Transportation, Regulation and growth

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Abstract

We study growth in an economy where "making" and "shipping" are jointly decisive for productivity. In our multi-industry, endogenous-innovation model, per-unit transportation costs act like an incompressible wedge: if firms can innovate in production but not in transport, the wedge prevents price declines from fully translating into scale and the economy stagnates; allowing innovation in transport re-opens self-sustaining growth and yields testable, industry-level predictions about entry, R&D allocation, and TFP dynamics. We map these wedges to regulatory accumulation: stacked compliance burdens in distribution and logistics raise effective per-unit costs (and can raise the cost of transportation R&D), echoing empirical findings that accumulated regulation depresses growth and productivity (Dawson & Seater 2013; Coffey, McLaughlin & Peretto 2020). We show how the model rationalizes quasi-experimental evidence from British Columbia's regulatory budgeting (Coffey & McLaughlin 2021) and new state-level evidence using State RegData (McLaughlin & Wong 2024), while remaining consistent with broader results on entrepreneurship and distributional consequences under heavier regulatory stocks.

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1 Introduction

Regulatory accumulation—the layering of rules over time—creates frictions whose macroeconomic importance is now well documented. Dawson and Seater (2013) show, using a long-run measure of U.S. federal regulation, that accumulation materially reduces aggregate output via TFP, capital, and labor channels. Coffey, McLaughlin, and Peretto (2020) use industry-panel evidence tied to RegData and find that accumulated federal restrictions have damped U.S. growth on the order of roughly 0.8 percentage points per year since 1980. These results establish a baseline: cumulative burdens, not just marginal rulemaking, can be a first-order determinant of growth.

Our paper offers a mechanism that connects those empirical facts to firm-level incentives: growth depends on an integrated system of production and transportation. If per-unit shipping costs are not themselves the object of innovation, they form an "incompressible" wedge that blocks the classic scale amplification of productivity improvements in making things; growth stalls even when production technologies improve. When transportation innovation is feasible and rewarded, firms reallocate R&D toward both "make" and "ship," restoring endogenous growth. This mechanism provides a structural, welfare-relevant lens on why cumulative frictions—including regulatory ones—can choke off innovation's translation into long-run prosperity.

The model's parameters map naturally to regulation. First, per-unit transport costs can be increased by accumulated requirements governing freight movement, routing, safety/inspection lags, and documentation—costs that are experienced every unit shipped. Second, regulation can raise the cost or lower the payoff of transportation-specific R&D, shifting firms' innovation portfolios away from the logistics margin. This mapping suggests a program of empirical tests using RegData as the measurement backbone (Al-Ubaydli & McLaughlin 2015).

Our framework also helps interpret quasi-experimental evidence where reductions in regulatory stocks boost performance. British Columbia's regulatory budgeting—which explicitly targeted the stock and required offsets—coincided with improved economic outcomes; estimates using RegData directly link marginal changes in the stock to year-over-year growth. In our model, such policy trims operate like temporary declines in per-unit shipping costs: they generate a short-run growth acceleration and a level gain in TFP even if the long-run growth rate is ultimately governed by the innovation technology and entry equilibrium.

State-level results reinforce the accumulation channel. McLaughlin and Wong (2024) exploit cross-state variation in accumulated restrictions (State RegData) and institutional

sclerosis to estimate causal effects of regulation on growth, finding meaningful negative impacts that scale with the stock. Our model predicts precisely that: higher per-unit wedges depress firm size and entry, reallocate R&D away from scalable margins, and reduce industry-weighted TFP growth.

Finally, the broader RegData literature points to important distributional and dynamism effects that our mechanism can accommodate. Heavy regulatory environments suppress business formation and small-firm employment growth (Bailey & Thomas 2017), and are associated with higher poverty and greater income inequality—consistent with cumulative frictions falling more heavily on marginal producers and low-income households (Chambers 2019; Chambers & O'Reilly 2020, 2022). By making the logistics wedge explicit, our model clarifies how these regressive effects can emerge through costs that bind per transaction, not only at entry.

Contributions. We (i) provide a tractable multi-industry growth model in which transport frictions are central, (ii) show that per-unit wedges can endogenously stall growth unless transport innovation is operative, (iii) map those wedges to regulatory accumulation in a way that generates clear, RegData-testable predictions, and (iv) align theory with evidence from British Columbia and U.S. states. The payoffs are both positive (explaining growth slowdowns under accumulation) and normative (identifying reforms—e.g., regulatory budgeting targeted at per-unit frictions—that raise welfare via level effects and, in some cases, higher steady-state innovation).

2 Model

In this section we set up the model and discuss its conceptual and analytical foundations.

2.1 Model structure

The production side of the economy consists of a representative competitive firm that combines intermediate goods to produce a homogeneous final good. To obtain a multi-industry structure, we model this production process as consisting of two vertically stacked layers.

At the bottom there is a Cobb-Douglas aggregator representing production of the final good as the assembly of differentiated components. Next up there is the second layer, in which each one of the differentiated components used in layer one is produced by a competitive firm operating a CES aggregator of specialized intermediate goods. Therefore,

in this model an industry consists of an endogenous set of specialized firms that supply the intermediate inputs used in the production of one of the differentiated components of the homogeneous consumption good. In both layer one and layer two of the consumption good sector, production is the assembly of differentiated inputs according to well-understood aggregators.

Each intermediate firm belongs to a specific industry endowed with its own industry-specific parameters. To keep things simple, we assume that the labor market is fully integrated with mobility across industries so that all firms in all industries face the same wage. Similarly, the financial market is fully integrated so that all firms in all industries face the same interest rate.

2.2 Household

The representative household has $L(t) = \Lambda e^{\lambda t}$ members, where $\Lambda > 0$ and $\lambda \in (-\infty, +\infty)$. In this setup, the size of the household, L, is population, where Λ is a scale factor and λ is the growth rate of the population. We do not restrict λ to be positive. Finally, to keep things simple, we abstract for labor-leisure choice so that the household's labor supply is L.

The household maximizes lifetime utility

$$U(t) = \int_t^\infty e^{-\rho(s-t)} L(s)\, u(s) ds, \quad \rho > \max\{0, \lambda\}, \tag{1}$$

where ρ is the discount rate. Instantaneous utility is

$$u = \log\left(\frac{Y}{p_G L}\right), \tag{2}$$

where Y is the household's expenditure on the consumption good whose price is p_G. The household's budget constraint is

$$\dot{A} = rA + \underbrace{\Pi_G + \sum_{j=1}^{J} \Pi_{G_j}}_{\text{both 0 in equilibrium}} + wL - Y., \tag{3}$$

where A is assets holding (equity shares in intermediate firms, see below), r is the rate of return on assets, w is the wage, Π_G and Π_{G_j} are the profits distributed to the household as dividends by the consumption good sector and its suppliers of differentiated components

$j = 1, ..., J$. These terms are zero in equilibrium.

The household chooses the time path of expenditure Y to maximize (1) subject to (2) and (3). The solution to this problem yields the Euler equation governing saving behavior,

$$r = r_A \equiv \rho + \frac{\dot{Y}}{Y} - \lambda. \tag{4}$$

We interpret r_A as the reservation rate of return on financial assets at which the household is willing to trade current for future consumption.

2.3 Consumption good producer

In layer one, a representative competitive firm produces the homogeneous consumption good with the Cobb-Douglas technology

$$G = J \prod_{j=1}^{J} G_j^{\varphi_j}, \quad \text{with} \sum_{j=1}^{J} \varphi_j = 1. \tag{5}$$

This is a well understood structure: the competitive layer-one firm spends on each component j a fraction $Y_j/Y = \varphi_j$ of its revenues, $Y = p_G G$. Moreover, in competitive equilibrium this producer makes zero profits, i.e., $\Pi_G = 0$.

The typical competitive layer-two firm operates the CES technology

$$G_j = N_j^{\chi_j - \frac{1}{\epsilon_j - 1}} \left(\int_0^{N_j} X_{ij}^{\frac{\epsilon_j - 1}{\epsilon_j}} di \right)^{\frac{\epsilon_j}{\epsilon_j - 1}}, \quad \chi_j > 0, \ \epsilon_j > 1, \quad j = 1, ..., J \tag{6}$$

where χ_j is the degree of increasing returns to variety, ϵ_j is the elasticity of product substitution, X_{ji} is the quantity of the non-durable intermediate good i supplied to industry j, and N_j is the mass of suppliers active in industry j. This structure is also well understood and delivers the demand system for intermediate goods in industry j,

$$X_{ij} = Y_j \frac{p_{X_{ij}}^{-\epsilon_j}}{\int_0^{N_j} p_{X_{kj}}^{1-\epsilon_j} dk}, \quad i \in [0, N_j], \quad j = 1, ..., J. \tag{7}$$

Each layer-two firm makes zero profits in competitive equilibrium. Therefore, $\sum_{j=1}^{J} \Pi_{G_j} = 0$.

2.4 Production, transportation, innovation

In this section we characterize intermediate firms and the technologies that they use to make things and to ship things. Understanding the tight interdependence between making things and shipping things is the main objective of the paper. We start with a simple description of the relationship that does not model explicitly the existence of different modes of transportation in order to focus on the model's general equilibrium structure and the dynamics of innovation and market structure that it produces. In section 8, we develop an extension that allows for different modes of transportation and characterize the industry-specific combination of modes that emerges in market equilibrium.

2.4.1 Technologies

Firm i in industry j operates four technologies: the manufacturing technology to make the good, the transportation technology to ship the good, the manufacturing innovation technology to figure out ways to improve the productivity of labor in making the good, and the transportation innovation technology to figure out ways to improve the productivity of labor in shipping the good.

2.4.2 Manufacturing

The firm produces its differentiated intermediate good with the technology

$$X_{ij} = Z_{ij}^{\theta_j} \left(L_{X_{ij}} - \phi_j \right) \quad 0 < \theta_j < 1, \ \phi_j > 0, \tag{8}$$

where X_{ij} is output, $Z_{ij}^{\theta_j}$ is labor productivity as a function of the stock of firm-specific manufacturing knowledge, $L_{X_{ij}}$ is total manufacturing labor and ϕ_j is overhead labor.

2.4.3 Transportation

The firm must ship its production to the customers. We restrict attention to a transportation technology with labor as the single input and assume in-house distribution; we can incorporate in the model a separate competitive market for distribution services without changing

the results. We posit the following transportation technology

$$L_{D_{ij}} = \left[\underbrace{\tau_j Z_{ji}^{-\theta_j}}_{\text{iceberg}} + \underbrace{(1+\tau_j)\,\eta_j S_j^{-\sigma_j}}_{\text{per-unit}} \right] X_{ij}, \quad \eta_j \geq 0,\ \sigma_j \geq 0 \tag{9}$$

where $L_{D_{ij}}$is the amount of labor required to ship X_{ij} units of good ij to the customer, η_j and σ_j are parameters and S_j is the stock of firm-specific transportation knowledge. Accordingly, the cost of shipping one unit to the customer consists of an iceberg component and a per-unit component. The iceberg component consists of the cost of producing the additional amount of the good needed to make up for the anticipated melting during shipping. The per-unit component consists of the cost of physically moving the good regardless of whether it melts on the way. Overall, the cost of shipping a good is the sum of the cost of producing the anticipated melted amount and the per-unit cost of delivering the (unmelted) good and the anticipated melted amount.

We allow for both the iceberg and the per-unit components to study their different implications and to connect our paper to the existing literature in which iceberg costs play a major role. To understand better this structure, it is useful to contrast the cases $\eta_j = 0$, $\tau_j > 0$ and $\eta_j > 0$, $\tau_j = 0$. When the distribution technology consists only of the iceberg cost, an increase in manufacturing productivity reduces the labor needed to produce the extra quantity that melts in transit, thereby reducing the cost of shipping. When the distribution technology consists only of the per-unit cost, in contrast, a reduction in production cost does not entail a reduction in distribution cost. To highlight the role of transportation even more, we study the cases $\sigma_j = 0$ and $\sigma_j > 0$ separately. With $\sigma_j = 0$, the per-unit cost is completely exogenous and chokes off endogenous growth even though the manufacturing innovation technology has the technical properties that would allow it. With $\sigma_j > 0$, the per-unit cost is endogenous and endogenous growth occurs because innovation in transportation *complements* innovation in manufacturing. To the best of our knowledge, this form of innovation-based dynamic complementarity is new to the literature.

The key to this formulation is that making things and shipping things are separate activities, each one with its own technology, but are also complementary. The iceberg component allows for a "spillover" from innovation in manufacturing to shipping, but such spillover is not the whole story. Our per-unit component entails an incompressible cost of shipping that has qualitatively important effects for innovation. "Incompressible" here means that the firm's effort in improving labor productivity in making things cannot drive to zero the

price of the good for the customer — as in traditional endogenous technological change models —because the price contains the per unit cost of shipping things, which does not fall proportionally to cost of making things. In this sense, our model articulates the idea that endogenous growth is a property of the integrated "manufacturing + transportation" system and cannot be understood by looking at one component in isolation from the other.

