Complementarity, Heterogeneity, and Multipliers: Utility for HANK.[I]

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Abstract

Complementarity (between consumption and work) is essential for heterogeneous-agent models' ability to generate realistic "multiplier" effects for aggregate demand shocks, while at the same time avoiding puzzling predictions. We show how parameterizing complementarity—in the spirit of Frisch's "utility acceleration"—separately from income effects is needed to achieve this. HANK models with complementarity can then deliver realistic fiscal multipliers while at the same time resolving both a "trilemma" (matching MPEs and MPCs) and a catch-22 "dilemma" (simultaneously resolving the forward guidance puzzle) emphasized in the literature. We prove this analytically in a tractable HANK model and illustrate it in a calibrated quantitative HANK model. Yet existing utility functions restrict either complementarity, or income effects—or both—and artificially imply that multipliers are exclusively a function of either. We propose two parametric functional forms where complementarity and the income effect are arbitrary and can be calibrated separately: a quasi-separable "GHH-CRRA" utility and a "CCRRA" (constant complementarity and RRA) function.

JEL Codes: D11, E32, E52, E62.

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PRELIMINARY AND INCOMPLETE: PLEASE DO NOT CIRCULATE

1 Introduction

Complementarity between consumption and hours worked has long been studied as both a salient feature of the data and a key modelling ingredient to resolve the consumption at retirement and consumption at unemployment puzzles (Hall (2009b); Hall and Milgrom (2008); Aguiar and Hurst (2005); Heckman 1974, Gourinchas and Parker 2002; French

[I]This encompasses, expands and supersedes "A GHH-CRRA Utility for Macro: Complementarity, Income, and Substitution" by Bilbiie. We grateful to Valerie Ramey for the impetus for writing this paper and to Adrien Auclert, Timo Boppart, Robert E. Hall, Per Krusell, Benjamin Moll, and Matt Rognlie for useful discussions. We gratefully acknowledge without implicating the support of the Janeway Institute and the Keynes Fund at Cambridge. All authors' affiliation: University of Cambridge.

2005; MaCurdy (1981); Gruber 1997; Browning and Crossley 2001). More recently, the same feature has been used in the context of fiscal multipliers, since the empirical literature has shown that there is a wide range estimates for the multiplier of a government spending shock (see Hall (2009a) for a review and Ramey; Ramey and Zubairy; Blanchard and Perotti; Perotti for key references). Since standard neoclassical and New Keynesian models were unable to produce positive consumption multipliers, complementarity in New Keynesian models appeared as a natural solution ((Bilbiie (2009, 2011); Monacelli and Perotti (2009))).

More recently, complementarity has been studied in the same context but in the presence of heterogeneity, i.e. in HANK models (Auclert et al. (2023)). This paper pointed to a tension in this class of models that they label a "trilemma": the impossibility for a standard HANK model with sticky prices to reproduce reasonable fiscal multipliers while at the same time matching the empirical estimates for Marginal Propensities to Consume (MPCs) and Marginal Propensities to Earn (MPEs). A different tension in this class of models pertaining to fiscal multipliers has been emphasized in Bilbiie (2025), and consists of a "Catch-22" dilemma: heterogeneity-based mechanisms—such as countercyclical inequality or countercyclical risk—that yield "multipliers" (aggregate-demand amplification) also worsen the forward-guidance puzzle (and generally make determinacy more challenging).

Our contribution is to show that complementarity parameterized *independently* helps resolve both of these issues: it allows HANK models to generate realistic multipliers while matching the MPC and MPE, and without being subject to the forward-guidance puzzle (and without imposing stringent determinacy requirements on Taylor rules). Firstly we prove these results analytically in a tractable HANK model. Secondly, we show that they apply in a calibrated quantitative HANK model. Finally, while our analytical proofs are done for arbitrary (nonparametric) utility functions and use a definition of Edgeworth complementarity that is parmeter-free (it builds on Frisch's "utility acceleration"), we propose two utility functions that are usable in applied work that satisfy the desirable properties we emphasize.

Analytically we study how some key macro business-cycle moments and co-movements ("multipliers") depend on a restricted set of parameters (elasticities) with and without heterogeneity. Without heterogeneity one composite parameter—a general-equilibrium elasticity of intertemporal substitution, in short **GE-EIS**—is key for multipliers. The GE-EIS is determined by the degree of complementarity between consumption and hours and by income effects on labor supply. With heterogenity, the MPC and the MPE are key moments, as emphasized by Auclert et al. (2023). We show that the ratio of these objects is a function of the income effect on labor supply and the constant consumption labor supply elasticity and is thus independent of complementarity. At the same time, we find increasing complementarity amplifies fiscal multipliers.

These results resolve the 'trilemma' by showing that, conditional on the average MPC, the ratio of the income effect and constant consumption labor supply elasticity can be chosen to match low empirical MPEs. While complementarity can be chosen to generate a particular level of fiscal multipliers. The "catch-22" is resolved similarly, the cyclicality of income inequality can be chosen as shown in Bilbiie (2025) to dampen the forward guidance puzzle. Then a high enough degree of complementarity will lead to amplification of

fiscal multipliers relative to the representative agent baseline.

Quantitatively, we confirm these results using a standard HANK model calibrated to match MPCs. We fix the income effect to match empirical estimates of the MPE. Then by varying the degree of complementarity we get a wide range of fiscal multipliers, thus matching all three facts from Auclert et al. (2023) due to separating the income effect on labour supply from complementarity.

Adjusting the model to allow for control of the cyclicality of income inequality enables us to study the "catch-22". We start from a baseline with pro-cyclical income inequality and thus dampening of forward guidance. We find that increasing complementarity increases the fiscal multiplier and can lead to fiscal multipliers greater than 1, i.e. amplification relative to the representative agent benchmark. This does not, however, come at the tradeoff of exacerbating the forward guidance puzzle as in Bilbiie (2025). Instead the forward guidance puzzle is actually further dampened as complementarity increases.

Finally we propose two utility functions where both complementarity and the income effect are free parameters that can be calibrated. Somewhat paradoxically but for good reasons, such utility functions do not readily exist in the literature; the analysis of *business cycles* followed for the most part King et al. (1988) (hereinafter KPR) to impose preference restrictions that deliver *long-run balanced growth*. Prominent exceptions from the business-cycle literature that introduced widely-used functional forms that are not consistent with balanced growth are *constant relative risk aversion* (*CRRA*) preferences as in MaCurdy (1981)'s analysis of labor supply and widely used e.g. in the New Keynesian literature (Woodford (2003); Gali, 2008); and Greenwood et al. (1988) (hereinafter *GHH*). Jaimovich and Rebelo (2009) propose a time-nonseparable functional form that nests KPR and GHH and still delivers balanced growth except in the latter limit case.[1]

The general preferences that we consider feature nonseparability between consumption and hours worked, with most of the (time-separable) functional forms mentioned above as special cases. As an existing literature has emphasized, such preferences are useful for several things, all of which can be seen as manifestations of the celebrated Barro and King (1984) observation regarding the difficulty of generating realistic business cycles with demand shocks. The literature thus used nonseparable preferences to look at the effects of governments spending shocks and fiscal multipliers (Bilbiie (2009, 2011); Monacelli and Perotti (2009); Auclert et al. (2023)), and for co-movement in response to investment-specific and news shocks (Beaudry and Portier (2006, 2014); Eusepi and Preston (2009, 2015); Furlanetto and Seneca (2014); Jaimovich and Rebelo (2009)).

Yet functional forms that are currently used in the literature restrict this key part of the model artificially. In particular, CRRA utility has arbitrary income effect, but no complementarity—so the GE-EIS is pinned down by the income effect only; KPR's nonseparable utility has arbitrary complementarity but unitary income effect (as required for balanced-growth); while GHH preferences have zero income/wealth effect on labor supply—so the GE-EIS is pinned down by consumption-hours complementarity only.[2]

[1] More recently, Boppart and Krusell (2016) argue that there has been a secular downward trend in hours worked in the data, propose a broad parametric class of utility functions consistent with that, and provide an excellent review of the literature from a balanced-growth restrictions standpoint.

[2] This is also counterfactual, let alone that, as will become clear below, in an efficient equilibrium it implies infinite EIS—just like linear utility does.

The first utility function takes the disarmingly simple, **quasi-separable** form:

$$U(C,N)=\frac{1}{1-\frac{\xi}{1-\gamma}}\left(\frac{C^{1-\gamma}}{1-\gamma}-h\frac{N^{1+\varphi}}{1+\varphi}\right)^{1-\frac{\xi}{1-\gamma}},$$

where φ will still stand for constant-consumption labor supply. To those seasoned in business-cycle research, although this is novel, it will look very familiar: it applies a GHH-type operator (with a twist) to a standard CRRA function of consumption C and hours N, rather than to a function that is affine in consumption (which is GHH *stricto sensu*). In other words, it is of the form $U(.)=F_G(F_M(C,N))$ where F_M is additively separable CRRA and F_G is essentially a power function (of the GHH type) applied to F_M: which justifies why we refer to $U(.)$ as *quasi-separable*. As discussed in detail and shown formally below, the function is a special (quasi-separable) sub-class of a broader parametric class proposed by Boppart and Krusell (2016) and used there for a different, complementary purpose focusing on long-run income effects: to replicate long-run and cross country evidence on the decline of hours worked through *strong* enough income effects.[3] Here, the focus is instead on business cycle facts and on how consumption-hours complementarity, income effects, and intertemporal substitution shape them. The second utility function is a (constant-)complementarity-augmented CRRA, CCRRA:

$$U(C,N)=\frac{C^{1-\sigma}-1}{1-\sigma}-hC^{-\kappa}\frac{N^{1+\psi}}{1+\psi}. \tag{1}$$

The remainder of the paper is devoted to formalizing the points mentioned above, thus explaining the virtues of utility functions with independently-parameterized complementarity for business-cycle macro research in the realm of (HA)NK models.

Other related literature. Aside from the contributions reviewed above, our paper is directly related to two streams of literature. The first consists of HANK models with a particular focus on fiscal multipliers, be they studying the effects if fiscal transfers, spending, or deficits: an incomplete list includes Oh and Reis (2012); Auclert et al. (2025); Bayer et al. (2023a,b); Broer et al. (2025, 2023); Cantore and Freund (2019); Caramp and Silva (2021); FerriÃšre and Navarro (2017); Hagedorn et al. (2018); Kaplan et al. (2017, 2014). The second stream consists of papers that used different functional forms for non-separable utility in order to focus on different business-cycle question: Hall and Milgrom (2008); Shimer (2009); Trabandt and Uhlig (2011)—all three are similar to our second, CCRRA function but restrict complementarity κ to be a specific function of σ.

2 Complementarity, Heterogeneity, and Utility: Preliminaries

This section sets the stage and introduce the analytical apparatus we use throughout, for the representative-agent case. It then introduces the two utility functional forms with arbitrary complementarity parameterized independently from the income effect.

[3]Special thanks to Timo Boppart and Per Krusell for having pointed out the relationship—that had initially escaped my attention—between the function we propose and their more general parametric class. In terms of the notation above, Boppart and Krusell consider a function $F_M(.)$ that is non-separable.

2.1 Analytical apparatus: complementarity, income effects, and multipliers

We begin by defining the key utility objects for our analysis, namely complementarity, income effect, Frisch vs constant-consumption labor elasticity, and elasticity of intertemporal substitution. Consider a household who maximizes the expected discounted value of future utility of the general non-separable form:

$$U\left(C_t, N_t\right),$$

where C_t is consumption and N_t are hours worked. We assume that utility is increasing in both arguments $U_C > 0, U_N < 0, U_{CN} = U_{NC} \neq 0$ and concave.[4]. The household works and receives labor income, saves by buying riskless, one-period, discount government bonds B_{t+1} that cost $(1 + I_t)^{-1}$ and deliver one unit of account next period (thus I_t is the nominal interest rate), receive profits D_t from the firms and pays lump-sum taxes T_t; the budget constraint is:

$$(1 + I_t)^{-1} B_{t+1} + P_t C_t = B_t + W_t N_t + D_t.$$

Maximizing lifetime utility subject to the budget constraint, taking prices as given, we obtain the labor supply, intratemporal optimality condition

$$-U_N\left(C_t, N_t\right) = U_C\left(C_t, N_t\right) \frac{W_t}{P_t} \tag{2}$$

and the Euler equation governing intertemporal substitution, a manifestation of the fundamental asset pricing theorem:

$$\begin{aligned} Q_{t,t+1} &= \beta \frac{U_C\left(C_{t+1}, N_{t+1}\right)}{U_C\left(C_t, N_t\right)} \\ \frac{1}{1 + R_t} &= E_t Q_{t,t+1} \end{aligned} \tag{3}$$

The consumption basket consists of a constant-elasticity of substitution aggregator of individual goods j, $C_t^{\frac{\theta-1}{\theta}} =$ where $\theta > 1$ is the elasticity of substitution. Each of the differentiated goods $j \in [0,1]$ is produced by one monopolistic firm who employs labor using a non-increasing returns to scale technology (we abstract from TFP shocks): $Y_t(j) = F(N_t) = N_t^{\alpha}(j)$ and receives a sales subsidy τ^s. Each firm maximizes profits $D_t(j) = (1 + \tau^s) P_t(j) Y_t(j) - W_t N_t(j)$ by choosing its own price $P_t(j)$, subject to the demand constraint $Y_t(j) = \left(P_t(j) / P_t\right)^{-\theta} C_t$, where P_t is the welfare price index $P_t^{1-\theta} =$ Regardless of firms' ability to set their prices, we can define average gross markup (the inverse of real marginal cost) as:

$$\frac{\mu_t}{1 + \tau^s} = \frac{F_N\left(N_t\right)}{W_t / P_t}, \tag{4}$$

[4]This implies $U_{CC} \leq 0; U_{NN} \leq 0$ and $U_{CC} U_{NN} - \left(U_{CN}\right)^2 \geq 0$ which insures that the Hessian of $U(.)$ is negative semidefinite. Strict concavity requires that the Hessian be negative definite, i.e. only the first and third inequality be satisfied with strong inequality. Quasi-concavity (which is implied by concavity) requires that the *bordered* Hessian be negative semidefinite, i.e. $2U_C U_N U_{CN} - U_{CC}\left(U_N\right)^2 - U_{NN}\left(U_C\right)^2 \geq 0$.

where the dynamics of markup μ_t depend on the specific price-setting scheme and will be discussed in the loglinearized version of the model below. In equilibrium the goods market clears: $Y_t = C_t + G_t$. The model is closed by specifying price-setting, and a monetary policy rule; for simplicity, we choose a Taylor rule that includes a response to expected inflation:

$$1 + I_t = \beta^{-1} \left(1 + E_t \Pi_{t+1}\right)^{\phi_\pi}.$$

In a flexible-price equilibrium which we shall denote by a star superscript (whereby all firms are able to adjust their prices), the only optimality condition of firms is the constant-markup rule:

$$\frac{\mu}{1+\tau^s} = \frac{F_N\left(N_t^*\right)}{\left(W_t/P_t\right)^*},$$

where $\mu = \theta/\left(\theta - 1\right)$ is the desired markup.

