Fiscal Theory of the Price Level in Small and Open Economies

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Abstract

We study the implications of the Fiscal Theory of the Price Level (FTPL) for the dynamics of an otherwise standard New Keynesian model of a small and open economy, paying special attention to the role played by the currency composition of government debt. We analyze the existence and uniqueness of a stationary equilibrium under interest rate rules, how the monetary and fiscal transmission mechanisms are altered when the FTPL is at play, and the extent to which alternative monetary and fiscal regimes influence the propagation of external shocks. We show that the share of debt denominated in foreign currency is crucial for understanding the role that the FTPL plays in these key aspects of open-economy dynamics.

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1 Introduction

The worldwide spike in inflation following the COVID pandemic, preceded by an unprecedented fiscal expansion in response to that shock, reignited the debate about fiscal and monetary policy interactions —especially as the amount of government debt has significantly increased almost everywhere in recent years (for instance, IMF, 2025 reports that close to 70% of the countries had, by 2024, heavier public debt burdens than before 2020). The Fiscal Theory of the Price Level (FTPL) has been one of the main theoretical frameworks for addressing these issues (foundational contributions include Leeper, 1991, Sims, 1994, and Woodford, 1994, with a detailed textbook treatment provided by Cochrane, 2023). Most studies in this literature have focused on closed-economy frameworks, and the scattered papers analyzing open economies (discussed below) omit a critical aspect of public finances in emerging and developing economies: government debt denominated in foreign currency. For instance, according to the Arslanalp and Tsuda (2014) database, in 2023, 30% of government debt in emerging countries was in foreign currency on average, with a standard deviation of 21%, while for developing economies, these values were 52% and 25%, respectively. Against this background, we study the implications of the FTPL for the dynamics of an otherwise standard New Keynesian model of a small open economy, with traded and non-traded goods, paying particular attention to the role played by the currency composition of government debt.

The FTPL focuses on the lifetime government budget constraint (a.k.a. the debt-valuation equation), which requires the discounted present value of current and expected primary surpluses to equal the real value of outstanding nominal debt. In a closed economy setup, where debt is fully denominated in local currency, a change in the net present value of primary surpluses (not compensated in the future) requires an opposite-sign movement in the price level to dilute (or revalue, depending on the required sign) the real burden of outstanding nominal debt.

An open economy setup broadens the analysis in several relevant dimensions. First, even if debt is fully denominated in domestic currency, FTPL considerations might induce non-trivial dynamics in the nominal exchange rate (as well as the real one, under nominal rigidities). Second, the larger the share of debt denominated in foreign currency, the smaller the amount of nominal debt that can be diluted via inflation, magnifying the effects on prices vis-à-vis a case with domestic-currency-only debt. Third, when part of the liabilities is in foreign currency, movements in the real exchange rate also affect the debt-valuation equation; particularly relevant for understanding the propagation of real shocks that would move the real exchange rate, even under monetary neutrality. Our goal is to understand the significance of these potentially distinct channels relative to the commonly adopted framework under passive fiscal policy; widely used in both academic and policy circles.

We set up an infinite horizon, small and open economy model with incomplete asset markets and a representative household. There is an exogenous endowment of tradables, while non-tradables are produced using labor under price rigidities that generate a New-Keynesian Phillips curve. Fiscal policy levies lump-sum taxes to finance debt denominated in both domestic and foreign currencies, while monetary policy sets a rule for the nominal interest rate. Different combinations of active or passive fiscal and monetary rules modify the dynamics of the economy.

We begin by studying the (local) existence and uniqueness of a stationary equilibrium under interest rate rules. Results resemble those obtained in closed-economy setups, almost independently of the currency composition of debt. In particular, under a Ricardian/passive fiscal rule, the Taylor principle (i.e., the monetary policy rate reacting more than proportionally to inflation) is enough to guarantee a unique stationary rational-expectations equilibrium; otherwise, multiplicity arises. Instead, if fiscal policy is non-Ricardian/active, the uniqueness of the equilibrium requires a passive

monetary policy (i.e., the policy rate moving less than proportionally to inflation).

We next derive a set of propositions to characterize the transmission of monetary and fiscal shocks. Dynamics not only vary depending on the fiscal and monetary regime (as studied elsewhere in closed-economy models), but our analysis also explores the consequences for both the real and nominal exchange rates. For example, while an increase in the policy rate leads to both a nominal and real appreciation under a passive fiscal setup, a nominal depreciation emerges if fiscal policy is active.

Moreover, we show that these differences are exacerbated in the presence of debt denominated in foreign currency. When fiscal policy is active, a smaller share of domestic-currency debt requires a larger change in the price level to compensate, magnifying also the effects on the exchange rate. These results speak directly to the empirical findings showing that an increase in the domestic monetary policy rate leads to an appreciation in developed economies, while the opposite seems to occur in emerging countries (see, for instance, Hnatkovska et al., 2016 and Bolhuis et al., 2024). Our analysis provides a novel interpretation of these differences based on FTPL considerations when foreign-currency debt is accounted for.

Finally, we study how alternative monetary and fiscal regimes alter the propagation of external shocks, such as surprises in trade-related income and the international cost of borrowing. We show that when debt is fully denominated in domestic currency, and as long as the policy rate responds to movements in inflation (even if the Taylor principle is not satisfied under a Ricardian fiscal regime), the particular fiscal/monetary configuration does not generate qualitatively different dynamics. In contrast, a larger fraction of debt in foreign currency significantly changes the dynamics, especially for variables related to the non-traded sector. In particular, while a jump in the nominal exchange rate helps dilute debt in domestic currency, it simultaneously increases the real burden of repaying dollar-denominated debt. Thus, a stronger non-traded inflation reaction is generally required, leading to changes in non-traded activity; to the extent that it might even reverse the sign originally induced by the real shock under consideration.

This paper relates to the recent literature that theoretically studies how the New Keynesian propagation mechanism differs under alternative fiscal and monetary configurations, such as Leeper and Leith (2016), Cochrane (2023), and Caramp and Silva (2023), among others. Our analysis complements these studies by considering the differences and similarities in an open-economy setting. It also highlights that, under sticky prices, FTPL effectively becomes a fiscal theory of the real exchange rate, especially if a fraction of government debt is denominated in foreign currency.

A few studies have previously analyzed how FTPL extends to open economies. Earlier examples include Loyo (1999), Dupor (2000), and Daniel (2001), while Bianchi (2021) is a more recent contribution. Many of these studies assume flexible prices, where purchasing power parity holds, and the real exchange rate plays no role. In contrast, by considering non-traded goods and sticky prices, we describe non-trivial dynamics for both the nominal and real exchange rates. Moreover, once the currency composition of debt is considered (which these previous studies omit), the real exchange rate plays a crucial role in the debt valuation equation. Finally, our analysis of how the propagation of real shocks in the context of FTPL consideration is also novel, to the best of our knowledge.

The rest of the paper is organized as follows. Section 2 describes the model, including the alternative fiscal and monetary configurations that we consider. The analysis of the existence and uniqueness of equilibria is presented in Section 3. The propagation of monetary shocks is discussed in Section 4, while Section 5 presents the analysis of fiscal innovations. Section 6 studies the impact of external shocks and the role played by the fiscal/monetary setup. Section 7 presents several extensions, and Section 8 concludes.

2 The Model

We use a standard New Keynesian model, for instance, Schmitt-Grohé and Uribe (2017, Ch. 9.16). This is an infinite-horizon, small and open economy with discrete time and uncertainty. Domestic households derive utility from final-consumption goods and disutility from labor. Financial markets include one-period, non-contingent bonds denominated in both domestic and foreign currencies (pesos and dollars, for simplicity). There is an exogenous endowment of tradables, satisfying the law of one price, with its dollar price determined abroad. In contrast, non-tradable goods have a monopolistically competitive structure, with a continuum of varieties, each produced by a monopolist that demands labor and faces a Calvo pricing friction. Final-consumption goods are composed of both tradables and non-tradables.

The fiscal authority spends an exogenous amount of non-traded goods, financed by both lump-sum and proportional-income taxes, as well as by issuing debt in both currencies. We assume that local-currency bonds are held exclusively by domestic households, while dollar-denominated debt can also be purchased by foreigners (who also lend to domestic households). Finally, monetary policy follows an interest-rate rule.

2.1 Households

The domestic representative household seeks to maximize

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{(c_t)^{1-\sigma}}{1-\sigma}-\chi\frac{(h_t)^{1+\varphi}}{1+\varphi}\right]\right\},$$

where c_t denotes aggregate consumption, h_t hours worked, while $\beta \in (0,1)$ and $\sigma, \varphi, \chi > 0$ are parameters capturing, respectively, the discount factor, risk aversion, inverse-Frisch elasticity of labor supply, and relative dis-utility of hours worked. In every period t, they face the following budget constraint in domestic currency:

$$P_t c_t + S_t D_{t-1}^{H*} + \frac{B_t}{R_t} + T_t = W_t h_t + \Sigma_t + B_{t-1} + S_t \frac{D_t^{H*}}{R_t^*}. \tag{1}$$

The left-hand side includes uses of income: final consumption (with price P_t), repayment of debt obligations in dollars decided at $t-1$ (D_{t-1}^{H*} is the amount in dollars to be repaid, and S_t is the nominal exchange rate), purchases of domestic-currency bonds B_t (with a gross nominal rate R_t), and lump-sum taxes T_t. The available resources on the right-hand side include labor income (W_t is the hourly nominal wage) and profits from the ownership of domestic companies (Σ_t), income from domestic bonds (B_{t-1}) purchased at $t-1$; and new debt in dollars (D_t^{H*}, with a gross rate R_t^*). Households also face No-Ponzi-game conditions (NPGC) for each financial asset.[1]

The optimality conditions characterizing the solution to the household problem are the budget constraint (1), both NPGC holding with equality (transversality conditions), and

$$w_t(c_t)^{-\sigma} = \chi(h_t)^{\varphi}, \quad (c_t)^{-\sigma} = \beta R_t E_t\left\{\frac{(c_{t+1})^{-\sigma}}{\pi_{t+1}}\right\}, \quad (c_t)^{-\sigma} = \beta R_t^* E_t\left\{\frac{(c_{t+1})^{-\sigma}\pi_{t+1}^S}{\pi_{t+1}}\right\},$$

[1] These, as well as transversality conditions for fiscal policy, are discussed in depth in Appendix A. However, as we work with a linearized version of the model and focus only on the local existence and uniqueness of equilibrium (as in most of the literature working with New-Keynesian stochastic models), most of the issues discussed in the appendix are not consequential for the rest of the analysis.

with $w_t \equiv W_t/P_t$, $\pi_t \equiv P_t/P_{t-1}$, and $\pi_t^S \equiv S_t/S_{t-1}$. These characterize the trade-offs between, respectively, consumption and hours worked, as well as current and future consumption through either type of financial asset.

2.2 Supply side

Final consumption goods are produced by competitive firms using the technology

$$c_t = \left[\omega^{1/\eta}\left(c_t^N\right)^{1-1/\eta} + (1-\omega)^{1/\eta}\left(c_t^T\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}},$$

where c_t^N and c_t^T denote, respectively, the consumption of non-tradables and tradables, with prices P_t^N and P_t^T in local currency. The parameters $\eta > 0$ and $\omega \in [0,1]$ are, respectively, the elasticity of substitution between goods and the relative weight of non-tradables in the final-consumption basket. Profit maximization leads to the following demands,

$$c_t^N = \omega\left(p_t^N\right)^{-\eta} c_t, \quad c_t^T = (1-\omega)\left(p_t^T\right)^{-\eta} c_t,$$

with $p_t^T \equiv P_t^T/P_t$ and $p_t^N \equiv P_t^N/P_t$. The ratio of these two captures the intra-temporal trade-off between both goods as a function of their relative price. For future reference, let $q_t \equiv P_t^T/P_t^N$ be the relative price between tradables and non-tradables.

Traded goods are a stochastic endowment y_t^T, sold at an international dollar price P_t^*, with $\pi_t^* \equiv P_t^*/P_{t-1}^*$. Both y_t^T and π_t^* are assumed to be stationary. The local price satisfies $P_t^T = S_t P_t^*$. Given its previous definition, p_t^T is also the real exchange rate (rer_t) in CPI units in this model.

Non-tradables are produced in two stages. First, a continuum of competitive firms produces non-traded goods by combining non-tradable varieties according to the Dixit-Stiglitz aggregator

$$y_t^N = \left[\int_0^1 \left(y_{jt}^N\right)^{\frac{\epsilon_N-1}{\epsilon_N}} \mathrm{d}j\right]^{\frac{\epsilon_N}{\epsilon_N-1}},$$

where y_{jt}^N is the demand for variety j, and $\epsilon_N > 1$ is the elasticity of substitution across varieties.

A monopolist for variety j produces using labor according to $y_{jt}^N = (h_{jt})^\alpha$, with $\alpha \in (0,1]$. It faces a Calvo problem in choosing prices: with probability θ_N it must keep the previous-period price, while with probability $1-\theta_N$ it can freely choose. Log-linearization around the zero-inflation steady state, applied to the monopolist's optimality conditions (see Appendix B.1), leads to the Phillips curve

$$\widehat{\pi}_t^N = \beta E_t\{\widehat{\pi}_{t+1}^N\} + \kappa\, \widehat{mc}_t^N,$$

where $\widehat{\cdot}$ denotes log-deviations from the steady state, mc_t^N are real marginal costs in non-traded units, and $\kappa \geq 0$ is a function of parameters $\beta, \theta_N, \alpha, \epsilon_N$.

2.3 Fiscal and monetary policy

The government budget constraint in period t is,

$$\frac{D_t^G}{R_t} + S_t\frac{D_t^{G*}}{R_t^*} + T_t = D_{t-1}^G + S_t D_{t-1}^{G*}. \tag{2}$$

Here, D_t^G and D_t^{G*} denote the repayment value at $t+1$ of non-contingent debt contracted at t in, respectively, domestic and foreign currencies. Notice that we assume the interest rate in dollars, R_t^*, is the same as that charged to households (we discuss the implications of this assumption below). We assume the government satisfies a transversality condition for each type of debt, as discussed in Appendix A.

A fiscal policy is defined as *Ricardian* (a.k.a. passive) if instruments are set such that (2) holds for any value of predetermined debt $(D_{t-1}^G, D_{t-1}^{G*})$ as well as for any possible path of the endogenous variables that affect the government's resource constraint (such as P_t^N, S_t, R_t, R_t^*). In turn, fiscal policy is *Non-Ricardian* (a.k.a. active) if this condition is not satisfied.

We consider two alternative policy setups. In the first, fiscal policy targets real lump-sum taxes defined in terms of the aggregate price level/CPI (P_t), while monetary policy targets CPI inflation (π_t). Instead, the other alternative assumes that lump-sum taxes are set in terms of non-tradables (P_t^N) and monetary policy is concerned with fluctuations in non-traded inflation (π_t^N). The former will be our baseline specification for numerical exercises, as it compares more realistically with actual policies observed in the real world. However, a setup centered around non-tradables has the advantage of delivering clean, algebraic results. For future reference, we refer to the former as either the baseline or CPI-based setup, while the latter is labeled as the NT-based policies.

Under the baseline, real lump-sum taxes are $\tau_t \equiv T_t/P_t$ and the real value of peso-denominated debt is $d_t^G \equiv D_t^G/P_t$. Instead, under the NT-based setup, these are $\tau_t \equiv T_t/P_t^N$ and $d_t^G \equiv D_t^G/P_t^N$. Thus, the government's budget constraint (2) in real terms for each case is, respectively,

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t \frac{d_t^{G*}}{R_t^*} + \tau_t, \quad \text{or} \quad \frac{d_{t-1}^G}{\pi_t^N} + q_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + q_t \frac{d_t^{G*}}{R_t^*} + \tau_t, \tag{3}$$

where $d_t^{G*} \equiv D_t^{G*}/P_t^*$ in either case.

Lump-sum taxes in each setup are determined by, respectively,

$$\tau_t - \overline{\tau} = \phi_T \left[\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{\tau}}{1-\beta} \right] + u_t^\tau, \quad \text{or} \quad \tau_t - \overline{\tau} = \phi_T \left[\frac{d_{t-1}^G}{\pi_t^N} + q_t \frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{\tau}}{1-\beta} \right] + u_t^\tau, \tag{4}$$

where $\overline{\tau}$ is the steady-state value of primary surpluses, and u_t^τ is an exogenous and stationary shock.

In terms of the currency-composition of debt, we assume the government maintains a constant share of debt denominated in dollars $\overline{\Omega} \in [0,1]$, where this ratio is computed at steady-state relative prices. That is, we assume either

$$\overline{\Omega} = \frac{\overline{rer}\, d_t^{G*}}{d_t^G + \overline{rer}\, d_t^{G*}}, \quad \text{or} \quad \overline{\Omega} = \frac{\overline{q}\, d_t^{G*}}{d_t^G + \overline{q}\, d_t^{G*}}, \tag{5}$$

for the baseline and NT-based setups, respectively; where $\overline{rer}$ and $\overline{q}$ denote the steady-state values of rer_t and q_t. From the perspective of a first-order approximation, such as the one we follow, this is a natural initial benchmark.

Overall, equations (3)-(5) characterize the evolution of the fiscal variables d_t^G, d_t^{G*}, and τ_t. Under a constant share of dollar debt, a Ricardian fiscal policy relies on the parameter ϕ_T. If $\phi_T = 0$ lump-sum taxes are exogenous and we have a Non-Ricardian configuration. In the other extreme, $\phi_T = 1$ generates a Ricardian policy, while values between 0 and 1 are also possible, as analyzed below.[2]

[2]Notice that we do not consider the possibility of outright default on debt, which is also the case in most of the related FTPL literature. An exception regarding default in the context of FTPL is Uribe (2006), who discusses a tension between

In terms of monetary policy, we assume a Taylor-type rule for the short-term interest rate in domestic currency, which in each policy setup is:

$$\left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} u_t^R, \quad \text{or} \quad \left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t^N}{\overline{\pi}}\right)^{\phi_\pi} u_t^R, \tag{6}$$

where $\overline{R}$ and $\overline{\pi}$ are steady state values for R_t and π_t (in steady state, $\pi^N = \pi$), u_t^R is an exogenous and stationary monetary shock, and $\phi_\pi \geq 0$. Following the definition proposed by Leeper (1991), an active monetary policy is characterized by $\phi_\pi > 1$, while a passive configuration occurs if $\phi_\pi < 1$. A relevant question, explored below, is which combination of parameters ϕ_π, ϕ_T, and $\overline{\Omega}$ delivers a unique stationary equilibrium.

2.4 Rest of the world

The interest rate for debt in dollars R_t^* is determined by

$$R_t^* = R_t^W \exp\left\{\psi\left(d_t^* - \bar{d}^*\right)\right\}. \tag{7}$$

where $d_t^* = d_t^{H*} + d_t^{G*}$ is the consolidated-net-foreign-debt position, R_t^W is an exogenous (stationary) process, and $\psi > 0$. Thus, locals are assumed to pay a premium for borrowing in dollars ($\exp\left\{\psi\left(d_t^* - \bar{d}^*\right)\right\}$) over the international rate (R_t^W). This elastic premium serves as a "closing device" (Schmitt-Grohe and Uribe, 2003), ensuring that linear dynamics in the real version of the model are stationary. In turn, as discussed below, assuming a premium elastic to the *consolidated* net-foreign debt, in tandem with specific assumptions about preferences, simplifies the dynamics of this model.

2.5 Equilibrium definition, main channels and calibration

A stationary rational-expectations equilibrium is a set of stochastic processes for endogenous allocations and prices, given initial conditions for predetermined variables and stochastic processes for exogenous variables, satisfying (i) households and firms optimality conditions, (ii) market clearing in all domestic markets, (iii) the government's budget constraint and its transversality conditions (as discussed in Appendix A), and (iv) the determination of the interest rate in dollars (7). Appendix B.2 lists all the equilibrium conditions, while section 3 below discusses the conditions for local existence and uniqueness. Here, we discuss some relevant equilibrium features that will prove useful in the analysis of the dynamics presented below.

By forward iteration of the government's budget constraint (3) in the baseline setup, we obtain (see Appendix A for details)

$$\left[\frac{(1-\overline{\Omega})}{\pi_t} + \frac{rer_t}{\overline{rer}}\overline{\Omega}\right] d_{t-1} = E_t\left\{\sum_{j=0}^{\infty} \frac{\tau_{t+j}}{rr_{t,t+j}}\right\} + h.o.t., \tag{8}$$

price level determination and default in a closed economy model; without explicitly exploring the choice between the two. Of course, there is a large literature on sovereign default in open-economy models (e.g., see the survey by Aguiar and Amador, 2014). However, given the global analysis characterizing those models, the combination of monetary and fiscal policy is generally omitted, as is the issue of the currency composition of debt (for instance, Espino et al., 2025 present a model with default as well as distortionary fiscal and monetary policies but exclude the possibility of peso-denominated debt). Moreover, discussing optimal policy choices in the context of FTPL should naturally consider time-inconsistency issues arising from incentives to dilute outstanding nominal debt with inflation, as shown by Calvo (1988) in a simple two-period model. These issues, while potentially relevant, are beyond the scope of the paper.

where $d_t \equiv d_t^G + rer_t d_t^{G*}$ is total government debt (in domestic consumption units), the real discount rate is $rr_{t,t+J} \equiv \prod_{j=0}^{J-1}(R_{t+j}/\pi_{t+1+j})$, while *h.o.t.* includes terms (related to valuation effects) that are zero under certainty and thus vanish up to first order.[3]

In the absence of dollar-denominated debt ($\overline{\Omega} = 0$), (8) is the familiar valuation equation from closed-economy FTPL models. For a given path of the real rate, a change in primary surpluses τ (either contemporaneous or expected) that is not compensated by an offsetting change in the surplus at some other time (thus changing the net-present value on the right-hand side) needs to be met by an opposite-sign change in π_t on the left-hand side for (8) to hold (as d_{t-1} is predetermined). If prices are fully flexible, the real rate is not affected by monetary policy; thus, current prices are pinned down by fiscal policy alone (monetary policy still determines *expected* inflation). Under sticky prices, an additional channel emerges as monetary policy affects the real rate. Still, the degree of accommodation of primary surpluses is key (governed in the model by the parameter ϕ_T in (4), i.e., how Ricardian fiscal policy is).

As we analyze below, the results from closed economy models are qualitatively similar in this open economy setup when all government debt is denominated in pesos ($\overline{\Omega} = 0$); although here we can also explore the exchange-rate consequences. When $\overline{\Omega} > 0$, several differences arise. For given values of d_{t-1} and rer_t, a particular change on the right-hand side of (8), either through movements in primary surpluses or the real rate, requires (*ceteris paribus*) a larger change in π_t on the left-hand side. Thus, we would expect inflation to be more volatile the larger the share of dollar-denominated debt. It turns out this is true as long as $\overline{\Omega}$ is not too large, for dynamics differ when the share of debt denominated in foreign currency is high enough, as we analyze below.

Another relevant difference arises from the real exchange rate rer_t on the left-hand side of (8). First, any shock inducing a real depreciation requires (for a given path of primary surpluses and the real rate) an increase in π_t. Of course, the same shock may also induce a direct effect on prices (both tradable and non-tradable inflation), and the final effect will depend on the policy configuration and the degree of price stickiness. Importantly, this channel is not present in the related literature, as most open-economy FTPL papers use one-good models.

In addition, if $\overline{\Omega} > 0$, a larger net present value of primary surpluses can be accommodated not only by increasing inflation but also by real appreciation (or a combination of both). Therefore, the co-movement between rer_t (as well as the nominal depreciation π_t^S) and π_t is influenced by FTPL considerations when a fraction of government debt is denominated in foreign currency. In particular, the observed sign of this correlation is not clear ex-ante, and it is likely dependent on the type of shock affecting the economy. This creates a role for fiscal policy as a determinant of the real exchange rate, different from other channels that might be present in standard models.

To analyze the exchange rate dynamics, it will prove useful to notice that, in the linearized version of the model, the uncovered interest rate parity (UIP) holds:

$$\widehat{R}_t = \widehat{R}_t^* + E_t\left\{\widehat{\pi}_{t+1}^S\right\}. \tag{9}$$

Combined with the real-exchange-rate definition, this can be iterated forward to obtain

$$\widehat{\pi}_t^S = \widehat{\pi}_t - \widehat{\pi}_t^* - \widehat{rer}_{t-1} + \sum_{j=0}^{\infty}\left[(\widehat{R}_{t+j}^* - \widehat{\pi}_{t+1+j}^*) - (\widehat{R}_{t+j} - \widehat{\pi}_{t+1+j})\right], \tag{10}$$

As can be seen, everything else equal, an increase in the monetary policy rate $\widehat{R}$ (either today or

[3] Under the NT-based setup, an analogous expression can be derived, replacing *rer* with q, and π with π^N.

expected) generates a nominal appreciation. The final effect, however, is determined by the endogenous response of the entire inflation path. When it falls, it reinforces the direct effect, and the nominal exchange rate definitely appreciates. Instead, if inflation increases, the final effect on $\widehat{\pi}_t^S$ is ambiguous. As we will analyze, fiscal policy being Ricardian or not, as well as the share of government debt denominated in dollars, are key to determining the final effect.

To sharpen the intuition, we will additionally assume $\sigma\eta = 1$ (i.e., equality of the intra- and inter-temporal elasticities of substitution, η and $1/\sigma$ respectively). In tandem with a closing device specified as in (7), this leads to c_t^T, d_t^*, and R_t^* being determined solely by the exogenous variables π_t^*, R_t^W, and y_t^T, with no influence from other domestic variables, particularly monetary and fiscal policy (see Appendix B.2.1).

As shown in appendix C, assuming also zero steady-state inflation (both domestic and foreign), the log-linearized equilibrium conditions under the NT-based policy setup can be reduced to

$$\widehat{\pi}_t^N = \beta E_t\left\{\widehat{\pi}_{t+1}^N\right\} + \widetilde{\kappa}\left(\eta\widehat{q}_t + \widehat{c}_t^T\right), \tag{11}$$

$$\widehat{R}_t = \widehat{R}_t^* + E_t\left\{\widehat{q}_{t+1} - \widehat{q}_t + \widehat{\pi}_{t+1}^N - \widehat{\pi}_{t+1}^*\right\}, \tag{12}$$

$$\widehat{R}_t = \phi_\pi\widehat{\pi}_t^N + \widehat{u}_t^R, \tag{13}$$

$$(1-\phi_T)\left[\widehat{d}_{t-1} - (1-\Omega)\widehat{\pi}_t^N + \Omega(\widehat{q}_t - \widehat{\pi}_t^*)\right] = \beta\left[\widehat{d}_t - (1-\Omega)(\widehat{R}_t) + \Omega(\widehat{q}_t - \widehat{R}_t^*)\right] + \widehat{u}_t^\tau. \tag{14}$$

Equation (11) is the New-Keynesian Phillips curve for non-traded inflation, which, in equilibrium, is driven by movements in $\widehat{q}_t$ and $\widehat{c}_t^T$ (the relevant determinants of non-traded demand).[4] In turn, equation (12) is the UIP condition, expressed in terms of the relative price $\widehat{q}_t$, while (13) is the Taylor rule, reacting only to non-traded inflation. Finally, equation (14) is the combination of the government's budget constraint and the rules for both the currency composition of debt and lump-sum taxes. Using the Taylor rule to replace the policy rate in the other equations, we are left with a system of three equations characterizing the dynamics of the $\widehat{\pi}_t^N, \widehat{q}_t, \widehat{d}_t$, given the exogenous external variables $\widehat{R}_t^W, \widehat{y}_t^T, \widehat{\pi}_t^*$ (recall that, under $\sigma\eta = 1$, $\widehat{R}_t^*$ and $\widehat{c}_t^T$ are driven solely by these external variables) and policy shocks $\widehat{u}_t^R$ and $\widehat{u}_t^\tau$.

Notice that when $\Omega = 0$, and in the absence of external shocks, this system resembles that of a closed-economy model, with $\widehat{\pi}_t^N$ and $\widehat{q}_t$ playing the roles of total inflation and aggregate demand, respectively, in the closed-economy counterpart. This implies that, as long as $\Omega = 0$, results from the closed-economy literature hold here as well, reinterpreting them in terms of non-traded inflation and the relative price/real exchange rate. If we also want to understand the behavior of the nominal exchange rate, based on the definition of q_t after log-linearization, we have $\widehat{\pi}_t^S = \widehat{q}_t - \widehat{q}_{t-1} - \widehat{\pi}_t^N + \widehat{\pi}_t^*$. This equivalence between closed and open economy setups may explain why there are fewer papers analyzing FTPL in the open economy. However, to the extent that a non-trivial fraction of government debt is denominated in foreign currency, or if we also care about the propagation of external shocks, our study provides a novel contribution.

