The gravity of urban leisure destinations: modelling non-commuting travel in a quantitative spatial model*

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PRELIMINARY AND INCOMPLETE

Abstract

Non-commuting trips represent the majority of trips in developed cities. To assess their importance, we develop a quantitative spatial equilibrium model that features non-commuting trips to leisure activities. The theoretical model predicts a gravity structural equation for non-commuting trips that depends on how attractive and accessible a destination is relative to any other destination in the city. Additionally, the theoretical model also predicts that the accessibility to leisure activities matters for the residential and labor choices of residents in a city. Using large-scale GPS data for the city of Budapest, we estimate the key parameters governing the gravity equation for non-commuting trips. Finally, we show how the model can be used to solve for the value of the accessibility to leisure activities. This allows us to assess its importance at determining the residential and labor choices of residents relative to more canonical variables such as amenities, rents, wages or commuting costs.

1 Introduction

Non-commuting trips represent the majority of trips in developed cities[1]. In spite of this, the literature of transport and urban economics pays disproportionate attention to commuting. The striking difference between the number of non-commuting trips in real life and their importance in the literature is the origin of this research question. Traveling is a costly activity both in terms of money and time. If the majority of trips in a city are undertaken due to

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[1]As an example, take the case of Barcelona, where according to the yearly travel diaries commuting trips represent only 30% of the total trips in the city.

non-commuting purposes but we do not model them, we might be missing most of benefits of traveling in a urban environment. Hence, the objective of this paper is to develop a theoretical framework that can be used to analyze non-commuting trips as well as their relative importance with respect to commuting trips.

To do so, we extend a quantitative spatial equilibrium model to include trips for leisure purposes, something that we call non-commuting trips to distinguish them of the usual trips undertaken to earn an income or purchase a product. Following the theoretical framework proposed in Anas 2007, we assume that residents enjoy spending their time in leisure activities in any location of the city. However, the presence of a temporal budget constraint forces them to prioritize some locations over others, creating a distribution of non-commuting trips. Additionally, we assume that utility exhibits marginal disutility in the time spent in a location per visit, a feature that allows to guarantee that some locations are visited multiple times by the same individual even though there exists a fixed monetary cost for every trip.

Our theoretical framework suggests a gravity-like equation that governs the number of non-commuting trips to a location in the city. Unsurprisingly, the number of non-commuting trips increases with the attractiveness of the destination, an unobservable variable akin to an amenity in a canonical quantitative spatial model, and it decreases with the generalized cost to reach it. Additionally, our structural equation shows that both the absolute values as well as the relative values of these variables matter for the number of non-commuting trips. To put it simply, a resident will travel more often to locations that are both attractive and accessible relative to her residence or workplace.

To empirically estimate this theoretical relationship, we use a PPML estimator with large-scale GPS data for the city of Budapest that allow us to observe the non-commuting trips for over 100,000 individuals. This section of the paper is still work in progress.

This setup creates an additional variable that we call accessibility to leisure that explains the residential and labor location choices of residents aside from rents, amenities or wages. The intuition is that when choosing where to live and work, residents now take into account the time and costs of accessing other locations of the city for leisure purposes. To measure this new variable, we show how our model can be used to uniquely determine its value provided that the distribution of non-commuting trips as well as the generalized trip costs are observed.

Finally, once the value of the accessibility to leisure is measured, we assess its importance in shaping the distribution of residential and labor choices relatives to the traditional values that can be found in a canonical quantitative spatial model. This section of the paper is still work in progress.

This paper contributes to the literature on quantitative spatial models, triggered by Ahlfeldt et al. 2015 and summarized in Redding and Rossi-Hansberg 2017. Our main contribution to this literature is the development and estima-

tion of a model that accounts for non-commuting trips, a feature only included in Miyauchi, Nakajima, and Redding 2021 for a urban environment and in Fajgelbaum et al. 2023 for a regional environment. While these two papers rely on additional idiosyncratic utility shocks to model non-commuting trips, our model builds on a constraint utility maximization problem. As explained in Anderson, Palma, and Thisse 1992, these two approaches are not independent and it remains to clarify under which assumptions they are actually equivalent.

