Stocks vs. Flows:
Second-Best Incentive Design for Stock-Generated Externalities

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Abstract

Conventional wisdom states that incentives targeting stocks and flows can lead to equivalent externalities correction. However, this may not hold when externalities are more directly produced from stocks ("stock-generated externalities") rather than flows of production, as is commonly modeled. Stocks that exhibit these external effects include trees and CO_2 absorption, shellfish and water cleaning, and livestock and methane emissions. I present a dynamic optimization model of privately-owned renewable resource extraction where the stock of natural capital jointly produces a private good and social benefits. In particular, I explore how incentives targeting measures of stock abundance and flows of production perform relative to the social optimum and a no policy benchmark. I present both theoretical and numerical results by calibrating the model to industry and scientific parameters from the Chesapeake Bay, where Eastern oyster (*C. virginica*) cultivation yields water purification benefits. I find that stock-targeted incentives have the potential to achieve the socially optimal allocations of resource abundance and environmental quality in the long run. While flow-targeting may achieve short run gains in environmental quality, the firm eventually reverts back to levels of extraction as if no policy occurred. This is likely driven by differences in revenue sources for the firm. Stock-targeting allows the firm to flexibly maximize profits over two sources of revenue: the natural resource stock and the private good. By contrast, flow-targeting solely changes the price of the private good, leading to similar profit maximization and extraction behavior as no-policy in steady state. The paper presents a theory for incentive design aimed at minimizing environmental damages through management of privately-owned living natural resources.

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1 Introduction

Conventional wisdom states that externalities scale with flows of production (Pigou, 1920; Ayres and Kneese, 1969). Previous models of environmental policy performance have therefore focused on controlling external damages stemming from firm activity (Gordon, 1954; Baumol, 1972; Nordhaus, 1992; Schmutzler and Goulder, 1997; Farzin and Tahvonen, 1996; Shortle and Horan, 2001; Golosov et al., 2014; van der Ploeg and Withagen, 2014; Folini et al., 2024). Damages from atmospheric CO_2 emissions, overfishing, and Superfund sites are classic examples of this type of "flow-generated externality" and policies targeting both stocks of inputs and flows of output can yield equivalent levels of environmental quality in this setting (Holtermann, 1976). However, more attention must be paid to a number of atypical externality production functions of major policy importance. These externalities are produced directly from stocks of inputs, which I call "stock-generated externalities."[1] Firm-level incentives targeting stocks and flows may not yield equivalent environmental outcomes when this type of externality is present. I attempt to theoretically highlight the issue of incentive design for stock-generated externalities in this paper.

Stocks of living, or renewable, natural resources are common sources of stock-generated externalities.[2] Trees, shellfish, and ungulates, among others, can produce external environmental effects such as atmospheric CO_2 absorption, surface water filtration, erosion mitigation, and greenhouse gas emissions in absence of human intervention.[3] Instrumentalizing living natural resources to combat environmental damages has not only been advocated for by researchers (Bricker et al., 2018; Dasgupta, 2021; Barrett et al., 2022; Zeng and Hausmann, 2022; Taylor et al., 2023) and the popular press (Depro et al., 2008; Wood, 2019; BBC, 2020; DePillis et al., 2024; Hotakainen, 2024), but incentive-based policies aimed at industries that privately hold these stocks are actively being pursued by non-governmental programs, firms, and governments at various levels (UN Framework Convention on Climate Change, 2016; U.S. Department of Agriculture, 2021; Sandi, 2025; Kampmann, 2024; Khalil, 2024; Aronoff and Rafey, 2025).

While an optimal policy would target a direct measure of the benefits or damages produced by the resource stock, in the real world policies are necessarily second-best (Karp, 2011). The first-best

[1] As a thought exercise, compare atmospheric greenhouse gasses (GHGs) generated from factory production versus cows. While many cows are destined for beef or dairy production, various species of cattle have existed independent of human intervention (Felius et al., 2014). All cattle regardless of ownership and species produce methane, a potent GHG, via enteric fermentation (Johnson and Johnson, 1995; Quinton, 2019). Therefore, external damages are "stock-generated," as they scale directly from the stock of cows and more indirectly from firm activity.

By contrast, GHG emissions from factory production would not occur without, well, factories. If factories stopped operating one day but the stock of machinery remained, assuming the machines were not running extraneously, GHG emissions would cease. Therefore, external damages are "flow-generated," as they are directly derived from the flow of production and indirectly from the stock of machinery.

Note that the model presented in this paper abstracts away from active "investments" in the stock through e.g. plantings, as in Chichilnisky et al. (1995) and . Investments in the natural resource stock may affect second-best policy performance and is the subject of future work.

[2] Another term for this type of resource is (living) natural capital (Fenichel and Abbott, 2014).

[3] These types of externalities are often called "ecosystem services" when the effects are positive.

requires a level of monitoring and administration that is infeasible for most agencies and programs. To put things into perspective, imagine a stand of trees owned by a lumber firm. A first-best incentive targeting CO_2 filtration would pay the firm based on the exact amount of carbon filtered and sequestered into either biomass or soil for every tree at every instant, specific to the species. Given that precise measurement is prohibitively expensive, proxy policy targets must be used (Jack et al., 2008).[4] The most common proxy implemented in incentive programs like the USDA's Conservation Reserve Program, those implemented as a part of REDD+, and Denmark's recently passed Livestock Tax, are approximations of the total resource stock (Jayachandran, 2013; U.S. Department of Agriculture, 2020; Kampmann, 2024; Izquierdo-Tort et al., 2025). Other policies, such as Maryland's Nutrient Credit Trading Program, have used observed flows of production as their main policy target (Sandi, 2025) and researchers have considered equivalents to the latter for the livestock industry (Funke et al., 2022).[5][6] Whether stocks *and* flows are close enough approximations to the direct external effect remains an open question.[7]

Can firm-level incentives targeting either stocks or flows yield equivalent levels of environmental quality when attempting to internalize stock-generated externalities? To answer this question, I present a dynamic optimization model of renewable resource extraction where a stock of privately-owned capital jointly produces a social or public benefit.[8] The model draws heavily from the seminal work of Dasgupta and Heal (1974) and Chichilnisky et al. (1995). I present a collection of centralized and decentralized equilibria meant to represent what happens in the theoretical optimum and more realistic policy settings: 1) the Social Planner's problem, 2) the no policy benchmark, otherwise known as business-as-usual (BAU), 3) a stock-targeted incentive, and 4) a flow-targeted incentive. The latter two scenarios are theoretical analogues to real-world, second-best policies that have been implemented or recently passed in forestry, fisheries, and livestock management. The model can be applied to other problems where privately-owned renewable resource stocks have a direct,

[4] In addition, incentive design must account for other potential sources of market failure, such as weak property rights and imperfect information among producers. Work by Jack, Jayachandaran, Sims and others, in addition to news coverage, highlights the need for careful ecosystem service incentive design to address these issues (Engel et al., 2008; Jack et al., 2008; Prager et al., 2016; Jack and Jayachandran, 2019; Teytelboym, 2019; Calle, 2020; McGrath, 2020; Wells et al., 2020).

[5] Note that the process of livestock rearing may induce other environmental externalities through deforestation, especially in countries with inconsistent enforcement such as Brazil (Assunção et al., 2023; Dominguez-Iino, 2025). Cattle ranching-driven deforestation has been associated with more remote farms with higher output, total acreage and forest cover based on a property-level analysis in the country (Skidmore et al., 2021). Given that deforestation likely scales with cattle *production* in this setting, this would be considered a flow-generated negative externality.

[6] While some researchers and agencies have stated that wood products and forest regrowth post-harvest maintain carbon neutrality or notable carbon sequestration benefits, recent work predicts that tree harvest may result in an additional 3.5-4.2 Gt of CO_2 annually (U.S. Forest Service, 2020; Peng et al., 2023; Taylor et al., 2023).

[7] There have also technical innovations to change the amount of external benefits or damages produced independent of the size of the natural resource stock, which occur in the form of breeding programs or changes in feed composition (Hawkins et al., 2001; Quinton, 2019; Wadsworth et al., 2019; Watson, 2021). Technical innovation in the presence of stock-generated externalities is not explicitly addressed in this paper and will be the focus of future work.

[8] Private ownership/leasing and extraction are common features in industries such as lumber, livestock and shellfish aquaculture (Butler and Wear, 2013; Tahvonen and Rautiainen, 2017; U.S. Department of the Interior, 2025; Washington State Department of Natural Resources, 2023). The issue of resource conservation under weak property rights, such as those that induce the common resource stock externality, in the presence of social benefits from the resource stock is the subject of future work.

external value to society through shoreline defense, biodiversity and groundwater conservation, among others.

There is a strong temptation to implement flow-targeting for stock-generated externalities, but one must resist. The theoretical results show a striking divergence in incentive performance. A stock-targeted incentive can yield the Social Planner's allocations, while flow incentives converge to BAU in the long run. All positive stock incentives, regardless of the marginal value of the subsidy, allow the firm to internalize the social value of the stock. A subset of stock subsidies that converge to the Social Planner's steady state values lead to the socially optimal level of biomass in the long run. However, given that consumption stems from extraction, policies targeting flows incentivize stock management only in the most indirect sense – the marginal unit extracted or conserved, rather than the entirety of the stock itself. Any impact that a flow policy has on social benefits provision in this setting are inherently dynamic. When the system settles into its steady state, firms revert to the same profit maximization behavior as if no policy existed. The steady state result is likely driven by differences in revenue sources for the firm. Stock-targeting allows for an additional degree of freedom beyond the private good by which to profit maximize. By contrast, a flow-targeted incentive solely changes the price of the private good, leading to the same profit maximization and extraction behavior as no-policy in steady state.

The model is then expanded to include pollution dynamics and environmental benefits provision. It is calibrated to scientific and industry parameters from the Chesapeake Bay, USA, where commercial Eastern oyster (*C. virginica*) cultivation is being used as a means to reduce fertilizer damages in the namesake watershed (Sandi, 2025). The calibration is used not only to explore the dynamic paths of natural resource accumulation and pollution but other general constraints a policymaker may experience. For example, the size of the incentive and the frequency of policy adjustments could vary based on pre-set fiscal spending periods or regime change (Battaglini and Coate, 2008).

The dynamic results support the theoretical narrative – policy target matters above all else. Sign and scale of a marginal incentive can matter substantially for natural resource accumulation, while the frequency of policy updates matters little. Positive marginal incentives on stocks lead to increases in resource accumulation along the entire time path. Stock-targeted incentives can achieve the social optimum when the value of the marginal incentive is structured to converge to the Social Planner's solution in steady state. Marginal flow disincentives can have short run, minor increases in resource accumulation while marginal flow incentives have the opposite effect, leading to excess short run extraction even when set at the "Pigouvian" values, as in the case of the Maryland's Nutrient Credit Trading Program. However, the effects are generally minor, with up to a 0.2% increase in biomass under marginal disincentives, and up to a 0.05% decrease in biomass under marginal incentives. Delayed adjustments to marginal incentives play very a minor role in changing dynamic firm behavior regardless of target (0.003-0.05% increase).

My primary contribution is a dynamic, decentralized theoretical model to evaluate the performance of both first- and second-best instruments when externalities are generated from stocks rather than

flows. Core literature modeling externality-correcting policies has typically focused on optimal (Nordhaus, 1992; Farzin and Tahvonen, 1996; Acemoglu et al., 2012; Golosov et al., 2014; Shapiro, 2016) or second-best policies for flow-generated externalities, such as greenhouse gas and criteria emissions.[9] The second-best can stem from distortionary taxation (Bovenberg and Goulder, 2002; Fullerton and Wolverton, 2005; Barrage, 2020), uncertainty/opaqueness of externality measurement (Shortle and Dunn, 1986; Schmutzler and Goulder, 1997; Cremer and Gahvari, 2002; Jacobsen et al., 2020), and heterogeneous externality production (Claassen and Horan, 2001; Xabadia et al., 2008; Knittel and Sandler, 2018). Incorporating exogenously growing capital further complicates modeling (Heal, 1998; Arrow et al., 2000), as does incorporating direct consumer valuation of the stock or, equivalently, the externality produced by the stock (Fenichel and Abbott, 2014). The theoretical work in this space has understandably incorporated simplifying assumptions for tractability. Some authors present policy dynamics in a centralized market (Tahvonen and Kuuluvainen, 1991; Chichilnisky et al., 1995) akin to the core work on flow-generated externalities, while others strictly model the producer side and/or assume a static social value of the externality in a dynamic setting (van Kooten et al., 1995; DePiper et al., 2017; Tahvonen and Rautiainen, 2017; Assmuth et al., 2021).

This paper presents a simple model that marries many of the key components from these different strands of literature by decoupling the consumer and producer's problems and incorporating a dynamically-adjusting social value of a renewable stock to more fully capture policy performance across time. Included policies are inspired by the imperfect measurement literature, where the policy targets are more or less imperfect proxies for externalities production, where I provide theoretical performance results of imperfect proxies for the first-best.[10] Further, I numerically evaluate the performance of other policy deviations from the first-best such as infrequent incentive updates, sign, and scale, in order to mimic some of the administrative constraints faced by policymakers.

I also contribute to empirical and policy work on incentives to internalize stock-generated externalities by providing a flexible theoretical framework underlying policy performance. In particular, there is a growing set of randomized agricultural programs aimed at internalizing the ecosystem benefits provided by natural resources either within or buffering privately-owned agricultural plots, otherwise known as Payments for Ecosystem Services (PES) (Jack et al., 2008; Alix-Garcia et al., 2015; Jayachandran et al., 2017; Jayachandran, 2023). In addition, there are a number of government- and privately-implemented programs incentivizing land conservation and water purification in lieu of (or in conjunction with) farm or aquacultural production or to offset ecosystem degradation (Teytelboym, 2019; U.S. Department of Agriculture, 2020, 2021; Frey et al., 2021; Sandi, 2025; Taylor et al., 2023; Aronoff and Rafey, 2025). For example, recent legislation passed by Denmark will tax producers of cows and pigs starting in the year 2030. The tax will be levied based on

[9] Dasgupta et al. (1980) also explore the dynamic implications of different tax policies within a centralized market based on their seminal work on non-renewable resource extraction (Dasgupta and Heal, 1974).

[10] The evaluation of price versus quantity instruments is not presented in this paper, but serves as one of many inspirations for the policy instruments explored here (Weitzman, 1974).

total stock held by producers, which proxies the total amount of methane emissions generated from the animals' physiological processes (Graham-McLay, 2024; Kampmann, 2024; Khalil, 2024; Sengupta and Fuente, 2024). These policies and programs are generally characterized as separate with respect to other natural resources. My point of view is that at their core, these programs attempt to address the same underlying problem highlighted in this paper. The hope is that the framework presented here may help inform researchers and policymakers focused on internalizing stock-generated externalities going forward.

2 Model

I begin with a stylized model of renewable natural resource extraction. The firm engages in costless harvest. Time is continuous and infinite. A representative consumer has preferences over both consumption of a private good C_t and the stock of natural capital S_t, where subscript t represents that the variables change over, but are not explicit functions of, time.

$$U(C_t, S_t) \tag{1}$$

Utility is assumed to be well-behaved and increasing in S_t ($U_C, U_S > 0$, $U_{CC}, U_{SS} \leq 0$). Preferences are assumed to be separable in C_t and S_t.[11][12] The representation of preferences in S_t is general enough to capture direct benefits generated from accumulation of the stock (e.g. through physiological processes) at every instant t and benefits from the change in the stock.

Production consists of harvesting H_t amount of the natural resource at every instant. The production function is denoted

$$F(H_t) \tag{2}$$

where $F_H > 0$ and $F_{HH} \leq 0$. The generality of these functional forms allows for modeling flexibility over different preferences and technology assumptions, such as under markets for commodities with decreasing returns to scale.[13]

Meanwhile, the natural resource changes exogenously through function

$$G(S_t) \tag{3}$$

which is logistic in time. This function, drawn from the population dynamics literature, is often parabolic in S_t such that $G(S_t) > 0$ for all $S_t \in [0, \bar{S}]$, with upper bound $\bar{S}$ denoting the population

[11] Separability is a common functional form assumption in environment and natural resources contexts (Tahvonen and Kuuluvainen, 1991; Bovenberg and Smulders, 1995; Heal, 1998).

[12] The version of the model with preferences over the stock of pollution is assumed to decrease in pollution stock and therefore implicitly increase in the stock of the natural resource. See Section 3.1.

[13] Note that increasing returns to scale technologies are not always found in extractive natural resources-based industries (Stier, 1980; Scuderi and Chen, 2019).

carrying capacity within a given geographic area.[14] Other versions may oscillate if there is some critical population $S_0 \geq 0$ such that growth is either negative or tends quickly towards zero, also called an Allee effect (Dennis, 1989) (see Appendix Figure A1 for illustrations of common population growth functions).[15] The model results do not fundamentally change in the presence of Allee effects, so I assume a parabolic function in S_t for ease of exposition.

