The Dilution Rate of Government Debt*

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Abstract

America's outstanding maturity structure is a combination of monetary and fiscal balance sheets. The Federal Reserve pairs open market operations with rate policy while the U.S. Treasury varies issuance duration with fiscal policy. This paper introduces the dilution rate of government debt, which measures newly-issued short-term to previously-issued long-term outstanding debt. The concept allows for dynamic maturity management analysis in linearized frameworks. I add a debt dilution rule to an otherwise standard Fiscal Theory of the Price Level model and show that shocks to debt dilution mirror fiscal policy shocks regardless of policy regime.

Keywords: Federal Reserve, U.S. Treasury, Debt maturity management, Fiscal Theory of the Price Level, U.S. inflation.

JEL Classification: E52, E58, E63, H63, C32.

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1 Introduction

The U.S. Federal Reserve (Fed) often deploys quantitative easing (QE) and interest rate cuts simultaneously during severe economic downturns, unleashing a powerful two-pronged approach to monetary stimulus. Slashing the federal funds rate reduces short-term borrowing costs for households, firms and the government, while purchasing long-term assets like U.S. Treasuries injects liquidity into financial markets.

Similarly, the U.S. Treasury tends to accompany expansionary fiscal policy with shorter issuance duration. Short-term debt was used extensively during the Great Financial Crisis (GFC) and again during the COVID pandemic. These effects are felt in the outstanding structure of U.S. debt.

Despite these fiscal and monetary policy marriages between balance sheets and policy tools, a tractable theory describing the direct impact of short-term–for–long-term asset swaps on inflation remains absent, so it is difficult to answer questions like 'does QE contribute to low inflation during recessions?,' 'does quantitative tightening (QT) counteract contractionary monetary policy?,' and 'how should the U.S. finance itself during a crisis?' The main issue arises largely due to the interaction between DSGE linearization and the household's no-arbitrage condition on debt objects of differing maturities.[1] Without additional assumptions on this no-arbitrage condition (e.g., risk premia), these debt objects are approximately perfect substitutes.[2] I overcome the need for additional assumptions and bypass the perfect substitution issue in this paper, developing and outlining theoretical predictions relating maturity management to inflation outcomes that survive linearization.

Cochrane (2001) and Leeper and Leith (2017) develop a general theory of debt management in a constant endowment setting, allowing for an arbitrarily large number of debt maturities to be freely issued by the government. They characterize the equilibrium price level as a weighted sum of current and expected future discounted primary surpluses but with a weight structure difficult to penetrate intuitively. Cochrane (2023) writes, "These formulas likely hide additional interesting

[1] Debt of various maturities can be used to complete markets in non-linear models like those in work by Angeletos (2002), Buera and Nicolini (2004) and Faraglia, Marcet, and Scott (2010). They are also useful insurance tools in non-linear, incomplete-markets models such as those in papers by Lustig, Sleet, and Yeltekin (2008), Debortoli, Nunes, and Yared (2017), and Faraglia et al. (2019)

[2] Woodford (2001) develops a second-best approach by examining a maturity structure consisting of perpetuities with exponentially-decaying coupons where the decay rate is held constant. Leeper and Zhou (2021) vary such a portfolio's rate of decay to compare welfare outcomes from policy across economies with different debt durations.

insights and special cases."

This paper investigates one such case. Limiting government issuance to two periods dramatically simplifies the general formulas in Cochrane's (2001) and Leeper and Leith's (2017) papers and allows for new, exciting perspectives on maturity management as a policy tool.

The main object of interest in this paper is the dilution rate of government debt (hereby referred to as the dilution rate), which measures the amount of outstanding short-term government debt relative to the amount of previously-outstanding, unmatured long-term debt. The name, inspired by concepts discussed in Cochrane's (2023) book, describes how the government dilutes the real value of its unmatured debt when it issues new debt maturing at the same date. The concept is similar to that of a publically-traded company diluting its current market capitalization by issuing additional stock, reducing price per share.

Theoretical models exist separating fiscal and monetary policy within a competitive, general equilibrium setting, beginning with work by Sargent and Wallace (1981). The Fiscal Theory of the Price Level (FTPL) initially developed by Leeper (1991), Sims (1994) and Woodford (1995) characterizes distinct regimes where fiscal and monetary policy commit to follow rules in an economy with nominal government debt. I embed the dilution rate into an otherwise standard FTPL model and assume a monetary-and-fiscal composite rule for its dynamics.

This exercise has two major consequences. First, so long as fiscal policy dilutes sufficiently strongly under a tax cut, the 'passive monetary/active fiscal' region extends well into the previous 'passive monetary/passive fiscal' region of the parameter space.

Second, this analysis allows for study of dilution shocks on inflation and debt dynamics. Such a shock, regardless of the region of the parameter space, mirrors a fiscal shock. A positive dilution rate shock is akin to a negative fiscal policy shock (surprise tax cut). One salient implication of this finding is that a central bank, through the use of debt dilution, can simultaneously shock the policy rate and dilution rate to synthesize coordinated monetary and fiscal policy. For instance, even in the 'passive monetary/active fiscal' determinacy region, a central bank that shocks the policy rate positively (rate hike) may neutralize inflationary pressures from monetary policy through a contemporaneous negative dilution rate shock (a surprise open-market sale).

I divide the paper into three sections. First, I introduce the dilution rate. Second, I embed the dilution rate in an otherwise standard FTPL model. Finally, I display and discuss determinacy

regions and IRFs to policy (monetary, fiscal, and dilution) shocks in various parameter regions.

2 The Dilution Rate of Government Debt

Consider the government's equilibrium flow government budget constraint (GBC) from papers by Cochrane (2001) and Leeper and Leith (2017) implied by the household budget constraint (HHBC), goods market clearing $C_t = Y_t$, and household optimization. Assume the special case where the government can freely borrow in 2 debt instruments and that its maximum available maturity is 2 periods to write the flow GBC as

$$\frac{B_{t-1}^{(t)}}{P_t} + \frac{Q_t^{(t+1)} B_{t-1}^{(t+1)}}{P_t} = \frac{Q_t^{(t+1)} B_t^{(t+1)}}{P_t} + \frac{Q_t^{(t+2)} B_t^{(t+2)}}{P_t} + s_t \tag{1}$$

where $B_t^{(t+j)}$ is outstanding j-period nominal debt with corresponding price $Q_t^{(t+j)}$, where P_t is the economy's aggregate price level, and where $s_t = \tau_t$ is government taxes.

Household optimization over $B_t^{(t+j)}$ determines the price of one-period debt and reveals the no-arbitrage condition $Q_t^{(t+2)} = Q_t^{(t+1)} \mathbb{E}_t Q_{t+1}^{(t+2)}$. Forward-iterate on the market value of debt to write the government's present value condition as

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t} + \mathbb{E}_t \frac{B_{t-1}^{(t+1)}}{r_{t,t+1} P_{t+1}}}_{MV(\text{Debt})/P_t} = \underbrace{\mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}}}_{\mathbb{E}[PV(\text{Prim. Spls.})]} \tag{2}$$

where the j-period real interest rate is defined to be $r_{t,t+j} = \beta^{-j} c_t^{-\sigma} / c_{t+j}^{-\sigma}$. The flow condition (2) says that the real market value of maturing and unmatured government debt must equal the expected discounted sum of current and future government primary surpluses.