Contrary to most of the papers in the literature of quantitative spatial equilibrium models, we model trip costs using the consumption leisure trade-off framework, a modeling approach well-rooted in the transport literature (Train and McFadden 1978). As in Hörcher and Graham 2024, we explicitly include a temporal budget constraint that allows us to endogenize the number of commuting trips to a destination, creating a difference between commutes and labor supply.

The paper is divided into five sections and one appendix. Section I consists in this introduction. In section II, we outline the main results of the theoretical model while we left most of the detailed mathematical derivation for the appendix. Section III consists in the estimation the non-commuting gravity equation as well as the construction of the accessibility to leisure activities variable. This section is still work in progress. Section IV includes some simulations with randomly generated data to illustrate the results in the previous section. Finally, section V concludes.

2 The model

In this section, we develop a quantitative spatial equilibrium model that features non-commuting trips. A city is divided into N locations or neighborhoods indexed by either i, j or k. Initially, we assume that each neighborhood is endowed with an an exogenous supply of housing H_i but this assumption can be easily relaxed to include an elastic supply equation. Contrary to most quantitative spatial models, we assume that residents care about their consumption allocation as well as their time allocation, which can be divided into working time or leisure time spent in any location of the city.

2.1 Residents maximization problem

The preferences of a resident ω who lives in i and commutes to j are represented by a Cobb-Douglas utility function defined over a consumption index C_{ij} and a leisure time index L_{ij}

$$U_{ij}(\omega) = C_{ij}^{\beta} L_{ij}^{1-\beta} s_{ij}(\omega) \tag{1}$$

where $s_{ij}(\omega)$ is an idiosyncratic amenity shock that captures the idea that residents have personal reasons for living and working in different locations. As

it is common in the literature, we assume that residents draw values for $s_{ij}(\omega)$ from independent Fréchet distributions

$$S_{ij}(s) = e^{-\lambda_i \lambda_j s^{-\theta}}, \qquad \lambda_i > 0,\ \lambda_j > 0,\ \theta > 1 \tag{2}$$

where λ_i captures the average amenity level from living in location i, λ_j captures the average amenity level from working in location j and θ controls the dispersion of amenities. The presence of idiosyncratic amenities shocks implies that different residents choose to live and work in different locations even though they face the same rents and wages. Since residents living and working in the same locations make the same choices, we suppress the dependence on ω except where important.

Following the literature, we assume that the consumption index C_{ij} takes a Cobb-Douglas form in consumption of a freely traded *numéraire* good g_{ij} and housing services h_{ij}

$$C_{ij} = g_{ij}^{\gamma} h_{ij}^{1-\gamma} \tag{3}$$

To model non-commuting trips, we assume that (sub)utility from a visit to location k for an i to j commuter can be represented by

$$u_{k|ij} = n_{k|ij} a_k l_{k|ij}^{\alpha}, \qquad 0 < \alpha < 1 \tag{4}$$

where $n_{k|ij}$ is the number of trips to location k, $l_{k|ij}$ is the time spent in a leisure activity at location k and a_k captures how attractive location k is. The parameter α captures diminishing marginal utility for the time spent in a location during a single visit. This guarantees that residents prefer to visit a location multiple times rather than spending the total time in a single longer visit.

Next, we assume that residents have love for variety in the sense that they prefer to allocate their time endowment to multiple locations rather to spend it all in a single one. To account for this fact, we nest (4) in a constant elasticity of substitution (CES) aggregator to build the leisure time index

$$L_{ij} = \left(\sum_k (n_{k|ij} a_k l_{k|ij}^{\alpha})^{\rho} \right)^{1/\rho}, \qquad 0 < \rho < 1 \tag{5}$$

where the parameter ρ captures to what extend total time spent at different locations can be seen as complimentary goods.

The total time endowment T of a resident can be spent either working or in leisure activities, which both require spending some time traveling

$$x_{ij}(W + t_{ij}) + \sum_k n_{k|ij}(l_{k|ij} + t_{k|ij}) = T$$

where x_{ij} is the number of commutes, W is the duration of a working day, t_{ij} is duration of the commute and $t_{k|ij}$ is the duration of the trip to location k for an i to j commuter.