The stock of the natural resource evolves by the equation

$$\dot{S} = G(S_t) - H_t \tag{4}$$

where $\dot{S}$ denotes the change in the natural resource stock with respect to time. Note that Equation 4 is nearly identical to classic wild capture fisheries models in bioeconomics (Gordon, 1954; Clark and Munro, 1975; Schaefer, 1991). Here I explicitly focus on a situation where the firm solely owns the stock of the natural resource within a fixed production area in order to capture the decisions by agents in industries where property rights are more secure, such as the lumber or aquaculture industries. Further, I do not focus on extensive margin effects due to the complexity of environmental characteristics of neighboring geographic areas, such as the presence of wild natural capital stock, water flow, and sediment porosity and permeability, among others. Depending on these characteristics and the production technology of the firm, expansion and contraction of production areas can lead to unclear changes in levels of the natural resource stock (both cultivated and wild).

2.1 The Planning Problem

The Social Planner's objective is to maximize the discounted present value of social welfare given choices in consumption C_t and the state of the natural capital stock S_t at every instant. ρ represents the social discount rate and private goods production $F(H_t)$ is equal to consumption for all t.

$$U_0 = \max_{\{C_t, S_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} U(C_t, S_t) dt \tag{5}$$

$$\dot{S} = G(S_t) - H_t$$
$$C_t = F(H_t)$$
$$S_0 > 0$$

The first order necessary conditions (FONCs) yield the following Euler equation, which is also the condition that equates marginal benefits and marginal costs of natural resource stock accumulation

[14] I do not consider time-varying carrying capacities in this paper, but time variance may play a role in determining the optimal harvest under the different policy scenarios (Carson et al., 2009).

[15] The parabolic form is called a pure compensation growth function, while a growth function that is both convex and concave but positive up to $\bar{S}$, i.e. a function with weak Allee effects such that $S_0 = 0$, is referred to as a depensation growth function. The functional form with strong Allee effects ($S_0 > 0$) is called a critical depensation growth function.

for each t.

$$G_S + \frac{U_S}{U_C}\frac{1}{F_H} = \rho - \frac{\dot{U}_C}{U_C} - \frac{\dot{F}_H}{F_H} \tag{6}$$

Y_X denotes the derivative of a function $Y(X_t)$ with respect to X_t, where t is suppressed for clarity. On the left hand side, the marginal benefits of stock accumulation include both private returns in the form of the marginal growth of the stock, as well as public returns, which is represented by the marginal rate of substitution between stock and consumption, scaled by the marginal product of harvest. On the right hand side is the marginal cost of stock accumulation, which includes both intertemporal consumption and production choices.

A steady state is defined as a set of allocations $\{C^*, H^*, S^*\}$ such that $\dot{S} = 0$ and $\dot{U}_C = \dot{F}_H = 0$. This gives us the following steady state condition.

$$G_{S^*} + \frac{U_{S^*}}{U_{C^*}}\frac{1}{F_{H^*}} = \rho \tag{7}$$

The social discount rate ρ plays a major role in determining the optimal level of the natural resource stock in a steady state equilibrium. Under constant discount rates, a myopic Social Planner implies $\rho \to \infty$ and a natural resource stock in steady state that is lower than when the population is at its maximum growth, also called its maximum sustainable yield (MSY). In other words, the Social Planner may make choices that deplete the stock in steady state in the presence of myopia. By contrast, a Social Planner that values the utility of future generations the same as those in the present is represented by $\rho = 0$. Under a discount rate of zero, S^* is higher than its maximum sustainable yield, which is the optimal result obtained under the Green Golden Rule (Beltratti et al., 1993). The choice of discount rates in the presence of environmental externalities is a topic of active debate (see Heal (2017) and Polasky and Dampha (2021) for comprehensive reviews). Given the scope of the problem modeled here, where the representative consumer maintains positive utility in entire stock of the natural resource, a discount rate closer to zero is more appropriate (Clark, 1973). Therefore, for the remainder of the Section 2, I assume a social discount rate ρ near zero.

Note that the Planning Problem is strongly reminiscent of Chichilnisky et al. (1995) and Heal (1998). I further break this model apart to explore a collection of incentive-based policy states of the world in order to understand how natural capital accumulation differs by policy.

2.2 The Decentralized Equilibrium Under No Policy

Let there be a world with a representative consumer and a representative firm, where the firm does not internalize the social benefits generated from a stock of natural capital that it owns, i.e. no incentive-based policy exists. I refer to this world as no policy or business-as-usual (BAU).

The firm's objective is to maximize the discounted present value of profits from private good

production subject to growth of the resource stock. θ_t is the internal rate of return at time t.[16]

$$\Pi_0 = \max_{\{H_t, S_t\}_{t=0}^{\infty}} \int_{t=0}^{\infty} e^{-\int_0^t \theta(\tau) d\tau} F(H_t) dt \tag{8}$$

$$\dot{S} = G(S_t) - H_t$$
$$S_0 > 0$$

Market prices are normalized to 1. The FONCs for the representative firm yield

$$G_S = \theta_t - \frac{\dot{F}_H}{F_H} \tag{9}$$

where the left hand side represents the marginal benefits with respect to stock accumulation and only consists of the firm's private benefits. The right hand side represents its marginal costs, which includes its internal rate of return and the growth rate of marginal production.

On the other side of the market, the consumer must now take into account the evolution of assets they hold in order to make choices over consumption. The consumer owns claims A_t on firm profits π_t, where instantaneous consumption choices help determine the evolution of the value of future claims $q_t \dot{A}$, reminiscent of (Lucas, 1978).[17]

$$q_t \dot{A} = A_t \pi_t - C_t \tag{10}$$

The consumer maximizes their present discounted value of utility

$$U_0 = \max_{\{C_t, A_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} U(C_t, S_t) dt \tag{11}$$

subject to Equation 10, where claims on assets in the initial period is positive ($A_0 > 0$). Notice that even though the consumer has preferences over the stock of the natural resource, it does not play a role in the consumer's problem directly and, therefore, is taken as given. In this setting, consumers have no say over how the firm manages their stock of the natural resource. Rather, they influence firm behavior strictly through the consumption of the private good.[18] The FONCs yield the following Euler equation

$$\frac{\dot{q}}{q_t} + \frac{\pi_t}{q_t} = \rho - \frac{\dot{U}_C}{U_C} \tag{12}$$

where the left hand side represents the marginal benefits of accumulating assets, and consists of

[16] The choice of a time-varying internal rate of return assumes no arbitrage along the equilibrium path and is a feature commonly observed in firms. See Gormsen and Huber (2023) for a recent overview of time-varying corporate discount rates.

[17] This is conceptually similar to a Lucas tree model under homogeneous agents without uncertainty and, potentially, with actual trees.

[18] The setting implies that while we may be in a world where "green goods" are for sale, the consumer cannot induce the firm to provide a higher level of the "green" characteristic specifically (Kotchen, 2005, 2006).

the growth rate of the price of future claims as well as firm profits normalized by the future claims price.

An equilibrium transition path exists if there are allocations $\{C_t, H_t, A_t, S_t\}_{t=0}^{\infty}$ and profits $\{\pi_t\}_{t=0}^{\infty}$ such that

1. The consumer and producer maximize their objectives subject to the evolution of claims on profits and the natural resource stock, respectively, and
2. Private good and asset markets clear, i.e.
   (a) $C_t = F(H_t) = \pi_t$
   (b) $A_t = 1$

for all t. Note that in order to ensure $2(b)$, the following no arbitrage condition must hold where the consumer's marginal benefit of asset accumulation is equal to the firm's internal rate of return.

$$\theta_t = \frac{\dot{q}}{q_t} + \frac{\pi_t}{q_t} \tag{13}$$

Along an equilibrium path Equation 9 can therefore be rewritten as

$$G_S = \rho - \frac{\dot{U}_C}{U_C} - \frac{\dot{F}_H}{F_H} \tag{14}$$

which resembles the Social Planner's Euler Equation (Equation 6). The main difference is that the marginal benefits from stock accumulation only consist of its private returns in the form of marginal stock growth because the firm does not internalize the social benefits generated from the natural capital stock.

A steady state market equilibrium is defined as a set of allocations $\{C^*, H^*, S^*\}$ and profits $\{\pi^*\}$ such that $\dot{S} = \dot{A} = 0$. This implies the following steady state equilibrium condition.

$$G_{S^*} = \rho \tag{15}$$

Given $G(S^*) = H^*$ and a consumer that is as patient as the Social Planner, Equation 15 suggests that the firm will aim to harvest at a level that effectively maximizes the growth of the natural resource stock in order to maximize profits, i.e. harvests at its maximum sustainable yield (MSY) denoted H_{MSY}.[19] This profit maximizing behavior at MSY echoes the classic steady state result in sole-owner fisheries bioeconomic models (Gordon, 1954; Clark and Munro, 1975).

Now that we have allocations under both the Social Planner and BAU as theoretical benchmarks, we can explore how social benefits under various policy targets may converge or differ from each.

[19] The value may be just below MSY due to impatience ($\rho \approx 0$).

2.3 Stock Incentives

Suppose the firm is provided an incentive at time $t = 0$ dependent on the total stock of the resource that it owns. There are no administrative costs to policy implementation. The firm's objective function is now to maximize the present discounted value of profits, which now consists of both the private good and any revenues (or costs) based on the total stock.

$$F(H_t) + \sigma(S_t) \tag{16}$$

$\sigma(S_t)$ is an exogenously-determined policy function that is differentiable in S_t and autonomous with respect to t.[20] The consumer's budget constraint now includes lump sum payments γ_t.[21]

$$q_t \dot{A} = A_t \pi_t - C_t - \gamma_t \tag{17}$$

An equilibrium transition path exists if there are allocations $\{C_t, H_t, A_t, S_t\}_{t=0}^{\infty}$, profits and taxes $\{\pi_t, \gamma_t\}_{t=0}^{\infty}$ such that

1. The consumer and producer maximize their own objectives subject to the evolution of claims on profits and the natural resource stock, respectively
2. Private good and asset markets clear, i.e.
   (a) $C_t = F(H_t) = \pi_t$
   (b) $A_t = 1$, and
3. The balanced budget condition $\gamma_t = \sigma(S_t)$ holds.

The FONCs give us the following condition for the equilibrium transition path.

$$G_S + \frac{\sigma_S}{F_H} = \rho - \frac{\dot{U}_C}{U_C} - \frac{\dot{F}_H}{F_H} \tag{18}$$

Compared to Equation 6, a stock incentive $\sigma(S_t)$ that will yield the Social Planner's allocations for all t exists if the marginal incentive σ_S is equal to the marginal rate of substitution between the natural resource stock and private good consumption. However, we may not be able to define a policy function that yields the Social Planner's path.

In order for σ_S to equal to the marginal rate of substitution between the natural resource stock and private good consumption, C_t must be expressed as a function of S_t, which is not possible for all t. For clarity, combining Equation 4 and the equivalence between production and consumption would lead to the following equation

$$C_t = F(G(S_t) - \dot{S})) \tag{19}$$

[20] In other words, $\sigma(\cdot)$ is not an explicit function of time.

[21] Note that I am agnostic to the direction of $\sigma(S_t)$ (and therefore γ_t). Rather, the theoretical results will yield a policy direction. See Proposition 1 broader implications.

which defines the relationship between consumption and the resource stock for all t. Given that $\dot{S}$ is endogenously determined by C_t, the right hand side of the equation is only known to the policymaker retrospectively. This implies knowledge of S_t may not be sufficient to determine contemporaneous C_t and a policy function $\sigma(S_t)$ that yields the Planner's allocations across all instants is not knowable in this setting. However, we can derive policies that yield the Planner's steady state allocations.

Suppose we are in steady state as defined in Section 2.2. Equation 18 reduces to

$$G_{S^*} + \frac{\sigma_{S^*}}{F_{H^*}} = \rho \tag{20}$$

Given $\dot{S} = 0$, we can now express C^* strictly as a function of S^*.

$$C^* = F(G(S^*)) \tag{21}$$

Equation 21 implies that a policy that takes the following functional form

$$\sigma_{S^*} = \frac{U_{S^*}}{U_{C^*}} = \frac{U_{S^*}}{U_{F(G(S^*))}} \tag{22}$$

can lead to the socially optimal levels of the natural resource stock in steady state, as well as the following proposition.

Proposition 1 *Suppose there is an incentive $\sigma(S_t)$ that is differentiable in S_t. If the marginal incentive in steady state takes the form*

$$\sigma_{S^*} = \frac{U_{S^*}}{U_{C^*}}$$

such that $C^* = F(G(S^*))$, then the level of the natural resource stock will converge to the social optimum.

Proposition 1 provides some clarity on policy design. First, the marginal incentive must eventually converge to the marginal rate of substitution between social benefits and consumption under the Social Planner. Trivially, a policymaker may set a constant marginal policy such that Equation 22 holds for all t in order to obtain levels of the natural resource stock that eventually converge to the social optimum. However, there may also be time-varying policies that achieve the same steady state level of the stock.

For illustration, suppose that we have the following consumer preferences

$$U(C_t, S_t) = \ln C_t + \ln S_t \tag{23}$$

with a linear production function $F(H_t) = H_t$, and a Pearl-Verhulst equation with an intrinsic

population growth rate $g = 1$ and a population carrying capacity $\bar{S}$.

$$G(S_t) = S_t \left(1 - \frac{S_t}{\bar{S}}\right) \tag{24}$$

Taken together, a steady state marginal policy that satisfies Proposition 1 will take the form

$$\sigma_{S^*} = \frac{U_{S^*}}{U_{C^*}} = \left(1 - \frac{S^*}{\bar{S}}\right) \tag{25}$$

When expressed in aggregate terms, this implies that the following functional form of $\sigma(S_t)$ will yield the socially optimal levels of the natural resource stock in steady state.

$$\sigma(S_t) = S_t \left(1 - \frac{S_t}{2\bar{S}}\right) \tag{26}$$

Assuming $S_t \leq \bar{S}$ for all t, Equation 26 suggests that a time-varying incentive that converges to the socially optimum closely maps to the growth of the natural resource.[22] This is encouraging for policymakers, as many natural resource management strategies already rely on biological stock assessments and estimations of the growth of the stock, updated at regular intervals (U.S. Department of Agriculture, 2022; National Oceanic and Atmospheric Administration, 2023). Note that a marginal policy function σ_S that yields the socially optimal level of the stock in steady state may not have an elementary antiderivative with respect to S_t (i.e. obtaining $\sigma(S_t)$ is not straightforward). However, policies that are defined on a marginal basis are often implemented in practice, so it may be sufficient from a policymaker's perspective to strictly obtain the functional form of the marginal incentive.

A broader implication of stock-targeting is that given $U_C, U_S > 0$ and by Equations 6 and 20, any positive marginal incentive $\sigma_S > 0$ will yield a natural capital stock that is higher than BAU across time. This further implies that as long as the stock-targeted policy is a subsidy, a policymaker stands to obtain some extra social benefit relative to doing nothing.[23]

The higher accumulation of resource stock in steady state relative to BAU is likely driven by differences in firm incentives. Under a policy that subsidizes accumulation of the stock, the firm now has two sources of revenue – private goods production and its inventory of the natural resource. This allows the an additional degree of freedom for the firm to forgo revenue from private production in favor of stock accumulation, analogous to the behavior of a multi-product firm. Increases in stock can now yield increased returns with respect to either source of revenue in the future. This then begs the question of whether a policy that incentivizes harvest may be able to achieve similar results. For that, we turn to our last policy.

[22] See Appendix Figure A2 for the equilibrium time paths of the natural resource stock under the social optimum and the stock incentive for this example policy.

[23] At least in this stylized world with no policy administration costs. Costs of administration can be accounted for in this model with an explicit policymaker's objective.

2.4 Flow Incentives

Now consider a world where the firm is incentivized at $t = 0$ based on some transformation of the flow of production – in this case the total harvest of the natural resource. The firm's objective is to now maximize the present discounted value of

$$F(H_t) + \tau(H_t) \tag{27}$$

where $\tau(H_t)$ is a twice differentiable function of H_t and is autonomous with respect to t. $\tau(H_t)$ may be either an increasing or decreasing function of H_t. As in Section 2.3, the consumer "pays" for the policy in a lump sum manner and the equilibrium path is defined as before, which yields the following condition along the equilibrium path.

$$G_S = \rho - \frac{\dot{U}_C}{U_C} - \frac{\dot{F}_H + \dot{\tau}_H}{F_H + \tau_H} \tag{28}$$

Equation 28 differs substantially from the Social Planner's transition path (Equation 6) in a couple of key ways. For one thing, S_t does not appear in any term other than the marginal growth of the stock, similar to BAU. Further, $\dot{\tau}_H$ implies that the incentive must be time-varying in order for the policy to have an effect on stock accumulation, whereas U_S/U_C could be constant across time.[24] This is likely because an incentive on harvest alters the price faced by producers for private good production, similar to the effect of a tax or subsidy on intertemporal consumption choices.

