Global Networks, Monetary Policy and Trade*

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Abstract

We develop a new framework to analyze the macroeconomic impact of trade distortions in global general equilibrium with incomplete markets. Our New Keynesian Open Economy (NKOE) model incorporates trade and production networks with full input-output linkages, sectoral heterogeneity in price rigidities, and cross-country heterogeneity in monetary policy. A key theoretical insight is that the dynamics of the inflation–output trade-off depend not only on network linkages but also on granularity. Inflation can be persistent and output losses are larger due to complementarities within the network, compared to two-country-one-sector or small open economy models. We introduce the NKOE Leontief inverse, which governs the propagation of tariff distortions across sectors, countries, and time. The overall macroeconomic impact of tariffs depends on endogenous monetary policy responses in both tariff-imposing and tariff-exposed countries, even without retaliation. The quantitative model -calibrated to 2025 tariffs, granular trade and production data, and empirically relevant elasticities- shows a stagflationary global response in the short run and trade diversion in the medium run. The magnitude of these effects depends on tariff size, retaliation, and policy response. Exchange rate movements depend on the nature of the shock: the dollar appreciates less or even depreciates under tariff threats, which are self-defeating as they cause deflation and lower output.

JEL Codes: E2, E3, E6, F1, F4

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Online Appendix available here.

1 Introduction

We introduce a new framework to analyze the macroeconomic consequences of protectionist trade policies. Our global New Keynesian open-economy (NKOE) model captures the complex interdependence of global trade, finance and production. The framework incorporates realistic structural features—including full international input-output (I-O) linkages, sector-specific nominal rigidities, and cross-country heterogeneity in monetary policy preferences—to provide a comprehensive assessment of both domestic and global effects of tariffs. The model allows trade to be unbalanced and assumes incomplete financial markets. Our core contribution is to delineate how the macroeconomic impact of tariffs can differ from standard small open economy (SOE) and closed-economy NK models when tariffs act as simultaneous demand and supply shocks and countries are allowed to borrow internationally to smooth the impact of these shocks, where bonds are traded under financial frictions present in international capital markets.

To capture realistic features of the global trade system together with commonly assumed frictions in the NK literature, we introduce five primitives:

(i) Consumers in each country make choices prior to the imposition of tariffs, revealing their biases toward home and foreign goods. This is captured by the *consumption share matrix* $\mathbf{\Gamma}$; to relate to standard small open economy (SOE) and two-country settings, we use scalar counterparts for home (H) and foreign (F) countries, denoted by γ_H and $\gamma_F = 1 - \gamma_H$.

(ii) Producers optimize their production by sourcing inputs globally. This is represented by the *I-O matrix* $\mathbf{\Omega}$, with scalar counterparts Ω_H and Ω_F capturing home and foreign input shares, respectively.

(iii) Goods from any country can be substituted—both in consumption and production—by goods within the same sector or across sectors. This is modeled through nested CES bundles. The *elasticities of substitution* (EoS) are given by the vector $\boldsymbol{\theta}$.

(iv) Nominal rigidities may induce a sluggish adjustment of prices, captured by the frequency of price adjustment at the sectoral level, denoted by $\mathbf{\Lambda}$.

(v) Monetary policy determines the price level. Central banks can respond to price changes according to a Taylor rule, with response coefficients captured by the diagonal matrix $\mathbf{\Phi}$ (or scalar ϕ_π). Alternative forms of monetary policy are also possible, like stabilizing consumption with a real rate rule or stabilizing nominal demand.

Our model yields two key novel insights. First, the persistence of inflation depends not only on network linkages, but also network granularity (i.e. having more than one sector). We show this insight with a dynamic general equilibrium object, the NKOE Leontief inverse, which captures propagation of trade distortions across sectors and across time. This allows us to understand the dynamics of inflation-output tradeoff. Inflation rises first, because a significant share of inflation comes from direct CPI and PPI effects that show up on impact. Given nominal rigidities, prices adjust slowly and hence the output decline comes later. Given inflation's persistence, a stagflationary impact can be observed for some time. Second, with network complementarities and world demand not being fixed, it is more difficult for output to increase in the tariff-imposing country compared to the SOE case. Our model with full global I-O linkages can deliver a bigger drop in output, even when compared to two-country general equilibrium. Our model is consistent with standard trade models in terms of tariffs' impact on exchange rate and trade balance. Exchange rate appreciates and trade balance improves under transitory tariffs. In the case of permanent tariffs the USD value of the trade balance remains unchanged. We show reversed tariff threats can generate deflation and a decline in output. If reversed tariff threats precede imposition of tariffs, then deflation and recession can precede stagflation. Exchange rate can also depreciate under tariff threats.

The effects of tariffs on prices and economic activity have been widely studied. For example, trade literature often assumes flexible prices, balanced trade with no international borrowing, and prefers static models given its long-run focus on productivity and welfare.[1] Open economy macro literature, on the other hand, relies mostly on SOE-NK models for a short-run analysis of transitory tariffs, ignoring intermediate inputs and supply chains.[2] We argue that, under both transitory and permanent tariffs, the dynamics of inflation-output trade-off, a key issue for the short-run approach, critically depends on the network structure and input complementarity—structural features that are only determined in the long-run.[3]

Let us start with a standard two-country (H and F) one-industry example to illustrate the intuition behind how the primitives shape the impact of tariffs. Suppose H places tariffs on F without retaliation. H is large and wants to manipulate its terms of trade. Under

[1]Note that this literature treats tariffs as permanent and works with exact hat-algebra in two-period models (see, for example, Costinot and Rodríguez-Clare, 2014).

[2]Early Keynesian literature studies the short-run impact of tariffs. See, for example, Mundell (1961), Eichengreen (1981), and Krugman (1982). This literature lacks the micro foundations of the modern-SOE-NK literature as in Galí and Monacelli (2005). Unfortunately, the early NK literature does not focus on tariffs but rather optimal exchange rate and monetary policies in SOEs. The paper by Barattieri et al. (2021) is an example who studied macro impact of tariffs in a SOE-NK model. In section 2, we overview the recent literature motivated by 2025 tariffs, focusing more on the normative aspects and discuss similarities and differences from our positive approach.

[3]The essential role of intermediate inputs and cross-border production chains in trade is well established (e.g., di Giovanni and Levchenko, 2010; Johnson, 2014).

flexible prices, with low home bias (γ_H) for the tariff imposing country H, H is a relatively sizable buyer of F's goods. With terms of trade improvement in its favor, H's consumption increases and its real exchange rate appreciates vis-a-vis F. We build on this standard case, adding production with endogenous labor supply and imported intermediate inputs. With endogenous labor supply, tariffs distort the labor-leisure choice. If production foreign bias Ω_H is high, where a large part of domestic production uses foreign intermediate inputs, then tariffs increase the cost of production and thereby act as a negative supply shock. These two forces (endogenous labor supply and intermediate inputs) dampen the original terms of trade gains and hence consumption can decline. Elasticity of substitution is important here: if both consumption goods and production inputs are highly substitutable within and across borders with a high θ, then tariffs can be expansionary, whereas if there is sufficient complementarity on the production side then tariffs may be contractionary.

Next, we introduce nominal rigidity (Λ) and monetary policy (ϕ_π or $\hat{M}_t$) to study the short-run effects of tariffs. As expected from standard New Keynesian theory, higher nominal rigidity (Λ) decreases inflation and amplifies the decline in output. While the terms of trade mechanism remains intact, if monetary policy targets inflation, consumption can get hit twice: once from higher domestic prices, akin to a consumption tax, next from higher interest rates.

The network setup granularizes these primitives and takes them to matrix scale with N countries and J industries. Instead of considering dependence on a single intermediate input, for example, we consider the full set of I-O linkages. Our model is summarized in a five-equation Global New Keynesian Representation: (i) the New Keynesian IS (NKIS) equation; (ii) the New Keynesian Phillips Curve (NKPC) for producer prices derived with Rotemberg costs; (iii) a definition of the consumption price vector, which deviates from producer prices due to exchange rate movements and tariff distortions; (iv) an Uncovered Interest Parity (UIP) condition that nests international arbitrage conditions; and (v) an equation of motion for external debt, which also incorporates the market-clearing condition. Together, these equations characterize the equilibrium and nest a broad class of NKOE models.

Our approach of writing down a N country-J sector model can be thought of as connecting two or more Rubbo (2023) economies under incomplete markets and trade imbalances. We do so by allowing representative households in countries to save in nominal local-currency, which are in net zero supply, and USD bonds, with which agents can save or dissave. We use portfolio adjustment costs to ensure that the steady-state level of debt is unique and this feature allows endogenous deviations from Uncovered Interest Parity (UIP). In the theoretical model, we linearize around a steady-state with non-zero debt that is consistent with the primitives consumption foreign bias and production foreign bias γ and Ω. In the quantitative

model, we discipline steady-state debt levels with real-life trade imbalances.

We derive a key object with the method of undetermined coefficients—the NKOE Leontief inverse, which relates the tariff-related distortions on both consumption (demand) and production (supply) to the dynamics of inflation-output trade-off. Using this object, we decompose the general equilibrium response to the tariff shock into channels where demand distortions propagate to the economy through expectations, price stickiness, and monetary policy, while supply distortions propagate to the economy through the network. Intuitively, if a given sector is central to production—either because it is widely used across industries (e.g., steel and aluminum) or due to its downstream importance (e.g., semiconductor chips)—it will carry significant weight in the standard Leontief inverse. If this sector also exhibits highly flexible (or rigid) prices—corresponding to a vertical (or horizontal) supply curve with fixed quantity (or highly elastic supply)—and is located in a country with relatively loose (or tight) monetary policy, the inflationary impact of a tariff on that sector will be amplified (or muted) by the network captured in the NKOE Leontief inverse.

Next, we explore how network granularity matters. Prior work, including Pasten et al. (2020) and Rubbo (2023), shows that having more than one sector ($J > 1$) matters for two reasons: parameter heterogeneity alters aggregate relationships like the New Keynesian Phillips Curve, and sector-specific shocks propagate differently depending on sectoral rigidity and network position. Our framework identifies a third and novel channel through which network granularity matters. When the number of sectors exceeds one, lagged sectoral prices affect current inflation through the NKOE Leontief inverse, $\mathbf{\Psi}^{NKOE}$, because this object is the coefficient matrix multiplying the lagged price vector in the solution for producer prices. When the number of sectors $J = 1$, $\mathbf{\Psi}^{NKOE}$ equals the identity matrix ($\boldsymbol{I}$) and the lagged price vector has no impact on contemporaneous inflation. However, when $J > 1$ $\mathbf{\Psi}^{NKOE} - \boldsymbol{I} \neq \mathbf{0}$, implying that past sectoral prices feed into current producer price inflation even in the absence of new shocks. As a result, certain more granular network setups, for example those incorporating buyer–seller level price setting, can induce longer and more sluggish inflation responses.

Our model provides a general solution to N-country models without requiring symmetry. In terms of pass-through of tariffs to prices and exchange rates, we work with several different versions. Under producer currency pricing, we have full pass-through versus dollar pricing where we have hybrid between local and producer currency pass-through.

We prefer to focus on the positive implications and refrain from driving normative results. The main reason is not knowing the tariff-imposing country's government objective function as this is the most realistic case for 2025 trade war. Still we can show that optimal tariff is bigger than zero for a country like the U.S. who can gain from manipulating terms of trade.

For the quantitative exercise, we use the non-linear version of our model. The sectoral heterogeneity in price setting is disciplined by estimates from Nakamura and Steinsson (2008) and we construct the sectoral level implemented tariffs from the WTO-IMF Tariff Tracker (WTO and IMF, 2025). The steady state network is calibrated using OECD's Inter-Country Input-Output (ICIO) tables (Yamano and et al., 2023), imposing no *a priori* assumptions on whether a good is purely final or tradable. This modeling flexibility ensures that the quantitative results are not driven solely by the overall share of material inputs in marginal costs, as is often the case in conventional NKOE models. Instead, this relationship arises endogenously from the global I-O structure, which in turn allows our framework to nest other models.[4]

The first quantitative exercise involves validating the model using the case of the tariffs imposed by the United States on China and other countries between February 2018 and September 2018. As shown in Figure 11, the model predicts a 3% nominal appreciation of the U.S. dollar (USD) against the Chinese yuan on impact, eventually reaching 4.2% nominal appreciation in the long run. This aligns with the observed 5.6% appreciation of the USD between June 2018 and December 2018. Real GDP loss reaches 0.1 percentage points, in the range of the estimate of Fajgelbaum et al. (2020), which found an aggregate real income loss of 0.04% of GDP. Finally, the model predicts an inflation impact of 0.27 percentage points, which is close to the 0.1-0.2 percentage point estimate of Barbiero and Stein (2025).

The next exercise, in Case 2, studies reversed tariff threats—scenarios in which a country announces tariffs but subsequently reverses the decision before implementation. These are tariffs deployed with an announcement in the present period, prompting trading partners to pledge retaliation in the future, while in fact all tariffs are withdrawn in the next period. This "threat" shock highlights the role of the exchange rate as a forward-looking variable particularly transparent. In a perfect foresight setting, when tariffs are announced today and reversed tomorrow through a subsequent announcement, agents optimize based on the entire sequence of announcements. When the threat of permanent tariffs leads to anticipation of retaliation and a full trade war, the exchange rate immediately adjusts to front-load the anticipated change in consumption behavior. In this scenario, the U.S. NEER appreciates by 1.7% on impact and then depreciates subsequently—even in the absence of a contemporaneous change in monetary policy. Real GDP falls by 0.4% and there is deflation. These outcomes are driven primarily by the expectations channel: agents "price in" a future in which the U.S.—a net importer relative to the rest of the world—imports fewer foreign

[4]For example, a model without intermediate inputs—where tariffs affect only demand—can be represented by collapsing the I-O matrix $\mathbf{\Omega}$. Likewise, a model with a single imported intermediate input and a final consumption good corresponds to a structure in which the columns of $\mathbf{\Omega}$ associated with final goods are zero vectors.

goods. Even before the mechanical price effects of actual tariffs materialize, anticipated trade distortions cause the exchange rate to adjust, generating deflation on impact in the absence of direct price increase effects from tariffs. Reversed tariff threats additionally worsen the trade balance as agents front-load imports, since the initial period before tariffs is a period in which it is cheaper to import than it will be in the future.

The next quantitative exercise, Case 3, predicts the impact of 2025 tariffs. The model predicts that U.S. real GDP increases slightly by 0.1%, while consumption declines by 0.2%. Inflation rises by 0.6 percentage points, and real wages fall by 0.3%. The U.S. trade balance improves by 0.5 percentage points of steady-state GDP. Abroad, the largest output declines occur in China (–2.1%), Mexico (–1.3%), and Canada (–0.9%). Inflation rises more strongly outside the U.S., led by China (+2.1 percentage points), Mexico (+0.9), and Canada (+0.8). The Euro Area and the rest of the world also contract, by 0.8% and 1.2%, respectively, with real wage declines of 2.8% and 3.2%.

Next, we consider two counterfactuals. In Case 4, we consider a version of Case 3 under non-reactive monetary policy- where nominal interest rates are held constant globally. As expected, global inflation is higher and more persistent in the absence of an interest rate reaction. The decline in output and employment is similar to Case 3. Case 5, as a counterfactual, examines an all-out trade war in which non-U.S. countries retaliate symmetrically to U.S. tariffs. This results in a larger global slowdown and a larger increase in inflation.

Put together, our theoretical and quantitative results contribute to the understanding of how the inflation output tradeoff evolves in the presence of tariffs. In a standard model with nominal rigidity, tariff-related inflation is seen as a one-time increase in prices and inflation precedes the slow decline in output. Our model qualifies this picture in two ways. First, the presence of a production network can make inflation more persistent. That is the "one-time" price increases take time to propagate through the network. Secondly, we show that while actual tariffs are generally inflationary and recessionary, due in great part due to direct effects, reversed tariff threats can be deflationary and recessionary. These two findings imply that in a context where reversed tariff threats precede the actual imposition of tariffs the order in which policymakers should expect to see inflation increases and output declines can change.

2 Literature

The literature on tariffs organizes around two key concepts: Terms of trade manipulation and Lerner symmetry (Lerner, 1936).[5] If a larger country wants to manipulate the terms of trade for its favor by restricting imports, exchange rate appreciation ends up offsetting this by restricting exports. There is a large empirical literature that shows the impact of tariffs is not fully offset by exchange rate movements. This literature also demonstrates that exchange rate pass-through to prices is much lower than tariff pass-through; the extent of tariff pass-through to border prices versus retail prices is subject of an extensive debate.[6] In addition, it is well-known in the two-country NKOE literature (e.g., Obstfeld and Rogoff, 1995; Clarida et al., 2002) that if exchange rate pass-through is less than full, domestic inflation (PPI) in open economies can differ from CPI inflation that includes imported goods. Thus foreign activity becomes important for domestic prices.

Several papers studied short-run impact of tariffs and trade barriers on the macroeconomy mostly focusing on normative implications, such as Auray et al. (2024a,b), Ambrosino et al. (2024). These papers highlight the importance of both demand and supply side and the former, like us, argue that if labor supply and intermediate inputs are added, the tariff outcome depends critically on the monetary policy stance. Focusing on the positive side, Werning et al. (2025) argues that tariffs are cost-push shocks, whereas Bianchi and Coulibaly (2025) finds that optimal monetary policy is expansionary as long as tariffs are rebated back to households and households do not internalize the impact of rebated tariff revenues. Bergin and Corsetti (2023) finds the opposite that optimal policy is contractionary. Monacelli (2025) focuses on optimal monetary policy and argues that it is expansionary. He makes a similar point to us that overall macro impact of tariffs depends on endogenous response of monetary policy. Our work differs from Monacelli (2025) in that we show how other countries' monetary policy responses are also important in shaping the home country inflation-output trade-off,

[5] See Erceg et al. (2018), Lindé and Pescatori (2019) and Costinot and Werning (2019) for modern treatments, laying out conditions under which the symmetry fails in different class of models. Jeanne and Son (2024) study exchange rate offset in a calibrated model.

[6] There is an active empirical debate on how much of the tariff is in the retail price faced by the consumer and how much of it impacts the marginal costs of both foreign and domestic firms. For example, for the 2018 tariffs, Amiti et al. (2019), Fajgelbaum et al. (2020), and Fajgelbaum and Khandelwal (2022), find complete pass-through of tariffs to consumer prices, whereas Cavallo et al. (2021) finds that the degree of pass-through from border to retailers and consumers is not complete. For categories like washing machines, the pass-through can be high (e.g., Flaaen et al., 2019). However, for more aggregated price indices that combine goods that are affected and unaffected by the tariffs, the pass-through is less clear-cut. Thus, the retailers absorbing a significant share of the cost, those raising their prices on goods competing with imports, and those increasing their prices on goods not directly exposed are hard to separate. Inventory "front-running," moving supply chains away, or studying the early months with sticky prices can also blur the picture on aggregate price increases and inflationary impulse.

even without retaliation. Auclert et al. (2025) argues that it is important to add intermediate inputs to standard SOE models studying tariffs macro impact. They argue that without taking the recession into account optimal tariffs cannot be calculated.

There is also renewed interest in studying optimal tariffs and trade imbalances. Similar to Auray et al. (2024a,b), Itskhoki and Mukhin (2025) highlight the importance of valuation effects for the determination of changes in steady state trade imbalances with tariffs in terms of nominal values. Costinot and Werning (2025) make the point that changes in real trade deficits with tariffs depend on the slope of Engel curves, which in turn depend on the extensive margin of trade.

Our work is related to the trade network literature connecting shocks to producers' marginal costs (see, for example, Caliendo and Parro, 2015), leading to amplification of factor price inflation. Cuba-Borda et al. (2025) presents an empirical examination of trade distortions within networks. Ho et al. (2022) develop a global NK model that relies on a real rate rule which fixes the path of consumption and solves the model numerically. Qiu et al. (2025) study monetary policy implications for a SOE with network linkages and characterize a monetary policy weighing mechanism that is related to upstreamness of an industry.

In relation to this literature, given its dynamic global general equilibrium framework and incomplete markets, our model offers a richer analytical characterization of tariff transmission by emphasizing the critical role of global I-O networks and monetary policy in shaping macroeconomic outcomes. Unlike models that impose fixed relative and aggregate consumption paths, our framework allows both demand and policy to respond endogenously, capturing the dynamic interactions between tariffs, exchange rates, and monetary policy.[7] It is also important to note that the decline in employment in the short-run with trade disruption depends both on the network and substitution and not only on existence of price rigidity in networks. For example, Rodríguez-Clare et al. (2020) study China trade shock with networks and wage rigidity upon which unemployment depends on. We show that even in flex-price world, output and employment can fall due to intermediate input shortage and endogenous labor supply. In addition, network and endogenous labor supply also interact with elasticity of substitution, where output and employment can decline under complementarities or they can expand under high elasticities.

We share with the closed economy network literature the importance of relative price changes in understanding the behavior of aggregate inflation (e.g., Pasten et al., 2020, 2024; Rubbo, 2023, 2024; Afrouzi and Bhattarai, 2023). Similar to our work, Afrouzi et al. (2024) also implement network adjusted heterogeneity in price stickiness across sectors. This liter-

[7] di Giovanni et al. (2023) and Silva (2024) focus on inflation instead of tariffs in 2-period global networks models, similar to Baqaee and Farhi (2024) who focus on tariffs.

ature builds on the broader network research by Long and Plosser (1983); Acemoglu et al. (2012); Atalay (2017); Liu (2019); Baqaee and Farhi (2019, 2022); Baqaee (2018); Carvalho et al. (2021b); Carvalho and Tahbaz-Salehi (2019), among others.

Roadmap. The remainder of this paper is organized as follows. Section 3 outlines our baseline New Keynesian open-economy model, detailing how we incorporate international production networks, nominal rigidity, and open-economy features. In Section 4, we solve for tariffs in the long run with flexible prices. In Section 5, we introduce nominal rigidities into the model to capture the dynamics in the short run. In Section 6 we focus on the question of how network granularity matters in our model and highlight how having more than one sector can lead to persistence of inflation. We introduce the data that we use in our quantitative exercises in Section 7 and present these results in Section 8. Finally, Section 9 concludes.

3 Modeling Framework

We develop a multi-country multi-sector New Keynesian model that incorporates nominal rigidities via Rotemberg costs, standard open-economy features such as portfolio adjustment costs, trade distortions and production networks.

Households optimize intertemporally, allocating consumption and labor supply while facing portfolio adjustment costs when holding foreign bonds. The production side follows a nested CES structure, with goods classified by sector and origin, and firms producing using labor and intermediate inputs. Prices are set in the producer's currency (PCP) and are subject to revenue-neutral tariffs. Monetary policy follows a Taylor rule (although we also solve the model under alternative rules). Exchange rates are endogenous. There are also endogenous deviations from Uncovered Interest Parity (UIP) arise due to portfolio adjustment costs; as a country's net debt increases, the effective interest rate it pays also rises.

3.1 Intertemporal problem.

The household in country n maximizes the present value of lifetime utility:

$$\max_{\{C_{n,t},L_{n,t},B^{US}_{n,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{n,t}^{1+\eta}}{1+\eta} \right]$$

subject to:

$$P^C_{n,t}C_{n,t} + T_{n,t} - B_{n,t} - \mathcal{E}^{US}_{n,t}B^{US}_{n,t} + \mathcal{E}^{US}_{n,t}P^{US}_{n,t}\psi(B^{US}_{n,t}/P^{US}_{n,t}) \leq$$
$$W_{n,t}L_{n,t} + \sum_i \Pi_{ni,t} - (1+i_{n,t-1})B_{n,t-1} - \mathcal{E}^{US}_{n,t}(1+i^{US}_{n,t-1})B^{US}_{n,t-1}$$

where $P^C_{n,t}$ is the price of the consumption bundle ($C_{n,t}$) at time t, β is the discount factor, σ is the intertemporal elasticity of substitution, χ denotes labor disutility weight and η captures the elasticity of labor. $\mathcal{E}^{US}_{n,t}$ is the exchange rate between country n and the U.S. An increase in $\mathcal{E}^{US}_{n,t}$ implies a depreciation of the local currency relative to the U.S. dollar. $W_{n,t}$ is the wage in country n at time t, $L_{n,t}$ is the quantity of labor supplied in country n, $i_{n,t}$ is the nominal interest rate in local currency bond $B_{n,t}$, and $i_{US,t}$ is the interest rate on the U.S. bond $B^{US}_{n,t}$, where these bonds are net foreign liabilities. The term $\psi(B^{US}_{n,t}/P^{US}_{n,t})$ represents a stationarity-inducing portfolio adjustment cost that ensures a unique steady-state level of real debt (i.e., debt denominated in USD, deflated by the U.S. consumer price level). Taxes and transfers are denoted by $T_{n,t}$. In our model, tariffs are revenue-neutral; tariff revenue is rebated back to domestic households in a lump-sum manner through the $T_{n,t}$ term.

Maximizing the household's lifetime utility subject to the present and future budget constraints yields the following standard first-order conditions (see Appendix A.1):

$$1 = \beta E_t\left[\left(\frac{C_{n,t+1}}{C_{n,t}}\right)^{-\sigma}\frac{P^C_{n,t}}{P^C_{n,t+1}}(1+i_{n,t})\right]\forall n \in N, \forall t \qquad \text{(Euler Equation)}, \quad (1)$$

$$\frac{1+i_{n,t}}{1+i^{US}_{n,t}} = E_t\left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}}\right]\frac{1}{1-\psi'(B^{US}_{n,t}/P^{US}_{n,t})} \qquad \text{(UIP) } n \in N-1. \quad (2)$$

The domestic bond is in net zero supply everywhere, and all countries save or dissave using U.S. bonds. In addition to the UIP condition, the rest of the arbitrage condition ensures that a country's bilateral exchange rates remain consistent with its exchange rates against the U.S. Finally, for completeness of notation, we define a country's exchange rate with itself:

$$\mathcal{E}_{n,m,t} = \frac{\mathcal{E}^{US}_{n,t}}{\mathcal{E}^{US}_{m,t}} \; \forall n \neq m \;\&\; m \neq US \; n, m \in N \quad (3)$$

$$\mathcal{E}_{n,n,t} = 1 \; \forall n \in N \quad (4)$$

We have $N \times N$ exchange rates, and along with the UIP condition, these two conditions uniquely determine the exchange rate.

3.2 Intratemporal problem.

We now turn to the household's intratemporal problem. The first part of the intratemporal problem is the standard labor-consumption tradeoff that determines labor supply:

$$\frac{W_{n,t}}{P^C_{n,t}} = \chi L^{\eta}_{n,t} C^{\sigma}_{n,t} \; \forall n \in N, \forall t \tag{5}$$

where $W_{n,t}$ is the wage in country n at time t.

Determining the intratemporal breakdown of consumption involves a nested CES structure. Outputs from different countries are first bundled into a country-sector consumption bundle, which is then aggregated into a country good:

$$C_{n,t} = \left[\sum_{i \in J} \Gamma^{\frac{1}{\theta^C_h}}_{n,i} C^{\frac{\theta^C_h - 1}{\theta^C_h}}_{n,i,t} \right]^{\frac{\theta^C_h}{\theta^C_h - 1}}. \tag{6}$$

Here, the index (n, i) captures the sector level (i) bundles in country n. $C_{n,i,t}$ is country n's consumption of industry bundle i and $\Gamma_{n,i}$ is the weight of the bundle i. θ^C_h is the elasticity that governs the substitution between different sectors in consumption (e.g., between automobiles and food in consumption). This bundle is then a combination of all goods of i procured by country n from countries $m \in N$ globally:

$$C_{n,i,t} = \left[\sum_{m \in N} \Gamma^{\frac{1}{\theta^C_{l,i}}}_{n,i,mi} C^{\frac{\theta^C_{l,i} - 1}{\theta^C_{l,i}}}_{n,i,mi,t} \right]^{\frac{\theta^C_{l,i}}{\theta^C_{l,i} - 1}}. \tag{7}$$

In this equation, we focus on country-sector varieties (mi) that form sectoral bundle (i) in country n, which we index with (n, i, mi). $\Gamma_{n,i,mi}$ is the weight of country m's good in this bundle (e.g., German automobiles –mi– in automobile bundle –i– for the U.S. consumers –n). $\theta^C_{l,i}$ is the elasticity of substitution between different country varieties in sector i. Prices and consumption levels of this object is indexed the same way. We can then express the relevant price levels in line with the CES structure:

$$P^C_{n,t} = \left[\sum_{i \in J} \Gamma_{n,i} (P^C_{n,i,t})^{1-\theta^C_h} \right]^{\frac{1}{1-\theta^C_h}}$$

$$P^C_{n,i,t} = \left[\sum_{m \in N} \Gamma_{n,i,mi} P^{1-\theta^C_{l,i}}_{n,mi,t} \right]^{\frac{1}{1-\theta^C_{l,i}}}$$

where $P^C_{n,i,t}$ is the local currency consumption price of the aggregated good basket i in country n at time t (We use the superscript C for denoting price bundles in the consumption side). We assume that prices are set in the producer's currency and then converted to the consumer's currency using the exchange rate under the producer currency pricing (PCP) assumption:

$$P_{n,mi,t} = \mathcal{E}_{n,m,t}(1 + \tau_{n,mi,t})P_{mi,t} \tag{8}$$

where $\mathcal{E}_{n,m,t}$ is the bilateral exchange rate, $\tau_{n,mi,t}$ is the tariff imposed by country n of country-sector mi and $P_{n,mi,t}$ is the price of mi good in country n.

Remark 1. Given the prices that end users see and the aggregation of consumer prices, tariffs serve as a distortionary wedge, similar to a consumption tax or tax on labor income, in the labor-consumption tradeoff given by equation (5).

$$C_{n,i,t} = \Gamma_{n,i}\left(\frac{P^C_{n,i,t}}{P^C_{n,t}}\right)^{-\theta^C_h} C_{n,t} \tag{9}$$

$$C_{n,mi,t} = \Gamma_{n,i,mi}\left(\frac{P_{n,mi,t}}{P^C_{n,i,t}}\right)^{-\theta^C_{l,i}} C_{n,i,t} \tag{10}$$

3.3 Production

Having defined the household's side, we now turn to the production side of the economy. Output in country n, sector i, at time t follows a CES production function:

$$Y_{ni,t} = A_{ni,t}\left[\alpha_{ni}^{1/\theta^P} L_{ni,t}^{\frac{\theta^P-1}{\theta^P}} + (1-\alpha_{ni})^{1/\theta^P}(X_{ni,t})^{\frac{\theta^P-1}{\theta^P}}\right]^{\frac{\theta^P}{\theta^P-1}} \forall n \in N, \forall i \in J, \tag{11}$$

where $Y_{ni,t}$ is the output of sector i in country n, $A_{ni,t}$ is the total factor productivity, θ^P governs the elasticity between the labor and intermediate bundle $X_{ni,t}$, and α_{ni} is the labor share.

All firms within a given country-sector combination are assumed to be identical, and each firm solves the following marginal cost minimization problem:

$$MC_{ni,t} = \min_{\{X_{ni,j,t}, L_{ni,t}\}} W_t L_{ni,t} + P^X_{ni,t} X_{ni,t} \quad \text{s.t.} \quad Y_{ni,t} = 1.$$

where $P^X_{ni,t}$ is the price of the intermediate bundle for country-sector ni (We use the superscript X for denoting prices for all bundles in the production side).

