Certain vs. Uncertain Timing: Financial Markets and Pricing Implications

Indira Puri*
Stern School of Business
New York University

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Abstract

While financial research has extensively explored risk over outcomes, it has focused less on risk over timing. I study such risk both theoretically and empirically. Theoretically, I prove that in certain textbook stochastic discount factor frameworks, the underlying space is not sufficiently rich to accommodate asset-specific uncertain timing. Expanding these frameworks results in a tradeoff between arguably desirable pricing properties and flexibility in pricing uncertain timing. Empirically, quotes and trades in options markets are consistent with a pricing of uncertain timing. In particular, comparing dividend-free American and European call options on the same underlying, offered by the same exchange, with identical contract specifications, the short side requires higher compensation for the American than the European, over and above pricing effects of an early exercise premium. These results are not fully explained by hedging considerations, re-investment risk, liquidity effects, informed trading,

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or contract specifications, but are consistent with markets pricing an aversion to timing risk. Together, these theoretical and empirical results suggest that the dedicated study of certain vs. uncertain timing may deepen pricing models and yield insights into empirical prices.

1 Introduction

Although risk aversion is now taken as a foundational building block in financial models, the notion of risk aversion was not always well-accepted. The St. Petersburg Paradox, introduced by Nicholas Bernoulli in 1713, posited a thought experiment on a gamble with infinite expected value but high variance. The fact that many people did not find the gamble appealing was evidence in favor of risk aversion, according to Bernoulli. But not all were satisfied. For example, his contemporary the Comte de Buffon argued that the response could instead be due to small probabilities being ignored; or to the improbability of such a gamble existing in reality due to capital constraints (Samuelson, 1977).

Today, however, risk aversion is taken as given, and implies a preference for certainty to uncertainty. To illustrate, suppose that all uncertainty is resolved immediately, and all payments made in 10 days. Treat the money as consumption, that is, as not moveable across time. Which do you prefer: \$2 for sure, or the gamble {\$1 with 50% probability and \$3 with 50% probability}? Risk aversion would say the certain outcome, \$2.

Now consider a different setting, in which money and time are flipped. Suppose that the amount of money is fixed, but there is uncertainty over the timing of payment. All uncertainty is still resolved immediately. You know you will receive \$10, but you don't know when you will receive it. Would you prefer that \$10 of consumption in two months for sure, or the gamble {50%, 1 month, 50% 3 months}?

This question clearly reflects a different kind of uncertainty - uncertainty over the time of payment rather than over the outcome. However, I show in this paper that pricing models treat these two types of risk very differently. In particular, certain textbook pricing frameworks have state spaces which are not sufficiently rich to price uncertain timing. An expansion of these frameworks trades off flexibility with other desirable properties. However, an empirical exploration in options markets suggests that uncertain timing may empirically be priced. My work does not take a strong stand on the direction of the pricing; instead, I show theoretically that price uncertain timing using canonical asset pricing approaches in a coherent and flexible way is not feasible; but market prices are consistent with a non-zero pricing of timing risk.

I start with a theoretical exploration, which yields five theorems on how the stochastic discount factor and other pricing frameworks treat uncertain timing. The-

orems 1 - 4 cumulatively make the following point: in the utility space, lotteries are probability measures. In the stochastic discount factor framework, assets are random variables. In making the move from probability measures to random variables, the choice of state space in the SDF framework becomes critical. Certain standard state spaces, such as consumption-based state spaces or arbitrary finite state spaces, are not sufficiently rich to allow for assets with uncertain timing. This does not mean that it cannot be done, but rather that the most straightforward textbook approaches have difficulty pricing uncertain timing. Theorem 5 considers models that go beyond existing SDF or utility approaches. It shows that any utility function that meets a number of arguably desirable properties cannot consistently price a preference for certain timing. In particular, this means that flexibility in pricing uncertain timing, even outside standard models, is limited: a collection of arguably desirable pricing properties will predict a preference for uncertain timing for at least some lotteries.

In detail, Theorem 1 considers a standard SDF framework, where the outcome of an asset is its payment, and the filtration consists of past realized consumption. The theorem shows that, under this framework - regardless of choice of the SDF - it is not possible to price uncertain timing separately from its marginals on the observable space. The intuition is that the space on which assets can be defined, within this canonical framework, is not sufficiently rich to allow for asset-specific timing risk.

A natural response is to expand the state space to allow for more than realized consumption. Theorem 2 considers arbitrary expansions of the state space. It shows that there is no parsimonious (finite-state) SDF framework that can coherently price uncertain timing.

The next possibility is to get rid of either parsimony or as outcomes as payoffs. Theorems 3 and 4 show that removing either assumption is sufficient to price assets with uncertain timing. However, the ability to price does not necessarily mean that other desirable pricing properties are met. Theorem 5 shows that any utility-based SDF that falls in a wide class with arguably desirable properties such as history independence, no probability distortion, and dynamic consistency – even thinking creatively beyond existing models – cannot consistently price a preference for certain timing.

Together, these theoretical results indicate that pricing assets with uncertain timing in a clean, straightforward, and fully flexible way may not be feasible if one wishes to preserve arguably desirable pricing properties.

The theory also includes extensions of these results to pricing uncertain timing with habit formation and to a reduced form pricing of uncertain timing through yield curves.

I next turn to an empirical exploration of certain vs. uncertain timing. Options markets provide an ideal setting in which to study the question in a relatively clean way. The reason is that American options can be exercised any time up to and including expiry, while European options can be exercised only at expiry. The short side of an American is therefore exposed to timing risk that the short side of an otherwise identical European is not.

The goal then becomes to identify the pricing due to this timing risk. To do so, I utilize results from prior literature showing that the long side empirically exercises early. Merton (1973), one of the classical theorems in derivatives, shows that American and European call options with no dividends in the lifetime of the option, should be priced identically. However, subsequent research finds an empirical tendency of the long side to exercise early, for example due to idiosyncracy, lack of sophistication, or capital constraints (ex. Finucane (1997); Poteshman and Serbin (2003); Bryzgalova et al. (2024)). I take these findings as given. I then ask - given that the long side empirically exercises early, how does the short side respond to that early exercise threat? Any pricing difference on the short side over and above pricing differences induced by the long side, would be consistent with a pricing of uncertain timing.

To test for the pricing of timing risk empirically, I therefore find otherwise identical American and European options, that is, offered by the same exchange, on the same underlying and with the same contract specifications. This is hard in general because most underlyings typically have only one option type – American or European – associated with them. However, options on the S&P 100 index have both American and European types with identical contract specifications, which are offered by the same exchange. I restrict attention to options that have no dividends in their lifetime. I then consider the difference in ask prices over and above the difference in bid prices, and find this difference to be positive and statistically significant, consistent with a pricing of uncertain timing. While the pricing turns out to be positive - consistent with a preference for certain timing - the broader finding is that these prices appear to be consistent with a non-zero pricing of uncertain timing.

A natural question is whether there are other channels which could also produce this empirical pricing result. I test for eight alternative plausible explanations. First,

the test above uses quote data, and executed prices in trades can sometimes differ. I therefore check whether trades in these otherwise identical American and European options are priced differently, and find that the American is more expensive, consistent with the quote-based results. Quantitatively, the American is 11 bp more expensive in quotes and 5.5% more expensive in executed trades. A second alternative explanation is that the options have liquidity differences. I therefore control for liquidity by restricting attention only to minutes and options which have similar bid-ask spreads (measured as %) and show that the results continue to hold. In addition, any symmetric liquidity premium gets differenced out in the specification, because the object of interest is the ask difference after controlling for the bid difference across the two contract types. A third possible explanation is that the difference in pricing represents an adversarial response to informed trading on the long side. Because past work shows that small trades, particularly in index options, are uninformed (Pan and Poteshman, 2006; Muravyev, 2016), I test for this by quantifying the proportion of small trades in both the American and European classes, and find small trades to be a similar proportion across the two classes, and to form the majority of trades in both classes. A fourth possible explanation is that the results are driven disproportionately by options with that are deeply in- or out- of the money; but the results appear in options that are close to at-the-money. A fifth possible explanation is that the difference reflects re-investment risk. However, the pricing difference appears even in short-dated options with only a few days to expiry, and in whose lifetime there are no FOMC meetings. A sixth possible explanation is that microstructure differences between the two options drive the results. But, by construction, the index options I consider have identical settlement time, contract size, strike prices, strike price intervals, position limits, margin rules, expiration date, and expiration months, and are offered by the same exchange – that is, they have identical contract specifications, except that one is American and the other European. A seventh possible explanation is that inventory pressures can cause bids to adjust different from asks - but a more flexible specification that allows for this, continues to find a pricing difference in asks over and above what can be explained by bids. An eighth possible explanation is that dynamic hedging of American options may make them more complex for the short side; to test for this, I consider options with gamma close to zero, and find that the pricing difference on the short side continues to hold. Cumulatively, these robustness checks, and the main specification, suggest an unwillingness to be short

the American over and above pricing of an early exercise premium on the long side, providing evidence consistent with an aversion to holding uncertain timing.

Finally, as a supplement to these empirical tests, I elicit willingness to pay for uncertain timing in surveys. These find a pricing of timing risk; and incentivized pricing narratives indicate that the majority of respondents price uncertainty in the time and money domains in a conceptually similar way.