Consider a long-run equilibrium ("steady-state") whereby all variables are constant. Focus for simplicity on the case where government spending is zero $G = 0 \rightarrow C = Y$. The intratemporal optimality condition (2) combined with labor demand (4) delivers the labor share:

$$\frac{-U_N}{U_C}\frac{N}{C} = \frac{WN}{PY} = \alpha\left(1 - \Phi\right), \tag{5}$$

where we defined the *steady-state distortion* following Woodford (2003), Chapter 6:

$$\Phi = 1 - \frac{1+\tau^s}{\mu} \in [0, 1]. \tag{6}$$

A value of 0 corresponds to an undistorted steady-state equilibrium (such as under an optimal subsidy $\tau^s = \mu - 1$) and the maximum of 1 obtains when the markup tends to infinity and the subsidy τ^s is fixed. Several other distortions such as imperfect labor markets and a wage markup, or distortionary taxation could be added; they would change the exact expression for Φ but neither its interpretation nor the limit result that under appropriately designed Pigouvian taxes it can be brought down to zero to restore efficiency. The reader can thus safely regard Φ as a general index of supply distortions in this economy from now onwards.

The following definitions outline the key parameters describing preferences. A complete nomenclature of preferences in the context of intertemporal substitution in consumption and hours worked is already outlined in Barro and King (1984) and extended in some dimensions relevant for the topic here by Bilbiie (2009, 2011). Rather than reiterate those definitions, we focus here on the minimal set of ingredients needed for this paper's point.

Definition 1 ***Complementarity*** *between consumption and hours occurs when:*

$$\kappa \equiv \frac{U_{CN}C}{-U_N} > 0.$$

According to this definition C and N are complements in the Edgeworth sense ($U_{CN} > 0$) when $\kappa > 0$, and substitutes otherwise; this is also related to the notion of "utility acceleration" introduced by Frisch (1959).

Definition 2 *The **income effect** on labor supply is:*

$$\gamma \equiv -\frac{U_{CC}C}{U_C} + \frac{U_{CN}C}{U_N}.$$

The income effect is most clearly seen by loglinearizing[5] (2) to obtain the *constant-consumption labor supply*:

$$\varphi n_t = w_t - \gamma c_t, \tag{7}$$
$$\text{where } \varphi \equiv \frac{U_{NN}N}{U_N} - \frac{U_{CN}N}{U_C}$$

is the constant-consumption labor supply (inverse) elasticity. γ measures the income effect because it tells how, at given relative price of consumption and leisure, the labor supply schedule shifts when income changes (a more detailed description is in the Appendix). There is a one-to-one mapping between the 'original' derivatives (and the respective curvatures) of the utility function and the newly introduced φ, γ and κ since $U_{NN}N/U_N = \varphi + \kappa\alpha\left(1-\Phi\right)$ and $-U_{CC}C/U_C = \gamma + \kappa$.

Notice that with nonseparable preferences the *Frisch* (constant-marginal-utility) elasticity of labor supply is different; in particular (see the Appendix for a derivation) Frisch labor supply is, denoting by λ_t the log-deviation of $U_C\left(.\right)$:

$$\left[\varphi + \frac{\kappa\gamma}{\gamma+\kappa}\alpha\left(1-\Phi\right)\right] n_t = w_t - \frac{\gamma}{\gamma+\kappa}\lambda_t. \tag{8}$$

The Frisch elasticity is the (inverse of the) term in square brackets, and it is always positive when U is concave. Frisch elasticity is lower than constant-consumption elasticity *if and only if* $\kappa > 0$, i.e. when consumption and hours are Edgeworth complements: when facing a higher wage, the household is willing to work relatively more hours to keep the level of consumption constant than it is to keep marginal utility of consumption constant. This distinction turns out to be crucial for the effects of shocks and co-movement.

The conditions for utility to be concave and quasiconcave and for consumption and leisure to be normal goods can be compactly summarized in terms of these parameters as (see Appendix A.2 for the formal derivation):

$$\varphi \geq 0; \gamma \geq 0; \kappa \geq -\frac{\gamma\varphi}{\varphi + \gamma\alpha\left(1-\Phi\right)}. \tag{9}$$

In other words, labor supply should be upward-sloping, the income effect should be positive, and the degree of substitutability of consumption and leisure cannot be too high.

The key parameter for business cycle analysis is the **general-equilibrium elasticity of intertemporal substitution GE-EIS**, defined formally as the partial derivative of relative demands of the consumption good at two different times with respect to the marginal rate of (intertemporal) substitution between the two goods, formally:

[5]To write the model using elasticities around the long-run equilibrium We take a first-order approximation to all optimality conditions, letting small case letters denote percentage deviations from steady state, e.g. for any variable X_t we have $x_t \equiv \left(X_t - X\right)/X$. The only exceptions from this are inflation and interest rates (which are already in percentage points), and government spending, whose steady-state value is assumed to be zero $G = 0$, therefore $g_t \equiv \left(G_t - G\right)/Y$.

Definition 3 *The elasticity of intertemporal substitution GE-EIS denoted by η is given by:*

$$\eta = \frac{d \ln (C_{t+1}/C_t)}{d \ln Q_{t,t+1}}$$

Since the marginal rate of intertemporal substitution $Q_{t,t+1}$ is equal to the (inverse) interest rate, the **GE-EIS** is generally equal to the elasticity of consumption (growth) to real interest rates; this, in turn, is fully determined by the income effect on labor supply and complementarity, as emphasized in our first Proposition.

Proposition 1 *The general-equilibrium elasticity of intertemporal substitution GE-EIS is determined by the income effect γ and complementarity κ as follows:*

$$\eta = (\gamma + \kappa\Phi)^{-1} > 0,$$

where Φ is the index of supply-side distortions.

The proof follows immediately by taking a local approximation of the Euler equation (3):

$$\frac{U_{CC}C}{U_C} c_t + \frac{U_{CN}N}{U_C} n_t = \frac{U_{CC}C}{U_C} E_t c_{t+1} + \frac{U_{CN}N}{U_C} E_t n_{t+1} + r_t. \tag{10}$$

The resource constraint $y_t = c_t + g_t$ and production function $n_t = \frac{1}{\alpha} y_t$ respectively substituted in the Euler equation (10) deliver:

$$y_t = E_t y_{t+1} - \eta r_t + \eta (\gamma + \kappa) (g_t - E_t g_{t+1}), \tag{11}$$

which illustrates that η in the Proposition governs intertemporal substitution in aggregate demand, or the inverse elasticity of aggregate demand to interest rate. Its being positive $\eta > 0$ follows from the restrictions (9), since $-\kappa < \gamma$ and $\Phi < 1$.[6] Notice that this definition of GE-EIS takes into account the equilibrium response of hours worked, and in particular how they are related through *general-equilibrium* forces to consumption.

As we shall see below, GE-EIS η is the main determinant of the effects of shocks, so it is important to understand its determinants. Start with the simplest case, of separable utility, i.e. no complementarity $\kappa = 0$: the GE-EIS is the standard, inverse of income effect on labor supply, γ^{-1} (likewise, in a non-distorted economy $\Phi = 0$). The intuition is straightforward: a given change in the marginal rate of intertemporal substitution (or interest rate) corresponds to a larger change in consumption today relative to tomorrow when γ is smaller. A smaller income effect means that equilibrium income needs to increase by relatively more to deliver a same shift in constant-consumption labor supply. In other words, intertemporal substitution and the income/wealth effect on labor supply are related one-to-one with separable preferences.

With complementarity (nonseparability) this is no longer the case, although as shown in the Theorem the GE-EIS still depends negatively on the income effect, for the same reasons outlined above. But complementarity weakens this link, and the intuition why is

[6] A similar expression is derived in Bilbiie, 2011 but focusing on the topic of fiscal multipliers and co-movement.

simple.[7] The "income effect" γ now comprises a part beyond mere curvature in consumption, that is due to complementarity: whenever income/consumption changes, at given marginal utility and with consumption a function of hours worked, the cross-derivative consumption-hours also matters. That is, the income effect depends on complementarity. But the key element is that this dependence is not one-to-one, except in a special case. Indeed, the transformation of hours into consumption on the production side is not efficient: there is a wedge between the marginal rate of substitution and the marginal rate of transformation, captured by Φ. When this wedge is nil (such as when a system of Pigouvian taxes and subsidies restores efficiency) complementarity does not affect the GE-EIS beyond the income effect; but when it is positive, complementarity needs to compensate for this inefficiency and induce the variation in hours that is necessary to deliver the consumption variation called for by intertemporal substitution.

Let us dissipate this potential source of confusion: complementarity does **not** imply more intertemporal substitution, and thus more effect of any shocks (such as monetary policy) that work almost exclusively through intertemporal substitution in this model. On the contrary—at given income effect, complementarity implies *less* elasticity of intertemporal substitution. But, as we shall study next, complementarity does imply more effects of some shocks, *despite* implying lower GE-EIS!

Another set of important implications concerns empirical estimates of the GE-EIS. The empirical literature estimating aggregate Euler equations finds very small value for the EIS, from Hall (1988) to e.g. Yogo (2004). And since with separable preferences the GE-EIS is the inverse of the income effect parameter (aka relative risk aversion) these low-EIS estimates are inconsistent with a wealth of micro evidence suggesting that income (wealth) effects on labor supply are typically small—and thus that γ is small; independent evidence on risk aversion (the other interpretation of this parameter) is in line with this.

Clearly, complementarity offers a way out of this conundrum through the mechanism described above: a low GE-EIS is consistent with a low income effect γ, if there is enough consumption-hours complementarity (high enough κ). An important caveat is that, as discussed above, this depends on the size of the supply–side distortion Φ: the lower this parameter (the closer the economy is to the efficient benchmark), the higher the degree of complementarity necessary to bring in line a low GE-EIS with low income effects. In the efficient limit $\Phi = 0$, there is evidently no way out of the conundrum. In other words, small income effects necessarily imply high elasticity of intertemporal substitution (and, as we shall see, high multipliers); in the limit, the case of GHH preferences $\gamma = 0$ artificially implies infinite GE-EIS (just like under linear separable utility)—a point also raised by Auclert et al. (2023), cautioning that the use of GHH preferences has consequences that are often not immediately obvious in quantitative models.

2.2 Complementarity and Multipliers without Heterogeneity: A Review

The foregoing clarifies how one key parameter in any macro model, the elasticity of intertemporal substitution, depends on income effects and complementarity under general

[7] Epstein and Zin (1989) disentangle intertemporal elasticity of substitution from *risk aversion* by using a very different, (recursive) utility that is non-separable over time.

non-separable utility functions. To solve for the full equilibrium and study the implication of this for business cycles, we need to go back to the supply side and specify how firms set prices. It is by now well understood that a model with flexible prices cannot yield (absent other assumptions) realistic business cycles—understood as co-movements between the key macro variables consumption, output, and hours worked as well as, in a version with physical capital, investment—in response to demand shocks: the *Barro-King critique*. The label "demand shocks" includes a large class of disturbances: interest-rate changes (be they driven by monetary policy or originating in the financial sector), changes in government spending, news shocks, or investment-specific technology shocks.[8]

Lets us restate the co-movement problem briefly here and refer the reader to the above papers (in particular Bilbiie (2009, 2011); Beaudry and Portier (2014); Eusepi and Preston (2015)) for a detailed treatment and discussion of other solutions. The local approximation to the labor demand condition of firms (4) yields:

$$w_t = -\left(1-\alpha\right) n_t + mc_t, \tag{12}$$

where $\alpha = F_{NN}N/F_N < 1$ is the degree of returns to scale in labor and mc_t is the log-deviation of real marginal cost (the inverse of average markups)—the dynamics of which will depend on the specific price-setting scheme. Replacing this in constant-consumption labor supply we obtain:

$$\gamma c_t = -\left(\varphi + 1 - \alpha\right) n_t + mc_t,$$

which is a key equation for the argument. Clearly, absent movements in either TFP or real marginal cost mc_t consumption and hours will co-move negatively—insofar as conditions of concavity and non-inferiority on preferences are satisfied.

Conversely, consumption and hours worked can co-move positively if and only if—absent TFP shocks—real marginal costs mc_t also move sufficiently. One example delivering the latter is, as is by now well understood in the literature, the assumption of sticky prices (labor demand may shift for other reasons, such as variable utilization). Adding the rest of the model (the approximated resource constraint and production function which combined deliver $n_t = \alpha^{-1}\left(c_t + g_t\right)$), we have the marginal cost schedule:

$$mc_t = \left(\gamma - 1 + \frac{\varphi + 1}{\alpha}\right) y_t - \gamma g_t. \tag{13}$$

Notice that while complementarity does not influence at all these conditions directly, it does influence via (11) the intertemporal substitution channel, which plays through marginal cost: Higher complementarity (more positive κ) means a larger effect on demand, so larger inflation which feeds back through supply. To understand this we need to complete the model: in addition to (11) and (13), we have to specify a price-setting scheme—which leads to an equation linking real marginal cost to inflation, a "Phillips

[8]This has been studied in turn, starting with Barro and King's seminal 1984 contribution, in Bilbiie (2009, 2011); Monacelli and Perotti (2009); Auclert et al. (2023) for government spending shocks; Beaudry and Portier (2006, 2014); Jaimovich and Rebelo (2009) for news shocks; Justiniano et al. (2010, 2011); Furlanetto and Seneca (2014) for investment-specific technology shocks; and Eusepi and Preston (2009, 2015) for generic demand (non-TFP) shocks.

curve"—and a Taylor rule specifying how the interest rate r_t responds to endogenous variables.

We first isolate the effect of complementarity by assuming that prices are completely fixed—as a consequence, we treat the interest rate r_t in (10) or (11) as exogenous. Under this simplifying assumption—previously used by Bilbiie (2011); Woodford (2011) to look at fiscal multipliers—output is entirely demand-determined and the model boils down to one equation, (11).[9] The effects of "demand" shocks and policies are then immediately derived as described in the following Proposition.

