Energy Markets and Macroeconomic Uncertainty: Quantifying the Sentiment Risk Channel

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Abstract

This paper investigates how shocks to investor sentiment about future macroeconomic prospects affect energy index returns. We develop an asset pricing model for the energy sector using Schroder and Skiadas (1999) continuous-time stochastic recursive preferences, which explicitly incorporate investors' attitudes toward long-run growth uncertainty. Our empirical strategy combines a latent factor method—applied to a comprehensive set of macroeconomic and sentiment indicators—with a democratic orthogonalization technique to isolate the distinct role of sentiment risk. We find that energy investors are significantly affected by uncertainty shocks to sentiments about the economic future. On average, sentiment risk accounts for 49.7% of the variation in energy index returns, a magnitude that underscores its critical role as a driver of energy market fluctuations. These results highlight the importance of integrating investor expectations and long-run risk attitudes into macroeconomic analyses of energy markets, with direct implications for understanding financial stability and investment behavior during periods of economic uncertainty.

Keywords: Energy Markets, Macroeconomic Uncertainty, Sentiment Risk, Recursive Utility, Asset Pricing, Energy Transition

JEL Classification: G12, Q43, E32, D81, C58

1 Introduction

The interaction between energy markets and the macroeconomy is profoundly shaped by expectations. Sentiment and attitudes toward long-run uncertainty are crucial indicators of how optimistic or pessimistic economic agents feel about future growth, investment, and consumption prospects. The importance of such expectations in dynamic economic analysis dates back at least to Pigou (1927) and continues to spark the interest of market participants, business managers, and policymakers (Barro, 1994; Biolsi and Du, 2020; Benhabib and Spiegel, 2018). Understanding these expectations is critical for analyzing how energy markets respond to–and propagate–broader macroeconomic shocks, a core focus of this conference.

A growing body of research underscores the significant role of investor sentiment in influencing energy markets. For instance, Qadan and Nama (2018) demonstrated the substantial impact of investor sentiment on oil prices, illustrating how fluctuations in sentiment indices can lead to significant changes in oil price dynamics. Utilizing sentiment indices developed by Baker and Wurgler (2006, 2007), Du and Zhao (2017) conducted a quasi-experimental study, revealing that investor sentiment had both statistically and economically significant effects on oil prices between 2003 and 2008. He and Zhou (2018) further examined the relationship between Baker and Wurgler investor sentiment and oil-specific demand shocks, highlighting the broader influence of investor sentiment on the energy sector. Assaf et al. (2021) focused on the dynamic interactions between uncertainty and energy markets, with a specific emphasis on the role of investor sentiments, using data from the University of Michigan's Consumer Sentiment Index. Li et al. (2021) employed news sentiment scores derived from crude oil news headlines using Natural Language Processing (NLP) techniques to analyze the impact of news sentiment on oil futures returns and volatility. Yang et al. (2019) integrated investor sentiment and leverage effects into predictive models to forecast crude oil futures volatility, utilizing sentiment indicators derived from news headlines. Lastly, Jawadi et al. (2024) analyzed the impact of investor sentiment on energy price volatility using high-frequency data and nonlinear models. These studies collectively enhance our understanding of how various sentiment indices influence energy markets.

However, a critical gap remains. While these studies establish a link between sentiment and price levels or volatility, they largely overlook the role of investors' fundamental *risk attitudes* related to long-run uncertainty. This is a crucial omission because the energy sector is inherently defined by investments with long time horizons and exposure to persistent macroeconomic risks. The transition to low-carbon energy sources, geopolitical shifts, and technological innovations all create profound uncertainty about the economy's long-term growth path. How investors' attitudes toward this long-run uncertainty are priced into energy assets remains under-explored.

This paper addresses this gap by analyzing how risk attitudes toward long-run macroeconomic prospects shape energy index returns in capital markets. We develop a pricing model grounded

in the recursive utility framework of Schroder and Skiadas (1999), which provides the theoretical foundation for incorporating long-run risks. The forward-looking feature of recursive utility allows sentiment to matter endogenously in consumer and investor decisions (Hansen, 2010, 2012; Cochrane, 2005; Sargent, 2007). In this context, sentiment about the future refers to changes in expectations regarding long-term growth (Kakeu and Byron, 2016; Hansen, 2010, 2012). Positive sentiment indicates an increase in expectations about future prospects, while negative sentiment reflects a decrease. Recursive utility provides a flexible approach to capturing how agents value the temporal resolution of uncertainty, with direct implications for their attitudes toward long-run risks (Epstein et al., 2014; Kreps and Porteus, 1978; Hansen et al., 2008; Ericson and Fuster, 2011; Kaltenbrunner and Lochstoer, 2010; Sargent, 2007; Strzalecki, 2013; Swanson, 2012).[1]

From an empirical perspective, our analysis integrates a latent factor method with a democratic orthogonalization technique to investigate an energy index risk-pricing model. This model identifies short-run consumption risk and, crucially, long-run sentiment risk as key drivers of energy index returns. By employing the democratic orthogonalization method developed by Löwdin (1970), we decompose the energy index returns into orthogonal components attributable to each risk type.[2] This method allows us to pinpoint the distinct contribution of each component, offering a clear understanding of each variable's role and quantifying how much of the fluctuation in energy asset values is driven by changing expectations about the macroeconomic future.

This paper makes several contributions to the literature at the intersection of energy economics and macrofinance. First, unlike previous research that primarily examined the connection between sentiment and energy prices, this study employs a recursive utility asset pricing model to analyze the dynamic relationship between priced sentiment risk and energy index returns, an area not previously explored. Second, by using data from the University of Michigan's Survey of Consumers, our model directly encapsulates long-term uncertainties about future economic growth prospects, thereby elucidating how risk attitudes toward the economy's long-term future impact energy in-

[1] A decision maker may prefer a late or early resolution of uncertainty based on their attitudes towards the correlation of payoffs across periods - long-run uncertainty (Duffie and Epstein, 1992b).

[2] Initially used in Quantum Chemistry, this statistical technique has been applied to empirically analyze asset pricing and macroeconomics models (Klein and Chow, 2013; Bessler et al., 2015; Bouaddi and Kakeu, 2023)

vestments. This has direct implications for understanding the macroeconomic consequences of the energy transition and other long-horizon shifts. Third, our empirical strategy employs both the latent factor method and the democratic orthogonalization method. The latent factor analysis technique allows a large panel of indicators on consumer future expectations to be summarized by a small number of estimated factors, an approach increasingly used in finance and macroeconomics (Chen et al., 2013; Bouaddi and Taamouti, 2013; Kakeu and Bouaddi, 2017; Thimme, 2017; Bernanke et al., 2005; Bai and Ng, 2002; Boivin and Giannoni, 2006; Forni et al., 2009; Bernanke and Boivin, 2003; Favero et al., 2005; Bai and Ng, 2002; Bouaddi and Taamouti, 2012; Ludvigson and Ng, 2009; Bouaddi and Kakeu, 2023).

To approximate the value function endogenously, we substitute it with a log-linear function of sentiment latent factors, consistent with Cochrane (2017, page 955), who recommends replacing the utility index with a function of quantifiable state variables. In estimating the short-run risk factor associated with consumption, we employ the Engle (2002) Dynamic Conditional Correlation model, similar to the method used by Bali and Engle (2010). The democratic orthogonalization method then allows us to determine the relative importance of sentiment and consumption risk factors.

The remainder of this paper is structured as follows. Section 2 presents the energy index pricing model that features sentiment risk factors. Section 3 describes the econometric methodology incorporating latent factor analysis. Section 4 quantifies the contributions of short-run consumption risks and sentiment risks to energy index returns and presents tests comparing the recursive utility model to a standard time-additive utility model. The last section offers concluding comments, with detailed mathematical tools provided in the Appendix.

2 Sentiments and the Investor's Pricing of the Energy Sector Index with Recursive Utility

This section develops an asset pricing framework that explicitly links energy market returns to investors' expectations about long-term macroeconomic prospects. In energy marketsâ€"characterized by long investment horizons, technological uncertainty, and exposure to climate and energy tran-

sition policiesâ€"understanding how investors price *long-run risk* is particularly crucial. Our empirical approach leverages the recursive utility asset pricing model by Duffie and Epstein (1992b), which provides the theoretical foundation for incorporating expectations about future economic growth into current asset valuations.

The recursive utility framework meticulously accounts for expectations about future prospects through adaptations in the value function, as elaborated by Hansen (2010, 2012); Cochrane (2005); Kakeu and Byron (2016); Sargent (2007). This approach is especially relevant for energy assets, whose values depend substantially on expectations about future energy demand, technological disruptions, and climate policies that may unfold over decades. The model characterizes the energy investor with stochastic recursive preferences in the style of Duffie and Epstein (1992b).

