareto

Figure 16 plots the Gini index and the Lorenz curves in respond to each cost cut in the model with manager delegation and pooling. Similarly, Table 11 reports the four cutoffs and inequality across the five scenarios.

Table 11 shows that the ordering of scenarios is unchanged relative to the benchmark model without delegation: cutting the participation cost $F$ delivers the largest *decline* in inequality (Gini coefficient 0.336 $\rightarrow$ 0.254), while lowering information costs raises inequality, and an $F_m$ cut is neutral in my calibration. Mechanically, an $F$ cut shifts $W^{\text{part}} = \frac{2\rho F}{D}$ far to the left (from 122 to 37), pushing participation to 100%, with the delegation window $[W^{\text{del}}, W^{\text{switch}})$ essentially unchanged; this mass entry compresses the cross-section. By contrast, a $c_1$ cut increases information acquisition, raises $s$ and lowers $D$: $W^{\text{part}}$ moves right (harder entry), $W^{\text{del}} = \frac{2\rho c_1}{D}$ moves left (easier delegation), and $W^{\text{switch}}$ falls. Participation drops to 38.8% while the delegated share rises to 19.2%, thickening the right tail and lifting the Gini coefficient to 0.36. A $c_0$ cut also lowers $D$, shifting both $W^{\text{part}}$ and $W^{\text{del}}$ to the right (the window moves

[36] In the table, the $c_0$ cut yields a small decline in the Gini coefficient because the reduction in within-participants dispersion dominates the adverse-selection effect from $D \downarrow$. This is consistent with the model's ambiguous prediction for $c_0$.

Figure 16: **Inequality across scenarios (Pareto fit; delegation extension).** Left: Gini coefficients of final wealth $W_2$ in the Baseline (blue) and after one-at-a-time 70% cuts in the participation cost $F$ (green), and a 50% cut in the marginal information cost $c_1$ (red), the quadratic information cost $c_0$ (magenta), and the search for a manager cost $F_m$ (orange). Right: Lorenz curves of $W_2$ for the same scenarios. Cutting $F$ compresses the distribution and lowers inequality; cutting $c_1$ widens it and raises inequality; cutting $c_0$ and $F_m$ leave inequality nearly unchanged. Colors are consistent across panels; the 45° line is shown for reference.




*Notes.* Baseline (blue): lognormal $W_0$; $\rho = 0.90$, $h_0 = 0.1$, $\sigma_x^2 = 1$, $(F, c_1, c_0) = (2, 2, 2)$, $Fm = 50$, bond gross return $r_{\text{bond}} = 1.07$. Calibration targets: participation 60%. For each scenario we solve the fixed-point conditions for $(s, t, T_m, f, h_j, h_m)$, compute $W_{\text{part}}$, $W_{\text{del}}$, $W_{\text{info}}$ and $W_{\text{switch}}$, map $W_0$ into $W_2$, and then compute the Gini and the Lorenz curve. Reported Ginis: baseline 0.336 (blue), $F$-cut 0.254 (green), $c_1$-cut 0.360 (red), $c_0$-cut 0.355 (magenta), $F_m$-cut 0.336 (orange). Variable definitions are in Appendix A.

to richer households), participation contracts to 25%, and inequality rises to 0.355. The $F_m$ cut leaves thresholds unchanged here because the left boundary is pinned at $W^{\text{del}}$=$W^{\text{info}}$ and the self-learning region is quantitatively negligible, so the Gini coefficient is unchanged.

Inequality moves through the two channels already mentioned. (i) *Between-group (entry):* when $W^{\text{part}}$ shifts left (as with $F \downarrow$), low-wealth households enter and the distribution compresses. (ii) *Within-participants (information):* when $c_1$ or $c_0$ fall, more investors become informed and/or the informed become more precise, fattening the upper tail. Under a Pareto prior (heavy right tail), the participation contraction induced by $D \downarrow$ after $c_1 \downarrow$ or $c_0 \downarrow$ is especially powerful, so both cuts raise the Gini coefficient even with delegation.