Proposition 2 *The effects of interest rate cuts ($-r_t$) and government spending increases (g_t) are given by, respectively:*

$$\textit{monetary: } \frac{dy_t}{d\left(-r_t\right)} = \eta = \left(\gamma + \kappa\Phi\right)^{-1}$$

$$\textit{fiscal: } \frac{dy_t}{dg_t} = \eta\left(\gamma + \kappa\right) = \frac{\kappa + \gamma}{\kappa\Phi + \gamma}$$

As the Proposition makes clear, the GE-EIS and its determinants, in particular the degree of complementarity, are key for *all* these effects; the rest of this Section discusses the intuition for each finding in turn.

Monetary policy and intertemporal substitution

The first part is very easy and follows immediately from our discussion in the previous section: the effect of a monetary expansion under fixed prices is given precisely by η, the elasticity of intertemporal substitution GE-EIS discussed at length above. This is particularly useful in the context of HANK models. Indeed, in such models standard CRRA separable preferences imply income effects in the cross-section that are not supported by the empirical evidence. As a solution to fix this issue several studies use GHH preferences, which has the other counterfactual implication that in an efficient equilibrium the GE-EIS becomes infinite (see Auclert et al. (2023) for an elaboration of this point and references in the context of heterogeneous-agent models). This paper thus provides one stone to kill two birds: minimize income effects (bringing them in line with cross-sectional evidence) while avoiding the issue created by using GHH.

The analysis above clarifies the general link between GE-EIS, income effects and complementarity that governs the effects of shocks and policies, and which can be useful in heterogeneous-agent models. In particular, the GE-EIS (and hence the effect of monetary policy) is *amplified* by the reduction of income effects on labor supply, at given complementarity. Whereas the GE-EIS and the effect of monetary policy are in fact **dampened**, *not amplified, by more complementarity*—at given income effect. GHH preferences mix the two because they eliminate income effects and feature a fortiori complementarity: but it is thus not the complementarity that delivers high GE-EIS, it is the lack of income effect.

[9] This amounts to assuming that prices are completemy fixed, or a Taylor rule of the form $i_t = E_t\pi_{t+1} + r_t$ where r_t is the exogenous part.

This is important to understand, because it is specific to monetary policy shocks: as we shall see next, the effects of other demand shocks are indeed amplified by complementarity in and of itself.

Fiscal multipliers

Fiscal multipliers are obtained by rewriting the expression from the Proposition:

$$\frac{dy_t}{dg_t} = 1 + \eta\kappa\left(1-\Phi\right) > 1 \text{ iff } \kappa > 0.$$

With fixed prices, a positive multiplier of consumption occurs if and only if consumption and hours worked are complements; this result and the analytical expression has been derived previously in Bilbiie (2011) (Proposition 3). The same caveat as for monetary policy applies: with zero income effects $\gamma = 0$ such as for GHH preferences the multiplier is simply given by Φ^{-1} and is artificially driven to infinity in the efficient limit (as also discussed by Auclert et al. (2023) for the special GHH case). More generally, the multiplier is increasing with the degree of complementarity (its derivative with respect to κ is proportional to $\gamma\left(1-\Phi\right) \geq 0$) and decreasing with the income effect (the derivative with respect to γ is proportional to $\kappa\left(\Phi-1\right) \leq 0$). We return to these issues below, both analytically and quantitatively.

2.3 Two Utility Functions for Applied Work: GHH-CRRA and CCRRA

The previous sections emphasized the role of C-N complementarity generally for generating realistic business cycles; we now move to a specific utility function featuring such complementarity that can be used in quantitative macro models; indeed, we shall propose *two* such function. Several utility functions with C-N complementarity do exist and have been extensively used in the macro literature; but as we shall see, they each restrict somewhat artificially some other parameter that is key for business cycles—in particular, the income effect on labor supply. Among the most prominent examples in the literature: GHH implies zero income/wealth effect (with the additional artificial implication of very large intertemporal elasticity of substitution—indeed infinite around an efficient equilibrium). While the non-separable utility function of KPR has unitary income effect—needed, of course, to induce balanced growth.

There is thus to the best of our knowledge no utility function with both flexible income effect on labor supply and consumption-hours complementarity (CRRA has the former but restricts the latter to zero; GHH and KPR always have the latter but restrict former to zero or one, respectively; and Jaimovich and Rebelo consists of a time-nonseparable flexible form that nests these last two).

Here, we introduce two (to the best of my knowledge) new utility functions that restrict none of this but still satisfy the regularity conditions (9) above (concavity, and non-inferiority). This is not *the* most general family of functions that satisfies those restrictions, although in principle that could be found too, by solving a system of partial differential equations formed by the restrictions; there are instead two examples that:

1. nests standard utility functions: indeed, for both functions, CRRA and GHH are special cases, and KPR is a limit case.

2. are time-separable, which differentiates them from Jaimovich-Rebelo preferences.

3. we can keep thinking of γ as income effect on labor and φ as inverse (constant-consumption) labor elasticity, but we now have a new free parameter that fully determines complementarity.

The functions are nested in the more general parametric class advanced by Boppart and Krusell to match evidence on the long-run decline of hours worked through strong income effects; indeed, the first (GHH-CRRA) can be viewed as the "quasi-separable" (in a sense to be defined precisely below) sub-class of that more general class.[10]

Both functions are disarmingly simple. The first function consists of applying a GHH-type operator to a CRRA function:

$$U\left(C_t, N_t\right) = \frac{1}{1-\frac{\xi}{1-\gamma}}\left(\frac{C_t^{1-\gamma}}{1-\gamma} - h\frac{N_t^{1+\varphi}}{1+\varphi}\right)^{1-\frac{\xi}{1-\gamma}} \tag{14}$$

defined for arbitrary income effect $\gamma \neq 1$ (whereas when $\gamma = 1$ one can use $U\left(C_t, N_t\right) = \left(\ln C_t - h\frac{N_t^{1+\varphi}}{1+\varphi}\right)^{1-\xi} / \left(1-\xi\right)$, see discussion below).

Some of the most widely used functions in business-cycle macro are nested: $\xi = 0$ is CRRA (e.g. Macurdy), and $\gamma = 0$ is GHH. And it can be easily shown that γ is the income effect while φ is the constant-consumption labor elasticity, as defined above (see Appendix for the derivations). The function is "quasi-separable" in that it consists of compounding two functions, $U\left(C, N\right) = F_G\left(F_M\left(C, N\right)\right)$ of which one $F_M\left(C, N\right)$ is CRRA-separable and the other F_G a GHH-type, power function.

The function is thus defined (with $1-\gamma$ dividing ξ) in order for the free parameter ξ's sign to entirely determine complementarity. Indeed, complementarity as defined above is given by:

$$\kappa = \xi \Upsilon$$

where $\Upsilon = \left[1 - \frac{1-\gamma}{1+\varphi}\alpha\left(1-\Phi\right)\right]^{-1} > 0$ and we used the expressions derived in the Appendix. It then follows immediately that the GE-EIS is $\eta = \left(\gamma + \xi\Phi\Upsilon\right)^{-1}$ and our previous general discussion still applies.

The income effect γ can be calibrated to match cross-section facts, which would presumably require a low γ. Or perhaps one would want to match long-run growth macro facts, which requires something different.[11]

[10] See the Appendix for a derivation of the GHH-CRRA function as the special quasi-separable sub-class of the Boppart-Krusell class.

[11] As mentioned in the Introduction, Boppart and Krusell (2016) recently argued that there is a downward long-run trend in hours worked and proposed a general class of utility functions that delivers that. Since it has arbitrary income effect, my quasi-separable function can also deliver that, for $\gamma > 1$. Indeed, just as CRRA utility, my function also implies (from intratemporal optimality) $hN_t^{1+\varphi} = \alpha\left(1-\Phi\right)C_t^{1-\gamma}$. From this it follows immediately that with an upward trend in C there can only be a downward trend in N if the income effect is strong enough, i.e. $\gamma > 1$.

Lastly, notice that the utility function proposed in (14) is not defined for $\gamma = 1$. But that is the exactly case that the macro literature has hitherto focused on because it makes income and substitution effects on labor cancel out, thus delivering *constant* hours along a growth path. That is: we already have an utility function with arbitrary complementarity—but with restricted income effect $\gamma = 1$, the KPR utility function: $U(C,N) = \xi^{-1}\left\{\left[C\exp\left(-\frac{N^{1+\varphi}}{1+\varphi}\right)\right]^{\xi} - 1\right\}$.[12]

The second function could be dubbed CCRRA, from constant-complementarity and relative risk aversion; it is a straightforward extension of CRRA where an extra multiplicative term captures the dependence of the marginal disutility of hours upon the consumption level:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - hC_t^{-\kappa}\frac{N_t^{1+\psi}}{1+\psi}. \tag{15}$$

As we show in the Appendix (where we derive the other key parameters above, from income effect to labor elasticities to relative risk aversion), κ then directly parameterizes the degree of complementarity.

3 Analytical Results in a Tractable HANK Model

In this section, we introduce a tractable HANK model with consumption-hours complementarity and decentralised labor supply. Our analytical results show that, when parameterising complementarity and the income effect separately, complementarity provides a solution for both the Auclert et al. (2023) "trilemma" and the Bilbiie (2025) "dilemma".

3.1 A Tractable HANK Model

The model follows the THANK framework introduced by Bilbiie (2020; 2025). The model features two household types. A fraction λ of households are hand-to-mouth (denoted by H), and the remaining $1-\lambda$ are savers (denoted by S). In each period, a fraction s of savers remain savers, and a fraction $1-s$ become hand-to-mouth. Similarly, a fraction h of hand-to-mouth households remain in the same state next period, with the remaining $1-h$ becoming savers.

Both types of household supply labor to firms in a decentralised labor market, earning wage W_t. Savers have access to financial markets and can save in one-period nominal bonds B_t with real return R_t, while hand-to-mouth consumers consume their disposable income. Disposable income is made up of labor income $W_t N_t^j$ and firm dividends D_t^j net of uniform lump-sum taxes T_t. Firms are owned by savers, and the profits are redistributed such that hand-to-mouth households earn a share τ^D of total dividends. The household

[12]While this is not strictly speaking a limit case of (14), it can be obtained as the limit of an equivalent function, defined over leisure instead of labor, and restricted to the CES form. See also Trabandt and Uhlig (2011) for a different parametric class of non-separable U, used there in the context of Laffer curves, that is consistent with balanced growth and has constant Frisch elasticity of labor supply.

budget constraints in real terms are:

$$C_t^S + B_{t+1} = \frac{W_t}{P_t} N_t^S + \frac{1-\tau^D}{1-\lambda} D_t - T_t + \frac{s}{1-\lambda}(1+R_t)B_t \tag{16}$$

$$C_t^H = \frac{W_t}{P_t} N_t^H + \frac{\tau^D}{\lambda} D_t - T_t + \frac{1-s}{\lambda}(1+R_t)B_t. \tag{17}$$

Each household type chooses consumption, hours, and savings to maximise expected lifetime utility. The first-order conditions with respect to hours and consumption for a household of type j are:

$$U_N(C_t^j, N_t^j) = -v_t^j \frac{W_t}{P_t} \tag{18}$$

$$U_C(C_t^j, N_t^j) = v_t^j \tag{19}$$

where v^j represents the Lagrange multiplier on j's budget constraint. Intratemporal optimality requires:

$$-U_N(C_t^j, N_t^j) = U_C(C_t^j, N_t^j)\frac{W_t}{P_t}. \tag{20}$$

The saver's intertemporal optimality condition is the self-insurance Euler equation:

$$U_C\left(C_t^S, N_t^S\right) = \beta \mathbb{E}_t \left\{(1+R_t)\left[sU_C\left(C_{t+1}^S, N_{t+1}^S\right) + (1-s)U_C\left(C_{t+1}^H, N_{t+1}^H\right)\right]\right\}. \tag{21}$$

We focus on the zero-liquidity limit in which equilibrium bond supply is zero.

Aggregate consumption and labor supply are defined as the weighted sum of savers' and hand-to-mouth consumption and hours, respectively:

$$C_t = \lambda C_t^H + (1-\lambda)C_t^S$$
$$N_t = \lambda N_t^H + (1-\lambda)N_t^S.$$

As in the RANK case above, the consumption basket again consists of a CES aggregate of individual goods k. For simplicity, each of the differentiated goods $k \in [0,1]$ is now produced by one monopolistic firm who employs labor using *linear technology*: $Y_t(k) = F(N_t) = N_t(k)$ and receives a sales subsidy τ^s. The government purchases G_t of the consumption good each period, and runs a balanced budget such that $T_t = G_t$. In equilibrium the goods market clears: $Y_t = C_t + G_t$.

