## Disclosure Statement

The author declares that he has no relevant or material financial interests that relate to the research described in this paper, titled “Long-Term Debt and the Effect of a Money-financed Fiscal Stimulus”.

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# Long-Term Debt and the Effect of a Money-financed Fiscal Stimulus *

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**Abstract**

This paper examines money-financed fiscal stimulus in a New Keynesian model with long-term debt, highlighting the roles of fiscal shock persistence, debt maturity, and the policy rule implementing money financing. Under a strict debt-targeting rule, longer maturities amplify stimulus when shocks are short-lived but dampen it when shocks are persistent. Strikingly, for empirically relevant maturities and persistence, money-financed stimulus can be less effective than debt-financed stimulus. These patterns persist at the zero lower bound, though sensitivity to maturity is weaker. Finally, we show that directly targeting money growth produces large and robust multipliers, restoring the conventional view that money financing is more stimulative than debt financing.

Keywords: Fiscal multipliers, Government spending shocks, tax shocks, New Keynesian model, Monetary and fiscal policies

JEL Classification: E31, E32, E52, E62, E63

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# 1 Introduction

Over the past decade, advanced economies have faced two reinforcing challenges: a secular decline in natural interest rates, which heightens the risk of the nominal rate hitting its lower bound, and a surge in public debt, especially in the aftermath of the COVID-19 pandemic, which constrains fiscal space. These developments have renewed interest in money-financed fiscal stimulus, in which the central bank directly creates money to finance fiscal expansion. The idea has gained further relevance amid growing concerns over 'fiscal dominance' in the United States, exemplified by political pressure on the Federal Reserve to lower rates to ease debt financing and sustain the debt level. The conventional view suggests that money-financed fiscal stimulus may be very effective and could be more than debt-financed stimulus because it is accompanied by money creation. Recently, Galí (2020) formalized this insight within a standard New Keynesian model featuring one-period debt. Yet this abstraction departs sharply from reality, where the average maturity of U.S. government debt is about six years. Intuitively, with long-term debt, fiscal space depends on monetary policy and the state of the economy, since bond prices fluctuate and thereby alter the realized return on government liabilities. This raises a significant question: how does long-term debt affect the effectiveness of money-financed fiscal stimulus?

In this paper, we extend the analysis by examining a money-financed fiscal stimulus in a model with long-term government debt. We find that its effectiveness depends on three factors: *debt maturity*, *fiscal shock persistence*, and the *policy rule* implementing money financing. Under a money-financing regime where monetary and fiscal authorities coordinate to stabilize the real value of debt, longer maturities *amplify* the stimulus when shocks are short-lived but *reduce* it when shocks are persistent. Strikingly, contrary to conventional wisdom, if debt maturity exceeds a critical threshold–which is the case for the real-world average debt maturity–a money-financed stimulus may become *less* effective than a debt-financed stimulus. When the zero lower bound binds, these patterns persist but with weaker quantitative sensitivity. Finally, we show that targeting money growth directly yields large and robust multipliers, largely independent of maturity and persistence, restoring the conventional view that money financing is more stimulative than debt financing.

More specifically, we construct a standard New Keynesian model with long-term debt. In the model, the government issues long-term debt, which the household purchases. The long-term debt is modeled as a portfolio of government bonds with a constant decaying

rate, following Woodford (2001). The government sets the lump-sum tax following a systematic rule. We consider a tax cut shock and a government spending shock. Under a money financing regime, we first consider the case where the central bank determines the money supply to finance the government deficit and keep the real value of government debt unchanged. Under a debt financing regime, the monetary policy conducts strict inflation targeting.

The interaction between monetary policy and the debt burden is central to our findings. Specifically, the price of long-term debt—and, consequently, the overall debt burden—is endogenous to monetary policy and the state of the economy, a mechanism we refer to as the *debt-revaluation* channel. When an unexpected deficit shock occurs, either due to tax cuts or increased government spending, the government can achieve unexpected capital gain or loss with the long-term debt in the period of shocks hitting, depending on the maturity of debt and the persistence of the shock. Therefore, under the money financing regime, the government budget can be balanced through two mechanisms: monetary creation or debt revaluation.

First, when the money supply is expanded to finance deficits, the nominal interest rate falls. Under sticky prices, this lowers the real interest rate and stimulates household consumption. At the same time, lower nominal rates raise the price of long-term bonds, increasing the government's debt burden. To offset this, the central bank issues additional money, further stimulating consumption. Importantly, with longer debt maturities, bond prices become more sensitive to future interest rate fluctuations, amplifying the expansionary effects of money-financed stimulus.

Second, when the price of long-term debt falls in response to an unexpected deficit shock, the government realizes a capital gain from the lower value of its outstanding liabilities, and the central bank correspondingly contracts the money supply, as less financing is required. The reduction in money supply further depresses bond prices and crowds out consumption under sticky prices, weakening the stimulative effect of the fiscal policy. Moreover, as bond prices continue to fall, the central bank must tighten the money supply even further, amplifying the contraction. This effect is especially pronounced with longer maturities, since a larger share of outstanding debt is exposed to revaluation, further dampening the effectiveness of money-financed stimulus.

We next examine how debt maturity and fiscal shock persistence interact to shape bond prices and the endogenous response of the money supply. When shocks are highly persistent, sustained deficits fuel expected inflation, raising nominal interest rates and

depressing long-term bond prices. This revaluation reduces the debt burden and contracts the money supply, with the effect amplified by longer maturities since bond values are more sensitive to future interest rates. By contrast, when persistence is low, deficits close quickly and long-term bond prices remain stable, limiting government capital gains from revaluation. In this case, monetary expansion dominates, and the money-financed multiplier rises with maturity. With high persistence, however, pronounced bond price fluctuations strengthen the debt-revaluation channel, causing the multiplier to decline as maturity lengthens. Strikingly, when both persistence and maturity exceed a threshold, money-financed multipliers can fall below their debt-financed counterparts. These mechanisms are formalized in Propositions 1 and 2 using a simplified model with an analytical characterization.

This mechanism has direct implications for the relative effectiveness of tax cuts and government spending, holding the size of fiscal stimulus constant. When fiscal shock persistence is low, the government spending multiplier exceeds the tax multiplier under a debt maturity consistent with U.S. estimates (six years). This result aligns with Galí (2020), who find that under short-term debt (one-period debt), the government spending multiplier under money financing is larger than the tax multiplier. However, when shock persistence is high, the government spending multiplier may fall below the tax multiplier under money financing. This outcome follows directly from the debt revaluation channel we propose. The key intuition is that government spending shocks typically raise interest rates more than tax cuts. A larger increase in nominal interest rates lowers long-term bond prices more sharply in a model with longer debt maturity, leading to a greater contraction in the money supply. Consequently, output declines more in response to government spending shocks than tax cuts.

We proceed to investigate how our findings depend on the policy rule and whether the robust, large stimulus effects of money financing can be restored. To this end, we extend the analysis to alternative monetary-fiscal arrangements consistent with money-financing. First, we relax the assumption that authorities strictly stabilize the real value of debt under money financing. We propose a generalized debt-targeting rule, which nests our baseline as a special case and allows a fraction of fiscal deficits to be financed by debt. We find that the impact of money-financed stimulus is dampened when the debt-financing share is large. With long-term debt, the multiplier is less contractionary (expansionary) than in the baseline under low (high) shock persistence. Second, we consider a money growth targeting rule, where money creation directly responds to changes

in government deficits. Under this rule, the money-financed multiplier is both large and robust across debt maturities and persistence levels, effectively restoring the conventional wisdom that money financing is more stimulative than debt financing.

Finally, we examine the effects of fiscal shocks when the economy is constrained by the zero lower bound (ZLB). First, our qualitative findings generally hold: both the tax and government spending multipliers decrease with longer debt maturity when shock persistence is low, while they increase when shock persistence is high. Second, quantitatively, the multipliers exhibit less variation as debt maturity changes. The key reason is that at the ZLB, the endogenous money supply from the debt revaluation channel has little effect since nominal interest rates are unresponsive to money supply changes.

**Literature Review** This paper is related to several strands of literature.

First, it contributes to the study of the effect of money-financed fiscal stimulus.[1] This concept dates back to Friedman (1948); Haberler (1952). Bernanke (2003) highlights its potential benefits in the context of Japan's liquidity trap, suggesting it as a policy option when conventional tools are constrained. Auerbach and Obstfeld (2005) extends this discussion using an analytical framework to examine the effectiveness of open market operations at the zero lower bound (ZLB). Buiter (2014) investigates the effects of money-financed transfers to households (a "helicopter drop"), which emphasizes the role of money's irredeemability as the key mechanism driving its expansionary impact on consumption. Galí (2020) analyzes the effect of the money-financed fiscal stimulus in the standard New Keynesian model. Following this work, Tsuruga and Wake (2019) explore the role of implementation lags of the fiscal policy, and Punzo and Rossi (2023) study redistribution effects in a borrower–saver framework under a money financing regime. This paper is mostly close to Galí (2020). We extend his analysis by introducing long-term government debt into the framework and highlighting how debt maturity, fiscal shock persistence, and the policy rule implementing the money-financing regime shape the effectiveness of money-financed fiscal stimulus. In contrast to his finding, we show that under a strict debt-targeting rule, fiscal stimulus in a money-financing regime can be less effective than in a debt-financing regime when debt maturity is calibrated to empirically realistic values. In addition, we demonstrate that a money growth targeting rule restores large and robust multipliers.

[1] This is related to the broad literature on the determinants of fiscal multiplier (e.g., Galí et al. (2007), Eggertsson (2011), Christiano et al. (2011), Davig and Leeper (2011), Bilbiie (2009) ).

Moreover, this paper is related to the literature on the role of long-term debt in fiscal and monetary policy.[2] Examples include studies of optimal debt management in real models (e.g., Angeletos (2002)), joint analyses of debt management and optimal monetary policy (e.g., Lustig et al. (2008); Faraglia et al. (2013)), and the fiscal theory of the price level (e.g., Cochrane (2001)). More recently, Teles and Tristani (2024) shows that under a sticky-information framework, the optimal financing of fiscal shocks depends critically on debt maturity. In contrast, we focus on how the effectiveness of money-financed fiscal stimulus varies with debt maturity.

The remainder of the paper is organized as follows. Section 2 introduces the baseline model. Section 3 presents a simplified version. Section 4 extends the analysis to the full dynamic model. Section 5 examines the role of different policy rules, and Section 6 considers the case of a liquidity trap. Finally, Section 7 concludes.

## 2 The Model Setup

**Households** There is a representative household that maximizes the expected lifetime utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t Z_t \left[ U\left( C_t, \frac{M_t}{P_t}, N_t \right) \right]$$

Subject to the budget constraint:

$$P_t C_t + P_t T_t + Q_t B_t + P_t^s B_t^s + M_t - M_{t-1} = W_t N_t + Div_t + B_{t-1}^s + (1 + \rho Q_t) B_{t-1}$$

Here, $\beta$ denotes the household's discount factor and $Z_t$ is a time preference shifter. The household consumes $C_t$, supplies labor $N_t$, and holds nominal money balances $M_t$. The price level of final goods is $P_t$, and the real balance is defined as $\mathcal{M}_t = \frac{M_t}{P_t}$. The household pays lump-sum taxes $T_t$ to the government and receives dividend income $Div_t$ from firms. It earns nominal wage income $W_t N_t$ and trades two types of bonds: a one-period bond $B_t^s$ in zero net supply, priced at $P_t^s = (1 + i_t)^{-1}$, where $i_t$ is the nominal interest rate, and a long-term bond $B_t$ priced at $Q_t$. Following Woodford (2001), the long-term bond

[2] Our focus is on how long-term debt shapes the effects of fiscal stimulus under a money-financing regime. More broadly, this connects to the extensive literature on monetary–fiscal policy coordination. See, for example, Leeper (1991), Sims (1994), Woodford (1994), Bassetto (2002), Bhattarai et al. (2014), and Bianchi et al. (2023) on the coordination of policy rules, and Benigno and Woodford (2003) and Schmitt-Grohé and Uribe (2004) on the coordination of optimal monetary and fiscal policy.

$B_t$ represents a portfolio of bonds with an average maturity $\frac{1}{1-\beta\rho}$. $\rho$ is the decaying rate. Solving the household's optimization problem gives rise to the first-order conditions:

$$\mathbb{E}_t\left(\beta\frac{Z_{t+1}}{Z_t}\frac{U_{c,t+1}}{U_{c,t}}\frac{1+i_t}{\Pi_{t+1}}\right)=1 \tag{2.1}$$

$$\mathbb{E}_t\left(\beta\frac{Z_{t+1}}{Z_t}\frac{U_{c,t+1}}{U_{c,t}}\frac{1+\rho Q_{t+1}}{Q_t}\frac{P_t}{P_{t+1}}\right)=1 \tag{2.2}$$

$$U_{c,t}\frac{W_t}{P_t}+U_{n,t}=0 \tag{2.3}$$

$$\frac{U_{m,t}}{U_{c,t}}-1+\mathbb{E}_t\left(\beta\frac{Z_{t+1}}{Z_t}\frac{U_{c,t+1}}{U_{c,t}}\frac{P_t}{P_{t+1}}\right)=0 \tag{2.4}$$

In these expressions, $U_{c,t}$, $U_{n,t}$, and $U_{m,t}$ denote the marginal utilities of consumption, labor, and real balances, respectively. $\Pi_t$ is defined as $P_t/P_{t-1}$. Equation (2.1) is the Euler condition for the one-period bond, and equation (2.2) is the intertemporal condition for the long-term bond. Equation (2.3) is the intratemporal condition equating the marginal rate of substitution between consumption and labor to the real wage. Equation (2.4) represents the optimality condition for holding real balances.

Following Galí (2020), we assume $h(\mathcal{M}/C)\equiv\frac{U_m}{U_c}$ is continuous and decreasing function and satisfies $h(\bar{\mathcal{M}})=0$ for some $0<\bar{\mathcal{M}}<\infty$. Let $\hat{z}_t\equiv-\ln\frac{Z_{t+1}}{Z_t}$. The log-linearized optimal conditions:

$$\hat{c}_t=\mathbb{E}_t\hat{c}_{t+1}-\frac{1}{\sigma}\left(\hat{i}_t-\mathbb{E}_t\hat{\pi}_{t+1}-\mathbb{E}_t\hat{z}_t\right) \tag{2.5}$$

$$\hat{q}_t=-\hat{i}_t+\rho\beta\mathbb{E}_t\hat{q}_{t+1} \tag{2.6}$$

$$\hat{w}_t=\varphi\hat{n}_t+\sigma\hat{c}_t \tag{2.7}$$

$$\hat{m}_t=\hat{c}_t-\eta\hat{i}_t \tag{2.8}$$

where $\sigma\equiv-\frac{U_{cc}}{U_cC}$, $\varphi=-\frac{U_{n,n}N}{U_n}$, and $\eta\equiv-\frac{1}{h'(\mathcal{M}/C)\mathcal{M}/C}\frac{1}{1+i}$. All variables with hats denote log-deviations from their steady-state values. In particular, $\hat{m}_t$ represents real balances and $\hat{w}_t$ denotes the real wage.

**Firms** Firms' problems are standard and are completely identical to the textbook New Keynesian model in Galí (2015). There is a representative, competitive final goods producer that aggregates a continuum of intermediate goods $Y_t(i)$, where $i\in[0,1]$, into a final output $Y_t$ using a CES production technology given by:

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.9}$$

where $\varepsilon$ denotes the elasticity of substitution between differentiated intermediate goods.

Profit maximization under perfect competition gives rise to the optimal demand for intermediate goods $Y_t(i)$:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \tag{2.10}$$

where $P_t(i)$ is the price of the intermediate good of variety $i$. The intermediate goods producers are monopolistically competitive with the production function

$$Y_t(i) = N_t(i)^{1-\alpha}$$

They set prices subject to nominal rigidity a la Calvo (1983). Specifically, each firm may reset its price with the probability $1-\theta$ in any given period while keeping its previous period price with the probability $\theta$. Formally, each firm solves the following problem:

$$\max_{P_t^*} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[ \frac{P_t^*}{P_{t+k}} Y_{t+k}(i) - W_{t+k} Y_{t+k}(i) \right] \right\},$$

subject to the demand function (2.10). The optimal condition is:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k}(i) \left( \frac{P_t^*}{P_{t+k}} - \frac{\epsilon}{\epsilon - 1} W_{t+k} \right) = 0 \tag{2.11}$$

The aggregate price dynamics are given by:

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta) \left( \frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \tag{2.12}$$

Combining equations (2.7), (2.11), and (2.12) as well as the market clearing conditions $C_t + G_t = Y_t$, we yield the log-linearized New Keynesian Phillips curve,

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_0 \hat{y}_t - \lambda \sigma \frac{g}{c} \hat{g}_t \tag{2.13}$$

Where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$ and $\kappa_0 = \lambda \left( \frac{\varphi+\alpha}{1-\alpha} + \sigma c^{-1} \right)$. $c$ and $g$ are steady state con-

sumption and government spending to GDP ratio, respectively.

**Government Authority** Now we begin to describe the coordination between monetary and fiscal policy. The government income includes lump-sum taxes $T_t$, the issuance of new debt $\frac{Q_t B_t}{P_t}$, and the seigniorage due to issuance of money $\frac{M_t - M_{t-1}}{P_t}$. On the expenditure side, the government spends on government purchases $G_t$, repays maturing debt $\frac{B_{t-1}}{P_t}$, and accounts for debt revaluation on outstanding bonds $\rho Q_t \frac{B_{t-1}}{P_t}$. Formally, the government budget constraint is given by:

$$(1 + \rho Q_t)\frac{B_{t-1}}{P_t} = \frac{Q_t B_t}{P_t} + (T_t - G_t) + \frac{M_t - M_{t-1}}{P_t}.$$

Unlike one-period bonds, long-term bonds introduce a debt revaluation term, $\rho Q_t \frac{B_{t-1}}{P_t}$. This term captures changes in the value of existing debt due to interest rate fluctuations. If $\rho = 0$, the debt revaluation term disappears, reducing the model to the one-period bond case as in Galí (2020). Conversely, a larger $\rho$ indicates a longer average maturity for long-term bonds, meaning a greater portion of the debt portfolio is exposed to interest rate fluctuations. Let the real debt be $\mathcal{B}_t \equiv \frac{Q_t B_t}{P_t}$ and $\hat{b}_t$ is the percentage deviation of $\mathcal{B}_t$ from its steady-state level. Log-linearize the government budget constraint around a steady state with zero inflation, no trend growth, and constant government purchases $G$, taxes $T$, and government debt $\mathcal{B}$, we have the dynamics of the budget constraint:

$$\beta^{-1}(\rho\beta\hat{q}_t - \hat{q}_{t-1} - \hat{\pi}_t + \hat{b}_{t-1}) = \hat{b}_t + \frac{\tau}{b}\hat{T}_t - \frac{g}{b}\hat{g}_t + \frac{\gamma}{b}(\hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t) \tag{2.14}$$

where $g = \frac{G}{Y}$, $\tau = \frac{T}{Y}$, $\gamma = \frac{M}{PY}$, and $b = \frac{QB}{PY}$, evaluated at the steady state.