The focus of the paper is on whether and how uncertain timing is priced within theory and in markets: any non-zero result would be interesting. The direction, though secondary from the perspective of this paper, also bears brief discussion. Either direction - or even no pricing of uncertain timing - could plausibly appear in financial markets. On the one hand, individuals who have been trained in finance may think through standard models and display a preference for uncertain timing. For example, my informal polls of finance professors at times find them trying to think through the yield curve or through other orthodox approaches. On the other hand, laboratory experiments on human subjects tend to show timing-risk aversion (Chesson and Viscusi, 2003; DeJarnette et al., 2020), while animal studies find the opposite pattern of timing-risk seeking (Kacelnik and Bateson, 1996). Which effect wins out is in financial markets is therefore in question, and our priors might suggest that financial market participants, being sophisticated and trained through financial models, would prefer uncertain timing. This makes it all the more surprising that the financial data is consistent with an aversion to timing risk.

The paper primarily contributes to a literature on theoretical properties of the stochastic discount factor, and to a debate over whether there is a right SDF to use (Hansen and Jagannathan (1991), Wachter (2006), Verdelhan (2010), van Binsbergen (2016), van Binsbergen and Koijen (2017), Campbell (2017), Verdelhan (2018), van Binsbergen (2020), among many others). It also contributes to a literature on empirical option pricing, particularly index options and empirical behavior surrounding American options (Muravyev, 2016; Bryzgalova et al., 2024). There are a number of other strands of literature the paper also speaks to, which are comprehensively reviewed in Section 6.

The paper proceeds as follows. Section 2 derives the theoretical results on asset pricing frameworks. Section 3 shows that options markets have prices that are consistent with a pricing of uncertain timing, and tests for robustness to various alternative explanations. Section 4 elicits willingness-to-pay and pricing narratives in

surveys and shows that the results are consistent with the market findings. Section 5 discussions potential implications across finance. Section 7 concludes. All proofs are in the appendix.

2 Pricing Uncertain Timing: Stochastic Discount Factors and Expansions

I start by defining the standard framework for Arrow-Debreu securities (Arrow and Debreu, 1954) and the SDF (as in Duffie (2001)).

Definition 1 (Arrow-Debreu Securities)**.**
Let $S = 1, 2, .., |S|$ be a finite set of states, $T = 1, 2, ...,$ a set of time periods, $S \times T$ with generic element (s, t) the set of state-time pairs, and $a_{s,t}$ an Arrow-Debreu security that pays 1 unit of consumption in state s at time t and 0 units of consumption in all other states and times.

Definition 2 (The Stochastic Discount Factor Framework)**.**
There exists a probability space $(\Omega, \{\mathcal{F}_t\}_{t\in T}, P)$, where Ω is a set of states, $\{\mathcal{F}_t\}$ a filtration, T is a set of times (typically the natural numbers), and P a probability measure. In this framework, assets are adapted processes: $X = (X_1, X_2, ...)$ where each X_t is a random variable measurable with respect to $\mathcal{F}_t$. Assume at time zero the existence of a stochastic discount factor $M = \{M_t\}_{t\in T}$: the price of asset X is given by $Price(X) = E_P[\sum_t M_t X_t]$.

To isolate the impact of uncertain timing, "assets" here are thought of as probabilistic consumption processes (similar to Epstein et al. (2014)) representing the entirety of consumption. We then think of the price as the time-zero consumption equivalent for the entire probabilistic stream.

I make explicit one assumption that tends to be common and implicit. A standard way of describing the stochastic discount factor is to say that there is substitution across time and over states. For example, the SDF is often used interchangeably with the phrase 'intertemporal marginal rate of substitution' (Hansen and Jagannathan, 1991; Cochrane and Hansen, 1992; Lustig and Verdelhan, 2012). Implicit in this phrase is the idea that something - typically consumption - is being substituted across states and time. This is formalized in the below statement of outcomes as payoffs.

Assumption (Outcomes as Payoffs)**.** *Fix any asset A. For any state and time (s,t), the value that A takes on in (s,t) is the amount of consumption it pays in that state.*

Finally, I formally define assets with uncertain timing.

Definition 3 (Assets with Uncertain Timing)**.**
Let R^+ be the set of nonnegative consumption amounts, and let $\mathcal{T}$ be a set of time periods. For any $x > 0$ and any $t \in T$, define the consumption stream

$$c^t(x) \;=\; (c_1, \ldots, c_T) \quad \textit{where} \quad c_s = \begin{cases} x, & s = t, \\ 0, & s \neq t. \end{cases}$$

Let $\delta_{c^t(x)}$ denote the Dirac measure on the stream $c^t(x)$. An asset with uncertain timing *delivering payoff x is a probability measure $(p_t)_{t\in\mathcal{T}}(x)$ over $\{\delta_{c^t(x)}\}_{t\in\mathcal{T}}$.*

Denote the family of all assets with uncertain timing by

$$A_{\text{Unc. Tim.}} = \Big\{ (p_t)_{t\in\mathcal{T}}(x) : \; x > 0, \; p_t \geq 0, \; \sum_{t\in\mathcal{T}} p_t = 1 \Big\}.$$

2.1 Pricing Uncertain Timing with Consumption-Based States

A first approach towards pricing uncertain timing is to think of states as outcome-based, where here, outcomes are consumption. This might seem like a reasonable approach because several early models such as Lucas (1978) and Mehra and Prescott (1985) use consumption-based state spaces; as do some later canonical models such as Campbell and Cochrane (1999).

As Theorem 1 shows, however, outcome-based states are not able to price assets with uncertain timing separately from their marginals.

Assumption (Outcome-Based States)**.** *Assume that $C = \mathbb{R}^+$ is the consumption space, then $\Omega = C^T$ and the filtration relies only on realizations of consumption up to and including time t: $\mathcal{F}_t = \{A \subseteq 2^{C^T} : \forall (c_0, .., c_T), (c'_0, ..., c'_T) \in A, \; c_i = c'_i \; \forall i \leq t\}$.*

Theorem 1. *Consider any stochastic discount factor framework under the assumptions of outcomes as payoffs, and outcome-based states. It is not possible to separately price assets A and A', where these assets are defined as follows:*

- *Asset A pays out 1 unit of consumption almost surely with respect to P, but the timing of payment is uncertain: it pays out 1 unit of consumption at period t_1 with probability p_A and at period $t_2 > t_1$ with probability $1 - p_A$.*
- *Asset A' is defined as a probability measure with respect to Ω, which pays out 1 unit of consumption at period t_1 with probability p_A, and independently pays out 1 unit of consumption at period t_2 with probability $1 - p_A$.*

The intuition is that both A and A' have the same observable consumption outcomes - therefore, without the ability to embed more information, they cannot be priced separately. A more general intuition is that the uncertain timing asset must be defined on a richer space than what is allowed by outcome-based states. It is not well-defined with respect to the filtration because it contains information beyond the history of consumption, via its perfect negative correlation across time. The uncertain timing asset therefore needs to be projected down into the filtration to be priced. Think of a three-dimensional object being projected into a two-dimensional space. In this case, what is lost by working on the less rich space where states depend only on the history of observed consumption and the SDF therefore prices the marginals, is the perfect negative correlation between time periods inherent in the uncertain timing asset.

More generally, while the states here are outcome-based, what is needed for the theorem is that the set of assets with uncertain timing is defined on a richer space than what the set of observable states can accommodate. This is proven formally in Theorem A.1 in Appendix A.1.1. These theorems makes explicit the critical role of the choice of state space in the SDF framework. In particular, the most straightforward textbook approaches do not have state spaces sufficiently rich to price asset-specific uncertain timing.

A natural reaction is to keep the SDF framework but to allow the state space to expand arbitrarily. We turn to these expansions of the state space next.

2.2 Pricing Uncertain Timing: Parsimony

We will now keep outcomes-as-payoffs but remove outcome-based states, instead allowing states to be arbitrary.

One appealing feature of the Arrow-Debreu approach is that while the set of states is finite, the cardinality of the assets that can be priced is uncountably infinite. To

see this, observe that the consumption payoff of an asset in a state can be any number in $\mathbb{R}^+$; therefore the set of payoffs with even two states is $\mathbb{R}^{+2}$, whose cardinality is uncountably infinite. But the number of state prices needed with two states is only two.

I formalize this particular appealing feature as *parsimony*: the cardinality of the set of states is finite (as in textbook introductions to the SDF, such as Campbell (2017) and Cochrane (2005)). As a result, an uncountably infinite set of assets can be priced with a finite-dimensional SDF.

Definition 4 (Parsimony)**.**
Say that a pricing framework as defined in Definition 2 is ***parsimonious*** *if the cardinality of* Ω *is finite.*

Keeping the SDF framework of Definition 2, but allowing the state space to expand arbitrarily, Theorem 2 shows that there is no parsimonious stochastic discount factor framework that allows for the coherent pricing of all assets with timing uncertainty. Coherence is a weak requirement, formalized below. It says, take any asset with uncertain timing and hold fixed all but two of its probabilities of paying in different time periods. If you change those probabilities, the price of the asset should change. The condition takes no position on whether the price increases or decreases - only that it is not the same but is responsive to changes in probabilities over timing of payment.

Definition 5 (Coherence)**.**
Let $p + x$ *be an asset with uncertain timing, whose probability of paying out in period* t *is* $p_x(t)$*. Pick* $t_1 < t_2$ *arbitrarily, where* $p_x(t_1) > 0$ *and* $p_x(t_2) > 0$*. Create a new asset with uncertain timing* p'_x *such that* $p'_x(t) = p_x(t)$ *for all* $t \neq t_1, t_2$*, and* $p'x(t_1) = p_x(t_1) + \epsilon$ *,* $p'_x(t_2) = p_x(t_2) - \epsilon$ *for some* $\epsilon < p_x(t_2)$*. Then the price of asset* p_x *should not be the same as the price of asset* p'_x*.*

Theorem 2. *There is no parsimonious stochastic discount factor framework with outcomes as payoffs that can coherently price assets with uncertain timing.*

2.3 Pricing Uncertain Timing, Ignoring Parsimony or Removing Outcomes as Payoffs

A next step is to either expand the state space in a way that ignores parsimony, or to allow assets to take on values that deviate from the amount of consumption they pay out in a given state and time. Theorems 3 and 4 show that, with either of these restrictions removed, there now exist (possibly trivial) SDF frameworks that can coherently price all assets with uncertain timing. However, these frameworks need not have other desirable pricing properties.