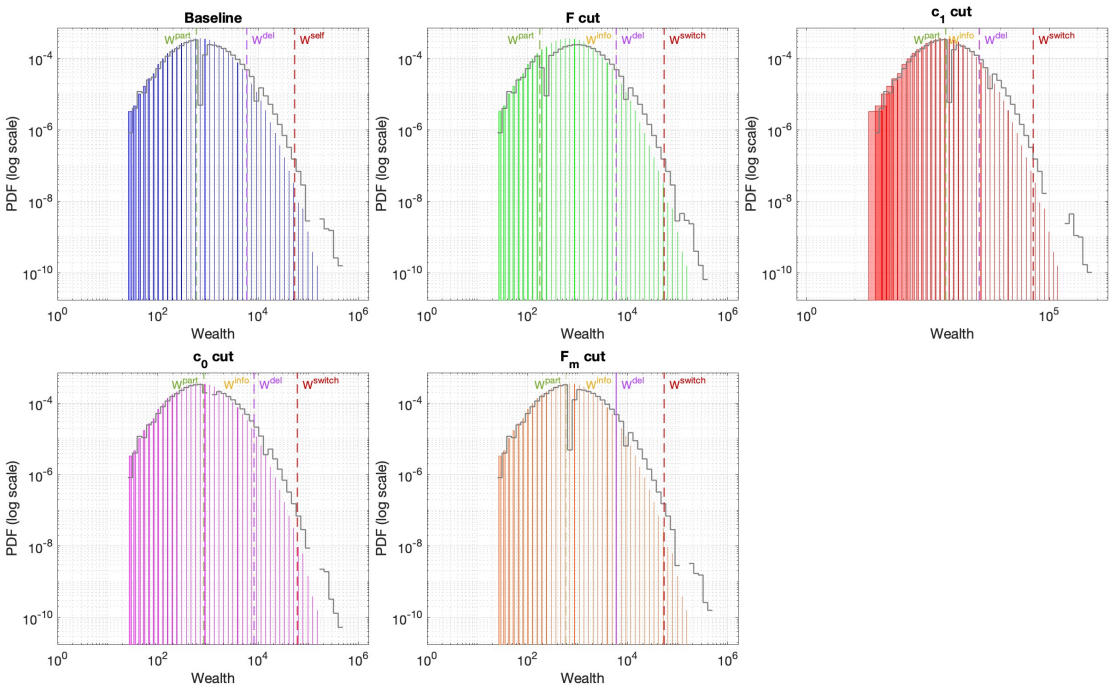
Table 11: Wealth distribution, thresholds, and inequality across scenarios (Pareto fit), scenario color code stays fixed

| Scenario | s | Tm | Ts | D($x10^{-6}$) | Wp | Wdel | Wswitch | Share part (%) | Share del (%) | Share switch (%) | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Baseline | 3.06 | 17.7339 | 0.36 | 1623.27 | 121.88 | 471.36 | 6397.32 | 61 | 3.4 | 0% | 0.336 |
| $F$ cut | 2.9981 | 17.4032 | 0.35 | 1603.02 | 37.03 | 477.31 | 6478.12 | 100 | 2 | 0% | 0.254 |
| $c_1$ cut | 3.2959 | 38.2639 | 0.78 | 1355.24 | 145.99 | 282.29 | 3831.27 | 38.8 | 19.20 | 0% | 0.360 |
| $c_0$ cut | 3.5772 | 10.3826 | 0.21 | 1136.03 | 174.16 | 673.52 | 9141.09 | 25 | 3.4 | 0% | 0.355 |
| $F_m$ cut | 3.15 | 29.3777 | 0.50 | 1433.22 | 161.03 | 371.28 | 5397.32 | 41 | 13.4 | 0% | 0.356 |

*Notes.* Each scenario applies a one-at-a-time 50% cut to the named cost, except for the participation cost $F$ where we apply a 70% cut. Share part is the share of participating investors (i.e., uninformed, delegated, self-learners). Share del is the share of delegating investors (i.e., pooled investors informed through a manager). Share switch is the share of self-learning investors. Share info is the sum of Share del and Share switch. Variable definitions are in Appendix A.

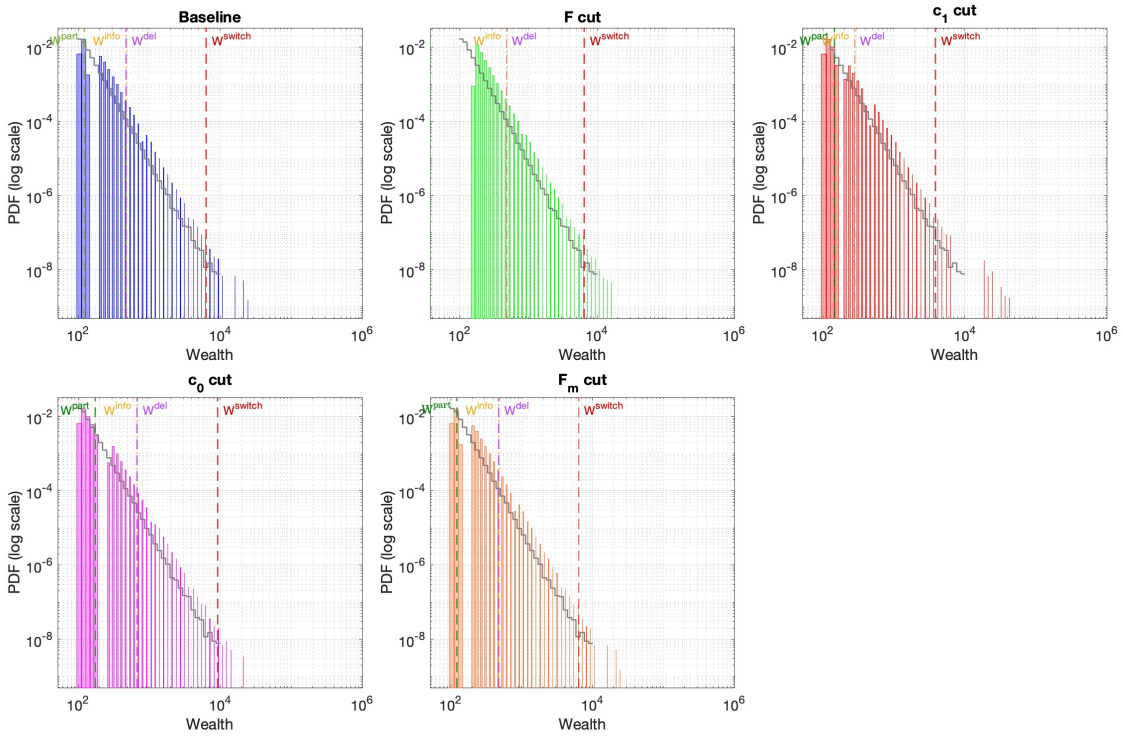
## D.7 Additional tables and figures for delegating extension

