The Nash-in-Kalai Model for Estimation with Dynamic Bargaining and Nontransferable Utility

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Abstract

This paper extends empirical dynamic bargaining models to accommodate the possibility of nontransferable utility. I leverage the Kalai proportional bargaining solution, which coincides with Nash bargaining under transferable utility, but is the only solution satisfying independence of irrelevant alternatives that is identified under nontransferable utility given unobserved information sets. In recursive problems, Kalai proportional bargaining enables a finite dependence-type cancellation of future states. For multilateral games, I propose the *Nash-in-Kalai* model, which aligns with Nash-in-Nash under transferable utility, but which enables valid general method of moments estimation even when agreements have foreseeable and nontransferable contracting externalities on future states. I apply this model to hospital–insurer contract panel data from West Virginia, where agreements had nontransferable contracting externalities on future deals reached during their tenure. Negotiators were substantially forward-looking, with an estimated annual discounting rate of 0.899.

1 Introduction

Prevailing empirical models of bargaining are either static, transferable utility (TU), or both. In static models, the world lasts for one period and negotiators' information is akin

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to perfect foresight. In TU models, negotiators possess an instrument (typically price) that helps one party precisely as much as it harms the other side.

In dynamic settings, TU corresponds to a strong, and often unrealistic, restriction that prices do not affect any future negotiation. For example, in industrial organization, TU precludes staggered contracting, because a price negotiated between a retailer and a supplier could otherwise have a foreseeable and nontransferable externality on how the supplier negotiates with other retailers while the contract remains in place. In labor economics, TU requires employers to fully match outside offers, because initial wages would otherwise have a nontransferable externality on the employee's income after departure. These restrictions ensure that the econometrician knows the effect of marginal price changes on future values, but prevent these models from accurately predicting effects that include important nontransferable utility (NTU) dynamics.

The Nash-in-Kalai proposal is motivated by data that shows real-world contracts are meaningfully staggered, which in multilateral models leads to NTU bargaining. In Dorn (2025a), I find that hospital–insurer contract lengths vary dramatically, from three to more than fifteen years. In order to fit such data into prevailing bargaining frameworks, a researcher faces the choice between three undesirable modeling assumptions: aggregate contracts over a long period and rule out discounting; aggregate contracts over a short period and implicitly impose myopia on long-term agreements; or accurately model contract timing but impose myopia systematically, ruling out forward-looking responses and biasing the policy estimates in that companion work upwards by 45% or more.

There are three related challenges to building an econometrically tractable model of bargaining with dynamic mechanisms and nontransferable utility. The first challenge is *uncertainty*: at the time of bargaining, the negotiators may have information about future utility that is not available to the econometrician. The second challenge is *recursiveness*: with multiperiod contracting, gains from trade are defined recursively in terms of the expected value of applying the bargaining solution to future recursively defined gains from

trade. The third challenge is *estimation*: practitioners generally wish to estimate bargaining parameters with general method of moments (GMM). It is therefore valuable for practice to derive a formula for expected payments that holds even when prices have foreseeable, internalized, recursively defined, and nontransferable externalities on future agreements.

The first contribution of this paper is to show that Nash bargaining weights may not be identified when negotiators are allowed to have unspecified information sets. Nash bargaining chooses an agreement to maximize a weighted product of gains from trade. For any bargaining weights $\tau_1, \tau_2 \in (0, 1)$, I provide a distribution of feasible utilities for which the outcome of choosing a price through Nash bargaining with weight τ_1 before learning the NTU Pareto frontier is observationally equivalent to the outcome of choosing a price through Nash bargaining with weight τ_2 after learning the Pareto frontier. As a result, even if the econometrician knows all parameters other than the Nash bargaining weight, there may be no way to identify the negotiators' bargaining weight without also knowing the information set under which bargaining occurs.

This paper studies the Kalai (1977) proportional bargaining solution, which can identify bargaining weights across information structures. The solution concept chooses the best agreement that splits gains from trade proportionally to fixed bargaining weights. This solution concept agrees with Nash for TU games. In NTU games, the solution permits identification via a moment on the realized share of gains from trade. I show that this is the only solution satisfying independence of irrelevant alternatives (IIA) with such a robust moment, or any general identification argument. The uniqueness result follows from Myerson (1981)'s characterization of concavity in more general social choice problems.

The second contribution of this paper is to show that recursively defined multiperiod Kalai proportional bargaining models possess a cancellation property akin to finite dependence. Under recursive Kalai proportional bargaining, agreements are unchanged if the true value of disagreement is replaced by the hypothetical value of two, or arbitrarily many, periods of disagreement. In this way, the econometrician can replace the complicated, uncertain,

and recursively defined path of disagreement with a more convenient alternative, such as remaining in impasse until the next agreement would be reached in equilibrium. The reason for the cancellation is as follows. Suppose players 1 and 2 choose an agreement path A in period t through Kalai proportional bargaining relative to an anticipated single-disagreement path $D1$, and the $D1$ path corresponds to a single period of disagreement followed by Kalai proportional bargaining relative to a two-disagreement path $D2$. If player 1's weight is τ, the per-period discounting rate is β, and the players have perfect information, then the agreement value function $V_{i,t}^{A} = v_{i,t}^{A} + \beta V_{i,t+1}^{A}$ is recursively chosen to solve

$$(1-\tau)\left(V_{1,t}^{A} - V_{1,t}^{D1}\right) = \tau\left(V_{2,t}^{A} - V_{2,t}^{D1}\right), \tag{1}$$

where the $D1$ path is recursively chosen to solve $(1-\tau)\left(V_{1,t+1}^{D1} - V_{1,t+1}^{D2}\right) = \tau\left(V_{2,t+1}^{D1} - V_{2,t+1}^{D2}\right)$. By scaling this condition by β and adding $v_{i,t}^{D1} - v_{i,t}^{D2} = 0$ to both sides, one obtains $(1-\tau)\left(V_{1,t}^{D1} - V_{1,t}^{D2}\right) = \tau\left(V_{2,t}^{D1} - V_{2,t}^{D2}\right)$. After adding this result to equation (1), one obtains that the agreement path also solves

$$(1-\tau)\left(V_{1,t}^{A} - V_{1,t}^{D2}\right) = \tau\left(V_{2,t}^{A} - V_{2,t}^{D2}\right). \tag{2}$$

By induction, the same conclusion holds if the true value of disagreement $D1$ is replaced by three ($D3$), four ($D4$), or arbitrarily many ($D\infty$) periods of disagreement. The same conclusion also holds under exogenous uncertainty through linearity of value functions. In static games, this property is called step-by-step: improving the value of disagreement through a first-step bargain over a smaller Pareto frontier has no effect on the resulting agreement. In recursive games, the resulting simplification is principled, and arises directly from the way in which the axioms of Kalai proportional bargaining discipline the outcome of any off-equilibrium bargain.

I leverage the step-by-step property to analyze identification of Kalai proportional bargaining parameters with multiperiod contracting. I focus on the case of transferable utility,

so that my results also apply to recursive Nash bargaining. I show that recursive models may not be nonparametrically identified: the forward-looking solution can have a myopic equivalent. I provide two sufficient conditions for identification. First, there is identification if there are no unobserved components of utility and there is variation in agreements conditional on the observed states. In this case, the discounting rate can be identified from the association of future states and starting prices. Alternatively, there is identification from an instrument satisfying an appropriate exclusion restriction, in particular only affecting value functions through inflation expectations. In this second case, the discounting rate can be identified from the instrument's joint association with future inflation and starting prices.

The third contribution of this paper is to show that the natural model of Kalai proportional bargaining in multilateral settings implies a moment on net present value (NPV) payments. I study a model of a Nash equilibrium in recursively-defined Kalai proportional bargains over expected NPV payoffs, which I call the *Nash-in-Kalai* bargaining model. In the model, gains from trade are calculated taking as given past outcomes and current strategies, but are recursively defined in terms of negotiations at future states that may be uncertain and influenced by the terms of the current contract. The solution concept coincides with Nash-in-Nash in TU models, such as the microfoundations of Nash-in-Nash and all prior empirical dynamic Nash-in-Nash models.

The Nash-in-Kalai model implies a formula for expected NPV payments that transparently extends the formula generated by TU Nash-in-Nash. Econometric errors can reflect either expectational error from uncertainty or unobserved components of utility, facilitating GMM estimation with appropriate instruments. This prediction carries through in the presence of recursive NTU value functions, unspecified information sets, and even nonstationary equilibria in continuous time. While the main restrictions used to generate this result — that negotiators must share a common discounting rate and unbiased beliefs about future conditions — are substantive, they are often plausible in settings in which the leading sources of uncertainty are shared, and are required to derive an unbiased moment on NPV transfers.

My empirical application, borrowed from the companion paper Dorn (2025a), applies an empirical model in this class using a public record administrative dataset from West Virginia on hospital–insurer contracts, which in other settings have been considered trade secrets. Critically, the data shows that contracts were staggered, with realized terms running from three years to more than one decade. Staggered contracts are only consistent with TU bargaining under myopia, because forward-looking negotiators will foresee the NTU externality of the current contract's terms on the market states at which both sides will negotiate with others while the contract remains in place. The empirical model and estimation strategy generally follow the GMM approach in Ho and Lee (2017), but allows negotiators to balance multiple periods of profits. In the spirit of my second identification result, the discounting rate β is identified by the predictable larger effect of increasing β on NPV transfers for firms with longer-lived contracts and faster price growth.

The estimated model clearly rejects a static view of the world. The existence of staggered contracts rejects a literally static model, and the estimated annual discounting rate of $\beta = 0.899$ rejects the null hypothesis of myopia needed to maintain TU bargaining. To quantify the consequences of incorrectly applying a static model to the true dynamic process, I compare the estimated bargaining parameters to the results of the prevailing static procedure that only uses one year of payments as an input, and assumes that all payments reflect recently-formed agreements. I find that the static Nash-in-Nash approach that uses only 2015 data would underestimate small insurers' bargaining weight by incorrectly assigning small insurers' high prevailing prices to bargaining weights rather than accumulated price growth under long-lived contracts.

The empirical content of the Kalai proportional solution is context-dependent. In settings in which empirical researchers maintain a TU Nash model, the Kalai solution is a generalization of their existing model. This situation includes empirical models of American healthcare and telecommunications (Ho and Lee, 2017, 2019; Crawford et al., 2018), leading models of search-on-the-job (Cahuc et al., 2006; Bagger et al., 2014; Bilal et al., 2022; Jarosch

et al., 2024), the microfoundations of static Nash-in-Nash (Collard-Wexler et al., 2019), and the smaller literature on dynamic Nash-in-Nash (Lee and Fong, 2013; Liu, 2021; Tiew, 2022; Deng et al., 2024). In these settings, empirical researchers implicitly maintain that the main role of a negotiated price is the transfer between the negotiating parties. Dynamic Nash and Kalai NTU extensions of these TU models are likely to have similar predictions, so that Kalai bargaining is likely to be an innocuous way to tractably capture dynamics. I find in my empirical application that this seems to hold for West Virginia hospital–insurer bargaining.

In settings in which researchers currently maintain an NTU model, the Kalai proportional solution will at least matter to interpretation. This includes many static bargaining models in which a negotiated upstream price has a direct and nontransferable effect on quantity supplied (Horn and Wolinsky, 1988; Grennan, 2013; Gowrisankaran et al., 2015; Ghili, 2022) as well as some stationary-equilibrium bargaining models that require calibration in practice (Gertler and Trigari, 2009; Gottfries, 2022).[1] The Kalai solution should remain innocuous as long as the dynamic Pareto frontier is roughly unchanged over time, so that homogeneous solutions like Nash bargaining will imply an approximately-fixed bilateral surplus share. Still, the interpretation of bargaining weights will change, and if the Pareto frontier fails comprehensiveness, then take-it-or-leave-it offers may correspond to interior Kalai weights. In such settings, the most innocuous use of the Kalai solution may be as an approximation to the dynamic NTU Nash solution that justifies the use of moments implied by NTU Kalai.

The Kalai solution is a substantive choice for dynamic models in which the threat of a changing Pareto frontier shape may be a relevant mechanism. For example, imagine two players negotiate a two-period agreement, knowing that the Pareto frontier will become more steep tomorrow. Under Nash bargaining with positive discounting, player 1 can improve their current agreement by committing to not bargain under disagreement. Kalai bargaining

[1] An aside on notation. I follow the coalitional bargaining literature and use "NTU" to refer to the generalization of TU bargaining problems to asymmetric or nonlinear Pareto frontiers (Mas-Colell and Hart, 1996). In matching problems, "NTU" is used to refer to extreme cases in which no compensation is possible, and "imperfectly transferable utility" is used to refer to cases that this paper would call NTU (Galichon et al., 2019). The matching NTU definition rules out bargaining, so I use the more general coalitional NTU definition.

ensures that such commitment will have exactly zero effect (Jarosch et al., 2024). In such a setting, dynamic Nash bargaining is particularly difficult to capture and will be at odds with a static bargaining model; the Kalai solution will be a priori plausible and much more tractable, but will impose a substantive restriction on behavior.

While this work is aimed at deriving a tractable solution concept for practice, my analysis leverages important theoretical work. My IIA uniqueness result is essentially a gentle extension of Myerson (1981)'s work. Kalai (1977) and Roth (1979) provide a more thorough treatment of step-by-step in Nash's bargaining problem; my characterization of the Nash-in-Kalai moment leverages applying this property to arrive at Binmore et al. (1989);s impasse point as an equivalent disagreement value. My work on (non)identification relates to work on dynamic discrete choice models (Abbring, 2010). The first set of identification conditions are motivated by a remark by Rust (1994) that in these settings, some identifying power may be derived from "agents who make different choices in the same state," and the second set is motivated by exclusion conditions in Magnac and Thesmar (2002) and Abbring and Daljord (2020), but is specifically motivated by inflation expectations. In dynamic discrete choice problems, the first set of conditions is known to be insufficient for identification (Rust, 1994) and the second condition has no natural analog. This analysis and model also indirectly to theoretical work on recursive bargaining in bilateral (Sorger, 2006; Flamini, 2020; Dutta, 2021) and multilateral (Do and Miklós-Thal, 2022; Deng et al., 2024) settings.

The plan of the paper is as follows. Section 2 reviews some key concepts for bilateral bargaining in utility space and provides some (non)identification results for NTU bargaining with possible uncertainty in a single-period world. Section 3 illustrates the step-by-step property's value in the context of recursive bilateral bargaining and derives sufficient conditions for identification with multiperiod agreements. Section 4 extends the setting to include strategic interactions with more than two agents, defines the Nash-in-Kalai model, and derives a moment on expected NPV transfers. Section 5 describes the empirical application. Section 6 concludes.

Notation. V and v (and occasionally W and w) refer to value functions, with lower-case being used if there is only one relevant input value and that value can be left implicit; A and D superscripts refer to paths following agreement and disagreement, respectively, with AI and DI referring to paths that involve entering impasse after any current agreement conclude; f refers to bargaining solutions, with τ referring to bargaining weights that index solutions; in theoretical sections, G and $\mathcal{G}$ refer to individual bargaining problems and sets of problems, respectively, while in the empirical application, $\mathcal{G}$ refers to a network of agreements; Pay refers to flow transfers between players, with those transfers sometimes governed by prices p; stars refer to equilibrium outcomes; u refers to flow utility before any such transfers; X or x refer to observable states; ϕ refers to observable inflation of prices or payments; ε and i refer to states that the players but not econometrician observe, with i referring to information that only affects value functions through its association with future transitions; primes refer to derivatives or next-period states; bars and hats refer to putative and moment-implied values of parameters, respectively; b refers to a structure of a given game: a set of variables, functions, and strategies σ that define play; $\Leftrightarrow$ refers to observational equivalence between specific structures b or the predictions of bargaining solutions f applied to a set of structures B; t refers to a time period, with a bargain of interest formed in t_0 and ending in period t^* or $t_0 + T - 1$ and a shared discounting rate β; ℓ refers to remaining length; r_i and R_{it} refer to i's per-bargain negotiation cost and number of new bargains, respectively; h refers to the game's state, which is always observed by all players and where the period-$t+1$ state h_{t+1} or h' is finalized at the end of period t; a refers to non-bargaining actions which are elements of in spaces A; $\mathbb{C}$ refers to bargaining outcomes (contracts) which are elements of spaces $\mathcal{C}$, and where 0 subscripts refer to null or disagreement contracts; the Δ_{ij} and Δ_{ijt_0} operators refer to the effect of the ij agreement on a given variable, with the latter operator being used when the effect depends on when disagreement begins; and other Greek letters refer to variables whose definition will only be relevant for a short time.