In order for a flow policy to produce socially optimal levels of the natural resource stock across time, it is sufficient to show that an incentive yields the Social Planner's allocations in steady state. Suppose that we are in a steady state as defined in previous sections. The Euler equation is

$$G_{S^*} = \rho + \frac{\dot{\tau}_{H^*}}{F_{H^*} + \tau_{H^*}} \tag{29}$$

where $\dot{\tau}_{H^*} \neq 0$. Given that the policy is a function of harvest H_t and not explicitly of t, harvest itself must also be time-varying. Note that the change in the marginal incentive with respect to time can also be expressed as

$$\dot{\tau}_H = \tau_{HH}\dot{H} \tag{30}$$

H^* in steady state implies $\dot{H} = 0$, which contradicts $\dot{\tau}_{H^*} \neq 0$. Equation 29 therefore reduces to

$$G_{S^*} = \rho \tag{31}$$

which is the steady state Euler equation under BAU or no policy.[25] This brings us to the following proposition.

[24] Note that $\tau_H \neq -F_H$ otherwise the path becomes undefined.

[25] The result trivially reduces to BAU in steady state if the policy $\tau(H_t)$ is not twice differentiable.

Proposition 2 *There does not exist an autonomous flow incentive $\tau(H_t)$ that can achieve the socially optimal allocations for all t. Further, the stock of the natural resource under such a policy will eventually converge to that under BAU, or no policy.*

Given that the model highlights the simplest case where the extensive margin channel is shut down and the firm is at its technological frontier, there is only one pathway for the policymaker to alter the choices of the firm. If the policymaker structures an incentive to target harvest, this changes the price firms receive for the private good. The firm's incentive will be to maximize their yields in order to maximize profits in the long run. This is exactly the same behavior we observe under BAU. In other words, the steady state level of benefits provided to society from the natural resource will be no better than the in absence of a policy in the long run. Unlike what we see under stock policies, this implies that a flow policy cannot fully internalize a stock-generated externality across time in this setting.

Note that the choice of an incentive $\tau(H_t)$ that is autonomous with respect to t drives this result. There may exist a non-autonomous flow policy that can satisfy Equation 29 for all t. However, time-dependence that does not otherwise violate other assumptions may also lead to a degenerate policy trajectory $\tau_H \to \infty$.

Proposition 2 does not rule out that there may be some t in which flow-targeting can yield improvements in stock accumulation over BAU. Comparing Equations 14 and 28, it must be the case that

$$\frac{\dot{F}_H + \dot{\tau}_H}{F_H + \tau_H} > \frac{\dot{F}_H}{F_H} \tag{32}$$

for some t in order for the firm to accumulate a higher amount of the natural resource stock relative to BAU. This is driven by two components – the marginal incentive τ_H and its change over time $\dot{\tau}_H$. Suppose for illustration that $\dot{F}_H > 0$ along the path. On the one hand, if $\dot{\tau}_H > 0$, then the firm may be incentivized to reduce harvest today if it anticipates that it can obtain higher returns from harvest in the future. However, an increase in the marginal incentive can simultaneously have a dampening effect on stock accumulation through progressively increasing current returns to harvest, so changes in stock accumulation relative to BAU may be minimal. On the other hand, if the policy is a marginal *dis*incentive on harvest $\tau_H < 0$, then the firm may reduce harvest at each instant as long as the marginal disincentive remains in place.

In order to gain clarity about the length of time these improvements are obtained and to explore other possible policy structures with respect to both flow and stock incentives, I apply more specific functional forms and pollution dynamics to the model. The choices are informed by the inorganic nitrogen emissions problem in the Chesapeake Bay, where cultivation of oysters for food consumption may provide water purification benefits.

3 Nutrient Pollution and Oyster Aquaculture in the Chesapeake Bay

Nutrient pollution in water bodies is an increasingly important problem faced by many coastal regions. Nutrient loads, defined as the total amount of substances such as nitrogen and phosphorus measured in a body of water passing a specific geographic location at a given point in time, often come from both point and non-point sources (Yang et al., 2022). Excess nutrient loads in surface water can lead to eutrophication, or the excess supply of organic matter such as algae and cyanobacteria (Pinckney et al., 2001; Taylor and Heal, 2023). Though high nutrient loads do not often cause direct physical harm to humans, eutrophication can lead to hypoxia (low dissolved oxygen) through the decay of excess organic matter and can increase the prevalence of toxic algae blooms (Anderson et al., 2008; Rabalais et al., 2009; Rabotyagov et al., 2014). Both of these channels can increase mortality in species that are both commercially important and necessary for a functioning aquatic ecosystem. While requirements to set standards for nutrient levels in ground and surface water were established in the Clean Water Act of 1972 and federal criteria were set through the Environmental Protection Agency in the 2000s and 2010s, enforcement at the state level has remained a challenge (U.S. Environmental Protection Agency, 2007, 2009). Efforts to control nutrient emissions from non-point sources, such as agricultural emissions from fertilizer use, has been under-explored or generally ineffective (Randall and Mulla, 2001; Tomczyk et al., 2023).

One approach to reducing ambient excess nutrients in estuaries and bays is by encouraging shellfish aquaculture. Shellfish habitat and cultivation areas coincide with common pathways for coastal nutrient runoff. Cultivation jointly produces not only private goods (i.e. the harvested shellfish) but positive environmental benefits in the form of nutrient and algae removal services which mitigates eutrophication. The mechanisms that link shellfish, such as oysters, to eutrophication mitigation are feeding and absorption. Bivalve shellfish consume particulate organic matter (POM), i.e. algae, through filter-feeding and release them as inert biodeposits, as well as absorb nutrients directly into shell and tissue (Dumbauld et al., 2009; Kellogg et al., 2013, 2014; Dalrymple and Carmichael, 2015; Darrow et al., 2017; Bricker et al., 2018; Fulweiler, 2019; Rullens et al., 2019; van der Schatte et al., 2020).

There have been many small-scale studies and reviews of nutrient sequestration and biodeposition from oyster and other shellfish cultivation, and the value of these services provided have ranged from $5,500 to $99,000 (2011 USD) on an annual per hectare basis (Songsangjinda et al., 2000; Porter et al., 2004; Grabowski et al., 2012; Rose et al., 2015; Humphries et al., 2016; Darrow et al., 2017; Yu and Gan, 2021).[26] For example, a study of Chesapeake Bay Eastern oyster (*C. virginica*) aquaculture estimated that a farm deploying a floating rack system with 286 oysters/m^2 per raft

[26] Note that improper management and/or design of oyster farms can have detrimental impacts on the surrounding ecosystem, though there is broad consensus that properly-managed farms provide net overall ecosystem benefits (Porter et al., 2004; Forrest et al., 2009; Dumbauld et al., 2009; Oo and Oo, 2016; Gentry et al., 2020; Barrett et al., 2022).

may be able to remove 331 kg/hectare of nitrogen and 47 kg/hectare of phosphorous per 12 to 24-month period (Higgins et al., 2011).[27] Based on a review by Rose et al. (2015), the costs of shellfish farming as a eutrophication mitigation approach tend to be far lower than comparable mechanical methods such as urban stormwater treatment facility upgrades.

There are few existing government programs incentivizing the production of nutrient removal services by oyster farmers, such as ones that incorporate oyster buybacks and reef restoration activities (Rutgers University Office of Communications, 2021; Native Oyster Restoration Alliance, 2025). A primary example is the Chesapeake Bay Nutrient Credit Trading Program, where a minority of watershed states incorporate oyster aquaculture activities into their credit markets. A total maximum daily load for nitrogen, phosphorus and sediment was initially set in 2010 for the Chesapeake Bay Watershed to limit the use of fertilizers by land-based farms upstream of the Bay.[28]

The Maryland Department of Environment and Agriculture implemented their nutrient credit trading program in 2018 (Sandi, 2025). In 2019, oyster aquaculture operations were included as voluntary participants under the Maryland program. Credits from oyster farming can be generated from harvest of up to four different size classes and two ploidy (diploid and triploid). Larger size classes, which is positively correlated with time held in the water, and more chromosomes are associated with larger water filtration benefits.

There are several market and biological factors that can affect the overall impact of the policy. First, a majority of the oysters cultivated in the region are intended for the fresh half-shell market. Based on industry accounts, the vast majority of half-shell consumers prefer oysters that are within the small or extra small size grades (approximately 2 to 3 inches in length). Therefore, incentives to harvest at larger size classes, which can generate larger filtration benefits, may be weak given the oyster consumer preferences. In addition, though more credits are generated from larger oysters, the per unit difference in the generation of each credit is small, which imply returns to growing larger oysters that may not be large enough to alter firm biomass accumulation behavior (Maryland Department of the Environment, 2023). Finally, denitrification has been more strongly associated with unharvested oyster reefs rather than harvested oyster aquaculture (Kellogg et al., 2014; Bruce et al., 2021; Ray et al., 2021).[29] Taken together, evidence suggests that policies aimed at reducing

[27] Common growing methods in the region are rack and bag systems, where the oysters are flipped or turned using manual labor or, primarily, tidal energy (University of Maryland, 2024). These methods prevent Eastern oysters from "cementing" to each other to form complex reef systems. The formation of oyster reefs, while ecologically valuable, can lead to both mortality through sedimentation and misshapen oyster meat, which farmers are generally unable to sell (Jordan-Cooley et al., 2011; Bruce et al., 2021). While some commercially-valuable oyster species like Pacific oysters (*C. gigas*) also form reef structures and so are subject to the same cultivation techniques, others such as Olympia oysters (*O. lurida*), endemic to the West Coast, do not use other individuals as substrate (Escapa et al., 2004; Brumbaugh and Coen, 2009).

[28] The jurisdictions for the watershed include Delaware, Maryland, New York, Pennsylvania, Virginia, West Virginia and the District of Columbia (U.S. Environmental Protection Agency, 2015).

[29] Note that while a 2021 meta-analysis found no significant difference between oyster aquaculture and reef culture with respect to NH_4^+, NO_x, and PO_4^{3-} removal, the same authors also concluded in a separate study that there are fundamental difficulties in study comparisons due to "methodological differences between studies, incomplete data reporting, and inconsistent measurements of environmental variables that may be used to predict denitrification"

nutrient pollution through oyster aquaculture may perform better when incentivizing oyster biomass accumulation rather than harvesting activities, as is currently implemented in the Chesapeake Bay Nutrient Credit Trading Program.[30]

I add more specificity to the model from Section 2 to include the functional forms and parameters from theoretical economic and scientific literature, as well as more specific pollution dynamics informed by this specific regional and regulatory context. There are other benefits to using this setting for calibration purposes. First, oysters are filter feeders, which means oyster farmers do not incur feed costs for their stock, similar to lumber and unlike the livestock and fin fish aquaculture industries. Given that the model abstracts away from variable input costs, the lack of feed costs and minimization of labor costs through production methods that utilize tidal energy for this industry allows for the parameters used to be more directly applicable to the model set up. Second, farms are typically of limited size and additional leases to expand cultivation are notoriously hard to obtain in this region (Metcalf, 2023). Therefore, limiting the scope to a fixed production area to evaluate intensive margin effects matches the industry's regulatory constraints. Finally, though Maryland's Nutrient Credit Trading Program is currently active, oyster farm participation has been low in terms of total credits generated within the time frame used to generate the parameters for the model, which allows for a relatively clean interpretation of the calibration results (Maryland Department of the Environment, 2023).

3.1 Model Characterization Under Nutrient Pollution

Given the generality of the model presented in Section 2, here I provide the functional forms fed into the numerical calibration as well as the parameters used as inputs to the model. First, let the consumer have preferences over consumption C_t and the stock of pollution D_t, where pollution is taken as given. I assume a CRRA utility function for the consumer that is separable in both consumption and the stock of pollution. The utility function for the representative consumer is defined as

$$U(C_t, D_t) = \log(C_t) - \frac{D_t^{1-\eta} - 1}{1 - \eta} \tag{33}$$

where $\eta < 1$. For simplicity, the representative consumer is modeled as having risk-neutral preferences over consumption C_t.

Private goods production is specified as a decreasing returns to scale Cobb-Douglas production

(Ray and Fulweiler, 2021; Ray et al., 2021).

[30] Note that extraction when oysters are at their largest may help remove more nitrogen relative to allowing deceased individuals to decay in the watershed (Bricker et al., 2020). The model in this paper assumes live biomass contributes to ecosystem benefits production in the broadest sense but does not explicitly address timing of harvest. There is recent work in forestry economics that incorporates timing of harvest in the presence of carbon storage incentives (see Tahvonen and Rautiainen (2017)), though more can be done to model the phenomenon in natural resource-based industries generally.

function

$$F(H_t) = H_t^{\alpha} \tag{34}$$

such that $\alpha \in (0, 1)$. This reflects the presence of decreasing returns to scale in extractive renewable natural resources industries, such as the oyster industry, and is based on findings in the production function estimation literature (Stier, 1980; Nautiyal and Singh, 1985; Scuderi and Chen, 2019). [31][32]

The growth function of the natural resource is a Pearl-Verhulst logistic population growth function

$$G(S_t) = gS_t\left(1 - \frac{S_t}{\bar{S}}\right) \tag{35}$$

where $g = r - m$ is the intrinsic population growth rate, equal to the recruitment rate r (total spat divided by total adult oysters) less the mortality rate m, and $\bar{S}$ is the environmental carrying capacity within the cultivation area (Schaefer, 1991). While the Pearl-Verhulst growth function abstracts away from other environmental factors that can affect population growth such as inter-species interactions and exogenous environmental pressures, the model contains enough of the key features of population growth to serve as an initial benchmark for modeling and is thus considered the workhorse in mathematical ecology (Berryman, 2003; Shorrocks, 2024). [33] Based on Equation 35, the level of the stock that attains maximum sustainable yield is $S_{MSY} = \bar{S}/2$ and the steady state level of harvest that attains maximum growth, or MSY, is $H_{MSY} = G(S_{MSY})$.

The primary addition to the model is the evolution of the the stock of pollution $\dot{D}$.

$$\dot{D} = ne - B(S_t) \tag{36}$$

ne represents emissions net of decay. $B(S_t)$ is the total amount of pollution removed by the stock of the natural resource at every instant.[34] Net emissions are defined as

$$ne = e_{enter} - e_{exit} \tag{37}$$

e_{enter} represents the total amount of exogenous emissions of inorganic nitrogen (N) that flow into the Chesapeake Bay watershed, measured in kilograms (kg) per year, and e_{exit} is the estimated total amount of inorganic N exiting the watershed through surface water transport. Given that the watershed system is complex and unstable, the equation below is used as a stylized back-of-the-envelope calculation for e_{exit} based on information reported in Lindsey et al. (2003), Sanford and

[31] This can be due to factors external to the firm. For example, a lumber company that fells a tree may not observe ex ante that the lumber is low quality due to parasite infestation, adverse weather events, etc. They may therefore need to discard timber that would otherwise have been sold with better quality wood.

[32] Technical change is not explored in this analysis.

[33] Other functions can be substituted in that account for other population growth characteristics, such as Allee effects, as mentioned in Section 2 (Liermann and Hilborn, 2001).

[34] This implies that D_t is a function of S_t. The model in Section 2 is therefore a generalization of the one presented here.

Pope (2013), Lee et al. (2014) and Clune et al. (2021).

$$e_{exit} = SW \cdot RE \cdot e_{enter} \tag{38}$$

SW is the proportion of water that enters the surface water network from either the groundwater system or from soil runoff, and RE is the proportion of surface water per year that exits the Bay via river export. ne therefore represents the total amount of inorganic N that remains in the watershed per year.

Finally, nitrogen filtration benefits are based on Jordan-Cooley et al. (2011), which presents a model of oyster reef population growth with filtration and sedimentation. Given that I focus on non-reef oyster aquaculture, I simplify their filtration function to such that sedimentation is no longer a factor obstructing oyster filtration. This is due to the prevailing aquaculture techniques of rack and bag culture, which prevent oysters from using other individuals as substrate (Ray et al., 2021). F_{max} is the maximum filtration rate.[35]

$$B(S_t) = F_{max} S_t \tag{39}$$

There are a couple of important ecological and biological points to flag. First, the Pearl-Verlhust population growth model, by construction, obscures away from age structure and how it may change under each policy scenario. This approximation can have implications for predicting environmental benefits from the stock, as different life stages imply different oyster filtration rates. Older/larger oysters are generally considered to provide higher levels of water cleaning benefits over their lifetimes (Maryland Department of the Environment, 2023).[36] Second, studies have shown that nitrogen can be re-released into the ecosystem through oyster mortality and decomposition if oysters are not harvested at the end of the lifecycle. While nitrogen contained in tissue is immediately released upon decomposition, total nitrogen that remains in the watershed depends on location of the cultivation area (closer to or further from the ocean). Further, nitrogen sequestration from shells that remain in the watershed can occur for centuries depending on whether the shells are buried in sediment (Kellogg et al., 2013; Malmquist, 2013).[37]

Given the lack of age structure in the Pearl-Verlhust model and the complexity of nutrient re-

[35] Filtration with sedimentation in Jordan-Cooley et al. (2011) is a function of oyster biomass

$$F(S_t) = \frac{F_{max}}{[Cg]_{max}} [Cg(S_t)] e^{\frac{[Cg]_{max} - Cg(S_t)}{[Cg]_{max}}}$$

where C is the maximum possible deposition rate, $g(S_t)$ represents sedimentation on deposition ($g(S_t) < 0 \ \forall \ t$), and $[Cg]_{max}$ is the maximum possible deposition given sedimentation for a given t. Without sedimentation (setting $g(S_t) = g_{max}$) yields

$$F(S_t) = F_{max}$$

The filtration function notation used above was reported in Lipcius et al. (2019).