The forward iteration outlined thus far is standard in models that examine fiscal policy's role in breaking Ricardian equivalence. Such forward-iteration is sufficient to express P_t alone as a function of current and expected future allocations when the debt takes the form of perpetuities with geometrically declining coupons, introduced in work by Woodford (2001).[3]

The government's present value condition (2) features an expectation including P_{t+1} within the price of unmatured long-term debt. Current and future debt variables $\left\{ \left\{ B_t^{(t+j)}, B_{t+1}^{(t+1+j)}, \cdots \right\}_j \right\}$ load

[3] This set of structures includes the 1-period-only case.

into expectations about future price levels recursively; P_{t+1}'s determination includes expectations about P_{t+2} and so on. I express the government's present value condition so that P_t is exclusively a function of current and expected future allocations. When expectations about future debt policy are uncorrelated with those about primary surpluses, the present value condition (2) becomes

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t}}_{\text{(Mat. Debt)}/P_t} = \underbrace{s_t + \mathbb{E}_t \sum_{i=1}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} \left(1 + \sum_{h=1}^{i} -\Psi_{t,t+h-1}^{-1}\right)}_{\mathbb{E}[PV(\text{Diluted Primary Surpluses})]} \tag{3}$$

where $\Psi_{t,t} = B_t^{(t+1)}/B_{t-1}^{(t+1)}$ is the economy's nominal dilution rate of government debt, following the language from Cochrane's (2023) textbook, and where $\Psi_{t,t+i} = \left(B_t^{(t+1)} \cdots B_{t+i}^{(t+i+1)}\right) / \left(B_{t-1}^{(t+1)} \cdots B_{t+i-1}^{(t+i+1)}\right)$ is the i-period ahead product of dilution rates.[4]

The dilution rate relates the time t stock of outstanding short-term debt to the time $t-1$ stock of outstanding long-term debt. For some, it may be easier to think of the dilution rate as relating short-term to long-term debt outstanding, holding the date of maturity constant.

The rewritten present value condition (3) includes P_t as a function exclusively of current and expected future allocations. Writing the condition this way also transforms the government's time t debt supply decision from $\left(B_t^{(t+1)}, B_t^{(t+2)}\right)$ to $\Psi_{t,t}$. Though the algebraically relevant term in this rewritten condition is not the dilution rate, but the inverse dilution rate $\Psi_{t,t}^{-1}$. The rewritten present value condition (3) collapses to the standard case when a government has no outstanding long-term debt $B_{t-1}^{(t+1)} = 0$. Additionally, it must be the case that the government is expected to issue a non-zero amount of short-term debt each period, or else $\Psi_{t,t+i}^{-1}$ becomes undefined for some i. Interpreting the inverse dilution rate directly is challenging, so I recommend thinking in terms of the (non-inverted) dilution rate and flipping the ratio when examining the algebra.

The government dilutes the real value of its unmatured long-term debt when it issues new short-term debt maturing at the same date, similar to a publicly traded company diluting its current market capitalization by issuing additional stock, reducing price per share.

To give a demonstration in the spirit of Cochrane's (2023) work, a government enters time t with previously-issued long-term debt outstanding $B_{t-1}^{(t+1)} > 0$. The outstanding debt's market value is $Q_t^{(t+1)} B_{t-1}^{(t+1)}$, which includes expectations over P_{t+1}. Absent other changes to current or expected

[4] Covariance terms make the rewritten present value condition (3) less straightforward while adding little expositional value. I relegate the general formula to its derivation in Appendix A.

policy, a budget-neutral increase in $B_t^{(t+1)}$ and decrease in $B_t^{(t+2)}$ increases period $t+1$'s maturing debt and P_{t+1} as a result according to the present value condition (2). Household demand for debt maturing at $t+1$ falls with its real (deflated) payout, so that $Q_t^{(t+1)}$ falls. Short-term issuance dilutes long-term debt through $Q_t^{(t+1)}$ which lessens government indebtedness at time t. P_t falls as a result according to the present value condition (2). The mechanism here reflects a trade-off in the timing of inflation: surprise dilution reduces current prices at the cost of higher future prices.[5]

Importantly, this intuition relies on expectations about future policy remaining constant. Because central banks and fiscal authorities respond to inflation, adding in policy rules flip this intuition on its head. I will introduce and discuss these implications in the next section.

In a steady state with $c_t = c$, $B_t^{(t+j)} = B^{(j)}$ and $s_t = s$, the rewritten present value condition (3) becomes

$$\frac{B^{(1)}}{P} = \left(\frac{1}{1-\beta}\right)\left(\frac{1}{1+\beta\Psi^{-1}}\right)s \tag{4}$$

The steady state present value condition (4) reveals a long-run restriction on the dilution rate that must hold to ensure equilibrium: $|\Psi| > \beta$. Or, equivalently, $|B^{(1)}| > \beta|B^{(2)}|$. There must exist a sufficiently large amount of short-term debt (assets) to ensure equilibrium determinacy. Such a condition is easily met in U.S. data where most outstanding debt (Treasuries + reserves) matures within two years.

3 The Baseline Model

In this section, I describe an economy augmented with rules governing monetary (interest rate) policy, fiscal (tax) policy, and jointly-implemented maturity (dilution) policy.

Before introducing the model, I define the economy's real dilution rate as

$$\psi_{t,t} = \frac{b_t^{(t+1)}}{b_{t-1}^{(t+1)}} = \frac{B_t^{(t+1)}P_{t-1}}{B_{t-1}^{(t+1)}P_t} = \frac{1}{\pi_t}\Psi_{t,t} \tag{5}$$

where $b_t^{(t+j)} = B_t^{(t+j)}/P_t$ represents real (deflated) outstanding debt at time t.

Additionally, for notational convenience in the rest of the paper, I define $a_t = \psi_{t,t}^{-1}$ as the inverse real dilution rate at time t.

[5] A similar exercise is outlined in Cochrane (2023) in a three-period economy.

The model includes a Fisher relation

$$Q_t^{(t+1)} = \beta \mathbb{E}_t \left[\frac{1}{\pi_{t+1}} \right] \tag{6}$$

where R_t is the economy's nominal interest rate.

Debt is priced according to

$$Q_t^{(t+1)} = \frac{1}{R_t} \tag{7}$$

and a no-arbitrage condition exists between short-term and long-term debt

$$Q_t^{(t+2)} = Q_t^{(t+1)} \mathbb{E}_t \left[Q_{t+1}^{(t+2)} \right] \tag{8}$$

Monetary policy follows a rate rule a la Taylor (1993)

$$R_t = \bar{R} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\alpha} \exp(z_t^R) \tag{9}$$

while fiscal policy follows a tax rule a la Leeper (1991)

$$\tau_t = \bar{\tau} \left(\frac{b_{t-1}^{(t)}}{\bar{b}} \right)^{\gamma} \exp(z_t^\tau) \tag{10}$$

Both policy authorities jointly adjust the dilution rate of government debt according to

$$a_t = \bar{a} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi} \left(\frac{b_{t-1}^{(t)}}{\bar{b}} \right)^{\delta} \exp(z_t^a) \tag{11}$$

where $\phi > 0$ regulates the strength to which the central bank engages in open market sales (purchases) when inflation rises (falls), and where $\delta > 0$ regulates the strength to which the fiscal authority lengthens (shortens) its issuance when outstanding debt rises (falls).[6]

Policy is conducted subject to the consolidated government budget constraint

$$\frac{b_{t-1}^{(t)}}{\pi_t} + \frac{Q_t^{(t+1)} a_t b_t^{(t+1)}}{\pi_t} = \tau_t + Q_t^{(t+1)} b_t^{(t+1)} + Q_t^{(t+2)} \mathbb{E}_t \left[a_{t+1} b_{t+1}^{(t+2)} \right] \tag{12}$$

[6]The components of the dilution rule are proxies for the previous, individual policy rules.

and policy disturbances are assumed to decay at a constant rate

$$z_t^R = \rho_R z_{t-1}^R + \varepsilon_t^R \tag{13}$$

$$z_t^\tau = \rho_\tau z_{t-1}^\tau + \varepsilon_t^\tau \tag{14}$$

$$z_t^a = \rho_a z_{t-1}^a + \varepsilon_t^a \tag{15}$$

where $\rho_R > 0$, $\rho_\tau > 0$, and $\rho_a > 0$. And where ε_t^R, ε_t^R, and ε_t^R are all mean-zero, normally-distributed shocks.

