Monetary Policy with Partial Fiscal Backing*

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Abstract

We show, thorough the lens of a new Keynesian model, partially unfunded government debt can notably improve social welfare when the economy is buffeted by demand and technology shocks. If monetary policy follows a conventional rule to stabilize price inflation and economic activity, the optimal share of the shocks dealt with by unfunded government debt is near one third, and is even higher facing a ZLB constraint. But unfunded debt becomes undesirable for society when a more inertial and effective monetary policy is employed. Nonetheless, unfunded debt can be welfare improving as a means for monetary policy to look through price markup shocks.

Keywords: automatic stabilizers, fiscal and monetary interactions, partially unfunded government debt.

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1 Introduction

Related literature:

- Bianchi, Faccini and Melosi (2023)
- Cochrane (2022)
- Smets and Wouters (2024)

The rest of the paper is organized as follows. Section 2 describes the model and its calibration, with demand and technology shocks buffeting the economy and with monetary policy following a Taylor rule, then Section 3 presents the model outcomes. Section 4 shows the outcomes when monetary policy follows instead a first-difference rule. Section 5 considers price and wage markup shocks buffeting the economy. Section 6 concludes. Appendices A and B investigate the robustness of the outcomes to leaning against fiscal inflation or unfunded debt, while Appendix C shows the equilibrium conditions of the optimal commitment policy.

2 The Model

2.1 Private sector

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t \tag{1}$$

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \varkappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t \tag{2}$$

$$\widetilde{\omega}_t \equiv \widetilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\hat{\omega}_t^n \tag{3}$$

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\left(\hat{\imath}_t - E_t\{\pi_{t+1}^p\} - \hat{r}_t^n\right) \tag{4}$$

$$\widetilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n \text{ and } \hat{y}_t^n \equiv \psi_{ya} a_t \tag{5}$$

$$\widetilde{\omega}_t \equiv \hat{\omega}_t - \hat{\omega}_t^n \text{ and } \hat{\omega}_t^n \equiv \psi_{\omega a} a_t \tag{6}$$

$$\hat{r}_t^n \equiv (1-\rho_z) z_t - \sigma\,(1-\rho_a)\,\psi_{ya} a_t \tag{7}$$

where the demand shock z_t and technology shock a_t follow $AR(1)$ processes with autoregressive coefficients respectively ρ_z and ρ_a

$\hat{\imath}_t = i_t - \rho$ and $\rho \equiv 1/\beta - 1 > 0$

$\varkappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$ and $\varkappa_w \equiv \lambda_w\left(\sigma + \frac{\varphi}{1-\alpha}\right)$

$\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)(1-\alpha)}{\theta_p(1-\alpha+\alpha\epsilon_p)}$ and $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$

$\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$

2.2 Government budget with long-term debt

Government bonds are perpetuities whose coupon declines at rate $\eta \in [0,1]$ in each period, and the price of the bond is Q_t.

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} + \beta^{-1} b \left(\eta\beta\hat{Q}_t - \hat{Q}_{t-1} - \pi_t^p\right) - \hat{\tau}_t \tag{8}$$

$$\hat{\imath}_t = \eta\beta E_t\left\{\hat{Q}_{t+1}\right\} - \hat{Q}_t \tag{9}$$

where $\hat{Q}_t \equiv (Q_t - Q)/Q$ and $Q \equiv \beta/(1-\eta\beta)$
average duration of the bond is $1/(1-\eta\beta)$
$\hat{b}_t \equiv (B_t - B)/Y$, $\hat{\tau}_t \equiv (T_t - T)/Y$, and $b \equiv B/Y$

2.3 Fiscal-led shadow economy

Copy of the above equations (1)-(9) but with superscript "F" added to all variables and with parameter $\gamma \in [0,1]$ multiplying each of the shocks z_t and a_t. Hence γ indicates the share of the shock that is dealt with by unfunded debt. That is, when $\gamma = 0$ ($\gamma = 1$) the policy response to the shock is *monetary-led* (*fiscal-led*), as shown next.

2.4 Policy rules with ZLB constraint

In the *fiscal-led* shadow economy, assume that the nominal interest rate is permanently pegged to its steady state (passive monetary policy) and that taxes do not respond to debt (active fiscal policy, so debt is unfunded and inflated away)[1]

$$\hat{\imath}_t^F = 0 \tag{10}$$

$$\hat{\tau}_t^F = \psi_\tau\hat{\tau}_{t-1}^F + (1-\psi_\tau)\,\psi_y\widetilde{y}_t^F \tag{11}$$

The policy rules in the *monetary-led* economy (active monetary and passive fiscal) are

$$\hat{\imath}_t = \max\left[-\rho, \hat{\imath}_t^*\right] \tag{12}$$

$$\hat{\imath}_t^* = \phi_i\hat{\imath}_{t-1}^* + (1-\phi_i)\left[\phi_p^M\left(\pi_t^p - \pi_t^{pF}\right) + \phi_y^M\left(\widetilde{y}_t - \widetilde{y}_t^F\right)\right] \tag{13}$$

$$\hat{\tau}_t = \psi_\tau\hat{\tau}_{t-1} + (1-\psi_\tau)\left[\psi_b^M\left(\hat{b}_{t-1} - \hat{b}_{t-1}^F\right) + \psi_y\widetilde{y}_t\right] \tag{14}$$

[1] This typical description of policy rules in a *fiscal-led* economy is not restrictive for our analysis. Specifically, Appendix A allows monetary policy to lean against fiscal inflation, $\hat{\imath}_t^F$ reacts to π_t^{pF}, and so unfunded debt becomes less desirable. Appendix B lets fiscal policy lean against unfunded debt, $\hat{\tau}_t^F$ reacts to $\hat{b}_{t-1}^F$, but the automatic stabilizer in the fiscal rule ensures the desirability of unfunded debt is not affected.

Fiscal policy is "passive" means that debt is funded by primary surpluses, so fiscal policy has to adjusts taxes enough to ensure the debt is repaid rather than inflated away, $\psi_b^M > \rho$. In fact, the fiscal rule (14) and government budget (8) together imply the dynamics of funded debt $\hat{b}_t$ depends on $\left(\beta^{-1} - \psi_b^M\right)\hat{b}_{t-1}$. The stabile dynamics of funded debt thus requires $\beta^{-1} - \psi_b^M < 1$, that is, $\psi_b^M > \beta^{-1} - 1 \equiv \rho$.

Note, there is no need to introduce fiscal policy shocks nor monetary policy shocks in the above policy rules as their implications for the optimal choice of γ would be trivial. In fact, given that taxes in the model are lump sum, fiscal shocks can be offset by funded debt, $\gamma = 0$. While the permanent nominal interest rate peg, (10), implies that monetary shocks can be assumed absent when debt is unfunded, $\gamma = 1$. Another implication of (10) is that the debt duration becomes irrelevant for the choice of γ, as $\hat{\imath}_t^F = 0$ implies $\hat{Q}_t^F = 0$ for all t, so $\hat{b}_t^F$ is independent of η.

2.5 Social welfare loss to evaluate policies

Model consistent welfare criterion

$$\mathbb{L} = \frac{1}{2}\left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) var\left(\widetilde{y}_t\right) + \frac{\epsilon_p}{\lambda_p} var\left(\pi_t^p\right) + \frac{\epsilon_w(1-\alpha)}{\lambda_w} var\left(\pi_t^w\right)\right] \tag{15}$$

The welfare outcomes obtained by the above policy rules (varying γ between zero and unity) are compared to the welfare achieved by the optimal policy under commitment (which is independent of γ). The equilibrium conditions of the optimal commitment policy are shown in Appendix C.

2.6 Baseline calibration

See Table 1. The calibration of the economy and of the Taylor rule are standard. The fiscal rule is calibrated to maximize social welfare when debt is unfunded, $\gamma = 1$. The bond coupon decay rate $\eta = 0.955$ is chosen so the average duration of the bond is 5 years, but the welfare outcomes are independent of η.

- The standard deviations of the shocks σ_z and σ_a are chosen so that conditional on each shock the ZLB binds 20 percent of the time if $\gamma = 0$ (funded debt), see Figure 1.
- Regarding the automatic stabilizer in the fiscal rule, i.e. the fiscal responsiveness to the output gap, set $\psi_y = 1$ so to maximize welfare if $\gamma = 1$ (unfunded debt), see Figure 2.
- Set $b = 4$ which corresponds to a debt target equal to 100% of annual GDP and it maximizes welfare if $\gamma = 1$, see Figure 3.

3 Partially Unfunded Debt and Economic Performance

- Figures 4 to 7 show the response of the economy with and without ZLB to shocks $-z_t$ and $+a_t$, i.e. shocks that cause a recession and that reduce the nominal interest rate when debt is funded. With funded debt ($\gamma = 0$) *inflation falls* the most, while unfunded debt ($\gamma = 1$) results in *overshooting of inflation*, regardless of the ZLB. But partially unfunded debt ($\gamma = 0.5$) *stabilizes inflation* which implies that it is welfare improving.

- Figures 8 and 9 show partially unfunded debt can improve social welfare, with and without ZLB. In Figure 8, conditional on each shock z_t and a_t, the optimal γ is around 0.3 without ZLB and increases to 0.4 with ZLB. The ZLB calls for relatively more unfunded debt, so more inflation pressure to stabilize the economy in downturns. In Figure 9, with funded debt ($\gamma = 0$) all three components of the welfare loss are larger when facing the ZLB; hence the ZLB implies more room for fiscal policy to help stabilize both inflation and output.