2.4.4 Manufacturing innovation

The firm accumulates manufacturing knowledge according to

$$\dot{Z}_{ij} = \alpha_j Z_j L_{Z_{ij}}, \quad \alpha_j > 0 \tag{10}$$

where $\dot{Z}_{ij}$ is the flow of new firm-specific manufacturing knowledge generated by $L_{Z_{ij}}$ units of R&D labor with productivity $\alpha_j Z_j$, where α_j is an exogenous parameter and Z_j is the stock of knowledge available to and useful to the firm in R&D operations. To obtain a tractable structure, we assume that this spillover consists of the average knowledge developed by all firms *in the industry*. In particular, we assume

$$Z_j = \int_0^{N_j} \frac{Z_{sj}}{N_j} ds,$$

where N_j is the mass of firms in industry j.

2.4.5 Transportation innovation

To identify as cleanly as possible the interdependence of manufacturing and shipping as a market equilibrium phenomenon, we want the transportation innovation technology to be as similar as possible to the manufacturing innovation technology. Therefore, the firm accumulates transportation-specific knowledge according to

$$\dot{S}_{ij} = \varsigma_j S_j L_{S_{ij}}, \quad \varsigma_j > 0 \tag{11}$$

where $\dot{S}_{ij}$ is the flow of new firm-specific transportation-specific knowledge generated by $L_{S_{ij}}$ units of R&D labor with productivity $\varsigma_j S_j$, where ς_j is an exogenous parameter and S_j is the stock of general-purpose transportation knowledge available to and useful to the firm in this activity. As for manufacturing, we assume that this spillover consists of the average

knowledge developed by all firms *in the industry* and assume

$$S_j = \int_0^{N_j} \frac{S_{kj}}{N_j} dk,$$

where N_j is the mass of firms in industry j.

3 Firm behavior and entry

In this section we characterize the decisions of intermediate firms and entrants.

3.1 Firm decisions

At time t, firm i maximizes the present discounted value of dividends,

$$V_{ij}(t) = \int_t^{\infty} e^{-\int_t^s [r(v)+\delta_j]v} \Pi_{ij}(s) ds, \quad \delta_j > 0 \tag{12}$$

where the firm's dividend flow is

$$\Pi_{ij} = \left[p_{X_{ij}} - w(1+\tau_j) \left(Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j} \right) \right] X_{ij} - w\phi_j - wL_{Z_{ij}} - wL_{S_{ij}} \tag{13}$$

and δ_j is a death shock.[1] With our assumption of a continuum of intermediate goods, each intermediate firm is atomistic. Therefore, the firm maximizes (12) subject to the demand curve (7), the innovation technology (10), the non-negativity constraints on R&D labor, $L_{Z_{ij}} \geq 0$ and $L_{S_{ij}} \geq 0$, taking as given the initial stock of manufacturing knowledge, $Z_{ij}(t) > 0$, and the initial stock of transportation knowledge, $S_{ij}(t) > 0$. The firm also takes as given the paths of spillovers, Z_j and S_j, and of the mass of firms, N. The solution to the firm's problem yields the maximized value of the firm.

To characterize the firm's behavior, we write the Current Value Hamiltonian

$$\begin{aligned} CVH_i &= \left[p_{X_{ij}} - wZ_i^{-\theta} - w\tau_j Z_{ij}^{-\theta_j} - w(1+\tau_j)\eta_j S_{ij}^{-\sigma_j} \right] X_{ij} - w\phi_j \\ &\quad - wL_{Z_{ij}} - wL_{S_{ij}} + v_{Z_{ij}} \alpha_j Z_j L_{Z_{ij}} + v_{S_{ij}} \varsigma_j S_j L_{S_{ij}}, \end{aligned}$$

where the costate variable, $v_{Z_{ij}}$, is the shadow value of the marginal unit of manufactur-

[1] The exit shock guarantees that the model has symmetric dynamics in a neighborhood of the steady state when population growth is zero. It also provides additional empirical discipline in quantitative exercises; see, e.g., Ferraro, Ghazi and Peretto (2020) and (2023).

ing knowledge and the costate variable, $v_{S_{ij}}$, is the shadow value of the marginal unit of transportation-specific knowledge. The firm's manufacturing knowledge stock, Z_{ij}, is the state variable that evolves according to equation (10) and the firm's transportation knowledge, S_{ij}, is the state variable that evolves according to equation (11). The product's price, $p_{X_{ij}}$, and R&D labor, $L_{Z_{ij}}$ and $L_{S_{ij}}$, are the control variables. Given the properties of our primitives, the first-order conditions of this problem are sufficient for maximization and deliver the following relations.

The firm's value-maximizing pricing decision is

$$p_{X_{ij}} = \frac{\epsilon_j}{\epsilon_j - 1} w \left(1 + \tau_j\right) \left(Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}\right). \tag{14}$$

Inverting the production technology (8) and using the price strategy (14) yields again the firm's conditional demand (15) for manufacturing and transportation labor

$$L_{X_{ij}} + L_{D_{ij}} = \frac{\epsilon_j - 1}{\epsilon_j} \frac{p_{X_{ij}} X_{ij}}{w} + \phi_j. \tag{15}$$

We can break it down in:

$$L_{X_{ij}} = \frac{\epsilon_j - 1}{\epsilon_j} \frac{Z_{ij}^{-\theta_j}}{\left(1 + \tau_j\right) \left(Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}\right)} \frac{p_{X_{ij}} X_{ij}}{w} + \phi_j;$$

$$L_{D_{ij}} = \frac{\epsilon_j - 1}{\epsilon_j} \frac{\tau_j Z_{ji}^{-\theta_j} + \left(1 + \tau_j\right) \eta_j S_{ij}^{-\sigma_j}}{\left(1 + \tau_j\right) \left(Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}\right)} \frac{p_{X_{ij}} X_{ij}}{w}.$$

The firm also employs labor in two R&D activities, $L_{Z_{ij}}$ and $L_{S_{ij}}$, to which we now turn.

Because the Hamiltonian is linear in $L_{Z_{ij}}$ and in $L_{S_{ij}}$, the investment decisions have a bang-bang structure. For $w < v_{Z_{ij}} \alpha_j K_j$, the value of the marginal unit of manufacturing knowledge is higher than its cost. Then, every agent in the economy wants to quit alternative employment to work in firm i's manufacturing-specific R&D. This allocation of labor cannot be an equilibrium. For $w > v_{Z_{ij}} \alpha_j K_j$, the value of the marginal unit of manufacturing knowledge is lower than its cost of production. The firm then does not invest in manufacturing knowledge accumulation and we have the corner solution $L_{Z_{ij}} = 0$. For $w = v_{Z_{ij}} \alpha_j K_j$, we have the interior solution given by the equality between marginal revenue and marginal cost of manufacturing knowledge accumulation. Similarly, for $w < v_{S_j} \varsigma_j S_j$, the value of the marginal unit of transportation knowledge is higher than its cost. Then, every agent in the

economy wants to quit alternative employment to work in firm i's transportation-specific R&D. This allocation of labor cannot be an equilibrium. For $w > v_{S_{ij}} \varsigma_j S_j$, the value of the marginal unit of transportation knowledge is lower than its cost of production. The firm then does not invest in transportation knowledge accumulation and we have the corner solution $L_{S_{ij}} = 0$. For $w = v_{S_{ij}} \varsigma_j S_j$, we have the interior solution given by the equality between marginal revenue and marginal cost of transportation knowledge accumulation.

Finally, value maximization yields the following pair of differential equation for the costate variables:

$$r + \delta_j = \theta_j w \left(1 + \tau_j\right) Z_i^{-\theta_j - 1} \frac{X_{ij}}{v_{Z_{ij}}} + \frac{\dot{v}_{Z_{ij}}}{v_{Z_{ij}}}; \tag{16}$$

$$r + \delta_j = \sigma_j w \left(1 + \tau_j\right) \eta_j S_{ij}^{-\sigma_j - 1} \frac{X_{ij}}{v_{S_{ij}}} + \frac{\dot{v}_{S_{ij}}}{v_{S_{ij}}}. \tag{17}$$

Associated to these equations are the terminal conditions:

$$\lim_{s \to \infty} e^{-\int_t^s [r(v) + \delta_j] dv} v_{S_{ij}}(s) S_{ij}(s) = 0;$$

$$\lim_{s \to \infty} e^{-\int_t^s [r(v) + \delta_j] dv} v_{Z_{ij}}(s) Z_{ij}(s) = 0.$$

Equation (16) defines the rate of return to manufacturing knowledge accumulation as the ratio between revenues from the manufacturing knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of manufacturing knowledge. The revenue from the marginal unit of manufacturing knowledge is the cost reduction it yields times the scale of production to which it applies. Similarly, equation (17) defines the rate of return transportation knowledge accumulation as the ratio between revenues from the transportation knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of transportation knowledge. The revenue from the marginal unit of transportation knowledge is the cost reduction it yields times the scale of production to which it applies. Using the price strategy (14) and solving forward these differential equations, we obtain the value of each unit of output of the two R&D units as the present discounted value of the revenue from the marginal cost reduction:

$$v_{Z_{ij}}(t) = \int_t^\infty e^{-\int_t^s [r(v) + \delta_j] dv} \frac{\theta_j \left(\epsilon_j - 1\right)}{\epsilon_j} \frac{p_{X_{ij}}(s) X_{ij}(s)}{Z_{ij}(s)} ds;$$

$$v_{S_{ij}}(t) = \int_t^\infty e^{-\int_t^s [r(v) + \delta_j] dv} \frac{\sigma_j \left(\epsilon_j - 1\right)}{\epsilon_j} \frac{p_{X_{ij}}(s) X_{ij}(s)}{Z_{ij}(s)} ds.$$

Accordingly, the total value produced by the manufacturing innovation unit is $v_{Z_{ij}} Z_{ij}$ and the total value produced by the transportation innovation unit is $v_{S_{ij}} S_{ij}$.

Peretto (2025) explains that under the restriction $1 > \theta_j (\epsilon_j - 1)$ the industry equilibrium is symmetric because (1) firms make identical price and R&D decisions, (2) we start them with identical values of initial firm-specific manufacturing and of transportation knowledge and, as we discuss below, (3) we assume that new firms enter at the average level of firm-specific manufacturing and transportation knowledge. Consequently, we can drop the subscript i from our firm-specific variables with the understanding that they denote the average across firms. For example, L_{X_j} denotes the average employment of labor in manufacturing across the N_j firms, L_{Z_j} denotes the average employment of labor in manufacturing-specific R&D across the N_j firms, and so on. In particular, since $Y_j = \int_0^{N_j} p_{X_{ij}} X_{ij} di$ we write $p_{X_j} X_j = Y_j / N_j$, where X_j is the average production across the N_j firms. Therefore, equation (15) yields

$$L_{X_j} + L_{D_j} = \frac{\epsilon_j - 1}{\epsilon_j} \frac{Y_j}{w N_j} + \phi_j, \tag{18}$$

where Y_j / N_j is average sales across the N_j firms.

We now take logs and time-derivative of the indifference conditions $w = v_{Z_{ij}} \alpha_j Z_j$ and $w = v_{S_{ij}} \varsigma_j S_j$, and use the demand curve (7), the price equation (14), the firm innovation technologies (10) and (11), and symmetry, to reduce equations (16) and (17) to:

$$r = r_{Z_j} \equiv \alpha_j \left[\frac{Y_j}{w N_j} \frac{\theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{Z_{ij}^{-\theta_j}}{Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}} - L_{Z_j} \right] + \frac{\dot{w}}{w} - \delta_j; \tag{19}$$

$$r = r_{S_j} \equiv \varsigma_j \left[\frac{Y_j}{w N_j} \frac{\sigma_j (\epsilon_j - 1)}{\epsilon_j} \frac{\eta_j S_{ij}^{-\sigma_j}}{Z_{ij}^{-\theta_j} + \eta_j S_{ij}^{-\sigma_j}} - L_{S_j} \right] + \frac{\dot{w}}{w} - \delta_j. \tag{20}$$

These expressions capture a core property of our theory of endogenous innovation in manufacturing and transportation built on the existence of increasing returns to scale internal to the firm: the rate of return to firm-specific innovation is a linear function of the size of the firm, given by the size of the market in which the firm operates times the market share that the firm secures. The market share channel captures another core property of our theory of endogenous innovation, which is built on the idea that firms compete in markets whose structure in the Industrial Organization sense — number of firms, average firm size, concentration and so on — is endogenous because of entry and exit.

3.2 Entry

An entrepreneur can create a new firm that starts with productivity equal to the industry average by using $\beta_j Y_j / w N_j$ units of labor.[2] The sunk entry cost, therefore, is industry-specific because it depends on the average size of the firms in the industry. In particular, we have $wL_{N_{ji}} = \beta_j Y_j / N_j$. Once in the market, the entrant firm solves a problem identical to the one outlined above for the incumbent firm. For each industry j, there exists an equilibrium with no entry when $V_j < \beta_j Y_j / N_j$ and there exists an equilibrium with entry when $V_j = \beta_j Y_j / N_j$. The associated rate of return to entry is

$$r = r_{N_j} \equiv \frac{N_j}{\beta_j Y_j} \left(\frac{Y_j}{\epsilon_j N_j} - w\phi_j - wL_{Z_j} - wL_{S_j} \right) + \frac{\dot{Y}_j}{Y_j} - \frac{\dot{N}_j}{N_j} - \delta_j. \tag{21}$$

The possibility of an equilibrium with no entry applies to each industry j independently of what happens in the other industries. Thus, this economy admits the coexistence of industries in no firm entry equilibrium with industries in free-entry equilibrium.