The energy investor is assumed to have preferences represented by a recursive functional Ã la Duffie and Epstein (1992b). Denoted by $J(t)$ the future utility index over consumption processes $\{c(s) : s > t\}$. Preferences are generated recursively by the following integral equation:

$$J(t) = E_t \left[\int_t^{\infty} f\Big(c(s), J(s)\Big) ds \right], \tag{1}$$

where E_t denotes the conditional expectation given the information available at time t.

The instantaneous utility $f(c,J)$ is assumed to be increasing and concave in consumption, and to satisfy certain regularity-type conditions. In addition, J must satisfy a transversality condition of the form

$$\lim_{t\to\infty} e^{-\nu t} E\left(|J(t)|\right) = 0, \tag{2}$$

for a suitable constant ν (Duffie and Epstein, 1992b, p.388).

These preferences culminate in an "aggregator" function for current utility at each moment t, denoted as $f(c(t),J(t))$. This function intricately melds current consumption, $c(t)$, with an index of future utility, $J(t)$. The index $J(t)$ further embodies prospective utility, echoing the sentiments of Koopmans (1960), which articulate expectations about future consumption prospects. Crucially, by weaving together current consumption and expectations regarding future consumption prospects,

recursive utility underlines the profound role of sentiment about future consumption in the consumer's decision-making process. This sentiment-encompassing both optimism and apprehension about future economic conditions-is dynamically molded by uncertainty shocks to the economy as well as shifts in the economic landscape, as underscored in Hansen (2010, 2012); Cochrane (2005); Kakeu and Byron (2016); Sargent (2007). For energy investors, these expectations are particularly influenced by factors such as the pace of the energy transition, geopolitical stability in energy-producing regions, and technological breakthroughs in renewable energy.

The energy investor's decision problem at time t is then formalized as follows:

$$J(W(t)) = \max_{\{c(s),\,(1-\omega(s))\}} E_t \left[\int_t^{\infty} f(c(s), J(s))\, ds \right], \tag{3}$$

subject to the wealth constraint:

$$dW(s) = -c(s)\, ds + W(s) \left[\omega(s) r(s) + (1-\omega(s)) \frac{d\lambda(s)}{\lambda(s)} \right], \tag{4}$$

where the investor's total stock of wealth at time t is given by $W(t) = B(t) + \lambda(t) S(t)$, where $W(t)$ is the total wealth at time t, $B(t)$ is the value of bonds held at time t, $\lambda(t)$ is the price of the energy sector index at time t, and $S(t)$ is the quantity of the energy sector index held at time t. In other words, the total wealth in this economy is held either in the form of bonds or in the form of the energy sector index.

As shown in Duffie and Epstein (1992a), the pricing equation for the energy sector index can be represented as follows:

$$\underbrace{\frac{1}{\lambda(t)} \frac{\mathrm{d}\lambda(t)}{\mathrm{d}t} E_t[\mathrm{d}\lambda(t)]}_{\text{Expected Return on Energy Sector Index}} \qquad -r(t) = - \underbrace{\frac{c(t) f_{cc}(t)}{f_c(t)} \sigma_{Sc}(t)}_{\text{Short-run Consumption Risk}} - \underbrace{\frac{J(t) f_{cJ}(t)}{f_c(t)} \sigma_{SJ}(t)}_{\text{Long-Run (Sentiment) Risk}}, \tag{5}$$

where $\frac{1}{\lambda(t)} \frac{\mathrm{d}\lambda(t)}{\mathrm{d}t} E_t[\mathrm{d}\lambda(t)]$ is the energy investor's expected instantaneous rate of return on the

energy sector index, with $\sigma_{Sc}(t)$ and $\sigma_{SJ}(t)$ representing the covariances of the index returns with consumption growth and changes in the future utility index, respectively.

This equation delineates that the equilibrium spread between the expected return on the energy sector index and the risk-free interest rate comprises two premia: a risk premium associated with consumption growth shocks and another with shocks to sentiment about the economy's future prospects. The first component suggests that an energy sector index becomes more appealing, requiring a lower premium if its returns inversely correlate with consumption growth rates ($\sigma_{Sc} < 0$). Conversely, a positive correlation necessitates a higher premium to offset consumption variability across different economic states.

The term $\left(\frac{1}{J(t)}\frac{\mathrm{d}J(t)}{\mathrm{d}t}\right)$ that enters the second covariance in pricing the energy sector index represents the growth rate of the future utility index, capturing uncertainty shocks to the long-run future prospects of the economy (Hansen, 2010, 2012; Cochrane, 2005; Sargent, 2007). This is particularly relevant for energy markets, where long-term expectations about climate policies, technological adoption, and resource availability create significant valuation uncertainty. Investors in the energy sector are particularly concerned with unexpected shocks to sentiment regarding the economy's future, as indicated by unexpected changes in the future utility index. This reflects consumers' revised expectations about future consumption prospects based on the information available at time t.

At any given moment t, the future utility index $J(t)$ is known with certainty. However, the growth rate of this index can experience unexpected shocks, described by the equation:

$$\frac{\mathrm{d}J(t)}{J(t)} = \frac{\widetilde{J}(t+dt)|_t - J(t)}{J(t)},$$

where $\widetilde{J}(t+dt)|_t$ represents the projected utility at time $t+dt$, conditional on the information available at time t. Positive shocks to sentiment about the future reflect an unexpected increase in expectations regarding future consumption prospects, while negative shocks indicate an unexpected decrease in these expectations. In the context of energy markets, such shocks could be triggered by

unexpected policy announcements (e.g., new climate regulations), technological breakthroughs, or geopolitical events affecting energy security. Additionally, worsening shocks to sentiment about future prospects can be interpreted as an unexpected increase in the probability of experiencing lower levels of consumption in the future.

The second risk premium component reflects the investor's stance towards changes in future growth prospects. A positive future utility index growth implies optimistic future prospects, while a negative trend suggests the opposite. This component's presence aligns with strategies to hedge against future economic uncertainties. The sign and magnitude of f_{cJ} play crucial roles here, influencing the premium based on the index's correlation with future prospect expectations. For instance, a negative f_{cJ} increases the premium for indexes positively correlated with optimistic future expectations ($\sigma_{SJ} > 0$), making these indexes less desirable during economic downturns. Conversely, indexes negatively correlated with future prospects ($\sigma_{SJ} < 0$) offer hedging opportunities against adverse economic shifts, potentially reducing the required risk premium for risk-averse investors.

This nuanced approach to pricing in the energy sector emphasizes the multifaceted nature of risk evaluation and the strategic positioning of energy indexes within an investor's portfolio, underlining the importance of both current economic conditions and future growth expectations in investment decision-making. The framework provides a structural way to understand how news about long-term macroeconomic trends–including the energy transition, climate policy developments, and technological innovations–gets incorporated into energy asset prices through investors' changing expectations about future economic prosperity.

2.1 A parametric example

The aggregator $f(c,J)$ encodes the fundamental trade-offs between current consumption and future utility that drive investment decisions in energy markets. To better understand how sentiment about the future, as well as short-run and long-run risk, affect the pricing of energy assets, we consider the parametric recursive utility specification developed by Schroder and Skiadas (1999).

This specification provides the flexibility needed to capture investors' attitudes toward the long-run uncertainties that characterize energy investments-including technological disruptions, climate policies, and resource availability. The conventional time-additive expected utility represents a special case of the more general Schroder and Skiadas (1999) recursive utility framework.

2.1.1 Aggregator of the Schroder and Skiadas (1999) recursive utility

To estimate equation (5), we rely on the aggregator of the Schroder and Skiadas (1999) parametric recursive utility given by:

$$f(c,J) = (1+\alpha)\left[\frac{c^{\gamma}}{\gamma}J^{\frac{\alpha}{1+\alpha}} - \beta J\right], \tag{6}$$

with $\alpha > -1$ and $\gamma < \min\left(1, \frac{1}{1+\alpha}\right)$. This parametric recursive utility is more general than the time-additive expected utility and simplifies both the exposition of the econometric model and the interpretation of the parameters to be estimated.