4 More Effective Monetary Policy

Replace the above Taylor rule (13) with a first-difference rule

$$\Delta \hat{\imath}_t^* = \phi_p^M \left(\pi_t^p - \pi_t^{pF} \right) + \phi_y^M \left(\Delta \hat{y}_t - \Delta \hat{y}_t^F \right) \tag{16}$$

where ϕ_p^M and ϕ_y^M are kept at their baseline values.

- Figures 10 shows the first-difference rule (16) can so improve welfare to make unfunded debt undesirable, with and without ZLB.

5 Facing Markup Shocks

Introduce price markup shock u_t and wage markup shock v_t. The persistence of these shocks is again 0.8, but their standard deviations are set to $\sigma_u = \sigma_v = 0.06\%$ which is small enough to avoid the ZLB in the model outcomes. Hence (1) and (2) become

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t + u_t \tag{17}$$

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \varkappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t + v_t \tag{18}$$

- Figures 11 shows unfunded debt can be welfare improving as a means for monetary policy to look through price markup shocks. In the left panel, facing price markup shocks the welfare loss is larger under the first-difference rule (16) relative to the Taylor rule (13). Intuitively, price markup shocks push price inflation and output in opposite directions, and so the strong reaction of rule (16) reduces price inflation volatility but at the cost of increased volatility of output. Welfare will also depend on how rule (16) affects wage inflation, as shown next.

- Figures 12 shows the response to shocks $-u_t$ and $-v_t$. Note, wage inflation tends to rise after $-u_t$ but tends to fall after $-v_t$. Facing either shock, the first-difference rule (16) raises output and so puts upward pressure on wage inflation. Rule (16) thus can make wage inflation more volatile facing price markup shocks but less volatile facing wage markup shocks.

6 Concluding Remarks

TBA

References

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Parameter	Description	Value
β	Discount factor	0.995
σ	Curvature of consumption utility	1
φ	Curvature of labor disutility	5
α	Index of decreasing returns to labor	0.25
ϵ_p	Elasticity of substitution of goods	9
ϵ_w	Elasticity of substitution of labour	4.5
θ_p	Calvo index of price rigidities	0.75
θ_w	Calvo index of wage rigidities	0.75
ϕ_i	Smoothing coefficient in Taylor rule	0.85
ϕ_p^M	Monetary response to price inflation	1.5
ϕ_y^M	Monetary response to output gap	0.125
ψ_τ	Smoothing coefficient in fiscal rule	0.85
ψ_b^M	Fiscal response to debt	0.1
ψ_y	Fiscal response to output gap	1
γ	Share unfunded debt	0
b	Debt-to-GDP target	4
η	Bond coupon decay rate	0.955
$\rho_{z,a}$	Persistence of shocks	0.8
σ_z	Std. deviation of demand shock	9.7%
σ_a	Std. deviation of technology shock	7.4%

Notes: Values are shown in quarterly rates.

Table 1: Baseline calibration.

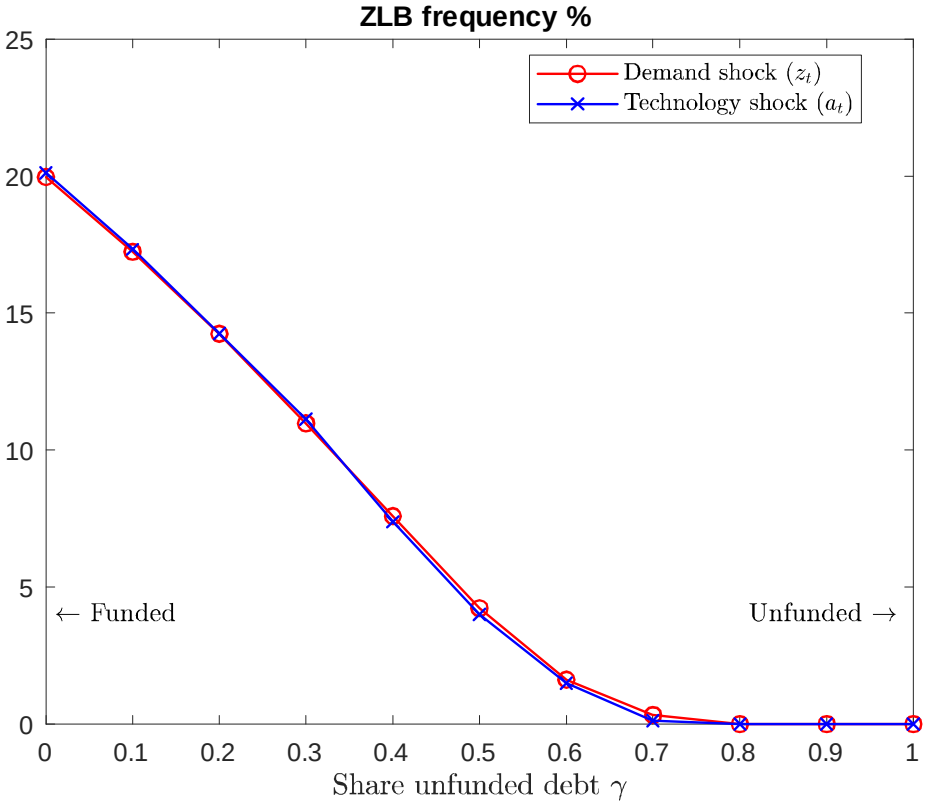


Figure 1: Partially unfunded debt can reduce the incidence of ZLB episodes.

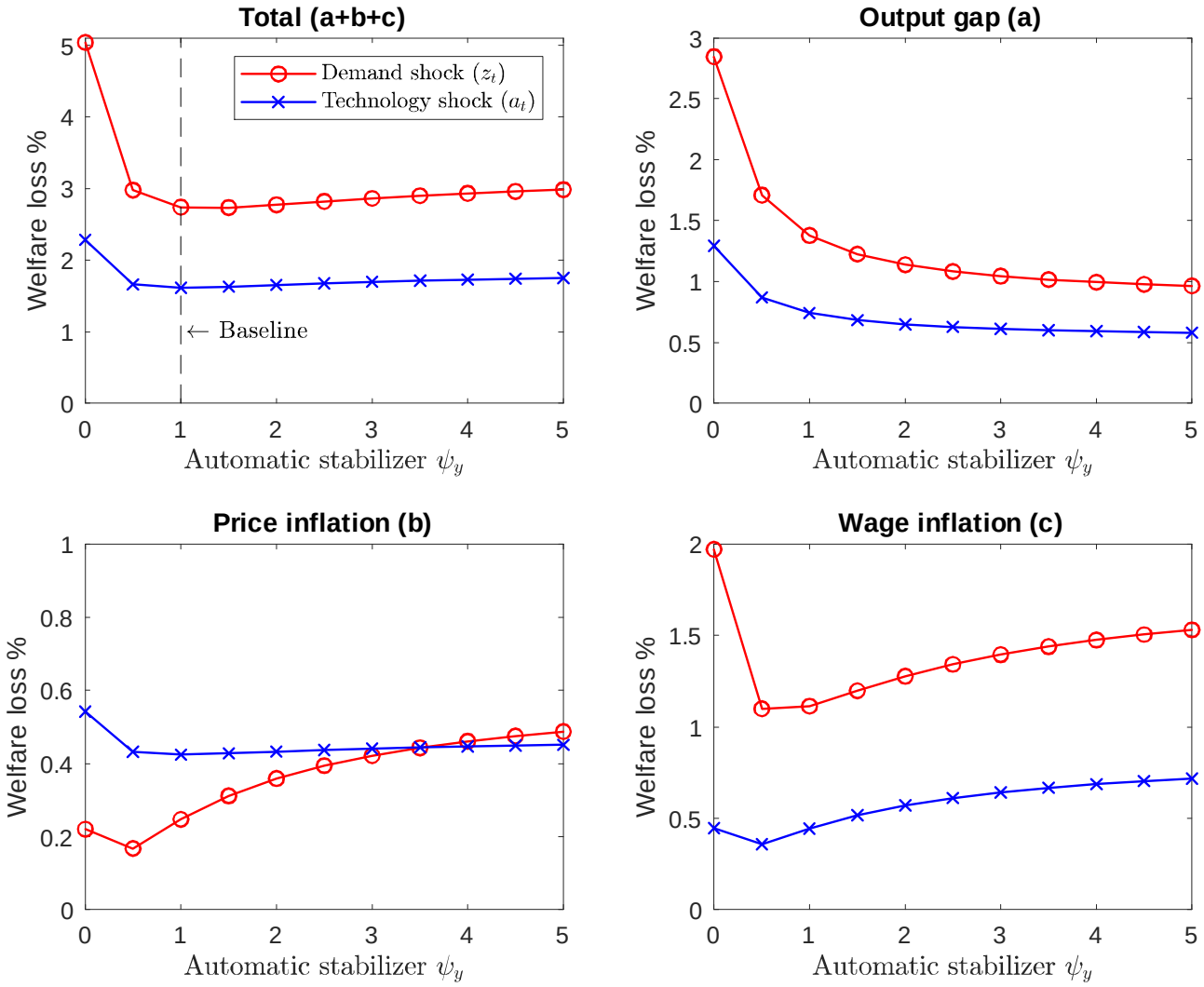


Figure 2: With unfunded debt ($\gamma = 1$), the optimal automatic stabilizer ψ_y in the fiscal rule is near 1.