Appendix B displays log-linearized versions of this model. I solve the model using the Gensys algorithm developed by Sims (2002).

4 Analysis

4.1 Calibration

I begin the analysis by calibrating the model according to Table 1.

Parameter	**Symbol**	**Value/Range**	**Description**
Fixed Parameters			
Discount factor	β	0.99	Discount factor
Steady-state inflation	$\bar{\pi}$	1.0	Steady state inflation
Steady-state dilution	$\bar{a}$	0.9	Steady state dilution rate
MP shock persistence	ρ_R	0.5	AR(1) coefficient for monetary shock
FP shock persistence	ρ_τ	0.5	AR(1) coefficient for fiscal shock
Debt shock persistence	ρ_a	0.5	AR(1) coefficient for debt policy shock
Policy Parameters (Varied)			
Monetary policy	α	[0.1, 2.0]	Taylor rule response to inflation
Fiscal policy	γ	[0.1, 2.0]	Tax response to real debt
Monetary dilution policy	ϕ	[0, 2.0]	Dilutive response to inflation
Fiscal dilution policy	δ	[0, 2.0]	Dilutive response to real debt
Structural Shocks (Standard Deviations)			
Monetary policy shock	ε_t^R	1.0	One standard deviation shock
Fiscal policy shock	ε_t^τ	1.0	One standard deviation shock
Debt policy shock	ε_t^a	1.0	One standard deviation shock

Table 1: Model Calibration and Parameter Ranges

4.2 Exploring the Parameter Space

A main result of FTPL is the existence of the 'passive monetary/active fiscal' policy regime. Absent long-term debt in this model, the 'passive monetary/active fiscal' regime would correspond to the region where $\alpha < 1$ (a relatively weak Taylor rule coefficient) and $\gamma < 1$ (fiscal policy that's relatively unresponsive to maturing debt). I vary the parameters α, γ, ϕ, and δ in order to explore the parameter space given long-term debt and a policy rule on dilution (11). Figure reveals that, without losses of determinacy in the 'active monetary/passive fiscal' or 'passive monetary/active fiscal' regions, the previously-indeterminant 'passive monetary/passive fiscal' region now also contains viable, bounded and existing equilibria.

4.3 Impulse Response Functions

How do shocks to debt dilution affect inflation and debt dynamics? Figures 2 and 3 show inflation and real debt IRFs when responding to policy rate shocks, tax shocks, and dilution rate shocks. Figure 2 shows these IRFs in the 'active monetary/passive fiscal' region while Figure 3 contains IRFs in the 'passive monetary/active fiscal' region.

I include IRFs conditional on dilutive responses from both policy authorities being relatively weak. Raising these parameters does not materially change the analysis. The effects of monetary and fiscal policy shocks on inflation and real debt are consistent with standard FTPL models with short-term debt. The monetary policy shock on inflation in the 'passive monetary/active fiscal' regime represents the 'stepping on a rake' effect discussed by Sims (2011) and Cochrane (2018). Responses by inflation and real debt to a shock to debt dilution mirror those to fiscal policy shocks regardless of the policy regime.

In the 'active monetary/passive fiscal' regime, a positive shock to inverse dilution (negative shock to dilution) is similar to a tax hike: real debt falls from added fiscal revenue while inflation is unaffected due to a strong monetary response.

In the 'passive monetary/active fiscal' regime, a positive shock to inverse dilution (negative shock to dilution) is again similar to a tax hike: real debt rises as inflation falls. Expectations on the future path of taxes rise as a response to dilution, driving inflation down - despite the intuition established in Section 2.