As a firm faces this problem, it chooses labor and the quantities of the intermediate good specific to the producing industry in the given country. This intermediate good bundle is constructed as follows. Intermediate goods from different countries are first bundled into a country-industry-good bundle. This bundle and the relevant relative demand condition are defined below:

$$X_{ni,j,t} = \left[\sum_{m \in N} \Omega_{ni,j,mj}^{\frac{1}{\theta_{l,j}^P}} X_{ni,mj,t}^{\frac{\theta_{l,j}^P - 1}{\theta_{l,j}^P}} \right]^{\frac{\theta_{l,j}^P}{\theta_{l,j}^P - 1}} \tag{12}$$

$$X_{ni,mj,t} = \Omega_{ni,j,mj} \left(\frac{P_{n,mj,t}}{P_{ni,j,t}^X} \right)^{-\theta_{l,j}^P} X_{ni,j,t} \tag{13}$$

Here, we index the sector bundle j for producer sector i in country n with (ni, j, t). $P_{ni,j,t}^X$ is the price index for this bundle, and $X_{ni,j,t}$ is the quantity. This bundle is formed by country varieties mj (e.g., Chinese steel –mj– in steel bundle –j– for the U.S. automobile industry – ni), which we index for (ni, mj, t). $\Omega_{ni,j,mj}$ captures the share of industry mj in bundle j for industry ni. $\theta_{l,j}^P$ governs the elasticity of substitution among different varieties within sector j in production side. The prices and intermediate inputs follow the same subscripts. Analogously, the intermediate bundle is constructed as follows:

$$\frac{X_{ni,j,t}}{X_{ni,t}} = \Omega_{ni,j} \left(\frac{P_{ni,j,t}^X}{P_{ni,t}^X} \right)^{-\theta_h^P} \quad \forall\ j \in J \tag{14}$$

$$X_{ni,t} = \left[\sum_{j \in J} \Omega_{ni,j}^{\frac{1}{\theta_h^P}} X_{ni,j,t}^{\frac{\theta_h^P - 1}{\theta_h^P}} \right]^{\frac{\theta_h^P}{\theta_h^P - 1}} \tag{15}$$

As we derive in detail in Appendix A.2, given the setup and definitions above, the firm's problem yields the following equilibrium conditions for the marginal cost $MC_{ni,t}$:

$$\frac{X_{ni,t}}{L_{ni,t}} = \frac{(1 - \alpha_{ni})}{\alpha_{ni}} \left(\frac{W_t}{P_{ni,t}^X} \right)^{\theta^P} \tag{16}$$

$$MC_{ni,t} = \frac{1}{A_{ni,t}} \left[\alpha_{ni} W_t^{1-\theta^P} + (1 - \alpha_{ni}) \left(\sum_j \Omega_{ni,j} (P_{ni,j,t}^X)^{1-\theta_h^P} \right)^{\frac{1-\theta^P}{1-\theta_h^P}} \right]^{\frac{1}{1-\theta^P}} \tag{17}$$

Within each country sector there is an infinite continuum of identical firms. Representa-

tive firm f in sector i of country n solves the following problem Rotemberg setup:

$$P^f_{ni,t} = \arg\max_{P^f_{ni,t}} \mathbb{E}_t \Bigg[\sum_{T=t}^{\infty} \text{SDF}_{t,T} \Bigg[Y^f_{ni,T}(P^f_{ni,T}) \left(P^f_{ni,T} - MC_{ni,T} \right) - \frac{(1-\vartheta_{ni})\delta_{ni}}{2} \left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1 \right)^2 Y_{ni,T} P_{ni,T} - \frac{\vartheta_{ni}\delta_{ni}}{2} \left(\frac{\mathcal{E}^{US}_{n,T-1} P^f_{ni,T}}{\mathcal{E}^{US}_{n,T} P^f_{ni,T-1}} - 1 \right)^2 Y_{ni,T} P_{ni,T} \Bigg] \Bigg]$$

where a bundler puts together the sectoral output as a CES bundle such that the demand function is $Y^f_{ni,t}(P^f_{ni,t}) = \left(\frac{P^f_{ni,t}}{P_{ni,t}} \right)^{-\theta^R} Y_{ni,t}$. A given country n's exchange rate vis-a-vis the US dollar is given by $\mathcal{E}^{\$}_{n,t}$. In this Rotemberg setup, $(1-\vartheta_{ni})\delta_{ni}$ captures the real cost of changing the price in producer currency and $\vartheta_{ni}\delta_{ni}$ captures the real cost of changing the price in the dominant currency (US dollar). ϑ_{ni} captures the share of prices that are rigid in the dominant currency as opposed to the producer currency. With this formulation our model combines producer currency pricing with dominant currency pricing (DCP). In this setup $\vartheta_{ni} \to 0$ would correspond to PCP and $\vartheta_{ni} \to 1$ would correspond to DCP, so with $\vartheta_{ni} \in (0,1)$ the model combines the two pricing schemes in a manner consistent with empirical evidence. In our quantitative results, we discipline ϑ_{ni} using the share of exports invoiced in the US dollar given by the empirical DCP literature.

As we show in Appendix A.2.1, this problem yields the following equilibrium condition:

$$\begin{aligned} &(1-\vartheta_{ni})(\Pi_{ni,t}-1)\Pi_{ni,t} + \vartheta_{ni} \left(\frac{\Pi_{ni,t}}{D^{US}_{n,t}} - 1 \right) \frac{\Pi_{ni,t}}{D^{US}_{n,t}} = \frac{\theta^R}{\delta_{ni}} \left[\frac{MC_{ni,t}}{P_{ni,t}} - \frac{\theta^R - 1}{\theta^R} \right] \\ &+ \beta\, \mathbb{E}_t \left[(1-\vartheta_{ni})(\Pi_{ni,t+1}-1)\Pi_{ni,t+1} + \vartheta_{ni} \left(\frac{\Pi_{ni,t+1}}{D^{US}_{n,t+1}} - 1 \right) \frac{\Pi_{ni,t+1}}{D^{US}_{n,t+1}} \right] \end{aligned} \tag{18}$$

where $\Pi_{ni,t}$ is gross inflation ($\Pi_{ni,t} = \frac{P_{ni,t}}{P_{ni,t-1}}$) and $D^{US}_{n,t}$ is gross depreciation of the producer's currency against the USD ($D^{US}_{n,t} = \frac{\mathcal{E}^{US}_{n,t}}{\mathcal{E}^{US}_{n,t-1}}$).

Equation (18) constitutes a country- and sector-specific forward-looking New Keynesian Phillips Curve, expressed in terms of nominal marginal cost deflated by the sector's producer price. As $\delta_{ni} \to 0$, prices become more flexible, leading to $\Pi_{n,t} = 1$ and $\frac{MC_{ni,t}}{P_{ni,t}} = \frac{\theta^R - 1}{\theta^R}$, which corresponds to the general pricing equation under monopolistic competition with steady state markups.[8]

[8] Assuming away DCP (setting $\vartheta_{ni} = 0$) would yield the standard non-linear New Keynesian Phillips Curve that is derived with Rotemberg adjustment costs: $(\Pi_{ni,t} - 1)\,\Pi_{ni,t} = \frac{\theta^R}{\delta_{ni}} \left(\frac{MC_{ni,t}}{P_{ni,t}} - \frac{\theta^R - 1}{\theta^R} \right) + \beta \mathbb{E}_t \left[(\Pi_{ni,t+1} - 1)\,\Pi_{ni,t+1} \right]$.

3.4 Balance of Payments and NIIP

We track the evolution of each country's net international investment position (NIIP) as follows:

$$\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}\left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}}C_{n,mj,t}\right)+\sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}}\left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}}X_{ni,mj,t}\right)+\mathcal{E}_{n,t}(1+i^{US}_{n,t-1})B^{US}_{n,t-1}$$
$$+\mathcal{E}_{n,t}P^{US}_{n,t}\psi(B^{US}_{n,t}/P^{US}_{n,t})=\sum_{i\in\mathcal{J}}P_{ni,t}Y_{ni,t}+\mathcal{E}_{n,t}B^{US}_{n,t}\quad\forall n\in N-1 \tag{19}$$

where we account for the fact that tariffs are modeled as revenue-neutral by dividing relevant prices by $(1+\tau_{n,mi,t})$, since end-user prices reflect the impact of tariffs just as they do the impact of exchange rates. The key point here is that, even tariff revenue is rebated back, both producers and consumers still see the tariff-distorted price when making their optimal consumption and production decisions.

Given market-clearing conditions and budget constraints, one country's budget constraint is redundant as an equilibrium condition. Thus, we omit that of the first country, which corresponds to the U.S. in our model. However, we still need to ensure that the market for USD bonds is closed:

$$B^{US}_{t}=\sum_{m}^{N-1}B^{US}_{m,t} \tag{20}$$

3.5 Definitions, Market Clearing, Policy and Equilibrium

We assume that all goods markets clear. Goods can be used as final (consumption) goods and as intermediate inputs in all countries. Therefore, we write the goods market-clearing condition for country-sector ni at time t as:

$$Y_{ni,t}=\sum_{n\in\mathcal{N}}C_{m,ni,t}+\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}X_{mj,ni,t}, \tag{21}$$

where country m is the consuming country and n is the producing country.

To close the model, we need to specify the market-clearing condition for labor, define aggregate inflation, and specify policy. Monetary policy in each follows a generalized rule that allows for interest rate smoothing and targeting of a generic price basket.

$$L_{n,t}=\sum_{i\in J}L_{ni,t} \tag{22}$$

$$P_{n,t}^T = \left(\prod_{m \in N} \prod_{j \in J} \left(P_{mj,t} \right)^{\Upsilon_{n,mj}^P} \right) \left(\prod_{m \in N \setminus \{n\}} \mathcal{E}_{n,m,t}^{\Upsilon_{n,m}^{\mathcal{E}}} \right) \tag{23}$$

$$\Pi_{n,t}^T = \frac{P_{n,t}^T}{P_{n,t-1}^T} \tag{24}$$

$$1 + i_{n,t} = (1 + i_{n,t-1})^{\rho_m^n} \left(\Pi_{n,t}^T \right)^{\phi_\pi^n} e^{\hat{M}_{n,t}} \quad \forall n \in N \tag{25}$$

Here, $\rho_m^n \in [0, 1)$ denotes the degree of interest rate smoothing or inertia, $\Pi_{n,t}^T$ is the gross inflation rate of the target price basket $P_{n,t}^T$. This generic representation allows us to accommodate different alternatives including CPI targeting, specific combination of producer prices, and/or exchange rate targeting.[9]

Definition 1. A non-linear competitive equilibrium for the model is a sequence of 11 endogenous variables $\{C_{nt}, C_{ni,t}, C_{n,mj,t}, X_{ni,mj,t}, X_{ni,j,t}, X_{ni,t}, Y_{ni,t}, L_{ni,t}, L_{n,t}, MC_{ni,t}, B_{n,t}^{US}\}_{t=0}^{\infty}$ and 12 prices $\{P_{ni,t}, P_{n,mi,t}, P_{n,t}^C, P_{n,t}^T, P_{ni,t}^C, P_{ni,t}^X, P_{ni,j,t}^X, \Pi_{n,t}^T, \Pi_{ni,t}, \mathcal{E}_{n,t}, i_{n,t}, W_{n,t}\}_{t=0}^{\infty}$ given exogenous processes $\{\tau_t, A_{ni,t}, \hat{M}_{n,t}\}_{t=0}^{\infty}$ such that equations (1)-(25) hold for all countries and time periods.

3.6 Linearized Model and Analytical Solution

We linearize the 25 equations above and define an approximated equilibrium in order to use the method of undetermined coefficients and solve the model analytically.[10]

Definition 2. A linearized competitive equilibrium for the model is a sequence of 11 endogenous variables $\{\hat{C}_{n,t}, \hat{C}_{ni,t}, \hat{C}_{n,mj,t}, \hat{X}_{ni,mj,t}, \hat{X}_{ni,j,t}, \hat{X}_{ni,t}, \hat{Y}_{ni,t}, \hat{L}_{ni,t}, \hat{L}_{n,t}, \widehat{MC}_{ni,t}, \hat{B}_{n,t}^{US}\}_{t=0}^{\infty}$ and 12 prices $\{\hat{P}_{nt}, \hat{P}_{n,t}^T, \hat{P}_{ni,t}, \hat{P}_{ni,t}^C, \hat{P}_{ni,t}^p, \hat{P}_{ni,j,t}^p, \hat{P}_{n,mi,t}, \pi_{n,t}^T, \pi_{ni,t}^p, \hat{\mathcal{E}}_{n,t}, \hat{i}_{n,t}, \hat{W}_{n,t}\}_{t=0}^{\infty}$ given exogenous processes $\{\hat{\tau}_t, \hat{A}_{ni,t}, \hat{M}_{n,t}\}_{t=0}^{\infty}$ such that equations (A.1)-(A.24) hold for all countries and time periods.

It is common to linearize open economy models around a steady state with net zero debt. We take a different approach (e.g., Obstfeld and Rogoff, 1995) and allow for asymmetry of the primitive parameters (i.e., home bias and imported intermediate input dependence) across countries, which implies a certain level of debt and net exports at the steady state that has to be consistent with these parameters. This level of steady state debt is then used to parametrize the portfolio adjustment costs that discourage deviations from steady-state levels of debt. In the quantitative section, we discipline the asymmetry of parameters and

[9] This notation nests different ways of combining producer prices including PPI targeting or the divine coincidence index of Rubbo (2023), which places more weight on sectors where prices are stickier.

[10] We denote the steady-state values with the bar notation.

the steady-state level of debt using the ICIO Table. Further details on this can be found in Appendix B.

To solve the model analytically, we make the following simplifying assumptions. The first simplifying assumption involves adopting elastic labor in the spirit of Golosov and Lucas (2007) preferences. That is we set $\chi = 1$ and $\eta = 0$, making labor infinitely elastic, which simplifies the intratemporal labor-leisure choice to $\hat{W}_{n,t} - \hat{P}_{n,t} = \sigma \hat{C}_{n,t}$. This simplification allows us to focus on consumption in our five-equation Global New Keynesian Representation and track aggregate output separately. Second simplifying assumption for analytical solution is to assume $\psi(B^{US}_{n,t}/P^{US}_{n,t}) \to 0$.[11] Third, for analytical simplicity we assume price rigidity in the producer prices, i.e., $\vartheta_{ni} = 0 \; \forall n \in N, i \in J$. Additionally, we assume that policy targets only a basket of producer prices and not the exchange rate, setting $\Upsilon^{\mathcal{E}}_{n,m} = 0 \; \forall n, m \in N$. In our analytical work, we use the weights that producer prices have in the consumption basket for the target basket such that $\hat{\boldsymbol{i}}_t = \boldsymbol{\Phi\Gamma}\hat{\boldsymbol{P}}^{\boldsymbol{P}}_{\boldsymbol{t}}$.[12] Finally, we set shocks other than tariffs to zero and introduce generalized elasticities that directly link the lowest-level bundles to the highest-level aggregates, such as:[13]

$$\hat{C}_{n,t} = \sum_{m \in N} \sum_{i \in J} \Gamma_{n,mi} \hat{C}_{n,mi,t}$$
$$\hat{C}_{n,mi,t} = -\theta^{C}_{l,i} \left(\hat{P}^{p}_{mi,t} + \hat{\mathcal{E}}_{n,m,t} + \tau_{n,mi,t} - \hat{P}^{C}_{ni,t} \right)$$

3.6.1 Vector and Matrix Notation

Given the number of countries and industries involved, we can utilize the matrix form to write the equilibrium conditions. To that end, let us consider the linearized producer price

[11] Portfolio adjustment costs serve as a stationarity-inducing device in the model. Because, these costs are numerically small, in analytical work we simplify them away.

[12] Our results do not hinge on using the $\boldsymbol{\Gamma}$ matrix here and one target a different mix of producer prices.

[13] To the first order, bundles presented in Sections 3.2 and 3.3 can be directly linked to the goods that form them. We can write these relations as:

$$\Gamma_{n,mi} = \Gamma_{n,i} \; \Gamma_{n,i,mi},$$
$$\Omega_{ni,mj} = (1 - \alpha_{ni}) \; \Omega_{ni,j} \; \Omega_{ni,j,mj},$$

inflation equation:

$$\pi^p_{ni,t} = \frac{\theta^P_{l,i}}{\delta_{ni}} \left(\underbrace{\alpha_{ni}\hat{W}_t + \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}(\hat{P}^p_{mj,t} + \hat{\mathcal{E}}_{n,m,t} + \hat{\tau}_{n,mj,t})}_{\widehat{MC}_{ni,t}} - \hat{P}^p_{ni,t} \right) + \beta\mathbb{E}_t\pi^p_{ni,t+1} \tag{26}$$

This can be expressed in vector and matrix notation as follows:

$$\underbrace{\boldsymbol{\pi}^P_t}_{NJ\times 1} = \underbrace{\boldsymbol{\Lambda}}_{NJ\times NJ}\Bigg(\underbrace{\boldsymbol{\alpha}}_{NJ\times N}\underbrace{\hat{\boldsymbol{W}}_t}_{N\times 1} + \underbrace{(\boldsymbol{\Omega}-\boldsymbol{I})}_{NJ\times NJ}\underbrace{\hat{\boldsymbol{P}}^P_t}_{NJ\times 1} + \underbrace{\boldsymbol{L}^P_{\mathcal{E}}}_{NJ\times N^2}\underbrace{\hat{\boldsymbol{\mathcal{E}}}_t}_{N^2\times 1} + \underbrace{\boldsymbol{L}^P_{\tau}}_{NJ\times N^2J}\underbrace{\hat{\boldsymbol{\tau}}_t}_{N^2J\times 1}\Bigg) + \beta\mathbb{E}_t\underbrace{\boldsymbol{\pi}^P_{t+1}}_{NJ\times 1} \tag{27}$$

where with some slight abuse of notation, we define the $\hat{\boldsymbol{\mathcal{E}}}_t$ as the $N^2\times 1$ vector of bilateral exchange rates, the $\hat{\boldsymbol{\tau}}_t$ as the $N^2J\times 1$ vector of tariff rates. In line with these vector representations, we also use $\mathbf{L}$ to denote loadings (i.e., how the subscript variable loads onto the superscript variable).[14] These expressions compactly describe how vector variables load onto a given equation and serve as partial derivatives. The matrix notation makes our expressions compact, generalizable, and useful for computational work.

Thus, keeping in mind the labor-leisure tradeoff and using the fact that the price level at time t is the past price level plus inflation, we can express producer prices in levels as:

$$\hat{\boldsymbol{P}}^P_t = \underbrace{[(1+\beta)\boldsymbol{I} + \boldsymbol{\Lambda}(\boldsymbol{I}-\boldsymbol{\Omega})]^{-1}}_{\boldsymbol{\Psi}_\Lambda}\left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\underbrace{\left(\hat{\boldsymbol{P}}^C_t + \sigma\hat{\boldsymbol{C}}_t\right)}_{\hat{\boldsymbol{W}}_t} + \boldsymbol{L}^P_{\mathcal{E}}\cdot\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^P_{\tau}\cdot\hat{\boldsymbol{\tau}}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}^P_{t+1}\right]$$

where $\boldsymbol{\Psi}_\Lambda$ is a stickiness-adjusted Leontief Inverse.

We can also express the CPI using these matrices. For analytical tractability, we define the $NJ\times 1$ dimensional CPI vector $\boldsymbol{P}^C_t$ such that $\boldsymbol{P}^C_{mi,t} = P^C_{m,t}$. With this, we can write the CPI as:

$$\hat{\boldsymbol{P}}^C_t = \boldsymbol{\Gamma}\cdot\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}}\cdot\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^C_{\tau}\cdot\hat{\boldsymbol{\tau}}_t,$$

where $\boldsymbol{\Gamma}$ is an $N\times NJ$ matrix.[15]

Finally, in the linearized model we define $V_{n,t} = (1+i^{US}_{n,t})B^{US}_{n,t}$ and linearize this variable.

[14] In particular, $(\boldsymbol{L}^P_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t)_{ni} = \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}\hat{\mathcal{E}}_{n,m,t}$ and $(\boldsymbol{L}^P_{\tau}\hat{\boldsymbol{\tau}}_t)_{ni} = \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}\hat{\tau}_{n,mj,t}$.

[15] Similar to the production case, $(\boldsymbol{L}^C_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t)_n = \sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\hat{\mathcal{E}}_{n,m,t}$ and $(\boldsymbol{L}^C_{\tau}\hat{\boldsymbol{\tau}}_t)_n = \sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\hat{\tau}_{n,mj,t}$.

As we do so, we stack the balance of payments equations together with the market clearing condition for U.S. bonds as we detail below.

3.6.2 Global New Keynesian Representation

With the vector and matrix notation established, the full set of linearized equilibrium conditions in Appendix A can be written in vector form as an equilibrium that satisfies the Blanchard-Kahn stability conditions. We use this representation both for interpretation and to solve the model using the method of undetermined coefficients.[16] This five-equation representation is similar in spirit to the canonical three-equation New Keynesian model, if that model were extended to a context with N open economies, including I-O linkages.

Definition 3. A linearized equilibrium comprises vector sequences $\{\hat{\boldsymbol{C}}_t, \hat{\boldsymbol{P}}_t^P, \hat{\boldsymbol{P}}_t^C, \hat{\boldsymbol{\mathcal{E}}}_t, \hat{\boldsymbol{V}}_t\}_{t_0}^{\infty}$ for a given sequence of $\{\hat{\boldsymbol{\tau}}_t\}_{t_0}^{\infty}$ and an initial condition for $\hat{\boldsymbol{V}}_0$ such that equations (28)-(32) hold:

$$\textbf{NKIS+TR:} \quad \sigma(\mathbb{E}_t\hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \boldsymbol{\Phi}\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P) - \mathbb{E}_t(\hat{\boldsymbol{P}}_{t+1}^C - \hat{\boldsymbol{P}}_t^C) \tag{28}$$

$$\textbf{CPI:} \quad \hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_{\tau}^C\hat{\boldsymbol{\tau}}_t \tag{29}$$

$$\textbf{NKPC:} \quad \hat{\boldsymbol{P}}_t^P = \boldsymbol{\Psi}_{\Lambda}\left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}_t^C + \sigma\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}_{\mathcal{E}}^P\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_{\tau}^P\hat{\boldsymbol{\tau}}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right] \tag{30}$$

$$\textbf{UIP+TR:} \quad \tilde{\boldsymbol{\Phi}}_1\mathbb{E}_t\hat{\boldsymbol{\mathcal{E}}}_{t+1} - \tilde{\boldsymbol{\Phi}}_2\hat{\boldsymbol{\mathcal{E}}}_t = \tilde{\boldsymbol{\Phi}}_3\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P) \tag{31}$$

$$\textbf{BoP:} \quad \beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{\Xi}_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{\Xi}_5\hat{\boldsymbol{\tau}}_t \tag{32}$$

where "TR" denotes that the Taylor rule has been substituted in, and $\mathbf{L}$ notation represents loadings (i.e., how the subscript variable loads onto the superscript variable as a linear combination of the entries of the vector variable, as detailed above), which also serve as partial derivatives. In the first and fourth of these equilibrium conditions, the Taylor rule is used to substitute out the nominal interest rate, where the diagonal matrix $\boldsymbol{\Phi}$ contains the Taylor rule's sensitivity to the basket of producer price inflation in the respective countries. For example, in the two-country case, we have $\boldsymbol{\Phi} = \begin{bmatrix} \phi_\pi & 0 \\ 0 & \phi_\pi^* \end{bmatrix}$. That is, we have $\hat{\boldsymbol{i}}_t = \boldsymbol{\Phi}\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P)$ and the first $N-1$ rows of $\tilde{\boldsymbol{\Phi}}_3\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P)$ load the vector form of interest rate differentials $\hat{\boldsymbol{i}}_t - \hat{i}_t^{US}$ for countries other than the first country in our system, the U.S.

The first of these equilibrium conditions is the Euler (New Keynesian IS; NKIS) equation, which is defined in terms of aggregate consumer prices. Intuitively, the impact of tariffs enters

[16] We depict prices in levels (e.g., $\hat{\boldsymbol{P}}_t^P$) rather than in first differences (e.g., $\boldsymbol{\pi}_t^P$) for two reasons in this representation. First, since prices appear both in levels and in first differences doing so allows us to write an equilibrium with 5 vector variables and 5 vector equations in a compact manner. Second, this representation is convenient for the algebra work we do with the method of undetermined coefficients.

the demand side through how tariffs load onto consumer prices.

The second equation defines the consumer price index (CPI). As the CPI and the producer price index (PPI) differ, with consumer prices being a weighted average of producer prices, exchange rates, and tariffs under our producer currency pricing assumption. Here, $\boldsymbol{L}_{\mathcal{E}}^{C}$ captures, in matrix form, how consumer prices of various goods are exposed to the exchange rate. The scalar analogy would be $(1-\gamma_H)$, where $\gamma_H \in [0,1]$ represents the home bias parameter for consumption. Similarly, $\boldsymbol{L}_{\tau}^{C}$ captures the share of goods exposed to tariffs.

The third equation is the New Keynesian Phillips Curve for producer price inflation, defined in levels for convenience in the analytical solution. The impact of the I-O network is captured in the stickiness-adjusted Leontief inverse term $\boldsymbol{\Psi}_{\Lambda}$. This term multiplies the diagonal matrix of stickiness parameters $\boldsymbol{\Lambda}$ and the matrix of nominal marginal costs. Additionally, $\boldsymbol{\Psi}_{\Lambda}$ multiplies both the vector of lagged producer prices $\hat{\boldsymbol{P}}_{t-1}^{P}$ and the discounted expectation of future producer prices $\beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^{P}$. In this setup, the exchange rate loads onto nominal marginal costs via the dependence of producers on imported intermediate inputs, which is captured by $\mathbf{L}_{\mathcal{E}}^{P}$. Similarly, tariffs have a direct impact, as they load onto the share of goods exposed to tariffs, captured by $\mathbf{L}_{\tau}^{P}$. If not for their additional impact on consumer prices, tariffs τ would be isomorphic to standard supply shocks in the New Keynesian context.

The fourth equation combines the UIP condition, exchange rate arbitrage conditions, and the definition of a country's exchange rate with itself (i.e., nesting linearized versions of equations (2), (3), and (4)). Here, the $\tilde{\boldsymbol{\Phi}}$ terms ensure that the ϕ_π terms for each country, along with the arbitrage conditions, are correctly loaded in each row.

The fifth equation combines market clearing for debt with the $N-1$ equations of motion for net debt, capturing the balance of payments as a function of prices, which reflect the terms of trade for each specific country-good variety, and the aggregate consumption vector.[17] This final equation describes how a country's net external position evolves in response to changes in good-specific terms of trade, as well as fluctuations in the interest rate and the balance sheet effect of debt via exchange rates. As such, it nests all the intratemporal relative demand conditions and pricing equations. Through this equation, debt responds to automatic debt dynamics and adjustments in exports following changes in the terms of trade.

This five-equation general representation can nest a broad class of open-economy New Keynesian models. For example, models with a bundle of intermediate inputs and a final good correspond to the case where $\boldsymbol{\Omega}$ involves $J=2$ and one of the columns of $\boldsymbol{\Omega}$ is a column of zeros. This representation is general for N-country New Keynesian models (e.g., Clarida

[17]The first $N-1$ rows contain linearized versions of equation (19), while the last row captures the bond market clearing condition given by equation (20). In Appendix B, we derive this equation of motion.

et al., 2002). However, by collapsing the number of countries to one and making the real rate exogenous, it reduces to a small open economy model reminiscent of Galí and Monacelli (2005).

As we show in Appendix B, the system above can be solved out with the method of undetermined coefficients as follows:

$$
\begin{aligned}
\underbrace{\hat{\boldsymbol{C}}_t}_{N\times 1} &= \underbrace{\boldsymbol{c}_p}_{N\times NJ}\hat{\boldsymbol{P}}^P_{t-1} + \underbrace{\boldsymbol{c}_v}_{N\times 1}\hat{V}_{t-1} + \underbrace{\boldsymbol{c}_\tau}_{N\times 1}\hat{\tau}_t \\
\underbrace{\hat{\boldsymbol{P}}^P_t}_{NJ\times 1} &= \underbrace{\boldsymbol{\Psi}^{NKOE}}_{NJ\times NJ}\hat{\boldsymbol{P}}^P_{t-1} + \underbrace{\boldsymbol{p}_v}_{NJ\times 1}\hat{V}_{t-1} + \underbrace{\boldsymbol{p}_\tau}_{NJ\times 1}\hat{\tau}_t \\
\underbrace{\boldsymbol{\pi}^P_t}_{NJ\times 1} &= \underbrace{(\boldsymbol{\Psi}^{NKOE} - \boldsymbol{I})}_{NJ\times NJ}\hat{\boldsymbol{P}}^P_{t-1} + \underbrace{\boldsymbol{p}_v}_{NJ\times 1}\hat{V}_{t-1} + \underbrace{\boldsymbol{p}_\tau}_{NJ\times 1}\hat{\tau}_t \\
\underbrace{\hat{\mathcal{E}}_t}_{1\times 1} &= \underbrace{\boldsymbol{e}_p}_{1\times NJ}\hat{\boldsymbol{P}}^P_{t-1} + e_v\hat{V}_{t-1} + e_\tau\hat{\tau}_t \\
\underbrace{\hat{V}_t}_{1\times 1} &= \underbrace{\boldsymbol{v}_p}_{1\times NJ}\hat{\boldsymbol{P}}^P_{t-1} + v_v\hat{V}_{t-1} + v_\tau\hat{\tau}_t
\end{aligned}
\tag{33}
$$

4 Tariffs in the Long Run Under Flexible Prices

The impact of tariffs on our main variables of interest, exchange rate, inflation, output, output gap, trade balance and consumption, are complex and dependent on the primitive parameters. In this section, we start with the flexible-price version of the model to establish intuition. In order to do so, we will focus on a two-country setup ($N = 2$) with an arbitrary number of industries J.

In order to understand the long-run impact of tariffs we consider a permanent increase in tariffs under flexible prices. As we detail in Appendix C.2, in our Global New Keynesian Representation this corresponds to setting $\rho \to 1$ and taking the limit of each of the diagonal entries of $\boldsymbol{\Lambda}$ to infinity (i.e. $\Lambda_{ni,ni} \to \infty$). Using this parametrization, with the method of undetermined coefficients we find:[18]

[18]The first order approximation is around a given steady state, whereas a permanent shock will lead to the system settling at a different steady state. As a result, in general the first-order solution based on an approximation around the initial steady state may not be valid when considering a permanent change that delivers the system to a new steady state. We confirm our approach is valid in two ways. First, under flexible prices the whole system can be written with first difference variables (e.g. $\Delta\hat{\mathcal{E}}_t$) and $\Delta\hat{\tau}_t$ serves as the exogenous shock variable instead of $\hat{\tau}_t$. This resulting system satisfies Blanchard-Kahn stability conditions and a permanent shock to $\hat{\tau}_t$ is a one-time shock to $\Delta\hat{\tau}_t$, for which there is global stability. Secondly, we verify these solutions with the quantitative model. and confirm with our non-linear solution detailed in

Proposition 1. *The first period impact of a permanent increase in tariffs under flexible prices on the endogenous variables is as follows:*

$$\begin{aligned}
\frac{\partial \hat{\boldsymbol{C}}}{\partial \hat{\tau}} &= \boldsymbol{c}_\tau = \left(\boldsymbol{\Gamma}\left(\boldsymbol{\alpha}\sigma - \Xi_4^{-1}(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^{C} + \boldsymbol{L}_{\mathcal{E}}^{P})\boldsymbol{\Xi}_2\right)\right)^{-1}\left[\Xi_4^{-1}\boldsymbol{\Gamma}(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^{C} + \boldsymbol{L}_{\mathcal{E}}^{P})\Xi_5 - \boldsymbol{\Gamma}(\boldsymbol{\alpha}\boldsymbol{L}_{\tau}^{C} + \boldsymbol{L}_{\tau}^{P})\right] \\
\frac{\partial \boldsymbol{\pi}^{P}}{\partial \hat{\tau}} &= \boldsymbol{p}_\tau = \boldsymbol{0} \\
\frac{\partial \hat{\mathcal{E}}}{\partial \hat{\tau}} &= e_\tau = \Xi_4^{-1}\Big[- \boldsymbol{\Xi}_2\boldsymbol{c}_\tau - \Xi_5\Big] \\
\frac{\partial \hat{V}}{\partial \hat{\tau}} &= v_\tau = 0 \\
\frac{\partial \boldsymbol{\pi}^{C}}{\partial \hat{\tau}} &= \boldsymbol{L}_{\mathcal{E}}^{C} e_\tau + \boldsymbol{L}_{\tau}^{C}
\end{aligned}$$

where we drop the time subscript, because under flexible prices, facing a permanent shock, the system instantly jumps to the new steady state and stays there thereafter.