Theorem 3. *There exists a parsimonious SDF framework without outcomes as payoffs that can uniquely price all assets with uncertain*

Theorem 4. *There exists a non-parsimonious stochastic discount factor framework with outcomes as payoffs that can coherently price the set of assets with uncertain timing.*

The proof of Theorem 3 is one of existence. It chooses a trivial SDF framework, e.g. one in which assets take on values that are exactly their prices, and the probability measure and SDF are meaningless. As this proof sketch suggests, existence is not the same as an interpretable pricing framework, or one with desirable properties. The proof of Theorem 4 similarly constructs an arbitrary SDF framework to meet the sole goal of being able to price uncertain timing – but without considering whether desirable pricing properties hold within that framework. We turn to one set of desirable properties next.

2.4 Pricing Uncertain Timing: Flexibility and Tradeoffs

I now show that a consistent preference for certain timing can interfere with arguably desirable properties, even if one thinks creatively beyond standard models. I start by switching from working with SDFs to working directly with utility functions, precisely defining the space and properties. I then prove the statement. For completeness, I also derive results on one popular non-recursive utility model and for negative shocks.

2.4.1 Space and Definitions

Per-period consumption lies in $C \subseteq \mathbb{R}^+ \cup \{0\}$ and time takes on values $T = 1, 2, ...$ The infinite dimensional space C^∞ represents deterministic consumption streams.

Vector $\vec{\bar{c}} \in C^{\infty}$ refers to receiving consumption $\bar{c}$ at each period t. Pair $(c,t) \in C \times T$ denotes receiving consumption c in period t, and zero consumption in all other periods. Let Δ be the set of simple lotteries over $C \times T$ endowed with the topology of weak convergence. A typical element of Δ is denoted m and a generic realization of m denoted $c(\vec{m})$. I study a complete and transitive ordering $\succeq$ over Δ.

Consumption lotteries, defined as lotteries that pay out uncertain consumption at a specified date $\bar{t}$ are denoted $q^{(\bar{t})} \in \Delta$, and time lotteries, defined as lotteries that pay out certain consumption $x \in C$ at an uncertain date, are denoted $p^{(x)} \in \Delta$. Notation $\vec{\bar{c}} + (x,t)$ denotes the deterministic consumption stream of $\bar{c}$ in each period, except in period t where consumption is $\bar{c}+x$. Notation $\vec{\bar{c}} + p^{(x)}$ denotes the lottery which, with probability $p^{(x)}(t)$, gives consumption vector $\vec{\bar{c}} + (x,t)$. Unless otherwise specified, for both time and consumption lotteries, all uncertainty is resolved immediately. This assumption is made to focus on the risk aspects of the models, rather than on resolution period effects.

Definition 6 (Prefer Certain to Uncertain Timing)**.**
Say that the preference $\succeq$ prefers certain to uncertain timing if, for any $\bar{c} \in C$, $x \in \mathbb{R}$, and time lottery $p^{(x)} \in \Delta$ with $\mathbb{E}\left[p^{(x)}\right] \in C \times T$, consumption stream $\vec{\bar{c}} + (x, \mathbb{E}\left[p^{(x)}\right]) \succ \vec{\bar{c}} + p^{(x)}$. Analogously, the preference prefers uncertain to certain timing or is indifferent over timing risk if the above holds with $\prec$ or $\sim$, respectively.

In words, given any constant consumption stream, the agent prefers certain to uncertain timing if, for time lottery $p^{(x)}$, they prefer to receive consumption at its deterministic expected time rather than play the time lottery.

Definition 6 imposes restrictions on behavior only at certainty; it says nothing about mean-preserving spreads.

The restriction to constant consumption is to focus on the pure timing uncertainty aspect of the preference, rather than on deterministic consumption variability. This definition and framework generalizes ones used in the decision theory literature (Ebert, 2020) to account for shocks of any sign and non-zero baseline consumption, as is typically the case in finance.

Definition 7 (Convex Time Discounting)**.**
Say that a representation U has convex discounting in the absence of risk if, for any deterministic $t, t', t'' \in T$ with $t'' = \frac{t+t'}{2}$, $\frac{1}{2}U((x,t)) + \frac{1}{2}U((x,t')) > U((x,t''))$.

The next set of definitions, regarding recursive preferences, are standard (Chew and Epstein (1990)).

Definition 8 (Utility Certainty Equivalent)**.**
Say that function $I : M([0,1]) \to [0,1]$ is a utility certainty equivalent operator if it is continuous, increasing with respect to first-order stochastic dominance, and has $I(x) = x$ for every $x \in \mathbb{R}_+$.

Definition 9 (Image Measure)**.**
Given a function $U : \Delta \to [0,1]$ and a probability measure $m \in \Delta$, define the image measure $m \cdot U^{-1} \in M([0,1])$ by letting $[m \cdot U^{-1}](B) \coloneqq m\left(\left\{c(\vec{m}) : U(c(\vec{m})) \in B\right\}\right)$ for all $B \in \mathcal{B}([0,1])$, where $\mathcal{B}$ is the Borel measure.

Definition 10 (Recursive Representation)**.**
Say that representation $U : \Delta \to [0,1]$ is recursive if it takes the form:

$$U(c, m) = W(c, I(m \cdot U^{-1})),$$

where $I : M([0,1]) \to [0,1]$ is a utility certainty equivalent operator, and $W : C \times [0,1] \to [0,1]$ is a continuous function, strictly increasing in its second argument.

Definition 11 (Kreps-Porteus Recursive Representation)**.**
Say that representation $U : \Delta \to [0,1]$ is a Kreps-Porteus recursive representation if it is a recursive representation and $I(m \cdot U^{-1}) = \phi^{-1}\mathbb{E}[\phi(m \cdot U^{-1})]$ for some increasing and continuous $\phi : \mathbb{R} \to \mathbb{R}$.

Assumption (Differentiability)**.** *Assume that the Kreps-Porteus function U is twice differentiable and that U' and U'' are well-defined at all but at most a finite number of consumption paths in C^∞.*

2.4.2 Theoretical Tradeoffs

The Kreps-Porteus class is characterized by dynamic consistency, history independence, and no probability distortion (Kreps and Porteus, 1978; Strzalecki, 2023). Dynamic consistency helps ensure that the willingness to pay for an asset remains consistent if nothing else changes. History independence is useful for tractability and to avoid having to impose a starting value for initial consumption. No probability

distortion allows for standard expectation taking. As a result, all three properties are arguably desirable.

Theorem 5 shows that it is not possible to have these properties alongside convex discounting and a preference for certain timing. Whereas the fact that, for positive shocks, certain standard utility functions predict a preference for uncertain timing is known (Chesson and Viscusi, 2003; DeJarnette et al., 2020), one might think that creatively going beyond these standard functions, it is possible to obtain a consistent preference for certain timing. My theorem below shows that this is not the case. Even thinking beyond any existing model in the literature, and including models that have never been written down, it is not possible to have full flexibility in pricing uncertain timing, if one wishes to do so in a debate-free way.

Theorem 5. *Suppose U is a differentiable Kreps-Porteus representation with convex time discounting. Then U does not satisfy a preference for certain to uncertain timing.*

2.4.3 Habit Formation

Theorem 6 shows that giving up history independence in one common way, e.g. via habit formation, also does not help accommodate a preference for certain timing.

Theorem 6. *The Campbell and Cochrane (1999) habit formation model does not satisfy a preference for certain timing.*

2.4.4 Positive vs. Negative Shocks

Corollaries 1 and 2, included for completeness, show that additive utility models (e.g., expected utility and hyperbolic discounting) predict a preference for uncertain timing for positive shocks and a preference for certain timing for negative shocks.[1] While the result for positive shocks is known (Chesson and Viscusi, 2003), decision theory derivations typically do not consider negative shocks or non-zero baseline consumption. Corollaries 1 and 2, in contrast, work with the full generality of Definition 6. The dichotomy between positive and negative shocks is one we will want to be aware of when testing for the pricing of uncertain timing in real-world markets, where shocks can take any sign.

[1] I thank John Campbell for suggesting I include this result.

Corollary 1. *Any expected utility model,* $U(p) = \int_{\vec{c(p)} \in p} \sum \delta^t u(c(p)_t) dp(\vec{c(p)})$ *prefers uncertain timing for all time lotteries* $p^{(x)}$ *when* $x > 0$*, and certain timing when* $x < 0$*.*

Corollary 2. *Any hyperbolic discounting model* $U(p) = \int_{\vec{c(p)} \in p} \sum \beta \delta^t u(c(p)_t) dp(\vec{c(p)})$ *prefers uncertain timing all time lotteries* $p^{(x)}$ *when* $x > 0$ *and* $p^{(x)}(t = 0) = 0$*, and certain timing when* $x < 0$ *and* $p^{(x)}(t = 0) = 0$*.*

2.5 Pricing Uncertain Timing with Yield Curves

So far, the theory has considered assets as having consumption-based payoffs – not moveable across time. It may be of interest to also examine what predictions result if assets instead have monetary payoffs and there exists a yield curve, with the market making risk-neutral adjustments with respect to the yield curve. As is standard, assume that yields are weakly positive. Theorem 7 shows that in these conditions a preference for uncertain timing persists with a flat or downward sloping concave yield curve, and may or may not persist with an upward sloping yield curve, depending on the curve's slope and curvature.

