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# “No One Can Borrow” – Reassessing the “Junior Can’t Borrow” Equity-Premium Puzzle Resolution

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## ABSTRACT

“Junior Can’t Borrow,” Constantinides, Donaldson, and Mehra’s (CDM) three-period exchange model, eliminates borrowing by the young, and, consequently, all generations to resolve the equity premium puzzle. This paper examines CDM’s resolution in an 80-period, OLG model with capital, fiscal policy, macro shocks and steeply rising borrowing costs. Imposing such costs on all generations and, thereby, eliminating the bond market produces, as in CDM, a large premium. But permitting even one generation to sell bonds without transactions costs dramatically raises the safe rate, restoring the puzzle. Hence, “No One Can Borrow” not “Junior Can’t Borrow” is CDM’s real message. Since zero or low marginal-cost borrowing is commonplace – be it via secured borrowing or friends and family loans, CDM’s “solution” isn’t plausible. As in representative-agent RBCs, macro risk is low in OLG-RBCs – not because aggregate shocks are small, but because they can be hedged via self-insurance (saving) and largely average out over agents’ lifetimes. Indeed, absent high marginal borrowing costs, agents, even older ones, are close to indifferent between holding shares and holding bonds, rendering their net bond demand (supply) curves highly elastic. This explains why even a single generation is willing to supply significant amounts of bonds at a low price (high yield).

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# 1 Introduction

The U.S. equity premium – the difference between mean real returns on U.S. stocks (the market portfolio) and short-term Treasuries (3-month T-Bills) – has averaged 6 percent on an annual basis since 1925. The risk-free real rate during this period has averaged 1 percent. Mehra and Prescott (1985)'s seminal paper features a realistically calibrated representative agent model with an equity premium of at most 0.35 percent and a risk-free rate of 4 percent. The authors dubbed the discrepancy between the observed and their model's risk premiums "the equity premium puzzle".

A large literature attempts to resolve the puzzle. Constantinides (1990), Abel (1990), Epstein and Zin (1991), Jermann (1998), Campbell and Cochrane (1999), Campbell (2001), Ju and Miao (2012), and Liu and Miao (2014) posited alternative preferences. Jermann (1998), Tallarini (2000), Boldrin, Christiano, and Fisher (2001), Mehra, Piguillem and Prescott (2011), and Liu and Miao (2014) considered capital frictions. Rietz (1988), Barro (2006), Gabaix (2012), and Wachter (2013) invoked rare disasters. Krueger and Kubler (2006) introduced significant stochastic depreciation. Farmer (2014) turned to non-fundamental shocks (sunspots). Finally, Barberis, Huang, and Santos (2001) resorted to behavioral models, including a number based on prospect theory.[1]

Our paper revisits Constantinides, Donaldson and Mehra (2002), henceforth CDM. CDM specifies a three-period OLG model of pure exchange in which the young face a hard, i.e., zero-borrowing constraint. This leaves only identical middle-aged agents to participate in the bond market since the oldest generation does not overlap for the extra period needed to contract intergenerationally. Absent their hard borrowing constraint, the young would borrow from (sell bonds to) the middle-aged. Hence, the bond price (yield) must be high (low) enough to entirely choke off demand. The resultant high (low) bond price (yield) produces a meaningful equity premium albeit in an economy without a bond market.

CDM is highly stylized, yet the authors suggest that their answer has general applicability, writing: *"For example, we may increase the number of generations from three to sixty, representing consumers of ages 20 to 80 in annual increments. In such a model, we expect that the youngest consumers are borrowing-constrained for a number of years and invest neither in equity nor in bonds ... [This change] will have pricing implications to the extent that the young investors who are currently infra-marginal in the equity and bond markets become marginal. We suspect that ... the essential message of our paper will survive."*

Our paper uses our 80-period, stochastic, GE OLG model – Hasanhodzic and Kotlikoff

[1] See Mehra's (2006) survey of the puzzle literature.

(2025) – to examine this conjecture.[2] Our model features soft borrowing constraints, modeled as increasing borrowing costs, that can be hardened to the extent desired. CDM's message does survive, but in a manner that renders it implausible. All generations need to be borrowing constrained for a realistic equity premium to emerge. In other words, "no one can borrow" rather than "junior can't borrow" is the appropriate generalization of CDM.

**Asset Demands and Rates of Return, Subsets of Generations Face Borrowing Costs**

| Model | Total Bond Demand* | Total Stock Demand | Avg Return on Capital (%) | Avg Risk-Free Rate (%) | Equity Premium (%) |
|---|---|---|---|---|---|
| **No Borrowing Costs** | 238.92 | 1077.64 | 3.91 | 3.74 | 0.17 |
| **Baseline Borrowing Costs on Ages 1-30** | 77.92 | 1070.82 | 3.92 | 3.67 | 0.26 |
| **Baseline Borrowing Costs on All Ages Except 18** | 32.78 | 1076.01 | 3.91 | 3.51 | 0.40 |
| **Baseline Borrowing Costs on Ages 1-70** | 24.06 | 1069.04 | 3.92 | 3.43 | 0.49 |
| **Baseline Borrowing Costs on All Ages Except 20** | 33.23 | 1077.57 | 3.90 | 3.47 | 0.43 |
| **Baseline Borrowing Costs on All Ages Except 40** | 24.50 | 1072.79 | 3.91 | 3.40 | 0.52 |
| **Baseline Borrowing Costs on All Ages Except 60** | 19.04 | 1070.79 | 3.92 | 3.25 | 0.67 |
| **Flattest Borrowing Costs on All Ages (slope 5)** | 2.76 | 1061.00 | 3.94 | 2.97 | 0.97 |
| **Flatter Borrowing Costs on All Ages (slope 15)** | 0.32 | 1061.06 | 3.94 | 1.63 | 2.31 |
| **Baseline Borrowing Costs on All Ages (slope 25)** | 0.12 | 1082.31 | 3.89 | 0.24 | 3.65 |
| **Steeper Borrowing Costs on All Ages (slope 45)** | 0.04 | 1076.56 | 3.91 | -2.42 | 6.32 |

*Note: Total bond supply is the negative of the total bond demand.

Table 1: Asset demands and rates of return when subsets of generations face borrowing costs in the baseline calibration with depreciation shocks and no Social Security.

With 80 periods, 79 generations can engage in mutual financial trade. As we show, the larger the number of borrowing constrained generations, the lower the risk-free rate and the greater the equity premium. However, permitting even a single generation to supply bonds no extra cost substantially raises the risk-free yield, thereby reintroducing the equity premium puzzle. As shown in Hasanhodzic and Kotlikoff (2025), a canonical OLG RBC entails very little macro risk, as experienced by individual agents, notwithstanding the incorporation of standard-sized TFP and depreciation shocks. Hence, each age-group's net demand (supply) for risk-free bonds is highly elastic.

Intuitively, agents are able to self-insure, i.e., save to tide themselves over in bad times. They can also count on good shocks offsetting bad shocks over their long lives.[3] The ability of

[2]The model's 80 periods represent real-life ages 20 to 100.

[3]This is true notwithstanding realistic serial correlation of shocks.

generations to share risk through the bond market in the absence of borrowing costs makes surprisingly little difference to economic welfare or the economy's functioning. Moreover, agents who don't face the model's extra costs of borrowing are relatively indifferent between holding shares and holding (selling) bonds. This explains why even just one generation is willing to supply a significant amount of bonds at a relatively low price (high yield).