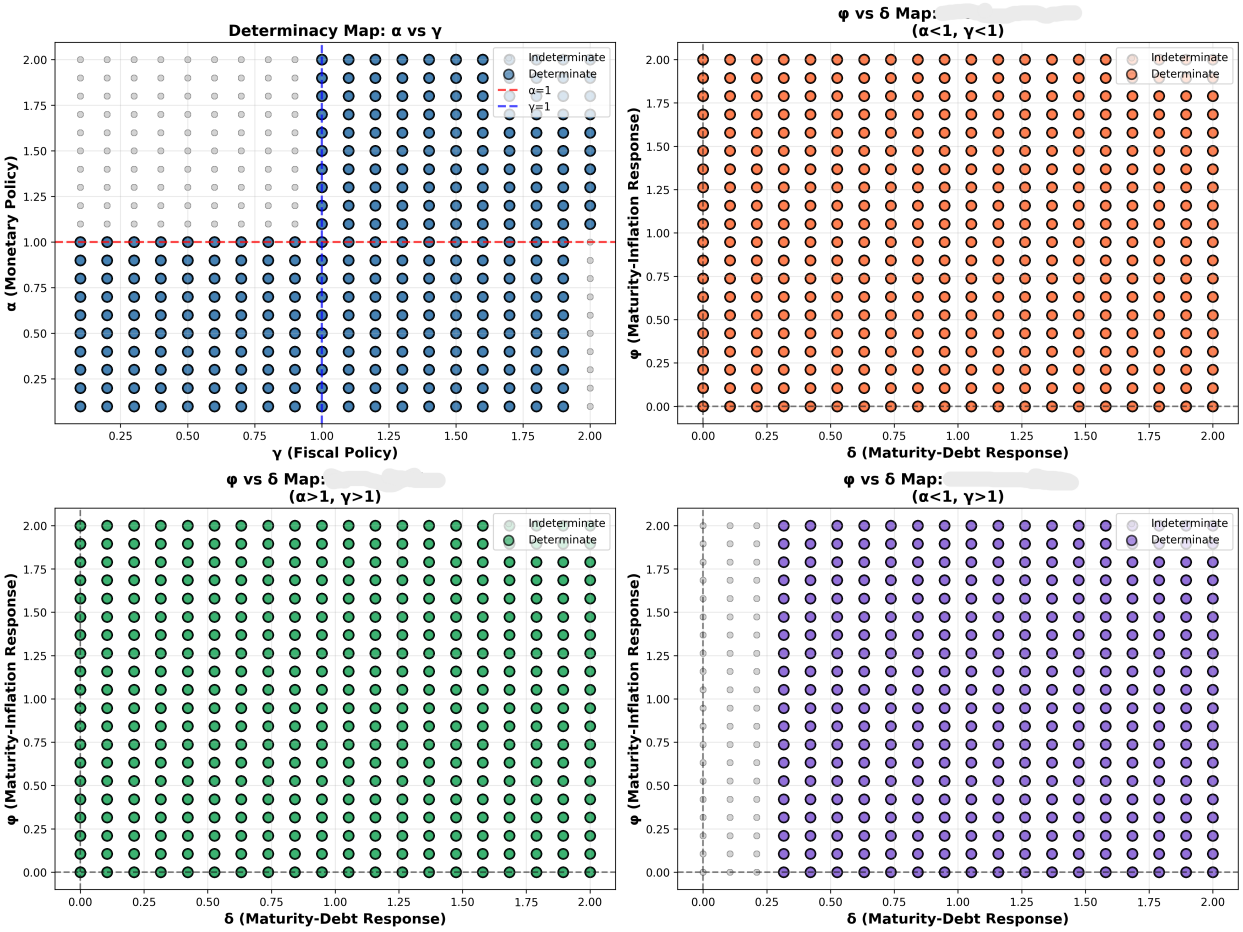


Figure 1: Determinacy Maps Across Policy Regimes

The figure displays determinacy regions across different policy parameter combinations. **Top-left panel:** Determinacy map in the (γ, α) space showing the interaction between fiscal and monetary policy stance. Blue circles indicate determinate equilibria, while gray circles represent indeterminate equilibria. The dashed lines at $\alpha = 1$ and $\gamma = 1$ delineate the boundaries between active and passive policy regimes. **Top-right panel:** Determinacy in the (δ, ϕ) space conditional on passive monetary and active fiscal policy ($\alpha < 1$, $\gamma < 1$). **Bottom-left panel:** Determinacy in the (δ, ϕ) space conditional on active monetary and passive fiscal policy ($\alpha > 1$, $\gamma > 1$). **Bottom-right panel:** Determinacy in the (δ, ϕ) space conditional on passive monetary and passive fiscal policy ($\alpha < 1$, $\gamma > 1$). The dilution parameters ϕ and δ capture the sensitivity of debt maturity structure to inflation and real debt levels, respectively. All parameter combinations are searched on a grid with $n = 20$ points per dimension.

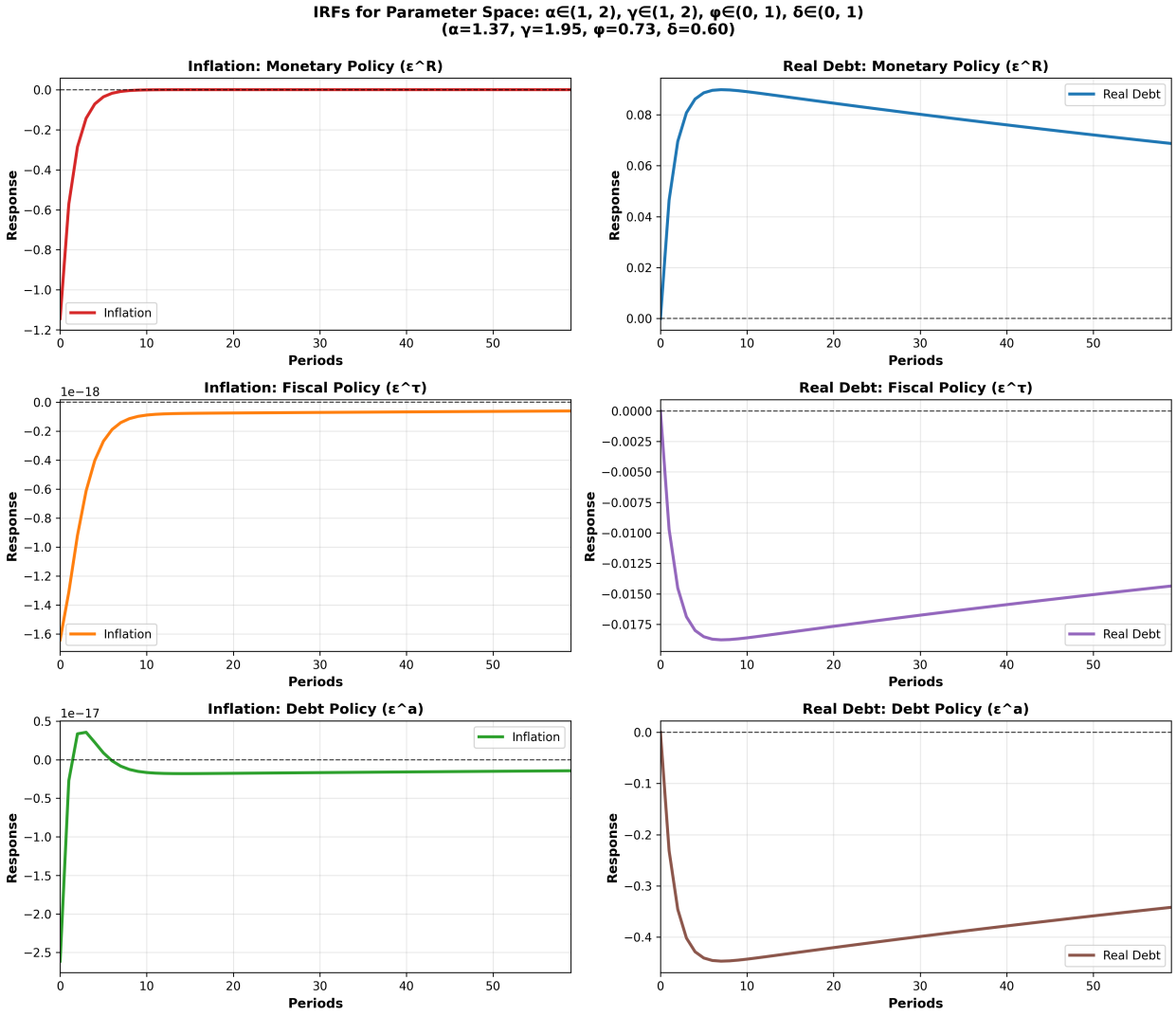


Figure 2: Impulse Response Functions: Active Monetary and Active Fiscal Policy

Notes: This figure displays impulse response functions (IRFs) for inflation and real debt following one standard deviation shocks to monetary policy (ε^R), fiscal policy (ε^τ), and debt maturity policy (ε^a). The parameter space corresponds to active monetary policy and passive fiscal policy: $\alpha \in (1,2)$, $\gamma \in (1,2)$, $\phi \in (0,1)$, $\delta \in (0,1)$. The specific parameter values are $\alpha = 1.37$, $\gamma = 1.95$, $\phi = 0.73$, $\delta = 0.60$. **Top row:** Responses to a monetary policy shock show negative inflation response and positive real debt response with slow convergence. **Middle row:** Fiscal policy shock generates negative inflation response and negative real debt response with persistent dynamics. **Bottom row:** Debt policy shock produces near-zero inflation response and negative real debt response.