Formally, let $T = 1, 2, ...$ be the set of times, the set of streams of cash as $(\mathbb{R})^T$ and the set of lotteries over cash streams as $\Delta\left(\mathbb{R}^T\right)$, with typical element p. Define the (risk-neutral) expected present value of a lottery over cash streams p as $\int_{(Y_1, Y_2, ...) \in p} \sum_t \frac{Y_t}{(1+y_t)^t} dp$, where y_t denotes the time t yield.

Theorem 7. *Consider any strictly positive (negative) cash payment with uncertain timing* p*, whose expected time of payment is* t*. Then the value of the uncertain timing lottery* p *is strictly higher (lower) than the value of receiving the cash payment at deterministic time* t *when the yield curve is flat.*

It is also strictly higher (lower) when the yield curve is downward sloping and concave; and it may or may not be higher (lower) for all other shapes of the yield curve, depending on the yield curve's slope and curvature.

3 Option Pricing and Uncertain Timing

3.1 Overview

Options constitute one of the world's largest and most actively traded asset classes. In the first quarter of 2024 alone, total notional value in options traded in the US

was 40 trillion dollars (Office of the Comptroller of the Currency, 2024), larger than the US national debt (Department of the Treasury, 2024), and over 10 times larger than annual social security benefits (Social Security Administration, 2024).

There are two types of commonly traded options: American and European. European options may be exercised only on the designated expiration date. American options can be exercised any time up to and including expiry. Merton's theorem (Merton, 1973), one of the classic theorems in derivatives pricing, shows that, for call options with no dividends in their lifetime, the value of an American is equal to the value of a European with the same strike price and expiration date. Merton's theorem relies on minimal assumptions such as no dominance and no arbitrage, and does not assume an underlying distribution for the price process.

Model-free tests of Merton's theorem have been limited because, in practice, assets typically have only one option type associated with them, American or European. To the extent that the theorem has been challenged, it is in the idea that the long side of the option may exercise early due to irrationality, transaction costs, or short sale constraints (Finucane, 1997; Poteshman and Serbin, 2003; Jensen and Pedersen, 2016). The idea that early exercise may be valuable for the long side of the option is called the early exercise premium.

This paper challenges Merton's theorem through a different channel: the short side of the American option may require compensation to hold timing risk. This unwillingness to hold timing risk is conceptually similar to standard risk aversion. Standard theory says the short side holds no compensatable risk because if an option is idiosyncratically exercised early, unlinked to market variables, there is no re-investment risk. If anything, under the conditions of Merton's theorem, so long as quotes are higher than the intrinsic value of the option – regardless of the underlying price process – the short side should welcome early exercise. However, the idea that timing uncertainty is inherently undesirable, whether for practical or psychological reasons, would predict that such risk is compensated.

I test for an aversion to timing risk over and above an early exercise premium by finding a setting with otherwise identical American and European options, and adopting a difference-in-difference approach.

Such a setting is hard to find in general, because most underlyings have only one option type (American or European) on them. However, there is (at least) one exception. The Chicago Board Option Exchange (CBOE) offers both American

and European options on the S&P 100, which have the same contract size, trading fees, settlement times, and other contract-specific details.[2] These otherwise identical American and European options will be the focus of analysis.

To isolate timing risk over and above an early exercise premium, I look at the following object:

$$(\text{ask}_{\text{American}} - \text{ask}_{\text{European}}) - (\text{bid}_{\text{American}} - \text{bid}_{\text{European}}) \tag{1}$$

Using this differences in differences approach accomplishes several goals. First, it captures the timing risk premium over and above the early exercise premium. This is because the early exercise premium is thought to arise on the long side as a valuation of flexibility (whether rational (Jensen and Pedersen, 2016) or irrational (Poteshman and Serbin, 2003)). Therefore, the second difference (i.e., the bid difference) captures the value of flexibility to the buy side.

The overall term can therefore be thought of as the premium in the ask of the American not attributable to higher pricing from the early exercise premium. This approach allows me to remain agnostic as to the source of the early exercise premium (an interesting but separate question), and focus only on the topic of this paper, which is an aversion to timing risk on the short side of an American, over and above the pricing in of desires for flexibility on the long side.

The differences-in-differences approach also allows me to control for unobservable differences between the American and European options. While all contract specifications are the same (by construction) and controls for liquidity are included (Section 3.3.1, below), it is possible that the option classes differ in unobservable ways. To the extent that these unobservable differences result in structurally higher prices in one options class or the other, those differences should show up in both the bid and

[2]There also exist both American and European options on the S&P 500; in particular, American-style contracts on SPDR and European-style contracts on the S&P 500 index. However, these differ in at least three ways that OEX and XEO do not. First, the most liquid of the S&P 500 options, SPY and SPX, have different contract sizes. In contrast, OEX and XEO have the same contract size. Second, all S&P 500 index options have different CBOE trading fees than SPY, meaning that even after matching on contract size, one would not be able to differentiate fees from pricing premia (CBOE, 2024*a*). This is because it is not possible to see participant identity in the data, and the fees differ by participant type. In contrast, OEX and XEO have the same CBOE trading fees for all participants. Third, SPY versus SPX (and other S&P 500 index options) settle 15 minutes apart; that difference in settlement time can contribute to what is known as a wildcard premium (Fleming and Whaley, 1994), further affecting pricing. In contrast, OEX and XEO have the same settlement time.

the ask. Subtracting out the bid difference therefore also allows me to control for any unobservable factors that would symmetrically affect the spread.

I then perform a number of robustness checks. For example, I test whether the result appears in trades: that is, whether traded American options are more expensive than traded Europeans, and find that they are. I similarly test for a number of alternative explanations for the result, including wildcard premia; hedging considerations; adversarial thinking; inventory considerations; and re-investment risk; among others.

This exercise contributes to a small literature on the pricing of American options relative to European ones, and in particular to papers testing for Merton's theorem. Prior papers in this literature focuses on the early exercise premium (Sung, 1995; Zivney, 1991; Harvey and Whaley, 1991; Li et al., 2024), typically using midpoints; I focus here on timing risk and separate out bid and ask quotes to identify an aversion to timing risk.[3]

3.2 Linkage to Theory

The motivating example in the introduction considered with certainty {\$10 in 2 months for sure} against receiving receiving that \$10 in a timing gamble, which was {50%, 1 month, 50% 3 months}.

The lottery corresponding to the options analysis is more egregious. In particular, from the perspective of the short side of an option, money can arrive or depart only at expiry for a European option but at any time up to and including expiry for an American option. As a result, an example that would conceptually correspond to the options comparison is \$x in, say, 7 days for sure ('Sure Later Timing'), against \$x in 1 day with some probability; 2 days with some probability; and so on, up to 7 days ('Uncertain Sooner Timing'). The second lottery time-dominates the first one, in that all payments are received or made weakly sooner.

[3]To my knowledge, there is one prior paper with a reduced-form comparison of American to European options that considers bids versus asks; it finds that most of the premium shows up on the ask side (Dueker and Miller, 2002). The paper hypothesizes that it may be because the options in question were about to be discontinued leading to wider bid-ask spreads. Timing uncertainty provides a more general way to rationalize why differences in American versus European options may show up in ask quotes - the short side requires compensation to bear timing risk. Further, the options that I consider here are continuously traded and not discontinued at any point in the sample.

Is $x positive or negative? When the short side owns the underlying (i.e., a covered call), $x is a positive cash flow akin to selling one's asset. When the short side does not own the underlying (i.e., a naked short call), $x will generally be negative. The options considered here are cash settled, which would generally indicate a negative value for $x if exercised, unless the short side re-enters the position; however, the data does not include trader identity or asset ownership, so it is not possible to say with certainty whether $x should be treated as positive or negative in this dataset.

The exercise is nevertheless informative because standard pricing models (e.g., Merton (1973)), do not account for a preference for certain timing. However, prices appear to be consistent with a preference for 'Sure Later Timing' over 'Uncertain Sooner Timing.'

As a follow-up question, one could ask which representative preference models the findings theoretically map to. When $x is positive, finding a preference for 'Sure Later Timing' relative to 'Uncertain Sooner Timing' goes against standard theories (see Theorem 5 and Corollary 1), which would predict a preference for uncertainty. In fact, it strongly goes against standard theory, because 'Uncertain Sooner Timing' strictly dominates 'Sure Later Timing.' When $x is negative, finding a preference for 'Sure Later Timing' would be in line with standard preference theories (Corollary 1), but not standard option pricing models which do not having a timing preference component.

3.3 Data

For the cleanest possible results, I use minute-by-minute quote data from CBOE, which offers both OEX and XEO.[4] I compare quotes within the same minute. CBOE treats both OEX and XEO identically for the purpose of fees, settlement dates and times, contract size, expiration dates, tick size, and other institutional features (CBOE, 2024*c*,*b*). The minute-by-minute quote data is available starting in 2012. Starting in 2020, the volume of contracts traded across these asset classes drops below 100,000; for example, in 2024 it was only on the order of 3,000 contracts. The sample I use is therefore Jan 2012 - Dec 2019.

[4]Past research has shown that minute-by-minute data is superior to end-of-day quotes (such as those offered by OptionMetrics), which are subject to end-of-day volatility (Andersen et al., 2017) and end-of-day quotes occurring after the settlement value of the underlying is determined (van Binsbergen et al., 2012). For completeness, I re-do analysis in OptionMetrics data, in Online Appendix B.