Proof. See Appendix B.4. □

Corollary 1. *Under flexible prices, a permanent shock has zero impact on producer price inflation when the central bank targets a basket of producer prices. Aggregate CPI inflation then reflects the change via the exchange rate and the direct effect of tariffs on consumer prices.*

This is because prices are flexible and the policy rule targets a mix of producer price inflation.[19] As a result, in response to a permanent shock, the entire adjustment is done by variables other than producer price inflation. Notably, consumer price inflation is not zero as relative prices have to adjust. Similarly the exchange rate and consumption respond to tariffs.

Corollary 2. *Under flexible prices, a permanent shock does not change the net debt/asset position of either country denominated in the U.S. Dollar, which is the currency in which both countries save.*

This follows from the fact that $\frac{\partial \hat{V}}{\partial \hat{\tau}} = 0$. Under flexible prices, a permanent shock does not change the trade balance of either country expressed in U.S. Dollars. Note that the balance of payments can be summarized as follows from the perspective of the first country,

Section 8 that the first-order analytical solution here is numerically the same as the non-linear solution.

[19] In Appendix C.2, we consider the case when the policy rule targets the entire CPI basket; in that case it is the level of consumer prices that remain unchanged in response to a permanent shock under flexible prices.

US, whose local currency debt is used to facilitate global savings:

$$\hat{V}_t = \beta^{-1}\hat{V}_{t-1} - \beta^{-1}(1-\beta)\widehat{NX}_t + \hat{i}_t$$

When the targeted producer price basket's inflation rate is zero, this implies that the interest rate will also be zero in deviation from the steady state. Similarly, since the net debt position does not move, we have $\hat{V}_t = \hat{i}_{n,t} = 0 \; \forall n, t$. Then we necessarily have that the USD value of net exports do not change. That is $\widehat{NX}_t = 0 \; \forall t$. This is in line with the exact local neutrality result of Costinot and Werning (2025) and with the finding of Itskhoki and Mukhin (2025) that the long-run trade balance is determined by the financial position of a country.[20] Note that this does not rule out changes in quantities; trade balance in terms of quantities or expressed as a share of GDP can change, while the U.S. dollar value of net exports will remain constant. The intuition here is that in the presence of a permanent shock and flexible prices, the tariffs do not present an intertemporal tradeoff. In line with the permanent income hypothesis, the entire adjustment is done in quantities, while debt is not utilized. As a result, the USD value of net exports does not change.

Remark 2. The impact on consumption and the exchange rate is jointly determined and dependent on the sensitivity of the balance of payments to aggregate consumption vectors and the terms of trade as captured by the $\boldsymbol{\Xi}$ coefficients.

This follows from the first and third equations in Proposition 1. From the third equation we see that, from the point of view of the home country that places tariffs on the foreign country, an increase in consumption is associated with appreciation (since a negative movement in $\hat{\mathcal{E}}_t$ is defined as appreciation). The intuition here is as follows. As the optimal tariff literature has argued, when a large country places tariffs it can tilt the price vector in its favor depending on the tariff-imposing country's share in the market-clearing condition of the relevant good and elasticities. That is if the tariff imposing country is a sizable buyer of the tariffed good, it can benefit from tariffs and end up with consumption greater than steady-state levels. These effects are captured by the $\boldsymbol{\Xi}$ terms in our notation. In the first equation of Proposition 1, we see that the direct effects captured by $\boldsymbol{L}_\tau^C$ and $\boldsymbol{L}_\tau^P$) have a negative sign in front of them, when one considers the entries for the home country, which is the first country in our notation. As a tax on consumption and production, these direct effects are associated with lower consumption. However, the general equilibrium effects captured by the other terms can result in higher consumption for the tariff-imposing country

[20] Itskhoki and Mukhin (2025) emphasize the gross position of the tariff-imposing country. While our modeling framework allows for countries to accumulate debt or assets in more than one currency, in our analytical and quantitative work we restrict countries to net saving/dissaving in the dollar. Even if when net debt is expressed in the currency of the home country, we still have valuation effects.

especially in the absence of retaliation.

4.1 Scalar Example with One Industry ($N = 2$ & $J = 1$)

Let us now consider the scalar case for additional intuition. In order to do so, we set $J = 1$ and assume away self use by each industry. Then the matrices at hand will look as follows, when expressed in terms of the primitives:[21]

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \Omega_H \\ \Omega_F & 0 \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} 1-\Omega_H & 0 \\ 0 & 1-\Omega_F \end{bmatrix}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} 1-\gamma_H & \gamma_H \\ \gamma_F & 1-\gamma_F \end{bmatrix}$$

$$\boldsymbol{\Psi} = (\mathbf{I} - \boldsymbol{\Omega})^{-1}, \quad \boldsymbol{L}_{\mathcal{E}}^{C} = \begin{bmatrix} \gamma_H \\ -\gamma_F \end{bmatrix}, \quad \boldsymbol{L}_{\tau}^{C} = \begin{bmatrix} \gamma_H L_{\tau}^{C} \\ \gamma_F L_{\tau}^{C} \end{bmatrix}, \quad \boldsymbol{L}_{\mathcal{E}}^{P} = \begin{bmatrix} \Omega_H \\ -\Omega_F \end{bmatrix}, \quad \boldsymbol{L}_{\tau}^{P} = \begin{bmatrix} \Omega_H L_{\tau}^{P} \\ \Omega_F L_{\tau}^{P} \end{bmatrix}$$

where L_{τ}^{C} and L_{τ}^{P} are dummy variables that take on the value 0 or 1, indicating whether a given country imposes tariffs on the other one. We use subscripts H and F to refer countries in the two country case.

The first case to consider involves symmetry in parameters and symmetric retaliatory tariffs by both sides. Where we employ symmetry, we drop subscripts such that $\Omega_H = \Omega_F = \Omega$ and $\gamma_H = \gamma_F = \gamma$.

Corollary 3. *Under symmetric parameters and retaliation, the impact of tariffs on consumption and the exchange rate is:*

$$\frac{\partial C_H}{\partial \tau} = \frac{\partial C_F}{\partial \tau} = -\frac{1}{\sigma}\left[\gamma + \frac{\Omega}{1-\Omega}\right] < 0$$

$$\frac{\partial \hat{\mathcal{E}}}{\partial \hat{\tau}} = \hat{\mathcal{E}}_{\tau} = 0$$

When parameters are symmetric and tariffs involve symmetric retaliation, the exchange rate response is zero. This, in turn, implies that a contraction in consumption by both countries is guaranteed. Import dependence both on the consumption side and production side sharpen this decline in consumption.

Next, we consider the case where parameters are asymmetric across the two countries but there is no retaliaton; tariffs are only placed by H on F.

Corollary 4. *Under asymmetric parameters and no retaliation, the impact of tariffs on*

[21] To make the notation easier to follow in the scalar case we simplify subscripts such that $\gamma_{H,F}$ becomes γ_H and $\Omega_{H,F}$ becomes Ω_H.

consumption is:

$$\frac{\partial \hat{C}_{H,t}}{\partial \tau_t} = -\underbrace{\frac{(\Omega_H + \gamma_H(1-\Omega_H))}{\sigma\,(1-\Omega_H)}}_{+}(e_\tau + 1)$$

$$= -\frac{(\Omega_H + \gamma_H(1-\Omega_H))}{\sigma(1-\Omega_H)}\ \underbrace{\frac{\Xi_{22}\big[\Omega_F + \gamma_F(1-\Omega_F)\big] + \Xi_4\sigma(1-\Omega_F) + \Xi_5\sigma(\Omega_F - 1)}{\Xi_{22}\big[\Omega_F + \gamma_F(1-\Omega_F)\big] - \Xi_{21}\big[\Omega_F + \gamma_H(1-\Omega_F)\big] + \Xi_4\sigma(1-\Omega_F)}}_{e_\tau}$$

With the home bias assumption under which γ_H and γ_F are less than $1/2$ and given boundary $\Omega < 1$ we can sign this expression. For tariffs to expand tariff-imposing home country's consumption a sufficiently large appreciation of the home country's currency is needed. In this version of our model this coressponds to a more than one for one appreciation (i.e. for $\frac{\partial \hat{C}_{H,t}}{\partial \hat{\tau}_t} > 0$ it must be that $-e_\tau > 1$).

Two observations are noteworthy here. The first is that the rest of the world's parameters matter beyond picking export and import elasticities, when considering tariffs by the home country on the foreign country. This is in contrast with the small open economy approach. The rest of the world's parameters are loaded onto consumption via the impact on the exchange rate. The Ξ coefficients capture the sensitivity of the balance of payments to changes in aggregate demand, and good-specific terms of trade (i.e. change in quantities induced by granular price changes, exchange rate and tariffs). As we show in Appendix B, these coefficients also contain the elasticities of substitution and the weights associated with the respective goods.

Secondly, the solution for the exchange rate turns into a complex object as soon as one leaves the case of symmetry combined with symmetric retaliation and one expands the Ξ terms to express them in terms of the γ, Ω and θ terms. In Online Appendix B.4, we show the complexity of this term under asymmetry. For intuition let us turn momentarily to the solution for the exchange rate under the symmetry assumption:

$$e_\tau = -\frac{\overbrace{\Omega + (1-\Omega)\gamma + \sigma\theta\big[1-(1-\Omega)\gamma\big]}^{+}}{\underbrace{2\Big(\Omega + (1-\Omega)\gamma + \underbrace{\sigma\theta\big[1-(1-\Omega)\gamma\big]}_{+}\Big)}_{+} - \underbrace{\sigma(1+\Omega)}_{+}} \tag{34}$$

The sign of the expression above depends on the subtraction in the denominator. While tariffs are generally assumed to appreciationary, this implies that there is a range for the

parameters θ, Ω and γ that result in depreciation. Setting $\sigma = 1$, we can see this analytically:

$$\theta \ < \frac{(1-\Omega)\left(\frac{1}{2}-\gamma\right)}{1-(1-\Omega)\gamma}$$

When the elasticity of substitution θ, Ω and γ are sufficiently low, clearing the balance of payments can require depreciation with $e_\tau > 0$. The intuition behind this result is that under this condition terms of trade gains are small and quantities are not responsive to prices. As a result a following. Ω and γ imply that the tariff-imposing country has a certain share in the market-clearing condition for the good that the tariff is placed on. When these terms are small terms of trade gains will be low. If additionally quantities cannot adjust easily

As is obvious in the expressions above, while the solution is linear in the state variables, it is not linear in the parameters. Since the solved out terms can involve mathematically long expressions, below we visualize how the solution changes in response to changes in the primitives at hand: θ, Ω_H, Ω_F, γ_H and γ_F. That is we initialize these parameters respectively at $\theta = 4$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$ and look at changes in home country's macroeconomic variables of interest in the period of impact for a 10% tariff imposed by the home country on the foreign country, as one varies one parameter at a time. Each primitive's contribution comes from comparing the baseline results to the case when that primitives is set to 0.

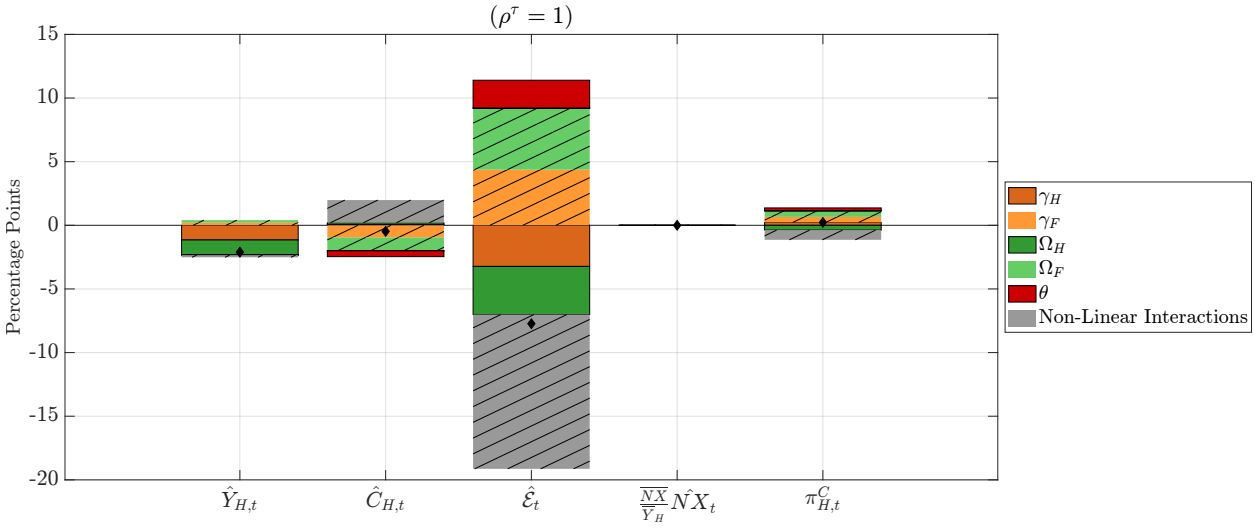
Figure 1 visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Specifically, to calculate contributions, we set each primitive of interest to 0, except for θ whose lower bound is set at 1.01, and recompute the outcome variables in that case.[22] Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section D, we plot bivariate plots that show these impacts are monotonic, except for the sign change in the exchange rate dependent on θ, which is delineated above and controlled for. That is why we can interpret these as contributions.

In Figure 1, we see that consumption is declining in both γ_H and Ω_H, while they are increasing in the foreign country's parameters. The exchange rate appreciates in response to tariffs. This appreciation is stronger as one lowers the home bias in consumption and production for the home country. The intuition here is that as once Ω_H and γ_H increase, H becomes a larger buyer of goods produced by F and thus one has a larger change in the relative demand for F's goods, which in turn leads to a larger appreciation. This appreciation

[22]Since θ can lead to a reversal of the exchange rate's sign, as explored above we set its lower bound at 1.01, so the contribution of θ can be interpreted as deviation from the Cobb-Douglas baseline.

is not large enough to flip the sign of consumption into positive territory. Output is mostly responsive to the elasticity of substitution, which allows both production and consumption to respond to prices in both countries.[23] Output is declining in γ_H and γ_H, while it is not significantly responsive to foreign country parameters.

Figure 1. Contribution of Primitives to Macro Aggregates Under Flexible Prices



NOTE: Figure 1 visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Each primitive's contribution is calculated by re-running the model with that primitive set to 0 one at a time and comparing the results to the baseline case. Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section D, we plot bivariate plots that show these impacts are monotonic and that is why we interpret these as contributions. Hatching emphasizes the foreign country's parameters and the non-linear interaction terms that involve the foreign country's parameters. Net exports are measured as a share of steady-state Nominal GDP to make its interpretation more intuitive. We initialize these parameters respectively at $\theta = 1.5$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$. The AR(1) persistence of the tariff shock is set at $\rho^\tau = 1$. This figure is consistent with our analytical work and simulations in Dynare.

5 Tariffs in the Short Run Under Sticky Prices

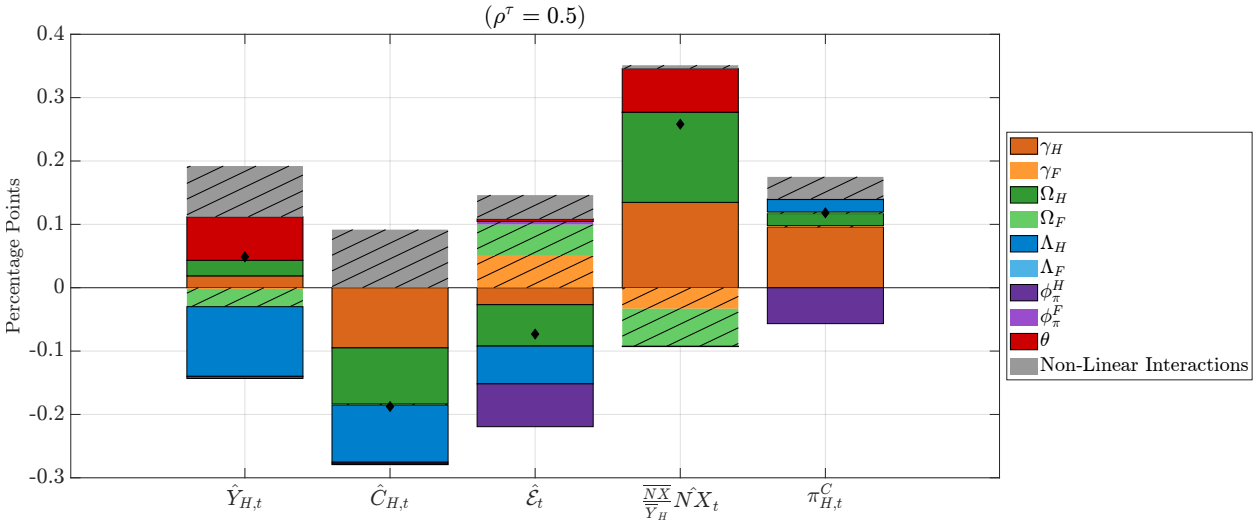
Having reviewed the impact of the first three of the five primitive factors, we now turn to the impact of the remaining two. That is, in this section, we add nominal rigidity in the form of sticky prices and policy. These additions change the impact of the first three primitives as well. To provide intuition, in the $N = 2$ & $J = 1$ case the primitives we are adding

[23] Additionally, while output is solved out from the five-equation representation, we can compute it based on the solution of other variables.

correspond to the following matrices and scalar objects:

$$\mathbf{\Lambda} = \begin{bmatrix} \Lambda_H & 0 \\ 0 & \Lambda_F \end{bmatrix}, \mathbf{\Phi} = \begin{bmatrix} \phi_\pi^H & 0 \\ 0 & \phi_\pi^F \end{bmatrix} \tag{35}$$

Figure 2. Contribution of Primitives to Macro Aggregates Under Sticky Prices



NOTE: Figure 2 visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Each primitive's contribution is calculated by re-running the model with that primitive set to 0 one at a time and comparing the results to the baseline case. Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section D, we plot bivariate plots that show these impacts are monotonic and that is why we interpret these as contributions. Hatching emphasizes the foreign country's parameters and the non-linear interaction terms that involve the foreign country's parameters. Net exports are measured as a share of steady-state Nominal GDP to make its interpretation more intuitive. We initialize these parameters respectively at $\theta = 1.5$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$. The AR(1) persistence of the tariff shock is set at $\rho^\tau = 0.5$. This figure is consistent with our analytical work and simulations in Dynare.

With these, to capture the core intuition of our model, we can build on the example in Figure 1 by constructing a similar figure for the sticky price version of the model. Figure 2 depicts the linear contributions of primitives to macro aggregates under sticky prices. Comparisons with Figure 1 are illustrative. In the presence of a transitory shock (AR(1) persistence $\rho^\tau = 0.5$), the trade balance temporarily improves and the exchange rate appreciation is smaller. Increases in elasticities of substitution incentivize production and net exports. Of the two newly added primitives, policy tends to put downward pressure on inflation and create additional appreciationary impact, because policy reacts to inflation by

raising interest rates. Of the two primitives, Λ, on the other hand, adds a positive impulse to inflation. As the home country's sectoral NKPC grows steeper, inflationary impact of tariffs increase and there is an added downward pressure on output and consumption.

5.1 Impact of Tariffs on Macroeconomic Aggregates

Having established intuition with the scalar case in Figure 2, we can next turn to the analytical solution for our model under sticky prices.

Proposition 2. *The first period impact of a transitory increase in tariffs ($0 < \rho < 1$) under sticky prices on the endogenous variables is as follows:*

$$\begin{aligned}
\frac{\partial \hat{\boldsymbol{C}}_t}{\partial \hat{\tau}_t} &= \boldsymbol{c}_\tau = \frac{1}{\sigma\,(1-\rho)}\Bigg[\Big[\sigma\,\boldsymbol{c}_p - (\boldsymbol{I} - \boldsymbol{L}^C_{\mathcal{E}}\boldsymbol{Z})\,\boldsymbol{\Phi}\boldsymbol{\Gamma} + \boldsymbol{\Gamma}(\boldsymbol{\Psi}^{NKOE} - \boldsymbol{I}) + \rho\,\boldsymbol{\Gamma}\Big]\boldsymbol{p}_\tau \\
&\qquad + (\sigma\,\boldsymbol{C}_2 + \boldsymbol{\Gamma}\,\boldsymbol{p}_v)v_\tau - (1-\rho)\,\boldsymbol{L}^C_\tau\Bigg] \\
\frac{\partial \boldsymbol{\pi}^P_t}{\partial \hat{\tau}_t} &= \boldsymbol{p}_\tau = \Big[\tilde{\boldsymbol{\Psi}}^{-1}_\Lambda - \beta\big(\rho\,\boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\Big]^{-1} \\
&\qquad \Big(\boldsymbol{\Lambda}\big(\boldsymbol{\alpha}\sigma\boldsymbol{c}_\tau + (\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}})e_\tau + (\boldsymbol{\alpha}\boldsymbol{L}^C_\tau + \boldsymbol{L}^P_\tau)\big) + \beta\,\boldsymbol{p}_v v_\tau\Big). \\
\frac{\partial \hat{\mathcal{E}}_t}{\partial \hat{\tau}_t} &= e_\tau = \frac{1}{(1-\rho)}\left[(\boldsymbol{e}_p - \boldsymbol{Z}\boldsymbol{\Phi}\boldsymbol{\Gamma})\boldsymbol{p}_\tau + e_v v_\tau\right] \\
\frac{\partial \hat{V}_t}{\partial \hat{\tau}_t} &= v_\tau = \beta^{-1}\Big[\boldsymbol{\Xi}_2\boldsymbol{c}_\tau + (\boldsymbol{\Xi}_3 + \boldsymbol{\Xi}_6\boldsymbol{\Phi}\boldsymbol{\Gamma})\boldsymbol{p}_\tau + \Xi_4 e_\tau + \Xi_5\Big] \\
\frac{\partial \boldsymbol{\pi}^C_t}{\partial \hat{\tau}_t} &= \boldsymbol{\Gamma}\boldsymbol{p}_\tau + \boldsymbol{L}^C_{\mathcal{E}} e_\tau + \boldsymbol{L}^C_\tau
\end{aligned}$$

where $\tilde{\boldsymbol{\Psi}}^{-1}_\Lambda = \boldsymbol{\Psi}^{-1}_\Lambda - \boldsymbol{\Lambda}\boldsymbol{\alpha}\boldsymbol{\Gamma}$. *In these expressions, expectations are solved out; however, for notational compactness we maintain the undetermined coefficients on the right-handside.*[24]

Proof. See Appendix B.6. □

The representation of the system in Proposition 2 captures key intuitions of our model. First, the impact on consumption and the exchange rate depends on tariffs' impact on the balance of payments equation v_τ. This is sensible, because this equation captures the intertemporal budget constraint.[25]

[24] In Appendix B.6, we show these coefficients in greater detail.

[25] To see this we can forward the relevant equation:

$$\hat{V}_t = E_t\left[\overline{V} + \sum_{k=1}^{\infty}\beta^{k-1}\Big((1-\beta)\widehat{NX}_{t+k} - \beta\hat{i}_{t+k}\Big)\right]$$

Remark 3. If tariff shocks are not persistent and/or they are not perceived to be persistent, the on-impact exchange rate response will be smaller. Relatedly, the ability of the tariff-imposing country to tilt the price vector in its favor and have a welfare-improving increase in consumption will be more limited.

When tariffs are placed, if terms of trade gains are large enough to overcome the inefficiencies in production, prices can move in a manner that favors the tariff imposing country. That is, in line with the flexible-price case above, simultaneously one can see a sufficiently large appreciation and consumption improve. This is dependent on the persistence of tariffs as evidenced by $(1-\rho)$ term in the denominator for the e_τ expression, as well as the market power that the tariff-imposing country has as a large buyer and how easy it is to substitute, both of which are captured by the Ξ terms in the balance of payments equation and in the expression for v_τ. In our model, the standard intuition regarding the signs of e_v and v_τ holds. Having a larger net debt position (having more net foreign assets) is depreciationary (appreciationary), so $e_v > 0$. Tariffs improve the trade balance and reduce net debt in the short run so $v_\tau < 0$. The signs of these two terms explain why tariffs are generally appreciationary on impact given that $e_\tau \approx \frac{e_v v_\tau}{(1-\rho)}$ significantly contributes to e_τ. In this expression we additionally see the role that policy plays through the $(-\boldsymbol{Z}\boldsymbol{\Phi}\boldsymbol{\Gamma})\boldsymbol{p}_\tau$) term; in a country that suffers positive (negative) tariff-related inflation, if policy is more reactive to there will be an additional appreciationary (depreciationary) impact from policy.

The second key aspect that Proposition 2 highlights is the role played by $\boldsymbol{\Psi}^{NKOE}$, which is the coefficient matrix multiplying the lagged price vector in the solution for producer prices, since producer prices feed into all other equations. We call this term the New Keynesian Open Economy Leontief Inverse, because it is a dynamic general equilibrium Leontief Inverse that is closely linked with the stickiness-adjusted Leontief Inverse. $\boldsymbol{\Gamma}(\boldsymbol{\Psi}^{NKOE} - \boldsymbol{I})$ appears in the first equation in Proposition 2, because expected consumer price inflation involves producer price inflation as it loads onto consumption prices. $\boldsymbol{\Psi}^{NKOE}$ additionally appears in the second equation in the propagation term alongside the stickiness-adjusted Leontief Inverse: $\left[\tilde{\boldsymbol{\Psi}}_\Lambda^{-1} - \beta\big(\rho\,\boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\right]^{-1}$. Notably, how $\boldsymbol{\Psi}^{\text{NKOE}}$ compares to $\tilde{\boldsymbol{\Psi}}_\Lambda^{-1}$ makes a difference and can amplify or mute entries in conjunction with the $\boldsymbol{\Lambda}$ matrix, containing price stickiness parameters.

Whereas a regular Leontief Inverse and the stickiness-adjusted Leontief Inverse captures propagation of pricess across sectors at a given point in time, $\boldsymbol{\Psi}^{NKOE}$ is a DGE object, and it can amplify or mute the importance of certain prices in the network as prices propagate across time, since this matrix is the matrix that multiplies the lagged price vector in the solution for producer prices. $\boldsymbol{\Psi}^{NKOE}$ is closely related to the stickiness-adjusted Leontief

where $\overline{V} = \lim_{k\to\infty} \hat{V}_{t+k}$.

Inverse, $\tilde{\boldsymbol{\Psi}}_\Lambda$. This is because $\boldsymbol{\Psi}^{NKOE}$, solves the following quadratic equation.

$$(\boldsymbol{I} - \beta\tilde{\boldsymbol{\Psi}}_\Lambda\boldsymbol{\Psi}^{NKOE})\boldsymbol{\Psi}^{NKOE} = \tilde{\boldsymbol{\Psi}}_\Lambda\left(\boldsymbol{\Lambda}(\boldsymbol{\alpha}\sigma\boldsymbol{C}_p) + \boldsymbol{\Lambda}(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C + \boldsymbol{L}_{\mathcal{E}}^P)\boldsymbol{e}_p + \beta(\boldsymbol{p}_v\boldsymbol{v}_p) + \boldsymbol{I}\right)$$

Via the terms that it involves, the NKOE Leontief inverse, which relates the tariff-related distortions on both consumption (demand) and production (supply) to the dynamics of the system inclusive of the impacts of policies. Intuitively, if a given sector is central to production- either because it is widely used across industries (e.g., steel and aluminum) or due to its downstream importance (e.g., semiconductor chips)- it will carry significant weight in the standard Leontief inverse. If this sector also exhibits highly flexible (or rigid) prices- corresponding to a vertical (or horizontal) supply curve with fixed quantity (or highly elastic supply)- the inflationary or deflationary impact of a tariff on that sector will be amplified (or muted) by the network captured in the NKOE Leontief inverse. In these cases this object resembles the stickiness-adjusted Leontief Inverse; however, in our context it additionally takes into account the impact of policy and demand's response to prices via $\boldsymbol{C}_p$, the impact of exchange rate and debt dynamics via $\boldsymbol{e}_p$ and $\boldsymbol{v}_p$. In that sense like the regular stickiness-adjusted Leontief Inverse can amplify or mute sectors based on $\boldsymbol{\Lambda}$, for the NKOE Leontief inverse it will also matter if a given sector is located in a country with relatively loose (or tight) monetary policy (e.g. via the impact on consumption and the exchange rate).

5.2 Decomposing Inflationary Impacts of Tariffs

To see the role that $\boldsymbol{\Psi}^{NKOE}$ plays, let us decompose the impact of tariffs on inflation. Combining the second and last equations of Proposition 2 yields the following decomposition of the effect of tariffs on consumer prices as we detail in C:

$$\begin{aligned}\frac{\partial\boldsymbol{\pi}_t^C}{\partial\hat{\tau}_t} = &\underbrace{\boldsymbol{L}_\tau^C}_{\text{Direct CPI effect}} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{L}_\tau^P}_{\text{Direct PPI effect}} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{\alpha}\boldsymbol{L}_\tau^C}_{\text{Direct Effect on Real Wages}} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{\alpha}\sigma\boldsymbol{c}_\tau}_{\text{Demand channel}} \\ &+ \underbrace{(\boldsymbol{\Gamma}(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C + \boldsymbol{L}_{\mathcal{E}}^P) + \boldsymbol{L}_{\mathcal{E}}^C)e_\tau}_{\text{ER channel}} + \underbrace{\beta\boldsymbol{\Gamma}\boldsymbol{\Lambda}^{-1}\boldsymbol{p}_v v_\tau}_{\text{Debt channel}} + \underbrace{\boldsymbol{\Gamma}(\left[\tilde{\boldsymbol{\Psi}}_\Lambda^{-1} - \beta\big(\rho\,\boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\right]^{-1}\boldsymbol{\Lambda} - \boldsymbol{I})\boldsymbol{H}_2}_{\text{Propagation with Stickiness}}\end{aligned} \tag{36}$$

where $\boldsymbol{H}_2 = \big(\boldsymbol{\Lambda}\boldsymbol{\alpha}\sigma\boldsymbol{c}_\tau + \boldsymbol{\Lambda}(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C + \boldsymbol{L}_{\mathcal{E}}^P)e_\tau + \boldsymbol{\Lambda}(\boldsymbol{\alpha}\boldsymbol{L}_\tau^C + \boldsymbol{L}_\tau^P) + \beta\,\boldsymbol{p}_v v_\tau\big)$.

In this decomposition the first term captures the direct effect on CPI. The second term captures the direct effect on PPI as it loads onto consumer prices, accounting for stickiness. The third term captures the direct effect on real wages; when tariffs are placed, they change the real wages perceived by the household and thereby distort the labor supply mechanism.

The fourth term, captures the impact of tariffs on producer prices via their impact on demand. The fifth term captures the impact of tariffs via the impact on the exchange rate, which takes into account how the exchange rate impacts both producer prices and consumer prices. The sixth term captures the impact of tariffs on producer prices via the impact on the net debt position. The final term captures propagation under price stickiness.