The discount parameter β is assumed to be positive. In a time-additive utility model, β would simplify to the rate of time preference. However, time preference is generally endogenous in non-additive recursive utility settings. The ratio $\frac{1}{1-\gamma}$ represents the elasticity of intertemporal substitution (EIS), while the parameter α captures the dependency of current utility on future utility J.[3]

The curvature of the aggregator $f(x,J)$ with respect to the future utility argument, J, characterizes preferences for the timing of resolution of uncertainty (Epstein et al., 2014; Kreps and Porteus, 1978; Duffie and Epstein, 1992b; Schroder and Skiadas, 1999; Strzalecki, 2013; Zhao, 2017), with a convex aggregator favoring early resolution and a concave aggregator favoring late resolution (Skiadas, 1998). This distinction is particularly relevant for energy investments, where uncertainty about future climate policies, technological breakthroughs, or resource discoveries may resolve over different time horizons. An additive temporal aggregator corresponds to indifference towards the timing of resolution. The sign of the product term $\gamma\alpha$ expresses the curvature of the aggregator with respect to the second argument, capturing attitudes towards the temporal resolution of

[3] When $\gamma = 0$, this aggregator becomes $f(x,J) = (1+\alpha J)\left[\log(x) - \frac{\beta}{\alpha}\log(1+\alpha J)\right]$.

uncertainty.[4]

As will be later evidenced in subsection 3.1, the parameter α highlights the role of sentiment risk related to future growth prospects of the economy in shaping energy index returns. This is especially important in energy markets, where valuations depend critically on expectations about long-term economic growth, energy demand patterns, and the pace of the energy transition. Additionally, the sign of the product parameter $\alpha\gamma$ helps understand consumers' attitudes towards the temporal resolution of uncertainty. A nonzero value of $\gamma\alpha$ indicates non-indifference towards the temporal resolution of uncertainty. A negative sign, $\gamma\alpha < 0$, signifies a preference for early resolution of uncertainty, whereas a positive sign, $\gamma\alpha > 0$, signifies a preference for late resolution of uncertainty. In energy investing, a preference for early resolution might characterize investors who are particularly averse to not knowing whether new climate regulations or technological disruptions will materialize. In the specific case where $\alpha = 0$, corresponding to indifference towards the timing of resolution, the aggregator of the standard time-additive expected utility is obtained as $f(x,V) = \frac{x^\gamma}{\gamma} - \beta J$.

Let us mention that in a deterministic setting, a world without uncertainty $(\sigma = 0)$, concerns for temporal resolution of uncertainty is not relevant ($\alpha = 0$), and the Schroder and Skiadas (1999) parametric recursive utility reduces to the aggregator of the time additive expected utility, that is $f(x,J) = \frac{x^\gamma}{\gamma} - \beta J$. However, energy markets operate in an environment of significant uncertainty, making the general recursive utility specification particularly appropriate for capturing how investors price energy assets in the face of evolving expectations about the long-term economic and policy landscape.

3 Empirical strategy based on latent factor method

This section describes our data, calibration approach, and presents details regarding the main estimation procedure for the energy index pricing equation. A combination of calibrated and estimated

[4]There is a connection between preferences for the timing of resolution of uncertainty and preferences for information (Skiadas, 1998).

parameters is used. Our empirical strategy is particularly suited for energy markets, where investor expectations about long-term factors such as climate policies, technological disruptions, and energy transition pathways play a crucial role in asset pricing but are inherently unobservable.

3.1 Estimation of the parameters

The estimation of the parameters α and γ requires the value function and its growth rate, which are not directly observable as they depend on expectations about future prospects of the economy. This challenge is particularly acute in energy markets, where valuations incorporate expectations about decades-long transitions and policy developments. To emphasize this point, Koopmans (1960) refers to the value function as the prospective utility. The value function can be estimated by using a small number of factors $F = (F_1, ..., F_L)$ called latent variables that capture expectations about future prospects of the economy. The estimation of latent factors is based on the procedure proposed by Bai and Ng (2002) and presented in Appendix A. Other studies that have used the latent factor approach include Bernanke et al. (2005); Bai and Ng (2002); Boivin and Giannoni (2006); Forni et al. (2009); Bernanke and Boivin (2003); Favero et al. (2005); Bouaddi and Taamouti (2012); Ludvigson and Ng (2009); Kakeu and Bouaddi (2017); Bouaddi and Taamouti (2013).

In general, there is no closed-form solution to the value function. An approximation of the value function is used for empirical purposes. The empirical estimate of models with a recursive utility framework is still difficult due to the latency of the value function (Chen et al., 2013; Thimme, 2017), and researchers must use approximations. While the utility index itself is not observable, one way to deal with the estimation is to substitute it as a function of quantifiable variables, such as latent factors, related to the state of the economy (Cochrane, 2017). In a recursive utility framework, the value function depends upon expectations about future consumption growth. Proxying the recursive utility index by using latent factor techniques that captures expectations about the future of the economy is well grounded following tradition in empirical works using recursive utility (Hansen, 2010, 2012; Cochrane, 2005; Chen et al., 2013; Kakeu and Bouaddi, 2017). For instance, Chen et al. (2013) use latent factor techniques to explicitly estimate the un-

observable continuation value of the future consumption plan in a discrete time Epstein and Zin (1989) recursive utility framework.

We use a Schroder and Skiadas (1999) continuous-time stochastic recursive preferences and we assume that the logarithm of the value function is a linear function of the estimated latent factors related to sentiment about the future.[5] We will use the Michigan consumer sentiment index which is a monthly assessment of consumer expectations about the future. The Michigan survey is a leading indicator that attempts to predict economic conditions a full year into the future. This approach allows us to capture the forward-looking nature of energy investments, where current prices reflect expectations about future demand patterns, policy environments, and technological developments that may take years to materialize.

Our approach uses a latent factor approximation of the value function which takes the following functional form:

$$J(t) = e^{\left(\theta + \sum_{i=1}^{L} \phi_i F_i(t)\right)}. \tag{7}$$

Equation (7) can be viewed as a way of log linearizing the model. Note that the differential form of equation (7) implies that changes in the value function can be approximated linearly by changes in the latent factors. Log-linear approximations are often used in macroeconomic and financial models (Ljungqvist and Sargent, 2004; Bansal and Yaron, 2004; Cochrane, 2005; Lau and Ng, 2007; Restoy and Weil, 2011).

The estimation of the latent factors is done in the first step of the econometric procedure. The approximated value function $\widetilde{J}(t) = J(\widetilde{F}_{1t}, ..., \widetilde{F}_{Lt})$ is a function of those estimated latent factors obtained from sentiment variables. The estimated value function $\widetilde{J}(t)$ is a consistent estimate of the value function $J(t)$ that is not observable.

When the value function is replaced by its approximate in the pricing equation, the estimation of the parameters θ, and $(\phi_i)_{i=1,..L}$ in the expression (7) are done simultaneously with the preference parameters α, β, and γ described in the expression (6). That is, this specification allows the value function to be endogenously and jointly estimated with the preference parameters in the

[5]The estimated factors are mutually orthogonal by construction.

structural model.[6]

In the initial step of the econometric procedure, we estimate the latent factors. The value function's approximation, denoted by $\tilde{J}(t) = J(\tilde{F}_1(t), ..., \tilde{F}_L(t))$, is based on these estimated latent factors. This approximation, $\tilde{J}(t)$, acts as a consistent estimate for the inherently unobservable actual value function, $J(t)$. By incorporating this approximate value function into the pricing equation, we simultaneously estimate the parameters δ and ϕ_i (for $i = 1, ..., L$) along with the preference parameters α, β, and γ, as previously outlined in expression (11). This methodology facilitates the simultaneous, endogenous estimation of both the value function and the preference parameters within the structural model framework.

Accordingly, the pricing equation simplifies to a formula for the net expected return on the energy sector index:

$$\frac{1}{\lambda(t)}\frac{1}{dt}E_t[d\lambda(t)] - r(t) = (1-\gamma)\sigma_{Sc}(t) - \sum_{i=1}^{L}\frac{\alpha}{1+\alpha}\phi_i F_i(t)\sigma_{SF_i}(t) \tag{8}$$

This equation elucidates the net expected return on the energy sector index, incorporating considerations of short-run consumption risk and long-run prospects risk. It provides a detailed framework for analyzing the expected returns of the energy sector index by examining the interplay of risk and return, influenced by current consumption patterns and expectations regarding future prospects. Movements in the latent factors should be traced to movements in sentiment indicators. The sentiment indicators are forward-looking indicators as they capture information about the state of the economy as well as expectations about future prospects of the economy. For energy markets, these expectations encompass not only general economic conditions but also sector-specific factors such as anticipated climate policies, technological breakthroughs in renewable energy, and geopolitical developments affecting energy security.