In steady state, we focus on the case where government spending is zero ($G = 0 \Rightarrow C = Y$) and consumption and hours are equal across household types ($C^j = C$ and $N^j = N$). The intratemporal optimality condition (20) combined with labor demand again delivers the labor share:

$$\frac{-U_N}{U_C}\frac{N}{C} = \frac{WN}{PY} = (1-\Phi) \tag{22}$$

To study the impact of the interactions between consumption-hours complementarity and household heterogeneity on fiscal and monetary transmission, we loglinearize the model around our equal-consumption zero-inflation steady state.[13] Loglinearising the intratemporal optimality condition (20) gives:

$$\underbrace{\left(\frac{U_{NN}N}{U_N}-\frac{U_{CN}N}{UC}\right)}_{\equiv\varphi} n_t^j = w_t - \underbrace{\left(\frac{U_{CN}C}{U_N}-\frac{U_{CC}C}{UC}\right)}_{\equiv\gamma} c_t^j$$

$$\varphi n_t^j = w_t - \gamma c_t^j \tag{23}$$

where φ is the (inverse) *constant-consumption* elasticity of labor supply to a change in the real wage. Due to the complementarity between consumption and hours, this labor supply elasticity differs from the Frisch *constant-marginal-utility* labor supply elasticity, which is derived below. Aggregate consumption and hours are given by $c_t = \lambda c_t^H + (1-\lambda)c_t^S$ and $n_t = \lambda n_t^H + (1-\lambda)n_t^S$, and both household types have the same preferences. The intratemporal optimality condition therefore also holds at the aggregate level.

$$\varphi n_t = w_t - \gamma c_t. \tag{24}$$

To derive the aggregate Euler equation, we can follow the steps Bilbiie (2025). First loglinearize the self-insurance Euler equation (21) to obtain:

$$\begin{aligned}\frac{U_{CC}C}{U_C}c_t^S + \frac{U_{CN}N}{U_C}n_t^S = s\frac{U_{CC}C}{U_C}\mathbb{E}_t c_{t+1}^S + (1-s)\frac{U_{CC}C}{U_C}\mathbb{E}_t c_{t+1}^H + s\frac{U_{CN}N}{U_C}\mathbb{E}n_{t+1}^S \\ + (1-s)\frac{U_{CN}N}{U_C}\mathbb{E}n_{t+1}^H.\end{aligned} \tag{25}$$

Loglinearising the hand-to-mouth budget constraint (17) gives:

$$c_t^H = n_t^H + \left(1-\frac{\tau^D}{\lambda}\right)w_t - t_t. \tag{26}$$

Combining this equation with the intratemporal optimality conditions (23) and (24) and the goods market clearing condition allows us to express hand-to-mouth consumption as a function of aggregate consumption and government spending:

$$c_t^H = \chi c_t + (\chi-1)\frac{\varphi}{\varphi+\gamma}g_t \tag{27}$$

$$\text{where } \chi \equiv 1+\varphi\left(1-\frac{\tau^D}{\lambda}\right).$$

As in Bilbiie (2025), χ represents the cyclicality of income inequality. A value of $\chi > 1$ represents countercyclical inequality, while $\chi < 1$ generates procyclical inequality. Using

[13] We use lowercase letters to denote variables in log deviations from steady state, $x_t \equiv \frac{X_t - X}{X}$. In the case of government spending, which is zero in steady state, the loglinearized variable is defined relative to steady-state output, $g_t \equiv \frac{G_t - G}{Y}$.

the intratemporal optimality equations, we can also express hand-to-mouth hours as:

$$n_t^H = \left[1 - (\chi - 1)\frac{\gamma}{\varphi}\right] c_t + \left[1 - (\chi - 1)\frac{\gamma}{\varphi + \gamma}\right] g_t. \tag{28}$$

The loglinearized government budget constraint and goods market clearing conditions are given by $t_t = g_t$ and $y_t = c_t + g_t = n_t$. Combining equations (27) and (28) with these conditions allows us to express the saver's consumption and hours as functions of aggregate variables:

$$c_t^S = \frac{1 - \lambda\chi}{1 - \lambda} c_t - \frac{\lambda}{1 - \lambda}(\chi - 1)\frac{\varphi}{\varphi + \gamma} g_t \tag{29}$$

$$n_t^S = \frac{1 - \lambda\left[1 - (\chi - 1)\frac{\gamma}{\varphi}\right]}{1 - \lambda} c_t + \frac{1 - \lambda\left[1 - (\chi - 1)\frac{\gamma}{\varphi + \gamma}\right]}{1 - \lambda} g_t. \tag{30}$$

Substituting these equations into the self-insurance Euler (25) allows us to obtain the aggregate Euler equation:[14]

$$\begin{aligned}
c_t = &\left[1 + \frac{(1-s)(\chi-1)\left[\gamma + \kappa\frac{\varphi + (1-\Phi)\gamma}{\varphi}\right]}{\gamma(1-\lambda\chi) + \kappa\left\{(1-\lambda\chi) - (1-\Phi)\left[(1-\lambda) + \lambda(\chi-1)\frac{\gamma}{\varphi}\right]\right\}}\right] \mathbb{E}_t c_{t+1} \\
&+ \frac{\lambda\gamma(\chi-1)\frac{\varphi}{\varphi+\gamma} + \kappa\left\{\lambda(\chi-1)\frac{\varphi}{\varphi+\gamma} + (1-\Phi)\left[(1-\lambda) + \lambda(\chi-1)\frac{\gamma}{\varphi+\gamma}\right]\right\}}{\gamma(1-\lambda\chi) + \kappa\left\{(1-\lambda\chi) - (1-\Phi)\left[(1-\lambda) + \lambda(\chi-1)\frac{\gamma}{\varphi}\right]\right\}}(g_t - \mathbb{E}_t g_{t+1}) \\
&+ \frac{(1-s)(\chi-1)\left[\gamma + \kappa\frac{\varphi + (1-\Phi)\gamma}{\varphi}\right]\frac{\varphi}{\varphi+\gamma}}{\gamma(1-\lambda\chi) + \kappa\left\{(1-\lambda\chi) - (1-\Phi)\left[(1-\lambda) + \lambda(\chi-1)\frac{\gamma}{\varphi}\right]\right\}} \mathbb{E}_t g_{t+1} \\
&- \frac{1-\lambda}{\gamma(1-\lambda\chi) + \kappa\left\{(1-\lambda\chi) - (1-\Phi)\left[(1-\lambda) + \lambda(\chi-1)\frac{\gamma}{\varphi}\right]\right\}} r_t.
\end{aligned}$$

[14] We have used the definitions of both κ and γ above, as well as the fact that $\frac{U_{CC}C}{U_C} = -(\gamma + \kappa)$ and $\frac{U_{CN}N}{U_C} = \kappa(1 - \Phi)$.

In the efficient limit with $\Phi = 0$, the aggregate Euler equation gives:

$$\begin{aligned} c_t = &\left[1 + \frac{(1-s)(\chi-1)\left(\gamma + \kappa\frac{\varphi+\gamma}{\varphi}\right)}{\gamma(1-\lambda\chi) - \kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}\right]\mathbb{E}_t c_{t+1} \\ &+ \frac{\lambda(\chi-1)\left(\gamma + \kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma} + \kappa(1-\lambda)}{\gamma(1-\lambda\chi) - \kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}(g_t - \mathbb{E}_t g_{t+1}) \\ &+ \frac{(1-s)(\chi-1)\left(\gamma + \kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma}}{\gamma(1-\lambda\chi) - \kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}\mathbb{E}_t g_{t+1} \\ &- \frac{1-\lambda}{\gamma(1-\lambda\chi) - \kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)} r_t. \end{aligned} \tag{31}$$

Using the aggregate Euler equation, we can immediately see the effect of real interest rate changes on aggregate demand, as shown in the Proposition below.

Proposition 3 *The static elasticity of aggregate demand to interest rates in the THANK model (in the efficient case) is given by:*

$$\eta^{HA} \equiv \frac{\partial c_t}{\partial(-r_t)} = \frac{1-\lambda}{\gamma(1-\lambda\chi) - \kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)} r_t.$$

This can be seen by taking the partial derivative of (31) with respect to the real interest rate. The slope of the aggregate Euler nests the RANK model GE-EIS for $\lambda = 0$:

$$\eta^{RANK} = \frac{1}{\gamma}.$$

The introduction of household heterogeneity amplifies the effect of real interest rate changes on consumption if $\eta^{HA} > \eta^{RANK}$. The condition for amplification can be expressed as:

$$\begin{aligned} \lambda(\chi-1)\left(\gamma + \kappa\frac{\varphi+\gamma}{\varphi}\right) &> 0 \\ \Rightarrow \chi &> 1. \end{aligned}$$

Hence, as in Bilbiie (2025), the addition of hand-to-mouth households amplifies monetary policy relative to the RANK model if and only if income inequality is countercyclical. Note that this condition is independent of consumption-hours complementarity κ.

We can also examine how η^{HA} changes with complementarity:

$$\frac{\partial^2 c_t}{\partial(-r_t)\partial\kappa} = \frac{(\chi-1)\frac{\varphi+\gamma}{\varphi}\lambda(1-\lambda)}{\left[\gamma(1-\lambda\chi) - \kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)\right]^2}.$$

The multiplier is increasing in κ when $\chi > 1$ and decreasing when $\chi < 1$. With countercyclical inequality, the rise in aggregate output due to a fall in real interest rates generates a larger rise in income for hand-to-mouth households, driving a larger rise in aggregate consumption. With complementarity between hours and consumption, this rise in consumption leads to an increase in labor supply, which increases output further. Hence, the combination of consumption-hours complementarity and countercyclical income inequality amplifies the effect of interest rate changes as if the aggregate MPC were higher.

Finally, it is of interest to study the conditions under which the heterogeneous-agent interest rate elasticity is positive ($\eta^{HA} > 0$), thus avoiding the "inverted aggregate demand" region—and ensuring that the "Taylor principle" remains a good guide for monetary policy to ensure determinacy in a version with a Phillips curve and a nominal interest rate rule. The "standard aggregate demand" logic $\eta^{HA} > 0$ prevails whenever the following condition is satisfied:

$$\lambda < \frac{1}{\chi + (\chi - 1)\frac{\kappa}{\gamma}\frac{\varphi+\gamma}{\varphi}}.$$

In the absence of complementarity ($\kappa = 0$), this condition is simply:

$$\lambda < \frac{1}{\chi}.$$

Therefore, with countercyclical inequality ($\chi > 1$), the introduction of complementarity $\kappa > 0$ tightens the condition on λ. With procyclical inequality ($\chi < 1$), the introduction of complementarity $\kappa > 0$ weakens further the condition on λ—but in that case, standard logic always prevails. On the contrary, in the procyclical-inequality case $\chi < 1$ when standard logic always prevails under separable preferences $\kappa = 0$, the inverted region may be brought by in case consumption and hours are substitutes, $\kappa < 0$.

3.2 Complementarity and the "Trilemma"

The analytical model outlined above offers a solution to the "trilemma" posed by Auclert et al. (2023). That is, when parameterising κ and γ separately, a model with heterogeneous households can generate positive fiscal multipliers, a high MPC, and a low MPE. To illustrate this solution, we focus on the 'TANK' limit of the model in which households do not move between states.[15]

To show the solution to the trilemma, we first show the relationship between individual MPEs and MPCs in our TANK model. This yields our next key result.

Proposition 4 *In the TANK model, the ratio of the individual marginal propensity to earn (MPE) to the marginal propensity to consume (MPC) is given by:*

$$\frac{MPE^j}{MPC^j} = \frac{WN}{C}\frac{\gamma}{\varphi},$$

which is independent of complementarity κ.

[15]The TANK limit is nested by the THANK model for $s = 1$. The use of this limit allows for a simpler expression for the fiscal multiplier but the solution to the trilemma does not materially differ from the THANK case.

This result is crucial in solving the "trilemma". The introduction of hand-to-mouth households allows the TANK model to replicate the high MPCs observed in the data. Setting a low value for γ (a small income effect) ensures that the MPE-MPC ratio is low. As this ratio is independent of the degree of Edgeworth complementarity, κ is entirely free to target the fiscal multiplier. This illustrates the importance of parameterizing κ and γ separately rather than using the single "complementarity index" introduced by Auclert et al. (2023), which, as we show below, is a convolution of the income effect and our Edgeworth complementarity.

To derive the result, we can follow the proof of Proposition 1 in Auclert et al. (2023) for our TANK model. Loglinearizing the first-order conditions (18) and (19) allows us to express the intratemporal optimality conditions as a 2x2 linear system:

$$\begin{pmatrix} \frac{U_{CC}C}{U_C} & \frac{U_{CN}N}{U_C} \\ \frac{U_{NC}C}{U_N} & \frac{U_{NN}N}{U_N} \end{pmatrix} \begin{pmatrix} c_t^j \\ n_t^j \end{pmatrix} = \begin{pmatrix} \hat{v_t}^j \\ \hat{v_t}^j + w_t. \end{pmatrix}$$

Inverting the matrix gives us equations for consumption and hours in terms of the wage rate and the Lagrange multiplier:

$$c_t^j = \frac{1}{\frac{U_{CC}C}{U_C}\frac{U_{NN}N}{U_N} - \frac{U_{CN}N}{U_C}\frac{U_{NC}C}{U_N}} \left[\left(\frac{U_{NN}N}{U_N} - \frac{U_{CN}N}{U_C} \right) \hat{v_t}^j - \frac{U_{CN}N}{U_C} w_t \right] \tag{32}$$

$$n_t^j = \frac{1}{\frac{U_{CC}C}{U_C}\frac{U_{NN}N}{U_N} - \frac{U_{CN}N}{U_C}\frac{U_{NC}C}{U_N}} \left[\left(\frac{U_{CC}C}{U_C} - \frac{U_{NC}C}{U_N} \right) \hat{v_t}^j + \frac{U_{CC}C}{U_C} w_t \right]. \tag{33}$$

In the context of our TANK model, the Auclert et al. (2023) 'complementarity index' is defined as:

$$CI \equiv \frac{\frac{\partial C^j(v^j, w)}{\partial w}}{W \frac{\partial N^j(v^j, w)}{\partial w}}.$$

Taking the derivative of first-order condition (19) with respect to the wage rate and rearranging yields

$$\begin{aligned} CI &= \frac{\frac{U_{CN}}{U_N}}{\frac{U_{CC}}{U_C}} = \frac{\frac{U_{CN}C}{-U_N}}{-\frac{U_{CC}C}{U_C}} = \frac{\frac{U_{CN}C}{-U_N}}{-\frac{U_{CC}C}{U_C} + \frac{U_{CN}C}{U_N} + \frac{U_{CN}C}{-U_N}} \\ &= \frac{\kappa}{\kappa + \gamma}, \end{aligned} \tag{34}$$

which shows that the CI is a convolution of our notion of Edgeworth complementarity and the income effect. It is the separate parameterization of these two forces that allows us to solve the "trilemma".