2 Starting Point: Single-Period Bilateral Bargaining

To most readers, the Nash-in-Kalai model will suggest several questions:

(1) What is Kalai proportional bargaining?

(2) Why is Kalai proportional bargaining only sometimes the same as Nash bargaining?

(3) Why is an alternative to Nash bargaining needed for dynamic games? and

(4) Why might Kalai proportional bargaining be the right alternative?

I answer the first two questions by reviewing some key concepts for static bargaining in utility space. I answer the last two questions with new results on identification for two agents negotiating over a price to apply for their world's single period.

2.1 What is Kalai Proportional Bargaining?

The Kalai proportional bargaining solution is a solution concept for Nash's bargaining problem. This work focuses on bilateral bargaining for simplicity.

A Nash bargaining problem G (for game) is a closed set V of feasible agreement values in $\mathbb{R}^2$ and a disagreement value $v^D \in V$. I use "$\geq$" and "$>$" to refer to the pointwise comparison: $v \geq v'$ if and only if $v_1 \geq v'_1$ and $v_2 \geq v'_2$, and analogously $v > v'$ means $v_1 > v'_1$ and $v_2 > v'_2$. I refer to a set of problems (V, v^D) as a family $\mathcal{G}$, and will make a few regularity assumptions.

Definition 1 (Regularity conditions)**.** *A game* (V, v^D) *satisfies* convexity *if for all* $v_1, v_2 \in V$ *and* $\lambda \in [0, 1]$, $\lambda v_1 + (1-\lambda)v_2 \in V$. *It satisfies* comprehensiveness *if* $v \in V$ *and* $v^D \leq v' \leq v$ *implies* $v' \in V$.

Convexity is an important regularity condition for bargaining games (Shimer, 2006), and generally reduces a bargaining problem to a Pareto frontier and disagreement value.

Comprehensiveness is sometimes called free disposal; comprehensiveness of the convex hull of the Pareto frontier and disagreement point ensure that the Kalai proportional solution is well-defined (Roth, 1979).

Within a bargaining game, there are many ways of choosing an allocation that makes both sides better off than disagreement. A particular bargaining solution is a function $f : \mathcal{G} \to \mathbb{R}^+$ such that $f(V, v^D) \in V$ and $f(V, v^D) \geq v^D$. I write that $f' = f$ on $\mathcal{G}$ if for all $(V, v^D) \in \mathcal{G}$, $f(V, v^D) = f'(V, v^D)$, and shorthand $f' = f$ if the two are equal on $\mathcal{G}$ when the family is clear from context.

Definition 2 (Kalai proportional bargaining)**.** *The* Kalai proportional bargaining solution *with player-1 weight* $\tau \in [0, 1]$ *is* $f_\tau^{(Kalai)}(V, v^D) = (\max \lambda : (\lambda\tau, \lambda(1-\tau)) \in V) * (\tau, 1-\tau)$.

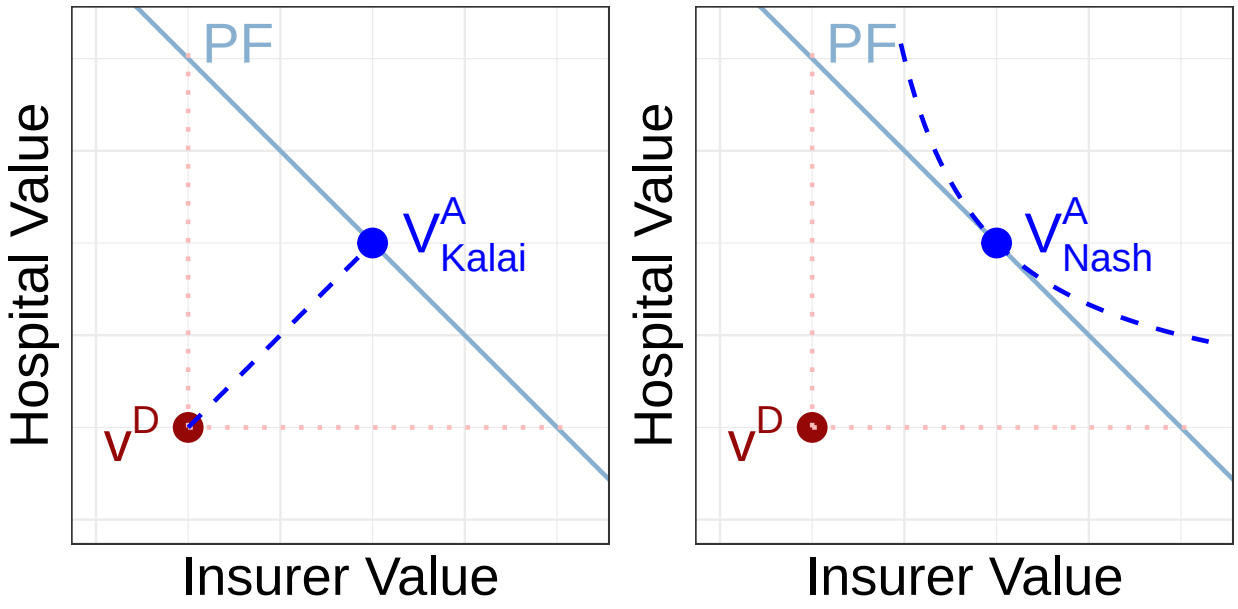


Figure 1: Left: the Kalai proportional bargaining solution for bargaining between an insurer (x axis) and hospital (y axis) with Pareto frontier PF in light blue and disagreement value v^D in red, and Pareto constrained indicated by dotted lines. The desired-surplus-split ray $v = v^D + x(1-\tau)/\tau$ is indicated by the dashed blue ray, and the resulting agreement is indicated by v^A_{Kalai}. Right: the Nash bargaining solution to the same game. The dashed line indicates the Nash bargaining indifference curve, and the resulting agreement is indicated by v^A_{Nash}.

The Kalai proportional solution for bargaining between a hospital (player 2) and insurer (player 1) is illustrated in Figure 1(a). Draw a dashed ray to indicate the values $v \in V$ that

yield positive gains from trade and the desired surplus split $\tau(v_2 - v_2^D) = (1-\tau)(v_1 - v_1^D)$. The solution V^A_{Kalai} is the best agreement along this desired-surplus-split ray.

2.2 Why are Kalai and Nash Bargaining Only Sometimes the Same?

Definition 3 (Nash bargaining)**.** *The Nash bargaining solution with player-one bargaining weight* $\tau \in [0,1]$ *is* $f_\tau^{(Nash)}(V, v^D) = \text{argmax}_{v \in V} \left(v_1 - v_1^D\right)^\tau (v_2 - v_2^D)^{1-\tau}$.

The Nash bargaining solution is illustrated in Figure 1(b). The Nash bargaining solution can be viewed as maximizing a joint utility function, with player one receiving greater weight. The joint utility function generates a joint indifference curve. In well-behaved problems with negotiations over a price, the Nash bargaining first-order condition at the agreed price p^*_{Nash} can be rewritten as:

$$\tau v_1'(p^*_{\text{Nash}})(v_2^* - v_2^D) = -(1-\tau)v_2'(p^*_{\text{Nash}})(v_1^* - v_1^D).$$

Kalai and Nash bargaining will have the same predictions when bargaining is TU: if the Pareto frontier is a line segment with a constant slope of negative one.

Definition 4 (Transferable utility)**.** *A family of games* $\mathcal{G}$ *satisfies* transferable utility *(TU) if for every* $(V, v^D) \in \mathcal{G}$*, there is a* $\lambda > 0$ *such that the set of* $v \in V$ *with* $v \geq v^D$ *is the convex hull of* v^D*,* $v^D + (\lambda, 0)$*, and* $v^D + (\lambda, v)$*.*

Equivalence follows by algebra.

Lemma 1 (Nash and Kalai coincide for TU games)**.** *Suppose* $\mathcal{G}$ *satisfies TU. Then* $f_\tau^{(Nash)} = f_\tau^{(Kalai)}$ *on* $\mathcal{G}$*.*

Proof. See Appendix A.3 for the short proof. □

Some important advantages of Nash bargaining are that it is weakly Pareto-optimal and satisfies independence of irrelevant alternatives and scale-invariance (Serrano, 2005).

Definition 5 (Bargaining properties)**.** *For $\alpha \in \mathbb{R}^d$ for $d = 1$ or 2, let $\alpha V = \{\alpha v : v \in V\}$. Also let $\mathcal{G}$ be the set of games that are convex and comprehensive. A bargaining solution f is (i)* weakly Pareto-optimal *(WPO) if for all $(V, v^D) \in \mathcal{G}$, there is no $v \in V$ such that $v > f(V, v^D)$; (ii) satisfies* independence of irrelevant alternatives *(IIA) if $(W, v^D), (V, v^D) \in \mathcal{G}$, $W \subseteq V$, and $f(V) \in W$ implies $f(W, v^D) = f(V, v^D)$; and (iii) is* scale-invariant *if for all diagonal 2-by-2 matrices Σ with positive diagonal elements and all $(V, v^D) \in \mathcal{G}$, $f(\Sigma V, \Sigma v^D) = \Sigma f(V, v^D)$.*

WPO is a minimal condition for a bargaining solution to be plausible. IIA is a necessary condition for f to maximize some notion of utility (Myerson, 1981).[2] I slightly abuse notation and write that if f satisfies IIA, $(V, v^D) \in \mathcal{G}$, and $f(V, v^D) \in W \subseteq V$, then $f(W, v^D) = f(V, v^D)$ even if W is not comprehensive. Scale invariance implies that if one modifies the scale of one side's gains from trade by a fixed constant, then the other side's outcome is unaffected.

Kalai proportional bargaining is not scale invariant, so it can only coincide with Nash bargaining within a family of games like TU that implicitly define a scale of relative utility. The value of scale invariance is more subtle than it seems at first glance: scaling player 2's utility by $\lambda > 0$ and the Kalai proportional bargaining weight ratio $\tau/(1-\tau)$ by λ^{-1} yields the same implied outcomes from the perspective of player 1, and evidence from the lab suggests the negotiators can be scale-varying in monetary games (Nydegger and Owen, 1974; Duffy et al., 2021). Instead, the most impactful consequence from losing scale-invariance that I have found is that the bargaining solution cannot be microfounded in terms of von Neumann-Morgenstern utility alone.[3] Two important conditions satisfied by Kalai proportional and not Nash bargaining solutions are concavity and step-by-step, which I discuss later.

This completes the review of static bargaining in utility space. On to dynamic games,

[2]The IIA axiom can be replaced by lower-level conditions that are outside the scope of this work (Thomson and Myerson, 1980).

[3]I present a microfoundation of the Nash-in-Kalai model in the companion note Dorn (2025b). The microfoundation requires some device that fixes relative utility scales: I follow Dutta (2012) and use revocation costs. This is the first microfoundation for either Nash-in-Kalai or Nash-in-Nash that allows the NTU case in which the two disagree.

where I show that scale-invariance makes identification of bargaining parameters more difficult.

2.3 Why is an Alternative to Nash Bargaining Needed?

Many empirical dynamic models include the possibility of uncertainty at the time of contracting. It is therefore often desirable to analyze a model in which agents have access to some, but potentially not all, information beyond the dataset at hand. It turns out that when the econometrician wishes to be agnostic about the information set available at the time of bargaining, there may not be a way to identify Nash bargaining weights under NTU.

It is easiest to quantify the role of uncertainty in econometric modeling in a single-period game. Consider the following class of games:

1. If $EA = 1$ (for ex ante), the players ($i = 1, 2$) have the opportunity to negotiate over a price $p \in \mathbb{R}$ using bargaining solution f.

2. The game type X is drawn from $Unif(0, 1)$ and revealed to the players.

3. If $EA = 0$, the players have the opportunity to negotiate over a price $p \in [0, 1]$.

4. If the sides have an agreement ($A = 1$), then player i receives utility $u_i(p \mid X) > 0$. If the sides have no agreement ($A = 0$), then both players receive v^D.

5. The econometrician observes the distribution P, the game type X, the utility functions u_1 and u_2, the disagreement payoff v^D, and the price p^*.

For example, when $EA = 1$, the sides may negotiate over a price whose effect on profits depends on uncertain demand shocks.

It would be convenient for empirical practice to have a moment of the form $E[g(p, X; \tau)] = 0$ to infer Nash bargaining weights. One type of moment might involve marginal costs (e.g. Gowrisankaran et al., 2015), which corresponds here to marginal utility. The econometrician

may then leverage the ex-ante or ex-post moments on putative bargaining weights $\bar{\tau}$:

$$m_{EA}(\bar{\tau}) = E\left[u_2'(p \mid X) - \frac{\bar{\tau}}{1-\bar{\tau}} \frac{E[u_2(p \mid X) - v_2^D]E[-u_1'(p \mid X)]}{E[u_1(p \mid X) - v_1^D]}\right]$$
$$m_{EP}(\bar{\tau}) = E\left[u_2'(p \mid X) - \frac{\bar{\tau}}{1-\bar{\tau}} \frac{\left(u_2(p \mid X) - v_2^D\right)(-u_1'(p \mid X))}{u_1(p \mid X) - v_1^D}\right].$$

The validity of these moments will depend on getting the timing of information right. If bargaining occurs ex ante before uncertainty over the shape of an NTU Pareto frontier is resolved, the players' expected gains from trade depend on making comparisons of the relative scales of their feasible utility across potential states of the world, so that affine transformation of one player's utility in some states can generate a different outcome for the other player. In contrast, if bargaining occurs ex post after the shape of the Pareto frontier is realized, the scale-invariance property of Nash bargaining ensures that the Nash solution is invariant to the such affine transformation. This creates ambiguity from the perspective of the econometrician that does not observe information sets: outcomes that are more favorable to player 1 could reflect a better bargaining weight, or they may reflect information on utility transformations being revealed in a way that systematically helps that player. As a result, bargaining weights estimated based on an incorrect conjecture of information timing can be systematically biased.

For an example of how unobserved information sets on utility nontransferability can introduce bias, consider games for which the observable state X parameterizes pre-transfer gains π_i and the marginal utility of price $\delta_i > 0$. With staggered contracts, a negotiated bilateral price may have internalized contracting externalities on other prices that are asymmetric, so that δ_1 may not equal δ_2.

Proposition 1 (Absence of a robust Nash moment condition)**.** *Consider the outcome of bargaining games with $EA = 0$, the bargaining solution $f_\tau^{(Nash)}$ for some $\tau \in (0,1)$, and utility $u_1(p \mid X) = \pi_1 - \delta_1 p$, $u_2(p \mid X) = \pi_2 + \delta_2 p$, and $v^D = 0$ for some random $\pi_i \in \mathbb{R}, \delta_i > 0$ with appropriate moment conditions. Suppose $\frac{E[\delta_1]}{E[\delta_2]} E\left[\pi_1\left(\frac{\delta_2}{\delta_1} - \frac{E[\delta_2]}{E[\delta_1]}\right)\right] \neq E\left[\pi_2\left(\frac{\delta_1}{\delta_2} - \frac{E[\delta_1]}{E[\delta_2]}\right)\right]$.*

Then $m_{EP}(\tau) = 0$*, but* $m_{EA}(\tau) \neq 0$*.*

Proof. By construction, $m_{EP}(\tau) = 0$. $m_{EA}(\tau)$ is well-defined under appropriate moment conditions. Note that $p^*_{EP} = (1-\tau)\frac{\pi_1}{\delta_1} - \tau\frac{\pi_2}{\delta_2}$. Then:

$$
\begin{aligned}
m_{EP}(\tau) &= E\left[\delta_2 - \frac{\tau}{1-\tau}\frac{E\left[(1-\tau)(\pi_2 + \pi_1(\delta_2/\delta_1)\right]E\left[\delta_1\right]}{\tau E\left[\pi_1 + \pi_2(\delta_1/\delta_2)\right]}\right] = E\left[\delta_2 - \frac{E\left[\pi_1\frac{\delta_2}{\delta_1} + \pi_2\right]E\left[\delta_1\right]}{E\left[\pi_1 + \pi_2(\delta_1/\delta_2)\right]}\right] \\
&= E\left[\delta_2 - \frac{E[\pi_2]E[\delta_1] + E[\pi_1]E[\delta_2] + E\left[\pi_1\left(\frac{\delta_2}{\delta_1} - \frac{E[\delta_2]}{E[\delta_1]}\right)\right]E\left[\delta_1\right]}{E[\pi_1] + E[\pi_2]\frac{E[\delta_1]}{E[\delta_2]} + E\left[\pi_2\left(\frac{\delta_1}{\delta_2} - \frac{E[\delta_1]}{E[\delta_2]}\right)\right]}\right] \\
&\neq E\left[\delta_2 - \frac{E[\pi_2]E[\delta_1]E[\delta_2]/E[\delta_2] + E[\pi_1]E[\delta_2] + E[\delta_2]E\left[\pi_2\left(\frac{\delta_1}{\delta_2} - \frac{E[\delta_1]}{E[\delta_2]}\right)\right]}{E[\pi_1] + E[\pi_2]\frac{E[\delta_1]}{E[\delta_2]} + E\left[\pi_2\left(\frac{\delta_1}{\delta_2} - \frac{E[\delta_1]}{E[\delta_2]}\right)\right]}\right] = 0.
\end{aligned}
$$

□

The term $E\left[\pi_1\left(\frac{\delta_2}{\delta_1} - \frac{E[\delta_2]}{E[\delta_1]}\right)\right]$ roughly captures the correlation of player one's pre-transfer gains with the realized Pareto frontier slope δ_2/δ_1. If such a correlation is large for one player and not the other, then a moment based on the implied ex-ante marginal costs will be biased if the prices are negotiated ex-post, regardless of the (interior) bargaining weight. A similar result can also be derived if prices are negotiated ex-ante, but some care must be taken to ensure $m_{EP}(\tau)$ is well-defined.

