A NEW THEORY OF CREDIT LINES (WITH EVIDENCE)*

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September 11, 2025

Abstract

We develop a dynamic model in which (latent) credit lines serve as a commitment device not to take on more debt, thereby curbing dilution incentives. Posting bundles that include credit lines allows lenders to compete more aggressively and borrowers to extract the maximum surplus. The model explains a number of empirical patterns, such as that the bulk of credit lines are not utilized and that they are bundled with loans. It also generates new predictions, notably that decreasing lender commitment increases borrower leverage and risk. We validate this prediction empirically via two natural experiments.

*For comments, thanks to Vladimir Asriyan, Ryan Bubb, Catherine Casamatta, Nico Crouzet, Peter DeMarzo, Willie Fuchs, Brett Green, Barney Hartman-Glaser, Zhiguo He, Florian Heider, Arvind Krishnamurthy, Uday Rajan, and audiences at the 2025 Adam Smith Conference, the 2024 BIS-CEPR-SCG-SFI Conference on Financial Intermediation, the 2024 EEA, the Fed Board, the 2023 FIFI Conference, the 2024 FIRS conference, the FT Webinar, the FTG Toronto, the 2024 IU Holden Conference, the 2024 Kentucky Finance Conference, LBS, the 2024 NBER Summer Institute, the NYU School of Law, the 2024 Jackson Hole Conference, the San Francisco Fed, the 2024 UBC Summer Conference, UCLA, the 2024 University of Kentucky Finance Conference, the University of Bonn, the University of Florida, the University of Lugano, the University of Mannheim, the University of Miami, USC, UT–Dallas, the 2025 Virtual Corporate Finance Seminar, and the 2024 WFA.

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1 Introduction

Households, firms, and sovereigns typically borrow in markets with multiple competing lenders. In such non-exclusive credit markets, borrowers have incentive to dilute existing debt, leading to excessive leverage and default risk (Bulow and Rogoff (1988), Fama and Miller (1972), and Hart and Moore (1995)) due to what Admati et al. (2017) call the "leverage ratchet effect," whereby debt today induces more debt tomorrow (Bolton and Jeanne (2007, 2009), Brunnermeier and Oehmke (2013), and DeMarzo and He (2021)). Although contractual tools like covenants and collateral can ostensibly mitigate the problem, they are often weak and can even backfire (Bizer and DeMarzo (1992), Attar et al. (2019b), and Donaldson, Gromb, and Piacentino (2018, 2021, 2025)).

In this paper, we show that a pervasive contract—the credit line—can address the problem. Even if never drawn, a large credit line can serve as a commitment device not to ratchet up leverage, thereby protecting existing lenders against dilution. The reason is that prospective lenders fear being diluted themselves if the credit line is drawn and, therefore, demand rates so high that a borrower does not take on new loans at all—the option to dilute new debt (via the credit line) deters dilution of old debt (via new debt). We show that a borrower and creditor in a non-exclusive market can use a simple contractual device—a debt–credit-line bundle—to replicate the outcome of an exclusive market. As such, lenders' commitment to lend—to honor the credit line—can make up for the borrower's lack of commitment not to borrow.

The model is consistent with evidence. It explains why credit lines are (i) bundled with loans (they improve loan terms), (ii) large (they dilute debt enough to deter it), (iii) often undrawn (they serve as a threat exercised off equilibrium), and

(iv) ostensibly prohibitively expensive for lenders if drawn (they are not actually extended in equilibrium). As lenders would prefer not to extend credit lines if they were drawn, it also makes the new prediction that a decrease in lender commitment should frustrate borrowing and increase leverage, a prediction we test using two natural experiments and find support for. Finally, we show that the patterns are stronger when dilution protection should matter most, proxied by low credit market concentration and high default risk.

The environment builds on the literature on dynamic capital structure in which a borrower raises debt dynamically from competing lenders (e.g., Admati et al. (2017), DeMarzo and He (2021)). Debt issuance has social benefits, due to, e.g., tax benefits or heterogeneous valuations, and costs, due to, e.g., financial distress. Individual and social incentives are not in general aligned due to non-exclusivity: After borrowing from one lender, the borrower could borrow from another, diluting the first.

We extend this framework by introducing lender competition not only in loan contracts but also in credit lines, namely options to borrow a pre-specified amount at a fixed price.[1]

We begin with two benchmarks. The first is exclusive competition. In this case, the borrower behaves as a monopolist: Anticipating the price impact of future borrowing, he chooses a static leverage level consistent with the trade-off theory (Kraus and Litzenberger (1973)). Credit lines are unnecessary.

The second benchmark is non-exclusive competition in loan contracts but without credit lines. In this case, the borrower gradually increases leverage, passing costs onto existing creditors. That is the leverage ratchet effect (Admati et al. (2017)).

[1]The framework can also capture dynamic monopoly in durable goods markets (Coase (1972)). There credit lines correspond to put options offered by the seller.

As the duration of exclusivity dt shortens, the effect intensifies, and the borrower's surplus converges to that under competitive pricing (DeMarzo and He (2021), Coase (1972)).

We then study our baseline environment with non-exclusivity and date-0 competition over bundles of loan contracts and credit lines. We first analyze the disciplining role of a credit line in place at any $t > 0$, then solve backwards for the equilibrium bundle offered at $t = 0$.

Our first main result shows how existing credit lines can deter new borrowing. Anticipating that new debt will trigger dilution via the draw down of a large credit line, lenders become less willing to lend. In equilibrium, this feedback can prevent debt issuance altogether, a mechanism we refer to as a "ratchet anti-ratchet effect."

Our second main result is a characterization of the equilibrium bundling outcome. When lenders compete via loan-credit line bundles at date 0, the borrower selects the bundle that replicates the equilibrium outcome under exclusivity. Each lender offers a bundle consisting of the loan that maximizes the borrower's lifetime utility paired with a credit line that deters future dilution, i.e. generates borrower commitment, despite remaining undrawn in equilibrium.

The credit line is an entry-deterrent reminiscent of the latent contracts in papers such as Parlour and Rajan (2001) and Attar et al. (2019a, 2019b). But, whereas in that literature lenders can use latent contracts to extract rents from borrowers, here they use credit lines to attract them, explaining why borrowers welcome credit lines in practice. Indeed, ex post, lenders would prefer not to honor credit lines, explaining why they must be signed by both parties, not just posted by lenders as in the latent contracts literature.

Credit lines here provide dilution protection akin to what covenants and col-

lateral do elsewhere in the literature on non-exclusivity (e.g., Donaldson, Gromb, and Piacentino (2018, 2025). The mechanism here has the advantage that it is self-enforcing and costless in equilibrium. For example, it does not require that existing lenders monitor covenant violations and is thus in full effect even if loans are sold off separately from credit lines, as is typical in the syndicated loan market.

Our third result relaxes the assumption of full lender commitment. We assume credit lines are revoked with probability $1 - \alpha$, motivated by empirical findings (Chodorow-Reich and Falato (2022)). We show that for high α, our main result holds approximately: The threat of future draw downs still deters dilution. For lower α, credit lines are less effective, but can still enforce some level of commitment.

We test this prediction using DealScan syndicated loan data. We proxy for revocation risk using lender-specific negative shocks, following Chodorow-Reich (2014) and Darmouni (2020). We find that increases in revocation risk are associated with greater borrowing, consistent with diminished deterrence power. In contrast, when overall lender liquidity declines, borrowing falls—a pattern documented in prior work. The difference suggests that the results are specific to the credit line channel.

We conclude with two extensions. First, we show that continuous-time competition over bundles leaves our result intact. Second, we introduce uncertainty in future financing needs and characterize the resulting menu of state-contingent bundles. In both cases, latent credit lines continue to enforce exclusivity.

This paper contributes to several literatures. In corporate finance, we show that the leverage ratchet effect can be offset via structured credit line offerings. In the literature on non-exclusive competition, we demonstrate that arbitrarily short exclusivity, combined with bundled menus, restores the exclusive outcome. In the literature on credit lines (e.g., Holmström and Tirole (1998)), we introduce a novel

function—commitment without drawdown. Finally, in dynamic durable goods settings, we show that put options can serve as credible commitment devices.

Layout. Section 2 presents the model; Section 3, the benchmarks; Section 4, the main results; Section 6, the empirical analysis. Section 7 concludes. Appendices include proofs, data construction, and supplementary figures and tables.

2 Model

There is a single borrower B and infinitely many lenders. Everyone has deep pockets and lives forever, discounting the future at rate ρ.

B's flow payoff is as follows:

$$v_t \mathrm{d}t = y\mathrm{d}t + p_t \mathrm{d}Q_t - c(Q_t)\mathrm{d}t, \tag{1}$$

where y is the cash flow over $[t, t+\mathrm{d}t)$, Q_t is the stock of outstanding debt, $\mathrm{d}Q_t$ is the new debt issued over $[t, t+\mathrm{d}t)$, p_t is the unit price of debt issued over $[t, t+\mathrm{d}t)$, and c is the cost of outstanding debt. All debt is perpetual, like consol bonds. B's lifetime utility from date t onward is

$$V_t = \int_0^\infty e^{-\rho s}\, v_{t+s}\, \mathrm{d}s. \tag{2}$$

The cost c captures expected coupon payments as well as any other debt-induced costs, such as distress costs. As such it depends on the stock—i.e. the total face value—of outstanding debt; the marginal cost of raising funds depends on the price as well, and equals $c'(Q_t)/p_t$. We suppose that $c(0) = 0$, so there is no cost of debt without any, $c' > \bar{c}'$ for a strictly positive constant $\bar{c}'$, so more debt is uniformly

more costly. We also assume that $c'' \leq 0$, which says that a dollar is less costly if B has a lot of debt than if he has a little, for example because he is less likely to repay once highly levered.[2]

Lenders' flow payoff from holding a unit of debt given stock Q_t is $\gamma(Q_t)\mathrm{d}t$, interpreted as the expected coupon payment. We assume that $\gamma' < 0$ and $\gamma(\infty) = 0$. The first assumption captures dilution: The more debt outstanding, the lower is the expected coupon payment. This amounts to a downward-sloping demand curve, which holds true in all the ratchet-effect-type models. The second assumption is that the expected coupon goes to zero as the stock of debt becomes large.[3] The value of the lenders' stream of coupons on a unit of debt is

$$\Gamma(Q_t) = \int_0^\infty e^{-\rho s}\gamma(Q_{t+s})\mathrm{d}s. \tag{3}$$

Assumption 1. *We impose the following conditions on c and γ:*

(i) Gains from trade: $\gamma(0) > c'(0)$.

(ii) First order approach: $\gamma(Q)Q - c(Q)$ is concave.

Part (i) ensures that there are gains from trade between B and the lenders; part (ii) that we can use the first-order approach freely.

[2]The specification is consistent with numerous microfoundations, for example that in DeMarzo and He (2021) or the following simple trade-off theory between tax benefits and distress costs: For stock of debt Q, let $\mathsf{DP}(Q)$ be the default probability, coupon the constant coupon rate, $\mathsf{TS} \times Q$ the tax shield, and $\mathsf{DC} \times Q$ the (ex post) distress costs, we have

$$c(Q) = \big(1 - \mathsf{DP}(Q)\big) \times \mathsf{coupon} + \mathsf{TS} \times Q + \mathsf{PD}(Q) \times Q \times \mathsf{DC}$$

and $\gamma(Q) = \big(1 - \mathsf{DP}(Q)\big) \times \mathsf{coupon}$. It is easy to see that c satisfies our assumptions whenever $\mathsf{DC} > \mathsf{coupon}$ and DP increases in Q.

[3]There is a discussion about an analogous assumption in the literature on durable goods monopolists; see, e.g., McAfee and Wiseman (2008), on what are refereed to as the "gap" and "non-gap" cases.

Financial markets. At each date t, lenders post menus of bundles of loans and credit lines $\big((p_t, \mathrm{d}Q_t), (\tilde{p}, \mathrm{d}\tilde{Q})\big)$ and B accepts at most one. (In the baseline, lenders are fully committed to contracts accepted by B, but not to others; in Section 5.2, we allow lenders to revoke accepted contracts with some probability.) [Maybe mention somewhere that these are annuities?]

Solution concept. The solution concept is Markov perfect equilibrium with state variable equal to B's balance sheet, i.e. his debt and credit lines: At each date, the lenders and B act—lenders post contracts and B chooses at most one and, if he has a credit line in place, chooses whether to draw it (in full)[4] or not—such that (i) they maximize their future lifetime payoffs given their beliefs, (ii) their beliefs are consistent, and (iii) B's balance sheet is a sufficient statistic for the history with respect to the actions—i.e. the (i) sequential rationality, (ii) equilibrium, and (iii) Markov conditions.

For a given credit line in place, whether it is drawn or not is a binary variable. That allows us to simplify notation by omitting the explicit dependence on the credit line; we denote the value function without the credit line in place (i.e. after it has been drawn) by V and with it (i.e. before it has been drawn) by $\tilde{V}$.

Later, we also introduce what we think is a mild equilibrium refinement that helps us to establish uniqueness (Assumption 3).

3 Benchmarks and Preliminaries

We start with two benchmarks without credit lines. The first is the exclusive allocation in which B can commit to borrow from only one lender, as in the classical

[4]It turns out that partial draw downs are not optimal in equilibrium anyway, a fact that follows shortly from the convexity of the value function. See footnote 7.

trade-off theory/static monopolist's problem. The second is the non-exclusive allocation in which B lacks commitment, as in the leverage ratchet effect/dynamic monopolist's problem.