The bond pricing equation with recursive utility can be used to avoid identification issues. Let

[6]To be clear, we use a two-step method of estimation. In the first step we estimate the factors (sentiment factors). In the second and final step, the latent factor variables are fixed at their estimated values from the first step, so that only the preferences parameters and value function of the structural model are estimated in the second step. In so doing the value function is endogenously estimated as a function of latent factors.

us introduce the stochastic process, $\{\Lambda(t) : t \geq 0\}$, defined as follows:

$$\Lambda(t) = e^{\int_0^t f_J(c(s),J(s))ds} \frac{f_c(c(t),J(t))}{f_c(c_0,J_0)}. \tag{9}$$

The expression shown in equation (9) is known as the Riesz representation of the utility gradient of the recursive utility.[7]

It can be shown that the stochastic process $\Lambda(t)$ satisfies the following equation:

$$\frac{1}{\Lambda(t)}\frac{1}{dt}E_t(d\Lambda(t)) = -r(t), \tag{10}$$

which becomes:

$$\begin{aligned} \underbrace{r(t)}_{Interest\ rate} &= -f_J(c,J) - \frac{1}{f_c(c,J)}E_t[df_c(c,J))] \\ &= -f_J(c,J) - \frac{cf_{cc}}{f_c}E_t\left[\frac{dc}{c}\right] - \frac{Jf_{cJ}}{f_c}E_t\left[\frac{dJ}{J}\right] - \frac{1}{2}\left[\frac{c^2 f_{ccc}}{f_c}\sigma_c^2 + 2\frac{Jcf_{ccJ}}{f_c}\sigma_{cJ} + \frac{J^2 f_{cJJ}}{f_c}\sigma_J^2\right] \end{aligned} \tag{11}$$

Using the latent factor approximation of the value function (7) in the equilibrium bond pricing can lead to:

$$\begin{aligned} r(t) &= \underbrace{-\left[\alpha\frac{c^\gamma}{\gamma}J^{-\frac{1}{1+\alpha}} - \beta(1+\alpha)\right]}_{Endogeneous\ Discount\ Rate} + \underbrace{(1-\gamma)E_t\left[\frac{dc}{c}\right]}_{Consumption\ growth} - \underbrace{\frac{\alpha}{1+\alpha}\left[\sum_{i=1}^{L}(\phi_i F_i(t)+\theta)\right]}_{Sentiment} \\ &\quad \underbrace{-\frac{1}{2}\left[(1-\gamma)(2-\gamma)\sigma_c^2\right]}_{Short-run\ Risk} \\ &\quad - \underbrace{\frac{1}{2}\left[2(\gamma-1)\frac{\alpha}{1+\alpha}\sum_{i=1}^{L}\phi_i(t)F_i(t)\sigma_{cF_i}(t) - \frac{\alpha}{(1+\alpha)^2}\sum_{i=1}^{L}\phi_i^2 F_i^2\sigma_{F_i}^2(t)\right]}_{Long-run\ Risk} \end{aligned} \tag{12}$$

This comprehensive framework allows us to quantify how much of energy asset returns can

[7]The utility gradient is computed as the Gateaux derivative of a dynamic utility. For more details, see Duffie and Skiadas (1994, page120) and Schroder and Skiadas (1999, page 76).

be attributed to changing expectations about the long-term economic and policy environmentâ€”a crucial consideration for understanding energy market dynamics in an era of significant structural change and policy uncertainty.

3.2 Data on Energy Index Returns

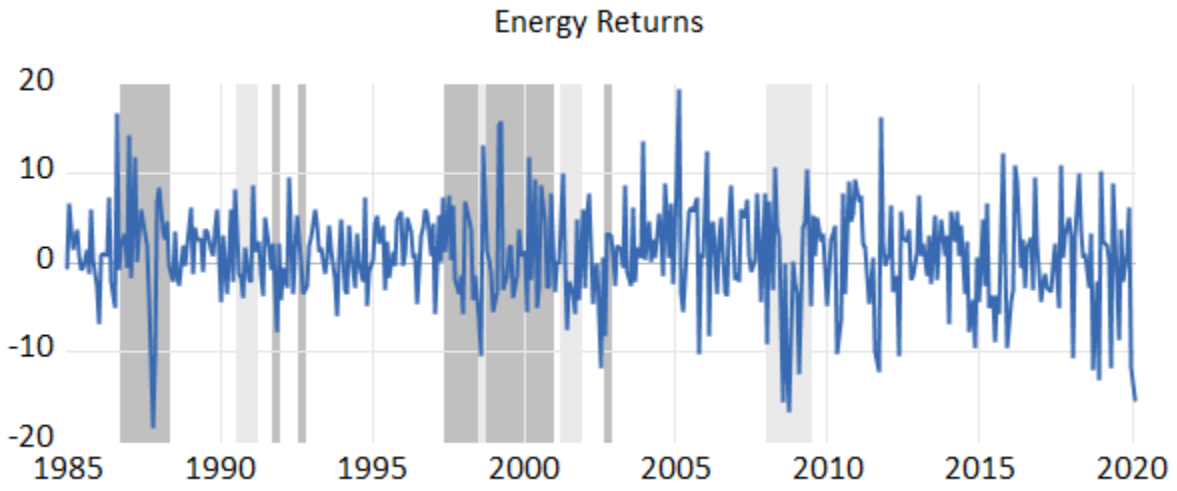


Figure 1: Energy industry index returns from December 1984 to February 2020

The data on energy industry returns used are collected from the Kenneth R. French Data Library. The description of the energy industry as well as their four-digit SIC codes, can be found in the Kenneth R. French Data Library.[8] The data cover the period from December 1984 to February 2020, encompassing several major energy market cycles including the oil price collapse of 1986, the dot-com boom and bust, the 2008 global financial crisis, and the shale revolution. It consists of the Fama/French monthly returns in percent on the energy industry's returns, constructed using value-weighted returns of all CRSP firms incorporated in the energy industry (US firms) listed on the NYSE, AMEX, and NASDAQ. This comprehensive coverage ensures our analysis captures the evolving risk-return profile of energy investments through periods of significant structural change

[8]The data is publicly available from `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

in global energy markets.

3.3 The Risk-Free Rate

For the risk-free rate of interest, we use the monthly equivalent of the annual rate on the one-month U.S. Treasury bill, taken from the CRSP database. The data ranges from December 1984 to February 2020, covering various monetary policy regimes and interest rate environments that have influenced energy investment decisions and the cost of capital for energy projects.

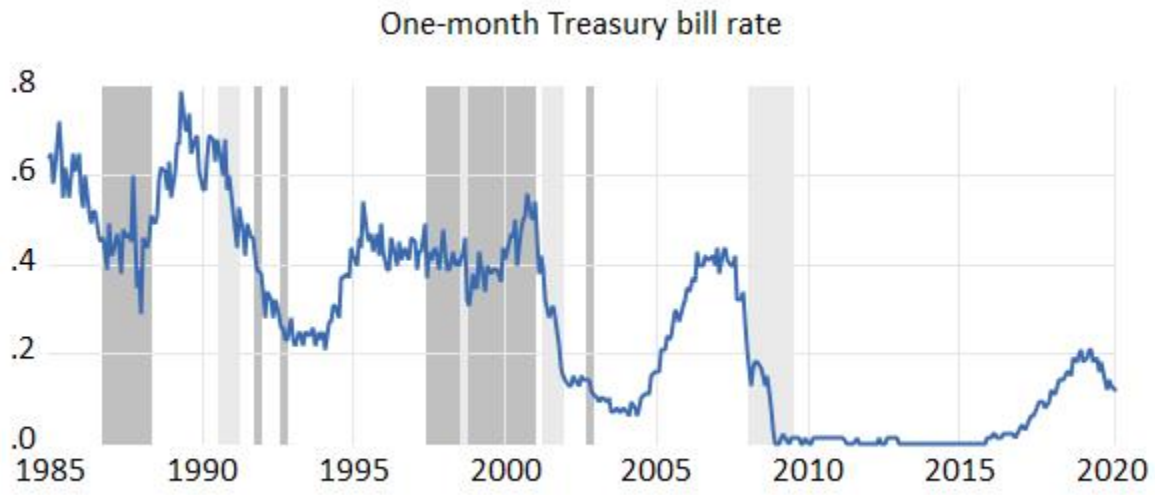


Figure 2: One-month U.S Treasury bill rate from December 1984 to February 2020

3.4 Description of the Data on Consumption per Capita

Consumption data come from the St. Louis Federal Reserve Bank and cover the period from December 1984 to February 2020. We used the real personal consumption expenditures of services and the real personal consumption expenditures of non-durable goods.[9] We also use the total civilian population to get the real consumption per capita.[10] We computed the per capita

[9]Data on real personal consumption expenditures of services and non-durable goods are from St. Louis Federal Reserve Bank website: https://fred.stlouisfed.org/series/PCESC96, https://fred.stlouisfed.org/series/PCENDC96, https://fred.stlouisfed.org/series/RKNANPUSA666NRUG.

[10]Data on civilian population are from St. Louis Federal Reserve Bank website: https://fred.stlouisfed.org/series/CNP16OV

real consumption as the sum of the real personal consumption expenditures of services and real personal consumption expenditures of non-durable goods over civilian population to get the per capita consumption level. This measure captures the fundamental consumption risks that investors seek to hedge through their energy investments. We then use the growth rate (log-difference) on real per capita personal consumption expenditures of services and non-durable goods to capture consumption dynamics relevant for asset pricing.