4 General equilibrium

We choose labor as the numeraire and set $w \equiv 1$. Assets market equilibrium requires:

- equalization of *all* rates of return, that is, $r = r_A = r_{Z_j} = r_{N_j}$ for all $j = 1, ..., J$;
- the value of the household's portfolio equals the value of securities issued by firms, $A = \sum_{j=1}^{J} N_j V_j$.

The standard no-arbitrage argument says that if any of the returns is below the reservation rate of return of savers, the associated activity is return dominated and shuts down. Consequently, the general equilibrium has two possible configurations.

1. Free entry holds in *all* industries.
2. Free entry fails in *at least one* industry.

[2] See Peretto and Connolly (2007) and Peretto (2025) for an interpretation of this assumption and alternative formulations of the sunk entry cost that leave the fundamental equilibrium properties of the theory unchanged. Also, in this class of general equilibrium models, an entrepreneur is any member of the representative household who chooses to assume the role by issuing equity in the financial market to raise the resources needed to hire labor to develop a new good and set up the firm that will bring it to the market.

We concentrate on configuration 1 and leave configuration 2 to future work in which we shall study structural slumps and a novel phenomenon specific to the multi-industry setting of this class of models, that is, a strong form of propagation of local events to the whole economy that we label *contagion*; see Peretto (2025) for the definition and analysis of these elements in models with no transportation.

When the free-entry condition holds in all industries, we have

$$A = \sum_{j=1}^{J} N_j V_j = \sum_{j=1}^{J} N_j \beta_j \frac{Y_j}{N_j} = \sum_{j=1}^{J} \beta_j \varphi_j Y = \beta Y,$$

where $\beta \equiv \sum_{j=1}^{J} \beta_j \varphi_j$. This is a nice aggregation result: the economy features a constant wealth/expenditure ratio despite the heterogeneity across industries. Substituting this result in the household budget constraint (3) and using the saving rule (4) yields

$$\frac{\dot{Y}}{Y} = \rho + \frac{\dot{Y}}{Y} - \lambda + \frac{wL - Y}{\beta Y}.$$

As expenditure growth cancels out, this equation yields

$$Y = \left[1 - (\rho - \lambda)\,\beta\right]^{-1} \cdot wL, \tag{22}$$

where we assume $1 > (\rho - \lambda)\,\beta$ to guarantee a positive solution. This result says that consumption expenditure is proportional to wage income. Recalling the normalization $w \equiv 1$, expenditure per capita becomes

$$y = y^* \equiv \left[1 - (\rho - \lambda)\,\beta\right]^{-1}. \tag{23}$$

Since expenditure per capita is constant, the saving rule (4) yields that the interest rate is $r = \rho$. Note that establishing this result does not require taking a stand on whether incumbent firms do or do not do in-house R&D.

Given this aggregation results, a self-contained system governs the dynamics of each industry when free entry holds in all industries. This property makes the model analytically tractable.

5 No innovation in transportation

In this section we study the model with no innovation in transportation. In particular, we set $\sigma_j = 0$ for all j so that every industry in the economy is at the corner solution $L_{S_j} = 0$ discussed in the previous section. The benefit of discussing this special case is that it reveals a fundamental property of the interdependence between making things and hipping things that the model captures. Specifically, the existence of the per-unit cost of shipping things kills endogenous growth in a model that seemingly posses all the elements required to deliver it. In other words, the introduction of transportation in an otherwise conventional model of innovation-led endogenous growth alters drastically the *qualitative* properties of the growth process that the model describes.

5.1 Industry-specific dynamics

The market size of industry j is $Y_j = \varphi_j Y^*$, where $Y^* = Ly^*$. The associated employment share is $L_j/L = \varphi_j$. We define industry-specific firm size, $x_j \equiv Y_j/N_j$, and solve equation (19) for

$$z_j \equiv \frac{\dot{Z}_j}{Z_j} = \alpha_j L_{Z_j} = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{x_j}{1 + q_j} - \rho - \delta_j, \tag{24}$$

where we define the composite variable

$$q_j \equiv \eta_j Z_j^{\theta_j} S_{ij}^{-\sigma_j} = \frac{Z_j^{\theta_j}}{S_{ij}^{\sigma_j}/\eta_j} = \frac{\text{labor productivity in manufacturing}}{\text{labor productivity in shipping}}, \tag{25}$$

which in this section reduces to $q_j = \eta_j Z_j^{\theta_j}$.

Equation (24) says that firms in the industry innovate if

$$x_j > x_{Z_j}(q_j) \equiv \frac{\epsilon_j (\rho + \delta_j)}{\alpha_j \theta_j (\epsilon_j - 1)} (1 + q_j).$$

We then substitute this result in equation (21) to write the dynamical system:

$$\frac{\dot{q}_j}{q_j} = \theta_j z_j = \theta_j \left[\frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{x_j}{1 + q_j} - \rho - \delta_j \right]; \tag{26}$$

$$\frac{\dot{x}_j}{x_j} = \rho + \delta_j - \frac{1}{\beta x_j} \left[\frac{x_j}{\epsilon_j} - \phi_j - \frac{\theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{x_j}{1 + q_j} + \frac{\rho + \delta_j}{\alpha_j} \right]. \tag{27}$$

The second equation applies if agents are willing to finance entry, that is, if

$$x_j > x_{N_j}(q_j) \equiv \frac{\phi_j - \frac{\rho+\delta_j}{\alpha_j}}{\frac{1}{\epsilon_j} - \frac{\theta_j(\epsilon_j-1)}{\epsilon_j(1+q_j)} - (\rho-\lambda)\beta}.$$

The associated rate of entry is

$$n_j \equiv \frac{\dot{N}_j}{N_j} = \frac{1}{\beta x_j}\left[\frac{x_j}{\epsilon_j} - \phi_j - \frac{\theta_j(\epsilon_j-1)}{\epsilon_j}\frac{x_j}{1+q_j} + \frac{\rho+\delta_j}{\alpha_j}\right] - \lambda - \rho - \delta_j.$$

Otherwise, we have:

$$\frac{\dot{x}_j}{x_j} = \rho + \delta_j - \frac{1}{\beta x_j}\left(\frac{x_j}{\epsilon_j} - \phi_j\right);$$

$$n_j = \frac{1}{\beta x_j}\left(\frac{x_j}{\epsilon_j} - \phi_j\right)\lambda - \rho - \delta_j$$

This system (26)-(24) does not have a steady state with endogenous growth if $\eta_j > 0$ because as long as firms invest and $z_j > 0$, the variable q_j grows and drags to zero the return to manufacturing innovation. The reason is that the cost of shipping the good to the customers does not shrink to zero in lockstep with the cost of making the good. The price of the good, therefore, cannot go to zero as in standard models of endogenous growth in which the economy moves all the way to infinite sales along an isoelastic demand curve. A steady state with endogenous growth exists only for $\eta_j = 0$, i.e., there is no per-unit transportation cost. Under this assumption, equation (26) degenerates to $\dot{q}_j = 0$ for any value of z_j and we are left with a traditional model of endogenous growth.

To characterize the dynamics, we look at the locus

$$\frac{\dot{x}_j}{x_j} \geq 0: \quad x_j \leq \frac{\left(\phi_j - \frac{\rho+\delta_j}{\alpha_j}\right)(1+q_j)}{\left[\frac{1}{\epsilon_j} - \beta(\rho+\delta_j)\right](1+q_j) - \frac{\theta_j(\epsilon_j-1)}{\epsilon_j}}$$

and construct the phase diagram in Fig 1. We have a region of path dependence and a region of trajectories that for given initial condition converge to the pair $(\bar{q}_j, \bar{x}_j)$ that solves:

$$x_j = \frac{(\rho+\delta_j)\epsilon_j}{\alpha_j\theta_j(\epsilon_j-1)}(1+q_j);$$

$$x_j = \frac{\phi_j - \frac{\rho+\delta_j}{\alpha_j}}{\frac{1}{\epsilon_j} - \frac{\theta_j(\epsilon_j-1)}{\epsilon_j(1+q_j)} - \beta(\rho+\delta_j)}.$$

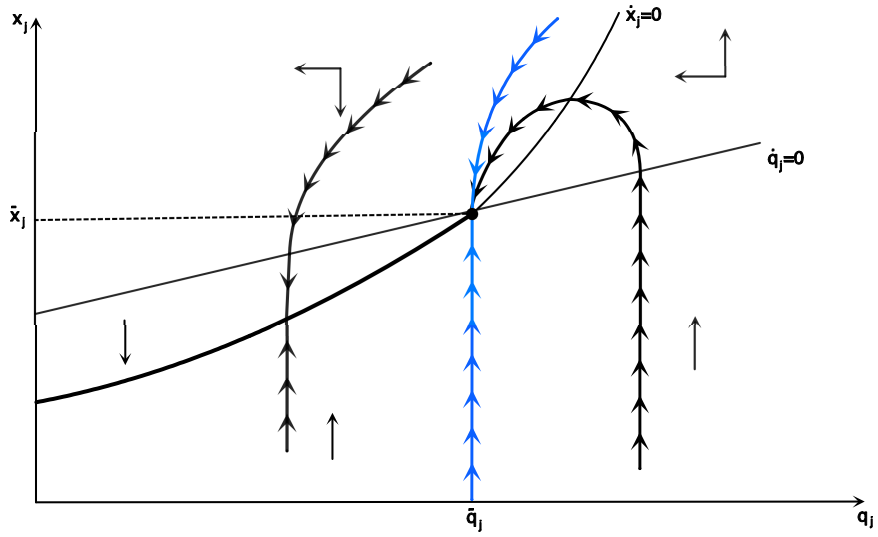


Figure 1: Equilibrium dynamics with no innovation in transportation.

We thus obtain:

$$\bar{x}_j = \frac{\epsilon_j \phi_j}{1 - \epsilon_j \beta \left(\rho + \delta_j\right)};$$

$$\bar{q}_j = \frac{\phi_j}{1 - \epsilon_j \beta \left(\rho + \delta_j\right)} \frac{\alpha_j \theta_j \left(\epsilon_j - 1\right)}{\rho + \delta_j} - 1.$$

The solution $\bar{x}_j$ is the steady state with no firm innovation of the model with no transportation cost (e.g., Peretto 2025).

The solution $\bar{q}_j$ says that the model with per-unit transportation cost adds to the theory two novel properties. First, the per-unit transportation cost causes stagnation because cost reduction is no longer an economically self-sustaining process. Second, the model exhibits an endogenous cost structure whereby in steady state the unit cost of manufacturing converges to

$$\bar{Z}_j^{-\theta_j} = \frac{\eta_j}{\bar{q}_j}.$$

Then, the iceberg transportation cost is

$$\tau_j \bar{Z}_j^{-\theta_j} = \tau_j \frac{\eta_j}{\bar{q}_j}$$

and accounts for the fraction

$$\frac{\tau_j \frac{\eta_j}{\bar{q}_j}}{\tau_j \frac{\eta_j}{\bar{q}_j} + \left(1 + \tau_j\right) \eta_j} = \frac{\tau_j}{\tau_j + \left(1 + \tau_j\right) \bar{q}_j}$$

of the unit expenditure on transportation. In turn, transportation accounts for the fraction

$$\frac{\tau_j \frac{\eta_j}{\bar{q}_j} + (1+\tau_j)\,\eta_j}{\tau_j \frac{\eta_j}{\bar{q}_j} + (1+\tau_j)\,\eta_j + \frac{\eta_j}{\bar{q}_j}} = \frac{\tau_j + (1+\tau_j)\,\bar{q}_j}{(1+\tau_j)\,(1+\bar{q}_j)}$$

of the total unit cost of producing and delivering the good to the customer.

The path-dependent trajectories obey the equation

$$\frac{\dot{x}_j}{x_j} = \rho + \delta_j - \frac{1}{\beta x_j}\left[\frac{x_j}{\epsilon_j} - \phi_j - \frac{\theta_j\,(\epsilon_j - 1)}{\epsilon_j}\frac{x_j}{1+q_j\,(0)} + \frac{\rho+\delta_j}{\alpha_j}\right],$$

where $q_j\,(0) = \eta_j\,[Z_j\,(0)]^{\theta_j}$. These dynamics yield that firm size converges to

$$x_j\,(q_j\,(0)) = \frac{\left(\phi_j - \frac{\rho+\delta_j}{\alpha_j}\right)[1+q_j\,(0)]}{\left[\frac{1}{\epsilon_j} - \beta\,(\rho+\delta_j)\right][1+q_j\,(0)] - \frac{\theta_j(\epsilon_j-1)}{\epsilon_j}}.$$

The other properties of these trajectories are similar to those discussed above for the trajectories that converge to $(\bar{q}_j, \bar{x}_j)$.