3.1 Benchmark: Full Commitment

We first consider the problem in which B can commit to future debt issuance; in particular, B can commit not to issue debt to other creditors, so debt dilution is not a concern.

At date 0, B chooses a debt level, Q_t for each t, to maximize the expected present value of his payoffs subject to the lenders' participation constraint:

$$\max_{(Q_t)_t} \int_0^\infty e^{-\rho t}(y + p_t \mathrm{d}Q_t - c(Q_t))\mathrm{d}t \tag{4}$$

$$\text{s.t.} \qquad p_t \leq \int_0^\infty e^{-\rho s}\gamma(Q_{t+s})\mathrm{d}s. \tag{5}$$

To see the solution, suppose that B issued debt at only one date, date τ, and never again. In that case, the integrands above are constants, and the problem reduces to marginal revenue equals marginal cost:

$$\big(p_\tau Q_\tau\big)' = \frac{c'(Q_\tau)}{\rho}. \tag{6}$$

This problem is the same for any date τ, so it is optimal to issue once at date 0, as delaying gains from trade is costly, and then keep debt constant, as formalized next:

Proposition 1 (Exclusive Benchmark)**.** *With commitment/exclusive competition,*

B issues debt only at $t = 0$ with quantity Q^e at price p^e, where Q^e solves

$$\gamma(Q^e) = c'(Q^e) - \gamma'(Q^e)Q^e \tag{7}$$

and

$$p^e = \frac{\gamma(Q^e)}{\rho}. \tag{8}$$

Here the borrower acts as a monopolist, setting marginal revenue equal to (the PV of) marginal cost. Substituting the price from the lenders' break-even condition, we have:

$$p^e = \frac{c'(Q^e)}{\rho} - \frac{\gamma'(Q^e)Q^e}{\rho}. \tag{9}$$

Since $\gamma' < 0$, it follows that the price is above the marginal cost. The term $\gamma'(Q^e)Q^e$ reflects that B takes into account that issuing one more unit of debt depresses the price for all debt—hence the $\gamma'(Q^e)$ is multiplied by Q^e.

3.2 Benchmark: No Commitment/Non-Exclusivity

We now consider the case in which B can issue debt continuously but cannot commit to his future issuance, a setting that resembles DeMarzo and He (2021). As allocations now need to be time-consistent, we can solve the recursive formulation of the problem, with value function

$$V_t = v_t \mathrm{d}t + e^{-\rho \mathrm{d}t} V_{t+\mathrm{d}t}. \tag{10}$$

Using that $V_t = V(Q_t)$ by the Markov property and the expression for v_t in equation (1), we have

$$V(Q) = \max_{\mathrm{d}Q} \Big\{ y\mathrm{d}t + p(Q+\mathrm{d}Q)\mathrm{d}Q - c(Q)\mathrm{d}t + e^{-\rho \mathrm{d}t} V(Q+\mathrm{d}Q) \Big\}, \tag{11}$$

where B takes the price function p as given. In the limit as $\mathrm{d}t \to 0$, the equation becomes[5]

$$\rho V(Q) = \max_{q} \Big\{ p(Q) + V'(Q) \Big\} q + y - c(Q), \tag{12}$$

for any smooth policy $q\mathrm{d}t = \mathrm{d}Q_t$. The objective is linear in the control q, so it must have coefficient zero:

$$p(Q) + V'(Q) = 0, \tag{13}$$

an equation that also appears in DeMarzo and He (2021).[6]

Proposition 2 (Non-exclusive Benchmark)**.** *In the limit as* $\mathrm{d}t \to 0$*, there is a unique smooth equilibrium. The value function is*

$$V(Q) = \frac{1}{\rho}\big(y - c(Q)\big), \tag{14}$$

[5]This follows from standard calculations, heuristically as follows. Substitute $V(Q+q\mathrm{d}t) = V(Q) + V'(Q)q\mathrm{d}t + O(\mathrm{d}t^2)$ into equation (11) and multiply through by $(1+\rho \mathrm{d}t) = e^{\rho \mathrm{d}t} + O(\mathrm{d}t^2)$ to get

$$V(Q) + \rho \mathrm{d}t V(Q) = \Big\{ y + p(Q+q\mathrm{d}t)q\mathrm{d}t - c(Q) \Big\}\mathrm{d}t(1+\rho\mathrm{d}t) + V(Q) + V'(Q)q\mathrm{d}t + O(\mathrm{d}t^2)$$

or

$$\rho V(Q) = y + p(Q+\mathrm{d}Q)q\mathrm{d}t - c(Q) + V'(Q)q\mathrm{d}t + \big\{ p(Q+q\mathrm{d}t)q\mathrm{d}t - c(Q) \big\}\rho\mathrm{d}t + O(\mathrm{d}t).$$

[6]There is the flavor of mixed-strategy equilibrium here, as the optimum is determined not by B's direct incentives but by the market's need to make him indifferent over his choice of control.

the price is

$$p(Q) = \frac{1}{\rho} c'(Q), \tag{15}$$

and the issuance policy is

$$q = \frac{\gamma(Q) - c'(Q)}{-c''(Q)/\rho}. \tag{16}$$

Equation (15) says that the price equals (the present value of) B's marginal cost. I.e. the price is competitive, per the Coase Conjecture on a durable goods monopolists (Coase (1972)). B would like to ration quantities to keep prices above marginal cost, but is always tempted to issue more.

The result also implies that V'' inherits the sign of $-c''$:

Corollary 1 (Convex value)**.** *The value function is convex, with* $V'' = -c''/\rho$.

We assume the equilibrium in Proposition 2 is played whenever credit lines are not available. DeMarzo and He (2021) given conditions for the analogous equilibrium in their environment to be unique, even allowing for policies with jumps.

Before moving on, we make an assumption on c and γ that controls the rate of change of Q_t:

Assumption 2 (Controlled Buybacks)**.** *Let* $(Q_t)_t$ *be as in Proposition 2. Then* $\gamma(Q_{t+s}) \leq \gamma(Q_t) \exp(k_0 + k_1 s)$ *for constants* k_0 *and* $k_1 < \rho$.

The assumption says that the rate of buy backs $-q$ cannot grow exponentially. That ensures that the value of debt goes to zero as the level goes to infinity and thus ensures lenders are not willing to pay much for a highly indebted borrower's new debt, even in anticipation of deleveraging. That helps with the proof of Proposition 3.

4 Results

We now study our baseline model, in which B lacks commitment and lenders compete in bundles. We assume here that lenders can offer loans at any date but bundles with credit lines only at date 0, but relax that in Section 5.1.

We show that when lenders can bundle credit lines with their loan offers, the commitment allocations of Section 3.1 obtain. We do so in three steps. First, we show that there is a ratchet effect for credit lines: B draws a credit line if and only if his debt is sufficiently high (Section 4.1). Second, we show that, as a result, credit lines can act as a self-deterring mechanism that prevent lenders to make loan offers, which we call the "ratchet-anti-ratchet effect" (Section 4.2). Third, we show that competition on bundles of loans and credit lines implement the commitment allocation (Section 4.3).

4.1 The Ratchet Effect for Credit Lines

Below, we show an analog of Admati et al.'s (2017) leverage ratchet effect, which says that drawing a credit line becomes more attractive as debt increases, a consequence of the concavity of c.

Before that, we define the *associated debt level* for a given credit line $(\tilde{p}, \mathrm{d}\tilde{Q})$, which we denote by $\mathcal{Q}$, as the debt level at which B is indifferent between drawing that credit line and never issuing debt again. Recalling that, by Proposition 2, he gets $V(\mathcal{Q})$ in that case, we have the following definition:

Definition 1 (Associated Debt Level)**.** *For a credit line* $(\tilde{p}, \mathrm{d}\tilde{Q})$*, an* associated debt level $\mathcal{Q}$ *is one such that B is indifferent between drawing* $(\tilde{p}, \mathrm{d}\tilde{Q})$ *with debt* $\mathcal{Q}$ *and*

never issuing debt again:[7]

$$\tilde{p}\mathrm{d}\tilde{Q} + V(\mathcal{Q} + \mathrm{d}\tilde{Q}) = V(\mathcal{Q}). \tag{17}$$

Using that $V(Q) = (y - c(Q))/\rho$, condition (17) can be re-written to reflect that the price of the credit line must exactly compensate B for the costs of a higher debt level at $\mathcal{Q}$,

$$\tilde{p} = \frac{1}{\rho}\frac{c(\mathcal{Q} + \mathrm{d}\tilde{Q}) - c(\mathcal{Q})}{\mathrm{d}\tilde{Q}}. \tag{18}$$

We now show that B draws a credit line whenever his debt exceeds its associated debt level:

Proposition 3 (Ratchet Effect)**.** *Let* $\mathrm{d}t \to 0$*. Consider any equilibrium in which B has a credit line* $(\tilde{p}, \mathrm{d}\tilde{Q})$ *in place with associated debt level* $\mathcal{Q} \leq Q^e$*. Then, B (weakly) prefers to draw the credit line if and only if* $Q \geq \mathcal{Q}$*.*

Intuitively, because c is concave, when B has more debt the cost of issuing at $\tilde{p}$ is relatively low for high Q. Hence B prefers to draw if $Q \geq \mathcal{Q}$, the indifference point, and not to otherwise.

We conclude this section with an assumption on the set of equilibria we study. We think it is mild, as it matters only if B is indifferent between drawing and not.

Assumption 3 (Tie-breaking Rule)**.** *Suppose B has a credit line* $(\tilde{p}, \mathrm{d}\tilde{Q})$ *and debt* Q *in place. If he is indifferent between drawing and not, he does not draw if and only if* Q *solves equation (17).*

[7] As foreshadowed in footnote 4, drawing is full or not at all. A partial draw down would be optimal only if $\tilde{p}\phi\mathrm{d}\tilde{Q} + V(Q + \phi\mathrm{d}\tilde{Q})$ had a maximum for $0 < \phi < 1$. But that function is convex in ϕ—$\big(\tilde{p}\phi\mathrm{d}\tilde{Q} + V(Q + \phi\mathrm{d}\tilde{Q})\big)'' = V''(Q + \phi\mathrm{d}\tilde{Q})\mathrm{d}\tilde{Q}^2 > 0$ since $V'' > 0$. So there cannot be an interior maximum; any interior critical point must be a minimum.

The assumption says that B breaks indifference toward not drawing only if he is indifferent between drawing the line (the LHS of (17)) and never borrowing again (the RHS of (17)). The next result says that (i) for any debt level, there is a credit line for which the condition is satisfied and that, given that credit line, (ii) there is no other debt level for which it is.

Lemma 1 (Existence and Uniqueness)**.** *For any debt level Q and size of credit line $\mathrm{d}\tilde{Q}$, there is a unique price such that equation (18) holds.*

For any credit line $(\tilde{p}, \mathrm{d}\tilde{Q})$, there is at most one debt level for which it does.

Combined with Proposition 3 and Assumption 3, the result says that B draws any credit line if and only if his debt level Q exceeds the credit line's associated debt level $\mathcal{Q}$.

4.2 The Ratchet-Anti-Ratchet Effect

Next, we use the result in Proposition 3 to show that for any debt level $\mathcal{Q} \leq Q^e$, there is an associated credit line $(\tilde{p}, \mathrm{d}\tilde{Q})$ with $\mathrm{d}\tilde{Q}$ large that makes lenders unwilling to post a loan contract that B is willing to accept. As a result, the debt level $\mathcal{Q}$ is an absorbing state for the process Q_t. The result follows from the ratchet effect in Proposition 3, i.e. if B takes on any debt he also takes on $\mathrm{d}\tilde{Q}$, and the assumption that $\gamma(\infty) = 0$, i.e., that lenders' value drops below B's in anticipation of his debt going up.

To see why, suppose first that B has a credit line with associated debt level $\mathcal{Q}$ (Definition 1) and $\tilde{p}$ above the lower bound in Proposition 3. So at $\mathcal{Q}$, B will choose to draw the line if he takes on any new debt $\mathrm{d}Q > 0$. Now B prefers *not* to take on the new debt at price p if his payoff from taking the loan and drawing the credit

line is less than that of just drawing the credit line:

$$p\mathrm{d}Q + \tilde{p}\mathrm{d}\tilde{Q} + V(\mathcal{Q} + \mathrm{d}Q + \mathrm{d}\tilde{Q}) \leq \tilde{p}\mathrm{d}\tilde{Q} + V(\mathcal{Q} + \mathrm{d}\tilde{Q}). \tag{19}$$

Rearranging gives an upper bound on the price p of new debt:

$$p \leq \frac{V(\mathcal{Q} + \mathrm{d}Q + \mathrm{d}\tilde{Q}) - V(\mathcal{Q} + \mathrm{d}\tilde{Q})}{\mathrm{d}Q}. \tag{20}$$

The inequality must hold for all $\mathrm{d}Q$. Thus, given V is convex (Corollary 1), for any p, it is necessary and sufficient that it holds as $\mathrm{d}Q \to 0$. That limiting condition is:

$$p \leq -V'(\mathcal{Q} + \mathrm{d}\tilde{Q}) = \frac{1}{\rho}c'(\mathcal{Q} + \mathrm{d}\tilde{Q}), \tag{21}$$

having used the expression for V in Proposition 2. Condition (21) says that B does not want to issue at a price below marginal cost. The twist is that the marginal cost is conditional on having drawn the line. Given B will draw, immediately increasing his debt by $\mathrm{d}\tilde{Q}$, it is the marginal cost at $\mathcal{Q} + \mathrm{d}\tilde{Q}$, not at just $\mathcal{Q}$, that matters.