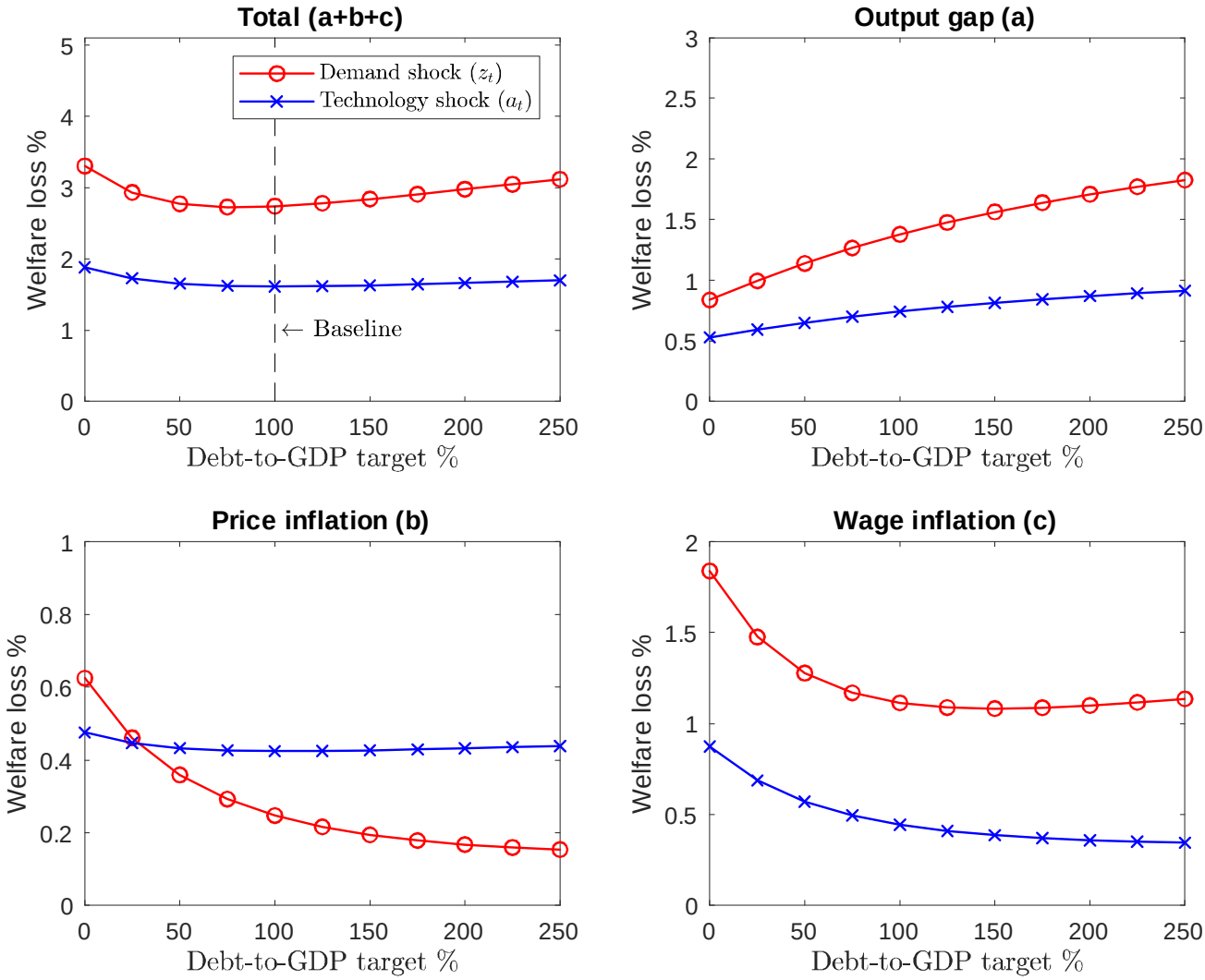


Figure 3: With unfunded debt ($\gamma = 1$), the optimal annual debt-to-GDP target $b/4$ is near 100%.

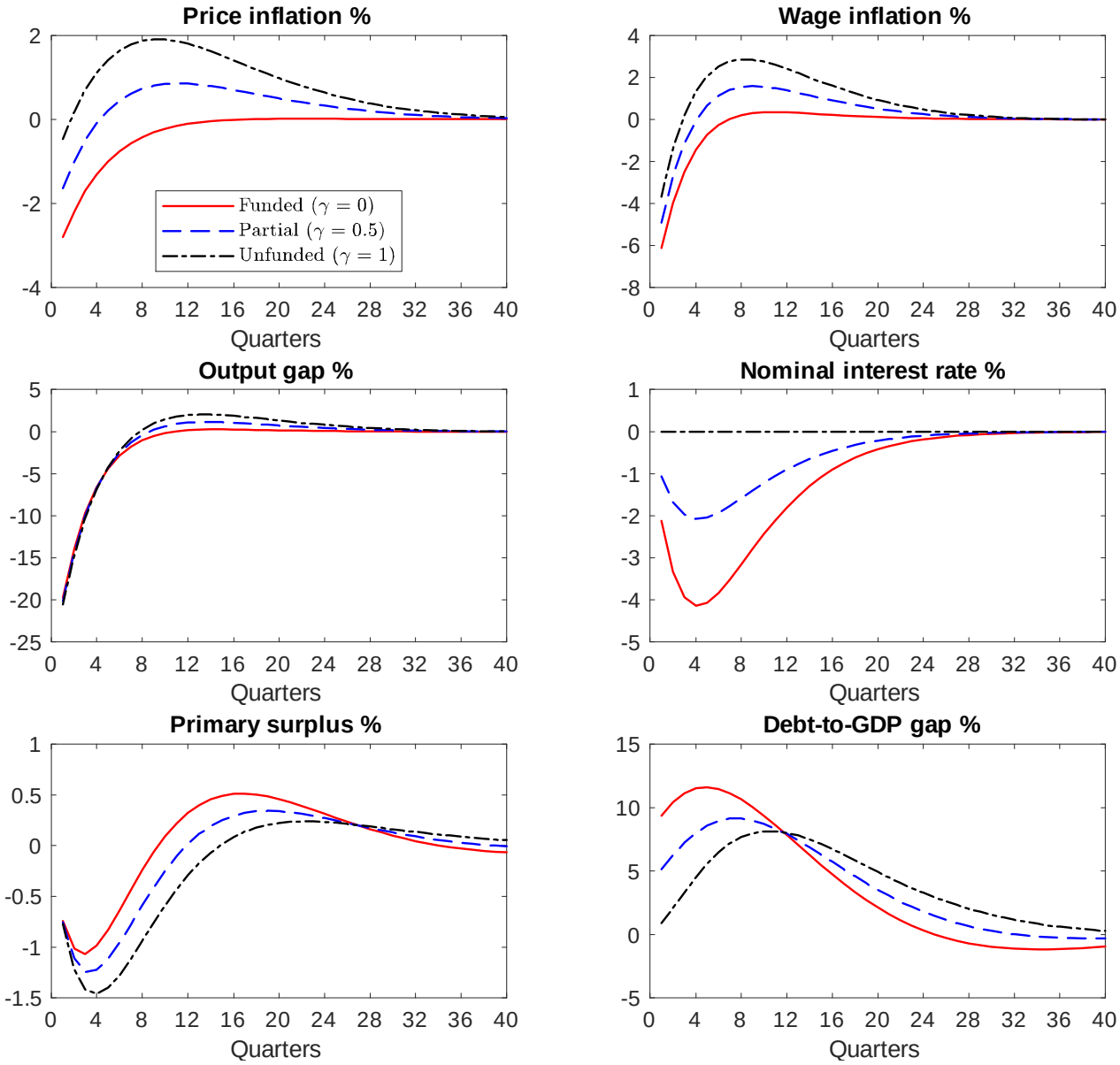


Figure 4: Partially unfunded debt can stabilize inflation facing demand shocks. Response to $-3sd$ demand shock without ZLB. Annual deviation from steady state.