5.2 Industry-specific TFP

In industry j, the price p_{G_j} is equal to the price index of the intermediates it uses. Hence,

$$\begin{aligned} p_{G_j} &= N_j^{-\chi_j}\left(\frac{1}{N_j}\int_0^{N_j} p_{X_{ij}}^{1-\epsilon_j} di\right)^{\frac{1}{1-\epsilon_j}} \\ &= \frac{\epsilon_j}{\epsilon_j - 1}\,(1+\tau_j)\,N_j^{-\chi_j} Z_j^{-\theta_j}\left(1+\eta_j Z_j^{\theta_j}\right). \end{aligned}$$

Accordingly, the industry's output is

$$G_j = \frac{Y_j}{p_{G_j}} = \frac{\varphi_j Y}{\frac{\epsilon_j}{\epsilon_j - 1}\,(1+\tau_j)}\frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+\eta_j Z_j^{\theta_j}}.$$

Noting that industry employment is $L_j = \varphi_j L$, industry output per worker is

$$\frac{G_j}{L_j} = \frac{\varphi_j Y}{\varphi_j L \frac{\epsilon_j}{\epsilon_j - 1}\,(1+\tau_j)}\frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+\eta_j Z_j^{\theta_j}} = \frac{y}{\frac{\epsilon_j}{\epsilon_j - 1}\,(1+\tau_j)}\frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+\eta_j Z_j^{\theta_j}},$$

We define industry TFP as

$$T_j \equiv \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+\eta_j Z_j^{\theta_j}}.$$

Accordingly, the industry growth rate of output per worker (industry growth for brevity) is

$$g_j = \frac{\dot{y}}{y} + \frac{\dot{T}_j}{T_j} = \chi_j n_j + \theta_j z_j - \frac{\eta_j \theta_j Z_j^{\theta_j}}{1+\eta_j Z_j^{\theta_j}} z_j = \chi_j n_j + \frac{\theta_j z_j}{1+\eta_j Z_j^{\theta_j}}.$$

This equation shows that as long as there is a per-unit transportation cost, endogenous growth driven by cost reduction is not possible.

In steady state, we have

$$g_j^* = \chi_j \lambda,$$

and the model determines only the level of TFP. In logs, we have

$$\log T_j(t) = \theta_j \log Z_j(0) + \chi_j \log N_j(0) + \log\left(\frac{\bar{Z}_j^{\theta_j}}{1+\eta_j \bar{Z}_j^{\theta_j}}\right) + \lambda t.$$

The key result here is that the only driver of TFP growth is the growth of the population via the weak scale effect. To highlight the role of the transportation parameters, we write the solution in the form

$$\log T_j(t) = \theta_j \log Z_j(0) + \chi_j \log N_j(0) + \log\left(\frac{\bar{q}_j}{1+\bar{q}_j}\right) - \log \eta_j + \lambda t.$$

Recalling that $\bar{q}_j$ does not depend on transportation parameters, this solution says that the per-unit transportation cost has a one-to-one negative effect on the steady-state path of the industry's TFP. Notice that the transportation parameter τ_j is absent from this expression. The reason is that the iceberg component of the transportation cost is proportional to the unit production cost and is thus fully absorbed by the firm's gross profit x_j/ϵ_j via the constant markup. In other words, that component of the transportation cost does not affect firm size and thus has no effect on the dynamics of TFP.

5.3 Aggregation and welfare

The price of the consumption good is the price index of the components that it embodies,

$$p_G = \prod_{j=1}^{J} \left(\frac{p_{G_j}}{\varphi_j} \right)^{\varphi_j} = \xi \times \prod_{j=1}^{J} T_j^{-\varphi_j}, \tag{28}$$

where we define the aggregate index of static cost drivers,

$$\xi \equiv \prod_{j=1}^{J} \left(\frac{\frac{\epsilon_j}{\epsilon_j - 1} (1 + \tau_j)}{\varphi_j} \right)^{\varphi_j}.$$

We then use the price index (28) to write GDP per capita

$$\frac{G}{L} = \frac{y}{p_G} = \frac{y}{\xi} \cdot \underbrace{\prod_{j=1}^{J} T_j^{\varphi_j}}_{\text{aggregate TFP} \equiv T}.$$

Accordingly, aggregate growth is

$$g = \frac{\dot{y}}{y} + \frac{\dot{T}_j}{T_j} = \frac{\dot{y}}{y} + \sum_{j=1}^{J} \varphi_j \frac{\dot{T}_j}{T_j} = \frac{\dot{y}}{y} + \sum_{j=1}^{J} \varphi_j \left(\chi_j n_j + \frac{\theta_j z_j}{1 + \eta_j Z_j^{\theta_j}} \right).$$

In steady state

$$g^* = \left(\frac{\dot{T}}{T} \right)^* = \sum_{j=1}^{J} \varphi_j^* \left(\frac{\dot{T}_j}{T_j} \right)^* = \sum_{j=1}^{J} \varphi_j \chi_j \lambda.$$

Thus, aggregate TFP growth is the weighted sum of industry growth, where the weights are the industry shares of GDP.

These results let us write individual utility as

$$\begin{aligned} u &= \log \left(\frac{y}{\xi} \right) + \sum_{j=1}^{J} \varphi_j \log T_j \\ &= \log y - \sum_{j=1}^{J} \varphi_j \log \left(\frac{\frac{\epsilon_j}{\epsilon_j - 1}}{\varphi_j} \right) - \sum_{j=1}^{J} \varphi_j \log (1 + \tau_j) + \sum_{j=1}^{J} \varphi_j \log T_j. \end{aligned}$$

The second line highlights the role of the transportation parameter τ_j. It has a one-for-one negative effect on flow utility because it adds to the cost of delivering to the customers

one unit of the differentiated good j. This is another nice aggregation result. The formula contains only objects that are in principle observable. For example, we can rewrite the first term as $\log(y/p_G) + \log(p_G/c)$, where the first term of this decomposition is the log of GDP per capita and the second term is the log of the fraction of the economy's price index accounted for by the static drivers of the cost of producing one unit of GDP. In the remainder of the paper we exploit this structure to study how the interdependence of manufacturing and transportation determines the welfare achieved by the representative household in market equilibrium.

6 Innovation in transportation

In this section we study the model with innovation in transportation. We set $\sigma_j > 0$ for all j so that the endogenous outcome $L_{S_j} > 0$ for all j can eventually occur in all industries.

6.1 Industry-specific dynamics

Recall the definition and interpretation of the variable $q_j = \eta_j S_j^{-\sigma_j} Z_j^{\theta_j}$. In general equilibrium, we obtain the following structure. Firm manufacturing innovation is

$$z_j = \alpha_j L_{Z_j} = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{1}{1+q_j} x_j - \rho - \delta_j, \tag{29}$$

where $L_{Z_j} > 0$ for

$$x_j > x_{Z_j}(q_j) \equiv \frac{\epsilon_j (\rho + \delta_j)}{\alpha_j \sigma_j (\epsilon_j - 1)} (1 + q_j).$$

Firm transportation innovation is

$$s_j \equiv \frac{\dot{S}_j}{S_j} = \varsigma_j L_{S_j} = \frac{\varsigma_j \sigma_j (\epsilon_j - 1)}{\epsilon_j} \frac{q_j}{1+q_j} x_j - \rho - \delta_j, \tag{30}$$

where $L_{S_j} > 0$ for

$$x_j > x_{S_j}(q_j) \equiv \frac{\epsilon_j (\rho + \delta_j)}{\varsigma_j \sigma_j (\epsilon_j - 1)} \frac{1+q_j}{q_j}.$$

We now use these equations to construct the dynamical system.

We take log and time-derivative of $q_j = \eta_j S_j^{-\sigma_j} Z_j^{\theta_j}$ to write

$$\begin{aligned}\frac{\dot{q}_j}{q_j} &= \theta_j z_j - \sigma_j s_j \\ &= \frac{\epsilon_j - 1}{\epsilon_j} \frac{\alpha_j \theta_j^2 - \varsigma_j \sigma_j^2 q_j}{1 + q_j} x_j - (\theta_j - \sigma_j)(\rho + \delta_j).\end{aligned}$$

We then use equations (29)-(30) and the expression for the rate of return to entry,

$$\rho = \frac{1}{\beta x_j}\left(\frac{x_j}{\epsilon_j} - \phi_j - L_{Z_j} - L_{S_j}\right) + \frac{\dot{x}_j}{x_j} - \delta_j,$$

to write

$$\frac{\dot{x}_j}{x_j} = \rho + \delta_j - \frac{1}{\beta}\left[\frac{1}{\epsilon_j} - \frac{\sigma_j q_j - \theta_j}{1 + q_j}\frac{\epsilon_j - 1}{\epsilon_j} - \frac{\phi_j - (\rho + \delta_j)\left(\frac{1}{\alpha_j} + \frac{1}{\varsigma_j}\right)}{x_j}\right]. \tag{31}$$

We thus have a reasonably simple industry-specific dynamical system in (q_j, x_j) space. We have two cases: $\theta_j > \sigma_j$ and $\theta_j < \sigma_j$. We do only the first since the second yields similar results.

The steady-state system is:

$$\dot{q}_j \geq 0: \quad x_j \geq x_j(q_j)_{\dot{q}_j = 0} \equiv \frac{(\theta_j - \sigma_j)(\rho + \delta_j)(1 + q_j)}{\frac{\epsilon_j - 1}{\epsilon_j}\left(\alpha_j \theta_j^2 - \varsigma_j \sigma_j^2 q_j\right)};$$

$$\dot{x}_j \geq 0: \quad x_j \leq x_j(q_j)_{\dot{x}_j = 0} \equiv \frac{\left[\phi_j - (\rho + \delta_j)\left(\frac{1}{\alpha_j} + \frac{1}{\varsigma_j}\right)\right](1 + q_j)}{\frac{1}{\epsilon_j} - \beta(\rho + \delta_j) + \frac{\epsilon_j - 1}{\epsilon_j}\theta_j + \left[\frac{1}{\epsilon_j} - \beta(\rho + \delta_j) - \frac{\epsilon_j - 1}{\epsilon_j}\sigma_j\right] q_j}.$$

The $\dot{q}_j = 0$ locus starts at

$$x_j(0)_{\dot{q}_j = 0} = \frac{(\theta_j - \sigma_j)(\rho + \delta_j)\epsilon_j}{(\epsilon_j - 1)\alpha_j \theta_j^2}$$

and is increasing in q_j with a vertical asymptote at

$$q_j = \frac{\alpha_j \theta_j^2}{\varsigma_j \sigma_j^2}.$$

The $\dot{x}_j = 0$ locus starts at

$$x_j\left(q_j\right)_{\dot{x}_j=0} = \frac{\phi_j - \left(\rho+\delta_j\right)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)}{\frac{1}{\epsilon_j} - \beta\left(\rho+\delta_j\right) + \frac{\epsilon_j - 1}{\epsilon_j}\theta_j}$$

and is increasing in q_j with a horizontal asymptote at

$$\lim_{q_j\to\infty} x_j\left(q_j\right)_{\dot{x}_j=0} = \frac{\phi_j - \left(\rho+\delta_j\right)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)}{\frac{1}{\epsilon_j} - \beta\left(\rho+\delta_j\right) + \frac{\epsilon_j - 1}{\epsilon_j}\sigma_j}.$$

Therefore, there is a unique steady state for

$$\frac{\phi_j - \left(\rho+\delta_j\right)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)}{\frac{1}{\epsilon_j} - \beta\left(\rho+\delta_j\right) + \frac{\epsilon_j - 1}{\epsilon_j}\theta_j} > \frac{\epsilon_j\left(\theta_j-\sigma_j\right)\left(\rho+\delta_j\right)}{\alpha_j(\epsilon_j-1)\theta_j^2}$$

and there are two steady states for

$$\frac{\phi_j - \left(\rho+\delta_j\right)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)}{\frac{1}{\epsilon_j} - \beta\left(\rho+\delta_j\right) + \frac{\epsilon_j - 1}{\epsilon_j}\theta_j} \leq \frac{\epsilon_j\left(\theta_j-\sigma_j\right)\left(\rho+\delta_j\right)}{\alpha_j(\epsilon_j-1)\theta_j^2}.$$

However, the smaller steady state is unstable (a saddle point). It follows that we can study the model with only one steady state without loss of generality. Figure 2 illustrates the dynamics.