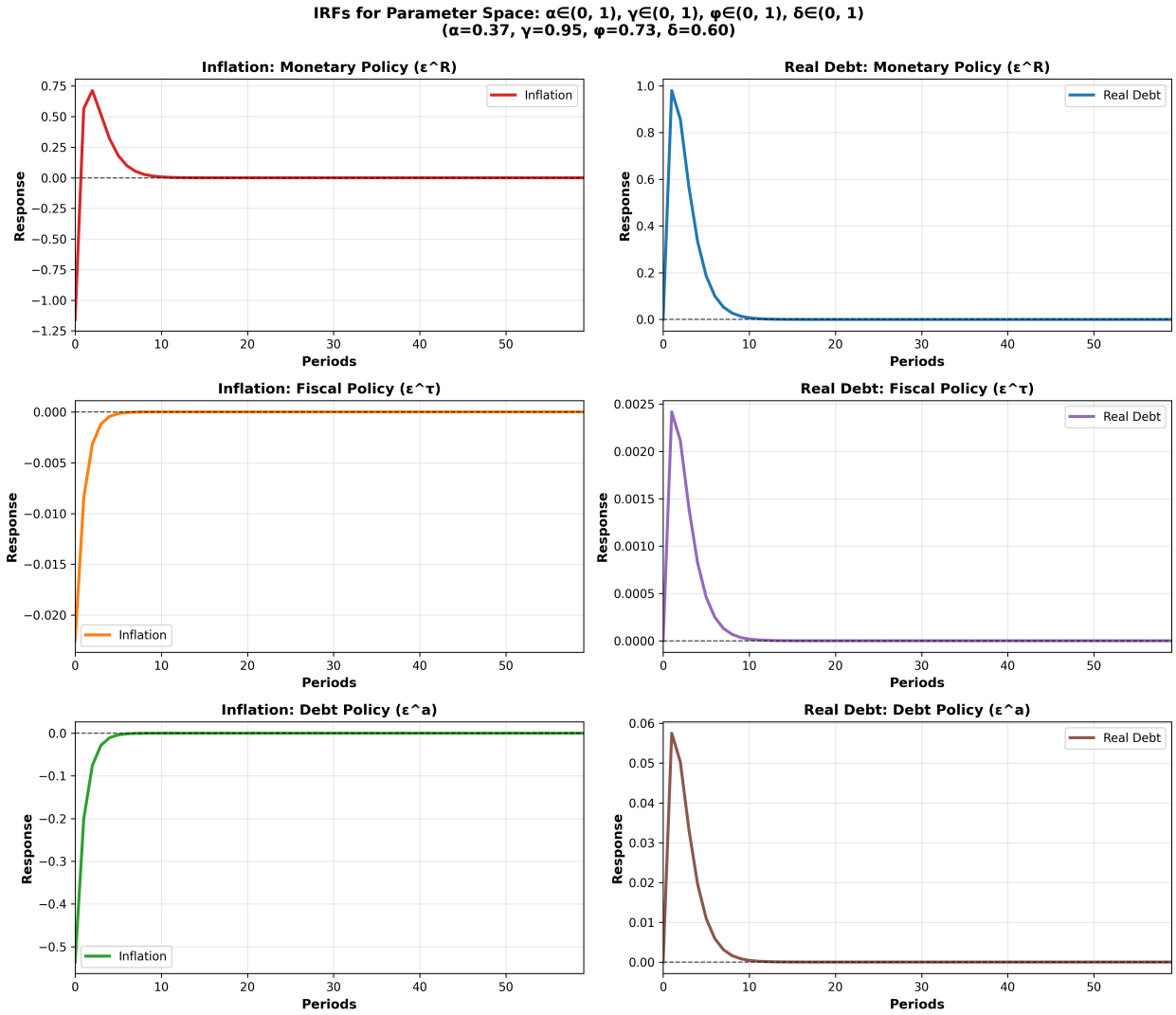


Figure 3: Impulse Response Functions: Passive Monetary and Passive Fiscal Policy

Notes: This figure displays impulse response functions (IRFs) for inflation and real debt following one standard deviation shocks to monetary policy (ε^R), fiscal policy (ε^τ), and debt maturity policy (ε^a). The parameter space corresponds to passive monetary policy and active fiscal policy: $\alpha \in (0,1)$, $\gamma \in (0,1)$, $\phi \in (0,1)$, $\delta \in (0,1)$. The specific parameter values are $\alpha = 0.37$, $\gamma = 0.95$, $\phi = 0.73$, $\delta = 0.60$. **Top row:** Responses to a monetary policy shock show positive inflation response and positive real debt response. **Middle row:** Fiscal policy shock generates small negative inflation response and small positive real debt response. **Bottom row:** Debt policy shock produces negative inflation response and positive real debt response.

In the U.S., monetary and fiscal policy both have tools to 'shock' the dilution rate of government debt. This section's results imply that these tools can be used to offset negative effects and/or amplify the effects from their individual policy shocks. One such example is that a central bank that shocks the policy rate positively (rate hike) neutralizes inflationary pressures from monetary policy through a contemporaneous negative dilution rate shock (a surprise open-market sale).

5 Conclusion

This paper introduces a new concept to the literature on public debt management. The dilution rate of government debt measures the amount of currently outstanding short-term debt to previously-issued, unmatured long-term debt. Such a concept is useful for three reasons. First, it can be widely applied to economic models with nominal debt. Second, it survives log-linear approximations common in much of modern macroeconomics. Third, it comes with clean interpretations and tractable algebra. Such an innovation improves on Woodford's (2001) constantly geometrically-declining structure because it allows for analysis of dynamic maturity responses to unexpected economic shocks.

I apply this concept an otherwise-standard FTPL model and investigate theoretical inflation and real debt dynamics from shocks to dilution. These results support the idea that surprise open market operations or changes to issuance duration may aid government branches achieve policy objectives.

Moving forward, it may be useful to apply the dilution rate of government debt to existing theories of optimal debt management, such as those with complete markets economies like Buera and Nicolini (2004) and incomplete markets economics like Lustig, Sleet, and Yeltekin (2008), Debortoli, Nunes, and Yared (2017) and Faraglia et al. (2019). Such a re-interpretation may connect those theories to yet-to-come linearized theories of optimal debt responses to shocks.

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A Appendix: Deriving the Rewritten Present Value Condition

This section derives the rewritten present value condition (3) from the text, as in the paper by Cochrane (2001). I begin with the well-known present value condition (2):

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t} + \mathbb{E}_t \frac{B_{t-1}^{(t+1)}}{r_{t,t+1} P_{t+1}}}_{MV(\text{Debt})/P_t} = \underbrace{\mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}}}_{\mathbb{E}[PV(\text{Prim. Spls.})]}$$

and subtract both sides by the market value of unmatured debt:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \frac{B_{t-1}^{(t+1)}}{r_{t,t+1} P_{t+1}}$$

then multiply the right-most term by $1 = B_t^{(t+1)} / B_t^{(t+1)}$ to write:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \left(\frac{B_{t-1}^{(t+1)}}{r_{t,t+1} B_t^{(t+1)}} \right) \frac{B_t^{(t+1)}}{P_{t+1}}$$

noticing that $B_t^{(t+1)}/P_{t+1}$ is $B_{t-1}^{(t)}/P_t$ iterated one period into the future.