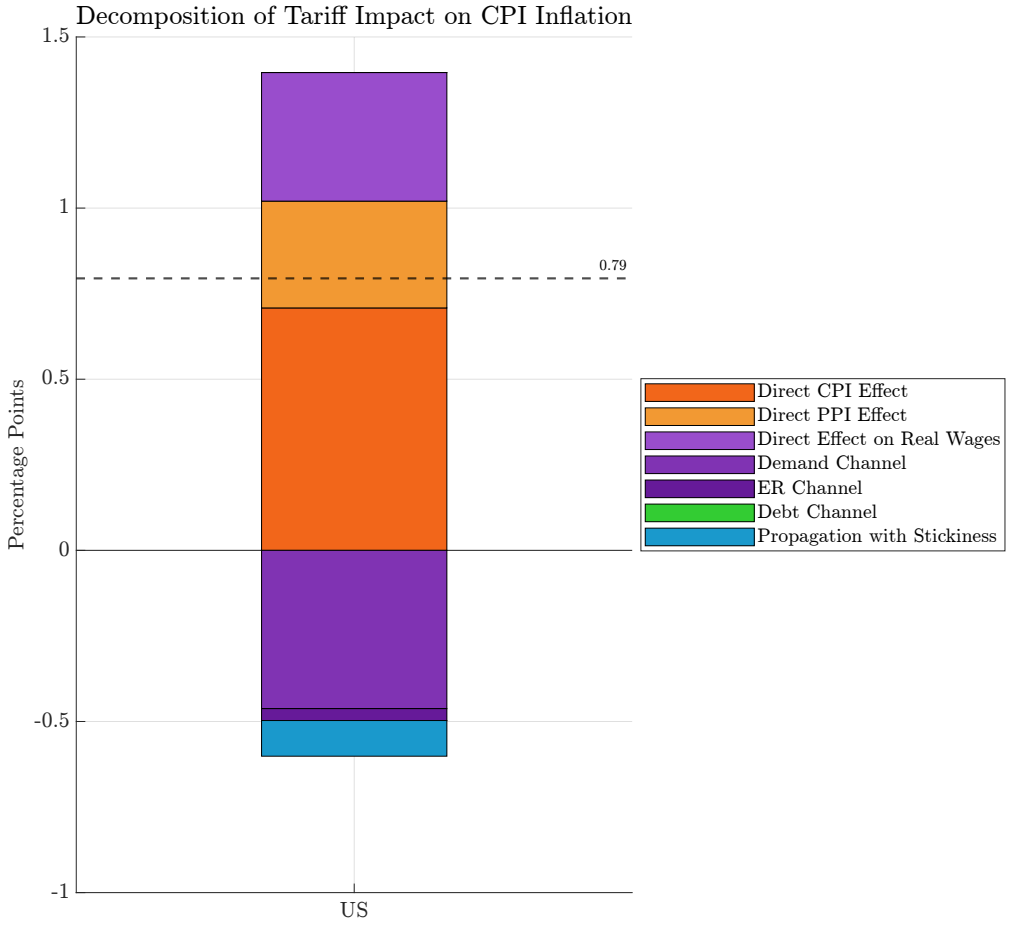
To illustrate how these channels operate and to build intuition around the model, let us consider an example based purely on the analytical solution above. Our objective here is not to conduct a full quantitative exercise- that is reserved for Section 8. Imagine dividing the world into two regions: the United States and the rest of the world. Suppose the United States imposes a 10% tariff on all goods and industries imported from the rest of the world with persistence $\rho^{\tau} = 0.5$. Agents in both regions anticipate that these tariffs will decay across time in line with ρ^{τ}. We use the parameter values described in greater detail in Section 8 and Table 2, except where simplifications of the analytical model apply (e.g., $\sigma = 1$, $\eta = 0$). The impact of this theoretical tariff shock is illustrated in Figure 3 below.

When this transitory tariff shock occurs, the direct impact on CPI and PPI generates an inflationary impulse of approximately 1 percentage point in the tariff-imposing country. The magnitude of these direct effects is related to the trade openness of the United States. We further observe direct effects on households' real wage, which distorts their labor supply decision. Beyond these direct effects, we also observe indirect effects.

In this example, demand declines in response to tariffs. This is for two reasons. First, policy responds to a mix of producer prices, which are increasing. As a result, with the central bank raising real interest rates to stabilize producer prices, demand will contract. Second reason behind the decline in demand is that the tariff shock is not persistent enough to generate a sufficiently large terms of trade gains to turn consumption positive. Exchange rate appreciation in this example adds a modest downward pressure on inflation as well. Finally, the propagation term, which combines the Leontief Inverses and the price stickiness matrix Λ adds a downward pressure on inflation on impact.

A core takeaway from Figure 3 is that for a reasonable parametrization, the direct impacts of tariffs drive a significant portion of the initial inflationary impact of tariffs. This is the context in which tariffs are often described as a "one time increase in prices." In our work below, we show that in the presence of production networks inflation can be more persistent precisely via the $\mathbf{\Psi}^{NKOE}$ term when more sectors are included in the model and the production network is thus modeled in a more granular manner.

Figure 3. Theoretical Example: Tariffs Without Retaliation



NOTE: Figure 3 visualizes the decomposition of CPI inflation in a two-country case, namely the U.S. and the rest of the world (RoW). We assume the US imposes an additional 10% tariff on RoW. Using Equation 36, we break down the different contributing effects. The dashed line represents the total effect, showing an inflation increase of 0.79% in the U.S.

5.3 Alternative Policy Formulations

In our Online Appendix, we solve the model under different policy regimes. In Section E.1.1 we start with the special case when there is a real rate rule that fixes consumption in all countries of interest. Next, in Section E.2, we develop the case when policy fixes nominal demand, and the pressure from tariffs is shared equally by the the aggregate price level and aggregate consumption within each country.

In line with our two main research questions, these cases serve two purposes. First, we consider these cases to see what happens to macroeconomic aggregates in response to tariffs if monetary policy targets quantities (e.g., consumption), or prices (e.g., inflation targeting),

or a mix of both (e.g., fixing nominal demand). Second, we explore how network propagation changes under different policy regimes. To capture propagation, we develop New Keynesian Open Economy Leontief Inverse matrices for these cases.

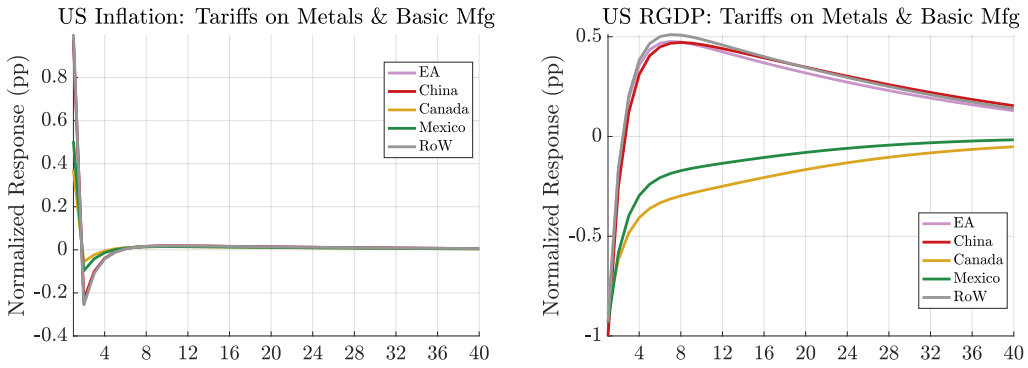
We find that network propagation is different under different policy regimes. Under a real rate rule that stabilizes consumption, tariffs lead to depreciation via expenditure switching and home and foreign monetary policies in the UIP equation. This is in marked contrast with the case when policy fixes nominal demand; this renders inflation in each sector and each country weakly positive, as tariffs act as a marginal cost shock and a marginal cost shock in one part of the network propagates as a marginal cost shock in all parts of the network. As these cases involve analytical simplifications they illustrate the different components that enter into the NKOE Leontief Inverse.

6 How Network Granularity Matters for Impact of Tariffs

Since we aim to understand how tariffs affect macroeconomic aggregates and how this answer changes in the presence of a production network, a natural question to ask is when having a more granular network (i.e. with more than one sector) matters?

In network models, having more than one sector ($J > 1$) can produce different results for macro aggregates in two related and established cases. First, when parameters are heterogenous at the granular level, aggregating the parameters and multiplying them will produce a different result compared to multiplying them at the granular level and then aggregating. Pasten et al. (2020) and Rubbo (2023) show this in the case of the New Keynesian Phillips Curve and in our notation this can be seen with Equation (26). where the granular price updating frequency multiplied with the granular weight will produce a different result if the frequencies are aggregated and then multiplied with an aggregated weight. Secondly, having more than one sector in the model can produce different results if one is studying shocks at the sectoral level and sectors different along various parameters (e.g. shocking a sector with a more vs. less vertical sectoral New Keynesian Phillips Curve). The current tariff context fits both of these conditions, which is why we find it appropriate the study this subject with a multi-sector model. Figure 4 constitutes an example of US tariffs on even the same sector across different countries can be different. This is because parameters are heterogenous at the country-sector level and shocks are sector-specific.

Figure 4. Impact of Sectoral Tariffs



NOTE: Figure 4 visualizes the impact of 10% ad valorem tariff applied to metals and basic manufacturing sector from different countries on inflation (left panel) and output (right panel).

Beyond these cases, our model yields a third way in which having $J > 1$ produces different results for macro aggregates: specifically, we find that lagged prices can matter for future inflation in the presence of a production network. This has to do with how propagation across time changes based on the number of sectors. To develop this point, consider Corollary 5:

Corollary 5. *When the number of sectors, $J = 1$, we have:*

$$\boxed{\boldsymbol{\Psi}^{NKOE} = \mathbf{I}, \quad \boldsymbol{p}_v = \mathbf{0}, \quad \boldsymbol{v}_p = \mathbf{0}, \quad v_v = 1}.$$

Proof. This follows from Proposition 2 and the fact that when $J = 1$, $N \times NJ$ matrices like $\boldsymbol{\alpha}$ and $\boldsymbol{\Gamma}$ are invertible. For details, see Appendix B.5. □

In order to understand the implications of Corollary 5, let us note that the solution to our Five-Equation Global NK Representation in (33), has a VAR(1) representation of the following type: $\boldsymbol{x}_t = \boldsymbol{A}\boldsymbol{x}_{t-1} + \boldsymbol{B}\,\epsilon_t$ where

$$\boldsymbol{x}_t \equiv \begin{bmatrix} \boldsymbol{\pi}_t^P \\ \hat{\boldsymbol{P}}_t^P \\ \hat{V}_t \\ \hat{\tau}_t \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \boldsymbol{\Psi}^{NKOE} - \boldsymbol{I} & \boldsymbol{p}_v & \boldsymbol{p}_\tau\,\rho \\ \mathbf{0} & \boldsymbol{\Psi}^{NKOE} & \boldsymbol{p}_v & \boldsymbol{p}_\tau\,\rho \\ \mathbf{0} & \boldsymbol{v}_p & v_v & v_\tau\,\rho \\ \mathbf{0} & \mathbf{0} & 0 & \rho \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{p}_\tau \\ \boldsymbol{p}_\tau \\ v_\tau \\ 1 \end{bmatrix}.$$

With the VAR(1) representation, we can derive analytical impulse response functions. For example, producer price inflation, j periods after tariffs with persistence ρ are imposed

can be written as:

$$\frac{\partial \boldsymbol{\pi}^P_{t+j}}{\partial \epsilon_t} \;=\; \boldsymbol{S}_\pi \, \boldsymbol{A}^j \, \boldsymbol{B} \text{ where } \boldsymbol{S}_\pi \equiv \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$$

Remark 4. It follows from Corollary 5 that the propagation matrix $\boldsymbol{A}$ looks different when the number of sectors, $J = 1$, versus $J > 1$.

$$\boldsymbol{A}_{J=1} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{p}_\tau \, \rho \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{p}_\tau \, \rho \\ \boldsymbol{0} & \boldsymbol{0} & 1 & v_\tau \, \rho \\ \boldsymbol{0} & \boldsymbol{0} & 0 & \rho \end{bmatrix}, \quad \boldsymbol{A}_{J>1} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{\Psi}^{NKOE} - \boldsymbol{I} & \boldsymbol{p}_v & \boldsymbol{p}_\tau \, \rho \\ \boldsymbol{0} & \boldsymbol{\Psi}^{NKOE} & \boldsymbol{p}_v & \boldsymbol{p}_\tau \, \rho \\ \boldsymbol{0} & \boldsymbol{v}_p & v_v & v_\tau \, \rho \\ \boldsymbol{0} & \boldsymbol{0} & 0 & \rho \end{bmatrix}$$

That is the number of sectors change how endogenous lagged variables help propagate shocks. Most importantly, the shape of $\boldsymbol{\Psi}^{NKOE}$ can lead to persistence. When $j = 1$, $\boldsymbol{\Psi}^{NKOE}$ collapses to the identity matrix and the impact of lagged prices on contemporaneous inflation $\boldsymbol{\Psi}^{NKOE} - \boldsymbol{I} = \boldsymbol{0}$. On the other hand, when $J > 1$, $\boldsymbol{\Psi}^{NKOE} - \boldsymbol{I} \neq \boldsymbol{0}$ and the lagged price vector can lead to persistence in inflation.

The degree of inflation persistence hinges on both the network setup and the price stickiness matrix $\boldsymbol{\Lambda}$. In Figure 3, we show that the on-impact contribution of the propagation term, which is part of the $\boldsymbol{B}$ vector in our notation above, can be negative as the Leontief Inverses get multiplied with the stickiness matrix. Beyond the period of impact, however, in some network setups $\boldsymbol{\Psi}^{NKOE}$ can lead to persistent inflation.

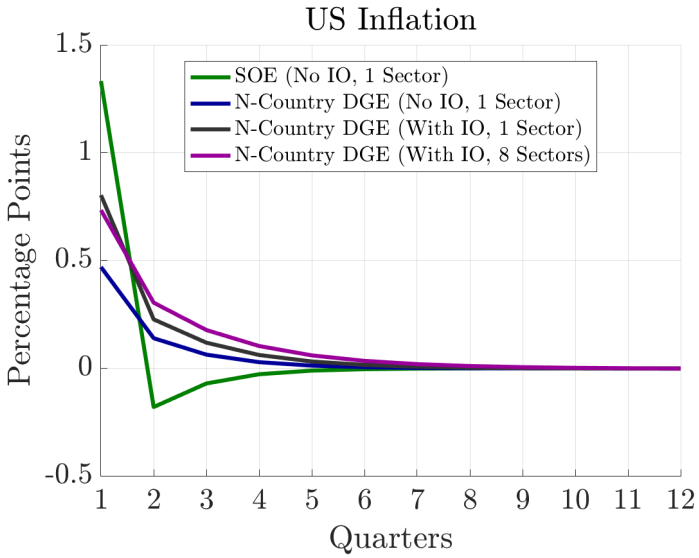
One key example of $\boldsymbol{\Psi}^{NKOE}$ leading to persistence in inflation is the case of price rigidity at the buyer-seller level. The trade literature has extensively studied how international trade at the granular level is dependent on matching of buyers and sellers; for example, Bocola and Bornstein (2023) demonstrates the importance of trade credit in this context. We do not model these matches with a matching function, but our framework is flexible enough to incorporate the case whereby contracts are signed between buyers and sellers such that the price ridigity cost (and prices) are at the bilateral level. That is instead of having producer prices $\hat{P}^P_t$ be an $NJ \times 1$ matrix, the relevant matrices can be adjusted to render it an $NNJ \times 1$ matrix, such that consumers in a given country might face a different price for the price of a good made in country n industry j compared to the price that buyers of the same good might face in a different country.

With this extension, our model can capture the fact that it takes time to adjust trade at the bilateral buyer-seller level. The intuition in this context is that, one buyer-seller pair updates a contract at a given point, it then prompts other buyer-seller pairs to update. This takes longer than each producing sector updating its price at once for all its buyers.

Algebraically this corresponds to more granular $\mathbf{\Omega}$ and $\mathbf{\Gamma}$, which in turn changes $\mathbf{\Psi}^{NKOE}$.

In Figure 5, in light of the theory, we show how the propagation of inflation changes across time in different network setups using the example of US tariffs placed on the rest of the world as another. The blue line shows the propagation of inflation in the absence of intermediate inputs (i.e. $\mathbf{\Omega} = \mathbf{0}$); inflation in this case is lower. The brown line depicts the case when $J = 1$ (i.e. $\mathbf{\Omega}$ is a, while the purple line depicts the case when $J = 8$ and prices are rigid at the buyer-seller level. Comparing the latter two cases, inflation on impact is slightly larger when $J = 1$; however, inflation persists for longer and at a higher level when $J > 1$ in line with Remark 4. The green line depicts the case when one takes the version of the model with one sector and without IO and converts it into an SOE model, by making the rest of the world arbitrarily larger; this case does not have inflation persistence. The initial impact on inflation is higher due to exchange rate passthrough, because the small open economy experiences depreciation as we note below.

Figure 5. Model Comparison for Inflation



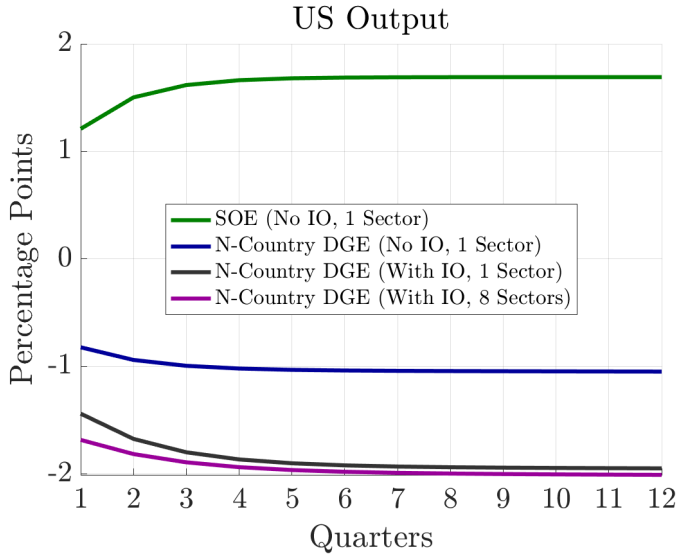
A related point can be made about output in our model. While the Five-Equation Global New Keynesian Representation does not directly track output as an endogenous variable, in Appendix B, we show that output can be expressed in terms of the endogenous variables of the five-equation system, which are solved in Proposition 2:

$$\hat{\boldsymbol{Y}}_{ni,t} = \mathbf{\Psi}_T \Bigg[\Big(\overline{\boldsymbol{Y}}^{ni-1} \mathbf{\Gamma}^\top \overline{\boldsymbol{C}} + \theta^P \mathbf{\Omega}^\top \boldsymbol{\alpha} \sigma \Big) \hat{\boldsymbol{C}}_t + \Big(\theta^C \overline{\boldsymbol{Y}}^{ni-1} \boldsymbol{T}_P^C + \theta^P \mathbf{\Omega}^\top \boldsymbol{T}_P^P \Big) \hat{\boldsymbol{P}}_t^P + \Big(\theta^C \overline{\boldsymbol{Y}}^{ni-1} \boldsymbol{T}_{\mathcal{E}}^C + \theta^P \mathbf{\Omega}^\top \boldsymbol{T}_{\mathcal{E}}^P \Big) \hat{\mathcal{E}}_t + \Big(\theta^C \overline{\boldsymbol{Y}}^{ni-1} \boldsymbol{T}_\tau^C + \theta^P \mathbf{\Omega}^\top \boldsymbol{T}_\tau^P \Big) \hat{\tau}_t \Bigg] \tag{37}$$

where overline notation denotes steady state values, while θ^C and θ^P are generalized elasticities of substitution respectively on the consumption and production side. $\boldsymbol{T}$ matrices capture how each specific price compares to the average price basket of a user. That is for users each granular good price is compared to the average consumer price basket, and for producers each intermediate input price is compared to the price of the overall intermediate input basket, and then with some elasticity (θ^C or θ^P) quantities respond.

This expression illustrates why the elasticity of substitution contributes largely to output in Figure 2 and why under complementarities, in the presence of a network the decline in output resulting from tariffs can be larger. Figure 6 makes this point in a manner similar to Figure 5. An additional point to highlight here is that, in the SOE case, when the rest of the world is made arbitrarily larger, the same set of parameters that otherwise yield a recession in the N-country DGE case can yield an expansion. This is because the rest of the world is able to easily increase their demand for the home country's goods since the tariffs imposed by the small open economy do not pose a negative demand shock for the rest of the world. Because the small open economy is not a large buyer of the rest of the world's goods, the shock can result in depreciation and an expansion of exports in line with the intuition established in Section 4.

Figure 6. Model Comparison for Output



7 Data

7.1 Input - Output Network

As the basis for consumption shares and intermediate input shares, we use the OECD Inter-Country Input-Output (ICIO) tables (Yamano and et al., 2023) for the year 2019.[26] We aggregate the ICI-O data to align with the country and industry groupings used in our analysis. we include the United States, Euro Area, China, Canada, and Mexico- reflecting the countries most affected by the tariff announcements as of April 2025- along with an aggregate entity representing the Rest of the World (RoW). On the industry side, we aggregate sectors into eight broad categories: agriculture, energy, mining, food, basic manufacturing, advanced manufacturing, residential services, and other services to match with sectoral rigidity data of Nakamura and Steinsson (2008) (see below).

We visualize the I-O network in Figure 7. The thickness of the edges in this network captures the input shares. The layout of the network was generated automatically using the edge-weighted spring embedded layout feature of Cytoscape. Global shocks could be carried over the links shown on this network. Strikingly, many Canadian and Mexican sectors are naturally grouped together with American industries. In contrast, the Chinese sectors are not very well integrated. This might be due to the fact that many Chinese goods imported by the U.S. could be for final consumption.

In Table 1, we show the basic stats for the U.S. industries. The U.S. economy heavily relies on services, with more than 75% GDP attributed to this sector. Most of the U.S. output is consumed domestically, with shares ranging from 80 to 99 %. The home share in consumption and intermediate inputs exhibit the lowest rates in manufacturing sectors. Interestingly, close to one third of consumer goods and intermediate inputs are sourced from foreign countries in advanced manufacturing. The energy sector's intermediate products are sourced at a higher level internationally. In Table D.2 of the Appendix, we provide a more detailed breakdown of the final and intermediate input shares at country-sector level.

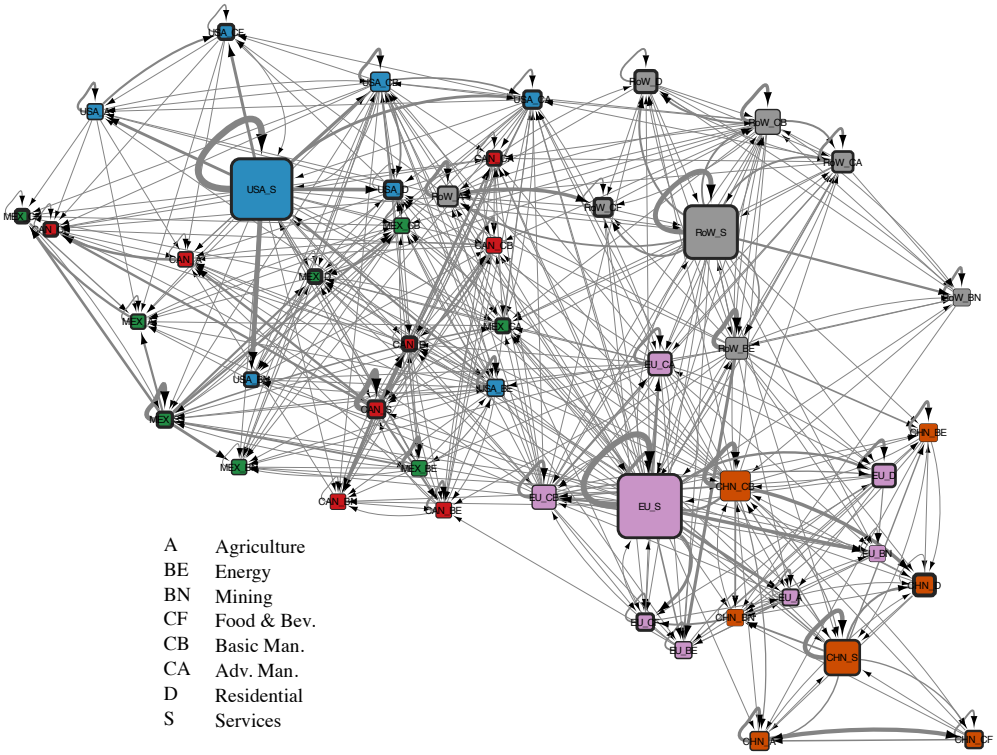
[26] Although the latest available data at the time of writing was for 2020, we use 2019 data to avoid distortions arising from the COVID-19 pandemic.

Table 1. Sector Statistics for USA (%)

Industry	Output Share	VA Share	Consumption Share	Output Home Share	Consumption Home Share	Intermediate Home Share
Agriculture	1.3	0.9	0.6	87.2	88.5	89.3
Energy	3.0	2.0	1.5	85.7	89.4	75.0
Mining	0.5	0.5	0.5	91.2	98.5	89.9
Food and Beverages	2.6	1.2	3.1	94.0	91.2	91.7
Basic Manufacturing	6.6	4.7	4.1	87.6	66.0	82.5
Advanced Manufacturing	6.2	5.1	8.2	81.7	67.0	66.9
Residential Services	6.4	6.1	7.7	99.9	99.9	99.5
Services	73.4	79.4	74.3	95.3	96.7	96.2

NOTES: The values are calculated from OECD ICI-O for year 2019 Yamano and et al. (2023). Output Share is the share of the sector in total U.S. output. VA share is the share of the sector in total U.S. GDP. Consumption share is calculated as the sector's weight in the household expenditure. Output Home Share represents the share of the output of the sector sold domestically. Consumption Home Share captures the share of domestic production in consumption and Intermediate Home Share captures the share of intermediate goods supplied domestically.

Figure 7. Visualizing the Input-Output Network



NOTE: Figure 7 visualizes the inter-country inter-industry I-O network. The color of the node represents the country. Size of the node represents the total output. The thickness of the edges show the share of inputs of target node coming from the source node (we do not show the edges smaller than 1%). The thickness of the borders of nodes represents the share of final goods in the output of the sector. The layout was generated automatically using the edge-weighted spring embedded layout using the openly available Cytoscape software.

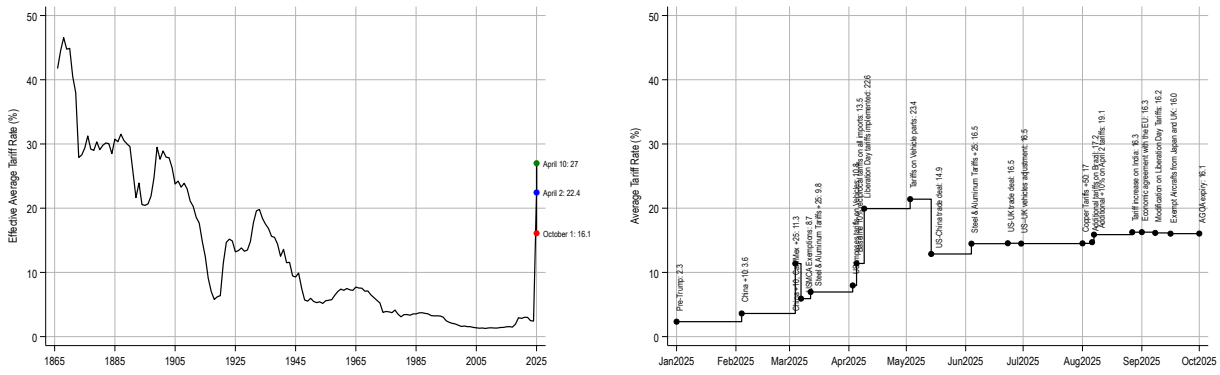
7.2 Tariffs

In the quantitative exercises that follow, we are motivated by the renewed interest among policymakers in using tariffs as a tool to manage external imbalances and exert geopolitical influence. This interest predates the second Trump presidency and reflects a broader global re-evaluation of trade policy not only for the standard terms of trade manipulation but also both for strategic and retaliatory purposes. In the quantitative section of our paper we solely focus on the tariffs announced in the early months of the second Trump administration.

As shown in Figure 8a, the tariffs proposed on April 2- referred to as "Liberation Day" by the administration- are projected to raise the effective U.S. tariff rate to 22.4%, the highest level in over a century. We obtain the country - sector levels tariffs from the WTO – IMF Tariff Tracker (WTO and IMF, 2025) at Harmonized System 6-digit level. We aggregate these tariff rates to ICI-O sectoral level by weighing them with the imports of the countries, provided in the same dataset. Figure 8b shows the implemented tariff rates since January 1, 2025 until June 20, 2025. The "liberation day tariffs," were announced on April 2, 2025 but with most tariffs going into effect on April 9th. Between these two dates, there was also a steep escalation between the U.S. and China tariffs to each other, resulting in tariff rates exceeding 125% for Chinese goods in the U.S.

Figure 8. Effective Tariff Rates

(a) Historic and Estimated, (%) **(b)** Since January 1, 2025, (%)



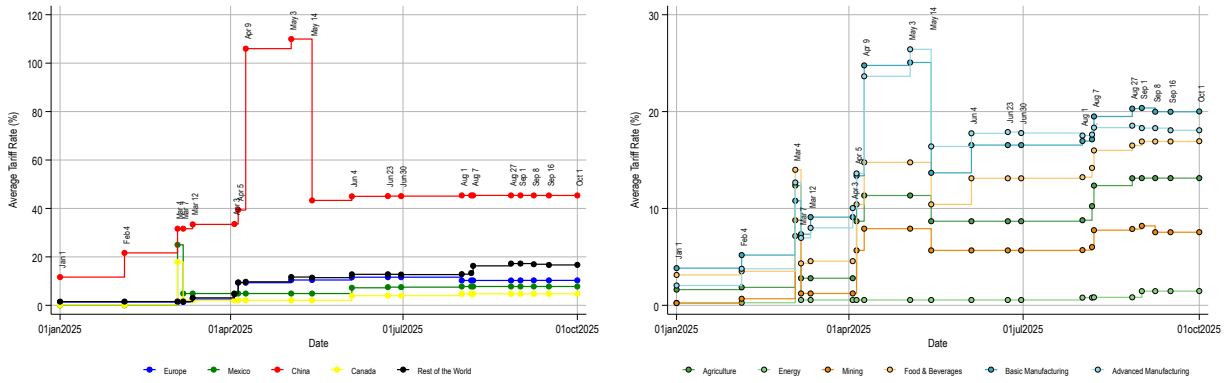
NOTE: Figure 8a visualizes effective tariff rate stands for customs duty revenue as a proportion of goods imports. Data from *Historical Statistics of the United States* Ea424-434, *Monthly Treasury Statement*, Bureau of Economic Analysis. Estimated effective tariff rates of for 2025 provided by Yale Budget Lab using the GTAP Model v7 (Corong et al., 2017). Figure 8b visualizes estimated effective tariff rates based on WTO - IMF Tariff Tracker (WTO and IMF, 2025). The dates here correspond to the actual implementation change of the tariffs. The data was accessed on October 20, 2025.

In Table D.1, we document the episodes of implemented tariff changes for the U.S. re-

ported by the WTO-IMF Tariff Tracker (WTO and IMF, 2025). We summarize the tariff rates at the country and sector level in Figure 9. The largest swings are observed for China with escalating tariff announcements with a moratorium on May 14, 2025 (Figure 9a). At the sectoral level, the tariffs are the highest for basic and advanced manufacturing goods. Figure D.2 in the Appendix shows the size of country-sector-level tariffs implemented in 2025 until the time of our writing in panel (a). Panel (b) focuses on the "Liberation Day" tariffs. Figure D.2a shows that the highest tariff rates are applied to the Chinese goods. Among Chinese sectors, basic manufacturing (e.g., textiles), food and beverages, and agriculture have the highest values with tariffs ranging from 45-50%. For most other countries, the tariffs started from very low levels but increased around 10-20% for many goods.We will use the most recent data (June 4, 2025) levels for our quantitative analysis. In Table D.2 of the Appendix, we provide detailed breakdown of the tariff rates as of October 1, 2025 and maximum tariff rate observed between January 1, 2025 and October 1, 2025.

Figure 9. Effective Country and Sector Level Tariff Rates

(a) Country Level Tariffs (%) **(b)** Sector Level Tariffs (%)



NOTE: Figure 9 visualizes estimated effective tariff rates at the country level and at the sectoral level based on WTO - IMF Tariff Tracker (WTO and IMF, 2025) between January 1, 2025 and October 1, 2025 (last available data as the manuscript was prepared). Both country level and sectoral level tariff rates are calculated as the weighted average of the 6-digit tariff rates by using the latest available import values reported in the dataset as weights.

