Innovation Spillovers Along the Supply Chain

Preliminary and Incomplete (Link to Most Recent Version)

Tasaneeya Viratyosin*

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Abstract

How does innovation propagate through production networks? I show that supply chain relationships impact R&D expenditure, and conversely, innovation plays a crucial role in relationship formation. I introduce a novel dynamic, heterogeneous-firm model that endogenizes both supply chain relationships and innovation. Product complementarity generates positive assortative matching, complementarity in R&D investment, and heterogeneous innovation spillover—innovation shocks by partners predominantly benefit highly innovative firms. I validate these predictions using exogenous variation in R&D tax credits and unanticipated increases in market value attributed to patent issuances and show that innovation shocks to supply chain partners predict increases in R&D, market value, and R&D productivity even after accounting for idea diffusion. Specifically, exploiting differences in firm supply chain networks and citation networks and exogenous variation firm-level from R&D tax credit, I find that a 1 p.p. increase in innovation from a firm's downstream network corresponds to a 1.26 p.p. increase in firm R&D intensity due to economic relationships and a 0.27 p.p. increase due to idea diffusion. Consistent with the model, increases to firm value due to partners' innovation accrue disproportionately to innovative firms. Finally, innovation feedback between sectors linked along the supply chain amplifies and prolongs the propagation of aggregate shocks, accounting for over half of the response of aggregate output to a discount rate shock.

Keywords: Innovation, supply chains, product complementarity, spillover effect

*Wharton School, University of Pennsylvania. Email: tviratyo@wharton.upenn.edu. Website: https://www.viratyosin.net/. I am immensely grateful to my advisors, Jules van Binsbergen, Winston Wei Dou, and Thomas Winberry, for their encouragement and many helpful comments and suggestions.

1 Introduction

Understanding the fundamentals behind economic growth and effectively fostering innovation have been central priorities for both academic researchers and policymakers. A vast literature has focused on how horizontal strategic interactions and technological spillovers between competitors impact innovation and rents (Grossman and Helpman [1993], Aghion and Howitt [1992], Aghion et al. [2005], Klette and Kortum [2004]). However, the role of vertical relationships within supply chains in shaping innovation and valuations remains relatively under-explored. Nevertheless, there are many examples of CEOs, firms, and investors emphasizing the importance of supply chain partners to strategic success—notable examples include Corning Glass's contribution to the success of Apple's first iPhone, and the significant boost in profits and valuations experienced by producers of computer processors amid recent advances in artificial intelligence.

Stylized Facts

Using data from over 80,000 annual observations of supply chain linkages between public U.S. firms spanning 1980 to 2018, I present evidence suggesting a strong relationship in innovation intensity between firms and their major supply chain partners. First, the correlation between a firm's annual R&D intensity (defined as R&D expenditure divided by the previous year's revenue) and the average R&D intensity of its reported customers (downstream partners) is 42 percent. This relationship is robust to including year-by-sector (3-digit SIC) fixed effects and firm fixed effects, suggesting that the variation in firm R&D intensity is not only explained by sector-level variations, but also depends on individual supply chain relationships.Second, I present evidence of positive assortative matching in innovation intensity across supply chain partners. Since sectors vary in average R&D intensity, I first sort firms with positive R&D expenditure by R&D intensity within each 3-digit SIC sector, and for each link, compare each firm's rank with the rank of their

upstream/downstream partner. I find that a 10 point increase in the rank of a firm's downstream partner is on average associated with a 1 point increase in rank of the firm itself.

Empirical Analysis

Quantifying innovation spillovers across supply chains poses several challenges. First, supply chain relationships are endogenous and evolve over time, with downstream firms selecting upstream inputs given prices set by upstream firms, to maximize their profits and valuations. Previous efforts to quantify spillover across supply chain partners often implicitly assume observed supply chain links are static, at least in the short run, or occur for reasons less related to innovation, such as geographic proximity. This assumption ignores the impact of innovation spillovers from potential supply chain partners, either due to expectations of a future relationship or due to indirect spillovers from current partners. Moreover, ignoring this force may lead researchers to overestimate the impact of switching costs on relationship persistence.

Given that firm innovation may respond not only to innovation by existing partners but also by potential future partners, I define a firm's supply chain network in a fiscal year to include not only currently observed partners, but potential partners which I define as competitors to current supply chain partners and current supply chain partners to competitors. To disentangle the reduced form effect of downstream and upstream innovation from existing supply chain partners and a firm's broader supply chain network, I consider the impact of not only sales-weighted average innovation by the firm's broader network, but also the impact of difference between average innovation intensity by the firm's currents partners and its broader network on firm innovation outcomes.

Second, technology spillover may occur across firms connected along the supply chain not only because these firms are economically linked through the production network but also due to idea diffusion where firms learn from their supply chain partners. To disentangle spillover through the supply chain network attributed to economic links versus idea diffusion, I exploit the fact

that economic links and citation links may not coincide. Specifically, I defined spillover from idea diffusion from the firm's upstream and downstream network by weighting innovation intensity by firm-specific citation weights, the extent to which a firm cites each current and potential supply chain partner over the previous 5 years. I similarly define idea diffusion from current partners as the extent citation-weighted innovation intensity by current partners exceeds its wider network.

Third, since firms connected along the supply chain and citation network may innovate in tandem for reasons unrelated to these relationships—for example, given increased innovation opportunities impacting both—I instrument spillover on exogenous variation in innovation due to firm-level variation in exposure to state and federal R&D tax credits. Largely, I find that economic relationships are as or more important than idea diffusion in explaining technology spillover across supply chain partners, and that firms respond to innovation by both existing and potential partners. Overall, I find that a 1 p.p. increase in innovation from a firm's downstream network corresponds to a 1.26 p.p. increase in firm R&D intensity due to economic relationships and a 0.27 p.p. increase due to idea diffusion. Each percentage point by which the innovation intensity of a firm's current partners exceeds its broader network corresponds to an additional 0.85 p.p. increase due to economic relationships and a 0.18 p.p. increase due to idea diffusion. Moreover, I show that for innovating firms, downstream spillover predicts both contemporaneous increases in market valuation as measured by Tobin's Q and R&D productivity, defined as the total number of patents or total patent valuation over the next 3 years per dollar R&D spent in the current period.

I find spillover is heterogeneous and dependent on firm innovation behavior: firms which do not innovate do not experience increases to market value over the period. I further validate these results by examining the impact of realizations to R&D expenditure in the form of patent issuance to the firm and corresponding market valuations as measured by Kogan et al. [2017]. I find similarly, that downstream innovation increases firm R&D and Tobin's Q, and the market value of

contemporaneous patents issued, with spillover largely attributable to economic links from a firm's wider downstream network. Spillover is similarly heterogeneous, with increases to Tobin's Q, future sales, and future capital expenditure largely experienced by innovating firms.

Model and Quantitative Exercise

Having established innovation propagation across supply chains in both directions due to both current and potential future economic relationships. I introduce a novel, dynamic heterogeneous firm model that endogenizes both the intensity of supply chain relationships and the innovation decisions of firms operating within a supply chain. In the model, firms in the downstream sector vary in product quality and compete monopolistically to produce differentiated goods using a variety of inputs supplied by the upstream sector. Both downstream intermediate goods and upstream inputs are imperfectly substitutable and vary in quality. Complementarity in quality endogenously induces both positive assortative matching and heterogeneous spillover consistent with the data.

Having established empirically that upstream innovation induces downstream innovation and vice versa, this introduces another wrinkle—reduced-form estimates will not capture the full extent of these dynamic feedback effects or how innovation reshapes strategic interactions among competing firms within the same sector. The model enables me to address this and quantify the general equilibrium effect of sector-level or aggregate shocks on innovation propagation attributable to sorting and feedback.

In the model, given the empirical evidence, I focus on the impact of economic relationships in the production network between supply chain partners rather than idea diffusion. A critical assumption of the model is that demand for a downstream firm's product increases, with complementarity, in both the quality of the downstream firm and the aggregate quality of its chosen upstream inputs. Referencing the previous example, this reflects the notion that a fast processor and a premium screen are more valuable within the same product. Downstream firms therefore

face a tradeoff: increasing upstream quality increases demand but is more expensive. Product complementarity across sectors endogenously produces positive assortative matching: relative to lower quality downstream firms, higher quality downstream firms demand a greater share of inputs from higher quality upstream firms. In addition, firms in both sectors have access to decreasing returns to scale quality-improving innovation technology and operate over multiple periods. Under product complementarity, realized and anticipated improvements in upstream quality strengthen downstream firms' incentives to innovate. Likewise, downstream quality improvements and innovation increase upstream returns from innovation. These strategic interactions create spillovers in innovation between upstream and downstream firms.

Furthermore, the model provides a useful framework to evaluate how spillovers from innovation and quality improvements—originating from both supply chain partners and industry competitors—vary across firms and impact equilibrium outcomes. Due to complementarity in quality along the supply chain and imperfect substitution between upstream inputs, in the short run, profits and innovation of higher quality downstream firms increase following a positive shock to upstream quality. In contrast, lower quality firms face higher input costs and benefit less from the complementarity, eroding profits and valuations. In the long run, increased downstream innovation, in turn, raises relative demand for higher quality inputs, disproportionately increasing demand and markups for upstream firms which produce high quality inputs.

This offers two important new insights. First, the propagation of upstream innovation shocks to downstream firms is not uniform across firms: the shock predominantly benefits high quality downstream firms and may negatively affect low quality firms. Second, while the innovation shock erodes prices and profits higher quality upstream competitors in the short run, in the long run, the responding increase in downstream innovation bolsters profits of high quality upstream firms. This dynamic offers a new mechanism for innovation spillover between industry peers: over longer

horizons, high quality upstream firms benefit from innovations by competitors—not due to technological spillover in the classical sense which arises from the non-rivalry of ideas—but because these innovations induce existing and prospective supply chain partners to innovate in tandem. Similarly, an unexpected shock to the quality of downstream firms, increases relative demand of higher quality upstream inputs, raising relative valuations and strengthening their incentives to innovate. Over the long run, this gradually increases downstream profits, particularly for higher quality downstream firms which benefit most from upstream innovation.

Taken in tandem, the model's insights suggest that when both sectors innovate, a shock triggering innovation in one or both sectors can be amplified through supply chain linkages. Indeed, following a transitory discount rate shock that increases innovation, the resulting higher levels of aggregate upstream and downstream quality sustains elevated levels of innovation and increases output even after the shock dissipates. This self-reinforcing dynamic is critical to the prolonged impact of the discount rate shock on output. Compared to counterfactuals where only one sector innovates and the quality distribution of the other sector remains fixed, simultaneous innovation across both sectors amplifies and extends the impact of the shock on output.

Literature Review

The paper contributes to several strands of literature. First, the paper sheds light on spillovers between supply chain partners and industry peers. As in Cohen and Frazzini [2008] and Barrot and Sauvagnat [2016], the paper studies spillover between supply chain partners. While these papers largely find positive spillover between firms linked in the production network, this paper provides new evidence and theory for heterogeneous spillover across the supply chain. Moreover, it sheds new light on how upstream innovation may increase input specificity and supplier market power from the perspective of downstream firms. The paper finds evidence of amplification and feedback of innovation across and within sectors, analogous to Dou et al. [2022]'s finding for financial distress.

The paper also contributes to the literature's understanding of technology spillover. In his seminal paper, Romer [1986], identified technology spillover through the non-rivalry as an important engine for economic growth. Because ideas are non-rival, firms do not internalize the positive externalities their innovation has on future innovation. Several papers including Bloom et al. [2013] quantify this impact using patent citation networks as proxies for technology diffusion. This paper introduces a novel mechanism for innovation spillover and amplification that does not rely on idea diffusion. Rather, firm innovation in one sector propagates both vertically and horizontally through supply chains, incentivizing supply chain partners to increase innovation. These innovations increase the aggregate quality of supply chain partners faced not only by innovating firms but also by industry peers.

2 Empirical Evidence

2.1 R&D Spillover

Using annual data on over 80,000 supply chain relationships between US public firms from 1980 to 2018 collected from Compustat and Factset and annual financial statement data from Compustat, I compare innovation for each firm with that of their supply chain partners. These relationships correspond to 46,532 supplier-year observations reporting an average of 5.6 customers and 24,657 customer-year observations reporting an average of 64.7 suppliers. In three-quarters of these relationships one or both parties report positive R&D expenditure and in over half both report positive R&D expenditure.

Two stylized facts provide compelling evidence of a connection between the innovation behavior of supply chain partners First, as demonstrated in Figure 1, there is a strong link between firm R&D intensity and the R&D intensity of its supply chain partner: across the sample, firm R&D

intensity and that of downstream partners display a 0.42 correlation. This relationship is robust to including firm-by-sector fixed effects, where sector is defined at the 3-digit SIC level. Second, within sector, observed matches exhibit positive assortative matching—firms are more likely to be partnered with similarly R&D intense firms. Figure 2 shows that firms' annual percentile ranking over R&D intensity is increasing in the corresponding rankings of their supply chain partners. Controlling by sector and year fixed effects, a 10 point increase in ranking by a firm corresponds to a 1 point increase in the average ranking of its upstream partners.

Since supply chains may evolve, innovation may spill over from not only current supply chain partners but also potential future partners. Forward looking R&D investment may respond to innovation by both existing partners and by potential partners in the anticipation of future supply chain relationships. I take a simple approach to identifying potential partners. For each firm, I define a firm's downstream network for that year to include not only current or observed downstream partners, but to also potential downstream partners, which I define as competitors of current downstream partners and the current downstream partners of competitors, where competitors are defined as firms operating in the same 3-digit SIC sector. I similarly define a firm's upstream network as not only a firm's current upstream partners, but also any competitors to their upstream partners or upstream partners of the firm's competitors. To identify the innovation spillover from a firm's downstream network and current downstream partners, I first define $DownSpill_{it}$ and $DownSpill0_{it}$ as the sales-weighted average R&D intensity of firm i's downstream network J'_{it} and current downstream partners $J_{it} \subseteq J'_{it}$, respectively. Because I am interested in spillover due to the economic relationship, I weigh the R&D intensity of each downstream firm by the average of their sales over the previous 5 years.

$$DownSpill_{it} = \frac{\sum_{j \in J'_{it}} Sales_{jt} RdIntensity_{jt}}{\sum_{j \in J'_{it}} Sales_{jt}}, \qquad DownSpill0_{it} = \frac{\sum_{j \in J_{it}} Sales_{jt} RdIntensity_{jt}}{\sum_{j \in J_{it}} Sale_{jt}}$$

Next, to isolate the innovation spillover from current relationships specifically, I define $DownstreamSpillCurre$ reflect the extent innovation intensity by of a firm's current downstream partners exceeds the innovation intensity of its downstream network, the wider set of both current and potential downstream partners.

$$DownSpillCurrent_{it} = DownSpill0_{it} - DownSpill_{it}$$

To estimate the impact of downstream innovation on firm R&D intensity, I regress firm R&D intensity $RdIntensity_{it}$ on contemporaneous spillover from i's downstream network and current partners, $DownSpill_{it}$ and $DownSpillCurrent_{it}$, respectively, controlling for firm and year fixed effects, δ_i and η_t, respectively, and vector of firm controls Γ.

$$RdIntensity_{it} = \beta_0 + \beta_1 DownSpill_{it} + \beta_2 DownSpillCurrent_{it} + \Gamma X_{it} + \delta_i + \eta_t + \epsilon_{it} \tag{2.1}$$

As shown in Column 1 of Table I, a 1 p.p. increase in innovation from current downstream partners alone corresponds to a 45 b.p. increase in firm R&D intensity. Column 2 of Table I presents the regression results of specification 2.1. Here, a 1 p.p. increase in innovation from a firm's downstream network corresponds to a 35 bp increase in firm R&D intensity. However, individual relationships are also important—each percentage point by which innovation from a firm's current downstream partners exceed its wider network corresponds to an additional 46 p.p. increase in a firm's innovation.

One challenge to identification is that technology spillover may occur across firms connected along the supply chain not only because these firms are economically linked through the production network but because firms learn from their supply chain partners—idea diffusion. To disentangle innovation spillover from supply chain partners due to economic links versus idea diffusion, I ex-

ploit the fact that while production networks and citation networks, which are often used as proxies for idea diffusion, may coincide, the intensity of a firm's connection to their supply chain partners through the citation network and its connection through the production network may differ. Specifically, for firms who filed patents in over the past 5 years, I similarly define downstream spillover from idea diffusion, $DownSpillCites0_{it}$ and $DownSpillCites_{it}$, as the citation weighted average of R&D intensity from current supply chain partners and potential supply chain partners, respectively, where $Cites_{kit}$ is the 5-year average number of citations by firm i of patents produced by firm i and similarly define $DownSpillCitesCurrent_{it}$ as the difference between $DownSpillCites0_{it}$ and $DownSpillCites_{it}$.

$$DownSpillCites_{it} = \frac{\sum_{j \in J'_{it}} Cites_{jit} RdIntensity_{jt}}{\sum_{j \in J'_{it}} Cites_{jit}}, \qquad DownSpillCites0_{it} = \frac{\sum_{j \in J_{it}} Cites_{jit} RdIntensity_{jt}}{\sum_{j \in J_{it}} Cites_{jit}}$$

Column 3 presents the regression results for the following equation.

$$RdIntensity_{it} = \beta_0 + \beta_1 DownSpill_{it} + \beta_2 DownSpillCurrent_{it} + \beta_3 DownSpillCites_{it} \tag{2.2}$$

$$+\beta_4 DownSpillCitesCurrent_{it} + \Gamma X_{it} + \delta_i + \eta_t + \epsilon_{it}$$

After accounting for idea diffusion, a 1 p.p. increase in innovation from a firm's downstream network corresponds to 0.34 p.p. increase in R&D intensity from economic links and 0.22 p.p. from idea diffusion. Each p.p. the firm's current downstream partners exceeds its downstream network corresponds to an additional 0.45 p.p. increase in R&D intensity from economic links and 0.07 p.p. from idea diffusion from current downstream partners.

Another threat to identification is that the observed regression coefficients may reflect the impact of correlated shocks, for example, arising from new research opportunities, which induce connected

firms to innovate more in tandem. To address this concern, following the literature on technology spillovers, I exploit firm-level variation in R&D user costs arising from changes in firm and state level tax credits, and instrument firm R&D expenditure on fluctuations in user costs. Appendix includes the first stage and provides additional detail on the construction of R&D user costs. Columns 4 and 5 of Table I repeat the analysis in Column 2 and 3 for spillovers from economic links alone and including idea diffusion, respectively, instrumenting downstream R&D intensity with exogenous variation in R&D user costs and including controls for a firm's own exposure to R&D user costs. As shown in Column 5, a 1 p.p. increase in innovation by a firm's downstream network results in a 1.26 p.p. and 0.27 p.p. increase in R&D intensity due to economic links and idea diffusion, respectively. Each p.p. of innovation by a firm's current downstream partners which exceeds its network, results in a 0.84 p.p. and 0.18 p.p. increase in R&D intensity due to economic links and idea diffusion, respectively.

Not only does downstream spillover through the supply chain increase firm innovation, but we can evaluate whether, consistent with the model, downstream innovation contributes to R&D productivity and firm valuation. I proxy for R&D productivity in two ways. First, I define R&D productivity, $RdProductivity1_{it}$, as the average number of patents filed over the subsequent three years, $t+1$ to $t+3$, divided by R&D expenditure in year t. Columns 1 and 2 of Table II present the impact of downstream innovation on log R&D productivity, where downstream innovation is instrumented using exogenous variation in R&D tax credits as before. The regression results in Column 2 suggest that while downstream innovation increases R&D productivity, this is largely attributable to the economic relationship rather than due to idea diffusion. A 1 p.p. increase in innovation by a firm's downstream network contributes to a 2.1 percent increase in patents per dollar of R&D spent. Each p.p. of innovation by a firm's current downstream partners which exceeds its downstream network, contributes an additional 1.3 percent. In contrast, downstream spillover

due idea diffusion does not significantly impact R&D productivity. Since patents vary in value, in Columns 3 and 4, I instead define R&D productivity, $RdProductivity2_{it}$, as the total value of patents filed over the subsequent three years, $t+1$ to $t+3$, divided by R&D expenditure in year t, using patent valuations estimated by Kogan et al. [2017]. As before, a 1 p.p. increase in innovation by a firm's downstream network corresponds to a 2.3 percent increase in R&D productivity. However, the effect of a firm's current downstream partners on R&D productivity as defined using patent valuations is no longer significant.