We now turn to whether lenders are willing to lend. A lender that anticipates B will draw the credit line is willing to offer a loan $(p, \mathrm{d}Q)$ if and only if its value exceeds the price, or

$$\Gamma(\mathcal{Q} + \mathrm{d}Q + \mathrm{d}\tilde{Q}) \geq p. \tag{22}$$

Comparing inequalities (21) and (22), we have that for $\mathrm{d}\tilde{Q}$ large enough, lenders will not be able to make an offer that B is willing to accept, a fact we use in the following result.[8]

Proposition 4 (Ratchet-Anti-ratchet)**.** *For any debt level $\mathcal{Q} \leq Q^e$, there exists an*

[8]The statement follows from the fact that as $Q \to \infty$, $c'(Q) \to \bar{c}' > 0$, whereas $\Gamma(Q) \to 0$.

associated credit line $(\tilde{p}, \mathrm{d}\tilde{Q})$ *with* $\mathrm{d}\tilde{Q}$ *large enough so that*

$$\Gamma(\mathcal{Q} + \mathrm{d}\tilde{Q}) \leq \frac{1}{\rho} c'(\mathcal{Q} + \mathrm{d}\tilde{Q}). \tag{23}$$

With such credit line in place, the value function is as follows:

$$\tilde{V}(Q) = \begin{cases} \frac{1}{\rho}\Big(\gamma(\mathcal{Q})(\mathcal{Q} - Q) + y - c(\mathcal{Q})\Big) & \text{if } Q \leq \mathcal{Q}, \\ \tilde{p}\mathrm{d}\tilde{Q} + V(Q + \mathrm{d}\tilde{Q}) & \text{otherwise,} \end{cases} \tag{24}$$

with V *as in Proposition 2.*

The main take-away from Proposition 4 is that any $\mathcal{Q} \leq Q^e$ can be supported as an absorbing state by a sufficiently large credit line.[9] Moreover, if B's debt is below $\mathcal{Q}$, then he issues debt to jump to the absorbing state $\mathcal{Q}$; when it is above $\mathcal{Q}$, B draws the credit line. At $Q = \mathcal{Q}$, B is indifferent between drawing or not, which given the tie-breaking rule, implies B will not draw (Assumption 3).

There are three key steps to the proof, which is in Appendix C.6. The first is to show that given a credit line with the properties in the proposition, $\mathcal{Q}$ is an absorbing state, in line with the preceding discussion. The second is a characterization of the value function given such a credit line is in place. The last is to show existence.

[9]Thus the credit line is part of the implementation of an optimal contract as in, e.g., DeMarzo and Fishman (2007), DeMarzo and Sannikov (2006), and Hartman-Glaser, Mayer, and Milbradt (Hartman-Glaser et al.). Unlike in that literature, the credit line is latent, consistent with low utlization (Section 6). That also contrasts with models in which credit lines are used for state-contingent liquidity insurance (e.g., Holmstrom and Tirole (1997) and Santos and Viswanathan (2020)).

4.3 Bundling

We now consider date 0, when lenders can offer bundles including loans and credit lines. The outcome is the exclusive contracting outcome of Proposition 1:

Proposition 5 (Credit Line Bundles)**.** *If lenders compete in bundles, B chooses a bundle with a loan and a credit line at date 0, never borrows again, and never draws the credit line.*

The loan coincides with exclusive outcome (p^e, Q^e) *in Proposition 1.*

The credit line $(\tilde{p}, \mathrm{d}\tilde{Q})$ *has associated debt level* $\mathcal{Q} = Q^e$ *(Definition 1).*

The equilibrium allocation is unique (under Assumption 3).

Proposition 5 builds on Proposition 4, which says that a credit line is a commitment device not to borrow in the future. At date 0, B can put a credit line in place and thereby commit not to borrow from anyone else. Thus if lenders can bundle loans with credit lines, and commit to such credit lines, then an instant of exclusivity—from date 0 to date $\mathrm{d}t$—is just as good as exclusivity forever. As a result, competition in bundles at date 0 can achieve the same outcome as exclusive competition.

As mentioned in the Introduction, this result contrasts with the literature on latent contracts (notably Parlour and Rajan (2001) and Attar et al. (2019a, 2019b)). Although our credit lines, never being drawn, resemble the latent contracts in that literature, the outcomes here do not resemble those there. With (i) exclusivity within periods, albeit arbitrarily short ones, and (ii) competition in bundles, not just loans, the non-competitive outcomes of that literature do not arise.

There is also a practical difference between credit lines in our model and latent (loan) contracts in the literature, namely that whereas a credit line is a contract agreed to between a borrower and a (potential) lender that must be honored, the

latent contract is just an offer from a lender that can be retracted. That matters, because, in our model, lenders would prefer not to honor credit lines. At the time that they would be drawn, B is so indebted that the price $\tilde{p}$ is too high for the lender to break even. Whereas credit lines, which are always available per the contract, can support a variety of outcomes, the analogous latent contracts demand a much stronger form of lender commitment.

5 Robustness and Extensions

In this section we relax several assumptions from the baseline model: We allow for (i) competition in loan-credit line bundles at all t, not just $t = 0$ (Section 5.1), (ii) revocation of the credit line by lenders with some probability (Section 5.2), and (iii) uncertainty through shocks to the gains from trade between B and the lender (Section 5.3).

5.1 Bundling at any Date

In our baseline model, we assumed that lenders compete by offering bundles of loans and credit lines at $t = 0$ and only loans thereafter, for $t > 0$. Here, we relax this assumption and show that our equilibrium is robust to competition in bundles at all dates.

Proposition 6 (Bundling for $t > 0$)**.** *We show that the outcome in Proposition 5 is robust to allowing competition in bundles for $t > 0$ in the following sense: If B has debt Q_0^e and a credit line $(\tilde{p}, \mathrm{d}\tilde{Q})$, as defined in Proposition 5 in place, then there is no bundle $\big((p_t, \mathrm{d}Q_t), (\tilde{p}_t, \mathrm{d}\tilde{Q}_t)\big)$ a lender can offer at $t > 0$ and make strictly positive expected profits.*

Proposition 7 (Bundling for $t > 0$)**.** *The equilibrium allocations in Proposition 5 are robust to allowing lenders to compete in bundles at all t.*

The result says that the equilibrium in Proposition 5 survives competition in debt-credit line bundles for all t.

To establish the result we consider a fictitious lender that has, by fiat, the power to enforce exclusivity in the future. We show that even that lender does not have a profitable deviation and, therefore, a lender without that power does not either. The intuition is the same as for Proposition 4: The initial credit line deters subsequent ones, just as it does subsequent debt contracts in the baseline, as any increase in Q triggers the draw down of the original credit line, diluting the new debt so much that it is loss making.

5.2 Credit Line Revocation

In our baseline model, we have assumed that lenders fully commit to credit lines; they are never revoked. Here we relax that assumption. We assume that, conditional on being drawn, a lender honors the credit line with probability α and defaults, lending nothing, with complementary probability. One motivation for this is that, as discussed in Section 4.3, the credit lines in our model are, by construction, loss making when drawn, so lenders would like to revoke them if they could; another is that the offering lenders could be distressed themselves and unable to honor their commitments (cf. Chodorow-Reich and Falato (2022)). Either way, as above, B draws the credit line $(\tilde{p}, \mathrm{d}\tilde{Q})$ if and only if his debt is (strictly) above the associated $\mathcal{Q}$, defined, by analogy with Definition 1, as

$$\alpha\big(\tilde{p}\mathrm{d}\tilde{Q} + V(\mathcal{Q} + \mathrm{d}\tilde{Q})\big) + (1-\alpha)V(\mathcal{Q}) = V(\mathcal{Q}). \tag{25}$$

Likewise, the condition for B not to want to borrow $\mathrm{d}Q > 0$ (in which case he draws on $(\tilde{p}, \mathrm{d}\tilde{Q})$) is analogous to inequality (19) with drawing the line replaced by drawing it with probability α: For $\mathrm{d}t \to 0$,

$$p\mathrm{d}Q + \alpha\Big(\tilde{p}\mathrm{d}\tilde{Q} + V(\mathcal{Q} + \mathrm{d}Q + \mathrm{d}\tilde{Q})\Big) + (1-\alpha)V(\mathcal{Q} + \mathrm{d}Q) \leq V(\mathcal{Q}), \tag{26}$$

or, using B's indifference condition in equation (25) to rewrite the RHS and rearranging,

$$p \leq -\alpha\frac{V(\mathcal{Q} + \mathrm{d}Q + \mathrm{d}\tilde{Q}) - V(\mathcal{Q} + \mathrm{d}\tilde{Q})}{\mathrm{d}Q} - (1-\alpha)\frac{V(\mathcal{Q} + \mathrm{d}Q) - V(\mathcal{Q})}{\mathrm{d}Q}. \tag{27}$$

Per the argument in Section 4.2 (cf. equation (20)), it is necessary and sufficient that the inequality holds for $\mathrm{d}Q \to 0$, or that

$$p \leq -\alpha V'(\mathcal{Q} + \mathrm{d}\tilde{Q}) - (1-\alpha)V'(\mathcal{Q}). \tag{28}$$

From here, we have the next result:

Lemma 2 (No New Debt with Revocation)**.** *Suppose B has debt $\mathcal{Q}$ and a revocable credit line $(\tilde{p}, \mathrm{d}\tilde{Q})$ in place, such that B is indifferent to drawing the line at $\mathcal{Q}$.*

For $\mathrm{d}t \to 0$, B prefers new debt (p, q) for some q to no loan if and only if

$$p \leq \frac{1}{\rho}\Big(\alpha c'(\mathcal{Q} + \mathrm{d}\tilde{Q}) + (1-\alpha)c'(\mathcal{Q})\Big). \tag{29}$$

The result says that B does not want to borrow at a price below marginal cost. The twist is that the marginal cost is conditional on drawing the line successfully with probability α.

We now turn to whether lenders are willing to lend. By the definition of Γ, a lender that anticipates that B will draw the credit line successfully with probability α is willing to offer a loan (p, q) if and only if

$$\alpha\Gamma(\mathcal{Q} + \mathrm{d}Q + \mathrm{d}\tilde{Q}) + (1-\alpha)\Gamma(\mathcal{Q} + \mathrm{d}Q) \geq p. \tag{30}$$

Just comparing inequalities (29) and (30) gives the next result:

Proposition 8 (Revocation)**.** *Consider the setting of Proposition 4. B does not take on new debt at any price lenders will lend at if and only if*

$$\alpha\Gamma(\mathcal{Q} + \mathrm{d}\tilde{Q}) + (1-\alpha)\Gamma(\mathcal{Q}) \leq \frac{1}{\rho}\Big(\alpha c'(\mathcal{Q} + \mathrm{d}\tilde{Q}) + (1-\alpha)c'(\mathcal{Q})\Big). \tag{31}$$

Re-writing (31) gives an expression for the debt levels that B can commit to with credit lines:

$$\Gamma(\mathcal{Q}) - \frac{c'(\mathcal{Q})}{\rho} \leq \frac{\alpha}{1-\alpha}\left(\frac{c'(\mathcal{Q} + \mathrm{d}\tilde{Q})}{\rho} - \Gamma(\mathcal{Q} + \mathrm{d}\tilde{Q})\right) \overset{\mathrm{d}\tilde{Q}\to\infty}{\to} \frac{\alpha}{1-\alpha}\frac{\bar{c}'}{\rho}, \tag{32}$$

having used the assumptions that $c'(\infty) = \bar{c}'$ and $\gamma(\infty) = 0$. That suggests that for α large, some commitment can be sustained; as we showed, the full commitment outcome is attained for $\alpha = 1$ with $\mathcal{Q} = Q^e$ (Proposition 5). As α gets smaller, the set of debt levels B can commit to with a credit line is reduced, approaching the non-exclusive outcome $\gamma(\mathcal{Q}) = c'(\mathcal{Q})$ of Proposition 2 as $\alpha \to 0^+$.

Intuitively, the possibility of revocation makes new lenders willing to pay more for new debt: They are less worried about being diluted by a draw down if the credit line might disappear. That makes it harder to deter new debt issuance and thus harder to commit not to dilute in the first place, a prediction that we test in

Section 6 (see Prediction 6).

5.3 Uncertainty

So far, we have assumed that there is no risk. We see that as a strength, highlighting how our theory of credit lines differs from others, which are generally based on insuring against adverse risks. Nonetheless, we relax that assumption here. One reason is that, with risk, B needs to take new debt to achieve the efficient outcome—the no-borrowing outcome in the baseline is no longer optimal. We argue, via an example, that our mechanism generates something that looks like relationship banking. Another reason is to show robustness: Our baseline analysis holds up fairly generally if, as in practice, the price of the credit line can be conditioned on the aggregate state.