Proposition 1 shows that under dynamic NTU Nash bargaining, a proposed moment's validity will often hinge on getting the structure information correct. This may just be an inconvenience: there can be a more clever strategy to recover identification. However, it turns out that no such clever identification strategy can exist in general.

The rest of this section focuses on a minimal condition for deriving a robust moment for estimation of bargaining parameters: whether a bargaining solution can generally be identified in these single-period games. I try to adapt notation from Magnac and Thesmar (2002)'s analysis of identification of dynamic discrete choice problems. In this section, a structure b is a value of EA, a distribution P, utility functions u, and strategies σ such that σ

is the output of applying a bargaining solution f to the generated bargaining problem. I write that structures b and b' are observationally equivalent if they generate the same observables: u_1, u_2, P and the joint distribution of (X, p^*). I shorthand observational equivalence as $b \Leftrightarrow b'$. I write that bargaining solutions f and f' are observationally equivalent if for every b with the bargaining solution f, there is a b' with the same EA, P, u, the bargaining solution f', and some σ' such that $b' \Leftrightarrow b$. In this case, I write $f \Leftrightarrow f'$. I abuse notation and write $f(b)$ for the set of feasible outcomes after replacing the bargaining solution in b with f, and write $f(b) \Leftrightarrow f'(b')$ if the intersection of $f(b)$ and $f'(b')$ is nonempty.

Definition 6 (Single-period identification)**.** *Let $\mathcal{F}$ be a family of bargaining solutions. I write that $\mathcal{F}$ is* single-period identified *if $b \Leftrightarrow b'$ implies $f(b) \Leftrightarrow f(b')$ for structures b, b' such that the convex hull of $\{(u_1(p \mid X), u_2(p \mid X))\} \cup \{0\}$ is comprehensive. I write that $\mathcal{F}$ is* single-period unidentified *if $\mathcal{F}$ is not single-period identified.*

If $\mathcal{F}$ is single-period unidentified, then an econometrician who mistakenly conjectures that bargaining is ex post ($EA = 0$) can infer a bargaining solution that has the wrong counterfactual prediction even with a known information structure.

For these single-period games, it is possible to construct a game for which incorrectly conjecturing EA yields exact predictions, but an arbitrarily wrong estimate of the Nash bargaining weight τ.

Proposition 2 (Nash bargaining weights may not be identified under NTU)**.** *Let $\mathcal{F}$ be a family of bargaining solutions that include $f_\tau^{(Nash)}$ and $f_{\tau'}^{(Nash)}$ for some distinct $\tau, \tau' \in (0, 1)$. Then $\mathcal{F}$ is single-period unidentified.*

Proof. Without loss of generality assume $\tau > \tau'$. Consider the utility functions from Proposition 1 with $\pi_1 = 1$, $\delta_1 = X^{-s}/\tau$, $\pi_2 = (1-\tau)X^s$, and $\delta_2 = 1$ for $\lambda = \sqrt{\frac{\tau - \tau'}{\tau(1-\tau')}}$, and either (i) the structure b with $EA = 0$ and $f = f_\tau^{(Nash)}$, (ii) the structure b' with $EA = 1$ and $f = f_{\tau'}^{(Nash)}$, or (iii) the structure b'' with $EA = 1$ and $f = f_\tau^{(Nash)}$.

By the scale-invariance of Nash bargaining, the structure b generates a constant price of zero. By construction of λ, the structure b' generates a constant price of zero. Therefore $b \Leftrightarrow b'$. But by inspection, $b' \not\Leftrightarrow b''$. Therefore $\mathcal{F}$ is single-period unidentified. □

The result shows that there is no clever combination of information, much less a moment, that can suffice to identify Nash bargaining weights in all cases. In more complicated games with multiple periods and interactions, constructing a valid moment to recover bargaining parameters is even more difficult.

The proof of Proposition 2 only uses the scale-invariance property of Nash bargaining: intuitively, bargaining before the realization of uncertainty requires comparing utility scales across states of the world. As a result, Proposition 2 establishes that there cannot be a robust moment for any family of bargaining solutions that contains two scale-invariant solutions with distinct interior predictions for TU games.

In the next subsection, I show that the Kalai proportional solution produces a robust moment for identification of bargaining weights, and in fact is the only family of bargaining solutions satisfying IIA that possess such a moment. The Kalai solution aligns with Nash bargaining in TU cases, which may be part of why Nash bargaining models have been successful in the empirical dynamic bargaining literature specifically in the case of TU.

2.4 Kalai Allows Identification Across Information Timing

Under Kalai proportional bargaining, there is a simple moment on payments that is robust to the structure of information.

Lemma 2 (Existence of a robust Kalai proportional moment)**.** *Consider a game* $(e, j, f_\tau^{\text{Kalai}})$ *such that the convex hull of the realized Pareto frontier and* v^D *is comprehensive almost surely. Write the realized price as* p^**. Then* $\tau E[u_2(p^* \mid X) - v_2^D] = (1-\tau)E[u_1(p^* \mid X) - v_1^D]$.

Proof. The comprehensiveness requirement ensures the Kalai proportional solution is well-defined across values of e. For games with $e = 1$, the claim is immediate. For games with

$e = 0$, the realized price satisfies $\tau(u_2(p^* \mid X)) = (1-\tau)(u_1(p^* \mid X))$ almost surely, so that the expected transfer holds by iterated expectations. □

Proposition 2 and Lemma 2 establish a trade-off. On the one hand, scale-invariant bargaining solutions like Nash bargaining may be theoretically advantageous. However, they cannot provide a moment on observable quantities that is robust to the structure of information. The Kalai proportional solution is not scale-invariant, but it provides such a robust moment.

One might wonder what other bargaining solutions also possess a robust moment for identification. It turns out that no other family of bargaining solutions that satisfy IIA can provide a moment that is robust to the structure of identification. The proof uses arguments from Myerson (1981). In particular, I make the following restriction.

Assumption 1 (IIA and span)**.** $\mathcal{F}$ is a family of bargaining solutions that satisfy IIA; are distinct in the sense that no separate $f, f' \in \mathcal{F}$ satisfy $f = f'$ for all G generated by single-period games; and span all Pareto-efficient outcomes in the sense that if v is Pareto optimal in some V, then for every $v^D < v$, there is an $f \in \mathcal{F}$ such that $f(V, v^D) = v$.

The distinctness assumption ensures that $f \Leftrightarrow f'$ only if the two solutions are the same, and is required for $\mathcal{F}$ to be single-period identified. The span assumption ensures the econometrician can always infer a bargaining weight from a Pareto-efficient agreement; a family that fails the span assumption is not amendable to empirical use.

Theorem 1 (IIA and single-period identification implies Kalai proportional bargaining)**.** *Suppose $\mathcal{F}$ is a family of bargaining solutions satisfying Assumption 1. Then $\mathcal{F}$ is single-period identified if and only if $\mathcal{F}$ is the family of Kalai proportional bargaining solutions.*

This result shows that no other family of bargaining solutions satisfying IIA can permit identification via a moment across information timing: any alternative family satisfying IIA would not even be identified.

The proof rests on several distinct claims, some of which may be of independent interest. I reach the full proof on Page 21.

An important condition for identification is *concavity*. Myerson (1981) defines concavity for more general social choice problems which lack a notion of disagreement. Bargaining solutions are a subset of social choice functions applied to a subset of social choice problems, so the concept can be extended to bargaining games.

Definition 7 (Concavity)**.** *For bargaining games $G = (V, v^D)$ and $G' = (W, u^D)$ and $\lambda \in [0,1]$, define $\lambda G + (1-\lambda)G' = \left(\lambda V + (1-\lambda)W, \lambda v^D + (1-\lambda)u^D\right)$. A bargaining solution f satisfies concavity if for all G, G' such that $\{G, G'\}$ satisfies convexity and comprehensiveness and all $\lambda \in [0,1]$, $f(\lambda G + (1-\lambda)G') \geq \lambda f(G) + (1-\lambda)f(G')$.*

Intuitively, the concavity property is that ex ante bargaining over expected values before the Pareto frontier is realized should produce weakly better outcomes for all players than the expected value of ex post bargaining after the Pareto frontier is realized.

Concavity is necessary for identification.

Lemma 3 (Single-period identification implies concavity)**.** *Suppose $\mathcal{F}$ is a family of bargaining solutions satisfying Assumption 1, and such that there is an $f \in \mathcal{F}$ that is not concave. Then $\mathcal{F}$ is single-period unidentified.*

Proof. At a high level, suppose there is an f, G, G' such that player two strictly prefers the ex post outcome: $\lambda f + 2(G) + (1-\lambda)f_2(G') > f_2(\lambda G + (1-\lambda)G')$. By WPO, player one must weakly prefer the ex ante outcome. By IIA and span (see Appendix A.3 for details), there is a related game for which the ex post outcome under f is observationally equivalent to the ex ante outcome under some $f' \in \mathcal{F}$ that is more favorable to player two. □

Concavity is unique to two classes of bargaining solutions: utilitarian bargaining and proportional bargaining.

Lemma 4 (Concavity and IIA implies utilitarian or egalitarian)**.** *Suppose f satisfies WPO, IIA, and concavity. Then either (i) $f \Leftrightarrow f_\tau^{\text{Kalai}}$ for some $\tau \in [0,1]$, or (ii) f is utilitarian in*

the sense that there is a $\tau \in [0,1]$ *such that* $f(V, v^D) \in \text{argmax}_{v \in V : v \geq v^D} \tau(v_1 - v_1^D) + (1 - \tau)(v_2 - v_2^D)$ *for all* V, v^D.

Proof. The claim is a gentle extension of Myerson (1981)'s Theorem 2, but requires care to handle disagreement constraints. The details are in Appendix A.3. □

Thomson (1994) provides more background on utilitarian bargaining. A simple property is that it cannot identify bargaining weights in single-period games.

Lemma 5 (Single-period identification implies not utilitarian)**.** *Suppose for* $\tau \in (0,1), \tau' \in [0,1]/\tau$*, there are* $f_\tau^{(Util)}, f_{\tau'}^{(Util)} \in \mathcal{F}$ *with* $f_\alpha^{(Util)}(V, v^D) \in \text{argmax}_{v \in V : v \geq v^D} \alpha(v_1 - v_1^D) + (1 - \alpha)(v_2 - v_2^D)$ *for all* V, v^D*. Then* $\mathcal{F}$ *is single-period unidentified.*

Proof. Without loss of generality assume that $\tau, \tau' < 1$. Write $f = f_\tau^{(Util)}$ and $f' = f_{\tau'}^{(Util)}$. Consider the games b and b' with $EA = 0$ and f and f', respectively, applied to $u_1(p \mid X) = 1 - p$ and $u_2(p \mid X) = \frac{\min\{\tau,\tau'\}/2}{1-\min\{\tau,\tau'\}/2} p$. Then $p^* = 0$ under both games, so $b \Leftrightarrow b'$. Now let b'', b''' be f and f', respectively, applied to $u_1(p \mid X) = 1 - p$ and $u_2(p \mid X) = \frac{(\tau+\tau')/2}{1-(\tau+\tau')/2} p$. Then $b'' \not\Leftrightarrow b'''$, so $f \not\Leftrightarrow f'$ and $\mathcal{F}$ is single-period unidentified. □

I am now ready to prove Theorem 1.

Proof of Theorem 1. Direction 1: if $\mathcal{F}$ is single-period identified, then it is the family of Kalai proportional solutions. Proof by contradiction. By Lemma 3, if $\mathcal{F}$ is single-period identified, then each element is concave. By Lemma 4 and IIA, each element is utilitarian and/or proportional. By Lemma 5, there is an $m > 0$ such that the game $EA = 0$, $u_1(p) = mp$, $u_2(p) = 1 - p$ has $p^* \in \{0, 1\}$ for all solutions in $\mathcal{F}$ that are not proportional. Then by the span assumption, for all $\tau \in (0,1)$, $f_\tau^{(Kalai)} \in \mathcal{F}$. Now suppose $f \in \mathcal{F}$ is utilitarian and not proportional. By single-period identification and $f_\tau^{(Kalai)} \in \mathcal{F}$ for interior τ, if $f \in \mathcal{F}$ is not proportional, f must predict gain-from-trade shares of either one or zero always. But then since f is utilitarian, f is also proportional. Therefore $\mathcal{F}$ only includes proportional solutions, and by the span assumption, $\mathcal{F}$ is the family of proportional solutions.

Direction 2: if every $f \in \mathcal{F}$ is proportional, then f is single-period identified. Let data be generated as in Section 2.3 from some $f \in \mathcal{F}$ with bargaining weight τ. Let $\hat{\tau}$ solve $\hat{\tau}E[u_2(p^* \mid X)] = (1-\hat{\tau})E[u_1(p^* \mid X)]$. By WPO and the existence of a p generating strict gains from trade, at least one of $E[u_1(p^* \mid X)]$ and $E[u_2(p^* \mid X)]$ must be strictly positive. Therefore $\hat{\tau}$ is unique. By Lemma 2, $\hat{\tau} = \tau$. It remains to show that no other bargaining solution in $\mathcal{F}$ is observationally equivalent. Suppose $f' \in \mathcal{F}/f$. Then by Lemma 2, f' generates a different moment. Therefore $f \not\Leftrightarrow f'$. □

The remainder of this work applies the Kalai proportional solution to a multiperiod world.

3 Multiperiod Bilateral Bargaining and Recursiveness

This section focuses on the added challenge of recursiveness in a multiperiod world. For simplicity, I focus on two agents Kalai proportionally bargaining over a TU Pareto frontier. I first show that the Kalai model is nonparametrically unidentified with multiperiod agreements. I then describe a static utility property called step-by-step, which simplifies recursive bargaining and allows me to provide sufficient conditions for identification.

In this TU setting, the Nash and Kalai proportional solutions coincide. As a result, the (non)identification results here also apply to TU Nash bargaining. However, the tool that I use to prove the sufficient conditions for identification, which is called step-by-step, will also be a benefit of the Kalai proportional solution in recursively defined NTU bargaining games.

3.1 The Multiperiod Kalai Model is Only Identified with Single-Period Agreements

Consider the following class of infinitely-lived TU bargaining games. Period t is as follows:

1. Players 1 and 2 learn their period utility shocks $\varepsilon_t = (\varepsilon_{t1}, \varepsilon_{2t})$, observable state x_t in a discrete space $\mathcal{X}$ with n distinct values in the support, unobservable information i_t,

and inflation rate $\phi_t > 0$. I write $h_t = (\varepsilon_t, x_t, i_t, \phi_t)$.

2. If the last period ended with a contract in place ($\ell_{t-1} > 1$), then the transfer is $Pay_t = \phi_t Pay_{t-1}$, the remaining length is $\ell_t = \ell_{t-1} - 1$, and I write $R_t = 0$ for no new negotiation.

3. If last period ended with an expiring or expired contract ($\ell_{t-1} \leq 1$), then the players can bargain by mutual assent and choose a starting transfer Pay_t for a T-period agreement (so that $\ell_t = T$ and $R_t = 1$) or can disagree (so that $Pay_t = 0$, $\ell_t = 0$, and $R_t = 0$).