The steady state $\left(q_j^*, x_j^*\right)$ is the solution of the pair of equations:

$$x_j = \frac{\left(\theta_j-\sigma_j\right)\left(\rho+\delta_j\right)\left(1+q_j\right)}{\frac{\epsilon_j-1}{\epsilon_j}\left(\alpha_j\theta_j^2 - \varsigma_j\sigma_j^2 q_j\right)};$$

$$x_j = \frac{\left[\phi_j - \left(\rho+\delta_j\right)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)\right]\left(1+q_j\right)}{\frac{1}{\epsilon_j} - \beta\left(\rho+\delta_j\right) + \frac{\epsilon_j-1}{\epsilon_j}\theta_j + \left[\frac{1}{\epsilon_j} - \beta\left(\rho+\delta_j\right) - \frac{\epsilon_j-1}{\epsilon_j}\sigma_j\right] q_j}.$$

Combining them, we obtain:

$$x_j^* = \frac{\left(\theta_j-\sigma_j\right)\left(\rho+\delta_j\right)\left(1+q_j^*\right)}{\frac{\epsilon_j-1}{\epsilon_j}\left(\alpha_j\theta_j^2 - \varsigma_j\sigma_j^2 q_j^*\right)}; \tag{32}$$

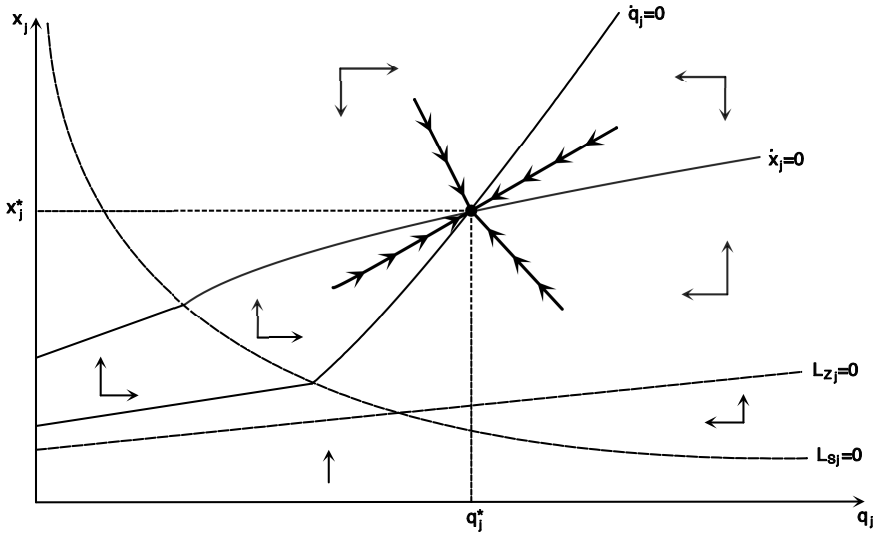


Figure 2: Equilibrium dynamics with innovation in transportation.

$$q_j^* = \frac{\frac{\left[\phi_j-(\rho+\delta_j)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)\right]^{\frac{\epsilon_j-1}{\epsilon_j}}\alpha_j\theta_j^2}{(\theta_j-\sigma_j)(\rho+\delta_j)} - \frac{1}{\epsilon_j} + \beta\left(\rho+\delta_j\right) - \frac{\epsilon_j-1}{\epsilon_j}\theta_j}{\frac{\left[\phi_j-(\rho+\delta_j)\left(\frac{1}{\alpha_j}+\frac{1}{\varsigma_j}\right)\right]^{\frac{\epsilon_j-1}{\epsilon_j}}\varsigma_j\sigma_j^2}{(\theta_j-\sigma_j)(\rho+\delta_j)} + \frac{1}{\epsilon_j} - \beta\left(\rho+\delta_j\right) - \frac{\epsilon_j-1}{\epsilon_j}\sigma_j}. \tag{33}$$

This is the industry's new labor productivity ratio relative to the case $\sigma_j = 0$ in which firms do not invest to improve the productivity of labor in transportation. It is worth stressing the absence of the parameter η_j from this solution. The reason is that changes in η_j are fully absorbed by the adjustment of the two stocks Z_j and S_j, which in equilibrium comove to produce the constant value q_j^* in equation (33) that does not depend on η_j. The insight follows form the definition of the variable q_j: it is the ratio between labor productivity in manufacturing to labor productivity in shipping; the no-arbitrage equilibrium of the market says that this ratio must be constant in steady state at a value determined by the parameters governing the rates of return to innovation in manufacturing and in transportation and the rate of return to entry.

6.2 Industry-specific cost and employment structure

The production cost, the iceberg transportation cost and the per-unit transportation cost go to zero asymptotically but remain in constant ratios. In particular, the iceberg transportation

cost accounts for the fraction

$$\frac{\tau_j Z_j^{-\theta_j}}{\tau_j Z_j^{-\theta_j} + (1+\tau_j)\,\eta_j S_j^{-\sigma_j}} = \frac{\tau_j}{\tau_j + (1+\tau_j)\,q_j^*}$$

of the unit expenditure on transportation. In turn, transportation accounts for the fraction

$$\frac{\tau_j Z_j^{-\theta_j} + (1+\tau_j)\,\eta_j S_j^{-\sigma_j}}{\tau_j Z_j^{-\theta_j} + (1+\tau_j)\,\eta_j S_j^{-\sigma_j} + Z_j^{-\theta_j}} = \frac{\tau_j + (1+\tau_j)\,q_j^*}{(1+\tau_j)\left(1+q_j^*\right)}$$

of the total unit cost of delivering the good to the customer. Note that in this case as well the steady-state value of the variable q does not depend on the transportation cost parameters τ_j and η_j. This is another manifestation of the model's scale invariance. However, q^* depends on the parameters of the transportation innovation technology, σ_j and ς_j, via the no-arbitrage principle at the heart of the model's market equilibrium.

We also have the employment structure:

$$L_{X_j}^* = \frac{\epsilon_j - 1}{\epsilon_j}\frac{x_j^*}{(1+\tau_j)\left(1+q_j^*\right)} + \phi_j;$$

$$L_{D_j}^* = \frac{\epsilon_j - 1}{\epsilon_j}\frac{\tau_j + (1+\tau_j)\,q_j^*}{w\,(1+\tau_j)\left(1+q_j^*\right)}x_j^*;$$

$$L_{Z_j}^* = \frac{\theta_j\,(\epsilon_j - 1)}{\epsilon_j}\frac{x_j^*}{1+q_j^*} - \frac{\rho+\delta_j}{\alpha_j};$$

$$L_{S_j}^* = \frac{\varsigma_j\sigma_j(\epsilon_j - 1)}{\epsilon_j}\frac{q_j^* x_j^*}{1+q_j^*} - \frac{\rho+\delta_j}{\varsigma_j};$$

$$L_{N_j}^* = (\lambda+\delta)\,\beta\varphi_j y^* L.$$

Note that while the pair $\left(q_j^*, x_j^*\right)$ does not depend on the iceberg component of the transportation cost, τ_j, the industry-specific allocation of labor to making things, $L_{X_j}^*$, and to shipping things, $L_{D_j}^*$, does depend on it simply because the iceberg cost contributes to the wedge between the cost of producing the good and the cost of delivering the good to the customer, where we think of delivering as making the good plus shipping the good. Another way to see this property is that while τ_j does not affect the industry's innovation and market structure dynamics, it obviously affects the price that the customer pays for the good. This implies that τ_j affects the utility that economic activity generates for the representative household. We now turn to this aspect of the model.

6.3 Industry-specific TFP growth

In industry j, the price p_{G_j} is equal to the price index of the intermediates it uses. Hence,

$$\begin{aligned} p_{G_j} &= N_j^{-\chi_j}\left(\frac{1}{N_j}\int_0^{N_j} p_{X_{ij}}^{1-\epsilon_j} di\right)^{\frac{1}{1-\epsilon_j}} \\ &= \frac{\epsilon_j}{\epsilon_j - 1}\left(1+\tau_j\right) N_j^{-\chi_j} Z_j^{-\theta_j}\left(1 + S_j^{-\sigma_j} Z_j^{\theta_j}\right) \\ &= \frac{\epsilon_j}{\epsilon_j - 1}\left(1+\tau_j\right) N_j^{-\chi_j} Z_j^{-\theta_j}\left(1+q_j\right). \end{aligned}$$

Accordingly, the industry's output is

$$G_j = \frac{Y_j}{p_{G_j}} = \frac{\varphi_j Y}{\frac{\epsilon_j}{\epsilon_j - 1}\left(1+\tau_j\right)} \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+q_j}.$$

Noting that industry employment is $L_j = \varphi_j L$, industry output per worker is

$$\frac{G_j}{L_j} = \frac{\varphi_j Y}{\varphi_j L \frac{\epsilon_j}{\epsilon_j - 1}\left(1+\tau_j\right)} \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+\eta_j Z_j^{\theta_j}} = \frac{y}{\frac{\epsilon_j}{\epsilon_j - 1}\left(1+\tau_j\right)} \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+\eta_j S_j^{-\sigma_j} Z_j^{\theta_j}},$$

where $Z_j^{\theta_j}$ is manufacturing productivity and $S_j^{\sigma_j}/\eta_j$ is transportation productivity. We define industry TFP as

$$T_j \equiv \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+q_j} = \frac{Z_j^{\theta_j} N_j^{\chi_j} S_j^{\sigma_j}}{S_j^{\sigma_j} + \eta_j Z_j^{\theta_j}},$$

Note that TFP is no longer a power function of manufacturing knowledge, Z_j, and is not a power function of transportation knowledge, S_j. The reason is that the per-unit transportation cost causes an interaction between production and distribution that results in industry TFP being concave in manufacturing productivity and concave in transportation productivity separately. Industry TFP, however, is homogeneous of degree one (i.e., linear) in manufacturing productivity and transportation productivity jointly. This property yields endogenous growth. In particular, the industry growth rate of output per worker (industry growth for brevity) is

$$g_j = \frac{\dot{y}}{y} + \frac{\dot{T}_j}{T_j} = \chi_j n_j + \theta_j z_j - \frac{q_j}{1+q_j}\frac{\dot{q}_j}{q_j}.$$

In steady state, $\dot{q}_j = \theta$ and therefore

$$g_j^* = \left(\frac{\dot{T}_j}{T_j}\right)^* = \chi_j \lambda + \theta_j z_j^*,$$

where:

$$z_j^* = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{x_j^*}{1 + q_j^*} - \rho - \delta_j;$$

$$s_j^* = \frac{\theta_j}{\sigma_j} z_j^*.$$

This equation says that in this model with heterogeneous industries the theory's scale invariance property holds industry by industry. Each industry-specific growth rate, g_j^*, is independent of the population scale factor, Λ, and of the industry employment share, φ_j, and depends on the population growth rate, λ, only through the Romer component, $\chi_j \lambda$, due to the weak scale effect. It is also independent of the transportation cost parameters τ_j and η_j.

6.4 Aggregate growth and welfare

Aggregation is the same as in the model with $\sigma_j = 0$ for all j, up to the dynamics of TFP. Thus, GDP per capita is again

$$\frac{G}{L} = \frac{y}{p_G} = \frac{y}{\xi} \cdot \underbrace{\prod_{j=1}^{J} T_j^{\varphi_j}}_{\text{aggregate TFP} \equiv T},$$

where ξ is the composite parameter defined in Section 5 and

$$T_j \equiv \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1 + q_j}.$$

Aggregate growth is

$$g = \frac{\dot{y}}{y} + \frac{\dot{T}}{T} = \frac{\dot{y}}{y} + \sum_{j=1}^{J} \varphi_j \left(\frac{\dot{T}_j}{T_j}\right)^* = \frac{\dot{y}}{y} + \sum_{j=1}^{J} \varphi_j \left(\chi_j n_j + \theta_j z_j - \frac{q_j}{1 + q_j} \frac{\dot{q}_j}{q_j}\right).$$

The difference is that here $\dot{q}_j \to 0$ for all j so that in steady state

$$g^* = \left(\frac{\dot{T}}{T}\right)^* = \sum_{j=1}^{J} \varphi_j \left(\frac{\dot{T}_j}{T_j}\right)^* = \sum_{j=1}^{J} \varphi_j \left(\chi_j \lambda + \theta_j z_j^*\right).$$

Thus, aggregate TFP growth is the weighted sum of industry growth, where the weights are the industry shares of GDP.

These results let us write individual utility as

$$u = \log\left(\frac{y}{\xi}\right) + \sum_{j=1}^{J} \varphi_j \log T_j.$$

This is another nice aggregation result. In the appendix we show that linearizing the dynamical system around the steady state we can compute

$$\frac{U^*}{\Lambda} = \frac{\log\left(\frac{y^*}{\xi}\right) + \varkappa}{\rho - \lambda} + \sum_{j=1}^{J} \frac{\varphi_j g_j^*}{(\rho - \lambda)^2} + \sum_{j=1}^{J} \frac{\varphi_j \Upsilon_j}{\rho - \lambda},$$

where:

$$\varkappa \equiv \chi_j \log \varphi_j + \chi_j \log y^* + \chi_j \log \Lambda;$$

$$\Upsilon_j \equiv \left[\theta_j - \frac{\theta_j + \chi_j}{\rho - \lambda + \nu_{x_j}}\right] \Delta_{x_j} - \left[\theta_j - \frac{\theta_j + \chi_j}{\rho - \lambda + \nu_{q_j}}\right] \Delta_{q_j};$$

$$\Delta_{x_j} \equiv x_j(0) - x_j^*;$$

$$\Delta_{q_j} \equiv q_j(0) - q_j^*.$$

This expression describes the welfare per capita produced by the aggregation of the J industry-specific transition paths, each one going from the initial point $(q_j(0), x_j(0))$ to the steady state $\left(q_j^*, x_j^*\right)$. In particular, in the expression for the composite factor Υ_j, the terms ν_{q_j} and ν_{x_j} are the eigenvalues of the linearized dynamical system attached to, respectively, q_j and x_j, and the terms Δ_{q_j} and Δ_{x_j} are the initial gaps of the variables q_j and x_j from their steady states; see the appendix for the derivation and the details.