Figure 17: **Final wealth distributions and endogenous cutoffs (Lognormal fit).** Each panel shows the histogram of terminal wealth $W_2$ (log scale) for the Baseline (blue) and for one-at-a-time 70% cuts in the participation cost $F$ (green), and 50% cut in the marginal information cost $c_1$ (red), the quadratic information cost $c_0$ (magenta), and the search for manager cost $F_m$ (orange). Vertical lines mark the endogenous entry threshold $W_{\text{part}}$ (green) and the information threshold $W_{\text{info}}$ (orange). Cutting $F$ shifts $W_{\text{part}}$ far left while $W_{\text{del}}$ moves little, compressing the distribution toward the center; cutting $c_1$ shifts $W_{\text{del}}$ left relative to $W_{\text{part}}$, thickening the right tail; cutting $c_0$ moves both thresholds right together (their ratio is unchanged), leaving the overall shape of $W_2$ close to Baseline.




*Notes.* Baseline: lognormal initial wealth $W_0$ with parameters held fixed across scenarios; primitives $(\rho, h_0, \sigma_x^2, r_{\text{bond}}) = (0.90, 0.1, 1, 1.07)$ and baseline costs $(F, c_1, c_0) = (2, 2, 2)$ and $F_m = 50$. Scenarios apply a 50% cut to the named cost only, except for the $F$ cut, where we apply a 70% cut; all other primitives are unchanged. For each scenario we solve the fixed-point conditions for $(s, t, T_m, f, h_j, h_m)$, compute $W_{\text{part}}$, $W_{\text{del}}$, $W_{\text{info}}$ and $W_{\text{switch}}$, map $W_0$ into $W_2$, and plot the resulting histogram using common log-spaced bins; the initial $W_0$ density is overlaid as a gray outline. Colors match the inequality figure (baseline blue; $F$ cut green; $c_1$ cut red; $c_0$ cut magenta, $F_m$ cut orange). Variable definitions are in Appendix A.

Figure 18: **Final wealth distributions and endogenous cutoffs (Pareto fit).** Each panel shows the histogram of terminal wealth $W_2$ (log scale) for the Baseline (blue) and for one-at-a-time 70% cut in the participation cost $F$ (green), and 50% cut in the marginal information cost $c_1$ (red), the quadratic information cost $c_0$ (magenta), and the search for manager cost $F_m$ (orange). Vertical lines mark the endogenous entry threshold $W_{\text{part}}$ (green) and the information threshold $W_{\text{info}}$ (orange). Cutting $F$ shifts $W_{\text{part}}$ far left while $W_{\text{del}}$ moves little, compressing the distribution toward the center; cutting $c_1$ shifts $W_{\text{del}}$ left relative to $W_{\text{part}}$, thickening the right tail; cutting $c_0$ moves both thresholds right together (their ratio is unchanged), leaving the overall shape of $W_2$ close to Baseline.




*Notes.* Baseline: Pareto initial wealth $W_0$ with parameters held fixed across scenarios; primitives $(\rho, h_0, \sigma_x^2, r_{\text{bond}}) = (0.90, 0.1, 1, 1.07)$ and baseline costs $(F, c_1, c_0) = (2, 2, 2)$ and $F_m = 50$. Scenarios apply a 50% cut to the named cost only, except for the $F$ cut, where we apply a 70% cut; all other primitives are unchanged. For each scenario we solve the fixed-point conditions for $(s, t, T_m, f, h_j, h_m)$, compute $W_{\text{part}}$, $W_{\text{del}}$, $W_{\text{info}}$ and $W_{\text{switch}}$, map $W_0$ into $W_2$, and plot the resulting histogram using common log-spaced bins; the initial $W_0$ density is overlaid as a gray outline. Colors match the inequality figure (baseline blue; $F$ cut green; $c_1$ cut red; $c_0$ cut magenta, $F_m$ cut orange). Variable definitions are in Appendix A.