The tax $\hat{T}_t$ is assumed to follow the rule:

$$\hat{T}_t = \psi_b \frac{b}{\tau}\hat{b}_{t-1} + \hat{\tau}_t \tag{2.15}$$

where $\hat{\tau}_t$ is the exogenous component following an AR(1) process:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + \varepsilon_{\tau,t} \tag{2.16}$$

The government spending $\hat{g}_t$ follows

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{g,t} \tag{2.17}$$

Combing equations (2.14) and (2.15) gives the following equation:

$$\hat{b}_t = \underbrace{\left(\beta^{-1} - \psi_b\right)\hat{b}_{t-1}}_{\text{tax stabilization}} + \underbrace{\beta^{-1}\left(\rho\beta\hat{q}_t - \hat{q}_{t-1} - \hat{\pi}_t\right)}_{\text{realized real return}} + \left(g_b\hat{g}_t - \tau_b\hat{\tau}_t\right) - \underbrace{\gamma_b\left(\hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t\right)}_{\text{seigniorage}}$$

where $g_b = \frac{g}{b}$, $\tau_b = \frac{\tau}{b}$, and $\gamma_b = \frac{\gamma}{b}$ denote the steady-state ratios of government spending, taxes, and real balances to government debt, respectively.

The dynamics of government debt are characterized by four components. The first component represents tax stabilization, which depends on the tax rule. A more aggressive tax response to past debt, i.e., a larger $\psi_b$, results in lower debt levels today. To ensure long-run stability of debt, we assume $\psi_b > \beta^{-1} - 1$. This condition aligns with the literature and is commonly referred to as passive (Ricardian) fiscal policy (e.g., Leeper (1991)).

The second component captures the realized real return on government debt, which is determined by bond prices and inflation. This mechanism is central to our analysis. With one-period bonds ($\rho = 0$), the term reduces to $-\hat{q}_{t-1} - \hat{\pi}_t$, the realized short-term real interest rate. In this case, the nominal return is predetermined and unaffected by current or future interest rate movements. With long-term bonds, however, an additional term, $\rho\beta\hat{q}_t$, arises, reflecting debt revaluation. A decline in bond prices generates a capital gain for the government, lowering realized returns and reducing the debt burden. This effect is amplified when maturity is longer (higher $\rho$), since a larger share of outstanding debt is exposed to revaluation. Finally, higher inflation erodes the real value of debt at all maturities.

The third component corresponds to government spending and tax income, while the fourth component represents seigniorage revenue.

Following Galí (2020), we compare the effect of the fiscal stimulus under two policy regimes: the debt financing regime and the money financing regime. Under the *debt financing* regime, the central bank follows a strict inflation-targeting rule. Formally,

$$\hat{\pi}_t = 0 \tag{2.18}$$

Under the *money financing* regime, the money supply adjusts endogenously to maintain a constant real debt level, i.e., $\hat{b}_t = 0$. It directly follows that the endogenous movement of tax revenue is constant given the tax rule. Therefore, the deficit is financed through money creation, inflation, and a reduction in the nominal return on government bonds. Then, we

can derive the money supply under the money-financed fiscal stimulus as follows:

$$\gamma_b \left(\hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t\right) = \beta^{-1}\left(\rho\beta\hat{q}_t - \hat{q}_{t-1} - \hat{\pi}_t\right) + \left(g_b\hat{g}_t - \tau_b\hat{\tau}_t\right) \tag{2.19}$$

Note that seigniorage revenue is equal to the money growth rate up to first-order: $\hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t = logM_t - logM_{t-1}$. From equation (2.19), it shows that the money supply is determined by fiscal deficits and, importantly, the price of long-term bonds.

# 3 A simplified Model

Before solving the full model, we simplify the model to analytically characterize the mechanism. In this section, we assume that prices are sticky only in the first period ($t = 1$) and fully flexible thereafter ($t \geq 2$). We first consider the impact of tax shocks. At first, we solve the equilibrium under the debt-financed fiscal stimulus. The solution under this regime is trivial due to the Ricardian equivalence. The tax shocks have no impact on the real variables. That is,

$$\hat{c}_t = 0 \tag{3.1}$$

Now, we proceed to solve the equilibrium under money-financed fiscal stimulus in two steps. In the first step, we solve for the inflation under the flexible price equilibrium (i.e., $t \geq 2$). Under the flexible price equilibrium, the lump-sum tax shock has no impact on the real variables because of the Ricardian equivalence as characterized by Galí (2020). That is, $\hat{c}_t = 0$. Combining equation (2.5), (2.6), (2.8), (2.19) and $\hat{c}_t = 0$, we solve the nominal interest rate $\hat{i}_t$ and inflation $\hat{\pi}_t$ for $t \geq 2$ given by:

$$\hat{i}_t = \phi_{i,\tau}\hat{\tau}_t \tag{3.2}$$

$$\hat{\pi}_t = \phi_m\hat{m}_{t-1} - \phi_\tau\hat{\tau}_{t-1} + \phi_r\hat{r}_{t-1} + \nu^{\pi}_{\tau,t} \tag{3.3}$$

The coefficients $\phi_{i,\tau} = \frac{\rho_\tau\tau_\gamma}{1+\eta-\eta\rho_\tau}$, $\phi_m = 1$, $\phi_\tau = \rho_\tau\tau_\gamma\left(1 - \frac{\eta}{1+\eta}\rho_\tau\right)^{-1}$, and $\phi_r = (\beta\gamma_b)^{-1}$, where $\tau_\gamma = \frac{\tau}{\gamma}$. The unexpected inflation cause by tax shock $\nu^{\pi}_{\tau,t}$ is

$$\nu^{\pi}_{\tau,t} = \left(\tau_b + \phi_{i,\tau}\gamma_b\eta - \frac{\phi_{i,\tau}\rho}{1-\rho\beta\rho_\tau}\right)\frac{1}{\beta^{-1}+\gamma_b}(-\varepsilon_{\tau,t}), t \geq 2 \tag{3.4}$$

The detailed derivations can be found in the Appendix A.2. This solution shows that inflation depends on the persistence of tax shocks. Consider a persistent tax cut introduced in period 1. First, such a shock raises future inflation, as reflected in the term $-\phi_\tau \hat{\tau}_{t-1}$. The mechanism is straightforward: money creation finances the tax cut, thereby generating inflation. Second, the effect becomes stronger with greater persistence, since the implied sequence of tax cuts expands as $\rho_\tau$ increases (with $\phi_\tau$ rising in $\rho_\tau$).

Having solved the flexible price equilibrium ($t \geq 2$), we can derive the solution in period 1. The on-impact tax multiplier is given by:

$$m_\tau = \frac{c}{\tau} \frac{\tau_b + \gamma_b \eta \phi_{i,\tau} - \frac{\rho}{1-\rho\beta\rho_\tau}\phi_{i,\tau}}{-\rho \frac{\sigma(1+\phi_r)-1}{1+\eta} + \gamma_b \frac{1+\eta\sigma\left(1+\phi_r\right)}{1+\eta} + (\beta^{-1}+\gamma_b)\kappa_0 c} \tag{3.5}$$

As price becomes fully flexible, $\kappa_0$ converges to infinity, and the tax multiplier goes to zero. Let the denominator be denoted by $\Gamma_0(\rho)$. The function $\Gamma_0(\rho)$ is decreasing in the bond decay rate $\rho$. We assume $\Gamma_0(1) > 0$, which guarantees a positive denominator for all maturities. This condition is satisfied under reasonable parameter calibrations. The solution (3.5) shows that the on-impact tax multiplier under the money financing depends on the shock persistence $\rho_\tau$ and the debt maturity $\rho$. Proposition 1 presents the first key results.

**Proposition 1.** *There exists a threshold of tax shock persistence, $\bar{\rho}'_\tau$, such that if the tax persistence $\rho_\tau$ is larger (smaller) than $\bar{\rho}'_\tau$, then the money-financed tax multiplier $m_\tau$ is decreasing (increasing) in maturity $\rho$.*

*Proof.* See Appendix A.3.1. □

Proposition 1 states that the shock persistence and the debt maturity is the key to determine the on-impact tax multiplier. To understand this result, we first forward the bond pricing equation and express it as follows:

$$\hat{q}_t = -\mathbb{E}_t \sum_{j=0}^{\infty} (\rho\beta)^j \hat{i}_{t+j} = \mathbb{E}_t \sum_{j=0}^{\infty} (\rho\beta)^j \Big[ \underbrace{-\hat{r}_{t+j}}_{\text{real interest rate}} \underbrace{- \hat{\pi}_{t+j+1}}_{\text{inflation}} \Big] \tag{3.6}$$

Equation (3.6) states that the price of long-term bonds depends on the expected path of the nominal interest rate. If the debt maturity is longer (larger $\rho$), the price of the long-term bond is more sensitive to future interest rate fluctuations.

Now we begin to analyze how the central bank and the government coordinate to balance the fiscal budget.

Consider an economy subject to a persistent tax cut shock. As discussed earlier, money creation finances the persistent deficit, generating inflation and thereby reducing the price of long-term bonds. However, because prices are sticky in the initial period, monetary expansion lowers the real interest rate and boosts consumption, which raises the price of long-term bonds. The net effect on bond prices in the first period depends on the relative strength of these opposing forces: when the shock persistence is high, the future inflationary effect dominates, leading to a decline in bond prices; when persistence is low, the impact of the current real interest rate dominates, increasing bond prices. Furthermore, the magnitude of the future inflationary effect on the current bond price is amplified with longer debt maturity, as a greater portion of government liabilities becomes exposed to future interest rate fluctuations.

Given this relationship between shock persistence, debt maturity, and bond prices, we now examine how long-term bond fluctuations endogenously affect the money supply. From equation (2.19), a decline (rise) in bond prices induces a contraction (expansion) of the money supply. Under sticky prices, this monetary contraction (expansion) reduces (raises) output.

Bringing these insights together, the mechanism can be summarized as follows. In response to a persistent tax cut, the central bank expands the money supply to finance the deficit, lowering real interest rates and raising future inflation. When persistence exceeds a threshold, the inflationary effect dominates, depressing bond prices. The decline in bond prices then induces a contraction in the money supply, offsetting the initial expansion and dampening the stimulative effect of money-financed tax cuts. Longer maturities amplify this contraction, as a larger share of government debt is exposed to revaluation, thereby reducing the multiplier. By contrast, when persistence is low, long-term bond prices rise, which further expands the money supply and raises output—an effect that is amplified with longer maturities.

Figure 1 visually illustrates the findings in Proposition 1. The solid blue line represents the one-period bond case studied in Galí (2020). Our analysis extends this framework from a one-dimensional black line to a two-dimensional plane. The shaded light yellow region in the upper portion of the plane denotes cases where the tax multiplier decreases with debt maturity, whereas the lower light green region represents cases where the tax multiplier increases with maturity. The green solid line indicates the threshold value. The figures show that both cases are plausible under reasonable calibration of the tax shock persistence.

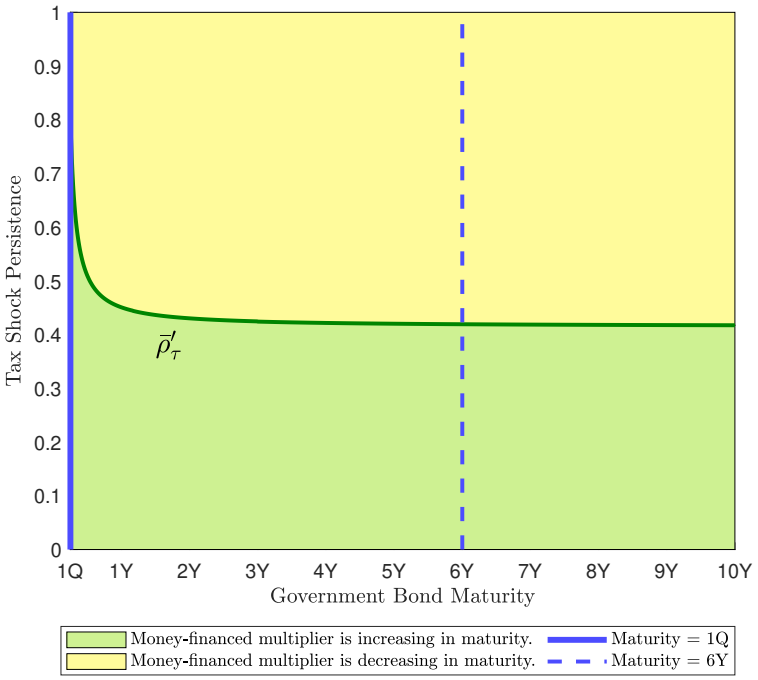



Figure 1: The figure illustrates the property of the money-financed tax multiplier in a simplified model. The green region indicates parameter combinations where the tax multiplier increases with the maturity of government bonds, while the yellow region corresponds to cases where the tax multiplier decreases with bond maturity. The solid green line marks the threshold level of tax shock persistence, denoted by $\rho'_\tau$. The solid blue line represents the case in which government bonds have a maturity of one quarter, whereas the dashed blue line reflects the empirically observed average maturity of six years. The remaining parameters are calibrated as in Section 4.1.

**Money Financing v.s. Debt Financing** In the following analysis, we compare the stimulus effects of tax cuts under money financing and debt financing. Proposition 2 summarizes the findings.

**Proposition 2.** *The relative effectiveness of money-financed versus debt-financed tax cuts depends on the persistence of the shock and the maturity of government debt.*

1. *If tax shock persistence is smaller than $\bar{\rho}''_\tau$, the money-financed tax multiplier is always larger than the debt-financed multiplier.*

2. *If tax shock persistence is larger than $\bar{\rho}''_\tau$ and debt maturity is greater than $\bar{\rho}_{m,\tau}$, the money-financed tax multiplier is smaller than the debt-financed multiplier.*

*Proof.* See appendix A.3.2. □

Proposition 2 yields two key insights. The first states that when tax cut persistence is below a certain threshold, the money-financed tax multiplier is always larger than

the debt-financed multiplier, irrespective of debt maturity. This result is consistent with Galí (2020), who finds a similar outcome under one-period bonds. Our analysis extends this conclusion to long-term bonds when tax cut persistence is low. This follows directly from Proposition 1 and the findings of Galí (2020): when tax cut persistence is low, the money-financed tax multiplier rises as maturity lengthens. Since Galí (2020) establishes that the money-financed tax multiplier exceeds the debt-financed multiplier under one-period debt, Proposition 1 implies that this relationship holds for all maturity levels.

The second insight challenges conventional wisdom as outlined in Galí (2020). It asserts that if tax persistence is sufficiently high and debt maturity is large enough, the money-financed tax multiplier can be smaller than the debt-financed multiplier. This result follows directly from Proposition 1. When tax persistence is sufficiently high, the money-financed tax multiplier decreases with maturity. If maturity is large enough, the money-financed multiplier declines to a point where it falls below the debt-financed multiplier.

Figure 2 illustrates the parameter space comparing the effects of money-financed and debt-financed tax multiplier across different combinations of tax shock persistence and debt maturity. The shaded red region in the upper portion of the figure represents the parameter space where money-financed tax cuts are less effective than debt-financed tax cuts. The red dashed line denotes the threshold value $\bar{\rho}''_\tau$ in the Proposition 2. The green solid line represents the threshold value $\bar{\rho}'_\tau$ from Proposition 1. Finally, the blue solid line corresponds to the case of one-period bonds, while the blue dashed line represents the case where the average maturity is six years, consistent with the empirical estimates in the literature (e.g., Bianchi et al. (2023)).

Several insights emerge from Figure 2. First, under one-period debt, the money-financed tax multiplier is always larger than the debt-financed tax multiplier, regardless of the level of tax shock persistence. Second, in a setting consistent with empirical estimates, where the average debt maturity is six years, the money-financed tax multiplier falls below the debt-financed tax multiplier when tax shock persistence exceeds roughly 0.6.

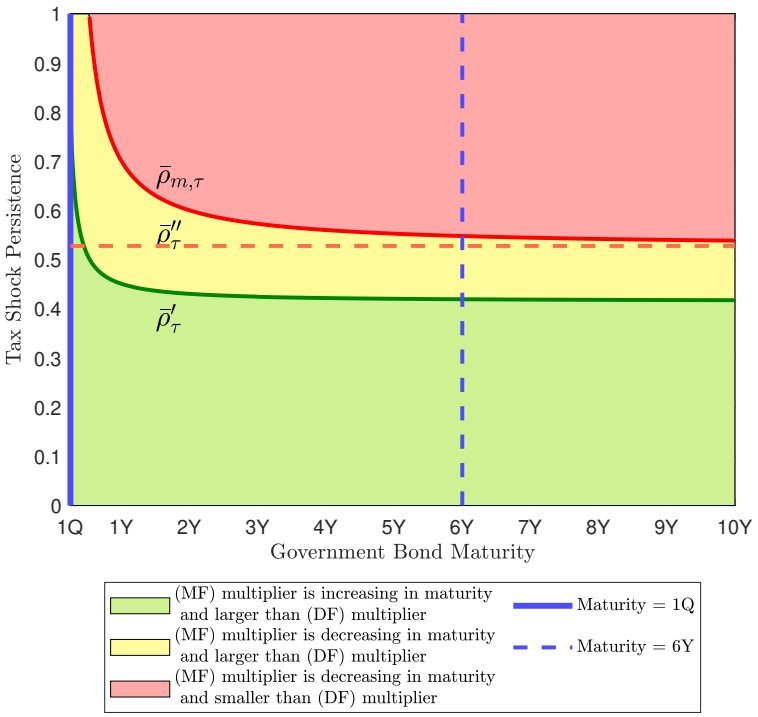



Figure 2: The figure illustrates the property of the money-financed tax multiplier in a simplified model. The green region indicates parameter combinations where the tax multiplier increases with the maturity of government bonds, while both the yellow region and red region correspond to cases where the tax multiplier decreases with bond maturity. In the green and yellow region, the money-financed tax multiplier exceeds the debt-financed counterpart. In contrast, the red region represents cases where the money-financed tax multiplier is smaller than the debt-financed multiplier. The green solid line, the orange dashed line, and the red solid line denote the threshold value.

# 4 Numerical Analysis

Our previous analysis builds on the assumption that the nominal rigidities are only persistent for one period, and therefore, we only focus on the on-impact effect of the fiscal stimulus, which is not consistent with the standard New Keynesian model. In the following analysis, we explore the effects under the fully dynamic model.

## 4.1 Calibration

The calibration of most parameters follows Galí (2020). We set the discount factor $\beta$ to 0.995, implying a steady state annual real interest rate of about 2%. The parameter $\varphi$ is set to 5. Hence, the Frisch elasticity of labor supply is 0.2. Parameter $\alpha = 0.25$. We set the elasticity of substitution across intermediate goods $\epsilon$ to 9, corresponding to a steady state

markup of 12.5%. The probability that firms can't reset the price ($\theta$) is 0.75, which means firms will adjust the price once a year on average. We set the inverse velocity of money to $\gamma = 1/3$, the elasticity of money demand with respect to the nominal interest rate to $\eta = 7$, and the tax response to debt coefficient to $\psi_b = 0.02$. Furthermore, we assume that real balances and consumption are separable in the utility function, i.e., $U_{c,m} = 0$. The steady-state debt-to-output ratio (on a quarterly basis) is set to $b = 2.4$, while the steady-state government spending-to-output ratio is fixed at 0.2. In the following exercises, we vary the persistence of fiscal shocks, $\rho_\tau$ and $\rho_g$, as well as the debt maturity through $\rho$.