We finish this section with Table 1, which describes the calibration of the parameters. Those shared with the model in Schmitt-Grohé and Uribe (2017, Ch. 9.16) are taken from them. The parameters $\overline{R}^*$, $\overline{d}^*$, χ, and $\overline{\tau}$ are set endogenously in steady state to obtain shares of primary surplus and trade balance to GDP of 5%,[5] as well as values for $\overline{h}$ and $\overline{rer}$ that generate a relative size of non-

[4] The slope of teh Phillips curve is $\widetilde{\kappa} \equiv \kappa\left(\frac{(1+\varphi)}{\alpha} - 1 + \sigma\right)$.

[5] Notice that, considering a time period to be a quarter, a ratio of primary surplus to GDP of 5% implies a ratio of debt to

tradable GDP of 57% (as implicit in the calibration of Schmitt-Grohé and Uribe, 2017). The other parameters related to monetary and fiscal policy (ϕ_π, ϕ_T, and $\overline{\Omega}$) are set to specific values described below, depending on the particular exercise we implement.

Table 1: Baseline Calibration

β	σ	φ	η	ω	α	ϵ_N	θ_N
0.9694	2	0.5	0.5	0.6	0.75	6	0.7
ψ	$\overline{\pi}$	$\overline{\pi}^*$	$\frac{\overline{\tau}}{\overline{gdp}}$	$\overline{h}$	$\frac{\overline{tb}}{\overline{gdp}}$	$\overline{rer}$	
0.000034	1	1	0.05	0.73	0.05	1	

3 Existence and Uniqueness

In this section, we explore the determination of a locally-stationary equilibrium. We first provide an analytical characterization based on the simplified policy setup. The system (11)-(14), after eliminating the policy rate, contains one predetermined/state variable ($\widehat{d}_t$) and two non-predetermined/jumping variables ($\widehat{q}_t$, $\widehat{\pi}_t^N$). Thus, the existence of a unique equilibrium requires one stable eigenvalue. Notice that government debt $\widehat{d}_t$ only features in equation (14), which can be written as

$$(1-\phi_T)\widehat{d}_{t-1} = \beta \widehat{d}_t + \widehat{o}_t,$$

where $\widehat{o}_t$ collects all other terms. Clearly, $\widehat{d}_t$'s eigenvalue is $(1-\phi_T)/\beta \geq 0$. Therefore, as long as $\widehat{o}_t$ is stationary, $\widehat{d}_t$ is non-explosive if its eigenvalue is less than one, which occurs if ϕ_T is large enough ($\phi_T > 1-\beta$). Alternatively, for a relatively small ϕ_T ($\phi_T \leq 1-\beta$), $\widehat{d}_t$ could still be stationary if the endogenous behavior of variables in $\widehat{o}_t$ generates another stable eigenvalue that offsets the explosive dynamics that $\widehat{d}_t$ would otherwise display. This, in turn, depends on the Taylor rule parameter ϕ_π being small enough ($\phi_\pi < 1$).

As it turns out, this intuition also carries over to the CPI-based policy setup. The following proposition, proven in appendix C.2 for the N-policy setup and in appendix D for the baseline setup, provides a formal characterization:

Proposition 1 *Under either the N- or the CPI-based policy setup, if either $\theta_N > 0$ and $\Omega \in [0,1]$, or $\theta_N = 0$ and $\Omega \in [0,1)$, the linearized model has*

- *A unique stable equilibrium if either $\phi_\pi > 1$ and $\phi_T > 1-\beta$ (active monetary, passive fiscal), or $\phi_\pi < 1$ and $\phi_T < 1-\beta$ (active fiscal, passive monetary).*
- *No stable equilibrium if $\phi_\pi > 1$ and $\phi_T < 1-\beta$ (active fiscal and monetary).*
- *A continuum of stable equilibria if $\phi_\pi < 1$ and $\phi_T > 1-\beta$ (passive fiscal and monetary).*

In turn, if $\Omega = 1$ and $\theta_N = 0$, the same results hold except that the combination $\phi_\pi < 1$ and $\phi_T < 1-\beta$ (active fiscal, passive monetary) also generates a continuum of stable equilibria.

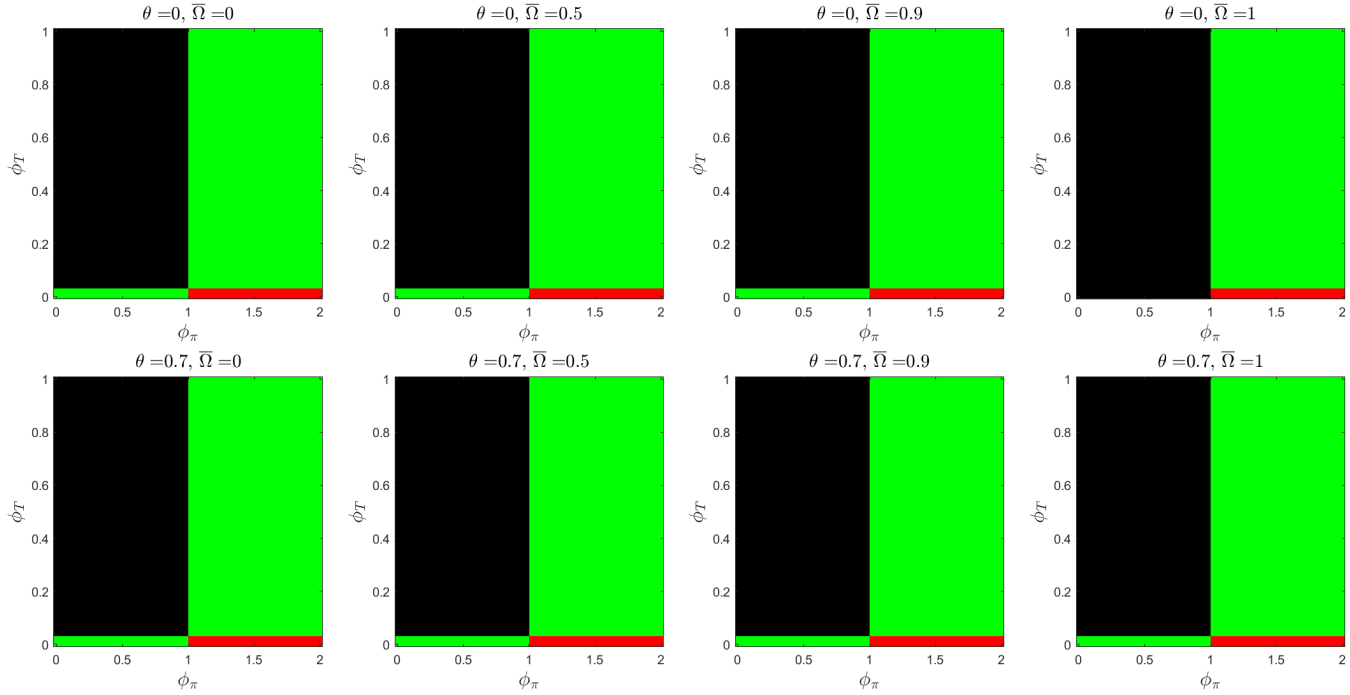
quarterly GDP in steady state equal to $5\%/(1-\beta) \approx 163\%$, equivalent to a share of debt to *annual* GDP of $163\%/4 = 40.75\%$. According to the IMF's Global Debt Database (https://www.imf.org/external/datamapper/GG_DEBT_GDP@GDD/), 40% is the 1/3rd percentile of General Government Debt's distribution across countries for 2024; implying a conservative calibration.

In line with the previous intuition, determinacy arises either with $\phi_T < 1-\beta$ and $\phi_\pi < 1$ (active fiscal, passive monetary) or $\phi_T > 1-\beta$ and $\phi_\pi > 1$ (passive fiscal, active monetary). Instead, if $\phi_T < 1-\beta$ and $\phi_\pi > 1$ (both active), there are no equilibria, while $\phi_T > 1-\beta$ and $\phi_\pi > 1$ (both passive) generate multiplicity.

The only exception to this pattern appears when prices are flexible ($\theta_N = 0$) and debt is fully dollarized ($\overline{\Omega} = 1$). In such a case, active fiscal policy is irrelevant for price determination under a passive monetary policy because, under flexible prices, the real exchange rate (the only relevant price affecting the current value of outstanding debt if $\overline{\Omega} = 1$) is independent of monetary policy. Thus, at least locally, in the case of $\phi_T < 1-\beta$ and $\phi_\pi < 1$, there is also multiplicity if $\theta_N = 0$ and $\overline{\Omega} = 1$. However, as long as prices are sticky, even under 100% dollar-denominated debt, an active fiscal policy combined with passive monetary policy can affect the real exchange rate; thus, the equilibrium is uniquely pinned down.

Figure 1 shows the results under the baseline parameterization. Each graph displays whether the particular combination of ϕ_π (horizontal axis) and ϕ_T (vertical axis) delivers a unique equilibrium (green), no equilibrium (red), or multiplicity/indeterminacy (black). Those in the upper panel correspond to cases with flexible prices ($\theta_N = 0$), while the others show cases under sticky prices ($\theta_N = 0.7$). Each column corresponds to a particular share of dollar-denominated government debt $\overline{\Omega}$.

Figure 1: Existence and uniqueness in the CPI-based policy setup.



Notes: Green: Uniqueness. Red: No stable equilibrium. Black: Multiplicity

Overall, these results show that the share of debt denominated in dollars is generally irrelevant in determining the conditions under which an equilibrium exists and is unique. However, in the following sections we show that, provided determinacy, the dynamics are indeed significantly altered by the currency composition of debt in cases in which fiscal policy is Non-Ricardian/active.

4 The Monetary Transmission Mechanism

This section analyzes the dynamics generated by an i.i.d. shock to the monetary policy rate, u_t^R in equation (6). We first study the case with government debt fully denominated in pesos ($\overline{\Omega} = 0$), and then discuss the role of its currency composition ($\overline{\Omega} \in (0, 1]$). In both cases, we first present the analytical results using the NT-based policy assumptions and then the numerical results based on the baseline setup and parameterization, as well as the intuition behind the results.

4.1 Only domestic currency debt

In the NT-based setup, we can analytically characterize the responses of the relevant variables at the moment the shock hits in two polar cases: active monetary and passive fiscal, with $\phi_\pi > 1$ and $\phi_T > 1 - \beta$, and active fiscal and passive monetary, with $\phi_\pi = \phi_T = 0$. These results are derived in Appendix C.3.1 and C.3.2, and the following propositions summarize the findings:

Proposition 2 *Under the NT-based policy setup with $\phi_\pi > 1$ (active monetary) and $\phi_T > 1 - \beta$ (passive fiscal), an i.i.d. positive shock to u_t^R induces a contemporaneous fall in non-traded inflation $\widehat{\pi}_t^N$, the relative price of tradables $\widehat{q}_t$ and non-traded output $\widehat{y}_t^N$, as well as a nominal appreciation ($\widehat{\pi}_t^S$ falls). This holds for any value of $\overline{\Omega} \in [0, 1]$.*

Proposition 3 *Under the NT-based policy setup with $\phi_\pi = 0$ (passive monetary), $\phi_T = 0$ (active fiscal) and $\overline{\Omega} = 0$ (only domestic-currency debt), an i.i.d. positive shock to u_t^R leads to a contemporaneous increase in non-traded inflation $\widehat{\pi}_t^N$ and a nominal depreciation $\widehat{\pi}_t^S$. The relative price of tradables $\widehat{q}_t$ falls if $\widetilde{\kappa}\eta + \beta > 1$, otherwise it increases after the shock.[6] Non-traded output $\widehat{y}_t^N$ follows the same pattern as $\widehat{q}_t$.*

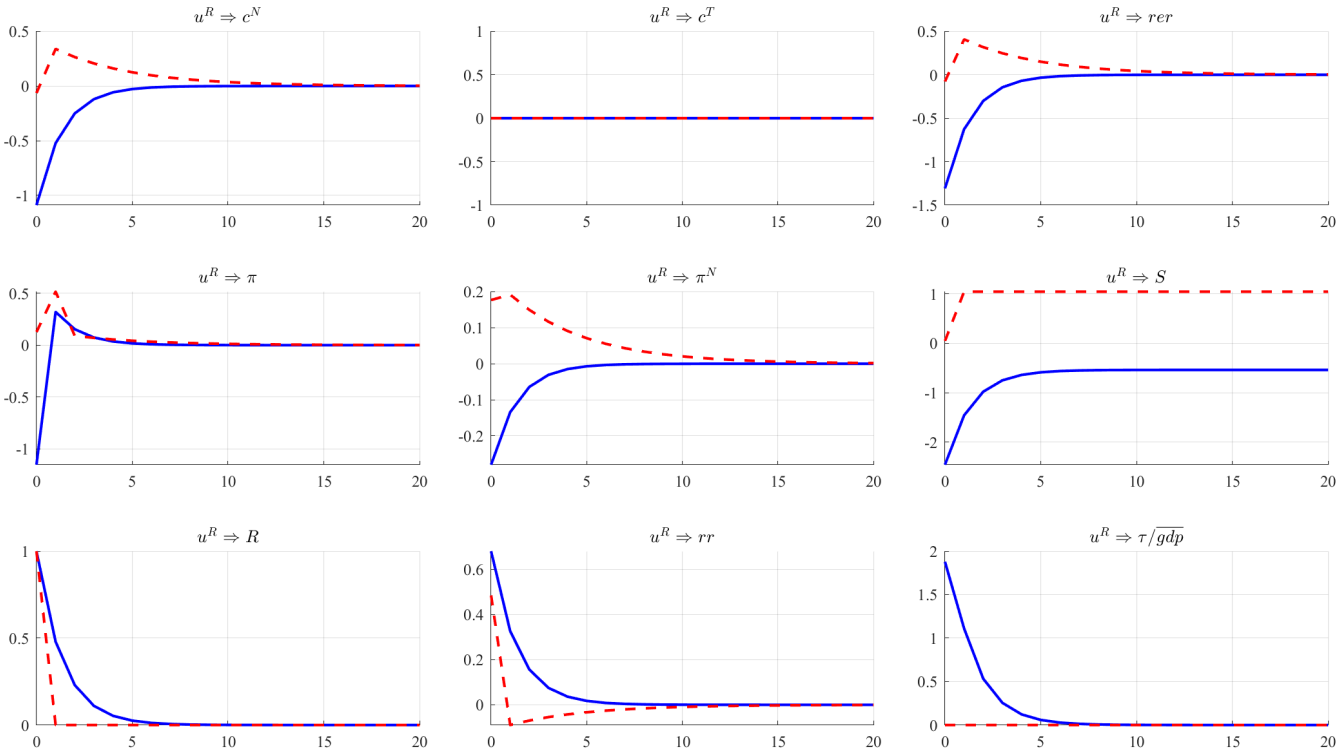
The cases analyzed in these propositions are also shown for the baseline setup and calibration in Figure 2. The blue lines display the case with active monetary policy (with the usual parameter for a Taylor rule of $\phi_\pi = 1.5$) and a fully Ricardian/passive fiscal policy ($\phi_T = 0$). From the household's perspective, the shock induces (*ceteris paribus*, given the same prices) an increase in the real interest rate. This leads to an inter-temporal substitution effect and, as the net present value of after-tax real and financial income falls, a negative wealth effect as well. Both of them tend to decrease current aggregate consumption.

As shown by Caramp and Silva (2023) in a closed economy setup, the size of the wealth effect heavily depends on the fiscal policy response to the shock. From the perspective of the lifetime government budget constraint (8), the increase in the real rate induces a reduction in the net present value of primary surpluses. If fiscal policy is Ricardian ($\phi_T = 1$), lump-sum taxes need to increase to counteract this effect. In turn, higher taxes induce an even deeper negative income effect for households, further decreasing desired consumption. According to Caramp and Silva (2023), this extra wealth effect can account for almost the totality of the contraction in consumption induced by the higher policy rate in closed-economy models.

In the open economy, the required change in taxes when $\phi_T = 1$ is also present; as shown in the figure. This translates into a reduction in desired demand for both traded and non-traded goods (provided homothetic preferences). The contraction in c^N induces a fall in non-traded inflation (π^N). In turn, by (10), the increase in the policy rate induces a nominal appreciation, as long as it is not compensated by an increase in inflation (indeed, π^N falls). Overall, *rer* appreciates. Instead, c^T does not move in equilibrium under $\sigma\eta = 1$.

[6] Under our baseline calibration, $\widetilde{\kappa}\eta + \beta > 1$.

Figure 2: Responses to a monetary-policy-rate shock, Ricardian vs. Non-Ricardian.



Notes: non-traded consumption (c^N, equal to output y^N), traded consumption (c^T), real exchange rate (rer), total inflation (π), non-traded inflation (π^S), nominal exchange rate (S), monetary-policy rate (R), ex-ante real interest rate (rr), and lump-sum taxes as a share of steady-state GDP ($\tau/\overline{gdp}$). The shock is an increase in u_t^R, with zero persistence, normalized to increase R by 1% on impact. Solid-blue lines correspond to the case with $\phi_\pi = 1.5$ and $\phi_T = 1$, while dashed-red lines assume $\phi_\pi = \phi_T = 0$; all with $\overline{\Omega} = 0$. All responses are in percentage-points deviations relative to the steady state.

After the initial period, the nominal appreciation is partially offset, as the UIP condition (9) requires a depreciation going forward if R increases today (i.e., the shock induces exchange-rate overshooting).[7] Overall, the shock contracts activity in the non-traded sector and reduces both traded and non-traded inflation.

In turn, as anticipated by Proposition 3, dynamics are quite different if fiscal policy is Non-Ricardian/active ($\phi_T = 0$) and the interest rate is not responsive to inflation ($\phi_\pi = 0$), as shown by the dashed-red lines in Figure 2. In this case, while the real rate increases (*ceteris paribus*) following the shock, lump-sum taxes remain constant. This has two important consequences. First, the negative wealth effect brought about by a Ricardian policy (through tax increases, as previously discussed) is absent; therefore, c^N only marginally drops at the moment the shock is realized. This extends the result in Caramp and Silva (2023) to this small and open economy setup.

Second, by the FTPL mechanics, the price level needs to increase to satisfy the government lifetime constraint equation (8), which leads to an increase in both non-traded prices and the nominal exchange rate. In terms of S, the initial-period response is only marginally positive. But we also know

[7] This also explains why, even though the shock is i.i.d., the policy rate slowly converges back to the steady state and not instantaneously. Recall that the Taylor rule (6) for the baseline setup is specified in terms of total inflation π, not just π^N. Thus, the overshooting behavior of S generates total inflation that is above the steady state even after the shock materializes, leading to analogous dynamics for R in the period following the realization of the shock. This adds an extra push downwards to c^N (via the same channels previously discussed), and thus, the effect of the shock does not simply vanish after the initial period (as would be the case in the closed-economy version).

from UIP (9) that a further depreciation is expected if R increases today, so the new higher level for S is achieved with a delay; eliminating the overshooting dynamics. As discussed in the introduction, this could rationalize the empirical finding in the literature that, for emerging countries, an increase in the policy rate tends to be followed by a nominal depreciation instead of an appreciation.

In terms of π^N, there are two channels at play. On the one hand, the fall in c^N pushes non-traded prices downward; though this channel should be small, as the negative wealth effect induced by taxes is absent. On the other hand, the FTPL channel requires prices to increase. If prices were fully flexible, this would materialize instantaneously. Under sticky prices, this builds up over time; therefore, additional non-traded inflation is expected in the future. Therefore, from the perspective of the New-Keynesian Phillips curve, the initial contraction in demand for non-tradables is compensated for by the forward-looking channel that anticipates higher future inflation. As a result, π^N rises from the moment the shock hits, with an additional increase in the subsequent period.

These dynamics for π^N and S also induce a different behavior for the real exchange rate: it initially decreases and, in the following periods, increases above the steady state, as S jumps even further. This leads to an additional substitution effect in favor of c^N, as traded goods become relatively more expensive if *rer* increases.[8]

The dynamics of c^N can also be understood from the behavior of the ex-ante real rate rr. In the Ricardian case (solid-blue line), the real-rate path converges back to the steady state monotonically from above, explaining why consumption displays a similar path back to the steady state with the opposite sign (due to intertemporal substitution). In turn, in the Non-Ricardian case (dashed-red line), the real rate falls below its steady state value after the initial increase due to the expected higher inflation, converging to the steady state from below. This is consistent with the dynamics displayed by c^N in the dashed-red lines. Of course, explanations based on *rer* or *rr* are two sides of the same coin, for these two variables are linked in equilibrium by the UIP condition expressed in real terms.

In the robustness appendix E.1, we explore the role of the monetary response parameter ϕ_π for a given fiscal regime when $\overline{\Omega} = 0$. In the Ricardian case ($\phi_T = 1$), only marginal differences arise when comparing the baseline $\phi_\pi = 1.5$ against a value just above the determinacy region ($\phi_\pi = 1.01$), with responses being relatively muted for lower values of ϕ_π. Instead, in the Non-Ricardian regime ($\phi_T = 0$), a monetary response parameter marginally below that required for determinacy ($\phi_\pi = 0.99$) significantly alters the dynamics. In particular, the policy rate remains above the steady state for much longer, despite the shock being i.i.d. This, in turn, implies a persistently higher path for the real rate, which, through the FTPL channel, induces additional (and more persistent) non-traded inflation, while the nominal depreciation continues even after the initial periods. At the same time, the initial response of non-traded consumption and the real exchange rate is marginally smaller; however, its convergence back to the steady state from above is much slower.

Overall, under this alternative policy configuration, an increase in the nominal rate leads to an increase in inflation and an expansion in output, similar to the effects shown, for instance, by Cochrane (2023) in a closed-economy setting. Our analysis extends these previous results to open economies, leading to a better understanding of the dynamics of both the nominal and real exchange rates.

4.2 The role of foreign currency debt

We next turn to cases with a positive share of dollar-denominated government debt. The following proposition is developed in Appendix C.3.2:

[8] Again, c^T is not affected in equilibrium, as the several effects offset each other if $\sigma\eta = 1$.

Proposition 4 *Under the NT-based policy setup with $\phi_\pi = \phi_T = 0$ (passive monetary, active fiscal), following an i.i.d.* ***positive*** *shock to u_t^R, the sign of the contemporaneous response of the variables of interest depends on the value of $\overline{\Omega}$ relative to two cutoff values ($\overline{\Omega}_{\text{cut},1} > \overline{\Omega}_{\text{cut},2}$), as detailed in columns one to five in Table 2. Within each range defined by these cutoffs, a marginally larger value of $\overline{\Omega}$ increases these response, as seen in the last column of Table 2.*

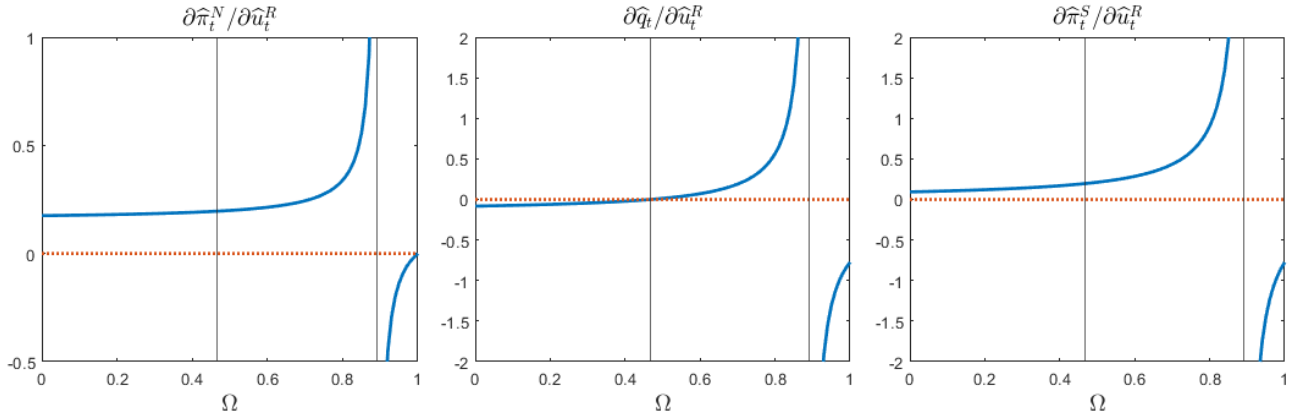
Table 2: The impact of monetary shocks in the NT-based setup, with $\phi_T = \phi_\pi = 0$, and the role of $\overline{\Omega}$

	Values of $\overline{\Omega}$				Derivative
Response of	$0 \leq \overline{\Omega} < \overline{\Omega}_{\text{cut},2}$	$\overline{\Omega}_{\text{cut},2} < \overline{\Omega} < \overline{\Omega}_{\text{cut},1}$	$\overline{\Omega}_{\text{cut},1} < \overline{\Omega} < 1$	$\overline{\Omega} = 1$	w.r.t. $\overline{\Omega}$
$\widehat{\pi}_t^N$	+	+	-	0	+
$\widehat{q}_t$	-	+	-	-	+
$\widehat{\pi}_t^S$	+	+	-	-	+
$\widehat{y}_t^N$	-	+	-	-	+

Notes: Columns two to five display the sign of the contemporaneous response of the variable indicated in the first column after a **positive** shock to $\widehat{u}_t^R$, for different ranges of $\overline{\Omega}$. The last column indicates how the response of that specific variable is modified for a marginally larger value of $\overline{\Omega}$. The table assumes that $\widetilde{\kappa}\eta + \beta > 1$, which is only relevant to for the effect on $\widehat{q}_t$ and $\widehat{y}_t^N$ in the case with $0 \leq \overline{\Omega} < \overline{\Omega}_{\text{cut},2}$. If instead $\widetilde{\kappa}\eta + \beta \leq 1$, these signs are the opposite, is detailed in Appendix C.3.2.

Figure 3 helps visualize these results by plotting the contemporaneous effect of $\widehat{u}_t^R$ on $\widehat{\pi}_t^N$, $\widehat{q}_t$, and $\widehat{\pi}_t^S$ for different values of $\overline{\Omega}$, using the baseline parameterization. As can be seen, for a sufficiently low ratio of dollar denominated debt ($\overline{\Omega} < \overline{\Omega}_{\text{cut},1}$), the contemporaneous responses of the four variables display qualitatively the same sign as those analyzed under $\overline{\Omega} = 0$. As $\overline{\Omega}$ increases, the initial responses of non-traded inflation and the nominal depreciation are larger, as long as $\overline{\Omega} < \overline{\Omega}_{\text{cut},1}$. The same can be observed for the relative price: its response is less negative for larger values of $\overline{\Omega}$, and it eventually turns positive in cases with $\overline{\Omega}_{\text{cut},2} < \overline{\Omega} < \overline{\Omega}_{\text{cut},1}$.

Figure 3: The contemporaneous effect of $\widehat{u}_t^R$ under the N-policy setup depending $\overline{\Omega}$



Note: Each line plots the contemporaneous effect of an **increase** in $\widehat{u}_t^R$ on $\widehat{\pi}_t^N$, $\widehat{q}_t$ and $\widehat{\pi}_t^S$, respectively, as a function of $\overline{\Omega}$ (recall $\widehat{y}_t^N$ is proportional to $\widehat{q}_t$). Vertical lines indicate the cutoff values $\overline{\Omega}_{\text{cut},1}$ and $\overline{\Omega}_{\text{cut},2}$ (recall $\overline{\Omega}_{\text{cut},2} < \overline{\Omega}_{\text{cut},1}$). These are computed under the baseline parametrization, which generates $\overline{\Omega}_{\text{cut},1} = 0.8921$ and $\overline{\Omega}_{\text{cut},2} = 0.4663$.