7 Applications

In this section we study the dynamic effects of changes in parameters. We focus on the parameters of the transportation technology. We also consider the fixed operating cost because the recent literature on business dynamism and the productivity slowdown (see, e.g., xxx, xxx, and xxx) documents that it plays a central role in explaining the trends in the data. In our model, it plays the same central role with the added insight that it also works through its effects on the dynamic interdependence of manufacturing and shipping.

7.1 Steady-state comparative statics

We start from the innovation rates:

$$z_j = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j} \frac{1}{1+q_j} x_j - \rho - \delta_j;$$

$$s_j = \frac{\varsigma_j \sigma_j (\epsilon_j - 1)}{\epsilon_j} \frac{q_j}{1+q_j} x_j - \rho - \delta_j.$$

We use the fact that $\theta_j z_j = \sigma_j s_j$ and the equations above to write the $\dot{x}_j = 0$ equation

$$\rho = \frac{1}{\beta x_j} \left(\frac{x_j}{\epsilon_j} - \phi_j - L_{Z_j} - L_{S_j} \right) - \delta_j$$

in the form

$$\phi_j + \left(\frac{1}{\alpha_j} + \frac{\theta_j}{\varsigma_j \sigma_j} \right) z_j = \left[\frac{1}{\epsilon_j} - \beta (\rho + \delta_j) \right] x_j.$$

Next, we use the manufacturing innovation rate to write

$$\frac{z_j + \rho + \delta_j}{\frac{\alpha_j \theta_j (\epsilon_j - 1)}{\epsilon_j (1+q_j)}} = x_j.$$

We substitute this expression in the equation above to obtain the function

$$q_j = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{1 - \epsilon_j \beta (\rho + \delta_j)} \frac{\phi_j + \left(\frac{1}{\alpha_j} + \frac{\theta_j}{\varsigma_j \sigma_j} \right) z_j}{z_j + \rho + \delta_j} - 1.$$

This function starts at

$$q_j (0) = \frac{\alpha_j \theta_j (\epsilon_j - 1)}{1 - \epsilon_j \beta (\rho + \delta_j)} \frac{\phi_j}{\rho + \delta_j} - 1$$

and has the horizontal asymptote

$$\lim_{z_j \to \infty} q_j = \frac{\theta_j (\epsilon_j - 1)}{1 - \epsilon_j \beta (\rho + \delta_j)} \left(1 + \frac{\alpha_j \theta_j}{\varsigma_j \sigma_j}\right) - 1.$$

For ϕ_j large, the function is decreasing. Since this function comes from the entry process, we call it the free-entry locus.

Now we take the ratio of the innovation rates to write

$$q_j = \frac{\theta_j \alpha_j}{\varsigma_j \sigma_j} \frac{\frac{\theta_j}{\sigma_j} z_j + \rho + \delta_j}{z_j + \rho + \delta_j}.$$

This function starts at

$$q_j (0) = \frac{\alpha_j \theta_j}{\varsigma_j \sigma_j}$$

and has the horizontal asymptote

$$\lim_{\hat{Z}_j \to \infty} q_j = \frac{\alpha_j \theta_j^2}{\varsigma_j \sigma_j^2}.$$

For $\theta_j > \sigma_j$ the function is increasing. Since this function comes from the definition of the variable q_j, we call it the cost-structure locus.

With this system we can study the effects of parameters in (z_j, q_j) space; see Figure 3. We study the effects of an increase in ϕ_j in red and the effects of an increase in ς_j in blue. Notably absent from this steady-state solution is η_j. We discuss its role below.

The increase in ϕ_j shifts up the free-entry locus and does not move the cost-structure locus. Therefore, both q_j^* and z_j^* rise. Equation (29) then says that x_j^* rises as well. The industry growth rate is $g_j^* = \chi_j \lambda + \theta_j z_j^*$ and thus rises.

The increase in ς_j shifts down both the free-entry locus and the cost-structure locus. Therefore, both q_j^* and z_j^* fall. Equation (29) then says that x_j^* falls as well. The industry growth rate is $g_j^* = \chi_j \lambda + \theta_j z_j^*$ and thus falls. This reflects the reallocation of R&D effort from manufacturing innovation to transportation innovation.

7.2 Comparative dynamics, η_j

The effect of η_j is interesting and reveals something important about the role of transportation in the growth process precisely because η_j does not enter the determination of the steady state in Figure 3. Let η_j^o be the old value and $\eta_j < \eta_j^o$ be the new value. At time $t = 0$, we

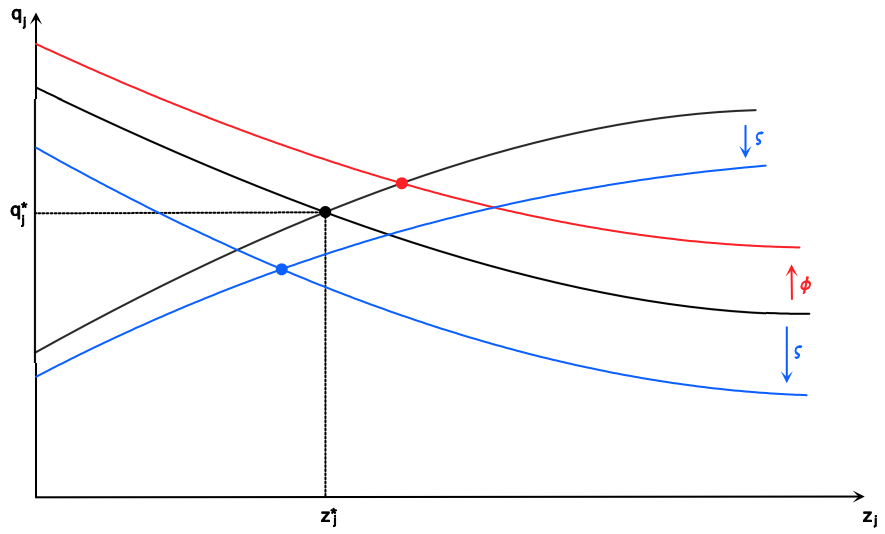


Figure 3: Steady state growth and transportation cost structure, with increase in ϕ_j in red and increase in ς_j in blue.

have a displacement of q_j from q_j^* to the initial condition

$$q_j(0) = \eta_j \cdot [S_j(0)]^{-\sigma_j} [Z_j(0)]^{\theta_j} < q_j^*.$$

By scale invariance, x_j^* does not change. Also, y^* does not change. Therefore, x_i time $t = 0$ remains at x_j^*. The industry then starts a transition from the point $\left(q_j(0), x_j^*\right)$ and returns to the steady state $\left(q_j^*, x_j^*\right)$. Figure 4 illustrates the process. Throughout the process, since q_j grows, we have $\theta_j z_j > \sigma_j s_j$, which means that labor productivity in manufacturing grows faster than labor productivity in transportation.

Industry growth at time $t = 0$ is

$$g_j(0) - g_j^* = \underbrace{\chi_j [n_j(0) - \lambda]}_{\text{positive}} + \underbrace{\theta_j \left[z_j(0) - z_j^*\right]}_{\text{positive}} - \underbrace{\frac{q_j(0)}{1 + q_j(0)}}_{\text{positive}} \cdot \underbrace{[\theta_j z_j(0) - \sigma_j s_j(0)]}_{\text{positive}}.$$

We rearrange this expression as

$$g_j(0) - g_j^* = \underbrace{\chi_j [n_j(0) - \lambda]}_{\text{positive}} + \underbrace{\theta_j \left[\frac{z_j(0)}{1 + q_j(0)} - z_j^*\right]}_{\text{ambiguous}} + \underbrace{\frac{q_j(0)}{1 + q_j(0)}}_{\text{positive}} \cdot \underbrace{\sigma_j s_j(0)}_{\text{positive}}.$$

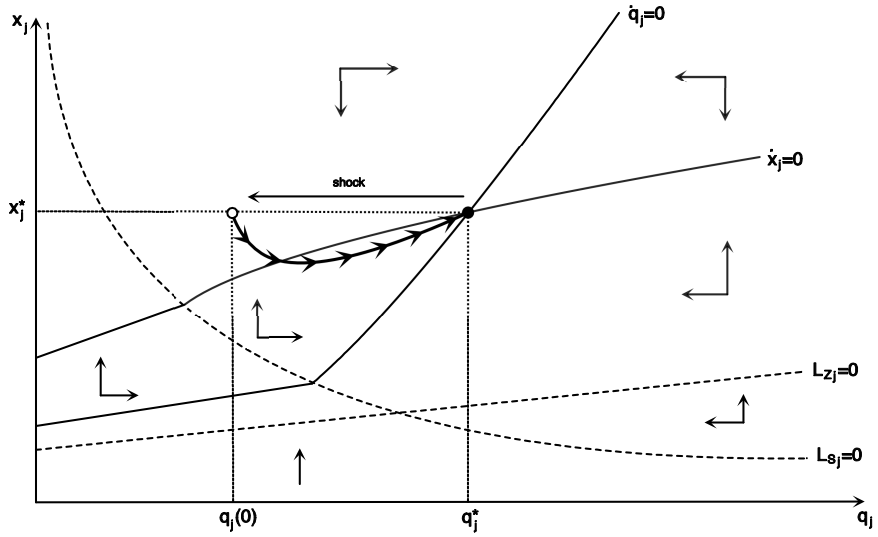


Figure 4: Transitional dynamics in response to an unanticipated, immediate and permanent fall of η_j.

This suggests that a sufficient condition for a positive initial jump is

$$\frac{z_j(0)}{1+q_j(0)} \geq z_j^* \Rightarrow \frac{\frac{\alpha_j\theta_j(\epsilon_j-1)}{\epsilon_j}\frac{x_j^*}{1+q_j(0)} - \rho - \delta_j}{1+q_j(0)} \geq z_j^*, \tag{34}$$

where $q_j(0) = \eta_j [S_j(0)]^{-\sigma_j} [Z_j(0)]^{\theta_j}$. If this condition holds, the industry growth rate jumps up and then returns gradually to g_j^*, either monotonically or following a hump-shaped path. It's worth stressing this result.

In industries where condition (34) holds, an unanticipated, immediate and permanent fall of the per-unit transportation cost parameter from η_j^o to η_j causes an initial fall of q_j to $q_j(0) < q_j^*$ with unchanged firm size, followed by a transition with q_j returning monotonically to q_j^* and firm size initially falling gradually and then reversing course and returning to x_j^*. The industry growth rate, g_j, jumps up and then returns gradually to g_j^*, either monotonically or following a hump-shaped path.

The components of the growth rate are:

$$z_j = \frac{\alpha_j\theta_j(\epsilon_j-1)}{\epsilon_j}\frac{1}{1+q_j}x_j - \rho - \delta_j;$$

$$s_j = \frac{\varsigma_j\sigma_j(\epsilon_j-1)}{\epsilon_j}\frac{q_j}{1+q_j}x_j - \rho - \delta_j;$$

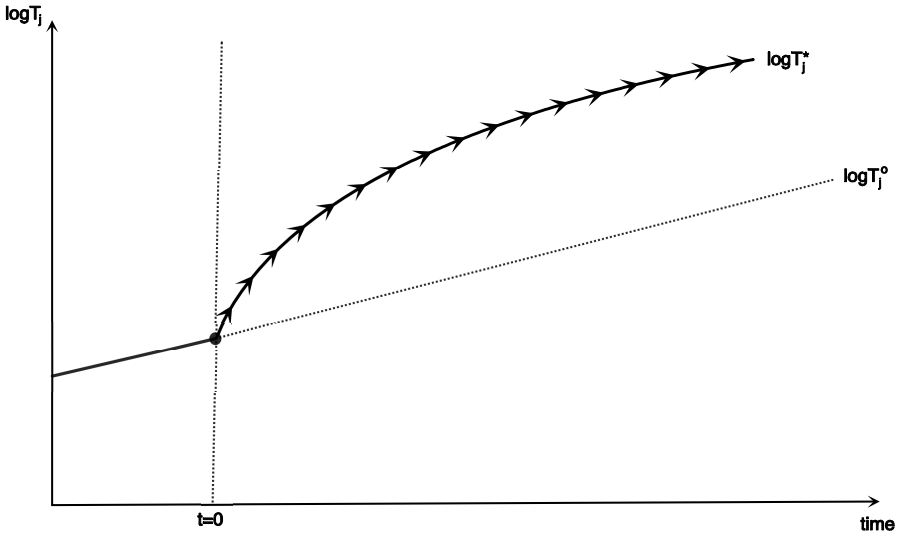


Figure 5: Transitional dynamics in response to an unanticipated, immediate and permanent fall of η_j. The growth acceleration locks in a permanent gain in the level of TFP.

$$\frac{\dot{q}_j}{q_j} = \theta_j z_j - \sigma_j s_j = \frac{\epsilon_j - 1}{\epsilon_j} \frac{\alpha_j \theta_j^2 - \varsigma_j \sigma_j^2 q_j}{1 + q_j} x_j - (\theta_j - \sigma_j)(\rho + \delta_j);$$

$$n_j = \lambda + \frac{1}{\epsilon_j \beta} - \frac{\sigma_j q_j - \theta_j}{1 + q_j} \frac{\epsilon_j - 1}{\epsilon_j \beta} - \frac{\phi_j - (\rho + \delta_j)\left(\frac{1}{\alpha_j} + \frac{1}{\varsigma_j}\right)}{\beta x_j} - (\rho + \delta_j).$$

The first equation says that z_j jumps up at $t = 0$ and then returns gradually to z_j^*. The second equation says that s_j jumps down at $t = 0$ and then returns gradually to s_j^*. The third equation says that $\dot{q}_j/q_j$ jumps up at $t = 0$ and then returns gradually to 0. The fourth equation says that n_j jumps up at $t = 0$ and then returns gradually to λ. Under the sufficient condition (34) discussed above, the net effect is a jump up at time $t = 0$ of the industry growth rate g_j followed by a gradual return to g_j^*.