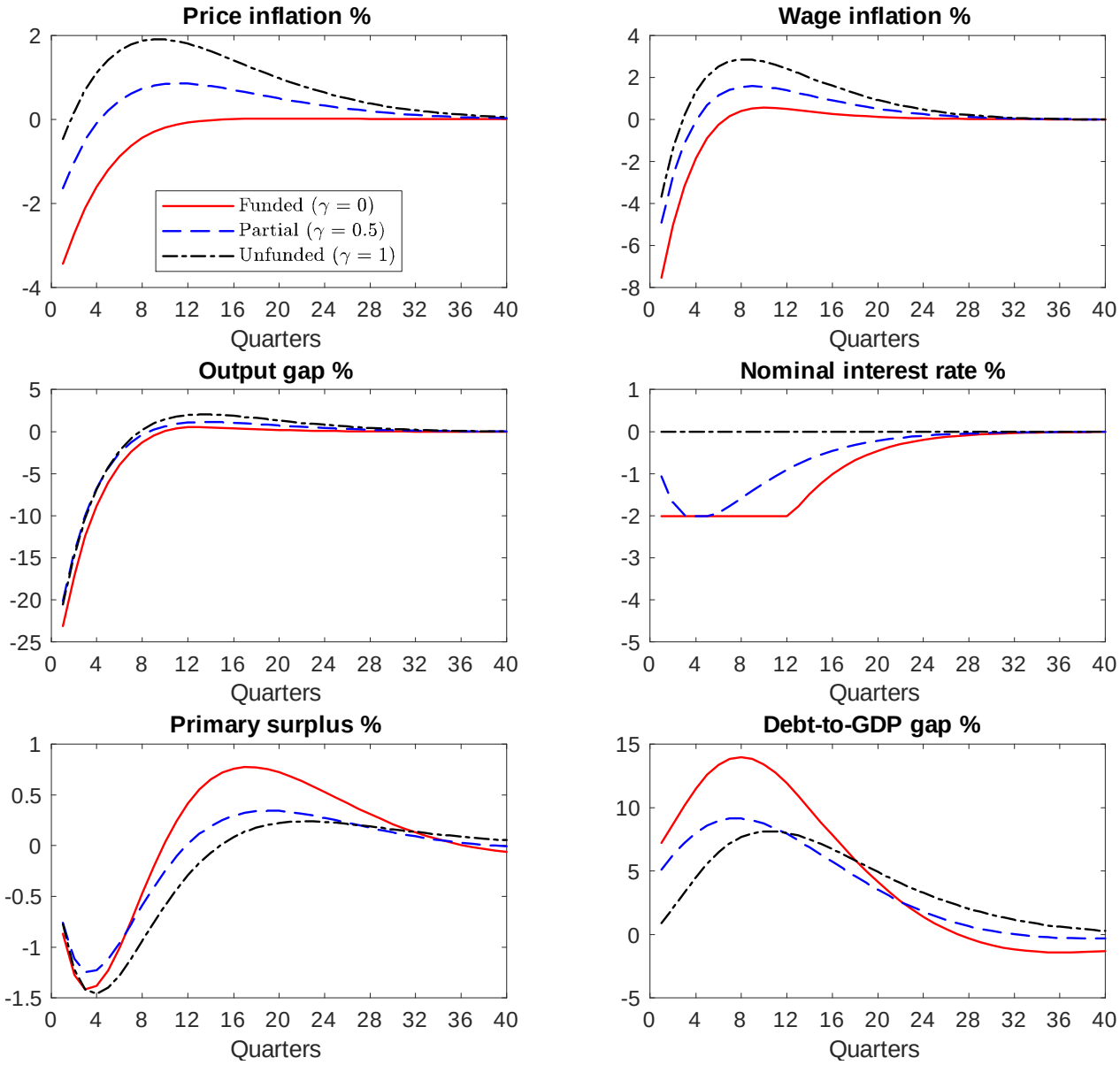


Figure 5: Response to $-3sd$ demand shock with ZLB. Annual deviation from steady state.

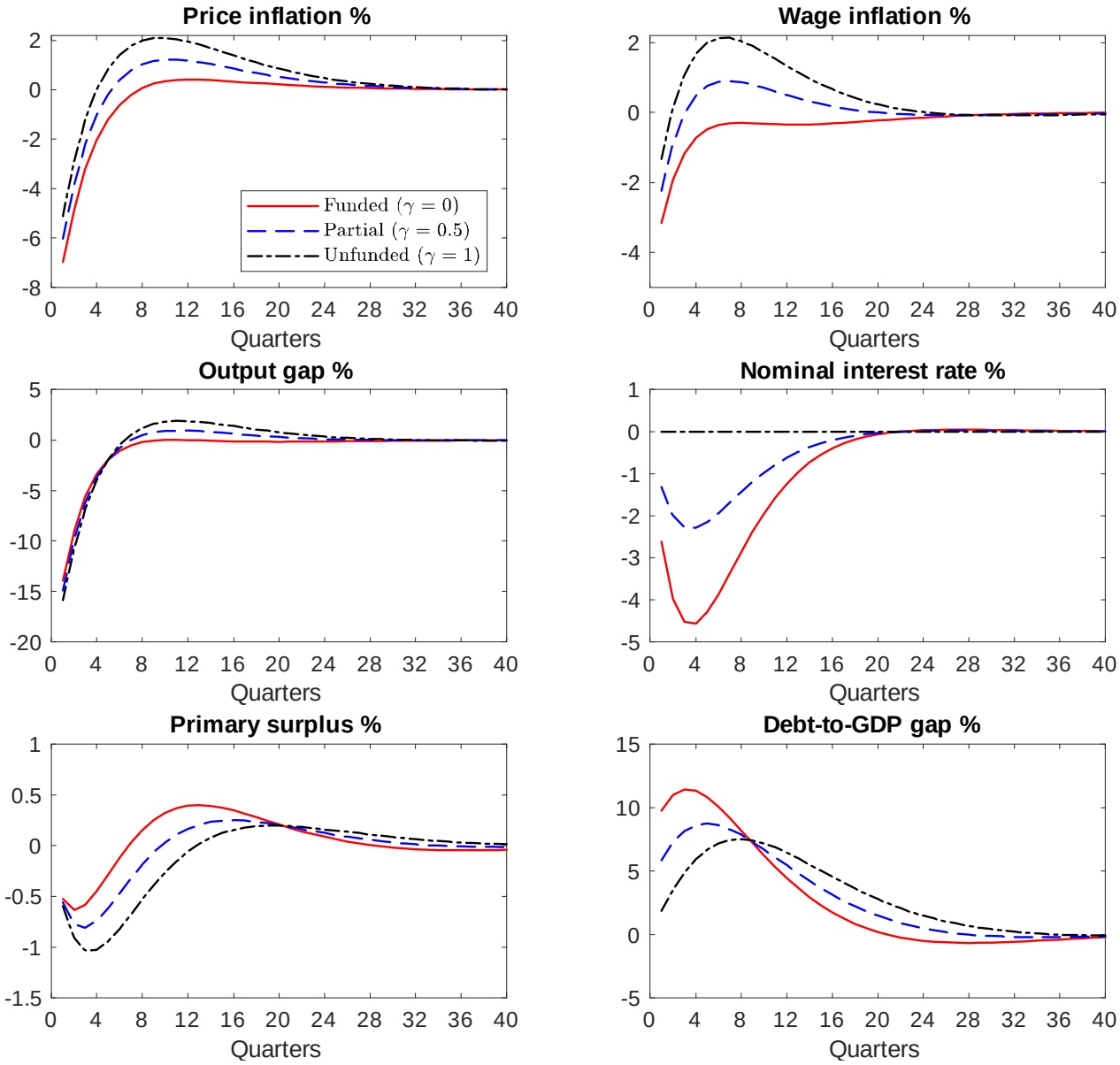


Figure 6: Partially unfunded debt can stabilize inflation facing technology shocks. Response to $+3sd$ technology shock without ZLB. Annual deviation from steady state.