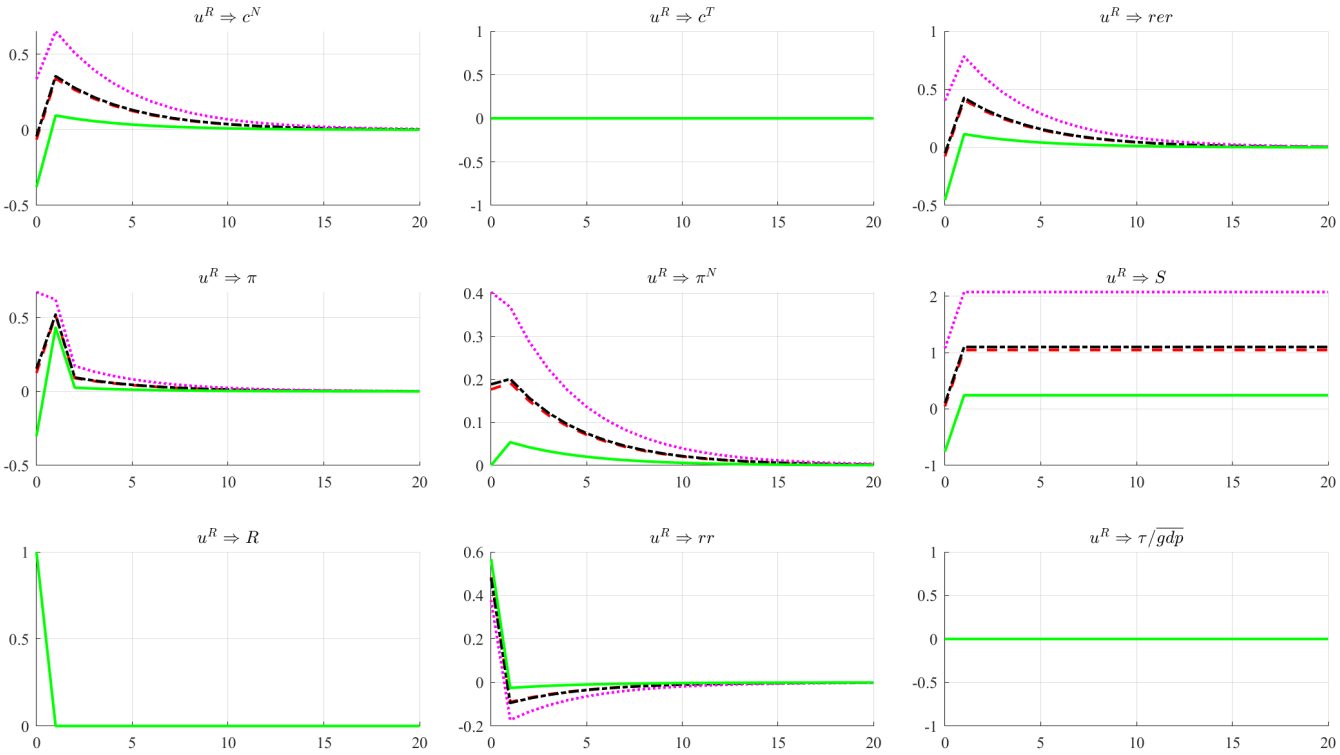
These results are in line with the discussion of the FTPL equation (8) in section 2.5: whenever there is a need to compensate for a change in the net-present value of primary surpluses, a smaller fraction of debt denominated in pesos requires a larger rise in inflation to dilute its value. As a result, the initial fall in the relative price and non-traded output that we observed when debt was fully

denominated in domestic currency is compensated, and it may even be positive for relatively larger shares of dollar-denominated debt.

However, this is not monotonic: for values $\overline{\Omega} > \overline{\Omega}_{\text{cut},1}$, all initial responses flip their sign.[9] This can be intuitively understood if we first consider the case of $\overline{\Omega} = 1$. From the left-hand side of the FTPL equation (8), we see that a real appreciation is required to compensate for the drop in the net-present value of surpluses that the shock generates. As non-traded prices are sticky, this cannot materialize through an increase in π^N; therefore, an initial nominal appreciation is induced. When $\overline{\Omega} = 1$, the initial response of π^N is, in fact, exactly zero. However, for relatively smaller values of $\overline{\Omega}$, the real appreciation exerts downward pressure on non-traded inflation, inducing an even larger real appreciation.

Overall, there are two forces behind how $\overline{\Omega}$ affects the dynamics. On the one hand, the relatively smaller share of debt denominated in pesos induces a larger inflation reaction (non-traded in the NT-based setup, aggregate inflation in the CPI-based configuration), which also pushes inflation upward by the UIP condition (10). On the other hand, as the share of dollar denominated debt gets larger, debt dilution can also materialize through real appreciation, which in turn requires a fall in the nominal exchange rate, as non-traded prices are sticky. In turn, the real appreciation also pushes non-traded inflation downwards by the Phillips curve (11).

Figure 4: Responses to a monetary-policy-rate shock, Non-Ricardian, different values of $\overline{\Omega}$.



Notes: The figure is analogous to Figure 2, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for $\overline{\Omega}$: In dashed-red $\overline{\Omega} = 0$, in dashed-dotted black $\overline{\Omega} = 0.5$, in dotted magenta $\overline{\Omega} = 0.9$, and in solid-green $\overline{\Omega} = 1$.

To conclude the analysis, Figure 4 compares the responses in the CPI-based policy setup for alter-

[9] If $\overline{\Omega} = \overline{\Omega}_{\text{cut},1}$, there is a vertical asymptote (the initial response tends to infinity when approaching $\overline{\Omega}_{\text{cut},1}$ from the left, and to minus infinity from the right).

native values of the share of dollar-denominated debt.[10] All cases displayed assume $\phi_\pi = \phi_T = 0$, considering alternative values for $\overline{\Omega}$ equal to 0, 0.5, 0.75, and 1. Except for the case of $\overline{\Omega} = 1$, we can see that a higher share of dollar-denominated debt yields larger responses for both non-traded inflation and the nominal exchange rate at the moment the shock hits, in line with the results in Proposition 4 for cases with $\overline{\Omega} < \overline{\Omega}_{\text{cut},1}$. We can also see the real exchange rate and non-traded consumption increasing with larger shares of dollar-denominated debt, starting from initially negative values for relatively smaller values of $\overline{\Omega}$. The figure also allows us to see that these differences are magnified in the periods following the impact of the shocks.

Additionally, we see that for $\overline{\Omega} = 1$ the initial responses of both the nominal and real exchange rates (as well as non-trade activity) change sign, in line with Proposition 4. However, we see that in this version of the model, this change in sign only materializes during the impact's period. Afterwards, the responses have the same sign as those displayed for smaller values of $\overline{\Omega}$, although with a smaller magnitude, even smaller than in the case with $\overline{\Omega} = 0$. Once again, these results emphasize the importance of accounting for the currency composition of debt in understanding the effects of monetary policy innovations under active fiscal policy.

5 The Effects of Fiscal Shocks

We next turn to the dynamics generated by a fiscal shock, u^τ in the policy rule (4). Under a Ricardian/passive fiscal policy, a change in taxes today induces an opposite-sign modification in future taxes, such that the net-present value on the right-hand side of (8) is unaltered; therefore, it has zero effects. Instead, the shock induces non-trivial aggregate dynamics under non-Ricardian/active fiscal policy. For the NT-based policy, Appendix C.3.2 provides a proof for the following results:

Proposition 5 *Under the NT-based policy setup with $\phi_\pi = \phi_T = 0$ (passive monetary, active fiscal), following an i.i.d. **negative** shock to u_t^T, the sign of the contemporaneous response of the variables of interest depends on the value of $\overline{\Omega}$ relative to the cutoff value $\overline{\Omega}_{\text{cut},1}$, as detailed Table 3. Moreover, a marginally larger value of $\overline{\Omega}$ increases these response, as seen in the last column of Table 3.*

Table 3: The impact of fiscal shocks in the NT-based setup, with $\phi_T = \phi_\pi = 0$, and the role of $\overline{\Omega}$

	Values of $\overline{\Omega}$		Derivative
Response of	$0 \leq \overline{\Omega} < \overline{\Omega}_{\text{cut},1}$	$\overline{\Omega}_{\text{cut},1} < \overline{\Omega} \leq 1$	w.r.t. $\overline{\Omega}$
$\widehat{\pi}_t^N$	+	-	+
$\widehat{q}_t$	+	-	+
$\widehat{\pi}_t^S$	+	-	+
$\widehat{y}_t^N$	+	-	+

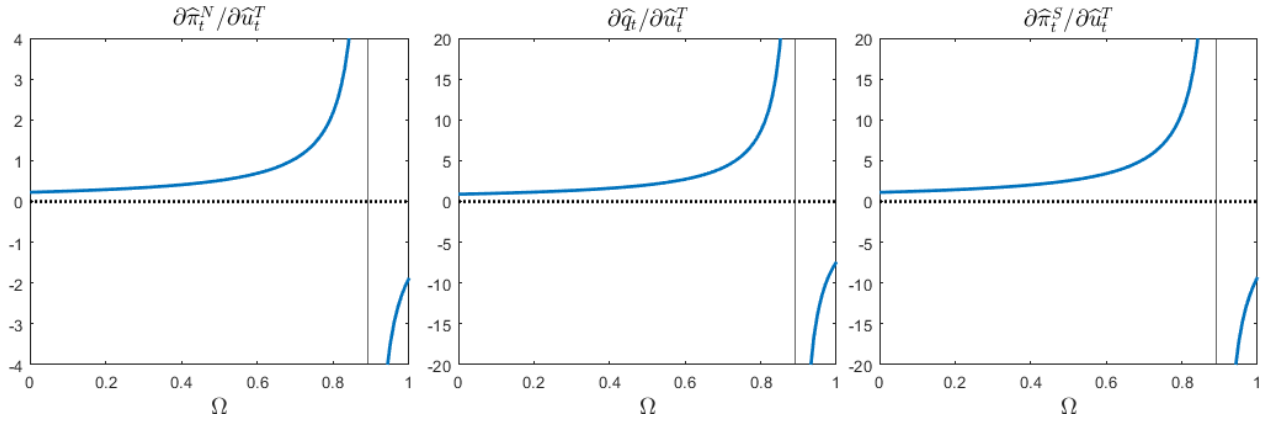
Notes: This is analogous to Table 3, for the case of a **negative** shock to $\widehat{u}_t^R$.

Figure 5 helps visualize these results by plotting the contemporaneous effect of $\widehat{u}_t^R$ on $\widehat{\pi}_t^N$, $\widehat{q}_t$, and $\widehat{\pi}_t^S$ for different values of $\overline{\Omega}$, using the baseline parameterization. Beginning with the case of only peso-denominated debt ($\overline{\Omega} = 0$), if the policy rate is fixed, the FTPL equation (8) indicates that a surprise reduction in primary surpluses requires either an increase in the price level today, an increase in expected inflation, or a combination of both. This is achieved by both a nominal depreciation and

[10] While we were not able to find an algebraic expression for the cutoff values $\overline{\Omega}_{\text{cut},1}, \overline{\Omega}_{\text{cut},2}$ in the CPI-based policy setup, we can approximate them numerically to $\overline{\Omega}_{\text{cut},1} = 0.945$ and $\overline{\Omega}_{\text{cut},2} = 0.735$.

an increase in π^N, the latter being quantitatively smaller and spread over time due to sticky prices. As a result, a real depreciation materializes.

Figure 5: The contemporaneous effect of $\widehat{u}_t^T$ under the N-policy setup depending $\overline{\Omega}$



Note: Each line plots the contemporaneous effect of an **drop** in $\widehat{u}_t^T$ on $\widehat{\pi}_t^N$, $\widehat{q}_t$ and $\widehat{\pi}_t^S$, respectively, as a function of $\overline{\Omega}$ (recall $\widehat{y}_t^N$ is proportional to $\widehat{q}_t$). Vertical lines indicate the cutoff value $\overline{\Omega}_{\text{cut},1}$ ($\overline{\Omega}_{\text{cut},2}$ is not relevant for the fiscal shock). This is computed under the baseline parametrization, which generates $\overline{\Omega}_{\text{cut},1} = 0.8921$.

This real depreciation leads to an expansion in non-traded activity through intra-temporal substitution, as non-traded goods become relatively cheaper. Moreover, an additional expansionary channel arises from the positive wealth effect that the tax reduction generates. While the real interest rate is reduced (as expected inflation rises), this is not enough to compensate for the increase in the net present value of after-tax income. Overall, total output rises in equilibrium.

Turning to cases where a fraction of the debt is denominated in dollars, as long as $\overline{\Omega} < \overline{\Omega}_{\text{cut},1}$, the dynamics are magnified by a larger share $\overline{\Omega}$. Again, this can be explained by the additional inflation that is required to dilute the relatively smaller outstanding amount of peso-denominated debt. In such a process, nominal depreciation and increases in non-traded inflation reinforce each other: the real exchange rate depreciates, which puts upward pressure on π^N according to the Phillips curve, while the additional inflation also increases the nominal exchange rate through the UIP condition. This, in turn, explains the vertical asymptote in $\overline{\Omega} = \overline{\Omega}_{\text{cut},1}$.

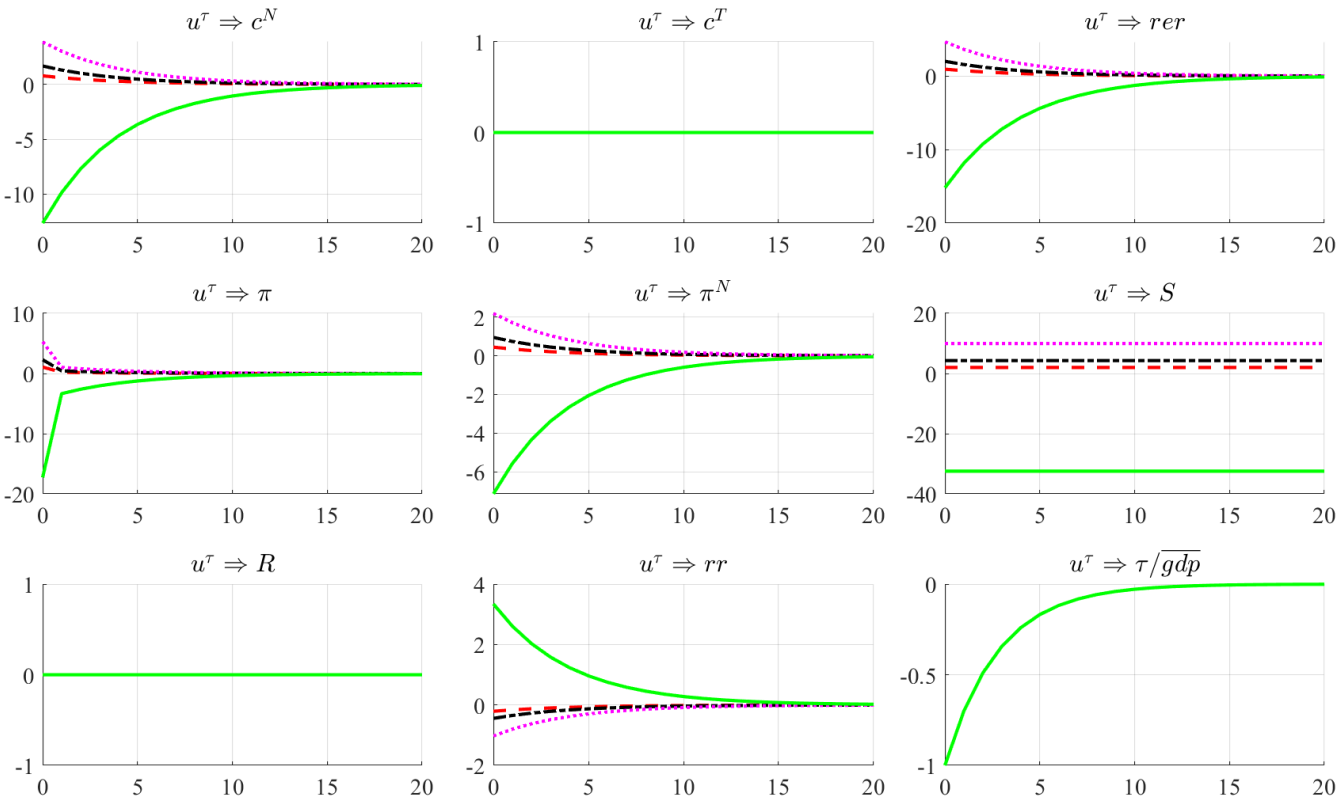
Instead, when $\overline{\Omega} > \overline{\Omega}_{\text{cut},1}$, as we previously discussed, the other potential way of diluting debt obligations dominates: a real appreciation helps reduce the real burden of dollar denominated debt. This triggers a nominal appreciation and a fall in non-traded inflation that, again, reinforce each other for values of $\overline{\Omega}$ approaching $\overline{\Omega}_{\text{cut},1}$ from above.

Figure (6) shows the effects of a fall in lump-sum taxes, normalized to represent 1% of steady-state GDP (with an autocorrelation of 0.7), for different values of $\overline{\Omega}$ under the CPI-policy framework and the baseline parameterization. In all cases, the policy rate is kept fixed ($\phi_\pi = 0$) and $\phi_T = 0$.[11] The results for the initial period replicate those described in Proposition 5. Here we can also see that these initial effects are quite persistent, except for the behavior of the nominal exchange rate S, which directly jumps to its new higher level without further movements.

In summary, even in the case of fiscal shocks, the share of dollar-denominated government debt is

[11] In the robustness appendix E.1, we see that if $\phi_T = 0$, but the monetary-response parameter is close to one, non-trivial differences arise; similar to those described before in the case of the monetary shock. In particular, if $\phi_\pi = 0.99$, the policy rate remains high for a longer period, increasing non-traded inflation and the nominal exchange rate even further, resulting in the policy rate rising even more persistently. At the same time, the initial increases in both c^N and *rer* are smaller than with $\phi_\pi = 0$, but its convergence back to the steady state is much slower.

Figure 6: Responses to a fiscal shock, Non-Ricardian, different values of $\overline{\Omega}$.



Notes: The figure is analogous to Figure 4, except that is shows responses to a drop in lump-sum taxes, normalized to represent 1% of steady-state GDP, with an autocorrelation of 0.7. All cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for $\overline{\Omega}$: In dashed-red $\overline{\Omega} = 0$, in dashed-dotted black $\overline{\Omega} = 0.5$, in dotted magenta $\overline{\Omega} = 0.75$, and in solid-green $\overline{\Omega} = 1$.

relevant in determining the final dynamics. In particular, a larger share of dollar-denominated debt magnifies the effects of both shocks (as long as it is not large enough). Moreover, in tandem with those related to monetary policy shocks, these results provide an interesting testable implication that future work can use to empirically analyze the FTPL channels from cross-country variation in the share of dollar-denominated government debt.

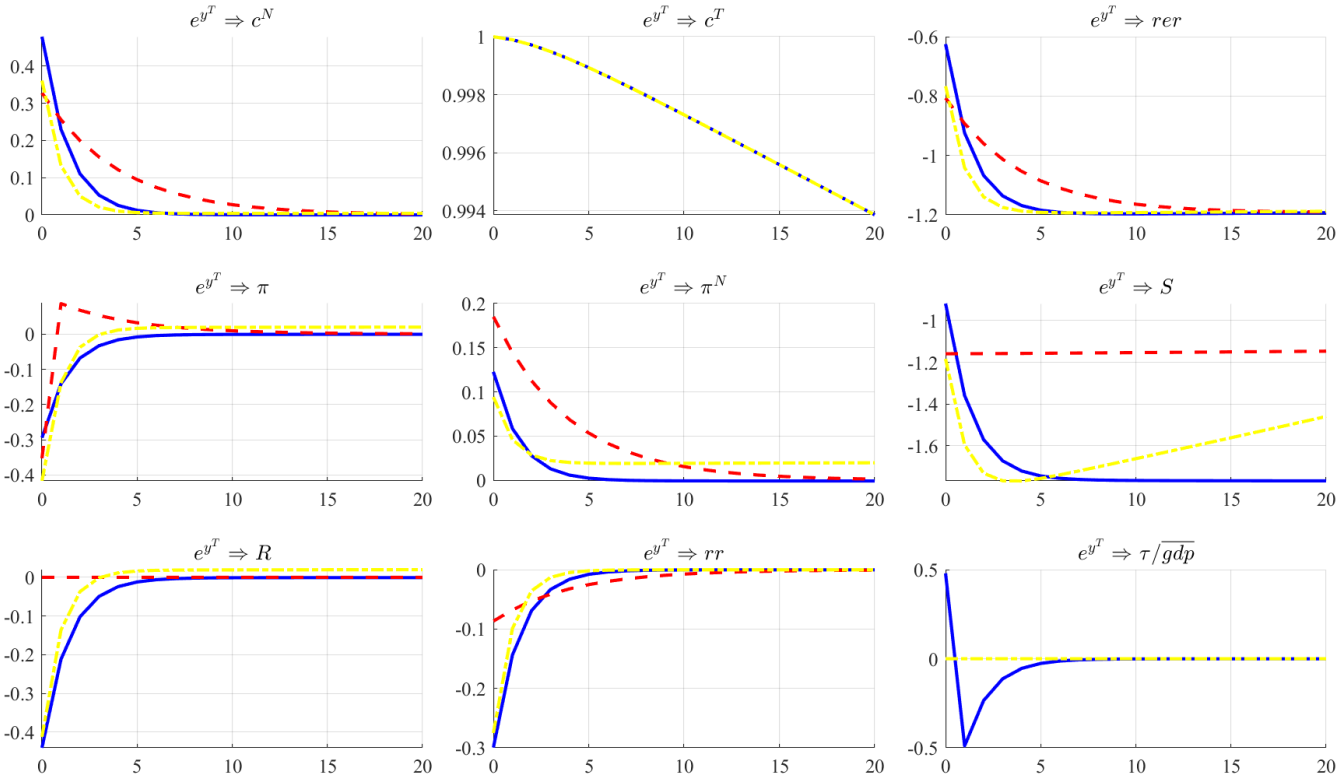
6 Dynamics Induced by Real Shocks

In this section, we study the effects of shocks to traded output y^T (which could also be interpreted as terms-of-trade shocks in this simple setup) and to the world interest rate R^W (shocks to π^* induce effects similar to those generated by R^W). For each of them, we first describe the propagation under Ricardian/passive fiscal policy, then analyze the role played by a Non-Ricardian setup, and finally study the role played by the currency composition of government debt. Unfortunately, the algebraic approach using the NT-based policy setup is not sufficiently clear to derive clean results in the case of real shocks: even if disturbances are i.i.d., as long as they affect tradable consumption and the net-foreign-debt position, endogenous persistence arises, complicating the algebra. For that reason, all results are generated using the CPI-based framework with the baseline calibration.

6.1 Traded output

Figure 7 displays the dynamics triggered by an increase in y^T, with an autocorrelation of 0.7, normalized to increase traded consumption by 1%, assuming $\overline{\Omega} = 0$. Before analyzing a specific policy regime, we should note that the shock induces a positive wealth effect, increasing the desired consumption of both types of goods. The increase in c^T is quite persistent, as the interest rate in dollars (R^*) is calibrated to have minor elasticity to the country's net-foreign debt position; thus, convergence back to the steady state is quite slow.[12] Additionally, the higher demand for non-tradables increases their relative price, leading to a real appreciation. If prices were fully flexible, this would materialize instantaneously. However, since non-traded prices are sticky, it takes time to reach their maximum appreciated value. Moreover, the real appreciation arises from a relatively small increase in non-trade inflation and, more significantly, from a nominal appreciation.

Figure 7: Responses to a traded-output shock, Ricardian vs. Non-Ricardian.



Notes: The figure show the responses to a positive shock to y^T, with an autocorrelation of 0.7, normalized to increase c^T by 1% at the moment the shock hits. All cases feature $\overline{\Omega} = 0$. In solid blue lines $\phi_T = 1$ and $\phi_\pi = 1.5$. in dashed-red lines $\phi_T = 0$ and $\phi_\pi = 0$, while in dashed-dotted yellow lines $\phi_T = 0$ and $\phi_\pi = 0.99$. See Figure 2 for variables' definitions.

The specific dynamics are shaped by the fiscal and monetary regimes. The blue lines display the case with a Ricardian/passive tax response ($\phi_T = 1$) and an active Taylor rule for the interest rate ($\phi_\pi = 1.5$). Under such a configuration, the policy rate falls following the reduction in inflation. This, in turn, induces an even further appreciation: by the UIP condition (9), a fall in the domestic rate requires a further expected appreciation. In addition, as the policy rate falls by more than overall inflation, the real rate also decreases. From the perspective of households, this further increases non-traded consumption (c^N) on impact through inter-temporal substitution.

[12] As this is a transitory shock, R^* persistently falls as an improvement in the current account is generated, causing the elastic premium to decrease.

In turn, fiscal policy is also affected by the shock through two different channels. First, the reduction in inflation increases the real value of the outstanding nominal debt. Second, the reduction in the real rate increases the net present value of taxes. The former dominates in equation (8), as can be seen from the fact that lump-sum taxes increase in response to the shock.

The dashed-red lines in Figure 7 show the responses to the same shock under a Non-Ricardian fiscal policy ($\phi_T = 0$) and a constant policy rate ($\phi_\pi = 0$). While responses are qualitatively similar to those in the previous policy configuration, two quantitative differences emerge from the policy rate remaining fixed. First, the further nominal appreciation in the period following the shock that we described in the blue lines disappears, which in turn eliminates the persistence of the initial fall in overall inflation. Second, as the nominal interest rate does not change but expected inflation is positive, the drop in the real rate is much smaller than in the previous policy configurations.

Moreover, the smaller reduction in the real rate limits the intertemporal substitution that influences non-trade consumption, and therefore c^N increases by less on impact. This also implies that the path of c^N in the periods after the shock hits converges more slowly to the steady state. In turn, as π^N is forward-looking under Calvo prices, the initial increase in non-traded inflation is larger, anticipating the expected higher demand in the future.

In terms of fiscal policy, as lump-sum taxes are fixed, the right-hand side of the FTPL equation (8) increases as the real rate falls. This implies that, relative to the case with Ricardian policy, prices need to fall even more. We have already described the reasons for which this is not going to happen through π^N, which implies that the initial nominal appreciation is larger.

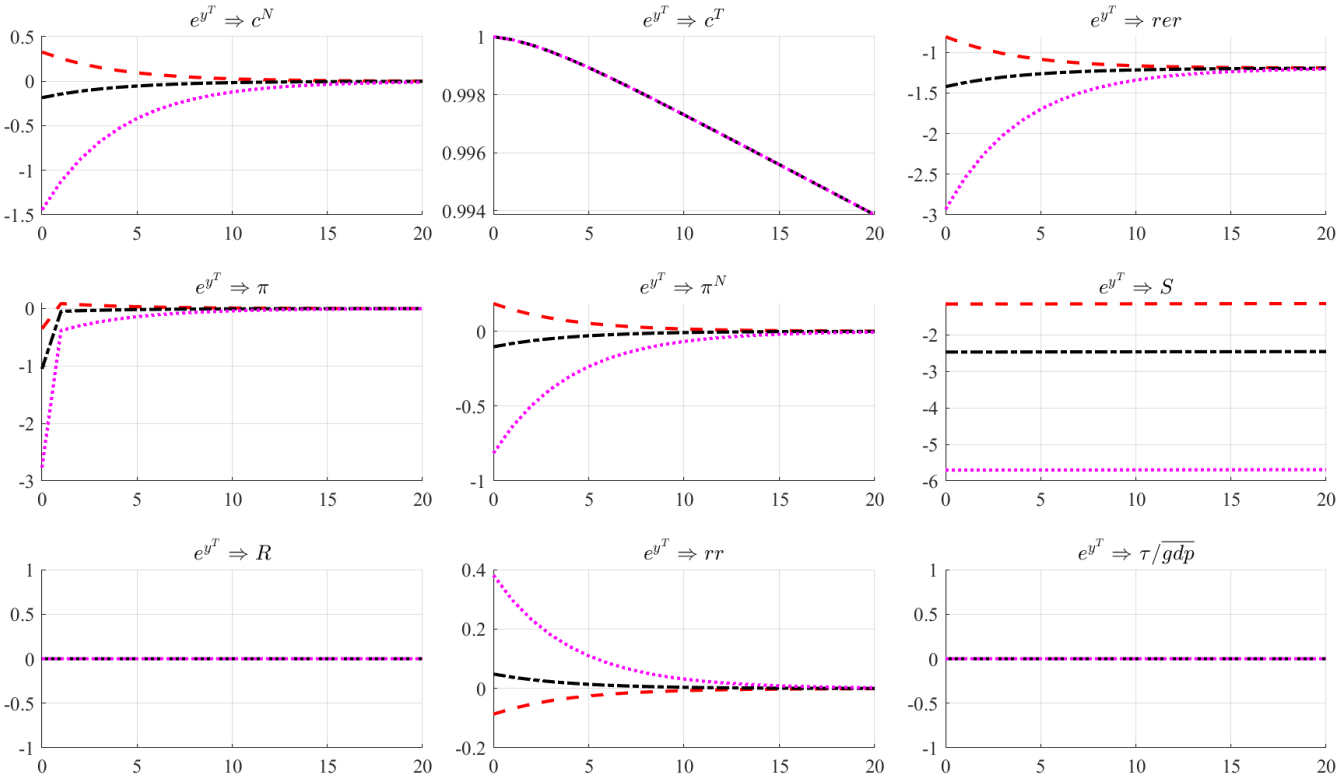
Notice that there are two differences that distinguish the dashed-red lines from the solid-blue ones: both taxes and interest rates are kept constant if $\phi_T = \phi_\pi = 0$. In order to better understand the relative importance of each of them, Figure 7 includes an additional case (in dashed-dotted yellow lines) where taxes still do not move ($\phi_T = 0$), but the policy rate increases with inflation, using $\phi_\pi = 0.99$ (just below the value required for equilibrium uniqueness under an active fiscal policy). In this third case, the dynamics of both the nominal and real interest rates, non-traded inflation and consumption, as well as the real exchange rate, are similar to those under a Ricardian/passive fiscal policy. Thus, to a large extent, the differences between the solid-blue and dashed-red lines are driven by the different behavior of the policy rate. Still, the FTPL channel continues to play a role. As the real rate drops even further in the dashed-dotted yellow lines, it further increases the right-hand side of the FTPL equation (8). Thus, an initially larger nominal appreciation is still required to compensate if taxes do not change.