Figure 5 illustrates the importance of the growth acceleration: it locks in a permanent cumulated gain in the level of TFP despite the fact that the steady-state growth rate does not change with the fall of η_j. Discounting and integrating translates this gain into a welfare gain.

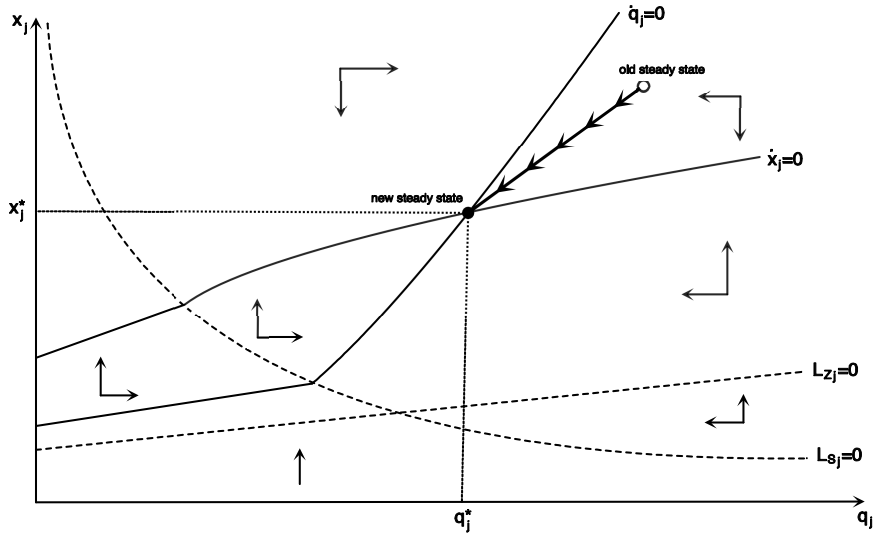


Figure 6: Transitional dynamics in response to an unanticipated, immediate and permanent rise of ς_j.

7.3 Comparative dynamics, ς_j

The negative steady-state growth effect of ς_j discussed above can be deceptive. The steady-state growth rate falls but the transition is very important because industry TFP,

$$T_j = \frac{Z_j^{\theta_j} N_j^{\chi_j}}{1+q_j},$$

rises throughout it through the gradual fall of q_j. The transitional growth rate relative to the new steady state is

$$g_j - g_j^* = \underbrace{\chi_j \left(n_j - \lambda\right)}_{\text{positive}} + \underbrace{\theta_j \left(z_j - z_j^*\right)}_{\text{ambiguous}} + \underbrace{\left(-\frac{q_j}{1+q_j}\frac{\dot{q}_j}{q_j}\right)}_{\text{positive}}.$$

Figure 6 illustrates the transition for this exercise. Since firm size shrinks, the term n_j rises above the steady-state value λ. Similarly, the last terms is positive since q_j shrinks. A sufficient condition for a growth acceleration is

$$\theta_j \left(z_j - z_j^*\right) = \theta_j \frac{\alpha_j \theta_j \left(\epsilon_j - 1\right)}{\epsilon_j} \left(\frac{x_j}{1+q_j} - \frac{x_j^*}{1+q_j^*}\right) \geq 0$$

throughout the transition.

If this is the case, then we have a dynamic tradeoff between the steady-state effect of ς_j and the transitional effect that yields a cumulative increase in TFP relative to the baseline path. In this exercise, in particular, we have the solution for welfare

$$\frac{U^*}{\Lambda} = \frac{\log\left(\frac{y^*}{c^*}\right) + \varkappa}{\rho - \lambda} + \sum_{j=1}^{J} \frac{\varphi_j g_j^*}{(\rho - \lambda)^2} + \sum_{j=1}^{J} \frac{\varphi_j \Upsilon_j}{\rho - \lambda},$$

where $\Delta_{x_j} \equiv x_j^o - x_j^*$ and $\Delta_{q_j} \equiv q_j^o - q_j^*$. This expression describes the welfare per capita produced by the aggregation of the J industry-specific transition paths, each one going from the initial steady state $\left(q_j^o, x_j^o\right)$ to the new steady state $\left(q_j^*, x_j^*\right)$.

This exercise says that to gauge the overall effect of a change in the model's environment that causes a reallocation of R&D from one innovation activity to the other, we must account for the transition dynamics — simply looking at the steady-state growth rate can be grossly misleading. In particular, the baseline steady-state path yields welfare per capita

$$\frac{U^o}{\Lambda} = \frac{\log\left(\frac{y^*}{c^*}\right) + \varkappa}{\rho - \lambda} + \sum_{j=1}^{J} \frac{\varphi_j g_j^o}{(\rho - \lambda)^2}.$$

Hence, we have that the effect on welfare per capita of the change in ς_j is

$$\frac{U^* - U^o}{\Lambda} = \frac{1}{\rho - \lambda} \sum_{j=1}^{J} \varphi_j \left[\underbrace{\Upsilon_j}_{\text{positive}} + \underbrace{\frac{g_j^* - g_j^o}{\rho - \lambda}}_{\text{negative}} \right].$$

This is the analytical characterization of the dynamic tradeoff discussed above. This calculation is easily implemented in a quantitative exercise.

Figure 7 illustrates the underlying dynamics. The TFP growth rate jumps up and then gradually decreases, going below g_j^o in finite time and then converging to $g_j^* < g_j^o$. The initial acceleration cumulates into a TFP gain relative to the baseline path due to the falling transportation costs. Eventually, however, the reallocation of labor from manufacturing innovation to transportation innovation produces the permanent reduction of the growth rate. The reason is that, as we have seen, the growth rate of labor productivity in transportation is not a driver of steady-state growth. If the transitional effect Υ_j is large enough, despite the steady-state growth deceleration, the economy is better off. Figure 8 plots the time path of the level of the log of TFP to illustrate this property. The welfare calculation above compares two areas of finite size because it multiplies the log of TFP by the subjective discount

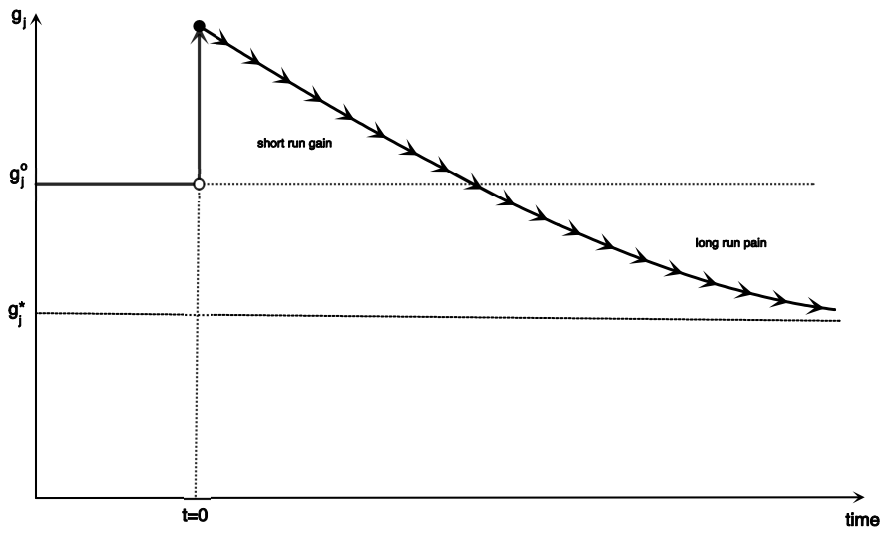


Figure 7: Dynamic tradeoff due to the unanticipated, immediate and permanent increase of ς_j. Cumulating and discounting the initial acceleration and the subsequent deceleration yields the welfare change $U^* - U^o$.

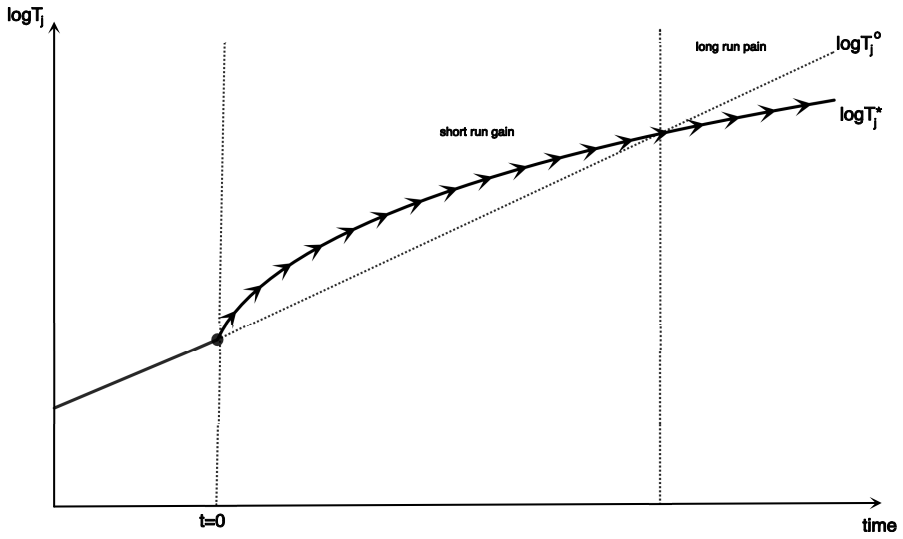


Figure 8: Dynamic tradeoff due to the unanticipated, immediate and permanent increase of ς_j. Cumulated effects on $\log T_j$. Discounting the time path and integrating yields the welfare change $U^* - U^o$.

factor from the household welfare function.

7.4 Summary and a question

This section has illustrated, through a battery of comparative statics and comparative dynamics exercises, the power and usefulness of our model of innovation-led endogenous growth with transportation. When we compare these results to the dynamics generated by the version of the model with no innovation in transportation, we zoom in on the essence of the mechanism driving our results and identify a simple and, to our knowledge, novel insight: because of the existence of per-unit shipping costs, to understand economic growth we must think in terms of the integrated system "production + transportation" because serving the market consists of making things and shipping things, not just making things. Here things stands for goods and services broadly construed; it ranges from physical objects, like gadgets and people, all the way to intangible services (e.g. legal consultations online) and information (e.g., sending scientific papers through the internet in digital format).

With this understanding in mind, we ask the following question. Can we interpret a parameter like η_j as a factor affected by regulation? If the answer is yes, then the model that we thoroughly analyzed in this section provides a powerful conceptual and analytical tool for the study of the effects of regulations on economic growth. Another parameter that can depend on regulations is ς_j, the productivity of labor in generating transportation-specific knowledge. We can think of this term as capturing the effect of regulations on the cost of innovation in transportation. Note that these are industry-specific parameters. Thus, the model is built to handle easily industry-specific regulations. In the next section, we develop a full microfoundation of this mapping, extending the model to allow for different modes of transportation.

8 Modes of transportation and regulation

The model has heterogenous industries with industry-specific parameters. The previous section has studied the effects of changes in two transportation parameters, η_j and ς_j. In this section we provide a deeper microfoundation of transportation that allows for different modes, in line with the salient facts discussed in the Introduction. Also, in line with the conceptual question that we posed in the summary of our exercises, we introduce regulation as a prime mover of the choice of the mix of modes that firms use to ship the things that

they make to their customers.

We define transportation as a technology that produces total distance covered by the firm's goods, D_{ij}, by mixing two modes that we call road and water transport. Specifically, the technology is the CRS function

$$D_{ij} = S_{ij}^{\sigma_j} \cdot F\left(\frac{L_{D_{ij}}^r}{R_r}, \frac{L_{D_{ij}}^w}{R_w}\right).$$

where $F(\cdot,\cdot)$ is a generic CRS production function with standard properties, $L_{D_{ij}}^r$ is labor allocated to road transportation, R_r is regulations that reduce the productivity of labor in road transpiration, $L_{D_{ij}}^w$ is labor allocated to water transportation, and R_w is regulations that reduce the productivity of labor in water transpiration. The firm's total expenditure on shipping is

$$C_{ij} = S_{ij}^{-\sigma_j} \cdot w\left(L_{D_{ij}}^r + L_{D_{ij}}^w\right).$$

It is important to note that the two inputs in this problem have the same market price, w, so that the only user-cost difference to the firm is that they have different effective prices, wR^r and wR^w. In this sense, allocating labor to road transport rather than water transport matters to the firm because (1) the two modes are *not* perfect substitutes, indeed our technology allows for both substitutability and complementarity, and (2) they have different productivity of labor due to regulation. This property has desirable implications for our purposes.