4. *Utility and transition.* Player i gets flow utility $1\{\ell_t > 0\}(u_i(x_t) + \varepsilon_i + Pay_t(2i-3) - r_i R_t)$, where r_i is player i's new-contract negotiating cost. The new state h_{t+1} is drawn from some fixed distribution $P(h' \mid h)$ and the econometrician observes (Pay, ℓ, x).

I discuss negotiation cost timing in Section 4.1.4. I implicitly impose that h follows a Markov process.

I proceed assuming certain regularity conditions.

Assumption 2 (Magnac and Thesmar (2002))**.** $E[\varepsilon_{t,i} \mid x] = 0$, the agents calculate value using the true law of motion $P(h' \mid h)$. Further, $P(\varepsilon', x', \phi', i' \mid h) = P(\varepsilon' \mid x, \phi) P(x', \phi', i' \mid x, \phi, i)$.

These are essentially the regularity conditions maintained by Magnac and Thesmar, but allowing the possibility of unobserved information about future states.

For simplicity, I follow Collard-Wexler et al. (2019) and focus on games in which agreements are always formed. Disagreement based on ε introduces a selection bias that can be corrected with distributional knowledge, but which is outside the scope of this work. Even in the case of $T = 1$ and no unobserved utility, it is impossible to separately identify the levels of $u_i(x)$ from r_i: one can increase u_i and r_i to achieve the same real outcome. With

T-period agreements, it turns out that one can only hope to identify

$$\tilde{u}_i(x) \equiv u_i(x) - r_i / \sum_{t=1}^{T} \beta^{t-1}. \tag{3}$$

Similarly, even without unobserved utility, payment data cannot separately identify the components of

$$\tilde{Pay}(x) \equiv -\tau \tilde{u}_1(x) + (1-\tau)\tilde{u}_2(x). \tag{4}$$

I implicitly assume that the researcher has access to instruments that can identify bargaining weights from $\tilde{Pay}$. In Section 2, these instruments are the realized utility functions.

A structure b is a set of current-period effective utility functions $\tilde{u}_i(x)$, information i, next-period value functions $V_i(Pay, h, \ell)$ for $0 \leq \ell' \leq T$ and $i = 1, 2$, discounting rate $\beta \in (0, 1)$, and new-contract choice functions $d^*(h) = (\ell^*(h), Pay^*(h))^T$ such that if $\ell > 0$,

$$\begin{aligned} V_i(Pay, h, \ell) &= 1\{\ell > 0\} \left(u_i(x) + \varepsilon_i + (2i-3)Pay\right) + \beta E[V_i(\phi' Pay', h', \ell - 1) \mid h], \\ V_i(Pay^*(h), h, 0) &= \max\left\{V\left(Pay^*(h), h', \ell^*(h)\right) - r, \quad \beta E[V_i(0, h', 0) \mid h]\right\}. \end{aligned}$$

A structure b is a *recursive Kalai solution* with bargaining weight τ if $V_i\left(Pay^*(h), h, \ell^*(h)\right) - r_i$ always reflects player i's outcome from applying the Kalai proportional bargaining solution to Nash's bargaining problem for Pareto frontiers of feasible $(V_1 - r_1, V_2 - r_2)$, disagreement point $\beta E[V_i(Pay, h', 0) \mid h]$, and player-one bargaining weight τ.

Definition 8 (Multiperiod identification)**.** *I write that two structures b, b' are* observationally equivalent *if $P(Pay', \ell', x', \phi' \mid Pay, \ell, x, \phi)$ are the same under b and b', in which case I write $b \Leftrightarrow b'$. For a family of structures B, I write that* traditional bargaining parameters are identified *under B if $b \Leftrightarrow b'$ implies that $\tilde{Pay}(x)$ are the same functions under b and b'. If traditional bargaining parameters are not identified, I write that B is* unidentified*. I write that β* is identified *under B if $b \Leftrightarrow b'$ implies that β is the same under b and b'. If traditional*

bargaining parameters are identified under B, I write that all bargaining parameters are identified *under B if β is also identified and I write that* only β is not identified *under B if β is not identified.*

With single-period Kalai proportional agreements, traditional bargaining parameters (but not the discounting rate β) are identified.

Proposition 3 (Identification of traditional bargaining parameters with single-period contracts)**.** *Let $B^{(1)}$ be the set of structures generated by recursive Kalai bargaining satisfying Assumption 2 with $T = 1$, and such that $\ell^*(h) = 1$ for all (h) in the support of the game. Then only β is not identified under $B^{(1)}$.*

Proof. First, I show that traditional bargaining parameters are identified. By Assumption 2, agreement and disagreement have the same next-period expected value. For any given τ, agreements Pay_t^* must satisfy

$$\begin{aligned} 0 &= -\tau\left(u_2(x) + \varepsilon_2 + Pay_t - r_2\right) + (1-\tau)u\left(\tilde{u}_1(x) + \varepsilon_1 - Pay_t^* - r_1\right) \\ &= \tilde{Pay}(x) - \tau\varepsilon_2 + (1-\tau)\varepsilon_1 - Pay_t^* \\ E[Pay_t^* \mid x] &= \tilde{Pay}(x) + E\left[-\tau\varepsilon_2 + (1-\tau)\varepsilon_1 \mid x\right] = \tilde{Pay}(x), \end{aligned}$$

with the final line following by Assumption 2. Therefore traditional bargaining parameters are identified.

Next, I show that β is not identified. Take $u_1(x) = u_2(x) = 1$, $r = 1/2$, and $\varepsilon = 0$ constant. Let b be the Kalai proportional solution with $\tau = 1/2$ and $\beta = 0$ and let b' be the solution with $\tau = 1/2$ and $\beta = 1/2$. By inspection, $b \Leftrightarrow b'$. Therefore β is not identified. □

With multiperiod agreements, even traditional bargaining parameters may not be identified. This is unsurprising given the lack of identification in dynamic discrete choice models (Rust, 1994).

Proposition 4 (Potential nonidentification and static representation with multiperiod contracts)**.** *Suppose $T \geq 2$, let $B^{(T)}$ be the set of structures generated by recursive Kalai models satisfying Assumption 2 with contract length T and such that $\ell^*(h) = 1$ for all (h) in the support of the game. Then $B^{(T)}$ is unidentified.*

Proof. Let data follow from some structure b with $\beta > 0$, $\varepsilon = 0$, and no uncertainty, so that d^* and Pay^* are deterministic conditional on x. Let b' be a structure with $\beta = 0$, $\tilde{Pay}(x) = E[Pay_t^* \mid x, \ell_t = T]$, and $\varepsilon = 0$. Then $b \Leftrightarrow b'$. □

This work provides sufficient conditions for identification. The arguments depend on an essential tool for dynamic recursive bargaining problems: the step-by-step property for static games.

3.2 The Step-by-Step Property for Recursive Bargaining

To illustrate the step-by-step property, consider Kalai proportional bargaining over two-period agreements with no unobservable utility ($\varepsilon = 0$), constant inflation ($\phi_t = 0$), and interior weights ($\tau \in (0, 1)$).

The sides choose a starting payment $Pay^*(x)$ to satisfy the recursively defined equality:

$$\frac{\tau}{1-\tau} = \frac{V_1\left(Pay^*(x), x, \varepsilon = 0, \phi = 1, \ell = 2\right) - \beta E\left[V_1(Pay^*(x'), x', 0, 1, 2) \mid x\right]}{V_2\left(Pay^*(x), x, \varepsilon = 0, \phi = 1, \ell = 2\right) - \beta E\left[V_2(Pay^*(x'), x', 0, 1, 2) \mid x\right]}. \tag{5}$$

This is a complicated expression to work with: expectations over x' given x affect the bargaining outcome through the expected value of one-period disagreement followed by bargaining at some x'; the one-period disagreement bargain at x' is recursively defined in terms of the conditional expected value of two-period disagreement followed by bargaining at some x''.

The solution turns out to be the same if the negotiators replace the value of one-period disagreement with the value of two-period disagreement. Hypothetically, imagine a cognitively simpler model in which the sides bargain over a payment Pay^{**} subject to disagreeing for two periods and then returning to equilibrium. This disagreement point is implausible

but useful: under two-period commitment, the associated function $Pay^{**}(x)$ would satisfy:

$$\frac{\tau}{1-\tau} = \frac{V_1\left(Pay^{**}(x), x, 0, 1, 2\right) - \beta^2 E\left[V_1(Pay^{*}(x''), x'', 0, 1, 2) \mid x\right]}{V_2\left(Pay^{**}(x), x, 0, 1, 2\right) - \beta^2 E\left[V_2(Pay^{*}(x''), x'', 0, 1, 2) \mid x\right]}. \tag{6}$$

This model would be much easier to solve, because $V_i(Pay, x, 0, 1, 2) = u_i(x) + \beta E[u_i(x') \mid x] + (1+\beta)(2i-3)Pay + \beta^2 E\left[V_i(Pay^{*}(x''), x'', 0, 1, 2) \mid x\right]$, yielding the two-period problem:

$$\frac{\tau}{1-\tau} = \frac{u_1(x) + \beta E[u_1(x') \mid x] - (1+\beta)Pay^{*}(x)}{u_2(x) + \beta E[u_2(x') \mid x] + (1+\beta)Pay^{*}(x)}. \tag{7}$$

Equation (7) is the problem that the players would solve if they formed a two-period agreement relative to two-period disagreement in a two-period world. Intuitively, this cognitively simpler model imposes a disagreement rule that exhibits a finite-dependence-type cancellation of outcomes beginning in the third period. This finite dependence cancellation allows me to solve the outcome in closed-form:

$$(1+\beta)Pay^{*}(X) = (1-\tau)\left(u_1(x) + \beta E[u_1(x') \mid x]\right) - \tau\left(u_2(x) + \beta E[u_2(x') \mid x]\right). \tag{8}$$

This equivalence will follow even if the sides are forward-looking and follow single-period disagreement.

The equivalence of Pay^{*} and Pay^{**} under Kalai proportional bargaining is as follows. Write $\gamma_i(x)$ for the (possibly random) gains from trade achieved by negotiation at some state h. By Equation (5), $\gamma_1(x) = \frac{\tau}{1-\tau}\gamma_2(x)$ for all x. Then, by expanding Equation (5) and applying the law of iterated expectations,

$$\frac{\tau}{1-\tau} = \frac{V_1\left(Pay^{*}(x), x, 0, 1, 2\right) + \beta\frac{\tau}{1-\tau}E\left[\gamma_2(x) \mid x\right] - \beta^2 E\left[V_1(Pay^{*}(x''), x'', 0, 1, 2) \mid x\right]}{V_2\left(Pay^{*}(x), x, 0, 1, 2\right) + \beta E\left[\gamma_2(x) \mid x\right] - \beta^2 E\left[V_2(Pay^{*}(x''), x'', 0, 1, 2) \mid x\right]}.$$

By algebra that I omit for brevity, this constraint is the same as if the future gains were set to zero, which yields the two-period-disagreement problem in Equation (6) and the two-

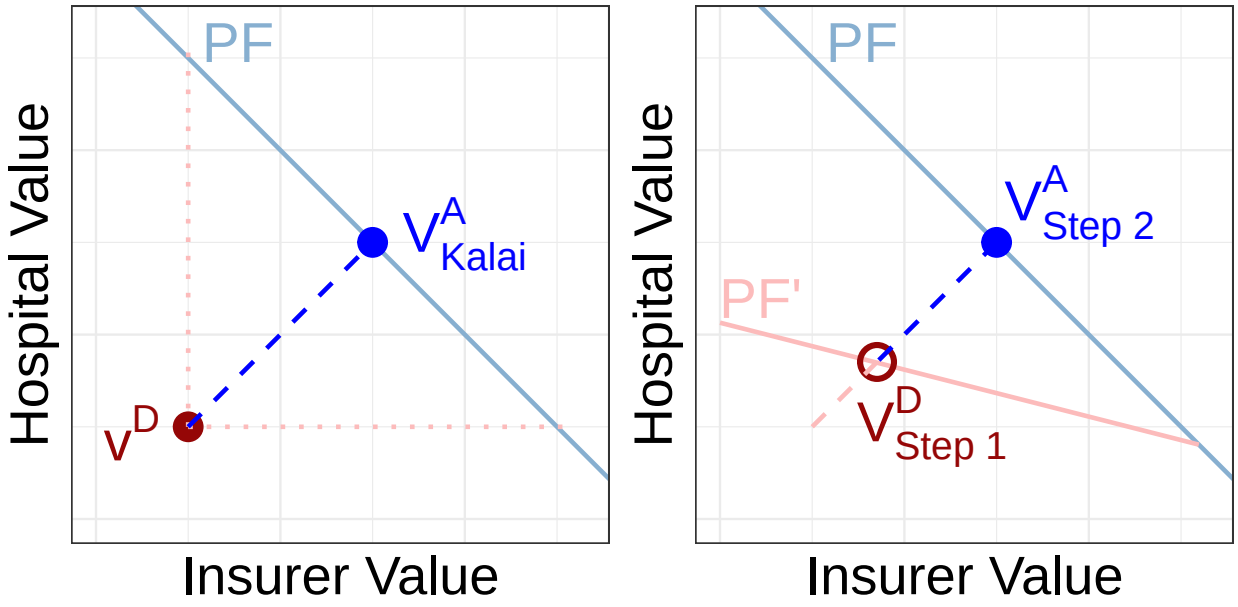


Figure 2: Left: Recall the Kalai proportional bargaining solution for choosing an agreement V^A_{Kalai} (blue) on a Pareto frontier PF relative to the value of disagreement v^D (red), with proportional split illustrated by dashed blue line. Right: illustrating the step-by-step property for a final agreement $V^A_{\text{Step 2}}$ (blue) recursively defined through bargaining relative to a first-step agreement $V^D_{\text{Step 1}}$ (red), chosen through applying the bargaining solution to negotiations over some smaller Pareto frontier PF' (pink, with dashed line indicating gains) relative to the full disagreement point v^D (omitted).

period-disagreement outcome in Equation (8).

This is the step-by-step property in action. Under Kalai proportional bargaining, it is known that the outcome of bargaining in one shot is the same outcome as recursively bargaining relative to a first-step agreement over a smaller Pareto frontier. Imagine negotiating the price of a new car by first negotiating over the hypothetical price of a car with no upgrades, and then negotiating over price and any upgrades, relative to the outcome of the constrained bargain. The step-by-step property ensures that this process will generate the same agreement as negotiating over the full surplus at once. Mathematically, if $v^D \in T \subseteq V$, then $f(V, v^D) = f(V, f(T, v^D))$. Kalai (1977) and Roth (1979) show that this property, which is easy to misstate in practical terms, is essentially unique to Kalai proportional bargaining.

Figure 2 illustrates the step-by-step property. Recall the intuition of the Kalai proportional solution as choosing the best agreement along a surplus-splitting ray. In Figure 2(b),

instead of negotiating over the full agreement, the sides first negotiate over a smaller Pareto frontier (pink) to reach a first-step agreement. The first-step agreement then becomes the disagreement point in the second-step bargain over the full surplus, leading to a final agreement. This recursive process generates the same outcome as one-shot bargaining over the full surplus Figure 2(a), because the first-step agreement corresponds to moving the disagreement value upwards along the same surplus-splitting ray.

The step-by-step property is useful for simplifying recursive bargaining problems. Abstract from uncertainty and suppose the sides choose an agreement with value v^A relative to the value of disagreeing once and achieving value v^{D_1}, with v^{D_1} recursively chosen next period through bargaining relative to the two-disagreement value v^{D_2}. By step-by-step, the one-shot problem of bargaining v^A relative to v^{D_2} produces the same outcome as recursively bargaining v^A relative to the first-step agreement v^{D_1}, with v^{D_1} recursively bargained relative to v^{D_2}. The same intuition extends in the presence of uncertainty via iterated expectations. Applying the step-by-step property, the value of one disagreement can be replaced by two, three, or infinitely-lived disagreement without changing the equilibrium agreement. The resulting simplification allows the econometrician to choose an equivalent disagreement point with cancellation properties that are qualitatively similar to finite dependence in dynamic discrete choice (DDC) models (Arcidiacono and Miller, 2011).

Importantly, the simplification does not restrict the ability of players to respond to disagreement. Instead, the simplification arises because the Kalai solution disciplines the effect of off-equilibrium surplus division on equilibrium outcomes. The degree to which the step-by-step property simplifies the resulting recursive model depends on the given context.

I now leverage this property to prove sufficient conditions for nonparametric identification with multiperiod agreements.

3.3 Identification of Kalai with Multiperiod Agreements

I provide two alternative sufficient conditions for identification.