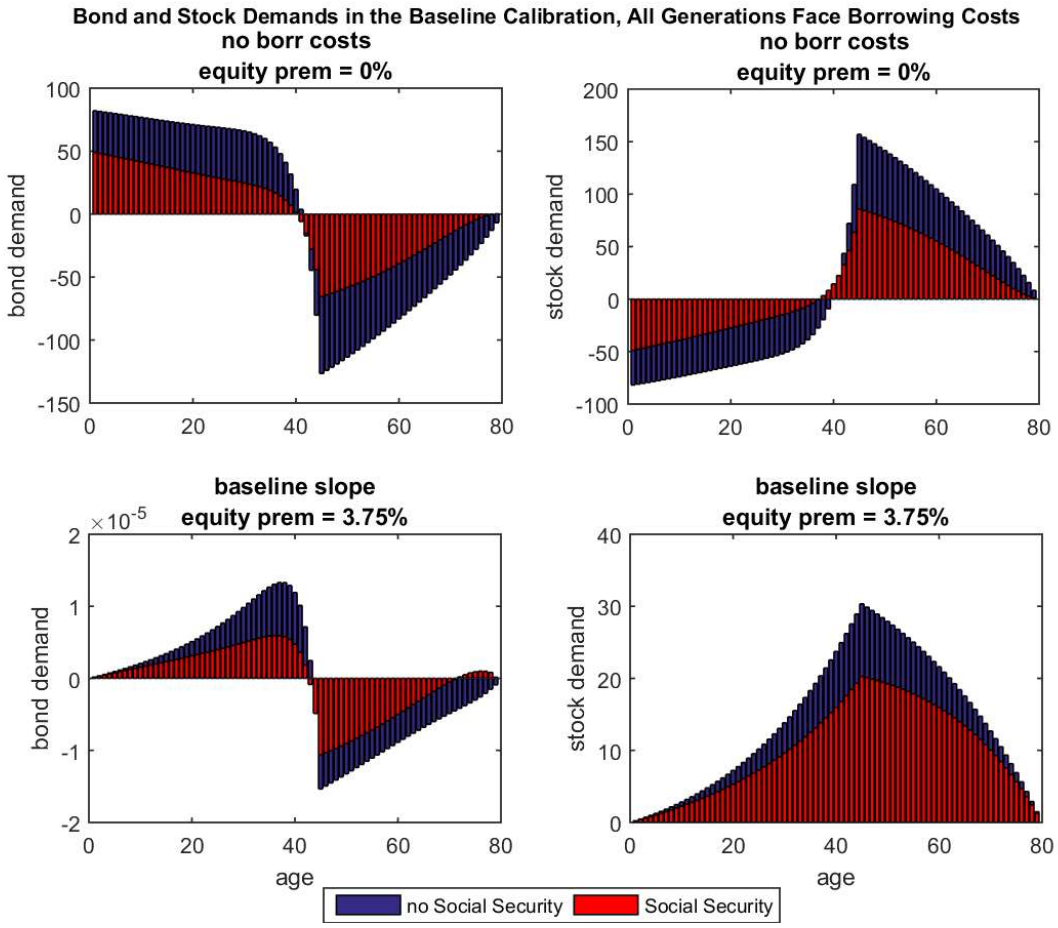



Figure 1: Bond and stock demands under the baseline calibration, shown with and without borrowing costs and with and without Social Security. When borrowing costs are present, they apply to all generations. The risk-free rate reported at the top of each panel refers to the no–Social Security case.

Consider Table 1. Once even a single generation can borrow at no extra cost the premium collapses. For example, if generation 40 (real-life age 60) can borrow cost free, the risk premium falls to 0.52 percent. The reduction in the premium is essentially invariant to which generation is unconstrained. If it's generation 20 (age 40), the premium falls to 0.43 percent. If it's generation 60 (age 80), the premium falls to 0.67 percent. Even allowing the oldest nine generations (ages 90 and above) to costlessly borrow, while making it prohibitively costly

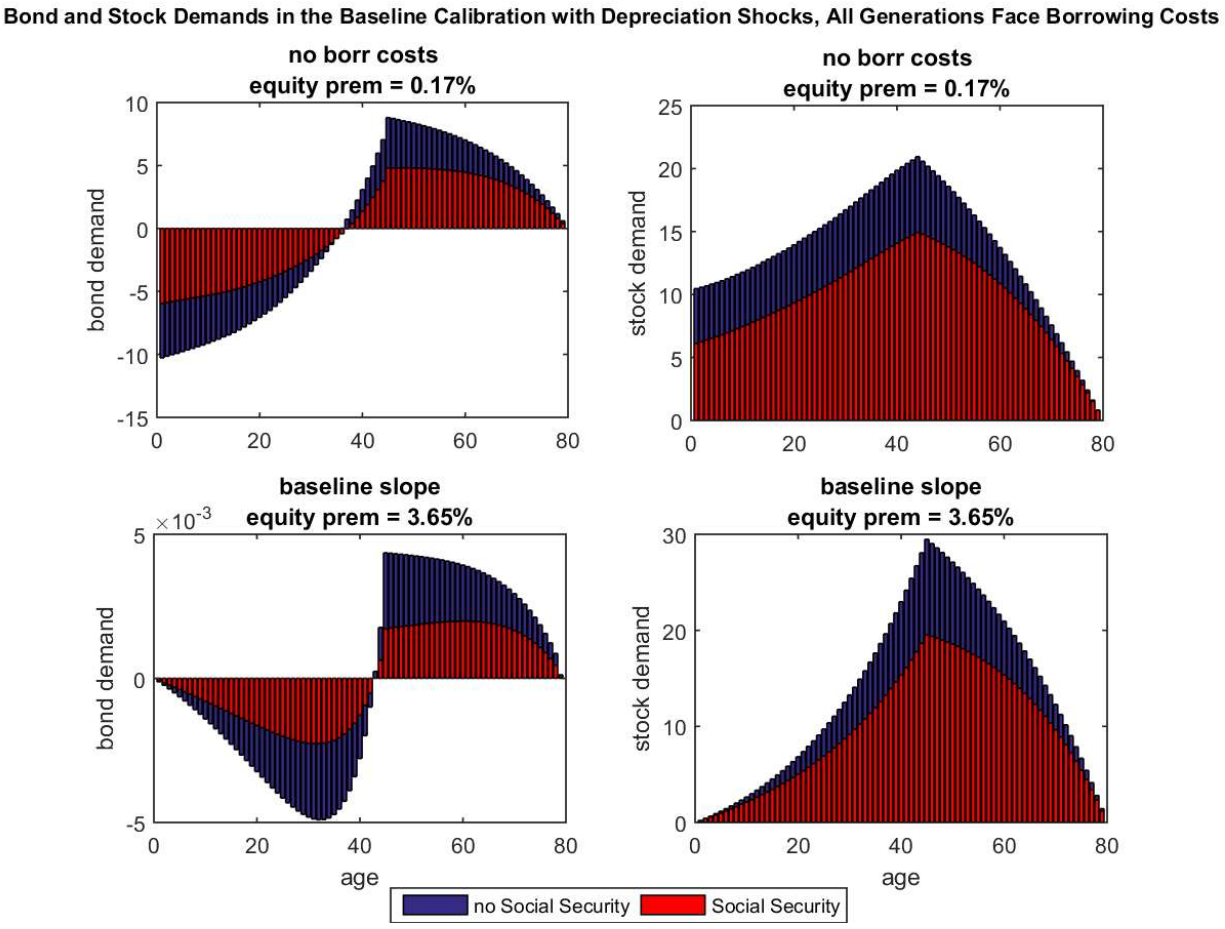



Figure 2: Bond and stock demands under the baseline calibration with depreciation shocks, shown with and without borrowing costs and with and without Social Security. When borrowing costs are present, they apply to all generations. The risk-free rate reported at the top of each panel refers to the no–Social Security case.

for generations 1 through 70 (ages 20 to 90) to borrow, drops the premium to 0.49 percent. When constraints are limited to the first 30 generations (ages 20 to 50), the premium is just 0.26 percent, virtually indistinguishable from the case of no borrowing constraints, which features a premium of 0.17 percent.