We now assume that gains from trade depend on an observable state s_t at date t, e.g., c or γ could be state dependent. Now there are two value functions for each state s: $\tilde{V}_s$ with the credit line in place and V_s without it. For a given credit line, the associated debt level is defined as in Definition 1, except now the price, value functions, and the associated debt level are indexed by the state s. As in our baseline, we now consider a credit line $\mathrm{d}\tilde{Q}$ with state-dependent price given by

$$\tilde{p}_s \mathrm{d}\tilde{Q} + V_s(\mathcal{Q}_s + \mathrm{d}\tilde{Q}) = V_s(\mathcal{Q}_s). \tag{33}$$

As in Section 4, we maintain the assumption that credit lines can be offered only at $t = 0$ and that only loans are allowed thereafter.[10]

For example, suppose that uncertainty is captured by a single shock that arrives

[10]That simplifies the proof here but we think that an analog of Proposition 7, on the robustness of the equilibrium allocation to competition in bundles for $t > 0$, can be extended to this setting.

at Poisson rate λ, so things start in state s_0 until a random time τ when they transition to state s_1 and they stay there ever after. Suppose that creditors' flow payoff is γ_s in state s and nothing else depends on the state. And suppose that $\mathcal{Q}_s$ is the optimum, so

$$(\gamma_s(\mathcal{Q}_s)\mathcal{Q}_s)' = c'(\mathcal{Q}_s). \tag{34}$$

The optimal strategy is to only issue $\mathrm{d}Q_0 = \mathcal{Q}_{s_0}$ and $\mathrm{d}Q_\tau = \mathcal{Q}_{s_1} - \mathcal{Q}_{s_0}$.

We suppose that, in equilibrium, B supports the issuance strategy via a credit line with price $\tilde{p}_s$, dependent on the state s and, possibly, the debt level Q_t, as discussed below; its size $\mathrm{d}\tilde{Q}$ need not dependent on anything. We show conditions under which that is indeed an equilibrium in Proposition 9. For $t > \tau$, everything is as in the baseline. For $t = \tau$, B issues new debt $\mathcal{Q}_{s_1} - \mathcal{Q}_{s_0}$ at a price denoted by p_τ, to be determined below. For $t < \tau$, we have that $\tilde{V}$ solves (the limit of)

$$\tilde{V}_{s_0} = (y-c)\mathrm{d}t + e^{-\rho \mathrm{d}t}\Big((1-\lambda \mathrm{d}t)\tilde{V}_{s_0} + \lambda \mathrm{d}t\big(p_\tau(Q_{s_1} - Q_{s_0}) + \tilde{V}_{s_1}\big)\Big), \tag{35}$$

where we hope the short-hand notations are self-explanatory.

The key twist relative to the baseline is what happens at time τ, since, given B has outstanding debt, the market price at date τ will generally not serve to equate the marginal benefit and marginal cost in equation (34).[11] Thus raising $\mathrm{d}Q_\tau$ from the market could undermine the optimum.

One straightforward way to avoid the problem is, in the spirit of Arrow–Debreu, to assume that that debt issued at date 0 is contingent on the state. So the face value changes automatically from $\mathcal{Q}_{s_0}$ to $\mathcal{Q}_{s_1}$. We stick to that interpretation in

[11] As ever, at price p_τ, B will maximize $p_\tau \mathrm{d}Q_\tau - c(Q_{s_0} + \mathrm{d}Q_\tau)/\rho$ and lenders $\gamma(Q_{s_0} + \mathrm{d}Q_\tau)\mathrm{d}Q_\tau/\rho - p_\tau \mathrm{d}Q_\tau$. The first order conditions imply $p_\tau = c'(Q_{s_0} + \mathrm{d}Q_\tau)/\rho$ and $p_\tau = \big(\gamma(Q_{s_0} + \mathrm{d}Q_\tau)\mathrm{d}Q_\tau\big)'/\rho$. Equating the expressions for p_τ yields equation (34) only if $Q_{s_0} = 0$.

our formal analysis. But there could also be a more realistic way to address the problem: To allow the price of the credit line to vary with the debt level, as touched on above. That could induce B to increase his debt with his initial lender, perhaps by drawing partially on the line, with the line adjusting to deter new debt from the market. That would resemble the long-term/captive relationships borrowers have a single lender empirically.

The next result shows the robustness of our baseline analysis to uncertainty.

Proposition 9 (Uncertainty)**.** *Let $\mathcal{Q}_s$ be the optimal debt level in state s with associated credit line $\left((\tilde{p}_s, \mathrm{d}\tilde{Q})\right)_s$ given by equation* (33)*. For $\mathrm{d}\tilde{Q}$ sufficiently large, there is an equilibrium in which (i) the ratchet effect obtains (in that B draws if and only if $Q > \mathcal{Q}_s$ in state s) and (ii) the exclusive equilibrium allocation is implemented.*

As in the baseline environment without uncertainty, the proof relies on the cost of another dollar of debt decreasing as debt increases and on gains from trade being negative for high debt. With uncertainty, the debt level and the credit line price adjust to ensure that the analogous conditions hold state by state: When new gains from trade arise, the credit line is repriced to ensure that B issues the new debt with its pre-existing (original) lender.

6 Empirical Analysis

Here we summarize and test our main empirical predictions. We use data on loan originations from Dealscan and data on firm capital structure and characteristics from Capital IQ, Compustat, and CRSP. Our sample ranges from 2002 through 2019, unless otherwise stated. See Appendix B for details on the data and variable construction.

Prediction 1. *Credit lines are bundled with loans.*

This follows from Proposition 5. Bundling gives the borrower better loan terms.

Empirically, we find that indeed 72% of term loans in our sample are bundled with a credit line at origination.

Prediction 2. *Credit lines are more likely to be bundled with loans for borrowers with high dilution risk.*

This also follows from Proposition 5, albeit less directly. Bundling is a commitment device not to dilute; therefore, it should be valuable only to the extent to which lenders are at risk of dilution.

Empirically, we use two proxies for dilution risk. The first is default risk, since there is no dilution without default. The second is lender competition, since there is no dilution without competing lenders. We measure default risk using Dealscan's borrower market segment classification and competition via the Herfindahl–Hirschman index (HHI); see Appendix B. Figure 1 shows that that indeed bundling is more likely for risky firms by plotting the share of term loans bundled with a credit line across borrower market segments in our sample. Figure 2 shows that bundling is also more likely in competitive markets by plotting a bin scatter of the regression for lender l in state s in year t,

$$\text{bundling propensity}_{lst} = \alpha_{lt} + \beta \text{HHI}_{st} + \varepsilon_{lst}, \tag{36}$$

which reveals a negative relationship between bundling and market concentration for a given lender. Here the outcome variable is the proportion of term loan deals that include a credit line among deals that lender l originates in state s in year t; HHI_{st} is the syndicated loan market HHI in state s in year t; and α_{lt} is a lender-year

fixed effect.

Prediction 3. *Credit lines are large.*

This follows from Proposition 4. To deter issuance, credit lines must be large enough to deplete the gains from trade between B and the lenders if drawn.

Empirically, we find that indeed undrawn credit lines are, on average, 80% of book equity, 20% of market equity, and, at origination, 1.7 times the size of the term loan that they are bundled with. Table 1 reports the distribution of these ratios for our sample.

Prediction 4. *Credit lines are larger for borrowers with high dilution risk.*

This also follows from Proposition 4, albeit less directly, by the same rationale as for Prediction 2.

Empirically, we proxy for default risk via cost of debt and lender competition as above (see Prediction 2). Figure 3 shows that indeed undrawn credit lines are larger relative to book or market equity for riskier firms, plotting these ratios across quartiles of interest expense over total liabilities by firm-year for our sample.[12] Figure 4 shows that firms' credit lines are also larger at origination relative to their bundled term loans in more competitive markets by plotting a bin scatter of the regression for lender l in state s in year t,

$$\left(\frac{\text{credit line volume}}{\text{term loan volume}}\right)_{lst} = \alpha_{lt} + \beta \text{HHI}_{st} + \varepsilon_{lst}, \tag{37}$$

which reveals a negative relationship between the relative size of the bundled credit line and market concentration for a given lender. Here the outcome variable is the

[12]For this analysis, we proxy for default risk using cost of debt rather than Dealscan market classifications, as measuring undrawn credit lines requires Capital IQ capital structure data instead of Dealscan origination data.

average volume of lender l's credit lines relative to the bundled term loans in state s in year t; HHI_{st} is the syndicated loan market HHI in state s in year t; and α_{lt} is a lender-year fixed effect.

Prediction 5. *Credit lines are rarely drawn.*

This follows from Proposition 4. In equilibrium credit lines are not drawn, but are merely an off-equilibrium threat.

Empirically, we find that indeed 45% of borrowers utilize none of their credit line and the median utilization is 16%. Even in crisis times, when theories of credit lines as liquidity insurance suggest high utilization, the majority (60%) of credit lines are not used. Figure 5 and Table 2 report the distribution of credit line utilization in our sample overall, during the Great Financial Crisis in 2008, and at the onset of the COVID-19 pandemic in Q1 2020.

Prediction 6. *Credit lines deter subsequent borrowing.*

This follows from Proposition 5. Credit lines allow borrowers to commit not to dilute.

Empirically, we test the prediction using two natural experiments. First, we use quasi-random variation in credit lines due to the Supplementary Leverage Ratio (SLR), which increased costs of credit lines for some banks in 2018 (see Appendix B). In Table 3 we estimate the following regression for firm i's deal d in year t,

$$\text{Y}_{idt} = \alpha_i + \beta \text{SLR}_{idt} \times \text{post}_t + \gamma \text{SLR}_{idt} + \delta \text{post}_t + \theta X_{idt} + \varepsilon_{idt}. \tag{38}$$

Here, Y_{idt} is an indicator for whether the term loan is bundled with a credit line in deal d (Columns 1–2) or whether firm i borrows again within one year (Columns 3–4); SLR_{idt} equals 1 if any lenders in the deal are subject to the SLR; post_t equals 1 if

t is 2018 or later; X_{idt} includes the average number of deals that lenders participating in deal d engage in during year t to capture cross-sectional differences in lenders; α_i is a firm fixed effect. We restrict our sample to non-refinancing loan packages originated from 2010 to 2020 to focus on new borrowing decisions around the SLR shock. We first note, akin to a "first stage" showing relevance, that firms borrowing from affected banks are less likely to use credit lines (Columns 1-2). We then find that indeed such firms are more likely to borrow within a year (Columns 3-4).

Our second empirical approach uses quasi-random variation in the likelihood that credit lines are revoked as, per Proposition 8, the borrower in our model takes more debt when revocation is more likely. We proxy for an increase in revocation risk with a negative shock to lender health (see Chodorow-Reich and Falato (2022)). Following Chodorow-Reich (2014) and Darmouni (2020), we construct an overall lender health shock and, analogously, a credit line lender health shock for each borrower (see Appendix B). In Table 4 we estimate the following regression for firm i,

$$\text{new debt}_i = \alpha + \beta \text{shock CL}_i + \gamma \text{shock}_i + \delta X_i + \varepsilon_i, \tag{39}$$

where the outcome variable is an indicator for borrowing in the syndicated loan market in the crisis period; controls X_i include the number of syndicates a firm borrowed from and a credit line usage indicator during the normal period. The definition of crisis and normal periods are described in Appendix B. We find that indeed borrowers take on more debt following an increase in the likelihood of credit line revocation (a negative shock to credit line lenders), so that $\beta > 0$. However, we also replicate the finding in the literature that a negative shock to all lenders decreases borrowing, $\gamma < 0$, suggesting our finding is specific to credit lines.

Prediction 7. *Rates on credit lines are relatively low ($\tilde{p}$ is relatively high).*

This follows from re-writing the indifference condition (18), substituting for V, recalling that c is concave, and using the condition for $\Gamma(Q^e + \mathrm{d}\tilde{Q})$ in Proposition 4:

$$\tilde{p} = \frac{1}{\rho}\frac{c(Q^e + \mathrm{d}\tilde{Q}) - c(Q^e)}{\mathrm{d}\tilde{Q}} > \frac{1}{\rho}c'(Q^e + \mathrm{d}\tilde{Q}) > \Gamma(Q^e + \mathrm{d}\tilde{Q}). \tag{40}$$

That says that, conditional on the line being drawn, the price $\tilde{p}$ that the lender commits to pay is higher than its value to the lender $\Gamma(Q^e + \mathrm{d}\tilde{Q})$.

Empirically, we find that there is no difference between the median rate on the credit line and that on the associated term loan (Figure 6 and Table 5 report the within-package difference in rate spreads between drawn credit lines and term loans in our sample, in basis points). That indeed seems low given that in our sample drawing increases leverage on average by 20% (cf. Prediction 3; see Appendix B for our construction of leverage). We confirm as much by estimating the counterfactual rate if leverage were to increase by that amount, which corresponds to an absolute increase in leverage of 0.04 in our data. Using Schwert's (2020) estimate of the semielasticity of interest rates w.r.t. leverage of 1.36, we calculate an implied increase of about 5.44% ($= 0.04 \times 1.36$). That is indeed larger than the difference of zero we observe.