[36] Benefits by age class can differ based on location and other conditions (Dalrymple and Carmichael, 2015).

[37] Shells contain approximately half of the total nitrogen sequestered in an oyster (Kellogg et al., 2013; Dalrymple and Carmichael, 2015).

release, Equation 39 is interpreted as an upper bound approximation of the benefits provided by the natural resource stock. Note that the paths of pollution accumulation in this model are partially attenuated by the lower bound estimate of carrying capacity $\bar{S}$ (see description of parameters in Table 1) which greatly determines natural resource stock accumulation and, therefore, the change in nutrient pollution in the watershed.

The complete model with nutrient pollution dynamics under the Social Planner's problem is presented below.

$$U_0 = \max_{\{C_t, S_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \left(\log(C_t) - \frac{D_t^{1-\eta} - 1}{1-\eta} \right) dt \tag{40}$$

$$\dot{S} = g S_t \left(1 - \frac{S_t}{\bar{S}} \right) - H_t$$

$$\dot{D} = ne - F_{max} S_t$$

$$C_t = H_t^{\alpha}$$

$$S_0 > 0, D_0 > 0$$

The Euler Equation under the Social Planner is thus

$$g \left(1 - \frac{2S_t}{\bar{S}} \right) - \phi_t F_{max} \frac{C_t}{\alpha H_t^{\alpha-1}} = \rho - \left(\frac{\dot{C}}{C_t} \right)^{-1} - \frac{[\dot{H_t^{\alpha-1}}]}{H_t^{\alpha-1}} \tag{41}$$

$\phi_t < 0$ represents the shadow cost of nitrogen pollution stock accumulation. ϕ_0 is calibrated to \$10/kg N at $t = 0$ based on the upper bound of the social cost of nitrogen reported in Keeler et al. (2016). $-\phi_t F_{max} = U_S$ is therefore the marginal value to society of an additional kg of oysters accumulated with respect to the nitrogen filtration benefit it provides at time t. Note that the trajectory of the stock of pollution may not reach a steady state. This is due to fact that, first, the level of pollution is taken as given by the consumer where the only method of control in this setting is natural resource stock accumulation, and second, there are biological constraints in the system that can prevent a steady state level of pollution to be reached, even under the Social Planner. The rest of the results follow as in Section 2.[38]

Equation 41 also suggests use of the following time-varying marginal subsidy with respect to the stock.

$$\sigma_S = -\phi_t F_{max} \left(g S_t \left(1 - \frac{S_t}{\bar{S}} \right) \right)^{\alpha} \tag{42}$$

Note that we cannot pin down an exact functional form for $\sigma(S_t)$ because Equation 42 does not yield an elementary antiderivative with respect to the stock. However, given that the marginal policy function is a monotonic transformation of the stock's growth function, which is commonly modeled by stock assessment biologists, the marginal incentive can be implementable if S_t is known.

[38] Other additions to this model can include costs to natural resource accumulation or harvest. The addition of these costs do not fundamentally alter the conclusions of this paper.

The parameters used for numerical calibration are presented in Table 1, along with the sources for each parameter where applicable. Reports of individual oysters of commercial harvest size (76 mm or approximately 3 inches shell length) are converted to kg of wet weight biomass using Mann et al. (2009).[39] Carrying capacity is approximated as the maximum commercial harvest of Eastern oysters between 1993 and 2022 and serves as a lower bound of the environmental carrying capacity. Filtration rate F_{max} was converted from the effect size reported in Cornwell et al. (2023) for the Chesapeake Bay Program Partnership to annual N per acre per year removed by 300 grams of oyster dry weight per square meter.

Table 1: Parameters and Sources

Parameters		Estimate	Source
Recruitment rate	r	0.4349	Maryland Department of Natural Resources (2020)
Mortality rate	m	0.1	Doering et al. (2021)
Carrying capacity	$\bar{S}$	20,411,640 kg	Tarnowski (2023)
Nitrogen loadings	e_{enter}	131,541,680 kg/yr	Clune et al. (2021)
Surface water	SW	0.5	Clune et al. (2021)
River export	RE	0.72	Lee et al. (2014)
Filtration rate	F_{max}	5.29 kg/kg/yr	Cornwell et al. (2023)
Social discount rate	ρ	2%	Council of Economic Advisers (2024)
Output elasticity	α	0.8	Industry Sources
Pollution curvature	η	0.75	
Initial harvest	H_0	$H_{MSY} \cdot 0.75$	
Initial stock	S_0	$S_{MSY} \cdot 0.45$	
Initial pollution	D_0	10,000,000 kg	

The harvest technology employed in this model does not provide a one-to-one comparison with the production inputs reported in previous production function estimation studies of the oyster industry (Scuderi and Chen, 2019; Beckensteiner et al., 2021), so I choose $\alpha = 0.8$ as an informed starting point based on discussions with industry professionals.[40] The pollution curvature parameter and initial values are chosen to illustrate the theoretical results graphically.[41]

[39] All dry weight measures are converted to wet weight by multiplying dry weight in g by 100. One standard bushel of oysters weighs approximately 45 to 60 lbs and contains between 100 to 150 oysters. Conservatively, the smallest average market size oyster (76 mm shell length) weighs 4.8 oz unshucked. Given that an oyster shell weighs 1 to 2.5 oz, 1 g of oyster dry weight is equivalent to 3.5274 oz wet weight, which is the approximate wet weight of shucked oyster of market size (Grizzle et al., 2017).

[40] The value of α can also vary with environmental and regulatory conditions, which I do not explore in this paper.

[41] For example, choosing $H_0 > G(S_0)$ will lead to population collapse and, therefore, uninformative transition paths.

3.2 Numerical Analysis

All numerical calibration results are generated in Matlab 2025a using `ode45`. I use the equilibrium conditions and functional forms stated above as inputs. The system is allowed to optimize over 100 years ($T = 100$) for ease of graphical display and is expected to continue along its steady state trajectories as $t \to \infty$. Initial guesses for the transition path of harvest are made under each policy scenario.[42] The guesses are chosen flexibly based on total surplus maximization in present values over the following functional forms and start at the same initial harvest H_0: linear, exponential, and sigmoidal. The specific functional forms and total surplus results are presented in Appendix Section B.

BAU and the flow incentive yield a closed form solution for the stock in steady state under a 2% discount rate at $S_T^{bau} = 9,596,296.406$ kg, where the superscript *bau* indicates allocations under no policy and, therefore, the flow incentive. Extraction at the end of the graphical time period is then $H_T^{bau} = 1,702,760.055$ kg. The time paths under the Social Planner do not have closed form solutions in steady state. However, the theoretical results suggest that S_T^* under the Social Planner is greater than under no policy or the flow incentive, so I set $H_T^* = H_T^{bau} \cdot 0.8$ for illustrative purposes. The transition paths are adjusted from their initial guesses over 25 iterations with a step size of 0.000001.

3.2.1 Theoretical Benchmarks

In order to understand how policy deviations may help or hinder environmental benefits provision across time, it is first important to know how the stocks of the natural resource, pollution, harvest and further, welfare evolve under the theoretical benchmarks. Figures 1–4 present the time paths of the aformentioned states, choices, and the utility of a representative consumer.

Figure 1 presents the evolution of the natural resource stock across time, where the blue line represents the trajectory under the social optimum and the green line is the equilibrium path under no policy. The level of the natural resource stock under the social optimum diverges away from the no policy benchmark quickly and converges to the steady state level of the stock at approximately 14.7 million kg of biomass within 40 years. The pollution stock (Figure 2) similarly diverges between the two benchmarks, where the stock of pollution under the social optimum is lower than no policy for all instants except $t = 0$, though the trajectories more noticeably diverge at $t = 8$. This is driven by differences in harvest (Figure 3), where the level of harvest under no policy is much higher across time than under the social optimum. The trajectories align with the results of Sections 2.1 and 2.2, where the level of harvest in steady state under the social optimum will be less than the profit-maximizing level under BAU. Lower levels of harvest result in a natural resource stock that is higher than under the firm's maximum sustainable yield $S^* > S_{MSY}$ in

[42] The trajectories are allowed to adjust based on initial values for H_0, S_0, D_0 and end value H_T^*. Paths under different initial value guesses and H_T^* in particular do not fundamentally change the following results.

steady state. Figure 4 shows the trajectories of utility in both current (Panel (a)) and present values (Panel (b)). In both figures, the level of utility for a representative consumer is consistently higher across time under the social optimum due to the avoided disutility pollution, in spite of the lower levels of consumption across time.

The following sections explore how proxy policies, either incentives on stock accumulation or harvest, along with deviations in terms of sign, scale, and time variation, can affect dynamic environmental benefits provision. Given the correspondence between pollution removal, the stock of the natural resource, and harvest, I primarily focus on the trajectories of the natural resource stock. The time paths for pollution stock, harvest, and surplus under each policy scenario are presented in Appendix Figures A3–A20.

3.2.2 Policies that Vary on Sign and Scale

Based on Equation 18, a stock-targeting policy must be a subsidy in order to influence the firm to conserve more of the natural resource than under no policy across time. Figure 5 presents the time paths of the natural resource stock under different constant per unit subsidies targeting stock accumulation relative to the no policy and socially optimal benchmarks. The policies vary by size of the subsidy to illustrate how scale can affect stock accumulation. The yellow dotted line ("Small") represents the natural resource stock trajectory under a constant per unit policy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD, while the red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. Finally, the purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the social optimum multiplied by a factor of two. The values of per unit subsidies are chosen for illustrative purposes.

Consistent with the predictions in Section 2.3, increasing the scale of a stock subsidy increases accumulation of the stock of biomass by the firm. However, the relationship appears to be non-linear, given that order of magnitude increases in the value of the subsidy yield lower relative increases in biomass accumulation. While a "very large" subsidy may lead to gains in the natural resource stock in excess of the social optimum, the gains in consumer surplus (Figure 6 Panel (a)) combined with the gains in producer surplus (Figure 6 Panel (b)) indicate that total welfare may be lower relative to a policy that yields a time path of producer surplus that more closely mimics that under the Social Planner (see Section 3.2.3).

Given that Section 2.4 supposes that either a tax on harvest or a subsidy that increases with respect to time may incentivize a firm to accumulate more natural resource stock, I employ constant per unit harvest taxes that mimic the constant per unit stock subsidies exercise. Figure 7 Panel (a) presents the time paths of the natural resource stock under different constant per unit taxes targeting harvest relative to BAU. The policies take on the negative values of the policies utilized in Figures 5 and 6 except for the "very large" tax, which is equal to 20,230,284.25 USD, or the

negative of the value of stock accumulation at $t = 0$ under the social optimum multiplied by a factor of five. The purpose of the different "very large" monetary value under flow-targeting is to yield more meaningful deviations in the time path relative to no policy which, as shown in Figure 7 7 Panel (b), still tracks closely with the no policy path of stock accumulation despite the increased size of the tax (within 0.2% of the no policy benchmark).

In general, a flow-targeted tax does little to change firm behavior, which supports the mechanism proposed in Section 2.4 where given a fixed production area, firms may not deviate from their profit-maximizing level of harvest even in the presence of disincentives to produce the private good. Taxes generally push the firm to harvest slightly less and accumulate higher amounts of the natural resource in the short run, with larger taxes encouraging more conservation. This supports the interpretation of the results in Section 2.4, where the sign of τ_H can play a role in the total benefits provided to society from the natural resource stock. Such a policy comes at a major cost to firms, which experience large negative profits across time without substantial gains in consumer surplus, especially at values of taxes that would yield even somewhat noticeable increases in biomass relative to no policy (Figure 8).

3.2.3 Time-Varying Policies

Figures 9 – 12 present the time paths of the natural resource stock under both instantaneously updating incentives and policy deviations through time variation. First, Proposition 1 proposes that for a stock policy, any marginal incentive that equals the marginal rate of substitution between the stock and consumption in steady state can yield time paths of the natural resource stock that converge to the social optimum. Figure 9 shows these trajectories under two such policies – one that varies with respect to time ("Time-Varying") and one that is set at the steady state value of stock accumulation for all t ("Steady State"). The time paths under each policy coincide closely with the social optimum, where the steady state marginal incentive yields a time path that is within 0.002% of the social optimum. The higher initial levels of biomass under a time-varying stock subsidy is primarily from the approximation of the marginal rate of substitution, where the initial values of the policy do not fully account for the effects of harvest on the growth of the natural resource (Appendix Figure A7). However, the difference between stock accumulation under a time-varying stock subsidy and the social optimum is small (within 0.015% of the social optimum).

Second, I implement marginal incentives with "chunky" subsidies, which are discontinuously updated every five, ten, 25, or 50 years (the purple, red, green, and dark blue dashed lines, respectively), highlighted in Figure 10. The firm has perfect foresight and expects the policies to be updated at these intervals from time $t = 0$. The lags in monetary values of incentives are to mimic potential political economy or administrative constraints that could prevent instantaneous updating by a policymaker. Similar to the trajectories in Figure 9, holding policies constant for a given number of years does not make a notable difference in stock accumulation by the firm,

though lags can somewhat affect the level of biomass held by the firm. The small but progressively increasing levels of the stock relative to the social optimum by policy lag are driven by the fact that the value of the marginal incentive is higher than under an instantaneously-updating policy for each time step, where the value of the policy that updates every 50 years is the largest for the longest amount of time (Appendix Figure A11). However, the differences in stock accumulation are minimal, ranging from within approximately 0.02% to 0.06% relative to the social optimum.

I further explore how time variation in flow policies may affect biomass accumulation. While the model does not yield a functional form for an optimal flow incentive, Equation 32 suggests a flow policy that increases over time may affect firm behavior more than a static alternative. However, the size of these changes in the policy with respect to time plays an unclear role in natural resource stock accumulation. Therefore, I assume that a hypothetical policymaker has made a "best guess" at policy structure by setting it to the ideal policy under the Social Planner ($\tau_H = U_S/U_C$), which is analogous to the monetary value of the oyster harvest instrument used in the Maryland Nutrient Credit Trading Program.[43] Based on Proposition 2, this value does not imply that optimal allocations will be attained. As shown in Figure 11, the transition paths of stock accumulation under no policy and the flow incentive tightly coincide, as expected in theory. It appears that a flow subsidy produces lower biomass accumulation relative to no policy (within 0.05%) in the short run, supporting the hypothesis in Section 2.4 that small instantaneous increases in the subsidy may induce a firm to harvest more today if it does not foresee large returns to conserving biomass for future production.[44] This is further supported by the results in Figure 12, which implement the "chunky" policy updates employed in Figure 10. Small instantaneous increases in a flow policy from the five and 10 year policy updates result in lower levels of biomass accumulation relative to larger increases under the 25 and 50 year lags in policy adjustments, though the paths coincide closely with the time-varying flow policy with instantaneous updates. The results together suggest that time variation in either stock- or flow-targeted policies may not play a strong role in changing firm accumulation of natural resources and, therefore, environmental benefits provision.

4 Conclusion

Economists have assumed that internalizing environmental externalities can be adequately done by targeting flows of production. However, this does not seem to be the case when externalities are generated from stocks of living natural resources. I employ a dynamic optimization model of renewable natural resource extraction using an infinite horizon, continuous time framework to evaluate how differences in policy targets contribute to the efficacy of such programs. I find that

[43] H_T can be expressed as a function of S_T given $F(H_T)$ is monotonic. Therefore $\tau(H_t)$ can be expressed as a function of strictly H_T.

[44] Note that the trajectory of the stock under no policy and flow-targeting have not fully reached a steady state. However, the trajectories tend towards the profit maximizing level of the stock as t increases, and deviations away from no policy under flow-targeting are estimated to converge back to no policy within 140 years.

policies that do not target the stock of the natural resource may provide little additional social benefit in the long run. Subsidies that target accumulation of the natural resource stock, which is a closer approximation of the environmental benefits production, provide yield higher social gains relative to no policy and can converge to the social optimum under appropriately-designed policies.

Numerically, this conclusion is explored through counterfactual policies to address inorganic nitrogen pollution in surface and groundwater. The model is calibrated to inorganic nitrogen pollution and hydrological parameters from the Chesapeake Bay, in addition to biological and industry-informed parameters for Eastern oyster production associated water purification benefits. I vary these policies by sign, scale, and lags in policy updates, conditional on policy target. The dynamic results support those derived in theory with respect to the targets of each policy, where the time path of all calibrated flow policies closely track the paths under no policy. Further, I find that the scale and sign of the policies matter substantially for the trajectory of allocations under stock-targeting but lags in policy updates matter little regardless of target. While taxes incentivize firms to conserve the natural resource under flow-targeting in the short run, due to substantial negative returns across time relative to gains in consumer surplus, the benefits of such a policy appear to be minimal.