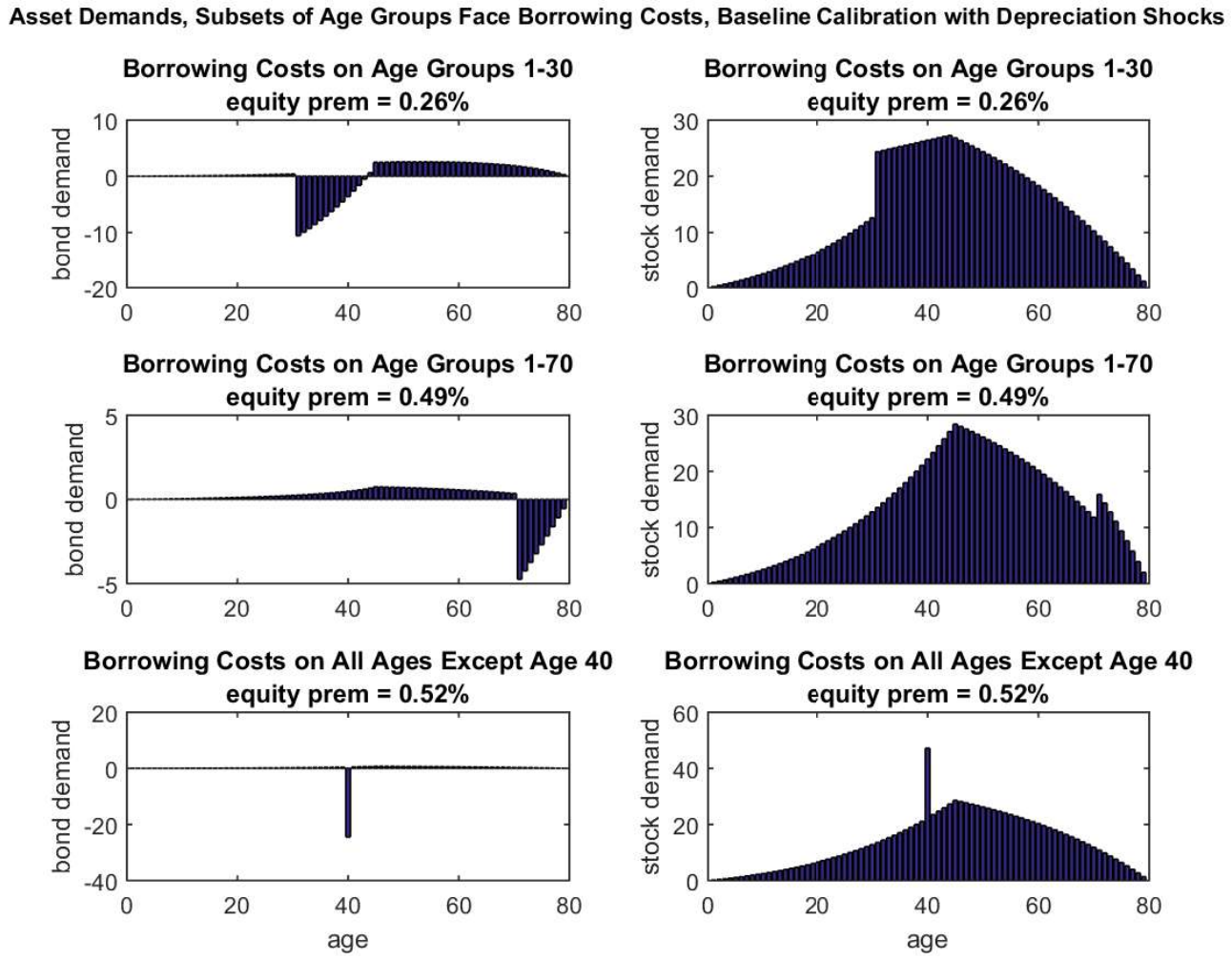



Figure 3: Bond and stock demands where subsets of generations face borrowing costs in the baseline calibration with depreciation shocks.

Table 1, as well as Figures 1–3, show the relationship between equity premium values and the aggregate demand for / aggregate supply of bonds by age. What happens if we add depreciation shocks to our baseline calibration? Absent borrowing costs, the equity premium is, to repeat, 0.17 percent. The supply of bonds, in this case, is significant, equaling 238.92 – roughly one fourth of the economy's capital (total stock holdings). When all age groups face borrowing costs, the private supply of bonds falls by three orders of magnitude, from

238.92 to 0.12. With minimal supply of bonds, the bond price skyrockets and the safe rate drops from 3.74 percent to 0.24 percent.

Interestingly, the case where all generations except the last nine are constrained looks much like the case where only a single generation is unconstrained. In the former, bond supply equals 24.06, the risk-free rate averages 3.43 percent, and the equity premium is 0.49 percent. In the latter, bond supply ranges from 19.04 to 33.23, the risk-free rate from 3.25 to 3.47 percent, and the equity premium from 0.43 to 0.67 percent.[4] Notably, when generation 60 is the only supplier, bond supply (19.04) is less than two-thirds of what it is when generation 20 alone supplies (33.23). This is precisely what the model predicts: with stochastic depreciation, the young are the natural suppliers, both better diversified and less exposed, and therefore willing to issue more bonds at lower prices (higher yields).

**Our Model**

Agents work for 45 years spending their remaining 25 years in retirement. The economy faces either TFP shocks or TFP plus depreciation shocks. The TFP shocks are serially correlated. The impact of these shocks on particular age groups at a point in time depends on the economy's history of shocks. Agents invest in the economy's two assets – capital and one-period, risk-free bonds, which are in zero net supply. We've limited our model to CDM's elements to maintain comparability. In particular, we haven't entertained idiosyncratic labor earnings shocks as in Harenberg and Ludwig (2019). Doing so would not likely alter our conclusions as they too report a very small equity premium.[5]

Preferences are isoelastic with a relative risk aversion coefficient of 2.[6] Production is Cobb-Douglas. Total factor productivity follows a trend-stationary process with normally distributed innovations, consistent with standard macroeconomic growth frameworks. Fiscal policy is limited to a pay-as-you-go Social Security system financed by a fixed payroll tax. Social Security's inclusion does not, however, meaningfully alter our findings. In particular, it makes no material difference to the equity premium reflecting, as emphasized in Hasanhodzic and Kotlikoff (2025), its inability to significantly share risk across generations.

CDM study a three-period, partial equilibrium, pure exchange model. In their framework, aggregate risk is captured by a joint process over total income and the wages of the middle-aged. Both models feature similarly calibrated macroeconomic shocks. But the im-

[4]The range covers the cases where only generation 20, 40, or 60 is unconstrained.

[5]We also conducted our analysis assuming idiosyncratic age-specific mortality. This produced a realistic age distribution and a lifetime age-consumption profile featuring higher consumption when young than old. Adding demographics made virtually no difference to our findings. We also simulated our model with age-specific borrowing costs. Specifically, we considered major declines in marginal borrowing cost with age. Here, also, we found little difference in our results. These results are available upon request.

[6]Increasing risk aversion to 5 does not alter our conclusions.

pact of macroeconomic shocks is very different in a 3-period than in an 80-period model. First, long periods mean agents face a higher probability of experiencing extreme outcomes. Second, long periods dramatically limit agents' ability to pool risk over time. This dynamic, emphasized in Hasanhodzic and Kotlikoff (2013, 2018, 2025), is central to understanding why the equity premium puzzle is so puzzling. The observed premium is large, but agents in our vanilla OLG RBC as well as those in vanilla representative-agent RBC models are able to cope effectively with macro shocks. Or as Weil (1989, pp. 402–403) puts it, "there is simply not enough aggregate risk in the model to replicate the risk premium for plausibly risk-averse consumers."

In our main calibration, we introduce capital depreciation shocks. These are not necessary to produce an equity premium. But, unlike TFP shocks, which raise or lower wage rates and real capital returns by the same percentage, depreciation shocks differentially impact agents of different ages, allowing the model to match the empirically observed pattern of increasing bond holdings with age. With only TFP shocks, the young – asset-poor and highly exposed to labor income volatility – are the primary demanders of safe bonds. Holding labor income is effectively equivalent to holding equity, prompting them to hedge by acquiring assets that perform well in downturns. The old, who rely more on asset returns and face less consumption volatility due to accumulated wealth, supply these bonds. Once capital depreciation shocks are introduced, the risk shifts. These shocks expose the principle of capital investment to uncertainty, making the old more vulnerable. In this case, they become the primary demanders of safe bonds. The young, in contrast, are both more diversified – because labor income remains an important component of their resources – and less exposed, given their longer investment horizons. As a result, they act as natural suppliers of bonds. The outcome is a more realistic term structure of risk and asset allocation across cohorts.

**Borrowing Costs**

We assume soft but steeply rising borrowing costs permitting agents to access very small amounts of credit at modest rates, while facing a significant cost for larger borrowing. The ubiquity of tiered-credit pricing in real-world financial markets reinforces this modeling choice. For example, credit cards frequently feature teaser rates, penalty rates, and zero-interest periods for fully paid balances.

Our approach builds on a large body of theoretical and empirical literature. Milde and Riley (1988) propose a screening mechanism in which lenders offer higher rates on larger loans, with borrowers signaling quality through their willingness to pay. Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) endogenize this relationship by tying higher interest rates to greater default risk, especially for larger unsecured loans. Wilcox (1989) models rising bor-

rowing costs as liquidity constraints that restrict consumption. In collateral-based models, Kiyotaki and Moore (1997) and Iacoviello (2005) connect reductions in collateral values to tighter borrowing limits, mimicking a graduated cost schedule. Iacoviello and Pavan (2013) apply this logic to a life-cycle framework with cyclical credit access, allowing borrowing conditions to tighten during downturns. Camara and Sangiacomo (2022) introduce interest-coverage constraints, where credit availability is tied to expected earnings relative to interest payments – another soft borrowing mechanism.

Scott (1996) documents substantial heterogeneity in unsecured borrowing rates in U.K. credit markets. He shows that deposit rates are typically less than half the rate on personal loans, which are themselves lower than credit card and retail credit rates. He recommends modeling borrowing opportunities with an upward-sloping interest rate schedule. Additional evidence from Connolly and McGregor (2011) links the expansion of household debt in the 1990s and 2000s to increasing effective borrowing costs. Caspi, Eshel, and Segev (2024) show that rising mortgage rates drive down consumption, consistent with soft cost constraints rather than hard borrowing caps.[7]

Beyond empirical relevance, adopting a soft constraint permits greater modeling flexibility. First, by varying the slope of the cost function, we can simulate a wide spectrum of credit conditions across cohorts or policy regimes. Hard constraints appear as a limiting case – via very steep cost functions – while flatter profiles capture more relaxed borrowing environments. This structure enables us to explore how marginal borrowing conditions affect aggregate outcomes across time and demographic groups. These different slope parameterizations are illustrated in figure 4.

Second, soft constraints are computationally tractable in large-scale models. Our projection-method (see Marcet (1988) and Judd et al. (2011)) algorithm approximates consumption functions as polynomials. Smooth borrowing cost ensures differentiability, which is critical for numerical stability and algorithmic convergence.

Although our model abstracts from explicit default to keep the analysis of interest rate dynamics tractable, default risk is implicitly embedded in our formulation. As in Milde and Riley (1988), larger borrowing costs reflect the real resource costs of verifying creditworthiness. The larger the loan, the more costly it is for lenders to screen the borrower. This verification process – credit scores, tax documentation, employment verification, interviews, appraisals – becomes progressively more intensive with loan size. At the same time, borrowers incur increasing effort in meeting documentation requirements and substantiating their identity and repayment ability.