Dynamics are significantly altered once we allow for a positive share of debt denominated in foreign currency. Figure 8 shows four cases, all of which have $\phi_T = 0$ and $\phi_\pi = 0$, but with different values of $\overline{\Omega}$: 0 (dashed red), 0.5 (dashed-dotted black), and 0.75 (dotted magenta).[13] With the exception of the case with fully dollar denominated debt (which we analyze below), larger values of $\overline{\Omega}$ result in further nominal and real appreciations, while non-traded inflation and consumption now fall. What triggers these differences?

In the presence of foreign denominated debt, the real appreciation generated by the shock (which has a real origin, present even under flexible prices) tends to reduce the real value (in domestic consumption units) of outstanding dollar-denominated debt if $\overline{\Omega} > 0$. For a given path of lump-sum taxes (as $\phi_T = 0$), satisfying the FTPL equation (8) requires either an increase in the real value of outstanding peso-denominated debt (which can materialize if the price level drops as the shock hits), a reduction in the net-present value of primary surpluses (which requires the real rate to increase),

[13] In Appendix E.2, a similar figure is displayed with $\phi_\pi = 0.99$, where it can be seen that the differences are magnified.

Figure 8: Responses to a traded-output shock, Non-Ricardian, different values of $\overline{\Omega}$.



Notes: The figure is analogous to Figure 7, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for $\overline{\Omega}$: In dashed-red $\overline{\Omega} = 0$, in dashed-dotted black $\overline{\Omega} = 0.5$, in dotted magenta $\overline{\Omega} = 0.75$.

or a combination of both. These two alternatives help each other in achieving the goal: a higher real rate pushes c^N downwards, which in turn reduces non-traded inflation today, helping to dilute the real value of outstanding debt in pesos. At the same time, the real-exchange rate appreciates even further, which also adds an intra-temporal substitution channel that further reduces c^N and thus π^N. Moreover, these effects are magnified for larger shares $\overline{\Omega} > 0$.

The figure does not include the case of $\overline{\Omega} = 1$, which is shown in Appendix E.2, as this case significantly alters the scale of the graphs. However, as hinted at by previous results, if government debt is fully denominated in dollars, the dynamics have the opposite sign. In particular, this means that a real *depreciation* is induced, which might seem counterintuitive. What is behind this result?

As we mentioned, the flexible-price effect triggered by this shock is a drop in both *rer* and R^*. If nothing else were to happen, this is inconsistent with the FTPL valuation equation (8) when $\overline{\Omega} = 1$: the *rer* falling reduces the real value of outstanding debt (left-hand side), while the drop in R^* (which tends to lower *rr* through UIP) increases the net present value of primary surpluses (right-hand side); thus, the equation does not hold. Therefore, either the real exchange rate should not appreciate, the real interest rate should increase, or a combination of both. However, to obtain a larger real rate (given that the nominal rate is fixed), additional expected non-trade inflation is required, which contradicts a real appreciation unless the nominal exchange rate jumps even further. This explains why not only do responses have the opposite sign, but the responses are much larger in absolute value.[14]

In conclusion, while in the case of government debt fully denominated in pesos, FTPL considera-

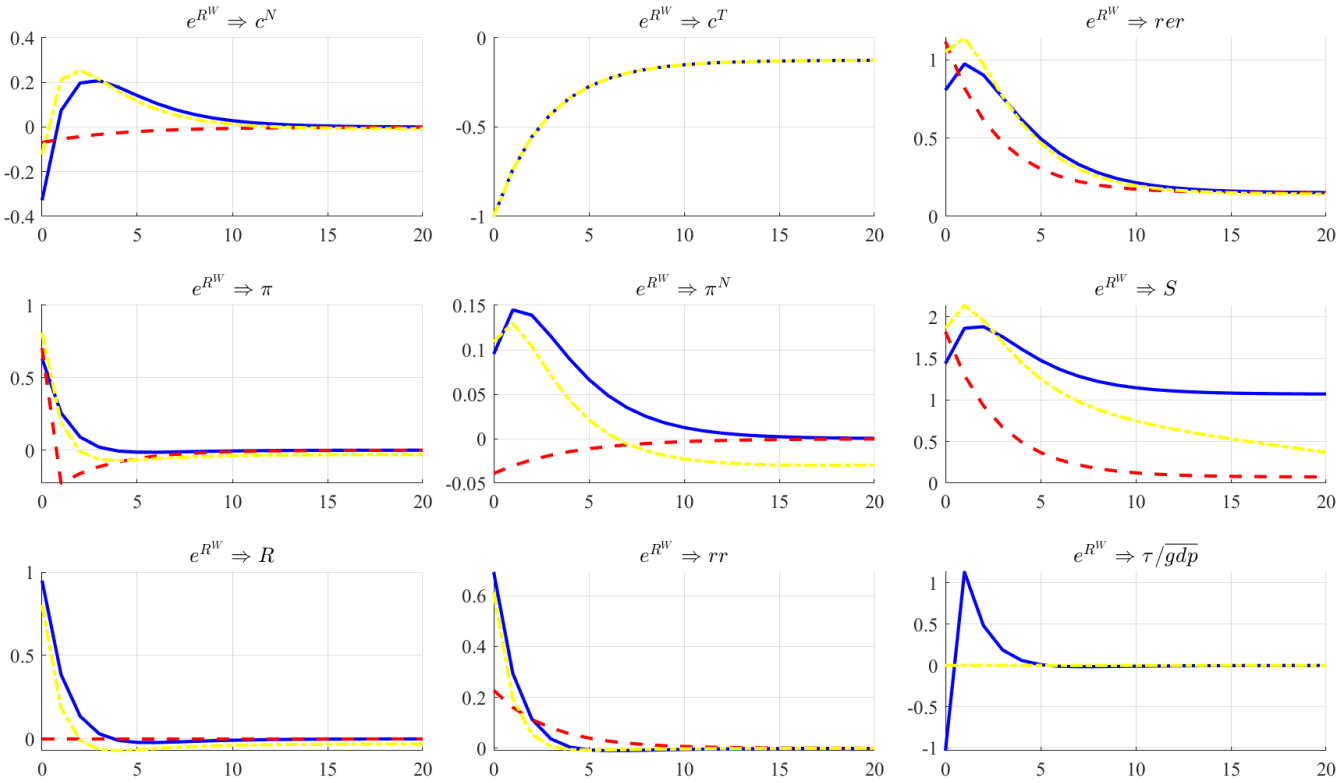
[14] One can numerically verify that this happens whenever $\overline{\Omega}$ is larger than a threshold analogous to $\overline{\Omega}_{\text{cut},1}$, which equals 0.945 in this case, as we previously mentioned.

tions do not qualitatively alter the propagation of shocks to y^T; this is not the case if a fraction of the debt is denominated in foreign currency. In particular, a positive shock might lead to a contraction in the non-trade sector, even though the real exchange rate is appreciating. This happens because, here, the appreciation is not mainly led by a larger demand in non-tradables, but it is produced by the FTPL channel that requires non-traded prices to fall, which materializes via a lower non-traded demand.[15]

6.2 World interest rate

We now turn to the analysis of the consequences generated by an increase in the world interest rate R^W. Figure 9 displays the dynamics of a shock with a persistence of 0.7, normalized to drop c^T by 1% on impact, under the assumption of $\overline{\Omega} = 0$. The flexible-price mechanism behind the propagation works as follows. The increase in the international cost of borrowing induces an intertemporal substitution effect and a negative wealth effect (as the country is a net-foreign borrower in our calibration), both of which reduce the consumption of both types of goods. In particular, the contraction in the demand for c^N reduces its relative price, leading to a real depreciation. In a world with sticky prices, the final outcome hinges on the fiscal and monetary configuration.

Figure 9: Responses to a world interest-rate shock, Ricardian vs. Non-Ricardian.



Notes: The figure show the responses to a positive shock to R^W, with an autocorrelation of 0.7, normalized to decrease c^T by 1% at the moment the shock hits. All cases feature $\overline{\Omega} = 0$. In solid blue lines $\phi_T = 1$ and $\phi_\pi = 1.5$. in dashed-red lines $\phi_T = 0$ and $\phi_\pi = 0$, while in dashed-dotted yellow lines $\phi_T = 0$ and $\phi_\pi = 0.99$. See Figure 2 for variables' definitions.

The solid-blue lines in Figure 9 correspond to the passive-fiscal, active-monetary setup ($\phi_T = 1$

[15] As shown in Appendix E.2, having a policy rate that reacts to inflation with $\phi_\pi = 0.99$ only magnifies these differences, as the drop in the policy rate in such a case generates a path for the nominal exchange rate that appreciates even further, which requires an even larger and more persistent fall in π^N (and thus c^N).

and $\phi_\pi = 1.5$). We can see that c^N falls on impact, and the real depreciation materializes through the nominal exchange rate. We can also see that c^N rises above its steady state level in the periods following the realization of the shock, which also explains why non-trade inflation increases further after a few periods. This occurs because there are two effects at play. On one hand, the homotheticity of preferences would imply a fall in c^N following the contraction in c^T; on the other hand, the increase in *rer* makes non-tradables relatively cheaper. Thus, initially, the former dominates, while the latter is more relevant in the following periods.

Another complementary view of this same effect comes from the dynamics of the domestic real rate *rr*. While it increases following the shock, it returns to the steady state at a faster rate than the world interest rate R^W.[16] In turn, this implies that the real exchange rate (via the UIP condition) also takes longer to converge back to the steady state after the initial jump. As this relatively more depreciated path implies cheaper non-traded goods, c^N increases after the initial drop, also increasing π^N as firms anticipate this future higher demand in the presence of Calvo frictions.

The dashed-red lines in Figure 9 correspond to the case of Non-Ricardian fiscal policy ($\phi_T = 0$) and a constant policy rate ($\phi_\pi = 0$). In that case, notice that c^N converges from below to the steady state, which shows how the previously analyzed dynamics (blue lines) in the periods following the shock were heavily influenced by the path of the policy rate. This, in turn, induces a fall in π^N, while the nominal exchange rate falls monotonically after the initial period.

From the FTPL perspective, this initial jump in the nominal exchange rate materializes to compensate for the fact that the increase in R^W puts upward pressure on the relevant discount rate. As lump-sum taxes are fixed, the net present value of primary surpluses drops, requiring a jump in the price level to compensate on the left-hand side of (8). Overall, the dynamics under this alternative configuration are quite different.

However, as we saw in the analysis of a y^T shock, these differences are mainly generated by the assumption of a constant policy rate. Indeed, the dashed-dotted yellow lines in Figure 9 illustrate the case of an active fiscal policy ($\phi_T = 0$) but with a monetary-policy rate sensitive to inflation ($\phi_\pi = 0.99$). We can see that the dynamics are much closer to the case of a Ricardian fiscal policy in the solid-blue lines.

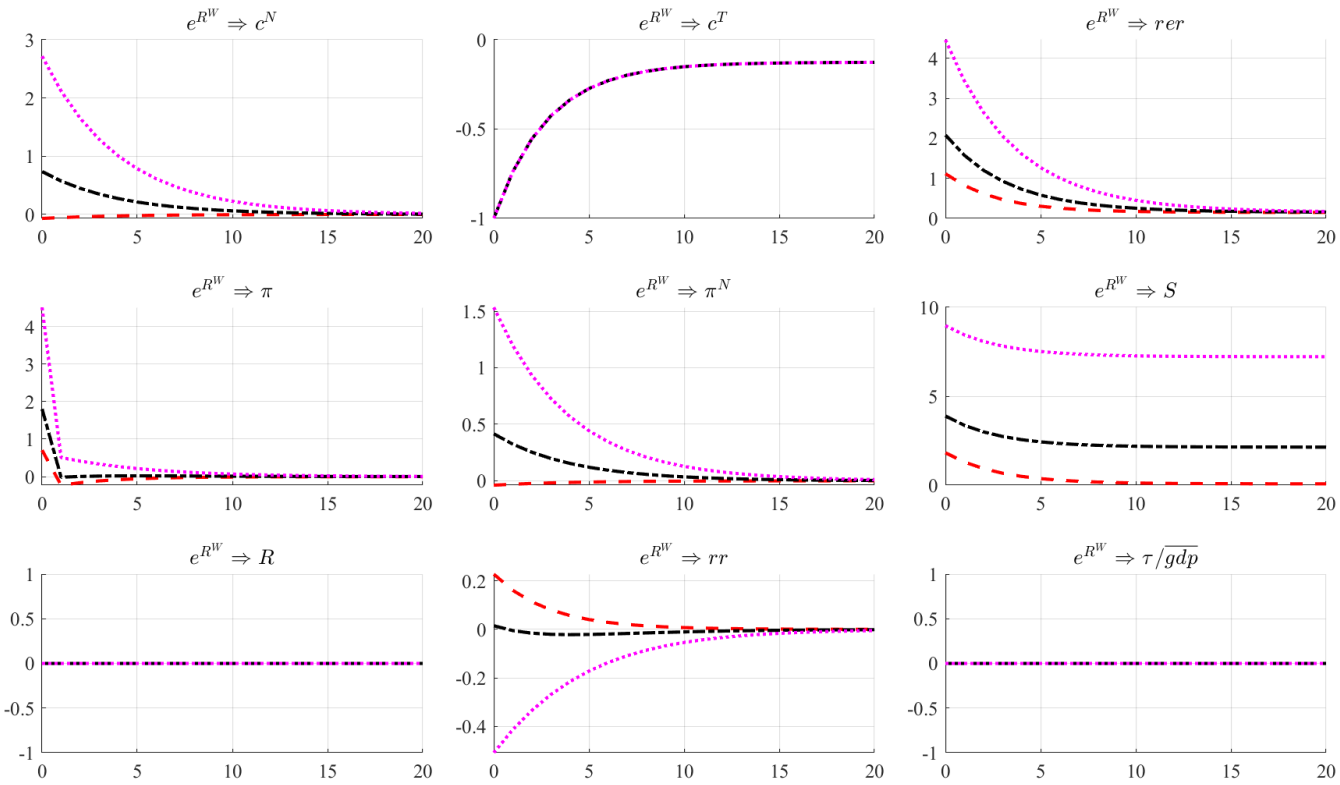
Figure 10 displays the dynamics under the active-fiscal, passive-monetary setup ($\phi_\pi = 0 = \phi_T = 0$), for different values of the share of government debt denominated in dollars, $\overline{\Omega}$.[17] As the shock induces a real depreciation, it increases the real value (in domestic units) of outstanding dollar-denominated debt. In order to compensate, either the price level should increase to dilute the burden of debt obligations in pesos, or the real rate needs to fall to increase the net-present value of primary surpluses (which is brought about by higher expected inflation), or a combination of both. As a result, the shock is more inflationary for higher ratios $\overline{\Omega}$. In turn, non-traded consumption expands for larger shares of dollar-denominated debt: while the fall in c^T pushes pressure to contract c^N, the real depreciation compensates; and the latter dominates as a higher $\overline{\Omega}$ induces a larger increase in *rer*.

The case of $\overline{\Omega} = 1$ is included in Appendix E.2. As expected from our previous analysis, the responses flip signs in this case, particularly inducing a real appreciation and a contraction in non-traded consumption. The effect on *rer* follows from the same arguments presented for the shock to y^T: the increase in the interest rate reduces the net present value of primary surpluses, which requires a real appreciation for the valuation equation (8) to hold. W

[16] While that evolution of R^W is not shown in the figure, its autocorrelation of 0.7 implies that, for instance, five periods after the shock hits, it is still at a value of 0.08 above the steady state, whereas *rr* is virtually zero at that point.

[17] Appendix E.2 also includes a case with $\phi_T = 0$ and $\phi_\pi = 0.99$, showing more exacerbated differences relative to Figure 10.

Figure 10: Responses to a world interest-rate shock, Non-Ricardian, different values of $\overline{\Omega}$.



Notes: The figure is analogous to Figure 9, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for $\overline{\Omega}$: In dashed-red $\overline{\Omega} = 0$, in dashed-dotted black $\overline{\Omega} = 0.5$, in dotted magenta $\overline{\Omega} = 0.75$.

Overall, we see that the dynamics induced by a shock to the world interest rate are also significantly different when fiscal policy is active. Moreover, the currency composition of the debt is key to determining the results.

7 Extension

TBA

8 Conclusions

TBA

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Supplementary Appendix

A Transversality conditions

Define $rr_{t,t+J}$ such that $rr_{t,t} \equiv 1$ and $rr_{t,t+J} \equiv \prod_{j=0}^{J-1} \frac{R_{t+j}}{\pi_{t+1+j}}$ for $J \geq 1$. Under perfect foresight, this would be the cumulative (up to period J) ex-ante real rate, from the perspective of time t. In a stochastic world, this is the cumulative rate for a given history of realizations from t to $t+J$.

Let household's real assets be $b_t \equiv B_t/P_t$ and $d_t^{H*} \equiv D_t^{H*}/P_t^*$. We assume they face two No-Ponzi-game conditions (NPGC) for each financial asset,

$$\lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} rer_{t+J} \frac{d_{t-1+J}^{H*}}{\pi_{t+J}^*} \right\} \leq 0, \quad \lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} \frac{b_{t-1+J}}{\pi_{t+J}} \right\} \geq 0. \tag{A.1}$$

The discussion below clarifies why we impose one for each asset an instead of one for total financial wealth.

In real terms, the household's budget constraint (1) is

$$c_t + rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} + \frac{b_t}{R_t} + \tau_t = w_t h_t + \sigma_t + \frac{b_{t-1}}{\pi_t} + rer_t \frac{d_t^{H*}}{R_t^*}, \tag{A.2}$$

where $\sigma_t \equiv \Sigma_t/P_t$. Defining $sav_t \equiv w_t h_t + \sigma_t - c_t - \tau_t$, the previous can be written as,

$$rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t} = rer_t \frac{d_t^{H*}}{R_t^*} - \frac{b_t}{R_t} + sav_t.$$

From the real exchange rate's definition, $rer_t = rer_{t+1} \frac{\pi_{t+1}}{\pi_{t+1}^S \pi_{t+1}^*}$. Replacing this on the first-term on the right-hand side, and then multiplying and dividing by π_{t+1} the second-term, yields

$$rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t} = \frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*} - \frac{\pi_{t+1}}{R_t} \frac{b_t}{\pi_{t+1}} + sav_t,$$

Adding and subtracting $\frac{\pi_{t+1}}{R_t} rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*}$ on the right hand side, we get

$$rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t} = \frac{\pi_{t+1}}{R_t} \left(rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*} - \frac{b_t}{\pi_{t+1}} \right) + sav_t + \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{\pi_{t+1}}{R_t} \right) rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*}$$

Let $so_{t+1}^H \equiv \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{\pi_{t+1}}{R_t} \right) rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*}$. The expectation $E_t\{so_{t+j}^H\}$ captures expected valuation changes due to deviations from perfect-foresight (or complete markets in a stochastic setting) uncovered interest rate parity which, in a non-linear stochastic model, generally does not hold because of covariance/premium terms. Notice also that this term arises only because we have more than one non-contingent asset and financial markets are incomplete; otherwise perfect-foresight non-arbitrage relationships would hold and $so_{t+1} = 0$. However, this term is zero up to a first order of approximation, for in the non-stochastic steady state valuation effects are nil.

Let $a_t^H \equiv rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t}$. Thus, we can write

$$a_t^H = \frac{a_{t+1}^H}{rr_{t,t+1}} + sav_t + so_{t+1}^H.$$

Replacing forward a_{t+1}^H yields

$$a_t^H = \frac{1}{rr_{t,t+1}}\left(\frac{a_{t+2}^H}{rr_{t+1,t+2}} + sav_{t+1} + so_{t+2}^H\right) + sav_t + so_{t+1}^H = \frac{a_{t+1}^H}{rr_{t,t+2}} + \sum_{j=0}^{1}\frac{\left(sav_{t+j} + so_{t+1+j}^H\right)}{rr_{t,t+j}},$$

where we have used the property $rr_{t,t+J} = rr_{t,t+N} \cdot rr_{t+N+1,t+J}$ for $0 \leq N < J$. Continuing to iterate forward up to an arbitrary period J, we get

$$a_t^H = \frac{a_{t+J}^H}{rr_{t,t+J}} + \sum_{j=0}^{J-1}\frac{\left(sav_{t+j} + so_{t+1+j}^H\right)}{rr_{t,t+j}}$$

Finally, applying expectations conditional on time t's information set on both sides and then taking the limit for J as it approaches infinity,

$$a_t^H = \lim_{J\to\infty} E_t\left\{\frac{a_{t+J}^H}{rr_{t,t+J}}\right\} + \sum_{j=0}^{\infty} E_t\left\{\frac{\left(sav_{t+j} + so_{t+1+j}^H\right)}{rr_{t,t+j}}\right\}$$

Recall that $a_t^H = rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t}$. Given the NPGC (A.1), the household optimal plan needs to satisfy the transversality condition

$$\lim_{J\to\infty} E_t\left\{\frac{1}{rr_{t,t+J}}\left(rer_{t+J}\frac{d_{t-1+J}^{H*}}{\pi_{t+J}^*} - \frac{b_{t-1+J}}{\pi_{t+J}}\right)\right\} = 0, \tag{A.3}$$

otherwise, if this limit were negative, consumption could be increased in every period, thereby increasing utility. Thus, it is not optimal to choose a plan in which this limit in (A.3) is not equal to zero. For this condition to hold, and given that the NPGC are imposed *individually* for each asset, both of the following transversality conditions need to hold:

$$\lim_{J\to\infty} E_t\left\{\frac{1}{rr_{t,t+J}} rer_{t+J}\frac{d_{t-1+J}^{H*}}{\pi_{t+J}^*}\right\} = 0, \quad \lim_{J\to\infty} E_t\left\{\frac{1}{rr_{t,t+J}}\frac{b_{t-1+J}}{\pi_{t+J}}\right\} = 0. \tag{A.4}$$

Regarding the government, its period t resource constraint in real terms is,

$$\frac{d_{t-1}^G}{\pi_t} + rer_t\frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t\frac{d_t^{G*}}{R_t^*} + sp_t, \tag{A.5}$$

where sp_t denotes the real primary surplus (equal simply to τ_t). Following similar steps as in the household case, this can be written as

$$\frac{d_{t-1}^G}{\pi_t} + rer_t\frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{\pi_{t+1}}{R_t}\frac{d_t^G}{\pi_{t+1}} + \frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} rer_{t+1}\frac{d_t^{G*}}{\pi_{t+1}^*} + sp_t$$

Adding and subtracting $\frac{1}{rr_{t,t+1}} rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*}$ on the right hand side, we get

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{1}{rr_{t,t+1}} \left(\frac{d_t^G}{\pi_{t+1}} + rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*} \right) + sp_t + \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{1}{rr_{t,t+1}} \right) rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*}$$

Let $so_{t+1}^G \equiv \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{1}{rr_{t,t+1}} \right) rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*}$. Similar to so_t^H, this represents valuation effects due to market incompleteness. Defining also, $a_t^G \equiv \frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*}$, we get

$$a_t^G = \frac{1}{rr_{t,t+1}} a_{t+1}^G + so_{t+1}^G + sp_t$$

Replacing forward a_{t+1}^G up until arbitrary period J we get

$$a_t^G = \frac{a_{t+J}^G}{rr_{t,t+J}} + \sum_{j=0}^{J-1} \frac{\left(sp_{t+j} + so_{t+1+j}^G \right)}{rr_{t,t+j}}$$

Finally, applying expectations conditional on information at time t on both sides, and then taking the limit for J up to infinity,

$$a_t^G = \lim_{J \to \infty} E_t \left\{ \frac{a_{t+J}^G}{rr_{t,t+J}} \right\} + \sum_{j=0}^{\infty} E_t \left\{ \frac{\left(sp_{t+j} + so_{t+1+j}^G \right)}{rr_{t,t+j}} \right\}$$

It follows that we need to impose the condition $\lim_{J \to \infty} E_t \left\{ \frac{a_{t+J}^G}{rr_{t,t+J}} \right\} = 0$ which, from the definition of a_t^G, is equivalent to

$$\lim_{J \to \infty} E_t \left\{ \frac{1}{rr_{t,t+J}} \left(\frac{d_{t+J-1}^G}{\pi_{t+J}} + rer_{t+J} \frac{d_{t-1+J}^{G*}}{\pi_{t+J}^*} \right) \right\} = 0. \tag{A.6}$$

While the household's TVC (A.4) follows from optimization, as we have discussed, the government's TVC does not. For that reason, the transversality condition on government debt is part of the controversy surrounding FTPL in the literature.

In closed economy models with representative agents, the transversality condition for the government is not an extra requirement, for it coincides with the household's transversality condition, which is an optimality condition. Here, as in equilibrium $b_t = d_t^H$ (assuming that only domestic households hold peso bonds), the part of (A.6) corresponding to the debt in pesos is equivalent to the household's TVC for its holding of domestic assets (i.e., the second equation in (A.4)), which is again an optimality condition given our assumptions.

However, if in an open economy we allow *both* domestic and foreign agents to hold government bonds, the household's TVC related to foreign debt is not the same as that of the government's. This is discussed by earlier open-economy contributions such as Dupor (2000) and Daniel (2001). In particular, notice that a government not satisfying the TVC, but which is still facing an NPGC condition (which is natural to assume), is wasting resources that could be used to either finance additional spending or reduce taxes at some point. Therefore, welfare could be improved if policy is constrained to satisfy the transversality condition with equality. As such, Daniel (2001) introduces the concept of "no-surplus fiscal policy" to describe schemes in which governments in a multi-country

model do not waste resources in this manner.

In our case, assuming that

$$\lim_{J\to\infty} E_t\left\{\frac{1}{rr_{t,t+J}} rer_{t+J}\frac{d^{G*}_{t-1+J}}{\pi^*_{t+J}}\right\} = 0, \tag{A.7}$$

in tandem with the household's TVC in pesos, as previously discussed, it is implied that we focus on "no-surplus fiscal policies."

After imposing these transversality conditions, the lifetime government budget constraint is

$$\frac{d^G_{t-1}}{\pi_t} + rer_t\frac{d^{G*}_{t-1}}{\pi^*_t} = \sum_{j=0}^{\infty} E_t\left\{\frac{\left(sp_{t+j} + so^G_{t+1+j}\right)}{rr_{t,t+j}}\right\}$$

Using the definition of the share of government debt in dollars Ω_t in (5), plus the constant-currency composition rule ($\Omega_t = \overline{\Omega}$), we obtain

$$\left[\frac{1-\overline{\Omega}}{\pi_t} + \frac{rer_t}{\overline{rer}}\frac{\overline{\Omega}}{\pi^*_t}\right] d_{t-1} = d_{t-1} = E_t\left\{\sum_{j=0}^{\infty}\frac{sp_{t+j}}{rr_{t,t+j}}\right\} + h.o.t.,$$

where $d_t \equiv d^G_t + \overline{rer}d^{G*}_t$, and also $h.o.t. \equiv \sum_{j=0}^{\infty} E_t\left\{so^G_{t+1+j}/rr_{t,t+j}\right\}$. This is equation (8) in the text.