7 Conclusion

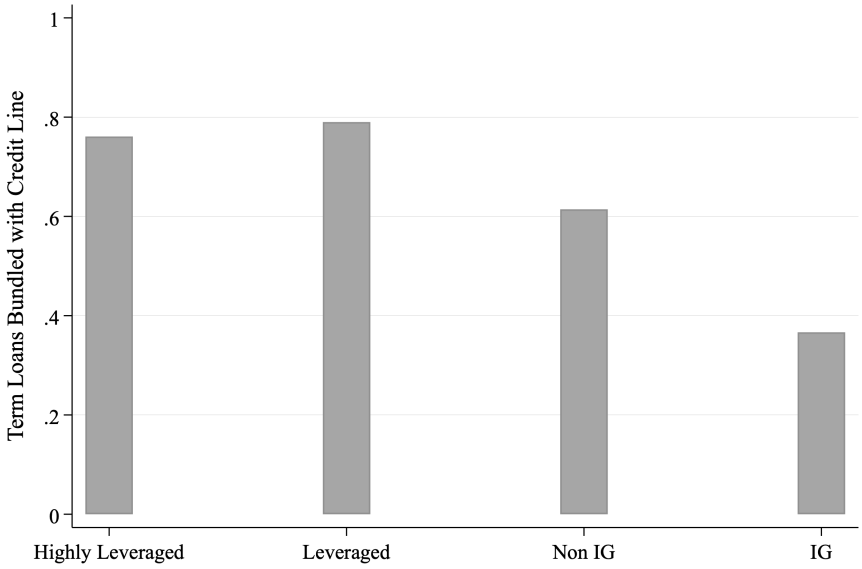
We study a model that suggests that credit lines play a heretofore overlooked role. They can mitigate debt dilution. The theory suggests that the option to borrow—viz. a credit line—is valuable even if it is never exercised, explaining why credit lines are ubiquitous but rarely drawn. It also underscores how and why credit lines should be bundled with loans, a pervasive practice never previously studied in the

theory literature.

Our paper contrasts with recent corporate finance papers on the leverage ratchet effect, suggesting that including credit lines in the contracting space makes the ratchet effect self-deterring, undoing its negative effects. It also contrasts with the literature on latent contracts, suggesting that the kinds of outcomes stressed in that literature might not obtain in dynamic environments under limited commitment.

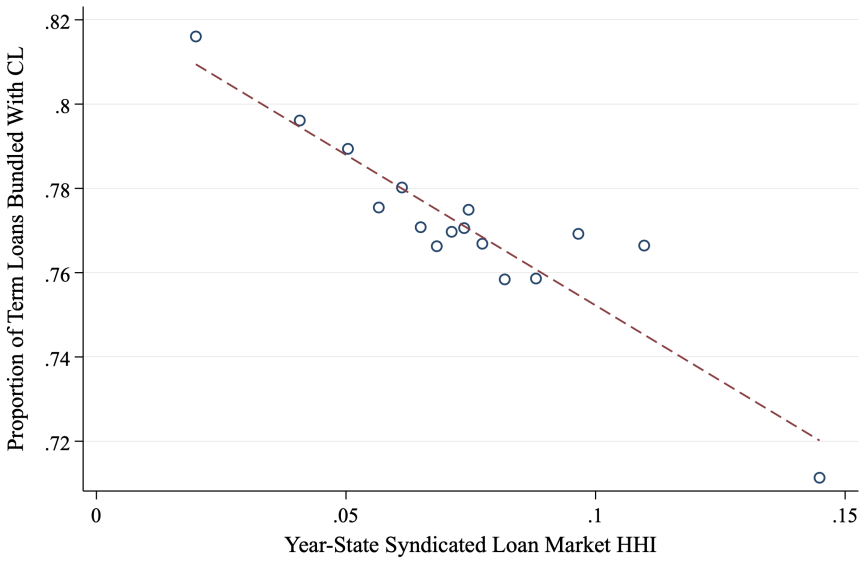
A Tables and Figures

Figure 1: Bundling Propensity is Increasing in Firm Riskiness



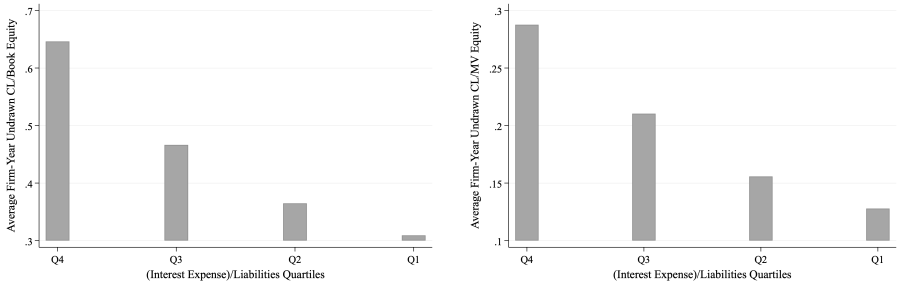
This figure shows how the proportion of term loans that are bundled with a credit line vary by firm riskiness, as measured by Dealscan's classification of firms' market segments. The loans to the safest borrowers are Investment Grade ("IG"), then "Non-IG", "Leveraged", and "Highly Leveraged" respectively, where the distinction between the latter three categories depends on pricing thresholds. Data are from Dealscan, covering US C&I syndicated loans from 2002 through 2019 and excluding financials.

Figure 2: Bundling Propensity is Increasing in Competition



This figure plots a bin scatter of the following regression for lender l, bundling propensity$_{lst} = \alpha_{lt} + \beta \text{HHI}_{st} + \varepsilon_{lst}$, where bundling propensity$_{lst}$ is the proportion of term loan deals that include a credit line among deals that lender l originates in state s in year t; HHI$_{st}$ is the syndicated loan market HHI in state s in year t; and α_{lt} is a lender-year fixed effect. Data are from Dealscan, covering US C&I syndicated loans from 2002 through 2019 and excluding financials.

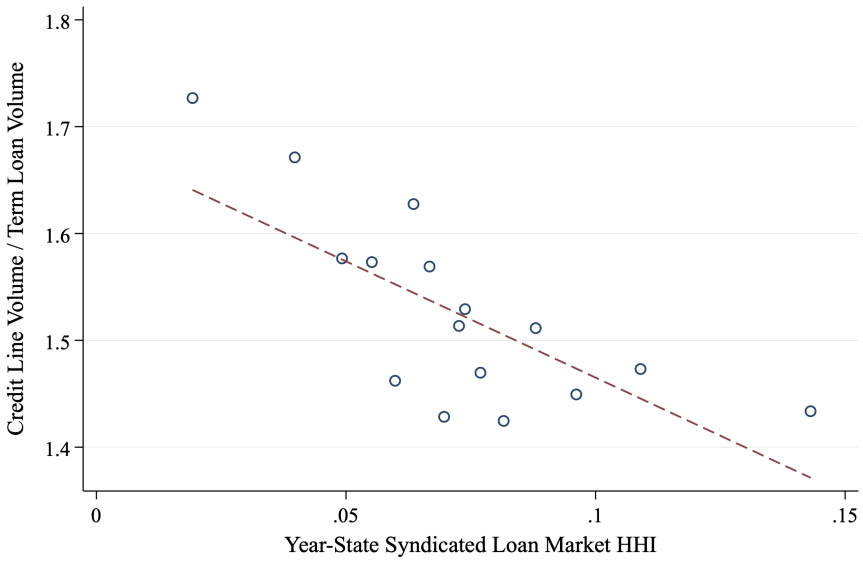
Figure 3: Credit Line Size is Increasing in Firm Riskiness



(a) As Share of Book Equity (b) As Share of Market Equity

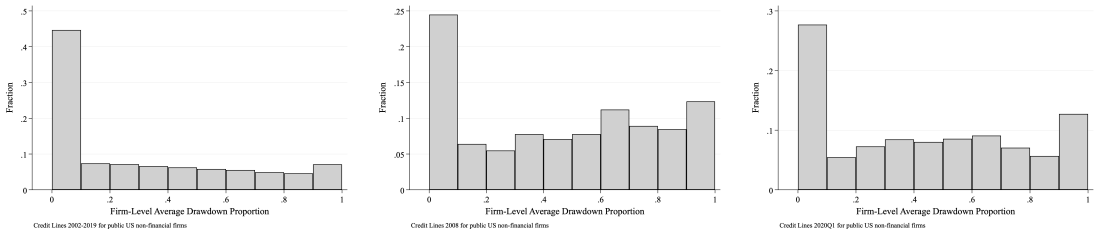
This figure plots the average firm-year undrawn CL as a share of (a) book equity and (b) market equity, by quartiles of (firm interest expense)/(total liabilities), and where Q4 represents firms with the highest interest expense. Data are annual at the firm level from Capital IQ and Compustat from 2002 through 2019, covering US public non-financial firms. Stock price data comes from CRSP.

Figure 4: Credit Line Size is Increasing in Competition



This figure plots a bin scatter of the following regression specification for lender l, $\left(\frac{\text{credit line volume}}{\text{term loan volume}}\right)_{lst} = \alpha_{lt} + \beta \text{HHI}_{st} + \varepsilon_{lst}$, where $(\text{credit line volume/term loan volume})_{lst}$ is the average volume of lender l's credit lines relative to the bundled term loans in state s in year t HHI_{st} is the syndicated loan market HHI in state s in year t; and α_{lt} is a lender-year fixed effect. Data are from Dealscan, covering US C&I syndicated loans from 2002 through 2019 and excluding financials. Average credit line volume is winsorized at 2%.

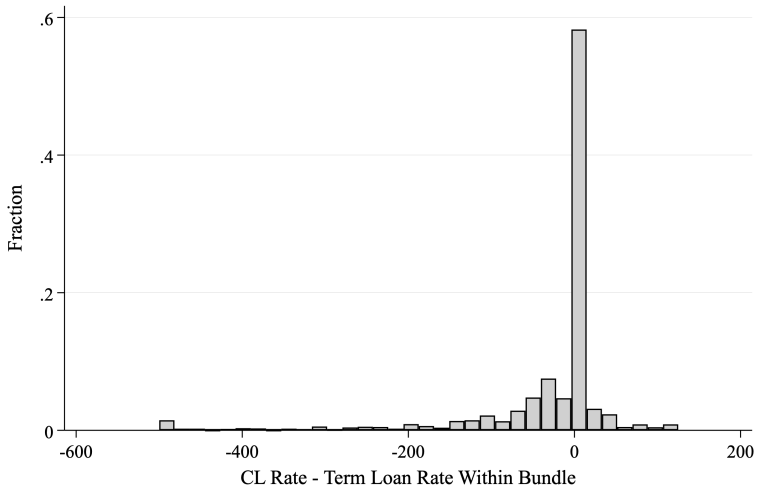
Figure 5: Credit Lines are Rarely Drawn



(a) Unconditional (b) Crisis: 2008 (c) Crisis: Covid-19

Panel (a) shows the annual utilization of credit lines from 2002 through 2019, panel (b) shows the annual utilization of credit lines during 2008, panel (c) shows the quarterly utilization of credit lines during 2020 Q1. Data are from Capital IQ and Compustat, covering US public non-financial firms.

Figure 6: Rates on Credit Lines are Relatively Low



This figure plots the rate spread for credit lines (when drawn) minus the rate spread for term loans within a package, in basis points. Data are at the loan package level from Dealscan from 2002 through 2019, covering US C&I syndicated loans and excluding financials. Differences are winsorized at 1%.

Table 1: Credit Lines Are Large

	Mean	p1	p10	p25	p50	p75	p90	p99
Credit Line / Term Loan	1.725	0.029	0.088	0.167	0.480	1.519	4.000	18.301
Undrawn CL /Book Equity	0.808	0.000	0.032	0.110	0.233	0.438	0.814	4.872
Undrawn CL /Market Equity	0.208	0.000	0.015	0.046	0.106	0.223	0.418	1.641

The table reports various measures of the size of firms' undrawn credit lines. Data for the first row are at the loan package level from Dealscan from 2002 through 2019, and for the remainder of the table are annual at the firm level from Capital IQ and Compustat from 2002 through 2019, covering US public non-financial firms.

Table 2: Credit Lines are Rarely Drawn

	mean	p10	p25	p50	p75	p90
2002–2019	0.296	0.000	0.000	0.170	0.548	0.836
2008 GFC	0.460	0.000	0.104	0.474	0.752	0.927
2021Q1 Covid	0.425	0.000	0.045	0.416	0.707	0.953

The table reports the distribution of credit line utilization unconditionally (2002 through 2019) and during crisis times (2008 and 2020Q1). Data are from Capital IQ and Compustat covering US public non-financial firms, and are annual at the firm level for the first two rows and quarterly at the firm level for the Covid period.

Table 3: Credit Lines Deter Subsequent Borrowing: At Origination

	Bundling Propensity		Subsequent Borrowing	
	(1)	(2)	(3)	(4)
SLR × Post	-0.09^{***}	-0.10^{***}	0.07^{**}	0.07^{*}
	(0.03)	(0.04)	(0.03)	(0.04)
SLR	-0.03	0.02	0.02	-0.05
	(0.02)	(0.04)	(0.02)	(0.05)
Post	0.02	0.01	-0.05^{***}	-0.12^{***}
	(0.02)	(0.03)	(0.02)	(0.04)
Avg Lender Deals	0.00	-0.00	-0.00	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Firm FE	No	Yes	No	Yes
Observations	6220	4101	6220	4101
Adjusted R^2	0.004	0.565	0.002	0.061

The table considers regressions of the form, $\mathrm{Y}_{idt} = \alpha_i + \beta \mathrm{SLR}_{idt} \times \mathrm{post}_t + \gamma \mathrm{SLR}_{idt} + \delta \mathrm{post}_t + \theta X_{idt} + \varepsilon_{idt}$. Y_{idt} is an indicator variable that is 1 if the term loan is bundled with a credit line in deal d in columns (1) and (2); and an indicator variable that is 1 if firm i borrows again within 1 year of deal d in columns (3) and (4); SLR_{it} is 1 if any lenders are subject to the SLR at the time deal d is originated; post_t is an indicator variable that is 1 if t is 2018 or later; X_{idt} includes the average number of deals that lenders participating in deal d engage in during year t to capture cross-sectional differences in lenders; α_i is a firm fixed effect. Standard errors are clustered at the firm level and reported in parentheses. Data are at the loan package level from Dealscan cover US non-financial C&I syndicated loans from 2010 through 2020, and refinancings are excluded. One, two, and three stars indicate statistical significance at the 10%, 5%, and 1% level, respectively.