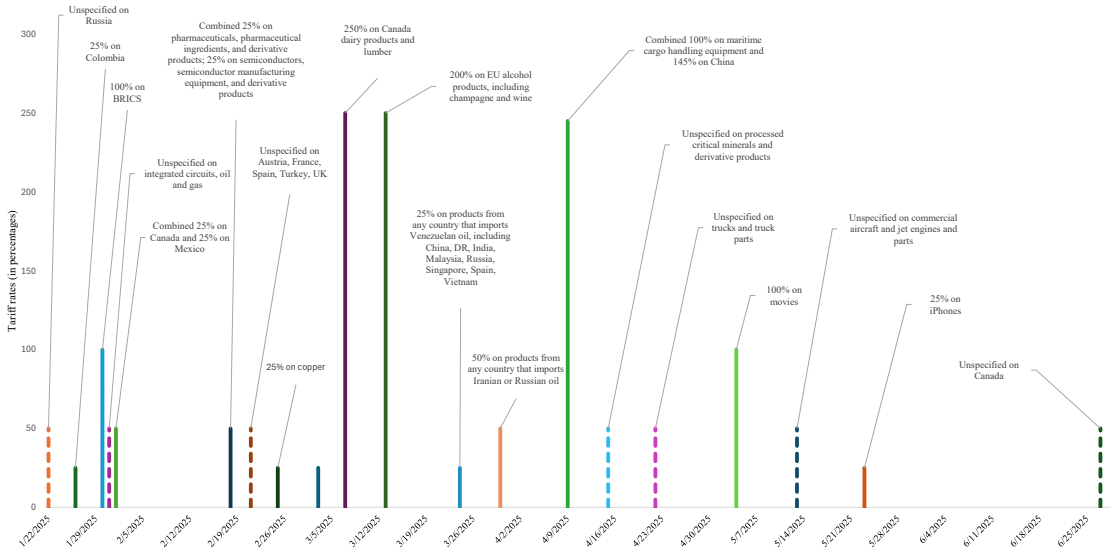
According to both the St Louis Fed[27] and the Tax Foundation[28], the 2018 tariffs affected $376 billion of goods from China, which is around 1.66% of the 2018 U.S. GDP. As of June 2025, most of the tariffs enacted on the "Liberation Day" have been halted via an injunction by the U.S. Court of International Trade. Those not affected still represent $500 billion

[27] https://www.stlouisfed.org/on-the-economy/2025/may/what-have-we-learned-us-tariff-increases-2018
[28] https://taxfoundation.org/research/all/federal/trump-tariffs-trade-war/

worth of U.S. imports, or 1.68% of the 2024 U.S. GDP. If all of the "Liberation Day" tariffs were to come into effect again, they would represent $2.3 trillion worth of U.S. imports, which is 7.7% of 2024 U.S. GDP.

The tariff rates changed considerably with very frequent announcements, repeals, threats, deals, and various negotiations. In Figure 10, we show some of the tariff threats, which include non-implemented tariffs and some announcements with uncertain future implementation. In Appendix D, we also show tariffs announcements (Appendix Figure D.1a) and impementations (Appendix Figure D.1) by date. This also leads to a great deal of uncertainty surrounding which tariffs will be implemented in the end. That is why we also model the tariff threats in our quantitative section.

Figure 10. Tariff Threats - not implemented and future implementation uncertain



NOTE: Figure 10 visualizes tariff threats between January 20, 2025 and June 30, 2025. The data for the tariff threats, implementations, and planned implementations were compiled from three main sources. The core of the data is from the Trade Compliance Resource Hub Trump 2.0 Tariff Tracker (https://www.tradecomplianceresourcehub.com/2025/06/27/trump-2-0-tariff-tracker/#updates). It presents a list from Reed Smith's International Trade and National Security team that tracks the latest threatened and implemented U.S. tariffs as of June 27th. This list is cross-referenced with Tax Foundation's Trump Trade War timeline as of June 17th (https://taxfoundation.org/research/all/federal/trump-tariffs-trade-war/), and a corresponding list from the PBS news article detailing a timeline of Trump's tariff actions as of May 26th (https://www.pbs.org/newshour/economy/a-timeline-of-trumps-tariff-actions-so-far). The tariffs that classified as "threats" are those that –as of June 30th - had not been implemented and were unlikely to be implemented based on available information. These threats were identified by extensive look into past and latest news, as well as the use of large language models. We created the data as of June 27, 2025. This website curates the all the tariff announcements by the U.S.

As a validation exercise, we also model the trade war between United States on China and other countries with tariffs imposed from February 2018 to September 2018. In this period, the U.S. implemented tariffs ranging from 10% to 25% to China, 10% tariff to aluminum, 25% to iron and steel, 30% to solar and 20 to 50 % tariffs to washers with some exceptions at country levels. In return, Canada, China, European Union, Mexico, Russia and Turkey retaliated with tariffs ranging from 5 to 20%. We obtained the detailed tariff data for this episode from Fajgelbaum et al. (2020) and trade values to calculate the weighted tariff rates from USITC website.[29]

8 Quantitative Results

We now return to the non-linear model without any simplifications. As the model is non-linear, we solve it with Dynare (Adjemian et al., 2011) under three alternative solution methods: first-order approximation, second-order approximation, and MIT shocks under perfect foresight. For small shocks, these methods yield nearly identical impulse response functions. However, our preferred solution approach, which we report below, employs MIT shocks under perfect foresight, because of the sizeable nature of the trade shocks at hand and given the non-linearities in the model, especially with regards to complementarities on the production side.

8.1 Calibration

The quantitative model incorporates a permanent real capital account wedge in each country to treat the year 2018 as the steady state to which the economy eventually returns. These wedges are added for the following reason. If a country has a trade deficit at the steady state, this requires that the country have positive net foreign assets that pay interest to finance this deficit (e.g., past trade surpluses finance the steady-state deficit). However, in practice, the United States has persistently maintained trade deficits and negative net foreign assets. In order to treat consumption and I-O tables for a given year (e.g., 2018) as the steady state and at the same time embed a realistic net foreign asset (NFA) position for all relevant country blocks, one needs to reconcile steady state algebra with real-life data. The real permanent capital account wedges help with reconciling the two. These wedges can be interpreted as a persistent difference in the patience of nations or alternatively can be thought of as a persistent exogenous difference in the interest paid on assets versus liabilities

[29] Exports: https://dataweb.usitc.gov/trade/search/TotExp/HTS, Imports: https://dataweb.usitc.gov/trade/search/GenImp/HTS.

that render having trade deficits and net debt at the steady state possible.

The calibration parameters are summarized in Table 2. The model employs sector-specific Calvo parameters based on the empirical estimates in Nakamura and Steinsson (2008), adjusted to a quarterly frequency. The production and intratemporal consumption structures are similar to those in Çakmaklı et al. (2025) and di Giovanni et al. (2023). On the production side, firms combine labor and intermediate input bundles to produce goods. Based on Atalay (2017), we set the elasticity of substitution between labor and intermediates $\theta^P = 0.6$. Intermediate input bundles are composed of sectoral bundles, which are assumed to be complements. Following Boehm et al. (2019) and Baqaee and Farhi (2024), we set this elasticity to $\theta_h^P = 0.2$. Each sectoral bundle consists of varieties sourced from different countries. In our baseline specification, we set the Armington elasticity across countries at the sectoral level to $\theta_{li}^P = 0.6$. On the intratemporal consumption side, we follow Baqaee and Farhi (2024) and assume Cobb–Douglas preferences across sectors, setting the sectoral elasticity to $\theta_h^C = 1$. Boehm et al. (2023) estimate short-run trade elasticities of approximately 0.76 and long-run elasticities around 2. In line with this and with di Giovanni et al. (2023), for the aggregation of varieties within sectoral consumption and production bundles (e.g. for the substitutability between cars from country A and country B), we allow for higher substitutability and conduct sensitivity analyses with θ_{li}^C and θ_{li}^P in the range between 0.6 and 2. In the baseline calibration, both parameters are set to 1.5.

Additionally, for realism, we incorporate monetary policy inertia by modifying the baseline Taylor rule. Specifically, Equation (25) is replaced with the following specification:

$$1 + i_{n,t} = (1 + i_{n,t-1})^{\rho_m^n} (\Pi_{n,t})^{\phi_\pi^n} \quad \forall n \in N$$

Here, ρ_m^n captures the degree of interest rate smoothing (or policy inertia), ϕ_π^n is the weight place on inflation in the Taylor rule. This specification is applied to all countries $n \in N$ in the model.

For the United States, we set $\rho_m^{\text{US}} = 0.82$ and $\phi_\pi^{\text{US}} = 1.29$, based on the estimates provided by Carvalho et al. (2021a). Following Clarida et al. (2000), we use $\rho_m^n = 0.95$ and $\phi_\pi^{EA} = 1$ for the rest of the world and the Euro Area, respectively. For other countries in the rest of the world, we assume $\phi_\pi^n = 0.2$, except for Mexico, where we use a slightly higher value of $\phi_\pi^{\text{MX}} = 0.3$. These ϕ_π values are calibrated using a model-consistent interpretation of the long-run average of quarterly inflation rates. Specifically, following the logic in Clarida et al. (2000), we set $\phi_\pi^n = \frac{1-\rho_m^n}{\overline{\pi}_n^C}$, where $\overline{\pi}_n^C$ denotes the long-run average of quarterly CPI inflation in country n. Using quarterly data from 2002Q2 to 2024Q4 and setting $\rho_m^n = 0.95$, we calibrate the inflation response coefficients accordingly. This calibration captures the

empirical observation that central banks in many countries outside the United States are less responsive to inflation fluctuations and are therefore less likely to adhere strictly to a Taylor rule.

Table 2. Parameter values

Parameter	Explanation	Value	Source
σ	Intertemporal EoS	2	e.g., Itskhoki and Mukhin (2021)
η	Elasticity of Labor	1	e.g., Itskhoki and Mukhin (2021)
ψ	Reactivity of UIP to Debt	0.001	Standard
ρ_m^n	Inertia in Taylor Rule for $n \neq US$	0.95	Clarida et al. (2000)
ρ_m^{US}	Inertia in Taylor Rule for U.S.	0.82	Carvalho et al. (2021a)
ϕ_π^{US}	Weight on inflation in Taylor Rule for U.S.	1.29	Carvalho et al. (2021a)
λ_n	Sector specific price rigidities		Nakamura and Steinsson (2008)
θ^P	EoS between intermediates and VA	0.6	Atalay (2017)
θ_h^C	Intratemporal EoS of consumption among sectors	1	di Giovanni et al. (2023)
θ_h^P	EoS among intermediate inputs	0.2	Baqaee and Farhi (2019); Boehm et al. (2019)
θ_{li}^C	Sector level consumption bundle EoS	0.6-2	di Giovanni et al. (2023)
θ_{li}^P	Sector level input bundle EoS	0.6-2	di Giovanni et al. (2023)

NOTES: "EoS" is the elasticity of substitution.

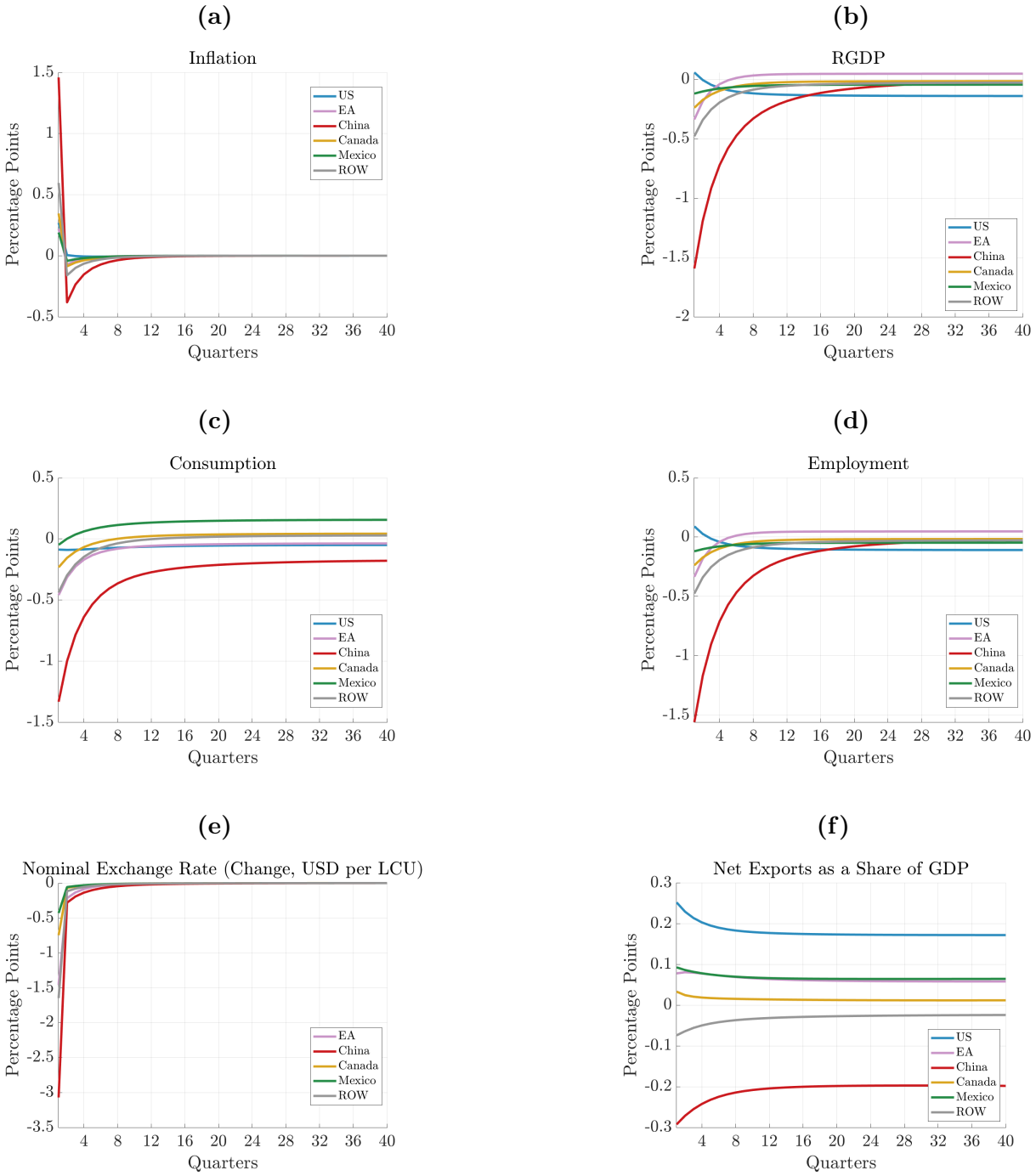
Finally, as the existing policy environment has started to feature permanently higher tariffs (e.g. as evidenced by the agreement between the US and EU), we diverge from the the analytical model's specification of tariffs following an AR(1) process (i.e., $\tau_t = \rho^\tau \tau_{t-1} + \epsilon_t^\tau$). In the cases below, we instead feed into the model a tariff shock that raises tariffs to a higher level for 100 periods, which correspond to 25 years in our calibration. Quantitatively, for our horizon of interest this produces results that are similar to an AR(1) tariff shock with near-permanent persistence, $\rho^\tau = 0.999$. We find that this approach produces impulse responses that are more realistic since they do not involve a decaying process in our horizon of interest.

8.2 Case 1: 2018's Trade War

We begin by validating the model using the case of the tariffs imposed by the United States on China and other countries between February 2018 and September 2018 (See Section 7.2 for details of the data). As shown in Figure 11, the model predicts a 3% nominal appreciation of the U.S. dollar (USD) against the Chinese yuan on impact, eventually reaching 4.2% nominal appreciation in the long run. This aligns with the observed 5.6% appreciation of the USD between June 2018 and December 2018. Real GDP loss reaches 0.1 percentage points. This is in the range of the estimate of Fajgelbaum et al. (2020), which found an aggregate real income loss of 0.04% of GDP. Finally, the model predicts an inflation impact

of 0.27 percentage points, which is close to the 0.1-0.2 percentage point estimate of Barbiero and Stein (2025).

Figure 11. Case 1: Impact of 2018's Trade War



NOTE: Figure 11 visualizes simulated responses to the 2018 U.S. tariff package targeting China. Impulse responses are computed with MIT shocks.

Table 3. On-Impact Response of Variables in Case 1: 2018's Trade War

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	0.06%	-0.34%	-1.59%	-0.24%	-0.12%	-0.48%
C_n	-0.09%	-0.46%	-1.33%	-0.23%	-0.05%	-0.44%
π_n	0.27%	0.22%	1.46%	0.35%	0.19%	0.60%
i_n	0.00%	0.22%	0.29%	0.07%	0.06%	0.12%
$\Delta\mathcal{E}_n$	0.00%	-1.46%	-3.07%	-0.75%	-0.43%	-1.65%
RER_n	0.00%	-1.51%	-1.92%	-0.67%	-0.51%	-1.33%
L_n	0.09%	-0.33%	-1.56%	-0.24%	-0.12%	-0.48%
$\frac{W_n}{P_n}$	-0.09%	-1.25%	-4.17%	-0.70%	-0.22%	-1.34%
$\frac{NX_n}{NGDP_n^{ss}}$	0.25%	0.07%	-0.29%	0.03%	0.09%	-0.07%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.06%	-0.02%	-0.31%	-0.01%	0.00%	-0.13%

NOTE: First-period impact of the U.S. tariffs in 2018. Effects are reported in deviation from the pre-tariff steady state. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

8.3 Case 2: Reversed Tariff Threats

As seen in Figure 10, there have been many tariff threats that are not implemented or uncertain to be implemented. In this section, we apply our model to the case of reversed tariff threats- scenarios in which a country announces future tariffs but subsequently reverses the decision before implementation. This case also incorporates retaliation: specifically, the United States announces in period 1 that tariffs will be imposed in period 2, prompting other countries to announce retaliatory measures for the same period. However, when period 2 arrives, it is announced that no tariffs will be levied by either side.

This scenario not only mimics the reality of how tariffs were introduced in 2025 but also it allows us to isolate the role of the expectations channel. In standard scenarios, when a tariff is introduced, as is the case in Cases 1, 3,4 and 5, under price stickiness it is the case that inflation shows up first and the decline in GDP takes some time. This is because of the theoretical mechanisms we explored in Section 5. A significant portion of the inflationary impacts of tariffs come from direct effects, which are present when tariffs are first placed, while the indirect effects especially those involving output take longer when prices take time to adjust. That is why, with a standard model, monetary policymakers might expect after tariffs are placed that the worst is over on inflation after a one-time increase in prices and

that there is more to worry with regards to output. A world with production networks and reversed tariff threats differs from this expectation in two ways. First, if actual tariffs are preceded by reversed tariff threats, there could be some deflation initially as we show below and output might begin to contract before inflation starts ticking up. Second, in setups with production networks, inflation can be more persistent even if reversed tariff threats delay direct effects.

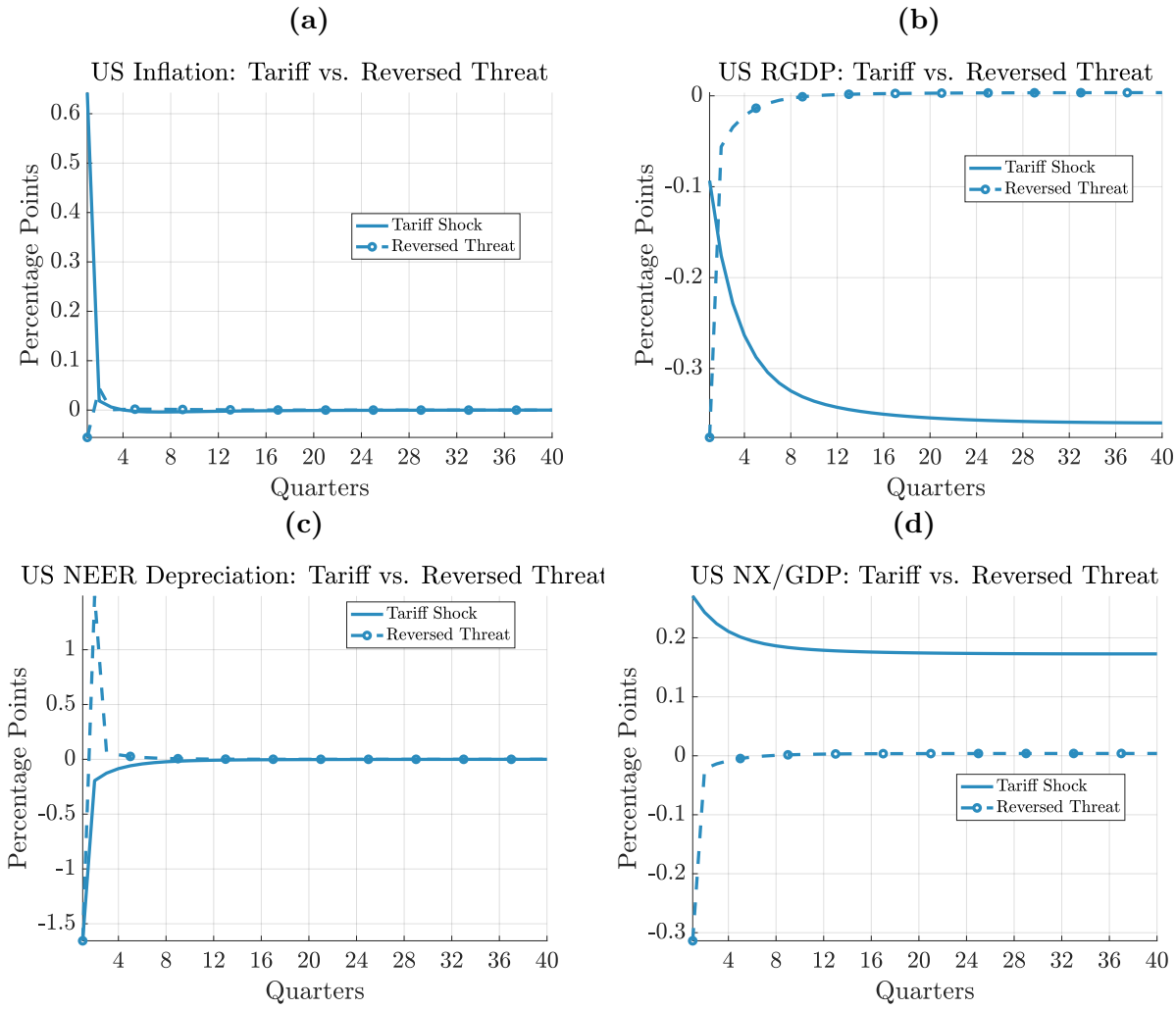
To analyze the effects of reversed tariff threats, we construct two impulse responses under perfect foresight. First, we simulate the all-out tariff war shock examined in Case 5, assuming it is both announced and implemented in the first period of the simulation. Second, we simulate the same shock- identical in magnitude- but announced to take effect in the second period, only to be withdrawn before implementation. The impulse response to the reversed tariff threat is then obtained by shifting the first (implemented) impulse response forward by one period and subtracting it from the second (announced-but-not-implemented) response. This approach isolates the effect of the anticipatory behavior triggered by the announcement, net of the effects of actual implementation. Importantly, we observe that from the second period onward, the quantity variables in both simulations converge and remain nearly identical. This reflects the fact that agents discount the future and adjust quantities in response to the announcement, but not to the same extent as they would if the shock were immediate and fully realized. Our approach here is inspired by the *fake news* algorithm of Auclert et al. (2021), in which agents receive information about a future increase in income and optimize accordingly, only to later discover that the anticipated change does not materialize. While Auclert et al. (2021) employ this construct as a computational device for solving models in sequence space, we interpret and apply it literally to study the macroeconomic implications of trade policy reversals.

Figure 12 compares the impact on inflation, real GDP, and U.S. NEER appreciation in Case 5 (Tariff Shock) to the reversed tariff threat scenario. First striking observation is that reversed tariff threats are deflationary, whereas actual tariffs are inflationary. The intuition here is closely linked to the decomposition of inflation in Figure 3. When the direct inflationary effects of tariffs are absent, indirect effects that are deflationary can overcome inflationary indirect effects as we see here.

Second, a future in which the United States demands fewer goods from abroad prompts an immediate appreciation of the USD, as the intertemporal budget constraint adjusts and agents incorporate expectated future income streams into current allocations and pricing. In this scenario, the U.S. trade-weighted nominal effective exchange rate appreciates by 1.7% on impact. In contrast, quantity variables respond more gradually. When agents realize in the second period that the shock will not materialize, they reoptimize, resulting in a

partial recovery. In line with this, although tariffs are never actually implemented, we see that real GDP declines by 0.4 percentage points. Notably this decline is larger than the decline in the event of actual tariffs, because in the case of the latter direct effects on prices start encouraging demand for domestic goods, whereas in the reversed tariff threat case anticipated supply distortions discourage production. The period before tariffs actually take place is a period in which it is cheaper to import than it will be in the future. As a result, one can simultaneously see appreciation and frontloaded net imports leading to a worsening of the trade balance as is the case here.

Figure 12. Case 2: Impact of Reversed Tariff Threats



NOTE: Figure 12 visualizes simulated response to reversed tariff announcements. Tariffs are announced in the first period, with retaliation expected, and later canceled in the second period.

It is notable that, once tariffs are reversed, the U.S. dollar depreciates: agents had previously priced in a future in which the U.S. would reduce demand for foreign goods, but

upon receiving new information in the second period that this scenario would not materialize, the exchange rate response is reversed. Expectations-linked overshooting is interesting since this does not happen with regular tariffs. A more realistic interpretation of the observed and somewhat sustained U.S. dollar depreciation in response to tariffs requires accounting for a large uncertainty (VIX) shock and policy volatility more than our simple one period on-off tariff threat exercise, or other shocks such as fiscal uncertainty, that are outside the scope of our paper.

Overall, this exercise demonstrates that the expectations channel, emphasized in our theoretical analysis, plays a central role. Reversed tariff announcements operate similarly to domestic demand shocks, particularly when announcements are perceived as credible. Importantly, the macroeconomic distortion introduced through the expectations channel does not dissipate immediately with the reversal announcement. Variables exhibit persistence, and the economy does not return to steady state instantaneously.

8.4 Case 3: 2025's Trade War

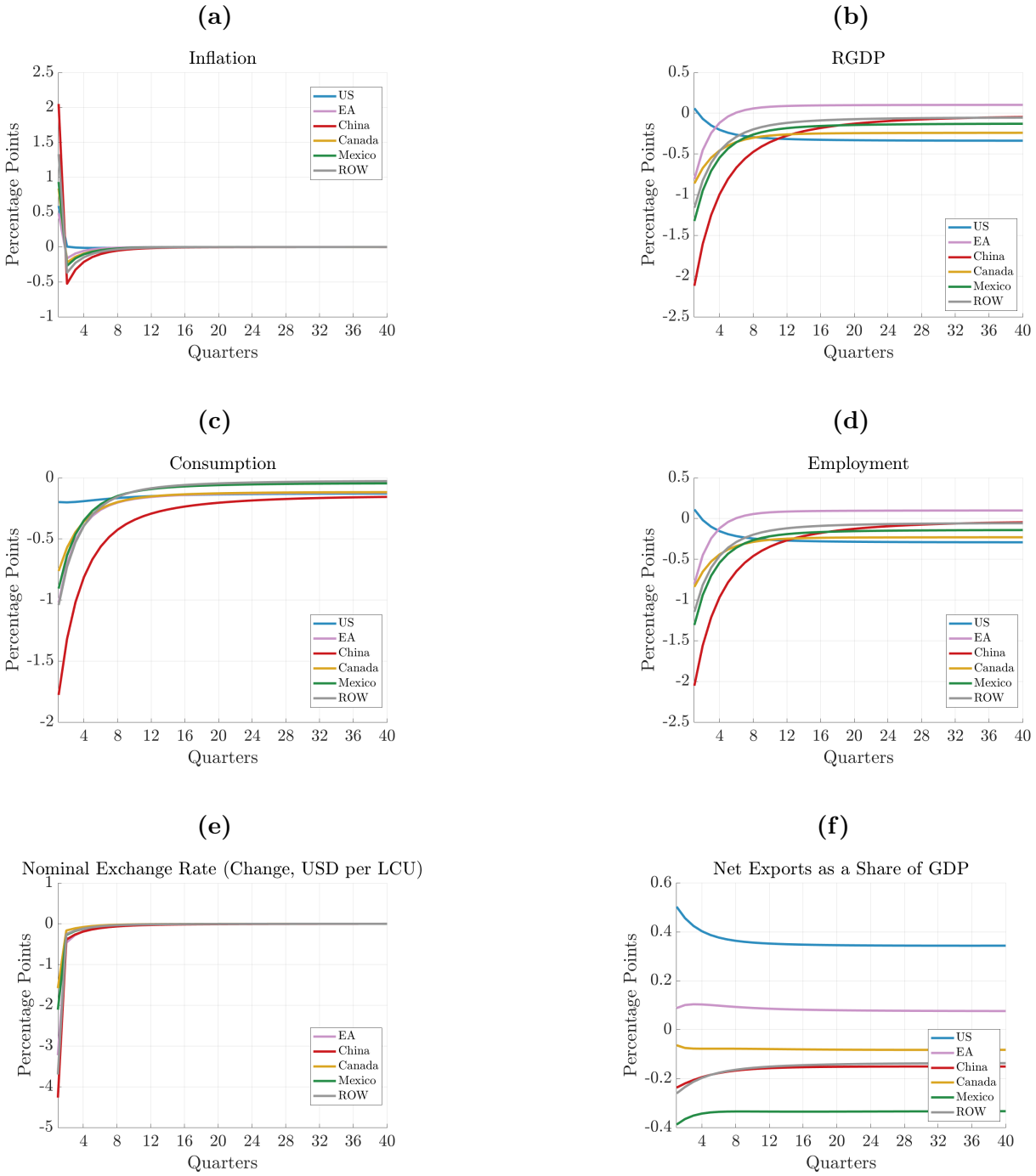
In 2025, the United States announced several rounds of tariffs targeting Mexico, Canada, Europe, China, and many other countries. In Section 7.2, We explained in detail the tariffs announced, implemented, changed and limited retaliation from others happened so far as of the time of our writing.

As shown in Figure 13 and Table 4, predicts a contraction in U.S. real GDP reaching 0.4 percent in the long-run. This is accompanied by almost 0.2% decrease in consumption, a 0.5 percentage point increase in net exports as a share of steady-state GDP (i.e. an improvement in trade deficit), and a 0.3% decline in real wages. Inflation rises by 0.6 percentage points. Additionally, the U.S. trade-weighted nominal effective exchange rate (NEER) appreciates by 3.3%.

The effects are pronounced for China, Mexico, and Canada. China's real GDP contracts by 2.12%, while Mexico's declines by 1.32% and Canada's by 0.86%. Labor market impacts are also substantial, with employment falling by 2.05% in China, 1.31% in Mexico, and 0.84% in Canada. Net exports decline by 0.24%, 0.38%, and 0.06% of steady-state GDP, respectively. Inflation rises by 2.05 percentage points in China, 0.93 percentage points in Mexico, and 0.84 percentage points in Canada. The Euro Area (EA) experiences a decline in real GDP of 0.81%, while the rest of the world (RoW) contracts by 1.16%. Consumption falls by 1.02% in the EA and 1.04% in the RoW. Inflation rises by 0.48 percentage points in the EA and 1.33 percentage points in the RoW. Employment declines by 0.80% and 1.15%, respectively. Real wages fall by 2.81% in the EA and 3.20% in the RoW. Net exports as

a share of steady-state GDP increase slightly in the EA (0.07%) but decline in the RoW (–0.26%).

Figure 13. Case 3: Impact of 2025 Tariffs



NOTE: Figure 11 visualizes simulated responses to the 2025 U.S. tariff package, targeting China, Canada, Mexico, Europe and the RoW. Impulse responses are computed with MIT shocks.

In this scenario, the tariff shock is inflationary on impact for all countries. Then in non-US countries there is a period of deflation. This occurs because US tariffs constitute a negative demand shock for the rest of the world and monetary policy reacts with a lag. In our model, DCP contributes to this dynamic as well. Other countries have a meaningful share of their exports invoiced in USD and since the model expects USD appreciation, the on-impact inflation seen in non-US countries is driven not only by supply distortions but also by exchange rate passthrough. It is notable that this tariff shock is a negative shock for the real GDP of all countries except for the Euro Area, which tends to benefit from trade diversion in the long run.