The first sufficient condition for identification is based on a remark by Rust (1994), who writes that in dynamic discrete choice models with no observed components of utility, some partial identification may come from "agents who make different choices in the same state." In recursive Kalai bargaining games, I show that this condition permits nonparametric identification.

Proposition 5 (Identification without unobservables)**.** *Let $B^{(T,1)}$ be the set of $b \in B^{(T)}$ such that $n > 1$, there are no unobserved components of utility ($\varepsilon = 0$), there is no inflation ($\phi = 0$), and there are at least $1 + T(n-1)$ combinations of (Pay, x) in the support of the data. Then if $T > 1$ and appropriate rank conditions hold (Appendix A.1), all bargaining parameters are identified under $B^{(T,1)}$.*

Proof. See Appendix A.3 □

Proposition 5 implies that only some dynamic structures have a static representation as in Proposition 4. For example, payment variation that reflects information should not be mapped to a static utility shock, because the resulting utility shocks would need to be correlated with future information. Note also that $1 + T(n-1)$ is an unnecessarily large number of information states, and in fact would suffice to identify nonexponential discounting rates via ordinary least squares.

The second sufficient condition is motivated by empirical practice, which tends to favor GMM estimation. I show that an instrument satisfying an exclusion restriction can suffice to identify the discounting rate.

Proposition 6 (Identification from inflation expectations)**.** *Suppose $T > 1$, payments are strictly positive almost surely, and ζ_t is a real-valued observed coarsening of i_t satisfying the exclusion restriction $\{x_t, \varepsilon_t\}_{t=t_0}^{t_0+T-1} \perp\!\!\!\perp \zeta_{t_0} \mid x_{t_0}, \phi_{t_0}$ and the relevance condition that for all $t = 1, \ldots, T-1$, $E\left[\prod_{0<s\leq t}(1+\phi_{t_0+s}) \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0}\right]$ is strictly increasing in ζ_{t_0}. Then if there are at least $n + (T-1)$ values of (x, ϕ, ζ) in the support such that appropriate rank conditions hold (Appendix A.1), all bargaining parameters are identified under $B^{(T)}$.*

Proof. More generally, identification follows from payments that are constructed based on a moment on any time weighting over payments and gains. See Appendix A.3 for details. □

I view this second condition as more plausible in empirical use. The key restriction is that ζ_t affects bargaining only through expectations about inflation, and there is a clear direction of payments so that inflation expectations affect starting payments. Empirical practice also can call for interactions across negotiations by different players.

4 Multiperiod, Multilateral Nash-in-Kalai Bargaining

I now propose a model of many agents negotiating staggered and interacting bilateral contracts in a changing and uncertain world. This will call for a model in which in every subgame, strategies form a Nash equilibrium over expected values, and at every negotiation, the contract formed by a given pair always follows from applying a pair-specific bargaining solution to the expected value functions at the current state of the world. Based on the results for simpler two-agent games, I propose *Nash-in-Kalai* bargaining.

4.1 Model Timing

I study dynamic markets of the following form.

There are n players, indexed $i = 1, \ldots, n$. The world is infinitely-lived with periods indexed by t. The discounting rate per unit of time is $\beta \in [0, 1)$. I allow there to be $m \geq 1$ periods per unit of time, in which case the per-period discounting rate is $\beta^{1/m}$. Myopia corresponding to $\beta = 0$.

The timing in each period t is as follows:

1. *Potential pre-bargaining actions are taken.* Given the state h_t, each player i chooses actions $a_{it}^{(1)}$ in some action space $A_{it}^{(1)}(h_t)$. I write the resulting vector of actions as $a_t^{(1)}$.

2. *Bargaining and potential simultaneous actions are taken.* The actions from stage 1 are announced to all players. Then, at the same time:

   (a) For $i < j$ pairs with more than one feasible contract, new contracts $\mathbb{C}_{0,ijt}$ in the feasible new-contract spaces $\mathcal{C}_{ijt}\left(a_t^{(1)}\right)$ are chosen through bargaining relative to the null contract $\mathbb{C}_{0,ijt}$. I write the resulting contract state as $\mathbb{C}_t$, and the number of new contracts formed by firm i as R_{it}.

   (b) Each player i chooses actions $a_{it}^{(2b)}$ in some action space $A_{it}^{(2b)}(h_t)$. I write the resulting vector of actions as $a_t = \left((a_{1t}^{(1)}, a_{1t}^{(2b)}), \ldots\right)$.

3. *Flow profits are formed.* Player i earns flow profits of

$$u_i\left(\mathbb{C}_t, a_t, x_t, \varepsilon_t\right) + \sum_{j \neq i} Pay_{ji}\left(\mathbb{C}_t, a_t, h_t\right) - r_i R_i\left(\mathbb{C}_t, h_t\right),$$

   where $r_i \geq 0$ is the cost of validating a new contract.

4. *Information is revealed.* The new state h_{t+1} is revealed to all players. This state includes the chosen action and contract states a_t and $\mathbb{C}_t$, which are observed by the econometrician; h_t; the next period's non-bargaining action spaces A_{it+1}, null contracts $\mathbb{C}_{0,ijt+1}$, contract space functions $\mathcal{C}_{ijt+1}(a_t^{(1)})$ (either a single renewal contract or a set including the null contract), and other observable states in x_{t+1}, which is observed by the econometrician; and the unobservable states $\varepsilon_{t+1}, i_{t+1}$.

This setup generalizes the two previous sections, so that all previous nonidentification arguments carry over.

Lemma 6 (The many-agent model generalizes previous games)**.** *The games in Section 2.3 and Section 3 are special cases of this model.*

Proof. See Appendix A.3. □

Within such a game, strategies are a set of functions $\sigma = \left\{\sigma_i^{(1)}, \sigma_{ij}^{(2a)}, \sigma_i^{(2b)}\right\}$ such that $\sigma_i^{(1)}(h_t) \in A_{it}^{(1)}(h_t)$, $\sigma_{ij}^{(2a)}(a_t^{(1)}, h_t) \in \mathcal{C}_{ijt}\left(a_t^{(1)}\right)$, and $\sigma_i^{(2b)}\left(a_t^{(1)}, h_t\right) \in A_{it}^{(2b)}(a_t^{(1)}, h_t)$.

Definition 9 (Nash-in-Kalai equilibrium)**.** *A* Nash-in-Kalai equilibrium *is a game and strategies σ of the form of this section such that (i) value functions are calculated under these strategies,*[4] *(ii) the strategies $\sigma_i^{(1)}$ and $\sigma_i^{(2b)}$ satisfy the appropriate notion of equilibrium induced by this game and the value functions generated by these strategies, and (iii) there are a set of bargaining weights $\tau_{ij} \in [0,1]$ such that for every feasible pair of $a_t^{(1)}, h_t$, the strategy $\sigma_{ij}^{(2a)}(a_t^{(1)}, h_t)$ taking other strategies as given solves the Kalai proportional bargaining problem over value functions with bargaining weight τ_{ij} on player $j > i$ and disagreement contract $\mathbb{C}_{0,ijt}(h_t)$.*

For generality, I allow the σ_i strategies to follow notions of equilibrium besides Markov-perfect. As a result, the notion of equilibrium is broad enough to include punishment for disagreement. In my empirical application, I assume that disagreement is short-lived.

I discuss each stage in further detail.

4.1.1 Potential Pre-Bargaining Actions

I allow actions to be taken before bargaining in Stage 1. This may include decisions like whether or not to allow a contract to renew. Whether actions are allowed before or after bargaining is typically not substantively important: the framework could allow both at the cost of further notation.

[4] In particular, if the state is h and for conciseness I write $\sigma_{-i}(2)(a^{(1)}, h)$ for the set of $\sigma_{jk}^{(2a)}(a^{(1)}, h)$ strategies for $j, k \neq i$ and the set of $\sigma_j^{(2b)}$ strategies for $j \neq i$ and abuse notation by writing $a_{-i}^{(2)} = \left(\{\mathbb{C}_{jk}\}_{j,k\neq i}, a_{-i}^{(2b)}\right)$ for the resulting outcome,

$$
\begin{aligned}
V_i^{(1)}\left(a_i^{(1)}, a_{-i}^{(1)}, h\right) &= E\left[V_i^{(2)}\left(\sigma_{ij}^{(2a)}(a_i^{(1)}, a_{-i}^{(1)}), h)\}, \sigma_i^{(2b)}(a_i^{(1)}, a_{-i}^{(1)}), h), \sigma_{-i}^{(2)}((a_i^{(1)}, a_{-i}^{(1)}), h), a^{(1)}, h\right)\right] \\
V_i^{(2)}\left(\{\mathbb{C}_{ij}\}, a_i^{(2b)}, a_{-i}^{(2)}, a^{(1)}, h\right) &= E\left[\begin{array}{ll} & u_i(\mathbb{C}, a, x_t, \varepsilon_t) + \sum_{j\neq i} Pay_{ji}(\mathbb{C}, a, h) - r_i R_i(\mathbb{C}, h) \\ + & V_i^{(4)}((\mathbb{C}, a, h, x', \varepsilon', i')) \end{array}\right] \qquad (9) \\
V_i^{(4)}(h') &= \beta^{1/m} E\left[V_i^{(1)}\left(\sigma_i^{(1)}(h'), \sigma_{-i}^{(1)}(h'), h'\right)\right].
\end{aligned}
$$

4.1.2 Contracts are Bargained

The core of my analysis is the bargaining Stage 2a.

I assume that contracts play a role in payments $Pay_{ijt} = -Pay_{jit}$ from player j to player i. A contract can have other characteristics: a fixed end date or an auto-renew clause; a benchmark rule for updating prices while the contract remains in place; restrictions on future actions; or per-period transfers as a function of realized agreements (Olssen and Demirer, 2022; Ho and Lee, 2024).

In defining the Nash-in-Kalai equilibrium, I require that negotiators share rational expectations and a constant per-period discounting rate. Shared expectations are necessary to negotiate over an expected transfer, rational expectations are necessary to construct a valid moment based on the realized transfer, and a constant per-period discounting rate is necessary for the players to bargain in terms of consistently defined net present values. Empirical models with asymmetric information or irrational expectations are frontier topics (Larsen and Zhang, 2021; Chen et al., 2024) that are outside the scope of this work, and alternative discounting rules could be accommodated with additional notation.

I also assume that a pair chooses a contract taking as given both future strategies and all strategies for other simultaneous actions. The assumption on future strategies is to my knowledge innocuous. The restriction on simultaneous strategies is the standard passive beliefs assumption that if i or j defects and changes their behavior, neither party adjusts their behavior in, or expectations about, other strategic decisions taken at the same time (Lee et al., 2021). This decision is relatively innocuous for dynamic models in which short-lived disagreement effectively constrains the feasible value of contract deviations.

4.1.3 Potential Simultaneous Actions are Taken

In Stage 2b, I allow strategic actions to be made at the same time as bargaining. This allows the Nash-in-Kalai model to nest Nash-in-Nash models in which Nash bargaining happens at the same time as other strategic decisions like price-setting (Ho and Lee, 2017;

Crawford et al., 2018). It is often most convenient to include occasional actions like premium-setting in this stage, so that repeated disagreement does not lead to new internalized effects of future actions.

4.1.4 Flow Profits are Formed

The representation of profits in Stage 3 as the sum of a pre-transfer flow profit function and negotiated transfers generalizes many, but not all, conceivable dynamic markets. For example, latent utility shocks ε can be captured in the unobserved information i_{t-1}. In my empirical application, I assume that payments under the null contract are zero. I write the model in this section more generally to include disagreement transfers, for example through out-of-contract purchasing (Prager and Tilipman, 2022).

I model a negotiation cost borne after bargaining succeeds. Real negotiation costs are borne both at the stage of preparing for negotiations (Gooch, 2019; ECG, 2020; Fletcher, 2020; Beier, 2020) and at the stage of carefully checking the terms of a potential agreement (STD TAC and Moss, 2014; PMMC, 2019; Fletcher, 2020). I model the bargaining friction as only the ex post cost to validate a potential agreement. Some work includes a sunk negotiation cost (Prager and Tilipman, 2022). Sunk costs can prevent firms from forming Pareto-efficient contracts. In a static model, sunk costs have the advantage of not entering into payments. In a forward-looking model, future sunk costs enter into current payments in a challenging way.

In characterizing the Nash-in-Kalai model, I implicitly assume that the convex hull of feasible value functions is comprehensive. If the bargaining game fails comprehensiveness, then the Nash-in-Kalai model may be poorly behaved for some bargaining weights, and a hypothetical dynamic Nash-in-Nash solution may depart substantially from existing work. For example, consider traditional models of double marginalization with negotiation of an upstream supplier price before a retailer sets downstream prices. The retailer's preferred price generates positive gains for the supplier, so that a Kalai weight that gives the retailer

a 100% share of gains is Pareto inefficient. In this case, a Nash-in-Kalai equilibrium with large retailer weight will be poorly behaved. Conversely, the dynamic Nash bargaining model with large retailer weight involves subtle supplier disagreement constraints based on expected future prices that are not present in a static model.

4.1.5 Information is Revealed

The information updating Stage 4 is written in a generic way in order to accommodate many types of markets. Note that I include the $\mathbb{C}_{ijt}$ contract outcome in h_{t+1} but not how off-equilibrium contracts are reached. As a result, I implicitly rule out asymmetric punishments for contract deviations based on the source of the deviation.

4.2 Nash-in-Kalai Yields a Moment on Transfers

I now show that the dynamic Nash-in-Kalai model yields a moment on expected NPV transfers that generalizes the static Nash-in-Nash moment on current transfers. The moment delivers key tractability properties.

I maintain some regularity conditions.

Assumption 3. (Regularity conditions)
For any state h, the distribution of $(x', i', \varepsilon') \mid h, a_t, \mathbb{C}_t$ follows the law of motion of $(x', i', \varepsilon') \mid (x, i, \varepsilon)$. There is a uniform transversality condition: if $\mathcal{F}_r(h_{t+1})$ is the set of feasible states in period $t + r \geq t$ after state h_{t+1} is reached at the end of period t, then

$$\lim_{r \to \infty} \sup_{h_{t+1}, h_{t+r} \in \mathcal{F}_r(h_{t+1})} \sup_i \beta^{r/m} \left| V_i^{(4)}(h_{t+r}) \right| = 0.$$

By repeated application of the step-by-step property, the econometrician (and negotiators) can replace the value of disagreement with the value of impasse (Binmore et al., 1989). In the ij impasse point, i and j continually attempt to reach an agreement, but surprise all actors and continue to disagree without abandoning negotiations. This concept roughly cap-

tures a dynamic version of static Nash-in-Nash disagreement: everyone behaves anticipating ij to reach an agreement, but ij always surprise the market and disagree. The impasse point is uniquely-defined regardless of whether disagreement is unilateral or bilateral, because the history includes the contract outcome $\mathbb{C}_{ijt}$ but not how off-equilibrium contracts were reached. The impasse point is typically defined in discrete time, even when bargaining is conducted in continuous time.

The dynamic Nash-in-Kalai bargaining model yields a moment on expected NPV transfers, and as a result the econometrician can construct moments on transfers for estimation. The moment naturally generalizes the static Nash-in-Nash bargaining moment to incorporate multiple periods of gains from trade. To characterize the moment, it is useful to define ij's gains from trade at equilibrium non-ij actions as:

$$\begin{aligned}
[\Delta_{ij} u_{it}] &\equiv u_i\left(\mathbb{C}_{ijt}, \mathbb{C}_{-ijt}, a_t, x_t, \varepsilon_t\right) - u_i\left(\mathbb{C}_{0,ijt}, \mathbb{C}_{-ij}, a_t, x_t, \varepsilon_t\right) \\
[\Delta_{ij} Pay_{it,-j}] &\equiv \sum_{k \neq j} Pay_{ki}\left(\mathbb{C}_{ijt}, \mathbb{C}_{-ijt}, a_t, x_t, \varepsilon_t\right) - Pay_{ki}\left(\mathbb{C}_{0,ijt}, \mathbb{C}_{-ij}, a_t, x_t, \varepsilon_t\right),
\end{aligned}$$

write the j-to-i flow payment from a single period of a contract under TU Nash-in-Nash as:

$$Pay_{NiN,t} \equiv -\tau_{ij}\left([\Delta_{ij} u_{it}] + [\Delta_{ij} Pay_{it,-j}]\right) + (1 - \tau_{ij})\left([\Delta_{ij} u_{jt}] + [\Delta_{ij} Pay_{jt,-i}]\right),$$

and write the effect of the first agreement in t_0 on subsequent disagreement payoffs as:

$$\left[\Delta_{ijt_0} u_{it}^D\right] \equiv u_i\left(\mathbb{C}_{0,ij}, \mathbb{C}_{-ijt}^{AI}, a_t^{AI}, x_t, \varepsilon_t\right) - u_i\left(\mathbb{C}_{0,ij}, \mathbb{C}_{-ijt}^{DI}, a_t^{DI}, x_t, \varepsilon_t\right),$$

where AI and DI reflect the path of contracts and actions if ij enter their impasse point at the first opportunity to depart from equilibrium after the conclusion of one and zero agreements, respectively. Define $\left[\Delta_{ijt_0} Pay_{it}^D\right]$ and $\left[\Delta_{ijt_0} R_{it}^D\right]$ similarly.