Next, in Columns 5 and 6 of Table II, I evaluate the impact of downstream innovation on Tobin's Q for firms which invest in R&D expenditure, replacing the dependent variable with $logTobinQ_{it}$. While a 1 p.p. increase in innovation by a firm's downstream network corresponds to a 1.2 percent increase in Tobin's Q, this is largely attributable to spillover from economic relationships rather than idea diffusion. Moreover, the impact of innovation by firm's current downstream partners alone on Tobin's Q is insignificant. Taken together this suggests that for innovative firms, variation in firm valuation from downstream relationship arises, not due to idea diffusion, but economic relationships with the downstream network as a whole, not only existing partners. Finally, I evaluate the impact of downstream innovation on Tobin's Q for non-innovative firms, instead, defined as firms which do not invest in R&D, focusing only on the impact of economic links, since the firms are unlikely to produce patents. For these firms, downstream innovation, by either its wider downstream network or current downstream partners, has no significant impact on valuation, suggesting that a firm's own innovation activity impacts the transmission of innovation shocks from downstream partners.

These results are robust to defining $DownSpill_{it}$ and $DownSpillCurrent_{it}$using equal-weighted rather than sales-weighted R&D intensity and are robust to bootstrapping standard errors.

To identify the impact of innovation spillover from upstream firms I similarly define $UpSpill_{it}$, $UpSpillCurrent_{it}$, $UpSpillCites_{it}$, and $UpSpillCitesCurrent_{it}$ to reflect the impact of innovation

shocks through the supply chain and through idea diffusion by firms upstream to firm i in year t. Tables III and IV present the analogous results for the impact of upstream spillover.

2.2 Patent Valuations

In this section, instead of looking at variation in R&D investment, I focus on variation in the realized outcomes from past R&D spending, measured by the value of patents issued to firms. For this, I lean on the data of patent valuations from Kogan et al. [2017] which uses daily firm-level increases to equity prices upon the issuance of new patents to estimate the market value of each individual patents. A key assumption is that each patent issuance reflects new information on the realization of firm innovation not previously reflected in the firm's market value. For each firm i, I define a firm's innovation shock, $InnovShock_{it}$, as the total market value of all patents issued to firm i in period t, $PatentValue_{it}$, divided by the market value of firm i as of the beginning of the period, $MarketValue_{it-1}$, winsorized at the 1 percent level.

$$InnovShock_{it} = \frac{PatentValue_{it}}{MarketValue_{it-1}}$$

Figure **??** presents the distribution of firm innovation shocks for the set of firms who were issued at least one patent during the period. On average these firms experience a 1.7 percent increase to market value attributable to issued patents each month. The distribution is naturally right skewed with the firm's innovation shock reflecting the change in the firm's market value attributed to patent issuances.

I define firm i's exposure to downstream innovation shocks by current partners, $DownSpill0_{it}$ as the sales-weighted average innovation shock experienced in year t by its current downstream firms $j \in J_t$, where J_t is these set of firms which invest in R&D known to be downstream of firm i as of the beginning of year t. I similarly define $DownSpill_{it}$ as the sales-weighted average of innovation

shocks experienced by all firms in $i's$ downstream network $J'_t \supset J_t$ where J'_t is defined as in the previous section to include both current and potential downstream partners. As before, I define $DownSpillCurrent_{it}$ to measure the extent which innovation shocks by a firm's current partners exceed that of its downstream network.

$$DownSpill0_{it} = \frac{\sum_{j \in J_t} Sales_{jt} InnovShock_{jt}}{\sum_{j \in J_t} Sales_{jt}}, \qquad DownSpill_{it} = \frac{\sum_{j \in J'_t} Sales_{jt} InnovShock_{jt}}{\sum_{j \in J'_t} Sales_{jt}}$$

$$DownSpillCurrent_{it} = DownSpill0_{it} - DownSpill_{it}$$

As in the previous section, to define spillover from downstream firms $j \in J_{it}$ to firm i due to idea diffusion, I weigh downstream innovation shocks by i's dependence on each firm j as a source of technology spillover using the 5-year average of citations of patents filed by firm i of patents filed by firm j. I similarly define $DownSpillCitesCurrent_{it}$ as the difference between $DownSpillCites0_{it}$ and $DownSpillCites_{it}$.

$$DownSpillCites_{it} = \frac{\sum_{j \in J'_{it}} Cites_{jit} InnovShock_{jt}}{\sum_{j \in J'_{it}} Cites_{jit}}, \qquad DownSpillCites0_{it} = \frac{\sum_{j \in J_{it}} Cites_{jit} InnovShock_{jt}}{\sum_{j \in J_{it}} Cites_{jit}}$$

To understand how technology shocks from downstream supply chain partners due to economic links and idea diffusion impact future R&D, I regress future log R&D expenditure, $LogR\&D_{it+1}$ on $DownSpill_{it}$ and $DownSpillCurrent_{it}$ to reflect spillover through economic links and $DownSpillCites_{it}$ and $DownSpillCitesCurrent_{it}$ to reflect spillover from idea diffusion. I include firm and year fixed effects, δ_i and η_t, respectively, and include vector of controls, Γ, for period t and $t-1$ log R&D and log sales.

$$LogR\&D_{it+1} = \beta_0 + \beta_1 DownSpill_{it} + \beta_2 DownSpillCurrent_{it} + \beta_3 DownSpillCites_{it} \qquad (2.3)$$

$$+\beta_4 DownSpillCitesCurrent_{it} + \Gamma X_{it} + \delta_i + \eta_t + \epsilon_{it}$$

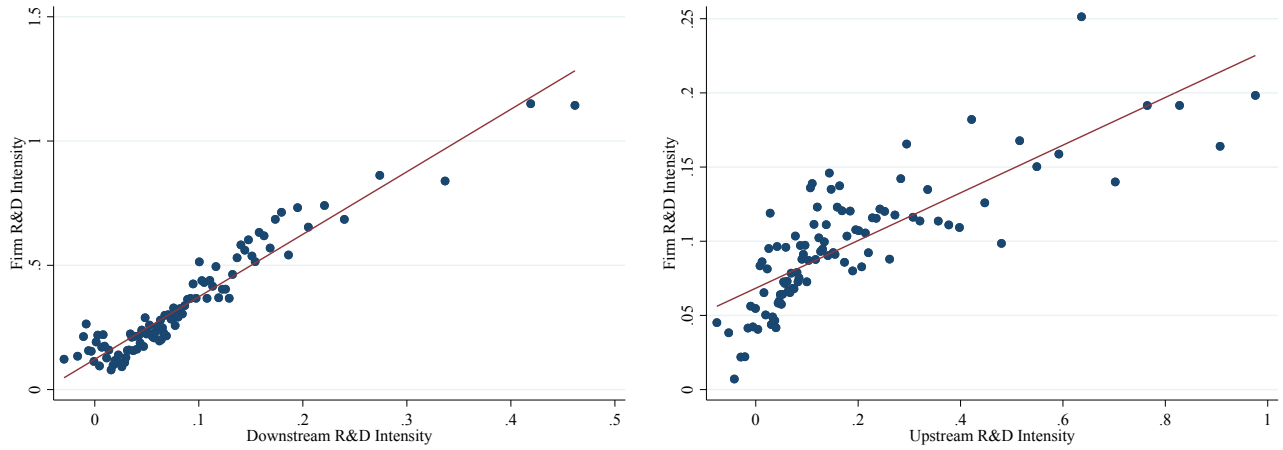
Columns 1 and 2 of Table V present the regression results. A 1 p.p. increase in the average firm value of its downstream network due to innovation, correspond to a 0.91 percent increase in R&D from the economic relationship and a 0.42 percent increase from idea diffusion. Each additional p.p. increase to firm value from current partners corresponded to a 0.12 percent increase in R&D due to idea diffusion. Columns 3 and 4 consider whether returns to innovation increase with productivity improvements by supply chain partners, by replacing the dependent variable of Equation 2.2 with, $InnovShock_{it}$, the increase to firm i's market value attributable to patents in period t. A 1 p.p. increase in average value of a firm's downstream network attributable to innovation corresponds to a 0.20 p.p. increase in firm value attributable to patents issued in period t due to economic links and a 0.23 p.p. increase from idea diffusion. Finally, columns 5 and 6 consider the impact on Tobin's Q. Spillover from the downstream network through economic links is both economically and statistically significant. Each p.p. increase in downstream value attributable to patents corresponds to a 1.72 percent increase in firm value. In contrast, each p.p. increase in a firm's own valuation attributable to patents corresponds to only 0.68 percent increase to firm value. Table VI presents the analogous results for spillover from upstream partners, with qualitatively similar results.

While the previous results demonstrate that economic links are critical to technology spillover from supply chain partners, the following results suggest a firm's own innovation decisions also impact spillover. Table VII interacts $DownSpill_{it}$ and $DownSpillCurrent_{it}$ with a dummy $PositiveR\&D_{it}$ if the firm itself invests in R&D and considers the impact of technology spillover on changes to firm value over period t, log sales in period $t+1$, and capital expenditure in period $t+1$. Columns 1 and 2 suggest that the positive impact of technology spillover accrue only to firms which themselves innovation. In fact, downstream innovation shocks negatively predict future capital expenditure for

firms which do not innovate.

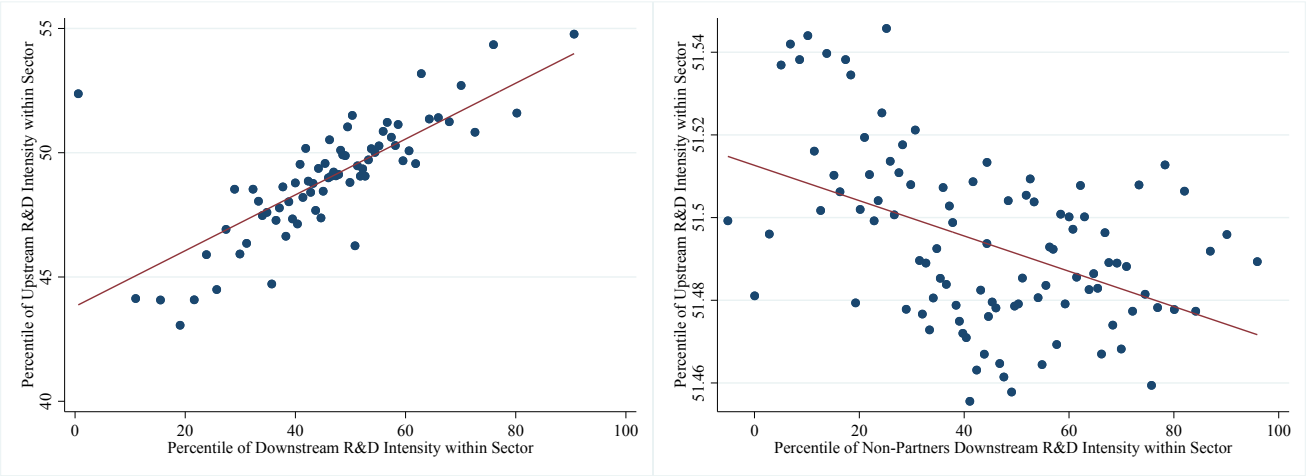
2.3 Tables and Figures

FIGURE 1 – Firm R&D Intensity compared to Average R&D Intensity of Downstream and Upstream Partners



Note:

FIGURE 2 – Firm R&D Intensity within Sector Compared to R&D Intensity of Observed Partners



Note:

Table I – Impact of Downstream R&D on Firm R&D Intensity

	(1) RdIntensity		(2) RdIntensity		(3) RdIntensity		(4) RdIntensity		(5) RdIntensity	
DownSpill0	0.453***	(0.037)								
DownSpill			0.350***	(0.048)	0.337***	(0.049)	1.278***	(0.152)	1.263***	(0.152)
DownSpillCurrent			0.461***	(0.037)	0.454***	(0.038)	0.853***	(0.140)	0.849***	(0.140)
DownSpillCites					0.222***	(0.064)			0.273***	(0.064)
DownSpillCitesCurrent					0.080	(0.060)			0.178***	(0.059)
r2	0.869		0.869		0.869		0.868		0.868	
n	19276		19276		19276		19276		19276	
F	1215		915		613		595		449	
Sample	R&D¿0		R&D¿0		R&D¿0		R&D¿0		R&D¿0	
FE	Firm, Year		Firm, Year		Firm, Year		Firm, Year		Firm, Year	
Instrumented	No		No		No		Yes		Yes	

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Note:

Table II – Impact of Downstream R&D on Firm R&D Productivity and Valuation

	(1) RdProductivity1	(2) RdProductivity1	(3) RdProductivity2	(4) RdProductivity2	(5) TobinsQ	(6) TobinsQ	(7) TobinsQ
DownSpill	2.141***	2.157***	2.233***	2.280***	1.160***	1.204***	-0.948
	(0.426)	(0.427)	(0.831)	(0.831)	(0.391)	(0.392)	(0.992)
DownSpillCurrent	1.326***	1.335***	0.735	0.760	-0.025	-0.000	-1.106
	(0.409)	(0.410)	(0.798)	(0.798)	(0.378)	(0.378)	(0.991)
DownSpillCites		-0.163		-0.471*		-0.324**	
		(0.128)		(0.250)		(0.130)	
DownSpillCitesCurrent		-0.015		-0.063		-0.253**	
		(0.098)		(0.191)		(0.118)	
r2	0.816	0.816	0.767	0.767	0.676	0.676	0.712
n	4285	4285	4285	4285	10406	10406	6688
F	23	18	15	12	49	37	14
Sample	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D==0
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year
Instrumented	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Note:

Table III – Impact of Upstream R&D on Firm R&D Intensity

	(1) RdIntensity	(2) RdIntensity	(3) RdIntensity	(4) RdIntensity	(5) RdIntensity
UpSpill0	0.018***				
	(0.003)				
UpSpill		0.020**	0.019**	0.008**	0.008**
		(0.009)	(0.009)	(0.003)	(0.003)
UpSpillCurrent		0.018***	0.018***	0.003***	0.003***
		(0.003)	(0.003)	(0.001)	(0.001)
UpSpillCites			0.015		0.021
			(0.015)		(0.015)
UpSpillCitesCurrent			0.007		0.012
			(0.010)		(0.010)
r2	0.910	0.910	0.910	0.909	0.909
n	9114	9114	9114	9114	9114
F	598	448	299	294	221
Sample	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D¿0
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year
Instrumented	No	No	No	Yes	Yes

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Note:

Table IV – Impact of Upstream R&D on Firm R&D Productivity and Valuation

	(1) RdProductivity1	(2) RdProductivity1	(3) RdProductivity2	(4) RdProductivity2	(5) TobinsQ	(6) TobinsQ	(7) TobinsQ
UpSpill	0.038***	0.037***	0.313***	0.309***	0.254***	0.251***	0.112***
	(0.010)	(0.010)	(0.029)	(0.029)	(0.028)	(0.028)	(0.022)
UpSpillCurrent	0.000	0.000	0.003	0.004	0.039***	0.038***	0.018*
	(0.003)	(0.003)	(0.007)	(0.007)	(0.006)	(0.006)	(0.010)
UpSpillCites		-0.006		-0.293**		0.121	
		(0.045)		(0.129)		(0.076)	
UpSpillCitesCurrent		-0.037		-0.265***		0.048	
		(0.024)		(0.070)		(0.048)	
r2	0.743	0.743	0.838	0.839	0.753	0.753	0.755
n	4257	4257	4257	4257	3655	3655	4440
F	17	13	49	39	34	26	17
Sample	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D==0
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year
Instrumented	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Note:

Table V – Impact of Downstream Technology Shocks on Firm R&D and Valuation

	(1) LogR&D(t+1)	(2) LogR&D(t+1)	(3) InnovShock	(4) InnovShock	(5) ChgTobinsQ	(6) ChgTobinsQ
DownSpill	0.990***	0.909**	0.238***	0.200***	1.751***	1.722***
	(0.354)	(0.354)	(0.070)	(0.069)	(0.463)	(0.464)
DownSpillCurrent	0.073	0.039	0.014	-0.002	-0.002	-0.016
	(0.051)	(0.052)	(0.010)	(0.010)	(0.061)	(0.063)
DownSpillCites		0.416***		0.235***		-0.090
		(0.065)		(0.013)		(0.085)
DownSpillCitesCurrent		0.124**		0.056***		0.081
		(0.056)		(0.011)		(0.069)
InnovShock					0.663***	0.677***
					(0.045)	(0.046)
r2	0.975	0.975	0.698	0.706	0.546	0.546
n	16356	16356	17499	17499	7636	7636
F	3887	2930	516	458	477	383
Sample	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D¿0
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Note:

TABLE VI – Impact of Upstream Technology Shocks on Firm R&D and Valuation

	(1) LogR&D(t+1)	(2) LogR&D(t+1)	(3) InnovShock	(4) InnovShock	(5) ChgTobinsQ	(6) ChgTobinsQ
UpSpill	0.327*	0.144	0.227***	0.206***	0.437**	0.437**
	(0.197)	(0.201)	(0.045)	(0.046)	(0.205)	(0.206)
UpSpillCurrent	0.016	-0.025	0.015**	0.017*	0.039	0.013
	(0.030)	(0.039)	(0.007)	(0.009)	(0.033)	(0.042)
UpSpillCites		0.215***		0.023*		-0.019
		(0.052)		(0.012)		(0.056)
UpSpillCitesCurrent		0.015		-0.013		0.057
		(0.047)		(0.011)		(0.047)
InnovShock					0.463***	0.468***
					(0.047)	(0.047)
r2	0.978	0.978	0.809	0.809	0.525	0.525
n	8238	8238	8551	8551	3051	3051
F	2391	1803	389	305	162	130
Sample	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D¿0	R&D¿0
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Note:

TABLE VII – Heterogenous Impact of Downstream and Upstream Technology Shocks on Firm Sales, Valuation, and Capital Expenditure

	(1) ChgTobinsQ	(2) LogSale(t+1)	(3) LogCapX(t+1)	(4) ChgTobinsQ	(5) LogSale(t+1)	(6) LogCapX(t+1)
Spill	0.157	-0.528	-1.753***	0.320	0.155	-0.503
	(0.567)	(0.539)	(0.668)	(0.241)	(0.183)	(0.470)
SpillCurrent	0.129	-0.050	-0.082	-0.080	0.106***	0.006
	(0.116)	(0.111)	(0.138)	(0.052)	(0.035)	(0.090)
Spill X Positive R&D	1.952***	1.003*	3.057***	0.019	-0.229	0.500
	(0.588)	(0.577)	(0.715)	(0.267)	(0.191)	(0.490)
SpillCurrent X Positive R&D	-0.132	-0.022	0.139	0.124**	-0.008	0.003
	(0.128)	(0.128)	(0.158)	(0.058)	(0.039)	(0.101)
InnovShock	0.671***	0.038	-0.013	0.409***	-0.016	0.087
	(0.042)	(0.049)	(0.061)	(0.039)	(0.027)	(0.070)
Positive R&D	0.078**	-0.113***	-0.198***	0.257***	-0.085***	0.026
	(0.033)	(0.028)	(0.035)	(0.049)	(0.027)	(0.070)
r2	0.530	0.975	0.946	0.493	0.991	0.950
n	10571	24899	24899	5014	12488	12488
F	468	2780	1009	174	3776	992
Partner	Downstream	Downstream	Downstream	Upstream	Upstream	Upstream
Sample	All	All	All	All	All	All
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Note:

3 Model

To characterize and quantify innovation spillovers across supply chain partners, this section introduces a novel dynamic, heterogenous agent model in which both supply chain relationships and innovation are endogenous.

In the model, downstream firms produce differentiated goods using a variety of intermediate inputs produced by upstream firms. Product complementarity in the quality of inputs along the supply chain endogenously creates positive assortative matching in the intensity of the economic relationship between individual upstream and downstream firms. These relationships are critical to understanding the spillover of innovation by supply chain partners to profits: due to the complementarity along the supply chain, innovation by supply chain partners largely benefit firms producing higher quality products. Moreover, quality improvements by supply chain partners not only impact current profits, but change firm incentives to innovate, resulting in complementarity in innovation between upstream and downstream firms. This feedback is particularly consequential when evaluating the impact of a discount factor shock. Increased innovation following the discount factor shock, results in a higher aggregate quality levels for both upstream and downstream sectors, which incentivize higher levels of innovation and result in greater output even after the shock dissipates.

3.1 Agents and Technology

I introduce a discrete time, infinite horizon model, with two monopolistically competitive intermediate sectors and a perfectly competitive final good sector oriented vertically along the production network.

1. The upstream sector is composed of measure 1 of atomistic, monopolistically competitive firms indexed by $k \in K = [0, 1]$ which supply differentiated inputs to the downstream sector and

vary in quality $z_{kt}^S \sim \Phi_t^S$, where Φ_t^S is the distribution of quality of upstream firms in time t.

2. The downstream sector is composed of measure 1 of atomistic, monopolistically competitive firms indexed by $i \in I = [0,1]$ which supply differentiated goods to the final goods sector and vary in quality $z_{it}^C \sim \Phi_t^C$ where Φ_t^C is the distribution of quality of downstream firms in time t.

3. The final goods sector is composed of a continuum of firms which produce using the downstream intermediate good and labor and operate perfectly competitively to produce the final good.