The results can be explained by the source of revenues for the firm under different targets. First, under a stock-targeting, the producer effectively operates as a multi-product firm, where it produces both the private good and an environmental service proxied by the natural capital stock held in inventory. This allows for the firm to forego production of the private good in favor of maximizing revenues from the stock of the natural resource, which leads to increases in natural resource conservation. Second, under flow-targeting, the firm obtain revenues from one source, which is private goods production, which is the same singular revenue source as the no policy scenario. Under no policy, the firm's incentives are to maximize their sustainable yield with respect to the stock, which will occur where the growth of the natural resource is at its highest. Thus, with a flow-targeted policy, a firm has little incentive to deviate from this behavior, particularly in steady state. Though a flow policy may have some bite if it is time varying, the scale and sign of the policy seem to matter more for stock accumulation than how it changes over time. This is because increases in natural resource stock in anticipation of higher future returns to harvest can be simultaneously attenuated by increases in current returns. Based on both the theoretical and numerical results, policymakers partnering with industries that extract renewable natural resources that produce these types of social values should implement policies that directly target the accumulation of those stocks, as in many PES programs, rather than those that target harvest.

There are several limitations to this work, as well as potential avenues for model extension. First, the theoretical and numerical results are purely ordinal, where the stock-targeted incentive dominates a flow-targeted incentive in the included scenarios. While the model is calibrated to parameters that are reflective of the realities of the region and industry of focus, there are important economic and ecological considerations I have abstracted away from in order to provide clarity with respect

to policy performance rankings. Therefore interpretations of dollar values and levels of benefits provided in this paper should be taken with caution.

Second, the model does not account for the costs of policy administration, which can influence the relative attactiveness of some policy targets versus others. Based on the numerical exercise, results suggest that in the presence of positive program implementation costs, the benefits of a flow subsidy may not be enough to warrant program implementation. Further, the costs of stock subsidy can also outweigh its benefits if a policymaker wants to obtain levels of the natural resource, and thereby environmental quality, that are higher than the social optimum.

Third, I assume one large "plot" over which production occurs, which implies that the results strictly capture intensive margin effects, and strong assumptions are made about the ecological characteristics of the modeled region. It is important to recognize that flow incentives may affect natural resource accumulation in ambiguous ways along the extensive margin. These policies may, in fact, allow for an increase in natural capital stock if the geographic areas over which firm production occurs has sparse or no natural capital stock prior to firm entrance. However, the effects of any flow-targeted policies aimed at increasing natural capital stock through the extensive margin will be attenuated through the firm profit maximization via harvest, as highlighted in this paper.

Relatedly, the analysis does not capture effects on the total stock of the natural resource (i.e. those that are both wild and cultivated) and has implications for the total benefits provided. The model implicitly assumes away natural resource restoration efforts that may occur in the modeled region, which provide water purification benefits in and of themselves, as in the case of the numerous oyster reef restoration projects occurring in the Chesapeake (National Oceanic and Atmospheric Administration, 2024). Given that the purpose of the paper is to understand policy performance, the introduction of wild resource stock obscures away from the purpose of the paper at hand. In addition, wild stock must be considered if extensive margin effects are incorporated, which adds another layer of complication. Firm activity may increase the stock of natural capital in places where the stock is sparse ex ante, but can also lead to decreases in areas that have an abundance of natural capital through production practices that effectively "spread out" the stock in order to maximize private goods production along cost minimization or quality dimensions. This is potentially the case in the oyster industry, where rack and bag aquaculture may displace densely-populated naturally occurring reef structures if incentives to encourage additional aquaculture leases are present.

Further, the analysis focuses on a class of policy functions that are not an explicit function of time. This restriction is particularly important for the flow incentive, where it may be the case that a policy that is a function of time can achieve the socially optimal transition paths. Such a policy may need to be monotonically increasing in time to affect the firm harvest choices in the socially optimal direction. In addition to the reasons stated in Section 2.4, considering again the problem of government administrative costs and budget constraints, a subsidy that is an increasing function of time would be fiscally impractical, and is therefore not a class of incentives featured in this paper. Other important limitations include the lack of firm heterogeneity, potential nonlinearities

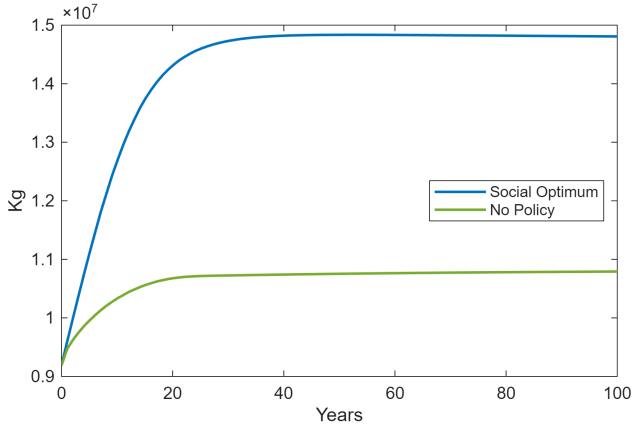
in the environmental benefits function, and the presence of other goods that may be substitutes or complements to either the private good or social benefit modeled here (Starrett, 1972; Sudit and Whitcomb, 1976; Kotchen, 2005; Xabadia et al., 2008; Jordan-Cooley et al., 2011).

Finally, technological change in either production, the growth of the natural resource stock or environmental benefits provision is absent. Agricultural research institutions often involve themselves in the development of innovations on all three dimensions (Nell, 2002; Dégremont et al., 2015; Villanueva-Fonseca et al., 2017; Quinton, 2019; Wadsworth et al., 2019; Watson, 2021). In future work, I plan to incorporate technical change along both of these dimensions in order to understand the role it may play in optimal policy design to internalize stock-generated externalities.

The model presented in this paper emphasizes the importance of living natural resources as sources of external benefits or damages in and of themselves. While the current paper focuses on privately-owned natural resources as instruments to improve environmental quality, the general setting can be extended to include other natural resources management problems such as single or multi-species conservation, natural shoreline protection, and groundwater management. The broader hope is that the model presented in this paper serves as useful tool for both policymakers and researchers as we continue to explore both market-based and regulatory tools to minimize damages to the environment.

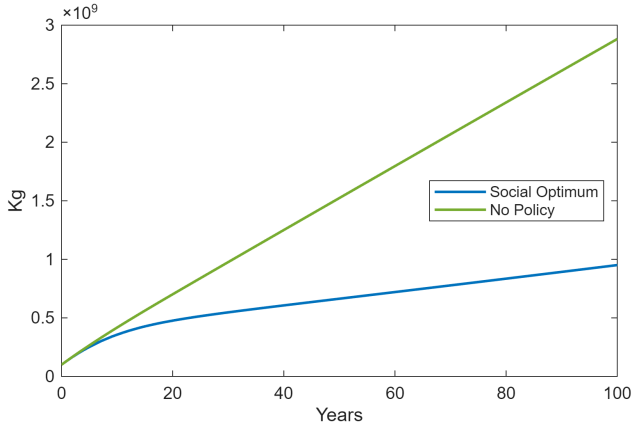
5 Figures

Figure 1: Natural Resource Stock Trajectories Under Theoretical Benchmarks



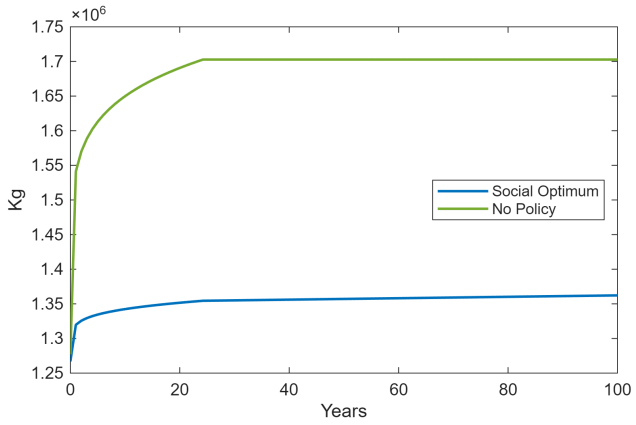
The lines represent the levels of the natural resource stock over time in kilograms under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy benchmark.

Figure 2: Pollution Stock Trajectories Under Theoretical Benchmarks



The lines represent the levels of inorganic nitrogen stock over time in kilograms under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy benchmark.

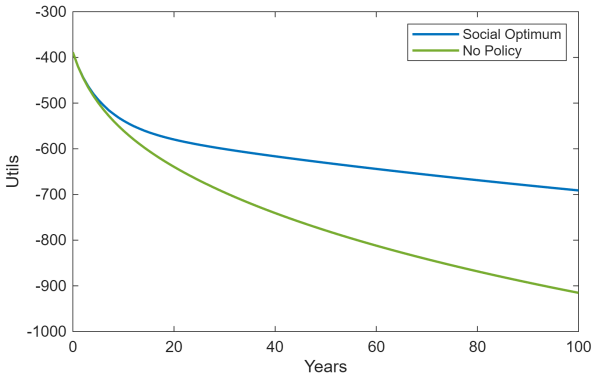
Figure 3: Harvest Trajectories Under Theoretical Benchmarks



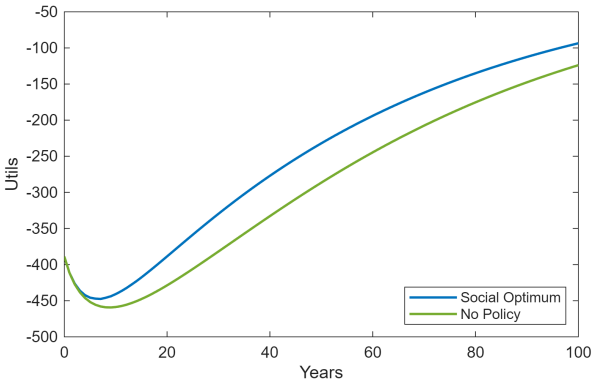
The lines represent the levels of harvest over time in kilograms under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy benchmark.

Figure 4: Utility Trajectories Under Theoretical Benchmarks

(a) Current Values

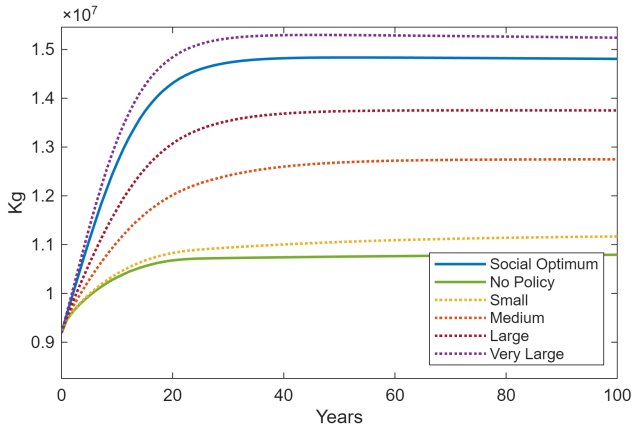


(b) Present Values



The lines represent the levels of utility over time for a representative consumer under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy benchmark.

Figure 5: Natural Resource Stock Trajectories Under Constant Marginal Stock Subsidies

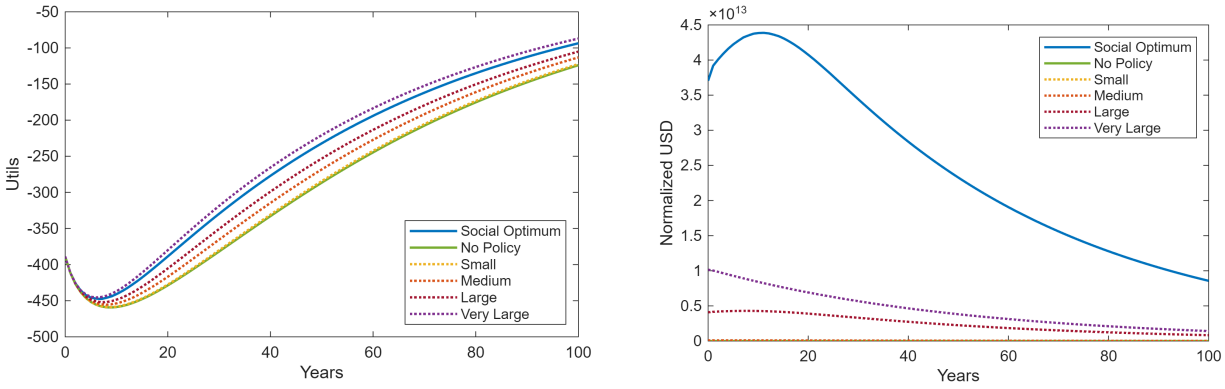


The lines represent the levels of utility of a representative consumer over time under the two theoretical benchmarks. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy benchmark. The yellow dotted line ("Small") represents the trajectory under a constant per unit subsidy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD. The red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. The purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of two.

Figure 6: Surplus Trajectories in Present Values Under Constant Marginal Stock Subsidies

(a) Consumer Surplus

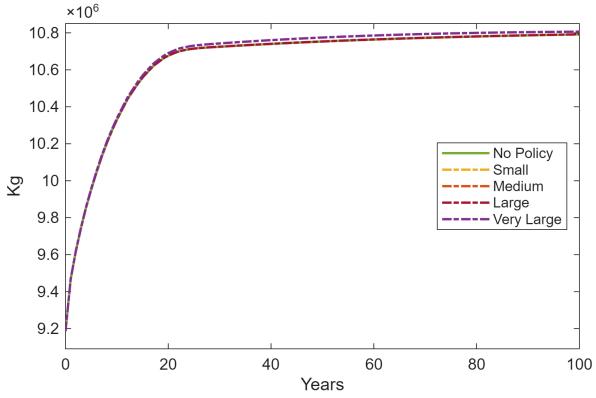
(b) Producer Surplus



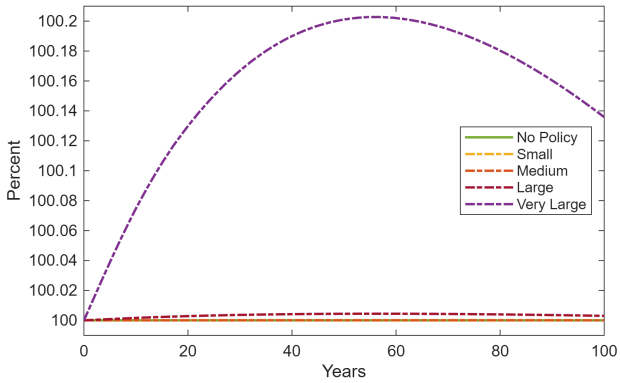
The lines represent the levels of consumer and producer surplus in present values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy benchmark. The yellow dotted line ("Small") represents the trajectory under a constant per unit subsidy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD. The red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. The purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of two.

Figure 7: Natural Resource Stock Under Constant Marginal Flow Taxes

(a) Natural Resource Stock Trajectories



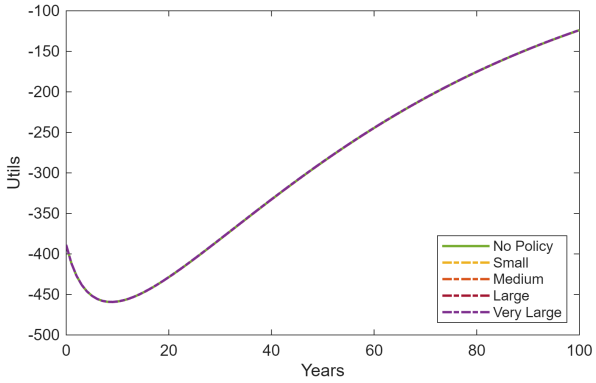
(b) Deviations from No Policy



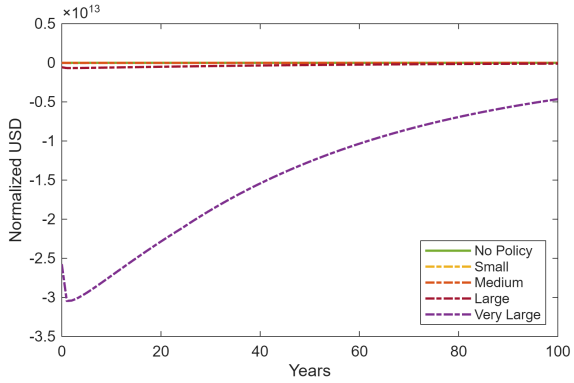
The lines represent the levels of the natural resource in kilograms over time under the no policy benchmark in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green line represents the trajectory under the no policy benchmark. The yellow dot-dashed line ("Small") represents the trajectory under a constant per unit tax set at 1000 USD. The orange dot-dashed line ("Medium") are the levels under a constant per unit tax at 10,000 USD. The red dot-dashed line ("Large") are the levels under a constant per unit tax at 445,000 USD. The purple dot-dashed line ("Very Large") are the levels under a constant per unit tax at 20,230,284.25 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of five.