## 4.2 A money-financed tax cut

We now examine the effect of a money-financed tax cut in the full model. To measure the effects of the tax cut, we use the cumulative tax multiplier, which measures the total change in output over 5 years relative to the total change in taxes. Formally, it is defined as:

$$M_\tau = -\frac{\sum_{t=0}^{T} \Delta Y_t}{\sum_{t=0}^{T} \Delta T_t} \tag{4.1}$$

Figure 3 presents the five-year cumulative tax multiplier across different levels of tax persistence. The results indicate that under the full model, when tax shock persistence is low, the stimulus effect of a tax cut increases with debt maturity. However, when tax shock persistence is high, the stimulus effect declines as debt maturity lengthens. Consequently, for sufficiently persistent shocks and long debt maturities, the tax stimulus under money financing may generate a smaller effect than under debt financing.

Figure 4 further illustrates the regions corresponding to different cumulative tax multiplier properties. The green-shaded area represents parameter values where the tax multiplier increases with debt maturity. The yellow and red-shaded regions indicate cases where the tax multiplier decreases with debt maturity. Specifically, in the green and yellow region, the money-financed tax multiplier remains larger than the debt-financed multiplier, while in the red region, it becomes smaller. Notably, in the case of one-period government bonds (represented by the blue solid line), the tax multiplier is larger under money financing than under debt financing, consistent with Galí (2020). However, when the government bond maturity aligns with real-world estimates of six years, the tax multiplier under money financing can fall below that under debt financing if tax shock persistence exceeds a reasonable threshold (approximately 0.75).

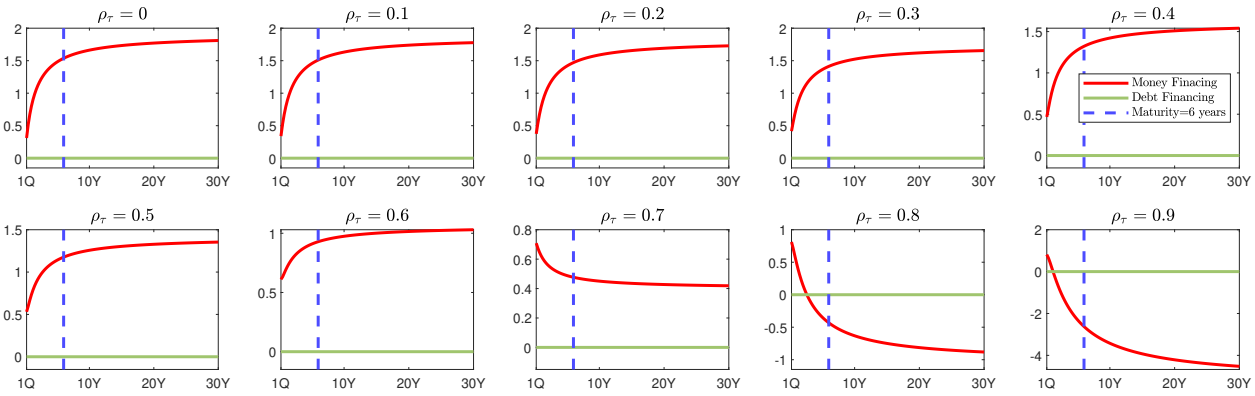



Figure 3: This figure presents the 5-year cumulative tax multiplier under money-financing and debt-financing, plotted against government debt maturity. The red solid line shows money financing, the green solid line shows debt financing, and the blue dashed line indicates the average debt maturity in the data. Tax shock persistence ($\rho_\tau$) ranges from 0.1 to 0.9.

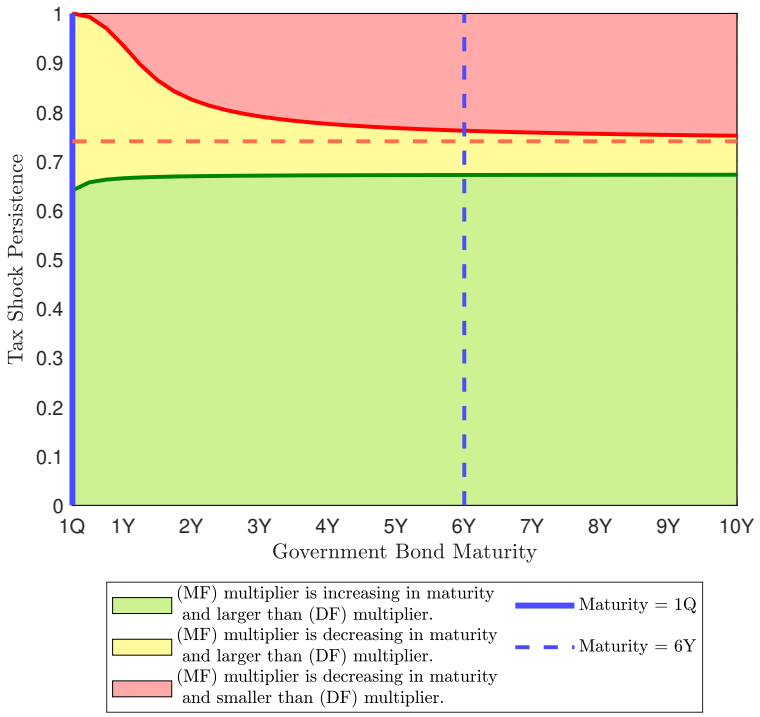



Figure 4: The figure illustrates the property of the money-financed tax multiplier in the full model. The green region marks parameter combinations where the multiplier rises with debt maturity, while the yellow and red regions correspond to cases where it falls. In the green and yellow region, the money-financed multiplier exceeds the debt-financed multiplier; in the red region, it is smaller. The green solid, orange dashed, and red solid lines denote threshold values. The blue solid line marks a debt maturity of one quarter, and the blue dashed line marks six years.

To illustrate the underlying mechanism, we present impulse response functions for

key macroeconomic variables following a tax cut shock under different levels of shock persistence. Figure 5 shows the responses to a low-persistence tax shock ($\rho_\tau = 0.2$, panel A) and a high-persistence tax shock ($\rho_\tau = 0.8$, panel B). The red dotted line represents money financing with a debt maturity of six years, the blue dashed line corresponds to money financing with a debt maturity of one quarter, and the green solid line denotes the case of debt financing.

Panel A of Figure 5 shows that under money financing, a tax cut prompts the central bank to expand the money supply to stabilize debt, especially when shock persistence is low. The resulting expansion lowers nominal interest rates and raises bond prices, reducing real rates under sticky prices and stimulating consumption. This effect is stronger when debt maturity is longer (six years versus one quarter), as the larger increase in bond prices raises the real debt burden. In response, the central bank expands the money supply further, evidenced by the larger increase in money growth on impact under the six-year maturity, thereby amplifying the stimulative effect on consumption.

However, under high shock persistence, Panel B of Figure 5 shows a sharper contraction in the money supply and output. Persistent deficits raise expected inflation, which stabilizes the real value of debt but also pushes up nominal interest rates and depresses long-term bond prices, consistent with the simple model. The government benefits from this fall in bond prices, as it reduces the real value of outstanding liabilities and helps finance the deficits. Consequently, the money supply contracts, dampening output. This pattern reflects Proposition 2: when persistence is high and debt maturity is long, the inflationary effect dominates, bond prices fall, and the stimulative effect of money financing is reversed. By contrast, the contraction is weaker when debt maturity is short, since short-term debt is less exposed to future interest rate fluctuations. This is evident in the figure, where consumption still expands when the maturity is only one quarter.

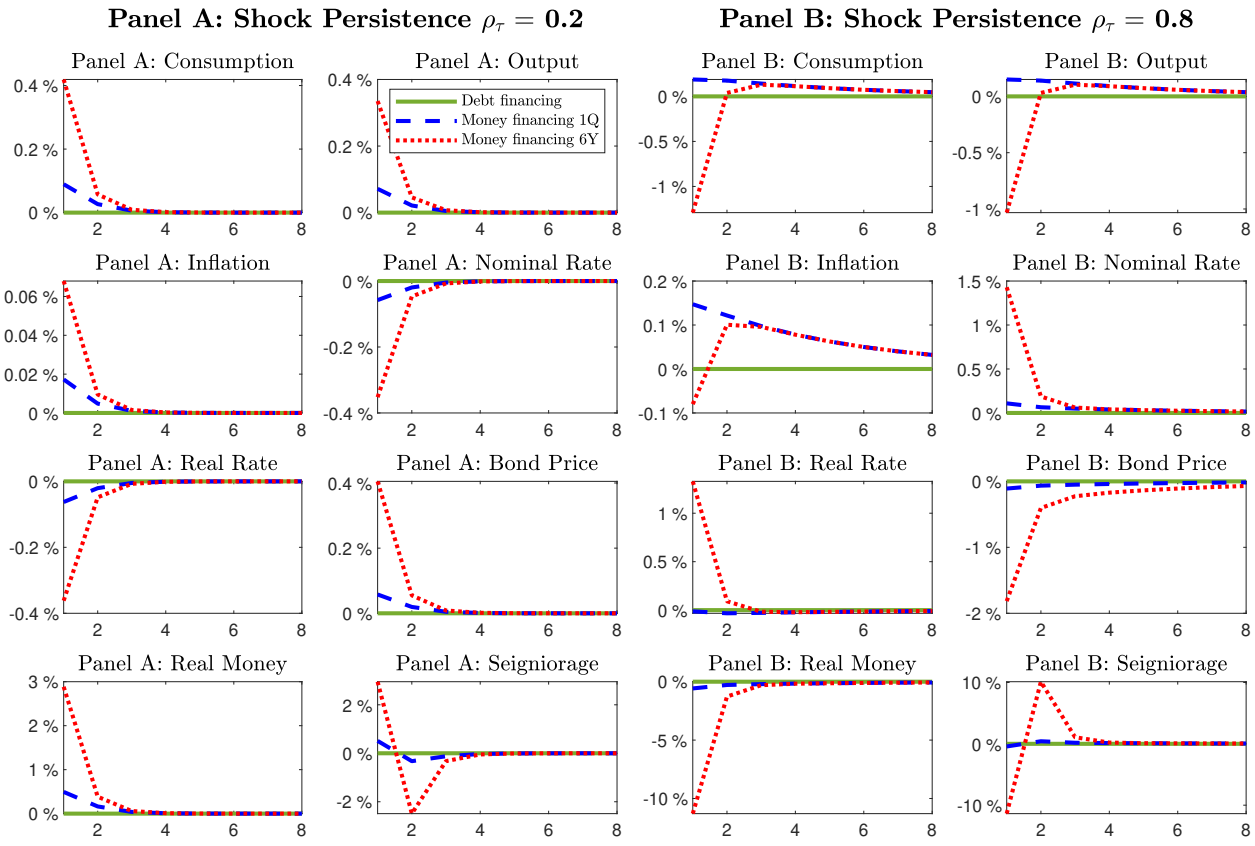



Figure 5: This figure presents the impulse response functions to the 1% tax shock with tax persistence $\rho_\tau = 0.2$ (left panel) and $\rho_\tau = 0.8$ (right panel). The red dotted line corresponds to the money financing regime when the government debt maturity is 6 years. The blue dashed line represents the money-financed regime when the debt maturity is 1 quarter. The green solid line represents the debt financing regime.

## 4.3 A money-financed increase in government purchases

Next we turn to analyze the effect of the money-financed government purchases under the full model. Here, we introduce the cumulative government spending multiplier defined as $M_g = \frac{\sum_{t=0}^{T} \Delta Y_t}{\sum_{t=0}^{T} \Delta G_t}$.

Figure 6 illustrates the five-year cumulative government spending multiplier across different levels of fiscal shock persistence. When persistence is low, the multiplier increases with debt maturity; however, when persistence is high, the multiplier declines as maturity lengthens.

Figure 7 further delineates these relationships by identifying parameter regions that govern the effect of debt maturity on the government spending multiplier. The green-shaded region represents cases where the multiplier increases with maturity, whereas the

yellow and red-shaded regions indicate cases where it decreases. In the green and yellow region, the money-financed multiplier remains higher than the debt-financed counterpart, while in the red region, it falls below the debt-financed multiplier. Consistent with the findings for tax multipliers, when government debt consists of one-period bonds, money financing always yields a higher spending multiplier than debt financing. However, when debt maturity extends to six years, the money-financed multiplier can be lower than the debt-financed multiplier if shock persistence exceeds roughly 0.75.

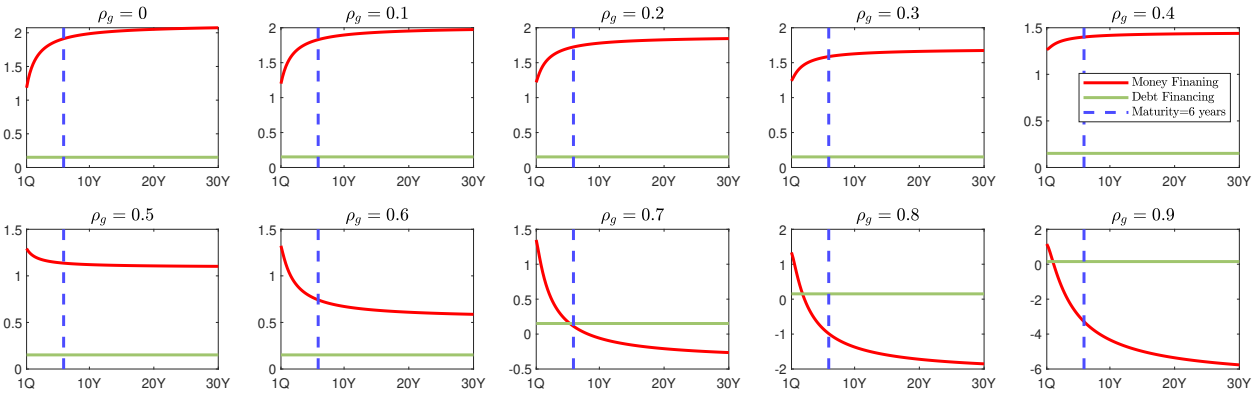



Figure 6: This figure presents the 5-year cumulative government spending multiplier under money financing and debt financing, plotted against government debt maturity. The red solid line shows money financing, the green solid line shows debt financing, and the blue dashed line indicates the average debt maturity in the data. Spending shock persistence ($\rho_g$) ranges from 0.1 to 0.9.

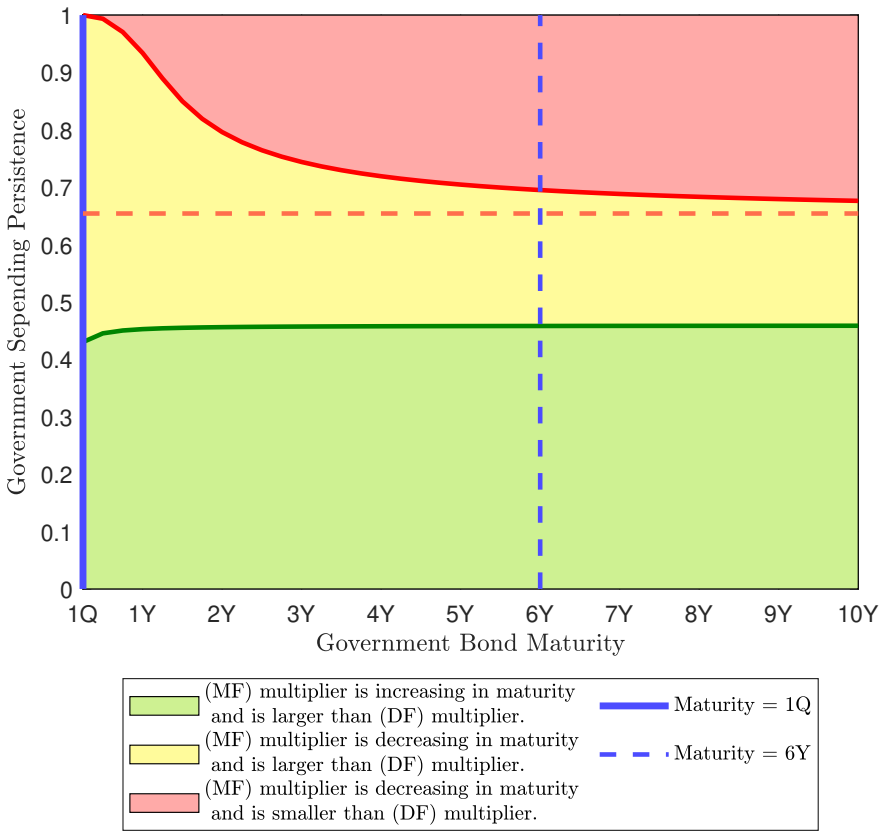



Figure 7: The figure illustrates the property of the money-financed government spending multiplier in the full model. The green region marks parameter combinations where the multiplier rises with debt maturity, while the yellow and red regions indicate where it falls. In the green and yellow region, the money-financed multiplier exceeds the debt-financed counterpart; in the red region, it is smaller. The green solid, orange dashed, and red solid lines denote threshold values. The blue solid line marks a debt maturity of one quarter, and the blue dashed line marks 6 years.

Figure 8 represents impulse response functions to a government spending shock under money financing and debt financing. The panel A of Figure 8 represents the case of the low level of shock persistence ($\rho_g = 0.2$) and the panel B of Figure 8 represents the case of the high level of shock persistence ($\rho_g = 0.8$). The red dotted line plots the case with the average debt maturity to be 6 years, while the blue dashed line corresponds to 1 quarter. The green solid line is the case of debt financing.

Panel A of Figure 8 illustrates that under debt financing, consumption is crowded out due to the rise of the real interest rate, a well-documented mechanism in the literature. In contrast, under money financing, consumption is crowded in, resulting in a government spending multiplier above unity. This occurs because the central bank expands the money supply to finance government spending, lowering both nominal and real interest rates,

which further stimulates consumption.

Moreover, the stimulative effect of government spending is stronger when the average debt maturity is longer (six years) rather than shorter (one quarter) when the shock persistence is low. The key mechanism is that with longer maturity, bond prices rise more under money financing. This increases the real debt burden, prompting the central bank to expand the money supply further, thereby amplifying the consumption response.

When shock persistence is high, rising deficits generate persistent inflation expectations, which in turn raise nominal interest rates. This depresses long-term bond prices, allowing the government to gain from the decline in the market value of its outstanding debt. The effect is amplified when debt maturity is longer, as bond prices fall more sharply, prompting a larger contraction in the money supply. The resulting reduction in money balances dampens output. Under these conditions, the money-financed spending multiplier can fall below the debt-financed multiplier, reversing conventional wisdom.