Data on dividends paid by companies in the S&P 100 comes from Compustat, whose series starts in 2002; data on end-of-day levels of the S&P 100 also comes from Compustat. Data on Federal Open Market Committee meeting dates comes from the Federal Reserve Board. For robustness, Appendix B.2 also collects dividend data by hand and shows that it maps well to Compustat.

3.3.1 Cleaning

Following standard data cleaning procedure for options data (Muravyev and Pearson, 2020), I drop any quotes with a bid less than \$0.05 or where the bid-ask spread is negative.

Both OEX and XEO options settle to the closing price on the day of exercise, regardless of the time in the day when the option is exercised. Timing uncertainty is therefore not present within a given day, but rather over different days. To study timing uncertainty, I therefore drop quotes where the option expires on the same day as the quote.

While most contract specifications are held fixed by CBOE, there are two features that differentiate the American and European options that should be controlled for. The first and most important is dividends. When a stock is known to start paying a dividend on a particular date, the stock price adjusts downwards by the dividend amount on that date (Hull, 2018). The S&P 100 is a weighted average of stock prices. Consequently, if for example an American and European option expire on a date with a dividend payment, the holder of the American option may be willing to pay a premium - say, up to the anticipated dividend amount - to exercise their option before the stock price update for the dividend occurs. We should therefore restrict our attention to options in whose lifetime, including at initiation and expiry, there are no dividend considerations for any of the stocks in the S&P 100. This restriction substantially reduces the sample, but is also an important step towards identification, by removing the standard explanation for American option premia.[5]

I therefore focus attention on dates and options where, were a market participant to enter the option on that date, they would not face any dividend considerations for

[5]Li et al. (2024) also consider the midpoints of options I study here, and find that at longer horizons, OEX is more expensive than XEO 86% of the time, consistent with an aversion-to-timing-uncertainty interpretation. However, they (1) do not correct for dividend concerns and (2) do not adjust for liquidity differences, and (3) do not separate the early exercise premium on the long side from an aversion to holding timing risk on the short side.

the lifetime of the option. To find such dates, I first find the set of dates on which S&P 100 dividends paid are zero.[6] I then fill in holidays and weekends (for which dividends paid are also zero, because these are not trading days).

The second feature that should be controlled for is liquidity: in this case, the American option is the more traded product, perhaps because it was introduced before the European version (Harvey and Whaley, 1991). Following Amihud and Mendelson (1986) and Bryzgalova et al. (2022), I use the bid-ask spread, that is, (ask - bid)/midpoint, as the illiquidity measure. Restricting this is a little tricky because if, for example, the price premium for the American option arises from sellers demanding compensation for timing uncertainty, imposing a tight bid-ask spread means that it will look as though American bids are higher than European bids in the sample - although actually what is happening is that American asks are higher than European asks and the data restriction imposes tight bid-ask spreads. Thus, we want some liquidity threshold to compensate for liquidity premia, but not such a tight liquidity threshold that we are no longer able to differentiate between movement on the bid and on the ask side. I use a 5% threshold in the main text, e.g. restrict attention to minutes where the difference in bid-ask spreads between the OEX and XEO quotes is less than 5%. For robustness, Table 3 re-performs the analysis for other thresholds and also without liquidity corrections. Regardless of the level of liquidity correction (or its removal), the difference in asks is larger than the difference in bids.

Finally, following Constantinides et al. (2013) and Ni et al. (2008), to focus on credible quotes, I drop options with a moneyness (defined as (underlying-strike)/strike) of below -0.2 or above 0.2. As is the case in other option markets, these tend to be less traded than those within the 0.2 moneyness band; here, after applying all filters above, options which are outside this moneyness band have only 5 contracts traded across the entire sample.

The notional value of all options in the final cleaned dataset is $3.65 billion.

3.4 Summary Statistics

Table 1 provides summary statistics for moneyness, days to expiry, and bid-ask spreads for all options in the analysis dataset post cleaning (Section 3.3.1). The bid-ask spreads are similar because of the imposed liquidity controls. Because com-

[6]Compustat calculates dividends based on the ex-dividend date, which is also the relevant date for dividend-based American option strategies.

panies within the S&P 100 pay out dividends at different times (Harvey and Whaley, 1991), in this data, the longest lived contracts with strictly zero dividend considerations have a lifetime from quote date to expiry of seven days. The median and mean quotes are in-the-money, and as a further robustness check (Section 3.7), I restrict consideration only to options that are in-the-money and find that the baseline results continue to hold. Appendix B.1 provides further descriptive statistics for these options.

TABLE 1: Summary Statistics for OEX and XEO Options

	Both	**OEX**	**XEO**
Bid-Ask Spread (%):			
Mean		6.27%	6.16%
Median		2.90%	2.69%
Standard Deviation		11.22%	11.30%
Moneyness:			
Mean	0.09		
Median	0.08		
Standard Deviation	0.06		
N. Contracts by Days to Expiry:			
1	743,713		
2	240,515		
3	112,668		
4	68,476		
5	11,036		
7	26,577		

Note: This table lists summary statistics for the main sample of OEX and XEO options, cleaned according to the procedure in Section 3.3. The bid-ask spread is calculated as $100 * (ask - bid)/midpoint$. Days to expiry is the number of days between the quote date and the expiration date of the option. Moneyness is calculated as $(underlying - strike)/strike$. The top line follows Equation 1, e.g. the ask difference between the American and European less the bid difference between the American and European.

3.5 Baseline Results in Quotes

Table 2, Column 1 provides the baseline results. The mean ask difference (OEX - XEO) is \$0.33; midpoint \$0.28; and bid \$0.22; all are statistically significant at the 1% level. The ask difference less the bid difference is a statistically significant \$0.11 on average, which corresponds in raw form to 11 basis points as a percentage of the midpoint option price. Because these tend to be options with only a few days to expiry, to satisfy the no-dividends-in-lifetime requirement (Table 1), annualizing the 11 basis points in any way results in a higher number. From the perspective of

standard pricing theory for options (e.g., Merton's theorem), the interesting part of this empirical exercise is that timing risk is priced; the direction and the method of annualization, if any, is in a sense secondary.

These results indicate that (1) the market prices American call options without dividend considerations higher than otherwise identical European options, and (2) that this higher pricing comes both from the long and the short side, but more so from the short side.

Figure 1 plots the average ask, bid, and midpoint difference, aggregated by week, over time. This figure shows that the ask difference is usually higher than the bid difference across all time periods considered. This indicates that the averages reported in Table 2 are not the result of a few positive outliers, but rather arise from a largely consistently positive ask difference less bid difference.

Timing uncertainty is the primary feature that distinguishes OEX from XEO in this dataset because, by construction, we have controlled for other contract characteristics: quote time, tick size, liquidity, type of settlement, tick size, expiration date, etc. One supply side story consistent with this evidence is that sellers require compensation for bearing the timing risk inherent in American options. The timing risk here arises from the long side possibly exercising early. I take no position on why the long side might exercise early, and do not observe empirical early exercise in the data. To check for one crude proxy for possibility of early exercise and whether it correlates with additional compensation required by the short side, I consider moneyness; the more in-the-money an option is, the more tempted the long side may be to exercise. Figure 2 groups the ask difference − bid difference by moneyness bands, and finds that this difference-in-difference is approximately monotonic in moneyness. This pattern is consistent with the short side of the American being more averse to uncertain timing the more risk there is of early exercise.

The robustness sections, next, shows that these baseline results continue to hold in various subsamples of the data.

3.6 Replicating Results in Trades

Here I test for whether the quote-based results - of an unwillingness to be short an American over and above an unwillingness to be short European - appear in executed trades. The separation between the short and long side will not be as clean as for

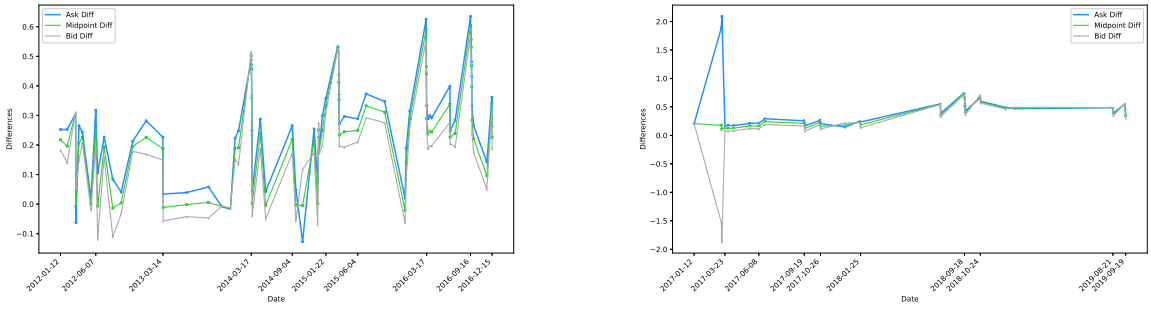
TABLE 2: Pricing Differences between Dividend-Free American (OEX) and European (XEO) Options

	Baseline	In the Money	No Fed Meetings	Positive Open Interest
Ask Difference - Bid Difference	\$0.11***	\$0.11***	\$0.11***	\$0.05***
	(0.04)	(0.04)	(0.04)	(0.02)
Ask Difference	\$0.33***	\$0.34***	\$0.33***	\$0.22***
	(0.03)	(0.03)	(0.03)	(0.02)
Midpoint Difference	\$0.28***	\$0.28***	\$0.28***	\$0.19***
	(0.02)	(0.02)	(0.02)	(0.02)
Bid Difference	\$0.22***	\$0.23***	\$0.22***	\$0.16***
	(0.03)	(0.03)	(0.03)	(0.03)
Ask Difference Quartiles:				
25%	\$0.20	\$0.20	\$0.20	\$0.00
50%	\$0.30	\$0.30	\$0.30	\$0.20
75%	\$0.50	\$0.50	\$0.50	\$0.40
Midpoint Difference Quartiles:				
25%	\$0.15	\$0.15	\$0.15	\$0.025
50%	\$0.25	\$0.25	\$0.25	\$0.15
75%	\$0.40	\$0.40	\$0.40	\$0.30
Bid Difference Quartiles:				
25%	\$0.10	\$0.10	\$0.10	\$0.00
50%	\$0.20	\$0.20	\$0.20	\$0.10
75%	\$0.40	\$0.40	\$0.40	\$0.30
Number of Observations	1,202,985	1,169,096	1,202,985	205,223
Number of Unique Days	101	101	101	98

*Standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01.*

Note: This table shows the difference in quoted prices (OEX (American) - XEO (European)) between otherwise identical OEX and XEO options in dollar terms. Standard errors are clustered by date and by contract, where a contract is a (strike price, expiration date) pair. The baseline column performs this analysis on the data cleaned as described in Section 3.3. The in-the-money column restricts the cleaned data to only in-the-money options. The no-fed-in-lifetime column restricts the cleaned data to only options in whose lifetime there is no Fed meeting. The positive open interest column restricts the cleaned data to only option quotes and minutes for which both OEX and XEO have strictly positive open interest.