Representative Household

Given wages w_t and dividends d_{it}^C and d_{kt}^S from upstream and downstream firms, a representative household supplies labor L inelastically and consumes the final good with utility $u(C_t)$ where

$$C_t \leq w_t L + \int_0^1 d_{it}^C di + \int_0^1 d_{kt}^S dk$$

Final Goods Sector

Final goods, Y_t, are produced by perfectly competitive firms with Cobb Douglas production, using labor L_t and intermediate inputs $\left\{x_{it}^C\right\}_{i \in I}$.

$$Y_t = \frac{\epsilon}{\epsilon - 1} \left(\int_0^1 z_{it} \left(x_{it}^C\right)^{\frac{\epsilon-1}{\epsilon}} di \right) L_t^{\frac{1}{\epsilon}}$$

Intermediate goods produced by the downstream sector are assumed to be substitutes with elasticity $\epsilon \geq 1$. Given wages w_t and downstream prices $\left\{p_{it}^C\right\}_{i \in I}$, the final good producers choose downstream

intermediate goods and labor to solve a static profit maximization problem.

$$\max_{L_t, \left\{x_{it}^C\right\}_{i\in[0,1]}} Y_t - \int_0^1 p_{it}^C x_{it}^C di - w_t L_t$$

Downstream Sector

Firms in the downstream sector indexed by i vary in quality z_{it}^C and supply differentiated intermediate goods to the final good sector. Each intermediate good is produced using $\psi^C z_{it}^C$ of the final good, consistent with higher quality blueprints being more costly to produce, and measure 1 of the upstream input. Quantity x_{it}^C is produced according to Leontief production function

$$x_{it}^C = \min\left\{\frac{y_{it}^C}{\psi^C z_{it}^C}, x_{it}^S\right\}$$

where y_{it}^C is the quantity of final good used, $x_{it}^S = \int_0^1 x_{ikt}^S dk$ is the total quantity sourced from upstream firms, and x_{ikt}^S is the quantity demanded by downstream firm i of firm k's upstream input.

The downstream firm faces demand $x_{it}^C = \left(\frac{z_{it}}{p_{it}^C}\right)^{\epsilon} L_t$ from final good producers. Critically, demand is increasing in overall quality z_{it}, which is an aggregate of the firm's own quality z_{it}^C and the quality of its chosen basket of inputs $\bar{z}_{it}^S$.

$$z_{it} = \left(\kappa_C \left(z_{it}^C\right)^{\frac{\nu-1}{\nu}} + \kappa_S \left(\bar{z}_{it}^S\right)^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}}$$

where products along the supply chain are assumed to be complementary ($\nu < 1$). Intuitively, this reflects the idea that demand for a product depends on the quality of the underlying components which are complementary to each other (a superior screen to a phone is most appreciated with a fast processor and an efficient battery).

The quality of basket of inputs is a CES aggregate of inputs sourced from the upstream sector

$$\bar{z}_{it}^{S} = \left[\int_0^1 \left(w_{ikt} z_{kt}^{S} \right)^{\frac{\zeta-1}{\zeta}} dk \right]^{\frac{\zeta}{\zeta-1}}$$

where z_{kt}^{S} is the quality of upstream firm k in time t and $\zeta > 1$, reflecting a preference for variety over substitutable inputs. Weight $w_{ikt} = \frac{x_{ikt}^{S}}{x_{it}^{S}}$ is the share of the basket allocated to inputs from firm k chosen by downstream firm i. Note that $\int_0^1 w_{ikt} dk = 1$.

The downstream firm indexed by $i \in I$ takes upstream prices $\left\{p_{kt}^{S}\right\}_{k \in K}$ and qualities $\left\{z_{kt}^{S}\right\}_{k \in K}$ as given and chooses unit price p_{it}^{C}, share of final goods y_{it}^{C}, and upstream inputs $\left\{x_{ikt}^{S}\right\}_{k \in K}$ to maximize profits

$$\pi^{C}\left(z_{it}^{C} \mid \Phi_t^{C}, \Phi_t^{S}\right) = \max_{p_{it}^{C}, y_{it}^{C}, \left\{x_{ikt}^{S}\right\}_{k \in K}} p_{it}^{C} x_{it}^{C} - y_{it}^{C} - \int_0^1 p_{kt}^{S} x_{ikt}^{S} dk \tag{3.1}$$

Since production is Leontief, the profit maximization problem of the downstream profits can instead be expressed as a choice over unit price p_{it}^{C} and upstream weights $\{w_{ikt}\}_{k \in K}$,

$$\pi^{C}\left(z_{it}^{C} \mid \Phi_t^{C}, \Phi_t^{S}\right) = \max_{p_{it}^{C}, \{w_{ikt}\}_{k \in K}} x_{it}^{C}\left(p_{it}^{C} - \psi^{C} z_{it}^{C} - \bar{p}_{it}^{S}\right) \text{ st } \int_0^1 w_{ikt} dk = 1 \tag{3.2}$$

where $w_{ikt} = x_{ikt}^{S} / x_{it}^{C} \in [0,1]$ and $\bar{p}_{it}^{S} = \int_0^1 w_{ikt} p_{kt}^{S} dk$.

Upstream Firms

Upstream firms indexed by $k \in K$ vary in quality z_{kt}^{S} and produce using linear production at unit cost $\psi^{S} z_{kt}^{S}$ (again consistent with high quality products being more costly to produce). Quantity x_{kt}^{S} is produced with linear production function

$$x_{kt}^{S} = \frac{y_{kt}^{S}}{\psi^{S} z_{kt}^{S}}$$

where y_{kt}^S is the quantity of the final good used in production. Upstream firms similarly compete monopolistically and choose single price p_{kt}^S to maximize profits in period t given downstream demand $x_{kt}^S = \int_0^1 x_{ikt}^S di$. Hence they solve profit maximization problem

$$\pi^S\left(z_{kt}^S \mid \Phi_t^C, \Phi_t^S\right) = \max_{p_{kt}^S} \left(p_{kt}^S - \psi^S z_{kt}^S\right) x_{kt}^S \tag{3.3}$$

Innovation

Firms operate over multiple periods and discount the future given discount rate $\beta \in (0,1)$and exogenous probability of death υ. Intertemporally, downstream firm's quality z_{it}^C is endogenous. A downstream firm in period t with quality z_{it}^C realizes period $t+1$ potential quality

$$\hat{z}_{it+1}^C = z_{it}^C + z_{it}^C d\Theta_{it}^C$$

where $d\Theta_{it}^C$ is drawn from Poisson distribution with mean $\alpha^C \left(\frac{\eta_{it}^C}{z_{it}^C}\right)^\xi$, η_{it}^C is innovation spending by the firm of the final good, and $\xi \in (0,1)$, reflecting decreasing returns to innovation. Note that both upstream and downstream profits depend not only on their own quality, but also jointly on the upstream and downstream distribution Φ_t^S and Φ_t^C. Hence, the value function of downstream firm i in period t can be denoted as $V^C\left(z_{it}^C \mid \Phi_t^C, \Phi_t^S\right)$ and similarly the upstream firm k in period t denoted as $V^S\left(z_{kt}^S \mid \Phi_t^C, \Phi_t^S\right)$.

Since downstream profits are not necessarily monotonic in quality z_{it}^C, value function $V^C\left(z_{it+1}^C \mid \Phi_{t+1}^C, \Phi_{t+1}^S\right)$ may also be non-monotonic. To ensure that future value$V^C\left(z_{it+1}^C \mid \Phi_{t+1}^C, \Phi_{t+1}^S\right)$ is increasing in innovation η_{it}^C, firms can choose next period's quality $z_{it+1}^C \in \left[z_{it}^C, \hat{z}_{it+1}^C\right]$ to maximize firm valuation such that

$$z_{it+1}^C \in \arg \max_{z_{it+1}^C \in \left[z_{it}^C, \hat{z}_{it+1}^C\right]} V^C\left(z_{it+1}^C \mid \Phi_{t+1}^C, \Phi_{t+1}^S\right)$$

Let the innovation intensity be $i_{it}^C = \eta_{it}^C / q_{it}^C$. Given discount rate $\beta \in (0,1)$, and probability of survival $1-\upsilon$, the downstream value function can be expressed as

$$V^C\left(z_{it}^C \mid \Phi_t^C, \Phi_t^S\right) = \max_{i_{it}^C} \pi^C\left(z_{it}^C \mid \Phi_t^C, \Phi_t^S\right) - i_{it}^C z_{it}^C + \beta\left(1-\upsilon\right) E_t V^C\left(z_{it+1}^C \mid \Phi_{t+1}^C, \Phi_{t+1}^S\right)$$

Upstream firms with quality z_{kt}^S can similarly invest η_{kt}^S in quality improving innovation and solve the following value function

$$V^S\left(z_{kt}^S \mid \Phi_t^C, \Phi_t^S\right) = \max_{i_{kt}^S} \pi^S\left(z_{kt}^S \mid \Phi_t^C, \Phi_t^S\right) - i_{kt}^S z_{kt}^S + \beta\left(1-\upsilon\right) E_t V^S\left(z_{kt+1}^S \mid \Phi_{t+1}^C, \Phi_{t+1}^S\right)$$

where $i_{kt}^S = \eta_{kt}^S / q_{kt}^S$ is the innovation intensity of the upstream firm.

Aggregation

In each period, t, the following markets clear.

1. Final good market

$$C_t + \int_0^1 \psi^C z_{it}^C di + \int_0^1 \psi^S z_{kt}^S dk + \int_0^1 i_{it}^C z_{it}^C di + \int_0^1 i_{kt}^S z_{kt}^S dk = Y_t$$

2. Labor market

$$L_t = L$$

3. Market for intermediate goods produced by downstream firms

$$x_{it}^C = x_{it}^C \qquad \forall i \in I$$

4. Market for inputs produced by upstream firms

$$\int_0^1 x_{ikt}^S di = x_{kt}^S \qquad \forall k \in K$$

Stationary equilibrium and exogenous entry and exit

Every period, fraction υ of firms in each sector exit with residual value 0 and are replaced with measure υ of upstream and downstream firms which begin with quality z_{min}^Cor z_{min}^S, respectively. The stationary equilibrium can be characterized by distributions Φ^C and Φ^S, quantities $\left\{x_{it}^C\right\}_{i\in I}, \left\{x_{ikt}^S\right\}_{i\in I,k\in K}, \left\{x_{kt}^S\right\}_{k\in K}$, prices $\left\{p_{it}^C\right\}_{i\in I}, \left\{p_{kt}^S\right\}_{k\in K}$, and innovation intensities $\left\{i_{it}^C\right\}_{i\in I}, \left\{i_{kt}^S\right\}_{k\in K}$ such that

1. Each downstream firm $i \in I$ with productivity z_{it}^C chooses price p_{it}^C, quantity x_{it}^C, and upstream allocations$\left\{x_{ikt}^S\right\}_{k\in K}$ to solve its static profit maximization problem.

2. Each upstream firm $k \in K$ with productivity z_{kt}^S chooses price p_{kt}^S and quantity x_{kt}^S to solve its static profit maximization problem.

3. Innovation policy functions $i^C\left(z_{it}^C \mid \Phi^C, \Phi^S\right)$ and $i^S\left(z_{kt}^S \mid \Phi^C, \Phi^S\right)$ solve value functions

$$V^C\left(z_{it}^C \mid \Phi^C, \Phi^S\right) = \pi^C\left(z_{it}^C \mid \Phi^C, \Phi^S\right) - i_{it}^{C*} z_{it}^C + \beta\left(1-\upsilon\right) E_t V^C\left(z_{it+1}^C \mid \Phi^C, \Phi^S\right)$$

$$V^S\left(z_{kt}^S \mid \Phi^C, \Phi^S\right) = \pi^S\left(z_{kt}^S \mid \Phi^C, \Phi^S\right) - i_{kt}^{S*} z_{kt}^S + \beta\left(1-\upsilon\right) E_t V^C\left(z_{kt+1}^S \mid \Phi^C, \Phi^S\right)$$

4. Distributions, Φ^C and Φ^S, evolve according to innovation policy functions, $i^{C*}\left(z_{it}^C \mid \Phi^C, \Phi^S\right)$ and $i^{S*}\left(z_{kt}^S \mid \Phi^C, \Phi^S\right)$, and given exogenous probably of exit υ.

3.2 Static Equilibrium

The downstream static profit maximization problem in Equation 3.2 provides insight into how demand for upstream inputs vary endogenously with both downstream quality, z_i^C, and upstream quality, z_k^S. For this subsection, I drop the period t subscript since all choices and state variables in the static problem occur in the same period. This is best illustrated in two steps: (1) Given target bundle quality, $\bar{z}^S$, how are weights $\{w_k\}_{k\in K}$ across upstream inputs chosen and what is the resulting price $\bar{p}^S$? (2) Given current downstream quality z_i^C, and a price schedule across potential upstream qualities, what is the firm's optimal bundle quality $\bar{z}_i^S$ and the resulting profits $\pi^C\left(z_i^C\right)$?

Definition 1. [Price Minimization Problem] Given upstream prices $\left\{p_k^S\right\}_{k\in K}$ and qualities $\left\{z_k^S\right\}_{k\in K}$, function g characterizes the minimum price needed to achieve target upstream bundle quality $\bar{z}^S$

$$g\left(\bar{z}^S\right) = \min_{\{w_k\}_{k\in K}} \int_0^1 w_k p_k^S dk \text{ st } \left[\int_0^1 \left(w_k z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} = \bar{z}^S \text{ and } \int_0^1 w_k dk = 1$$

where w_k is the relative weight allocated to upstream firm k's product and μ is the multiplier on constraint $\int_0^1 w_k dk = 1$.

First, Definition 1, formally characterizes the price minimization problem of the downstream firm which chooses weights $\{w_k\}_{k\in K}$ to minimize the bundle price $\bar{p}^S = g\left(\bar{z}^S\right)$ for targeted quality $\bar{z}^S$. Critically because of the Leontief production function, downstream firms face capacity constraint $\int_0^1 w_k dk = 1$ with shadow price μ. The constraint is consistent with the idea that, because of the production technology, a firm cannot simply add more inputs in a single unit to create higher quality, meaning that downstream firms are constrained by the upstream quality distribution. Without it, the maximum achievable $\bar{z}^S$is unbounded.

Proposition 2. *[Optimal Allocation of Upstream Inputs] Given upstream prices and qualities, down-*

stream firm i will chose optimal weights such that

$$\frac{w_{ik}}{w_{ij}} = \left(\frac{p_k^S + \mu_i}{p_j^S + \mu_i}\right)^{-\zeta} \left(\frac{z_k^S}{z_j^S}\right)^{\zeta-1}$$

where μ_i is the shadow price of capacity constraint $\int_0^1 w_{ik} = 1$ associated with its desired upstream bundle quality.

If $\zeta > 1$, then μ_i is increasing in $\bar{z}_i^S$. If upstream prices are increasing in quality, then w_{ik}/w_{ij} is increasing in μ_i and hence in $\bar{z}_i^S$.

Moreover, by taking first order conditions of the price minimization problem, for downstream firm i and upstream firms $j, k \in K$, Proposition 2 characterizes relative weights w_{ik}/w_{ij}. Critically, if upstream inputs are imperfectly substitutable, $\zeta > 1$, then $p_k^S > p_j^S$ then relative allocation w_{ik}/w_{ij} is increasing in the multiplier on the capacity constraint, μ_i. Notably, μ_i is increasing in $\bar{z}_i^S$—when the downstream firm wants to achieve a higher bundle quality, the capacity constraint tightens. If upstream prices are increasing in quality, this induces the firm to allocate greater weight to the higher quality input $z_k^S > z_j^S$. The capacity constraint is critical to creating allocations which vary with $\bar{z}_i^S$, and, as shown later, creating positive assortative matching. Without the constraint, if $\mu_i = 0$, then all downstream firms would choose the same relative weights invariant to $\bar{z}_i^S$. Product substitutability across upstream inputs also plays an important role: as ζ decreases and upstream inputs becomes less substitutable, relative weights w_{ik}/w_{ij} increase and sorting becomes more pronounced.

Proposition 3. *[Convexity of Upstream Price Schedule] If $\zeta > 1$, then $g', g'' > 0$.*

To understand firm i's optimal choice of upstream quality $\bar{z}_i^S$,given downstream quality z_i^C, I first establish Proposition 3—when upstream inputs are imperfectly substitutable, $\zeta > 1$, not only is price function g increasing but also convex in $\bar{z}^S$. Intuitively, this is because, to achieve a higher

$\bar{z}_i^S$, the downstream firm must allocate greater weight to higher quality inputs, but because $\zeta > 1$, with diminishing returns.

Proposition 4. *[Optimal Upstream Bundle Quality] Suppose $\epsilon > 1$, $\nu < 1$, and $g', g'' > 0$. Then firms choose $\bar{z}_i^S$ such that $MR\left(\bar{z}_i^S\right) = g'\left(\bar{z}_i^S\right)$ where $\Psi_i = \psi^C z_i^C + g\left(\bar{z}_i^S\right)$ is defined as the per unit cost and $MR\left(\bar{z}_i^S\right) = \frac{\epsilon}{\epsilon-1}\Psi_i \kappa_S \left(\bar{z}_i^S\right)^{-\frac{1}{\nu}} (z_i)^{\frac{1-\nu}{\nu}}$ is the per unit marginal return to increasing$\bar{z}_i^S$. If $z_i^C > z_m^C$, then $\bar{z}_i^S > \bar{z}_m^S$.*

In the second step, given price schedule $g\left(\bar{z}_i^S\right)$, I consider firm i's optimal choice of upstream bundle $\bar{z}_i^S$ given its own quality z_i^C. Defining unit costs as $\Psi_i = \psi^C z_i^C + g\left(\bar{z}_i^S\right)$, and substituting optimal price $p_i^C = \frac{\epsilon}{\epsilon-1}\Psi_i$, the downstream profit function $\pi^C\left(z_i^C\right)$ can be expressed as

$$\pi^C\left(z_i^C\right) = \max_{\bar{z}_i^S} \left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon} \frac{1}{\epsilon-1} L\left(z_i\right)^{\epsilon} \left(\Psi_i\right)^{1-\epsilon} \tag{3.4}$$

Per Proposition 4, the downstream firm will choose upstream quality $\bar{z}_i^S$ by setting the marginal return from increasing $\bar{z}_i^S$, $MR\left(\bar{z}_i^S\right)$ equal to marginal cost $g'\left(\bar{z}_i^S\right) > 0$.

$$MR\left(\bar{z}_i^S\right) = g'\left(\bar{z}_i^S\right)$$

Intuitively, when increasing upstream quality $\bar{z}_i^S$, the downstream firm faces a trade off: higher quality inputs increase joint quality z_i and hence demand, but is increasingly more expensive (since g convex). Due to complementarity, ($\nu < 1$), between upstream in downstream inputs to joint quality z_i , the marginal return to increasing upstream quality $\bar{z}_i^S$, $MR\left(\bar{z}_i^S\right)$, is greater when downstream quality$z_i^C$is higher.

$$\frac{dMR\left(\bar{z}_i^S\right)}{dz_i^C} = \frac{\epsilon}{\epsilon-1} z_i^{-1} \kappa_S \left(\frac{z_i^S}{z_i}\right)^{-\frac{1}{\nu}} \psi^C + \frac{1-\nu}{\nu} p_i^C z_i^{-2} \kappa_S \kappa_C \left(\frac{z_i^S}{z_i}\right)^{-\frac{1}{\nu}} \left(\frac{z_i^C}{z_i}\right)^{-\frac{1}{\nu}}$$

Hence, provided $\epsilon > 1$ and $\nu < 1$, higher quality downstream firms choose higher quality upstream bundles. This result is most pronounced when downstream products become less substitutable (ϵ declines) or if complementarity increases (ν falls). If $\zeta > 1$ and upstream prices are increasing in quality, this is sufficient to prove positive assortative matching—higher quality downstream firms allocate create unit weights to products by higher quality upstream firms.

Proposition 5. *[Upstream Optimal Price] Suppose $\zeta > 1$.*

(1) Upstream firm will choose optimal price

$$p_k^S = \frac{1}{\zeta - 1}\bar{\mu}_k + \frac{\zeta}{\zeta - 1}\psi^S z_k^S$$

where sufficient statistic$\bar{\mu}_k$ reflects capacity constraints $\{\mu_i\}_{i \in I}$ faced the firm's customers.

$$1 = \int_0^1 \frac{x_i^C w_{ik}}{x_k^S} \left[\frac{1}{\zeta} + \left(\frac{\zeta - 1}{\zeta} \right) \left(\frac{\psi^S z_k^S + \mu_i}{\psi^S z_k^S + \bar{\mu}_k} \right) \right]^{-1} di$$

(2) p_k^S is increasing in z_k^S.