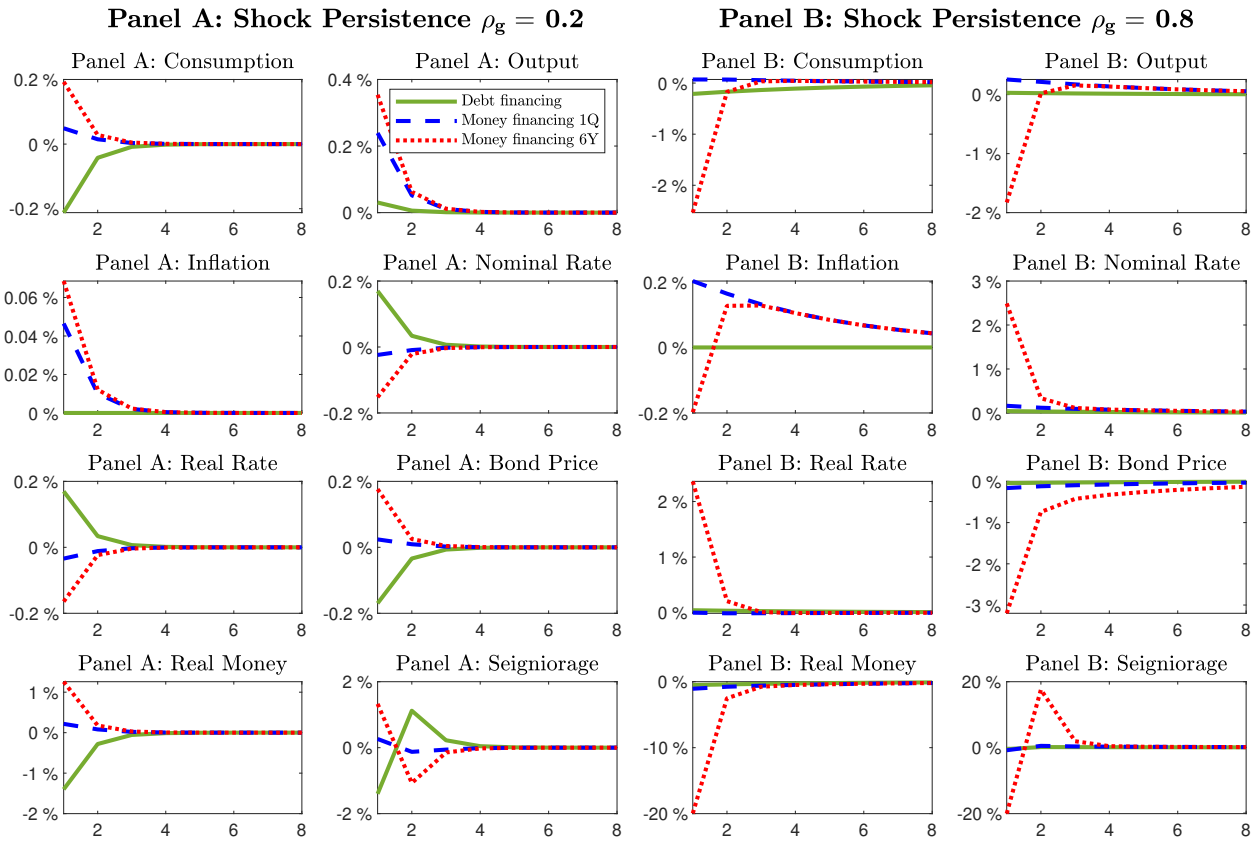



Figure 8: This figure presents the impulse response functions to the 1% government spending shock with persistence $\rho_g = 0.2$ (left panel) and $\rho_g = 0.8$ (right panel). The red dotted line corresponds to the money-financed policy with a 6-year government bond maturity. The blue dashed line represents the money-financed policy with a 1-quarter government bond maturity. The green solid line represents the debt finance policy.

**Comparing the money-financed tax multiplier and government spending multiplier** We now compare the effects of money-financed tax cuts and government spending on output. Figures 9 present impulse responses for both shocks under a short debt maturity (one quarter). Consistent with Galí (2020), the money-financed government spending multiplier exceeds the tax multiplier, reinforcing conventional wisdom.

However, when debt maturity extends to six years (Figures 10), the relative effectiveness of the two policies depends on fiscal shock persistence. With low persistence ($\rho_\tau = \rho_g = 0.2$), government spending still has a slightly larger effect than tax cuts (panel A). In contrast, with high persistence ($\rho_\tau = \rho_g = 0.8$), the results reverse—tax cuts become more effective than government spending (panel B).

The key intuition is that, holding other factors constant, government spending shocks raise the discount factor more than tax cuts, a result consistent across both RBC and New

Keynesian models. The associated increase in the nominal interest rate lowers bond prices and contracts the money supply, with longer debt maturity amplifying this effect. As seen in Figure 10 (Panel B), government spending shocks lead to a larger contraction in money supply compared to tax cuts, ultimately resulting in lower output.

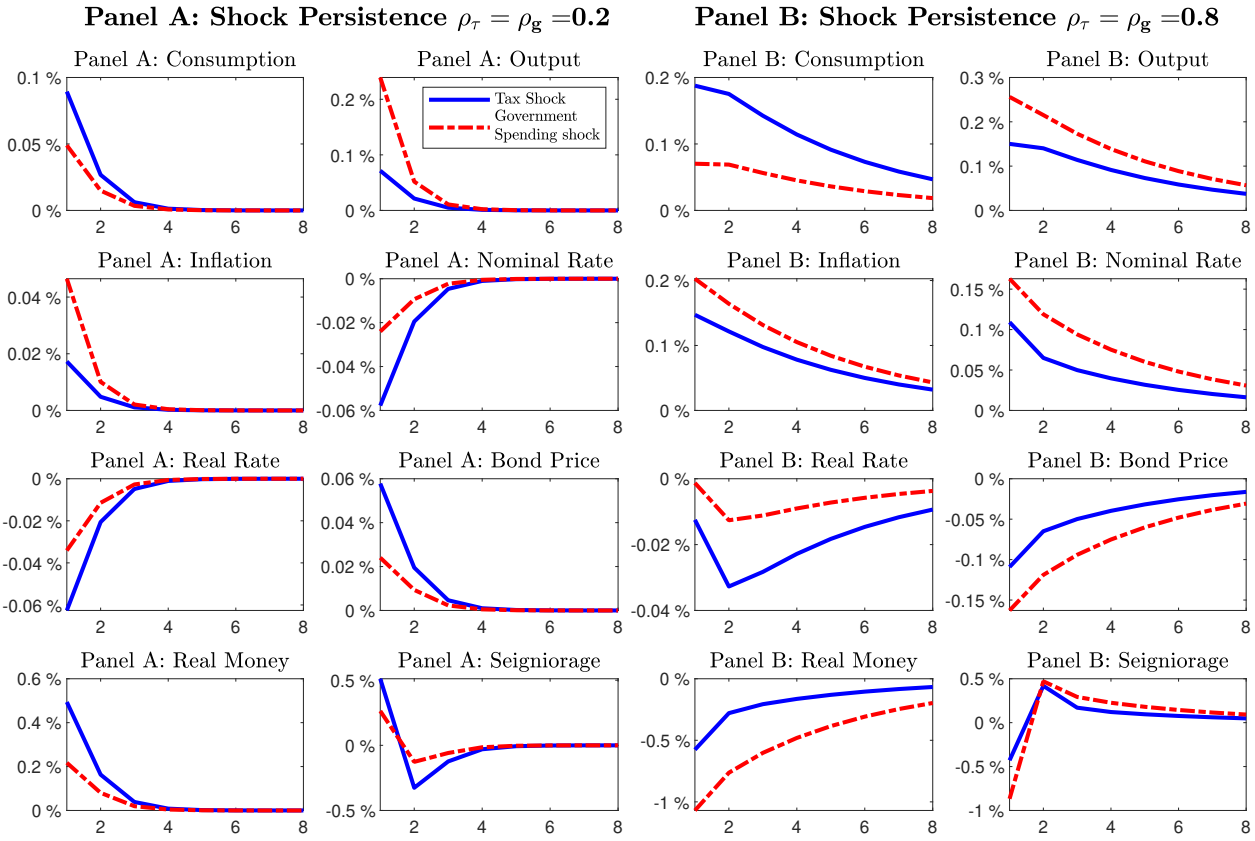



Figure 9: This figure shows the impulse response functions to 1% tax shock (blue solid line) and 1% government spending shock (red dash-dotted line), under fiscal shock persistence of 0.2 (left panel) and 0.8 (right panel). The government bond maturity is 1 quarter.

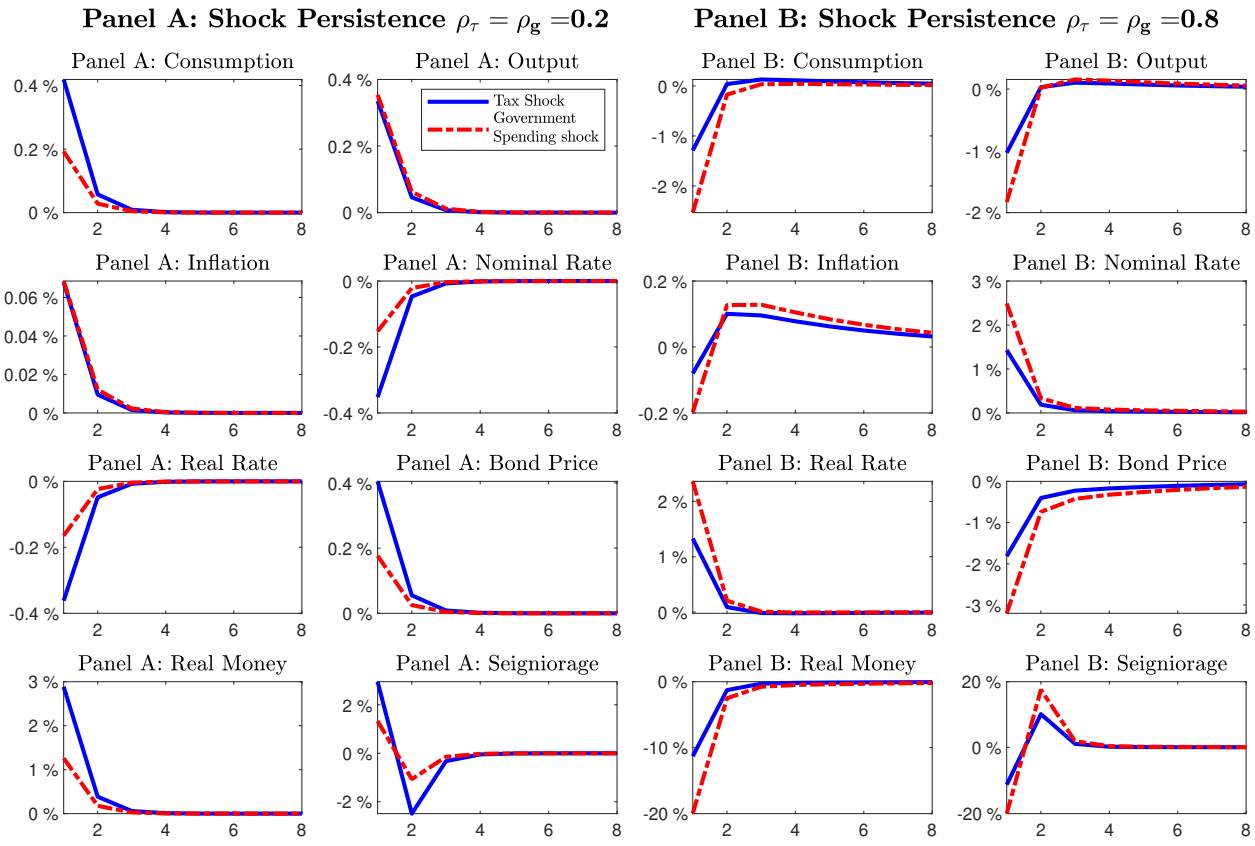



Figure 10: This figure shows the impulse response functions to 1% tax shock (blue solid line) and 1% government spending shock (red dash-dotted line) with fiscal shock persistence 0.2 (left panel) and 0.8 (right panel). The government bond maturity is 6 years.

# 5 Alternative Policy Rules

The analysis in the preceding sections assumes strict debt targeting under the money-financing regime. That is, the money supply is determined to keep the real value of debt constant. In this section, we relax this assumption and consider two alternative policy rules in the spirit of a money financing regime.

**A flexible debt targeting rule** First, we allow the coordinated monetary–fiscal authority to accommodate a partial increase in debt to finance higher government deficits. Formally, debt is targeted according to the following generalized rule:

$$\hat{b}_t = (\beta^{-1} - \psi_b)\hat{b}_{t-1} - \psi_s(\tau_b\hat{\tau}_t - g_b\hat{g}_t)$$

where the parameter $\psi_s$ governs the share of the current-period fiscal deficit financed through debt. We focus on the case $\psi_s > 0$, with smaller values of $\psi_s$ implying that a smaller fraction of the deficit is debt-financed. This formulation nests our baseline model as a special case: when $\psi_s = 0$, strict debt targeting is restored, yielding $\hat{b}_t = 0$.

Figure 11 illustrates that even when part of the deficit is financed through debt, the central insight of our analysis remains intact. The money-financed fiscal multiplier tends to rise with debt maturity when fiscal shocks are short-lived, but declines with maturity when shocks are more persistent. The figure also highlights that the multiplier depends on the share of the deficit covered by debt issuance. When persistence is low, the multiplier is relatively smaller when debt financing plays a larger role (high $\psi_s$). When persistence is high, the multiplier remains smaller at short maturities but becomes larger at long maturities if debt financing accounts for a greater share of the deficit. These results are consistent with Proposition 2: when the money-financed multiplier exceeds the debt-financed multiplier, increasing the share of the deficit financed by money raises the overall multiplier, and vice versa. The corresponding impulse response functions are reported in Appendix B.1.

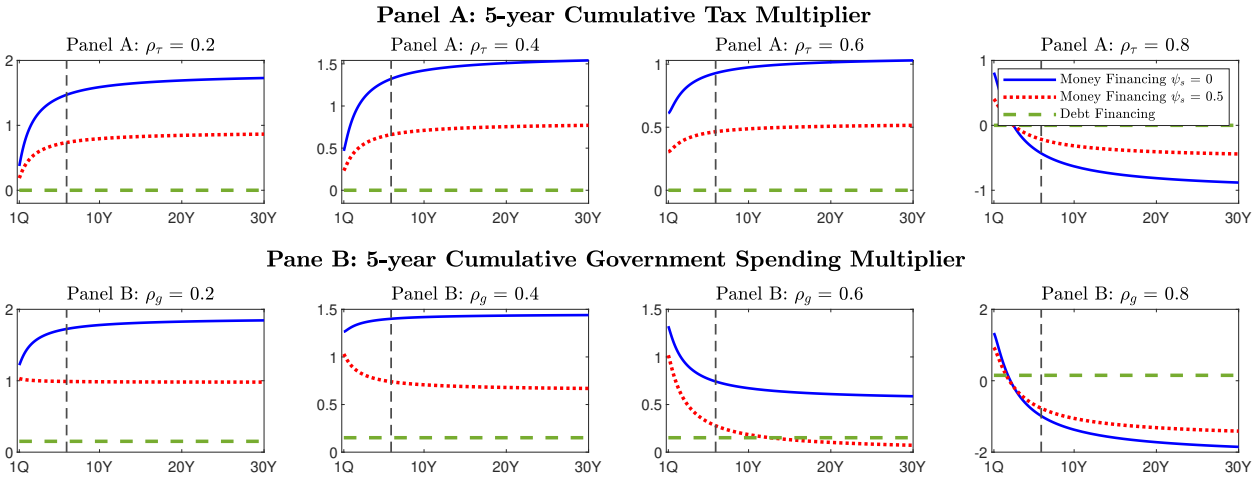



Figure 11: This figure reports the money-financed tax multiplier (Panel A) and government spending multiplier (Panel B) under a flexible debt targeting rule. Results are shown across different values of fiscal shock persistence ($\rho_\tau, \rho_g$), debt maturities, and the share of the fiscal deficit financed through debt ($\psi_s$).

**A money growth targeting rule** Second, we consider a money growth targeting rule, where the growth rate of money supply is tied directly to the size of government deficits.

Formally, the rule is specified as:

$$\log(M_t) - \log(M_{t-1}) = \psi_m(g_b \hat{g}_t - \tau_b \hat{\tau}_t)$$

where $\psi_m \in [0, 1]$ denotes the fraction of the deficit financed through money creation.

Figure 12 reports the 5-year cumulative tax and spending multipliers under this rule. In contrast to the debt-targeting framework, the money-financed fiscal multiplier is essentially invariant to changes in debt maturity and fiscal shock persistence. Moreover, it is consistently larger than its debt-financed counterpart, thereby restoring the conventional wisdom that money financing is more stimulative. The mechanism is straightforward: when monetary policy targets money growth directly, the supply of money becomes insulated from debt maturity effects, ensuring that money financing maintains its expansionary impact. The corresponding impulse response functions are reported in Appendix B.2.

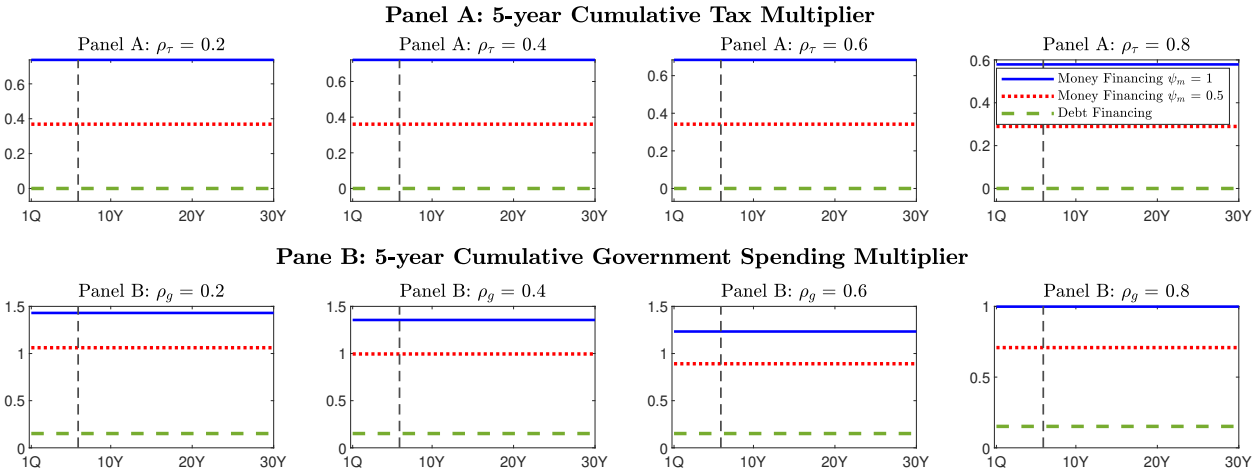



Figure 12: This figure reports the money-financed tax multiplier (Panel A) and government spending multiplier (Panel B) under a money growth targeting rule. Results are shown across different values of fiscal shock persistence ($\rho_\tau, \rho_g$), debt maturities, and the share of the fiscal deficit financed through money creation ($\psi_m$).

# 6 The effect of a money-financed fiscal stimulus in a liquidity trap

In this section, we examine the effects of money-financed fiscal stimulus, with a particular focus on the role of debt maturity when the nominal interest rate reaches the zero lower

bound.