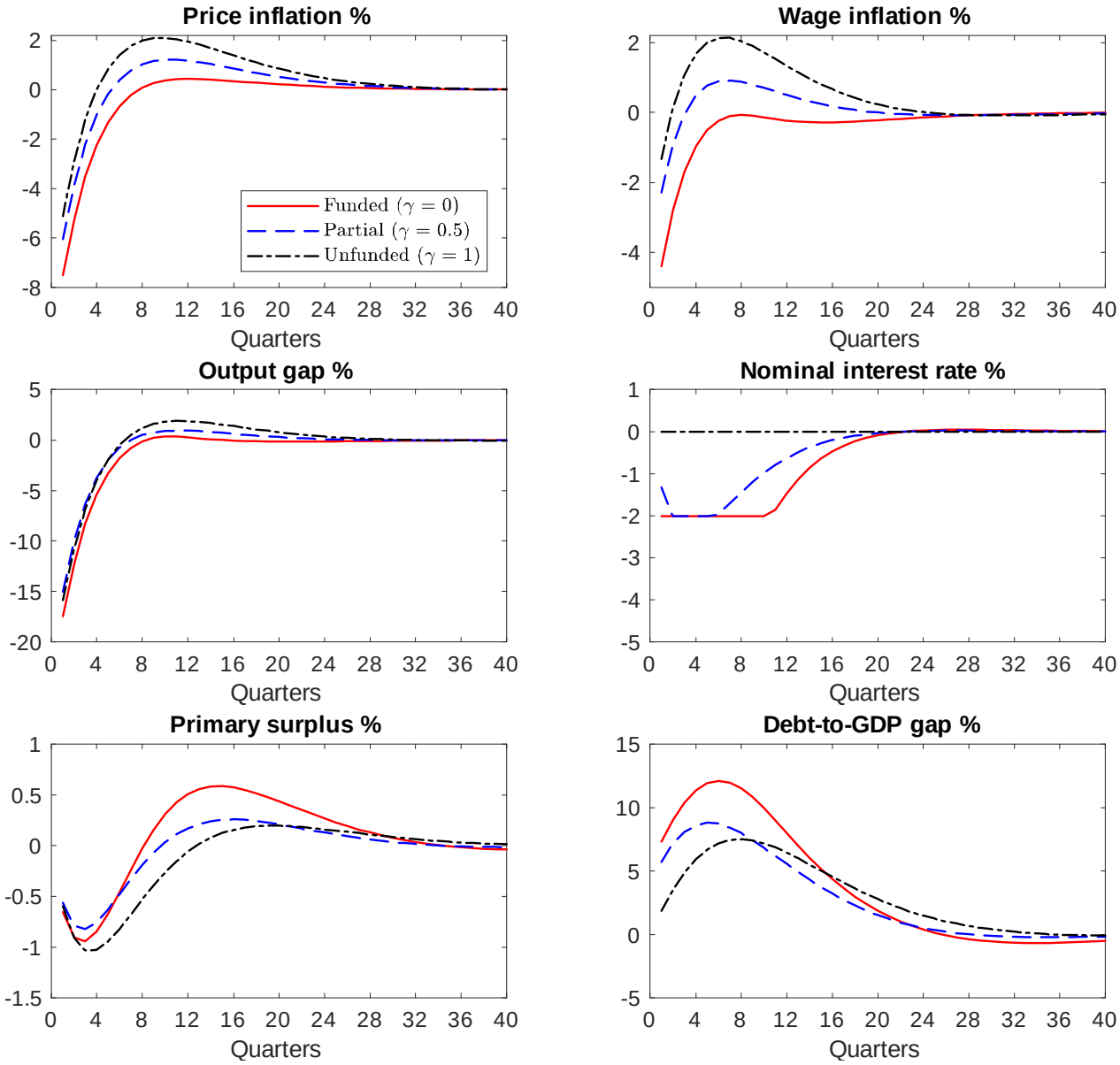


Figure 7: Response to $+3sd$ technology shock with ZLB. Annual deviation from steady state.

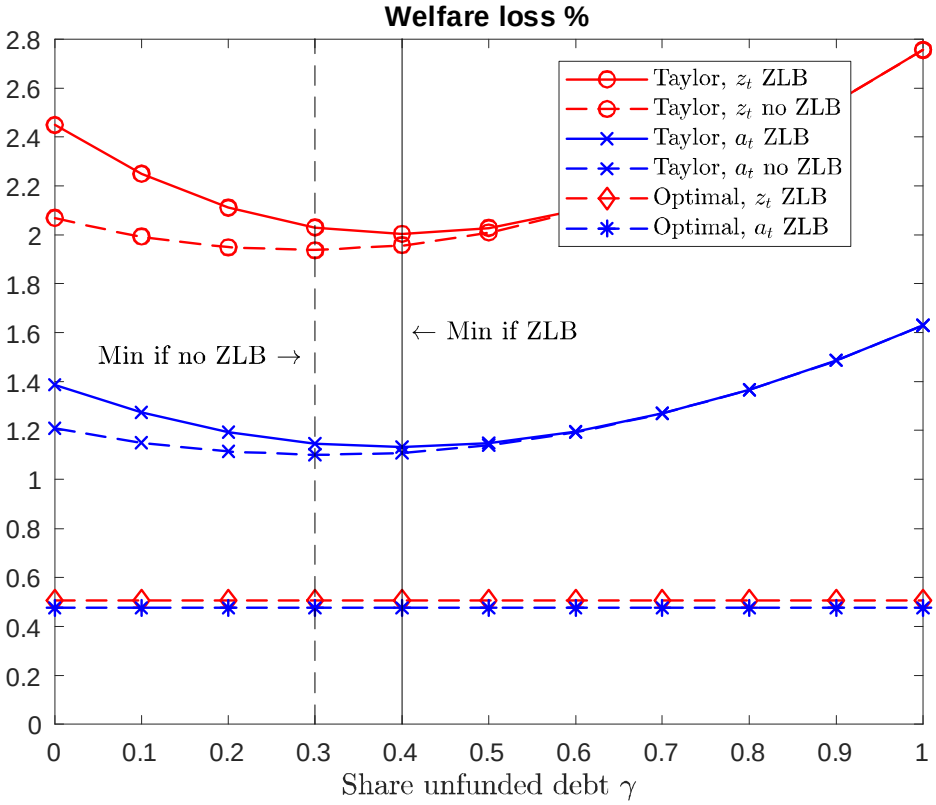


Figure 8: Partially unfunded debt can improve social welfare, with and without ZLB.

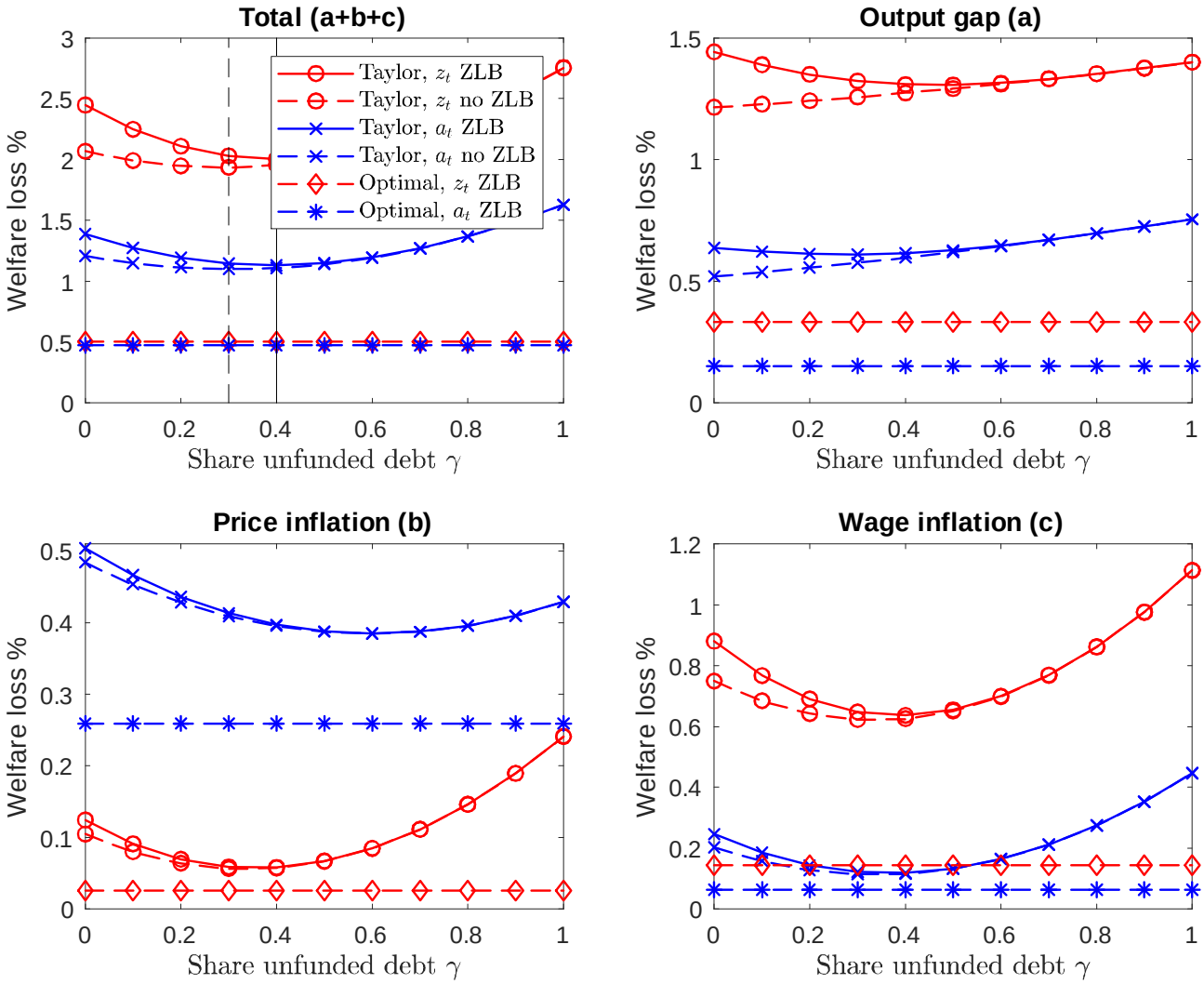


Figure 9: Partially unfunded debt and welfare components, with and without ZLB.