Apply Section 2's definition of the nominal dilution rate to write:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \left(\frac{\Psi_{t,t}^{-1}}{r_{t,t+1}} \right) \frac{B_t^{(t+1)}}{P_{t+1}}$$

and forward-iterate once on this condition:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \left(\frac{\Psi_{t,t}^{-1}}{r_{t,t+1}} \right) \left[\sum_{i=0}^{\infty} \frac{s_{t+1+i}}{r_{t+1,t+1+i}} - \left(\frac{\Psi_{t+1,t+1}^{-1}}{r_{t+1,t+2}} \right) \frac{B_{t+1}^{(t+2)}}{P_{t+2}} \right]$$

which simplifies to:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \Psi_{t,t}^{-1} \sum_{i=0}^{\infty} \frac{s_{t+1+i}}{r_{t,t+1+i}} + \mathbb{E}_t \left(\frac{\Psi_{t,t+1}^{-1}}{r_{t,t+2}} \right) \frac{B_{t+1}^{(t+2)}}{P_{t+2}}$$

and continuing this forward-iteration reveals:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \Psi_{t,t}^{-1} \sum_{i=0}^{\infty} \frac{s_{t+1+i}}{r_{t,t+1+i}} + \mathbb{E}_t \Psi_{t,t+1}^{-1} \sum_{i=0}^{\infty} \frac{s_{t+2+i}}{r_{t,t+2+i}} - \mathbb{E}_t \Psi_{t,t+2}^{-1} \sum_{i=0}^{\infty} \frac{s_{t+3+i}}{r_{t,t+3+i}} + \cdots - \cdots$$

so that I can write:

$$\frac{B_{t-1}^{(t)}}{P_t} = s_t + \left(\mathbb{E}_t \frac{s_{t+1}}{r_{t,t+1}} - \Psi_{t,t}^{-1} \mathbb{E}_t \frac{s_{t+1}}{r_{t,t+1}} \right) + \left(\mathbb{E}_t \frac{s_{t+2}}{r_{t,t+2}} - \Psi_{t,t}^{-1} \mathbb{E}_t \frac{s_{t+2}}{r_{t,t+2}} + \mathbb{E}_t \Psi_{t,t+1}^{-1} \frac{s_{t+2}}{r_{t,t+2}} \right) + \cdots$$

where, starting with the term $\mathbb{E}_t \Psi_{t,t+1}^{-1} \frac{s_{t+2}}{r_{t,t+2}}$, expectations of future debt policy may be correlated with that of future primary surpluses. If we allow this covariance to be zero, we can freely factor $\Psi_{t,t+i}^{-1}$ out of every $\mathbb{E}_t \Psi_{t,t+i-1}^{-1} \frac{s_{t+i}}{r_{t,t+i}}$ term and group terms by s_{t+i} to write equation (3):

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t}}_{\text{(Mat. Debt)}/P_t} = \underbrace{s_t + \mathbb{E}_t \sum_{i=1}^{\infty} \frac{s_{t+i}}{r_{t,t+i}} \left(1 + \sum_{h=1}^{i} -\Psi_{t,t+h-1}^{-1} \right)}_{\mathbb{E}[PV(\text{Diluted Primary Surpluses})]} \tag{3}$$

such that the household TVC holds and that the long-run condition $|B_t^{(t+1)}| > \beta |B_t^{(t+2)}|$ as $t \to \infty$ holds.

B Appendix: Displaying the Log-Linearized Model

This section displays the steps taken to prepare the linearized model for analysis. The first step is to linearize the equations described in Section 3 of the text:

$$Q_t = -\mathbb{E}_t \left[\pi_{t+1}\right]$$

$$Q_t = -R_t$$

$$R_t = \alpha \pi_t + z_t^R$$

$$\tau_t = \gamma B_t + z_t^\tau$$

$$a_t = \phi \pi_t + \delta B_t + z_t^a$$

$$B_t - \pi_t + \frac{\beta a}{\pi} \left(a_t + B_{t+1} - \pi_t - E_t \left[\pi_{t+1}\right]\right)$$

$$= \left(\frac{\pi \tau}{b}\right) \tau_t + \beta \left(B_{t+1} - \mathbb{E}_t \left[\pi_{t+1}\right]\right) + \frac{\beta^2 a}{\pi} \left(\mathbb{E}_t \left[a_{t+1}\right] + \mathbb{E}_t \left[B_{t+2}\right] - \mathbb{E}_t \left[\pi_{t+1}\right] - E_t \left[\pi_{t+2}\right]\right)$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^\tau = \rho_\tau z_t^\tau + \varepsilon_{t+1}^\tau$$

$$z_{t+1}^a = \rho_a z_t^a + \varepsilon_{t+1}^a$$

where, at the risk of displaying potentially confusing notation, $B_t = b_{t-1}^{(t)}$

Now define $x_t = B_{t+1}$ to write this system as:

$$\mathbb{E}_t \left[\pi_{t+1}\right] = \alpha \pi_t + z_t^R$$

$$\tau_t = \gamma B_t + z_t^\tau$$

$$a_t = \phi \pi_t + \delta B_t + z_t^a$$

$$B_t - \pi_t + \frac{\beta a}{\pi} \left(a_t + B_{t+1} - \pi_t - E_t \left[\pi_{t+1}\right]\right)$$

$$= \left(\frac{\pi \tau}{b}\right) \tau_t + \beta \left(B_{t+1} - \mathbb{E}_t \left[\pi_{t+1}\right]\right) + \frac{\beta^2 a}{\pi} \left(\mathbb{E}_t \left[a_{t+1}\right] + \mathbb{E}_t \left[x_{t+1}\right] - \mathbb{E}_t \left[\pi_{t+1}\right] - E_t \left[\pi_{t+2}\right]\right)$$

$$x_t = B_{t+1}$$

$$z^R_{t+1} = \rho_R z^R_t + \varepsilon^R_{t+1}$$

$$z^\tau_{t+1} = \rho_\tau z^\tau_t + \varepsilon^\tau_{t+1}$$

$$z^a_{t+1} = \rho_a z^a_t + \varepsilon^a_{t+1}$$

After substituting in policy rules, we have:

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \alpha\pi_t + z^R_t$$

$$B_t - \pi_t + \frac{\beta a}{\pi}\left(\left(\phi\pi_t + \delta B_t + z^a_t\right) + B_{t+1} - \pi_t - E_t\left[\pi_{t+1}\right]\right)$$

$$= \left(\frac{\pi\tau}{b}\right)\left(\gamma B_t + z^\tau_t\right) + \beta\left(B_{t+1} - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + \frac{\beta^2 a}{\pi}\left(\mathbb{E}_t\left[\phi\pi_{t+1} + \delta B_{t+1} + z^a_{t+1}\right] + \mathbb{E}_t\left[x_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+1}\right] - E_t\left[\pi_{t+2}\right]\right)$$

$$x_t = B_{t+1}$$

$$z^R_{t+1} = \rho_R z^R_t + \varepsilon^R_{t+1}$$

$$z^\tau_{t+1} = \rho_\tau z^\tau_t + \varepsilon^\tau_{t+1}$$

$$z^a_{t+1} = \rho_a z^a_t + \varepsilon^a_{t+1}$$

Simplify the GBC slightly to write:

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \alpha\pi_t + z^R_t$$

$$B_t - \pi_t + \frac{\beta a}{\pi}\left(\left(\phi\pi_t + \delta B_t + z^a_t\right) + B_{t+1} - \pi_t - E_t\left[\pi_{t+1}\right]\right)$$

$$= \left(\frac{\pi\tau}{b}\right)\left(\gamma B_t + z^\tau_t\right) + \beta\left(B_{t+1} - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + \frac{\beta^2 a}{\pi}\left(\phi\mathbb{E}_t\left[\pi_{t+1}\right] + \delta B_{t+1} + \rho_a z^a_t + \mathbb{E}_t\left[x_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+1}\right] - E_t\left[\pi_{t+2}\right]\right)$$

$$w_t = \pi_{t+1}$$

$$x_t = B_{t+1}$$

$$z^R_{t+1} = \rho_R z^R_t + \varepsilon^R_{t+1}$$

$$z^\tau_{t+1} = \rho_\tau z^\tau_t + \varepsilon^\tau_{t+1}$$

$$z^a_{t+1} = \rho_a z^a_t + \varepsilon^a_{t+1}$$

Define the forecast errors η^π_{t+1} and η^x_{t+1} such that:

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \pi_{t+1} - \eta^\pi_{t+1}$$

$$\mathbb{E}_t\left[x_{t+1}\right] = x_{t+1} - \eta^x_{t+1}$$

and substitute these into the system to write:

$$\pi_{t+1} - \eta^\pi_{t+1} = \alpha\pi_t + z^R_t$$

$$B_t - \pi_t + \frac{\beta a}{\pi}\left(\left(\phi\pi_t + \delta B_t + z^a_t\right) + B_{t+1} - \pi_t - \pi_{t+1} + \eta^\pi_{t+1}\right) = \left(\frac{\pi\tau}{b}\right)\left(\gamma B_t + z^\tau_t\right)$$

$$+\beta\left(B_{t+1} - \pi_{t+1} + \eta^\pi_{t+1}\right) + \frac{\beta^2 a}{\pi}\left(\phi\left(\pi_{t+1} - \eta^\pi_{t+1}\right) + \delta B_{t+1} + \rho_a z^a_t + x_{t+1} - \eta^x_{t+1} - \pi_{t+1} + \eta^\pi_{t+1} - E_t\left[\pi_{t+2}\right]\right)$$

$$x_t = B_{t+1}$$

$$z^R_{t+1} = \rho_R z^R_t + \varepsilon^R_{t+1}$$

$$z^\tau_{t+1} = \rho_\tau z^\tau_t + \varepsilon^\tau_{t+1}$$

$$z^a_{t+1} = \rho_a z^a_t + \varepsilon^a_{t+1}$$

Next we can substitute the expression for π_{t+1} into the GBC for π_{t+2} to write:

$$\pi_{t+1} = \alpha\pi_t + z_t^R + \eta_{t+1}^\pi$$

$$B_t - \pi_t + \frac{\beta a}{\pi}\left((\phi\pi_t + \delta B_t + z_t^a) + B_{t+1} - \pi_t - \pi_{t+1} + \eta_{t+1}^\pi\right) = \left(\frac{\pi\tau}{b}\right)(\gamma B_t + z_t^\tau)$$

$$+\beta\left(B_{t+1} - \pi_{t+1} + \eta_{t+1}^\pi\right) + \frac{\beta^2 a}{\pi}\left(\phi\left(\pi_{t+1} - \eta_{t+1}^\pi\right) + \delta B_{t+1} + \rho_a z_t^a + x_{t+1} - \eta_{t+1}^x - \pi_{t+1} + \eta_{t+1}^\pi\right.$$

$$\left.-\alpha\left(\pi_{t+1} - \eta_{t+1}^\pi\right) + \rho_R z_t^R\right)$$

$$x_t = B_{t+1}$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^\tau = \rho_\tau z_t^\tau + \varepsilon_{t+1}^\tau$$

$$z_{t+1}^a = \rho_a z_t^a + \varepsilon_{t+1}^a$$

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \pi_{t+1} - \eta_{t+1}^\pi$$

$$\mathbb{E}_t\left[x_{t+1}\right] = x_{t+1} - \eta_{t+1}^x$$

Now we can group terms in the GBC to write the system out as

$$\pi_{t+1} = \alpha\pi_t + z_t^R + \eta_{t+1}^\pi$$

$$\pi_{t+1}\left(\beta - \frac{\beta a}{\pi} + \frac{\beta^2 a}{\pi} + \frac{\beta^2 a\alpha}{\pi} - \frac{\beta^2 a\phi}{\pi}\right) + B_{t+1}\left(\frac{\beta a}{\pi} - \beta - \frac{\beta^2 a\delta}{\pi}\right) + x_{t+1}\left(-\frac{\beta^2 a}{\pi}\right)$$

$$= \pi_t\left(1 - \frac{\beta a\phi}{\pi} + \frac{\beta a}{\pi}\right) + B_t\left(\frac{\pi\tau\gamma}{b} - 1 - \frac{\beta a\delta}{\pi}\right) + x_t\left(0\right) + z_t^R\left(\frac{\beta^2 a\rho_R}{\pi}\right) + z_t^\tau\left(\frac{\pi\tau}{b}\right) + z_t^a\left(\frac{\beta^2 a\rho_a}{\pi} - \frac{\beta a}{\pi}\right)$$

$$+\eta_{t+1}^\pi\left(\beta - \frac{\beta a}{\pi} + \frac{\beta^2 a}{\pi} + \frac{\beta^2 a\alpha}{\pi} - \frac{\beta^2 a\phi}{\pi}\right) + \eta_{t+1}^x\left(-\frac{\beta^2 a}{\pi}\right)$$

$$x_t = B_{t+1}$$

$$z^R_{t+1} = \rho_R z^R_t + \varepsilon^R_{t+1}$$

$$z^\tau_{t+1} = \rho_\tau z^\tau_t + \varepsilon^\tau_{t+1}$$

$$z^a_{t+1} = \rho_a z^a_t + \varepsilon^a_{t+1}$$

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \pi_{t+1} - \eta^\pi_{t+1}$$

$$\mathbb{E}_t\left[x_{t+1}\right] = x_{t+1} - \eta^x_{t+1}$$

Noticing that $\frac{\pi\tau}{b} = (1-\beta)(1+\frac{\beta a}{\pi})$, we can rewrite this system as:

$$\pi_{t+1} = \alpha\pi_t + z^R_t + \eta^\pi_{t+1}$$

$$\pi_{t+1}\left(\beta - \frac{\beta a}{\pi} + \frac{\beta^2 a}{\pi} + \frac{\beta^2 a\alpha}{\pi} - \frac{\beta^2 a\phi}{\pi}\right) + B_{t+1}\left(\frac{\beta a}{\pi} - \beta - \frac{\beta^2 a\delta}{\pi}\right) + x_{t+1}\left(-\frac{\beta^2 a}{\pi}\right)$$

$$= \pi_t\left(1 - \frac{\beta a\phi}{\pi} + \frac{\beta a}{\pi}\right) + B_t\left(\gamma\left(1-\beta\right)\left(1+\frac{\beta a}{\pi}\right) - 1 - \frac{\beta a\delta}{\pi}\right) + x_t\left(0\right) + z^R_t\left(\frac{\beta^2 a\rho_R}{\pi}\right) + z^\tau_t\left(1-\beta\right)\left(1+\frac{\beta a}{\pi}\right)$$

$$+ z^a_t\left(\frac{\beta^2 a\rho_a}{\pi} - \frac{\beta a}{\pi}\right) + \eta^\pi_{t+1}\left(\beta - \frac{\beta a}{\pi} + \frac{\beta^2 a}{\pi} + \frac{\beta^2 a\alpha}{\pi} - \frac{\beta^2 a\phi}{\pi}\right) + \eta^x_{t+1}\left(-\frac{\beta^2 a}{\pi}\right)$$

$$x_t = B_{t+1}$$

$$z^R_{t+1} = \rho_R z^R_t + \varepsilon^R_{t+1}$$

$$z^\tau_{t+1} = \rho_\tau z^\tau_t + \varepsilon^\tau_{t+1}$$

$$z^a_{t+1} = \rho_a z^a_t + \varepsilon^a_{t+1}$$

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \pi_{t+1} - \eta^\pi_{t+1}$$