The Nash-in-Kalai moment extends TU Nash-in-Nash as follows.

Theorem 2 (Nash-in-Kalai Moment)**.** *Consider a dynamic Nash-in-Kalai equilibrium that*

satisfies Assumption 3. Suppose players $i < j$ form a contract in a subgame time t_0 that remains in place through the (potentially random) terminal time t^*. Then the expected NPV of realized j-to-i payments $Pay_{ijt}(\mathbb{C}_t, a_t, x_t, \varepsilon_t)$ is equal to the sum of the expected NPV of Nash-in-Nash flow transfers, a negotiation cost transfer, and an impasse repricing transfer term:

$$\mathbb{E}_{t_0}\left[\sum_{t=t_0}^{t^*} \beta^{\frac{t-t_0}{m}} Pay_{ijt}\right] = \text{Pay}_{NiN} + \text{Pay}_{NC} + \text{Pay}_{IRT}, \tag{10}$$

where the expected NPV of static Nash-in-Nash transfers ($\tilde{Pay}$ in Equation (4)) is:

$$\text{Pay}_{NiN} = \mathbb{E}_{t_0}\left[\sum_{t=t_0}^{t^*} \beta^{\frac{t-t_0}{m}} Pay_{NiN,t}\right], \tag{11}$$

the negotiation cost transfer Pay_{NC} *is equal to* $\tau_{ij} r_i - (1-\tau_{ij}) r_j$*, and the impasse repricing transfer* Pay_{IRT} *is:*

$$\text{Pay}_{IRT} = E_{t_0}\left[\sum_{t=t_0}^{\infty} \beta^{\frac{t-t_0}{m}} \left\{ \begin{array}{r} -\tau_{ij}\left(\left[\Delta_{ijt_0} u_{it}^D\right] + \left[\Delta_{ijt_0} Pay_{it}^D\right] - r_i\left[\Delta_{ijt_0} R_{it}^D\right]\right) \\ +(1-\tau_{ij})\left(\left[\Delta_{ijt_0} u_{jt}^D\right] + \left[\Delta_{ijt_0} Pay_{jt}^D\right] - r_j\left[\Delta_{ijt_0} R_{jt}^D\right]\right) \end{array} \right\}\right]. \tag{12}$$

Proof. The claim follows by repeated application of the step-by-step property. For details, see Appendix A.3. □

In many empirical applications, the realized transfers Pay_{ijt} can be observed; the expected flow Nash-in-Nash transfers in Pay_{NiN} depend on estimable demand functions, a small number of bargaining parameters, and unobserved payoff shocks ε that can be assumed to satisfy some moment; and Pay_{NC} depends on only a few parameters.

The term Pay_{IRT} accounts for the fact that in future periods, the Nash-in-Nash disagreement point differs from the Nash-in-Kalai impasse point. The static Nash-in-Nash gains used in Pay_{IRT} are calculated relative to disagreement under the non-ij contracts formed in equilibrium, while the Nash-in-Kalai gains are calculated relative to disagreement un-

der the non-ij contracts formed when continually expecting expecting ij to exit impasse. Under myopia ($\beta = 0$) or single-period contracts, Pay_{IRT} is equal to zero. If negotiation costs are also equal to zero, then Equation (10) reduces to the TU Nash-in-Nash transfer $-\tau_{ij}\left([\Delta_{ij}u_{it_0}] + [\Delta_{ij}Pay_{it_0,-j}]\right) + (1-\tau_{ij})\left([\Delta_{ij}u_{jt_0}] + [\Delta_{ij}Pay_{jt_0,-i}]\right)$.

Theorem 2 holds even in a nonstationary equilibrium. When the market is in steady state, the predicted transfer is especially simple.

Corollary 1 (Steady state Pay_{IRT} is zero)**.** *Consider a Nash-in-Kalai equilibrium such that in every subgame in which a pair bargains, all subsequent actions a and non-ij contracts $\mathbb{C}_{-ij}$ are the same on the agreement and disagreement path. Then in every bargain,* $\text{Pay}_{IRT} = 0$.

Proof. This holds by inspection and recalling the independence assumption of the distribution of $(x', \varepsilon') \mid h$. □

Corollary 1 is particularly useful for analyzing theoretical markets. Real-world markets are not in steady state, but so long as a market is close to steady state, Corollary 1 implies that approximating Pay_{IRT} to zero will produce an approximately valid moment. This is the strategy I follow in my empirical application.

5 Empirical Application

I apply the proposed empirical model to data on hospital–insurer bargaining from West Virginia. I prove a high-level summary here. For more detail on the data and setting, see Dorn (2025c). For more detail on the estimation procedure, see the companion paper Dorn (2025a).

The main dataset is panel information on contract formation and expiration by hospital-insurer-year from West Virginia between 2005 and 2015. I also have 2016 inpatient data on visits by county of residence and age for leading insurers. I focus on six modeled insurers: Highmark Blue Cross Blue Shield (BCBS), the leading insurer; the Health Plan of the Upper

Ohio Valley (HPUOV), a regional insurer with substantial market share in some regions; and the for-profit insurers Aetna, Carelink/Coventry, Cigna, and United. The state's leading hospital care providers were Charleston Area Medical Center, a multicampus hospital system in Charleston with a small affiliate nearby; the West Virginia University Health System, which was statewide with a flagship hospital in Morgantown and acquired some hospitals in this era; and the Mon Health system, whose flagship was also in Morgantown and which merged with Charleston Area Medical Center in 2022.

The empirical model is a dynamic extension of the static Ho and Lee (2017) model: hospitals agree to receive reduced payments in exchange for an insurer steering enrollees to the hospital; wider networks make insurance plans a higher-quality good when selling to consumers; and some consumers become sick and need care at a hospital. I focus on inpatient care. I assume that all sick patients receive care somewhere and always choose a provider within their insurer's network. I differentiate between a set of medium and large insurers that are modeled, and a tail of "nonmodeled" insurers.

The specific profit functions and payments for hospital i (with generic hospitals h) and insurer j (with generic insurers n) are:

$$u_{it} = -c_i \sum_{n \in \mathcal{G}_{it}} D^H_{int}, \quad u_{jt} = D^M_{jt} \left(\phi_{jt} - \eta_j \right), \quad \text{and} \quad Pay_{ijt} = D^H_{ijt} p_{ijt},$$

where c_i is hospital i's cost per unit of care, D^H_{hnt} is the units of care profited by hospital h to consumers with insurance n in period t (assumed to be zero without an agreement), $\mathcal{G}_{it}$ is the network of firms that have an agreement with firm i in period t, D^M_{jt} is the number of lives insured by insurer j in period t, ϕ_{jt} is the insurer's premiums per life insured, η_j are noninpatient costs to the insurer, and p_{ijt} is the contracted per-unit price.

There are known to be contracting externalities in this setting. Imagine a simple single-period world with one monopolist insurer and two hospitals. The insurer simultaneously sets premiums ϕ and negotiates with the two hospitals over a price per unit of care. After

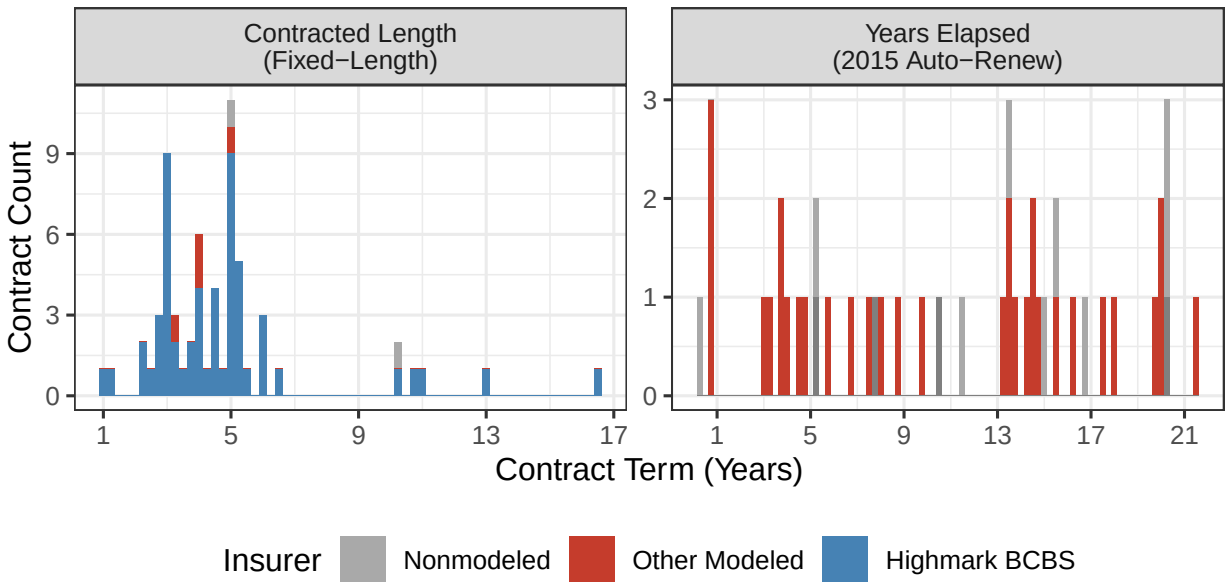


Figure 3: Contracts are multiyear. Distribution of reported contract lengths for fixed-term contracts in general (left panel) and auto-renew contracts with reported formation dates as-of fiscal 2015 (right panel). Colors indicates insurer. The two nonmodeled contracts correspond to Wheeling–Pittsburgh Steel.

premiums are set, consumers choose insurance. Consumers all become sick, and then flip a fair coin to learn their preferred hospital. Consumers go to their preferred hospital if possible, the other hospital if forced, and do not purchase insurance if the network is empty. When bargaining with hospital 2, the insurer knows that part of the value of the hospital is diverting patients away from hospital 1. As a result, a higher anticipated hospital-1 price leads to a higher hospital-2 price, generating contracting externalities. These contracting externalities have been modeled through anticipated simultaneous agreements, for example in the TU Ho and Lee (2017) model that I build on.

When contracts are staggered and negotiators are forward-looking, contracting externalities are internalized. Imagine that hospital 1 and hospital 2 bargain in alternating periods. When the insurer negotiates with hospital 1, they know that their negotiated price will have a positive effect on the price they negotiate with hospital 2 in the next period. These effects are asymmetric, yielding a NTU model and all of the issues of the previous sections.

Real-world contracts are meaningfully staggered. Figure 3, borrowed from the companion

paper Dorn (2025a), shows that for both fixed-length contracts associated with larger insurers and auto-renew contracts associated with smaller insurers, contracts consistently remained in place for three years or longer. Many contracts remained in place for a decade or more. Different lengths imply some degree of staggering. Appendix Figure 4 shows that contracts would be staggered even within a given year. As a result, if negotiators are forward-looking, the bargaining model will be NTU.

I estimate a Nash-in-Kalai model, leveraging Theorem 2 for estimation. I model bargaining weights as featuring an interaction of insurer j and hospital i effects:

$$\log\left(\tau_{ij}/\left(1-\tau_{ij}\right)\right) = \tau_j + \tau^{Size}\log(Size_i),$$

where δ_j is an insurer fixed effect, τ^{Size} is a hospital-size coefficient, and $Size_i$ is a measure of hospital system size from the start of the dataset. I estimate hospital demand based on Highmark Blue Cross Blue Shield (BCBS) patients, who are in-network at every hospital, and estimate insurer demand as a function of network quality using Affordable Care Act regulations to control for premiums.

The key new parameter to estimate is the annual discounting rate β. Hospitals and insurers had predictably different variation in contract lengths and payment rates (Dorn, 2025a), so I use insurer and hospital-size-group indicator variables as instruments in the spirit of Proposition 6. Intuitively, the insurer instruments mainly identify τ_j, and the hospital-size-group instruments identify the discounting rate β from inflation expectations, controlling for market power (as identified through the hospital and insurer demand estimates) and potential bargaining weight heterogeneity (through $Size_i$).

I estimate three bargaining models. The *Only-2015* model represents estimates based on the standard approach of taking all data in a market over a short period and estimating a Nash-in-Nash model treating all contracts as short-lived. This model is TU, so the Only-2015 model can be interpreted as a Nash-in-Nash or Nash-in-Kalai model. The *Myopic* model uses

the same demand estimates, but estimates bargaining parameters for new contracts in all years. The model is also TU under myopia, so that this model can also be interpreted as a Nash-in-Nash or Nash-in-Kalai model. Finally, the *Forward-Looking* model uses the same bargains and estimated demand functions as the Myopic model, but allows for negotiators to be forward-looking under specifically the Nash-in-Kalai model.

	Parameter				
	β	τ_{BCBS}	τ_{HPUOV}	τ_{FP}	$-\tau^{Size}$
Only-2015	·	0.487**	-7.54	0.694***	3.354
(Nash/Kalai)	(·)	(0.191)	(17.204)	(0.175)	(22.875)
Myopic	·	0.876***	0.825***	0.861***	1.037***
(Nash/Kalai)	(·)	(0.012)	(0.232)	(0.034)	(0.199)
Forward-Looking	0.899***	0.854***	0.877***	0.889***	0.989***
($\text{Pay}_{IRT} = 0$)	(0.03)	(0.006)	(0.026)	(0.005)	(0.028)
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 1: Estimated bargaining and patience weights for the only-2015 (first row), myopic (second row), and preferred forward-looking (third row) bargaining models. The MCO τ_j bargaining weights represent the estimated bilateral share τ_{ij} evaluated for a hospital i with size equal to the average bargain value, and include estimated heterogeneity between Highmark BCBS (BCBS), the regional insurer HPUOV, and the modeled for-profit insurers (FP). The estimated forward-looking model overwhelmingly rejects myopia, and the only-2015 model produces implausible estimated bargaining weights. For additional parameter estimates, see Appendix Table 2.

The estimated bargaining parameters are in Table 1. The variation in contract lengths visible in Figure 3 rejects a static model like the Only-2015 model, and I find that the the failure to explicitly model the variation in contract length can be substantive. In this case, the Only-2015 model estimates that the regional insurer HPUOV's bargaining weight is negative. This behavior is in part driven by the large estimated role of hospital size, which is associated with contract structure: if I fix $\tau^{Size} = 0$, the estimated bargaining weights for Highmark BCBS, HPUOV, and the for-profit insurers drop to 0.365, 0.278, and 0.16, respectively.

Next, I turn to the Myopic and Forward-Looking models that have accurate timing. These models predict limited heterogeneity in insurer bargaining weights, but important heterogeneity in hospital bargaining weights. The bargaining weights are similar whether β is allowed to be above zero or not, because demand is fairly correlated across time. Nevertheless, the forward-looking model overwhelmingly rejects the null hypothesis of myopia, estimating an (inflation-adjusted) annual discounting rate of 0.899. This parameter estimate indicates that the negotiators are likely to respond to beliefs about future conditions when choosing their starting price, indicating that there can be important dynamic incentives that the previous static literature could not model. Further, these models find similar bargaining weights for BCBS and the smaller for-profit insurers. The Only-2015 model's estimates with no heterogeneity are interior, but mistakenly estimate very small bargaining weights for for-profit insurers, likely due to misinterpreting the smaller insurers' large realized price increases as a lack of bargaining power, rather than a foreseeable dynamic that forward-looking negotiators would offset in NPV terms.

6 Conclusion

This paper introduces the Nash-in-Kalai solution for bargaining problems with nontransferable utility. In the presence of uncertainty over nontransferable Pareto frontiers, Nash bargaining and other scale-invariant bargaining solutions are unidentified, and only one family of bargaining solutions satisfying independence of irrelevant alternatives is identified: The Kalai proportional bargaining solution. The Kalai proportional solution's unique step-by-step property implies a valuable equivalent representation of the recursive Kalai proportional bargaining problem. I leverage this representation to provide sufficient conditions for identification of Nash-in-Kalai bargaining parameters with multiperiod agreements. The Nash-in-Kalai model admits a moment on expected NPV transfers in a wide degree of generality. As a result, the model can be applied to settings with many firms negotiating interacting agree-

ments at different times. My empirical application rejects both static and myopic models: I find that a static model incorrectly assigns foreseeable and discounted price dynamics to bargaining weights, and a myopic model incorrectly rules out forward-looking responses.