(3) If all upstream firms are homogeneous in quality z_k^S, then $\mu_i = 0$ for all i and $p_k^S = \frac{\zeta}{\zeta - 1}\psi^S z_k^S$.

Next, I consider the static upstream profit maximization problem. Upstream firm k which produces differentiated product with quality z_k^S chooses prices p_k^S to maximize profits subject to decreasing demand function $x_k^S = \int_0^1 x_{ik}^S dk$. However, because downstream firms face a capacity constraint, elasticity of demand varies with quality z_i^C—higher quality downstream firms are less elastic.

$$-\frac{dx_{ik}^S / x_{ik}^S}{dp_k^S / p_k^S} = \zeta \frac{p_k^S}{p_k^S + \mu_i}$$

Indeed, firm k's demand elasticity can be characterized as a quantity-weighted average of demand elasticities from all downstream firms: $\frac{dx_k^S / x_k^S}{dp_k^S / p_k^S} = \int_0^1 \frac{x_{ik}^S}{x_k^S} \left(\frac{dx_{ik}^S / x_{ik}^S}{dp_k^S / p_k^S} \right) di$. Hence, the upstream firm which

solves the profit maximization problem in Equation 3.3 will choose optimal price p_k^S such that

$$\frac{1}{\zeta} = \int_0^1 \frac{x_{ik}^S}{x_k^S} \left(\frac{p_k^S - \psi^S z_k^S}{p_k^S + \mu_i} \right) di \tag{3.5}$$

where p_k^S can be characterized as

$$p_k^S = \frac{1}{\zeta - 1} \bar{\mu}_k + \frac{\zeta}{\zeta - 1} \psi^S z_k^S \tag{3.6}$$

a function of sufficient statistic$\bar{\mu}_k$, which per Proposition 5, reflects an aggregate of capacity constraints $\{\mu_i\}_{i \in I}$ faced by the firm's customers. Recall that μ_i reflects the extent the downstream firm is constrained by the upstream quality distribution. If overall, the upstream distribution lags in quality and downstream firms are more constrained, this will lead to greater prices, p_k^S, and markups, particularly for high quality upstream firms to which the downstream firms will target their demand. Observe that if $\mu_i = 0$ for all firms, either because there is no capacity constraint or because all upstream firms have equal quality, this returns to the standard result, $p_k^S = \frac{\zeta}{\zeta - 1} \psi^S z_k^S$. Finally, for $\zeta > 1$, as it follows from 3.5 that upstream prices p_k^S are increasing in quality z_k^S.

Corollary 6. *[Positive Assortative Matching] Suppose $\epsilon, \zeta > 1$ and $\nu < 1$. If $z_m^C > z_i^C$ and $z_k^S > z_j^S$, then $w_{ik}/w_{ij} > w_{mk}/w_{mj}$.*

This leads to a key result: the minimal assumptions of substitutability between firms within the same sector, $\epsilon, \zeta > 1$, and complementarity in input quality across sectors, $\nu < 1$, can produce positive assortative matching. Specifically, higher quality downstream firms will demand relatively greater quantities from higher quality upstream firms, and analogously higher quality upstream firms will sell more to higher quality downstream firms, consistent with the positive assortative matching observed in the data. Moreover, as discussed previously, positive sorting hinges on the

capacity constraint binding implied by the production technology, and becomes most pronounced when complementarity between inputs across sectors increases (ν declines) and when substitutability within sectors declines (ϵ and ζ fall).

3.3 Downstream Profits and Upstream Technology Spillover

This section first examines how downstream profits $\pi^C\left(z_i^C \mid \Phi^C, \Phi^S\right)$ vary with its own downstream quality z_i^C and second examines how changes to the upstream quality impact downstream profits.

Definition 7. Given downstream quality z_i^C, upstream quality $\bar{z}_i^S$, and price schedule $g\left(\bar{z}_i^S\right)$, I define the elasticity of downstream profits to its own quality as $\varphi^C\left(z_i^C\right) = \frac{d\pi_i^C/\pi_i^C}{dz_i^C/z_i^C}$ and the cross elasticity of downstream profits to upstream quality as $\hat{\varphi}^C\left(z_i^C\right) = \frac{d\pi_i^C/\pi_i^C}{d\bar{z}_i^S/\bar{z}_i^S}$.

Proposition 8. *[Downstream Profits] Suppose $\epsilon > 1$, $\nu < 1$, g convex. For given upstream prices $\left\{p_k^S\right\}_{k\in K}$ and upstream distribution Φ^S, there exists threshold $z^{C*} < z^{C**}$ where $\varphi\left(z^{C*}\right) = 1$ and $\varphi\left(z^{C**}\right) = 0$ such that*

(0) Elasticity φ^C is the sum of a positive demand effect and a negative cost effect

$$\varphi^C\left(z_i^C\right) = \underbrace{\epsilon \frac{dz_i/z_i}{dz_i^C/z_i^C}}_{demand} + \underbrace{(1-\epsilon)\frac{d\Psi_i/\Psi_i}{dz_i^C/z_i^C}}_{cost}$$

where $\frac{dz_i/z_i}{dz_i^C/z_i^C} = \kappa_C\left(\frac{z_i^C}{z_i}\right)^{\frac{\nu-1}{\nu}}$ and $\frac{d\Psi_i/\Psi_i}{dz_i^C/z_i^C} = \psi^C\frac{z_i^C}{\Psi_i}$.

(1) For $z_i^C < z^{C*}$, such that $\varphi^C\left(z_i^C\right) > 1$, downstream profits $\pi^C\left(z_i^C\right)$ are increasing and convex in z_i^C

(2) For $z^{C*} < z_i^C < z^{C**}$, such that $0 < \varphi^C\left(z_i^C\right) < 1$, downstream profits are increasing and concave in z_i^C.

(3) For $z_i^C > z^{C**}$, such that $\varphi^C\left(z_i^C\right) < 0$, downstream profits are decreasing z_i^C.

First, from the downstream profit function characterized in equation 3.4, I examine how downstream profits evolve with downstream quality for a given upstream distribution Φ^S and corresponding prices $\left\{p_k^S\right\}_{k\in K}$. Contrary to standard models, per Proposition 8, downstream profits are only monotonically increasing in quality z_i^C up to maximum z^{C**}. Indeed, the elasticity of downstream profits to downstream quality φ^C can be expressed as the sum of two forces. First, increasing z_i^C increases demand, but with diminishing returns due to complementarity between z_i^C and its chosen upstream bundle. Second, due to linear input costs ψ^C, increasing z_i^C increases unit costs Ψ_i at least linearly (since g is convex). For z_i^C small, the first dominates and φ^C and profits are increasing. However, as shown in see in Figure 5, as z_i^C increases, the second dominates, and downstream profits level out and then decline. In this region, since the upstream distribution is fixed, the increased demand from greater joint quality, z_i, is not sufficient to offset the increased costs of production. Consequently, if downstream firms take the upstream distribution and prices as fixed, they will not innovate beyond z^{C**} where $\varphi^C\left(z^{C**}\right) = 0$.

Moreover, by examining $d\varphi^C/dz_i^C$, I show that as firms approach threshold z^{C**}, downstream profits are concave, suggesting declining returns to quality-improving innovation. Indeed, downstream profits are concave as long as $\varphi^C\left(z_i^C\right) < 1$. However, when downstream quality is sufficiently small such that $z_i^C < z^{C*}$where $\varphi\left(z^{C*}\right) = 1$, profits are convex in quality. In this region, the downstream firm's quality lags sufficiently behind the upstream quality distribution to the extent that the higher input costs associated with higher upstream quality diminish downstream profits, thereby discouraging innovation.[1]

Proposition 9. *[Upstream Technology Spillover] Suppose $\epsilon > 1$, $\nu < 1$, and g convex and all firms have upstream quality $z_k^S = \bar{z}^S$, then*

[1] A policy which introduced lower quality upstream inputs would increase innovation for these firms. While some downstream firms may lag behind, as discussed in the subsequent section, upstream firms would not innovate such that upstream quality outstrips all downstream firms.

(0) Cross elasticity $\hat{\varphi}$ is the sum of a positive demand effect and a negative cost effect

$$\hat{\varphi}^C\left(z_i^C\right) = \underbrace{\epsilon \frac{dz_i/z_i}{d\bar{z}^S/\bar{z}^S}}_{demand} + \underbrace{(1-\epsilon)\frac{d\Psi_i/\Psi_i}{d\bar{z}^S/\bar{z}^S}}_{cost}$$

where $\frac{dz_i/z_i}{d\bar{z}^S/\bar{z}^S} = \kappa_S\left(\frac{\bar{z}^S}{z_i}\right)^{\frac{\nu-1}{\nu}}$ *and* $\frac{d\Psi_i/\Psi_i}{d\bar{z}^S/\bar{z}^S} = g'\left(\bar{z}^S\right)\frac{\bar{z}^S}{\Psi_i}$.

(1) Threshold z^{C**} increases with upstream quality $\bar{z}^S$.

(2) Elasticity $\varphi^C\left(z_i^C\right)$ increases with $\bar{z}^S$ and cross elasticity $\hat{\varphi}^C\left(z_i^C\right)$ increases with z_i^C.

(3) There exists threshold z^{C***} such that for $z_i^C > z^{C***}$, profits are increasing in upstream quality ,$\hat{\varphi}^C\left(z_i^C\right) > 0$, and decreasing otherwise.

Second, I consider how upstream quality improvements impact downstream profits. For this section, I consider downstream profits under the special case where all upstream firms have homogeneous quality z_k^S, and thus the aggregate bundle quality and price is $\bar{z}^S = z_k^S$ and $\bar{p}^S = \frac{\zeta}{\zeta-1}\psi^S\bar{z}^S$. Three observations described in Proposition 9 become apparent: First, for $\nu < 1$ and $\epsilon > 1$, as upstream quality $\bar{z}^S$ increases, $\frac{dz_i/z_i}{dz_i^C/z_i^C}$ increase and $\frac{d\Psi_i/\Psi_i}{dz_i^C/z_i^C}$ decrease, increasing $\varphi\left(z_i^C\right)$ and threshold, z^{C**}, at which downstream profits are maximized: upstream quality constrains the set of economically feasible downstream z_i^C. Secondly, upstream quality impacts downstream profits. Specifically, $\hat{\varphi}^C$ is increasing in z_i^C , meaning upstream technology spillover is heterogeneous, predominantly benefiting higher quality downstream firms. The cross elasticity of downstream profits to upstream quality, $\hat{\varphi}^C$, can be expressed as the sum of the positive demand effect, where increases to upstream quality increase joint quality z_i and hence demand, and the negative cost effect, since higher quality inputs are more expensive. Indeed, for lower quality downstream firms the second force dominates, leading to the third observation: for $z_i^C < z^{C***}$ where $\hat{\varphi}^C\left(z^{C***}\right) = 0$, upstream technology improvements reduce profits, and increase profits otherwise.

While the above analysis simplifies the problem to the case where all upstream firms have the same quality, the results are qualitatively similar when the upstream distribution is not degenerate and downstream firms have a choice over upstream basket quality$\bar{z}_i^S$. The appendix considers the case where the firms in the upstream distribution can be of either high or low quality. Intuitively, when the proportion over the higher quality upstream input increases, the maximum achievable $\bar{z}_i^S$ increases, and the price function $g\left(\bar{z}_i^S\right)$ flattens. Higher quality downstream firms who had previously been reaching for quality, benefit from lower prices and optimally choose a higher quality basket $\bar{z}_i^S$. In contrast, lower quality firms not only face higher input prices for their previously chosen basket, but also optimally shift to a lower quality basket and thus face lower demand.

Upstream Innovation Shock

To understand the impact of innovation by supply chain partners first without two-way feedback, this section considers the impact of an unexpected permanent increase to upstream quality in the special case where only downstream firms innovate and upstream firms can be of either low or high quality: $z_{kt}^S \in \left\{z_0^S, z_1^S\right\}$.

In period 0, fraction ϕ^S of upstream firms are of high quality $z_1^S > z_0^S$. Upstream firms do not innovate but with probability υ are replaced with an identical firm of the same quality and the downstream distribution $\Phi_t^C$is at its stationary equilibrium. In period 1, ϕ^S increases permanently to $\hat{\phi}^S$ such that the firm weighted average of z_k^S increases by 10 percent.

Figure 5 plots profit function $\pi^C\left(z_{it}^C \mid \Phi_t^C, \Phi_t^S\right)$ in period 0 when the economy is at the original stationary distribution (black), upon the realization of the shock in period 1 (blue), and finally, when the economy has settled into its new stationary distribution (red). The secondary axis plots the percent change from period 0. As described earlier, profits are initially increasing and convex in quality z_{it}^C, then increasing and concave in z_{it}^C, until profits reach their maximum and begin to decline in z_{it}^C. Following a positive upstream quality shock, downstream profits increase for the

highest quality firms, but may decrease for lower quality firms. Moreover, the maximum over π^C increases, as does the downstream quality which achieves this maximum, and the slope of the profit function in the preceding region increases, increasing returns to innovation. Profits at each z_{it}^C fall slightly once the economy has reached its new equilibrium since the higher quality downstream distribution enables suppliers to charge higher markups.

We can see this clearly in Figure 6 which plots the transition path as expressed in percentage changes for aggregate moments in the economy and for relatively high and low z_{it}^C and z_{kt}^S firms for 30 periods. First, average z_t^S increases unexpectedly in period 1. Aggregate downstream profits increase, but spillover to downstream firms is not uniform: the highest quality downstream firms benefit most, increasing both profits and R&D. Profits of relatively lower quality downstream firms fall as their previous basket becomes more expensive. These downstream firms still increase R&D, but to a lesser extent than higher quality downstream firms. Aggregate R&D initially increases eventually falling to a level higher than before the shock. Higher quality firms experience a greater increase to firm valuations, reflecting both increased profits today and higher returns to innovation.

While average upstream profits increase after the shock, profits and valuations of high quality upstream firms initially fall as these firms now compete with more high quality firms. As downstream firms innovate, however, average z_t^C increases, and these firms shift their demand toward higher quality inputs, increasing profits and firm valuation for these high quality upstream firms. This provides another mechanism for horizontal spillover: in the long run, high quality upstream suppliers benefit from competitor innovations which induce downstream firms to innovate.

While average z_t^S increase only once in period 1, average z_t^C and z_t increases as downstream firms innovate, capturing the feedback effect of an upstream quality improvement on downstream innovation. Finally, aggregate output Y_t increases immediately following the shock, and continues to increase as z_t^C increases, overshooting slightly before converging to the stationary equilibrium.

This upward drift would not occur if not for the innovation response of downstream firms. The slight overshooting occurs because the increase to downstream quality eventually enables upstream firms to charge higher markups, resulting in lower quantities.

3.4 Upstream Profits and Downstream Technology Spillover

This section instead first examines how upstream profits $\pi^S\left(z_k^S \mid \Phi^C, \Phi^S\right)$ vary with its own upstream quality z_k^S and second examines how changes to the downstream quality impact upstream profits.

Definition 10. Define the elasticity of upstream profits to its own quality as $\varphi^S\left(z_k^S\right) = \frac{d\pi_k^S/\pi_k^S}{dz_k^S/z_k^S}$. If all downstream firms have homogenous quality $z_i^C = z^C$, then define the cross elasticity of downstream profits to upstream quality $\hat{\varphi}^S\left(z_k^S\right) = \frac{d\pi_k^S/\pi_k^S}{dz^C/z^C}$.

Proposition 11. *[Upstream Profits] Suppose $\epsilon, \zeta > 1$ and $\nu < 1$ and all downstream firms have homogeneous quality $z_i^C = z^C$ with shadow price $\mu_i = \mu$, quantity $x_i^C = x^C$, and aggregate $\mathcal{A}_{i0} = \mathcal{A}_0$.*

(0) Elasticity φ^S can be expressed as follows

$$\varphi^S\left(z_k^S\right) = (\zeta - 1)\left[1 + \frac{\psi^S z_k^S}{\mu}\right]^{-1}$$

(1) Profits are increasing in quality, if $1 + \frac{\psi^S z_k^S}{\mu} > 0$ and decreasing otherwise. If $\mu < 0$, then profits are increasing provided $z_k^S < z^{S**} = -\frac{\mu}{\psi^S}$.

(2) As $z_k^S \to z^{S**}$, $\pi^S\left(z_k^S\right)$ is concave.

First, I consider how upstream profits vary with its own quality, z_k^S, in the special case where all downstream firms have homogeneous quality $z_i^C = z^C$ and hence identical shadow price $\mu_i = \mu$. Substituting optimal price p_k^S it is straight forward to show upstream profits as a function of its

own quality z_k^S and downstream moments x^C, $\mathcal{A}_0$, and μ.

$$\pi^S\left(z_k^S\right) = \frac{1}{\zeta - 1}\left(\frac{\zeta}{\zeta - 1}\right)^{-\zeta}\left(\frac{x^C}{\mathcal{A}_0}\right)\left(\frac{z_k^S}{\mu + \psi^S z_k^S}\right)^{\zeta - 1} \tag{3.7}$$

Examining elasticity $\varphi^S\left(z_k^S\right)$ as characterized in Proposition 11 provides insight into how upstream profits and therefore innovation evolves with upstream quality, z_k^S. As before, now the downstream distribution acts as a limit on the range of upstream quality which is economically feasible. Observe that upstream profits are increasing in quality, $\varphi^S > 0$, provided μ is positive, meaning that downstream firms would prefer higher quality inputs, or upstream quality is sufficiently small such that $\psi^S z_k^S < -\mu$. Indeed, if the downstream distribution is fixed, upstream firms will not innovate beyond z^{S**} where $z^{S**} = -\frac{\mu}{\psi^S}$. Furthermore, $d\varphi^S/dz_k^S < 0$, suggesting that for $\mu > 0$ or z_k^S approaching z^{S**}, upstream profits are concave in quality. As with downstream firms, as upstream firm quality increases, returns from innovation decline.

Proposition 12. *[Downstream Technology Spillover] Suppose $\epsilon, \zeta > 1$ and $\nu < 1$ and all downstream firms have homogenous quality $z_i^C = z^C$. Then*

(0) Cross elasticity $\hat{\varphi}^S$ can be decomposed into a aggregate demand effect and a reallocation effect

$$\hat{\varphi}^S\left(z_k^S\right) = \underbrace{\frac{d\left(\frac{x^C}{\mathcal{A}_0}\right)}{dz^C}\frac{z^C}{\left(\frac{x^C}{\mathcal{A}_0}\right)}}_{demand} - \underbrace{\left(\zeta - 1\right)\frac{z^C}{\mu + \psi^S z_k^S}\frac{d\mu}{dz^C}}_{reallocation}$$

(1) Threshold z^{S**} increases with downstream quality z_i^C.

(2) Elasticity $\varphi^S\left(z_k^S\right)$ increases with z^C and cross elasticity $\hat{\varphi}^S\left(z_k^S\right)$ increases with z_k^S

(3) There exists threshold z^{S***} such that for $z_i^S > z^{C***}$, profits are increasing in downstream quality ,$\hat{\varphi}^S\left(z_k^S\right) > 0$, and decreasing otherwise.

Second, I consider how upstream profits vary with downstream quality z^C, again considering the special case where all downstream firms have homogeneous quality as discussed in Proposition 12. As before, I discuss three observations. First, the range of economically feasible upstream qualities, for which upstream profits are increasing in upstream quality, is increasing in downstream quality z^C. Recall that μ reflect the extent the downstream firms are constrained. Hence as z^C increases so does μ. If μ is negative, then threshold z^{S**} also increases. Second, technology spillover from downstream firms is heterogeneous, with profits increasing more for higher quality upstream firms. Indeed, cross elasticity $\hat{\varphi}^S$ of upstream profits to downstream quality can be decomposed into a positive aggregate demand effect, where downstream innovation increases demand for inputs equally, and a reallocation effect. Under the reallocation effect, when downstream quality improves, complementarity means that downstream firms shift relative demand to higher quality inputs, predominantly benefitting higher quality upstream firms, and leading to the impact of downstream spillover as characterized by cross elasticity $\hat{\varphi}^S$ to be increasing in upstream quality. Indeed, thirdly, for sufficiently low quality downstream firms, the negative effect of reallocation dominates the positive demand effect. Consequently for these firms, downstream technology improvement propagates negatively, reducing profits.

Downstream Innovation Shock

As before, to understand the impact of downstream innovation on upstream firms without two-way feedback, this section considers the impact of an unexpected permanent increase to downstream quality in the special case where only upstream firms innovate and downstream firms have quality $z^C_{it} \in \left\{z^S_0, z^S_1\right\}$.

In period 0, fraction ϕ^C of upstream firms have higher quality $z^C_1 > z^C_0$. Downstream firms do not innovate but with probability υ are replaced with an identical firm of the same quality and the upstream distribution $\Phi^S_t$is at its stationary equilibrium. In period 1, ϕ^C increases permanently to

$\hat{\phi}^C$ such that the firm weighted average of z_{kt}^C increases by 10 percent.