[7]See also Altonji and Siow (1987), Wirjanto (1995), and Fernandez-Corugedo (2002) for models where interest rates rise with the amount borrowed in various contexts.

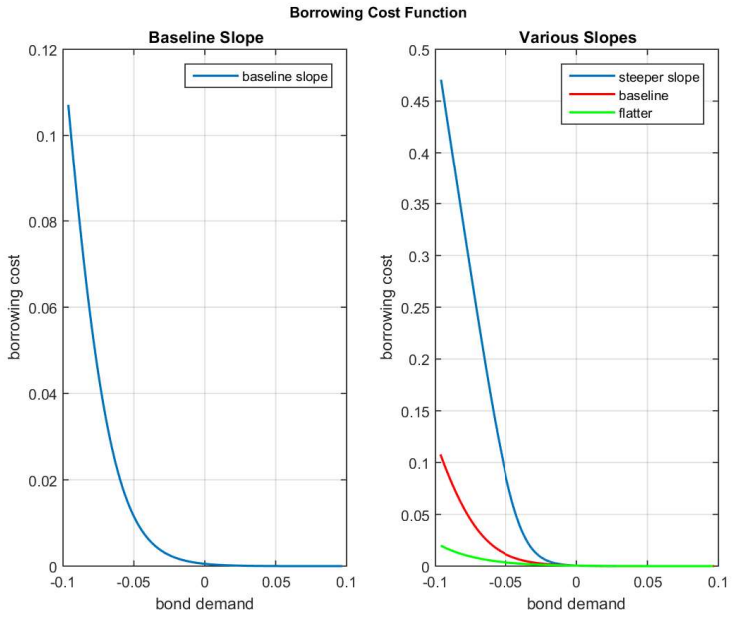



Figure 4: Borrowing cost function with different slopes for a fixed level of assets. The x-axis displays bond demands. Since the asset level is fixed, the function is increasing in bond shares.

Our framework echoes a broader literature that introduces frictions into financial intermediation. Krishnamurthy and Vissing-Jorgensen (2012) emphasize the convenience yield on safe assets, such as Treasuries, which depresses their returns relative to corporate bonds. Goodfriend (2005) and Goodfriend and McCallum (2007) incorporate loan production technologies into DSGE models, in which loan creation depends on monitoring effort and collateral quality. Their work highlights how real resource costs associated with credit delivery shape the transmission of monetary policy and the pricing of financial contracts.

As expected, a steeper borrowing cost schedule reduces gross bond supply, lowers the risk-free rate, and increases the equity premium. In our baseline plus capital depreciation case, the average marginal borrowing cost is roughly 14 times the average risk-free rate. This aligns with real-world patterns. Credit card companies commonly charge 20 to 30 percent interest, while risk-free rates have often been in the 1 to 2 percent range. Recall that in this equilibrium, the risk-free rate is 0.30 percent and the equity premium is 3.65 percent. Thus, when all generations face substantial marginal borrowing costs, they are indifferent between lending at a very

When the slope of the borrowing cost function is reduced, the ratio of borrowing to lending rates falls to about 1.3. The risk-free rate rises to 1.63 percent, while the equity premium falls to 2.31 percent. With a still flatter cost schedule, the ratio drops to 0.25, the

risk-free rate increases to 2.97 percent, and the equity premium falls further to 0.97 percent.

Another measure of the plausibility of these borrowing costs is the use of real resources are consumed in the process of intermediation. Mehra, Piguillem, and Prescott (2013) estimate that financial intermediation absorbs roughly 1 to 2 percent of GDP. Their approach embeds intermediation costs into a standard neoclassical growth model by imposing a fixed cost wedge between the gross return on capital and the net return to savers. The model includes two sectors – one with and one without financial intermediation. Rather than modeling the institutional structure of banking, they treat these costs as real resource losses required to allocate capital. By calibrating the cost wedge to match historical averages for the equity premium (about 6 percent) and the risk-free rate (roughly 1 to 2 percent), the model implies intermediation costs in the range of 1 to 2 percent of GDP.

This figure is consistent with empirical data. Philippon (2015) estimates that the unit cost of financial intermediation in the US has averaged around 2 percent of intermediated assets over the past century, which translates to 1.5 to 2 percent of GDP. National accounts data from the Bureau of Economic Analysis report value added from banking and credit intermediation services at 1.5 to 2.5 percent of GDP, though this includes services beyond lending. The OECD and IMF also measure banking-sector value added at close to 2 percent of GDP in most advanced economies.

In our baseline calibration with depreciation shocks, total borrowing costs average about 0.2 percent of output. That's roughly one tenth of the estimate discussed earlier. But this isn't surprising. The central point of the paper is that, to generate a realistic equity premium, bond demand has to fall sharply. With almost no borrowing, total borrowing costs naturally shrink as well.

Another way to judge whether these costs are reasonable is to look at their effect on welfare. The impact is very small, which suggests the costs we impose are not implausibly high. This result is consistent with earlier findings by Lucas (1987), Krusell and Smith (1999), and Hasanhodzic and Kotlikoff (2013, 2018, 2025), who show that realistically sized aggregate shocks don't have large microeconomic consequences. In our model, even when agents are unconstrained, they don't demand much in the way of safe assets. So, major changes in bond pricing shows up as small changes in utility.

To solve the model, we extend the algorithm developed in Hasanhodzic and Kotlikoff (2013, 2018, 2025) by incorporating borrowing costs. That algorithm, in turn, builds on the foundational work of Judd, Maliar, and Maliar (2009, 2011), who advanced a numerically stable and accurate solution method—extending the earlier approach of Marcet (1988)—to address the curse of dimensionality in large-scale dynamic models.

The paper proceeds as follows. Sections 2 and 3 present the model and its calibration.

Section 4 further develops the main findings. Section 5 concludes.[8]

# 2 The Model

Our baseline model features $G = 80$ overlapping generations with shocks to total factor productivity. Our main model adds capital depreciation shocks to the baseline. Each agent works through age $R = 45$, dies at age $D = 80$, and maximizes expected lifetime utility. There is an increasing cost of supplying bonds, i.e. of borrowing. Firms maximize static profits.

## 2.1 Endowments and Preferences

Workers supply 1 unit of labor each period. Assuming a hump-shaped age-earnings profile as in Hansen (1983) and Krueger and Kubler (2006), does not change the results. Hence, total labor supply equals $R$. Utility is time-separable and isoelastic, with risk aversion coefficient $\gamma$:

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}. \tag{1}$$

## 2.2 Financial Markets

Households invest in either risky capital or one-period safe bonds. Investing 1 unit of consumption in bonds at time $t$ yields $1 + \bar{r}_t$ units in period $t + 1$. The safe rate of return, $\bar{r}_t$, is indexed by $t$ since it is known at time $t$ although it is received at time $t + 1$. The total demand for assets of household age $g$ at time $t$ is denoted by $\theta_{g,t}$, and its share of assets invested in bonds is denoted by $\alpha_{g,t}$. Households enter period t with $\theta_{g-1,t-1}$ in assets, which corresponds to the total assets they demanded in the prior period. The aggregate supply of capital in period $t$, $K_t$, is the sum of assets period-$t$ agent asset holdings, i.e.

$$K_t = \sum_{g=1}^{G} \theta_{g,t-1}. \tag{2}$$

[8]For details of the solution algorithm, see the Computational Appendix of Hasanhodzic and Kotlikoff (2025). Numerical accuracy is high. Mean absolute Euler equation errors are on the order of $10^{-3}$–$10^{-4}$. Deviations from the portfolio choice first-order conditions, which theoretically equal zero, are at most on the order of $10^{-4}$ and, in most cases, vanish to the seventh decimal place. As an additional check, the Den Haan and Marcet (1990) moment conditions fall well within the acceptance interval across all model variants.

Bonds are in zero net supply, hence by being short (long) bonds, households are borrowing (lending) to each other.

## 2.3 Technology

Production is governed by

$$Y_t = z_t K_t^{\psi} L_t^{1-\psi}, \tag{3}$$

where $z$ is total factor productivity, $K_t$ is capital input, $\psi$ is capital's share of output, and $L_t$ is labor demand. The firm takes factor prices as given and chooses inputs ($K$ and $L$) to maximize profits:

$$\Pi_t = z_t K_t^{\psi} L_t^{1-\psi} - w_t L_t - r_t^{\text{gross}} K_t, \tag{4}$$

with $w_t$ denoting the real wage and $r_t^{\text{gross}}$ the gross rental rate of capital.

Profit maximizing implies:

$$w_t = z_t(1-\psi)\left(\frac{\sum_{g=1}^{G}\theta_{g,t-1}}{R}\right)^{\psi}, \tag{5}$$

$$r_t^{\text{gross}} = z_t\psi\left(\frac{\sum_{g=1}^{G}\theta_{g,t-1}}{R}\right)^{\psi-1}, \tag{6}$$

If depreciation $\delta_t$ applies to capital, the net return to capital (what owners earn) is given by

$$r_t = z_t\psi\left(\frac{\sum_{g=1}^{G}\theta_{g,t-1}}{R}\right)^{\psi-1} - \delta_t, \tag{7}$$

where depreciation $\delta_t \sim \mathcal{N}(\mu_\delta, \sigma_\delta^2)$, as in Ambler and Paquet (1994).