To introduce the zero lower bound (ZLB) constraint, we assume that the economy experiences an adverse demand shock that drives the natural interest rate into negative territory. Specifically, the demand shock $\hat{z}_t$ follows the process $\hat{z}_t = -0.01 < \ln(\beta)$ for $t = 1, 2, \ldots, T$. After period T, the demand shocks vanish and the natural interest rate return back to its positive value. The trajectory of this shock is fully anticipated at $t = 1$. Since the shock lowers the natural interest rate below zero, the nominal interest rate would also decline. However, the ZLB constraint prevents it from falling below zero. In log-deviation terms, this constraint is expressed as $\hat{i}_t \geq \ln(\beta)$.

In the context of money-financed fiscal policy, the ZLB leads to an excess supply of money over demand. Consequently, the standard money market equilibrium condition (2.8) is replaced by the following complementary conditions:

$$(\hat{i}_t - \ln\beta)(\hat{m}_t - \hat{c}_t + \eta\hat{i}_t) = 0 \tag{6.1}$$

$$\hat{i}_t \geq \ln\beta \tag{6.2}$$

$$\hat{m}_t \geq \hat{c}_t - \eta\hat{i}_t \tag{6.3}$$

In the case of debt-financed fiscal policy, the monetary policy rule (2.18) is modified as follows:

$$\hat{\pi}_t(\hat{i}_t - \ln\beta) = 0 \tag{6.4}$$

$$\hat{i}_t \geq \ln\beta \tag{6.5}$$

During ZLB periods, where $\hat{i}_t = \ln\beta$, the monetary authority is unable to lower the nominal interest rate below zero to stabilize inflation at zero.

To isolate the impact of fiscal shocks under the zero lower bound, we compute the difference between the impulse responses with and without a fiscal shock in the same model. Figures 13 and 14 illustrate the cumulative tax and government spending multipliers under money financing, considering different shock persistence levels and debt maturities. Several key findings emerge.

First, the tax and government spending multiplier under money-financing are largely invariant to the maturity at the zero lower bound when the shock persistence is low. To further understand this result, Figure 15 presents the effect of tax cut shocks (Panel A) and government spending shocks (Panel B) when the shock persistence is low ($\rho_\tau = \rho_g = 0.2$). Comparing the case of the one-period debt (blue line) and long-term debt (red line), we

find that the difference is insignificant. The reason is that when the nominal interest rate is constrained by the ZLB, changes in the money supply do not affect the interest rate, and thus the endogenous money supply plays no role in the dynamics. Moreover, when the shock persistence is low, the shocks die out quickly before the zero lower bound ends, and therefore, the difference is minimal even if the economy exits the liquidity trap.

Second, when shock persistence is high, both the tax and government spending multipliers decline more significantly with longer debt maturity. This finding is qualitatively consistent with results under normal times. Figure 16 illustrates the effects of tax cut shocks (Panel A) and government spending shocks (Panel B) when $\rho_\tau = \rho_g = 0.8$. In this case, the shocks persist beyond the liquidity trap, continuing to influence the economy even after it returns to normal conditions. As previously explained, under normal circumstances, a longer debt maturity dampens the stimulative effect of fiscal expansion. Once the economy exits the ZLB, consumption and inflation are lower with longer-maturity debt than with shorter-term debt. Under rational expectations, households and firms anticipate future consumption and inflation dynamics when making decisions. Consequently, in a model with longer debt maturity, expected future inflation and consumption is smaller, leading to weaker consumption responses during the ZLB episode.

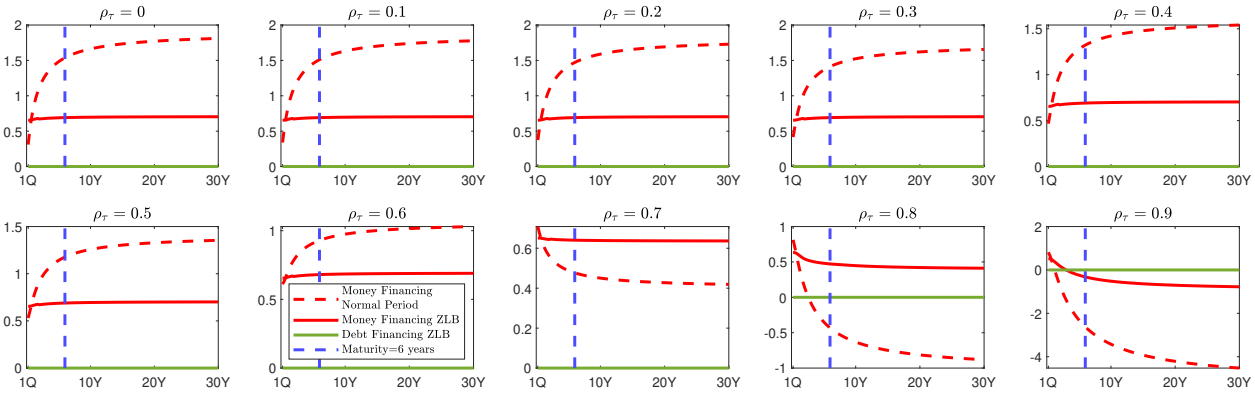



Figure 13: The figure plots the five-year money-financed tax multiplier against debt maturity during normal times (red dashed line) and at the zero lower bound (red solid line). For comparison, the debt-financed multiplier at the zero lower bound is shown in green (solid line).

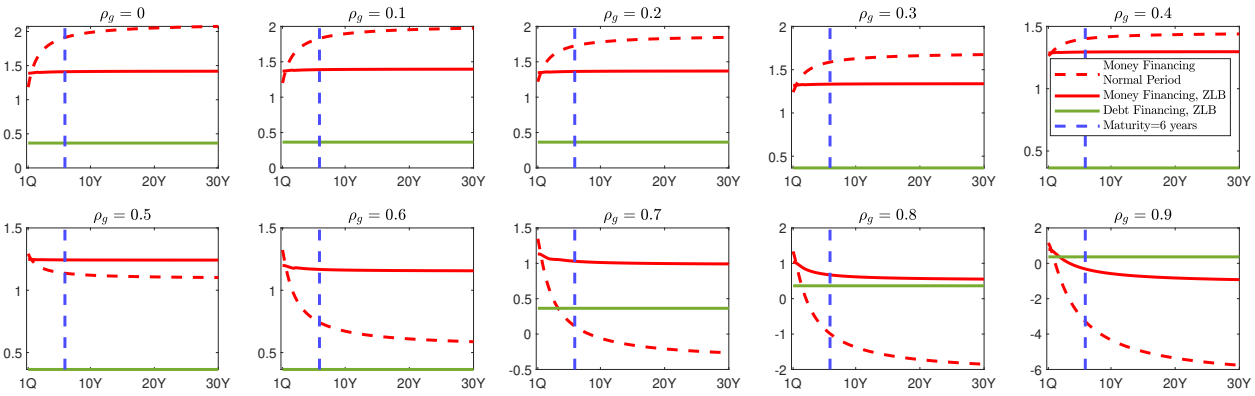



Figure 14: The figure plots the five-year money-financed government spending multiplier against debt maturity during normal times (red dashed line) and at the zero lower bound (red solid line). For comparison, the debt-financed multiplier at the zero lower bound is shown in green (solid line).

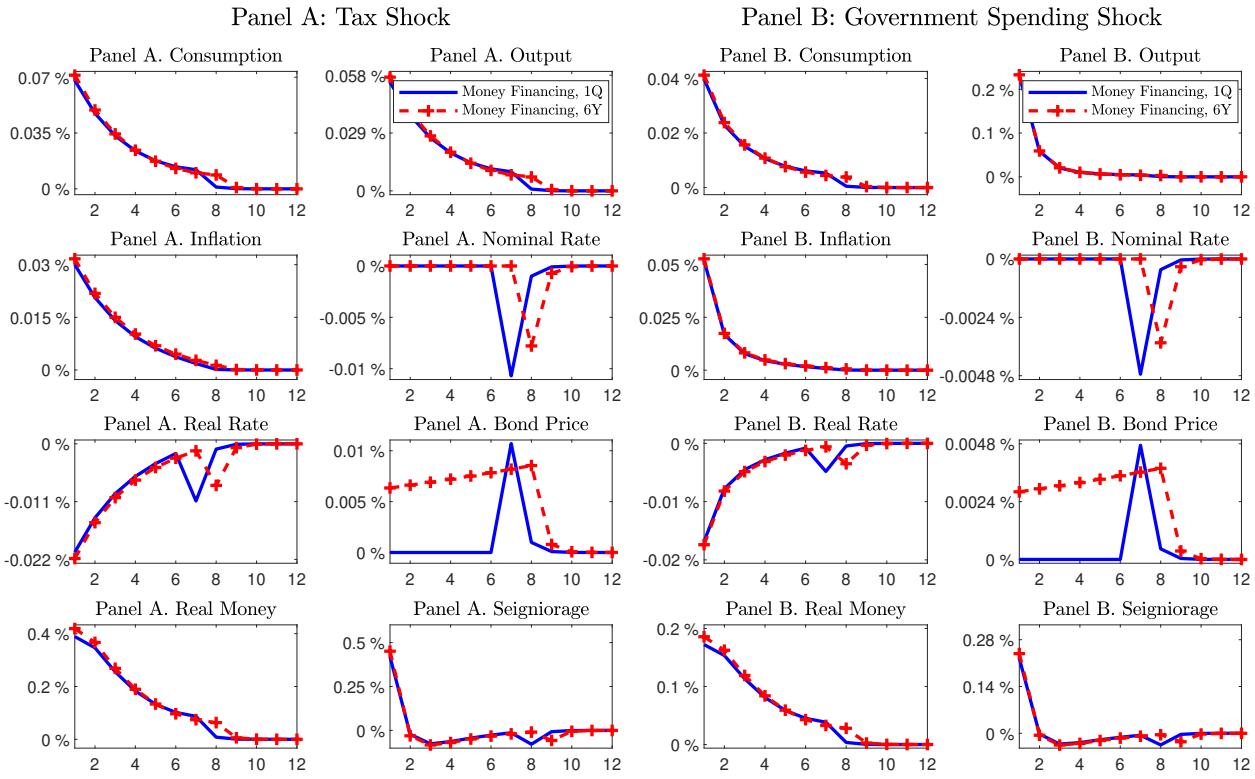



Figure 15: The figure shows the effect of a money-financed fiscal shock. Panel A shows the response to a tax shock, and Panel B shows the response to a government spending shock. The blue solid line corresponds to one-quarter bond maturity, and the red dashed line with cross markers corresponds to six-year bond maturity. Shock persistence is set at 0.2.

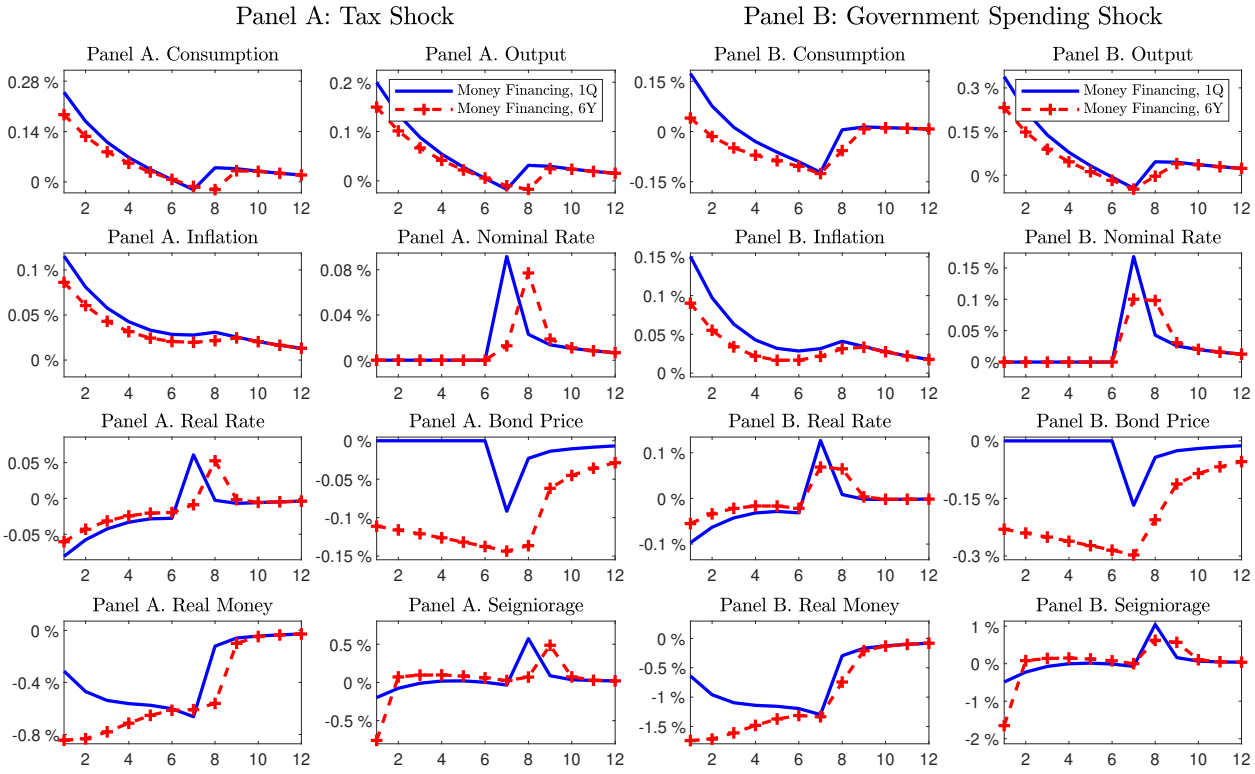



Figure 16: The figure shows the effect of a money-financed fiscal shock. Panel A shows the response to a tax shock, and Panel B shows the response to a government spending shock. The blue solid line corresponds to one-quarter bond maturity, and the red dashed line with cross markers corresponds to six-year bond maturity. Shock persistence is set at 0.8.

# 7 Concluding Remarks

This paper studies the effects of money-financed fiscal stimulus in a New Keynesian model with long-term debt, highlighting the roles of debt maturity, fiscal shock persistence, and the policy rule implementing money financing. Under a strict debt-targeting rule, we find that when shocks are short-lived, longer debt maturities amplify the stimulus in normal times. By contrast, when shocks are persistent, longer maturities reduce effectiveness, and for empirically relevant debt structures, money-financed stimulus can even be less effective than debt-financed stimulus.

These qualitative patterns persist when the zero lower bound binds, though the quantitative sensitivity to debt maturity is weaker. Importantly, directly targeting money growth generates large and robust multipliers, largely independent of debt maturity or shock persistence, restoring the conventional view that money-financed stimulus is more

effective than debt-financed stimulus.

In this analysis, we assume a fixed debt maturity structure when evaluating the effects of fiscal stimulus. An interesting avenue for future research is to examine the government's optimal debt management strategy when the central bank engages in money financing and how this, in turn, influences the effectiveness of money-financed fiscal stimulus.

# References


Angeletos, G.-M. (2002). Fiscal policy with noncontingent debt and the optimal maturity structure. *The Quarterly Journal of Economics 117*(3), 1105–1131. 1

Auerbach, A. J. and M. Obstfeld (2005). The case for open-market purchases in a liquidity trap. *American Economic Review 95*(1), 110–137. 1

Bassetto, M. (2002). A game–theoretic view of the fiscal theory of the price level. *Econometrica 70*(6), 2167–2195. 2

Benigno, P. and M. Woodford (2003). Optimal monetary and fiscal policy: A linear-quadratic approach. *NBER macroeconomics annual 18*, 271–333. 2

Bernanke, B. (2003). *Some thoughts on monetary policy in Japan*. Board of Governors of the Federal Reserve System. 1

Bhattarai, S., J. W. Lee, and W. Y. Park (2014). Inflation dynamics: The role of public debt and policy regimes. *Journal of Monetary Economics 67*, 93–108. 2

Bianchi, F., R. Faccini, and L. Melosi (2023). A fiscal theory of persistent inflation. *The Quarterly Journal of Economics 138*(4), 2127–2179. 2, 3

Bilbiie, F. O. (2009). Nonseparable preferences, fiscal policy puzzles, and inferior goods. *Journal of Money, Credit and Banking 41*(2-3), 443–450. 1

Buiter, W. H. (2014). The simple analytics of helicopter money: Why it works — always. *Economics 8*(1), 20140028. 1

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics 12*(3), 383–398. 2

Christiano, L., M. Eichenbaum, and S. Rebelo (2011). When is the government spending multiplier large? *Journal of Political Economy 119*(1), 78–121. 1

Cochrane, J. H. (2001). Long-term debt and optimal policy in the fiscal theory of the price level. *Econometrica 69*(1), 69–116. 1

Davig, T. and E. M. Leeper (2011). Monetary–fiscal policy interactions and fiscal stimulus. *European Economic Review 55*(2), 211–227. 1

Eggertsson, G. B. (2011). What fiscal policy is effective at zero interest rates? *NBER macroeconomics annual 25*(1), 59–112. 1

Faraglia, E., A. Marcet, R. Oikonomou, and A. Scott (2013). The impact of debt levels and debt maturity on inflation. *The Economic Journal 123*(566), F164–F192. 1

Friedman, M. (1948). A monetary and fiscal framework for economic stability. *The American Economic Review 38*(3), 245–264. 1

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press. 2

Galí, J. (2020). The effects of a money-financed fiscal stimulus. *Journal of Monetary Economics 115*, 1–19. 1, 1, 2, 2, 2, 3, 3, 3, 4.1, 4.2, 4.3, A.2.2

Galí, J., J. D. López-Salido, and J. Vallés (2007). Understanding the effects of government spending on consumption. *Journal of the european economic association 5*(1), 227–270. 1

Haberler, G. (1952). The Pigou Effect Once More. *Journal of Political Economy 60*(3), 240–240. 1

Leeper, E. M. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics 27*(1), 129–147. 2, 2

Lustig, H., C. Sleet, and Ş. Yeltekin (2008). Fiscal hedging with nominal assets. *Journal of Monetary Economics 55*(4), 710–727. 1

Punzo, C. and L. Rossi (2023). A money-financed fiscal stimulus: Redistribution and social welfare. *Journal of Money, Credit and Banking 55*(2-3), 595–617. 2024/01/15. 1

Schmitt-Grohé, S. and M. Uribe (2004). Optimal fiscal and monetary policy under imperfect competition. *Journal of Macroeconomics 26*(2), 183–209. 2

Sims, C. A. (1994). A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. *Economic theory 4*, 381–399. 2

Teles, P. and O. Tristani (2024). The monetary financing of a large fiscal shock. *Journal of Monetary Economics 147*, 103630. 1

Tsuruga, T. and S. Wake (2019). Money-financed fiscal stimulus: The effects of implementation lag. *Journal of Economic Dynamics and Control 104*, 132–151. 1

Woodford, M. (1994). Monetary policy and price level determinacy in a cash-in-advance economy. *Economic theory 4*, 345–380. 2

Woodford, M. (2001). Fiscal requirements for price stability. *Journal of Money, Credit and Banking 33*(3), 669–728. 1, 2

# A Model Solution and Proofs

## A.1 Equilibrium Conditions

The equilibrium conditions include the Euler equation:

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{z}_t) \tag{A.1}$$

bond pricing

$$\hat{q}_t = -\hat{i}_t + \rho\beta \mathbb{E}_t \hat{q}_{t+1} \tag{A.2}$$

money demand

$$\hat{m}_t = \hat{c}_t - \eta \hat{i}_t \tag{A.3}$$

Phillips curve

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_o \hat{y}_t - \lambda\sigma \frac{g}{c} \hat{g}_t \tag{A.4}$$

resource constrain

$$c\hat{c}_t + g\hat{g} = \hat{y}_t \tag{A.5}$$

In order to stabilize the debt, $\hat{b}_t = (\beta^{-1} - \psi_b)\hat{b}_{t-1}$, the monetary rule follows

$$\gamma_b(\hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t) = -(\tau_b \hat{\tau}_t - g_b \hat{g}_t) + \beta^{-1}(\rho\beta\hat{q}_t - \hat{q}_{t-1} - \hat{\pi}_t) \tag{A.6}$$

The tax and government spending AR(1) process

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + \varepsilon_{\tau,t} \tag{A.7}$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{g,t} \tag{A.8}$$

For convenience, substitute market-clearing condition (A.5) into the Euler equation. Let $\tilde{y}_t \equiv c^{-1}\hat{y}_t$. The Euler equation becomes

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{z}_t) + g_c(1 - \rho_g)\hat{g}_t \tag{A.9}$$

Let $\kappa \equiv \kappa_0 c^{-1}$, the Phillips curve becomes

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \tilde{y}_t - \lambda\sigma \frac{g}{c} \hat{g}_t \tag{A.10}$$

## A.2 The Simplified Model Solution

For convenience, we assume the price is sticky at $t = 1$ and becomes flexible after $t \geqslant 2$. This assumption allows monetary policy to have real effects and keeps the model tractable. In addition, we set the demand shock $\hat{z}_t = 0$ in this section and ignore the ZLB constraint.