Finally, it is important to note that the transversality condition for government debt in dollars (A.7) is not only relevant for describing the government problem but also for guaranteeing that the balance of payments is sustainable. Consolidating the household and government's budget constraints (A.2) and (A.5), using the market clearing condition for non-traded goods, the following represents the balance of payments in this model, expressed in domestic-consumption units,

$$rer_t\frac{d^*_{t-1}}{\pi^*_t} = rer_t\frac{d^*_t}{R^*_t} + tb_t$$

where $d^*_t \equiv d^{G*}_t + d^{H*}_t$ is the countries' net-foreign lending position, and $tb_t \equiv rer_t(y^T_t - c^T_t)$ is the trade balance. As before, this can be written as

$$rer_t\frac{d^*_{t-1}}{\pi^*_t} = \frac{\pi_{t+1}}{\pi^S_{t+1}R^*_t}rer_{t+1}\frac{d^*_t}{\pi^*_{t+1}} + tb_t$$

or,

$$rer_t\frac{d^*_{t-1}}{\pi^*_t} = \frac{1}{rr_{t,t+1}}rer_{t+1}\frac{d^*_t}{\pi^*_{t+1}} + \left(\frac{\pi_{t+1}}{\pi^S_{t+1}R^*_t} - \frac{1}{rr_{t,t+1}}\right)rer_{t+1}\frac{d^*_t}{\pi^*_{t+1}} + tb_t$$

Defining $so_{t+1} \equiv \left(\frac{\pi_{t+1}}{\pi^S_{t+1}R^*_t} - \frac{1}{rr_{t,t+1}}\right)rer_{t+1}\frac{d^*_t}{\pi^*_{t+1}}$, iterating forward up to the period J, applying expectations conditional on information at time t on both sides, and taking the limit for J as it approaches infinity, we get

$$rer_t\frac{d^*_{t-1}}{\pi^*_t} = \lim_{J\to\infty} E_t\left\{\frac{1}{rr_{t,t+J}}rer_{t+J}\frac{d^*_{t-1+J}}{\pi^*_{t+J}}\right\} + \sum_{j=0}^{\infty} E_t\left\{\frac{(tb_{t+j} + so_{t+1+j})}{rr_{t,t+J}}\right\}$$

It follows that for the balance of payments to be sustainable, we need,

$$\lim_{J\to\infty} E_t\left\{\frac{1}{rr_{t,t+J}} rer_{t+J}\frac{d^*_{t-1+J}}{\pi^*_{t+J}}\right\}=0,$$

Therefore, as $d^*_t \equiv d^{G*}_t + d^{H*}_t$, the households optimal TVC for dollar assets, along with the assumption of the "no-surplus" policy previously discussed, is equivalent to requiring the sustainability of the balance of payments'. In other words, in the open economy, arguing about whether the TVC condition for the government's debt needs to hold is equivalent to discussing the balance-of-payment's sustainability.

Still, it should be highlighted that this discussion is more pertinent to a **global** equilibrium analysis, which previous open-economy papers have only tackled in deterministic models. Instead, in this paper, as we approximate the solution by linearization around a non-stochastic steady state, the **local** existence and uniqueness requirements implicitly assume that these transversality conditions hold. Moreover, the valuation terms previously described vanish up to a first order of approximation. Future research could be devoted to studying the requirements for global existence and uniqueness in stochastic small open economy models.

B Equilibrium characterization

B.1 Production of non-tradable varieties and Calvo

A monopolist producing the variety j uses labor according to $y^N_{jt} = (h_{jt})^\alpha$, with $\alpha \in (0,1]$. It internalizes the demand for j (obtained from the maximization of the competitive firms aggregating non-trade goods), given by

$$y^N_{jt} = \left(\frac{P^N_{jt}}{P^N_t}\right)^{-\epsilon_N} y^N_t, \text{ for all } j.$$

In addition, it faces a Calvo problem in choosing its price P^N_{jt}: with probability θ_N, it is forced to set $P^N_{jt} = P^N_{jt-1}$, while with probability $1-\theta_N$, it can freely choose a price $\widetilde{P}^N_{jt}$. Using well-known aggregation results (e.g. Schmitt-Grohé and Uribe, 2017, Ch. 9.16), in particular that all firms able to choose set the same price $\widetilde{P}^N_t$, the following characterize the dynamics of non-traded prices,

$$p^N_t mc^N_t = \frac{w_t}{\alpha}(y^N_t)^{\left(\frac{1}{\alpha}-1\right)}, \tag{B.1}$$

$$f_t = (\widetilde{p}^N_t)^{1-\epsilon_N} y^N_t \frac{(\epsilon_N-1)}{\epsilon_N} + \theta_N E_t\left\{\chi_{t,t+1}\left(\frac{\widetilde{p}^N_t}{\widetilde{p}^N_{t+1}}\right)^{1-\epsilon_N}(\pi^N_{t+1})^{\epsilon_N} f_{t+1}\right\}, \tag{B.2}$$

$$f_t = (\widetilde{p}^N_t)^{-\epsilon_N} y^N_t mc^N_t + \theta_N E_t\left\{\chi_{t,t+1}\left(\frac{\widetilde{p}^N_t}{\widetilde{p}^N_{t+1}}\right)^{-\epsilon_N}(\pi^N_{t+1})^{1+\epsilon_N} f_{t+1}\right\}, \tag{B.3}$$

$$1 = \theta_N(\pi^N_t)^{\epsilon_N-1} + (1-\theta_N)(\widetilde{p}^N_t)^{1-\epsilon_N}. \tag{B.4}$$

Equation (B.1) equates marginal costs to the ratio between wages and the marginal product of labor, where mc^N_t is the real marginal cost in non-traded units. Equations (B.2)-(B.3) provide a recursive representation to the optimal choice of price $\widetilde{P}^N_t$ for those allowed to chose (where $\widetilde{p}^N_t \equiv \widetilde{P}^N_t / P^N_t$), equating the net present value of marginal revenues in those states in which the price $\widetilde{P}^N_t$ still holds

(called f_t in this notation) to that of marginal costs.[18] The last condition (B.4) relates the optimal price chosen by those allowed to do so and non-traded inflation $\pi_t^N \equiv P_t^N / P_{t-1}^N$.

These equations can be log-linearized around the zero-inflation steady state (see Schmitt-Grohé and Uribe, 2017, Ch. 9.16) to obtain,

$$\widehat{\pi}_t^N = \beta E_t\{\widehat{\pi}_{t+1}^N\} + \kappa \cdot \widehat{mc}_t^N,$$

where $\widehat{\cdot}$ denotes log-linear approximation, and $\kappa = \frac{(1-\theta_N)(1-\beta\theta_N)}{\theta_N\left(\frac{\epsilon_N}{\alpha}+1-\epsilon_N\right)}$.

B.2 Equilibrium conditions

Besides the transversality conditions in Appendix A, the following characterize the equilibrium,

$$w_t (c_t)^{-\sigma} = \chi (h_t)^{\varphi}, \tag{Eq.1}$$

$$(c_t)^{-\sigma} = \beta R_t E_t \left\{ \frac{(c_{t+1})^{-\sigma}}{\pi_{t+1}} \right\}, \tag{Eq.2}$$

$$(c_t)^{-\sigma} = \beta R_t^* E_t \left\{ \frac{(c_{t+1})^{-\sigma} \pi_{t+1}^S}{\pi_{t+1}} \right\}, \tag{Eq.3}$$

$$c_t = \left[\omega^{1/\eta} \left(c_t^N\right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(c_t^T\right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}}, \tag{Eq.4}$$

$$c_t^N = \omega \left(p_t^N\right)^{-\eta} c_t. \tag{Eq.5}$$

$$c_t^T = (1-\omega) \left(rer_t\right)^{-\eta} c_t, \tag{Eq.6}$$

$$p_t^N mc_t^N = \frac{w_t}{\alpha} (y_t^N)^{\left(\frac{1}{\alpha}-1\right)}, \tag{Eq.7}$$

$$f_t = (\widetilde{p}_t^N)^{1-\epsilon_N} y_t^N \frac{(\epsilon_N - 1)}{\epsilon_N} + \theta_N E_t \left\{ \beta \frac{(c_{t+1})^{-\sigma}}{(c_t)^{-\sigma}\pi_{t+1}} \left(\frac{\widetilde{p}_t^N}{\widetilde{p}_{t+1}^N} \right)^{1-\epsilon_N} (\pi_{t+1}^N)^{\epsilon_N} f_{t+1} \right\}, \tag{Eq.8}$$

$$f_t = (\widetilde{p}_t^N)^{-\epsilon_N} y_t^N mc_t^N + \theta_N E_t \left\{ \beta \frac{(c_{t+1})^{-\sigma}}{(c_t)^{-\sigma}\pi_{t+1}} \left(\frac{\widetilde{p}_t^N}{\widetilde{p}_{t+1}^N} \right)^{-\epsilon_N} (\pi_{t+1}^N)^{1+\epsilon_N} f_{t+1} \right\}, \tag{Eq.9}$$

$$1 = \theta_N (\pi_t^N)^{\epsilon_N - 1} + (1-\theta_N)(\widetilde{p}_t^N)^{1-\epsilon_N}. \tag{Eq.10}$$

$$R_t^* = R_t^W \exp\left\{\psi \left(d_t^* - \bar{d}^*\right)\right\}, \tag{Eq.11}$$

$$d_t^* = d_t^{*G} + d_t^{*H}, \tag{Eq.12}$$

$$y_t^N = (h_t)^{\alpha}, \tag{Eq.13}$$

$$y_t^N = (\Delta_t)^{\alpha} c_t^N, \tag{Eq.14}$$

$$\Delta_t = \theta_N \Delta_t (\pi_t^N)^{\frac{\epsilon_N}{\alpha}} + (1-\theta_N)(\widetilde{p}_t^N)^{-\frac{\epsilon_N}{\alpha}}, \tag{Eq.15}$$

$$q_t = rer_t / p_t^N, \tag{Eq.16}$$

[18] In these, $\chi_{t,t+h} \equiv \beta^h \frac{(c_{t+h})^{-\sigma} P_t}{(c_t)^{-\sigma} P_{t+h}}$ is the households' stochastic discount factor for nominal claims.

$$rer_t = rer_{t-1}\frac{\pi_t^S \pi_t^*}{\pi_t}, \tag{Eq.17}$$

$$p_t^N = p_{t-1}^N \frac{\pi_t^N}{\pi_t}, \tag{Eq.18}$$

$$\frac{d_{t-1}^*}{\pi_t^*} = \frac{d_t^*}{R_t^*} + (y_t^T - c_t^T). \tag{Eq.19}$$

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t \frac{d_t^{G*}}{R_t^*} + \tau_t, \quad \text{or} \quad \frac{d_{t-1}^G}{\pi_t^N} + q_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + q_t \frac{d_t^{G*}}{R_t^*} + \tau_t, \tag{Eq.20}$$

$$\tau_t - \overline{\tau} = \phi_T \left[\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{\tau}}{1-\beta} \right] + u_t^\tau, \quad \text{or} \quad \tau_t - \overline{\tau} = \phi_T \left[\frac{d_{t-1}^G}{\pi_t^N} + q_t \frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{\tau}}{1-\beta} \right] + u_t^\tau, \tag{Eq.21}$$

$$\overline{\Omega} = \frac{\overline{rer} d_t^{G*}}{d_t^G + \overline{rer} d_t^{G*}}, \quad \text{or} \quad \overline{\Omega} = \frac{\overline{q} d_t^{G*}}{d_t^G + \overline{q} d_t^{G*}}, \tag{Eq.22}$$

$$\left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} u_t^R, \quad \text{or} \quad \left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t^N}{\overline{\pi}^N}\right)^{\phi_\pi} u_t^R, \tag{Eq.23}$$

All variables were described previously, except for Δ_t, which captures the potential inefficiency induced by price dispersion in the presence of sticky prices. However, as we will assume zero steady-state inflation, it is not relevant up to first order.

Overall, we have 2 endogenous variables:

$$\begin{array}{cccccccccccc} w_t & c_t & h_t & R_t & \pi_t & R_t^* & \pi_t^S & c_t^N & c_t^T & p_t & p_t^T & rer_t \\ mc_t^N & y_t^N & f_t & \widetilde{p}_t^N & \pi_t^N & d_t^G & d_t^{G*} & \tau_t & d_t^* & d_t^{*H} & \Delta_t & \end{array}$$

and 5 exogenous variables:

$$y_t^T \quad \pi_t^* \quad R_t^W \quad u_t^\tau \quad u_t^R$$

For $x_t = \{y_t^T, \pi_t^*, R_t^W, u_t^R\}$ we assume the following AR(1) process in logs

$$\log\left(\frac{x_t}{\overline{x}}\right) = \rho_x \log\left(\frac{x_{t-1}}{\overline{x}}\right) + \sigma_x \varepsilon_t^x,$$

where ε_t^x is an i.i.d. shock, $\rho_x \in [0,1)$ and $\sigma_x > 0$. Instead, for u_t^τ we assume

$$\frac{(u_t^\tau - \overline{u}^\tau)}{\overline{gdp}} = \rho_\tau \frac{(u_{t-1}^\tau - \overline{u}^\tau)}{\overline{gdp}} + \sigma_\tau \varepsilon_t^\tau,$$

where $\overline{gdp} = \overline{y}^N + \overline{y}^T$ is real GDP in steady state, $\rho_\tau \in [0,1)$, and $\sigma_\tau > 0$.

B.2.1 Tradable block

Combining equations (Eq.3), (Eq.6), (Eq.16) and (Eq.17) we get

$$(c_t^T)^{-\sigma} (rer_t)^{-\sigma\eta} = \beta R_t^* E_t \left\{ \frac{(c_{t+1}^T)^{-\sigma} (rer_{t+1})^{-\sigma\eta} rer_{t+1}}{\pi_{t+1}^* rer_t} \right\},$$

If, in addition, we assume $\eta\sigma = 1$, this yields

$$(c_t^T)^{-\sigma} = \beta R_t^* E_t \left\{ \frac{(c_{t+1}^T)^{-\sigma}}{\pi_{t+1}^*} \right\}, \tag{Eq.24}$$

Equations (Eq.24), (Eq.11), and (Eq.19) form a system for the endogenous variables c_t^T, d_t^*, and R_t^*, which can be solved as a function of the exogenous variables π_t^*, R_t^W, y_t^T alone. Thus, in this setup, the tradable block is isolated from non-tradables, as well as from monetary and fiscal policies. The intuitive reason for this result is that any effect on the real exchange rate (rer_t), originating from exogenous variables different from π_t^*, R_t^W, y_t^T, induces both intra- and inter-temporal substitution effects. By the former, an increase in rer_t produces (*ceteris paribus*) a desire to substitute away from tradables, as these become more expensive; an effect determined by the intra-temporal elasticity of substitution η. By the latter, a temporary increase in rer_t produces (*ceteris paribus*) a fall in the real return of saving in tradables, increasing desired tradable consumption today (an effect determined by the intra-temporal substitution elasticity $1/\sigma$). Thus, if $\eta\sigma = 1$ the two offset each other.

C Model under the non-traded policy setup

This section describes the simplified version of the model used to obtain algebraic results. The first part derives the log-linearized version and its minimalist representation. The second discusses the conditions for the (local) existence and uniqueness of a stationary equilibrium. The third characterizes the solution and shows how both policy shocks affect the economy and how those results depend on the fiscal/monetary policy setup.

Relative to the general model with equilibrium conditions listed in Appendix B.2, these are the additional simplifying assumptions:

- Equal inter- and intra-temporal elasticity of substitution: $\eta = 1/\sigma$.
- Zero inflation in steady state: $\overline{\pi} = \overline{\pi}^N = \overline{\pi}^* = \overline{\pi}^S = 1$.
- Lump-sum transfers are set in terms of non-traded goods: $\tau_t \equiv T_t / P_t^N$.
- Constant currency composition of government debt: $\overline{\Omega} = \overline{q} d_t^{G*} / (d_t^G + \overline{q} d_t^{G*})$.
- Taylor rule in terms of non-trade inflation: $\left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t^N}{\overline{\pi}^N}\right)^{\phi_\pi} u_t^R$.

C.1 Log-linearization

Recall from section B.2.1, c_t^T, d_t^*, and R_t^* are a function of the exogenous variables π_t^*, R_t^W, y_t^T alone. Thus, in what follows, we treat them as given in accounting for the dynamics of the other variables.

C.1.1 Fiscal and monetary policy

Assuming a rule for real lump-sum transfers defined as $\tau_t \equiv T_t / P_t^N$, the government budget constraint in terms of non-tradables is

$$\frac{d_{t-1}^G}{\pi_t^N} + q_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + q_t \frac{d_t^{G*}}{R_t^*} + \tau_t,$$

where $d_t^G \equiv D_t^G / P_t^N$ and $d_t^{G*} \equiv D_t^{G*} / P_t^*$. Considering the rule for the currency composition of debt.

$$\overline{\Omega} = \frac{\overline{q} d_t^{G*}}{d_t^G + \overline{q} d_t^{G*}},$$

and defining $d_t \equiv d_t^G + \overline{q} d_t^{G*}$, the budget constraint is

$$\left[\frac{(1-\overline{\Omega})}{\pi_t^N} + \frac{\overline{\Omega}}{\pi_t^*} \frac{q_t}{\overline{q}} \right] d_{t-1} = \left[\frac{(1-\overline{\Omega})}{R_t} + \frac{\overline{\Omega}}{R_t^*} \frac{q_t}{\overline{q}} \right] d_t + \tau_t$$

To log-linearize, define $A_t \equiv \left[\frac{(1-\overline{\Omega})}{\pi_t^N} + \frac{q_t}{\overline{q}} \frac{\overline{\Omega}}{\pi_t^*} \right]$, $B_t \equiv \left[\frac{(1-\overline{\Omega})}{R_t} + \frac{q_t}{\overline{q}} \frac{\overline{\Omega}}{R_t^*} \right]$, $\widehat{A}_t \equiv \log(A_t/\overline{A})$, and $\widehat{B}_t \equiv \log(B_t/\overline{B})$. Also, as we focus on cases with $\overline{d}, \overline{\tau} > 0$, let $\widehat{d}_t \equiv \log(d_t/\overline{d})$ and $\widehat{\tau}_t \equiv \log(\tau_t/\overline{\tau})$. Thus, applying the change of variables, the constraint is

$$\overline{A} e^{\widehat{A}_t} \overline{d} e^{\widehat{d}_{t-1}} = \overline{B} e^{\widehat{B}_t} \overline{d} e^{\widehat{d}_t} + \overline{\tau} e^{\widehat{\tau}_t}.$$

Taking a first-order approximation

$$\overline{A}\overline{d}(\widehat{A}_t + \widehat{d}_{t-1}) = \overline{B}\overline{d}(\widehat{B}_t + \widehat{d}_t) + \overline{\tau}\widehat{\tau}_t.$$

In a steady state with $\overline{\pi} = \overline{\pi}^* = \overline{\pi}^S = 1$, then $R = R^* = 1/\beta$, so we have

$$\overline{A} = 1, \quad \overline{B} = \beta, \quad \overline{d}(1-\beta) = \overline{\tau}.$$

Therefore, the first-order approximation simplifies to

$$\widehat{A}_t + \widehat{d}_{t-1} = \beta(\widehat{B}_t + \widehat{d}_t) + (1-\beta)\widehat{\tau}_t. \tag{C.1}$$

Similarly, the rule for lump-sump taxes can be written as

$$\tau_t - \overline{\tau} = \phi_T \left(A_t d_{t-1} - \frac{\overline{\tau}}{1-\beta} \right) + u_t^\tau \quad \Rightarrow \quad \overline{\tau} e^{\widehat{\tau}_t} - \overline{\tau} = \phi_T \left(\overline{A} e^{\widehat{A}_t} \overline{d} e^{\widehat{d}_{t-1}} - \frac{\overline{\tau}}{1-\beta} \right) + u_t^\tau,$$

or

$$e^{\widehat{\tau}_t} - 1 = \phi_T \left(\frac{1}{(1-\beta)} e^{\widehat{A}_t + \widehat{d}_{t-1}} - \frac{1}{1-\beta} \right) + \frac{1}{(1-\beta)} \widehat{u}_t^\tau,$$

where $\widehat{u}_t^\tau \equiv u_t^\tau (1-\beta)/\overline{\tau}$. Taking a first order approximation,

$$\widehat{\tau}_t = \phi_T \frac{1}{(1-\beta)} (\widehat{A}_t + \widehat{d}_{t-1}) + \frac{1}{(1-\beta)} \widehat{u}_t^\tau. \tag{C.2}$$

Thus, combining (C.1) and (C.2), we can write

$$(1-\phi_T)(\widehat{d}_{t-1} + \widehat{A}_t) = \beta(\widehat{d}_t + \widehat{B}_t) + \widehat{u}_t^\tau.$$

Moreover, from the definitions of A_t and B_t, we can log-linearize to obtain,

$$\begin{aligned} \widehat{A}_t = & \quad -(1-\overline{\Omega})\widehat{\pi}_t^N + \Omega(\widehat{q}_t - \widehat{\pi}_t^*) = -\widehat{\pi}_t^N + \Omega(\widehat{q}_t + \widehat{\pi}_t^N - \widehat{\pi}_t^*), \\ \widehat{B}_t = & \quad -(1-\overline{\Omega})\widehat{R}_t + \Omega(\widehat{q}_t - \widehat{R}_t^*) = -\widehat{R}_t + \Omega(\widehat{R}_t + \widehat{q}_t - \widehat{R}_t^*). \end{aligned}$$

Thus, the log-linearized version of the government debt's equation is

$$(1-\phi_T)\left[\widehat{d}_{t-1}-\widehat{\pi}_t^N+\Omega(\widehat{q}_t+\widehat{\pi}_t^N-\widehat{\pi}_t^*)\right]=\beta\left[\widehat{d}_t-\widehat{R}_t+\Omega(\widehat{R}_t+\widehat{q}_t-\widehat{R}_t^*)\right]+\widehat{u}_t^\tau, \tag{C.3}$$

Finally, log-linearizing the Taylor rule,

$$\widehat{R}_t=\phi_\pi\widehat{\pi}_t^N+\widehat{u}_t^R. \tag{C.4}$$

Overall, equations (C.3) and (C.4) charcaterize the policy block.

C.1.2 The rest of the model

The log-linearized version of the equations not related to policy or traded variables is

$$\widehat{w}_t-\widehat{p}_t^N-\sigma\widehat{c}_t^N=\varphi\widehat{h}_t,\quad \widehat{p}_t^N+\widehat{mc}_t^N=\widehat{w}_t+\left(\frac{1}{\alpha}-1\right)\widehat{y}_t^N,\quad \widehat{y}_t^N=\alpha\widehat{h}_t,\quad \widehat{y}_t^N=\widehat{c}_t^N,\quad \widehat{c}_t^N=\eta\widehat{q}_t+\widehat{c}_t^T,$$

$$0=\omega\widehat{p}_t^N+(1-\omega)\,\widehat{rer}_t,\quad \widehat{q}_t=\widehat{rer}_t-\widehat{p}_t^N,\quad \widehat{p}_t^N=\widehat{p}_{t-1}^N+\widehat{\pi}_t^N-\widehat{\pi}_t,$$

$$\widehat{rer}_t=\widehat{rer}_{t-1}+\widehat{\pi}_t^S+\widehat{\pi}_t^*-\widehat{\pi}_t,\quad \widehat{R}_t=\widehat{R}_t^*+E_t\left\{\widehat{\pi}_{t+1}^S\right\},\quad \widehat{\pi}_t^N=\beta E_t\{\widehat{\pi}_{t+1}^N\}+\kappa\cdot\widehat{mc}_t^N.$$

The first four can be used to write marginal costs as

$$\widehat{mc}_t^N=\left(\frac{1+\varphi}{\alpha}-1+\sigma\right)\widehat{c}_t^N,$$

In addition, using $\widehat{c}_t^N=\eta\widehat{q}_t+\widehat{c}_t^T$, the Phillips curve for non-tradables can be written as

$$\widehat{\pi}_t^N=\beta E_t\left\{\widehat{\pi}_{t+1}^N\right\}+\widetilde{\kappa}\left(\eta\widehat{q}_t+\widehat{c}_t^T\right), \tag{C.5}$$

where $\widetilde{\kappa}\equiv\kappa\left(\frac{(1+\varphi)}{\alpha}-1+\sigma\right)$, which is positive if we consider $\sigma>1$.

From $0=\omega\widehat{p}_t^N+(1-\omega)\,\widehat{rer}_t$ and the equations describing the evolution of $\widehat{p}_t^N$ and $\widehat{rer}_t$, we can write inflation as

$$\widehat{\pi}_t=\omega(\widehat{p}_{t-1}^N+\widehat{\pi}_t^N)+(1-\omega)(\widehat{rer}_{t-1}+\widehat{\pi}_t^S+\widehat{\pi}_t^*)=\omega(\widehat{\pi}_t^N)+(1-\omega)(\widehat{\pi}_t^S+\widehat{\pi}_t^*),$$

where the last equality follows from $0=\omega\widehat{p}_{t-1}^N+(1-\omega)\,\widehat{rer}_{t-1}$.

In addition, from $\widehat{q}_t=\widehat{rer}_t-\widehat{p}_t^N$ and the equations describing the evolution of $\widehat{p}_t^N$ and $\widehat{rer}_t$,

$$\widehat{q}_t=\widehat{q}_{t-1}+\widehat{\pi}_t^S+\widehat{\pi}_t^*-\widehat{\pi}_t^N.$$

In turn, this can be used to re-write the UIP condition as

$$\widehat{R}_t=\widehat{R}_t^*+E_t\left\{\widehat{q}_{t+1}-\widehat{q}_t+\widehat{\pi}_{t+1}^N-\widehat{\pi}_{t+1}^*\right\}. \tag{C.6}$$

C.1.3 Summary

Equations (C.3), (C.4), (C.5), (C.6) can be further combined to eliminate $\widehat{R}_t$

$$\widehat{\pi}_t^N=\beta E_t\left\{\widehat{\pi}_{t+1}^N\right\}+\widetilde{\kappa}\left(\eta\widehat{q}_t+\widehat{c}_t^T\right), \tag{C.7}$$

$$\phi_\pi \widehat{\pi}_t^N + \widehat{u}_t^R = \widehat{R}_t^* + E_t\left\{\widehat{q}_{t+1} - \widehat{q}_t + \widehat{\pi}_{t+1}^N - \widehat{\pi}_{t+1}^*\right\}, \tag{C.8}$$

$$(1-\phi_T)\left[\widehat{d}_{t-1} - \widehat{\pi}_t^N + \Omega(\widehat{q}_t + \widehat{\pi}_t^N - \widehat{\pi}_t^*)\right] = \beta\left[\widehat{d}_t - (1-\Omega)(\phi_\pi \widehat{\pi}_t^N + \widehat{u}_t^R) + \Omega(\widehat{q}_t - \widehat{R}_t^*)\right] + \widehat{u}_t^\tau. \tag{C.9}$$

This system characterizes the dynamics of the three endogenous variables $\widehat{\pi}_t^N, \widehat{q}_t, \widehat{d}_t$, given the exogenous external variables $\widehat{R}_t^W, \widehat{y}_t^T, \widehat{\pi}_t^*$ (recall from section B.2.1 that $\widehat{R}_t^*$ and $\widehat{c}_t^T$ are driven solely by these external variables) and policy shocks $\widehat{u}_t^R$ and $\widehat{u}_t^\tau$.

C.2 Existence and uniqueness

To study the determination of a stationary equilibrium in the linearized system, it is enough to consider the dynamics in the absence of shocks, which yields the following three equations

$$\widehat{\pi}_t^N = \beta \widehat{\pi}_{t+1}^N + \widetilde{\kappa}\eta \widehat{q}_t, \tag{C.10}$$

$$\phi_\pi \widehat{\pi}_t^N = \widehat{q}_{t+1} - \widehat{q}_t + \widehat{\pi}_{t+1}^N, \tag{C.11}$$

$$(1-\phi_T)\left[\widehat{d}_{t-1} - \widehat{\pi}_t^N + \Omega(\widehat{q}_t + \widehat{\pi}_t^N)\right] = \beta\left[\widehat{d}_t - (1-\Omega)\phi_\pi \widehat{\pi}_t^N + \Omega \widehat{q}_t\right]. \tag{C.12}$$

We divide the analysis into three cases: the baseline with sticky prices ($\theta_N \in (0,1)$), flexible prices with a positive fraction of peso-denominated debt ($\theta_N = 0$ and $\Omega \in [0,1)$), and flexible prices with debt completely denominated in foreign currency ($\theta_N = 0$ and $\Omega = 1$).

C.2.1 Case A: $\theta_N \in (0,1)$

Equations (C.10)- (C.12) can be re-arranged as

$$\widehat{\pi}_{t+1}^N = \frac{1}{\beta}\widehat{\pi}_t^N - \frac{\widetilde{\kappa}\eta}{\beta}\widehat{q}_t,$$

$$\widehat{q}_{t+1} = \left(\phi_\pi - \frac{1}{\beta}\right)\widehat{\pi}_t^N + \left(1 + \frac{\widetilde{\kappa}\eta}{\beta}\right)\widehat{q}_t,$$

$$\widehat{d}_t = (\Omega - 1)\left(\frac{1-\phi_T}{\beta} - \phi_\pi\right)\widehat{\pi}_t^N + \Omega\left(\frac{1-\phi_T}{\beta} - 1\right)\widehat{q}_t + \frac{(1-\phi_T)}{\beta}\widehat{d}_{t-1}.$$

Defining $w_t \equiv [\widehat{\pi}_t^N, \widehat{q}_t, \widehat{d}_{t-1}]'$, the system can be written as

$$w_{t+1} = \Psi w_t$$

where Ψ has the form

$$\Psi = \begin{pmatrix} \frac{1}{\beta} & -\frac{\widetilde{\kappa}\eta}{\beta} & 0 \\ \left(\phi_\pi - \frac{1}{\beta}\right) & \left(1 + \frac{\widetilde{\kappa}\eta}{\beta}\right) & 0 \\ (\Omega-1)\left(\frac{1-\phi_T}{\beta} - \phi_\pi\right) & \Omega\left(\frac{1-\phi_T}{\beta} - 1\right) & \frac{(1-\phi_T)}{\beta} \end{pmatrix} \equiv \begin{pmatrix} \Psi_{\pi^N\pi^N} & \Psi_{\pi^N q} & 0 \\ \Psi_{q\pi^N} & \Psi_{qq} & 0 \\ \Psi_{d\pi^N} & \Psi_{dq} & \Psi_{dd} \end{pmatrix}.$$

In this system, $\widehat{d}_t$ is a predetermined/state variable, while $\widehat{\pi}_t^N$ and $\widehat{q}_t$ are non-predetermined/jumping. Local equilibrium uniqueness thus requires one stable and two non-stable eigenvalues of Ψ. If there are more than two stable eigenvalues, there are multiple local equilibria. Otherwise, there is no locally stationary equilibrium.