Total factor productivity, $z$, obeys

$$\ln(z_{t+1}) = \rho \ln(z_t) + \epsilon_{t+1}, \tag{8}$$

where $\epsilon_{t+1} \sim \mathcal{N}(0, \sigma^2)$.

## 2.4 Borrowing Costs

The model's borrowing cost function is that proposed by Chen and Mangasarian (1996). The function smoothly rises for negative bond holdings and is essentially zero when bond holdings are close to zero or positive (see Figure 4).

To borrow the amount of $\alpha\theta$, households must pay a cost $f(\alpha)\theta$, where

$$f(\alpha) = 0.2\left(-b\alpha - 1 + \frac{1}{5}\ln(1 + \mathrm{e}^{5b\alpha+5})\right) \tag{9}$$

and $b$ is the parameter described in Section 3 governing slope of $f$. Since $f$ is increasing in bond shares ($\alpha$), for a given level of assets ($\theta$) the marginal borrowing cost is increasing in the total amount borrowed ($\alpha\theta$).

Specifying the borrowing costs as $f(\alpha)\theta$ rather than $f(\alpha\theta)$ ensures that the model remains scalable.[9] This specification also makes economic sense. With $2\theta$ in assets and some $\alpha$, the marginal costs would be the same as with $\theta$ in assets and that same alpha, since the extra assets could be used as collateral. This is in line with Goodfriend (2005) and Goodfriend and McCallum (2007), where collateral is modeled as a valuable input in loan production because it enables a bank to enforce the repayment of loans with less monitoring (i.e., the greater is the borrower's collateral, the more productive is the intermediary's monitoring effort).

## 2.5 Social Security

Social Security benefits are financed by a wage tax, $\tau$, and provided to all retirees on a per-capita basis. Transfers to the elderly are denoted by $H_t$.

## 2.6 Household Problem

The economy's state is ($s$, $z$, $\delta$), with $s = (x_1,\ldots,x_{G-1})$ denoting the set of age-specific holdings of cash-on-hand.[10] Households of age $g$ in state $(s, z, \delta)$ maximize expected remaining lifetime utility given by

$$V_g(s, z, \delta) = \max_{c,\theta,\alpha}\left\{u(c) + \beta \mathrm{E}\left[V_{g+1}(s', z', \delta')\right]\right\} \tag{10}$$

[9] If all of "level" variables double, borrowing costs also double.

[10] Note that $x_G$, the cash on hand of the oldest generation is not included in the state vector. When the depreciation shock, $\delta$, is zero, the value of $x_G$ can be inferred from the other state variables. When $\delta$ is random, this is no longer the case. Now the initial value of $x_G$ (or equivalently the initial value of $\delta$) is needed to fully characterize the economy's initial-period state vector. But we still exclude $x_G$ from the state vector because it provides no addition information about the economy's future evolution. Also, we can directly calculate $x_G$ and, thus, the consumption of the old for periods beyond the first.

subject to

$$c_{1,t} = \ell_1(1-\tau_t)w_t - \theta_{1,t} + (1-\ell_1)H_t, \tag{11}$$

$$c_{g,t} = \ell_g(1-\tau_t)w_t + \left[\alpha_{g-1,t-1}(1+\bar{r}_{t-1}) + (1-\alpha_{g-1,t-1})(1+r_t)\right]\theta_{g-1,t-1} - \theta_{g,t} \tag{12}$$
$$- f(\alpha)\theta_{g-1,t-1} + (1-\ell_g)H_t,$$
$$\text{for } \ 1 < g < G, \text{ and}$$

$$c_{G,t} = \ell_G(1-\tau_t)w_t + \left[\alpha_{G-1,t-1}(1+\bar{r}_{t-1}) + (1-\alpha_{G-1,t-1})(1+r_t)\right]\theta_{G-1,t-1} \tag{13}$$
$$- f(\alpha)\theta_{G-1,t-1} + (1-\ell_G)H_t,$$

where $c_{g,t}$ is the consumption of a $g$-year old at time $t$, $\tau_t$ is the payroll tax financing government spending and transfers to the elderly, $H_t$ is the benefit given to the elderly, $\ell_i$ is labor supply of generation $i$, and (11)–(13) are budget constraints for age group 1, those between 1 and $G$, and that for age group $G$.[11]

## 2.7 Equilibrium

At time t, the economy's state is $(s_t, z_t)$, with $s_t = (x_{1,t},\ldots,x_{G-1,t})$ denoting the set of age-specific holdings of cash-on-hand. Given the initial state of the economy $s_0$, $z_0$, $\delta_0$, where $s_0 = (x_{1,-1},\ldots,x_{G-1,-1})$, the recursive competitive equilibrium is defined as follows:

**Definition.** The recursive competitive equilibrium is governed by the collection of the value functions and the household policy functions for total savings $\theta_g(s,z,\delta)$, the share of savings invested in bonds $\alpha_g(s,z,\delta)$, consumption $c_g(s,z,\delta)$ for each age group $g$, the choices for the representative firm $K(s,z,\delta)$ and $L(s,z,\delta)$, as well as the pricing functions $r(s,z,\delta)$, $w(s,z,\delta)$, and $\bar{r}(s,z,\delta)$ such that:

1. Given the pricing functions, the value functions (10) solve the recursive problem of the households subject to the budget constraints (11)–(13), and $\theta_g$, $\alpha_g$, and $c_g$ are the associated policy functions for all $g$ and for all dates and states.

2. Wages and rates of return on capital satisfy (5) and (7), i.e., at each point, for given $w$ and $r$ the firm maximizes profits.

3. All markets clear: Labor and capital market clearing conditions are implied by $L_t = R$

[11] Recall that $\ell_i = 1$ for $i \leq R$ and equals 0 otherwise. Also recall that labor market clearing condition requires that $\sum_{i=1}^{G} \ell_i = R$, where $R$ is the total labor supply.

and (2). Since bonds are in zero net supply, bond market clearing requires

$$\sum_{g=1}^{G} \alpha_g(s,z,\delta)\theta_g(s,z,\delta) = 0. \tag{14}$$

Market clearing conditions in labor, capital, and bond markets and satisfaction of household budgets imply market clearing in consumption.

4. The government balances its budget, i.e.,

$$\tau_t = \frac{H_t(G-R)}{w_t R}. \tag{15}$$

Finally, for all age groups $g = 1, \ldots, G-1$, optimal intertemporal consumption and investment choice satisfies

$$\begin{aligned} 1 = \beta \mathrm{E}_z \big[ \big(1 + r(s',z',\delta') + \alpha_g(s,z,\delta)(\bar{r}(s,z,\delta) - r(s',z',\delta')) \\ - f(\alpha_g(s,z,\delta))\big) \frac{u'(c_{g+1}(s',z',\delta'))}{u'(c_g(s,z,\delta))} \big] \end{aligned} \tag{16}$$

and

$$0 = \mathrm{E}_z \left[ u'(c_{g+1}(s',z',\delta'))(\bar{r}(s,z,\delta) - r(s',z',\delta') - f'(\alpha_g(s,z,\delta))) \right], \tag{17}$$

where $\mathrm{E}_z$ is the conditional expectation of $z'$ given $z$, and $f'(\alpha) = 0.2b\left(-1 + \frac{\mathrm{e}^{5b\alpha+5}}{1+\mathrm{e}^{5b\alpha+5}}\right)$ is the derivative of $f$ given by (9). Note that the endogenous part of the state next period, $s'$, is determined by the asset demands chosen the period before.

# 3 Calibration

The parameters are calibrated as follows.

## 3.1 Endowments and Preferences

The risk aversion parameter $\gamma$ is set to 2. Agents work for 45 periods and live for 80. Each period represents a year. The quarterly subjective discount factor, $\beta$, is set at 0.99, which is standard in the macroeconomics literature.

## 3.2 Technology

**Baseline Calibration**

Hansen (1985) estimates the quarterly autocorrelation coefficient of the TFP process at $\rho = 0.95$, with the standard deviation of the innovation, $\sigma_\varepsilon$, ranging from 0.007 to 0.01. Prescott's (1986) estimates are 0.9 for $\rho$ and 0.00763 for $\sigma$. Our assumed quarterly values for $\rho$ and $\sigma$ are 0.95 and 0.01, respectively. On an annual basis they are 0.814 and 0.019, respectively, generating a mean TFP value of 0.997 with a standard deviation of 0.033 and a coefficient of variation of 3.29 percent. The resulting correlation coefficient between the wage and the rate of return on capital is 0.544.