### A.2.1 Flexible Price Model Solution

We first solve the model under flexible price assumption. Two useful lemmas that facilitate solving the model.

**Lemma 1.** *Let $\hat{i}^m_{t-1,t} \equiv \omega\hat{q}_t - \hat{q}_{t-1}$ be the nominal return realized for the long-term bond between $t-1$ and $t$, where $\omega = \rho\beta$ is the effective discount factor for the bond. The following equations hold:*

$$\Delta\mathbb{E}_t\hat{i}^m_{t-1,t} = -\sum_{j=0}^{\infty}\omega^{j+1}\Delta\mathbb{E}_t\hat{i}^m_{t+j,t+j+1}$$

$$\mathbb{E}_t\hat{i}^m_{t,t+1} = \hat{i}_t$$

*where $\Delta\mathbb{E}_t \equiv \mathbb{E}_t - \mathbb{E}_{t-1}$ is the expectation update from time $t-1$ and $t$.*

*Proof.* By the definition of $\hat{i}^m_{t,t+1}$

$$\hat{q}_t = -\hat{i}^m_{t,t+1} + \omega\hat{q}_{t+1} = -\sum_{j=0}^{\infty}\omega^j\hat{i}^m_{t+j,t+j+1}$$

Therefore,

$$\Delta\mathbb{E}_t\hat{i}^m_{t-1,t} = \Delta\mathbb{E}_t\left(\omega\hat{q}_t - \hat{q}_{t-1}\right) = -\sum_{j=0}^{\infty}\omega^{j+1}\mathbb{E}_t\hat{i}^m_{t+j,t+j+1}$$

The last equal sign is maintained because $\hat{q}_{t-1}$ is determined in $t-1$ and $\mathbb{E}_t\hat{q}_{t-1} = \mathbb{E}_{t-1}\hat{q}_{t-1}$. By the bond price equation (A.2),

$$\mathbb{E}_t\hat{i}^m_{t,t+1} = \mathbb{E}_t\left(\omega\hat{q}_{t+1} - \hat{q}_t\right) = \hat{i}_t$$

the proof is made. □

**Lemma 2.** *Under the flexible price $t \geqslant 2$, the inflation solution can be represented as*

$$\hat{\pi}_t = \phi_m\hat{m}_{t-1} - \phi_\tau\hat{\tau}_{t-1} + \phi_g\hat{g}_{t-1} + \phi_r\hat{r}_{t-1} + \nu^\pi_{\tau,t} + \nu^\pi_{g,t} \tag{A.11}$$

*where $\phi_m$, $\phi_\tau$, $\phi_g$ and $\phi_r$ are some constant coefficient and $\nu^\pi_{g,t}$ and $\nu^\pi_{\tau,t}$ are linear functions of the tax shock and the government spending shock respectively.*

*Proof.* Denote state variable $\hat{s}_{t-1} = (\hat{m}_{t-1}, \hat{\tau}_{t-1}, \hat{g}_{t-1}, \hat{q}_{t-1})$ and the exogenous shocks $\varepsilon_t = (\varepsilon_{\tau,t}, \varepsilon_{g,t})$. The decision rule of $(\hat{\pi}_t, \hat{m}_t)$ takes form:

$$\hat{\pi}_t = \boldsymbol{h}_\pi \hat{s}_{t-1} + \boldsymbol{\psi}_\pi \varepsilon_t \tag{A.12}$$

$$\hat{m}_t = \boldsymbol{h}_m \hat{s}_{t-1} + \boldsymbol{\psi}_m \varepsilon_t \tag{A.13}$$

where $\boldsymbol{h}_\pi, \boldsymbol{\psi}_\pi, \boldsymbol{h}_m, \boldsymbol{\psi}_m$ are some constant coefficients. By lemma (1),

$$\mathbb{E}_{t-1}\left[(\rho\beta\hat{q}_t - \hat{q}_{t-1} - \hat{\pi}_t)\right] = i_{t-1} - \mathbb{E}_{t-1}\hat{\pi}_t = \hat{r}_{t-1}$$

Take expectations $\mathbb{E}_{t-1}$ on the monetary policy(A.6),

$$\mathbb{E}_{t-1}\Big[\gamma_b(\hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t)\Big] = -\tau_b\rho_\tau\hat{\tau}_{t-1} + g_b\rho_g\hat{g}_{t-1} + \beta^{-1}\hat{r}_{t-1} \tag{A.14}$$

Substitute (A.12) and (A.13) into (A.14), we have

$$(\boldsymbol{h}_\pi + \boldsymbol{h}_m)\gamma_b\hat{s}_{t-1} = \gamma_m\hat{m}_{t-1} - \tau_b\rho_\tau\hat{\tau}_{t-1} + g_b\rho_g\hat{g}_{t-1} + \beta^{-1}\hat{r}_{t-1}$$

Let $h_\pi(q)$ and $h_m(q)$ be the coefficient of decision $\hat{\pi}_t$ and $\hat{m}_t$ on state $\hat{q}_{t-1}$ respectively. Provided that $h_\pi(q) + h_m(q) \neq 0$, which will be verified latter, the $\hat{q}_{t-1}$ can be represented by the linear combination of $\hat{m}_{t-1}$, $\hat{\tau}_{t-1}$, $\hat{g}_{t-1}$ and $\hat{r}_{t-1}$. Therefore, the inflation $\hat{\pi}_t$ (and other endogenous variables) can be represented by $(\hat{m}_{t-1}, \hat{\tau}_{t-1}, \hat{g}_{t-1}, \hat{r}_{t-1})$. □

We now solve inflation dynamics for $t \geqslant 2$ under the flexible price assumption.

**Flexible price solution** By Philips curve (A.10), the output $\tilde{y}_t$ is

$$\tilde{y}_t = \frac{\lambda\sigma}{\kappa}g_c\hat{g}_t = \underbrace{\frac{\sigma c^{-1}(1-\alpha)}{\alpha + \varphi + \sigma c^{-1}(1-\alpha)}}_{\chi_g} g_c\hat{g}_t \tag{A.15}$$

$\chi_g$ is the government spending multiplier. By market clearing condition (A.5),

$$\hat{c}_t = \tilde{y}_t - g_c\hat{g}_t = -\underbrace{(1-\chi_g)g_c}_{\chi_c}\hat{g}_t \tag{A.16}$$

By the Euler equation (A.9), the real interest rate is

$$\hat{r}_t = \sigma\Big[\mathbb{E}_t(\tilde{y}_{t+1}) - \tilde{y}_t + (1-\rho_g)g_c g_t\Big] = \sigma \underbrace{(1-\rho_g)(1-\chi_g)g_c}_{\chi_r}\hat{g}_t \tag{A.17}$$

The real interest only responds to the government spending shock.

By lemma (2), we consider find the coefficient $\phi_m, \phi_\tau, \phi_g, \phi_r$ and $\nu^\pi_{\tau,t}, \nu^\pi_{g,t}$. The solving process takes two steps. First, we shut down the government spending shock $\hat{g}_t = 0, \forall t$ and solve $\phi_m, \phi_\tau, \phi_r$ and $\nu^\pi_{\tau,t}$. Second, we mute the tax shock $\hat{\tau}_t = 0, \forall t$ and solve the $\phi_g$ and $\nu^\pi_{g,t}$.

First step $\hat{g}_t = 0$. For $t \geqslant 2$, the inflation solution is

$$\hat{\pi}_t = \phi_m \hat{m}_{t-1} - \phi_\tau \hat{\tau}_{t-1} + \phi_r \hat{r}_{t-1} + \nu^\pi_{\tau,t}, \tag{A.18}$$

Combining $\hat{c}_t = \hat{r}_t = 0$, (A.18) and the money demand (A.3), we have

$$\hat{m}_t = \frac{\eta\phi_\tau}{1+\eta\phi_m}\hat{\tau}_t, \quad t \geqslant 2 \tag{A.19}$$

Substitute (A.18) and (A.19) into (A.14),

$$\beta^{-1}\hat{r}_{t-1} - \tau_b\rho_\tau\hat{\tau}_{t-1} - \gamma_b\Big(\frac{\eta\phi_\tau}{1+\eta\phi_m}\rho_\tau\hat{\tau}_{t-1} - \hat{m}_{t-1} + \phi_m\hat{m}_{t-1} - \phi_\tau\hat{\tau}_{t-1} + \phi_r\hat{r}_{t-1}\Big) = 0$$

By undetermined coefficient method, we solve the $\phi_m, \phi_\tau, \phi_r$:

$$\phi_m = 1, \quad \phi_\tau = \rho_\tau\tau_\gamma\Big(1 - \frac{\eta}{1+\eta}\rho_\tau\Big)^{-1}, \quad \phi_r = \gamma_b^{-1}\beta^{-1} \tag{A.20}$$

where $\tau_\gamma \equiv \tau_b \cdot \gamma_b^{-1} = \frac{\tau}{\gamma}$. The nominal interest rate is given by

$$\hat{i}_t = \mathbb{E}_t\hat{\pi}_{t+1} = \hat{m}_t - \phi_\tau\hat{\tau}_t = -\underbrace{\frac{\rho_\tau\tau_\gamma}{1+\eta-\eta\rho_\tau}}_{\phi_{i,\tau}}\hat{\tau}_t \tag{A.21}$$

Take $\Delta\mathbb{E}_t$ on the monetary policy.

$$\beta^{-1}\Delta\mathbb{E}_t(\hat{i}^m_{t,t-1} - \hat{\pi}_t) - \tau_b\varepsilon_{\tau,t} - \gamma_b\big(\Delta\mathbb{E}_t\hat{m}_t + \Delta\mathbb{E}_t\hat{\pi}_t\big) = 0 \tag{A.22}$$

By money demand (A.3) and (A.21),

$$\Delta\mathbb{E}_t\hat{m}_t = \Delta\mathbb{E}_t(-\eta\hat{i}_t) = \eta\phi_{i,\tau}\varepsilon_{\tau,t} \tag{A.23}$$

By lemma (1),

$$\Delta\mathbb{E}_t(\hat{i}^m_{t-1,t} - \hat{\pi}_t) = -\sum_{j=0}^{\infty}\omega^{j+1}\Delta\mathbb{E}_t\hat{i}^m_{t+j,t+j+1} - \Delta\mathbb{E}_t(\hat{\pi}_t)$$

$$\Delta\mathbb{E}_t\hat{i}^m_{t+j,t+j+1} = \Delta\mathbb{E}_t\big(\mathbb{E}_{t+j}\hat{i}^m_{t+j,t+j+1}\big) = \Delta\mathbb{E}_t\big(\hat{i}_{t+j}\big) = \Delta\mathbb{E}_t\big(\hat{r}_{t+j} + \hat{\pi}_{t+j+1}\big) = \Delta\mathbb{E}_t\big(\hat{\pi}_{t+j+1}\big)$$

Therefore,

$$\Delta\mathbb{E}_t(\hat{i}^m_{t-1,t} - \hat{\pi}_t) = -\sum_{j=1}^{\infty}\omega^{j}\Delta\mathbb{E}_t\big(\hat{\pi}_{t+j}\big) - \Delta\mathbb{E}_t(\hat{\pi}_t) \tag{A.24}$$

For $j \geqslant 1$, by (A.18) and (A.21),

$$\Delta\mathbb{E}_t\big(\hat{\pi}_{t+j}\big) = \Delta\mathbb{E}_t\Big(\hat{m}_{t+j} - \phi_\tau\hat{\tau}_{t+j}\Big) = -\phi_{i,\tau}\rho_\tau^{j-1}\varepsilon_{\tau,t} \tag{A.25}$$

Substitute (A.23) - (A.25) into (A.22), we have

$$\nu^{\pi}_{\tau,t} \equiv \Delta\mathbb{E}_t\hat{\pi}_t = \left(\tau_b + \phi_{i,\tau}\gamma_b\eta - \frac{\phi_{i,\tau}\rho}{1-\rho\beta\rho_\tau}\right)\frac{1}{\beta^{-1}+\gamma_b}(-\varepsilon_{\tau,t}) \tag{A.26}$$

Second step $\hat{\tau}_t = 0$. The procedure is similar. However, $\hat{r}_t = \sigma\chi_r\hat{g}_t$ and $\hat{c}_t = -\chi_c\hat{g}_t$ are not zero given a government spending shock.

$$\phi_g = \gamma_b^{-1}\Big(1 - \frac{\eta\rho_g}{1+\eta}\Big)^{-1}\left\{g_b\rho_g + \frac{\gamma_b}{1+\eta}\Big[\chi_c + \eta\sigma\chi_r(1+\phi_r)\Big]\right\} \tag{A.27}$$

$$\nu^{\pi}_{g,t} = \Delta\mathbb{E}_t\pi_t = \left[g_b + \gamma_b\big(\chi_c + \psi_{i,g}\eta\big) - \frac{\phi_{i,g}\rho}{1-\rho\beta\rho_g}\right]\frac{1}{\beta^{-1}+\gamma_b}\varepsilon_{g,t} \tag{A.28}$$

where $\phi_{i,g}$ is the nominal interest rate response to the government shock

$$\phi_{i,g} \equiv \frac{\partial\hat{i}_t}{\partial\hat{g}_t} = \frac{-\chi_c}{1+\eta} + \frac{\phi_g}{1+\eta} + \frac{1+\phi_r}{1+\eta}\sigma\chi_r \tag{A.29}$$

Finally, we verify $h_\pi(q) + h_m(q) \neq 0$. By (A.16), (A.19) and (A.29), the money demand is independent of $\hat{q}_{t-1}$,, i.e, $h_m(q) = \frac{\partial\hat{m}_t}{\partial\hat{q}_{t-1}} = 0$. By lemma (1), the real interest rate can be

written as

$$\begin{aligned}\hat{r}_{t-1} &= \hat{i}_{t-1} - \mathbb{E}_{t-1}\hat{\pi}_t \\ &= \mathbb{E}_{t-1}\hat{i}^m_{t-1,t} - \mathbb{E}_{t-1}\hat{\pi}_t \\ &= \mathbb{E}_{t-1}(\omega\hat{q}_t) - \hat{q}_{t-1} - \phi_m\hat{m}_{t-1} + \phi_\tau\hat{\tau}_{t-1} - \phi_r\hat{r}_{t-1}\end{aligned}$$

By bond pricing equation (A.2), (A.21) and (A.29),

$$\hat{q}_t = -\sum_{j=0}^{\infty}\omega^j\mathbb{E}_t\hat{i}_{t+j} = -\sum_{j=0}^{\infty}\omega^j\mathbb{E}_t(\phi_{i,\tau}\hat{\tau}_{t+j} + \phi_{i,g}\hat{g}_{t+j})$$

This implies that the under flexible case, $\hat{q}_t$ is independent of $\hat{q}_{t-1}$. Thus,

$$\hat{r}_{t-1} = -\frac{1}{1+\phi_r}\hat{q}_{t-1} + \frac{1}{1+\phi_r}\Big[\mathbb{E}_{t-1}(\omega\hat{q}_t) - \phi_m\hat{m}_{t-1} + \phi_\tau\hat{\tau}_{t-1}\Big] \tag{A.30}$$

Substitute into (A.30) into (A.11), we get $h_\pi(q) = -\frac{\phi_r}{1+\phi_r}$. This verifies the condition.

**A summary of flexible price solution** Under the flexible price, the dynamics of inflation is

$$\hat{\pi}_t = \phi_m\hat{m}_{t-1} - \phi_\tau\hat{\tau}_{t-1} + \phi_g\hat{g}_{t-1} + \phi_r\hat{r}_{t-1} + \nu^\pi_{\tau,t} + \nu^\pi_{g,t} \tag{A.31}$$

The coefficients are given by (A.20) and (A.27). The innovations $\nu^\pi_{\tau,t}$ and $\nu^\pi_{g,t}$ are given by (A.26) and (A.28). The dynamics of nominal interest rate is

$$\hat{i}_t = -\phi_{i,\tau}\hat{\tau}_t + \phi_{i,g}\hat{g}_t \tag{A.32}$$

The coefficient $\phi_{i,\tau}$ and $\phi_{i,g}$ are given by (A.21) and (A.29). The dynamics of money is

$$\hat{m}_t = \hat{c}_t - \eta\hat{i}_t = \eta\phi_{i,\tau}\hat{\tau}_t - (\chi_c + \eta\phi_{i,g})\hat{g}_t \tag{A.33}$$

The bond price is given by

$$\hat{q}_t = -\sum_{j=0}^{\infty}(\rho\beta)^j\mathbb{E}_t\hat{i}_{t+j} = \frac{\phi_{i,\tau}}{1-\rho\beta\rho_\tau}\hat{\tau}_t - \frac{\phi_{i,g}}{1-\rho\beta\rho_g}\hat{g}_t \tag{A.34}$$

### A.2.2 Time 1 On-Impact Multiplier

In this section, we solve $t = 1$ on-impact multiplier. The basic idea is we represent $\hat{m}_1$ $\hat{q}_1$ and $\hat{\pi}_1$ in output $\tilde{y}_1$ using Euler equation, money demand, bond pricing and the Phillips curve, then substitute them into the money supply equation and solve the equilibrium $\tilde{y}_1$. Similarly, we solve for the money-financed tax multiplier and the government spending multiplier separately. Note that because the price is flexible after $t \geqslant 2$, firms do not consider the aggregate price level when setting the time 1 price. Therefore, the expected inflation term is dropped from the time 1 Philips curve

$$\hat{\pi}_1 = \kappa \tilde{y}_1 - \lambda \sigma \frac{g}{c} \hat{g}_1 \tag{A.35}$$

**The on-impact tax multiplier** By (A.35), the inflation is $\hat{\pi}_1 = \kappa \tilde{y}_1$. By Euler equation (A.9), $\hat{r}_1 = -\sigma \tilde{y}_1$. By money demand (A.3) and market clear (A.5),

$$\hat{m}_1 = \tilde{y}_1 - g_c \hat{g}_1 - \eta \left( \hat{r}_1 + \mathbb{E}_1 \hat{\pi}_2 \right)$$

Using the time 2 inflation solution (A.31), the money $\hat{m}_1$ can be written as

$$\hat{m}_1 = \frac{1 + \eta\sigma(1 + \phi_r)}{1 + \eta} \tilde{y}_1 + \eta \phi_{i,\tau} \hat{\tau}_1 \tag{A.36}$$

Combine the expression of $\hat{m}_1$ and inflation $\hat{\pi}_2$, we solve the nominal rate

$$\hat{i}_1 = \hat{r}_1 + \mathbb{E}_1 \hat{\pi}_2 = -\phi_{i,\tau} \hat{\tau}_1 - \frac{\sigma(1 + \phi_r) - 1}{1 + \eta} \tilde{y}_1 \tag{A.37}$$

By bond pricing equation (A.2) and time 2 bond price(A.34),

$$\hat{q}_1 = \frac{\phi_{i,\tau}}{1 - \rho\beta\rho_\tau} \hat{\tau}_1 + \frac{\sigma(1 + \phi_r) - 1}{1 + \eta} \tilde{y}_1 \tag{A.38}$$

Substituting (A.36) - (A.38) into money supply (A.6), the time 1 tax multiplier is

$$m_\tau = \frac{c}{\tau} \left\{ \frac{\tau_b + \gamma_b \eta \phi_{i,\tau} - \frac{\rho}{1 - \rho\beta\rho_\tau} \phi_{i,\tau}}{-\rho \frac{\sigma(1+\phi_r)-1}{1+\eta} + \gamma_b \frac{1 + \eta\sigma\left(1+\phi_r\right)}{1+\eta} + (\beta^{-1} + \gamma_b)\kappa} \right\} \tag{A.39}$$

For convenience, we denote the numerator and denominator in the bracket as $\Gamma_\tau(\rho, \rho_\tau)$ and $\Gamma_0(\rho)$ respectively. Because the debt-financed tax multiplier is 0, the difference between the money-financed tax multiplier and the debt-financed multiplier is $\frac{\Gamma_\tau(\rho,\rho_\tau)}{\Gamma_0(\rho)}$, scaled by the constant $c/\tau$.