Notice that Ψ has a block lower triangular structure of the form

$$\Psi = \begin{pmatrix} \Psi_{11} & 0 \\ \Psi_{21} & \Psi_{22} \end{pmatrix},$$

and thus $det(\Psi - \lambda I) = det(\Psi_{11} - \lambda I) \times det(\Psi_{22} - \lambda I)$. This means that we can analyze the eigenvalues of Ψ_{11} and Ψ_{22} separately. The eigenvalue of Ψ_{22} is simply equal to $\frac{1-\phi_T}{\beta}$, which is less than one if $1 - \beta < \phi_T$.

In terms of Ψ_{11}, we need to solve $det(\Psi_{11} - \lambda I) = 0$:

$$det(\Psi_{11} - \lambda I) = \begin{vmatrix} \Psi_{\pi^N \pi^N} - \lambda & \Psi_{\pi^N q} \\ \Psi_{q\pi^N} & \Psi_{qq} - \lambda \end{vmatrix} = \left(\frac{1}{\beta} - \lambda\right)\left(1 + \frac{\widetilde{\kappa}\eta}{\beta} - \lambda\right) + \left(\phi_\pi - \frac{1}{\beta}\right)\frac{\widetilde{\kappa}\eta}{\beta} = 0$$

$$\implies \beta\lambda^2 - (1 + \beta + \widetilde{\kappa}\eta)\,\lambda + (1 + \phi_\pi \widetilde{\kappa}\eta) = 0 \tag{C.13}$$

The eigenvalues of Ψ_{11} are then the solutions to the quadratic polynomial:

$$\begin{aligned} \lambda_1 &= \frac{(1 + \beta + \widetilde{\kappa}\eta) - \sqrt{(1 + \beta + \widetilde{\kappa}\eta)^2 - 4\beta(1 + \phi_\pi \widetilde{\kappa}\eta)}}{2\beta}, \\ \lambda_2 &= \frac{(1 + \beta + \widetilde{\kappa}\eta) + \sqrt{(1 + \beta + \widetilde{\kappa}\eta)^2 - 4\beta(1 + \phi_\pi \widetilde{\kappa}\eta)}}{2\beta}. \end{aligned} \tag{C.14}$$

By Vieta's formulas, it follows that:

$$\lambda_1 \lambda_2 = \frac{1 + \phi_\pi \widetilde{\kappa}\eta}{\beta}$$

Since $\beta \in (0, 1)$, $\widetilde{\kappa}\eta > 0$, and $\phi_\pi \geq 0$, the product $\lambda_1 \lambda_2 > 1$. Also, as $\lambda_2 > 0$, so is $\lambda_1 > 0$. In addition, as $\lambda_2 > \lambda_1$ by definition, then $\lambda_2 > 1$. Thus, at least one of the eigenvalues of Ψ_{11} is greater than 1.

Regarding the first eigenvalue, notice that

$$\lambda_1 = \frac{(1 + \beta + \widetilde{\kappa}\eta) - \sqrt{(1 + \beta + \widetilde{\kappa}\eta)^2 - 4\beta(1 + \phi_\pi \widetilde{\kappa}\eta)}}{2\beta}$$

$$= \frac{(1 + \beta + \widetilde{\kappa}\eta) - \sqrt{(1 + \beta + \widetilde{\kappa}\eta)^2 - 4\beta(1 + \widetilde{\kappa}\eta) - 4\beta\widetilde{\kappa}\eta(\phi_\pi - 1)}}{2\beta}$$

If $\phi_\pi > 1$:

$$\lambda_1 > \frac{(1 + \beta + \widetilde{\kappa}\eta) - \sqrt{(1 + \beta + \widetilde{\kappa}\eta)^2 - 4\beta(1 + \widetilde{\kappa}\eta)}}{2\beta} = \frac{(1 + \beta + \widetilde{\kappa}\eta) - \sqrt{(-1 + \beta + \widetilde{\kappa}\eta)^2}}{2\beta} = 1,$$

so $\lambda_1 > 1$ if $\phi_\pi > 1$. Conversely, if $\phi_\pi < 1$

$$\lambda_1 < \frac{(1 + \beta + \widetilde{\kappa}\eta) - \sqrt{(1 + \beta + \widetilde{\kappa}\eta)^2 - 4\beta(1 + \widetilde{\kappa}\eta)}}{2\beta} = \frac{(1 + \beta + \widetilde{\kappa}\eta) - \sqrt{(-1 + \beta + \widetilde{\kappa}\eta)^2}}{2\beta} = 1$$

thus, $\lambda_1 < 1$ when $\phi_\pi < 1$. And recall that we have already shown that $\lambda_1 > 0$.

Overall, of the three eigenvalues of Ψ, one of them must be greater than one, another is greater than one if $\phi_T < 1 - \beta$, and the last is greater than one if $\phi_\pi > 1$. Therefore, we have

- Unique stable equilibrium: either $\phi_\pi > 1$ and $\phi_T > 1 - \beta$ (active monetary, passive fiscal) or

$\phi_\pi < 1$ and $\phi_T < 1 - \beta$ (active fiscal, passive monetary).

- No stable equilibrium: $\phi_\pi > 1$ and $\phi_T < 1 - \beta$ (active fiscal and monetary).
- Continuum of stable equilibria: $\phi_\pi < 1$ and $\phi_T > 1 - \beta$ (passive fiscal and monetary).

Finally, notice that these results are independent of the share of dollar-denominated debt Ω.

C.2.2 Case B: $\theta_N = 0$ & $\Omega \in [0, 1)$

If prices are flexible, $\theta_N = 0$, $\kappa \to \infty$, so the system (C.10)- (C.12) reduces to $\widehat{q}_t = 0$ (i.e. money is neutral, so the real exchange rate is not affected by monetary policy) and

$$\widehat{\pi}^N_{t+1} = \phi_\pi \widehat{\pi}^N_t,$$

$$\widehat{d}_t = (\Omega - 1)\left(\frac{1 - \phi_T}{\beta} - \phi_\pi\right)\widehat{\pi}^N_t + \frac{(1 - \phi_T)}{\beta}\widehat{d}_{t-1},$$

Defining $z_t \equiv [\widehat{\pi}^N_t, \widehat{d}_{t-1}]'$, the system can be written as

$$z_{t+1} = \Phi z_t$$

where Φ:

$$\Phi = \begin{pmatrix} \phi_\pi & 0 \\ (\Omega - 1)\left(\frac{1-\phi_T}{\beta} - \phi_\pi\right) & \frac{(1-\phi_T)}{\beta} \end{pmatrix}$$

Φ is also lower triangular, and thus the eigenvalues are ϕ_π and $\frac{(1-\phi_T)}{\beta}$. Uniqueness now requires one stable and one unstable eigenvalue, for $\widehat{d}_t$ is predetermined, while $\widehat{\pi}^N_t$ is not. Therefore, we have

- Unique stable equilibrium: either $\phi_\pi > 1$ and $\phi_T > 1 - \beta$ (active monetary, passive fiscal) or $\phi_\pi < 1$ and $\phi_T < 1 - \beta$ (active fiscal, passive monetary).
- No stable equilibrium: $\phi_\pi > 1$ and $\phi_T < 1 - \beta$ (active fiscal and monetary).
- Continuum of stable equilibria: $\phi_\pi < 1$ and $\phi_T > 1 - \beta$ (passive fiscal and monetary).

precisely as with sticky prices, and once again independent of Ω (as long as it is less than one).

C.2.3 Case C: $\theta_N = 0$ & $\Omega = 1$

In this case, the system (C.10)- (C.12) reduces to $\widehat{q}_t = 0$ and

$$\widehat{\pi}^N_{t+1} = \phi_\pi \widehat{\pi}^N_t,$$

$$\widehat{d}_t = \frac{(1 - \phi_T)}{\beta}\widehat{d}_{t-1},$$

which is a system of two autonomous equations. Therefore, we have

- Unique stable equilibrium: $\phi_\pi > 1$ and $\phi_T > 1 - \beta$ (active monetary, passive fiscal)
- No stable equilibrium: $\phi_\pi > 1$ and $\phi_T < 1 - \beta$ (active fiscal and monetary) .

– Continuum of stable equilibria: $\phi_\pi < 1$ and $\phi_T > 1 - \beta$ (passive fiscal and monetary) or $\phi_\pi < 1$ and $\phi_T < 1 - \beta$ (active fiscal, passive monetary).

Thus, with debt fully denominated in dollars and flexible prices, fiscal policy can only be passive (and monetary policy active) to uniquely pin down the stationary equilibrium.

C.2.4 Summary

The results from this section can be summarized by the following proposition:

Proposition 1 *Under the simplifying assumptions, if either $\theta_N > 0$ and $\Omega \in [0, 1]$, or $\theta_N = 0$ and $\Omega \in [0, 1)$, the linearized model has*

– *A unique stable equilibrium if either $\phi_\pi > 1$ and $\phi_T > 1 - \beta$ (active monetary, passive fiscal), or $\phi_\pi < 1$ and $\phi_T < 1 - \beta$ (active fiscal, passive monetary).*

– *No stable equilibrium if $\phi_\pi > 1$ and $\phi_T < 1 - \beta$ (active fiscal and monetary).*

– *A continuum of stable equilibria if $\phi_\pi < 1$ and $\phi_T > 1 - \beta$ (passive fiscal and monetary).*

In turn, if $\Omega = 1$ and $\theta_N = 0$, the same results hold except that the combination $\phi_\pi < 1$ and $\phi_T < 1 - \beta$ (active fiscal, passive monetary) also generates a continuum of stable equilibria.

C.3 The effect of policy shocks in two special cases

In this sub-section, we characterize the solution of the linearized model (C.7)-(C.9) conditional on uniqueness, assuming only i.i.d. policy shocks, in two polar cases: Active monetary and passive fiscal, with $\phi_\pi > 1$ and $\phi_T > 1 - \beta$, and active fiscal and passive monetary, with $\phi_\pi = \phi_T = 0$. The goal is to characterize the derivatives of $\widehat{\pi}_t^N, \widehat{q}_t$ with respect to $\widehat{u}_t^R, \widehat{u}_t^T$, and how these depend on Ω.

C.3.1 Solution when $\phi_\pi > 1$ and $\phi_T > 1 - \beta$

In this case, equations (C.7) and (C.8) are enough to characterize the evolution of $\widehat{\pi}_t^N$ and $\widehat{q}_t$ (as in standard NK models with passive fiscal policy). Excluding non-policy shocks, these are

$$\widehat{\pi}_t^N = \beta E_t \left\{ \widehat{\pi}_{t+1}^N \right\} + \widetilde{\kappa} \eta \widehat{q}_t, \tag{C.15}$$

$$\phi_\pi \widehat{\pi}_t^N + \widehat{u}_t^R + \widehat{q}_t = E_t \left\{ \widehat{q}_{t+1} + \widehat{\pi}_{t+1}^N \right\}. \tag{C.16}$$

Clearly, fiscal shocks and the currency composition of debt are irrelevant for the equilibrium behavior of these variables. Thus,

$$\frac{\partial \widehat{\pi}_t^N}{\partial u_t^T} = 0, \quad \frac{\partial \widehat{q}_t}{\partial u_t^T} = 0.$$

As both variables are purely forward looking, the solution takes the form

$$\pi_t^N = N u_t^R, \quad q_t = M u_t^R.$$

A shocks are assumed to be i.i.d., $E_t \left\{ \widehat{\pi}_{t+1}^N \right\} = E_t \left\{ \widehat{q}_{t+1} \right\} = 0$ in the proposed solution. Replacing in (C.15) and (C.16),

$$N u_t^R = \widetilde{\kappa} \eta M u_t^R, \tag{C.17}$$

$$\phi_\pi N u_t^R + \widehat{u}_t^R + M u_t^R = 0. \tag{C.18}$$

Thus, N and M need to solve

$$\begin{cases} N = \widetilde{\kappa}\eta M, \\ 0 = \phi_\pi N + 1 + M, \end{cases} \implies \begin{cases} M = -\frac{1}{\phi_\pi \widetilde{\kappa}\eta + 1} < 0, \\ N = -\frac{\widetilde{\kappa}\eta}{\phi_\pi \widetilde{\kappa}\eta + 1} < 0, \end{cases}$$

Recall also that the nominal depreciation is $\widehat{\pi}_t^S = \widehat{q}_t + \widehat{\pi}_t^N - \widehat{q}_{t-1}$. Overall, we have

$$\frac{\partial \widehat{\pi}_t^N}{\partial u_t^R} < 0, \quad \frac{\partial \widehat{q}_t}{\partial u_t^R} < 0, \quad \frac{\partial \widehat{\pi}_t^S}{\partial u_t^R} < 0.$$

Moreover, recall that non-traded consumption $\widehat{c}_t^N$ and output $\widehat{y}_t^N$ are proportional to $\widehat{q}_t$, so any conclusions regarding the latter translate into the former as well. Moreover, as total inflation is a weighted average of $\widehat{\pi}_t^N$ and $\widehat{\pi}_t^S$, and both react with the same sign to the monetary shock, aggregate inflation also falls after a monetary tightening.

The following proposition summarizes the results for the monetary shock:

Proposition 2 *Under the NT-based policy setup with $\phi_\pi > 1$ (active monetary) and $\phi_T > 1 - \beta$ (passive fiscal), an i.i.d. positive shock to u_t^R induces a contemporaneous fall in non-traded inflation $\widehat{\pi}_t^N$, the relative price of tradables $\widehat{q}_t$ and non-traded output $\widehat{y}_t^N$, as well as a nominal appreciation ($\widehat{\pi}_t^S$ falls). This holds for any value of $\overline{\Omega} \in [0, 1]$.*

C.3.2 Solution when $\phi_\pi = \phi_T = 0$

Equations (C.7)-(C.9) without external shocks and imposing $\phi_\pi = \phi_T = 0$ can be written as

$$\widehat{\pi}_t^N = \beta E_t \left\{ \widehat{\pi}_{t+1}^N \right\} + \widetilde{\kappa}\eta \widehat{q}_t, \tag{C.19}$$

$$\widehat{q}_t + \widehat{u}_t^R = E_t \left\{ \widehat{q}_{t+1} + \widehat{\pi}_{t+1}^N \right\}, \tag{C.20}$$

$$\widehat{d}_{t-1} - \widehat{\pi}_t^N + \Omega(\widehat{q}_t + \widehat{\pi}_t^N) = \beta \left[\widehat{d}_t - \widehat{u}_t^R + \Omega(\widehat{q}_t + \widehat{u}_t^R) \right] + \widehat{u}_t^\tau. \tag{C.21}$$

We seek the solution of $\widehat{\pi}_t^N, \widehat{q}_{t+j}$ as a function of $\widehat{u}_t^R, \widehat{u}_t^\tau, \widehat{d}_{t-1}$. We proceed with the following steps:

- Combine the (C.19)-(C.20) to obtain a second-order difference equation for $\widehat{\pi}_t^N$. Find the solution for $E_t\{\widehat{\pi}_{t+j}^N\}$ for $j > 0$, as a function of $\widehat{\pi}_t^N$ and $\widehat{u}_t^R$.
- Characterize the solution for $\widehat{q}_t$ as a function of $\widehat{\pi}_t^N$ and $\widehat{u}_t^R$.
- Solve forward the government budget constraint (C.21) and use the previous expectations to replace the relevant terms. This yields the desired solution for $\widehat{\pi}_t^N$.
- Based on the previous result, find also the solution for $\widehat{q}_t$ and $\widehat{\pi}_t^S$.

This strategy is similar to that in Leeper and Leith (2016) and Tan (2017), although here the algebra is more tedious when $\Omega > 0$, which is not present in their closed-economy setup.

Obtaining $E_t\{\widehat{\pi}_{t+j}^N\}$ for $j > 0$

Using (C.19) to eliminate $\widehat{q}_t$ and $\widehat{q}_{t+1}$ in (C.20) yields

$$\frac{\widehat{\pi}_t^N - \beta E_t\left\{\widehat{\pi}_{t+1}^N\right\}}{\widetilde{\kappa}\eta} + \widehat{u}_t^R = E_t\left\{\frac{\widehat{\pi}_{t+1}^N - \beta\widehat{\pi}_{t+2}^N}{\widetilde{\kappa}\eta}\right\} + E_t\left\{\widehat{\pi}_{t+1}^N\right\}.$$

Rearranging,

$$\beta E_t\left\{\widehat{\pi}_{t+2}^N\right\} - (1+\beta+\widetilde{\kappa}\eta)E_t\left\{\widehat{\pi}_{t+1}^N\right\} + \widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R = 0. \tag{C.22}$$

This is a second-order difference equation. We look for solutions for $E_t\left\{\widehat{\pi}_{t+1}^N\right\}$ of the form

$$E_t\left\{\widehat{\pi}_{t+1}^N\right\} = \lambda\widehat{\pi}_t^N + \Lambda\widehat{u}_t^R,$$

where λ, Λ are unknown scalars that we need to find. With this solution, using the law of iterated expectations (LIE) and shocks being i.i.d.,

$$E_t\left\{\widehat{\pi}_{t+2}^N\right\} = E_t\left\{\lambda\widehat{\pi}_{t+1}^N + \Lambda\widehat{u}_{t+1}^R\right\} = \lambda E_t\left\{\widehat{\pi}_{t+1}^N\right\} = \lambda\left(\lambda\widehat{\pi}_t^N + \Lambda\widehat{u}_t^R\right) = \lambda^2\widehat{\pi}_t^N + \lambda\Lambda\widehat{u}_t^R$$

Replacing both expectations in (C.22)

$$\beta(\lambda^2\widehat{\pi}_t^N + \lambda\Lambda\widehat{u}_t^R) - (1+\beta+\widetilde{\kappa}\eta)(\lambda\widehat{\pi}_t^N + \Lambda\widehat{u}_t^R) + \widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R = 0.$$

Thus, λ and Λ need to satisfy

$$\beta\lambda^2 - (1+\beta+\widetilde{\kappa}\eta)\lambda + 1 = 0, \quad \beta\lambda\Lambda - (1+\beta+\widetilde{\kappa}\eta)\Lambda + \widetilde{\kappa}\eta = 0$$

Notice that the first equation is precisely the quadratic polynomial (C.13), under the assumption $\phi_\pi = 0$. Thus, the solution λ_1 in (C.14) satisfies the requirement $|\lambda_1| < 1$ that is needed for a stationary solution in this case. A related result is that, as λ_1 solves the quadratic equation,

$$(1+\beta+\widetilde{\kappa}\eta)\lambda_1 = \beta(\lambda_1)^2 + 1 \quad \Longrightarrow \quad \widetilde{\kappa}\eta = \frac{(1-\lambda_1)(1-\beta\lambda_1)}{\lambda_1}.$$

To compute Λ,

$$\beta\lambda\Lambda - (1+\beta+\widetilde{\kappa}\eta)\Lambda + \widetilde{\kappa}\eta = 0 \implies \Lambda = \frac{\widetilde{\kappa}\eta}{(1+\beta+\widetilde{\kappa}\eta) - \beta\lambda_1} = \ldots$$

$$\frac{\widetilde{\kappa}\eta}{(1+\beta+\widetilde{\kappa}\eta) - \frac{(1+\beta+\widetilde{\kappa}\eta) - \sqrt{(1+\beta+\widetilde{\kappa}\eta)^2 - 4\beta}}{2}} = \frac{\widetilde{\kappa}\eta}{\frac{(1+\beta+\widetilde{\kappa}\eta)}{2} + \frac{\sqrt{(1+\beta+\widetilde{\kappa}\eta)^2 - 4\beta}}{2}} = \frac{\widetilde{\kappa}\eta}{\beta\lambda_2} = \widetilde{\kappa}\eta\lambda_1$$

where the last equality uses Vieta's formula $\lambda_1\lambda_2 = \frac{1}{\beta}$. Putting all together, the solution for the expectation $E_t\left\{\widehat{\pi}_{t+1}^N\right\}$ is

$$E_t\left\{\widehat{\pi}_{t+1}^N\right\} = \lambda_1\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right). \tag{C.23}$$

From this,

$$E_t\left\{\widehat{\pi}_{t+j}^N\right\} = E_t\left\{E_{t+j-1}\left\{\widehat{\pi}_{t+j}^N\right\}\right\} = E_t\left\{\lambda_1\left(\widehat{\pi}_{t+j-1}^N + \widetilde{\kappa}\eta\widehat{u}_{t+j-1}^R\right)\right\} =$$
$$\lambda_1 E_t\left\{\widehat{\pi}_{t+j-1}^N\right\} = \ldots = (\lambda_1)^{j-1}E_t\left\{\widehat{\pi}_{t+1}^N\right\} = (\lambda_1)^j\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right), \tag{C.24}$$

where the first equality follows from the LIE, the second from (C.23) evaluated at $t+j-1$, the third from $\widehat{u}_t^R$ being i.i.d., the fourth repeats the procedure up to $t+1$, and the last uses (C.23) directly.

Obtaining $\widehat{q}_t$

From the Phillips curve (C.19)

$$\widehat{q}_t = \frac{\widehat{\pi}_t^N - \beta E_t\left\{\widehat{\pi}_{t+1}^N\right\}}{\widetilde{\kappa}\eta} = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\widehat{\pi}_t^N - \beta\lambda_1\widehat{u}_t^R. \tag{C.25}$$

Using the government budget constraint

Equation (C.21) can be arranged as

$$\widehat{d}_{t-1} = \beta\widehat{d}_t + aux_t,$$

where $aux_t \equiv \widehat{\pi}_t^N(1-\Omega) - \widehat{q}_t\Omega(1-\beta) - \widehat{u}_t^R\beta(1-\Omega) + \widehat{u}_t^\tau$. Iterating the equation forward, taking expectations and imposing the transversality condition yields

$$\widehat{d}_{t-1} = aux_t + \sum_{j=1}^{\infty}\beta^j E_t\{aux_{t+j}\}. \tag{C.26}$$

Notice that, by its definition and (C.25), aux_t can be written as

$$aux_t = \widehat{\pi}_t^N(1-\Omega) - \left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\widehat{\pi}_t^N - \beta\lambda_1\widehat{u}_t^R\right]\Omega(1-\beta) - \widehat{u}_t^R\beta(1-\Omega) + \widehat{u}_t^\tau = \ldots$$
$$\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\widehat{\pi}_t^N - \beta\left[1-\Omega\left(1+(1-\beta)\lambda_1\right)\right]\widehat{u}_t^R + \widehat{u}_t^\tau.$$

Also, as shocks are i.i.d., using (C.24) we can compute

$$E_t\{aux_{t+j}\} = \left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]E_t\{\widehat{\pi}_{t+j}^N\} = \ldots$$
$$\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right](\lambda_1)^j\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right),$$

for $j>0$. Thus

$$\sum_{j=1}^{\infty}\beta^j E_t\{aux_{t+j}\} = \left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right)\sum_{j=1}^{\infty}(\lambda_1\beta)^j = \ldots$$
$$\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{\beta\lambda_1}{(1-\beta\lambda_1)}\left(\widehat{\pi}_t^N + \widetilde{\kappa}\eta\widehat{u}_t^R\right)$$

Plugging these back into (C.26),

$$\begin{aligned}\widehat{d}_{t-1} &= \left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\widehat{\pi}_t^N-\beta\left[1-\Omega\left(1+(1-\beta)\lambda_1\right)\right]\widehat{u}_t^R+\widehat{u}_t^\tau+\\ &\quad\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{\beta\lambda_1}{(1-\beta\lambda_1)}\left(\widehat{\pi}_t^N+\widetilde{\kappa}\eta\widehat{u}_t^R\right)\\ &= \underbrace{\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{1}{(1-\beta\lambda_1)}}_{\equiv A}\widehat{\pi}_t^N+\widehat{u}_t^\tau\\ &\quad-\underbrace{\left\{\beta\left[1-\Omega\left(1+(1-\beta)\lambda_1\right)\right]-\widetilde{\kappa}\eta\left[1-\Omega\left(1+(1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right]\frac{\beta\lambda_1}{(1-\beta\lambda_1)}\right\}}_{\equiv B}\widehat{u}_t^R\end{aligned}$$

Thus, the solution for $\widehat{\pi}_t^N$ is

$$\widehat{\pi}_t^N=\frac{1}{A}(\widehat{d}_{t-1}-\widehat{u}_t^\tau)+\frac{B}{A}\widehat{u}_t^R. \tag{C.27}$$

Also, from (C.25),

$$\begin{aligned}\widehat{q}_t &= \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\left(\frac{1}{A}\widehat{d}_{t-1}+\frac{B}{A}\widehat{u}_t^R-\frac{1}{A}\widehat{u}_t^\tau\right)-\beta\lambda_1\widehat{u}_t^R\\ &= \underbrace{\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta A}}_{\equiv C}\left(\widehat{d}_{t-1}-\widehat{u}_t^\tau\right)+\underbrace{\left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\frac{B}{A}-\beta\lambda_1\right]}_{\equiv D}\widehat{u}_t^R.\end{aligned} \tag{C.28}$$

Moreover, recall that non-traded consumption $\widehat{c}_t^N$ and output $\widehat{y}_t^N$ are proportional to $\widehat{q}_t$, so any conclusions regarding the latter translate into the former as well.

Finally, the nominal depreciation is $\widehat{\pi}_t^S=\widehat{q}_t+\widehat{\pi}_t^N-\widehat{q}_{t-1}$, thus

$$\widehat{\pi}_t^S=\underbrace{\left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}+1\right]\frac{1}{A}}_{\equiv F}\left(\widehat{d}_{t-1}-\widehat{u}_t^\tau\right)+\underbrace{\left[\left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}+1\right)\frac{B}{A}-\beta\lambda_1\right]}_{\equiv G}\widehat{u}_t^R-\widehat{q}_{t-1} \tag{C.29}$$

Given these results, we next explore the role of Ω. First, we characterize the solution when $\Omega=0$, which is analogous to a closed economy setup under active fiscal policy, such as Cochrane (2023), Leeper and Leith (2016), among others. Then, we study the case with $\Omega>0$; our novel contribution.