Table 4: Credit Lines Deter Subsequent Borrowing: Revocation

	(1)	(2)	(3)	(4)
Shock	-0.16^{***}	-0.17^{***}	-0.17^{***}	-0.18^{***}
	(0.05)	(0.05)	(0.05)	(0.05)
Shock CL		0.03^{***}	0.02^{**}	0.03^{***}
		(0.01)	(0.01)	(0.01)
Number of Syndicates			0.03^{***}	0.04^{***}
			(0.01)	(0.01)
Pre CL Indic				-0.03^{**}
				(0.01)
Constant	0.20^{***}	0.19^{***}	0.19^{***}	0.18^{***}
	(0.04)	(0.04)	(0.04)	(0.04)
Observations	4883	4883	4883	4883
Adjusted R^2	0.002	0.003	0.009	0.010

The table reports the effect of credit line revocation risk on firm borrowing the regression, new $\text{debt}_i = \alpha + \beta \text{shock}_i + \gamma \text{shock CL}_i + \delta X_i + \varepsilon_i$. The outcome variable is an indicator for borrowing in the syndicated loan market in the crisis period. The construction of the shocks as well as definition of crisis and normal periods are described in Appendix B. The controls include the number of syndicates firm i borrowed from during normal period (capturing borrowers' access to/need for credit) and an indicator variable tracking if firm borrowed CL in normal period (capturing general differences between borrowers that do/do not use credit lines). Observations are at the firm level. Robust standard errors are reported in parentheses. Two and three stars indicate statistical significance at the 5% and 1% level, respectively.

Table 5: Rates on Credit Lines Are Relatively Low

	mean	p10	p25	p50	p75	p90
CL Rate - Term Loan Rate	-33.663	-122.195	-25.000	0.000	0.000	4.779
Observations	2870					

The table reports the rate spread for credit lines (when drawn) minus the rate spread for term loans within a package, (including the utilization fee) in basis points. Data are at the loan package level from Dealscan from 2002 through 2019 covering US C&I syndicated loans and excluding financials, and are winsorized at 1%.

B Details on Empirical Methodology

Here we describe details omitted from Section 6.

B.1 Data

For information on loan originations, we start with the universe of US C&I syndicated loans in DealScan. Our sample ranges from 2002 through 2019, unless otherwise stated. We classify US C&I loans to be loans that are originated in the US and for which the deal purpose is listed as "general purpose" or "working capital". We exclude loans to financials.

We retrieve information on firm capital structure using Capital IQ data from 2002 through 2019, and keep Annual filings (as opposed to other interim filings such as on quarter ends). We keep the latest available filing for each financial period. We keep firms that merge to Compustat, resulting in a sample of US non-financial public firms. Additionally, we use quarterly data from Capital IQ for the first quarter of 2020 to examine credit line usage at the onset of the the Covid-19 market disruption.

Finally, we observe firms' yearly market value of equity by merging our sample of US non-financial public firms to monthly CRSP equity data, and by taking the year-average of the market capitalization of the firm for the corresponding year.

B.2 Variable Construction

Borrower risk. We proxy for borrower risk using Dealscan's borrower market segment classifications, which range from safest (Investment Grade) to riskiest (Highly Leveraged). Dealscan's borrower market segment classifications depend on pricing thresholds as follows. Investment grade: Loan to borrowers rated BBB-/Baa3 or

higher with pricing thresholds based on market trends which change over time. Non Investment grade: A loan which is not considered Investment Grade. Leveraged: Loan to borrowers rated BB+/Ba1 or lower with pricing thresholds based on market trends which change over time. Highly Leveraged: Leveraged loan with pricing 100 bps higher than the pricing thresholds set forth for a leveraged loan.

Loan market competition. We proxy for local loan market competition with the Herfindahl-Hirschman Index (HHI) of the syndicated loan market for each state s and year t. The HHI is calculated as the sum of squared market shares for all lenders in our sample in state s and year t. In calculating lender market shares, we impute missing shares following Chodorow-Reich (2014) and Darmouni (2020), by calculating the average lender share of lead arrangers and participants separately for each syndicate structure during the time period, among syndicates that do not have missing lender shares.[13] We then fill in the missing lender shares with the average lender shares calculated for the corresponding syndicate structure.

Lender health shocks. For each borrower, we construct overall lender health shocks following Chodorow-Reich (2014) and Darmouni (2020) and, analogously, for lenders with credit lines outstanding to the borrower. Specifically, we define $\Delta L_{b,-i}$ as the decrease in a bank b's lending to firms $j \neq i$ in the crisis period vis-à-vis normal times:

$$\Delta L_{b,-i} := 1 - \frac{2\sum_{j\neq i} L_{b,j,\text{crisis}}}{\sum_{j\neq i} L_{b,j,\text{normal}}}, \tag{41}$$

[13]By syndicate structure we mean the number of lead arrangers and the number of other participants in a syndicate.

where $L_{b,j}$ is the effective number of loan facilities from b to j during normal and crisis times, defined as 10/2005–6/2007 and 10/2008–6/2009, respectively. The effective number of loan facilities is the number of loan facilities originated, with each weighted by the corresponding lender share. We again impute lenders' shares following Chodorow-Reich (2014) and Darmouni (2020). We calculate the average lender share of lead arrangers and participants separately for each syndicate structure during the time period surrounding the Global Financial Crisis, from 2004 through 2010, among syndicates that do not have missing lender shares, and fill in the missing lender shares with the average lender shares calculated for the corresponding syndicate structure. We exclude refinancings and amendments (except extensions) during crisis times. We restrict the sample to firms that borrowed during the normal period and banks that are present in both the normal and crisis period. We winsorize $\Delta L_{b,-i}$ at 2%.

We then construct the shocks for each borrower i's lenders and credit line lenders as weighted sums of $\Delta L_{b,-i}$ over lenders in a borrower i's last pre-crisis syndicate:

$$\text{Shock}_i = \sum_{b \in S} \alpha_b \Delta L_{b,-i} \qquad \text{Shock CL}_i = \sum_{b \in S} \alpha_b^{\text{CL}} \Delta L_{b,-i}, \tag{42}$$

where S is the set of lenders in borrower i's last pre-crisis syndicate. For a lender $b \in S$, α_b is its average share across all loan facilities in the syndicate and α_b^{CL} is its share within the credit line facility. If borrower i's last pre-crisis syndicate has no CL or a CL that matures prior to 2008, we set $\text{Shock CL}_i = 0$.

SLR deal. We classify a loan deal to be subject to the SLR if any of the banks that participate in the deal are subject to the SLR at the time when the loan is originated.

Cost of loan deal. We calculate the rate for the a term loan as the all-in-spread-drawn for the term loan. We calculate the rate of drawing the credit line as the all-in-spread-undrawn for the credit line plus the utilization fee. If there are multiple term loans or credit lines in a loan deal, we take the average of these fees across all credit lines or term loans respectively.

B.3 Institutional Details

SLR. The SLR was introduced as part of the US implementation of the Basel III framework. The SLR was adopted in September 2014, and became effective on January 1st, 2018. It is a non risk-based leverage ratio that depends on the banks' total leverage exposure, which includes a bank's on and off-balance sheet exposures. Banks with greater than $250B in total assets must maintain a tier 1 leverage ratio that is above a certain threshold (for instance, for GSIBs this threshold is 5%).

$$\text{SLR} \equiv \frac{\text{Tier 1 Capital}}{\text{Total leverage exposure}} \tag{43}$$

The SLR is considered to be the binding leverage ratio for these banks; undrawn credit lines, which are off-balance sheet exposures, contribute to the denominator of the ratio.

C Proofs

C.1 Proof of Proposition 1

Here we apply a guess-and-verify approach: We assume that it is optimal to set $\mathrm{d}Q_t = 0$ for all $t > 0$, solve for the optimal Q_0, and then show the B cannot benefit by issuing again. Recall that Assumption 1 ensures that our commitment problem has unique solution and thus we can use the first-order approach.

Step 1: Optimal issuance at date 0. The optimal date-0 issuance as if B never issues debt again, $Q_0 = \mathrm{d}Q_0$, solves

$$p_0 + \frac{\mathrm{d}p_0}{\mathrm{d}Q_0} Q_0 - \int_0^\infty e^{-\rho t} c'(Q_0) \mathrm{d}t = 0. \tag{44}$$

Using lenders' participation constraint and computing the integrals (which is easy for $Q_t \equiv Q_0$), we can rearrange to find an expression for the optimal Q_0:

$$p_0 = \frac{\gamma(Q_0)}{\rho} = \frac{c'(Q_0) - \gamma'(Q_0) Q_0}{\rho}. \tag{45}$$

Step 2: No issuance at date $\tau > 0$. Now we verify that the marginal benefit from having $\mathrm{d}Q_\tau \neq 0$ for some $\tau > 0$ is zero. To do so, we differentiate the objective

$$p_0 Q_0 + \int_0^\tau e^{-\rho t} (y - c(Q_0)) \mathrm{d}t + p_\tau \mathrm{d}Q_\tau + \int_\tau^\infty e^{-\rho t} (y - c(Q_\tau)) \mathrm{d}t = \tag{46}$$

$$= p_0 \mathrm{d}Q_0 + \frac{(y - c(Q_0))}{\rho} (1 - e^{-\rho\tau}) + e^{-\rho\tau} p_\tau \mathrm{d}Q_\tau + e^{-\rho\tau} \frac{(y - c(Q_\tau))}{\rho} \tag{47}$$

where $Q_\tau = Q_0 + \mathrm{d}Q_\tau$. The FOC is

$$\frac{\mathrm{d}p_0}{\mathrm{d}Q_\tau}Q_0 + e^{-\rho\tau}p_\tau + e^{-\rho\tau}\frac{\mathrm{d}p_\tau}{\mathrm{d}Q_\tau}\mathrm{d}Q_\tau - \frac{e^{-\rho\tau}}{\rho}c'(Q_0 + \mathrm{d}Q_\tau) = 0. \tag{48}$$

Now observe that $\mathrm{d}p_0/\mathrm{d}Q_\tau = e^{-\rho\tau}\mathrm{d}p_\tau/\mathrm{d}Q_\tau$, because

$$p_0 = \int_0^\tau e^{-\rho t}\gamma(Q_0)\mathrm{d}t + \int_\tau^\infty e^{-\rho t}\gamma(Q_\tau)\mathrm{d}t = \int_0^\tau e^{-\rho t}\gamma(Q_0)\mathrm{d}t + e^{-\rho\tau}p_\tau. \tag{49}$$

So, substituting from above and canceling the $e^{-\rho\tau}$, the FOC reads

$$\frac{\mathrm{d}p_\tau}{\mathrm{d}Q_\tau}Q_0 + p_\tau + \frac{\mathrm{d}p_\tau}{\mathrm{d}Q_\tau}\mathrm{d}Q_\tau - c'(Q_0 + \mathrm{d}Q_\tau) = 0. \tag{50}$$

Using $Q_\tau = Q_0 + \mathrm{d}Q_\tau$, we have

$$p_\tau + \frac{\mathrm{d}p_\tau}{\mathrm{d}Q_\tau}Q_\tau = \big(p_\tau Q_\tau\big)' = \frac{c'(Q_0 + \mathrm{d}Q_\tau)}{\rho}. \tag{51}$$

I.e. the equation for the static optimum (45). □

C.2 Proof of Proposition 2

The expression for V in equation (14) follows from substituting the equation from the optimal control (equation (13)) into the continuous-time HJB (equation (12)). The equation for p in (15), comes from differentiating the equation for V (equation (14)) and replacing V' with $-p$ from equation (13).

The issuance policy follows form the law of motion for the price,[14]

$$p(Q) = \gamma(Q) + p'(Q)q. \tag{52}$$

Using $p = c'/\rho$, differentiating and rearranging, gives the expression for q in the proposition. □

C.3 Proof of Corollary 1

The result follows from Proposition 2 and the assumption that $c'' < 0$. □

C.4 Proof of Proposition 3

We show the result via the following lemmata.

- In Lemma C.1, we rule out smooth equilibria in which B does not draw. That allows us to consider only drawing or jumping.

- In Lemma C.2, we rule out equilibria in which B does not draw today but draws later on. That allows us to consider either drawing immediately or never.

  At that point, we have the corollary that, given our focus on Markov equilibria, in any candidate equilibrium in which B does not draw, there must be an absorbing state.

- In Lemma C.3, we show that for Q high, B does not jump to any absorbing state $Q^+ > \mathcal{Q}$.