**The on-impact government spending multiplier** Following a similar procedure, the government multiplier is

$$m_g = \frac{c}{g}\left\{\frac{g_b + \gamma_b(\chi_c + \eta\phi_{i,g}) - \frac{\rho}{1-\rho\beta\rho_g}\phi_{i,g}}{-\rho\frac{\sigma(1+\phi_r)-1}{1+\eta} + \gamma_b\frac{\eta\sigma(1+\phi_r)+1}{1+\eta} + (\beta^{-1}+\gamma_b)\kappa} + \chi_g g_c\right\} \tag{A.40}$$

We denote the numerator in the bracket as $\Gamma_g(\rho, \rho_g)$. Note that the debt-financed multiplier is $\chi_g$ if price is flexible after $t \geqslant 2$. The difference between the money-financed government spending multiplier and debt-financed government spending multiplier is $\frac{\Gamma_g(\rho,\rho_g)}{\Gamma_0(\rho)}$, scaled by the constant $c/g$.

Throughout, we will maintain the assumption for all $\rho \in [0, 1)$, $\Gamma_0(\rho) > 0$. The assumption holds given Galí (2020) calibration.

## A.3 Proofs of the Propositions

### A.3.1 Proof of Proposition 1

*Proof.* The tax multiplier is decreasing in maturity if and only if

$$\frac{\partial}{\partial\rho}\left(\frac{\Gamma_\tau(\rho,\rho_\tau)}{\Gamma_0(\rho)}\right) = \left[\Gamma_0(\rho)\right]^{-2}\left[\frac{-\phi_{i,\tau}}{(1-\rho\beta\rho_\tau)^2}\Gamma_0(\rho) + \Gamma_\tau(\rho,\rho_\tau)\frac{\sigma(1+\phi_r)-1}{1+\eta}\right] \leqslant 0$$

That is

$$\tau_b\frac{\sigma(1+\phi_r)-1}{1+\eta} \leqslant \frac{\rho_\tau\tau_\gamma}{1+\eta-\eta\rho_\tau}\left\{\left[-\gamma_b\eta + \frac{\rho}{1-\rho\beta\rho_\tau}\right]\frac{\sigma(1+\phi_r)-1}{1+\eta} + \frac{\Gamma_0(\rho)}{(1-\rho\beta\rho_\tau)^2}\right\}$$

If $\rho_\tau = 0$, the right side is 0. Moreover, the right side is increasing in $\rho_\tau$ and goes to infinity provided $\rho_\tau$ is large enough. The threshold $\bar{\rho}'_\tau$, as the function of maturity $\rho$, is the solution when the equality holds. □

### A.3.2 Proof of Proposition 2

*Proof.* The money-financed tax multiplier is larger than the debt-financed multiplier if and only if $\Gamma_\tau(\rho, \rho_\tau) \geqslant 0$, that is

$$\rho \leqslant H(\rho_\tau) \equiv \frac{\tau_b + \gamma_b \eta \phi_{i,\tau}}{(\tau_b + \gamma_b \eta \phi_{i,\tau})\beta\rho_\tau + \phi_{i,\tau}} = \left(\beta\rho_\tau + \frac{1}{\tau_b/\phi_{i,\tau} + \gamma_b \eta}\right)^{-1}$$

Note that $H(0) = +\infty$. By (A.21), $\phi_{i,\tau}$ is increasing in tax persistence $\rho_\tau$, therefore, $H(\rho_\tau)$ is deceasing in $\rho_\tau$. The threshold tax persistence $\overline{\rho}''_\tau$ is defined as the solution of $H(\rho_\tau) = 1$. By definition, the money-financed multiplier is always larger than the debt-financed multiplier if $\rho_\tau \leqslant \overline{\rho}'_\tau$. If tax shock persistence $\rho_\tau > \overline{\rho}''_\tau$, then the money-financed tax multiplier is smaller than the debt-financed multiplier if maturity $\rho$ is larger than $\rho_{m,\tau} \equiv H(\rho_\tau)$. □

# B Alternative Policy Rules

## B.1 A flexible debt targeting rule

A flexible debt targeting rule is given by:

$$\hat{b}_t = (\beta^{-1} - \psi_b)\hat{b}_{t-1} - \psi_s(\tau_b \hat{\tau}_t - g_b \hat{g}_t)$$

Figures 17 and 18 present the impulse responses to a 1 percent tax shock under different levels of shock persistence ($\rho_\tau = 0.2$ and $\rho_\tau = 0.8$) and government bond maturities (one quarter and six years). Figures 19 and 20 show the impulse responses to a 1 percent government spending shock under the same persistence and maturity combinations.

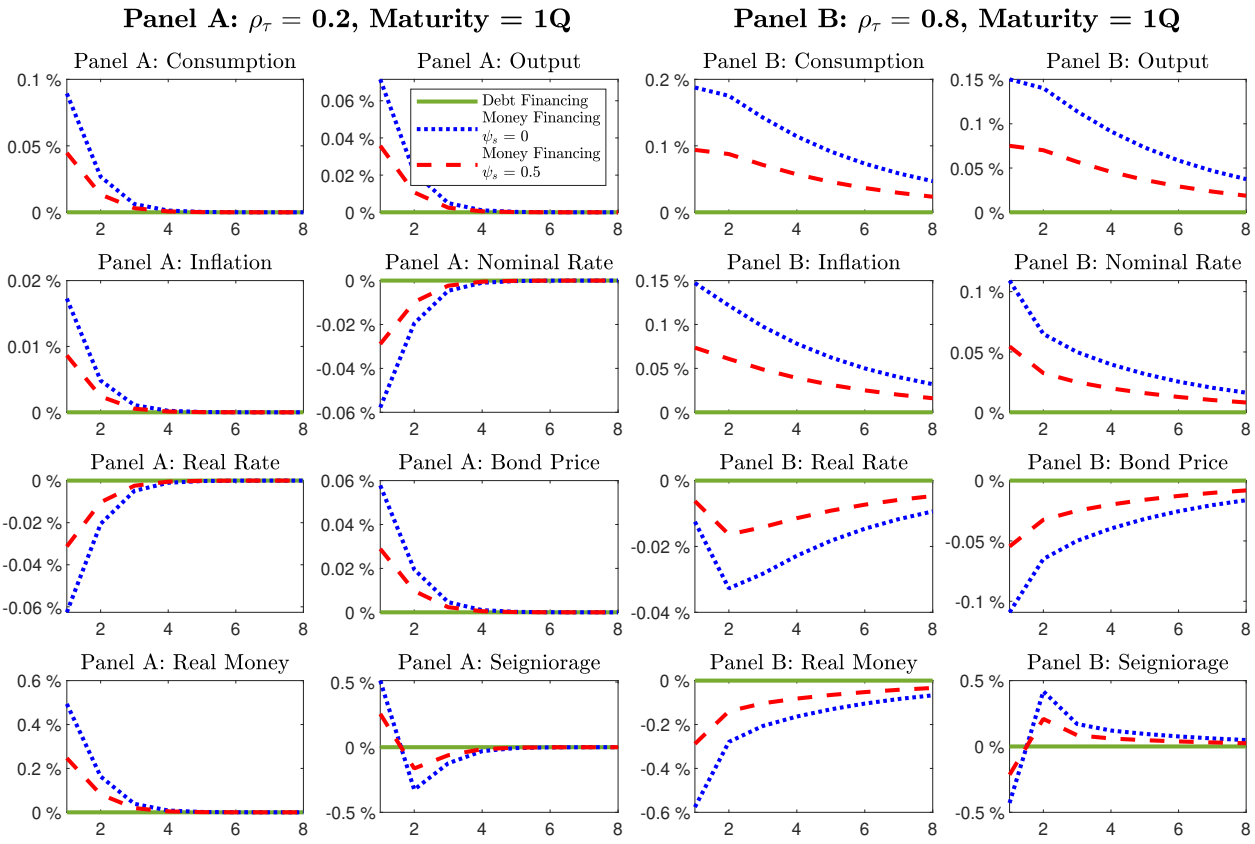



Figure 17: The figure presents the impulse response functions to 1% tax cut shock with tax persistence $\rho_\tau = 0.2$ (left panel) and tax persistence $\rho_\tau = 0.8$ (right panel). The government bond maturity is 1 quarter. The debt law of motion is generalized to allow for financing $\psi_s$ proportion of fiscal deficits. The blue dotted line corresponds to money financing with $\psi_s = 0$. The red dashed line represents money financing with $\psi_s = 0.5$. The green solid line represents the debt financing.

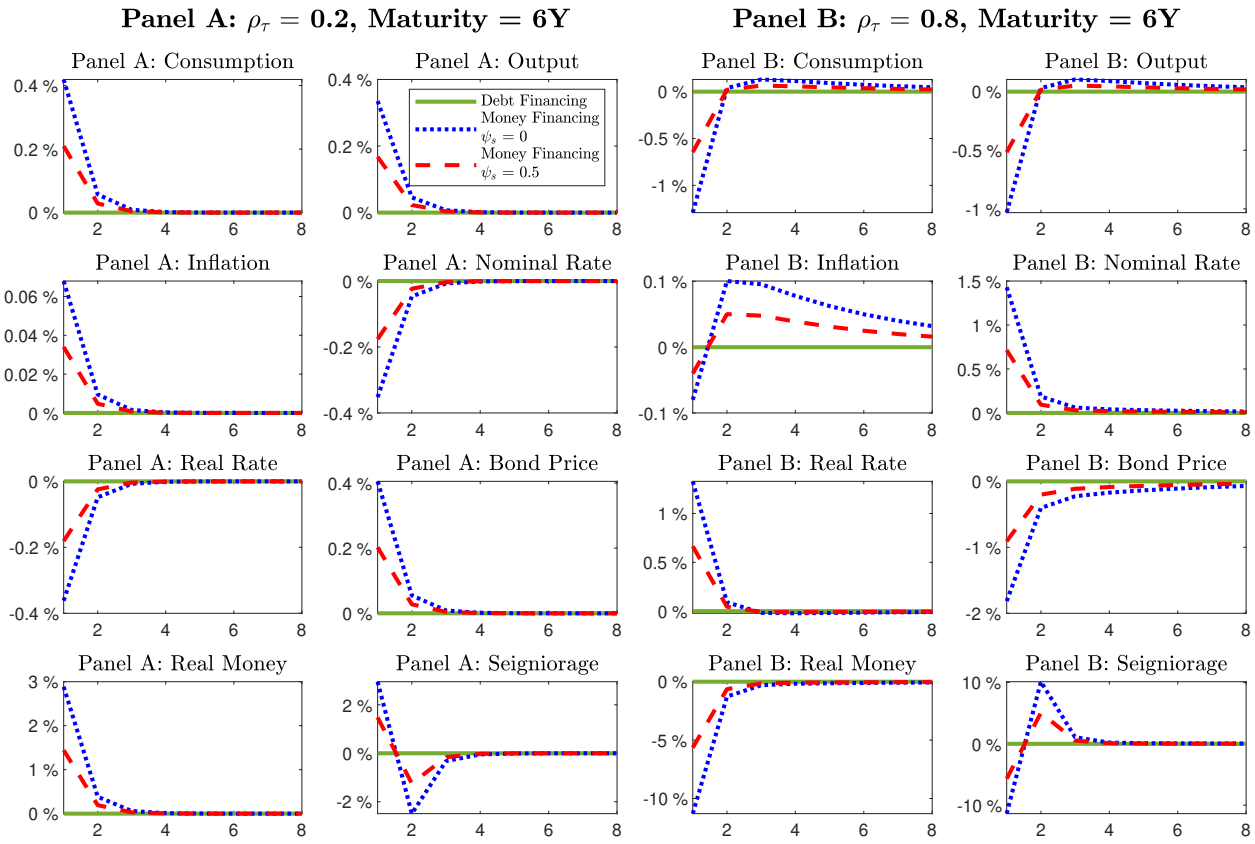



Figure 18: The figure presents the impulse response functions to 1% tax cut shock with tax persistence $\rho_\tau = 0.2$ (left panel) and tax persistence $\rho_\tau = 0.8$ (right panel). The government bond maturity is 6 years. The debt law of motion is generalized to allow for financing $\psi_s$ proportion of fiscal deficits. The blue dotted line corresponds to money financing with $\psi_s = 0$. The red dashed line represents money financing with $\psi_s = 0.5$. The green solid line represents the debt financing.

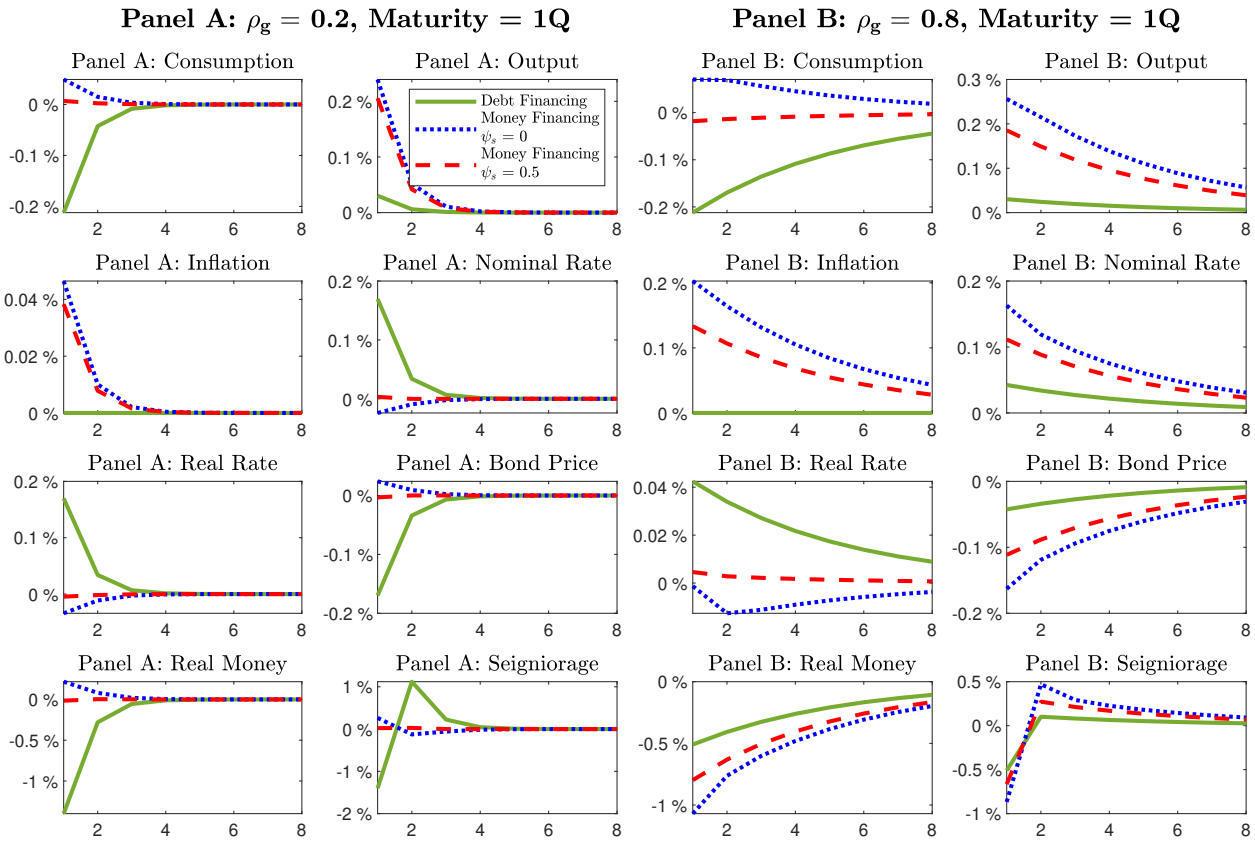



Figure 19: The figure presents the impulse response functions to 1% government spending shock with persistence $\rho_g = 0.2$ (left panel) and persistence $\rho_g = 0.8$ (right panel). The government bond maturity is 1 quarter. The debt law of motion is generalized to allow for financing $\psi_s$ proportion of fiscal deficits. The blue dotted line corresponds to money financing with $\psi_s = 0$. The red dashed line represents money financing with $\psi_s = 0.5$. The green solid line represents the debt financing.