Figure 7 plots the upstream profit function $\pi^S\left(z_{kt}^S \mid \Phi_t^C, \Phi_t^S\right)$ in time 0 at the original stationary equilibrium (black), upon realization of the shock time 1 (blue), and at the new stationary equilibrium (red). First, in all three cases, upstream profits are increase in z_{kt}^S, meaning that downstream quality is sufficiently high such that there are still returns to upstream innovation. The upstream profit function is also concave in z_{kt}^S, suggesting that as z_{kt}^S increases returns to innovation decline: the relative demand of downstream firms for inputs influence upstream returns to innovation. In period 1, downstream quality improvement increases profits most for the highest quality upstream firms, since not only do they experience an increase in quantity demanded, but downstream firms shift their allocation toward higher quality inputs. The upstream profit function not only increases, but for the same quality z_{kt}^S, the slope increases, inducing upstream firms to innovate more. As upstream firms innovate and the economy approaches its new stationary equilibrium, upstream markups decline, since the relative distance between the quality of upstream and downstream firms decline, leading profits at each quality level z_{kt}^S to fall slightly.

Figure 8 plots the transition path from the original stationary equilibrium to the new stationary equilibrium over 30 periods for aggregate outcomes and for relatively high and low z_{it}^C and z_{kt}^S firms. Similar to before, the permanent shock has both short term and long term effects. Upon realization of the positive shock to downstream firms, markups and aggregate upstream profits increase. In the cross section, high quality upstream firms benefit most, experiencing an increase to profits as demand for their product increases. For lower quality upstream firms, the impact of reallocation dominates, resulting in lower profits. Aggregate upstream innovation immediately increases, driven predominantly by the highest quality upstream firms, and then declines as the economy converges to its new stationary equilibrium. As upstream firms innovate, aggregate upstream quality z_t^S and output Y_t gradually increases. Markups decline as the distance between downstream and upstream

firms decrease.

While downstream firms do not innovate and average z_t^C increases only in time 0, aggregate downstream profits continue to increase as upstream firms innovate. In the cross section, profits at each quality z_{it}^C initially decline, because the upstream firms increase prices. As upstream firms innovate, however, this predominantly benefits high quality downstream firms who benefit most from upstream quality improvements. The model illustrates another mechanism for horizontal spillover by upstream firms. Competitor innovation, while initially eroding profits, may propagate positively in the long run if they induce supply chain partners to innovate.

3.5 Innovation Feedback and Discount Rate Shock

The preceding exercises considered the impact of a quality in one sector when only the sector innovates. Following a positive shock to quality in one sector, product complementarity increases aggregate profits and innovation in the other sector. Taken together, as the quality of upstream distribution improves downstream firms are incentivized to innovate, improving the quality of the downstream distribution, which incentives upstream firms to innovate, improving the quality of the upstream distribution, and so forth. Hence, if both sectors are allowed to innovate, a shock that induces one sector or both sectors to innovate may be amplified the supply chain.

Figure 9 plots the impact of an unexpected 1 p.p. increase to the discount factor. The shock is transitory with a decay rate of 10 percent and all agents are assumed to have perfect foresight. Three scenarios are considered: (1) the baseline case where both sectors can innovate(black), (2) only downstream firms innovate and the upstream distribution does not change from time 0 (red) (3) only upstream firms innovate and the downstream distribution does not change from time 0 (blue). In the baseline, the positive shock to the discount factor induces a sharp increase in both upstream and downstream innovation. Average upstream and downstream quality, profits, and

output continue to increase even after the shock dissipates. From the perspective of downstream firms the higher quality of the upstream distribution supports a higher level of innovation and downstream quality, which in turn induces upstream firms to innovate more even after the shock has resolved, perpetuating the impact of the discount factor shock over a longer horizon.

To understand the impact of feedback between the upstream and downstream distribution, the red and blue lines consider the same shock under the scenario where only the downstream and upstream firms, respectively, innovate and the other distribution is held fixed. First, when both sectors innovate in tandem in response to the shock the effect is amplified: the increase to output is greater than the sum of the increases when each sector innovates individually. Second, due to the self-reinforcing effect of the higher quality distributions, in the baseline case, R&D decays slower and output and profits continue to increase for several more periods, relative to when only one sector innovates.

Previous exercises showed that shocks to supply chain partners spill over heterogeneously depending on the firm's own quality. Figure 9 evaluates the impact of the same discount rate shock in the baseline case and subsequent upstream and downstream innovation on downstream and upstream profits for firms of relatively low, medium, and high quality z_{it}^C and z_{it}^S. First, while aggregate downstream profits increase, as upstream firms innovate, profits of high quality downstream firms increase since they benefit most due to the complementarity between the upstream distribution and their own quality, while profits for low quality downstream firms who face higher prices fall. As downstream innovation increases, overall demand initially increases even for low quality upstream firms. However, as the proportion of high quality downstream firms increase, the reallocation effect dominates, since these high quality downstream firms demand more high quality inputs than low quality inputs. In sum, for both sectors, the impact of the discount factor shock is heterogenous, benefitting higher quality firms most.