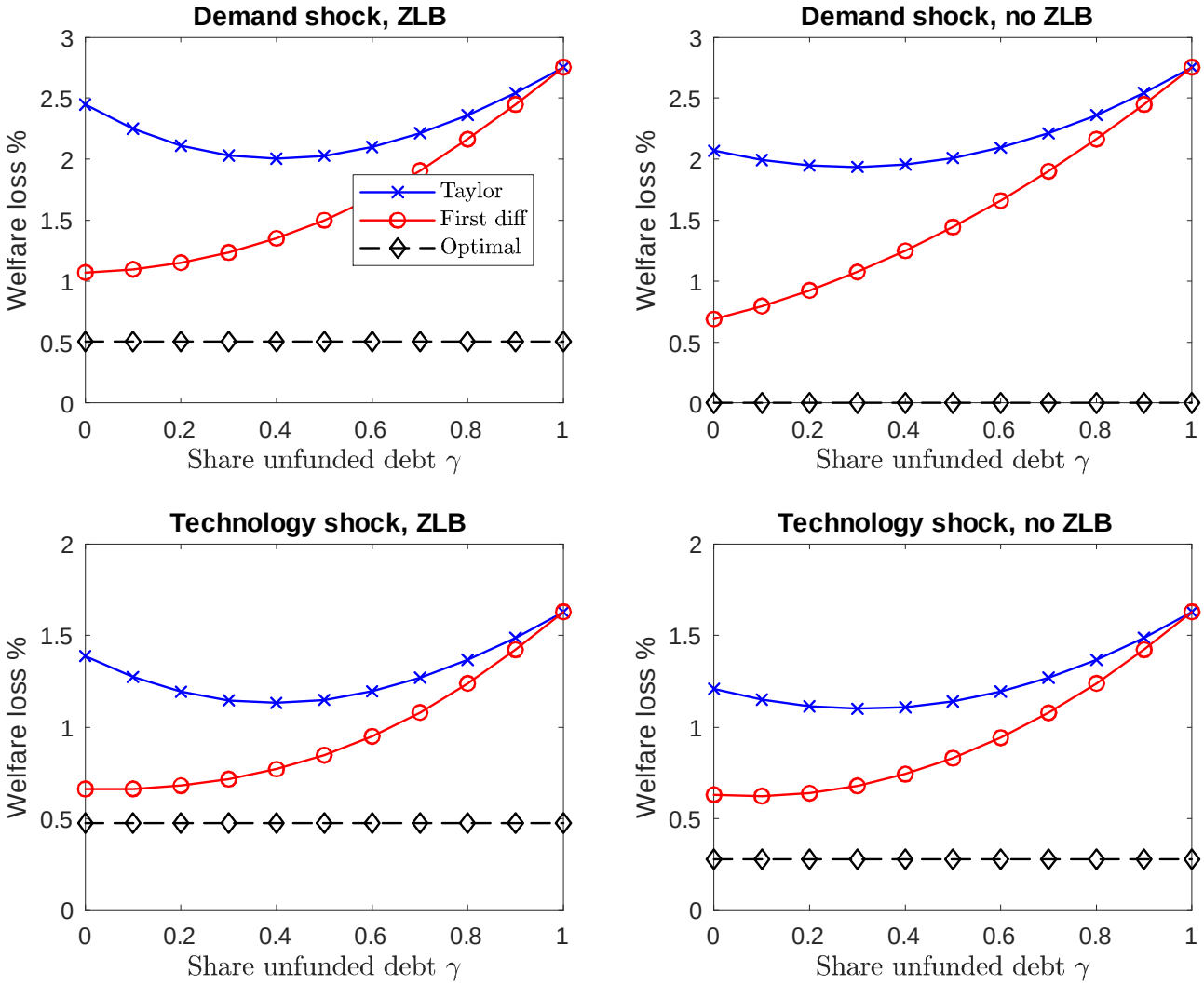


Figure 10: A more inertial monetary policy such as a first-difference rule can so improve welfare to make funded debt desirable, with and without ZLB.

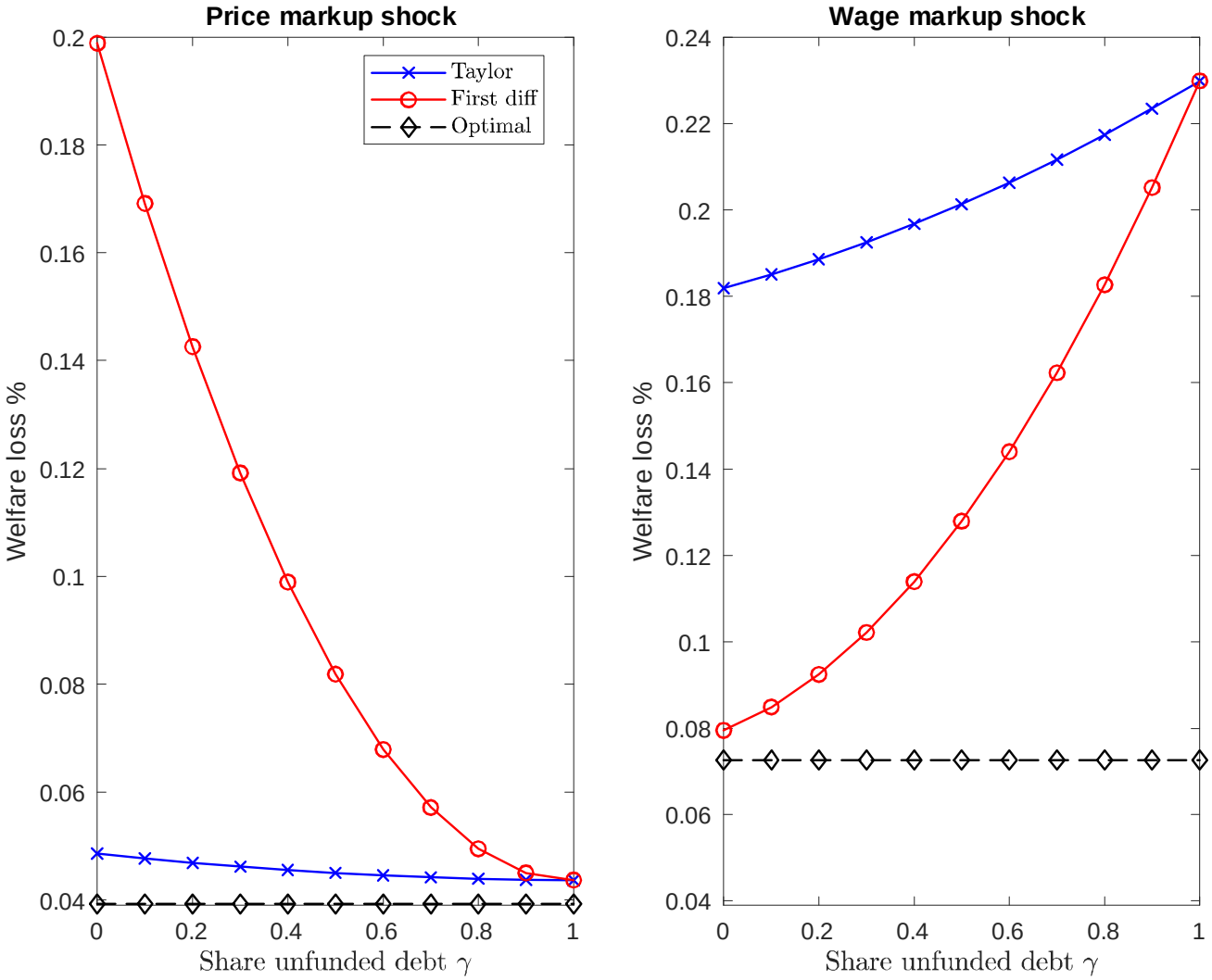


Figure 11: Unfunded debt can be welfare improving as a means for monetary policy to look through price markup shocks. Shock volatility small enough to avoid ZLB.

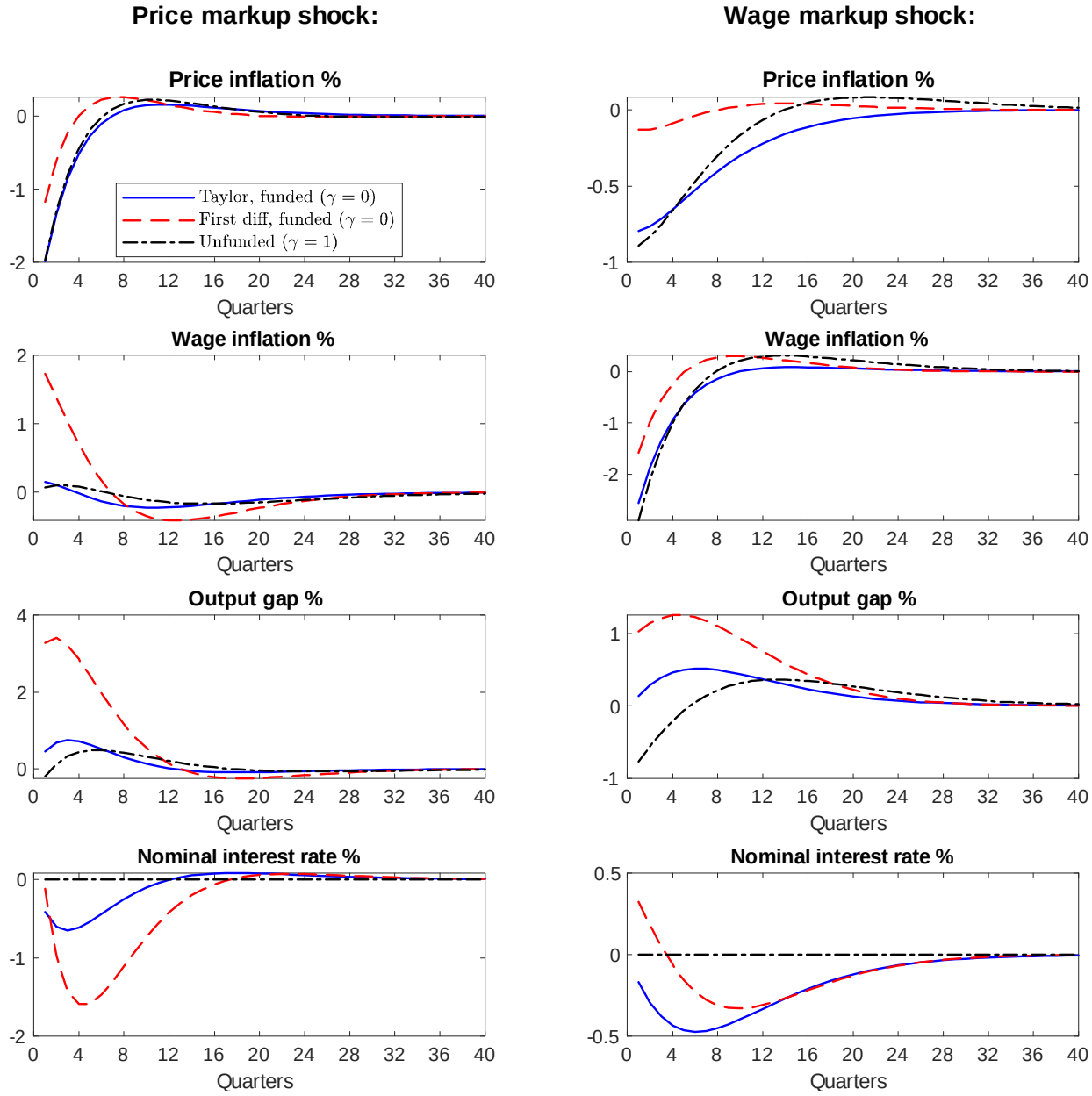


Figure 12: Monetary policy following a first-difference rule can make wage inflation more volatile facing price markup shocks but less volatile facing wage markup shocks. Response to $-3sd$ markup shock away from ZLB. Annual deviation from steady state.