Table 4. On-Impact Response of Variables in Case 3: 2025's Tariffs

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	0.06%	-0.81%	-2.12%	-0.86%	-1.32%	-1.16%
C_n	-0.20%	-1.02%	-1.78%	-0.76%	-0.91%	-1.04%
π_n	0.59%	0.48%	2.05%	0.84%	0.93%	1.33%
i_n	0.00%	0.49%	0.41%	0.17%	0.28%	0.27%
$\Delta\mathcal{E}_n$	0.00%	-3.23%	-4.27%	-1.58%	-2.11%	-3.70%
RER_n	0.00%	-3.33%	-2.88%	-1.34%	-1.77%	-2.99%
L_n	0.11%	-0.80%	-2.05%	-0.84%	-1.31%	-1.15%
$\frac{W_n}{P_n}$	-0.29%	-2.81%	-5.50%	-2.35%	-3.09%	-3.20%
$\frac{NX_n}{NGDP_n^{ss}}$	0.50%	0.07%	-0.24%	-0.06%	-0.38%	-0.26%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.11%	-0.01%	-0.47%	-0.01%	0.02%	-0.27%

NOTE: First-period outcomes of the 2025 unilateral U.S. tariff package. Tariff rates vary by country-sector; effects are reported in deviation from the steady state. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

8.5 Case 4: Non-Reactive Monetary Policy

A natural question to ask is how much of the observed results in Case 3 are driven by the specific monetary policy rule that the model assigns to various countries. After all, macroeconomic outcomes like inflation are ultimately policy driven. To that end we consider a second version of Case 4 in which, nominal interest rates are held constant across the globe. One interpretation of this exercise is that this is an attempt to understand the impact of

tariffs before policy weighs in.

Naturally this can raise questions about determinacy. In general, New Keynesian models suffer from indeterminacy if the nominal interest rate is held exogenous and does not respond to endogenous outcomes like inflation. To overcome this problem, we adopt the following exercise influenced by the HANK literature's use of real rate rules. When linearized, the policy rule used in our quantitative work is $i_{n,t} = \rho_m^n i_{n,t-1} + \phi_\pi^n \pi_{n,t}^C$. Setting $\rho_m^n = 1$. We can take the limit of $\phi_\pi^n \to 0$ and as long as $\phi_\pi > 0$, the system stays in the region of determinacy. With this approach, in Case 4 we depict how the results in Case 3 would change if the interest rate was numerically held constant.

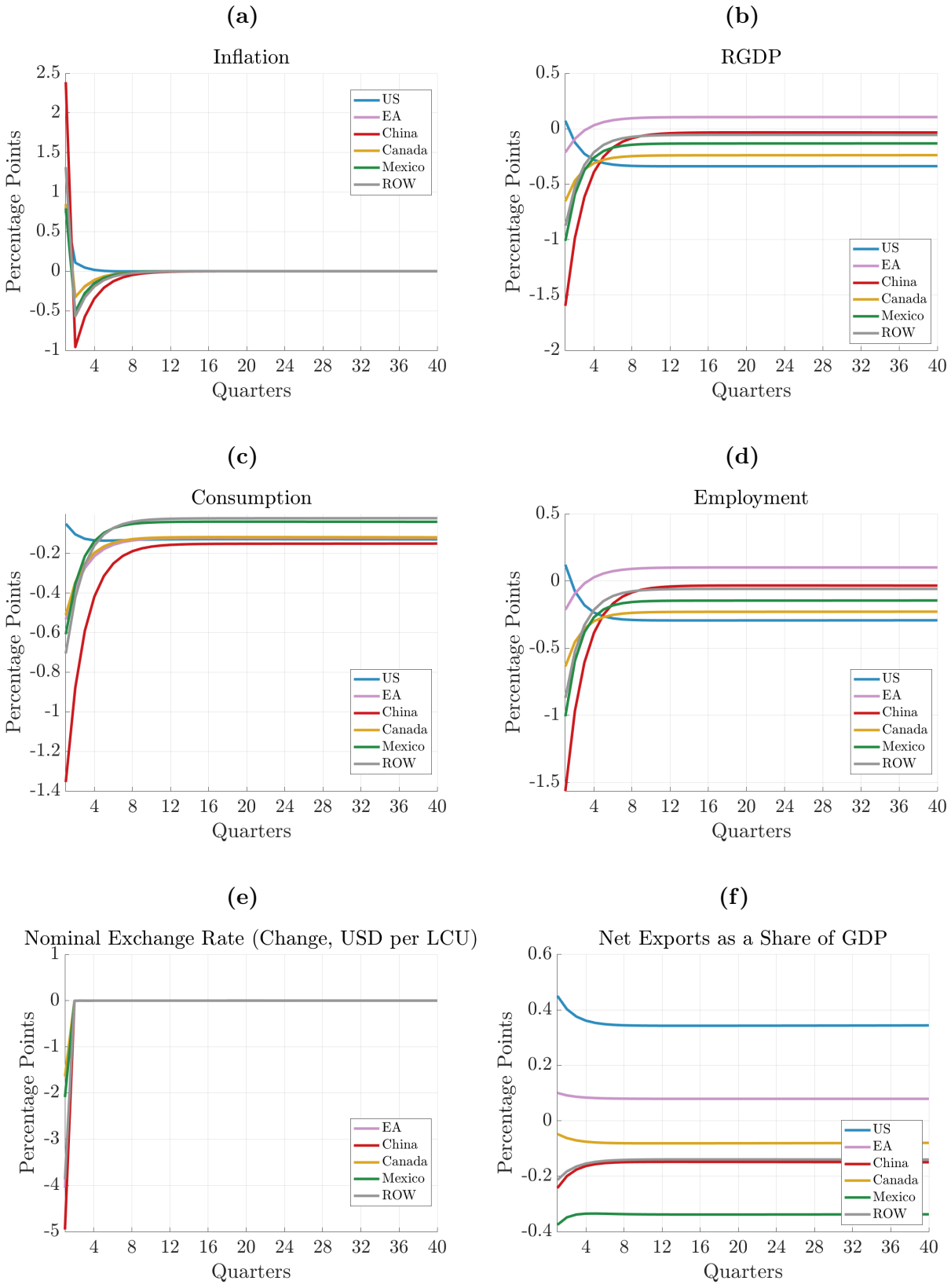
Intuitively, in this case inflation is higher on impact globally and inflation's persistence is higher in the US since in this case policy does not respond. Since policy is non-reactive the deflation in non-US countries in the second period onwards is larger as well. Beyond the changes in inflation, the results for other variables are largely similar to Case 3. This is not surprising since the baseline Taylor rule calibration already features a large degree of policy inertia.

Table 5. On-Impact Response of Variables in Case 4: 2025's Tariffs

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	0.07%	-0.21%	-1.60%	-0.65%	-1.01%	-0.88%
C_n	-0.05%	-0.53%	-1.35%	-0.51%	-0.61%	-0.71%
π_n	0.76%	0.82%	2.39%	0.85%	0.79%	1.32%
i_n	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
$\Delta\mathcal{E}_n$	0.00%	-4.05%	-4.96%	-1.64%	-2.09%	-3.87%
RER_n	0.00%	-4.00%	-3.42%	-1.55%	-2.05%	-3.34%
L_n	0.12%	-0.21%	-1.56%	-0.64%	-1.01%	-0.87%
$\frac{W_n}{P_n}$	0.02%	-1.28%	-4.21%	-1.65%	-2.21%	-2.26%
$\frac{NX_n}{NGDP_n^{ss}}$	0.44%	0.11%	-0.24%	-0.05%	-0.37%	-0.22%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.10%	-0.02%	-0.55%	-0.01%	0.02%	-0.30%

NOTE: First-period outcomes of the 2025 unilateral U.S. tariff package. Tariff rates vary by country-sector; effects are reported in deviation from the steady state. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

Figure 14. Case 4: Impact of 2025 Tariffs With Non-Reactive Monetary Policy



NOTE: Figure 14 visualizes simulated responses to the 2025 U.S. tariff package, targeting China, Canada, Mexico, Europe and the RoW in the case whereby monetary policy is non-reactive. Impulse responses are computed with MIT shocks.

8.6 Case 5: All-Out Trade War

We now turn to a counterfactual quantitative exercise exploring what would happen if non-US countries retaliated symmetrically to US tariffs. Case 3 was based on actual tariffs that have been placed and these involve a small degree of retaliation, but by and large these have been small. As a counterfactul we explore how the results would look under an an all-out symmetric tariff war. In this case, the United States imposes tariffs on all major trade partners at the same rates as specified in Case 3 and trade partners retaliate by imposing symmetric tariffs on U.S. exports.

China experiences a sizable contraction in GDP, declining by 1.4%, while consumption drops by 1.2%. The real exchange rate depreciates by 2.0%. Inflation rises by 1.3 percentage points, and employment declines by 1.3%. Real wages fall by 3.6%. The Euro Area experiences a moderate contraction: real GDP declines by 0.5%, consumption falls by 0.6%, and real wages decrease by 1.7%. Inflation rises by 0.3 percentage points. The euro depreciates by 1.6% against the U.S. dollar, while the real exchange rate declines by 2.0%. These exchange rate adjustments help absorb part of the external shock, limiting further output losses.

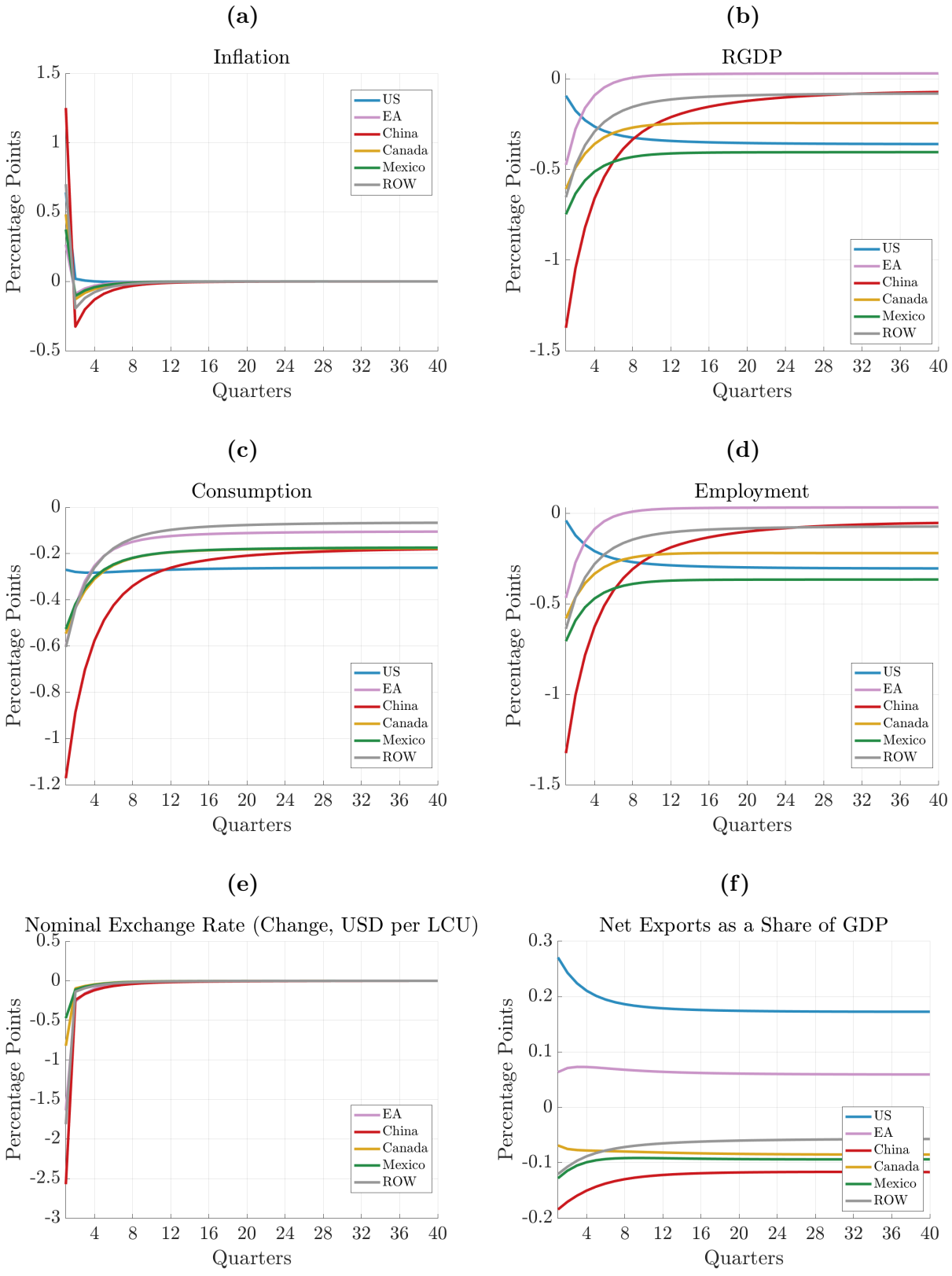
The United States experiences a mild contraction, with real GDP declining by 0.1% and consumption falling by 0.3%. Inflation rises by 0.6 percentage points, and employment decreases slightly by 0.0%. Real wages fall by 0.6%. Net exports as a share of steady-state GDP increase by 0.3 percentage points.

Canada and Mexico both experience moderate contractions. Real GDP declines by 0.6% in Canada and 0.8% in Mexico. Consumption falls by 0.6% and 0.5%, respectively. Employment decreases by 0.6% in Canada and 0.7% in Mexico. Inflation rises by 0.5 percentage points in Canada and 0.4 percentage points in Mexico. Nominal exchange rates depreciate by 0.8% for Canada and 0.5% for Mexico relative to the U.S. dollar, while real exchange rates fall by 1.0% and 0.7%, respectively. Real wages decline by 1.7% in Canada and 1.8% in Mexico.

The rest of the world (RoW) also experiences a contraction, with real GDP declining by 0.7%, consumption by 0.6%, and employment by 0.6%. Inflation rises by 0.7 percentage points. The nominal exchange rate depreciates by 1.8%, while the real exchange rate declines by 1.8%. Real wages fall by 1.8%. Net exports as a share of steady-state GDP decline by 0.1%.

Overall, the all-out tariff war scenario leads to a synchronized global slowdown. Output and employment contract across all regions, while inflation rises moderately. The largest adverse effects occur in China, followed by Mexico, the RoW, and the Euro Area, reflecting their higher trade exposure to the United States and each other.

Figure 15. Case 5: Impact of All-Out Tariff War



NOTE: Figure 15 visualizes an all-out tariff war scenario in which trade partners retaliate symmetrically. Impulse responses are calculated with MIT shocks.

Table 6. On-Impact Response of Variables in Case 5: All-Out Tariff War

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	-0.09%	-0.47%	-1.37%	-0.61%	-0.75%	-0.65%
C_n	-0.27%	-0.60%	-1.17%	-0.55%	-0.53%	-0.60%
π_n	0.64%	0.27%	1.25%	0.48%	0.37%	0.70%
i_n	0.00%	0.27%	0.25%	0.10%	0.11%	0.14%
$\Delta\mathcal{E}_n$	0.00%	-1.64%	-2.57%	-0.82%	-0.47%	-1.81%
RER_n	0.00%	-2.01%	-1.98%	-0.98%	-0.74%	-1.76%
L_n	-0.04%	-0.47%	-1.33%	-0.58%	-0.71%	-0.64%
$\frac{W_n}{P_n}$	-0.58%	-1.66%	-3.62%	-1.66%	-1.75%	-1.84%
$\frac{NX_n}{NGDP_n^{ss}}$	0.27%	0.05%	-0.19%	-0.06%	-0.11%	-0.12%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.06%	-0.01%	-0.27%	-0.01%	0.01%	-0.13%

NOTE: First-period outcomes from a global tariff war scenario with full retaliation. Tariff magnitudes and persistence match Case 2. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

8.7 Discussion

Our analytical and quantitative analyses allow us to engage with several central questions. Under what conditions are tariffs appreciationary or depreciationary for the nominal exchange rate? Under what conditions are tariffs inflationary or deflationary? And under what conditions tariffs can be contractionary? We know these answers from the model but here in the light of the quantitative results that takes into account non-linearities, we provide further discussion.

8.7.1 Trade Deficits and the Dollar

In our quantitative framework, we find that tariffs can lead to an appreciation of the currency of the tariff-imposing country on impact. However, once retaliation is introduced, the exchange rate response becomes sensitive to the relative hawkishness of central banks. For instance, in a scenario where the U.S. imposes tariffs and the rest of the world responds, the U.S. dollar (USD) may depreciate on impact if the rest of the world has higher ϕ_π parameters-leading to greater interest rate differentials in favor of non-USD currencies.

Figure 16. USD Exchange Rates against to Major Currencies, following 2018 tariff war and 2025 Inaugurations

(a) January 2018 – February 2020 **(b)** November 2024 – June 2025



NOTE: Figure 16 visualizes the USD vs. EUR, CNY, GBP, JPY, CAD, SEK, CHF normalized exchange rates during the 2018 Trade war episode between USA and China, between January 2018 and February 2020 (before Covid-19 pandemic) and during the 2025 Tariff episode, between the election of President Trump for his second term (November 2024) and the latest available date (June 30, 2025). Data Source: Bloomberg.

Other work, such as Jiang et al. (2025) and Itskhoki and Mukhin (2025), explain the observed depreciation of the dollar, since the beginning of April 2025, with the loss of safe heaven status or convenience yield, where the two are related as shown before (e.g., Kekre and Lenel, 2024). Pinter et al. (2025) highlight the importance of non-trade related orthogonal shocks that coincide with a deterioration in Treasury market liquidity. Another alternative for the observed dollar depreciation is that the impact of policy uncertainty embedded in tariff threats dwarfs the standard appreciationary effect of a regular tariff shock. In fact, the early appreciation followed by a depreciation of the dollar shown in Figure 12 as a result of tariff threats is similar to what is observed after inauguration in January 2018 and January 2025, as shown in Figure 16. Although the depreciation of the dollar, especially vis-a-vis classic safe haven currencies, is in excess of 10 percent at the time of this writing, it is still small viewed in a historical context as shown in Appendix Figure D.3. Interestingly, the initial appreciation of around 2% and then the depreciation of 4% plotted in this Appendix

figure are both in the ballpark of what we get with our tariff threat calibration.[30]

In our model, trade deficits and surpluses can only move during transition with transitory tariffs but not at the steady state. This is because tariffs do not change consumption-saving patterns permanently, and hence steady state net asset/liability positions. A similar argument has also been put forth by Obstfeld (2025).

8.7.2 Inflation-Output Trade-Off and Employment

Our analytical work and calibrations show that tariffs can be inflationary or deflationary for the country on which they are imposed. A more subtle question is whether tariffs can be deflationary for the tariff-imposing country itself, such as the United States. Within our modeling framework, and barring extreme parameterizations, the direct effect of tariffs, which mechanically exerts upward pressure on prices, dominates the deflationary forces from other channels. If inflation were to turn negative, monetary policy would reverse direction and cut interest rates, thereby supporting prices. Consequently, in both our analytical solution and baseline simulations (Cases 1, 2, and 3), tariffs are inflationary for the imposing country and output declines in the short-run and also in the long-run with retaliation. The key exception is Case 4, in which tariffs threats lead to deflation due to expectation channels.

Overall tariffs can create a stagflationary outcome with increasing inflation and declining employment and output. The response of monetary policy is critical here.

9 Conclusion

We develop a new global general equilibrium framework to study the macroeconomic impact of tariffs under trade imbalances. Our N-country-J-sector NKOE model incorporates full global I-O linkages, heterogeneity in sectoral price rigidities and in monetary policy responses across countries involved in a trade war. We formulate the model around five primitives composed of structural parameters (consumption shares, production shares, elasticities of substitution), frictions (nominal rigidities), and endogenous monetary policy response.

Our core contribution is to delineate how the economic impact of tariffs can differ by adding dynamics, monetary policy, international borrowing/lending, and unbalanced trade into a general trade and production network with nominal rigidities. In our environment, monetary policy changes the transmission of the tariff shock, both within a given economy and across different but connected economies through trade and debt. To analyze this transmission, we derive the NKOE Leontief inverse and decompose the effects of tariffs into

[30] We can match the observed movements in dollar better with a wedge in the UIP equation for idiosyncratic local risk factor linked to policy volatility as done in Kalemli-Özcan and Varela (2021).

direct and indirect channels- each of which maps directly onto structural components of the model. To show the importance of country connections through both trade and debt, we demonstrate that relative to the financial autarky baseline, the case with international risk sharing leads to dampening of the impact of the tariff shock and having it more evenly and smoothly distributed across time and countries. Our results highlight the inflationary and contractionary effects of tariff shocks in an environment with forward-looking agents, where these effects are further amplified through the expectations channel. Although our focus is on the effects of short-run tariff shocks, we show that the effects of permanent tariff shocks can also be contractionary: the inability to substitute between domestic and foreign inputs makes goods more expensive leading to a decline in output.

Our work yields two key implications, relevant both for scholars and policy makers. First, models that omit a multi-sector structure may underestimate the impact of tariffs on real economic quantities- such as output and employment- while overestimating their effect on inflation, especially under the assumption of balanced trade. Second, tariff threats carry real macroeconomic consequences- even when they are subsequently reversed. When agents expect future price increases, they begin to smooth consumption downward in anticipation. Because the exchange rate is forward-looking, it appreciates immediately in response to these expectations, but then reverses itself and depreciates when threat turns empty. In this way, tariff threats function as contractionary demand shocks, even in the absence of actual tariff implementation. A deeper understanding of both production network structures and expectation-driven dynamics- such as those modeled here- can help central banks navigate a policy environment in which tariffs, retaliation, and related threats are becoming increasingly common. As Federal Reserve Chair Jerome Powell recently emphasized: "We may find ourselves in the challenging scenario in which our dual-mandate goals are in tension....There aren't historical experiences we can consult here. So it may turn out that the tariff pass-through is less or more than we think. We are perfectly open to the idea that the pass-through will be less than we think, and, if so, that will matter for our policy.[31]" Our analysis can shed light on these pressing policy questions.

By theoretically unifying long- and short-run perspectives on the impact of trade barriers, our framework echoes foundational insights from classical economic literature, dating back to Hume (1758), which emphasized the price–specie flow mechanism. This mechanism illustrates how price levels adjust endogenously through trade flows, ultimately rendering trade restrictions self-defeating. Restrictions on exports and imports induce exchange rate movements that offset perceived gains. For countries imposing import restrictions, rising labor and input costs typically follow, forcing firms to reduce employment and scale back produc-

[31]Semiannual Monetary Policy Report to the Congress, June 24, 2025.

tion—ultimately undermining domestic economic performance. This core insight traces back even further to Gervaise (1720), underscoring the long-standing understanding that trade barriers distort price signals, resource allocation and economic growth.

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Appendix

A Approximated Linear Equilibrium Conditions

Before simplifications are introduced, linearized equilibrium conditions are as follows:[32]

$$E_t\hat{C}_{n,t+1} - \hat{C}_{n,t} = \frac{1}{\sigma}\left(\hat{i}_t - E_t\pi_{n,t+1}\right) \tag{A.1}$$

$$\hat{i}_{n,t} - \hat{i}_{US,t} = E_t\hat{\mathcal{E}}_{n,t+1} - \hat{\mathcal{E}}_{n,t} + \hat{\psi} \tag{A.2}$$

$$\hat{\mathcal{E}}_{n,m,t} = \hat{\mathcal{E}}^{US}_{n,t} - \hat{\mathcal{E}}^{US}_{m,t} \tag{A.3}$$

$$\hat{\mathcal{E}}_{n,n,t} = 0 \tag{A.4}$$

$$\hat{W}_{n,t} - \hat{P}^C_{n,t} = \eta\hat{L}_{n,t} + \sigma\hat{C}_{n,t} \tag{A.5}$$

$$\hat{C}_{nt} = \sum_{j\in J}\Gamma_{n,j}\hat{C}_{n,j,t} \tag{A.6}$$

$$\hat{C}_{n,j,t} = \sum_{m\in N}\Gamma_{n,j,mj}\hat{C}_{n,mj,t} \tag{A.7}$$

$$\hat{P}_{n,mj,t} = \hat{\mathcal{E}}_{n,m,t} + \hat{\tau}_{n,m,t} + \hat{P}_{mj,t} \tag{A.8}$$

$$\hat{C}_{n,j,t} = \hat{C}_{n,t} - \theta^C_h\left(\hat{P}^C_{n,j,t} - \hat{P}^C_{n,t}\right) \tag{A.9}$$

$$\hat{C}_{n,mj,t} = \hat{C}_{n,j,t} - \theta^C_{l,j}\left(\hat{P}^C_{n,mj,t} - \hat{P}^C_{n,j,t}\right) \tag{A.10}$$

$$\hat{X}_{ni,j,t} = \sum_{m\in N}\Omega_{ni,j,mj}\hat{X}_{ni,mj,t} \tag{A.11}$$

$$\hat{X}_{ni,mj,t} = \hat{X}_{ni,j,t} - \theta^P_{l,j}\left(\hat{P}_{n,mj,t} - \hat{P}^X_{ni,j,t}\right) \tag{A.12}$$

$$\hat{X}_{ni,t} = \sum_{j\in J}\Omega_{ni,j}\hat{X}_{ni,j,t} \tag{A.13}$$

$$\hat{X}_{ni,j,t} = \hat{X}_{ni,t} - \theta^P_h\left(\hat{P}^X_{ni,j,t} - \hat{P}^X_{ni,t}\right) \tag{A.14}$$

$$\hat{Y}_{ni,t} = \hat{A}_{ni,t} + \alpha_{ni}\hat{L}_{ni,t} + (1-\alpha_{ni})\hat{X}_{ni,t} \tag{A.15}$$

$$\widehat{MC}_{ni,t} = -\hat{A}_{ni,t} + \alpha_{ni}\hat{W}_{n,t} + (1-\alpha_{ni})\hat{P}^X_{ni,t} \tag{A.16}$$

$$\hat{X}_{ni,t} - \hat{L}_{ni,t} = \theta^P\hat{W}_{n,t} - \theta^P\hat{P}^X_{ni,t} \tag{A.17}$$

$$\pi_{ni,t} = \frac{\theta^R}{\delta_{ni}}\left(\widehat{MC}_{ni,t} - \hat{P}_{ni,t}\right) + \beta\mathbb{E}_t\pi_{ni,t+1} \tag{A.18}$$

$$\bar{B}^{US}\hat{B}^{US}_t = \sum_{m}^{N-1}\bar{B}^{US}_m\hat{B}^{US}_{m,t} \tag{A.19}$$

[32]Please note in this set of equilibrium conditions the highest layer of the intermediate input bundle is simplified away.

$$\bar{Y}_{ni}\hat{Y}_{ni,t} = \sum_{n\in\mathcal{N}} \bar{C}_{m,ni}\hat{C}_{m,ni,t} + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{X}_{mj,ni}\hat{X}_{mj,ni,t}, \tag{A.20}$$

$$\bar{L}_n\hat{L}_{n,t} = \sum_{i\in J} \bar{L}_{ni}\hat{L}_{ni,t} \tag{A.21}$$

$$\pi_{n,t} = \hat{P}^C_{n,t} - \hat{P}^C_{n,t-1} \tag{A.22}$$

$$\hat{i}_{n,t} = \phi_\pi \pi_{n,t} + \hat{M}_{n,t} \tag{A.23}$$

and:

$$\begin{aligned}
&\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{C}_{n,mj}(\hat{P}_{n,mj,t} + \hat{C}_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{X}_{ni,mj}(\hat{P}_{n,mj,t} + \hat{X}_{ni,mj,t}) \\
&+ \bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{i}^{US}_{n,t-1} + \hat{B}^{US}_{n,t-1}\right) \\
=&\sum_i \bar{P}_{ni}\bar{Y}_{ni}(\hat{P}_{ni,t} + \hat{Y}_{ni,t}) + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t} + \hat{B}^{US}_{n,t}),
\end{aligned} \tag{A.24}$$

where we denote the steady state (and limit) values with the bar notation.

B Relating the Balance of Payments to Prices

In this section, we show that the balance of payments can be captured by the following equation:

$$\beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \Xi_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{\Xi}_5\hat{\boldsymbol{\tau}}_t$$

To start with, we can rewrite the BoP as follows:

$$\begin{aligned}
&\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{C}_{n,mj}(\hat{P}_{n,mj,t} + \hat{C}_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{X}_{ni,mj}(\hat{P}_{n,mj,t} + \hat{X}_{ni,mj,t}) \\
&+ \bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{i}^{US}_{n,t-1} + \hat{B}^{US}_{n,t-1}\right) = \sum_i \bar{P}_{ni}\bar{Y}_{ni}(\hat{P}_{ni,t} + \hat{Y}_{ni,t}) + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t} + \hat{B}^{US}_{n,t}) \\
&\bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{i}^{US}_{n,t-1} + \hat{B}^{US}_{n,t-1}\right) = \overline{NX}_n\widehat{NX}_{n,t} + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t} + \hat{B}^{US}_{n,t})
\end{aligned}$$

Defining $\hat{V}_t$ as dollar-denominated debt inclusive of interest payments: $\hat{V}_t = B^{US}_{n,t}(1+i_t)$:

$$\bar{\mathcal{E}}_n\bar{V}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{V}^{US}_{n,t-1}\right) = \overline{NX}_n\widehat{NX}_{n,t} + \frac{\bar{\mathcal{E}}_n\bar{V}^{US}_n}{1+\bar{i}^{US}}(\hat{\mathcal{E}}_{n,t} + \hat{V}^{US}_{n,t} - \hat{i}^{US}_t)$$

WLOG $\bar{\mathcal{E}}_n = 1$. Also noting $(1 + \bar{i}_n^{US}) = \beta^{-1}$ and $\overline{NX} = (1-\beta)\bar{V}_n^{US}$:

$$\begin{aligned}
\bar{V}_n^{US}\left(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t-1}^{US}\right) &= (1-\beta)\bar{V}_n^{US}\widehat{NX}_{n,t} + \beta\bar{V}_n^{US}(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t}^{US} - \hat{i}_t^{US}) \\
\left(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t-1}^{US}\right) &= (1-\beta)\widehat{NX}_{n,t} + \beta(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t}^{US} - \hat{i}_t^{US}) \\
(1-\beta)\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t-1}^{US} &= (1-\beta)\widehat{NX}_{n,t} + \beta\hat{V}_{n,t}^{US} - \beta\hat{i}_t^{US} \\
\beta\hat{V}_{n,t}^{US} - \hat{V}_{n,t-1}^{US} &= (1-\beta)\hat{\mathcal{E}}_{n,t} - (1-\beta)\widehat{NX}_{n,t} + \beta\hat{i}_t^{US}
\end{aligned}$$

Our five-equation representation will stack these equations of motion for net debt for each country along with the market clearing condition for US bonds. To that end let us define net exports as follows:

$$\begin{aligned}
\underbrace{\overline{\boldsymbol{NX}}_n}_{N\times N}\underbrace{\hat{\boldsymbol{NX}}_{n,t}}_{N\times 1} &= \underbrace{\overline{\boldsymbol{Y}}^{NNJ}}_{N\times NJ}(\hat{\boldsymbol{P}}^P + \hat{\boldsymbol{Y}}_{ni,t}) - \underbrace{\overline{\boldsymbol{C}}}_{N\times N}(\hat{\boldsymbol{P}}_t^{C,\tau} + \hat{\boldsymbol{C}}_t) - \underbrace{\overline{\boldsymbol{X}}^{ni}}_{N\times NJ}(\hat{\boldsymbol{P}}_{ni,t}^{X,\tau} + \hat{\boldsymbol{X}}_{ni,t}) \\
\overline{\boldsymbol{NX}}_n\hat{\boldsymbol{NX}}_{n,t} &= \overline{\boldsymbol{Y}}^{NNJ}\left[(\hat{\boldsymbol{P}}_t^P + \hat{\boldsymbol{Y}}_{ni,t}) - \boldsymbol{\alpha}(\hat{\boldsymbol{P}}_t^{C,\tau} + \hat{\boldsymbol{C}}_t) - \boldsymbol{\Omega}(\hat{\boldsymbol{P}}_{ni,t}^{X,\tau} + \hat{\boldsymbol{X}}_{ni,t})\right] \qquad \text{(B.1)}
\end{aligned}$$

where the overline matrices contain appropriately mapped steady-state values. A key thing to note is that the price vectors here are ex-tariff indices as noted by the superscript. The second line follows from the fact that we can assume α share of output goes to labor income at the steady state, which is then spent on consumption. That is we have:

$$\underbrace{\overline{\boldsymbol{C}}}_{N\times N} = \underbrace{\overline{\boldsymbol{Y}}^{NNJ}}_{N\times NJ}\underbrace{\boldsymbol{\alpha}}_{NJ\times N}$$

B.1 Market-Clearing Condition

Consider the following scalar market-clearing condition for a generic good before linearization:

$$Y_{ni,t} = \sum_{m\in N} C_{m,ni,t} + \sum_{m\in N}\sum_{j\in J} X_{mj,ni,t}$$

This yields the following when linearized:

$$\overline{Y}_{ni}(\hat{Y}_{ni,t}) - \sum_{m\in N}\overline{C}_m\Gamma_{m,ni,t}(\hat{C}_{m,ni,t}) - \sum_{m\in N}\sum_{j\in J}\overline{Y}_{mj}\Omega_{mj,ni,t}(\hat{X}_{mj,ni,t}) = 0 \;\forall\; n\in N, i\in J$$

where we add the steady state values because Γ terms are reported as a share of the aggregate consumption basket at the steady state in our model and similarly Ω terms are as a share

of production.