For a one-time transfer T_t, Auclert et al. (2023) define the marginal propensity to consume and the marginal propensity to earn as:

$$MPC^j = \frac{dC_t^j}{dT_t} \tag{35}$$

$$MPE^j = -W \frac{dN_t^j}{dT_t}. \tag{36}$$

The MPE-MPC ratio can therefore be expressed as:

$$\begin{aligned}\frac{MPE^j}{MPC^j} &= -\frac{WN}{C}\frac{\frac{\partial n_t^j(\hat{v}_t{}^j, w_t)}{\partial \hat{v}_t{}^j}}{\frac{\partial c_t^j(\hat{v}_t{}^j, w_t)}{\partial \hat{v}_t{}^j}} \\ &= -\frac{WN}{C}\frac{\frac{\partial n_t^j(\hat{v}_t{}^j, w_t)}{\partial \hat{w}_t{}^j}}{\frac{\partial c_t^j(\hat{v}_t{}^j, w_t)}{\partial \hat{v}_t{}^j}}\frac{\frac{\partial n_t^j(\hat{v}_t{}^j, w_t)}{\partial \hat{v}_t{}^j}}{\frac{\partial n_t^j(\hat{v}_t{}^j, w_t)}{\partial w_t}}\end{aligned} \tag{37}$$

Taking the derivative of (33) with respect to $\hat{v}_t^j$ and w_t, we can write:

$$\begin{aligned}\frac{\frac{\partial n_t^j(\hat{v}_t{}^j, w_t)}{\partial \hat{v}_t{}^j}}{\frac{\partial n_t^j(\hat{v}_t{}^j, w_t)}{\partial w_t}} = \frac{\frac{U_{CC}C}{U_C} - \frac{U_{CN}C}{U_N}}{\frac{U_{CC}C}{U_C}} &= 1 - \frac{\frac{U_{CN}}{U_N}}{\frac{U_{CC}}{U_C}} \\ &= 1 - CI = \frac{\gamma}{\kappa + \gamma}\end{aligned} \tag{38}$$

In Auclert et al. (2023), the EIS and the Frisch elasticity are defined as:

$$Frisch \equiv \frac{\partial n_t^j(\hat{v}_t{}^j, w_t)}{\partial \hat{w}_t{}^j} = \frac{\frac{U_{CC}C}{U_C}}{\frac{U_{CC}C}{U_C}\frac{U_{NN}N}{U_N} - \frac{U_{CN}N}{U_C}\frac{U_{NC}C}{U_N}} \tag{39}$$

$$EIS \equiv -\frac{\partial c_t^j(\hat{v}_t{}^j, w_t)}{\partial \hat{v}_t{}^j} = \frac{\frac{U_{CN}N}{U_C} - \frac{U_{NN}N}{U_N}}{\frac{U_{CC}C}{U_C}\frac{U_{NN}N}{U_N} - \frac{U_{CN}N}{U_C}\frac{U_{NC}C}{U_N}}. \tag{40}$$

Substituting (38) and these definitions into (37) shows that Proposition 1 of Auclert et al. (2023) holds in the TANK model:

$$\frac{MPE^j}{MPC^j} = \frac{WN}{C}\frac{Frisch}{EIS}(1 - CI). \tag{41}$$

Combining the definitions of κ and γ with the steady-state intratemporal optimality condition yields:

$$\begin{aligned}\frac{U_{CC}C}{U_C} &= -(\gamma + \kappa) \\ \frac{U_{NN}N}{U_N} &= \varphi + \kappa(1 - \Phi) \\ \frac{U_{CN}N}{U_C} &= \kappa(1 - \Phi).\end{aligned}$$

Substitute these equations into the definitions (39) and (40) to obtain:

$$Frisch = \frac{-(\gamma + \kappa)}{-(\gamma + \kappa)\left[\varphi + (1 - \Phi)\right] + \kappa(1 - \Phi)\kappa} = \frac{1}{\varphi + (1 - \Phi)\frac{\kappa\gamma}{\gamma + \kappa}} \tag{42}$$

$$EIS = \frac{\frac{\varphi}{\gamma + \kappa}}{\varphi + (1 - \Phi)\frac{\kappa\gamma}{\gamma + \kappa}}. \tag{43}$$

Substituting into (41) shows the result in our Proposition 3 - the ratio of MPEs to MPCs is entirely independent of complementarity, κ:

$$\frac{MPE^j}{MPC^j} = \frac{WN}{C}\frac{\gamma}{\varphi}. \tag{44}$$

Having established that the TANK model can generate high MPCs alongside low MPEs, solving the "trilemma" requires the model to simultaneously generate 'moderate' positive fiscal multipliers. To derive the fiscal multiplier in the TANK model with complementarity, set $s = 1$ in the aggregate Euler equation (31). This yields the aggregate Euler equation for the TANK model:

$$\begin{aligned} c_t =& \mathbb{E}_t c_{t+1} + \frac{\lambda(\chi-1)\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma}+\kappa(1-\lambda)}{\gamma(1-\lambda\chi)-\kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}(g_t-\mathbb{E}_t g_{t+1}) \\ &- \frac{1-\lambda}{\gamma(1-\lambda\chi)-\kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)} r_t. \end{aligned} \tag{45}$$

Taking the partial derivative of the aggregate Euler equation with respect to government spending gives the (fixed real interest rate) fiscal multiplier:

$$\frac{\partial c_t}{\partial g_t} = \frac{\lambda(\chi-1)\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma}+\kappa(1-\lambda)}{\gamma(1-\lambda\chi)-\kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}.$$

The fiscal multiplier in the TANK model nests the RANK multiplier when $\lambda = 0$. Note that, as in Auclert et al. (2023), the RANK fiscal multiplier is inversely proportional to the 'complementarity index':

$$\frac{\partial y_t}{\partial g_t}^{RANK} = \frac{1}{1-CI}.$$

However, the complementarity index does not feature in the the TANK fiscal multiplier. Therefore, the CI is not the relevant combination of κ and γ to understand the role of complementarity in fiscal policy transmission in the model with heterogeneous households.

Provided that the elasticity of aggregate demand to interest rates (η^{HA}) is positive, government spending generates a positive consumption response in the TANK model if and only if:

$$\lambda(\chi-1)\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma}+\kappa(1-\lambda) > 0.$$

This implies a threshold value for χ above which fiscal multipliers are positive in the model:

$$\frac{\partial c_t}{\partial g_t} > 0 \text{ if } \chi > 1 - \frac{\kappa(1-\lambda)}{\lambda\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma}} \equiv \bar{\chi}.$$

In the TANK model without complementarity, the requirement for a positive fiscal multiplier is countercyclical inequality ($\chi > 1$). The introduction of complementarity ($\kappa > 0$) alters this threshold, as discussed in our next Proposition.

Proposition 5 *Introducing stronger complementarity between consumption and hours into the TANK model generates positive fiscal multipliers for more procyclical income inequality. That is, a higher value for κ reduces the χ threshold above which fiscal multipliers are positive.*

This result can be proved immediately by noting that the derivative of $\bar{\chi}$ with respect to κ is negative:

$$\frac{\partial \bar{\chi}}{\partial \kappa} = -\frac{(1-\lambda)\lambda\gamma\frac{\varphi}{\varphi+\gamma}}{\left[\lambda\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma}\right]^2} < 0.$$

Hence, the higher is the degree of Edgeworth complementarity ($\kappa \uparrow$), the lower is the χ threshold above which the fiscal multiplier is positive ($\bar{\chi} \downarrow$). Introducing complementarity amplifies fiscal policy, allowing us to achieve positive fiscal multipliers for lower values of χ than in the baseline TANK model.

As the MPE-MPC ratio is independent of κ, the separation of κ and γ therefore offers a solution to the "trilemma" posed by Auclert et al. (2023). A higher value of κ generates positive fiscal multipliers for more procyclical income inequality, while a lower value of γ allows the model to generate high MPCs alongside low MPEs.

3.3 Complementarity and the "Catch-22"

The introduction of consumption-hours complementarity also offers a solution to the "dilemma" posed by Bilbiie (2025).

Proposition 6 *The THANK model with complementarity resolves the Bilbiie (2025) "catch-22". With $\kappa > 0$, the model can generate positive fiscal multipliers without the forward guidance puzzle.*

To show how the THANK model resolves the "dilemma", first recall that the aggregate

Euler equation is given by:

$$
\begin{aligned}
c_t = & \underbrace{\left[1+\frac{(1-s)(\chi-1)\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)}{\gamma(1-\lambda\chi)-\kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}\right]}_{\equiv\tilde{\delta}}\mathbb{E}_t c_{t+1} \\
& + \frac{\lambda(\chi-1)\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma}+\kappa(1-\lambda)}{\gamma(1-\lambda\chi)-\kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}(g_t-\mathbb{E}_t g_{t+1}) \\
& + \underbrace{\frac{(1-s)(\chi-1)\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma}}{\gamma(1-\lambda\chi)-\kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}}_{=(\tilde{\delta}-1)\frac{\varphi}{\varphi+\gamma}}\mathbb{E}_t g_{t+1} \\
& - \frac{1-\lambda}{\gamma(1-\lambda\chi)-\kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}r_t.
\end{aligned}
$$

As shown by Bilbiie (2020), the THANK model exacerbates the forward guidance puzzle when the aggregate Euler exhibits compounding of 'good news' shocks ($\tilde{\delta}>1$), but solves the forward guidance puzzle when the aggregate Euler exhibits discounting ($\tilde{\delta}<1$). Complementarity therefore does not alter the conditions required to solve the forward guidance puzzle. The THANK model with complementarity exhibits discounting when:

$$
\frac{(1-s)(\chi-1)\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)}{\gamma(1-\lambda\chi)-\kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}<0.
$$

Conditional on the interest rate elasticity (η^{HA}) being positive, this requires procyclical income inequality ($\chi<1$). In the baseline (no complementarity) THANK model, this poses a "dilemma" between the forward guidance puzzle (which requires $\chi<1$) and positive fiscal multipliers (which require $\chi>1$). This "catch-22" is solved in the THANK model with complementarity between hours and consumption, which can generate positive fiscal mutlipliers alongside aggregate Euler equation discounting.

For a government spending shock of persistence ρ, the fiscal multiplier is given by:

$$
\frac{\partial c_t}{\partial g_t}=\frac{1}{1-\tilde{\delta}\rho}\frac{(1-\rho)\left[\lambda(\chi-1)\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma}+\kappa(1-\lambda)\right]+\rho(\tilde{\delta}-1)\frac{\varphi}{\varphi+\gamma}}{\gamma(1-\lambda\chi)-\kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}.
$$

For a one-time shock ($\rho=0$), the fiscal multiplier is the same as the TANK case:

$$
\frac{\partial c_t}{\partial g_t}=\frac{\lambda(\chi-1)\left(\gamma+\kappa\frac{\varphi+\gamma}{\varphi}\right)\frac{\varphi}{\varphi+\gamma}+\kappa(1-\lambda)}{\gamma(1-\lambda\chi)-\kappa\frac{\varphi+\gamma}{\varphi}\lambda(\chi-1)}.
$$

As shown in Proposition 5, a higher value of complementarity ($\kappa\uparrow$) lowers the threshold above which the fiscal multiplier is positive ($\bar{\chi}\downarrow$). Hence, while the baseline THANK

model ($\kappa = 0$) requires countercyclical income inequality and Euler equation compounding to generate a positive fiscal multiplier, the THANK with consumption-hours complementarity ($\kappa > 0$) can generate a positive fiscal multiplier even with *procyclical* income inequality and Euler equation *discounting* ($\chi < 1$ and $\tilde{\delta}$). This means that the model resolves the dilemma between a positive fiscal multiplier and alleviating the forward guidance puzzle.

4 Quantitative Analysis in a HANK Model

Having studied analytically the role of consumption-hour complementarity in New Keynesian models with heterogeneity we will illustrate the implications in quantitative circumstances. To do this we will setup a standard Heterogenous Agent New Keynsian Model. We will then show that varying complementarity changes the fiscal multiplier while not effecting the marginal propensity to earn conditional on a target of the marginal propensity to consume. Finally, we show how complementarity can resolve the "catch-22" by increasing the fiscal multiplier while maintaining discounting in the Euler equation.

4.1 HANK with consumption-hour complementarity

For comparability the modelling setup except for the utility function is taken from Auclert et al. (2023).

A continuum of households make choices over consumption, labor supply and savings in a liquid asset in discrete time t. They have idiosyncratic labor productivity e which evolves according to an exogenous Markov chain. The liquid asset earns a ex-post real interest rate of r_t and holdings are subject to a borrowing constraint of $\underline{a}$. The household earns a wage per efficiency unit of labor of w_t and faces a wage tax of τ_t^w as well as transfers T_t. Formally they solve

$$\begin{aligned} V_t(a,e) \quad &= \quad \max_{c,n,a'} U(c,n) + \beta \mathbb{E}\left[V_{t+1}(a',e')|e\right] \\ &\text{s.t.} \quad c + a' = (1-\tau_t^w) w_t e n + T_t + (1+r_t)a \\ &\qquad\quad a' \geq \underline{a} \end{aligned} \tag{46}$$

For $U(c,n)$ we use the functional form described earlier (14) in which ξ controls the degree of consumption-hours complementarity, γ the income effect on labor supply and φ the constant-consumption elasticity of labor supply.

The supply of liquid assets is made up of one-period real bonds issued by the government B_t and equity in the intermediate goods firms. Due to the assumption of certainty equivalence at the aggregate level, in equilibrium the two assets must offer the same ex-ante expected return. Letting p_t be the price of equity post distribution of dividends and d_t be the dividends themselves this no arbitrage condition is

$$1 + \mathbb{E}[r_t] = \mathbb{E}\left[\frac{d_{t+1} + p_{t+1}}{p_t}\right] \tag{47}$$

As we analyze responses to MIT shocks under perfect foresight for $t > 0$ the ex-ante expected return $\mathbb{E}[r_{t-1}]$ and ex-post returns r_t will always be equal. However this is not the case for $\mathbb{E}[r_{-1}]$ and r_0 as the shock can trigger unanticipated changes in the value of equity. We assume that in the non stochastic steady state households all hold the same ratio of bonds and equity[16] and thus experience the same ex-post interest rate. The ex-post interest rate is thus a weighted average of the ex-post return of equity and bonds

$$1 + r_t = \frac{p_{t-1}}{p_{t-1} + B_{t-1}} \frac{d_t + p_t}{p_{t-1}} + \frac{B_{t-1}}{p_{t-1} + B_{t-1}} (1 + \mathbb{E}[r_t]) \tag{48}$$

The supply side of the model follows the standard New Keynesian structure. The final consumption good is a constant-elasticity of substitution aggregator of individual goods $C_t = \int_0^1 C_t j^{\frac{\theta-1}{\theta}}$, where $\theta > 1$ is the elasticity of substitution. Each of the differentiated goods $j \in [0, 1]$ is produced by a monopolistic firm who operates a constant returns to scale production function in labor $Y_t(j) = ZN_t j - F$, where Z is an aggregate productivity term and F is a fixed cost of production. Varying F allows firm profit and equity, a key component of liquidity, to be changed independently of the markup and the elasticity of substitution. This property will also be useful in the later analysis of the economy without liquidity. These firms set their prices taking into account the downwards sloping demand function they face as well as quadratic price adjustment costs a la Rotemberg (1982) with parameter κ_p. Since the demand function and adjustment costs are the same for all firms, there exists a symmetric equilibrium in which they all set the same price and therefore have the same output. This gives rise to the following New Keynesian Philips curve.

$$\log(1 + \pi_t) = \left(w_t - \frac{\theta - 1}{\theta} \right) + \frac{1}{1 + \mathbb{E}[r_t]} \frac{Y_{t+1}}{Y_t} \log(1 + \pi_{t+1}) \tag{49}$$

And aggregate firm dividends are

$$d_t = Y_t - w_t N_t - \frac{\theta}{2\kappa_p} \log(1 + \pi_t)^2 Y_t \tag{50}$$

The government purchases an exogenous quantity of finals goods G_t, issues one-period real debt B_t and makes uniform constant transfers to households of T. It finances these three expenditures with the labor tax τ_t^w. The budget constraint is given by

$$B_t + \tau_t^w w_t N_t = (1 + \mathbb{E}[r_t]) B_{t-1} + G_t + T \tag{51}$$

We assume that government has a fiscal rule that implies increases in government expenditure today are paid for at rate of $1 - \rho_B$ each period.