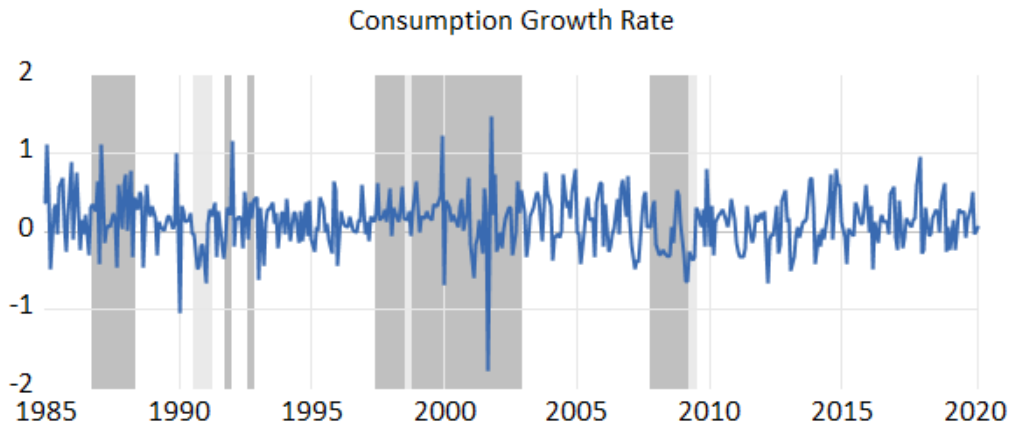


Figure 3: Real consumption per capita growth rate from December 1984 to February 2020

3.4.1 Estimation of the Latent Factors Related to the Panel of Sentiment Data

We use the University of Michigan's Survey of Consumers, which is closely watched by news media and decision makers for its forward-looking assessment of economic conditions.[11] The survey is based on responses regarding the general economic situation, unemployment, and savings capability (Curtin, 2007; Bram and Ludvigson, 1998; Dominitz and Manski, 2004; Ludvigson, 2004). The latent factor analysis is used to estimate a small number of latent factors from the panel of consumer sentiment indicators, spanning from February 1980 to December 2014. This period covers multiple business cycles and structural shifts in the economy, providing rich variation in sentiment dynamics. The panel of sentiment indicators forms a group of Indexes of "Index Consumer

[11] Other indicators include the Bank of England Systematic Risk Survey and The Bloomberg Consumer Comfort Index.

Sentiment" and "Index of Consumer Expectations" by age groups, regions, income and education level—a total of 32 indicators capturing diverse dimensions of economic expectations. These data are downloaded from the University of Michigan website.[12] The latent factors are estimated using the optimization program $(29) - (30)$ presented in Appendix 1. Using the information criteria proposed by Bai and Ng (2002), one factor is selected, representing 76% of the total variation of the panel of sentiment indicators. This high explanatory power suggests that a single dominant dimension captures most of the common variation in consumer expectations about future economic conditions.

3.5 Database on the University of Michigan's Survey of Consumers' Sentiment

Sentiment factors are derived from the monthly Survey of Consumers by the University of Michigan.[13] The survey on consumer expectations focuses on consumers' views about prospects for their own financial situation, their prospects about the general economy over the near term, and their prospects about the economy over the long term. The survey contains consumers' prospects about personal finances, savings and retirement, economic conditions, unemployment, prices, government expectations, household goods buying conditions, vehicle buying conditions, and home buying and selling conditions.[14] For energy market applications, these forward-looking expectations are particularly relevant as they influence investment decisions in long-lived energy assets and shape expectations about future energy demand patterns.

[12] `https://data.sca.isr.umich.edu/subset/subset.php`

[13] The database is available at: `https://data.sca.isr.umich.edu/subset/subset.php`

[14] In estimating the latent factors, we used an index instead of one sub-index for sentiment because the latter will represent only a subgroup of elements affecting the consumer sentiment. In the survey, there are indicators related to consumer perceptions about his/her financial conditions and indicators related to consumer perceptions about the global business conditions in the country and the world. Furthermore, there are also perception indicators about the business conditions in the short and long run. All these indicators, when taken separately, affect sentiment differently. Considering a sub-index may miss some essential features affecting sentiment about future prospects of the economy. Estimating a common factor that combines all these indicators is the only fundamental that better captures the actual state of the consumer sentiment.

3.6 Empirical Results

We estimated the parameters related to the energy index return pricing using an econometric approach that incorporates latent factor analysis on sentiment indicators. To select the optimal number of fundamental factors influencing the sentiment indicators, we applied the information criteria of Bai and Ng (2002). This criterion identified one factor, denoted as F without a subscript, suggesting that a single latent dimension adequately captures the common variation in consumer expectations relevant for energy asset pricing. More details about the econometric model are provided in Appendix B.

The pricing equation simplifies to a formula for the net expected return on the energy sector index: The term on the left-hand side is the energy index return, the first term on the right-hand side is the short-run consumption risk premium, and the second term is the sentiment risk premium.

$$\underbrace{\frac{1}{\lambda(t)}\frac{1}{dt}E_t[d\lambda(t)] - r(t)}_{\text{Energy Index Return}} = \underbrace{(1-\gamma)\sigma_{Sc}(t)}_{\text{Short-run Consumption Risk Premium}} \underbrace{-\sum_{i=1}^{L}\frac{\alpha}{1+\alpha}\phi_i F_i(t)\sigma_{SF_i}(t)}_{\text{Sentiment Risk Premium}} \tag{13}$$

Another important equation for identification is the bond pricing equation:

$$\begin{aligned} r(t) &= -\left[\alpha\frac{c^{\gamma}}{\gamma}J^{-\frac{1}{1+\alpha}} - \beta(1+\alpha)\right] + (1-\gamma)E_t\left[\frac{dc}{c}\right] - \frac{\alpha}{1+\alpha}\left[\sum_{i=1}^{L}(\phi_i F_i(t)+\theta)\right] \\ &\quad -\frac{1}{2}\left[(1-\gamma)(2-\gamma)\sigma_c^2\right] \\ &- \frac{1}{2}\left[2(\gamma-1)\frac{\alpha}{1+\alpha}\sum_{i=1}^{L}\phi_i(t)F_i(t)\sigma_{cF_i}(t) - \frac{\alpha}{(1+\alpha)^2}\sum_{i=1}^{L}\phi_i^2 F_i^2\sigma_{F_i}^2(t)\right] \end{aligned} \tag{14}$$

With the Schroder and Skiadas (1999) parametric recursive utility, the sign of the product of the parameters $\gamma\alpha$ is crucial for understanding investors' attitudes towards sentiment risk associated with long-run prospects. A negative sign for the product of the parameters, $\gamma\alpha < 0$, indicates that investors prefer an early resolution of uncertainty shocks to sentiment about future prospects. Consequently, they are averse to sentiment risks associated with future growth prospects.

	Coefficient	Std. Error	z-Statistic	Prob.
γ	0.201785	9.02×10^{-7}	223815.7	0.0000
ϕ	0.200092	3.34×10^{-7}	598617.3	0.0000
α	-0.532431	5.43×10^{-7}	-980840.2	0.0000
θ	1.296090	0.000608	2131.268	0.0000
β	0.041873	0.001481	28.26662	0.0000

Table 1: Estimation Results for the Recursive Utility Model

As shown in Table 1, the product of the estimated parameters $\gamma\alpha$ is negative, and the parameters α and γ are statistically significant. This suggests that energy investors prefer an early resolution of uncertainty related to sentiment about the long-term future, and thus, they are adverse to long-run risks associated with uncertainty shocks to sentiment about future growth prospects. This finding has important implications for energy investments, where long-term uncertainties about climate policies, technological disruptions, and resource availability create significant valuation challenges.

The parameters ϕ are related to the latent factor variables F. As shown in Table 1, estimates of the parameters related to the factors F are statistically significant. This suggests that energy index returns are significantly influenced by uncertainty shocks to sentiment about the future growth prospects of the economy. The statistical significance of all estimated parameters provides strong evidence for the relevance of the recursive utility framework in explaining energy asset returns.

The graph below shows the estimated value function over time, which captures the evolution of investors' expectations about future consumption prospects and serves as the empirical proxy for the unobservable continuation value in our recursive utility framework.

As mentioned earlier, the term $\left(\frac{1}{J(t)}\frac{dJ(t)}{dt}\right)$ in the second covariance of the energy sector index pricing equation represents the growth rate of the future utility index. This term captures uncertainty shocks to sentiment about future economic prospects (Hansen, 2010, 2012; Cochrane, 2005; Sargent, 2007).