The Nash-in-Kalai model is likely to be valuable beyond healthcare. The framework may prove useful for modeling media asset acquisition (Morrissette, 2023), programming disputes with time-bound programming (Hayes, 2023), disagreement with inertia costs (Handel, 2013), staggered team formation in the presence of aggregate wage limits (Mulholland and Jensen, 2019), and supply chain bargaining in the presence of inflation (Davidson, 1988; Baudendistel, 2023). A feasible application for future work is to extend popular econometric search-on-the-job models (Cahuc et al., 2006; Bagger et al., 2014; Bilal et al., 2022) beyond TU bargaining. The dynamic moment of Theorem 2 provides sufficient conditions for estimation, but an important avenue for future work is characterizing plausible conditions for statistical inference in this setting. Finally, an open question for future work is whether computational tools can render a feasible Nash-in-Nash model with staggered contracts when the econometrician is willing to take a stance on information timing.

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A Additional Content

A.1 Rank Conditions

Assumption 4 (Rank condition, Proposition 5)**.** Fix some x_0. The distinct values of (Pay, x) are such that the set of $P(X_{t_0+t} = x' \mid x_{t_0} = x, Pay_{t_0} = Pay)$ for initial periods t_0, $t = 0, \ldots, T-1$, and $x' \neq x_0$ and a constant are linearly independent.

This rank condition is chosen to emphasize simplicity over comprehensiveness. For example, when $T > 1$, this condition requires i_{t_0} to be potentially informative for x_{t_0+2} given (x_{t_0}, x_{t_0+1}). This rank condition is sufficient to identify period-dependent weights β_t via ordinary least squares, which imply the fixed per-period discounting rate β.

Assumption 5 (Rank condition, Proposition 6)**.** For each (x, ϕ) combination in the tuples, pick some $\zeta_0(x, \phi)$ such that $(x, \phi, \zeta_0(x, \phi))$ is a tuple in the support. Then (i) consider the set of (x, ϕ, ζ) tuples $\zeta \neq \zeta_0(x, \phi)$. Then the set of values of

$$\left\{ \begin{array}{ll} & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta\right] \\ - & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta_0(x, \phi)\right] \end{array} \right\}$$

are linearly independent. Further (ii) let Σ be the matrix whose rows correspond to values of x and whose columns correspond to $\sum_{t=t_0}^{T-1} \beta^t P(x_{t_0+t} = x' \mid x_{t_0} = x)$. Then Σ is invertible.

This assumption is also chosen to simplify the resulting identification argument.

A.2 Nash-in-Kalai Period Split

Definition 10. *Without loss of generality assume $t_0 = 1$. The original game's period t will correspond to period $t_s = 1+2^{s-1}(t-1)$. Let $m_s = m_1 * 2^{s-1}$; write $\hat{t}(t_s) = 1+\lfloor\{(t_s-1)2^{1-s}\}\rfloor$ for the mapping from transformed-game period to original-game period. If $1+2^{s-1}(\hat{t}(t_s)-1) = t_s$, then period t_s corresponds to an original-game period: let $A^{(1)}_{it_s}$, $A^{(2)}_{it_s}$, and $\mathcal{C}_{hnt_s}\left(a^{(1)}_{t_s}\right)$ in*

the transformed game be as in $A_{i\hat{t}(t_s)}$ *and* $\mathcal{C}_{hn\hat{t}(t_s)}\left(a^{(1)}_{\hat{t}(t_s)}\right)$ *in the original game. Otherwise, let the action spaces be singletons, and let* $\mathcal{C}_{hnt_s}\left(a^{(1)}_{t_s}\right)$ *be equal to* $\mathcal{C}_{hnt_s-1}\left(a^{(1)}_{t_s-1}\right)$ *(up to any necessary reindexing like for contract length) if* hn *failed to form an agreement in* t_s-1*. Let* x_{t_s} *and* ε_{t_s} *be updated at the end of a period* t_s *with* $1+2^{s-1}(\hat{t}(t_s+1)-1)=t_s+1$*; otherwise, let them be unchanged. Let other information be updated only in those periods, in which case let the information on* $(x_{t'_s},\varepsilon_{t'_s},i_{t'_s})$ *in future periods be the information on* $(x_{\hat{t}(t'_s)},\varepsilon_{\hat{t}(t'_s)},i_{\hat{t}(t'_s)})$ *in the original game.*

A.3 Additional Proofs

Proof of Lemma 1. Let $G\in\mathcal{G}$ be given. For v on the Pareto frontier of V, write $Pay(v)=\frac{v_2-v_2^D}{\bar{v}_2-v_2^D}$. Then $v_2^*+v_1^*=v_2^D+v_1^D+v$ and

$$\tau v_1'\left(Pay\left(f_\tau^{(Nash)}(V,v^D)\right)\right)(v_2^*-v_2^D)=\frac{\tau(f_\tau^{(Nash)}(V,v^D)_2-v_2^D)}{\bar{v}_2-v_2^D}$$
$$=(1-\tau)v_2'\left(Pay\left(f_\tau^{(Nash)}(V,v^D)\right)\right)(v_1^*-v_1^D)=\frac{(1-\tau)(f_\tau^{(Nash)}(V,v^D)_1-v_1^D)}{\bar{v}_2-v_2^D},$$

so that $\tau(v_2^*-v_2^D)=(1-\tau)(v_1^D+v-v_2^*-v_1^D)$. But $f_\tau^{(Kalai)}$ satisfies the same constraint. Therefore $f_\tau^{(Nash)}(V,v^D)=f_\tau^{(Kalai)}(V,v^D)$. □

Proof of Lemma 3. f_3 is not concave, so there exists an V,W,λ such that at least one player strictly prefers $\lambda f_3(V,v^D)+(1-\lambda)f_3(W,u^D)$ to $f_3(\lambda V+(1-\lambda)W,\lambda v^D+(1-\lambda)u^D)$. By strict inequality, it must be that $\lambda\in(0,1)$.

Without loss of generality, assume player 1 strictly prefers the ex post game. By WPO of the ex ante solution, player 2 weakly prefers the ex ante game. Write $v_{EA}=f\left(\lambda V+(1-\lambda)W,\lambda v^D+(1-\lambda)u^D\right)$, $v^*=f_3(V,v^D)$, and $w^*=f_3(W,u^D)$, then $v_{EA,1}<\lambda v_1^*+(1-\lambda)w_1^*$ and $v_{EA,2}\geq\lambda v_2^*+(1-\lambda)w_2^*$. A quirk is that the ex post choice may not be an ex ante outcome under any bargaining solution, so I produce a modified pair of games for which the ex post outcome is Pareto efficient (and so can be chosen) and the ex ante

prediction under f is unchanged by IIA.

Let v', w' be points in V, W such that $v' \geq v^D$, $w' \geq u^D$, $v'_1 \geq v^*_1$, $v'_2 \leq v^*_2$, $w'_1 \geq v^*_1$, $w'_2 \leq w^*_2$, and $\lambda v' + (1-\lambda)w' = v_{EA}$. Such a point must exist because $v_{EA} \in \lambda V + (1-\lambda)W$.

Recall that the comprehensive convex hull of $\{v_1, \ldots, v_k\}$ is the set of $v \in \mathbb{R}^2$ such that $v \leq \sum_j \lambda_j v_j$ for some nonnegative λ_j summing to one (Myerson, 1981).

Now let V' be the comprehensive convex hull of v', v^*, and v^D, and let W' be the comprehensive convex hull of w', w^*, and u^D. The Pareto frontier of $\lambda V' + (1-\lambda)W'$ is the convex hull of $\lambda v' + (1-\lambda)w'$ and $\lambda v^* + (1-\lambda)w^*$. Let $X \geq 1/2$ correspond to playing V' and let $X < 1/2$ correspond to playing W'. Also let the utility functions u corresponding to mapping $p \in [0,1]$ to points on the utility frontier, with $p = 1/3$ corresponding to v' and w' and $p = 2/3$ corresponding to v^* and p^*. (Note that it is possible for the frontier of utility to be a single point in one of these games.) Also let $\mathscr{P}$ be the singleton distribution of $Bernoulli(\lambda)$.

By the span constraint of Lemma 3, there exists an f' such that $f'(\lambda V' + (1-\lambda)W', \lambda v^D + (1-\lambda)u^D) = \lambda v' + (1-\lambda)w'$. Let b be a structure generated by $EA = 0$ bargaining under f, and let b' be a structure generated by $EA = 1$ bargaining under f'. Any equilibrium results in setting $p^* = 1/3$, so that $b \Leftrightarrow b'$. But f predicts setting $p^* = 2/3$ in the ex ante game, so that $f \not\Leftrightarrow f'$. Thus, $\mathcal{F}$ is not single-period identified. □

Proof of Lemma 4. For this proof, let $\mathcal{G}$ be the set of games (V, v^D) satisfying convexity and comprehensiveness, $v^D = 0$, and $V = \{v \in V : v \geq 0\}$. I write that the game $(\{0\}, 0) \in \mathcal{G}$, with $f(\{0\}, 0) = 0$ for any bargaining solution f.

I prove the weaker claim that if f satisfies WPO, IIA, and concavity for games in $\mathcal{G}$, then either f is proportional or f it utilitarian.

The proof generally follows Myerson (1981), so I proceed assuming the reader has a copy on hand. Let $\mathcal{G}$ be the set of games that satisfy Definition 1 and do not allow ex post losses, and let f be a bargaining solution that satisfies WPO, IIA, and this proof's weaker form of concavity. Myerson's Theorem 1 shows that because $\mathcal{G}$ is a convex combination, if f is

linear, then f is utilitarian.

Most of Myerson's results follow after some change in notation. His Lemma 1 through Lemma 6 follow after adjusting the notion of comprehensive convex hull to intersect with $\mathbb{R}^2_+$, and writing $M = \{f(G) : G \in \mathcal{G}\}$. Lemma 7 follows after adding a caveat that the claim only holds if $\zeta = \lambda x(1-\lambda)y \geq 0$, and with some care verify the claim holds for $y = 0$.

Myerson's Lemma 8 requires more modification. Because f is not utilitarian, it continuous to be the case that there is a $v, u = f(v)$ such that $p \cdot v > p \cdot u$, where p is constructed in Myerson's Lemma 4. Then by Myerson's Lemma 7, take $\lambda = \frac{1}{\|u\|}$ and mix the game $H(v)$ with zero to obtain a game, with associated solution c satisfying $\|c\| = 1$. Without loss of generality I write that $u = c$. Then it clear by inspection that $u - c \in M$, because $f(\{0\}, 0) = 0 = u - c$.

Myerson's Lemmas 9 and 10 follow immediately. Lemma 11 follows with the caveat that the claim only holds if $u + \lambda d \geq 0$. Myerson's Lemma 12 completes the proof by showing that if $x \in M$, then $x = u + (p \cdot x - p \cdot u)\, u$, which holds so long as $u + (p \cdot x - p \cdot u)\, u \geq 0$. But by construction in this extension, $p \cdot u = 1$, so that the only requirement is that $p \cdot x \geq 0$. But $x \in M$, so there is an 0 such that $x = F(V, 0)$, and $0 \in M \cap V$, so that by Myerson's Lemma 5, $p \cdot x \geq p \cdot 0 = 0$. Therefore $u + (p \cdot x - p \cdot u)u \geq 0$, so that the result of Myerson's Lemma 12 holds and if f is not utilitarian, then f is proportional. □

Proof of Proposition 5. By Proposition 3, all bargaining parameters are identified if $T = 1$, so I proceed assuming $T > 1$.

By applying the step-by-step property T times and then canceling future (exogenously evolving) states, every (Pay, x) combination observed in a period t_0 must satisfy:

$$E\left[\sum_{t=0}^{T-1} \beta^t \prod_{0<s\leq t} (1 + \phi_{t_0+s}) \mid x_{t_0}\right] Pay = E\left[\sum_{t=0}^{T-1} \beta^t \tilde{Pay}(x_{t_0+t}) \mid x_{t_0}\right].$$

By assumption, there are at least x pairs of (Pay, x) and (Pay', x) with $Pay \neq Pay'$

both in the support of $Pay^* \mid x$. Then for each such pair:

$$Pay = \sum_{t=0}^{T-1} \sum_{x'} \beta^t \frac{1-\beta}{1-\beta^T} \tilde{Pay}(x') P(x_{t_0+t} = x' \mid x_{t_0} = x, Pay_{t_0} = Pay).$$

Fix some x_0. Then:

$$Pay = \overbrace{\tilde{Pay}(x)}^{b_x} + \overbrace{\sum_{t=0}^{T-1} \sum_{x' \neq x_0} \beta^t \frac{1-\beta}{1-\beta^T} \left(\tilde{Pay}(x') - \tilde{Pay}(x_0) \right)}^{c_{tx'}} P(x_{t_0+t} = x' \mid x_{t_0} = x, Pay_{t_0} = Pay).$$

Write the vector $\mathbf{p}$ of values of Pay, writing $\mathbb{X}$ as the matrix whose rows are each distinct (Pay, x) combination and whose columns are a constant one followed by one column for each of the probabilities. Then by the rank condition (Assumption 4) on the $1 + T(n-1)$ pairs, the values of $b_x = \tilde{Pay}(x)$ and $c_{t,x'}$ are identified for all $t \geq 0, x' \neq x_0$ by $(\mathbb{X}^T\mathbb{X})^{-1}\mathbb{X}^T\mathbf{p}$. Dividing $c_{1,x'}/c_{0,x'}$ for some $x' \neq x_0$, which exists by the assumption $n > 1$, identifies β. Then dividing $c_{t,x'}$ by $\beta^t(1-\beta)/(1-\beta^T)$ for any $x' \neq x_0$ identifies $\tilde{Pay}(x') - \tilde{Pay}(x_0)$. But $\tilde{Pay}(x_0)$ is already identified, so all bargaining parameters are identified. □

Proof of Proposition 6. Consider the distribution in new-contract periods t_0. Condition on a pair of x_{t_0}, ϕ_{t_0} in the support. By applying the step-by-step property T times, canceling future (exogenously evolving) states, writing $\beta_t = \beta^t$, and applying iterated expectations, for every $(x_{t_0}, \phi_{t_0}, \zeta)$ in the support, the parameters must satisfy:

$$\begin{aligned} E\left[\sum_{t=0}^{T-1} \beta_t \left(\prod_{0<s\leq T} (1+\phi_{t_0+s}) \right) Pay^*_{t_0} \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta \right] &= E\left[\sum_{t=t_0}^{t_0+T-1} \beta^{t-t_0} \tilde{Pay}(x_t) \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta \right] \\ &= E\left[\sum_{t=t_0}^{t_0+T-1} \beta^{t-t_0} \tilde{Pay}(x_t) \mid x_{t_0}, \phi_{t_0} \right]. \end{aligned}$$

Let ζ_0 be as in Assumption 5, i.e. $(x, \phi, \zeta_0(x, \phi))$ is in the support of (x, ϕ, ζ). Then for all

(x, ϕ, ζ) in the support:

$$
\begin{aligned}
0 = & E\left[\sum_{t=0}^{T-1} \beta_t \left(\prod_{0<s\leq t} (1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta\right] \\
& - E\left[\sum_{t=0}^{T-1} \beta_t \left(\prod_{0<s\leq t} (1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta_0(x_{t_0}, \phi_{t_0})\right] \\
= & \sum_{t=0}^{T-1} \beta_t \left\{ \begin{array}{ll} & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta\right] \\ - & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0}, \phi_{t_0}, \zeta_{t_0} = \zeta_0(x_{t_0}, \phi_{t_0})\right] \end{array} \right\}.
\end{aligned}
$$

Then, writing $y_{x,\phi,\zeta} = E\left[Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta\right] - E\left[Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta_0(x, \phi)\right]$, I obtain

$$
y_{x,\phi,\zeta} = \sum_{t=1}^{T-1} \beta_t \left\{ \begin{array}{ll} & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta\right] \\ - & E\left[\left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0} = x, \phi_{t_0} = \phi, \zeta_{t_0} = \zeta_0(x, \phi)\right] \end{array} \right\}.
$$