[16] Households are indifferent between the ratio in steady state due to their equal returns and shocks being unanticipated

$$B_t - B_{ss} = \rho_B(B_{t-1} - B_{ss} + G_t - G_{ss}) \tag{52}$$

Given this rule τ_t^w is determined as the residual that ensures that the government budget constraint holds in all periods.

The monetary policy authority operates a real interest rate rule setting $(1 + \mathbb{E}[r_t]) = r_{ss}$.

Market Clearing. There are three markets that must clear. First the labor market, labor demanded by firms must equal labor supplied by households

$$N_t^d = N_t^s \tag{53}$$

Second, in the goods market production must equal private consumption, government consumption and price adjustment costs

$$Y_t = C_t + G_t + \frac{\theta}{2\kappa_p}\log(1 + \pi_t)^2 Y_t \tag{54}$$

Finally, in the asset market the sum of government bonds and firm equity must equal asset demand from households.

$$B_t + p_t = A_t \tag{55}$$

4.2 Calibration

In the calibration we again follow Auclert et al. (2023) with some small changes. We target a quarterly average MPC of 0.25. In order to be able to match this alongside the required asset demand we allow for permanent discount factor heterogeneity with one half of the agents having a discount factor of β_1 and the other half having a discount factor of $\beta_2 < \beta_1$.

We take the model period to be a quarter and set the annualized value of r to 2%. Unlike in Auclert et al. (2023) the Frisch elasticity of labor supply cannot be set directly and we will instead calibrate φ to target a value of 0.5. We also choose the disutility of labor h such that aggregate effective labor is 1 and aggregate labor productivity Z such that aggregate output is also 1.

As is standard we assume idiosyncratic labor productivity e is governed by an AR(1) process. We take the persistence of this process to be 0.966 as in McKay et al. (2016) and calibrate the variance of innovations to the process to match the standard deviation of log gross earnings of 0.92 estimated by Song et al. (2019). This process is discretized as an 11 point Markov chain. We set the steady-state labor tax $\tau^w = 0.334$ and the steady-state transfer to $T = 0.143 \cdot wN$ following Auclert and Rognlie (2018).

We set the value of firm equity equal to 0.85 times annual GDP as in Auclert et al. (2023) but we allow for a higher value of government debt to annual GDP of 1 compared to their value of 0.55. This change has a practical purpose in allowing the model to

match the MPC target for all values of complementarity. Additionally it is consistent with higher debt levels seen in many developed economies in recent years. Given tax revenue and interest payments, government spending G is determined residually as 0.168. The borrowing constraint is set so households cannot borrow $\underline{a} = 0$. As in Auclert et al. (2023) we set the slope of the Philips curve to $\kappa_p = 0.01$ but this is not important for our results which are with a constant r rule and focus on quantities not prices.

Finally we set the income effect on labor supply equal to $\gamma = 0.25$. From the analytical formula from 3 $\frac{MPE}{MPC} = \frac{WN}{C}\frac{\gamma}{\varphi}$ and assuming $\frac{WN}{C} \approx 1$, this implies a quarterly $MPE = 0.25 \cdot \frac{0.25}{2} = 0.03$. This lies within the range that Auclert et al. (2023) argued was supported by the empirical data. While this does imply a high value of the intertemporal elasticity of substitution without complementarity, as complementarity increases the intertemporal elasticity of substitution will decrease for a fixed γ.

4.3 MPEs, Multipliers and Complementarity

Having set up the model, we now study the quantitative effect of complementarity on fiscal multipliers and the marginal propensity to earn. To do this we re-calibrate the model for values of ξ between 0 and 1 and then calculate the average MPE and the fiscal multiplier. In each of these calibrations the average MPC is a target and is thus held constant as ξ varies.

We plot the results of this exercise in Figure 1. The fiscal multiplier is calculated as the impact response $\frac{dY_0}{dG_0}$ to an AR(1) government spending shock with a quarterly persistence of 0.9, the same as in Auclert et al. (2023). The annual MPE is calculated by taking the responses of wage income over a year to a one time transfer in the first quarter of that year. The green box is the range of values that Auclert et al. (2023) argued are consistent with the empirical evidence for both MPEs and multipliers and we maintain the same x-axis range that they had in order to highlight the difference.

As discussed in the analytical results the MPE is a function of the income effect, the constant consumption labor supply elasticity and the MPCs. Therefore given the MPC target and the Frisch target the γ chosen means that the model without complementarity has an MPE that lies within the range of empirical estimates. As the degree of complementarity is changed the value of the MPE changes little, going from 0.027 when $\xi = 0$ to 0.030 when $\xi = 1$. The changes in the MPE are because as ξ becomes positive a wedge is created between the constant consumption labor supply elasticity φ and the Frisch elasticity of labor supply. An alternative calibration strategy in which φ is fixed at a target value would eliminate most of the small changes in the MPE for different ξ. Thus empirically realistic MPEs can be matched by models with any degree of complementarity as long as the income effect on labor supply γ is sufficiently low.

Consumption-hours complementarity has a more significant effect on the multiplier, consistent with the analytical results. Starting at 1.24 when $\xi = 0$ and climbing to 2.79 when $\xi = 1$. In response to the government spending shock wages rise to induce the extra hours required to accommodate the increase in demand for final goods. When there is consumption-hours complementarity and hours rise, this increases the marginal utility of consumption today. Thus boosting consumption demand, further increasing output

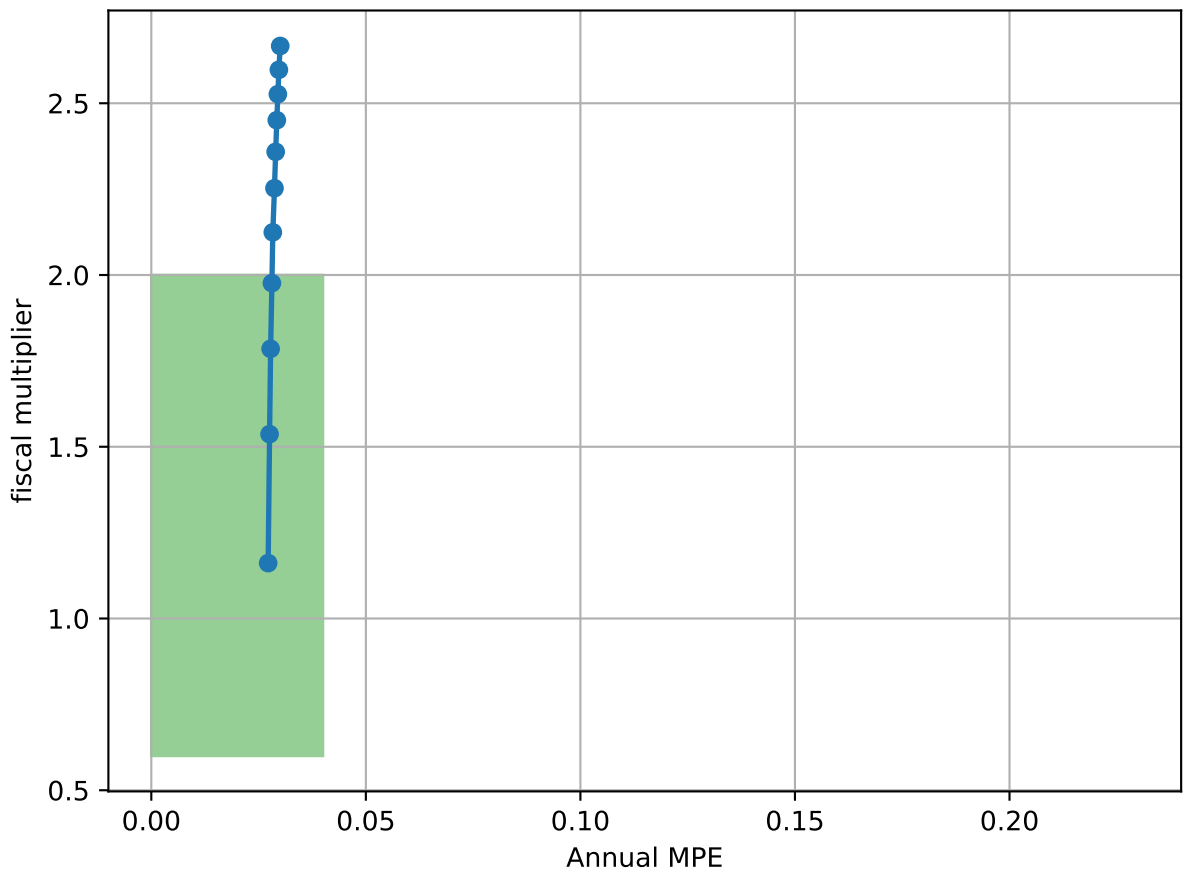


Figure 1: Fiscal Multipliers, MPEs and Complementarity

leading to a larger multiplier.

While the multipliers at the top end of complementarity are larger than justified by empirical evidence, it is possible to span much of the empirically reasonable cases by adjusting the degree of complementarity. It should be noted that the multipliers here are conditional on a constant r monetary policy rule. Under the more realistic case of a Taylor rule they would be smaller due to the countervailing monetary policy response.

This result differs from Auclert et al. (2023) who find that when varying their complementarity index, there is a trade-off between the MPE and the fiscal multiplier. As can be seen in 34 this is because when their complementarity index is varied two underlying parameters are varying, γ the income effect on labor supply and the degree of complementarity κ. It is the changes in γ which drive the changes in the MPE and κ the fiscal multipliers. Thus by allowing for separate parameterization of these two aspects of utility, their implications for MPEs and fiscal multipliers can be separated.

4.4 "Catch-22" and Complementarity

In this section we analyze the role of complementarity in breaking the "catch-22" of heterogeneity in New Keynesian models. This is the statement that if heterogeneity leads

to amplification of fiscal multipliers compared to the representative agent case this implies compounding in the Euler equation rather than discounting. This exacerbates the forward guidance puzzle relative to the representative agent case leading to larger responses today as a given interest change is moved further and further into the future.

To do this we make several changes to the model from before. Due to the key role of the cyclicality of income inequality in determining the effect of heterogeneity on multipliers and Euler equation discounting/compounding we change the payouts of firm dividends in order to control it. Instead of firm dividends being paid to owners of firm equity, we now assume that it is transferred to households according to a fixed rule. We choose the fixed cost F such that steady state profits are 0 and therefore the dividend rule does not effect the steady state of the model. The tax funded transfer from the previous section is also set to 0 so the only transfer that households receive is from dividends.

In the benchmark case of acyclical income inequality the transfer must to be proportional to steady state income wn. As n will depend on both assets and productivity this implies that the transfer must also depend on both. So the transfer rule we choose takes the following form.

$$T_t(e,a) = (w_{ss}n_{ss}(e,a))^{\alpha}/m \cdot d_t^{pay} \tag{56}$$

Where m is a normalising constant such that $\int (w_{ss}n_{ss}(e,a))^{\alpha}/m dD_{ss}(e,a) = 1$ where D_{ss} is the steady state distribution. α is a parameter controlling the cyclicality of income inequality, when $\alpha = 1$ dividend payments are proportional to steady state income and thus income inequality is acylical. Dividend payments d_t^{pay} are not the dividend from firms d_t but an intermediate quantity that takes into account changes in the distribution. As $D_t \neq D_{ss}(e,a)$, it will not always be the case that $\int (w_{ss}n_{ss}(e,a))/m dD_{ss}(e,a) = 1$ and so d_t^{pay} is chosen in order to ensure total dividends received by households equal total dividends from firms. That is it solves the following problem each period

$$\int (w_{ss}n_{ss}(e,a))/m d_t^{pay} dD_t(e,a) = d_t \tag{57}$$

gm

The only other change required such that the baseline case of acyclical inequality matches the representative agent fiscal multiplier of 1 is to make government spending shocks fully tax financed. This can be simply achieved by setting the ρ_B parameter in the government's debt issuance rule to 0. Therefore the government issues no additional debt in response to a shock to government spending.

This is not enough, however, to match the representative agent result of neither compounding or discounting in the Euler equation. To match that benchmark we have to go to the zero liquidity limit as in the results by Werning (2015). Since we have already removed equity from the asset supply the only further change is to set the steady state level of government debt to 0[17].

[17]For numerical reasons the level of debt cannot be set to exactly 0 but instead we set it to a small value for which the model remains numerically stable.

Figure 2 contains the results. The x-axis is the Euler equation discount factor which is measured from using the period 0 output effect of a change in interest rates at either period 0 or period 20. We define this discount factor as

$$\delta_{fg} = \left(\frac{\frac{\partial Y_0}{\partial r_{20}}}{\frac{\partial Y_0}{\partial r_0}} \right)^{\frac{1}{20}} \tag{58}$$

So if the response is the same no matter when the interest rate is shocked as in the representative agent case $\frac{\partial Y_0}{\partial r_{20}} = \frac{\partial Y_0}{\partial r_0}$ and thus $\delta_{fg} = 1$. However if the response decreases as the shock occurs further into the future then $\delta_{fg} < 1$. Additionally applying this measure to the analytical model we get $\delta_{fg} = \tilde{\delta}$. On the y axis is the contemporaneous response to a one period government spending shock in period 0. This is again the counterpart of the analytical government spending multiplier. The black dotted lines indicate the representative agent baseline, a fiscal multiplier of 1 and neither discounting nor compounding in the euler equation $\delta_{fg} = 1$.

In the yellow dash dot line we vary α, the parameter that controls the cyclicality of inequality while keeping $\xi = 0$. This line passes through the representative agent baseline of a fiscal multiplier of 1 and $\delta_{fg} = 1$ when $\alpha = 1$. Otherwise the curve lies in two quadrants, the upper right featuring amplification of fiscal multipliers relative to the representative agent and compounding in the Euler equation. As well as the bottom left featuring discounting in the Euler equation but dampening of fiscal multipliers relative to the representative agent. This is the quantitative version of the "catch 22", that there is a tradeoff between the desire for amplification of fiscal multipliers and discounting of the Euler equation in New Keynesian models with heterogeneity.