[14] That equation, which can be seen as the Black–Scholes differential equation for a derivative with price p written on an underlying Q following $\mathrm{d}Q_t = q_t \mathrm{d}t$, is the limit of the standard discounting formula $p(Q) = \gamma(Q)\mathrm{d}t + e^{-\rho \mathrm{d}t} p(Q + \mathrm{d}Q)$.

- In Lemma C.4, we show, symmetrically, that for Q high, B does not jump to any absorbing date $Q^- < \mathcal{Q}$ under a condition on prices, which we show holds in Lemma C.5 and Lemma C.6.

- Finally, in Lemma C.7, we show that B does not draw for $Q < \mathcal{Q}$.

The proposition follows because, at that point, we have that for $Q > \mathcal{Q}$, B either prefers to draw or to jump to absorbing state $\mathcal{Q}$ and B weakly prefers to draw at $\mathcal{Q}$ by the definition of the associated credit line.

Lemma C.1. *Suppose* $(\tilde{p}, \mathrm{d}\tilde{Q})$ *satisfies* (17)*. Suppose also that the policy is smooth and the value function* $\tilde{V}$ *is differentiable. B prefers to draw for* $Q > \mathcal{Q}$*.*

Proof. Suppose (in anticipation of a contradiction) that B strictly prefers not to draw at Q. By continuity, there is a neighborhood of Q for which he strictly prefers not to draw. By smoothness, near Q, $\tilde{V}$ satisfies the HJB as if there were no credit line (i.e. $\rho V(Q) = y + p(Q)q - c(Q) + V'(Q)q$). That equation has the solution $\rho\tilde{V}(Q) = y - c(Q)$. Given c is concave, from (17) we get, $\tilde{V}(Q) \leq \tilde{p}\mathrm{d}\tilde{Q} + V(Q + \mathrm{d}\tilde{Q})$. That is the desired contradiction. □

Lemma C.2. *Suppose* $(\tilde{p}, \mathrm{d}\tilde{Q})$ *satisfies* (17)*. If B prefers to draw at* t^+*, he prefers to draw at* $t < t^+$*.*

Proof. Suppose (in anticipation of a contradiction) that B prefers to draw at $t^+ = t + \mathrm{d}t$ but not at t. Keeping in mind that, by Lemma C.1, it suffices to focus on jumps and denoting Q_t by Q and Q_{t^+} by Q^+, that says:

$$\tilde{p}\mathrm{d}\tilde{Q} + V(Q + \mathrm{d}\tilde{Q}) < \tilde{V}(Q) \leq p^+(Q^+ - Q) + \tilde{p}\mathrm{d}\tilde{Q} + V(Q^+ + \mathrm{d}\tilde{Q}), \qquad (53)$$

where p is the issuance price and the discount rate ρ vanishes because $o(e^{-\rho \mathrm{d}t}) = o(\mathrm{d}t) < o(\mathrm{d}Q)$.

Since B draws immediately at t^+, we know that p and V are given by Proposition 2: $V(Q) = (y - c(Q))/\rho$ and $p = c'(Q)/\rho$. Substituting and rearranging, we have that

$$c(Q^+ + \mathrm{d}\tilde{Q}) - c(Q + \mathrm{d}\tilde{Q}) < c'(Q^+ + \mathrm{d}\tilde{Q})(Q^+ - Q). \tag{54}$$

There are three cases: $Q^+ = Q$, $Q^+ > Q$, and $Q^+ < Q$. In the first, the inequality says $0 < 0$, which is false. We rule out the other two in turn:

- If $Q^+ > Q$, then equation (54) says

$$\frac{c(Q^+ + \mathrm{d}\tilde{Q}) - c(Q + \mathrm{d}\tilde{Q})}{Q^+ - Q} < c'(Q^+ + \mathrm{d}\tilde{Q}). \tag{55}$$

  That contradicts the concavity of c, as desired.

- If $Q^+ < Q$, then equation (54) says

$$\frac{c(Q + \mathrm{d}\tilde{Q}) - c(Q^+ + \mathrm{d}\tilde{Q})}{Q - Q^+} > c'(Q^+ + \mathrm{d}\tilde{Q}). \tag{56}$$

  That also contradicts the concavity of c, as desired.

□

The analysis so far says that it suffices to compare drawing immediately to a sequence of jumps $\{Q_{t_n}\}_n$. But, by the Markov assumption, at each Q_{t_n}, B either jumps immediately or stays there forever. (Staying at Q_{t_n} for an interval and then jumping later would violate the Markov assumption, which implies that at Q_{t_n} you either jump or not.) Immediate jumps are telescoping—they cancel out. So it is

sufficient to compare drawing to jumping to an absorbing state. That is what we do in the next two lemmata.

Lemma C.3. *There is no absorbing state with $Q^+ > \mathcal{Q}$.*

Proof. Suppose (in anticipation of a contradiction) that Q^+ is an absorbing state. B must prefer not to draw at Q^+:

$$\frac{y - c(Q^+)}{\rho} \geq \tilde{p}\mathrm{d}\tilde{Q} + V(Q^+ + \mathrm{d}\tilde{Q}) \tag{57}$$

or, substituting $V(Q)$ from Proposition 2,

$$\frac{y - c(Q^+)}{\rho} \geq \tilde{p}\mathrm{d}\tilde{Q} + \frac{y - c(Q^+ + \mathrm{d}\tilde{Q})}{\rho} \tag{58}$$

or

$$\frac{1}{\rho}\frac{c(Q^+ + \mathrm{d}\tilde{Q}) - c(Q^+)}{\mathrm{d}\tilde{Q}} \geq \tilde{p}. \tag{59}$$

Combined with condition (18), that implies that

$$\frac{1}{\rho}\frac{c(Q^+ + \mathrm{d}\tilde{Q}) - c(Q^+)}{\mathrm{d}\tilde{Q}} \geq \frac{1}{\rho}\frac{c(\mathcal{Q} + \mathrm{d}\tilde{Q}) - c(\mathcal{Q})}{\mathrm{d}\tilde{Q}}, \tag{60}$$

which, given $Q^+ > \mathcal{Q}$, contradicts the concavity of c. □

Lemma C.4. *Suppose $\mathcal{P} < \gamma(Q^-)/\rho$ for $Q^- < \mathcal{Q}$, where $\mathcal{P}$ is the price at $\mathcal{Q}$. B prefers to jump to $\mathcal{Q}$ than to any possible absorbing state $Q^- < \mathcal{Q}$.*

Proof. We want to show that

$$\mathcal{P}(\mathcal{Q} - Q) + \tilde{V}(\mathcal{Q}) \geq p^-(Q^- - Q) + \frac{y - c(Q^-)}{\rho} \tag{61}$$

As B always has the option not to draw, his payoff from keeping the option $\tilde{V}$ must be at least high as that from exercising it:

$$\tilde{V}(\mathcal{Q}) \geq \tilde{p}\mathrm{d}\tilde{Q} + V(\mathcal{Q} + \mathrm{d}\tilde{Q}) = V(\mathcal{Q}), \tag{62}$$

where the last equality is condition (17). Thus, it suffices to show that inequality (61) holds for $V(\mathcal{Q})$ in place of $\tilde{V}(\mathcal{Q})$:

$$\mathcal{P}(\mathcal{Q} - Q) + V(\mathcal{Q}) \geq p^{-}(Q^{-} - Q) + \frac{y - c(Q^{-})}{\rho}. \tag{63}$$

Substituting in for V from Proposition 2,

$$\mathcal{P}(\mathcal{Q} - Q) + \frac{y - c(\mathcal{Q})}{\rho} \geq p^{-}(Q^{-} - Q) + \frac{y - c(Q^{-})}{\rho} \tag{64}$$

Now given that $Q - \mathcal{Q} > 0$, if $p^{-} > \mathcal{P}$, it suffices to show the inequality with $\mathcal{P}$ replaced with p^{-}:

$$p^{-}(Q - Q^{-}) - p^{-}(Q - \mathcal{Q}) \geq \frac{1}{\rho}\big(c(\mathcal{Q}) - c(Q^{-})\big) \tag{65}$$

or

$$p^{-}(\mathcal{Q} - Q^{-}) \geq \frac{1}{\rho}\big(c(\mathcal{Q}) - c(Q^{-})\big) \tag{66}$$

or, given $\mathcal{Q} > Q^{-}$,

$$p^{-} \geq \frac{1}{\rho}\frac{c(\mathcal{Q}) - c(Q^{-})}{\mathcal{Q} - Q^{-}}. \tag{67}$$

Recall that Q^{-} is a supposed absorbing state, so $p^{-} = \gamma(Q^{-})/\rho$. The inequality becomes

$$\gamma(Q^{-}) \geq \frac{c(\mathcal{Q}) - c(Q^{-})}{\mathcal{Q} - Q^{-}}, \tag{68}$$

a sufficient condition for which is

$$\gamma(Q^-) \geq c'(Q^-), \tag{69}$$

which is satisfied for Q^- below Q^e (cf. Proposition 1 and condition (17)).

□

Lemma C.5. *Suppose that Q^0 and Q^1 are both absorbing states with $Q^0 < Q^1 < Q^e$. B prefers to jump from $Q > Q^1$ to Q^1 than to Q^0.*

Proof. We want to show that

$$p^1(Q^1 - Q) + \frac{y - c(Q^1)}{\rho} > p^0(Q^0 - Q) + \frac{y - c(Q^0)}{\rho} \tag{70}$$

or, rearranging,

$$\frac{1}{\rho}\Big(c(Q^1) - c(Q^0)\Big) < p^0(Q - Q^0) - p^1(Q - Q^1) \tag{71}$$

$$= p^0(Q - Q^0) \underbrace{-p^0(Q - Q^1) + p^0(Q - Q^1)}_{=0} - p^1(Q - Q^1) \tag{72}$$

$$= p^0(Q^1 - Q^0) + (p^0 - p^1)(Q - Q^1). \tag{73}$$

That holds whenever

$$\frac{1}{\rho}\frac{c(Q^1) - c(Q^0)}{Q^1 - Q^0} < p^0 + (p^0 - p^1)\frac{Q - Q^1}{Q^1 - Q^0}. \tag{74}$$

Given c is concave, $p^0 = \gamma(Q^0)/\rho$, and the last term is positive ($p^0 > p^1$, $Q > Q^1$, and $Q^1 > Q^0$), a sufficient condition is that

$$c'(Q^0) < \gamma(Q^0), \tag{75}$$

which is satisfied since $Q^0 < Q^e$. □

Lemma C.6. *For $Q^- < \mathcal{Q}$, $p^- > \mathcal{P}$.*

Proof. Suppose (in anticipation of a contradiction) that $p^- < \mathcal{P}$. Given the arguments above, in particular the "corollary" argument following Lemma C.2), we know three things:

1. Q_t must jump down from $\mathcal{Q}$ (otherwise the price would be higher);
2. Q_t must jump to an absorbing state immediately, say $Q^\flat$ (otherwise the policy would either be smooth, violating Lemma C.1, or would stay at a point for some time before jumping, violating the Markov property);
3. the absorbing state must be below Q^-, $Q^\flat < Q^-$ (otherwise the price at $Q^\flat$ would be above that at Q^-, violating the hypothesis that $\mathcal{P} > p^-$).

B's policy must, of course, be optimal: At $\mathcal{Q}$, he must prefer to jump to $Q^\flat$ than to Q^-. But that contradicts Lemma C.5. We conclude that $p^- > \mathcal{P}$, as desired. □

Lemma C.7. *For $Q < \mathcal{Q}$, B prefers not to draw.*

Proof. Here we show that when $Q < \mathcal{Q}$, B prefers to jump to $\mathcal{Q}$ than to draw the credit line (which implies only that drawing the line is not optimal, not that jumping to $\mathcal{Q}$ is):

$$\tilde{p}\mathrm{d}\tilde{Q} + V\big(Q + \mathrm{d}\tilde{Q}\big) < p\left(\mathcal{Q}\right)\left(\mathcal{Q} - Q\right) + \tilde{V}(\mathcal{Q}). \tag{76}$$

As $V(\mathcal{Q}) \leq \tilde{V}(\mathcal{Q})$ from (62), it suffices to show the inequality holds with $\tilde{V}$ replaced by V. Substituting V from Proposition 2 as well and rearranging, it must be that

$$\tilde{p}\mathrm{d}\tilde{Q} < \frac{\gamma\left(\mathcal{Q}\right)}{\rho}\left(\mathcal{Q} - Q\right) + \frac{c\big(Q + \mathrm{d}\tilde{Q}\big) - c\big(\mathcal{Q}\big)}{\rho}. \tag{77}$$

The result follows from observing that the RHS above exceeds the LHS for all $Q < \mathcal{Q}$. To see that, observe first that it holds with equality at $Q = \mathcal{Q}$ and that the LHS does not depend on Q whereas the RHS is decreasing in it; that follows from differentiation:

$$\frac{c'\left(Q + \mathrm{d}\tilde{Q}\right)}{\rho} - \frac{\gamma\left(\mathcal{Q}\right)}{\rho} < \frac{c'\left(\mathrm{d}\tilde{Q}\right)}{\rho} - \frac{\gamma\left(Q^e\right)}{\rho} < 0, \tag{78}$$

given $\mathrm{d}\tilde{Q}$ is large, $\mathcal{Q} < Q^e$, and the definition of Q^e in equation (7). □

C.5 Proof of Lemma 1

Existence (of $\tilde{p} > 0$) follows immediately from c being increasing: Just set

$$\tilde{p} := \frac{1}{\rho} \frac{c(Q + \mathrm{d}\tilde{Q}) - c(Q)}{\mathrm{d}\tilde{Q}}. \tag{79}$$

Uniqueness follows immediately from c concave: $\big(c(Q + \mathrm{d}\tilde{Q}) - c(Q)\big)/\mathrm{d}\tilde{Q}$ is strictly monotonic, hence intersects the constant $\rho\tilde{p}$ at most once.