Case with $\Omega=0$

The solution simplifies considerably when $\Omega=0$. For instance,

$$\frac{1}{A}=(1-\beta\lambda_1)>0$$

where the inequality holds as $\beta,\lambda_1\in(0,1)$. Also, when $\Omega=0$, B simplifies to

$$B=\beta-\widetilde{\kappa}\eta\frac{\beta\lambda_1}{1-\beta\lambda_1}=\beta\left(1-\frac{\widetilde{\kappa}\eta\lambda_1}{1-\beta\lambda_1}\right)=\beta\lambda_1>0.$$

where we have used $\frac{\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1} = (1-\lambda_1)$ from the solution of the quadratic equation. Therefore,

$$\frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^R} = \frac{B}{A} > 0, \qquad \frac{\partial\widehat{\pi}_t^N}{\partial\widehat{u}_t^T} = -\frac{1}{A} < 0.$$

In terms of the relative price, $\widehat{q}_t$,

$$C = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta A} = \frac{(1-\beta\lambda_1)^2}{\widetilde{\kappa}\eta} > 0$$

$$D = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\frac{B}{A} - \beta\lambda_1 = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}(1-\beta\lambda_1)\beta\lambda_1 - \beta\lambda_1 = \frac{\lambda_1}{(1-\lambda_1)}(1-\beta\lambda_1)\beta\lambda_1 - \beta\lambda_1$$

where we used $\widetilde{\kappa}\eta = \frac{(1-\lambda_1)(1-\beta\lambda_1)}{\lambda_1}$ (as λ_1 solves the quadratic equation) and the previously found A and B. This can be further simplified to

$$D = \frac{\beta\lambda_1}{(1-\lambda_1)}\left[\lambda_1(1-\beta\lambda_1) - 1 + \lambda_1\right] = \frac{\beta\lambda_1^2}{(1-\lambda_1)}(1-\beta-\widetilde{\kappa}\eta)$$

where in the last step we have used again the quadratic formula $\beta(\lambda_1)^2 = (1+\beta+\widetilde{\kappa}\eta)\lambda_1 - 1$. Thus, $D < 0$ as long as $\widetilde{\kappa}\eta + \beta > 1$ (which holds in our baseline calibration). Summarizing, we have

$$\begin{array}{ll} \frac{\partial\widehat{q}_t}{\partial\widehat{u}_t^R} < 0 & \text{if} \quad \widetilde{\kappa}\eta + \beta > 1, \\ \frac{\partial\widehat{q}_t}{\partial\widehat{u}_t^R} > 0 & \text{if} \quad \widetilde{\kappa}\eta + \beta \leq 1, \end{array} \qquad \frac{\partial\widehat{q}_t}{\partial\widehat{u}_t^T} < 0.$$

In terms of the nominal depreciation, $\widehat{\pi}_t^S$,

$$F = \left[\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right]\frac{1}{A} > 0$$

$$G = \left(\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta} + 1\right)\frac{B}{A} - \beta\lambda_1 = \left(\frac{\lambda_1}{(1-\lambda_1)} + 1\right)\frac{B}{A} - \beta\lambda_1 = \frac{1}{(1-\lambda_1)}\frac{B}{A} - \beta\lambda_1$$

With this, using the solution for B/A in this case, $G > 0$ if

$$(1-\beta\lambda_1)\beta\lambda_1 > \lambda_1\beta(1-\lambda_1) \iff 1-\beta\lambda_1 > 1-\lambda_1 \iff 1 > \beta,$$

which always holds. Summarizing, we have

$$\frac{\partial\widehat{\pi}_t^S}{\partial\widehat{u}_t^R} > 0, \qquad \frac{\partial\widehat{\pi}_t^S}{\partial\widehat{u}_t^T} < 0.$$

The following proposition summarizes the results for monetary shocks

Proposition 3 *Under the NT-based policy setup with $\phi_\pi = 0$ (passive monetary) and $\phi_T = 0$ (active fiscal) and $\overline{\Omega} = 0$ (only domestic-currency debt), an i.i.d. positive shock to u_t^R leads to a contemporaneous increase in non-traded inflation $\widehat{\pi}_t^N$ and a nominal depreciation $\widehat{\pi}_t^S$. The relative price of tradables $\widehat{q}_t$ falls if $\widetilde{\kappa}\eta + \beta > 1$, otherwise it increases after the shock. Non-traded output $\widehat{y}_t^N$ follows the same pattern as $\widehat{q}_t$.*

Case with $0 < \Omega \leq 1$

The dynamics of non-traded inflation $\widehat{\pi}_t^N$ in the general case are given by,

$$A = \left[1 - \Omega\left(1 + (1-\beta)\frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\right)\right](1-\beta\lambda_1)^{-1}.$$

Using $\widetilde{\kappa}\eta = (1-\lambda_1)(1-\beta\lambda_1)/\lambda_1$ from the quadratic equation,

$$A = \left[1 - \Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right](1-\beta\lambda_1)^{-1}.$$

Thus, $A > 0$ as long as

$$\Omega < \frac{1-\lambda_1}{1-\beta\lambda_1} \equiv \Omega_{\text{cut},1},$$

otherwise $A < 0$. In the cutoff case with $\Omega = \Omega_{\text{cut},1}$, there is a vertical asymptote: $\frac{1}{|A|} \to \infty$.

Now consider B,

$$B = \beta\Big[1 - \Omega\big(1 + (1-\beta)\lambda_1\big)\Big] - \widetilde{\kappa}\eta\Big[1 - \Omega\Big(1 + (1-\beta)\frac{1-\beta\lambda_1}{\widetilde{\kappa}\eta}\Big)\Big]\frac{\beta\lambda_1}{1-\beta\lambda_1}.$$

The second term can be simplified to

$$\widetilde{\kappa}\eta\frac{\beta\lambda_1}{1-\beta\lambda_1}\Big[1 - \Omega - \Omega(1-\beta)\frac{1-\beta\lambda_1}{\widetilde{\kappa}\eta}\Big] = \frac{\beta\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}(1-\Omega) - \beta\lambda_1\Omega(1-\beta).$$

Substituting back in B,

$$\begin{aligned} B &= \beta\big[1 - \Omega(1 + (1-\beta)\lambda_1)\big] - \frac{\beta\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}(1-\Omega) + \beta\lambda_1\Omega(1-\beta) \\ &= \beta(1-\Omega) - \frac{\beta\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}(1-\Omega) = (1-\Omega)\beta\left[1 - \frac{\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1}\right] = (1-\Omega)\beta\lambda_1 \end{aligned}$$

where we have used $\frac{\lambda_1\widetilde{\kappa}\eta}{1-\beta\lambda_1} = (1-\lambda_1)$ from the solution of the quadratic equation. Thus, $B \geq 0$, with strict equality if $\Omega = 1$. These results about A and B are enough to characterize $\dfrac{-1}{A}$ and $\dfrac{B}{A}$, which determine the effects of both shocks on $\widehat{\pi}_t^N$. We present below a summary table.

Besides the signs of the effects of both shocks, we also study how these change for different values of Ω. For that, we need $\partial(B/A)/\partial\Omega$ and $\partial(-1/A)/\partial\Omega$. The former is

$$\frac{\partial B/A}{\partial\Omega} = \frac{A\frac{\partial B}{\partial\Omega} - B\frac{\partial A}{\partial\Omega}}{A^2}$$

whose sign is determined by the numerator's

$$\begin{aligned} A\frac{\partial B}{\partial\Omega} - B\frac{\partial A}{\partial\Omega} &= -\left[1 - \Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right](1-\beta\lambda_1)^{-1}\beta\lambda_1 + (1-\Omega)\beta\lambda_1\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)(1-\beta\lambda_1)^{-1} \\ &= \frac{\beta\lambda_1}{(1-\beta\lambda_1)}\left[-1 + \Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right) + (1-\Omega)\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right] = \frac{\beta\lambda_1}{(1-\beta\lambda_1)}\left[-1 + \left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right] \end{aligned}$$

$$= \frac{\beta\lambda_1}{(1-\beta\lambda_1)} \frac{(1-\beta)\lambda_1}{(1-\lambda_1)} > 0$$

In turn, the sign of $\frac{\partial(-1/A)}{\partial\Omega}$ depends upon that of

$$\frac{\partial A}{\partial \Omega} = -\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)(1-\beta\lambda_1)^{-1} < 0$$

Finally, notice that these two derivatives are independent of the value of Ω, so they hold regardless of whether Ω is above or below the cutoff value $\Omega_{\text{cut},1}$.

Overall, the response of $\widehat{\pi}_t^N$ to both shocks, and the role of Ω, can be summarized as:

Effects on $\widehat{\pi}_t^N$

	Values of Ω			Derivative
Effect of	$0 \leq \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega < 1$	$\Omega = 1$	w.r.t. Ω
$\widehat{u}_t^R$	+	-	0	+
$\widehat{u}_t^T$	-	+	+	-

Next, regarding $\widehat{q}_t$ we have

$$C = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta A},$$

whose sign equals that of $1/A$. In addition,

$$D = \frac{(1-\beta\lambda_1)}{\widetilde{\kappa}\eta}\frac{B}{A} - \beta\lambda_1 = \frac{\lambda_1}{(1-\lambda_1)}\frac{B}{A} - \beta\lambda_1.$$

As $B \geq 0$, $D < 0$ if $A < 0$ (i.e. $\Omega > \Omega_{\text{cut},1}$). Instead, if $A > 0$ (i.e. $\Omega < \Omega_{\text{cut},1}$), $D < 0$ if

$$B < \beta(1-\lambda_1)A \iff (1-\Omega)\beta\lambda_1 < \beta(1-\lambda_1)\left[1-\Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right](1-\beta\lambda_1)^{-1} \iff$$

$$(1-\Omega)\lambda_1(1-\beta\lambda_1) < 1-\lambda_1-\Omega(1-\beta\lambda_1) \iff \Omega(1-\beta\lambda_1)(1-\lambda_1) < 1-\lambda_1-\lambda_1+\beta(\lambda_1)^2.$$

From the quadratic formula $\beta(\lambda_1)^2 - \lambda_1 + 1 = \lambda_1(\beta + \widetilde{\kappa}\eta)$, so $D < 0$ if

$$\Omega(1-\beta\lambda_1)(1-\lambda_1) < \lambda_1(\beta + \widetilde{\kappa}\eta - 1).$$

Thus, if $\widetilde{\kappa}\eta + \beta > 1$ (as it holds in our baseline calibration), $D < 0$ if

$$\Omega < \frac{\lambda_1(\beta + \widetilde{\kappa}\eta - 1)}{(1-\beta\lambda_1)(1-\lambda_1)} \equiv \Omega_{\text{cut},2}$$

Instead, when $\widetilde{\kappa}\eta + \beta \leq 1$, we have $D < 0$ if

$$\Omega < \frac{\lambda_1(\beta + \widetilde{\kappa}\eta - 1)}{(1-\beta\lambda_1)(1-\lambda_1)} \leq 0$$

which does not hold, as we only consider $\Omega \in [0,1]$. Thus $D > 0$ if $\Omega < \Omega_{\text{cut},1}$ and $\widetilde{\kappa}\eta + \beta \leq 1$.

We next establish that $\Omega_{\text{cut},2} < \Omega_{\text{cut},1}$:

$$\Omega_{\text{cut},2} - \Omega_{\text{cut},1} = \frac{\lambda_1(\beta + \widetilde{\kappa}\eta - 1)}{(1-\beta\lambda_1)(1-\lambda_1)} - \frac{1-\lambda_1}{1-\beta\lambda_1} = \frac{\lambda_1(\beta + \widetilde{\kappa}\eta - 1) - (1-\lambda_1)^2}{(1-\lambda_1)(1-\beta\lambda_1)} = ...$$
$$\frac{\beta(\lambda_1)^2 - 2\lambda_1 + 1 - 1 + 2\lambda_1 - (\lambda_1)^2}{(1-\lambda_1)(1-\beta\lambda_1)} = \frac{(\beta-1)(\lambda_1)^2}{(1-\lambda_1)(1-\beta\lambda_1)} < 0.$$

Overall, we have obtained

$$\begin{array}{llll} & & D<0 & \text{if} \quad \Omega_{\text{cut},1} < \Omega \\ C>0 & \text{if} \quad \Omega < \Omega_{\text{cut},1} & D>0 & \text{if} \quad \Omega_{\text{cut},2} < \Omega < \Omega_{\text{cut},1} \text{ and } \widetilde{\kappa}\eta + \beta > 1 \\ C<0 & \text{if} \quad \Omega > \Omega_{\text{cut},1} & D<0 & \text{if} \quad \Omega < \Omega_{\text{cut},2} \text{ and } \widetilde{\kappa}\eta + \beta > 1 \\ & & D>0 & \text{if} \quad \Omega < \Omega_{\text{cut},1} \text{ and } \widetilde{\kappa}\eta + \beta \leq 1 \end{array}$$

Finally, the derivatives of both C and D with respect to Ω are those of $\frac{1}{A}$ and $\frac{B}{A}$, respectively, which we have already characterized. We can then summarize the responses of $\widehat{q}_t$ as follows:

Effects on $\widehat{q}_t$

	Values of Ω and $\widetilde{\kappa}\eta + \beta$				
	$0 \leq \Omega < \Omega_{\text{cut},2}$				Derivative
Effect of	$\widetilde{\kappa}\eta + \beta > 1$	$\widetilde{\kappa}\eta + \beta \leq 1$	$\Omega_{\text{cut},2} < \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega$	w.r.t. Ω
$\widehat{u}_t^R$	-	+	+	-	+
$\widehat{u}_t^T$	-	-	-	+	-

In terms of $\widehat{\pi}_t^S$, as the sign of both $\widehat{\pi}_t^N$ and $\widehat{q}_t$ following a fiscal shock $\widehat{u}_t^T$ is the same (independent of Ω), $\widehat{\pi}_t^S$ shares the same sign. The effect of the monetary shock $\widehat{u}_t^R$ also has the same sign on both $\widehat{\pi}_t^N$ and $\widehat{q}_t$ when $\Omega_{\text{cut},1} < \Omega$, the sign of the effect in $\widehat{\pi}_t^S$ is pinned down for those values of Ω. In the case with $\Omega < \Omega_{\text{cut},1}$, recall G can be written as

$$G = \frac{1}{(1-\lambda_1)}\frac{B}{A} - \beta\lambda_1.$$

With this, $G < 0$ if

$$B > \lambda_1\beta(1-\lambda_1)A \iff (1-\Omega)\beta\lambda_1 > \lambda_1\beta(1-\lambda_1)\left[1 - \Omega\left(\frac{1-\beta\lambda_1}{1-\lambda_1}\right)\right](1-\beta\lambda_1)^{-1} \iff$$

$$(1-\Omega)(1-\beta\lambda_1) > 1 - \lambda_1 - \Omega(1-\beta\lambda_1) \iff 1 - \beta\lambda_1 > 1 - \lambda_1 \iff \lambda_1(1-\beta) > 0,$$

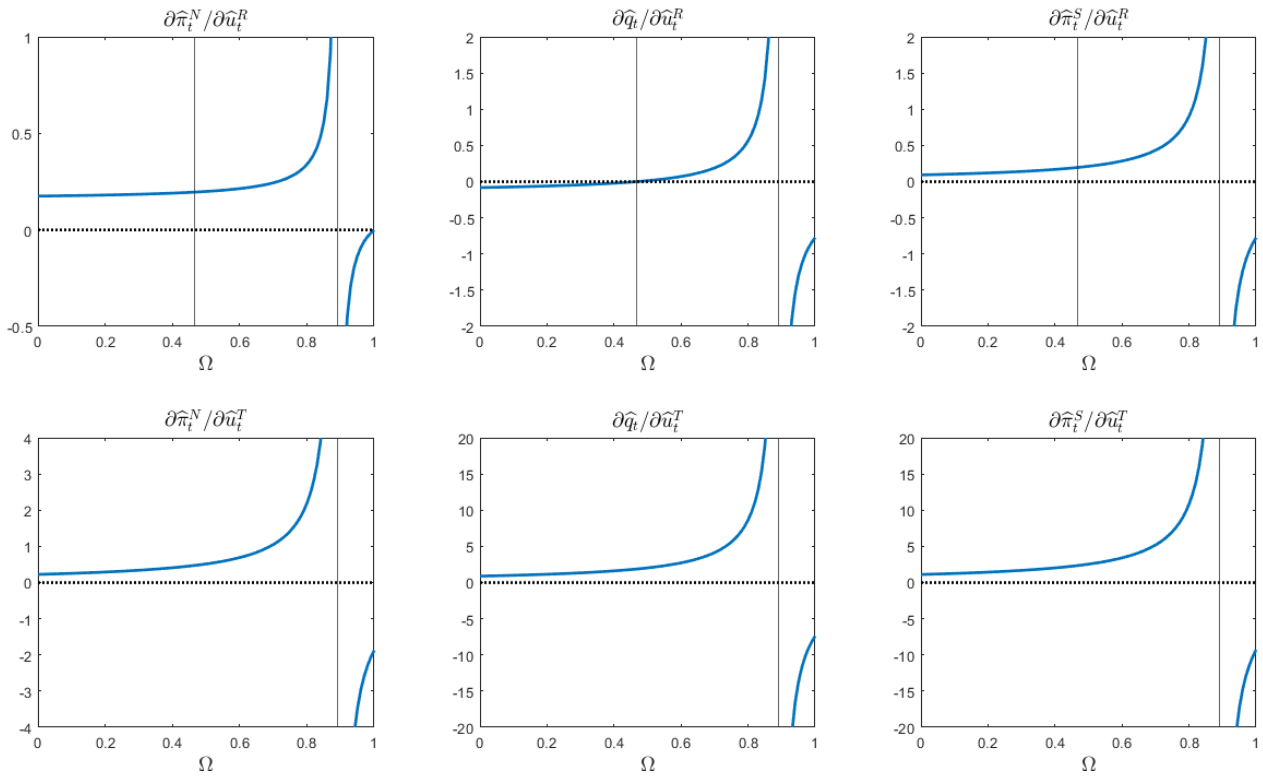
which always holds. Thus, the cutoff $\Omega_{\text{cut},2}$ is irrelevant for $\widehat{\pi}_t^S$. In addition, as with C and D, F and G vary with Ω as $\frac{1}{A}$ and $\frac{B}{A}$ do. Summarizing,

Effects on $\widehat{\pi}_t^S$

	Values of Ω		Derivative
Effect of	$0 \leq \Omega < \Omega_{\text{cut},1}$	$\Omega_{\text{cut},1} < \Omega \leq 1$	w.r.t. Ω
$\widehat{u}_t^R$	+	-	+
$\widehat{u}_t^T$	-	+	-

Figure 11 below provides a visual summary of the results using the baseline calibration. Finally, as total inflation is a weighted average of $\widehat{\pi}_t^N$ and $\widehat{\pi}_t^S$, and the sign of the response after either shock on both is the same, aggregate inflation displays the same pattern.

Figure 11: The effect fo policy shocks under the simplifying assumption



Note: vertical lines indicate the cutoff values $\Omega_{\text{cut},1}$ and $\Omega_{\text{cut},2}$ (recall $\Omega_{\text{cut},2} < \Omega_{\text{cut},1}$).

D Existence and uniqueness in the baseline policy setup

In the baseline setup, the equivalent linearized equations to the government budget constraint (C.3) and the Taylor rule (C.4) are

$$(1-\phi_T)\left[\widehat{d}_{t-1} - \widehat{\pi}_t + \Omega(\widehat{rer}_t + \widehat{\pi}_t - \widehat{\pi}_t^*)\right] = \beta\left[\widehat{d}_t - \widehat{R}_t + \Omega(\widehat{R}_t + \widehat{rer}_t - \widehat{R}_t^*)\right] + \widehat{u}_t^\tau,$$

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \widehat{u}_t^R.$$

Thus, in the absence of shocks, the relevant system of equations is:

$$\widehat{\pi}_t^N = \beta\widehat{\pi}_{t+1}^N + K\widehat{rer}_t, \tag{D.1}$$

$$\widehat{rer}_t = \widehat{rer}_{t-1} + \omega(\widehat{\pi}_t^S - \widehat{\pi}_t^N), \tag{D.2}$$

$$\widehat{\pi}_t = \omega(\widehat{\pi}_t^N) + (1-\omega)(\widehat{\pi}_t^S), \tag{D.3}$$

$$\widehat{R}_t = \widehat{\pi}_{t+1}^S, \tag{D.4}$$

$$(1-\phi_T)\left[\widehat{d}_{t-1} - (1-\Omega)\widehat{\pi}_t + \Omega\widehat{rer}_t\right] = \beta\left[\widehat{d}_t - (1-\Omega)\widehat{R}_t + \Omega\widehat{rer}_t\right], \tag{D.5}$$

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t, \tag{D.6}$$

where in the Phillips curve (D.1) we have defined $K \equiv \widetilde{\kappa}\frac{\eta}{\omega}$.

D.1 Case A: $\theta_N \in (0,1)$

Combining the equations (D.1)-(D.6) to eliminate $\widehat{\pi}_t$ and $\widehat{R}_t$, we get the following equations characterizing the dynamics of $\widehat{\pi}_t^N, \widehat{rer}_t, \widehat{\pi}_t^S, \widehat{d}_t$:

$$\widehat{\pi}_t^N = \beta \widehat{\pi}_{t+1}^N + K\widehat{rer}_t, \tag{D.7}$$

$$\widehat{rer}_t = \widehat{rer}_{t-1} + \omega(\widehat{\pi}_t^S - \widehat{\pi}_t^N), \tag{D.8}$$

$$\phi_\pi \left[\omega \widehat{\pi}_t^N + (1-\omega)\widehat{\pi}_t^S\right] = \widehat{\pi}_{t+1}^S, \tag{D.9}$$

$$\widehat{d}_t = \frac{(1-\phi_T)}{\beta}\widehat{d}_{t-1} - (1-\Omega)\left[\frac{(1-\phi_T)}{\beta} - \phi_\pi\right]\left[\omega \widehat{\pi}_t^N + (1-\omega)\widehat{\pi}_t^S\right] + \Omega\left[\frac{(1-\phi_T)}{\beta} - 1\right]\widehat{rer}_t, \tag{D.10}$$

Here, $\widehat{rer}_t$ and $\widehat{d}_t$ are predetermined/state endogenous variables, while $\widehat{\pi}_t^N$ and $\widehat{\pi}_t^S$ are non-predetermined/jumpi
Local equilibrium uniqueness thus requires 2 stable and 2 non-stable eigenvalues. If there are more than 2 stable eigenvalues, there are multiple local equilibria. Otherwise, there is no locally stationary equilibrium.

These equations can be re-arranged as

$$\widehat{\pi}_{t+1}^S = \phi_\pi \omega \widehat{\pi}_t^N + \phi_\pi(1-\omega)\widehat{\pi}_t^S \tag{D.11}$$

$$\widehat{\pi}_{t+1}^N = \frac{1}{\beta}(1+\omega K)\widehat{\pi}_t^N - \frac{1}{\beta}K\widehat{rer}_{t-1} - \frac{1}{\beta}\omega K \widehat{\pi}_t^S, \tag{D.12}$$

$$\widehat{rer}_t = \widehat{rer}_{t-1} + \omega\widehat{\pi}_t^S - \omega\widehat{\pi}_t^N, \tag{D.13}$$

$$\begin{aligned}\widehat{d}_t = \frac{1-\phi_T}{\beta}\widehat{d}_{t-1} + \Omega\left(\frac{1-\phi_T}{\beta} - 1\right)\widehat{rer}_{t-1} - \omega\left[\frac{1-\phi_T}{\beta} - (1-\Omega)\phi_\pi - \Omega\right]\widehat{\pi}_t^N - ... \\ - \left[(1-\Omega-\omega)\frac{1-\phi_T}{\beta} - (1-\Omega)\phi_\pi(1-\omega) + \Omega\omega\right]\widehat{\pi}_t^S\end{aligned} \tag{D.14}$$

Defining $w_t \equiv [\widehat{\pi}_t^S, \widehat{\pi}_t^N, \widehat{rer}_{t-1}, \widehat{d}_{t-1}]'$, the system can be written as

$$w_{t+1} = \Psi w_t$$

where

$$\Psi = \begin{pmatrix} \Psi_{\pi^S\pi^S} & \Psi_{\pi^S\pi^N} & 0 & 0 \\ \Psi_{\pi^N\pi^S} & \Psi_{\pi^N\pi^N} & \Psi_{\pi^N rer} & 0 \\ \Psi_{rer\pi^S} & \Psi_{rer\pi^N} & \Psi_{rerrer} & 0 \\ \Psi_{d\pi^S} & \Psi_{d\pi^N} & \Psi_{drer} & \Psi_{dd} \end{pmatrix}$$

Given that the matrix Ψ is a block lower triangular matrix of the form

$$\Psi = \begin{pmatrix} \Psi_{11} & 0 \\ \Psi_{21} & \Psi_{22} \end{pmatrix},$$

and thus $det(\Psi - \lambda I) = det(\Psi_{11} - \lambda I) \times det(\Psi_{22} - \lambda I)$. This means that we can analyze the eigenvalues of Ψ_{11} and Ψ_{22} separately. The eigenvalue of Ψ_{22} is simply equal to $\frac{1-\phi_T}{\beta}$, which is less than one if $1 - \beta < \phi_T$.

In terms of Ψ_{11}, notice that

$$det(\Psi_{11} - \lambda I) = \begin{vmatrix} \Psi_{\pi^S\pi^S} - \lambda & \Psi_{\pi^S\pi^N} & 0 \\ \Psi_{\pi^N\pi^S} & \Psi_{\pi^N\pi^N} - \lambda & \Psi_{\pi^N rer} \\ \Psi_{rer\pi^S} & \Psi_{rer\pi^N} & \Psi_{rerrer} - \lambda \end{vmatrix} =$$

$$(\Psi_{\pi^S\pi^S} - \lambda)\left[(\Psi_{\pi^N\pi^N} - \lambda)(\Psi_{rerrer} - \lambda) - \Psi_{rer\pi^N}\Psi_{\pi^N rer}\right] - (\Psi_{\pi^S\pi^N})\left[\Psi_{\pi^N\pi^S}(\Psi_{rerrer} - \lambda) - \Psi_{rer\pi^S}\Psi_{\pi^N rer}\right] =$$

$$\lambda^3 - (a_{\pi\pi} + a_{rerrer})\lambda^2 + (a_{\pi\pi}a_{rerrer} - a_{\pi rer}a_{rer\pi} - a_{\pi^S\pi}a_{\pi\pi^S})\lambda + (a_{\pi^S\pi}a_{\pi\pi^S}a_{rerrer} - a_{\pi^S\pi}a_{\pi rer}a_{rer\pi^S})$$

Thus, we need to compute the solutions to,

$$\lambda^3 - \left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)\lambda^2 + \left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \phi_\pi\frac{1}{\beta}(1+\omega K) - \omega\phi_\pi\frac{1}{\beta}\right)\lambda - \phi_\pi\frac{(1-\omega)}{\beta} = 0$$

In other words, the characteristic equation of matrix A takes the following form:

$$\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 = 0.$$

We are interested in determining *sufficient conditions* under which this polynomial has either:

- Exactly **two roots are outside** the unit circle (i.e., $|\lambda| > 1$), and **one is inside** ($|\lambda| < 1$).
- Exactly **two roots are inside** the unit circle, and **one is outside**.

To determine these conditions, we follow Woodford (2003, App. C).

Sufficient Conditions for 2 Roots Outside and 1 Root Inside

Any one of the following sets of conditions is sufficient to ensure this root configuration:

Condition Set 1:

$$\begin{cases} 1 + A_2 + A_1 + A_0 < 0, \\ -1 + A_2 - A_1 + A_0 > 0. \end{cases}$$

Condition Set 2:

$$\begin{cases} 1 + A_2 + A_1 + A_0 > 0, \\ -1 + A_2 - A_1 + A_0 < 0, \\ A_2^2 - A_0A_2 + A_1 - 1 > 0. \end{cases}$$

Condition Set 3:

$$\begin{cases} 1 + A_2 + A_1 + A_0 > 0, \\ -1 + A_2 - A_1 + A_0 < 0, \\ A_2^2 - A_0A_2 + A_1 - 1 < 0, \\ |A_2| > 3. \end{cases}$$

Sufficient Conditions for 2 Roots Inside and 1 Root Outside

Any one of the following sets of conditions is sufficient to ensure this alternative configuration:

Condition Set A:

$$\begin{cases} 1 + A_2 + A_1 + A_0 > 0, \\ -1 + A_2 - A_1 + A_0 > 0. \end{cases}$$

Condition Set B:

$$\begin{cases} 1 + A_2 + A_1 + A_0 < 0, \\ -1 + A_2 - A_1 + A_0 < 0, \\ A_2^2 - A_0 A_2 + A_1 - 1 > 0. \end{cases}$$

Condition Set C:

$$\begin{cases} 1 + A_2 + A_1 + A_0 < 0, \\ -1 + A_2 - A_1 + A_0 > 0, \\ A_2^2 - A_0 A_2 + A_1 - 1 < 0, \\ |A_2| > 3. \end{cases}$$

D.1.1 Case I: $\phi_\pi > 1$

We need 2 roots outside and 1 root inside the unit circle. First, we examine the first condition of each set:

$$1 + A_2 + A_1 + A_0 = (\phi_\pi - 1)\,\frac{\omega K}{\beta} > 0,$$

if $\phi_\pi > 1$. Thus, we now need to prove that:

$$-1 + A_2 - A_1 + A_0 < 0$$

where

$$-1 + A_2 - A_1 + A_0 = -2\left(1 + \frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\phi_\pi(1-\omega)}{\beta}\right) - (1+\phi_\pi)\frac{\omega K}{\beta} < 0$$

which is negative if for $\phi_\pi > 0$.

Finally, to show that the sufficient conditions for the existence of two roots outside and one root inside the unit circle are satisfied, we need to prove that:

$$A_2^2 - A_0 A_2 + A_1 - 1 > 0,$$

or,

$$A_2^2 - A_0 A_2 + A_1 - 1 < 0, \text{ and } |A_2| > 3.$$

We have

$$A_2^2 - A_0 A_2 + A_1 - 1 = \left[-\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)\right]^2 - \phi_\pi \frac{(1-\omega)}{\beta}\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)$$
$$+ \left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \phi_\pi \frac{1}{\beta}(1 + \omega K) - \omega\phi_\pi \frac{1}{\beta}\right).$$

Therefore, $A_2^2 - A_0 A_2 + A_1 - 1$ takes the form of a quadratic equation in ϕ_π:

$$A_2^2 - A_0 A_2 + A_1 - 1 = a\phi_\pi^2 + b\phi_\pi + c,$$

where $a = (1-\omega)^2(1-\frac{1}{\beta}) < 0$. In particular, this means that this quadratic equation is strictly concave in ϕ_π; a result that we will use momentarily.