$$\mathbb{E}_t\left[x_{t+1}\right] = x_{t+1} - \eta^x_{t+1}$$

Simplifying coefficients in the GBC and multiplying the GBC by π gets us:

$$\pi_{t+1} = \alpha\pi_t + z_t^R + \eta_{t+1}^{\pi}$$

$$\beta\left[\pi - a\left(1-\beta\left(1+\alpha-\phi\right)\right)\right]\pi_{t+1} + \beta\left[a\left(1-\beta\delta\right)-\pi\right]B_{t+1} - \left(\beta^2 a\right)x_{t+1}$$

$$= \left[\pi + \beta a\left(1-\phi\right)\right]\pi_t + \left[\gamma\left(1-\beta\right)\left(\pi+\beta a\right) - \pi - \beta a\delta\right]B_t + \left(0\right)x_t + \left[\beta^2 a\rho_R\right]z_t^R + \left[\left(1-\beta\right)\left(\pi+\beta a\right)\right]z_t^{\tau}$$

$$+ \left[\beta a\left(1-\beta\rho_a\right)\right]z_t^a + \beta\left[\pi - a\left(1-\beta\left(1+\alpha-\phi\right)\right)\right]\eta_{t+1}^{\pi} - \left(\beta^2 a\right)\eta_{t+1}^{x}$$

$$B_{t+1} = x_t$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^{\tau} = \rho_{\tau} z_t^{\tau} + \varepsilon_{t+1}^{\tau}$$

$$z_{t+1}^a = \rho_a z_t^a + \varepsilon_{t+1}^a$$

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \pi_{t+1} - \eta_{t+1}^{\pi}$$

$$\mathbb{E}_t\left[x_{t+1}\right] = x_{t+1} - \eta_{t+1}^{x}$$

Next we can write this system in canonical form:

$$\Gamma_0 y_{t+1} = \Gamma_1 y_t + \Psi\varepsilon_{t+1} + \Pi\eta_{t+1}$$

Where:

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \beta\left[\pi - a\left(1-\beta\left(1+\alpha-\phi\right)\right)\right] & \beta\left[a\left(1-\beta\delta\right)-\pi\right] & -\beta^2 a & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \alpha & 0 & 0 & 1 & 0 & 0 \\ \left[\pi + \beta a\left(1-\phi\right)\right] & \left[\gamma\left(1-\beta\right)\left(\pi+\beta a\right) - \pi - \beta a\delta\right] & 0 & \left[\beta^2 a\rho_R\right] & \left[\left(1-\beta\right)\left(\pi+\beta a\right)\right] & \left[\beta a\left(1-\beta\rho_a\right)\right] \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_R & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_\tau & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_a \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad \Pi = \begin{bmatrix} 1 & 0 \\ \beta\left[\pi - a\left(1-\beta\left(1+\alpha-\phi\right)\right)\right] & -\beta^2 a \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

And:

$$y_t = \begin{bmatrix} \pi_t \\ B_t \\ x_t \\ z_t^R \\ z_t^\tau \\ z_t^a \end{bmatrix} \quad , \quad \varepsilon_t = \begin{bmatrix} \varepsilon_t^R \\ \varepsilon_t^\tau \\ \varepsilon_t^a \end{bmatrix} \quad , \quad \eta_t = \begin{bmatrix} \eta_t^\pi \\ \eta_t^x \end{bmatrix}$$

Define Φ:

$$\Phi = [\Psi\ \bar{0}\ \Pi]$$

Where

$$\bar{0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'$$

Then we can rewrite the canonical form:

$$\Gamma_0 x_{t+1} = \Gamma_1 x_t + \Phi \xi_{t+1}$$

Where:

$$\xi_t = \begin{bmatrix} \varepsilon_t^R & \varepsilon_t^\tau & \varepsilon_t^a & 0 & \eta_t^\pi & \eta_t^x \end{bmatrix}'$$

C Appendix: Data Sources and Construction

This paper uses four main time series from 1949Q1–2022Q4: U.S. tax rate, Fed's policy rate, U.S. real debt dilution rate, and U.S. price levels. Their construction and data sources are below:

- U.S. Tax Rate, constructed as total U.S. receipts over U.S. GDP:
  - Receipts: Section 3, table T30200-Q, line 40 in the National Income and Product Accounts (NIPA) from the BEA and found at `https://apps.bea.gov/iTable/?isuri=1&reqid=19&step=4&categories=flatfiles&nipa_table_list=1`.
  - GDP: Gross domestic product. Section 1, table T10105-Q, line 1 in the National Income and Product Accounts (NIPA) from the BEA. Found at `https://apps.bea.gov/iTable/?isuri=1&reqid=19&step=4&categories=flatfiles&nipa_table_list=1`.
- Fed's Policy Rate, average NY Fed discount rates from 1949Q1–1954Q2 and average Fed Funds rates from 1954Q3–2022Q4:
  - NY Fed discount rates: National Bureau of Economic Research, Release: NBER Macrohistory Database. `https://fred.stlouisfed.org/series/M13009USM156NNBR`
  - Fed Funds effective rate: Source: Board of Governors of the Federal Reserve System (US), Release: H.15 Selected Interest Rates. `https://fred.stlouisfed.org/series/FEDFUNDS`
- U.S. Inflation (1943–2023)
  - Growth rate in annual GDP deflator. Section 1, table T10109-A, line 1 in the NIPA from the BEA. Found at `https://apps.bea.gov/iTable/?isuri=1&reqid=19&step=4&categories=flatfiles&nipa_table_list=1`
- U.S. Real Dilution Rate
  - Constructed as previous-period [(long-term debt outstanding)/(price level)] divided by [(short-term debt outstanding)/(price level)].
  - Long-term debt is Privately-Held Treasurys maturing in more than one quarter.

– Short-term debt calculated as (Reserves Outstanding) + (Privately-Held Treasurys maturing in less than one quarter).

  * Privately-Held Debt (1948Q4–2022Q4). Center for Research in Security Prices (CRSP) U.S. Treasury Database

  * Reserves Outstanding (1949Q1–2002Q4). Calculated as the average reserves outstanding/GDP from 2002–2022 multiplied by GDP from 1942–2001. Member Bank Reserve Account. Source: Center for Financial Stability Release: The Federal Reserve System's Weekly Balance Sheet Since 1914. `https://fred.stlouisfed.org/series/LDMB`

  * Reserves Outstanding (2003Q1–2022Q4). Calculated as (Federal Reserve Notes, Net of F.R. Bank Holdings) + (Deposits with F.R. Banks, Other Than Reserve Balances) + (Other Deposits at the Fed) + (Term Deposits Held by Depository Institutions) - (U.S. Treasury, Supplementary Financing Account) - (Treasury balance in TGA) + (Reverse Repurchase Agreements).

    · Federal Reserve Notes, Net of F.R. Bank Holdings. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WLFN`.

    · Deposits with F.R. Banks, Other Than Reserve Balances. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WDFOL`.

    · Other Deposits at the Fed. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WLODL`.

    · Term Deposits Held by Depository Institutions. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/TERMT`.

    · U.S. Treasury, Supplementary Financing Account. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WLSFAL`.

    · Treasury balance in TGA. Daily Treasury Statements, found at `https://fsapps.`

fiscal.treasury.gov/dts/issues.

- Reverse Repurchase Agreements. Table H.4.1.T5 on the Fed's weekly balance sheet and found at https://fred.stlouisfed.org/series/WLRRAL.