**Baseline Calibration with Depreciation Shocks**

The baseline calibration with depreciation shocks features both TFP and depreciation shocks. The TFP process is our baseline's. The quarterly value of the standard deviation of the depreciation shock, $\sigma_\delta$, is set at 0.011, implying an annual value of 0.045.[12] This value is chosen to reproduce the standard deviation of the return to national wealth,

$$r_t = \frac{W_{t+1} - W_t + C_t - E_t}{W_t}, \tag{18}$$

where $W_t$, $E_t$, $C_t$, and $r_t$ stand for time-t wealth, labor income, economy-wide consumption, and the rate of return on economy-wide wealth.[13]

Under this calibration, the wage has a standard deviation of 0.107 around a mean of 1.905, for a coefficient of variation of 5.62 percent. The correlation coefficient between the wage and the rate of return on capital is -5.46 percent, well within Davis and Willen's (2000) empirical estimates.

## 3.3 Borrowing Costs

In our main calibration, which excludes Social Security and incorporates both aggregate total factor productivity (TFP) shocks and stochastic depreciation shocks, the borrowing-cost function is parameterized such that the ratio of the marginal borrowing rate to the risk-free interest rate equals 14. This calibration captures the typical 10 to 20 percent

[12] We treat $Y$ (equation 3) as the net production function and set the mean value of depreciation at zero. Also note that our value for $\sigma_\delta$ is far higher than the 0.0052 quarterly estimate of Ambler and Paquet (1994).

[13] We measure total labor income assuming labor's share of proprietorship and partnership income is the same as its share of national income. Note, some components of the Federal Reserve wealth series are carried at book, which biases downward our estimate of the variability of the national wealth return.

spread of personal, unsecured credit over the risk-free rate.[14]

Table 2 presents the full parametrization of the borrowing cost function across all model calibrations.[15]

**Borrowing Cost Function Parameters**

| Borrowing Costs | rb | b (slope) | cm |
|---|---|---|---|
| **Baseline calibration (no depreciation shocks)** | | | |
| Baseline | 0.2 | 28 | 5 |
| Steeper | 0.2 | 45 | 5 |
| **Baseline calibration with depreciation shocks** | | | |
| Baseline | 0.2 | 25 | 5 |
| Steeper | 0.2 | 45 | 5 |
| Flatter | 0.2 | 15 | 5 |
| Flatest | 0.2 | 5 | 5 |

Table 2: Parametrization of the borrowing cost function under different calibrations.

### 3.4 Social Security

We set the payroll tax rate, $\tau$, at 15 percent, close to the combined employer plus employee 15.3 percent FICA tax. Annual benefits equal annual payroll tax revenues with all retirees receiving the same payment. This form of pay-go policy transfers wage shocks from young workers to the elderly.

## 4 Results

Turning to the results, we proceed in three steps. First, for both the baseline calibration and the version with depreciation shocks—each evaluated under three alternative borrowing-cost

[14] Adding Social Security to the model provides a consumption floor in old age, thereby reducing precautionary savings by the elderly. This spells, in turn, a higher equilibrium risk-free rate. Since the borrowing-cost function remains invariant to this policy change, the marginal borrowing rate is largely unaffected in absolute terms. Consequently, the ratio of the marginal borrowing cost to the risk-free rate declines to 2.6. In contrast, when we introduce subjective mortality beliefs into the baseline economy (without Social Security), households increase their demand for safe assets as a hedge against longevity risk. This additional demand raises bond prices and lowers the risk-free rate, while leaving the marginal borrowing cost unchanged. As a result, the borrowing cost-to-risk-free rate ratio rises substantially to 26.5. Notably, despite the large fluctuations in this ratio across policy environments, all values remain within empirically plausible bounds, particularly given the high spreads observed in real-world unsecured credit markets, especially in low interest rate environments.

[15] Figure 4 illustrates the importance of the shape of the borrowing cost function for a fixed level of assets under three different slope parameterizations – steeper, baseline, and flatter – corresponding to varying values of the parameter $b$.

specifications—we compare model-implied asset returns and aggregate variables with their empirical counterparts. Second, we contrast stochastic transition paths of the economy with their deterministic analogues. Third, we examine how shocks and Social Security shape the age profile of bond and stock demands.

## 4.1 Summary Statistics

**Summary Statistics of Rates of Return**

| | Mean Return to Capital (%) | Mean Safe Rate of Return (%) | S.D. Return to Capital (%) | S.D. Safe Rate of Return (%) | Equity Premium (%) | Sharpe Ratio |
|---|---|---|---|---|---|---|
| **BASELINE CALIBRATION** | | | | | | |
| **No Borrowing Costs** | | | | | | |
| No Social Security | 3.83 | 3.83 | 0.13 | 0.10 | 0.00 | 0.01 |
| Social Security | 4.80 | 4.80 | 0.17 | 0.14 | 0.00 | 0.01 |
| **Borrowing Costs** | | | | | | |
| No Social Security | 3.84 | 0.09 | 0.14 | 0.11 | 3.75 | 27.72 |
| Social Security | 4.80 | 1.05 | 0.17 | 0.15 | 3.75 | 21.59 |
| **Steeper Borrowing Costs** | | | | | | |
| No Social Security | 3.87 | -2.16 | 0.14 | 0.11 | 6.02 | 44.20 |
| Social Security | 4.79 | -1.24 | 0.17 | 0.14 | 6.03 | 34.80 |
| **BASELINE CALIBRATION WITH DEPRECIATION SHOCKS** | | | | | | |
| **No Borrowing Costs** | | | | | | |
| No Social Security | 3.91 | 3.74 | 4.63 | 0.41 | 0.17 | 0.04 |
| Social Security | 4.91 | 4.75 | 4.64 | 0.5 | 0.15 | 0.03 |
| **Borrowing Costs** | | | | | | |
| No Social Security | 3.89 | 0.24 | 4.62 | 0.36 | 3.65 | 0.79 |
| Social Security | 4.90 | 1.32 | 4.63 | 0.44 | 3.59 | 0.77 |
| **Steeper Borrowing Costs** | | | | | | |
| No Social Security | 3.91 | -2.42 | 4.62 | 0.36 | 6.32 | 1.37 |
| Social Security | 4.89 | -1.38 | 4.63 | 0.44 | 6.26 | 1.35 |
| **DATA BASED ON RETURN TO U.S. WEALTH** | | | | | | |
| | 6.512 | 1.083 | 4.886 | 2.195 | 5.429 | 1.111 |

Table 3: Summary statistics of rates of return in the model and in the data.

Table 3 reports the return to capital – measured as the real return to national wealth – and the safe rate – measured as the real return on Treasuries – for the years 1947 through 2015.

Over this period, the average risk premium equaled 5.43 percent. To construct annual real returns to national wealth, we combine National Financial Accounts data from the Federal Reserve with National Income Accounts data from the Bureau of Economic Analysis.

In the model without borrowing costs, the risk premium vanishes under the baseline calibration and rises only slightly, to 0.17 percent, once depreciation shocks are introduced. Adding borrowing costs, however, transforms the picture. Depending on the specification of the borrowing-cost function, the premium ranges from 3.59 to 6.32 percent, compared with 5.43 percent in the data. The baseline with depreciation shocks further matches the variability of the real return to U.S. wealth: the standard deviation of the return to capital lies between 4.62 and 4.64 percent in the model, against 4.89 percent in the data. As a result, the implied Sharpe ratios – between 0.77 and 1.37 – align closely with the empirical value of 1.11.

Without depreciation shocks, the baseline calibration understates the standard deviation of the returns to national wealth by an order of magnitude – 0.13 to 0.17 percent in the model vs. 4.89 percent in the data. However, as Table 4 shows, the variability of consumption in the model under the baseline calibration closely matches the data – 2.47 to 2.67 percent in the model vs. 3.11 percent in the data. Even when depreciation shocks are added, the model's consumption variability stays reasonably close to the data, now ranging from 4.41 to 6.06 percent. Similar observations can be made for output variability, which ranges from 3.69 to 3.82 percent in the baseline calibration and between 5.61 and 6.06 percent with depreciation shocks, compared to 3.33 percent in the data.

For the variability of investment, the discrepancy between the model and the data is more pronounced. Specifically, standard deviation of investment ranges from 2.41 to 6.00 percent in the model compared to 15.93 percent in the data. This discrepancy arises because investment in the model is smoothed by heterogeneous agents' savings decisions and lacks amplification mechanisms such as capital adjustment costs or firm-level investment shocks. Additionally, the depreciation shocks considered are i.i.d. and insufficiently persistent to generate large capital fluctuations.

Nevertheless, correlation of output with consumption matches the data pretty well in the absence of depreciation shocks. Specifically, the correlation of output with consumption ranges from -0.80 to -0.83 in the model compared to -0.79 in the data. Such negative correlation makes sense. It is due to life-cycle heterogeneity in consumption responses. When output falls, younger, working-age agents experience income losses and reduce consumption sharply. Although older, asset-holding agents dissave to partially smooth their consumption, their response is more muted. On net, aggregate consumption declines during recessions, resulting in a countercyclical consumption pattern and a negative correlation with output.