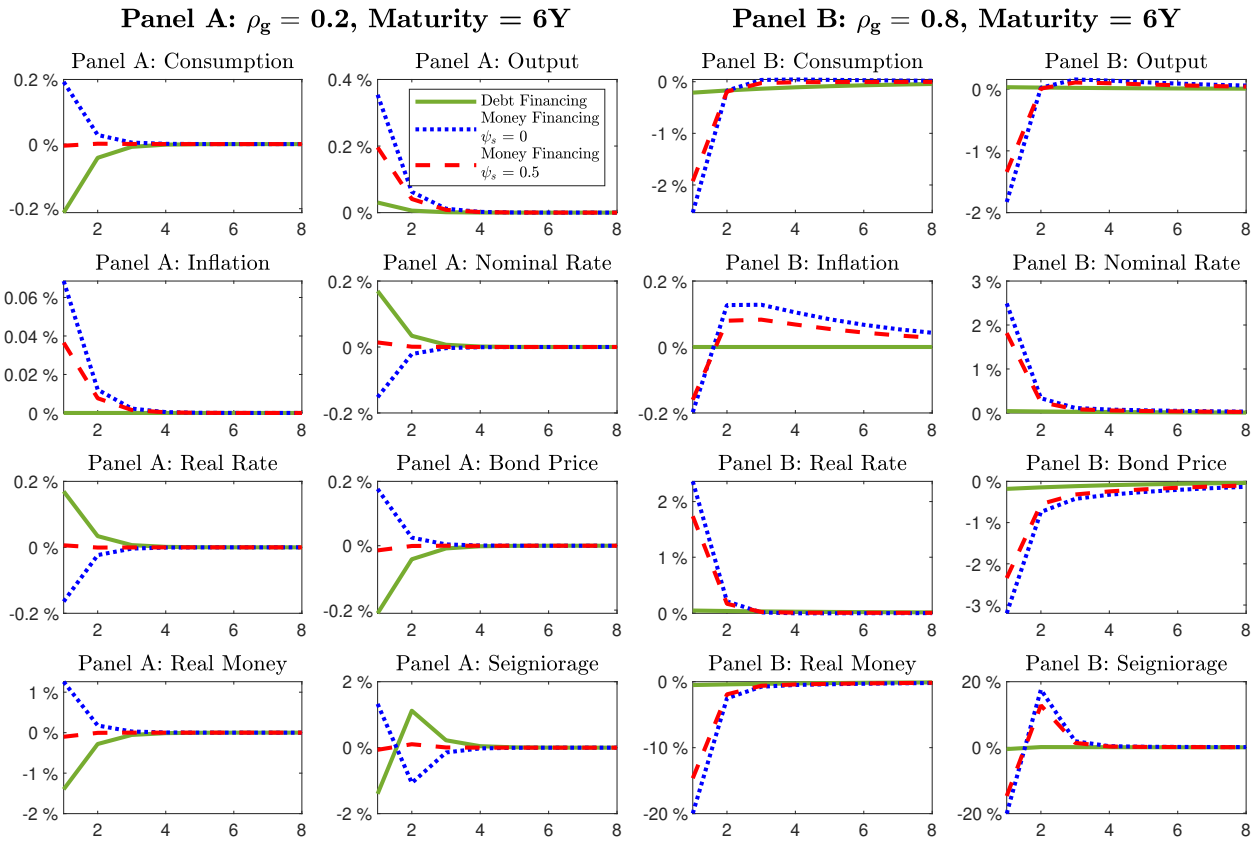



Figure 20: The figure presents the impulse response functions to 1% government spending shock with persistence $\rho_g = 0.2$ (left panel) and persistence $\rho_g = 0.8$ (right panel). The government bond maturity is 6 years. The debt law of motion is generalized to allow for financing $\psi_s$ proportion of fiscal deficits. The blue dotted line corresponds to money financing with $\psi_s = 0$. The red dashed line represents money financing with $\psi_s = 0.5$. The green solid line represents the debt financing.

## B.2 A money growth rule

A money growth rule is given by

$$\log(M_t) - \log(M_{t-1}) = \psi_m(g_b\hat{g}_t - \tau_b\hat{\tau}_t)$$

where $\psi_m \in [0, 1]$, represents the proportion of deficits financed by the seigniorage (money growth).

Figures 21 and 22 report the impulse responses to a 1 percent tax shock under the money growth targeting rule, for maturities of one quarter and six years, respectively. Likewise, Figures 23 and 24 display the impulse responses to a 1 percent government spending shock under the same rule.

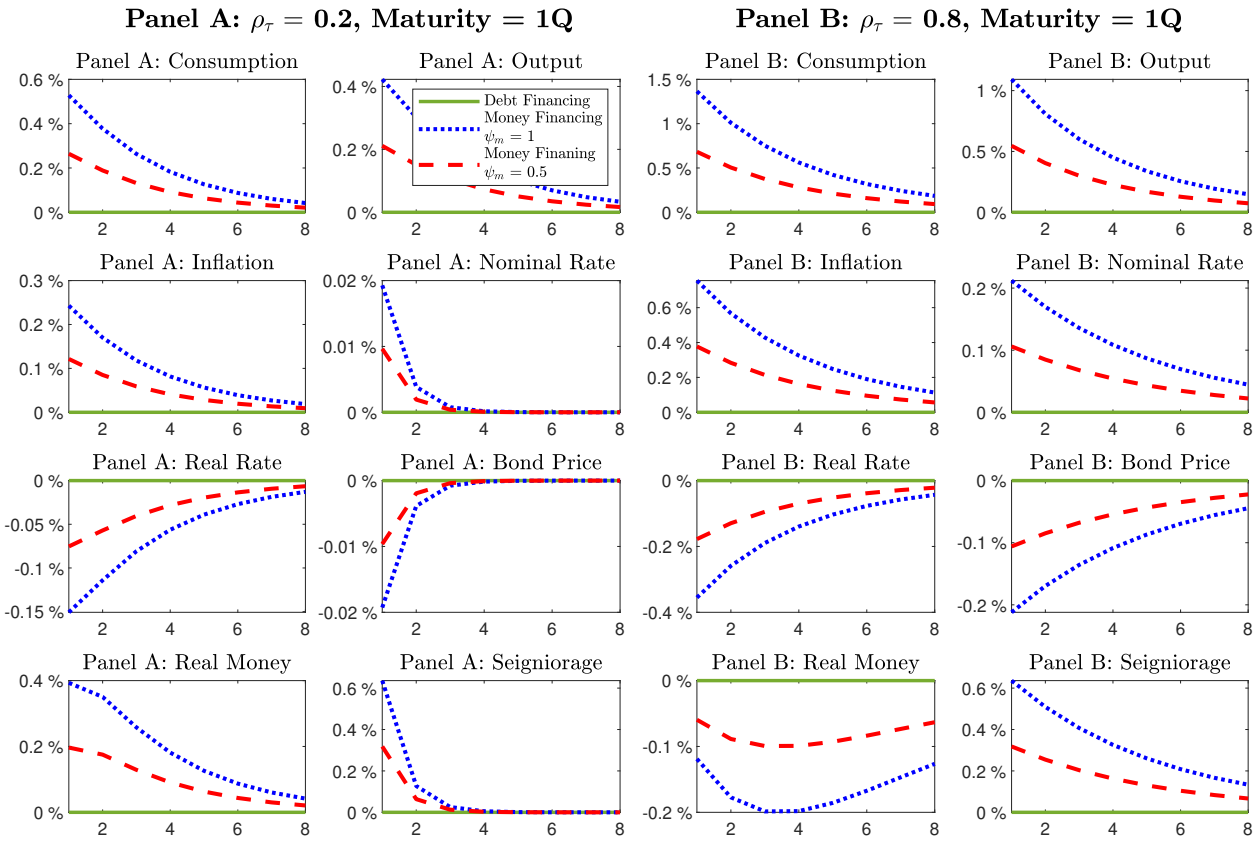



Figure 21: The figure presents the impulse response functions to 1% tax cut shock with tax persistence $\rho_\tau = 0.2$ (left panel) and tax persistence $\rho_\tau = 0.8$ (right panel). The government bond maturity is 1 quarter. The monetary rule is targeting fiscal deficits, where $\psi_m$ proportion of deficits is financed by the money creation. The blue dot line corresponds to money financing with $\psi_m = 1$. The red dashed line represents money financing with $\psi_m = 0.5$. The green solid line represents the debt financing.

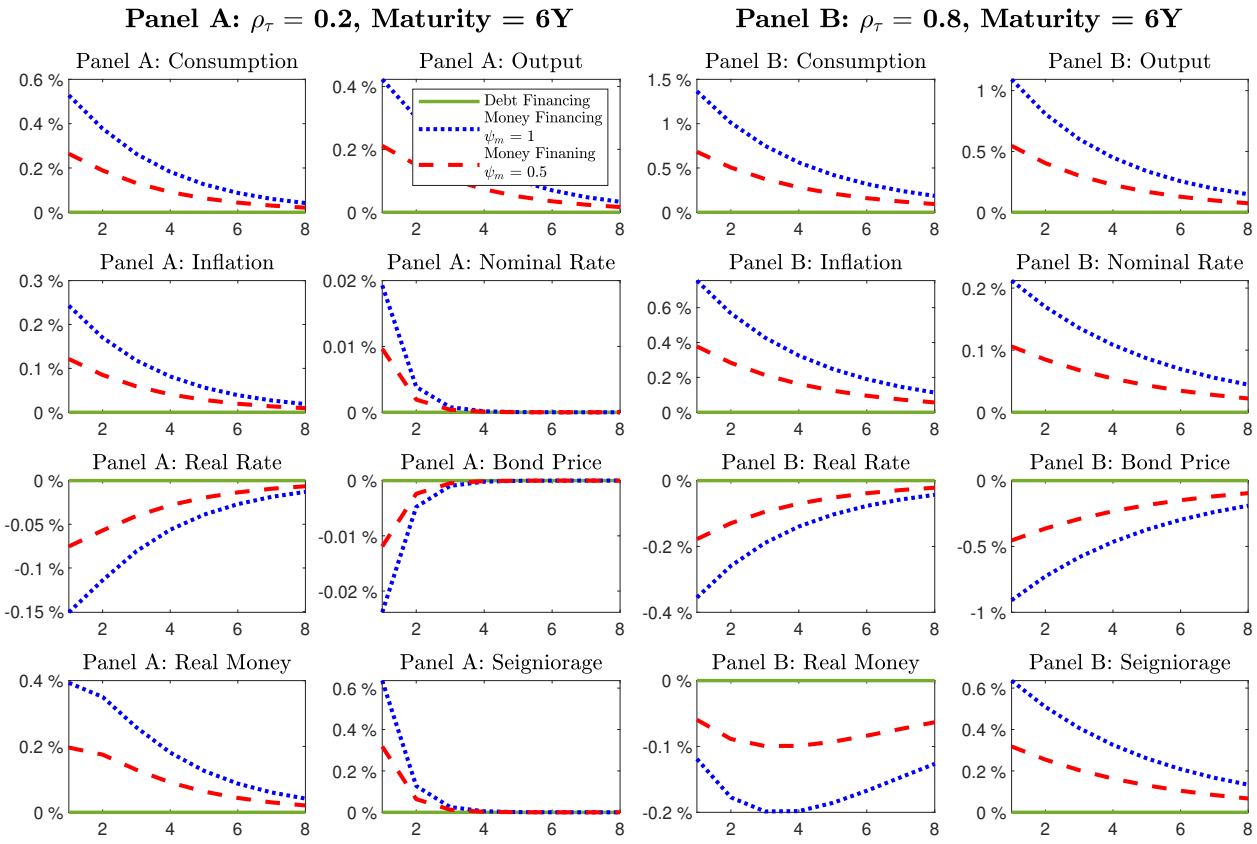



Figure 22: The figure presents the impulse response functions to 1% tax cut shock with tax persistence $\rho_\tau = 0.2$ (left panel) and tax persistence $\rho_\tau = 0.8$ (right panel). The government bond maturity is 6 years. The monetary rule is targeting fiscal deficits, where $\psi_m$ proportion of deficits is financed by the money creation. The blue dot line corresponds to money financing with $\psi_m = 1$. The red dashed line represents money financing with $\psi_m = 0.5$. The green solid line represents the debt financing.

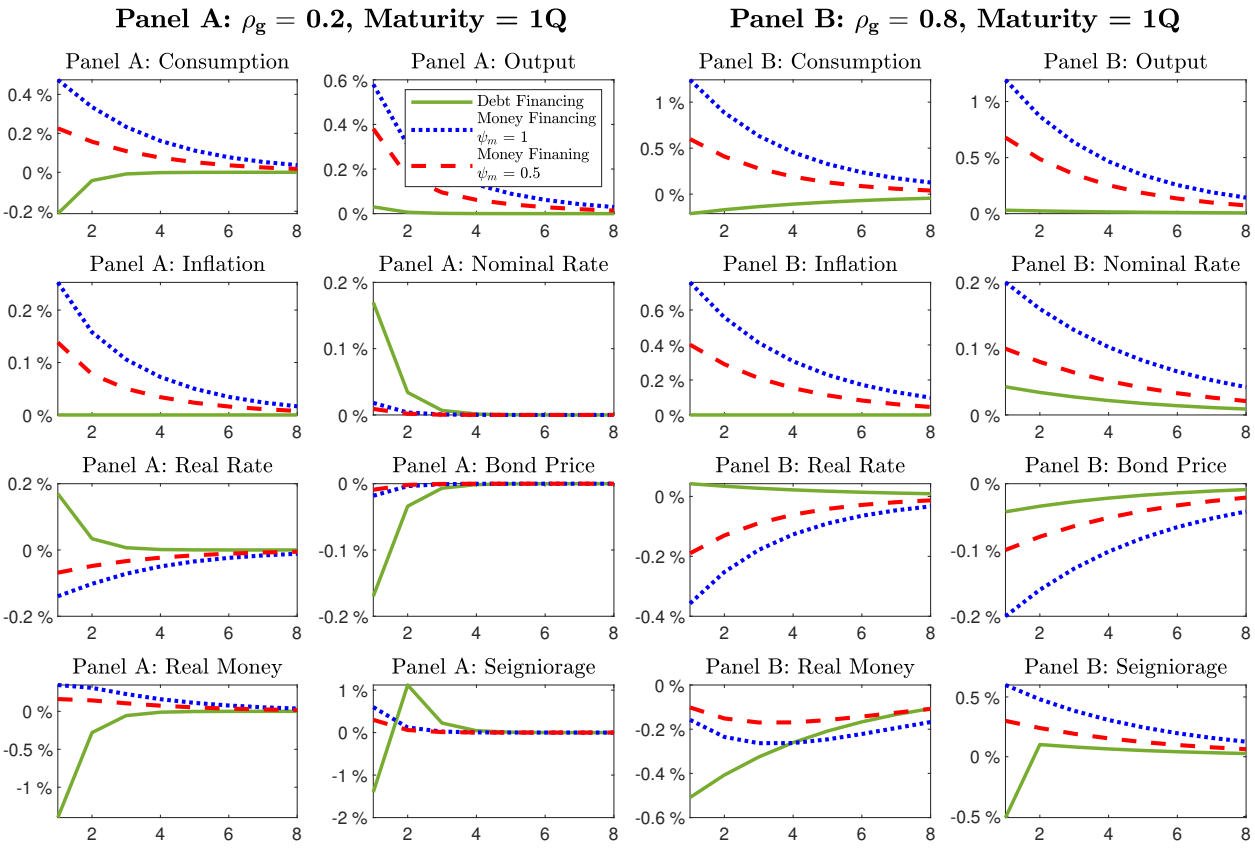



Figure 23: The figure presents the impulse response functions to 1% government spending shock with persistence $\rho_g = 0.2$ (left panel) and persistence $\rho_g = 0.8$ (right panel). The government bond maturity is 1 quarter. The monetary rule is targeting fiscal deficits, where $\psi_m$ proportion of deficits is financed by the money creation. The blue dot line corresponds to money financing with $\psi_m = 1$. The red dashed line represents money financing with $\psi_m = 0.5$. The green solid line represents the debt financing.

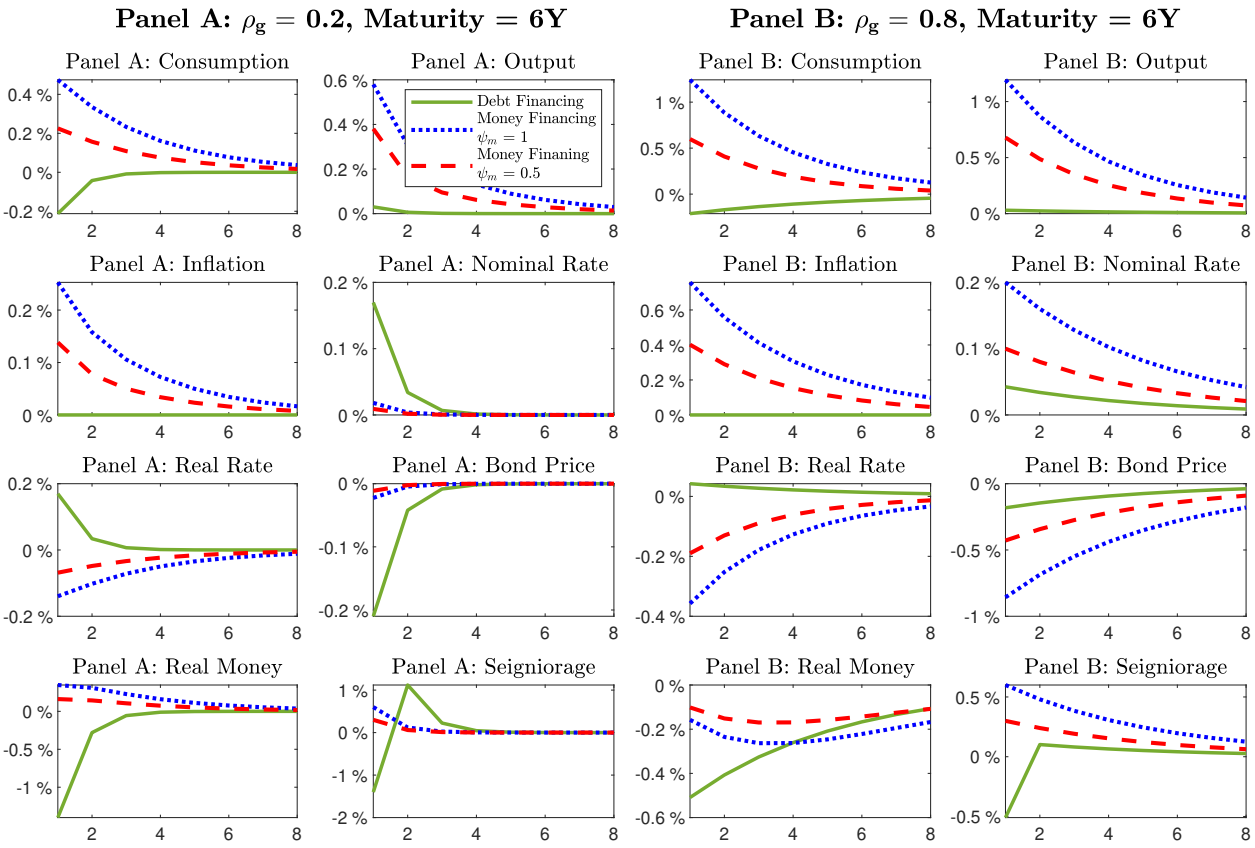



Figure 24: The figure presents the impulse response functions to 1% government spending shock with persistence $\rho_g = 0.2$ (left panel) and persistence $\rho_g = 0.8$ (right panel). The government bond maturity is 6 years. The monetary rule is targeting fiscal deficits, where $\psi_m$ proportion of deficits is financed by the money creation. The blue dot line corresponds to money financing with $\psi_m = 1$. The red dashed line represents money financing with $\psi_m = 0.5$. The green solid line represents the debt financing.