FIGURE 1: Ask Difference - Bid Difference is Positive Across Sample



(A) 2012-2016 (B) 2017-2020

Note: This figure plots average price differences for asks, bids, and midpoints, aggregated daily. Days with data are shown as points on each line. Because some dates have large magnitude differences, for ease of visual processing, Figure 1a focuses on the period 2012-2016 and Figure 1b 2017-2020. The figure shows that ask differences (in blue) are generally positive and higher than bid differences (in gray) over time.

TABLE 3: OEX - XEO Pricing Differences Under Different Liquidity Thresholds

	Baseline (Liquidity Difference < 5%)	Liquidity Difference < 1%	Liquidity Difference < 10%	No Liquidity Controls
Ask Difference - Bid Difference	\$0.11***	\$0.07***	\$0.13**	\$0.11*
	(0.04)	(0.00)	(0.06)	(0.06)
Ask Difference	\$0.33***	\$0.32***	\$0.34***	\$0.32***
	(0.03)	(0.02)	(0.03)	(0.03)
Midpoint Difference	\$0.28***	\$0.29***	\$0.27***	\$0.26***
	(0.02)	(0.02)	(0.02)	(0.02)
Bid Difference	\$0.22***	\$0.25***	\$0.21***	\$0.21***
	(0.03)	(0.02)	(0.04)	(0.04)
Ask Difference Quartiles:				
25%	\$0.20	\$0.20	\$0.20	\$0.10
50%	\$0.30	\$0.30	\$0.30	\$0.30
75%	\$0.50	\$0.50	\$0.50	\$0.40
Midpoint Difference Quartiles:				
25%	\$0.15	\$0.15	\$0.10	\$0.10
50%	\$0.25	\$0.25	\$0.25	\$0.25
75%	\$0.40	\$0.40	\$0.40	\$0.40
Bid Difference Quartiles:				
25%	\$0.10	\$0.10	\$0.10	\$0.10
50%	\$0.20	\$0.20	\$0.20	\$0.20
75%	\$0.40	\$0.40	\$0.40	\$0.40
Number of Observations	1,202,985	1,089,218	1,244,861	1,301,399
Number of Unique Days	101	101	101	101

*Standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01.*

Note: This table shows the difference in quoted prices (OEX (American) - XEO (European)) between otherwise identical OEX and XEO options in dollar terms, with different liquidity controls. Standard errors are clustered by date and by contract, where a contract is a (strike price, expiration date) pair. The baseline column performs this analysis on the data cleaned as described in Section 3.3. The second column restricts to options and dates or minutes where the the difference in bid-ask spread (measured as a percentage) between XEO and OEX is less than 1%; the third column less than 10%; and the fourth column makes no liquidity restrictions.

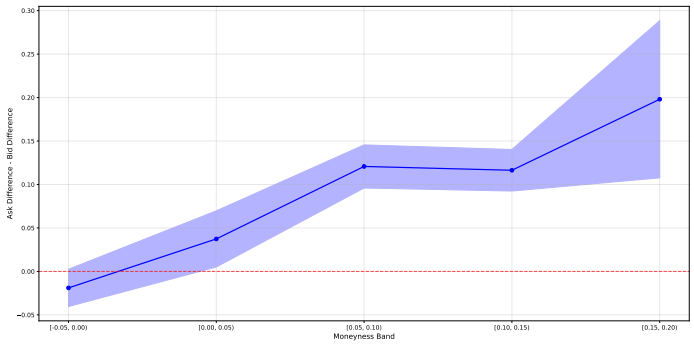


FIGURE 2: Ask Difference - Bid Difference by Moneyness

Note: This figure shows average and 95% confidence intervals for (Ask(OEX) - Ask(XEO)) - (Bid(OEX) - Bid(XEO)), with standard errors clustered at the date and contract level. Moneyness is defined as $(underlying - strike)/strike$. Moneyness bands are taken within 0.05 increments. For all options with a given moneyness band, the ask difference less bid difference is regressed on a constant, with standard errors clustered at the date and contract level. The resulting intercept estimate and its confidence intervals are shown.

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the quotes, but the fact that similar (in fact, quantitatively larger) results appear for trades is reassuring.

First, as in Section 3.3.1, consider only options in whose lifetime there are no dividends. I then match each XEO trade with an OEX trade with the same strike price and expiry, where both trades occur within 10 minutes of each other. If there are multiple such OEX trades, I take the one closest in time to the XEO trade. I then compare the traded price for the OEX and XEO trades. This method of cleaning yields 1189 trades in the sample across 62 days.

Following the matching procedure above finds that American options trade on average 3.75 cents higher than European options ($p < 0.05$, with standard errors clustered by date); as a percentage of the executed option price, this difference is on average 5.5%. The larger difference (as a percentage) for trades than for quotes comes from the average traded price being much lower than the average quoted price, as is typical for short-dated options where trading volume is heavily skewed towards cheaper contracts (Andersen et al., 2017).

3.7 Assessing Possible Explanations

Timing uncertainty is only one story that is consistent with the baseline results from quotes and trades; there may be others. In this section, I consider the ability of twelve alternative explanations and discuss their ability to capture the results.

3.7.1 Out-of-the-Money Anomalies.

The second column of Table 2 restricts analysis to the subsample of quotes that are in-the-money. The results continue to hold: on average, ask quotes for the American option are \$0.34 higher than for the European, midpoints \$0.28 higher, and bid quotes \$0.23 higher.

3.7.2 Strictly Positive Open Interest.

One concern may be that option quotes are not meaningful for options which do not have positive open interest. This is unlikely to explain the results because CBOE has designated market makers for option classes, who are required to uphold certain requirements, including minimum quote contract size requirements (CBOE, 2019). As a further explicit check, Column 4 of Table 2 restricts attention to options and minutes for which both OEX and XEO have strictly positive open interest.

3.7.3 Executed Prices.

Similarly, one may argue that quotes and executed prices differ. Section 3.6 computes the difference in traded prices for OEX and XEO and finds that dividend-free OEX call options trade at higher prices than otherwise identical XEO options. These trading results are consistent with the results from quotes.

3.7.4 Wildcard Premium Differences.

Fleming and Whaley (1994) document that some options have wildcard premiums, meaning that the settlement price of the contract is established before the last time at which the option holder can decide to exercise. This would affect a comparison of options on the S&P 500, because PM-settled SPX (European) options are exercised at the close of the trading day, but holders of SPY (American) have the option to exercise after the closing price of the underlying - so one is subject to a wildcard premium while the other is not. This is one of the reasons I chose not to use the S&P 500 for the reduced form comparison. A benefit of the options on the S&P 100 considered here is that both stop trading at the same time and have identical settlement specifications, and therefore there is no wildcard premium difference between them.

3.7.5 Re-Investment Risk.

If interest rates are expected to changing during the lifetime of the option, the could be an induced reason for aversion to uncertain timing. Here, however, most options are short dated, making interest rate considerations unlikely to play a first order role. As a further explicit check, Column 3 of Table 2 restricts attention to options in whose lifetime there are no FOMC meetings and finds identical results to the baseline.

3.7.6 Informed Trading

The next set of explanations explores the idea that the short side of the market may be strategically reacting to actions from the long side of the market. We will consider these explanations one by one, but they can also be answered by showing that the options we consider are traded largely by uninformed investors. If the trading is uninformed, that alleviates strategic concerns that might affect pricing.

There are several reasons to believe that the trades corresponding to our main sample are uninformed. First, past research shows that trading in index options is generally uninformed, with informed investors typically locating in single-name stock options, rather than index options (Pan and Poteshman, 2006; Chan et al., 2002). But second, we can perform an explicit check for uninformed trading by looking at trade size, because Muravyev (2016) shows that small option trades tend to be uninformed in that they do not predict market movements. In addition, small option trades have been shown to be one (noisy) proxy for retail trading in options, which past work finds to be idiosyncratic with suboptimal behavior (Barber and Odean, 2013; Bauer et al., 2009; Bryzgalova et al., 2022). Appendix B.4 checks the proportion of small trades (few contracts traded, small monetary amount transacted, or both) and shows that these small trades constitute over over 50% of trades corresponding to the main sample. Within buys - which is what the short side of the market would be responding to - small trades account for approximately 57-83% of trades, depending on the measure used. This proportion of small trades is similar across the American and European classes. From past research, we conclude that a large portion of trades corresponding to the main sample are uninformed, which means strategic responses to them are less likely to be driving the results.