Investors in the energy sector are concerned with unexpected shocks to sentiment regarding the economy's future. These shocks are reflected in unexpected changes in the future utility (value function) index. This value function reflects investors' revised expectations about future consump-

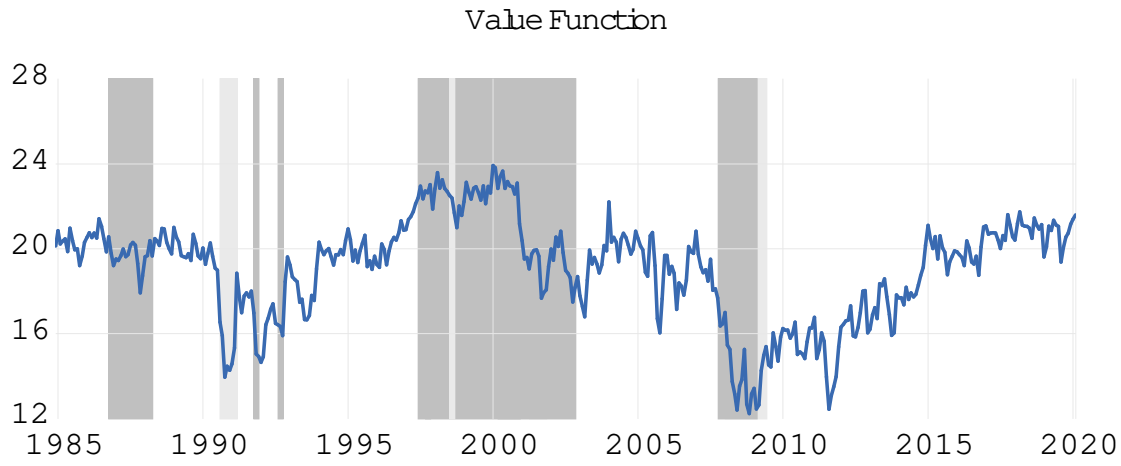


Figure 4: Estimated value function over time

tion prospects based on the information available at time t. Positive shocks to sentiment about the future indicate an unexpected increase in expectations regarding future consumption prospects, while negative shocks suggest an unexpected decrease in these expectations. For energy markets, these sentiment shocks can be triggered by developments such as unexpected climate policy announcements, technological breakthroughs in competing energy sources, or geopolitical events affecting energy security.

4 Decomposition of the Variance of the Expected Return on the Energy Sector Index: Impact of Short-Run Consumption Risk and Sentiment Risk

In this section, we aim to quantify the contributions of short-run consumption risks and sentiment risks to the returns of the energy index. This decomposition is particularly relevant for understanding how different types of risks drive energy asset valuations in an era of significant structural change and policy uncertainty. The energy sector faces unique challengesâ€”including climate policy developments, technological disruptions in renewable energy, and shifting global energy demand patternsâ€”that make the distinction between short-term consumption risks and long-term sentiment risks especially important for investors and policymakers. To achieve this decomposi-

tion, we employ the democratic orthogonalization method, originally developed by Löwdin (1970) in the context of Quantum Chemistry. This method has recently been introduced to the asset pricing and finance literature (Klein and Chow, 2013; Bessler et al., 2015). Here, we utilize this method to determine the relative importance of sentiment and risk factors in influencing variations in energy index returns.

First, we rewrite the optimal consumption path equation as follows:

$$\underbrace{\frac{1}{\lambda(t)}\frac{1}{dt}E_t[d\lambda(t)] - r(t)}_{\text{Expected Return on Energy Sector Index}} = \underbrace{x_{1,t}}_{\text{Short-run Consumption Risk}} + \underbrace{x_{2,t}}_{\text{Sentiment Risk}} \tag{15}$$

where

$$\begin{cases} x_{1,t} = (1-\gamma)\sigma_{Sc}(t) & (16) \\ x_{2,t} = \sum_{i=1}^{L} \frac{\alpha}{1+\alpha}\phi_i F_i(t)\sigma_{SF_i}(t) & (17) \end{cases}$$

4.1 Democratic decomposition of Löwdin (1970) applied to equation (15)

In a discrete time setting $t = 1, ..., T$, the right-hand side components of equation (15) are represented as the columns of the following matrix:

$$X = \begin{pmatrix} x_{1,1} & x_{2,1} \\ \vdots & \vdots \\ x_{1,t} & x_{2,t} \\ \vdots & \vdots \\ x_{1,T} & x_{2,T} \end{pmatrix} \tag{18}$$

If the columns of the matrix X are statistically independent or orthogonal, then the ratio of

the variance of the i^{th} column to the sum of the variances of all columns captures the contribution of that factor in explaining variations in expected consumption growth. However, in general, the columns of the matrix X may not be statistically independent or orthogonal. In energy markets, short-run consumption risks and long-run sentiment risks are often correlatedâ€"for example, during economic downturns when both current consumption and future growth expectations decline simultaneously. In such cases, it is possible to find an orthogonal equivalent of X whose columns are independent or orthogonal, with diagonal entries equal to the variances of the components of the columns of X_t.

The democratic decomposition method developed by Löwdin (1970) is a statistical technique that transforms a matrix of correlated variables into an information-equivalent matrix of uncorrelated variables. This method isolates the specific contribution of each component, allowing for a clear interpretation of individual relationships. Unlike the sequential orthogonalization approach, democratic decomposition is independent of the ordering of variables and considers the entire set of vector data simultaneously. This property is particularly valuable in our context, as it ensures that our results are not sensitive to whether we orthogonalize sentiment risk with respect to consumption risk or vice versa.

Applied to our energy index pricing framework, the democratic decomposition extracts standalone orthogonal components of short-run risk and sentiment risk while maintaining an optimal relationship with the underlying variables. The variances of orthogonal components derived from democratic decomposition are identical to those of the original variables. This ensures that the orthogonal components best resemble the original variables, enabling the computation of the relative contributions of short-run consumption risk and sentiment risk in explaining variations in energy index returns. This approach provides a robust foundation for quantifying how much of energy asset price movements can be attributed to changing expectations about long-term economic and energy market prospects.

We provide a brief overview of the democratic orthogonalization method developed by Löwdin (1970). Consider the general case where the components of the vector $X_t = (x_{1,t}, x_{2,t})'$ may exhibit

some correlation. To analyze the contribution of each component effectively, an orthogonal equivalent, denoted by $Z_t = (z_{1,t}, z_{2,t})'$, is sought. This equivalent's associated covariance matrix is diagonal, with diagonal entries equal to the variances of X_t's components.

Define the demeaned version of matrix X as:

$$\mathcal{X} = \begin{pmatrix} x_{1,1} - \bar{x}_1 & x_{2,1} - \bar{x}_2 \\ \vdots & \vdots \\ x_{1,t} - \bar{x}_1 & x_{2,t} - \bar{x}_2 \\ \vdots & \vdots \\ x_{1,T} - \bar{x}_1 & x_{2,T} - \bar{x}_2 \end{pmatrix} \tag{19}$$

The sample covariance matrix of $\mathcal{X}$, denoted $\widehat{\Omega} = \text{Cov}(\mathcal{X})$, can be factorized as:

$$\widehat{\Omega} = F \Lambda F' \tag{20}$$

$$F'F = I \tag{21}$$

$$\Lambda \text{ is a diagonal matrix.} \tag{22}$$

As demonstrated by Löwdin (1970), the democratically orthogonalized components are obtained by:

$$\mathcal{Z} = Y \Upsilon. \tag{23}$$

where

$$Y = \mathcal{X} F \Lambda^{-\frac{1}{2}} F', \tag{24}$$

$$\Upsilon = \text{Diag}(\widehat{\Omega}). \tag{25}$$

Here, $\Upsilon = \text{Diag}(\widehat{\Omega})$ is the diagonal matrix with its main diagonal equal to that of $\widehat{\Omega}$, and it is established that $Y'Y = I$.

The variances of the columns of matrix $\mathcal{Z}$ resulting from democratic decomposition are identical to the variances of the columns in the initial matrix X.

Using 200 bootstrap samples of $\mathcal{Z}$, we can compute the estimated cross-sectional variances of $\mathcal{Z}$ to analyze the relative contributions of sentiment risks to the energy index returns.

$$\begin{cases} \text{Sentiment Risk Impact} = \dfrac{s_{1,t}^2}{s_{1,t}^2 + s_{2,t}^2}, \quad t = 1, ..., T. & (26) \\ \\ \text{Short-run Consumption Risk Impact} = \dfrac{s_{2,t}^2}{s_{1,t}^2 + s_{2,t}^2}, \quad t = 1, ..., T. & (27) \end{cases}$$

4.2 Descriptive Statistics of Sentiment Risk Impact Proportions on Energy Index Returns

Table 2 displays the descriptive statistics of the short-run risk and the long-run risk. It shows that, on average, the sentiment risk component accounts for 49.71% of the energy index returns.