Appendices

To study the robustness of the model outcomes presented in the main text, Appendix A allows monetary policy to lean against fiscal inflation, while Appendix B lets fiscal policy lean against unfunded debt. Appendix C provides the equilibrium conditions of the optimal policy under commitment.

Appendix A Lean Against Fiscal Inflation

While in the main text monetary policy is assumed irresponsive to fiscal inflation, this appendix extends the model to allow monetary policy to lean against fiscal inflation. This extension shows that leaning against fiscal inflation can be self-defeating, increasing the volatility in the economy, and so unfunded debt becomes less desirable.

In the *fiscal-led* shadow economy, replace the nominal interest rate peg (10) with a rule less accommodative towards price inflation

$$\hat{\imath}_t^F = \phi_p^F \pi_t^{pF}, \tag{19}$$

where $\phi_p^F \in [0, 1)$ so monetary policy leans less than one-to-one against fiscal inflation, and thus remains passive. Accounting for such leaning against fiscal inflation, in the *monetary-led* economy the Taylor rule (13) becomes

$$\hat{\imath}_t^* = \phi_i \hat{\imath}_{t-1}^* + (1-\phi_i)\left[\phi_p^M\left(\pi_t^p - \pi_t^{pF}\right) + \phi_p^F \pi_t^{pF} + \phi_y^M\left(\widetilde{y}_t - \widetilde{y}_t^F\right)\right]. \tag{20}$$

When $\phi_p^F = 0$ the model simplifies to the case of a nominal interest rate peg, while to investigate the effects of leaning against fiscal inflation we set $\phi_p^F = 0.9$. Figure A.1 compares the responses of the economy after demand and technology shocks in the presence of unfunded debt, without leaning (red solid lines, $\phi_p^F = 0$) versus with leaning (blue dashed lines, $\phi_p^F = 0.9$). As the figure shows, the monetary policy that leans against fiscal inflation amplifies the volatility in price and wage inflation as well as output gap volatility. The monetary policy that leans against fiscal inflation, as a consequence, increases the welfare loss from demand and technology shocks.

Overall, therefore, a monetary policy that leans against fiscal inflation increases the volatility in the economy, and thereby makes unfunded debt less desirable for society than shown in the main text.

Appendix B Lean Against Unfunded Debt

In the main text fiscal policy is assumed irresponsive to unfunded debt, while this appendix extends the model so fiscal policy can lean against unfunded debt. As this extension shows, leaning against unfunded debt in the presence of an automatic fiscal stabilizer does not affect the volatility in the economy, so the desirability of unfunded debt is not affected.

In the *fiscal-led* shadow economy, replace the tax rule (11) with a rule less accommodative towards unfunded debt

$$\hat{\tau}_t^F = \psi_\tau \hat{\tau}_{t-1}^F + (1-\psi_\tau)\left(\psi_b^F \hat{b}_{t-1}^F + \psi_y \widetilde{y}_t^F\right), \tag{21}$$

where $\psi_b^F \in [0, \rho)$ thus fiscal policy leans less than ρ against unfunded debt, and so remains active. In fact this fiscal rule, together with the shadow economy's counterpart of the government budget (8), implies the dynamics of unfunded debt $\hat{b}_t^F$ depends on $\left(\beta^{-1} - \psi_b^F\right)\hat{b}_{t-1}^F$. The dynamics of unfunded debt requires $\beta^{-1} - \psi_b^M > 1$, that is, $\psi_b^F < \beta^{-1} - 1 \equiv \rho$. As fiscal policy does not adjust taxes enough to ensure the debt is repaid, the debt is unfunded and so has to be inflated away. With fiscal policy leaning against unfunded debt, in the *monetary-led* economy the fiscal rule (14) becomes

$$\hat{\tau}_t = \psi_\tau \hat{\tau}_{t-1} + (1-\psi_\tau)\left[\psi_b^M\left(\hat{b}_{t-1} - \hat{b}_{t-1}^F\right) + \psi_b^F \hat{b}_{t-1}^F + \psi_y \widetilde{y}_t\right]. \tag{22}$$

When $\psi_b^F = 0$ the model returns to the case of fiscal policy irresponsive to unfunded debt, while to study the effects of leaning against unfunded debt we set $\psi_b^F = 0.004$. Thus, ψ_b^F remains below $\rho \equiv 1/\beta - 1 \approx 0.005$, since $\beta = 0.995$ in the baseline calibration. Figure A.2 compares the responses of the economy after demand and technology shocks in the presence of unfunded debt, without leaning (red solid lines, $\psi_b^F = 0$) versus with leaning (blue dashed lines, $\psi_b^F = 0.004$). Despite fiscal policy leaning against unfunded debt, the responses of fiscal policy, inflation and the output gap are not not visibly different.

This neutrality of leaning against unfunded debt arises, intuitively, because of the automatic stabilizer in the fiscal rule. Recall, the fiscal responsiveness to the output gap, $\psi_y = 1$, is calibrated so to maximize social welfare when debt is unfunded, $\gamma = 1$. Figure A.2 shows also the response of the economy when leaning against unfunded debt occurs in the absence of an automatic fiscal stabilizer (black dash-dotted lines, $\psi_b^F = 0.004$ and $\psi_y = 0$). Removing the automatic stabilizer, fiscal policy becomes relatively irresponsive facing the demand and technology shocks, and so inflation and the output gap become more volatile.

In sum, the automatic stabilizer in the fiscal rule ensures leaning against unfunded debt is irrelevant for the volatility in the economy and, therefore, unfunded debt stays equally desirable for society as shown in the main text.

Appendix C Optimal Policy under Commitment with a ZLB Constraint

The problem of optimal policy with commitment is given by

$$\min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} \left(\pi_t^p \right)^2 + \frac{\epsilon_w (1-\alpha)}{\lambda_w} \left(\pi_t^w \right)^2 \right],$$

subject to (1)-(4) and $\hat{\imath}_t + \rho \geq 0$. Note the government budget does not constrain monetary policy, and fiscal policy is assumed to set $\hat{\tau}_t$ so to fully stabilize debt $\hat{b}_t = 0$ for all t.

Write the period Lagrangian

$$\begin{aligned}
\mathbb{L}_t = & \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} \left(\pi_t^p \right)^2 + \frac{\epsilon_w (1-\alpha)}{\lambda_w} \left(\pi_t^w \right)^2 \right] \\
& + m_{1t} \left[\widetilde{y}_t + \frac{1}{\sigma} \left(\hat{\imath}_t - \hat{r}_t^n \right) \right] - \beta^{-1} m_{1t-1} \left(\widetilde{y}_t + \frac{1}{\sigma} \pi_t^p \right) \\
& + m_{2t} \left(\pi_t^p - \varkappa_p \widetilde{y}_t - \lambda_p \widetilde{\omega}_t \right) - m_{2t-1} \pi_t^p \\
& + m_{3t} \left(\pi_t^w - \varkappa_w \widetilde{y}_t + \lambda_w \widetilde{\omega}_t \right) - m_{3t-1} \pi_t^w \\
& + m_{4t} \left(\widetilde{\omega}_t - \widetilde{\omega}_{t-1} - \pi_t^w + \pi_t^p + \Delta \hat{\omega}_t^n \right) \\
& + \beta E_t \mathbb{L}_{t+1}.
\end{aligned}$$

The Kuhn-Tucker conditions are

$$0 = \frac{\partial \mathbb{L}_t}{\partial \widetilde{y}_t} = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \widetilde{y}_t + m_{1t} - \beta^{-1} m_{1t-1} - \varkappa_p m_{2t} - \varkappa_w m_{3t} \tag{23}$$

$$0 = \frac{\partial \mathbb{L}_t}{\partial \pi_t^p} = \frac{\epsilon_p}{\lambda_p} \pi_t^p - \frac{1}{\beta \sigma} m_{1t-1} + m_{2t} - m_{2t-1} + m_{4t} \tag{24}$$

$$0 = \frac{\partial \mathbb{L}_t}{\partial \pi_t^w} = \frac{\epsilon_w (1-\alpha)}{\lambda_w} \pi_t^w + m_{3t} - m_{3t-1} - m_{4t} \tag{25}$$

$$0 = \frac{\partial \mathbb{L}_t}{\partial \widetilde{\omega}_t} = \frac{\partial \beta E_t \mathbb{L}_{t+1}}{\partial \widetilde{\omega}_t} - \lambda_p m_{2t} + \lambda_w m_{3t} + m_{4t} \tag{26}$$

$$0 = \frac{\partial \mathbb{L}_t}{\partial \hat{\imath}_t} \left(\hat{\imath}_t + \rho \right) = \frac{1}{\sigma} m_{1t} \left(\hat{\imath}_t + \rho \right), \quad m_{1t} \geq 0 \text{ and } \hat{\imath}_t + \rho \geq 0, \tag{27}$$

whereas the envelope condition gives

$$\frac{\partial \beta E_t \mathbb{L}_{t+1}}{\partial \widetilde{\omega}_t} = -\beta E_t m_{4t+1}.$$

The equilibrium conditions under the optimal policy are then (1)-(9) and (23)-(27).