**Standard Deviation of Percent Deviation from Trend and Correlations With Output, Model vs. Data**

| | SD (%) | | | Correlation of Output | |
|---|---|---|---|---|---|
| | Output | Cons | Inv | Cons | Inv |
| **BASELINE CALIBRATION** | | | | | |
| **No Borrowing Costs** | | | | | |
| No Social Security | 3.69 | 2.53 | 2.53 | -0.83 | 0.83 |
| Social Security | 3.79 | 2.41 | 2.41 | -0.80 | 0.80 |
| **Borrowing Costs** | | | | | |
| No Social Security | 3.73 | 2.67 | 2.68 | -0.82 | 0.82 |
| Social Security | 3.82 | 2.47 | 2.48 | -0.80 | 0.80 |
| **Steeper Borrowing Costs** | | | | | |
| No Social Security | 3.74 | 2.66 | 2.67 | -0.82 | 0.82 |
| Social Security | 3.82 | 2.48 | 2.48 | -0.80 | 0.80 |
| **BASELINE CALIBRATION WITH DEPRECIATION SHOCKS** | | | | | |
| **No Borrowing Costs** | | | | | |
| No Social Security | 6.06 | 5.05 | 5.00 | 0.27 | -0.27 |
| Social Security | 6.01 | 4.41 | 4.38 | 0.19 | -0.19 |
| **Borrowing Costs** | | | | | |
| No Social Security | 5.62 | 5.85 | 5.80 | 0.30 | -0.30 |
| Social Security | 5.61 | 4.95 | 4.92 | 0.23 | -0.23 |
| **Steeper Borrowing Costs** | | | | | |
| No Social Security | 5.62 | 5.84 | 5.79 | 0.30 | -0.30 |
| Social Security | 5.61 | 4.96 | 4.93 | 0.23 | -0.23 |
| **DATA** | | | | | |
| Data | 3.33 | 3.11 | 15.93 | -0.79 | 0.68 |

Table 4: Standard deviation of percent deviations from trend of U.S. real NNP and real personal consumption expenditures, 1929–2020, and standard deviation of percent deviations of output and aggregate consumption from their means in the model.

On the other hand, the baseline model's correlation between output and investment overstates the data slightly, ranging from 0.80 to 0.83, compared to 0.68 in the data. The relatively high correlation generated by the model is consistent with standard RBC dynamics, where investment is highly sensitive to productivity-driven fluctuations in the marginal product of capital. When TFP rises, the return to capital increases, incentivizing agents to reallocate resources toward investment, leading to a tightly linked, pro-cyclical relationship between output and investment. In the life-cycle setting, although agents differ in age and asset holdings, the aggregate investment response remains strongly procyclical because younger agents save more when productivity improves, and older agents' asset decumulation is generally not large enough to offset this effect. As a result, investment tracks output more closely in the model than in the data, where additional frictions, such as financial constraints, investment adjustment costs, or expectations-driven delays may dampen the responsiveness of investment to output.

Adding depreciation shocks flips the sign of correlation coefficients in the model relative to the data.[16] Specifically, the correlation between output and consumption flips from negative to positive, ranging from 0.19 to 0.32 in the model (in contrast to -0.79 in the data). This shift arises because depreciation shocks alter the capital stock directly and contemporaneously, creating wealth effects that drive consumption and output in the same direction. A positive depreciation shock increases capital erosion, reducing the capital stock and lowering output. In response to this negative wealth shock, agents – including older cohorts with limited time horizons — reduce consumption. Conversely, a negative depreciation shock slows capital erosion, boosts the capital stock and output, and leads agents to consume more. Because these shocks affect both output and consumption contemporaneously and in the same direction, the correlation between the two variables becomes positive.

When stochastic depreciation shocks are added to the model, the correlation between output and investment also flips sign, mirroring the behavior observed for consumption. Strikingly, for every calibration, the investment-output correlation becomes approximately the negative of the consumption-output correlation – for example, when the consumption-output correlation is 0.30, the investment-output correlation is -0.30. This symmetry reflects the fact that, in response to depreciation shocks, agents reallocate resources between consumption and investment in a way that keeps aggregate demand roughly stable. A positive depreciation shock reduces the effective capital stock, lowering output and tightening the intertemporal budget constraint. Agents, particularly those with short horizons, prioritize maintaining consumption, which leads to a decline in investment. Similarly, when depre-

[16]Note that depreciation shocks also induce a negative correlation between the wage and the rate of return on capital.

ciation is unexpectedly low, output and perceived wealth rise, prompting agents to both consume more and invest less in precautionary saving. As a result, consumption and output move together, while investment moves in the opposite direction. This stands in contrast to the data, where investment and output are strongly positively correlated (0.68), highlighting the degree to which depreciation shocks distort the co-movement structure of the model. These findings underscore how shifting the source of aggregate fluctuations from productivity to capital erosion alters how agents respond, and drives a breakdown in the model's ability to replicate empirical macro correlations.[17]

Table 3 also shows that our models' mean return to capital ranges from 3.72 percent to 4.90 percent, depending, crucially on the presence of Social Security. This falls short of the 6.51 percent mean return to U.S. wealth. But we could easily introduce additional government policies, such as government consumption financed by an income tax, that would crowd out capital and raise capital's return to match that of the data.[18]

## 4.2 Fluctuations

Figure 5 plots the evolution over 750 years of the capital stock, output, the wage, and the real return to capital for the baseline model without and with stochastic depreciation. Social Security and borrowing costs are in place in each case. Total factor productivity, $z$, has, to repeat, a mean of 1.0, a standard deviation of 0.033, and an annual autocorrelation coefficient of 0.814, all of which are standard values in the literature. In the baseline model, the standard deviations of capital, output, the wage, and the real return are 31.84, 4.46, 0.07, and 0.17 percent, respectively. The corresponding means are 801.89, 116.63, 1.74, and 4.80 percent.[19] Thus, the coefficients of variations of the economy's key macroeconomic variables are all small. Indeed, all four equal 0.04. This accords with Lucas's (1978) finding of very small macroeconomic risk albeit in a single-agent exchange economy.

Including stochastic depreciation, with mean zero and standard deviation of 0.046, lowers the capital stock's long-run average value to 769.03 and increases its standard deviation somewhat to 97.45. Long-run average output and the wage decrease to 114.29 and 1.70,

[17]Following Prescott (1986), we detrend real net national product and real personal consumption expenditures for the years 1929 to 2020 and form the standard deviation of the percent deviations from trend. Data on real net domestic investment is from 1947 to 2000 and we divide it by real net national product before detrending and forming standard deviation of deviations from trend. Our model abstracts from growth, so we simply form the standard deviation of our model's percentage deviation of annual output or consumption from their means.

[18]We intentionally abstract from other sources of crowding out in order to isolate the specific contribution of borrowing costs to the spread between the return to capital and the risk-free rate.

[19]The first 350 observations are excluded from these statistics to focus on long-run values.

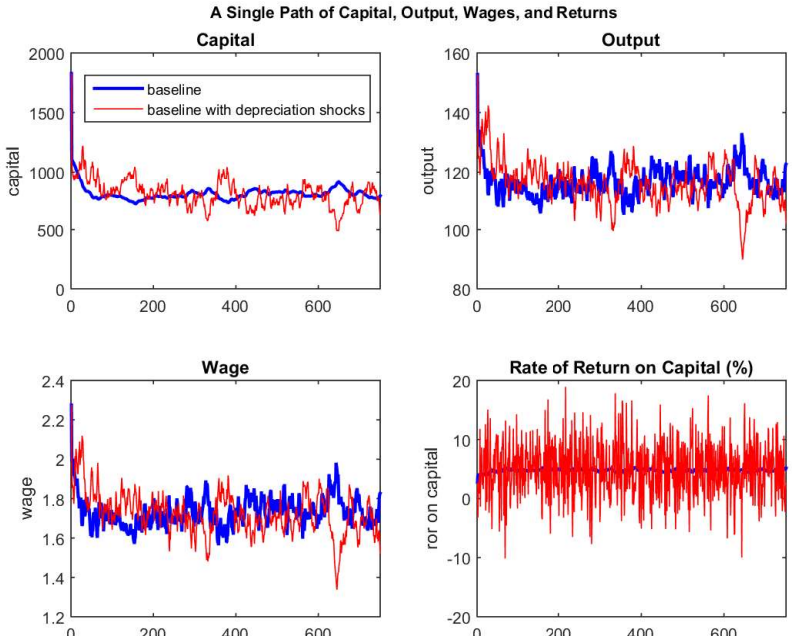



Figure 5: A single path of capital, output, wage, and return under the baseline calibration with and without depreciation shocks, with Social Security and borrowing costs.