C.6 Proof of Proposition 4

We prove the proposition in three steps. First, we show that with a credit line satisfying the conditions of the proposition in place, then $\mathcal{Q}$ is an absorbing state (Lemma C.8). Second, we use this result to characterize the value function when such a credit line is in place (Lemma C.9). Finally, we verify that such a credit line always exists (Lemma C.10).

Lemma C.8. *If a credit line $(\tilde{p}, \mathrm{d}\tilde{Q})$ satisfying the conditions of the Proposition is in place, then $\mathcal{Q}$ is an absorbing state.*

Proof. We show first that there is no issuance, i.e. no lender would buy at a price at which B would sell:

- Proposition 4 and the tie-breaking rule in Assumption 3 imply that lenders know B will draw for $Q > \mathcal{Q}$. Now the result follows from Assumption 2, inequalities (20) and (22), and the assumption that c' is bounded above zero as follows: $\Gamma(Q_t) = \int_0^\infty e^{-\rho s}\gamma(Q_{t+s})\mathrm{d}s \leq \int_0^\infty e^{-\rho s}\gamma(Q_t)e^{k_0+k_1 s}\mathrm{d}s = \gamma(Q_t)e^{k_0}/(\rho - k_1)$ with $\gamma(Q) \to 0$ as $Q \to \infty$. Thus, as condition (23) in the statement is satisfied, lenders do not lend at any price B would accept.

We now show that there are no buy backs, i.e. no lender would sell at a price B would buy:

- As $\mathcal{Q} \leq Q^e$ (the exclusive allocation debt level that maximizes gains from trade) it follows from Assumption 1.2 that $\gamma(\mathcal{Q}) > c'(\mathcal{Q})$ and thus there are also no gains from offering debt buybacks.

We conclude that $\mathcal{Q}$ must be an absorbing state—there is no issuance and no buyback—and so $\tilde{V}(\mathcal{Q}) = \dfrac{y - c(\mathcal{Q})}{\rho}$.

□

Lemma C.9. *If $\mathcal{Q}$ is an absorbing state, then the value function is as stated in Proposition 4.*

Proof. The result follows from what we have established already:

1. For $Q > \mathcal{Q}$, B draw the credit line by Proposition 3 and Assumption 3.

2. For $Q < \mathcal{Q}$, B jump to absorbing state $\mathcal{Q}$ by Proposition 3 (see Lemma C.7 in the proof).

3. $\mathcal{Q}$ is an absorbing state by Lemma C.8 above.

□

Lemma C.10. *Given $\tilde{V}$ above, a credit line satisfying the properties in the Proposition exists.*

Proof. In light of Lemma C.8 and Lemma C.9 above, the result follows immediately from Lemma 1. □

C.7 Proof of Proposition 5

First, suppose (in anticipation of a contradiction) that B takes up any bundle inducing a different outcome at date 0 in equilibrium. By Proposition 4, another lender can offer a bundle that implements Q^e and, by Proposition 1, it breaks even at p^e. By the definition of the full-commitment outcome, B is strictly better off. Thus, since B accepts at most one contract at each date, there is $\epsilon > 0$ such that the lender can offer $(p^e - \epsilon, Q^e)$ and both B and the lender (that was previously getting zero) are strictly better off. That is a profitable deviation and therefore a contradiction to the proposed equilibrium. Second, B is indifferent between drawing or not at all time, so he does not draw. □

C.8 Proof of Proposition 7

We show the result via the following lemmata.

- In Lemma C.11, we establish that if a fictitious lender that, by fiat, has the power to enforce exclusivity in the future cannot make positive expected profits, neither can one without that power. We denote the fictitious lender by L^*.

- In Lemma C.12, we show that such a lender L* indeed cannot profit by offering any debt-credit line bundle.

Lemma C.11. *Consider a (fictitious) lender L^* that can prevent all new lenders from entering in the future. (L^* has no power to prevent B from drawing existing credit lines or to prevent some lenders and not others from entering. The contract space is the same and there is no re-contracting.)*

If L^* cannot make positive profits, no other (real) lender can either.

Proof. By construction, when L* offers a contract it has a larger feasible set than any other lender. Therefore its profit must be weakly higher. The result follows. □

Lemma C.12. *Define the lender L^* as in Lemma C.11.*

If B takes new debt from L^*, he draws his existing credit line.

Proof. There are two cases: L* enforces exclusivity or does not.

- Case 1: L* enforces exclusivity. First observe that, by Proposition 2, B's continuation value V under non-exclusivity with no credit lines coincides with that if B never borrows again.[15] That says that his continuation value conditional on drawing the credit lines is V, as in the baseline, so the ratchet effect applies (Proposition 3): If B takes debt from L*, he will draw his original credit line $(\tilde{p}, \mathrm{d}\tilde{Q})$.

- Case 2: L* does not enforce exclusivity. Here the argument is the same: Per the baseline, B draws the existing line (Proposition 3).

The result now follows from the anti-ratchet effect (Proposition 4): Anticipating that B will draw his original credit line, there is no price at which L* will lend. □

[15] If B never borrows again and maintains debt Q forever, he gets $\int_0^\infty e^{-\rho t}\big(y - c(Q)\big)\mathrm{d}t = \big(y - c(Q)\big)/\rho$, which is the value $V(Q)$ in Proposition 2.

C.9 Proof of Lemma 2

The result follows from substituting $V' = -c'/\rho$ from equation (14) into equation (28). □

C.10 Proof of Proposition 8

Immediate from inequalities (29) and (30). □

C.11 Proof of Proposition 9

We take a guess-and-verify proof. We guess that the value function with the credit line $\left((\tilde{p}_s, \mathrm{d}\tilde{Q})\right)_s$ associated with debt levels $(\mathcal{Q}_s)_s$ is follows:

$$\tilde{V}_s(Q) = \begin{cases} \Gamma_s(\mathcal{Q}_s - Q) + V_s(\mathcal{Q}_s) & \text{if } Q \leq \mathcal{Q}_s, \\ \tilde{p}_s \mathrm{d}\tilde{Q} + V_s(Q + \mathrm{d}\tilde{Q}) & \text{otherwise,} \end{cases} \tag{80}$$

where Γ_s is the equilibrium price in state s.

Part (i): Ratchet effect. We first show that B draws if $Q > \mathcal{Q}_s$ and then that he does not if $Q \leq \mathcal{Q}_s$.

- *B draws if* $Q > \mathcal{Q}_s$. B prefers to draw if the value from doing so exceeds that from not:

$$\tilde{p}_s \mathrm{d}\tilde{Q} + V_s(Q + q + \mathrm{d}\tilde{Q}) \leq \tilde{V}_s(Q + q) \tag{81}$$

$$= \tilde{p}_s \mathrm{d}\tilde{Q} + V_s(Q + q + \mathrm{d}\tilde{Q}) \tag{82}$$

having substituted from our guess for the value function (equation 80) in the

last step. The above says that B is always indifferent, implying optimality.

- *B does not draw if* $Q < \mathcal{Q}_s$. It suffices to show that B prefers not to draw and then immediately to issue $\mathcal{Q}_s - Q$ at price Γ_s than to draw (and subsequently to follow the optimal strategy):

$$\Gamma_s(\mathcal{Q}_s - Q) + \tilde{V}\big(Q + (\mathcal{Q}_s - Q)\big) \geq \tilde{p}\mathrm{d}\tilde{Q} + V(Q + \mathrm{d}\tilde{Q}) \tag{83}$$

or, using the definition of the associated credit line in equation (33) to substitute for $\tilde{p}\mathrm{d}\tilde{Q}$, that

$$\Gamma_s(\mathcal{Q}_s - Q) + \tilde{V}(\mathcal{Q}_s) \geq -V_s(\mathcal{Q}_s + \mathrm{d}\tilde{Q}) + \tilde{V}_s(\mathcal{Q}_s) + V(Q + \mathrm{d}\tilde{Q}). \tag{84}$$

Given $\mathcal{Q}_s > Q$ in this case, that says that

$$\Gamma_s \geq -\frac{V_s(\mathcal{Q}_s + \mathrm{d}\tilde{Q}) - V_s(Q + \mathrm{d}\tilde{Q})}{\mathcal{Q}_s - Q}. \tag{85}$$

Since V_s is convex and $\mathrm{d}\tilde{Q} > 0$, it suffices to show that

$$\Gamma_s \geq -\frac{V_s(\mathcal{Q}_s) - V_s(Q)}{\mathcal{Q}_s - Q}, \tag{86}$$

which holds by the definition of $\mathcal{Q}_s$ as the social optimum—the gains from issuance $\Gamma_s(\mathcal{Q}_s - Q)$ exceed the decrease in the value function $V_s(Q) - V_s(\mathcal{Q}_s)$.

Part (ii): Optimality. The result that the optimum can be implemented follows from Proposition 4 and Proposition 5. □

References

Admati, A., P. DeMarzo, M. Hellwig, and P. Pfleiderer (2017). The leverage ratchet effect. *The Journal of Finance 73*(1), 145–198.

Attar, A., C. Casamatta, A. Chassagnon, and J.-P. Décamps (2019a). Contracting sequentially with multiple lenders: The role of menus. *Journal of Money, Credit and Banking 51*(4), 977–990.

Attar, A., C. Casamatta, A. Chassagnon, and J.-P. Décamps (2019b). Multiple lenders, strategic default, and covenants. *American Economic Journal: Microeconomics 11*, 98–130.

Bizer, D. S. and P. M. DeMarzo (1992, February). Sequential Banking. *Journal of Political Economy 100*(1), 41–61.

Bolton, P. and O. Jeanne (2007). Structuring and restructuring sovereign debt: The role of a bankruptcy regime. *Journal of Political Economy 115*(6), 901–924.

Bolton, P. and O. Jeanne (2009). Structuring and restructuring sovereign debt: The role of seniority. *The Review of Economic Studies 76*(3), 879–902.

Brunnermeier, M. and M. Oehmke (2013). The maturity rat race. *The Journal of Finance 68*(2), 483–521.

Bulow, J. and K. Rogoff (1988). The buyback boondoggle. *Brookings Papers on Economic Activity 19*(2), 675–704.

Chodorow-Reich, G. (2014). The employment effects of credit market disruptions: Firm-level evidence from the 2008–9 financial crisis. *The Quarterly Journal of Economics 129*(1), 1–59.

Chodorow-Reich, G. and A. Falato (2022). The loan covenant channel: How bank health transmits to the real economy. *The Journal of Finance 77*(1), 85–128.

Coase, R. (1972). Durability and monopoly. *Journal of Law and Economics 15*(1), 143–49.

Darmouni, O. (2020). Informational frictions and the credit crunch. *The Journal of Finance 75*(4), 2055–2094.

DeMarzo, P. and Z. He (2021). Leverage dynamics without commitment. *The Journal of Finance 76*(3), 1195–1250.

DeMarzo, P. M. and M. J. Fishman (2007). Optimal long-term financial contracting. *Review of Financial Studies 20*(6), 2079–2128.

DeMarzo, P. M. and Y. Sannikov (2006). Optimal security design and dynamic capital structure in a continuous-time agency model. *The Journal of Finance 61*(6), 2681–2724.

Donaldson, J. R., , D. Gromb, and G. Piacentino (2021). Collateral reallocation. Working paper, USC.

Donaldson, J. R., D. Gromb, and G. Piacentino (2018). The paradox of pledgeability. *Journal of Financial Economics,* forthcoming.

Donaldson, J. R., D. Gromb, and G. Piacentino (2025). Conflicting priorities: A theory of covenants and collateral. *The Journal of Finance 80*(3), 1739–1768.

Fama, E. F. and M. H. Miller (1972). *The Theory of Finance*. Holt, Rinehart and Winston.

Hart, O. and J. Moore (1995). Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management. *American Economic Review 85*(3), 567–85.

Hartman-Glaser, B., S. Mayer, and K. Milbradt. A theory of cash flow-based financing with distress resolution. *The Review of Economic Studies*.

Holmstrom, B. and J. Tirole (1997, August). Financial Intermediation, Loanable Funds, and the Real Sector. *The Quarterly Journal of Economics 112*(3), 663–91.

Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy 106*(1), pp. 1–40.

Kraus, A. and R. H. Litzenberger (1973). A state-preference model of optimal financial leverage. *The Journal of Finance 28*(4), 911–922.

McAfee, R. P. and T. Wiseman (2008). Capacity choice counters the Coase conjecture. *The Review of Economic Studies 75*(1), 317–332.

Parlour, C. A. and U. Rajan (2001). Competition in loan contracts. *American Economic Review 91*(5), 1311–1328.

Santos, J. A. C. and S. V. Viswanathan (2020, August). Bank syndicates and liquidity provision. Working Paper 27701, National Bureau of Economic Research.

Schwert, M. (2020). Does borrowing from banks cost more than borrowing from the market? *The Journal of Finance 75*(2), 905–947.