In the blue line we start from $\alpha = 0.95$ and change the degree of complementarity from $\xi = 0$ to $\xi = 1$. As ξ increases fiscal multipliers increase, going beyond 2 for $\xi = 1$. This is again due to the increase in hours due to a government spending shock increasing the marginal utility of consumption due to complementarity and therefore increasing household demand. However unlike with the catch-22 this does not come at the cost of euler equation compounding. Instead as we start from a point with euler equation discounting, the degree of discounting is amplified by the increases in complementarity. Thus complementarity allows for HANK models to have both amplification of fiscal multipliers while not exacerbating the forward guidance puzzle.

5 Conclusions

One of the very primitives of any macroeconomic model used for business-cycle and policy analysis is the utility function. In this paper, we first showed that some key macro facts pertaining to business cycles depend crucially on a restricted set of parameters that can be defined for fully general utility functions $U(C, N)$ of consumption and hours worked. In particular, a general-equilibrium elasticity of intertemporal substitution, a key parameter in any dynamic model, shapes the response of the economy to demand shocks and to changes in policy. With general nonseparable preferences, this elasticity depends on

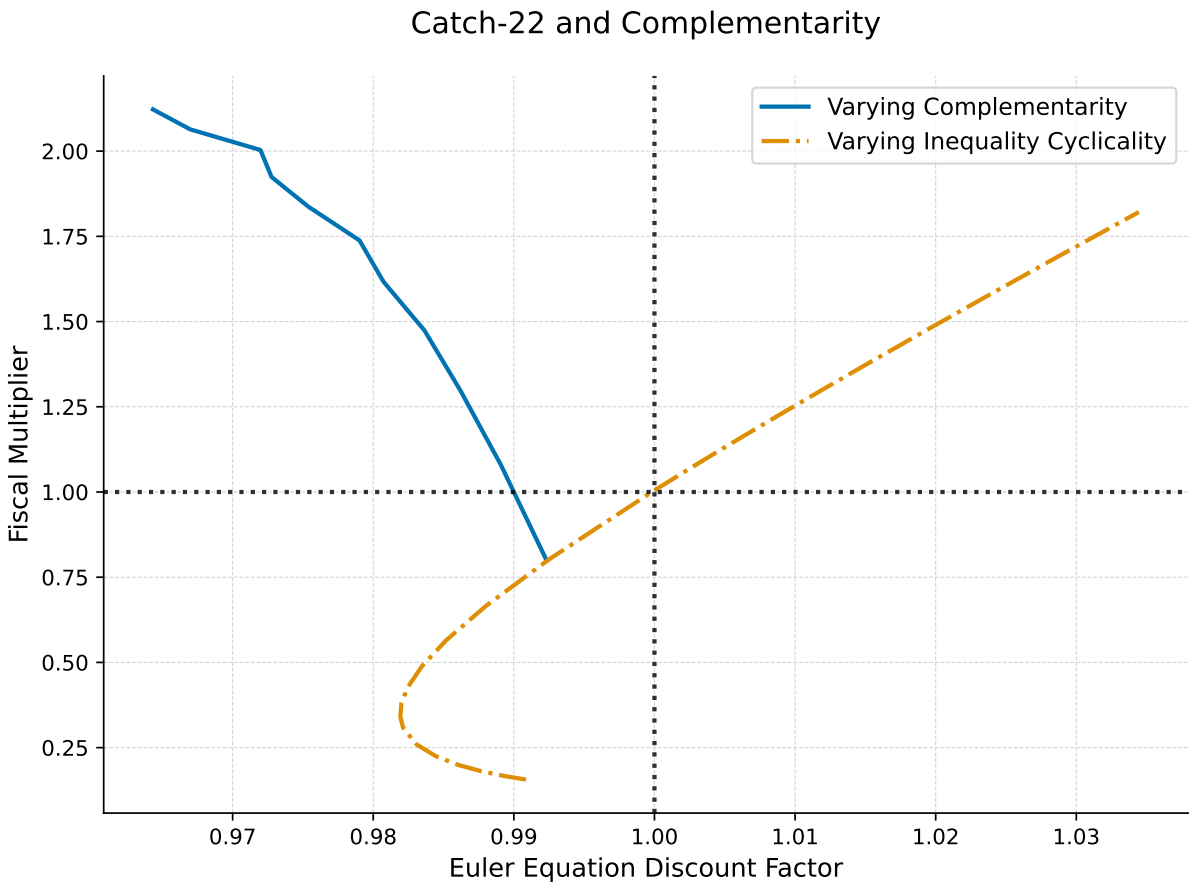


Figure 2: Catch-22 and Complementarity

the income effect on labor supply, and on the degree of complementarity of consumption and hours worked (as well as on some supply-side parameters).

The paper provides a general discussion of the role of all of these three parameters: complementarity, income effect, and their interaction with heterogeneity in determining key macroeconomic effects of demand shocks and policies, and business-cycle comovement. We have shown how complementarity between consumption and hours, parameterized independently of income effects, is key to resolve to outstandting puzzles pertaining to "multipliers" in heterogeneous-agent models.

Our analytical results are purposefully derived for a very general (arbitrary) utility function defined over consumption and hours, $U(C, N)$. Lastly, and of particular relevance for applied, quantitative work, we propose two parametric forms of utility functions that, with a minimal departure from already-used and familiar functions, allows parameterizing the degree of complementarity freely—while also allowing simultaneously for arbitrary income effects on labor supply, and constant-consumption labor supply elasticity. The first is a "CRRA meets GHH", quasi-separable function, and the second a constant-complementarity, CCRRA function; both can be employed in quantitative studies and for estimation in any business-cycle model, including models with heterogeneous agents where income and wealth effects can be calibrated based on cross-sectional data. We provided one such quantitative illustration of how a standard quantitative HANK model with our utility function can—by parameterizing complementarity—yield substantial fiscal multipliers while matching MPCs and MPEs, and also resolving the forward-guidance puzzle.

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Appendix

A Intertemporal utility maximization

An agent j chooses consumption, asset holdings and hours solving the standard intertemporal problem: max E_0, subject to the sequence of constraints:

$$B_t \leq Z_t + P_t D_t + W_t N_t - P_t C_t.$$

C_t, N_t are consumption and hours worked, B_t is the nominal value at *end of period* t of a portfolio of all state-contingent assets held (including shares in firms)—likewise for Z_t, beginning of period wealth. D_t is the real dividend payoff. Absence of arbitrage implies that there exists a stochastic discount factor $Q_{t,t+1}$ such that the price at t of a portfolio with uncertain payoff at $t+1$ is for state-contingent assets:

$$\frac{B_t}{P_t} = E_t \left[Q_{t,t+1} \frac{Z_{t+1}}{P_{t+1}} \right], \tag{59}$$

which iterated forward gives the fundamental pricing equation. The riskless gross short-term REAL interest rate R_t is a solution to:

$$\frac{1}{R_t} = E_t Q_{t,t+1} \tag{60}$$

Note for nominal assets we have the nominal interest rate $\frac{1}{I_t} = E_t \frac{P_t}{P_{t+1}} Q_{t,t+1}$

Substituting the no-arbitrage conditions (59) into the wealth dynamics equation gives the flow budget constraint. Together with the usual 'natural' no-borrowing limit for *each* state this implies the usual intertemporal budget constraint:

$$E_t \left[Q_{t,t+1} X_{t+1} \right] \leq X_t + W_t N_t - P_t C_t.$$

$$X_t = Z_t + P_t D_t$$

$$E_t \tag{61}$$

where

$$Y_{t+i} = D_{t+i} + \frac{W_{t+i}}{P_{t+i}} N_{t+i} \tag{62}$$

is income. Maximizing utility subject to this constraint gives the first-order necessary and sufficient conditions at each date and in each state:

$$\beta \frac{U_C \left(C_{j,t+1} \right)}{U_C \left(C_{j,t} \right)} = Q_{t,t+1}$$

along with (**??**) holding with equality (or alternatively flow budget constraint holding with equality and transversality conditions ruling out Ponzi games be satisfied: $\lim_{i \to \infty} E_t \left[Q_{t,t+i} Z_{t+i} \right] = 0$). Using (**??**) and the functional form of the utility function the short-term real interest rate must obey:

$$\frac{1}{R_t} = \beta E_t \left[\frac{U_C \left(C_{j,t+1} \right)}{U_C \left(C_{j,t} \right)} \right].$$

A.1 Frisch elasticity and labor supply

One distinguishing feature of non-separable preferences which turns out to be crucial for understanding the issue under study here is that the Frisch labor supply, i.e. labor supply for a constant level of marginal utility $U_C(C_t, L_t)$, is different from the constant-consumption labor supply (7). Frisch labor supply is obtained by treating marginal utility as a variable (which is indeed the Lagrange multiplier on the household budget constraint): $U_C(C_t, N_t) = \Lambda_t$, or in log-linearized form:

$$\frac{U_{CC}C}{U_C}c_t + \frac{U_{CN}N}{U_C}n_t = \lambda_t$$

Substituted in (7) to eliminate consumption $c_t = \left(\frac{U_{CC}C}{U_C}\right)^{-1}\lambda_t - \left(\frac{U_{CC}C}{U_C}\right)^{-1}\frac{U_{CN}N}{U_C}n_t$, this delivers

$$\frac{U_{CN}C}{U_N}\left(\frac{U_{NN}N}{U_{CN}C} - \frac{U_{CN}N}{U_{CC}C}\right)n_t = w_t + \left(1 - \frac{U_{CN}}{U_N}\frac{U_C}{U_{CC}}\right)\lambda_t. \tag{63}$$

or, using the parameters defined above and the newly defined (inverse) Frisch elasticity of labor supply φ_F replacing $\frac{U_{NN}N}{U_N} = \varphi + \kappa\frac{\alpha}{\mu}$ $U_{CC}C/U_C = -\kappa - \gamma$ we obtain the expression in text.

A.2 Concavity and non-inferiority restrictions

The conditions may be of practical use to researchers using such an utility function in applied work.[18] The restrictions implied by concavity and non-inferiority given in text (9) are obtained as follows. The three concavity requirements are equivalent to:$U_{CC} \leq 0 \leftrightarrow \kappa \geq -\gamma$; $U_{NN} \leq 0 \leftrightarrow \kappa\alpha(1-\Phi) \geq -\varphi$ and $U_{CC}U_{NN} - (U_{CN})^2 \geq 0 \leftrightarrow \kappa \geq -\frac{\gamma\varphi}{\varphi+\gamma\alpha(1-\Phi)}$. Quasi-concavity (which is implied by concavity) requires that the bordered Hessian be negative semidefinite, i.e. $2U_CU_NU_{CN} - U_{CC}(U_N)^2 - U_{NN}(U_C)^2 \geq 0 \leftrightarrow \gamma + \frac{\varphi}{\alpha(1-\Phi)} \geq 0$.

The good inferiority conditions can be obtained starting with the static budget constraint rewritten as:

$$C_t + W_t(1 - N_t) \leq E_t,$$

where E_t is 'full income' given by $E_t = W_t + B_t + D_t - T_t - (1+R_t)^{-1}B_{t+1}$. A good is inferior if its income elasticity of demand is negative at given prices. We find demands for leisure and consumption for a given level of full income E by solving the static optimization problem: $\max U(C, N)$ s.t. $C + W(1-N) \leq E$, which leads to:

$$\begin{aligned} WU_C(C,N) + U_N(C,N) &= 0 \\ C + W(1-N) - E &= 0 \end{aligned} \tag{64}$$

We apply the implicit function theorem to this system to study variations in consumption and leisure demand to changes in income E.The solution is (using $W = U_L/U_C$, the

[18]This section is draws on the conditions derived in Bilbiie (2011), but for utility defined over hours rather than leisure.

definitions of φ and $\tilde{\gamma}$ and the steady-state restriction (5)):

$$\frac{dC}{dE} = \frac{1}{\alpha\left(1-\Phi\right)\frac{\gamma}{\varphi}+1}$$
$$\frac{dN}{dE} = -\frac{\gamma}{\varphi}\frac{\alpha\left(1-\Phi\right)}{\alpha\left(1-\Phi\right)\frac{\gamma}{\varphi}+1}\frac{1}{W}$$

Non-inferiority of consumption requires $\frac{\partial C}{\partial E} \geq 0$, i.e.:

$$-\frac{\gamma\alpha\left(1-\Phi\right)}{\varphi} \leq 1. \tag{65}$$

Non-inferiority of leisure requires $\frac{\partial N}{\partial E} \leq 0$, so:

$$\frac{\frac{\gamma}{\varphi}\alpha\left(1-\Phi\right)}{\alpha\left(1-\Phi\right)\frac{\gamma}{\varphi}+1} \geq 0$$

$$-\frac{\varphi}{\gamma\alpha\left(1-\Phi\right)} \leq 1. \tag{66}$$

Neither good is inferior if and only if (65) and (66) hold *simultaneously*, i.e.:

$$\frac{\gamma\alpha\left(1-\Phi\right)}{\varphi} \geq 0. \tag{67}$$

Non-inferiority and quasi-concavity ($\gamma\alpha\left(1-\Phi\right)+\varphi \geq 0$) imply the first two restrictions in (9).

The concavity requirements outlined above imply $\kappa \geq -\min\left(\gamma, \frac{\varphi}{\alpha(1-\Phi)}, \frac{\gamma\varphi}{\varphi+\gamma\alpha(1-\Phi)}\right)$, while these non-inferiority restrictions imply the third condition on complementarity in (9), because $\gamma > \frac{\gamma}{1+\varphi^{-1}\gamma\alpha(1-\Phi)}$and $\frac{\varphi}{\alpha(1-\Phi)} > \frac{\gamma\varphi}{\varphi+\gamma\alpha(1-\Phi)}$.

B Utility Derivations: GHH-CRRA and C-CRRA

This Appendix contains the derivations for the two parametric utility functions we introduced.