Figure 8: Surplus Trajectories in Present Values Under Constant Marginal Flow Taxes

(a) Consumer Surplus



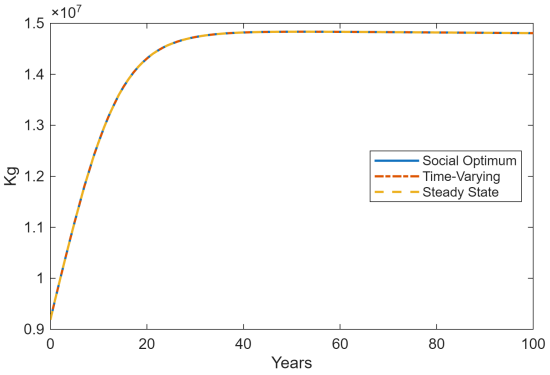
(b) Producer Surplus



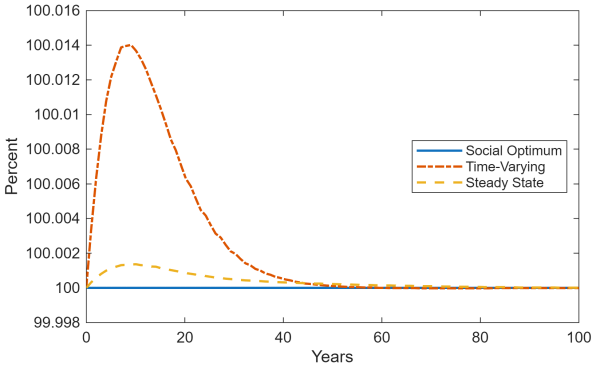
The lines represent the levels of consumer and producer surplus in present values under the no policy benchmark in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green line represents the trajectory under the no policy benchmark. The yellow dot-dashed line ("Small") represents the trajectory under a constant per unit tax set at 1000 USD. The orange dot-dashed line ("Medium") are the levels under a constant per unit tax at 10,000 USD. The red dot-dashed line ("Large") are the levels under a constant per unit tax at 445,000 USD. The purple dot-dashed line ("Very Large") are the levels under a constant per unit tax at 20,230,284.25 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of five.

Figure 9: Natural Resource Stock Under Stock Subsidies that Converge to Social Optimum

(a) Natural Resource Stock Trajectories



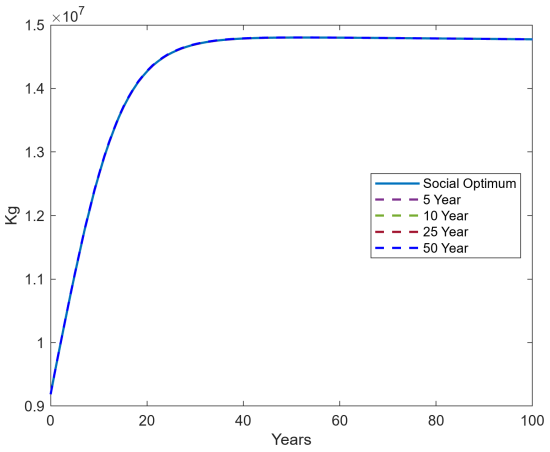
(b) Deviations from the Social Optimum



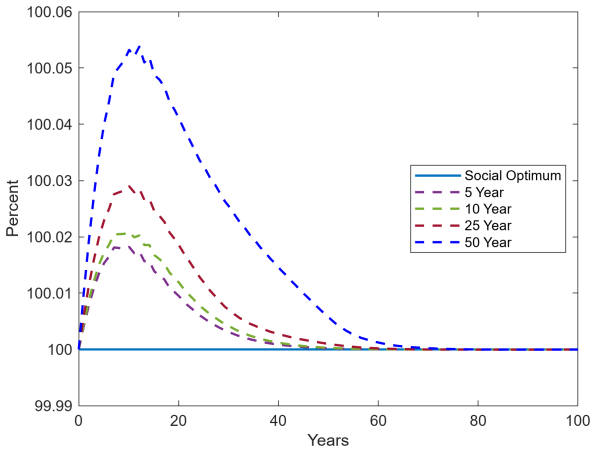
The lines represent the levels of the natural resource stock (Panel (a)) and the Deviations (Panel (b)) from the Social Optimum under policies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

Figure 10: Natural Resource Stock Under Stock Subsidies with Periodic Updates

(a) Natural Resource Stock Trajectories



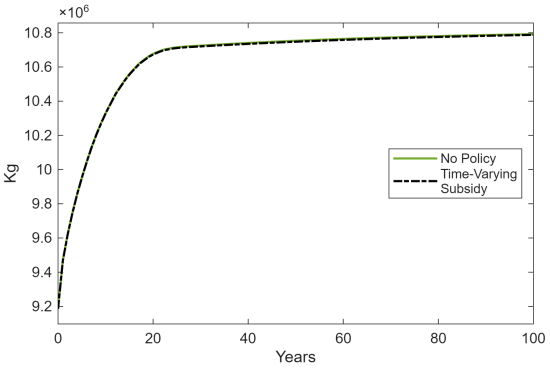
(b) Deviations from the Social Optimum



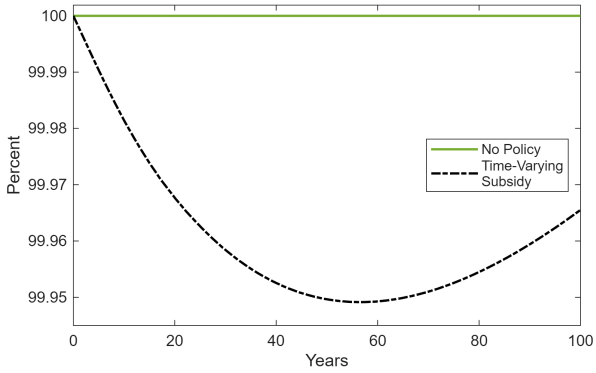
The lines represent the levels of the natural resource stock (Panel (a)) and the percentage deviations (Panel (b)) under a stock policy over time against policies with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure 11: Natural Resource Stock Under Time-Varying Flow Subsidies

(a) Natural Resource Stock Trajectories



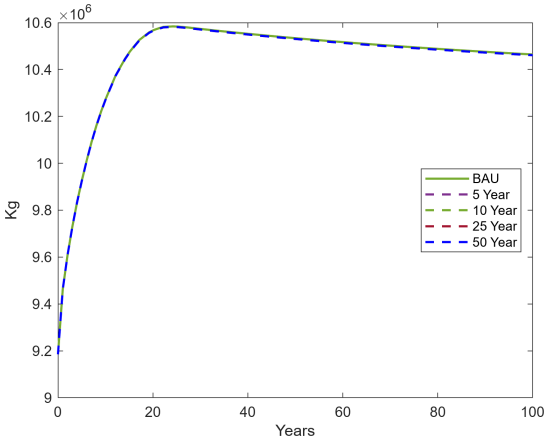
(b) Deviations from BAU



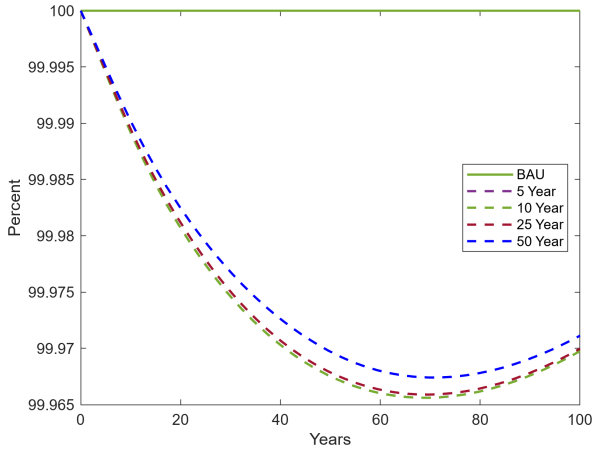
The lines represent the levels of the natural resource stock (Panel (a)) and the Deviations (Panel (b)) from no policy, or BAU. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under the no policy. The black dot-dashed line ("Time-Varying Subsidy") represents the trajectory under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner.

Figure 12: Natural Resource Stock Under Flow Subsidies with Periodic Updates

(a) Natural Resource Stock Trajectories



(b) Deviations from BAU



The lines represent the levels of the natural resource stock (Panel (a)) and the percentage deviations (Panel (b)) under a stock policy over time against subsidies that are updated periodically to to the value of stock accumulation as if under the Social Planner. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under no policy. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

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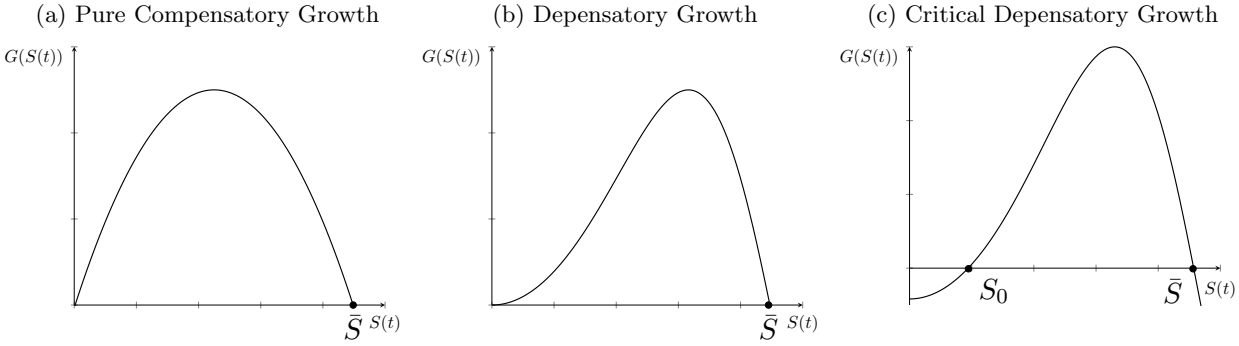
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Online Appendix for "Stocks versus Flows"

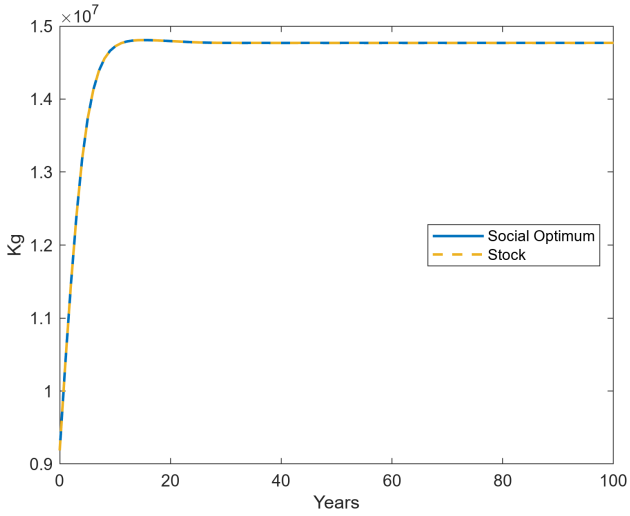
A Figures

Figure A1: Population Growth Functions



The figures represent different population growth functions for a single species within a fixed geographic area, i.e. habitat. Panel (a) represents pure compensatory growth, which is concave with respect to the total stock of biomass S_t across all levels of biomass, and where population growth converges to zero as the stock reaches the environmental carrying capacity $\bar{S}$, which is time-invariant under the current model. Panel (b) represents depensatory growth, which is concave up to a given level of biomass and then concave, converging to $\bar{S}$. Panel (c) represents critical depensatory growth, where growth is negative up to a critical threshold $S_0 > 0$, also referred to as the "Allee threshold." This indicates that the population may collapse at or below some positive level S_0.

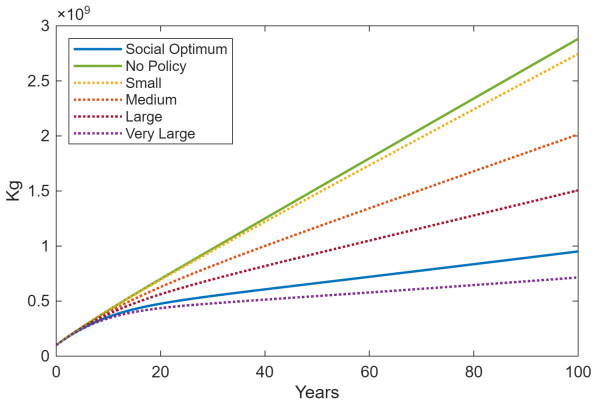
Figure A2: Natural Resource Stock Trajectories Under Log Linear Separable Utility



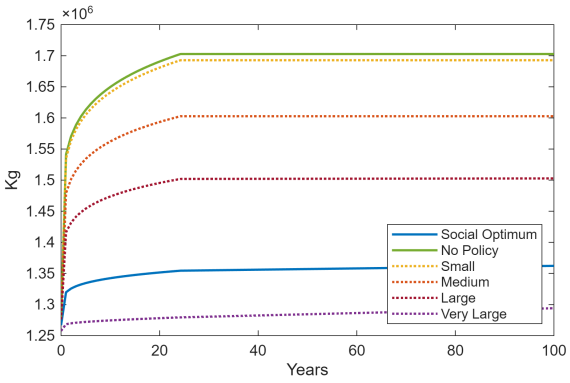
The lines represent the levels of the natural resource stock over time in kilograms under the Social Optimum versus a stock incentive, where the consumer has log linear separable utility with respect to both consumption and the natural resource stock and production is linear in harvest (Section 2.3). The blue solid line represents the trajectory under the Social Planner benchmark. The yellow dashed line represents the trajectory of the under the stock targeting policy in Equation 26.

Figure A3: Pollution and Harvest Trajectories Under Constant Marginal Stock Subsidies

(a) Pollution Stock



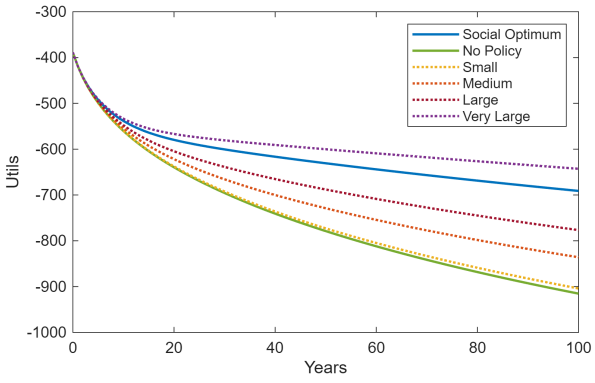
(b) Harvest



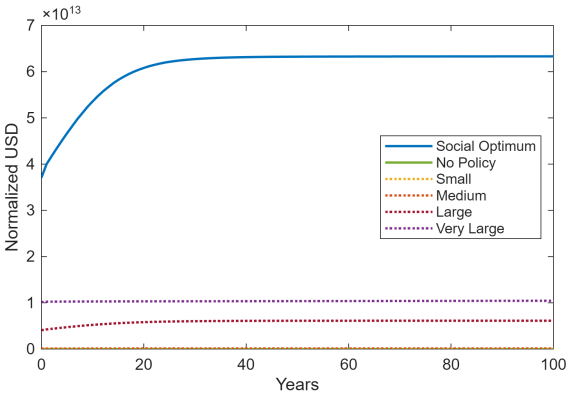
The lines represent the levels of inorganic nitrogen stock and harvest in kilograms under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy benchmark. The yellow dotted line ("Small") represents the trajectory under a constant per unit subsidy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD. The red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. The purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of two.

Figure A4: Surplus Trajectories in Current Values Under Constant Marginal Stock Subsidies

(a) Consumer Surplus



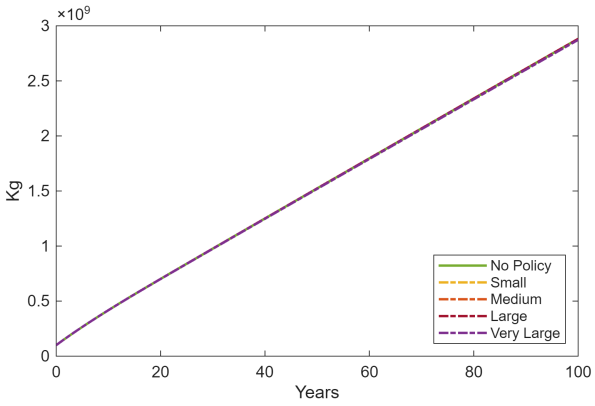
(b) Producer Surplus



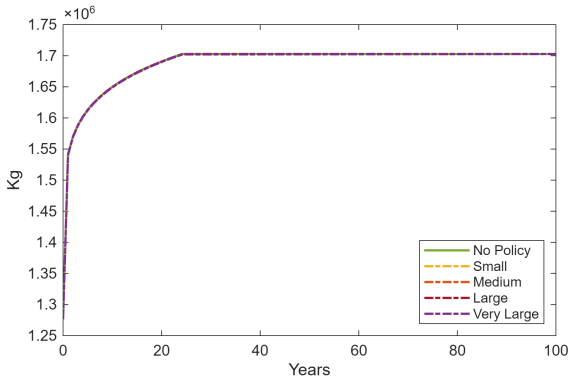
The lines represent the levels of consumer and producer surplus in current values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum, and the green line represents the trajectory under the no policy benchmark. The yellow dotted line ("Small") represents the trajectory under a constant per unit subsidy set at 1000 USD. The orange dotted line ("Medium") are the levels under a constant per unit subsidy at 10,000 USD. The red dotted line ("Large") are the levels under a constant per unit subsidy at 445,000 USD. The purple dotted line ("Very Large") are the levels under a constant per unit subsidy at 8,092,113.71 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of two.