3.7.7 Adversarial Thinking.

Because of the constraint that these options not have dividend considerations in their lifetime, these are generally short-dated options (Table 1). One might ask why the long side of an option would choose to enter an American rather than European; in particular, rationally a market participant should be willing to pay more for a short-dated American than European if either (1) the long side values the timing flexibility, or (2) the long side intends to exercise right away. If (2) holds, then the short side, anticipating this, may adjust their prices. However, standard theory predicts they would adjust prices in the opposite direction from what I find. This is because an American where the long side exercises immediately is equivalent to a European expiring today; but we know from both standard models and a large empirical literature that European options with more days to expiry are more expensive, because of a time value of money argument (Bakshi and Kapadia, 2003). This kind of adversarial thinking would therefore predict that American options trade at a discount relative to European options, the opposite of what I find here.

Another type of adversarial thinking might posit that options are exercised only on days disadvantageous to the short side of the option. Robustness check 'Re-Investment Risk,' above, shows that the result holds even when the most common market variables related to this, interest rate considerations, are controlled for. In addition, such considerations should be minimal only a few days to expiry, but the effect shows up for such options too. Finally, arguments of this form should be minimal if trading on the long side is uninformed, as discussed in Section 3.7.6, above.

3.7.8 Hedging Considerations.

One reason American options may be harder to hold for the short side than European ones is that, when there is uncertainty over the day of exercise, hedging becomes more complicated, and the short side of the American may have a practical or psychological aversion to dealing with this complexity, or may have budgeting needs that make changes in hedging difficult.

To show that there is a pricing of timing uncertainty over and above differential hedging considerations, Appendix B.3 re-performs the analysis for only options whose hedging requirements are not expected to change over their lifetime. In particular, it

re-conducts the test for options whose gamma is close to zero. This means that the amount of hedging needed for being short the option is not expected to change over the option's lifetime. Though the sample becomes an order of magnitude smaller, the qualitative result: a pricing premium on the short side, over and above that on the long side (Equation 1) continues to hold.

3.7.9 Asymmetric Changes in Bids and Asks - For Example Due to Inventory

Specification 1 treats asks and bids symmetrically. But, it is possible that the long side moves differently than the short side. For example, under some models (Ho and Stoll, 1981), if a market maker has accumulated excessive long positions due to rising bids, they might lower ask prices to encourage sales and rebalance inventory. The main specification does not allow for that flexible difference in changes in bids relative to asks. To allow for this flexibility, I run the regression:

$$\left(\text{ask}_{\text{American}} - \text{ask}_{\text{European}}\right)_{m,o} = \beta_0 + \beta_1 \left(\text{bid}_{\text{American}} - \text{bid}_{\text{European}}\right)_{m,o} + \epsilon_{m,o},$$

where m denotes the minute and o the option. We are interested in whether $\beta_0 > 0$ even after allowing β_1 to be flexible (e.g., not imposing that it is one). The regression yields a statistically significant $\beta_0 = 0.37$ ($p < 0.01$, clustering standard errors by date), showing that there is an unwillingness to be short the American over and above flexible pricing incorporation of the long side.

3.7.10 Further Liquidity Considerations

Figlewski (2022) shows theoretically that when the liquidity of the options market is highly variable, an American option can be more valuable than a European one because the American allows the long side to exercise at times when the market is relatively illiquid. However, the baseline analysis controls for liquidity, by construction (Section 3.3.1). In addition, the above argument is a reason why the long side may value the American, but not a reason for the short side to price the American at a premium; per the Section 3.7.7 above, standard option value arguments imply that the short side should be willing to offer options at a discount if they believe early exercise is likely. In addition, Section 3.7.8 above shows that the results hold even for options where the short side has no need to change their hedging requirements, e.g.

their own budgeting considerations are not affected.

3.7.11 Rare Disasters

Seo and Wachter (2019) document that index put options are routinely more expensive to purchase than predicted by Black-Scholes, and finds that a model including a small probability of a rare disaster provides a better approximation of prices. My findings do not contradict this, but neither can they be fully explained by rare disasters. First, the analysis is on call options which, all else equal, should be less appealing in the face of rare disasters; the analysis of Seo and Wachter (2019) focuses on put options, which provide insurance should a rare disaster occur. Second, the empirical test is reduced form and within-index options; it does not rely on Black-Scholes assumptions in any way.

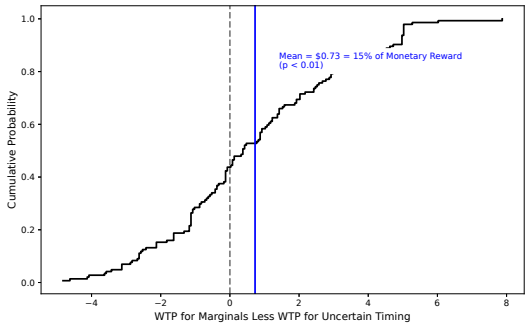
3.7.12 Early Resolution of Uncertainty

Ai et al. (2023) find evidence of a preference for early resolution of uncertainty using FOMC announcements. In the context of American options, uncertainty is resolved sooner if the long side chooses to exercise early. As a result, if the short side prefers early resolution, they should be more willing to be short the American than the European. But this is the opposite of what the data suggests (Table 2). In particular, here the short side requires additional compensation for the American than the European option.

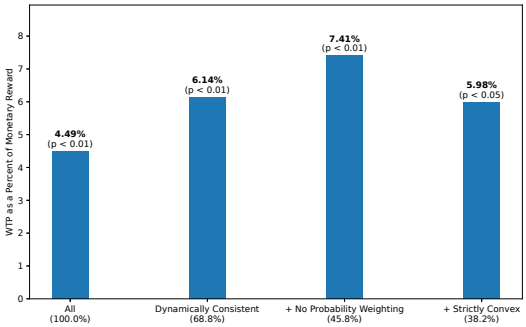
4 Willingness-to-Pay and Pricing Narratives

As a supplement to the market results, this section elicit willingness to pay for lotteries to further test the predictions of some of the theorems derived in this paper. Consistent with the market-based results, uncertain timing is priced. Further, it is priced in a manner that differs from pricing of the marginals. Finally, the pricing of uncertain timing holds even amongst those individuals who display arguably desirable pricing properties. An incentivized elicitation of the approach used to price finds that many respondents self-describe as pricing certainty in the time and money domains

FIGURE 3: Willingness-to-Pay Measures



(A) WTP for Marginals − WTP for Uncertain Timing (Theorem 1)



(B) WTP for Certain − Uncertain Timing and Theoretical Properties (Theorem 5)

Note: Panel (A) shows the cumulative density function for difference in willingness to pay for the marginals of the uncertain timing lottery less the willingness to pay for the uncertain timing lottery itself. Panel (A) also provides a label for the mean difference and the p-value of that difference. Panel (B) shows, as a percentage of the money at stake ($5), the difference in willingness to pay for certain and uncertain timing. The first bar shows this overall; the second within the subset of respondents who are dynamically consistent; the third bar within the subset of respondents who are dynamically consistent and do not distort probabilities; and the fourth bar within the subset of respondents who are dynamically consistent, do not distort probabilities, and display locally convex deterministic discounting. The percentages on the bottom of the x-axis show the fraction of respondents within each group. For example, 68.8% of respondents are dynamically consistent, while 38.2% satisfy all three properties. The p value is shown underneath the mean value for each bar.

similarly.[7]

4.1 Data and Methodology

The survey was run on Prolific in March 2025; IRB approval was obtained from the New York University Institutional Review Board, under ID IRB-FY2024-8127. Respondents received an average payment of $4.23, and the median time taken was 21 minutes. Willingness to pay was measured via standard incentivized multiple price lists (Holt and Laury, 2002), that is, a question and row were selected at random for 10% of respondents, who then received a bonus payment according to their response in that row. A series of eleven comprehension questions ensured that participants understood the difference between sure money and sure timing; sure money and uncertain timing; and uncertain money and sure timing (Online Appendix A). There were 144 respondents total.

The main questions to test for attitudes towards uncertain timing were elicitations of willingness to pay (WTP) today for the lotteries: (1) {50%, $5 in 2 days; 50%, $5

[7]Self-descriptions are often used in surveys (ex. Schotter (2003), Bailey et al. (2018)) – here I use them in the context of studying attitudes towards timing risk.

TABLE 4: Incentivized Narratives

Panel A: Sample Narratives	**Panel B: Classification**	
(1) I simply just went for less risks.	GENERAL PRICING RULE	51%
(2) I tried to play it relatively safe.	MONEY CERTAINTY PRICING, TIME UNCERTAINTY TOLERANT	17%
(3) I chose the safe option with no probability and was a guarantee up until \$4 when I changed to the probability option.	MONEY CERTAINTY PRICING, TIME CERTAINTY PRICING	17%
(4) For lotteries that include a 50% chance of receiving nothing, I elected to receive everything over a minimal amount. For lotteries that paid out and varied only in time, I elected to receive the deferred payment for anything less than the immediate award.	MONEY UNCERTAINTY TOLERANT, TIME UNCERTAINTY TOLERANT	11%
	MONEY UNCERTAINTY TOLERANT, TIME CERTAINTY PRICING	3%
(5) I chose to not look at the timeframe of when I would get the bonus because I knew that I would get it eventually. Instead, I only focused on the dollar amount that would be given to me. In the case of a lottery, I calculate the dollar amount by taking the expected payoff from the lottery.		

in 8 days}; (2) {50%, \$0 in 2 days; 50%, \$5 in 2 days}; (3) {50%, \$0 in 8 days; 50%, \$5 in 8 days}; and (4) \$5 in 5 days. The prediction of the framework described in Theorem 1 is that WTP for (2) + WTP for (3) = WTP for (1). The prediction of Definition 6 is that the WTP for (1) should be weakly lower than the WTP for (4).