Since the firm's value maximization problem is time separable, we can solve the transportation cost minimization problem separately, i.e., the firm solves the static transportation cost minimization problem taking the productivity term $S_{ij}^{\sigma_j}$ as given. With the technology above, the solution is the per-unit transportation cost function

$$S_{ij}^{-\sigma_j} \cdot C_j(wR_r, wR_w).$$

Since the wage w is common to the two modes, we write the associated cost shares in the form:

$$c_j^r(R_r, R_w) \equiv \frac{wL_{D_{ij}}^r}{w\left(L_{D_{ij}}^r + L_{D_{ij}}^w\right)} = \frac{\partial \log C_j(wR_r, wR_w)}{\partial \log(wR_r)};$$

$$c_j^w(R_r, R_w) \equiv \frac{wL_{D_{ij}}^w}{w\left(L_{D_{ij}}^r + L_{D_{ij}}^w\right)} = \frac{\partial \log C_{ij}(wR_r, wR_w)}{\partial \log(wR_w)}.$$

Note that $c_j^r(R_r, R_w) + c_j^w(R_r, R_w) = 1$. The shares have the standard properties of cost shares. In particular, they allow for substitutability or complementarity between the two modes. In the first case, we have $\partial c_j^r(R_r, R_w)/\partial R_r < 0$ with $\partial c_j^r(R_r, R_w)/\partial R_w > 0$ and $\partial c_j^w(R_r, R_w)/\partial R_r > 0$. In the second case, we have $\partial c_j^r(R_r, R_w)/\partial R_r > 0$ with $\partial c_j^r(R_r, R_w)/\partial R_w < 0$ and $\partial c_j^w(R_r, R_w)/\partial R_r < 0$. This simple structure, therefore, captures the notion that regulations that reduce the productivity of labor in one mode of transportation reduce that mode's share of use by firms. Moreover, these substitution effects are entirely industry specific.

An example can be useful. Let the transportation technology be the CES

$$D_{ij} = S_{ij}^{\sigma_j} \cdot \left[\varrho_j \left(\frac{L_{D_{ij}}^r}{R_r} \right)^{\frac{\omega_j - 1}{\omega_j}} + (1 - \varrho_j) \left(\frac{L_{D_{ij}}^w}{R_w} \right)^{\frac{\omega_j - 1}{\omega_j}} \right]^{\frac{\omega_j}{\omega_j - 1}}, \quad \omega_j > 0,\ 0 < \varrho_j < 1.$$

Then,

$$c_j^r(R_r, R_w) = \frac{(wR_r)^{1-\omega_j}}{(wR_r)^{1-\omega_j} + (wR_w)^{1-\omega_j}} = \frac{R_r^{1-\omega_j}}{R_r^{1-\omega_j} + R_w^{1-\omega_j}}.$$

This yields $c_j^r(R_r, R_w)$ increasing in R_r for $\omega_j < 1$ (complementarity) and decreasing in R_r for $\omega_j > 1$ (substitutability).

We now connect this structure to the simpler model that we studied in the previous sections. Our preferred interpretation is that this representation microfounds the industry-specific parameter η_j. In other words, recalling that in general equilibrium we set $w = 1$, we now think of η_j as the *endogenous* variable $\eta_j = C_j(R_r, R_w)$. The complete mapping is as follows. The firm must ship each unit of its good over a set distance that we normalize at 1 for simplicity and without loss of generality. Then, the cost minimization problem above says that the most efficient way to ship the good is a combination of road and water transportation that costs $S_{ij}^{-\sigma_j} C_j(R_r, R_w)$. The shares of the two modes are $c_j^r(R_r, R_w)$ and $c_j^w(R_r, R_w)$, which then yield the labor allocation $L_{D_{ij}}^r = c_j^r(R_r, R_w) L_{D_{ij}}$ and $L_{D_{ij}}^r = c_j^w(R_r, R_w) L_{D_{ij}}$, where $L_{D_{ij}} = L_{D_{ij}}^r + L_{D_{ij}}^w$ is the firm's total use of labor in transportation. The total $L_{D_{ij}}$ is determined as in the baseline model discussed in the previous sections.

This extension adds insight to the model but does not change the fundamental property that we uncovered by studying the simpler version that treats η_j as a exogenous parameter. The property is that η_j does not affect the steady state configuration of the model, i.e., it does not affect x_j^* and q_j^*, and therefore it does not affect z_j^* and g_j^*. As we discussed, the reason is that changes in η_j are fully absorbed by the adjustment of the two stocks Z_j

and S_j, which in equilibrium comove to produce the constant value q_j^* in equation (33) that does not depend on η_j. In this microfounded extension that yields the *endogenous* solution $\eta_j = C_j(R_r, R_w)$, the property still applies: regulations that change the value η_j have only transitional effects like those illustrated in Figure 4; they do not affect the steady-state values x_j^*, q_j^*, z_j^* and g_j^*. However, they do affect the industry-specific composition of transportation described by the cost share functions $c_j^r(R_r, R_w)$ and $c_j^w(R_r, R_w)$.

9 Appendix

We write the log of TFP in the form

$$\begin{aligned}\log T_j &= \theta_j \int_0^t z_j(k)\,dk + \chi_j \log \frac{N_j}{Y_j} + \chi_j \log Y_j - \log(1+q_j) \\ &= \theta_j \int_0^t z_j(k)\,dk - \chi_j \log x_j - \log(1+q_j) + \varkappa + \chi_j \lambda t,\end{aligned}$$

where

$$\varkappa \equiv \chi_j \log \varphi_j + \chi_j \log y^* + \chi_j \log \Lambda.$$

We approximate the log terms to write

$$\log T_j \simeq \theta_j \int_0^t z_j(k)\,dk - \chi_j x_j - q_j + \varkappa + \chi_j \lambda t.$$

Next, we linearize around the new steady state. In particular, we write:

$$z_j = \gamma_0 + \gamma_x x_j - \gamma_q q_j;$$

$$x_j(t) = x_j(0)\,e^{-\nu_{x_j} t} + x_j^*\left(1 - e^{-\nu_{x_j} t}\right) = \left[x_j(0) - x_j^*\right] e^{-\nu_{x_j} t} + x_j^* = \Delta_{x_j} e^{-\nu_{x_j} t} + x_j^*;$$

$$q_j(t) = q_j(0)\,e^{-\nu_{x_j} t} + q_j^*\left(1 - e^{-\nu_{x_j} t}\right) = \left[q_j(0) - q_j^*\right] e^{-\nu_{q_j} t} + q_j^* = \Delta_{q_j} e^{-\nu_{q_j} t} + q_j^*.$$

Now we work on the various terms separately. We start with

$$\begin{aligned}
\int_0^t z_j(k)\,dk &= \int_0^t \left[\gamma_{j0} + \gamma_{jx}x_j(k) - \gamma_{jq}q_j(k)\right]dk \\
&= \int_0^t \left(\gamma_{j0} + \gamma_{jx}x_j^* - \gamma_{jq}q_j^*\right)dk + \int_0^t \gamma_{jx}\Delta_{x_j}e^{-\nu_{x_j}k}dk - \int_0^t \gamma_{jq}\Delta_{q_j}e^{-\nu_{q_j}k}dk \\
&= z_j^* t + \gamma_{jx}\Delta_{x_j}\int_0^t e^{-\nu_x k}dk - \gamma_{jq}\Delta_{q_j}\int_0^t e^{-\nu_{q_j}k}dk \\
&= z_j^* t + \gamma_{jx}\Delta_{x_j}\left(1 - e^{-\nu_{x_j}t}\right) - \gamma_{jq}\Delta_{q_j}\left(1 - e^{-\nu_{q_j}t}\right).
\end{aligned}$$

We then write in compact notation

$$\begin{aligned}
\log T_j(t) &= \theta_j\Delta_{x_j} - \theta_j\Delta_{q_j} - \left(\theta_j + \chi_j\right)\Delta_{x_j}e^{-\nu_x t} + \left(\theta_j + \chi_j\right)\Delta_{q_j}e^{-\nu_q t} + \varkappa + \chi_j\lambda t \\
&= \theta_j\left(\Delta_{x_j} - \Delta_{q_j}\right) - \left(\theta_j + \chi_j\right)\left(\Delta_{x_j}e^{-\nu_x t} - \Delta_{q_j}e^{-\nu_q t}\right) + \varkappa + \chi_j\lambda t.
\end{aligned}$$

Finally, we have

$$\frac{U^*}{\Lambda} = \int_0^\infty e^{-(\rho-\lambda)t}\log\left(\frac{y^*}{c^*}\right)dt + \Gamma^*,$$

where

$$\begin{aligned}
\Gamma^* &= \sum_{j=1}^J \varphi_j\left[\int_0^\infty e^{-(\rho-\lambda)t}\left[\varkappa + g_j^* t + \theta_j\left(\Delta_{x_j} - \Delta_{q_j}\right) - \left(\theta_j + \chi_j\right)\left(\Delta_{x_j}e^{-\nu_x t} - \Delta_{q_j}e^{-\nu_q t}\right)\right]dt\right] \\
&= \sum_{j=1}^J \varphi_j\left[\int_0^\infty e^{-(\rho-\lambda)t}\varkappa dt\right] + \sum_{j=1}^J \varphi_j\left[\int_0^\infty e^{-(\rho-\lambda)t}g_j^* t dt\right] \\
&\quad + \sum_{j=1}^J \varphi_j\left[\int_0^\infty e^{-(\rho-\lambda)t}\theta_j\left(\Delta_{x_j} - \Delta_{q_j}\right)dt\right] \\
&\quad - \sum_{j=1}^J \varphi_j\left[\int_0^\infty e^{-(\rho-\lambda)t}\left(\theta_j + \chi_j\right)\Delta_{x_j}e^{-\nu_x t}dt\right] \\
&\quad + \sum_{j=1}^J \varphi_j\left[\int_0^\infty e^{-(\rho-\lambda)t}\left(\theta_j + \chi_j\right)\Delta_{q_j}e^{-\nu_q t}dt\right].
\end{aligned}$$

Collecting and rearranging terms, we reduce this expression to

$$\Gamma^* = \frac{\varkappa}{\rho-\lambda} + \sum_{j=1}^J \frac{\varphi_j g_j^*}{(\rho-\lambda)^2} + \sum_{j=1}^J \frac{\varphi_j}{\rho-\lambda}\underbrace{\left[\theta_j\left(\Delta_{x_j} - \Delta_{q_j}\right) - \frac{\left(\theta_j + \chi_j\right)\Delta_{x_j}}{\rho-\lambda+\nu_{x_j}} + \frac{\left(\theta_j + \chi_j\right)\Delta_{q_j}}{\rho-\lambda+\nu_{q_j}}\right]}_{\Upsilon_j}.$$

The last term in this expression is the cumulative change in TFP due to the transition from the initial point $\left(q_j^o, x_j^o\right)$ to the new steady state $\left(q_j^*, x_j^*\right)$.

Therefore, we can integrate and write welfare per capita

$$\frac{U^*}{\Lambda} = \frac{\log\left(\frac{y^*}{c^*}\right) + \varkappa}{\rho - \lambda} + \sum_{j=1}^{J} \frac{\varphi_j g_j^*}{(\rho - \lambda)^2} + \sum_{j=1}^{J} \frac{\varphi_j \Upsilon_j}{\rho - \lambda}.$$

10 References

Al-Ubaydli, O., & McLaughlin, P. A. (2015). RegData: A numerical database on industry-specific regulations for all U.S. industries and federal regulations, 1997–2012. Regulation & Governance, 9(1), 45–62.

Bailey, J. B., & Thomas, D. W. (2017). Regulating away competition: The effect of regulation on entrepreneurship and employment. Journal of Regulatory Economics, 52(3), 237–254.

Chambers, D. (2019). Regulation and poverty: An empirical examination of the relationship between the incidence of federal regulation and poverty rates. Public Choice, 180(1-2), 131–144.

Chambers, D., & O'Reilly, C. (2020). Regulation and income inequality in the United States: 1970–2008. Public Choice, 182(1-2), 63–89.

Chambers, D., & O'Reilly, C. (2022). Regulation and income inequality: Updated evidence. Southern Economic Journal, 88(3), 774–803.

Coffey, B., & McLaughlin, P. A. (2021). Regulation and economic growth: Evidence from British Columbia's experiment in regulatory budgeting. Mercatus Center Working Paper.

Coffey, B., McLaughlin, P. A., & Peretto, P. (2020). The cumulative cost of regulations. Journal of Regulatory Economics, 58(1), 1–34.

Dawson, J. W., & Seater, J. J. (2013). Federal regulation and aggregate economic growth. Journal of Economic Growth, 18(2), 137–177.

McLaughlin, P. A., & Wong, J. T. H. (2024). The causal effect of regulations on economic growth: Evidence from the U.S. states. Mercatus Center Working Paper.