In vector form we can write the market-clearing condition for goods as follows:

$$\underbrace{\overline{\boldsymbol{Y}}^{ni}}_{NJ\times NJ}\underbrace{\hat{\boldsymbol{Y}}_{ni,t}}_{NJ\times 1}=\underbrace{\overline{\boldsymbol{C}}_n}_{NJ\times NJ}\underbrace{\boldsymbol{\Gamma}^M}_{NJ\times NNJ}\underbrace{\hat{\boldsymbol{C}}_t^{nmj}}_{NNJ\times 1}+\overline{\boldsymbol{Y}}^{ni}\underbrace{\boldsymbol{\Omega}^M}_{NJ\times NJNJ}\underbrace{\hat{\boldsymbol{X}}_t^{nimj}}_{NJNJ\times 1}$$

where overline notation indicates appropriately scaled diagonal matrices thatContain steady-state values.

Aggregate consumption and the intermediate input bundles are CES bundles; as such they have appropriately defined price indices and relative demand conditions. However, in our context there is no such object for output. In essence relating granular quantities, $\hat{\boldsymbol{C}}_t^{nmj}$ and $\hat{\boldsymbol{X}}_t^{nimj}$ respectively to aggregates $\hat{\boldsymbol{C}}_t^{n}$ and $\hat{\boldsymbol{X}}_t^{ni}$ via CES structure and the relative demand conditions we want to be able to substitute $\hat{\boldsymbol{Y}}_{ni,t}$ in the BoP equation, using only prices, exchange rate, aggregate consumption and tariffs.

Note key identities:

$$\sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\,\hat{C}_{n,mj,t}=\hat{C}_{n,t},\qquad\sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\big(\hat{P}^P_{mj,t}+\hat{\mathcal{E}}_{n,m,t}\big)=\hat{P}^{C,\tau}_{n,t},$$

$$\sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\big(\hat{P}^P_{mj,t}+\hat{\mathcal{E}}_{n,m,t}+\hat{\tau}_{n,mj,t}\big)=\hat{P}^{C}_{n,t}.$$

$$\sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}\,\hat{X}_{ni,mj,t}=\frac{\overline{X}_{ni}}{\overline{Y}_{ni}}\,\hat{X}_{ni,t},\qquad\sum_{m\in N}\sum_{j\in J}\frac{\overline{X}_{ni,mj}}{\overline{X}_{ni}}\big(\hat{P}^P_{mj,t}+\hat{\mathcal{E}}_{n,m,t}\big)=\hat{P}^{X,\tau}_{ni,t},$$

$$\sum_{m\in N}\sum_{j\in J}\frac{\overline{X}_{ni,mj}}{\overline{X}_{ni}}\big(\hat{P}^P_{mj,t}+\hat{\mathcal{E}}_{n,m,t}+\hat{\tau}_{n,mj,t}\big)=\hat{P}^{X}_{ni,t}.$$

$$\Omega_{ni,mj}=\frac{\overline{X}_{ni,mj}}{\overline{Y}_{ni}},\qquad\overline{X}_{ni}=(1-\alpha_{ni})\,\overline{Y}_{ni}.$$

By the relative demand conditions we have:

$$\begin{aligned}\hat{C}_{n,mi,t}&=\hat{C}_{n,t}+\theta^C(\hat{P}^C_{n,t}-(\hat{P}^P_{mi,t}+\hat{\mathcal{E}}_{n,m,t}+\hat{\tau}_{n,mi,t}))\\ \hat{X}_{mj,ni,t}&=\hat{X}_{mj,t}+\theta^P(\hat{P}^X_{mj,t}-(\hat{P}^P_{ni,t}+\hat{\mathcal{E}}_{m,n,t}+\hat{\tau}_{m,ni,t}))\end{aligned}$$

Vectorizing:

$$\hat{\boldsymbol{C}}_t^{nmj} = \underbrace{\boldsymbol{S}_1}_{NNJ\times N}\hat{\boldsymbol{C}}_{n,t} + \theta^C(\boldsymbol{S}_1\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{nmi})$$
$$\hat{\boldsymbol{X}}_t^{nimj} = \underbrace{\boldsymbol{S}_2}_{NJNJ\times NJ}\hat{\boldsymbol{X}}_{ni,t} + \theta^P(\boldsymbol{S}_2\hat{\boldsymbol{P}}_{ni,t}^X - \hat{\boldsymbol{P}}_{ni,mj,t}^X)$$

Plugging these into the market-clearing condition:

$$\begin{aligned}\overline{\boldsymbol{Y}}^{ni}\hat{\boldsymbol{Y}}_{ni,t} = \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\left(\boldsymbol{S}_1\hat{\boldsymbol{C}}_{n,t} + \theta^C\left(\boldsymbol{S}_1\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{nmi}\right)\right)\\ + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\left(\boldsymbol{S}_2\hat{\boldsymbol{X}}_{ni,t} + \theta^P\left(\boldsymbol{S}_2\hat{\boldsymbol{P}}_{ni,t}^X - \hat{\boldsymbol{P}}_{ni,mj,t}^X\right)\right)\end{aligned}$$

Defining these granular price variables:

$$\underbrace{\hat{\boldsymbol{P}}_t^{nmi}}_{NNJ\times 1} = \underbrace{\boldsymbol{S}_3}_{NNJ\times NJ}\underbrace{\hat{\boldsymbol{P}}_t^P}_{NJ\times 1} + \underbrace{\boldsymbol{S}_4}_{NNJ\times 1}\hat{\mathcal{E}}_t + \underbrace{\boldsymbol{S}_5}_{NNJ\times 1}\hat{\tau}_t$$
$$\underbrace{\hat{\boldsymbol{P}}_{ni,mj,t}^X}_{NJNJ\times 1} = \underbrace{\boldsymbol{S}_6}_{NJNJ\times NNJ}\underbrace{\hat{\boldsymbol{P}}_t^{nmi}}_{NNJ\times 1}$$

where $\boldsymbol{S}$s are just selector matrices that map these variables to appropriate scale by "selecting" the right value from the vector on the right.

Inserting granular price variables into the last expression:

$$\begin{aligned}\overline{\boldsymbol{Y}}^{ni}\hat{\boldsymbol{Y}}_{ni,t} &= \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\left(\boldsymbol{S}_1\hat{\boldsymbol{C}}_{n,t} + \theta^C\left(\boldsymbol{S}_1\hat{\boldsymbol{P}}_t^C - \left(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t\right)\right)\right)\\ &\quad + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\left(\boldsymbol{S}_2\hat{\boldsymbol{X}}_{ni,t} + \theta^P\left(\boldsymbol{S}_2\hat{\boldsymbol{P}}_{ni,t}^X - \boldsymbol{S}_6\left(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t\right)\right)\right)\\ &= \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_1\hat{\boldsymbol{C}}_{n,t} + \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_1\hat{\boldsymbol{P}}_t^C - \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P - \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_4\hat{\mathcal{E}}_t - \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_5\hat{\tau}_t\\ &\quad + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_2\hat{\boldsymbol{X}}_{ni,t} + \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_2\hat{\boldsymbol{P}}_{ni,t}^X - \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P\\ &\quad - \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_4\hat{\mathcal{E}}_t - \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_5\hat{\tau}_t \qquad \text{(B.2)}\end{aligned}$$

Note that the $\boldsymbol{\Gamma}$ matrix contains steady-state values divided by aggregate consumption such that:

$$\underbrace{\hat{\boldsymbol{C}}_{n,t}}_{N\times 1} = \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ}\underbrace{\hat{\boldsymbol{C}}_t^{nmj}}_{NNJ\times 1}$$

$$\hat{\boldsymbol{P}}_t^C = \underbrace{\boldsymbol{\Gamma}}_{N\times NJ}\underbrace{\hat{\boldsymbol{P}}_t^P}_{NJ\times 1} + \boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^C\hat{\tau}_t$$

$$\hat{\boldsymbol{P}}_t^C = \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ}\underbrace{\hat{\boldsymbol{P}}_t^{nmi}}_{NNJ\times 1} = \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ}\left(\underbrace{\boldsymbol{S}_3}_{NNJ\times NJ}\underbrace{\hat{\boldsymbol{P}}_t^P}_{NJ\times 1} + \underbrace{\boldsymbol{S}_4}_{NNJ\times 1}\hat{\mathcal{E}}_t + \underbrace{\boldsymbol{S}_5}_{NNJ\times 1}\hat{\tau}_t\right)$$

That is we can define our loading notation in the main 5 equation representation:

$$\underbrace{\boldsymbol{\Gamma}}_{N\times NJ} = \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ}\underbrace{\boldsymbol{S}_3}_{NNJ\times NJ}$$
$$\boldsymbol{L}_{\mathcal{E}}^C = \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ}\underbrace{\boldsymbol{S}_4}_{NNJ\times 1}$$
$$\boldsymbol{L}_{\tau}^C = \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ}\underbrace{\boldsymbol{S}_5}_{NNJ\times 1}$$

Remark 5.

$$\boxed{\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_1 = \boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}} \qquad \text{and} \qquad \boxed{\boldsymbol{\Omega}^M\boldsymbol{S}_2 = \boldsymbol{\Omega}^\top}$$

where $\overline{\boldsymbol{C}}$ is $N\times 1$ and $\overline{\boldsymbol{C}}_n$ is that matrix mapped to the $NJ\times 1$ context.

Continuing from (B.2) and using Remark 5

$$\begin{aligned}\overline{\boldsymbol{Y}}^{ni}\hat{\boldsymbol{Y}}_{ni,t} = {}& \boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_{n,t} + \theta^C\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{P}}_t^C - \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P - \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_4\hat{\mathcal{E}}_t - \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_5\hat{\tau}_t \\ & + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^\top\hat{\boldsymbol{X}}_{ni,t} + \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^\top\hat{\boldsymbol{P}}_{ni,t}^X - \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P - \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_4\hat{\mathcal{E}}_t - \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_5\hat{\tau}_t\end{aligned}$$

Now grouping variables:

$$\begin{aligned}\overline{\boldsymbol{Y}}^{ni}\hat{\boldsymbol{Y}}_{ni,t} = {}& \boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_{n,t} + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^\top\hat{\boldsymbol{X}}_{ni,t} \\ & + \theta^C\left(\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{P}}_t^C - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_4\hat{\mathcal{E}}_t - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_5\hat{\tau}_t\right) \\ & + \theta^P\left(\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^\top\hat{\boldsymbol{P}}_{ni,t}^X - \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P - \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_4\hat{\mathcal{E}}_t - \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_5\hat{\tau}_t\right) \\ = {}& \boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_{n,t} + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^\top\hat{\boldsymbol{X}}_{ni,t} \\ & + \theta^C\left(\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{P}}_t^C - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\left(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t\right)\right) \\ & + \theta^P\left(\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^\top\hat{\boldsymbol{P}}_{ni,t}^X - \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\left(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t\right)\right)\end{aligned}$$

Multiplying by $\overline{\boldsymbol{Y}}^{ni-1}$ on the left:

$$\begin{aligned}\hat{\boldsymbol{Y}}_{ni,t} = &\left(\overline{\boldsymbol{Y}}^{ni}\right)^{-1}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_{n,t} + \boldsymbol{\Omega}^\top\hat{\boldsymbol{X}}_{ni,t} \\ &+ \theta^C\left(\overline{\boldsymbol{Y}}^{ni}\right)^{-1}\left(\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{P}}_t^C - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\left(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t\right)\right) \\ &+ \theta^P\left(\boldsymbol{\Omega}^\top\hat{\boldsymbol{P}}_{ni,t}^X - \boldsymbol{\Omega}^M\boldsymbol{S}_6\left(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t\right)\right)\end{aligned} \tag{B.3}$$

This expression is intuitive in that, for each entry, it compares the average price index relevant to a user—whether on the consumption or production side—with the bilateral price that user actually faces for a specific good. The resulting deviation induces an adjustment in quantities, governed by the corresponding demand elasticity.

Using the mapping identities

$$\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_3 = \boldsymbol{\Omega}^\top, \qquad \boldsymbol{L}_{\mathcal{E}}^X \equiv \boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_4, \qquad \boldsymbol{L}_{\tau}^X \equiv \boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_5,$$

the θ^P-block can be written compactly as

$$\theta^P\overline{\boldsymbol{Y}}^{ni}\Big(\boldsymbol{\Omega}^\top\hat{\boldsymbol{P}}_{ni,t}^X - \left(\boldsymbol{\Omega}^\top\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^X\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^X\hat{\tau}_t\right)\Big),$$

Then:

$$\begin{aligned}\overline{\boldsymbol{Y}}^{ni}\hat{\boldsymbol{Y}}_{ni,t} = &\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_t + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^\top\hat{\boldsymbol{X}}_{ni,t} \\ &+ \theta^C\left(\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{P}}_t^C - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_4\hat{\mathcal{E}}_t - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_5\hat{\tau}_t\right) \\ &+ \theta^P\overline{\boldsymbol{Y}}^{ni}\Big(\boldsymbol{\Omega}^\top\hat{\boldsymbol{P}}_{ni,t}^X - \left(\boldsymbol{\Omega}^\top\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^X\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^X\hat{\tau}_t\right)\Big)\end{aligned}$$

We can also write:

$$\begin{aligned}\hat{\boldsymbol{Y}}_{ni,t} = &\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_t + \boldsymbol{\Omega}^\top\hat{\boldsymbol{X}}_{ni,t} \\ &+ \theta^C\overline{\boldsymbol{Y}}^{ni-1}\left(\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{P}}_t^C - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t)\right) \\ &+ \theta^P\Big(\boldsymbol{\Omega}^\top\hat{\boldsymbol{P}}_{ni,t}^X - \left(\boldsymbol{\Omega}^\top\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^X\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^X\hat{\tau}_t\right)\Big)\end{aligned} \tag{B.4}$$

Substituting in $\hat{\boldsymbol{W}}_t = \hat{\boldsymbol{P}}_t^C + \sigma\hat{\boldsymbol{C}}_t$, $\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^C\hat{\tau}_t$, $\hat{\boldsymbol{P}}_t^{C,\tau} = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t$,$\hat{\boldsymbol{P}}_{ni,t}^X = \left(\boldsymbol{\Omega}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^P\hat{\tau}_t\right)$ and $\hat{\boldsymbol{P}}_{ni,t}^{X,\tau} = \left(\boldsymbol{\Omega}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P\hat{\mathcal{E}}_t\right)$:

$$\begin{aligned}\hat{\boldsymbol{Y}}_{ni,t} = &\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_t + \boldsymbol{\Omega}^{\top}\hat{\boldsymbol{X}}_{ni,t} \\ &+ \theta^{C}\overline{\boldsymbol{Y}}^{ni-1}\left(\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\left(\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^{P} + \boldsymbol{L}_{\mathcal{E}}^{C}\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^{C}\hat{\tau}_t\right) - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\left(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^{P} + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t\right)\right) \\ &+ \theta^{P}\Big(\boldsymbol{\Omega}^{\top}\left(\boldsymbol{\Omega}\hat{\boldsymbol{P}}_t^{P} + \boldsymbol{L}_{\mathcal{E}}^{P}\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^{P}\hat{\tau}_t\right) - \left(\boldsymbol{\Omega}^{\top}\hat{\boldsymbol{P}}_t^{P} + \boldsymbol{L}_{\mathcal{E}}^{X}\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^{X}\hat{\tau}_t\right)\Big)\end{aligned}$$

$$\begin{aligned}\hat{\boldsymbol{Y}}_{ni,t} = &\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_t + \boldsymbol{\Omega}^{\top}\hat{\boldsymbol{X}}_{ni,t} \\ &+ \theta^{C}\overline{\boldsymbol{Y}}^{ni-1}\Big(\left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{\Gamma} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_3\right]\hat{\boldsymbol{P}}_t^{P} + \left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{L}_{\mathcal{E}}^{C} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_4\right]\hat{\mathcal{E}}_t + \left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{L}_{\tau}^{C} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_5\right]\hat{\tau}_t\Big) \\ &+ \theta^{P}\Big(\left[\boldsymbol{\Omega}^{\top}\boldsymbol{\Omega} - \boldsymbol{\Omega}^{\top}\right]\hat{\boldsymbol{P}}_t^{P} + \left[\boldsymbol{\Omega}^{\top}\boldsymbol{L}_{\mathcal{E}}^{P} - \boldsymbol{L}_{\mathcal{E}}^{X}\right]\hat{\mathcal{E}}_t + \left[\boldsymbol{\Omega}^{\top}\boldsymbol{L}_{\tau}^{P} - \boldsymbol{L}_{\tau}^{X}\right]\hat{\tau}_t\Big) \qquad \text{(B.5)}\end{aligned}$$

B.2 Substituting out $\hat{X}_{ni,t}$

We have the following expressions from relative demand on the construction of the production bundle and production function:

$$\begin{aligned}\hat{X}_{ni,t} &= \hat{L}_{ni,t} + \theta(\hat{W}_t - \hat{P}_{ni,t}^{X}) \\ \hat{Y}_{ni,t} &= \alpha_{ni}\hat{L}_{ni,t} + (1-\alpha_{ni})\hat{X}_{ni,t}\end{aligned}$$

Solving for labor in the first equation and plugging it into the second:

$$\hat{X}_{ni,t} = \hat{Y}_{ni,t} + \theta^{P}\alpha_{ni}(\hat{W}_t - \hat{P}_{ni,t}^{X})$$

Vectorizing we have:

$$\hat{\boldsymbol{X}}_{ni,t} = \hat{\boldsymbol{Y}}_{ni,t} + \theta^{P}(\boldsymbol{\alpha}\hat{\boldsymbol{W}}_t - \tilde{\boldsymbol{\alpha}}\hat{\boldsymbol{P}}_{ni,t}^{X})$$

where the $\tilde{\boldsymbol{\alpha}}$ matrix, which is $NJ \times NJ$ is an appropriately scaled version of the $\boldsymbol{\alpha}$, which is $NJ \times N$. Putting this expression together with (B.4) and solving for $\hat{\boldsymbol{Y}}_{ni,t}$:

$$\begin{aligned}\hat{\boldsymbol{Y}}_{ni,t} = \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega}^{\top})^{-1}}_{\boldsymbol{\Psi}_T}\Bigg[&\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_t \\ &+ \theta^{C}\overline{\boldsymbol{Y}}^{ni-1}\Big(\left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{\Gamma} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_3\right]\hat{\boldsymbol{P}}_t^{P} + \left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{L}_{\mathcal{E}}^{C} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_4\right]\hat{\mathcal{E}}_t + \left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{L}_{\tau}^{C} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_5\right]\hat{\tau}_t\Big)\end{aligned}$$

$$+\theta^P\Big(\left[\boldsymbol{\Omega}^\top\boldsymbol{\Omega}-\boldsymbol{\Omega}^\top\right]\hat{\boldsymbol{P}}_t^P+\left[\boldsymbol{\Omega}^\top\boldsymbol{L}_{\mathcal{E}}^P-\boldsymbol{L}_{\mathcal{E}}^X\right]\hat{\mathcal{E}}_t+\left[\boldsymbol{\Omega}^\top\boldsymbol{L}_{\tau}^P-\boldsymbol{L}_{\tau}^X\right]\hat{\tau}_t+\boldsymbol{\Omega}^\top(\boldsymbol{\alpha}\hat{\boldsymbol{W}}_t-\tilde{\boldsymbol{\alpha}}\hat{\boldsymbol{P}}_{ni,t}^X)\Big)\Bigg]$$

We can further substitute in $\hat{\boldsymbol{W}}_t=\hat{\boldsymbol{P}}_t^C+\sigma\hat{\boldsymbol{C}}_t=\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P+\boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t+\boldsymbol{L}_{\tau}^C\hat{\tau}_t+\sigma\hat{\boldsymbol{C}}_t$, $\hat{\boldsymbol{P}}_t^C=\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P+\boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t+\boldsymbol{L}_{\tau}^C\hat{\tau}_t$, $\hat{\boldsymbol{P}}_t^{C,\tau}=\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P+\boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t$,$\hat{\boldsymbol{P}}_{ni,t}^X=\Big(\boldsymbol{\Omega}\hat{\boldsymbol{P}}_t^P+\boldsymbol{L}_{\mathcal{E}}^P\hat{\mathcal{E}}_t+\boldsymbol{L}_{\tau}^P\hat{\tau}_t\Big)$ and $\hat{\boldsymbol{P}}_{ni,t}^{X,\tau}=\Big(\boldsymbol{\Omega}\hat{\boldsymbol{P}}_t^P+\boldsymbol{L}_{\mathcal{E}}^P\hat{\mathcal{E}}_t\Big)$:

$$\begin{aligned}
\hat{\boldsymbol{Y}}_{ni,t}=&\underbrace{(\boldsymbol{I}-\boldsymbol{\Omega}^\top)^{-1}}_{\boldsymbol{\Psi}_T}\Bigg[(\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}+\theta^P\boldsymbol{\Omega}^\top\boldsymbol{\alpha}\sigma)\hat{\boldsymbol{C}}_t\\
&+\theta^C\overline{\boldsymbol{Y}}^{ni-1}\Big(\underbrace{\left[\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\boldsymbol{\Gamma}-\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_3\right]}_{\boldsymbol{T}_P^C}\hat{\boldsymbol{P}}_t^P+\underbrace{\left[\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\boldsymbol{L}_{\mathcal{E}}^C-\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_4\right]}_{\boldsymbol{T}_{\hat{\mathcal{E}}}^C}\hat{\mathcal{E}}_t+\underbrace{\left[\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\boldsymbol{L}_{\tau}^C-\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_5\right]}_{\boldsymbol{T}_{\tau}^C}\hat{\tau}_t\Big)\\
&+\theta^P\boldsymbol{\Omega}^\top\Big(\underbrace{\left[\boldsymbol{I}-(\boldsymbol{\Omega}^\top)^{-1}\boldsymbol{\Omega}+\boldsymbol{\alpha}\boldsymbol{\Gamma}-\tilde{\boldsymbol{\alpha}}\boldsymbol{\Omega}\right]}_{\boldsymbol{T}_P^P}\hat{\boldsymbol{P}}_t^P\\
&\quad+\underbrace{\left[\boldsymbol{L}_{\mathcal{E}}^P-(\boldsymbol{\Omega}^\top)^{-1}\boldsymbol{L}_{\mathcal{E}}^X+\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C-\tilde{\boldsymbol{\alpha}}\boldsymbol{L}_{\mathcal{E}}^P\right]}_{\boldsymbol{T}_{\hat{\mathcal{E}}}^P}\hat{\mathcal{E}}_t\\
&\quad+\underbrace{\left[\boldsymbol{L}_{\tau}^P-(\boldsymbol{\Omega}^\top)^{-1}\boldsymbol{L}_{\tau}^X+\boldsymbol{\alpha}\boldsymbol{L}_{\tau}^C-\tilde{\boldsymbol{\alpha}}\boldsymbol{L}_{\tau}^P\right]}_{\boldsymbol{T}_{\tau}^P}\hat{\tau}_t\Big)\Big)\Bigg]\\
=&\boldsymbol{\Psi}_T\Bigg[(\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}+\theta^P\boldsymbol{\Omega}^\top\boldsymbol{\alpha}\sigma)\hat{\boldsymbol{C}}_t\\
&+\theta^C\overline{\boldsymbol{Y}}^{ni-1}\Big(\boldsymbol{T}_P^C\hat{\boldsymbol{P}}_t^P+\boldsymbol{T}_{\hat{\mathcal{E}}}^C\hat{\mathcal{E}}_t+\boldsymbol{T}_{\tau}^C\hat{\tau}_t\Big)\\
&+\theta^P\boldsymbol{\Omega}^\top\Big(\boldsymbol{T}_P^P\hat{\boldsymbol{P}}_t^P+\boldsymbol{T}_{\hat{\mathcal{E}}}^P\hat{\mathcal{E}}_t+\boldsymbol{T}_{\tau}^P\hat{\tau}_t\Big)\Bigg]
\end{aligned}$$

Rearranging:

$$\begin{aligned}
\hat{\boldsymbol{Y}}_{ni,t}=\boldsymbol{\Psi}_T\Bigg[&\Big(\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}+\theta^P\boldsymbol{\Omega}^\top\boldsymbol{\alpha}\sigma\Big)\hat{\boldsymbol{C}}_t\\
&+\Big(\theta^C\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{T}_P^C+\theta^P\boldsymbol{\Omega}^\top\boldsymbol{T}_P^P\Big)\hat{\boldsymbol{P}}_t^P\\
&+\Big(\theta^C\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{T}_{\mathcal{E}}^C+\theta^P\boldsymbol{\Omega}^\top\boldsymbol{T}_{\mathcal{E}}^P\Big)\hat{\mathcal{E}}_t\\
&+\Big(\theta^C\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{T}_{\tau}^C+\theta^P\boldsymbol{\Omega}^\top\boldsymbol{T}_{\tau}^P\Big)\hat{\tau}_t\Bigg]
\end{aligned}\tag{B.6}$$

This expression tells us, in terms of market clearing that, output gets shifted by aggregate demand (directly and indirectly via the impact of wages and thereby the substitution between goods and intermediate inputs) and by goods-specific terms of trade captured by the $\boldsymbol{T}$ matrices.

Returning to the net exports equation:

$$\overline{\boldsymbol{NX}}_n \hat{\boldsymbol{NX}}_{n,t} = \overline{\boldsymbol{Y}}^{NNJ} \left[(\hat{\boldsymbol{P}}_t^P + \hat{\boldsymbol{Y}}_{ni,t}) - \boldsymbol{\alpha}(\hat{\boldsymbol{P}}_t^{C,\tau} + \hat{\boldsymbol{C}}_t) - \boldsymbol{\Omega}(\hat{\boldsymbol{P}}_{ni,t}^{X,\tau} + \hat{\boldsymbol{X}}_{ni,t}) \right]$$

Substituting out $\hat{\boldsymbol{X}}_{ni,t}$:

$$\overline{\boldsymbol{NX}}_n \hat{\boldsymbol{NX}}_{n,t} = \overline{\boldsymbol{Y}}^{NNJ} \left[(\boldsymbol{I} - \boldsymbol{\Omega})\hat{\boldsymbol{Y}}_{ni,t} + \hat{\boldsymbol{P}}_t^P - \boldsymbol{\alpha}(\hat{\boldsymbol{P}}_t^{C,\tau} + \hat{\boldsymbol{C}}_t) - \boldsymbol{\Omega}(\hat{\boldsymbol{P}}_{ni,t}^{X,\tau} + \theta^P(\boldsymbol{\alpha}\hat{\boldsymbol{W}}_t - \tilde{\boldsymbol{\alpha}}\hat{\boldsymbol{P}}_{ni,t}^X)) \right]$$

We can, once again, substitute in $\hat{\boldsymbol{W}}_t = \hat{\boldsymbol{P}}_t^C + \sigma\hat{\boldsymbol{C}}_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^C\hat{\tau}_t + \sigma\hat{\boldsymbol{C}}_t$, $\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^C\hat{\tau}_t$, $\hat{\boldsymbol{P}}_t^{C,\tau} = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t, \hat{\boldsymbol{P}}_{ni,t}^X = \left(\boldsymbol{\Omega}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^P\hat{\tau}_t\right)$ and $\hat{\boldsymbol{P}}_{ni,t}^{X,\tau} = \left(\boldsymbol{\Omega}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P\hat{\mathcal{E}}_t\right)$:

$$\begin{aligned}
\overline{\boldsymbol{NX}}_n \hat{\boldsymbol{NX}}_{n,t} = \overline{\boldsymbol{Y}}^{NNJ} \Big[& (\boldsymbol{I} - \boldsymbol{\Omega})\hat{\boldsymbol{Y}}_{ni,t} + \hat{\boldsymbol{P}}_t^P - \boldsymbol{\alpha}\left(\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t + \hat{\boldsymbol{C}}_t\right) \\
& - \boldsymbol{\Omega}\Big(\boldsymbol{\Omega}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P\hat{\mathcal{E}}_t + \theta^P\left[\boldsymbol{\alpha}\left(\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^C\hat{\tau}_t + \sigma\hat{\boldsymbol{C}}_t\right) - \tilde{\boldsymbol{\alpha}}\left(\boldsymbol{\Omega}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^P\hat{\tau}_t\right)\right]\Big) \\
= \overline{\boldsymbol{Y}}^{NNJ} \Big[& (\boldsymbol{I} - \boldsymbol{\Omega})\hat{\boldsymbol{Y}}_{ni,t} \\
& + \left[\boldsymbol{I} - \boldsymbol{\alpha}\boldsymbol{\Gamma} - \boldsymbol{\Omega}^2 - \theta^P\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{\Gamma} - \tilde{\boldsymbol{\alpha}}\boldsymbol{\Omega}\right)\right]\hat{\boldsymbol{P}}_t^P \\
& - \left[\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C + \boldsymbol{\Omega}\boldsymbol{L}_{\mathcal{E}}^P + \theta^P\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}_{\mathcal{E}}^P\right)\right]\hat{\mathcal{E}}_t \\
& - \left[\theta^P\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{L}_{\tau}^C - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}_{\tau}^P\right)\right]\hat{\tau}_t \\
& - \left[\boldsymbol{I} + \theta^P\sigma\boldsymbol{\Omega}\right]\boldsymbol{\alpha}\hat{\boldsymbol{C}}_t \Big]
\end{aligned}$$

We can plug in $\hat{\boldsymbol{Y}}_{ni,t}$ from (B.6):

$$\overline{\boldsymbol{NX}}_n \hat{\boldsymbol{NX}}_{n,t} = \boldsymbol{\Xi}_P\,\hat{\boldsymbol{P}}_t^P + \boldsymbol{\Xi}_{\mathcal{E}}\,\hat{\mathcal{E}}_t + \boldsymbol{\Xi}_{\tau}\,\hat{\tau}_t + \boldsymbol{\Xi}_C\,\hat{\boldsymbol{C}}_t, \tag{B.7}$$

$$\boldsymbol{\Xi}_P = \overline{\boldsymbol{Y}}^{NNJ}\Bigg\{\underbrace{\boldsymbol{\Psi}_\Delta\Big(\theta^C\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{T}_P^C + \theta^P\boldsymbol{\Omega}^\top\boldsymbol{T}_P^P\Big) - \theta^P\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{\Gamma} - \tilde{\boldsymbol{\alpha}}\boldsymbol{\Omega}\right)}_{\text{Change in Quantities (ToT)}} + \underbrace{\Big[\boldsymbol{I} - \boldsymbol{\alpha}\boldsymbol{\Gamma} - \boldsymbol{\Omega}^2\Big]}_{\text{Change in Prices (Valuation)}}\Bigg\}, \tag{B.8}$$

$$\boldsymbol{\Xi}_{\mathcal{E}} = \overline{\boldsymbol{Y}}^{NNJ}\Bigg\{\underbrace{\boldsymbol{\Psi}_\Delta\Big(\theta^C\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{T}_{\mathcal{E}}^C + \theta^P\boldsymbol{\Omega}^\top\boldsymbol{T}_{\mathcal{E}}^P\Big) - \theta^P\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}_{\mathcal{E}}^P\right)}_{\text{Change in Quantities (ToT)}} - \underbrace{\Big[\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C + \boldsymbol{\Omega}\boldsymbol{L}_{\mathcal{E}}^P\Big]}_{\text{Change in Prices (Valuation)}}\Bigg\}, \tag{B.9}$$

$$\boldsymbol{\Xi}_\tau = \overline{\boldsymbol{Y}}^{NNJ}\Bigg\{\underbrace{\boldsymbol{\Psi}_\Delta\Big(\theta^C\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{T}_\tau^C + \theta^P\boldsymbol{\Omega}^\top\boldsymbol{T}_\tau^P\Big) - \Big[\theta^P\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{L}_\tau^C - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}_\tau^P\right)\Big]}_{\text{Change in Quantities (ToT)}}\Bigg\}, \tag{B.10}$$

$$\boldsymbol{\Xi}_C = \overline{\boldsymbol{Y}}^{NNJ}\Bigg\{\underbrace{\boldsymbol{\Psi}_\Delta\Big(\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}} + \theta^P\sigma\boldsymbol{\Omega}^\top\boldsymbol{\alpha}\Big) - \Big[\boldsymbol{I} + \theta^P\sigma\boldsymbol{\Omega}\Big]\boldsymbol{\alpha}}_{\text{Aggregate Demand Effect}}\Bigg\}. \tag{B.11}$$

where

$$\boldsymbol{\Psi}_\Delta = (\boldsymbol{I} - \boldsymbol{\Omega})(\boldsymbol{I} - \boldsymbol{\Omega}^\top)^{-1}.$$

Recall that the equation of motion for net debt for a given country is $\beta\hat{V}_{n,t}^{US} - \hat{V}_{n,t-1}^{US} = (1-\beta)\hat{\mathcal{E}}_{n,t} - (1-\beta)\widehat{NX}_{n,t} + \beta\hat{i}_t^{US}$. To avoid a redundant equation, one of the equations of motion for net debt is replaced with a market clearing condition for US bonds. Then stacking these expressions we arrive at the fifth equation capturing the balance of payments:

$$\beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}_t^P + \Xi_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{\Xi}_5\hat{\boldsymbol{\tau}}_t$$

where $\Xi_1 = 1$ in the case of the two-country model; aggregating this yields the fifth equation in the five-equation representation.