In addition, to measure correlations of the answers to these questions with (a) dynamic consistency, (b) no probability distortion, and (c) convex discounting, that is, some of the properties at odds with a preference for certain timing in Theorem 5, the survey included additional standard questions for deterministic time discounting (Sprenger, 2015) and probability distortion vs. complexity aversion (Puri, 2025); these questions are described in detail in Online Appendix A.

The order of the questions was randomized within the survey for each participant. Directly after the uncertain timing question, the participant was surprised with a question which asked them to describe how they made their prior choices. The question was made incentive compatible by having, for 5% of respondents, a new participant read the original participant's description, and having the new participant attempt to replicate the original participant's choices on the timing risk questions.

If the new participant correctly replicated the original participant's choice on a randomly selected question, both earn a bonus.

4.2 Results

The spread between the willingness to pay for the marginals vs. the willingness to pay for uncertain timing with the same marginals, is depicted in Figure 3a. In contrast with the framework described in Theorem 1, these direct elicitations reveal an average difference of \$0.73 in willingness-to-pay, equivalent to 15% of the \$5 monetary award in the uncertain timing lottery.

Figure 3b shows that the statistically significant willingness to pay for certain timing remains even after applying properties that financial economists have historically found appealing; namely, dynamic consistency, no probability distortion, and locally convex discounting.[8]

Table 4, Panel A provides examples of incentivized narratives about how respondents approached uncertain timing and substitution across time and money. Table 4, Panel B classifies these into general choice rules (e.g., no differentiation between time and money in approach to choice); stated certainty preference in both domains; certainty preference in money but uncertainty tolerant over time; etc. The classification was done using Large Language Models, using the prompts described in Online Appendix A. Overall, 68% of respondents report pricing narratives of either a general pricing rule across domains or a higher WTP for certainty in both domains.

5 Potential Applications Across Finance

Future work can explore the empirical pricing of uncertain timing across a variety of financial markets.

Bond Spreads. Callable and convertible bonds are subject to timing uncertainty. Pricing models for these bonds typically conceptualize them as a standard bond plus an option (Brealey et al., 2020). Just as, in Section 3, options prices differ from

[8]Online Appendix A performs a robustness check in which the sample is restricted only to high financial literacy respondents who satisfy monotonicity conditions, and shows that the results continue to hold.

predictions of classical theorems, consistent with a pricing of uncertain timing (Section 3), corporate bond spreads may differ as well.

Corporate Decisions. Companies often have uncertainty not only over the magnitude of cash flows on an uncertain project, but also when they will occur. One puzzle in the corporate literature is that companies appear to use hurdle rates that are higher than their weighted average cost of capital (Graham, 2022). It may be the case that one reason for these high hurdle rates is that the cost of capital estimate does not account for the uncertain timing of return inherent in some projects.

Mortgage-Backed Securities. In mortgages, prepayment risk is at heart uncertainty over the time at which the mortgage will be paid back. In practice, prepayment is often predicted using interest rate factors, but research shows that much of the pricing of MBS reflects something beyond interest rates (Boyarchenko et al., 2019). It may be the case that, as with the options markets in this paper, MBS are pricing (possibly difficult to observe) uncertain timing of mortgage prepayment.

Policy Investment Decisions. A number of policy decisions involve uncertainty both over the outcome and over the timeline of implementation. For example, infrastructure projects include both types of uncertainty. A pricing of uncertain timing may therefore be relevant in policy contexts.

Convenience Yields. In times of fiscal shutdown or economic crisis, both the likelihood of default and, the probability that payments might be delayed, increase. It may be the case that convenience yields decrease in such instances due not only to outcome uncertainty but also due to uncertain timing.

6 Connections to Literature

The contribution of this paper spans financial subfields. In showing that the stochastic discount factor framework with certain canonical state spaces is unable to accommodate all assets with uncertain timing, the paper contributes to a literature on the stochastic discount factor and its properties (Ross (1978), Harrison and Kreps (1979),

Hansen and Singleton (1982), Hansen and Jagannathan (1991), Duffie (2001), Hansen and Scheinkman (2017)).

By theoretically showing that pricing frameworks with standard properties do not accommodate timing uncertainty, but empirical prices are consistent with a preference for certain timing, the paper also contributes to literature that thinks about how to model the representative investor; a topic extensively thought about and discussed by, for example, Campbell and Cochrane (1999); Lettau and Ludvigson (2001); Daniel et al. (2002); Lewellen and Nagel (2006), and Verdelhan (2010), among many others.

In showing a possible friction between theory and empirical pricing, the paper complements work on empirical puzzles relative to standard theories. One such recent example is work on the term structure of the equity premium (van Binsbergen et al., 2012; van Binsbergen and Koijen, 2017), which finds that standard theory does not predict well the pricing of dividend strips.

In studying empirical pricing of options, the paper contributes to a literature on actual prices of options and how these either adhere to, or deviate from, classical theory. Past work has studied a willingness to pay for flexibility on the long side (Hull, 2018), which my specification explicitly distinguishes from; as well as frictions across markets (Bryzgalova et al., 2024); and the identification and behavior of retail trades (Bryzgalova et al., 2022). Here, I study the short side of an index option market and find evidence that equilibrium pricing is consistent with an aversion to bearing timing risk.

Because pricing appears throughout finance, the paper also has implications for how to think about discounting risky cash flows (Graham, 2022), when the time of project completion or return may be uncertain; welfare implications associated with climate finance, where the timing of subsidy or disaster may be uncertain (Bolton and Kacperczyk, 2020; Stroebel and Wurgler, 2021; Pastor et al., 2024); valuation in venture capital and private equity, where the timing of return is uncertain (Da Rin et al., 2013; Metrick and Yasuda, 2021; Gourier et al., 2024); the pricing of corporate bonds, where the timing of conversation or call is uncertain (Brealey et al., 2020); and more generally, has implications for financial decisions in any setting with uncertain timing. In particular, one implication is that an approach towards evaluating uncertain timing with standard pricing models or frameworks - as typically done when needed, for example the pricing of callable bonds draws on standard option pricing theories - may or may not accurately reflect empirical market treatment.

The paper also complements a large literature on the elasticity of intertemporal substitution, a component of preferences whose calibration is thought to be first order for financial decisions (Hansen and Singleton, 1982; Vissing-Jorgensen, 2002; Woodford, 2003). It also complements a smaller literature on the timing of uncertainty resolution (Epstein et al., 2014). Here, I show that no matter what the calibration of the EIS, and even when all uncertainty is resolved immediately, it is theoretically challenging to flexibly price uncertain timing.

The finance literature has not historically considered whether standard pricing frameworks treat uncertain timing accurately. This paper shows that such attitudes are relevant empirically and theoretically. In particular, pricing models with standard properties make strong predictions over uncertain timing, while real-world markets with sophisticated participants feature pricing consistent with compensation for bearing uncertain timing. This is not, however, the first paper to think about these topics in the world; there is a small but rich interdisciplinary literature. In biology, Kacelnik and Bateson (1996) show that foraging animals like pigeons and rats are temporally risk seeking. In decision theory, Chesson and Viscusi (2003) show that expected utility is temporally risk-seeking over gains, but unincentivized experiments seem to show the opposite. Onay and Onculer (2007) generalize these results to time-separable preferences, and show that temporal risk aversion may be partially accommodated by a model that includes probability weighting. DeJarnette et al. (2020) leverage this past work to show that temporal risk aversion is axiomatically incompatible with a property they introduce called stochastic impatience, across a wide range of utility functions, and perform incentivized experiments supporting certain timing; they and Dillenberger et al. (2024) also derive results on both properties within Epstein-Zin. Ebert (2020) introduces terminology and discusses possible ways to formalize temporal attitudes.

In addition to contributing to the finance literature, the paper contributes to this interdisciplinary literature by bringing the topic of timing risk to applications and theoretical frameworks relevant for finance. On the theoretical side, this is a first paper across the interdisciplinary literature to study temporal risk within the stochastic discount factor framework central to asset pricing. In addition, I expand the theory to apply beyond commonly-used models. Second, on the empirical side, I provide some of the first real-world evidence of prices being consistent with a preference for certain timing in large financial markets. Third, I elicit willingness-to-pay measures

which supplement the market results.

Finally, the paper also has policy implications for the recent discussion around discount rates, especially as regards pricing of longer-term government policies (Joskow et al., 2023). Whereas prior work has calibrated a long-run discount rate parameter (Lucas, 2023), the present work shows that in certain canonical SDF treatments, such calibration, and its application, simultaneously pins down not only deterministic discounting but also (perhaps inadvertently) prices uncertain timing.

7 Conclusion

This paper explored the pricing of certain vs. uncertain timing both under pricing frameworks with standard or historically desirable properties, and empirically in financial markets. While certain textbook SDF approaches are not rich enough to accommodate the pricing of uncertain timing, uncertain timing appears to be priced in options markets. Expanding beyond these straightforward textbook approaches allows for the pricing of uncertain timing, but full flexibility in the method of pricing conflicts with other desirable properties such as dynamic consistency, the future mattering less than the present, and history independence.

These findings suggest that better understanding both theoretical and empirical components of uncertain timing may deepen financial models and understanding of pricing in financial markets. For example, in many settings, uncertainty in outcomes and uncertainty over time are intertwined. Future work may wish to seriously examine the uncertain timing aspect of empirical settings such as corporate bond pricing, investment decisions, and policy decisions, among others.

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