B.1 The GHH-CRRA Utility Function

Marginal utilities for the nonseparable utility (14) are:

$$U_C = C^{-\gamma}\left(\frac{C^{1-\gamma}}{1-\gamma} - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}}; U_N = -hN^{\varphi}\left(\frac{C^{1-\gamma}}{1-\gamma} - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}}$$

The intratemporal optimality condition thus implies

$$\frac{-U_N}{U_C} = hC^{\gamma}N^{\varphi} = \frac{(1+s)\,F_N}{\mu}; h\frac{N^{1+\varphi}}{C^{1-\gamma}} = \alpha\left(1-\Phi\right)$$

Cross-derivatives are hence:

$$U_{CC} = -\gamma C^{-\gamma-1}\left(\frac{C^{1-\gamma}}{1-\gamma} - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}} - \frac{\xi}{1-\gamma}\left(C^{-\gamma}\right)^2\left(\frac{C^{1-\gamma}}{1-\gamma} - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}-1}$$

$$U_{CN} = h\frac{\xi}{1-\gamma}C^{-\gamma}N^{\varphi}\left(\frac{C^{1-\gamma}}{1-\gamma} - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}-1}$$

$$U_{NN} = -h\varphi N^{\varphi-1}\left(\frac{C^{1-\gamma}}{1-\gamma} - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}} + \frac{\xi}{1-\gamma}\left(hN^{\varphi}\right)^2\left(\frac{C^{1-\gamma}}{1-\gamma} - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\frac{\xi}{1-\gamma}-1}$$

Using these expressions, it can be shown directly that φ and γ correspond to constant-consumption labor supply and, respectively, income effect on labor supply as defined in text; furthermore, we obtain complementarity as:

$$\kappa = \frac{U_{CN}C}{-U_N} = \frac{\xi}{1 - \frac{1-\gamma}{1+\varphi}h\frac{N^{1+\varphi}}{C^{1-\gamma}}}.$$

Concavity requires $\kappa \geq -\frac{\gamma\varphi}{\varphi+\gamma\alpha(1-\Phi)}$ so $\xi \geq -\frac{\gamma\varphi}{1+\varphi}\left(1 + \frac{1-\alpha(1-\Phi)}{\varphi+\gamma\alpha(1-\Phi)}\right)$.

With log utility in consumption we have instead:

$$U_C = \frac{1}{C}\left(\ln C - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\xi};\; U_N = -hN^{\varphi}\left(\ln C - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\xi}$$

and $U_{CN} = hN^{\varphi}\xi\frac{1}{C}\left(\ln C - h\frac{N^{1+\varphi}}{1+\varphi}\right)^{-\xi-1}$ implying complementarity $\kappa = \frac{U_{CN}C}{-U_N} = \frac{\xi}{U}$ where $U = \ln C - h\frac{N^{1+\varphi}}{1+\varphi}$ and $hCN^{\varphi} = \frac{(1+s)F_N}{\mu}$ implies $N^{1+\varphi} = \frac{\alpha(1-\Phi)}{h}$.

The function (14) as a quasi-separable sub-class of the general Boppart-Krusell parametric class

The Boppart-Krusell parametric class is, see their equation 8:

$$U(C,N) = \frac{C^{1-\sigma}\left[1 - a\left(NC^{\frac{\nu}{1-\nu}}\right)^b\right]^d - 1}{1-\sigma}. \tag{68}$$

It can be immediately verified that to obtain our function in (14) we can thus set:

$$\frac{1-\sigma}{d} = 1-\gamma;\; b = 1+\varphi$$
$$\nu = \frac{\gamma-1}{\gamma+\varphi};\; d = 1 - \frac{\xi}{1-\gamma};\; a = h\frac{1-\gamma}{1+\varphi}$$

which delivers:

$$U(C,N) = \frac{(1-\gamma)^{1-\frac{\xi}{1-\gamma}}\left[\frac{C^{1-\gamma}}{1-\gamma} - h\frac{N^{1+\varphi}}{1+\varphi}\right]^{1-\frac{\xi}{1-\gamma}} - 1}{(1-\gamma)\left(1-\frac{\xi}{1-\gamma}\right)}$$

i.e. our function in (14) up to a constant.

Deriving the general $(\gamma, \varphi, \kappa)$ parameters for the general Boppart-Krusell class

While, as illustrated above, it is immediate—given the (14) function proposed here—to find the parameter values that deliver it as a sub-class of the Boppart-Krusell class, it is more difficult to do the reverse. That is, to start with the general Boppart-Krusell class and try to find parameter restrictions that capture the key parameters emphasized here compactly: complementarity, income effect, and constant-consumption labor supply.

To that end, let us calculate our generally-defined parameters for the Boppart-Krusell class. Notice that the intratemporal optimality condition in steady-state is (after some tedious algebra):

$$\frac{-U_N}{U_C}\frac{N}{C} = \left[\frac{1-\sigma}{d}\frac{\left(NC^{\frac{\nu}{1-\nu}}\right)^{-b}}{ab} - \left(\frac{\nu}{1-\nu}+\frac{1-\sigma}{bd}\right)\right]^{-1} = \alpha\left(1-\Phi\right).$$

More tedious algebra computing the second derivatives delivers complementarity, income effect, and constant-consumption labor elasticity as:

$$\begin{aligned}
\kappa &\equiv \frac{U_{CN}C}{-U_N} \\
&= -b\frac{\nu}{1-\nu} - \frac{1-\sigma}{d}\frac{d+\frac{\nu}{1-\nu}\alpha\left(1-\Phi\right)}{1+\frac{\nu}{1-\nu}\alpha\left(1-\Phi\right)}; \\
\gamma &\equiv -\frac{U_{CC}C}{U_C} + \frac{U_{CN}C}{U_N} \\
&= 1 + b\frac{\nu}{1-\nu} + \left(b\frac{\nu}{1-\nu}+\frac{1-\sigma}{d}\right)\frac{\nu}{1-\nu}\alpha\left(1-\Phi\right); \\
\varphi &\equiv \frac{U_{NN}N}{U_N} - \frac{U_{CN}N}{U_C} \\
&= b - 1 + \left(b\frac{\nu}{1-\nu}+\frac{1-\sigma}{d}\right)\alpha\left(1-\Phi\right)
\end{aligned} \tag{69}$$

Evidently, the expressions in this general case are complicated functions of the deep utility parameters, and not much progress can be made without imposing further restrictions.

Treating κ, γ, φ as given, we thus have three restrictions, and the Boppart-Krusell class has four parameters that are of the essence here (b, d, ν, σ). So we need one more restriction to hope to solve this system uniquely. Before imposing it, notice that by combining the last two, we already solve for one of parameters:

$$\nu = \frac{\gamma - 1}{\gamma + \varphi},$$

the sign of which tells us if income effects dominate substitution ($\gamma > 1$) or not, and is the key parameter in Boppart and Krusell.

One particularly appealing additional restriction (which de facto imposes quasi-separability as defined here) is:

$$b\frac{\nu}{1-\nu} + \frac{1-\sigma}{d} = 0, \tag{70}$$

which implies that the function in square brackets becomes separable in C and N—what we call "quasi-separable". Replacing this extra restriction in our equations we obtain:

$$b = 1 + \varphi$$

and

$$d = 1 - \frac{\kappa\left(1 + \frac{\gamma-1}{1+\varphi}\alpha\left(1-\Phi\right)\right)}{1-\gamma}$$

which replaced back in the extra restriction gives:

$$1 - \sigma = (1-\gamma)\, d.$$

Rewriting the Boppart-Krusell class for this quasi-separable case we obtain:

$$U(C,N) = \frac{\left[C^{1-\gamma} - aN^{1+\varphi}\right]^{d} - 1}{d\,(1-\gamma)}$$

which, redefining d so as to have a parameter whose sign governs C-N complementarity (regardless of whether $\gamma \gtrless 1$) delivers exactly the function (14) proposed here.

B.2 The CCRRA (Constant Complementarity and Relative Risk Aversion) Utility Function

An alternative functional form allowing to parameterize complementarity directly is a different type of deviation from CRRA:[19]

Marginal utilities are:

$$U_C = C^{-\sigma} + \frac{h}{1+\psi}\kappa C^{-\kappa-1}N^{1+\psi}; U_N = -hC^{-\kappa}N^{\psi}$$

The intratemporal optimality condition thus implies

$$\frac{-U_N}{U_C} = \frac{hC^{-\kappa}N^{\psi}}{C^{-\sigma} + \frac{h}{1+\psi}\kappa C^{-\kappa-1}N^{1+\psi}} = \frac{(1+s)\,F_N}{\mu}; \frac{hC^{-\kappa}N^{1+\psi}}{C^{1-\sigma} + \frac{h}{1+\psi}\kappa C^{-\kappa}N^{1+\psi}} = \alpha\left(1-\Phi\right)$$

Cross-derivatives are hence:

$$\begin{aligned} U_{CC} &= -\sigma C^{-\sigma-1} - \frac{h}{1+\psi}\kappa\left(\kappa+1\right)C^{-\kappa-2}N^{1+\psi} \\ U_{CN} &= h\kappa C^{-\kappa-1}N^{\psi} \\ U_{NN} &= -h\psi C^{-\kappa}N^{\psi-1} \end{aligned}$$

[19] This can be rewritten as (using $\frac{h}{1+\psi} = \frac{a}{1-\sigma}$): $U(C,N) = \frac{C^{1-\sigma}\left(1-a(C^{x}N)^{1+\psi}\right)-1}{1-\sigma}$, where $\kappa = -\left(1-\sigma+x\left(1+\psi\right)\right)$.

Using these expressions, we obtain that complementarity is directly parameterized by:

$$\kappa = \frac{U_{CN}C}{-U_N}.$$

More tedious algebra computing the second derivatives delivers income effect, and constant-consumption labor elasticity as:

$$\gamma = \sigma - \kappa - \kappa \frac{\sigma - 1 - \kappa}{1 + \psi} \alpha \left(1 - \Phi\right)$$

use $\frac{hC^{-\kappa}N^{1+\psi}}{C^{1-\sigma} + \frac{h}{1+\psi}\kappa C^{-\kappa}N^{1+\psi}} = \alpha\left(1-\Phi\right) \rightarrow$

$$\begin{aligned}
\gamma &\equiv -\frac{U_{CC}C}{U_C} + \frac{U_{CN}C}{U_N} \\
&= \frac{\sigma C^{-\sigma} + \frac{h}{1+\psi}\kappa\left(\kappa+1\right)C^{-\kappa-1}N^{1+\psi}}{C^{-\sigma} + \frac{h}{1+\psi}\kappa C^{-\kappa-1}N^{1+\psi}} - \kappa && (71) \\
&= \frac{\left(\sigma-\kappa\right)C^{-\sigma} + \frac{h}{1+\psi}\kappa C^{-\kappa-1}N^{1+\psi}}{C^{-\sigma} + \frac{h}{1+\psi}\kappa C^{-\kappa-1}N^{1+\psi}} = \left(\sigma-\kappa\right) + \left(1-\sigma+\kappa\right)\frac{\frac{h}{1+\psi}\kappa C^{-\kappa-1}N^{1+\psi}}{C^{-\sigma} + \frac{h}{1+\psi}\kappa C^{-\kappa-1}N^{1+\psi}} && (72) \\
&= \left(\sigma-\kappa\right) + \left(1-\sigma+\kappa\right)\frac{\kappa}{1+\psi}\alpha\left(1-\Phi\right) && (73) \\
&= \sigma\left(1 - \frac{\kappa}{1+\psi}\alpha\left(1-\Phi\right)\right) - \kappa\left(1 - \frac{1+\kappa}{1+\psi}\alpha\left(1-\Phi\right)\right) && (74) \\
\varphi &\equiv \frac{U_{NN}N}{U_N} - \frac{U_{CN}N}{U_C} = ... && (75) \\
&= \psi - \kappa\alpha\left(1-\Phi\right) && (76)
\end{aligned}$$

Several notes are in order: The RRA coefifcient or utility curvature is

$$\begin{aligned}
-\frac{U_{CC}C}{U_C} &= \gamma + \kappa = \sigma - \kappa x \alpha\left(1-\Phi\right) \\
&= \sigma - \kappa\frac{\sigma-1-\kappa}{1+\psi}\alpha\left(1-\Phi\right)
\end{aligned}$$

Concavity requires $\kappa \geq -\frac{\gamma\varphi}{\varphi+\gamma\alpha(1-\Phi)}$ so ...., let $\tilde{\alpha} = \alpha\left(1-\Phi\right)$

$$\kappa \geq -\frac{\psi - \kappa\tilde{\alpha}}{\frac{\psi-\kappa\tilde{\alpha}}{\left(\sigma-\kappa\right)+\left(1-\sigma+\kappa\right)\frac{\kappa}{1+\psi}\tilde{\alpha}} + \tilde{\alpha}}$$

What Auclert et al defined as "CI" is:

$$CI = \frac{\kappa}{\kappa+\gamma} = \frac{\kappa}{\sigma - \kappa\frac{\sigma-1-\kappa}{1+\psi}\tilde{\alpha}}$$

A GHH-like case with zero income effects is "nested" ($\gamma = 0$) for

$$\sigma = \kappa \left(1 - \frac{\tilde{\alpha}}{1 + \psi - \kappa\tilde{\alpha}}\right)$$

It is possible to have over-compensating income effect $\gamma > 1$:

$$\begin{aligned}(\sigma - \kappa) + (1 - \sigma + \kappa)\frac{\kappa}{1+\psi}\alpha(1-\Phi) > 1 \rightarrow \\ (\sigma - \kappa - 1)\left(1 - \frac{\kappa}{1+\psi}\alpha(1-\Phi)\right) > 0 \rightarrow \\ \sigma > 1 + \kappa\end{aligned}$$

Balanced growth is restored with

$$\sigma = 1 + \kappa$$

This boils down to the KPR class:

$$\frac{C^{1-\sigma} - 1}{1-\sigma} - \frac{h}{1+\psi}C^{1-\sigma}N^{1+\psi} = \frac{C^{1-\sigma}\left(1 - \frac{h}{1+\psi}N^{1+\psi}\right) - 1}{1-\sigma}$$

where $\sigma > 1$ implies complementarity and <1 substitutability (concavity $\kappa \geq -\frac{\psi - \kappa\tilde{\alpha}}{\psi - \kappa\tilde{\alpha} + \tilde{\alpha}}$)

Finally, the inverse Frisch elasticity for CCRRA is:

$$\begin{aligned}\varphi + \frac{\kappa\gamma}{\gamma+\kappa}\alpha(1-\Phi) &= \psi - \kappa\tilde{\alpha} + \kappa\tilde{\alpha}\frac{1 + (\sigma - \kappa - 1)\left(1 - \frac{\kappa}{1+\psi}\tilde{\alpha}\right)}{\sigma - (\sigma - 1 - \kappa)\frac{\kappa}{1+\psi}\tilde{\alpha}} \\ &= \psi - \kappa^2\tilde{\alpha}\frac{1}{\sigma - (\sigma - 1 - \kappa)\frac{\kappa}{1+\psi}\tilde{\alpha}} \\ &= \frac{(1+\psi)\sigma - (\sigma - 1)\kappa\tilde{\alpha}}{\sigma - (\sigma - 1 - \kappa)\frac{\kappa}{1+\psi}\tilde{\alpha}} - 1\end{aligned}$$