From the expression above and from intuition, we can see that a higher elasticity of substitution makes the balance of payments more reactive to changes in prices. More broadly, we see net exports react to the aggregate demand stance of countries and the terms of trade in each sector.

Stacking the final expression above for different countries n, alongside a market-clearing condition for U.S. bonds, yields the fifth equation in the five-equation Global New Keynesian Representation.

B.3 Scalar Example ($N = 2$,$J = 1$

Let us consider a scalar example ($N = 2$,$J = 1$), whereby we simplify away the home country's use of its own goods as intermediate inputs. With steady-state consumption normalized to 1, we can express steady-state values for variables like $C_{H,H,t}$ and $X_{F,H,t}$ in terms of home bias in consumption $(1-\gamma_H)$ and imported input dependence Ω_H, which is transformed into $\Psi_H = \frac{1}{1-\Omega_H}$.[33] Thus, when linearized, we have the following equations:

$$
\begin{aligned}
\hat{Y}_{H,t} &= (1-\Omega_H)(1-\gamma_H)\hat{C}_{H,H,t} + (1-\Omega_F)\gamma_F\hat{C}_{F,H,t} + \Omega_F\hat{X}_{F,H,t} \\
\hat{Y}_{F,t} &= (1-\Omega_F)(1-\gamma_F)\hat{C}_{F,F,t} + (1-\Omega_H)\gamma_H\hat{C}_{H,F,t} + \Omega_H\hat{X}_{H,F,t} \\
\hat{X}_{H,F,t} &= \theta^P(1-\Omega_H)(\hat{P}^C_{H,t} + \sigma\hat{C}_{H,t} - (\hat{P}^P_{F,t} + \hat{\tau}^H + \hat{\mathcal{E}}_t)) + \hat{Y}_{H,t} \\
\hat{X}_{F,H,t} &= \theta^P(1-\Omega_F)(\hat{P}^C_{F,t} + \sigma\hat{C}_{F,t} - (\hat{P}^P_{H,t} + \hat{\tau}^F - \hat{\mathcal{E}}_t)) + \hat{Y}_{F,t} \\
\hat{C}_{H,H,t} &= -\theta^C\left(\hat{P}^P_{H,t} - \hat{P}^C_{H,t}\right) + \hat{C}_{H,t} \\
\hat{C}_{H,F,t} &= -\theta^C\left(\hat{P}^P_{F,t} + \hat{\tau}^H + \hat{\mathcal{E}}_t - \hat{P}^C_{H,t}\right) + \hat{C}_{H,t} \\
\hat{C}_{F,F,t} &= -\theta^C\left(\hat{P}^P_{F,t} - \hat{P}^C_{F,t}\right) + \hat{C}_{F,t} \\
\hat{C}_{F,H,t} &= -\theta^C\left(\hat{P}^P_{H,t} + \hat{\tau}^F - \hat{\mathcal{E}}_t - \hat{P}^C_{F,t}\right) + \hat{C}_{F,t} \\
\hat{P}^C_{H,t} &= (1-\gamma_H)\hat{P}^P_{H,t} + \gamma_H(\hat{P}^P_{F,t} + \mathcal{E}_t + \hat{\tau}^H_t) \\
\hat{P}^C_{F,t} &= (1-\gamma_F)\hat{P}^P_{F,t} + \gamma_F(\hat{P}^P_{H,t} - \mathcal{E}_t + \hat{\tau}^F_t) \\
\widehat{\overline{NX}\,NX}_t &= \left[1-(1-\Omega_H)(1-\theta^P)\right]\hat{P}^P_{H,t} + \left[-1+(1-\Omega_F)(1-\theta^P)\right]\hat{P}^P_{F,t} \\
&\quad + (\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (1-\Omega_F)(\hat{C}_{F,t} + \theta^P\hat{P}^C_{F,t}) - (1-\Omega_H)(\hat{C}_{H,t} + \theta^P\hat{P}^C_{H,t}) - \Omega_F\hat{\mathcal{E}}_t
\end{aligned}
$$

These equations can express net exports as a share of prices, which can then be plugged into the following balance of payments equation:

$$\hat{V}_t = \beta^{-1}\hat{V}_{t-1} - \frac{(1-\beta)}{\beta}\widehat{NX}_t + \hat{i}_t$$

The rest of the model, as covered by the other four equations of the Global NK representation would be as follows:

$$
\begin{aligned}
\sigma(E_t\hat{C}_{H,t+1} - E_t\hat{C}_{H,t}) &= \hat{i}_t - E_t\pi^C_{H,t+1} \\
\sigma(E_t\hat{C}_{F,t+1} - E_t\hat{C}_{F,t}) &= \hat{i}_t - E_t\pi^C_{F,t+1}
\end{aligned}
$$

[33] Note $\overline{NX} = (1-\beta)\overline{V} = -(1-\Omega_H)(1-\gamma_H) + (1-\Omega_F)(1-\gamma_F)$.

$$\pi^P_{H,t} = \Lambda_H \left(\alpha_H(\hat{P}^C_{H,t} + \sigma C_{H,t}) + \Omega_H \left(\hat{P}^P_{F,t} + \hat{\mathcal{E}}_t + \hat{\tau}^H_t\right) - \hat{P}^P_{H,t}\right) + \beta \mathbb{E}_t \pi^P_{H,t+1}$$
$$\pi^P_{F,t} = \Lambda_F \left(\alpha_F(\hat{P}^C_{F,t} + \sigma C_{F,t}) + \Omega_F \left(\hat{P}^P_{H,t} + \hat{\tau}^F_t - \hat{\mathcal{E}}_t\right) - \hat{P}^P_{F,t}\right) + \beta \mathbb{E}_t \pi^P_{F,t+1}$$
$$\pi^C_{H,t} = \hat{P}^C_{H,t} - \hat{P}^C_{H,t-1}$$
$$\pi^C_{F,t} = \hat{P}^C_{F,t} - \hat{P}^C_{F,t-1}$$
$$\pi^P_{H,t} = \hat{P}^P_{H,t} - \hat{P}^P_{H,t-1}$$
$$\pi^C_{P,t} = \hat{P}^P_{F,t} - \hat{P}^P_{F,t-1}$$
$$\hat{i}_{H,t} - \hat{i}_{F,t} = E_t \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t$$
$$\hat{i}_{H,t} = \phi^H_\pi \pi^P_{H,t}$$
$$\hat{i}_{F,t} = \phi^F_\pi \pi^F_{H,t}$$

C Decomposing the Impact on Inflation

Starting with the second equation of Proposition 2 and combining with the CPI definition, we can write:

$$\frac{\partial \boldsymbol{\pi}^P_t}{\partial \hat{\tau}_t} = \boldsymbol{p}_\tau = \left[\tilde{\boldsymbol{\Psi}}^{-1}_\Lambda - \beta\big(\rho\, \boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\right]^{-1} \Big(\boldsymbol{\Lambda}\big(\boldsymbol{\alpha}\sigma \boldsymbol{c}_\tau + (\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}})e_\tau + (\boldsymbol{\alpha}\boldsymbol{L}^C_\tau + \boldsymbol{L}^P_\tau)\big) + \beta\, \boldsymbol{p}_v v_\tau\Big)$$

$$\rightarrow \frac{\partial \boldsymbol{\pi}^C_t}{\partial \hat{\tau}_t} = \boldsymbol{\Gamma} \underbrace{\left[\tilde{\boldsymbol{\Psi}}^{-1}_\Lambda - \beta\big(\rho\, \boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\right]^{-1} \boldsymbol{\Lambda}}_{\boldsymbol{H}_1} \underbrace{\big(\boldsymbol{\alpha}\sigma \boldsymbol{c}_\tau + (\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}})e_\tau + (\boldsymbol{\alpha}\boldsymbol{L}^C_\tau + \boldsymbol{L}^P_\tau) + \beta \boldsymbol{\Lambda}^{-1} \boldsymbol{p}_v v_\tau\big)}_{\boldsymbol{H}_2} + \boldsymbol{L}^C_{\mathcal{E}} e_\tau + \boldsymbol{L}^C_\tau$$

$$\frac{\partial \boldsymbol{\pi}^C_t}{\partial \hat{\tau}_t} = \boldsymbol{\Gamma}\Big(\boldsymbol{H}_2 + (\boldsymbol{H}_1 - \boldsymbol{I})\boldsymbol{H}_2\Big) + \boldsymbol{L}^C_{\mathcal{E}} e_\tau + \boldsymbol{L}^C_\tau$$

This is the desired decomposition:

$$\frac{\partial \boldsymbol{\pi}^C_t}{\partial \hat{\tau}_t} = \underbrace{\boldsymbol{L}^C_\tau}_{\text{Direct CPI effect}} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{L}^P_\tau}_{\text{Direct PPI effect}} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{\alpha}\boldsymbol{L}^C_\tau}_{\text{Direct Effect on Real Wages}} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{\alpha}\sigma \boldsymbol{c}_\tau}_{\text{Demand channel}}$$
$$+ \underbrace{(\boldsymbol{\Gamma}(\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}}) + \boldsymbol{L}^C_{\mathcal{E}})e_\tau}_{\text{ER channel}} + \underbrace{\beta\boldsymbol{\Gamma}\boldsymbol{\Lambda}^{-1}\boldsymbol{p}_v v_\tau}_{\text{Debt channel}} + \underbrace{\boldsymbol{\Gamma}(\left[\tilde{\boldsymbol{\Psi}}^{-1}_\Lambda - \beta\big(\rho\, \boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\right]^{-1}\boldsymbol{\Lambda} - \boldsymbol{I})\boldsymbol{H}_2}_{\text{Propagation with Stickiness}}$$

D Additional Results

Table D.1. U.S. Tariff events from the WTO-IMF Tariff Tracker.

Event Date	Average Tariff (%)	Event Label	Event Description
1/1/2025	2.3	Pre-Trump	The baseline tariff rates for U.S. imports from China have been updated to reflect actual tariff rates applied per tariff line, based on data from the U.S. Census for 2024. These were then compared with the Most Favored Nation (MFN) tariff rates for 2024 to identify pre-existing tariff hikes before the start of 2025. The resulting tariff rates were rounded to the nearest 0.5%. For other exporters, the baseline tariff rates are a combination of MFN and preferential rates for 2024.
2/4/2025	3.6	China +10	On February 4, 2025, the United States imposed an additional 10% tariff on all imports from China.
3/4/2025	11.3	China +10	On March 3, 2025, the United States further increased tariffs from 10% to 20% on all imports from China.
3/4/2025	11.3	Can/Mex +25	On March 4, 2025, the United States implemented an additional 25% tariff on imports from Canada and Mexico. Energy resources from Canada will have a lower 10% tariff.
3/7/2025	8.7	USMCA Exemptions	Effective on 7 March 2025 the United States announced an exemption for all imports complying with the United States-Mexico-Canada Agreement (USMCA). Compliance rate has been estimated using 2023 imports notification submitted by the U.S. to WTO's IDB. Additionally, tariffs on potash imports have been reduced from 25% to 10%.
3/12/2025	9.7	Steel & Alum. Tariffs +25	On March 12, 2025, the United States imposed additional duties on steel and aluminum imports. Specifically, a 25% tariff was applied to steel and aluminum imports, with the exception of Russian Federation, which faced a 200% tariff on aluminum.
4/3/2025	10.7	U.S. tariffs on Vehicles	Effective April 3, 2025, the United States imposed new tariffs on vehicle imports. Additional 25% tariff was applied to vehicles from all countries.
4/5/2025	13.4	Baseline 10% reciprocal tariffs	On April 05, 2025, the United States imposed a baseline additional 10% tariff on imports (there are exemptions) from all countries, except for Canada, Mexico, and countries subject to rates set forth in Column 2 of the HTSUS (Russian Federation, Cuba and Belarus, which is a WTO Observer).
4/9/2025	22.6	Liberation Day tariffs implemented	On April 9, 2025, the United States imposed additional tariffs of 34% on imports from China. On April 9, 2025, the United States increased the additional tariffs from 34% to 84% on imports from China. On April 10, 2025, the United States increased the additional tariffs from 84% to 125% on imports from China. The increased tariffs on imports from the other 55 countries with implementation date on April 9, 2025, were suspended effective April 10, 2025, for 90-days until July 9, 2025.
5/3/2025	23.3	Tariffs on Vehicle parts	Effective May 3, 2025, new tariffs were imposed on vehicle part imports. A 25% tariff was applied to vehicles' parts from all countries.
5/14/2025	14.9	U.S.-China trade deal	U.S. and China agreed to a trade deal that reduces 125% tariffs to 10%.
6/4/2025	16.5	Steel & Alum. Tariffs +25	U.S. doubled tariffs on foreign steel and aluminum imports to 50%. This applies to all trading partners except the UK.
6/23/2025	14.54	US-UK trade deal	The United States and the United Kingdom have reached a trade agreement that imposes a 10% import tariff on vehicle parts from the UK. For other countries, US imposed additional tariffs on specific consumer and household items made with steel, depending on how much steel they contain.
6/30/2025	14.48	US–UK vehicles adjustment	The United States and the United Kingdom have reached a trade agreement that establishes a 10% US import tariff on vehicles from the UK, with a quota limit of 100,000 vehicles.
8/1/2025	14.52	Copper Tariffs +50%	The United States implemented an extra 50% tariff on semi-finished copper products and related items under Section 232, citing reasons related to national security. US also imposed additional tariffs to Canada.

8/6/2025	14.70	Additional tariffs on Brazil	The United States introduced an extra 40% tariff on the majority of imports from Brazil, to take effect seven days later, while allowing exemptions and special provisions for certain products.
8/7/2025	15.88	Additional +10% on April 2 tariffs	The United States issued a new Executive Order that changes the "liberation day" tariffs for specific trading partners, and imposes an extra 10% tariff on partners not included on the list. US also implements agreements with South Korea and Japan.
8/27/2025	16.26	Tariff increase on India	The United States increased tariffs on exports from India to 50%, doubling the earlier rate of 25% and affecting a wide variety of products.
9/1/2025	16.27	Economic agreement with the EU	A new tariff schedule for the US-EU agreement is being applied retroactively from August 1, 2025.
9/8/2025	16.15	Modifications on Liberation Day Tariffs	The United States updated the liberation day tariffs.
9/16/2025	16.04	Exempt Aircrafts from Japan and UK	The United States granted an exemption from tariffs for the "Aircraft" product category on imports coming from the UK and Japan.
10/1/2025	16.05	AGOA expiry	The African Growth and Opportunity Act (AGOA) has officially ended, concluding 25 years during which goods imported into the U.S. from sub-Saharan African countries received tariff-free treatment.

NOTE: The tariff events are described by WTO - IMF Tariff Tracker (WTO and IMF, 2025). Note that this table only includes the actual implemented tariffs but do not include the tariffs to be implemented until October 20, 2025.

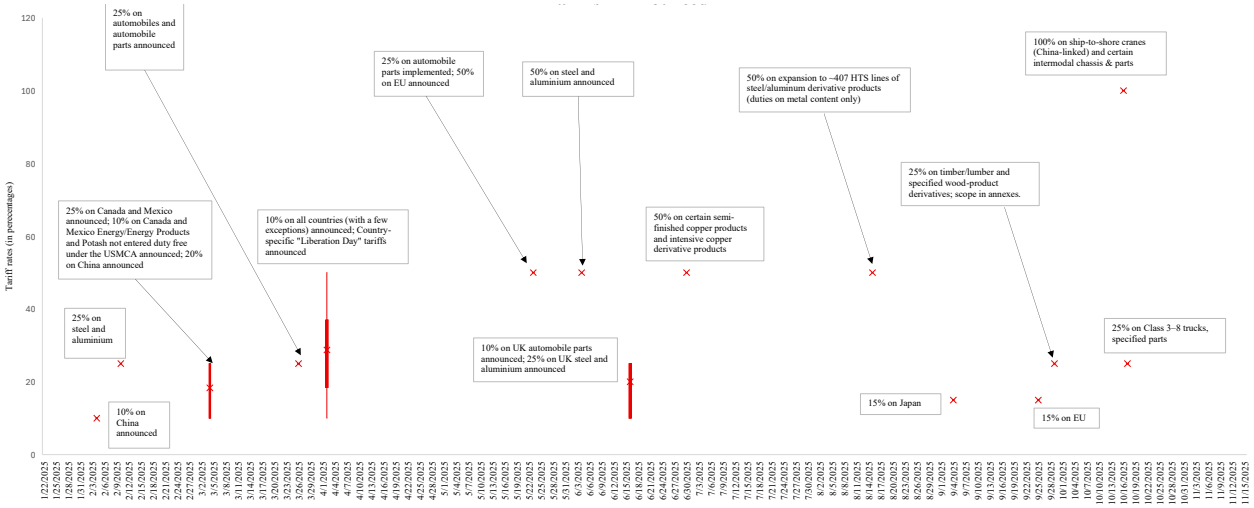
Table D.2. Sectoral Shares and Tariffs for the U.S.

Country	Industry	World Share (%)	U.S. Share (%)	U.S. Import Share (%)	U.S. Final Share (%)	U.S. Int Share (%)	U.S. Curr. Tariff (%)	U.S. Max. Tariff (%)	Ret. Curr. Tariff (%)	Ret. Max. Tariff (%)
USA	Agriculture	7.0	89.1	0.0	88.5	89.3	0.0	0.0	0.0	0.0
USA	Energy	15.0	79.0	0.0	89.4	75.0	0.0	0.0	0.0	0.0
USA	Mining	11.1	94.8	0.0	98.5	89.9	0.0	0.0	0.0	0.0
USA	Food & Bev.	13.1	91.3	0.0	91.2	91.7	0.0	0.0	0.0	0.0
USA	Basic Man.	11.2	77.4	0.0	66.0	82.5	0.0	0.0	0.0	0.0
USA	Adv. Man.	13.1	67.0	0.0	67.0	66.9	0.0	0.0	0.0	0.0
USA	Resid. Serv.	13.1	99.7	0.0	99.9	99.5	0.0	0.0	0.0	0.0
USA	Services	29.1	96.5	0.0	96.7	96.2	0.0	0.0	0.0	0.0
EUU	Agriculture	13.3	2.1	19.6	2.3	2.1	9.5	9.5	0.0	0.0
EUU	Energy	14.0	1.7	8.2	2.0	1.6	13.2	13.2	0.0	0.0
EUU	Mining	13.9	0.7	14.0	0.4	1.2	13.8	13.8	0.0	0.0
EUU	Food & Bev.	23.9	2.6	29.8	2.8	2.1	13.2	13.2	0.0	0.0
EUU	Basic Man.	23.2	7.6	33.7	12.4	5.5	6.5	6.5	0.0	0.0
EUU	Adv. Man.	29.2	8.8	26.7	8.7	9.0	10.4	15.2	0.0	0.0
EUU	Resid. Serv.	28.5	0.1	35.7	0.1	0.2	0.0	0.0	0.0	0.0
EUU	Services	30.7	1.5	42.2	1.4	1.7	0.0	0.0	0.0	0.0
CHN	Agriculture	31.7	0.4	3.9	0.4	0.4	26.2	126.6	14.0	94.6
CHN	Energy	17.6	0.1	0.4	0.1	0.1	17.6	17.6	12.7	106.7
CHN	Mining	21.4	0.1	1.4	0.1	0.1	22.1	70.8	8.5	106.1
CHN	Food & Bev.	24.1	0.8	9.3	0.8	0.8	25.7	124.1	11.8	113.3
CHN	Basic Man.	38.0	5.3	23.4	8.6	3.8	29.3	98.8	9.1	107.8
CHN	Adv. Man.	32.1	9.0	27.4	8.9	9.2	30.8	82.3	9.4	113.6
CHN	Resid. Serv.	27.2	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0
CHN	Services	12.9	0.3	9.1	0.3	0.3	0.0	0.0	0.0	0.0
CAN	Agriculture	1.1	1.7	15.3	1.8	1.6	2.2	25.0	4.0	4.0
CAN	Energy	2.3	5.9	28.2	2.0	7.4	1.0	10.0	0.0	0.0
CAN	Mining	3.1	1.4	26.9	0.5	2.6	0.5	25.0	2.3	2.3
CAN	Food & Bev.	1.4	1.2	13.9	1.1	1.4	1.9	24.5	5.8	5.8
CAN	Basic Man.	1.2	2.3	10.3	1.5	2.7	12.9	25.0	9.0	9.0
CAN	Adv. Man.	1.1	2.3	6.8	2.3	2.2	5.9	25.0	6.7	6.7
CAN	Resid. Serv.	1.9	0.1	44.5	0.1	0.2	0.0	0.0	0.0	0.0
CAN	Services	2.1	0.4	11.1	0.3	0.5	0.0	0.0	0.0	0.0
MEX	Agriculture	1.0	1.7	15.3	1.8	1.6	1.8	25.0	0.0	0.0
MEX	Energy	1.4	1.5	7.1	0.5	1.8	0.3	25.0	0.0	0.0
MEX	Mining	1.7	0.1	1.7	0.0	0.2	0.2	25.0	0.0	0.0
MEX	Food & Bev.	2.0	0.8	9.6	0.9	0.8	14.0	25.0	0.0	0.0
MEX	Basic Man.	1.0	1.3	5.7	1.2	1.3	12.4	25.0	0.0	0.0
MEX	Adv. Man.	2.1	6.3	19.1	6.3	6.3	7.7	25.0	0.0	0.0
MEX	Resid. Serv.	1.1	0.0	7.6	0.0	0.0	0.0	0.0	0.0	0.0
MEX	Services	1.1	0.3	8.6	0.3	0.3	0.0	0.0	0.0	0.0
ROW	Agriculture	45.9	5.0	46.0	5.2	5.0	18.3	18.3	0.0	0.0
ROW	Energy	49.8	11.8	56.1	5.9	14.0	0.1	0.1	0.0	0.0
ROW	Mining	48.8	2.9	56.0	0.5	6.1	9.0	10.0	0.0	0.0
ROW	Food & Bev.	35.5	3.2	37.4	3.2	3.2	18.8	18.8	0.0	0.0
ROW	Basic Man.	25.5	6.1	26.9	10.3	4.2	17.7	18.5	0.0	0.0
ROW	Adv. Man.	22.4	6.6	20.0	6.7	6.4	16.7	17.3	0.0	0.0
ROW	Resid. Serv.	28.0	0.0	12.0	0.0	0.1	0.0	0.0	0.0	0.0
ROW	Services	24.2	1.0	29.0	1.0	1.0	0.0	0.0	0.0	0.0

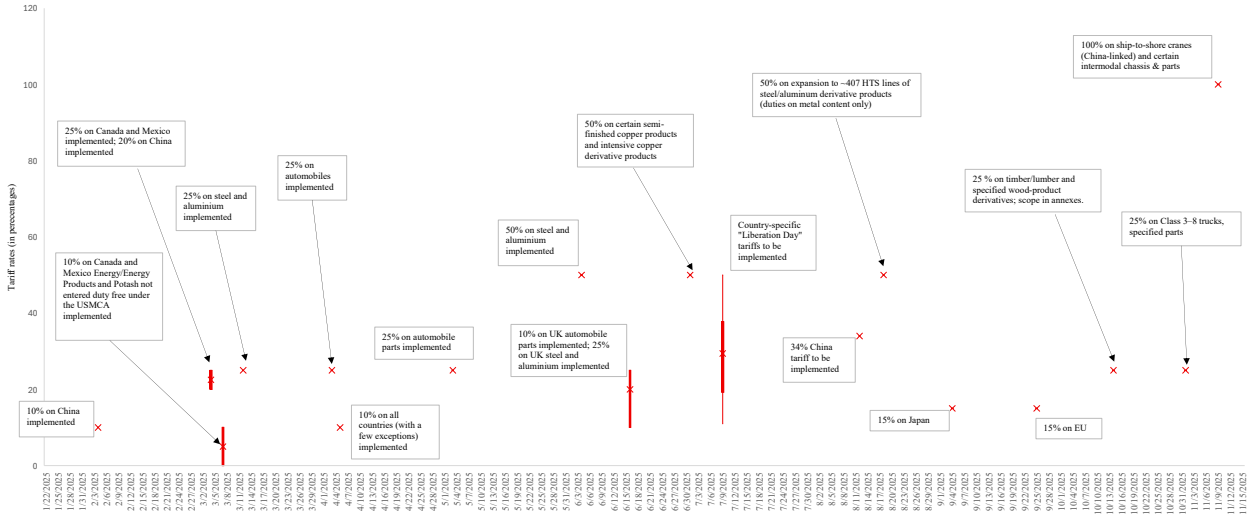
NOTE: Share data is obtained from OECD ICI-O Tables (Yamano and et al., 2023) and tariff data is obtained from WTO - IMF Tariff Tracker database (WTO and IMF, 2025). World Share is the share of the industry in the world in that industry, U.S. Share is the share of the industry in both U.S. final goods and intermediate goods, U.S. Import Share is the share of the industry in the U.S. imports in that industry, U.S. Final Share is the share in the final good consumption in that industry, U.S. Int. Share is the intermediate use share in that industry, U.S. Curr. Tariff is the tariff as of June 30, 2025, U.S. Max Tariff is the maximum tariff observed since January 1, 2025. Ret. Curr. Tariff and Ret. Max. Tariff are the retaliatory tariff levels that countries adapted against the U.S. industries.

Figure D.1. Tariff Announcements and Implementations

(a) Tariff Announcements



(b) Tariffs - Implemented (and to be Implemented)

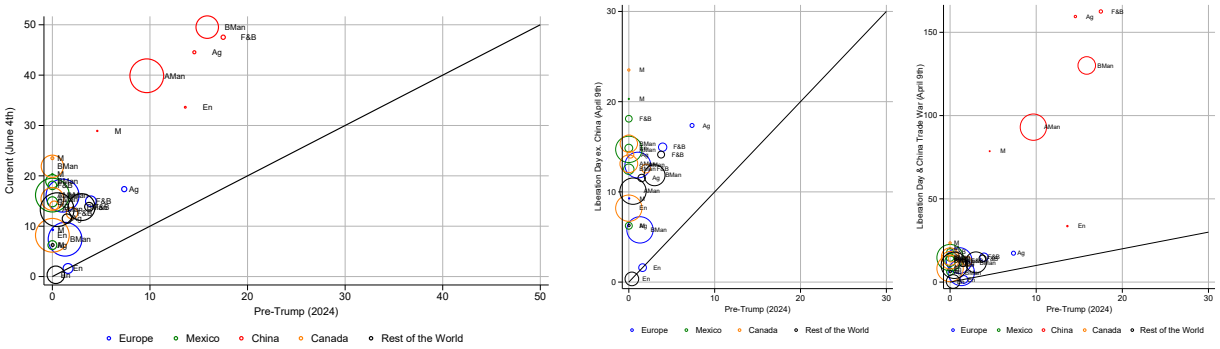


NOTE: Figure D.1 visualizes tariff announcements and implementations between January 20, 2025 and June 30, 2025. The data for the tariff threats, implementations, and planned implementations were compiled from three main sources. The core of the data is from the Trade Compliance Resource Hub Trump 2.0 Tariff Tracker (https://www.tradecomplianceresourcehub.com/2025/06/27/trump-2-0-tariff-tracker/#updates). It presents a list from Reed Smith's International Trade and National Security team that tracks the latest threatened and implemented U.S. tariffs as of June 27th. This list is cross-referenced with Tax Foundation's Trump Trade War timeline as of June 17th (https://taxfoundation.org/research/all/federal/trump-tariffs-trade-war/), and a corresponding list from the PBS news article detailing a timeline of Trump's tariff actions as of May 26th (https://www.pbs.org/newshour/economy/a-timeline-of-trumps-tariff-actions-so-far). The tariffs that classified as "threats" are those that –as of June 30th —had not been implemented and were unlikely to be implemented based on available information. These threats were identified by extensive look into past and latest news, as well as the use of large language models. We created the data as of June 27, 2025. This website curates the all the tariff announcements by the U.S.

Figure D.2. Effective Country-Sector Level Tariff Rates

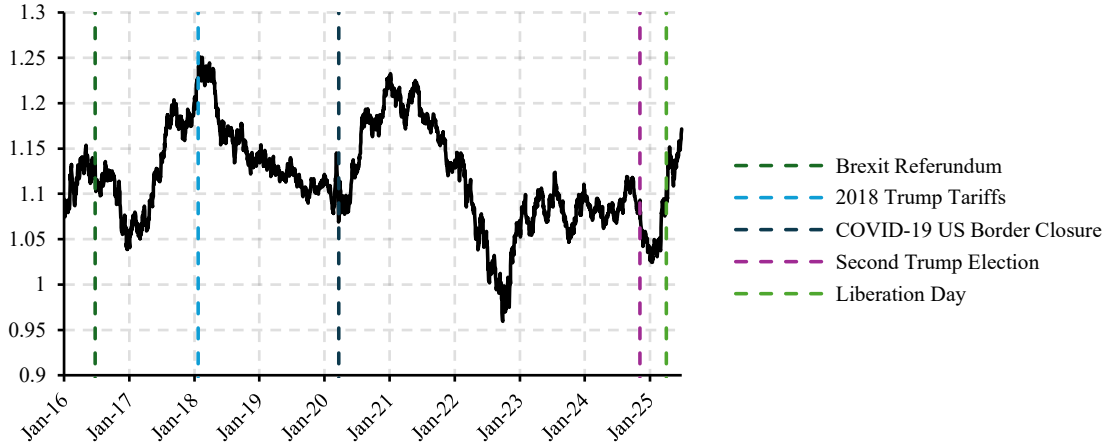
(a) As of June 4, 2025 (%)

(b) As of the "Liberation Day", (%)



NOTE: Figure D.2a visualizes estimated effective tariff rates at the country sector level based on WTO - IMF Tariff Tracker (WTO and IMF, 2025) as of the last available day (June 4, 2025) when we accessed the data on June 20, 2025. Figure D.2b visualizes estimated effective tariff rates at the country sector level when the tariffs announced on the "liberation day" and extra tariffs on China went into effect. In the left panel, we remove the Chinese sectors. In the right panel, we show all country-sector combinations. Bubble size corresponds to the U.S. imports from that country-sector pair for the last available data at WTO. The colors code for countries: Canada, China, euro area, Mexico and the Rest of the World. Sectoral Acronyms are Ag: Agriculture, En: Energy, M: Mining, F&B: Food & Beverages, BMan: Basic Maufacturing, AMan: Advanced Manufacturing.

Figure D.3. USD - Euro Exchange Rate 2016-2025



NOTE: Figure D.3 visualizes the USD Euro Exchange Rate from 2015 to 2025. The vertical lines indicate different events. Source: Bloomberg.