By the relevance condition (so that each difference in expectations is nonzero for all $\zeta \neq \zeta_0(x, \phi)$), the assumption that we have at least $T-1$ such pairs with $\zeta \neq \zeta_0(x, \phi)$, and the rank condition (Assumption 5(i)), the parameters β_t are identified for all $t \geq 1$ by least squares. Then the moments:

$$
E\left[\sum_{t=0}^{T-1} \beta_t \left(\prod_{0<s\leq t} (1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0}\right] = E\left[\sum_{t=t_0}^{T-1} \beta_t \tilde{Pay}(x_t)\right]
$$

identify $\tilde{Pay}(x_t)$ by the rank condition (Assumption 5(ii)) by writing $\mathbf{p}$ for the vector whose elements are $E\left[\sum_{t=0}^{T-1} \beta_t \left(\prod_{0<s\leq t}(1+\phi_{t_0+s})\right) Pay^*_{t_0} \mid x_{t_0} = x\right]$, taking Σ as in Assumption 5, and identifying the vector of $\tilde{Pay}(x)$ values via $\Sigma^{-1}\mathbf{p}$. Thus, all bargaining parameters are identified. (In the net present value case, the discounting rate β is identified as β_1.) □

Proof of Lemma 6. For Section 2.3, take some arbitrary $\beta > 0$. Abuse notation and let the observable state x_t include both the single-period state x and an indicator for whether the period t is 1, 2, or at least as large as three. Let ε_t be a singleton and take $r_i = 0$. Let a

contract include a price applied to two periods. Let $u_{it} = Pay_{it} = 0$ for periods 1 and ≥ 3. If bargaining succeeds in period 1, let $u_{1,2}(\mathbb{C}_2, a_2, x_2, \varepsilon_2) = \beta^{-1} v_1^D$, $Pay_{12,2}(\mathbb{C}_2, a_2, h_2) = \beta^{-1}\left(u_1(p \mid X) - v_1^D\right)$, and $u_{2,2}(\mathbb{C}_2, a_2, h_2, \varepsilon_2) = \beta^{-1}\left(u_2(p \mid X) - \beta Pay_{12,2}(\mathbb{C}_2, a_2, h_2)\right)$. If bargaining fails in period 1, let $u_{i,2}(\mathbb{C}_2, a_2, h_2, \varepsilon_2) = \beta^{-1} v_i^D$ for any potential agreement and $Pay_{12,2}(p) = p$ for all potential non-null agreements, so that there is never bargaining in period 2. If $EA = 1$, let x_t be revealed at the end of period 1; if $EA = 0$, let x_t be revealed before the start of period 1. Let strategies be Markov and follow bargaining solution f. This structure reproduces the desired net present values of agreement and disagreement in period 1.

For Section 3, let the observable state x_t also include the observable inflation rate ϕ_t and remaining length ℓ_t. The specifications for β, Pay, and r_i are natural. Note that to align notation, the Nash-in-Kalai game has $u_t(\mathbb{C}_t, a_t, h_t, \varepsilon_t) = 1\{\ell_t > 0\}(u_i(x_t) + \varepsilon_{t,i})$. Let strategies be Markov. This structure is clearly equivalent to the structure of Section 3. □

Proof of Theorem 2. Consider a period t_0 Stage 2a subgame in which $i < j$ form a new contract with positive probability. Condition on the event in which they form a contract. Let t^* be the (potentially random) final period in which ij remain in their current contract. Note that $\mathbb{C}^{AI}_{-ijt} = \mathbb{C}_{-ijt}$ and $a_t^{AI} = a_t$ for periods $t = t_0, \ldots, t^*$. It is convenient to write u_{it}^A for the path of u_{it} in equilibrium, u_{it}^{AI} for the path if ij agree for periods t_0 through t^* and then enter impasse, and so on.

It is useful to rewrite the claim in a more compact, but less interpretable, way. The claim can be written as:

$$
\begin{aligned}
0 = E_{t_0}\left[\sum_{t=t_0}^{t^*} \beta^{\frac{t-t_0}{m}} \left\{ \begin{array}{r} -\tau_{ij}\left(u_{it}^A + Pay_{it}^A - r_i R_{it}^A - u_{it}^{DI} - Pay_{it}^{DI} + r_i R_{it}^{DI}\right) \\ +(1-\tau_{ij})\left(u_{jt}^A + Pay_{jt}^A - r_j R_{jt}^A - u_{jt}^{DI} - Pay_{jt}^{DI} + r_j R_{jt}^{DI}\right) \end{array} \right\}\right] \\
+ E_{t_0}\left[\sum_{t=t^*+1}^{\infty} \beta^{\frac{t-t_0}{m}} \left\{ \begin{array}{r} -\tau_{ij}\left(u_{it}^{AI} + Pay_{it}^{AI} - r_i R_{it}^{AI} - u_{it}^{DI} - Pay_{it}^{DI} + r_i R_{it}^{DI}\right) \\ +(1-\tau_{ij})\left(u_{jt}^{AI} + Pay_{jt}^{AI} - r_j R_{jt}^{AI} - u_{jt}^{DI} - Pay_{jt}^{DI} + r_j R_{jt}^{DI}\right) \end{array} \right\}\right].
\end{aligned}
$$

Recall that by definition of the AI path, the A and AI paths are the same in periods t_0 through t^*. Therefore the claim reduces to showing

$$0 = E_{t_0}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(A,0)}+Pay_{it}^{(A,0)}-r_iR_{it}^{(A,0)}-u_{it}^{(D,\infty)}-Pay_{it}^{(D,\infty)}+r_iR_{it}^{(D,\infty)}\right)\\+(1-\tau_{ij})\left(u_{jt}^{(A,0)}+Pay_{jt}^{(A,0)}-r_jR_{jt}^{(A,0)}-u_{jt}^{(D,\infty)}-Pay_{jt}^{(D,\infty)}+r_jR_{jt}^{(D,\infty)}\right)\end{array}\right\}\right].$$

I now add some notation for this proof alone to keep track of paths. Let $t^{(A,d)}$ be the (potentially random) period in which ij reach their $d+1^{\text{st}}$ agreement in equilibrium. Let the superscript (A,d) refer to the path in which all players remain in equilibrium until Stage 2 of period $t^{(A,d)}$, then ij defect to disagreement, and then all players follow equilibrium play. Note that the subscripts A and (A,∞) refer to the same path, and the subscripts AI and $(A,0)$ refer to the same path. Similarly, let $t^{(D,d)}$ be the period in which ij reach their $d+1^{\text{st}}$ agreement in the subsubgame in which they disagree at the first d opportunities to negotiate, and let (D,d) refer to the path in which ij disagree at every opportunity to negotiate until period $t^{(D,d)}$. Note that the subscripts DI and (D,∞) are the same.

The Kalai proportional bargaining solution requires that:

$$0 = E_{t_0}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(A,\infty)}+Pay_{it}^{(A,\infty)}-r_iR_{it}^{(A,\infty)}-u_{it}^{(D,0)}-Pay_{it}^{(D,0)}+r_iR_{it}^{(D,0)}\right)\\+(1-\tau_{ij})\left(u_{jt}^{(A,\infty)}+Pay_{jt}^{(A,\infty)}-r_jR_{jt}^{(A,\infty)}-u_{jt}^{(D,0)}-Pay_{jt}^{(D,0)}+r_jR_{jt}^{(D,0)}\right)\end{array}\right\}\right]$$

and for any finite $t^{(A,d)}$ and $t^{(D,d)}$,

$$\begin{aligned}0 &= E_{t^{A,d}}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(A,d+1)}+Pay_{it}^{(A,d+1)}-r_iR_{it}^{(A,d+1)}-u_{it}^{(A,d)}-Pay_{it}^{(A,d)}+r_iR_{it}^{(A,d)}\right)\\+(1-\tau_{ij})\left(u_{jt}^{(A,d+1)}+Pay_{jt}^{(A,d+1)}-r_jR_{jt}^{(A,d+1)}-u_{jt}^{(A,d)}-Pay_{jt}^{(A,d)}+r_jR_{jt}^{(A,d)}\right)\end{array}\right\}\right]\\ &= E_{t^{D,d+1}}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(D,d)}+Pay_{it}^{(D,d)}-r_iR_{it}^{(D,d)}-u_{it}^{(D,d+1)}-Pay_{it}^{(D,d+1)}+r_iR_{it}^{(D,d+1)}\right)\\+(1-\tau_{ij})\left(u_{jt}^{(D,d)}+Pay_{jt}^{(D,d)}-r_jR_{jt}^{(D,d)}-u_{jt}^{(D,d+1)}-Pay_{jt}^{(D,d+1)}+r_jR_{jt}^{(D,d+1)}\right)\end{array}\right\}\right].\end{aligned}$$

By the Transverality condition,

$$
\begin{aligned}
0 &= E_{t_0}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(A,0)}+Pay_{it}^{(A,0)}-r_iR_{it}^{(A,0)}-u_{it}^{(D,\infty)}-Pay_{it}^{(D,\infty)}+r_iR_{it}^{(D,\infty)}\right)\\ +(1-\tau_{ij})\left(u_{jt}^{(A,0)}+Pay_{jt}^{(A,0)}-r_jR_{jt}^{(A,0)}-u_{jt}^{(D,\infty)}-Pay_{jt}^{(D,\infty)}+r_jR_{jt}^{(D,\infty)}\right)\end{array}\right\}\right]\\
&+\sum_{d=0}^{\infty}E_{t_0}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(A,d+1)}+Pay_{it}^{(A,d+1)}-r_iR_{it}^{(A,d+1)}-u_{it}^{(A,d)}-Pay_{it}^{(A,d)}+r_iR_{it}^{(A,d)}\right)\\ +(1-\tau_{ij})\left(u_{jt}^{(A,d+1)}+Pay_{jt}^{(A,d+1)}-r_jR_{jt}^{(A,d+1)}-u_{jt}^{(A,d)}-Pay_{jt}^{(A,d)}+r_jR_{jt}^{(A,d)}\right)\end{array}\right\}\right]\\
&+\sum_{d=0}^{\infty}E_{t_0}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(D,d+1)}+Pay_{it}^{(D,d+1)}-r_iR_{it}^{(D,d+1)}-u_{it}^{(D,d)}-Pay_{it}^{(D,d)}+r_iR_{it}^{(D,d)}\right)\\ +(1-\tau_{ij})\left(u_{jt}^{(D,d+1)}+Pay_{jt}^{(D,d+1)}-r_jR_{jt}^{(D,d+1)}-u_{jt}^{(D,d)}-Pay_{jt}^{(D,d)}+r_jR_{jt}^{(D,d)}\right)\end{array}\right\}\right]\\
&=E_{t_0}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(A,0)}+Pay_{it}^{(A,0)}-r_iR_{it}^{(A,0)}-u_{it}^{(D,\infty)}-Pay_{it}^{(D,\infty)}+r_iR_{it}^{(D,\infty)}\right)\\ +(1-\tau_{ij})\left(u_{jt}^{(A,0)}+Pay_{jt}^{(A,0)}-r_jR_{jt}^{(A,0)}-u_{jt}^{(D,\infty)}-Pay_{jt}^{(D,\infty)}+r_jR_{jt}^{(D,\infty)}\right)\end{array}\right\}\right]\\
&+\sum_{d=0}^{\infty}E_{t_0}\left[E_{t^{A,d}}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(A,d+1)}+Pay_{it}^{(A,d+1)}-r_iR_{it}^{(A,d+1)}-u_{it}^{(A,d)}-Pay_{it}^{(A,d)}+r_iR_{it}^{(A,d)}\right)\\ +(1-\tau_{ij})\left(u_{jt}^{(A,d+1)}+Pay_{jt}^{(A,d+1)}-r_jR_{jt}^{(A,d+1)}-u_{jt}^{(A,d)}-Pay_{jt}^{(A,d)}+r_jR_{jt}^{(A,d)}\right)\end{array}\right\}\right]\right]\\
&+\sum_{d=0}^{\infty}E_{t_0}\left[E_{t^{D,d+1}}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(D,d+1)}+Pay_{it}^{(D,d+1)}-r_iR_{it}^{(D,d+1)}-u_{it}^{(D,d)}-Pay_{it}^{(D,d)}+r_iR_{it}^{(D,d)}\right)\\ +(1-\tau_{ij})\left(u_{jt}^{(D,d+1)}+Pay_{jt}^{(D,d+1)}-r_jR_{jt}^{(D,d+1)}-u_{jt}^{(D,d)}-Pay_{jt}^{(D,d)}+r_jR_{jt}^{(D,d)}\right)\end{array}\right\}\right]\right]\\
&=E_{t_0}\left[\sum_{t=t_0}^{\infty}\beta^{\frac{t-t_0}{m}}\left\{\begin{array}{r}-\tau_{ij}\left(u_{it}^{(A,0)}+Pay_{it}^{(A,0)}-r_iR_{it}^{(A,0)}-u_{it}^{(D,\infty)}-Pay_{it}^{(D,\infty)}+r_iR_{it}^{(D,\infty)}\right)\\ +(1-\tau_{ij})\left(u_{jt}^{(A,0)}+Pay_{jt}^{(A,0)}-r_jR_{jt}^{(A,0)}-u_{jt}^{(D,\infty)}-Pay_{jt}^{(D,\infty)}+r_jR_{jt}^{(D,\infty)}\right)\end{array}\right\}\right],
\end{aligned}
$$

which is the desired claim. □

A.4 Tables and Figures

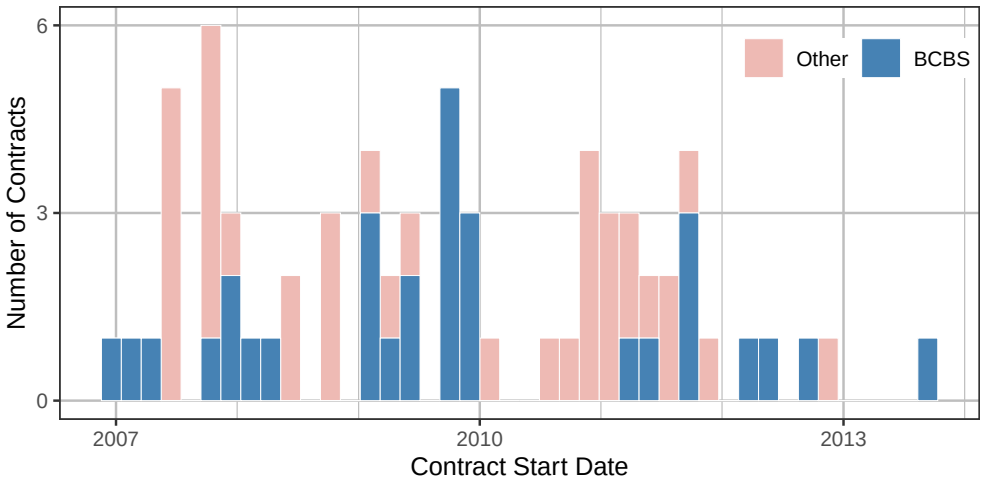


Figure 4: Contracts are formed at different times. Histogram of contract start dates for contracts used in the estimation sample and introduced in 2007–2014 for Highmark BCBS (blue) and other modeled insurers (pink). Vertical lines indicate January 1 of a given year. Contracts were not systematically introduced on the same dates.

Table 2: Additional estimated bargaining parameters. BCBS parameters correspond to Highmark BCBS. "Data" corresponds to average difference between MLR-implied costs per life and estimated average inpatient payments per life insured, and would exactly set the MLR moment to zero for the myopic and forward-looking models. The r^M net negotiation costs are close to their starting point of \$10,000 and may weakly identified or unidentified.

	Parameter (τ^{Size} Estimated)							
	η_{BCBS}	η_{HPUOV}	η_{Aetna}	$\eta_{UnitedHealth}$	η_{Cigna}	$\eta_{Carelink}$	r^M_{yBCBS}	r^M_{nBCBS}
Only-2015	3657***	3404***	3658***	2008***	4627***	3139***	10000***	9999***
(Nash/Kalai)	(45)	(85)	(116)	(29)	(32)	(39)	(2614)	(1441)
Myopic	4640***	4036***	3659***	3197***	4624***	3139***	10000***	10000***
(Nash/Kalai)	(14)	(650)	(37)	(374)	(26)	(463)	(1444)	(1)
Forward-Looking	4638***	3631***	3660***	3284***	4626***	3140***	9999***	9999***
($\text{Pay}_{IRT} = 0$)	(130)	(302)	(37)	(69)	(30)	(45)	(29)	(65)
Data	3600	3356	3554	1999	4635	3114		

Note: *p<0.1; **p<0.05; ***p<0.01