	Sentiment Risk Impact
Mean	0.497158
Median	0.497196
Maximum	0.502694
Minimum	0.488703

Table 2: Descriptive Statistics of the Sentiment Risk Impact on Energy Index Returns

The table shows that the average impact of long-run sentiment risk on energy index returns is approximately 49.71%. This indicates that, on average, nearly half of the energy index returns can be attributed to sentiment risk. This finding is particularly significant for energy markets, where long-term uncertainties about climate policies, technological disruptions, and energy transition pathways create substantial valuation challenges. The median value, which is very close to the mean at 49.72%, suggests that the distribution of sentiment risk impact is symmetrical and not driven by outliers. The maximum observed impact of sentiment risk is about 50.27%, showing

the upper limit of sentiment risk contribution to energy index returns during periods of heightened uncertainty about future economic and energy market prospects. The minimum observed impact is around 48.87%, indicating the lower bound of sentiment risk contribution during more stable periods.

These statistics provide a clear overview of the extent to which sentiment risk influences the returns on the energy sector index, highlighting its significant role. The nearly equal split between sentiment risk and short-run consumption risk underscores the importance of considering both current economic conditions and long-term expectations when analyzing energy market dynamics. For policymakers and investors navigating the energy transition, this decomposition suggests that nearly half of energy asset price movements reflect changing expectations about the long-term futureâ€"emphasizing the critical role of policy certainty and stable long-term frameworks in energy investment decisions.

4.3 Comparison with a Time-Additive Expected Utility Model

This section provides an empirical comparison of the recursive utility model, which includes sentiments, with the time-additive utility model, which excludes sentiments. This comparison is particularly relevant for energy markets, where the long-term nature of investments and exposure to persistent policy and technological uncertainties make the distinction between these utility specifications potentially significant. We address the question of how well the recursive utility model explains energy market data relative to the competing time-additive utility specification, and whether incorporating long-run sentiment risk provides meaningful improvements in explaining energy asset returns.

For the time-additive expected utility, the utility function is represented by the aggregator $f(c,J) = U(c) - \beta J$, where $U(c) = \frac{c^\gamma}{\gamma}$. This corresponds to the case where $\alpha = 0$, effectively eliminating the role of future utility expectations and sentiment about long-term prospects in the pricing equation. This specification assumes that investors are indifferent to the timing of resolution of uncertainty and do not price long-run risks associated with future economic growth prospects.

With the time-additive expected utility, the pricing equation for the energy index returns becomes:

$$\frac{1}{\lambda(t)}\frac{1}{dt}E_t[d\lambda(t)] - r(t) = (1-\gamma)\sigma_{Sc}(t)$$

This simplified pricing equation includes only the short-run consumption risk premium, omitting the sentiment risk component that captures investors' concerns about long-term economic prospects. For energy assets with decades-long investment horizons and significant exposure to climate policies and technological transitions, this omission could lead to substantial misspecification.

The estimated parameters for the time-additive pricing equation are shown in Table 3.

	Coefficient	Std. Error	z-Statistic	Prob.
γ	1.429604	0.485415	2.945119	0.0032

Table 3: Estimation Results for the Time-Additive Utility Model

The parameter γ represents the relative risk aversion coefficient. In the time-additive utility model, it is estimated to be approximately 1.43, which falls within the conventional range of risk aversion estimates in the literature. The standard error of γ is 0.485415, indicating reasonable precision of the estimate. The z-statistic of 2.945119, with a corresponding p-value of 0.0032, suggests that γ is statistically significant at conventional levels.

However, comparing this result with the recursive utility estimates in Table 1 reveals important differences. The time-additive model produces a substantially higher estimate of risk aversion ($\gamma = 1.43$) compared to the recursive utility model ($\gamma = 0.20$). This discrepancy suggests that the time-additive specification may be attributing to risk aversion what is actually investors' aversion to long-run uncertainty about future economic prospects. These results indicate that while the time-additive model has a significant parameter estimate for γ, the recursive utility model with sentiments offers a more comprehensive and economically plausible explanation by incorporating additional sentiment risk factors that are particularly relevant for energy investments facing long-term structural uncertainties.

4.3.1 Tests of the Difference Between the Expected Return of the Energy Sector Index Implied by (*Recursive Utility*) and (*Standard Time-Additive Utility*)

To determine whether the model with recursive utility is quantitatively different from the one with time-additive utility, we computed the energy index return rates implied by equation (13) (*recursive utility*) and equation (28) (*time-additive utility*). We then used a kernel non-parametric density estimation to analyze the difference between the energy index returns implied by these two equations. The results are shown in Figure 5.

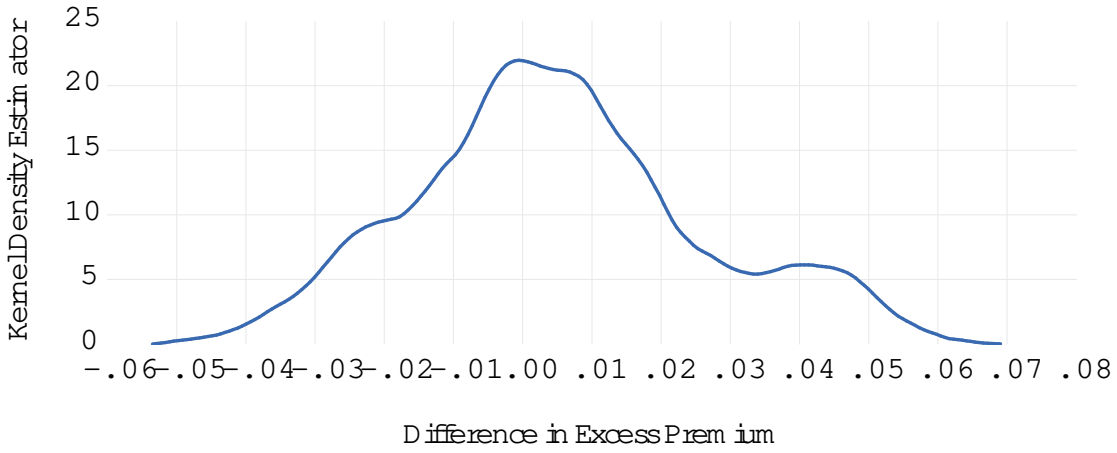


Figure 5: Density distribution of the difference between energy index returns implied by recursive utility and time-additive utility models

If there were no meaningful difference between the two models, the density would be degenerate at zero, with the entire mass concentrated at zero. However, the graph shows that the density is not degenerate and most of the mass is located away from zero. This indicates that the expected returns implied by the recursive utility model are significantly different from those implied by the time-additive utility model.

The systematic deviation from zero provides strong evidence that incorporating sentiment risk and long-run uncertainty through recursive utility meaningfully changes the predicted returns on energy assets. This finding has important implications for energy investment analysis and policy

evaluation: models that ignore long-run sentiment risk may misprice energy assets, particularly during periods of heightened uncertainty about climate policies, technological disruptions, or long-term energy demand patterns. The significant difference between the two models underscores the value of using recursive utility for analyzing energy markets, where investments are inherently long-term and exposed to persistent uncertainties about the future energy landscape.

The rejection of the time-additive specification in favor of the recursive utility model suggests that energy investors do indeed care about the timing of resolution of uncertainty and price assets accordingly. This has concrete implications for understanding how energy markets respond to news about long-term developments such as climate policy announcements, technological breakthroughs in competing energy sources, or shifts in long-term energy demand forecasts. For policymakers designing energy transition frameworks and for investors allocating capital to long-lived energy assets, these results highlight the importance of considering how expectations about the distant future get incorporated into current asset prices through the sentiment risk channel.

5 Concluding Remarks

This paper provides an empirical analysis of the impact of sentiment risk on energy index returns using the framework of Schroder and Skiadas (1999) continuous-time stochastic recursive preferences. By combining the latent factor method with the democratic orthogonalization technique, we extracted meaningful insights from a large set of macroeconomic indicators and quantified the role of sentiment risk in shaping energy index returns. Our approach is particularly well-suited for energy markets, where long investment horizons and exposure to structural uncertainties make the incorporation of long-run expectations essential for understanding asset pricing dynamics.

Our findings demonstrate that energy investors with recursive preferences are sensitive to uncertainty shocks related to sentiments about the future growth prospects of the economy. The significant influence of sentiment risk, which accounts for 49.7% of the variation in energy index returns, underscores the importance of incorporating sentiment and long-run expectations into investment decision-making processes. This result suggests that market participants should con-

sider both current economic conditions and future growth expectations when assessing energy investments, particularly during periods of significant structural change such as the ongoing energy transition.