3.6 Figures

FIGURE 3 – Upstream Allocation

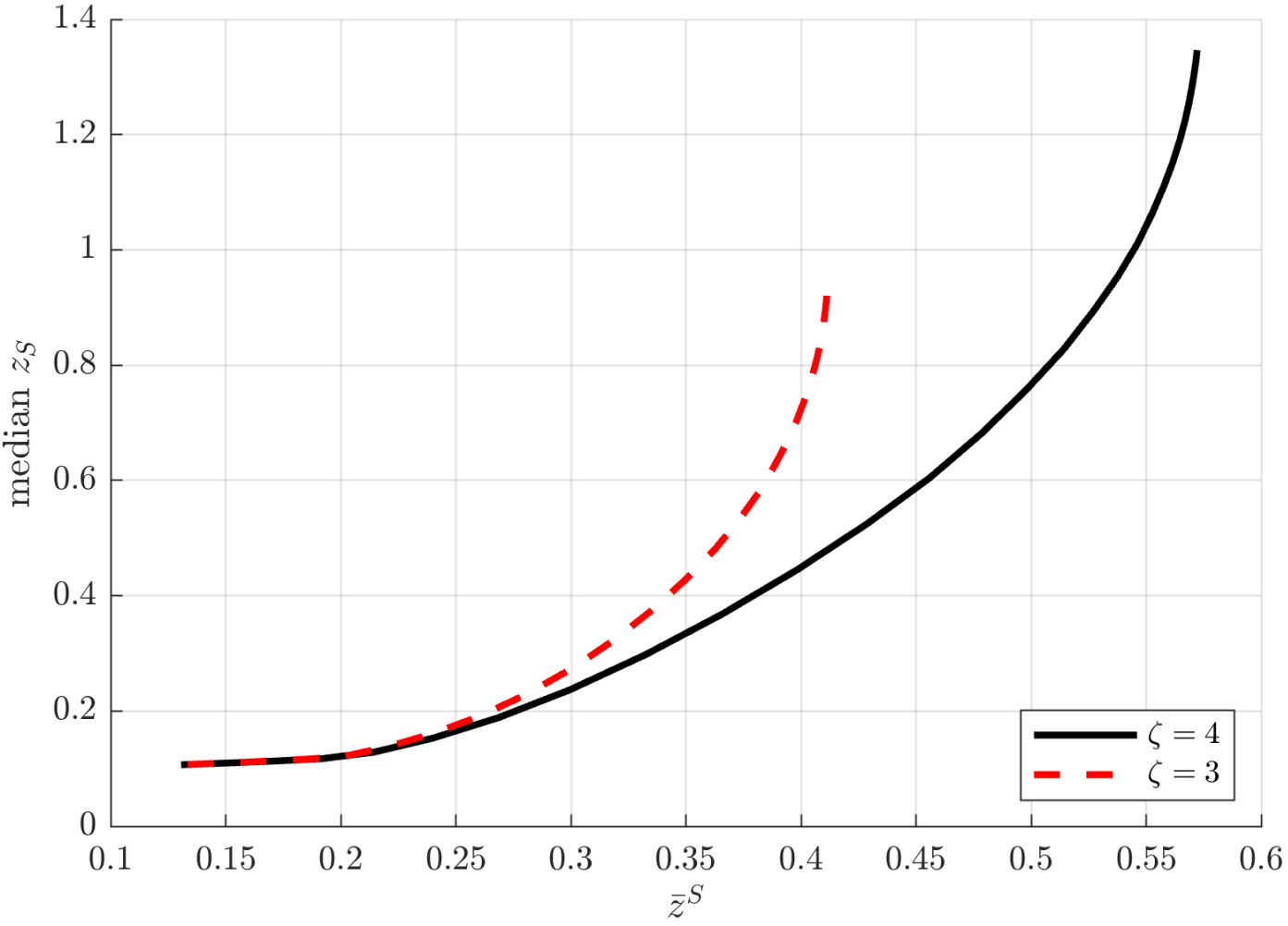


FIGURE 4 – Positive Assortative Matching

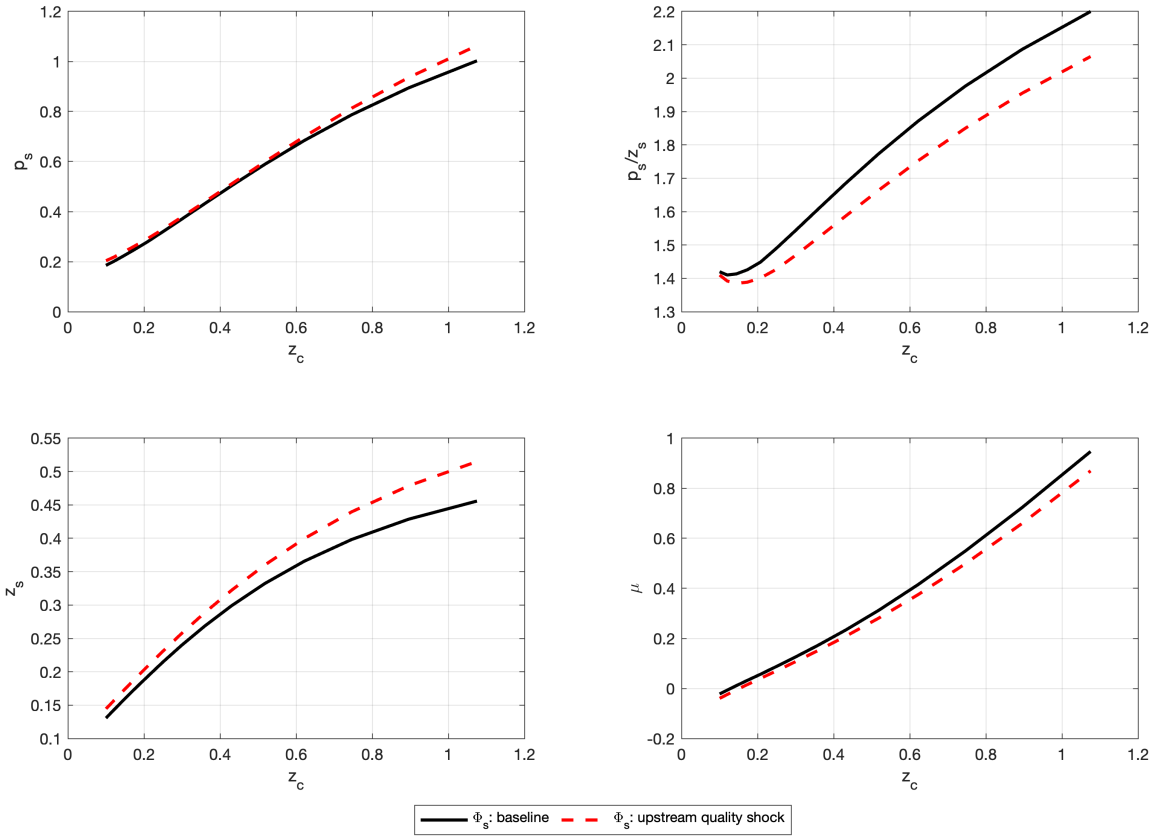


FIGURE 5 – Upstream Quality Shock: Stationary Equilibria

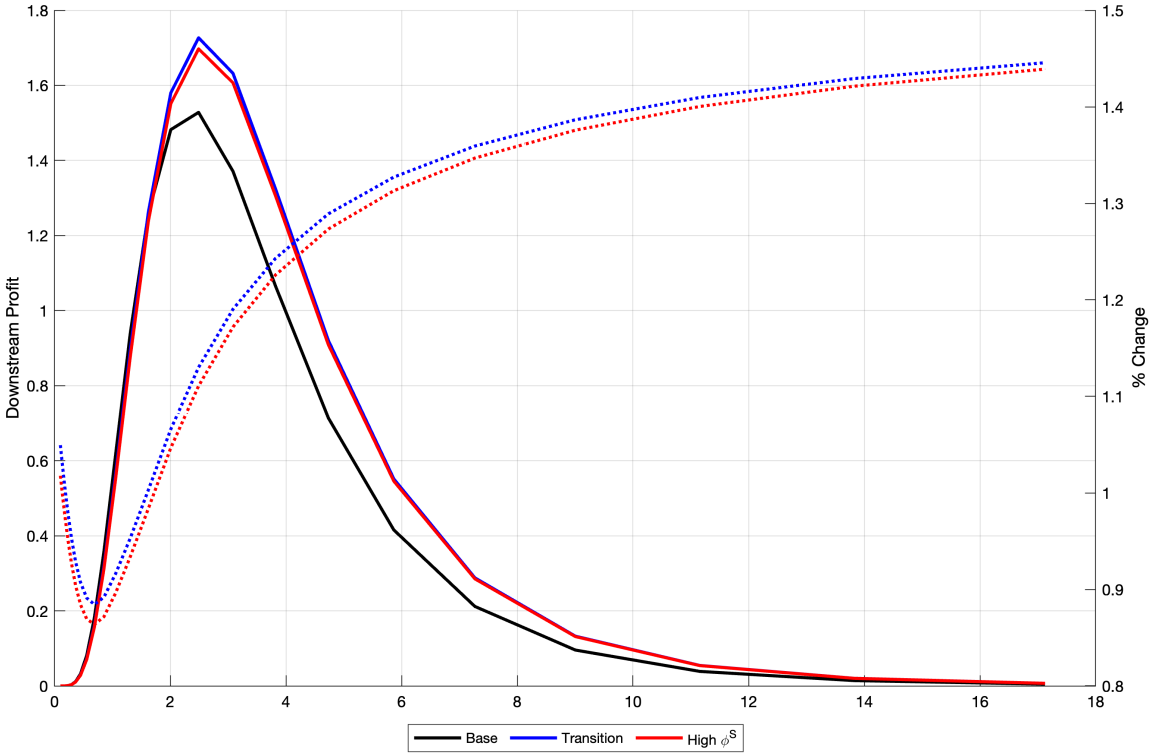


FIGURE 6 – Upstream Quality Shock: Transition Path

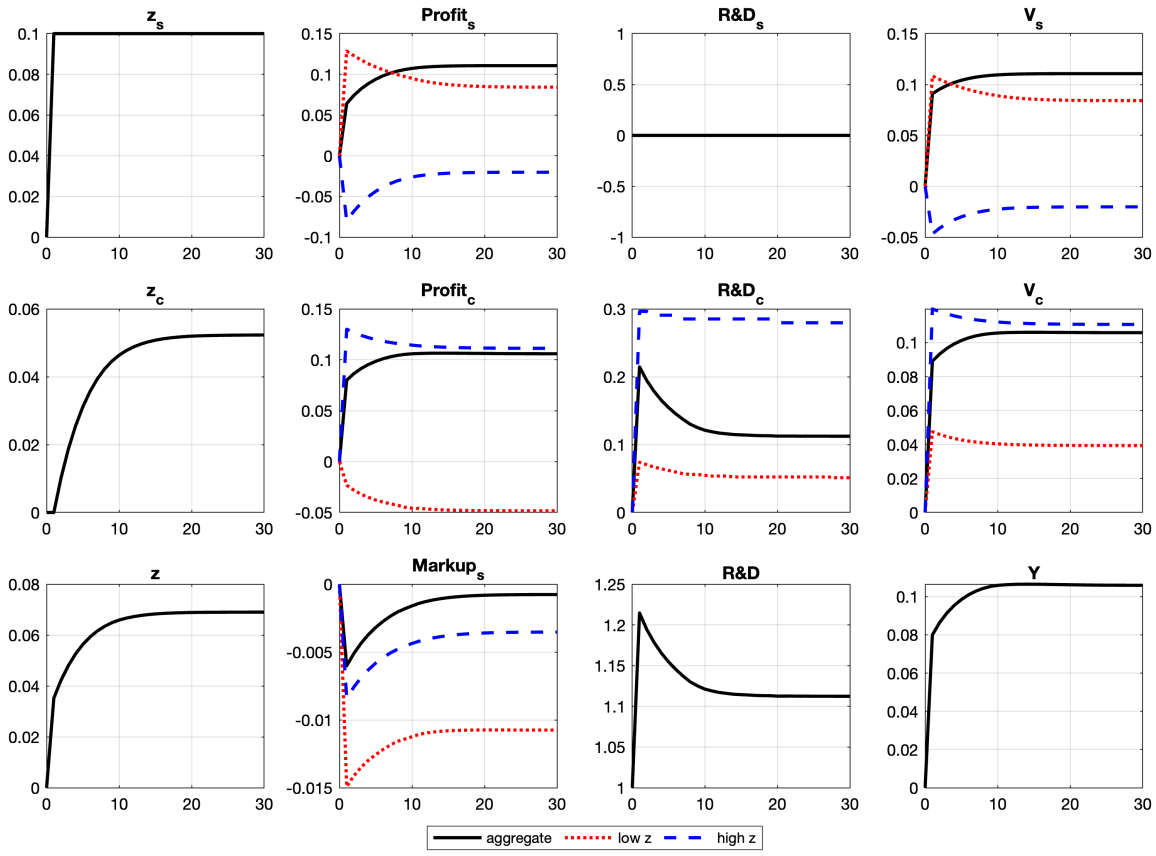


FIGURE 7 – Downstream Quality Shock: Stationary Equilibria

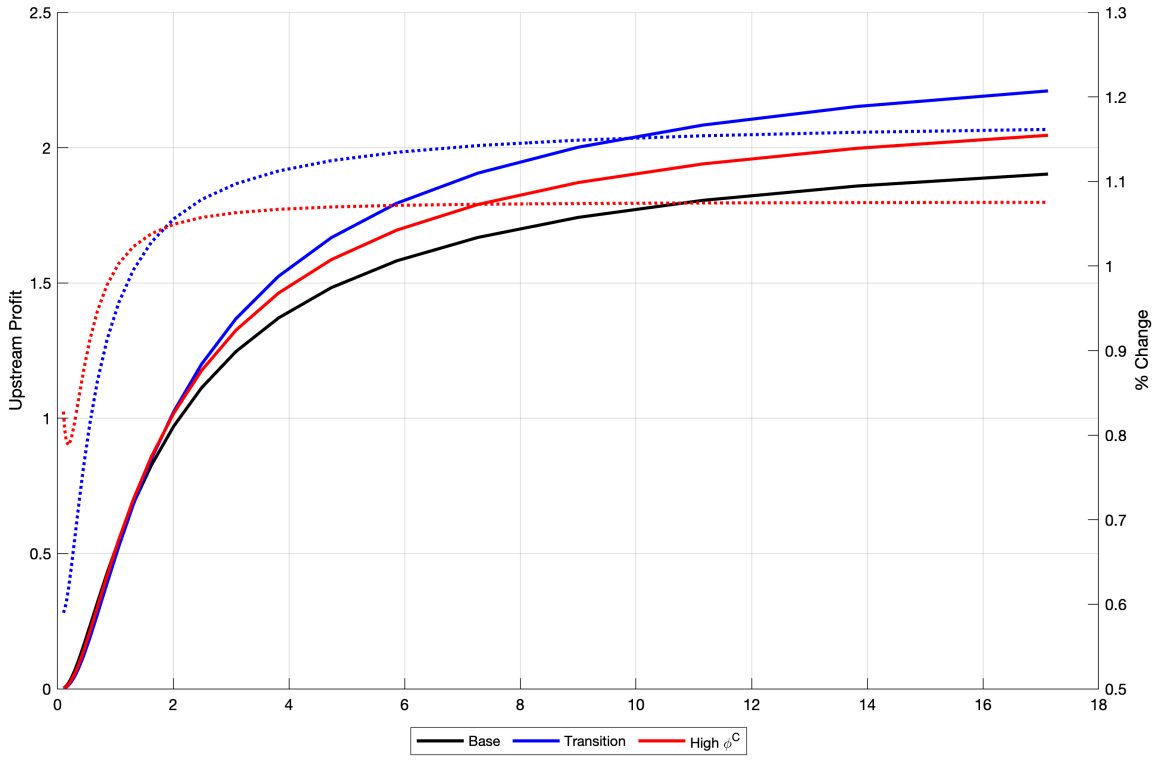


FIGURE 8 – Downstream Quality Shock: Transition Path

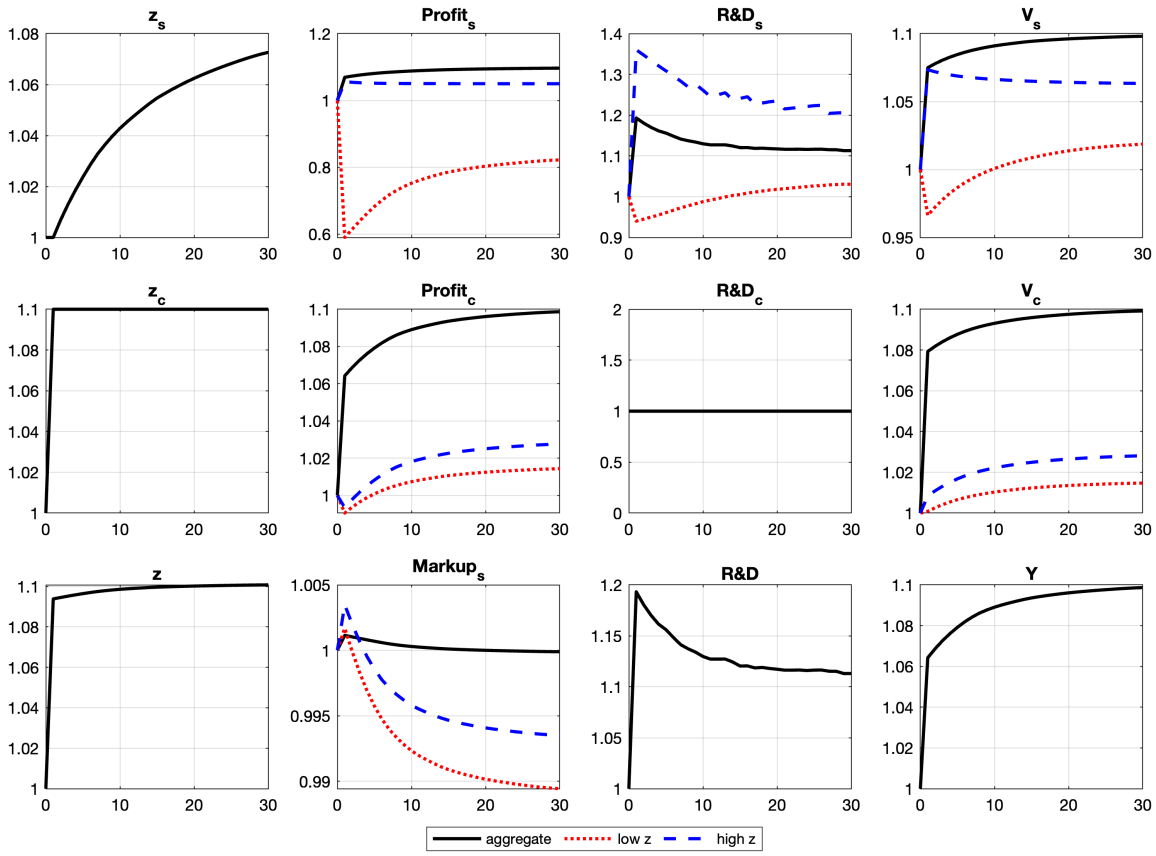


FIGURE 9 – Shock to Discount Factor: Transition Path

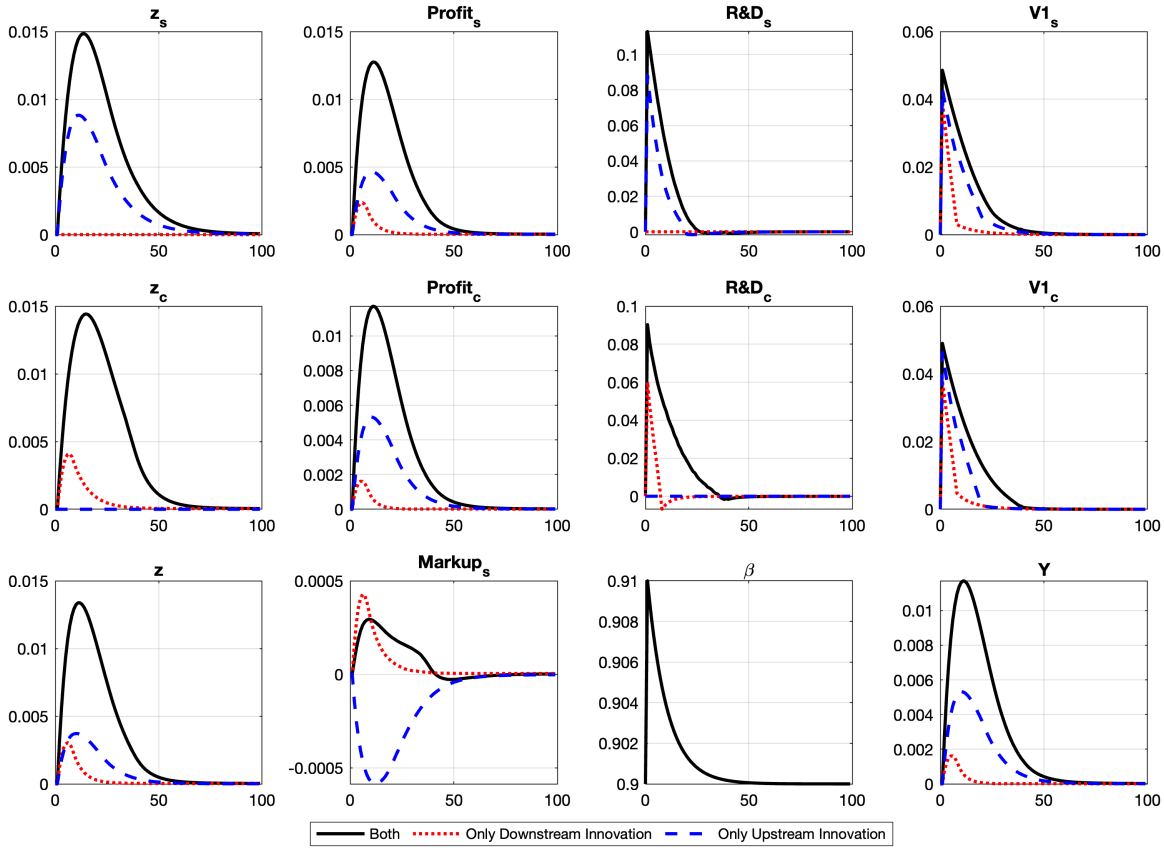
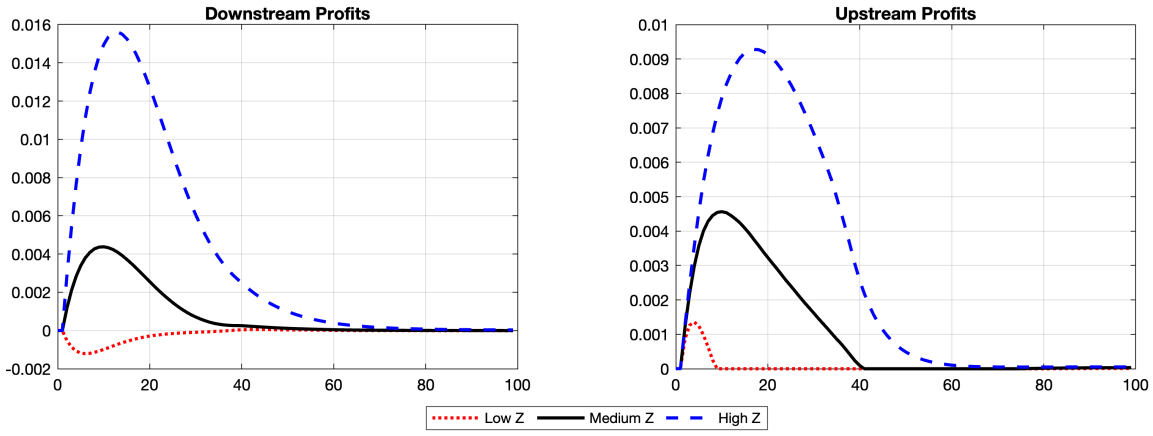


FIGURE 10 – Shock to Discount Factor: Transition Path for Different z_{it}^{C} and z_{kt}^{S}



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Supplemental Online Appendix

NOT FOR PUBLICATION

A Model

A.1 Price minimization problem g

A.1.1 Solution to price minimization problem

The price minimization problem is

$$g\left(\bar{z}_i^S\right) = \min_{\{w_{ik}\}_{k\in K}} \underbrace{\int_0^1 w_{ik} p_k^S dk}_{\bar{p}_i^S} \text{ st } \left[\int_0^1 \left(w_{ik} z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} = \bar{z}_i^S \text{ and } \int_0^1 w_{ik} = 1$$

Let λ_i and μ_i be the multipliers on each constraint. Then

$$p_k^S + \mu_i = \lambda_i \left(\bar{z}_i^S\right)^{\frac{1}{\zeta}} \left(z_k^S\right)^{\frac{\zeta-1}{\zeta}} \left(w_{ik}\right)^{-\frac{1}{\zeta}} \iff w_{ik} = \lambda_i^{\zeta} \left(\bar{z}_i^S\right) \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu_i\right)^{-\zeta}$$

Summing across k, gives us the following relationships between $\bar{p}_i^S$, μ_i, and λ_i:

$$\bar{p}_i^S + \mu_i = \lambda_i \bar{z}_i^S$$

$$1 = \lambda_i^{\zeta} \bar{z}_i^S \mathcal{A}_{0i}$$

$$\bar{p}_i^S = \left(\mathcal{A}_{0i}\right)^{-1} \mathcal{A}_{1i}$$

where $\mathcal{A}_{0i} = \int_0^1 \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu_i\right)^{-\zeta} dk$ and $\mathcal{A}_{1i} = \int_0^1 p_k^S \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu_i\right)^{-\zeta} dk$ are functions of μ_i. Thus the weight to firm k can be expressed as

$$w_{ik} = \left(\mathcal{A}_{0i}^{-1}\right) \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu_i\right)^{-\zeta}$$

Thus for any $\bar{z}_i^S$ we can find μ_i which solves

$$\bar{z}_i^S = \left(\mathcal{A}_{i0}\right)^{\frac{1}{\zeta-1}} \left[\tilde{p}^S + \mu_i\right]^{\frac{\zeta}{\zeta-1}} = \left[\left(\mathcal{A}_{i0}\right)^{\frac{1}{\zeta}} \mu_i + \left(\mathcal{A}_{i0}\right)^{\frac{1}{\zeta}-1} \mathcal{A}_{i1}\right]^{\frac{\zeta}{\zeta-1}}$$

The multiplier on the constraint takes values $\mu_{i0} \in \left(-\bar{p}_{\min}^S, \infty\right)$ where $\bar{p}_{\min}^S = \min_{k\in K} p_k^S$. $\bar{z}_i^S$ and $\bar{p}_i^S$ are monotonically increasing in μ_{i0} (proven later). Note, if Φ^S is discrete with probability weights $\left\{\phi_k^S\right\}_{k\in K}$ for qualities $\left\{z_k^S\right\}_{k\in K}$, then $w_{ik} = \omega_{ik}/\phi_k^S$ where ω_{ik}^S is the allocation to type k.

A.1.2 Domain of g

Observe that g has domain $\bar{z}^S \in \left[0, \bar{z}_{\max}^S\right]$ where

$$\bar{z}_{\max}^S = \max_{\{w_{ik}\}_{k\in K}} \left[\int_0^1 \left(w_{ik} z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} \text{ st } \int_0^1 w_{ik} dk = 1$$

which has solution

$$\bar{z}_{\max}^S = \left[\int_0^1 \left(z_k^S\right)^{\zeta-1} dk\right]^{\frac{1}{\zeta-1}} \qquad w_{k,\max} = \left(\bar{z}_{\max}^S\right)^{1-\zeta} \left(z_k^S\right)^{\zeta-1}$$

Since the downstream firm must choose measure 1 of the upstream input, and firms are atomistic, $\bar{z}^S \to 0$ if the firm allocates $w_{ik} = 1$ to goods from a single firm. However, with the minimum per unit price $\bar{p}^S_{\min} = \min\left\{p^S_k\right\}_{k\in K}$. At that price, if the set of firms which offer the minimum prices is not atomistic, the firm might be able to achieve a higher $\bar{z}^S$, which it would strictly prefer. Indeed, the relevant domain of g is $\bar{z}^S \in \left[\bar{z}^S_{\min}, \bar{z}^S_{\max}\right]$ where

$$\bar{z}^S_{\min} = \max_{\{w_{ik}\}_{k\in K}} \left[\int_0^1 \left(w_{ik} z^S_k\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} \text{ st } \int_0^1 w_{ik} p^S_k dk = \bar{p}^S_{\min} \text{ and } \int_0^1 w_{ik} dk = 1$$

This has solution

$$\bar{p}^S_{\min} = \left(\mathcal{A}_{0i}\right)^{-1} \mathcal{A}_{1i}$$

where $\mathcal{A}_{0i} = \int_0^1 \left(z^S_k\right)^{\zeta-1} \left(p^S_k + \mu_i\right)^{-\zeta} dk$ and $\mathcal{A}_{1i} = \int_0^1 p^S_k \left(z^S_k\right)^{\zeta-1} \left(p^S_k + \mu_i\right)^{-\zeta} dk$ and $\bar{z}^S_{\min} = \left(\bar{p}^S_i + \mu_i\right)^{\frac{\zeta}{\zeta-1}} \left(\mathcal{A}_{0i}\right)^{\frac{1}{\zeta-1}}$.

A.1.3 g is convex, $\bar{p}^S$ and $\bar{z}^S$ increasing in μ

Proposition. *Define price minimization function g as*

$$g\left(\bar{z}^S_i\right) = \min_{\{w_{ik}\}_{k\in K}} \underbrace{\int_0^1 w_{ik} p^S_k dk}_{\bar{p}^S_i} \text{ st } \left[\int_0^1 \left(w_{ik} z^S_k\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} = \bar{z}^S_i \text{ and } \int_0^1 w_{ik} dk = 1$$

If $\zeta > 1$, then g is convex.

Proof. Define $\bar{z}^S\left(w_i\right)$ and $\bar{p}^S\left(w_i\right)$as the aggregate quality and price achieved given allocation $w_i = \{w_{ik}\}_{k\in K}$.

$$\bar{z}^S\left(w_i\right) = \left[\int_0^1 \left(w_{ik} z^S_k\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} \qquad \bar{p}^S\left(w_i\right) = \int_0^1 w_{ik} p^S_k dk$$

Let $m\left(w_i\right) = \int_0^1 \left(w_{ik} z^S_k\right)^{\frac{\zeta-1}{\zeta}} dk$. The Hessian matrix of m is diagonal matrix with elements

$$\frac{1-\zeta}{\zeta^2} \left(w_{ik}\right)^{\frac{\zeta-1}{\zeta}-2} \left(z^S_k\right)^{\frac{\zeta-1}{\zeta}}$$

Since for $\zeta > 1$, all elements are non positive, m is quasi-concave. Hence monotonic increasing transformation $\bar{z}^S\left(w_i\right)$=$\left[m\left(w_i\right)\right]^{\frac{\zeta}{\zeta-1}}$ is quasi concave. Since $\bar{z}^S\left(w_i\right)$ is homogeneous of degree 1 and quasi concave, it is also concave. The inverse of the downstream firm's price minimization problem can be characterized by quality maximization problem $h = g^{-1}$

$$h\left(\bar{z}^S_i\right) = \max_{\{w_{ik}\}_{k\in K}} \left[\int_0^1 \left(w_{ik} z^S_k\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} \text{ st } \int_0^1 w_{ik} p^S_k dk = \bar{p}^S_i \text{ and } \int_0^1 w_{ik} = 1$$

It is straightforward to show h is also concave. Consider $\bar{z}^S_0 = h\left(\bar{p}^S_0\right)$ and $\bar{z}^S_1 = h\left(\bar{p}^S_1\right)$ where $\bar{p}^S_1 > \bar{p}^S_0$ with optimal allocations w_0 and w_1, respectively. Trivially, $\bar{z}^S_1 > \bar{z}^S_0$. Consider $\lambda \in (0,1)$ and allocation $w_\lambda = (1-\lambda) w_0 + \lambda w_1$. Since $\bar{z}^S$ is concave

$$\bar{z}^S > (1-\lambda) \bar{z}^S_0 + \lambda \bar{z}^S_1$$

We can also verify

$$\sum_{k\in K} w_{\lambda k} = (1-\lambda)\sum_{k\in K} w_{0k} + \lambda \sum_{k\in K} w_{1k} = 1$$

$$\bar{p}^S_\lambda = \sum_{k\in K} \lambda p_k w_{\lambda k} = (1-\lambda)\sum_{k\in K} p_k w_{0k} + \lambda \sum_{k\in K} p_k w_{1k} = (1-\lambda)\,\bar{p}^S_0 + \lambda \bar{p}^S_1$$

Hence h is concave. Since $g = h^{-1}$, g is convex. ■

Corollary. *On domain $\bar{z}^S \in \left[\bar{z}^S_{\min}, \bar{z}^S_{\max}\right]$, price function $\bar{p}^S = g\left(\bar{z}^S\right)$ is monotonically increasing and convex.*

Proof. A convex function achieves at most 1 minimum. Since this occurs at $\bar{z}^S_{\min}$, g is monotonically increasing and convex on domain $\bar{z}^S \in \left[\bar{z}^S_{\min}, \bar{z}^S_{\max}\right]$ ■

Proposition. *If $\zeta > 1$, then $\frac{d\bar{p}^S}{d\mu} > 0$*

Proof. Recall that $\bar{p}^S = (\mathcal{A}_0)^{-1}\mathcal{A}_1$. Thus

$$\frac{d\bar{p}^S}{d\mu} = \zeta\,(\mathcal{A}_0)^{-1}\left[\bar{p}^S\,(\mathcal{A}_2) - (\mathcal{A}_3)\right]$$

where $\mathcal{A}_0 = \int_0^1 \left(z^S_k\right)^{\zeta-1}\left(p^S_k+\mu\right)^{-\zeta} dk$ and $\mathcal{A}_2 = \int_0^1 \left(z^S_k\right)^{\zeta-1}\left(p^S_k+\mu\right)^{-\zeta-1} dk$
and $\mathcal{A}_3 = \int_0^1 p^S_k \left(z^S_k\right)^{\zeta-1}\left(p^S_k+\mu\right)^{-\zeta-1} dk$. Since $\zeta > 1$, and $\mathcal{A}_0 > 0$, it is sufficient to show that $\bar{p}^S\,(\mathcal{A}_2) - (\mathcal{A}_3) > 0$. Define

$$\tilde{W} = \int_0^1 \left(z^S_k\right)^{\zeta-1}\left(p^S_k+\mu\right)^{-\zeta-1} dk$$

$$\tilde{w}_{ik} = \tilde{W}\left(z^S_k\right)^{\zeta-1}\left(p^S_k+\mu\right)^{-\zeta-1}$$

Then

$$\bar{p}^S\,(\mathcal{A}_2) - (\mathcal{A}_3) = \int_0^1 \left(\bar{p}^S - p^S_k\right)\left(z^S_k\right)^{\zeta-1}\left(p^S_k+\mu\right)^{-\zeta-1} dk = \tilde{W}\int_0^1 \left(\bar{p}^S - p^S_k\right)\tilde{w}_{ik} dk$$

Observe that since $\zeta > 1$

$$\bar{p}^S = \int_0^1 p^S_k w_{ik} dk > \int_0^1 p^S_k \tilde{w}_{ik} dk$$

Thus $\bar{p}^S\,(\mathcal{A}_2) - (\mathcal{A}_3) > 0$ ■

Proposition. *If $\zeta > 1$, then $\frac{d\bar{z}^S}{d\mu} > 0$*

Proof. Note that $\bar{z}^S = (\mathcal{A}_0)^{\frac{1}{\zeta-1}}\left[(\mu+\bar{p})\right]^{\frac{\zeta}{\zeta-1}}$ where $\mathcal{A}_0 = \int_0^1 \left(z^S_k\right)^{\zeta-1}\left(p^S_k+\mu\right)^{-\zeta} dk$. As $\zeta > 1$, and $\mathcal{A}_0 > 0$, it is sufficient to show that $\mu + \bar{p}$ is increasing in μ. Since $\frac{d\bar{p}^S}{d\mu}$, it follows that $\frac{d\bar{z}^S}{d\mu}$. ■