Given the alternative conditions listed above, it is sufficient to show that there is no $\phi_\pi > 1$ such that both $A_2^2 - A_0A_2 + A_1 - 1 < 0$ and $|A_2| < 3$ hold simultaneously. This can be shown as follows. First, notice that

$$|A_2| = \frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1.$$

Since $\beta \in (0,1)$, $1 + \frac{1}{\beta} > 2$, thus to prove $|A_2| > 3$, it is sufficient to show that:

$$(1-\omega)\phi_\pi + \frac{\omega K}{\beta} \geq 1$$

which holds if

$$\phi_\pi \geq \frac{1 - \omega\frac{K}{\beta}}{1-\omega}.$$

Instead, to obtain $A_2^2 - A_0A_2 + A_1 - 1 > 0$, it is sufficient to show that:

$$\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)^2 - \phi_\pi\frac{(1-\omega)}{\beta}\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right) + ... \\ (\phi_\pi - 1)\frac{\omega K}{\beta} + \frac{\phi_\pi}{\beta}(1-\omega) \quad \text{(D.15)}$$

is positive.

For cases in which $\frac{1-\frac{\omega K}{\beta}}{1-\omega} < 1$, then

$$|A_2| > 3 \quad \forall\, \phi_\pi \in (1, +\infty)$$

In contrast, for cases with $\phi_\pi > 1$ and $\frac{1-\frac{\omega K}{\beta}}{1-\omega} \geq 1$, we know that for $\phi_\pi \in \left(1, \frac{1-\frac{\omega K}{\beta}}{1-\omega}\right)$, it is not guaranteed that $|A_2| > 3$. Thus, we must show that within this interval, the condition $A_2^2 - A_0A_2 + A_1 - 1 > 0$ holds (i.e. that (D.15) is positive). Notice that, if $\phi_\pi = 1$,

$$\left(\frac{1}{\beta} + (1-\omega) + \frac{\omega K}{\beta} + 1\right)^2 - \frac{(1-\omega)}{\beta}\left(\frac{1}{\beta} + (1-\omega) + \frac{\omega K}{\beta} + 1\right) + \frac{1}{\beta}(1-\omega) = \\ = (1-\omega) + \frac{\omega K}{\beta} + 1 + \frac{\omega}{\beta} + \frac{1}{\beta}\frac{(1-\omega)}{\frac{1}{\beta} + (1-\omega) + \frac{\omega K}{\beta} + 1} > 0.$$

Instead, if $\phi_\pi = \frac{1-\frac{\omega K}{\beta}}{1-\omega}$,

$$\left(\frac{1}{\beta}+(1-\omega)\frac{1-\frac{\omega K}{\beta}}{1-\omega}+\frac{\omega K}{\beta}+1\right)^2-\frac{1-\frac{\omega K}{\beta}}{1-\omega}\frac{(1-\omega)}{\beta}\left(\frac{1}{\beta}+(1-\omega)\frac{1-\frac{\omega K}{\beta}}{1-\omega}+\frac{\omega K}{\beta}+1\right)$$
$$+\left(\frac{1-\frac{\omega K}{\beta}}{1-\omega}-1\right)\frac{\omega K}{\beta}+\frac{1-\frac{\omega K}{\beta}}{1-\omega}\frac{(1-\omega)}{\beta}=$$
$$=2+\frac{\omega K}{\beta^2}+\left[\left(\frac{1-\frac{\omega K}{\beta}}{1-\omega}-1\right)\frac{\omega K}{\beta}+\frac{1-\frac{\omega K}{\beta}}{1-\omega}\frac{(1-\omega)}{\beta}\right]\frac{1}{\frac{1}{\beta}+(1-\omega)\frac{1-\frac{\omega K}{\beta}}{1-\omega}+\frac{\omega K}{\beta}+1}>0$$

We can now establish the stability conditions for all values of $\phi_\pi > 1$. The expression $A_2^2 - A_0A_2 + A_1 - 1$ has been shown to be a strictly concave function of ϕ_π (i.e., a downward-opening parabola). Also, on the interval where $\phi_\pi \in \left(1, \frac{1-\frac{\omega K}{\beta}}{1-\omega}\right)$, which is the region where the condition $|A_2| > 3$ is not guaranteed, the function equals a strictly positive value at the boundary points of this interval (i.e., at $\phi_\pi = 1$ and $\phi_\pi = \frac{1-\frac{\omega K}{\beta}}{1-\omega}$). A fundamental property of strictly concave functions dictates that:

Lemma D.1 *1 Let f be a strictly concave function on the closed interval $[a, b]$. If $f(a) > 0$ and $f(b) > 0$, then $f(x) > 0$ for all x in the open interval (a, b).*

Then, it follows that $A_2^2 - A_0A_2 + A_1 - 1 > 0$ holds for all ϕ_π in the specified range.

We have shown that, for the case where $\phi_\pi > 1$, the sufficient conditions are satisfied to be in a scenario with two roots outside the unit circle and one inside.[19] Given that $\widehat{d}_t$ and $\widehat{rer}_t$ are predetermined variables, and $\widehat{\pi}_t^S$ and $\widehat{\pi}_t^N$ are jumping variables, we require two eigenvalues with a modulus of less than one and two with a modulus greater than one in order to ensure the *local uniqueness of equilibrium*. This implies that the remaining eigenvalue must lie inside the unit circle:

$$det(\Psi_{22} - \lambda I) = |\Psi_{dd} - \lambda|$$

Thus, $\lambda_4 = \Psi_{dd} = \frac{(1-\phi_T)}{\beta}$ is in modulus less than one if $\phi_T > 1 - \beta$.

D.1.2 Case II: $\phi_\pi < 1$

Now, we need 2 roots inside and 1 root outside the unit circle. In contrast to the previous case, we now have that:

$$1 + A_2 + A_1 + A_0 = (\phi_\pi - 1)\frac{\widetilde{\kappa}}{\beta}\frac{\eta}{\omega} < 0$$

As previously stated, for $\phi_\pi > 0$ we know that

$$-1 + A_2 - A_1 + A_0 < 0$$

Now, for the case where $\phi_\pi \in (0, 1)$, it is sufficient to prove that for all the intervals

$$A_2^2 - A_0A_2 + A_1 - 1 > 0$$

[19] This holds whether $A_2^2 - A_0A_2 + A_1 - 1 > 0$ or $A_2^2 - A_0A_2 + A_1 - 1 < 0$; we avoid discussion of certain non-generic boundary cases.

If $\phi_\pi = 0$:

$$\left(\frac{1}{\beta} + \frac{\omega K}{\beta} + 1\right)^2 - \frac{\omega K}{\beta} = \frac{1}{\beta} + \left(1 - \frac{1}{\frac{1}{\beta} + \frac{\omega K}{\beta} + 1}\right)\frac{\omega K}{\beta} + 1 > 0.$$

We previously proved that the quadratic function is positive when ϕ_π is valued at 1. Therefore, given that the quadratic function is a strictly concave function, we can state that:

$$A_2^2 - A_0 A_2 + A_1 - 1 > 0 \quad \forall\, \phi_\pi \in (0, 1)$$

Therefore, we have shown that when $\phi_\pi < 1$, the sufficient conditions are met to establish the existence of two roots inside the unit circle and one outside. Accordingly, the remaining eigenvalue must lie outside the unit circle, which holds if $\phi_T < 1 - \beta$.

Overall, we were able to prove that the same conditions for existence and uniqueness identified for the NT-based policy setup hold in the baseline alternative, as long as prices are sticky

D.2 Case B: $\theta_N = 0$

From the system (D.7)-(D.10), when $\theta_N = 0$, $K \to \infty$, and thus $\widehat{rer}_t = 0$ and $\widehat{\pi}_t^N = \widehat{\pi}_t^S$. Thus, we are left with

$$\widehat{\pi}_{t+1}^N = \phi_\pi \widehat{\pi}_t^N, \tag{D.16}$$

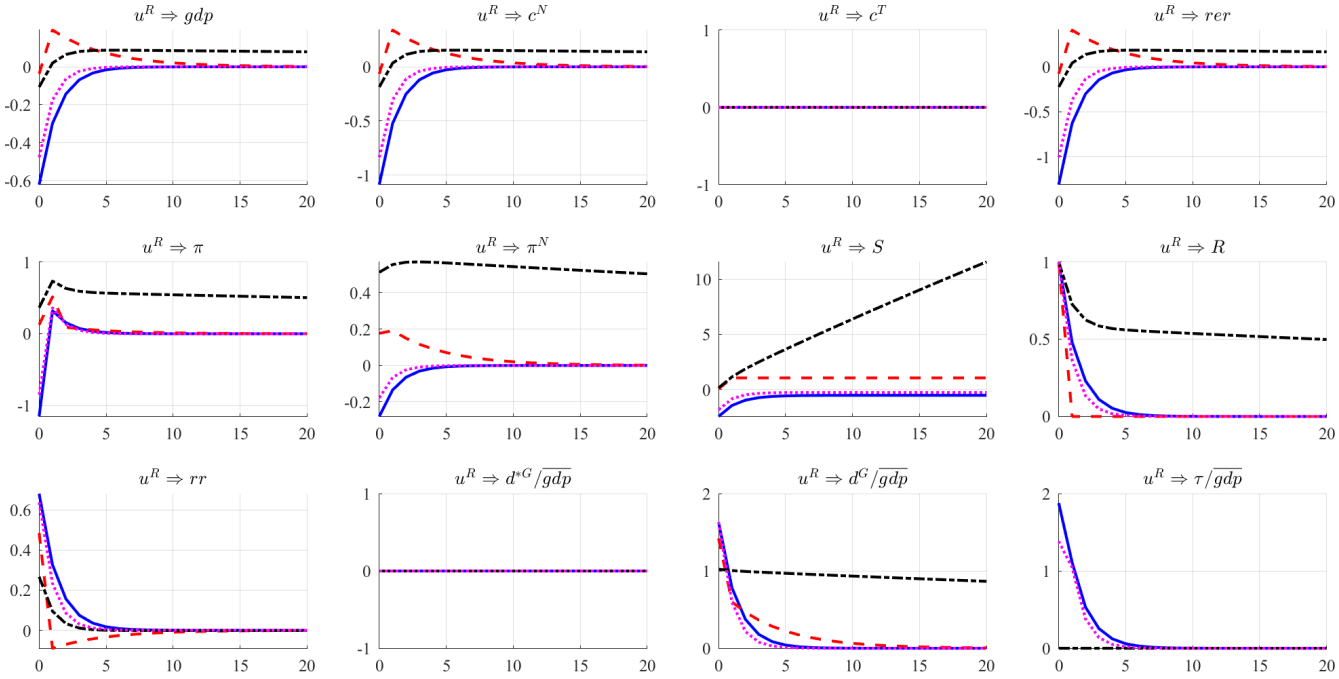
$$\widehat{d}_t = (1 - \Omega)\left[\phi_\pi - \frac{(1 - \phi_T)}{\beta}\right]\pi_t^N + \frac{(1 - \phi_T)}{\beta}\widehat{d}_{t-1}, \tag{D.17}$$

This is the same system already analyzed in Appendix C.2.2 for the NT-based policy setup. Therefore, the same conclusions hold.

E Robustness Exercises

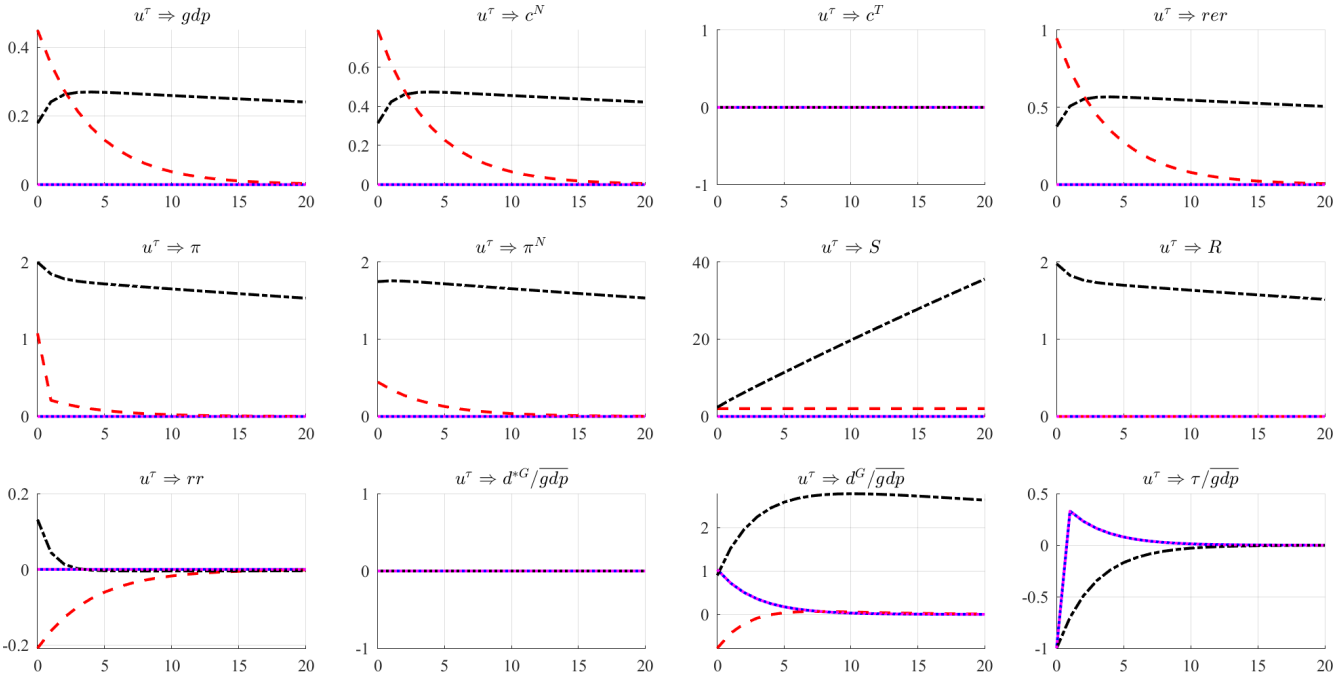
E.1 Monetary and fiscal shocks, the role of ϕ_π

Figure D.1: Responses to a monetary-policy-rate shock, Ricardian vs. Non-Ricardian, the role of ϕ_π.



Notes: The shock is an increase in u_t^R, with zero persistence, normalized to increase R by 1% on impact. Solid-blue and dashed red lines are the same as in Figure 2 for comparison (i.e. respectively, $\phi_\pi = 1.5, \phi_T = 1$, and $\phi_\pi = 0, \phi_T = 0$). In addition, the dashed-dotted black lines correspond to the case of $\phi_\pi = 0.99, \phi_T = 0$, while dotted-magenta lines use $\phi_\pi = 1.01, \phi_T = 1$. In all cases, $\overline{\Omega} = 0$. See Figure 2 for variables' definitions.

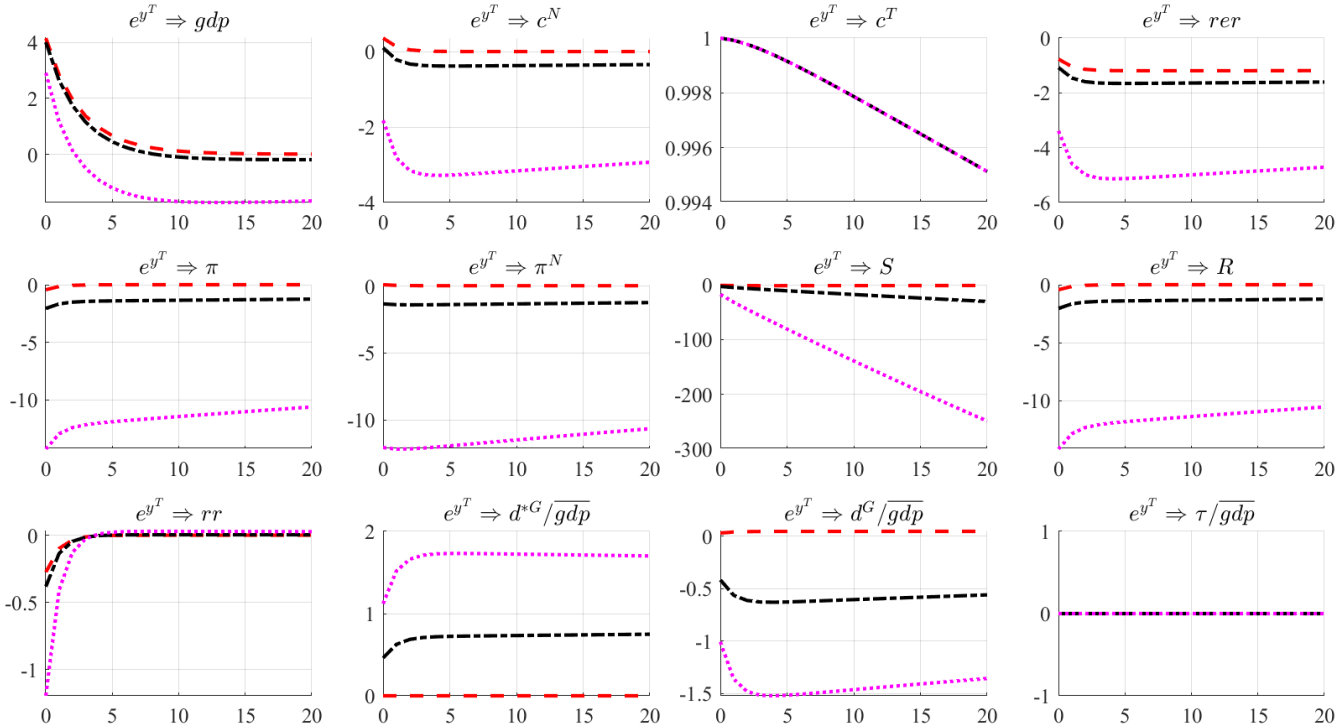
Figure D.2: Responses to a monetary-policy-rate shock, Ricardian vs. Non-Ricardian, the role of ϕ_π.



Notes: The shock is a drop in lump-sum taxes, normalized to represent 1% of steady-state GDP, with an autocorrelation of 0.7. All cases feature $\overline{\Omega} = 0$, differing in the fiscal and monetary configuration: solid-blue lines use $\phi_\pi = 1.5$ and $\phi_T = 1$, dashed-red lines $\phi_\pi = 0$,$\phi_T = 0$, dashed-dotted black lines correspond to the case of $\phi_\pi = 0.99$ and $\phi_T = 0$, and dotted-magenta lines use $\phi_\pi = 1.01$ and $\phi_T = 1$. See Figure 2 for for variables' definitions.

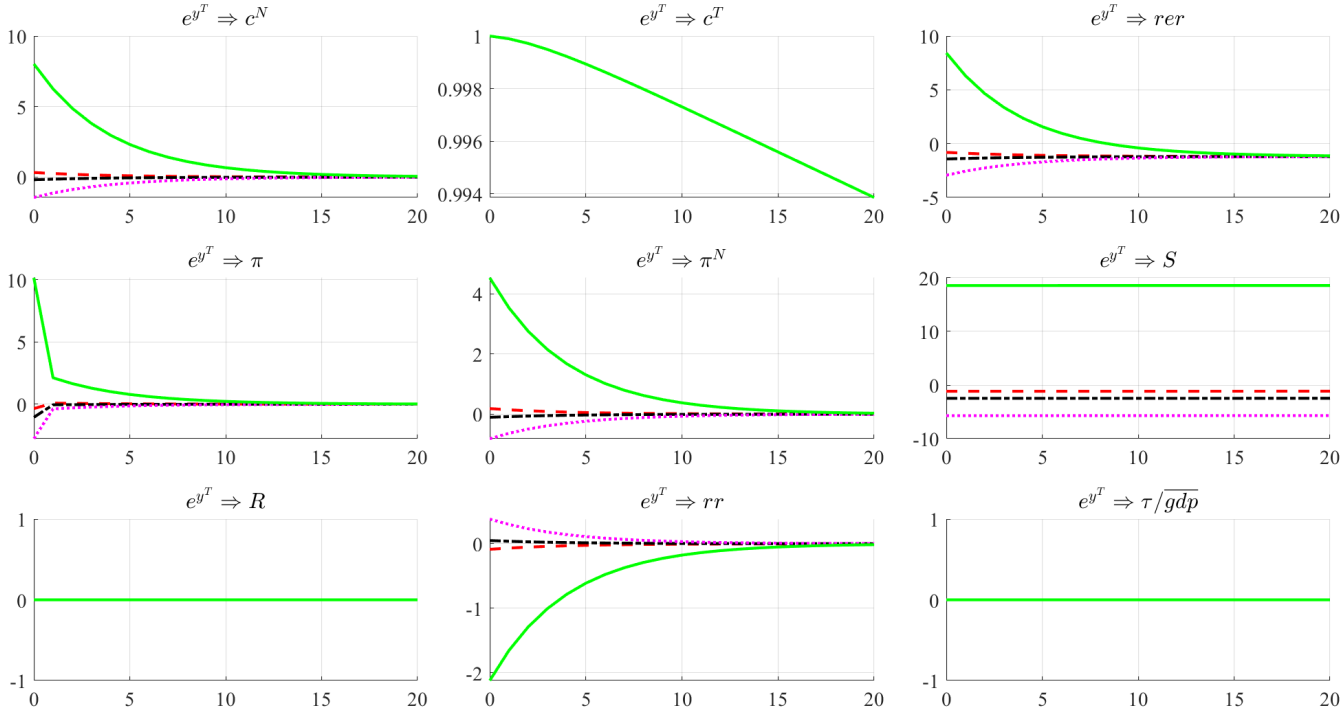
E.2 The effect of real shocks, alternative policy configurations

Figure D.3: Responses to a traded-output shock, Non-Ricardian, different values of $\overline{\Omega}$, with $\phi_\pi = 0.99$.



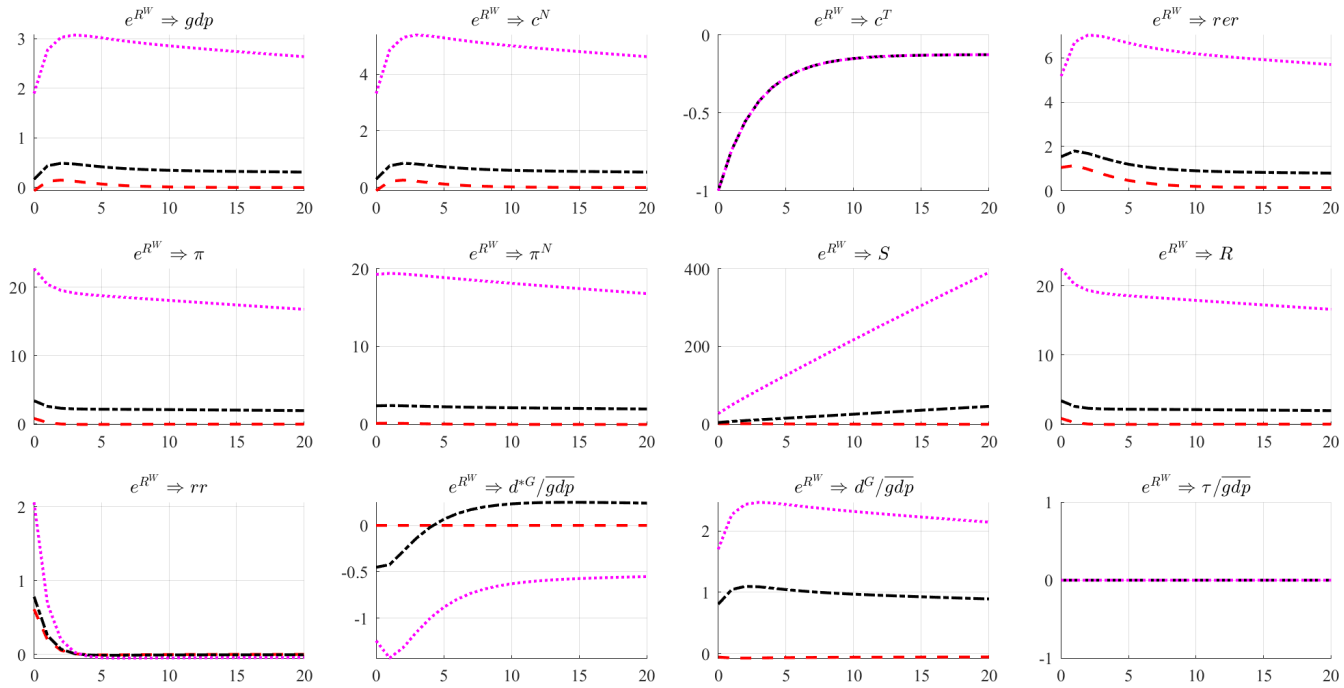
Notes: The figure is analogous to Figure 8, but using a value for $\phi_\pi = 0.99$.

Figure D.4: Responses to a traded-output shock, Non-Ricardian, different values of $\overline{\Omega}$.



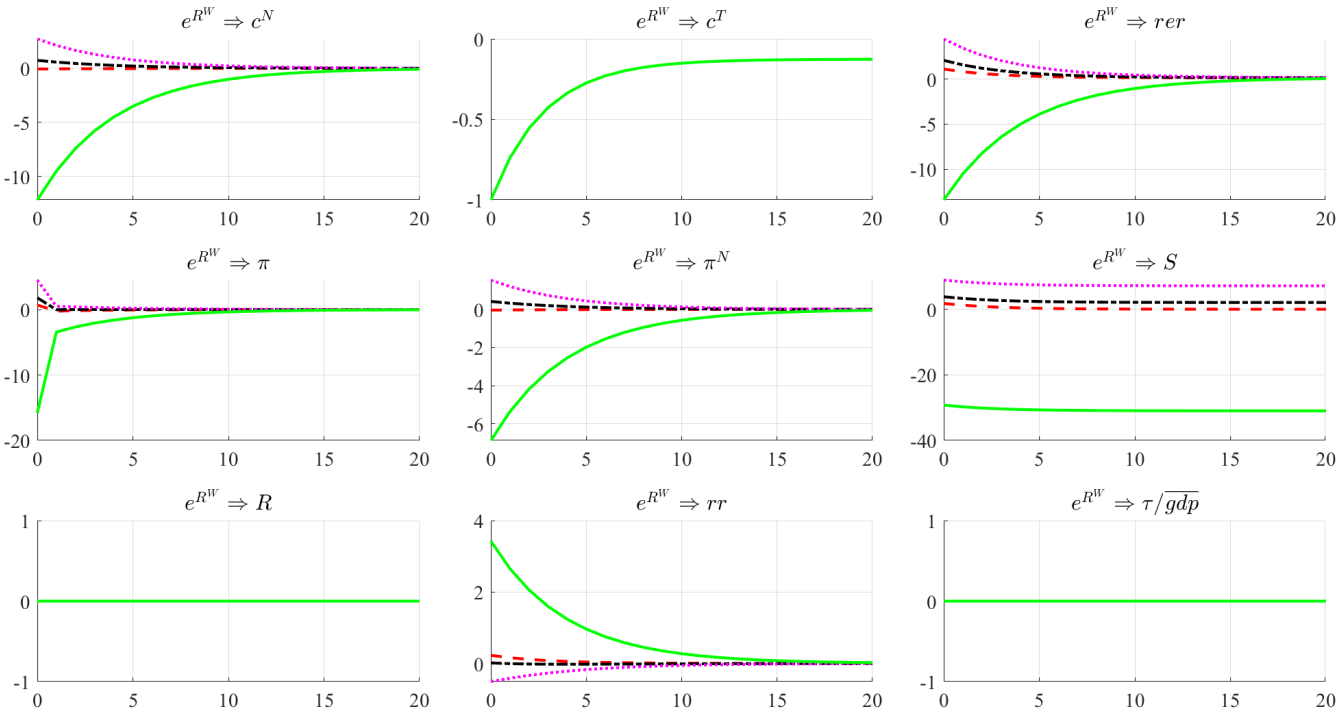
Notes: The figure is analogous to Figure 8, adding also the case of $\overline{\Omega} = 1$ in solid-green lines.

Figure D.5: Responses to a world interest-rate shock, Non-Ricardian, different values of $\overline{\Omega}$, with $\phi_\pi = 0.99$.



Notes: The figure is analogous to Figure 10, but using a value for $\phi_\pi = 0.99$.

Figure D.6: Responses to a world interest-rate shock, Non-Ricardian, different values of $\overline{\Omega}$.



Notes: The figure is analogous to Figure 10, adding also the case of $\overline{\Omega} = 1$ in solid-green lines.