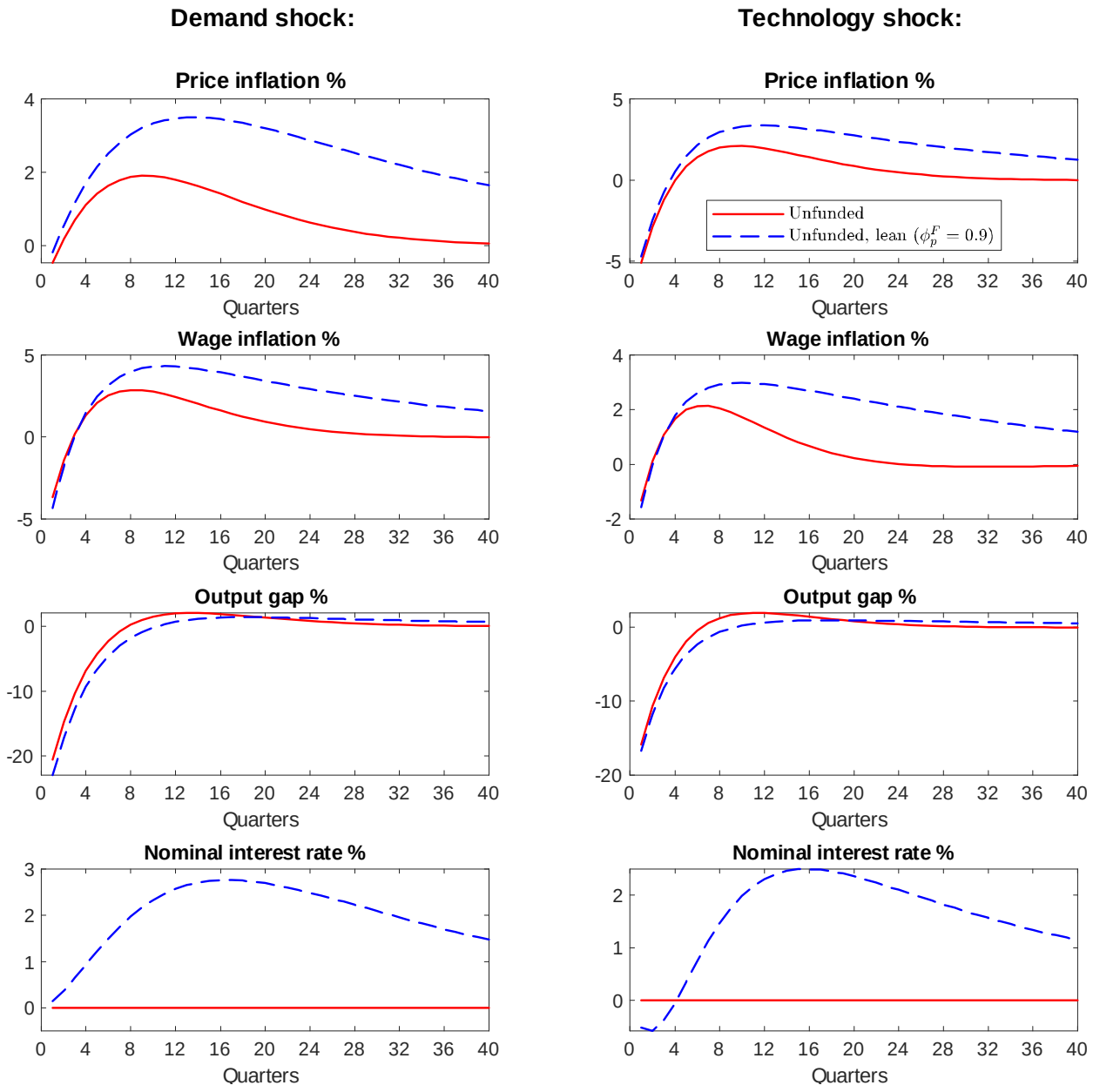


Figure A.1: With unfunded debt ($\gamma = 1$), monetary policy leaning against fiscal inflation can amplify volatility in both inflation and the output gap. Response to $-3sd$ demand shock or $+3sd$ technology shock away from ZLB. Annual deviation from steady state.

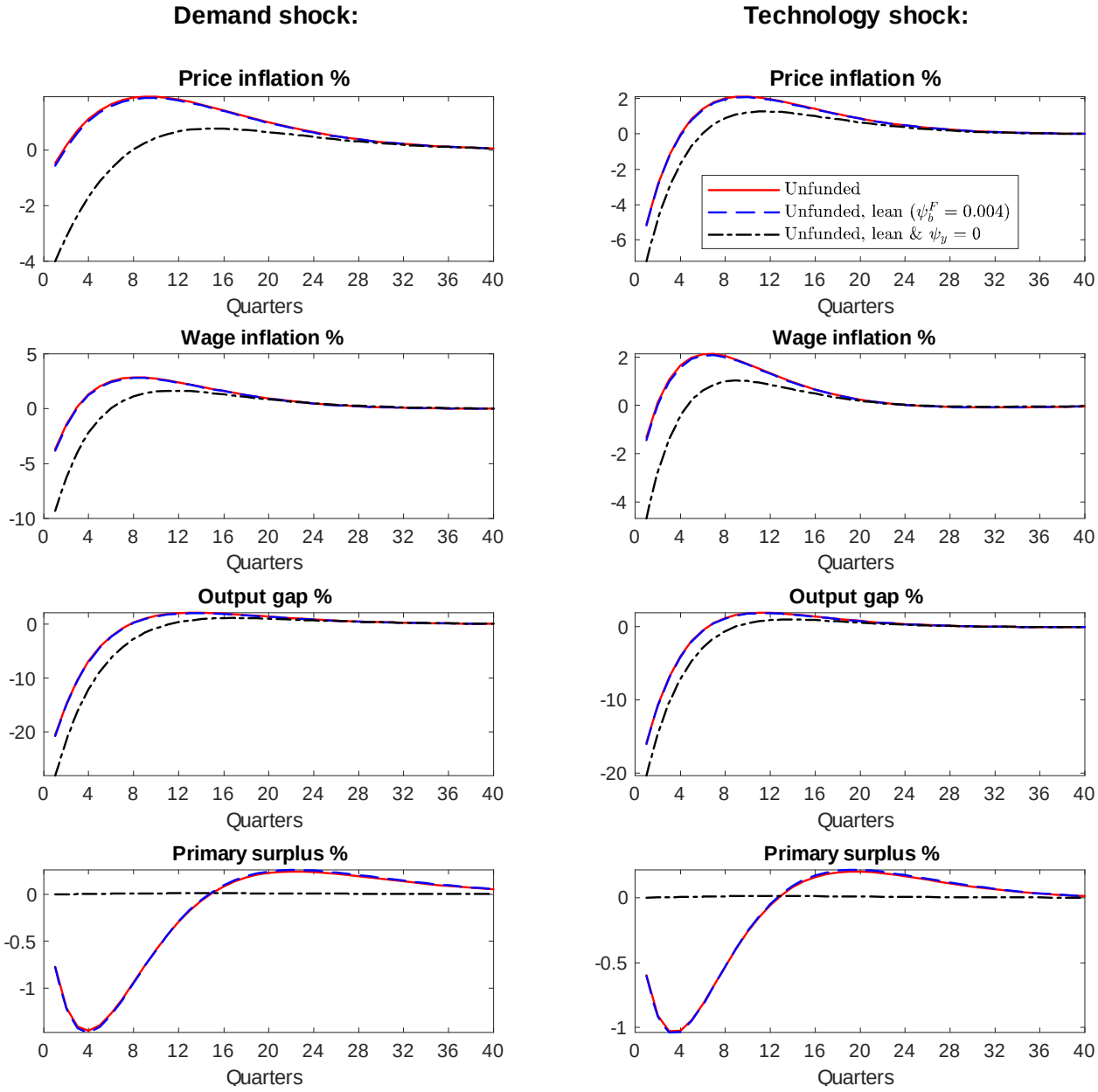


Figure A.2: With unfunded debt ($\gamma = 1$), the automatic fiscal stabilizer ψ_y can make leaning against unfunded debt irrelevant. Response to $-3sd$ demand shock or $+3sd$ technology shock away from ZLB. Annual deviation from steady state.