Figure A5: Pollution and Harvest Trajectories Under Constant Marginal Flow Taxes

(a) Pollution Stock



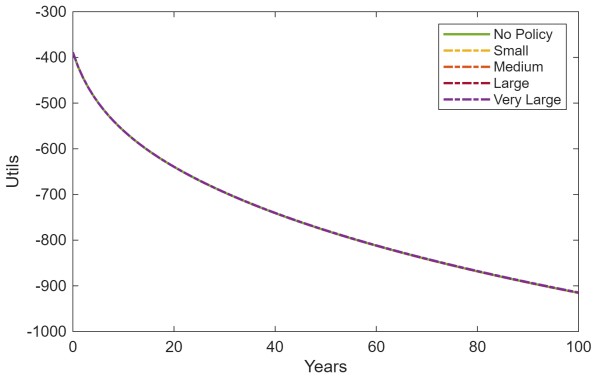
(b) Harvest



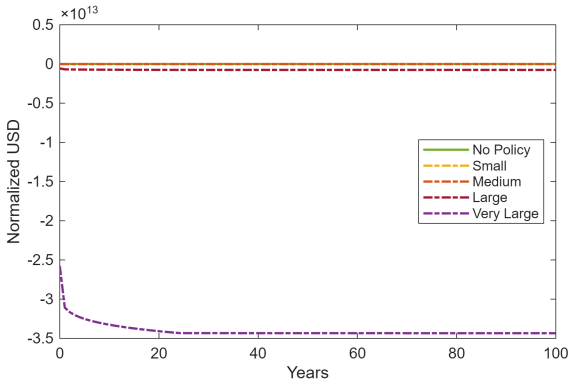
The lines represent the levels of inorganic nitrogen stock and harvest in kilograms under the no policy benchmark in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green line represents the trajectory under the no policy benchmark. The yellow dot-dashed line ("Small") represents the trajectory under a constant per unit tax set at 1000 USD. The orange dot-dashed line ("Medium") are the levels under a constant per unit tax at 10,000 USD. The red dot-dashed line ("Large") are the levels under a constant per unit tax at 445,000 USD. The purple dot-dashed line ("Very Large") are the levels under a constant per unit tax at 20,230,284.25 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of five.

Figure A6: Surplus Trajectories in Current Values Under Constant Marginal Flow Taxes

(a) Consumer Surplus

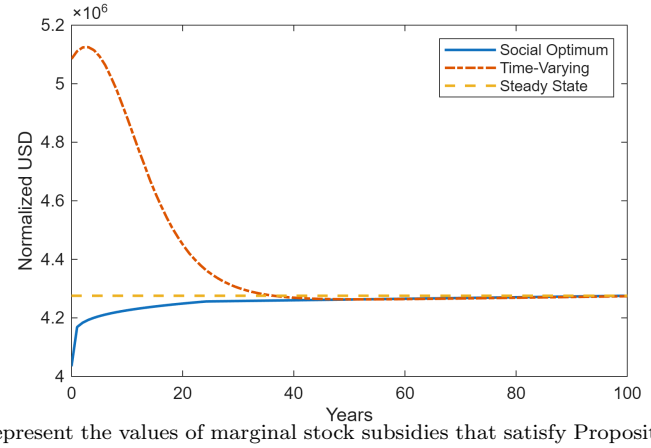


(b) Producer Surplus



The lines represent the levels of consumer and producer surplus in current values under the no policy benchmark in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green line represents the trajectory under the no policy benchmark. The yellow dot-dashed line ("Small") represents the trajectory under a constant per unit tax set at 1000 USD. The orange dot-dashed line ("Medium") are the levels under a constant per unit tax at 10,000 USD. The red dot-dashed line ("Large") are the levels under a constant per unit tax at 445,000 USD. The purple dot-dashed line ("Very Large") are the levels under a constant per unit tax at 20,230,284.25 USD, which is the value of stock accumulation at $t = 0$ under the Social Optimum multiplied by a factor of five.

Figure A7: Trajectories of Marginal Stock Subsidies that Converge to Social Optimum

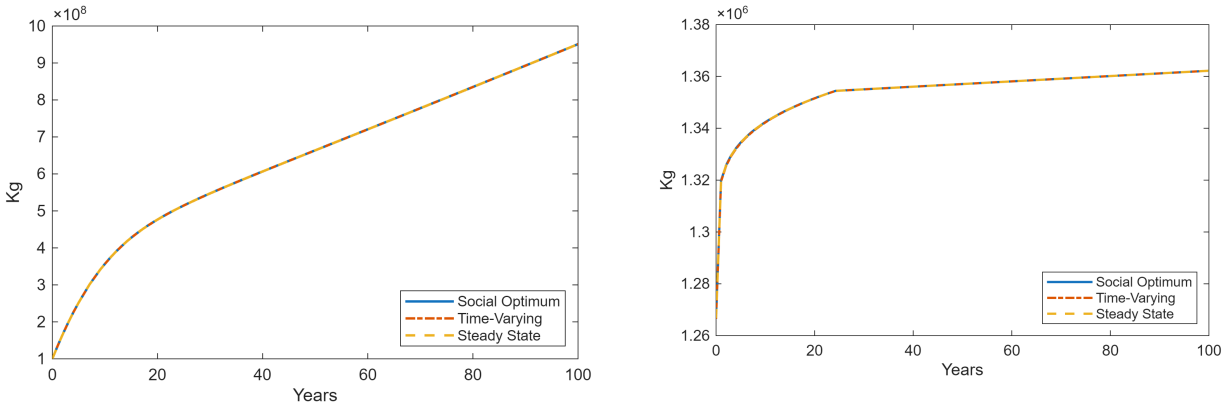


The lines represent the values of marginal stock subsidies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

Figure A8: Pollution and Harvest Trajectories Under Stock Subsidies that Converge to Social Optimum

(a) Pollution Stock

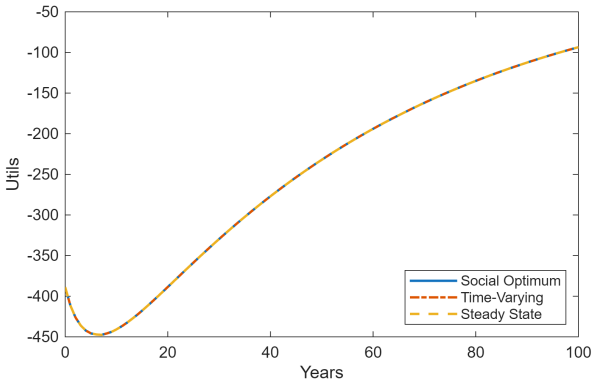
(b) Harvest



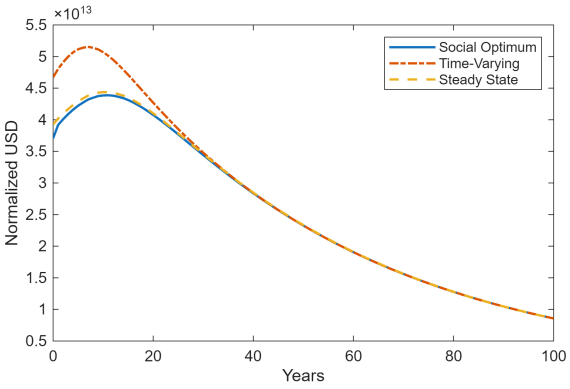
The lines represent the levels of inorganic nitrogen stock and harvest in kilograms under policies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

Figure A9: Surplus Trajectories in Present Values Under Stock Subsidies that Converge to Social Optimum

(a) Consumer Surplus



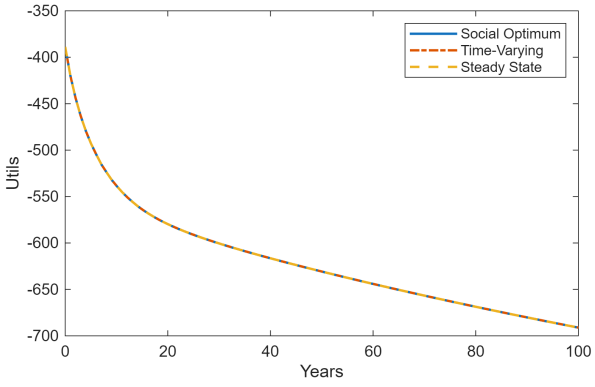
(b) Producer Surplus



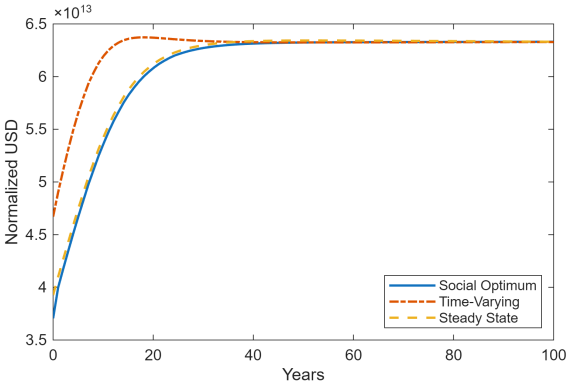
The lines represent the levels of consumer and producer surplus in present values under policies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

Figure A10: Surplus Trajectories in Current Values Under Stock Subsidies that Converge to Social Optimum

(a) Consumer Surplus

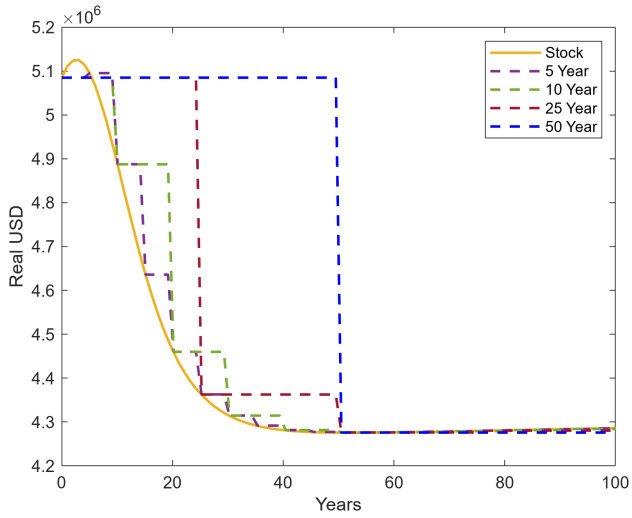


(b) Producer Surplus



The lines represent the levels of consumer and producer surplus in current values under policies that satisfy Proposition 1. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line represents the trajectory under the Social Optimum. The orange dot-dashed line ("Time-Varying") represents the trajectory under a time-varying stock subsidy. The yellow dashed line ("Steady State") represents the trajectory under a constant per unit subsidy set at the steady state value of natural resource stock accumulation under the Social Optimum.

Figure A11: Trajectories of Marginal Stock Subsidies with Periodic Updates

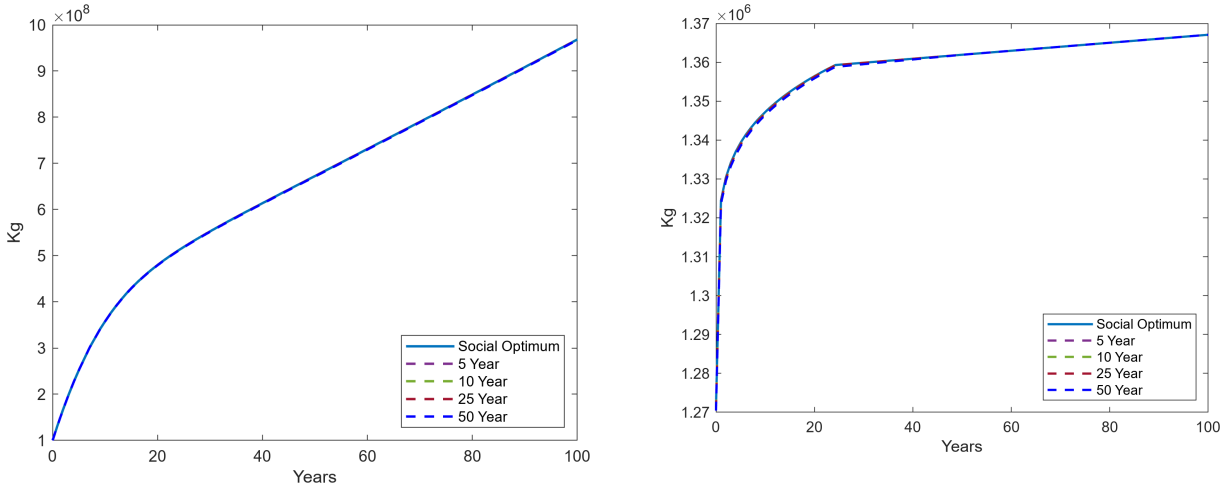


The lines represent the value of the stock subsidy over time with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The yellow solid line represents the trajectory under a time-varying subsidy set that satisfies Proposition 1. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A12: Pollution and Harvest Trajectories Under Stock Subsidies with Periodic Updates

(a) Pollution Stock

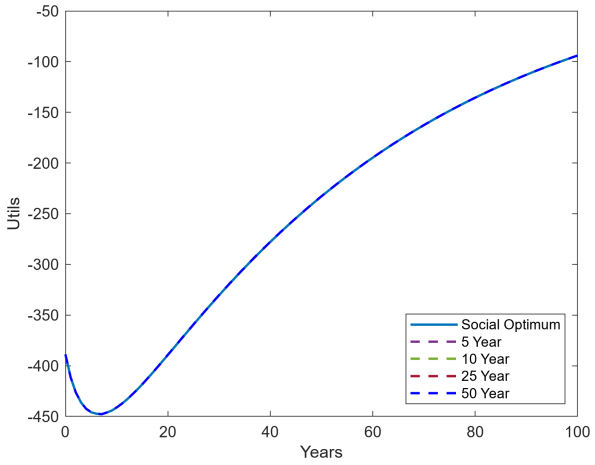
(b) Harvest



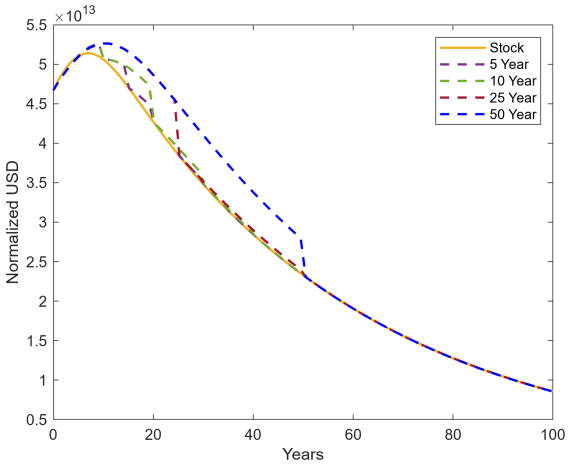
The lines represent the levels of inorganic nitrogen stock and harvest in kilograms under the two theoretical benchmarks in addition to per-unit incentives with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line in Panel (a) represents the trajectory of consumer surplus under the Social Optimum. The yellow solid line in Panel (b) represents the trajectory of producer surplus under a stock subsidy that satisfies Proposition 1. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A13: Surplus Trajectories in Present Values Under Stock Subsidies with Periodic Updates

(a) Consumer Surplus



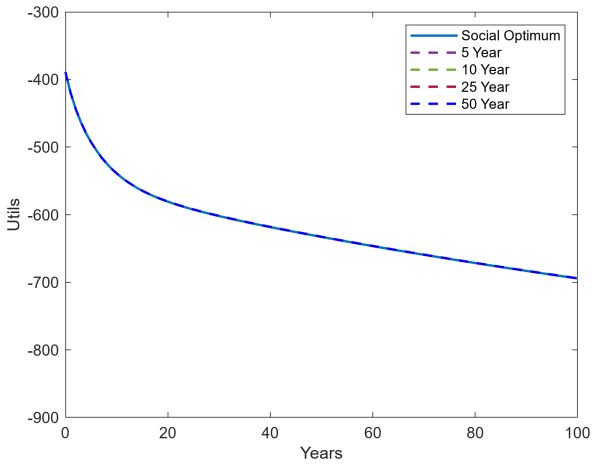
(b) Producer Surplus



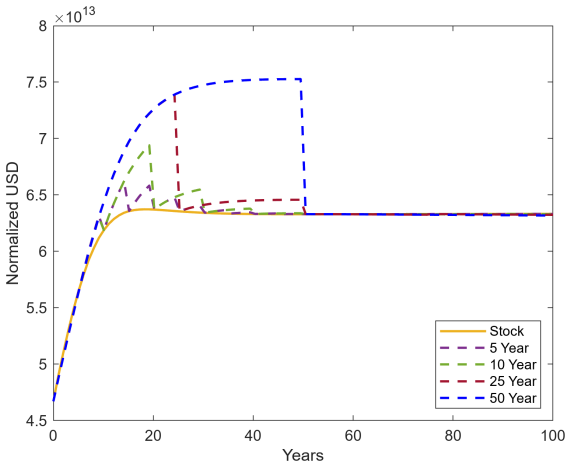
The lines represent the levels of consumer and producer surplus in present values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line in Panel (a) represents the trajectory of consumer surplus under the Social Optimum. The yellow solid line in Panel (b) represents the trajectory of producer surplus under a stock subsidy that satisfies Proposition 1. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A14: Surplus Trajectories in Current Values Under Stock Subsidies with Periodic Updates