A.2 Positive Assortative Matching

Proposition. *If $\zeta > 1$, $p^S_k > p^S_j$, and $\bar{z}^S_m > \bar{z}^S_i$, then $w_{ik}/w_{ij} > w_{mk}/w_{mj}$*

Proof. The first order condition of the price minimization problem, given target $\bar{z}^S_i$, gives

$$p^S_k + \mu_i = \lambda_i \left(\bar{z}^S_i\right)^{\frac{1}{\zeta}}\left(z^S_k\right)^{\frac{\zeta-1}{\zeta}}\left(w_{ik}\right)^{\frac{-1}{\zeta}}$$

Thus for firms $k, j \in K$

$$\frac{w_{ik}}{w_{ij}} = \left(\frac{z_k^S}{z_j^S}\right)^{\zeta-1} \left(\frac{p_k^S + \mu_i}{p_j^S + \mu_i}\right)^{-\zeta}$$

If $p_k^S > p_j^S$, then $\frac{w_{ik}}{w_{ij}}$ increasing in μ_i. Since $\bar{z}_i^S$ increasing in μ_i, then $\frac{w_{ik}}{w_{ij}}$ increasing in $\bar{z}_i^S$. ■

Proposition. *Suppose $\epsilon > 1$ and $\nu < 1$, and price function $g'\left(\bar{z}^S\right) > 0$ and $g''\left(\bar{z}^S\right) > 0$. If $z_m^C > z_i^C$, then $\bar{z}_m^{S*} > \bar{z}_i^{S*}$. (The optimal upstream basket is increasing in downstream quality).*

Proof. The downstream firm solve profit function

$$\pi^C\left(z_i^C\right) = \max_{\bar{z}_i^S} \left(\frac{\epsilon - 1}{\epsilon}\right)^{\epsilon} \frac{1}{\epsilon - 1} L\left(z_i\right)^{\epsilon} \left(\Psi_i\right)^{1-\epsilon}$$

where $z_i = \left(\kappa_C \left(z_i^C\right)^{\frac{\nu-1}{\nu}} + \kappa_S \left(\bar{z}_i^S\right)^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}}$ and $\Psi_i = \psi^C z_i^C + g\left(\bar{z}_i^S\right)$. Optimal basket choice $\bar{z}_i^S$ satisfies

$$\epsilon \frac{\bar{z}_i^S}{z_i} \frac{dz_i}{d\bar{z}_i^S} + (1 - \epsilon) \frac{\bar{z}_i^S}{\Psi_i} \frac{d\Psi_i}{d\bar{z}_i^S} = 0$$

This equation can be rewritten as $F = \epsilon F_1 + (1 - \epsilon) F_2 = 0$ where

$$F_1 = \frac{\bar{z}_i^S}{z_i} \frac{dz_i}{d\bar{z}_i^S} = \kappa_S \left(\frac{\bar{z}_i^S}{z_i}\right)^{\frac{\nu-1}{\nu}} \qquad \text{and} \qquad F_2 = \frac{\bar{z}_i^S}{\Psi_i} \frac{d\Psi_i}{d\bar{z}_i^S} = \frac{\bar{z}_i^S}{\psi^C z_i^C + g\left(\bar{z}_i^S\right)} g'\left(\bar{z}_i^S\right)$$

Since $\nu < 1$, $\frac{dF_1}{d\bar{z}_i^S} < 0$ and $\frac{dF_1}{dz_i^C} > 0$. In addition, $\frac{dF_2}{d\bar{z}_i^S} > 0$, since $g', g'' > 0$[2], and $\frac{dF_2}{dz_i^C} < 0$. Recall $\epsilon > 1$. Hence $\frac{dF}{d\bar{z}_i^S} < 0$ and $\frac{dF}{dz_i^C} > 0$. Thus by the Implicit Function Theorem, $\frac{d\bar{z}_i^S}{dz_i^C} = -\frac{dF/dz_i^C}{dF/d\bar{z}_i^S} > 0$. ■

Proposition. *Upstream firms choose $p_k^S = \frac{1}{\zeta-1}\bar{\mu}_k + \frac{\zeta}{\zeta-1}\psi^S z_k^S$ where $\bar{\mu}_k$ solves*

$$1 = \int_0^1 \frac{x_i^C w_{ik}}{x_k^S} \left[\frac{1}{\zeta} + \left(\frac{\zeta - 1}{\zeta}\right) \left(\frac{\psi^S z_k^S + \mu_i}{\psi^S z_k^S + \bar{\mu}_k}\right)\right]^{-1} di$$

Proof. Upstream firm k with quality z_k^S faces demand $x_k^S = \int_0^1 x_{ik}^S di$ where $x_{ik}^S = w_{ik} x_i^C$ and

$$x_{ik}^S = x_i^C \left(z_k^S\right)^{\zeta-1} \left(\mathcal{A}_{i0}\right)^{-1} \left(p_k^S + \mu_i\right)^{-\zeta}$$

Upstream firms take $x_i^C, \mathcal{A}_{i0}$, and μ_i as given for all $i \in I$ and set the ratio of revenue to gross margin equal to elasticity of demand

$$\frac{p_k^S}{p_k^S - \psi^S z_k^S} = -\frac{dx_k^S / x_k^S}{dp_k^S / p_k^S}$$

The price elasticity it faces is a quantity-weighted average of downstream price elasticity of demand

$$\frac{dx_k^S / x_k^S}{dp_k^S / p_k^S} = \int_0^1 \frac{x_{ik}^S}{x_k^S} \left(\frac{dx_{ik}^S / x_{ik}^S}{dp_k^S / p_k^S}\right) di$$

[2]Since g convex, elasticity of price with respect to upstream quality $\frac{dg\left(\bar{z}_i^S\right)/g\left(\bar{z}_i^S\right)}{d\bar{z}_i^S/\bar{z}_i^S}$ is increasing in upstream quality $\bar{z}_i^S$

where elasticity of demand of i for firm k's input is increasing in p_k^S and decreasing in μ_i. That is, demand by higher quality downstream firms is less responsive to price.

$$-\frac{dx_{ik}^S/x_{ik}^S}{dp_k^S/p_k^S} = \zeta \frac{p_k^S}{p_k^S + \mu_i}$$

Hence the optimal price satisfies

$$\frac{1}{\zeta} = \int_0^1 \frac{x_{ik}^S}{x_k^S} \left(\frac{p_k^S - \psi^S z_k^S}{p_k^S + \mu_i} \right) di$$

Substituting $p_k^S = \frac{1}{\zeta - 1}\bar{\mu}_k + \frac{\zeta}{\zeta - 1}\psi^S z_k^S$, then

$$1 = \int_0^1 \frac{x_{ik}^S}{x_k^S} \left(\frac{1}{\zeta} + \left(\frac{\zeta - 1}{\zeta} \right) \frac{\mu_i + \psi^S z_k^S}{\bar{\mu}_k + \psi^S z_k^S} \right)^{-1} di$$

■

Proposition. *p_k^S is increasing in z_k^S.*

Proof. Recall that upstream firms choose p_k^S such that

$$\frac{1}{\zeta} = \int_0^1 \frac{x_{ik}^S}{x_k^S} \left(\frac{p_k^S - \psi^S z_k^S}{p_k^S + \mu_i} \right) di$$

Note $x_k^S = \int_0^1 x_{ik}^C di$ where $x_{ik}^C = w_{ik} x_i^C$. Thus the previous equation can be rewritten as follows where $F = 0$.

$$F = \int_0^1 x_i^C w_{ik}^C \left[\frac{1}{\zeta} \left(p_k^S - \psi^S z_k^S \right)^{-1} - \left(p_k^S + \mu_i \right)^{-1} \right] di$$

Recall that $w_{ik} = \left(z_k^S \right)^{\zeta - 1} \left(\mathcal{A}_{i0} \right)^{-1} \left(p_k^S + \mu_i \right)^{-\zeta}$. Substituting $\frac{dw_{ik}}{dz_{ik}^S} = (\zeta - 1) \frac{w_{ik}}{z_k^S}$, $\frac{dw_{ik}}{dp_k^S} = -\zeta \frac{w_{ik}}{p_k^S + \mu_i}$, and $F = 0$, we can evaluate $\frac{dF}{dz_k^S}$ and $\frac{dF}{dp_k^S}$.

$$\frac{dF}{dz_k^S} = \int_0^1 x_i^C w_{ik} \frac{\psi^S}{\zeta} \left(p_k^S - \psi^S z_k^S \right)^{-2} di$$

Since $p_k^S - \psi^S z_k^S > 0$, $\frac{dF}{dz_k^S} > 0$. Next, $\frac{dF}{dp_k^S} = -\mathcal{I}_1 - \zeta \mathcal{I}_2$ where

$$\mathcal{I}_1 = \int_0^1 x_i^C w_{ik} \left[\frac{1}{\zeta} \left(p_k^S - \psi^S z_k^S \right)^{-2} - \left(p_k^S + \mu_i \right)^{-2} \right] di$$

$$\mathcal{I}_2 = \int_0^1 x_i^C w_{ik} \left[\frac{1}{\zeta} \left(p_k^S - \psi^S z_k^S \right)^{-1} \left(p_k^S + \mu_i \right)^{-1} - \left(p_k^S + \mu_i \right)^{-2} \right] di$$

By Jensen's Inequality $\mathcal{I}_1 > 0$. Then $\mathcal{I}_2 > \mathcal{I}_2 - \zeta \mathcal{I}_1$ where

$$\mathcal{I}_2 - \zeta \mathcal{I}_1 = \left(p_k^S - \psi^S z_k^S \right) F + (\zeta - 1) \int_0^1 x_i^C w_{ik} \left(p_k^S + \mu_i \right)^{-2} di > 0$$

Note $F = 0$ and $p_k^S + \mu_i > 0$ and $\zeta > 1$. Hence $\frac{dF}{dp_k^S} < 0$. Thus, by Implicit Function Theorem, $\frac{dp_k^S}{dz_k^S} = -\frac{dF/dz_k^S}{dF/dp_k^S} > 0$ ■

Proposition. *[Positive Assortative Matching] Suppose $\epsilon, \zeta > 1$ and $\nu < 1$. If $z_m^C > z_i^C$ and $z_k^S > z_j^S$, then $w_{ik}/w_{ij} > w_{mk}/w_{mj}$.*

Proof. Since $\zeta > 1$, $p_k^S > p_j^S$. Since $\epsilon > 1$ and $\nu < 1$, if $z_m^C > z_i^C$ then optimal basket $\bar{z}_m^S > \bar{z}_i^S$ so $\mu_m > \mu_i$. Thus $w_{ik}/w_{ij} > w_{mk}/w_{mj}$. ■

A.3 Downstream Profits

Definition. Given downstream quality z_i^C, chosen upstream quality $\bar{z}_i^S$, and price schedule $g\left(\bar{z}_i^S\right)$, define unit costs faced by the downstream firm as

$$\Psi_i = \psi^C z_i^C + g\left(\bar{z}_i^S\right)$$

Substituting demand $x_i^C = \left(\frac{z_i}{p_i^C}\right)^\epsilon L$ and optimal price $p_i^C = \frac{\epsilon}{\epsilon-1}\Psi_i$, profits for a downstream firm with blueprint quality z_i^C can be expressed as

$$\pi^C\left(z_i^C\right) = \left(\frac{\epsilon-1}{\epsilon}\right)^\epsilon \frac{1}{\epsilon-1} L\left(z_i\right)^\epsilon \left(\Psi_i\right)^{1-\epsilon}$$

The elasticity of downstream profits to its own quality, $\varphi^C\left(z_i^C\right) = \frac{d\pi_i^C/\pi_i^C}{dz_i^C/z_i^C}$, can be expressed as

$$\varphi^C\left(z_i^C\right) = \epsilon\frac{dz_i/z_i}{dz_i^C/z_i^C} + (1-\epsilon)\frac{d\Psi_i/\Psi_i}{dz_i^C/z_i^C}$$

where $\frac{dz_i/z_i}{dz_i^C/z_i^C} = \kappa_C\left(\frac{z_i^C}{z_i}\right)^{\frac{\nu-1}{\nu}}$ and $\frac{d\Psi_i/\Psi_i}{dz_i^C/z_i^C} = \psi^C\frac{z_i^C}{\Psi_i}$.

The cross elasticity of downstream profits to upstream quality, $\hat{\varphi}^C\left(z_i^C\right) = \frac{d\pi_i^C/\pi_i^C}{d\bar{z}_i^S/\bar{z}_i^S}$, can be expressed as

$$\hat{\varphi}^C\left(z_i^C\right) = \epsilon\frac{dz_i/z_i}{d\bar{z}_i^S/\bar{z}_i^S} + (1-\epsilon)\frac{d\Psi_i/\Psi_i}{d\bar{z}_i^S/\bar{z}_i^S}$$

where $\frac{dz_i/z_i}{d\bar{z}_i^S/\bar{z}_i^S} = \kappa_S\left(\frac{\bar{z}_i^S}{z_i}\right)^{\frac{\nu-1}{\nu}}$ and $\frac{d\Psi_i/\Psi_i}{d\bar{z}_i^S/\bar{z}_i^S} = g'\left(\bar{z}_i^S\right)\frac{\bar{z}_i^S}{\Psi_i}$.

If all downstream firms choose $\bar{z}_i^S = \bar{z}^S$ with price $g\left(\bar{z}^S\right) = \frac{\zeta}{\zeta-1}\psi^S\bar{z}^S$, then $\frac{d\Psi_i/\Psi_i}{d\bar{z}_i^S/\bar{z}_i^S} = \frac{\frac{\zeta}{\zeta-1}\psi^S\bar{z}^S}{\Psi_i}$.

Proposition. *Suppose $\epsilon > 1$, $\nu < 1$, g convex, and $\epsilon + (1-\epsilon)\psi^C < 0$. For given upstream prices $\left\{p_k^S\right\}_{k\in K}$ and upstream distribution Φ^S, there exists threshold $z^{C*} < z^{C**}$ such that*

1. For $z_i^C < z^{C*}$, downstream profits $\pi^C\left(z_i^C\right)$ are increasing and convex in z_i^C
2. For $z^{C*} < z_i^C < z^{C**}$, downstream profits are increasing and concave in z_i^C.
3. For $z_i^C > z^{C**}$, downstream profits are decreasing z_i^C.

Proof. As $z_i^C \to \infty$, $\frac{dz_i/z_i}{d\bar{z}_i^S/\bar{z}_i^S} \to 0$, $\frac{d\Psi_i/\Psi_i}{d\bar{z}_i^S/\bar{z}_i^S} \to 1$, and $\varphi^C\left(z_i^C\right) \to (1-\epsilon) < 0$. Thus there exists z^{C**} where $\varphi^C\left(z^{C**}\right) = 0$. Moreover, for $z_i^C < z^{C**}$, $\pi^C\left(z_i^C\right)$ are increasing in z_i^C, and decreasing otherwise. The second derivative of profits can be expressed as

$$\frac{d^2\pi^C}{d\left(z_i^C\right)^2} = \frac{\pi^C}{z_i^C}\left[\frac{\varphi^C\left(z_i^C\right)}{z_i^C}\left[\varphi\left(z_i^C\right)-1\right] + \frac{d\varphi^C\left(z_i^C\right)}{dz_i^C}\right]$$

where

$$\frac{d\varphi^C\left(z_i^C\right)}{dz_i^C} = \epsilon\kappa_C\frac{\nu-1}{\nu}\left(\frac{z_i^C}{z_i}\right)^{-\frac{1}{\nu}} + (1-\epsilon)\frac{\frac{\psi^C z_i^C}{g(\bar{z}^S)}}{\left(1+\frac{\psi^C z_i^C}{g(\bar{z}^S)}\right)^2}$$

Since $\epsilon > 1$ and $\nu < 1$, $\frac{d\varphi^C\left(z_i^C\right)}{dz_i^C} < 0$. As $z_i^C \to z^{C**}$, $\varphi\left(z_i^C\right) \to 0$. Hence $\frac{d^2\pi^C}{d\left(z_i^C\right)^2} < 0$. Observe that for $\varphi\left(z_i^C\right)$ sufficiently large, such that $\frac{d^2\pi^C}{d\left(z_i^C\right)^2} > 0$, establishing lower threshold z^{C*}. Intuitively, when Φ^S is held constant, there are decreasing returns to increasing z_i^C. However, increasing z_i^C is accompanied by linearly increasing blueprint cost. For $z_i^C > z^{C**}$, the latter dominates the former. ■

Proposition. *Suppose $\epsilon > 1$, $\nu < 1$, and g convex and all firms use bundle quality $\bar{z}^S$ with price $g\left(\bar{z}^S\right) = \frac{\zeta}{\zeta-1}\psi^S\bar{z}^S$*

1. Threshold z^{C**} increases with upstream quality $\bar{z}^S$.
2. Elasticity $\varphi^C\left(z_i^C\right)$ increases with $\bar{z}^S$ and cross elasticity $\hat{\varphi}^C\left(z_i^C\right)$ increases with z_i^C.
3. There exists threshold z^{C***} such that for $z_i^C > z^{C***}$, profits are increasing in upstream quality ,$\hat{\varphi}^C\left(z_i^C\right) > 0$, and decreasing otherwise.

Proof. (1) If the upstream distribution is degenerate then, all downstream firms use bundle $\bar{z}^S$ with price $\bar{p}^S = \frac{\zeta}{\zeta-1}\psi^S\bar{z}^S$. Since $\nu < 1$, $\frac{dz_i/z_i}{d\bar{z}_i^S/\bar{z}_i^S}$ increases with $\bar{z}^S$ and $\frac{dz_i/z_i}{d\bar{z}_i^S/\bar{z}_i^S}$ decrease with $\bar{z}^S$. Since $\epsilon > 1$, threshold z^{C**} such that $\varphi^C\left(z^{C**}\right) = 0$ must increase.

(3) Note that

$$\frac{d\varphi^C\left(z_i^C\right)}{d\bar{z}^S} = \epsilon\frac{1-\nu}{\nu}\kappa_C\kappa_S\left(\frac{z_i^C}{z_i}\right)^{\frac{\nu-1}{\nu}}\left(\frac{\bar{z}^S}{z_i}\right)^{\frac{\nu-1}{\nu}}\frac{1}{\bar{z}^S} + (\epsilon-1)\frac{\frac{\zeta}{\zeta-1}\psi^C\psi^S z_i^C}{\left(\Psi_i\right)^2} > 0$$

$$\frac{d\hat{\varphi}^C\left(z_i^C\right)}{dz_i^C} = \epsilon\frac{1-\nu}{\nu}\kappa_C\kappa_S\left(\frac{z_i^C}{z_i}\right)^{\frac{\nu-1}{\nu}}\left(\frac{\bar{z}^S}{z_i}\right)^{\frac{\nu-1}{\nu}}\frac{1}{z_i^C} + (\epsilon-1)\frac{\frac{\zeta}{\zeta-1}\psi^C\psi^S\bar{z}^S}{\left(\Psi_i\right)^2} > 0$$

(2) As $z^C \to 0$, $\hat{\varphi}^C \to 1-\epsilon < 0$. Since $\hat{\varphi}$ is continuous and increasing in z_i^C, there exists z^{C***} such that for $z_i^C > z^{C***}$, $\hat{\varphi}^C\left(z_i^C\right) \geq 0$, and $\hat{\varphi}^C\left(z_i^C\right) > 0$ otherwise. ■

A.4 Upstream Profits

Definition. Define the elasticity of upstream profits to its own quality as $\varphi^S\left(z_k^S\right) = \frac{d\pi_k^S/\pi_k^S}{dz_k^S/z_k^S}$. If all downstream firms have homogenous quality $z_i^C = z^C$, then define the cross elasticity of downstream profits to upstream quality $\hat{\varphi}^S\left(z_k^S\right) = \frac{d\pi_k^S/\pi_k^S}{dz^C/z^C}$.

Proposition. *Suppose $\epsilon, \zeta > 1$ and $\nu < 1$ and all downstream firms have homogenous quality $z_i^C = z^C$ with shadow price $\mu_i = \mu$, quantity $x_i^C = x^C$, and aggregate $\mathcal{A}_{i0} = \mathcal{A}_0$.*

(1) Profits are increasing in quality, if $1 + \frac{\psi^S z_k^S}{\mu} > 0$ and decreasing otherwise. Define z^{S**} as this threshold.