The methodology applied in this study offers a robust approach to disentangling the contributions of different risk factors in financial markets. The use of latent factor analysis and democratic orthogonalization provides a clear interpretation of the roles of sentiment and risk, allowing for a more nuanced understanding of the drivers of energy index returns. Our empirical strategy successfully addresses the challenge of measuring the unobservable value function by leveraging forward-looking sentiment indicators, providing a template for future research at the intersection of asset pricing and energy economics.

Future research could extend this analysis by exploring the impact of sentiment risk across different economic sectors and geographic regions. The energy transition creates distinct challenges and opportunities across various energy subsectors (renewables vs. fossil fuels) and regions with different energy resource endowments and policy frameworks. Additionally, further investigation into the dynamic interactions between sentiment risk and other macroeconomic variables could provide deeper insights into the complex mechanisms driving market behavior. Particularly promising would be research examining how climate policy announcements, technological breakthroughs in energy storage or renewables, and geopolitical developments affecting energy security influence sentiment risk premia in energy assets.

In summary, this study highlights the critical role of sentiment risk in the energy sector and contributes to the broader literature at the intersection of recursive utility asset pricing and energy markets. By acknowledging the importance of sentiment risk in energy markets, investors and policymakers can better navigate the complexities inherent in making informed decisions about long-term energy investments and transition pathways. Our findings emphasize that nearly half of energy asset price movements reflect changing expectations about the long-term economic and energy landscape, underscoring the importance of policy certainty and stable long-term frameworks for facilitating efficient energy investment decisions.

6 Appendix

Appendix A: Sentiment Latent Factor Optimization Problem

The basic factor model can be expressed in a matrix form as

$$Z = F\Lambda' + E, \tag{28}$$

where Z denotes the observed matrix of time series, F is a $T \times r$ matrix of r unobservable common factors, Λ is a $N \times r$ matrix of factor loadings and E is $T \times N$ matrix of idiosyncratic errors that are uncorrelated with the components of F.

We particularly focus on the case where a high-dimensional panel of N time series depends on a relatively small number of r common latent factors. This dimensionality reduction approach is particularly valuable for energy market applications, where multiple sentiment indicators may capture different aspects of expectations about future energy demand, policy developments, and technological disruptions. The estimates of the latent factors are obtained by solving the following minimization program

$$\min_{F,\Lambda} Trace \frac{(Z - F\Lambda')(Z - F\Lambda')'}{NT} \tag{29}$$

subject to

$$\frac{F^{'}F}{T} = I_r, \tag{30}$$

where I_r is r a dimensional identity matrix. Because $F\Lambda' = FAA^{-1}\Lambda' = F^*\Lambda^{*\prime}$ for any invertible $r \times r$ matrix A, the factors and the factor loadings are not jointly identified. Thus, the normalization (30) is an identification constraint. It can be shown that each estimated factors $\widetilde{F}$ are the eigenvectors (multiplied by $\sqrt{T}$) associated with the largest eigenvalues of the matrix $\frac{ZZ^{'}}{TN}$. A convergence result by Bai and Ng (2002) says that as both T and N grow to infinity, the estimated latent variables $(\widetilde{F})$ converge to their true scaled counterpart (F).

To select the number of factors we use the following criterion (see Bai and Ng (2002)):

$$IC_{p2}(r) = \ln\left(\frac{trace\left(\widehat{E}\widehat{E}'\right)}{NT}\right) + r\frac{N+T}{NT}\ln\left(C_{NT}^2\right)$$

where $\hat{E}$ is the matrix of errors in the sentiment factor model and $C_{NT} = \min\left(\sqrt{N}, \sqrt{T}\right)$ and $\widehat{E} = Z - \widehat{F}\widehat{\Lambda}'$. This information criterion identified one dominant factor explaining 76% of the variation in our sentiment indicators, suggesting that a single latent dimension adequately captures the common variation in consumer expectations relevant for energy asset pricing.

Appendix B: The Estimation of the Model Parameters

Our approach uses a latent factor approximation of the value function which takes the following functional form:

$$J(t) = e^{\left(\theta + \sum_{i=1}^{L}\phi_i F_i(t)\right)}. \tag{31}$$

This log-linear approximation provides a tractable way to incorporate the unobservable value function into our empirical framework, allowing us to capture how expectations about future economic and energy market prospects influence current energy asset prices.

The econometric model for estimating equation (13) is given by:

$$\underbrace{\frac{1}{\lambda(t)}\frac{1}{dt}E_t[d\lambda(t)] - r(t)}_{\text{Energy Index Return}} = \underbrace{(1-\gamma)\sigma_{Sc}(t)}_{\text{Short-run Consumption Risk Premium}} \underbrace{-\sum_{i=1}^{L}\frac{\alpha}{1+\alpha}\phi_i F_i(t)\sigma_{SF_i}(t)}_{\text{Sentiment Risk Premium}} \tag{32}$$

Another important equation for identification is the bond pricing equation:

$$\begin{aligned} r(t) &= -\left[\alpha\frac{c^\gamma}{\gamma}J^{-\frac{1}{1+\alpha}} - \beta(1+\alpha)\right] + (1-\gamma)E_t\left[\frac{dc}{c}\right] - \frac{\alpha}{1+\alpha}\left[\sum_{i=1}^{L}(\phi_i F_i(t) + \theta)\right] \\ &\quad -\frac{1}{2}\left[(1-\gamma)(2-\gamma)\sigma_c^2\right] \\ &\quad - \frac{1}{2}\left[2(\gamma-1)\frac{\alpha}{1+\alpha}\sum_{i=1}^{L}\phi_i(t)F_i(t)\sigma_{cF_i}(t) - \frac{\alpha}{(1+\alpha)^2}\sum_{i=1}^{L}\phi_i^2 F_i^2\sigma_{F_i}^2(t)\right] \end{aligned} \tag{33}$$

The conditional mean of the factor F_t is modeled as an ARMA(p,q). That is

$$F_t = \tau + \sum_{j=1}^{p} \psi_j F_{t-j} + \sum_{k=1}^{q} \rho_k \varepsilon_{F,t-k} + \varepsilon_{F,t} \tag{21}$$

where $\varepsilon_{F,t}$ is the regression error term or the unexpected part of the sentiment factor. Following the literature in time series, we optimally selected the lags p and q using the Schwartz criterion. This approach allows us to capture the persistence in sentiment factors, which is particularly relevant for energy markets where expectations about long-term developments evolve gradually.

The variance-covariance matrix of the regression errors is given by

$$\Omega_t = \begin{pmatrix} \sigma_{c,t}^2 & \sigma_{cF,t} \\ \sigma_{cF,t} & \sigma_{F,t}^2 \end{pmatrix}$$

We follow Bollerslev et al. (1988) who proposed a general model for estimating the conditional covariance matrix Ω_t termed VEC model. The VEC(1,1) model is given by

$$h_t = \Gamma + A\xi_{t-1} + Bh_{t-1}$$

where

$$h_t = vech(\Omega_t) = \begin{pmatrix} \sigma_{c,t}^2 \\ \sigma_{cF,t} \\ \sigma_{F,t}^2 \end{pmatrix},$$

$$\xi_t = vech\left(\varepsilon_t \varepsilon_t'\right),$$

where $\varepsilon_t = \left(\varepsilon_{c,t}, \varepsilon_{F,t}\right)'$, $vech(\cdot)$ is an operator that stacks the lower triangular part of a $N \times N$ matrix, and the sizes of Γ, A and B are $\frac{N(N+1)}{2}$ dimensional constant vectors. The total number of parameters is $2\left(\frac{N(N+1)}{2}\right)^2 + \frac{N(N+1)}{2}$. In our case with $N = 2$, the total number of parameters is 21.

To reduce the number of parameters in the VEC model, Bollerslev et al. (1988) suggest the diagonal VEC (DVEC) model where A and B are restricted to be diagonal matrices. The result-

ing restricted model comprehends $\frac{3N(N+1)}{2}$ parameters. That is for $N = 2$, the total number of parameters is 9.

The DVEC model can be rewritten as

$$\Omega_t = \overline{\Gamma} + \overline{A} \odot \left(\varepsilon_{t-1}\varepsilon'_{t-1}\right) + \overline{B} \odot \Omega_{t-1}$$

where $\odot$ is the Hadamard product (element by element product) and $\overline{\Gamma}, \overline{A}$ and $\overline{B}$ are $N \times N$ matrices given by

$$\Gamma = diag(vech\left(\overline{\Gamma}\right)),$$
$$A = diag(vech\left(\overline{A}\right)),$$
$$B = diag(vech\left(\overline{B}\right)),$$

where $\overline{A}$ and $\overline{B}$ are diagonal matrices.

We estimate the model by quasi-maximum likelihood estimator. We use bootstrapped t-statistics and p-values for coefficients significance to ensure robust inference given the complexity of our model specification and the potential for non-standard distributional properties in energy market returns.

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