(a) Consumer Surplus



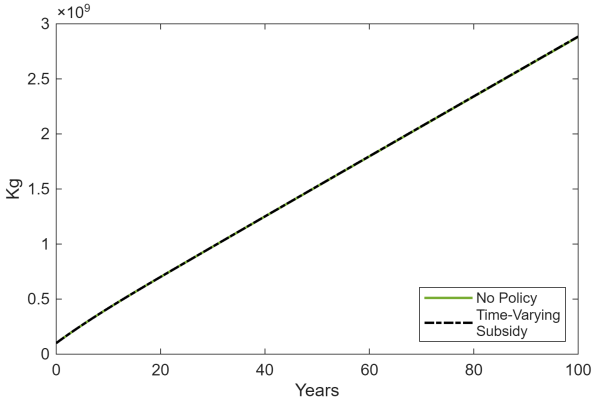
(b) Producer Surplus



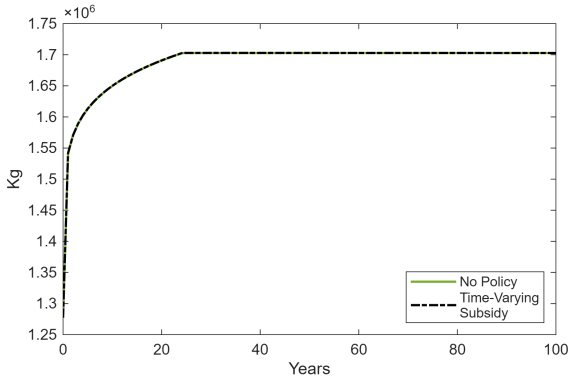
The lines represent the levels of consumer and producer surplus in current values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The blue solid line in Panel (a) represents the trajectory of consumer surplus under the Social Optimum. The yellow solid line in Panel (b) represents the trajectory of producer surplus under a stock subsidy that satisfies Proposition 1. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A15: Pollution and Harvest Trajectories Under Flow Subsidies that Converge to Social Optimum

(a) Pollution Stock



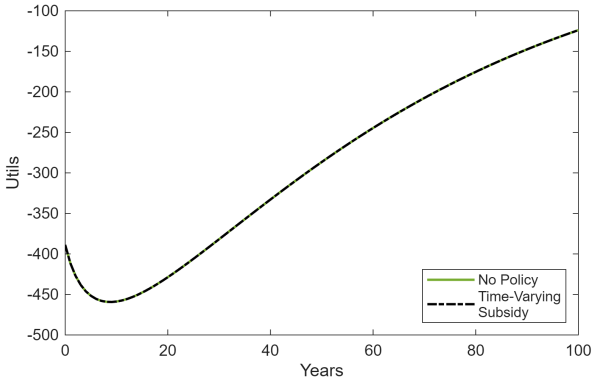
(b) Harvest



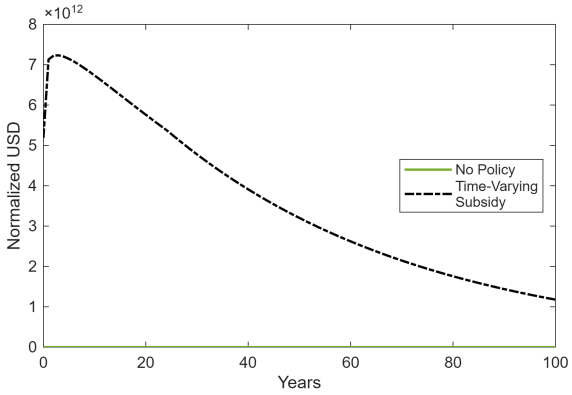
The lines represent the levels of inorganic nitrogen stock and harvest in kilograms under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under the no policy. The black dot-dashed line ("Time-Varying Subsidy") represents the trajectory under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner.

Figure A16: Surplus Trajectories in Present Values Under Flow Subsidies that Converge to Social Optimum

(a) Consumer Surplus

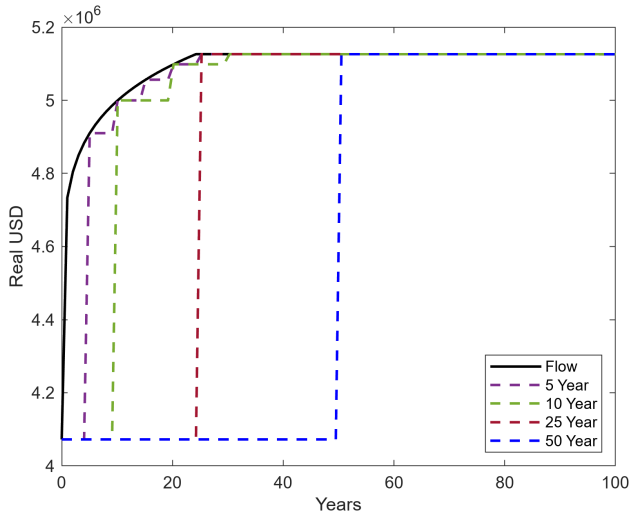


(b) Producer Surplus



The lines represent the levels of consumer and producer surplus in present values under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line represents the trajectory under the no policy. The black dot-dashed line ("Time-Varying Subsidy") represents the trajectory under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner.

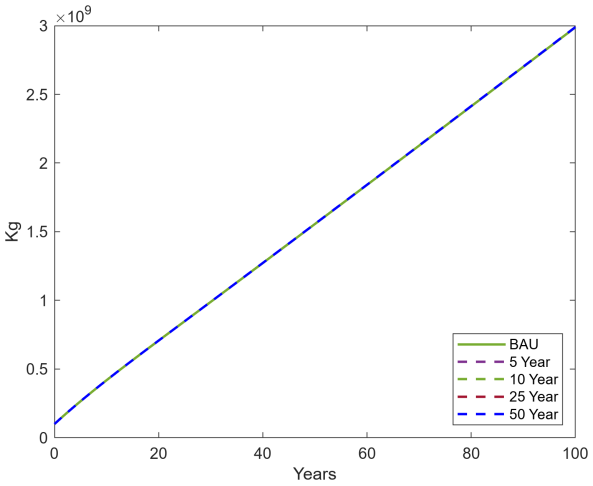
Figure A17: Trajectories of Marginal Flow Subsidies with Periodic Updates



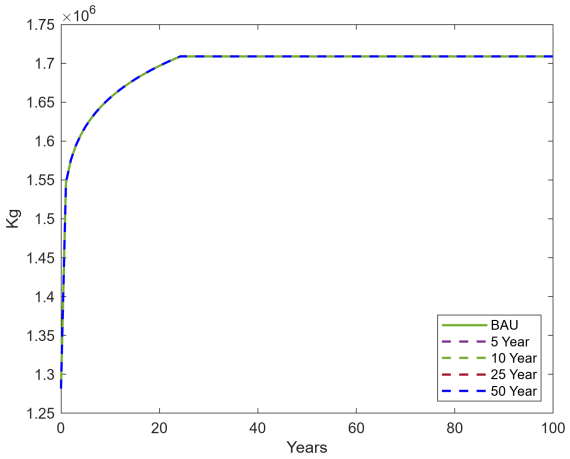
The lines represent the value of the stock subsidy over time with different periodic updates. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The black solid line represents the trajectory under a time-varying subsidy set at the value of stock accumulation as if under the Social Planner. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A18: Pollution and Harvest Trajectories Under Flow Subsidies with Periodic Updates

(a) Pollution Stock



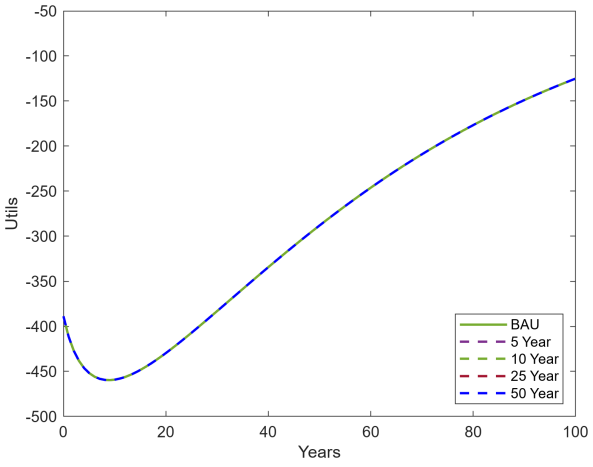
(b) Harvest



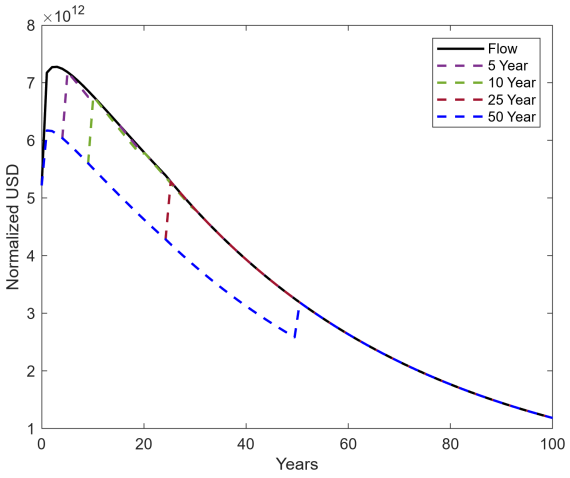
The lines represent the levels of inorganic nitrogen pollution and harvest in kilograms under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line in Panel (a) represents the trajectory of consumer surplus under no policy. The black solid line in Panel (b) represents the trajectory of producer surplus under a time-varying flow subsidy. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A19: Surplus Trajectories in Present Values Under Flow Subsidies with Periodic Updates

(a) Consumer Surplus



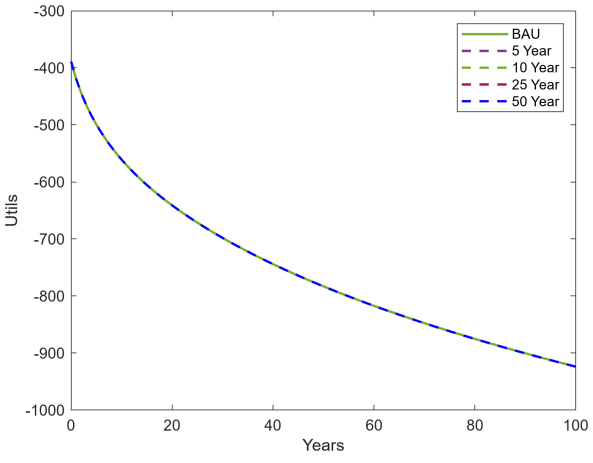
(b) Producer Surplus



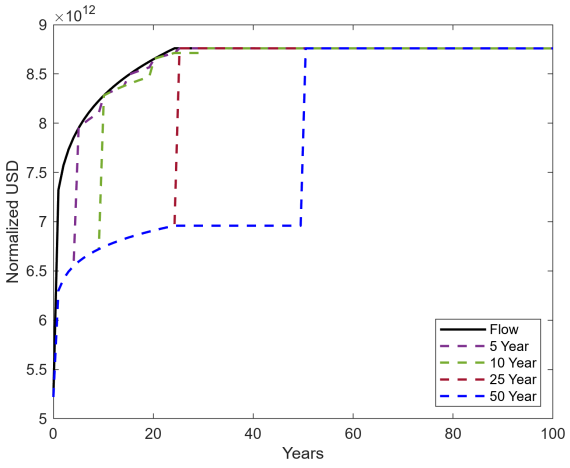
The lines represent the levels of consumer and producer surplus in present values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line in Panel (a) represents the trajectory of consumer surplus under no policy. The black solid line in Panel (b) represents the trajectory of producer surplus under a time-varying flow subsidy. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

Figure A20: Surplus Trajectories in Current Values Under Flow Subsidies with Periodic Updates

(a) Consumer Surplus



(b) Producer Surplus



The lines represent the levels of consumer and producer surplus in current values under the two theoretical benchmarks in addition to different constant per-unit incentives. The trajectories are generated from the model specification with pollution dynamics (Section 3.1) and t is in years. The green solid line in Panel (a) represents the trajectory of consumer surplus under no policy. The black solid line in Panel (b) represents the trajectory of producer surplus under a time-varying flow subsidy. The purple dashed line ("5 Year") is the trajectory under a policy that updates every five years. The green dashed line ("10 Year") is the trajectory under a policy that updates every ten years. The red dashed line ("25 Year") is the trajectory under a policy that updates every 25 years. The blue dashed line ("50 Year") is the trajectory under a policy that updates every 50 years.

B Harvest Trajectories and Total Surplus

An initial guess of the trajectory for harvest must be provided in order to calibrate the model. The system adjusts over subsequent iterations based on the evolution of the state variables (the stocks of the natural resource and pollution) and the costate equations (the Euler equation). However, the model does not provide direct insight into the shape of the time path for the control variable of the system. Therefore, I provide the following guesses of the trajectory: linear, exponential, and sigmoidal. The functional forms of each guess are below.

$$H_t = \begin{cases} H_0 + \left(\frac{|H_t - H_0|}{T}\right) t & \text{if } H_0 < H_t \\ H_0 - \left(\frac{|H_t - H_0|}{T}\right) t & \text{otherwise} \end{cases} \tag{43}$$

$$H_t = \begin{cases} H_0 + \left(\frac{|H_t - H_0|}{(0.25T)^\chi}\right) t^\chi & \text{if } t \leq 0.25T \ \& \ H_0 < H_t \\ H_0 - \left(\frac{|H_t - H_0|}{(0.25T)^\chi}\right) t^\chi & \text{if } t \leq 0.25T \ \& \ H_0 > H_t \\ H_t & \text{if } t > 0.25T \end{cases} \tag{44}$$

$$H_t = \begin{cases} H_0 + |H_t - H_0| \left(\frac{1}{1+\psi e^{\omega t}}\right) & \text{if } H_0 < H_t \\ H_0 - |H_t - H_0| \left(\frac{1}{1+\psi e^{\omega t}}\right) & \text{otherwise} \end{cases} \tag{45}$$

The exponential time path is piecewise based on $t \lesseqgtr 0.25T$ in order to allow flexibility in the time to convergence, which in the current model is set at 25 years after policy implementation. The curvature parameter $\chi = 0.15$ reflects quick convergence to the steady state and is chosen based on conversations with industry professionals. Similarly, $\psi = 50$ and $\omega = -0.4$ determine the speed and timing of convergence under a sigmoidal trajectory.

Each time path is given the same H_0, while the ending values H_t depend on theoretical predictions. BAU and flow incentives are given $H_t = H_{MSY}$, while the system under the Social Planner and stock incentive are given $H_t = 0.8H_{MSY}$. The system is then calibrated to each initial guess, and the present discounted values of consumer and producer surplus are calculated to find the trajectory that maximizes total surplus. Using parameters and initial values in Table 1, the following surpluses were calculated for the main policy calibrations.

Table B1: Harvest Trajectories and Surpluses

Policy	Initial Guess	Producer Surplus	Consumer Surplus	Total Surplus
Social Planner	Linear	3,373,956.37	-24,490.76	3,349,465.61
	Exponential	3,477,056.73	-25,210.19	3,451,846.53
	Sigmoid	3,453.650.69	-24,946.73	3,428,703.96
BAU	Linear	3,626,551.43	-25,493.32	3,601,058.11
	Exponential	4,128,596.12	-29,344.84	4,099,251.28
	Sigmoid	4,014,456.99	-27,801.29	3,986,655.69
Stock	Linear	3,373,860.87	-24,489.49	3,349,371.38
	Exponential	3,476,926.41	-25,208.71	3,451,717.69
	Sigmoid	3,453,532.18	-24,945.39	3,428,586.79
Flow	Linear	3,628,154.99	-25,504.93	3,602,650.05
	Exponential	4,128,750.16	-29,346.82	4,099,403.33
	Sigmoid	4,014,971.70	-27,805.87	3,987,165.83

The exponential trajectory for harvest maximizes total surplus under all scenarios. The calibrations under time varying policy updates and policies under different signs and scales similarly choose exponential paths as the surplus maximizing initial guess. Therefore, I use exponential paths as the primary initial guess for the system in the results presented in Section 3.2.