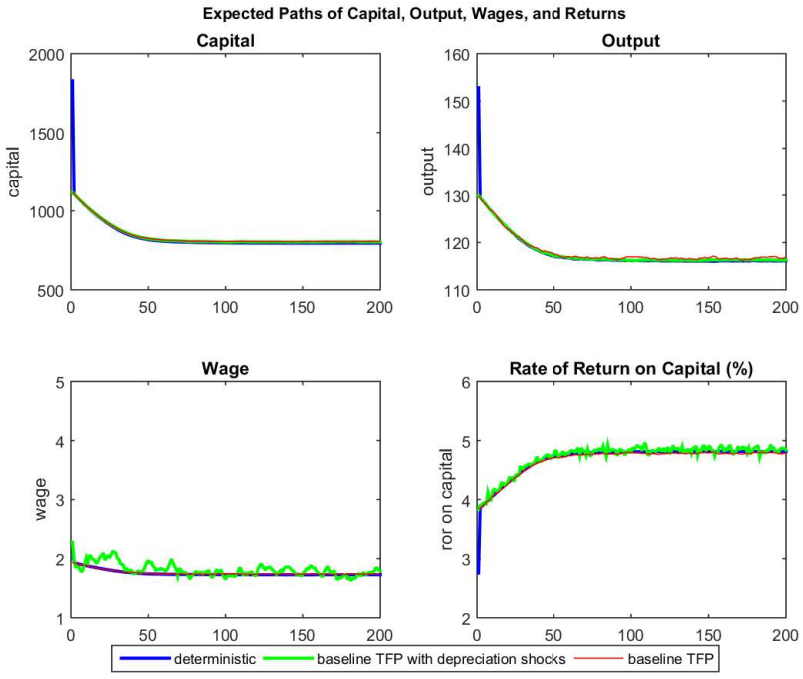



Figure 6: Expected path of capital, output, wage, and return in deterministic model and under the baseline calibration with and without depreciation shocks, with Social Security and borrowing costs. Expectations are computed over 10,000 simulated paths.

respectively. The rate of return on capital is directly hit by depreciation shocks, hence its standard deviation jumps dramatically – to 4.63 percent around a mean of 4.90 percent.

Figure 6 presents the expected paths of capital, output, wages, and returns, where expectations are taken over 10,000 simulated histories (each with independently drawn shock paths). For comparison, the figure also reports the corresponding deterministic transition paths. The results show that the stochastic paths closely track their deterministic counterparts, reinforcing our conclusion that aggregate macroeconomic risk, from the perspective of agents, is quantitatively minor.

## 4.3 Bond and Stock Demands

Recall again Figures 1 and 2, first presented in Section 1, which plot average bond demands by age without and with depreciation shocks, and without and with Social Security. In Figure 1, which abstracts from depreciation shocks, the life-cycle pattern is inverted. Younger households are net demanders of bonds, while older households supply them. This outcome reflects the absence of depreciation risk. When assets are not subject to stochastic depreciation, capital accumulation is relatively safe (principle cannot be lost), and older cohorts have little incentive to hold risk-free bonds for precautionary purposes. Instead, they act as suppliers of bonds, effectively providing safe assets to younger households.

By contrast, younger cohorts, who face uncertain future labor income and long horizons over which to smooth consumption, actively demand bonds. For them, bonds serve as a vehicle for intertemporal consumption smoothing in the early stages of the life cycle, before labor income peaks. In equilibrium, this configuration leads to positive bond demand among the young, matched by bond supply from older cohorts.

When depreciation shocks are introduced (Figure 2), the pattern reverses. Young households now supply bonds, not demand them. The reason is diversification: they hold both labor and capital income, and these two streams are imperfectly correlated. This gives them a hedge that allows them to assume risk and supply bonds. Older households, by contrast, rely heavily on their accumulated capital. When depreciation shocks hit, their principal is at risk of being eroded. To protect against this, they demand bonds as a safe asset. In equilibrium, the roles flip: the young become the natural suppliers of bonds, while the old, seeking protection from capital risk, become the demanders.

Across all panels, Social Security compresses the life-cycle profile of bond holdings. Without it, the bond market must carry the full load of intergenerational borrowing/lending and risk sharing. With it, a portion of these roles is absorbed by transfers. The effect of Social Security is therefore to flatten and reduce the amplitude of age-specific bond positions,

regardless of whether depreciation shocks or borrowing constraints are present.[20] In other words, in equilibrium, Social Security lowers demand for safe assets, as transfers substitute for private saving. As a results, bond demand falls, bond prices drop, the risk-free rate rises, and the equity premium narrows.

Comparing the top and bottom panels of each figure highlights the effect of borrowing costs. First, borrowing costs compress the overall scale of bond trading. Second, they reshape the age profile of holdings. Specifically, as Figure 2 shows, with costs (bottom-left), the youngest supply almost nothing, since borrowing is prohibitively expensive. The middle-aged provide most of the safe asset supply to the elderly demanders.[21]

Comparing the top-left and bottom-left panels of Figure 2 highlights how borrowing costs reshape stock demand by age. Without costs (top), the young can borrow freely and use leverage to hold substantial stock positions. Their labor income provides diversification, making them natural holders of risky capital. With costs (bottom), this mechanism is shut down; the young can no longer borrow to invest in equity, and their stock demand falls sharply. The middle-aged and older cohorts still accumulate stock out of savings, but overall holdings are much smaller. This mirrors the bond-demand patterns on the left-hand-side of the figure, where borrowing costs also compressed market scale and shifted the margin away from the young.[22]

# 5 Conclusion

Simulating a sizable equity premium in standard macroeconomic models has proved difficult, hence the appellation "equity premium puzzle". The Constantinides, Donaldson, and Mehra (2002) resolution – eliminating borrowing (bond supply) by the young – produces a realistic premium. But their model is highly stylized, featuring pure exchange and just three generations. This paper examines CDM's answer within a large-scale stochastic OLG model (Hasanhodzic and Kotlikoff, 2025) with capital, fiscal policy, and realistic macro-shocks. Our US-calibrated model features 80 overlapping generations with isoelastic preferences.

[20] Interestingly, in the bottom-left panel of Figure 1, when Social Security is in place, the young no longer need to hold as many safe assets, as they do without it. Supply from the middle-aged and the younger old now exceeds demand, so the very old flip from suppliers to buyers.

[21] Steeper borrowing costs leave the age profile of bond positions unchanged but compress their scale.

[22] Figure 1 tells the same story. The top-left panel of that figure shows the lifecycle pattern of bond positions, while the top-right depicts the corresponding pattern of stock holdings. Taken together, they reflect two sides of the same trade. The young enter with little wealth and a strong desire for safety. They demand bonds, using the proceeds to short stocks. In effect, they are engineering portfolios that tilt away from risk and toward security, despite their limited resources. This is the mirror image of their stock profile: small or negative equity positions matched by positive bond demand.

The model's TFP and depreciation shocks are calibrated to replicate the economy's fluctuations. Fiscal policy comprises pay-go Social Security. As for computation, we adopt Marcet's (1988) projection algorithm to avoid the curse of dimensionality.[23]

In our model, as in CDM, restricting borrowing raises the equity premium puzzle. But full puzzle resolution, i.e., producing a realistic premium, requires closing down the bond market entirely, i.e., preventing not just the young, but *all* generations from borrowing. Permitting even one of our 80 generations, regardless of age, to borrow (supply bonds), reduces the equity premium far below that observed. Thus, "No One Can Borrow" rather than "Junior Can't Borrow" is the natural generalization of CDM.

But shutting down borrowing entirely, which we operationalize via a rapidly rising borrowing-cost function, renders CDM's resolution implausible. Yes, many households face steeply rising borrowing costs depending on the nature of their borrowing. Both others can and do borrow at low transactions costs. Yet others face substantial borrowing costs, but costs that are largely fixed, not sharply rising with the amount borrowed. Examples here range from mortgaging a home purchase to borrowing from one's parents. Analyzing CDM's proposition in a large-scale stochastic OLG model required positing rapidly increasing marginal borrowing costs. But assuming they are universal for modeling purposes and pretending they are universal in the real world are two different things. CDM's resolution of the equity premium puzzle carries over to a realistic OLG framework, but only if no one can borrow. But this assumption appears a bridge far too far.

Thus, the puzzle remains. Indeed, our model extends it to the realm of large-scale OLG-RBC models. As in representative-agent models, macro risk is small in OLG-RBC settings. It is not small because TFP or depreciation shocks are small. Our model's macro shocks match or exceed those recorded in the data. Macro risks are small because these shocks can be well hedged by self-insuring – saving against bad times – and by playing the averages – relying on bad times not lasting forever. Consequently, absent borrowing costs, agents, no matter their age, have highly elastic net bond demand (supply) curves. Thus, they are willing to hold bonds rather than equities but only if their sure return is close to the expected return on equities. Equivalently, they are willing to hold stock rather than bonds, i.e., they are willing to supply bonds, but only if the equity premium is close to zero.

Our OLG-RBC model not only extends the equity premium puzzle, it delivers an important policy message. Macro shocks are not large enough to materially alter the economy's behavior and transition path. Indeed, the average transition path of macroeconomic variables is essentially coincident with their deterministic counterparts. Thus, policies that undermine

[23]Judd, Maliar, and Maliar (2009, 2011) expand on its use.

the well-being of future generations to the benefit of current generations in a deterministic setting will produce, on average, the same generation-specific gains and losses. Indeed, for purposes of assessing the expected generational impact of policy changes, one can simply rely on deterministic modeling.

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