Proposition 13. *(2) As $z_k^S \to z^{S**}$, $\pi^S\left(z_k^S\right)$ is concave.*

Proof. Since all downstream are identical, upstream firm k with quality z_k^S optimally chooses $p_k^S = \frac{1}{\zeta-1}\mu + \frac{\zeta}{\zeta-1}\psi^S z_k^S$ and face demand

$$x_k^C = \left(z_k^S\right)^{\zeta-1} x^C \left(\mathcal{A}_0\right)^{-1} \left(\frac{\zeta}{\zeta-1}\left[\mu + \psi^S z_k^S\right]\right)^{-\zeta}$$

Upstream profits can be expressed as a function of its own quality z_k^S and downstream moments x^C, $\mathcal{A}_0$, and μ.

$$\pi^S\left(z_k^S\right) = x_k^C\left(p_k^S - \psi^S z_k^S\right) = \frac{1}{\zeta-1}\left(\frac{\zeta}{\zeta-1}\right)^{-\zeta}\left(\frac{x^C}{\mathcal{A}_0}\right)\left(\frac{z_k^S}{\mu + \psi^S z_k^S}\right)^{\zeta-1}$$

Thus elasticity of profits to upstream quality $\varphi^S\left(z_k^S\right) = (\zeta-1)\left[1 + \frac{\psi^S z_k^S}{\mu}\right]^{-1}$ is positive provided $1 + \frac{\psi^S z_k^S}{\mu} > 0$. Since

$$\frac{d\varphi^S\left(z_k^S\right)}{dz_k^S} = -1(\zeta-1)\left[1 + \frac{\psi^S z_k^S}{\mu}\right]^{-2}\psi^S$$

and $\zeta > 1$, for $1 + \frac{\psi^S z_k^S}{\mu} > 0$, $\frac{d\varphi^S\left(z_k^S\right)}{dz_k^S} < 0$ and the profit function is concave in quality. ■

Proposition. *If $\zeta > 1$ and $\mu_i > \psi^S z_k^S$, then elasticity $\frac{d\left(\frac{x_i^C}{\mathcal{A}_{i0}}\right)/\left(\frac{x_i^C}{\mathcal{A}_{i0}}\right)}{dz_i^C/z_i^C} > 0$*

Proof. The demand effect can be expressed as the difference between $\frac{dx_i^C/x_i^C}{dz_i^C/z_i^C}$ and $\frac{d\mathcal{A}_{i0}/\mathcal{A}_{i0}}{dz_i^C/z_i^C}$

$$\frac{d\left(\frac{x_i^C}{\mathcal{A}_{i0}}\right)/\left(\frac{x_i^C}{\mathcal{A}_{i0}}\right)}{dz_i^C/z_i^C} = \frac{dx_i^C/x_i^C}{dz_i^C/z_i^C} - \frac{d\mathcal{A}_{i0}/\mathcal{A}_{i0}}{dz_i^C/z_i^C}$$

Each term can be expressed as

$$\frac{dx_i^C/x_i^C}{dz_i^C/z_i^C} = \epsilon\left[\frac{dz_i/z_i}{dz_i^C/z_i^C} - \frac{dp_i^C/p_i^C}{dz_i^C/z_i^C}\right] = \epsilon\left[\frac{\kappa_C\left(z_i^C\right)^{\frac{\nu-1}{\nu}}}{\kappa_C\left(z_i^C\right)^{\frac{\nu-1}{\nu}} + \kappa_S\left(\bar{z}_i^S\right)^{\frac{\nu-1}{\nu}}} - \frac{\psi^C z_i^C}{\psi^C z_i^C + \bar{p}_i^S}\right]$$

$$\frac{d\mathcal{A}_{i0}/\mathcal{A}_{i0}}{dz_i^C/z_i^C} = -\zeta\left(\int_0^1 \left(z_k^S\right)^{\zeta-1}\left(\frac{\zeta}{\zeta-1}\right)^{-\zeta}\left(\mu_i + \psi^S z_k^S\right)^{-\zeta-1} dk\right)\left(\frac{d\mu_i}{dz_i^C}\right)\left(\frac{z_i^C}{A_{i0}}\right)$$

Note $\frac{dx_i^C/x_i^C}{dz_i^C/z_i^C}$ always positive. If $\mu_i > \psi^S z_k^S$ then $\frac{d\mathcal{A}_{i0}/\mathcal{A}_{i0}}{dz_i^C/z_i^C} < 0$. Thus $\frac{d\left(\frac{x_i^C}{\mathcal{A}_{i0}}\right)/\left(\frac{x_i^C}{\mathcal{A}_{i0}}\right)}{dz_i^C/z_i^C} > 0$. ■

Proposition. *Suppose $\epsilon, \zeta > 1$ and $\nu < 1$ and all downstream firms have homogenous quality $z_i^C = z^C$. Then*

1. Threshold z^{S**} increases with downstream quality z_i^C.
2. Elasticity $\varphi^S\left(z_k^S\right)$ increases with z^C and cross elasticity $\hat{\varphi}^S\left(z_k^S\right)$ increases with z_k^S
3. There exists threshold z^{S***} such that for $z_i^S > z^{C***}$, profits are increasing in downstream quality ,$\hat{\varphi}^S\left(z_k^S\right) > 0$, and decreasing otherwise.

Proof. Recall that since $\epsilon, \zeta > 1$ and $\nu < 1$, z^C is increasing in both $\bar{z}^S$ and μ. Hence z^{S**} increases.

Cross elasticity of upstream profits to downstream quality can be characterized as the sum the demand effect which impacts all upstream firms equally and the reallocation effect which benefits higher quality firms.

$$\hat{\varphi}^S\left(z_k^S\right) = \underbrace{\frac{d\left(\frac{x^C}{\mathcal{A}_0}\right)}{dz^C}\frac{z^C}{\left(\frac{x^C}{\mathcal{A}_0}\right)}}_{demand} - \underbrace{\left(\zeta - 1\right)\frac{z^C}{\mu + \psi^S z_k^S}\frac{d\mu}{dz^C}}_{reallocation}$$

Since $\frac{d\mu}{dz^C} > 0$, $\frac{d\varphi^S\left(z_k^S\right)}{dz^C} > 0$ and $\frac{d\hat{\varphi}^S\left(z_k^S\right)}{dz_k^S} > 0$

$$\frac{d\varphi^S\left(z_k^S\right)}{dz^C} = \left(\zeta - 1\right)\left[1 + \frac{\psi^S z_k^S}{\mu}\right]^{-2}\frac{\psi^S z_k^S}{\left(\mu\right)^2}\frac{d\mu}{dz^C} > 0$$

$$\frac{d\hat{\varphi}^S\left(z_k^S\right)}{dz_k^S} = \left(\zeta - 1\right)\psi^S\frac{z^C}{\left(\mu + \psi^S z_k^S\right)^2}\frac{d\mu}{dz^C} > 0$$

Observe that for $1 + \frac{\psi^S z_k^S}{\mu} > 0$, the demand effect is positive, and for sufficiently high z_k^S, $\hat{\varphi}^S\left(z_k^S\right) > 0$. Hence for $z_k^S > z^{S***}$, where $\hat{\varphi}^S\left(z^{S***}\right) = 0$, upstream profits are increasing in downstream quality. ■

A.5 Upstream firm quality $z_k^S \in \left\{z_0^S, z_1^S\right\}$

Suppose the upstream distribution is discrete with K qualities. Given allocation ω where $\omega_k = \phi_k w_k$, upstream basket quality and price can be expressed as

$$\bar{z}^S\left(\omega\right) = \left[\sum_{k\in K}\phi_k^S\left(z_k^S\frac{\omega_k}{\phi_k^S}\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \qquad \bar{p}^S\left(\omega\right) = \sum_{k\in K}\omega_k p_k^S$$

Consider the special case where upstream firms can be either high or low quality: $z_k^S \in \left\{z_0^S, z_1^S\right\}$, with fraction ϕ^S of upstream firms having higher quality $z_1^S > z_0^S$. Then

$$\bar{z}^S\left(\omega\right) = \left[\left(\phi_0^S\right)^{\frac{1}{\zeta}}\left(\omega_0 z_0^S\right)^{\frac{\zeta-1}{\zeta}} + \left(\phi_1^S\right)^{\frac{1}{\zeta}}\left(\omega_1 z_1^S\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}$$

$$\bar{p}^S\left(\omega\right) = \omega_0 p_0^S + \omega_1 p_1^S$$

Define $m\left(\omega\right) = \left(\phi_0^S\right)^{\frac{1}{\zeta}}\left(\omega_0 z_0^S\right)^{\frac{\zeta-1}{\zeta}} + \left(\phi_1^S\right)^{\frac{1}{\zeta}}\left(\omega_1 z_1^S\right)^{\frac{\zeta-1}{\zeta}}$. Then

$$\frac{dm\left(\omega\right)}{d\phi_1} = -\frac{1}{\zeta}\left(\frac{\omega_0 z_0^S}{\phi_0^S}\right)^{\frac{\zeta-1}{\zeta}} + \frac{1}{\zeta}\left(\frac{\omega_1 z_1^S}{\phi_1^S}\right)^{\frac{\zeta-1}{\zeta}} > 0 \iff \frac{\omega_1 z_1^S}{\phi_1^S} > \frac{\omega_0 z_0^S}{\phi_0^S}$$

Hence there exists threshold $\omega_1^{**} = \frac{z_0^S}{z_0^S + z_1^S \frac{\phi_0^S}{\phi_1^S}}$ such that for $\omega_1 > \omega_1^{**}$, $\frac{dm(\omega)}{d\phi_1} > 0$. Since $\zeta > 1$, $\frac{d\bar{z}^S(\omega)}{d\phi_1} > 0$.

If prices do not change with ϕ_1^S, then neither does $\bar{p}^S(\omega)$. Note that $\bar{z}^S$ is monotonic in ω_1. Thus as ϕ_1^S increases, there exists threshold $\bar{z}^{S**} = \bar{z}^S([1-\omega_1^{**}, \omega_1^{**}])$ such basket $\bar{z}_i^S$ becomes less expensive if $\bar{z}_i^S > \bar{z}^{S**}$ and vice versa. Since z_i^C is increasing in optimal basket $\bar{z}_i^S$, there exists threshold z^{C**} such that for $z_i^C > z^{C**}$. Since downstream firm i can achieve the original basket $\bar{z}_i^S$ at lower cost profits under its original ω serve as a lower bound, and any changes to ω must improve profits. For these firms, profits are increasing in ϕ_1^S: $\frac{d\pi^C(z_i^C)}{d\phi_1^C} \geq 0$. Note that

$$\frac{d\log \pi^C}{d\log \phi_1^S} = \epsilon \kappa_S \left(\frac{\bar{z}_i^S}{z_i}\right)^{\frac{\nu-1}{\nu}} \frac{d\bar{z}_i^S}{d\phi_1^C} \frac{\phi_1^S}{\bar{z}_i^S} + (1-\epsilon) \frac{\bar{p}_i^S}{\Psi_i} \frac{d\bar{p}_i^S}{d\phi_1^S} \frac{\phi_1^S}{\bar{p}_i^S}$$

Downstream profits are increasing in ϕ_1^S provided

$$\frac{d\bar{z}_i^S}{d\bar{p}_i^S} = \frac{\frac{d\bar{z}_i^S}{d\phi_1^S}}{\frac{d\bar{p}_i^S}{d\phi_1^S}} > \left(\frac{\epsilon - 1}{\epsilon}\right) \frac{z_i}{\Psi_i \kappa_S} \left(\frac{\bar{z}_i^S}{z_i}\right)^{\frac{1}{\nu}} > 0$$

where

$$\frac{d\bar{z}_i^S}{d\phi_1^C} = \frac{1}{\zeta - 1} \left[\bar{z}_i^S\right]^{\frac{1}{\zeta}} \left[\left(\frac{\omega_1 z_1^S}{\phi_1^S}\right)^{\frac{\zeta-1}{\zeta}} - \left(\frac{\omega_0 z_0^S}{\phi_0^S}\right)^{\frac{\zeta-1}{\zeta}}\right]$$

$$\frac{d\bar{p}_i^S}{d\phi_1^S} = \omega_0 \frac{dp_0^S}{d\phi_1^S} + \omega_1 \frac{dp_1^S}{d\phi_1^S}$$

If $\omega_1 < \omega_1^{**}$, then $\frac{d\bar{z}_i^S}{d\phi_1^S} < 0$: as ϕ_1^S increase firm i chooses lower basket quality $\bar{z}_i^S$. Note that $\frac{dp_1^S}{d\phi_1^S} < 0$ and $\frac{dp_0^S}{d\phi_1^S} > 0$: as the share of high quality upstream firms increase prices of competing high quality firms fall. In this region, if $\frac{d\bar{p}_i^S}{d\phi_1^S} > 0$ then $\frac{d\bar{z}_i^S}{d\bar{p}_i^S} < 0$ so $\frac{d\pi^C(z_i^C)}{d\phi_1^S} < 0$.

A.6 Sorting

Measuring Relationships Persistence

Defining Δ_{ik} as the expected percent of the relationship that is lost from period t to $t+1$

$$\Delta_{ikt} = E\left[\max\left(\frac{w_{ikt} - w_{ikt+1}}{w_{ikt}}, 0\right)\right]$$

If neither firms innovate, if the economy is at its stationary equilibrium, then

$$\Delta_{ikt} = 1 - \left(1 - \phi^S\right)\left(1 - \phi^C\right)$$

where ϕ^S and ϕ^C is the exogenous probability the firm exists. If the firm innovates, then Δ_{ik} increases. Then

$$\Delta_{ikt} = \underbrace{\left[1 - \left(1 - \phi^S\right)\left(1 - \phi^C\right)\right]}_{1-bothsurvive} + \underbrace{\left(1 - \phi^C\right)\left(1 - \phi^S\right)}_{bothsurvive} \sum_{z_{it+1}^C} \sum_{z_{kt+1}^S} \pi^C\left(z_{it+1}^C \mid z_{it}^C\right) \pi^S\left(z_{it+1}^S \mid z_{it}^S\right) \max\left(\frac{w_{ikt} - w_{ikt+1}}{w_{ikt}}, 0\right)$$

This measure can be aggregated to the firm and sector level

$$\Delta_{it}^{C} = \int_0^1 w_{ikt}\Delta_{ikt}dk$$

$$\Delta_{t}^{C} = \int_0^1 \Delta_{it}^{C}di$$

Exogenous Switching Costs

Alternatively, suppose instead of being fully flexible, downstream firms cannot choose fraction τ of their suppliers. Then

$$\tilde{w}_{ik} = \tau + (1-\tau)\, w_{ik}$$

Suppose firms act disjointly and choose w_{ik} as if solving the original problem. Then

$$\tilde{z}^{S}(w_k) = \left[\int_0^1 \left(\tau + (1-\tau)\, w_k\right)^{\frac{\zeta-1}{\zeta}} \left(z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}}$$

$$\tilde{p}^{S}(w_k) = \tau \int_0^1 p_k^S dk + (1-\tau)\int_0^1 w_k p_k^S dk$$

Recall $w_{ik} = \left(\mathcal{A}_{0i}^{-1}\right)\left(z_k^S\right)^{\zeta-1}\left(p_k^S+\mu_i\right)^{-\zeta}$. Then

$$\tilde{w}_{ik}\left(p_k^S\right) = \tau \underbrace{\left(z_k^S\right)^{\zeta-1}\left(p_k^S\right)^{-\zeta}} + (1-\tau)\underbrace{\left(\mathcal{A}_{0i}^{-1}\right)\left(z_k^S\right)^{\zeta-1}\left(p_k^S+\mu_i\right)^{-\zeta}}$$

Hence demand can be expressed as

$$x_k = \int_0^1 x_{ik}di = \int_0^1 \tilde{w}_k x_i^C di$$

$$x_k^S = \left(z_k^S\right)^{\zeta-1}\tau\left(p_k^S\right)^{-\zeta}\int_0^1 x_i^C di + \left(z_k^S\right)^{\zeta-1}(1-\tau)\int_0^1 \left(\mathcal{A}_{0i}^{-1}\right)x_i^C\left(p_k^S+\mu_i\right)^{-\zeta}di$$

and upstream firms solve

$$\max x_k^S\left(p_k^S - \psi^S z_k^S\right)$$

$$x_k^S + \frac{dx_k^S}{dp_k^S}\left(p_k^S - \psi^S z_k^S\right) = 0$$

$$\frac{dx_k^S}{dp_k^S} = -\zeta\tau\left(z_k^S\right)^{\zeta-1}\left(p_k^S\right)^{-\zeta-1}\int_0^1 x_i^C di - \zeta(1-\tau)\left(z_k^S\right)^{\zeta-1}\int_0^1 \left(\mathcal{A}_{0i}^{-1}\right)x_i^C\left(p_k^S+\mu_i\right)^{-\zeta-1}di$$

Then p_k^S solves

$$\tau\left(p_k^S\right)^{-\zeta}\int_0^1 x_i^C di + (1-\tau)\int_0^1 \left(\mathcal{A}_{0i}^{-1}\right)x_i^C\left(p_k^S+\mu_i\right)^{-\zeta}di$$

$$-\zeta\left(p_k^S - \psi^S z_k^S\right)\left[\tau\left(p_k^S\right)^{-\zeta-1}\int_0^1 x_i^C di + (1-\tau)\int_0^1 \left(\mathcal{A}_{0i}^{-1}\right)x_i^C\left(p_k^S+\mu_i\right)^{-\zeta-1}di\right] = 0$$

Recall that previously

$$\bar{z}^S_{\max} = \left[\int_0^1 \left(z^S_k\right)^{\zeta-1} dk\right]^{\frac{1}{\zeta-1}} \qquad w_{k,\max} = \left(\bar{z}^S_{\max}\right)^{1-\zeta}\left(z^S_k\right)^{\zeta-1}$$

and $w_{k,\min} = \frac{1}{\phi_S}\mathbf{1}\left\{p^S_k = \min\left\{p^S\right\}\right\}$.

B Empirical Evidence

B.1 Construction of R&D tax credits

Since supplier and customer firms R&D investment decisions may coincide for many reasons, including changes to the technology space, this poses an identification challenge. In order to identify the spillover from the existing supplier customer relationship, I exploit plausibly exogenous firm-level to the user cost of R&D investment arising from differential firm exposure to state and federal R&D tax credits, following Wilson [2009] Bloom et al. [2013], and others. I use a two stage approach, where I first instrument each firm's R&D intensity on its state and federal user cost of R&D, and then regress the firm's R&D intensity on its customers' predicted R&D intensity.

Firms are differentially exposed to state R&D tax credits due to differences in state-level corporate income taxes and R&D subsidies rates in conjunction with firm-level differences in exposure to each state. I follow Bloom, et. al. 2013, and assume each firm's state user cost of R&D , ρ_{sit}, is proportional to a weighted average of the state-level user cost of capital, $\tilde{\rho}_{st}$, weighted by, θ_{kst}, the firm's R&D exposure to state k. To measure θ_{kst}, I use the the proportion of unique inventors located in state k corresponding to patents produced by the firm i in the previous 5 years. That is, the firm's state user cost of R&D, ρ^s_{it}, is the weighted

$$\rho^s_{it} = \log\left(\sum_{k=1}^{50}\theta_{kst}\tilde{\rho}_{st}\right)$$

While federal corporate tax rates and subsidies vary across time, firms also have different federal R&D user costs cross sectionally due to non-linearities in the percentage of R&D which qualifies for the credit. Since the goal of this tax credit was to induce additional R&D investment, the subsidy was applied to R&D only above the firm's tax applicable base, which at different points in time was a function of either past R&D or past sales. Moreover, the subsidy was not available to R&D exceeding twice the base. These rules, among others, induced differences in a firm's user cost of R&D arising from federal R&D tax credits, ρ^f_{it}, due the firm's past sales and R&D differences, as well as changes to the tax code over time. I largely follow Bloom et al. [2013] and Arque-Castells and Spulber [2022] in their estimation of firms' R&D user cost from federal R&D tax credits. I use Wilson [2009] and Arque-Castells and Spulber [2022] estimates of state and federal level corporate tax rates and R&D subsidy rates. With the state and federal user costs in hand, in the first stage, I project each firm's R&D expenditure and R&D intensity as predicted by the state and federal user costs.

$$lxrd_{it} = \gamma_0 + \gamma_{1t}\rho^s_{it} + \gamma_{2t}\rho^f_{it} + \eta_{it} \Rightarrow \widehat{lxrd_{it}}$$

$$\widehat{lrdintensity}_{it} = \frac{\widehat{lxrd_{it}}}{lsale_{it}}$$

As before, I define the firm's instrumented customer R&D intensity as the average predicted R&D intensity of its customers $k = 1, ..., N$

$$\widehat{customer_lrdintensity}_{it} = \frac{1}{N}\sum_{k=1}^{N} \widehat{lrdintensity}_{kt}$$

In the second stage, to estimate the role of customer R&D on a supplier's R&D, I regress a firm's R&D intensity on the predicted customer R&D intensity. I control for the firm's own exposure to state and federal R&D user costs and, to capture changes to the industry environment which may impact innovation, I include for industry (SIC3) by year fixed effects δ_{it}. Standard errors are bootstrapped.

$$lrdintensity_{it} = \beta_0 + \beta_1 \widehat{customer_lrdintensity}_{it} + \beta_2 \rho^s_{it} + \beta_3 \rho^f_{it} + \delta_{it} + \epsilon_{it}$$