Similarly, the monetary budget constraint for an i to j commuter is

$$g_{ij} + r_i h_{ij} + \sum_k n_{k|ij} c_{k|ij} = x_{ij}(w_j W - c_{ij})$$

where r_i is the rent price for a unit of housing in location i, c_{ij} is the monetary commuting cost and $c_{k|ij}$ is the monetary trip cost to k for an i to j commuter.

The Cobb-Douglas utility function implies that a fraction β of the time endowment T is always spent working to earn an income that allows residents to consume both housing and the *numéraire*. The remaining time is thus spent in non-commuting trips for leisure activities. Additionally, the Cobb-Douglas consumption index implies that residents always spend a fraction $(1 - \gamma)$ of their income in housing services while the rest is spent in consumption of the *numéraire* good

$$g_{ij} = \gamma v_{ij} \beta T \tag{6}$$

$$h_{ij} = \frac{(1-\gamma) v_{ij} \beta T}{r_i} \tag{7}$$

where $v_{ij} = \frac{w_j W - c_{ij}}{T + t_{ij}}$ is the net hourly wage. This variable can also be interpreted as the value of time (VoT) for an i to j commuter since it captures the rate at which time can be transformed into money to be used in consumption.

Next, utility maximization implies that the time spent in a location k for a leisure activity is

$$l_{k|ij} = \frac{\alpha}{1-\alpha} \frac{\tau_{k|ij}}{v_{ij}} \tag{8}$$

where $\tau_{ij} = t_{k|ij} v_{ij} + c_{k|ij}$ is the generalized trip cost to location k for an i to j commuter. As it can be seen from inspection of (8), the duration of the visit diminishes as the net hourly wage increases. The intuition driving this result is that residents with a large hourly wage are less affected by the fix monetary cost $c_{k|ij}$ that must be paid regardless of the length of the visit, which means that they can visit the location more often. On the other hand, individuals with

a small net hourly wage choose to spend a longer time to compensate the fact that they cannot afford to pay the fix cost multiple times.

The number of non-commuting trips to a location k follows a gravity structure in both the attractiveness parameter a_k and the generalized cost $\tau_{k|ij}$ to reach it

$$n_{k|ij} = \frac{a_k^{\frac{\rho}{1-\rho}} \tau_{k|ij}^{-\frac{1-\alpha\rho}{1-\rho}}}{\sum_t a_t^{\frac{\rho}{1-\rho}} \tau_{k|ij}^{\frac{-\rho(1-\alpha)}{1-\rho}}} \sum_t n_{t|ij} \tau_{t|ij} \tag{9}$$

As in any gravity equation, what determines the number of non-commuting trips to a location are the relative value of the variables. In this way, a location k is visited more often when it has a high value for a_k and a low generalized cost $\tau_{k|ij}$ relative to the remaining ones.

Summing across locations in (8) and using the fact residents have $(1-\beta)T$ units of time available, we can find that residents spent a fraction α of this time in leisure activities

$$\sum_k n_{k|ij} l_{k|ij} = \alpha(1-\beta)T \tag{10}$$

while the remaining fraction of time is spent either traveling or working to pay for the non-commuting trips

$$\sum_k n_{k|ij} \tau_{k|ij} = (1-\alpha)(1-\beta) v_{ij} T \tag{11}$$

which allow us to pin down the total number of trips in (39).

Finally, indirect utility for a resident ω who commutes i to j is

$$V_{ij}(\omega) = \kappa r_i^{\beta(\gamma-1)} A_{ij}^{\frac{(1-\beta)(1-\rho)}{\rho}} v_{ij}^{\beta+(1-\alpha)(1-\beta)} s_{ij}(\omega) \tag{12}$$

where $A_{ij} = \left(\sum_k a_k \tau_{k|ij}^{-\frac{\rho(1-\alpha)}{1-\rho}} \right)$ and $\kappa \equiv \alpha^{(1-\beta)\alpha}(1-\alpha)^{(1-\beta)(1-\alpha)}(1-\beta)^{(1-\beta)}\gamma^{\beta\gamma}(1-\gamma)^{\beta(1-\gamma)}\beta^{\beta}T$.

We interpret the term A_{ij} as an accessibility measure to leisure activities. This variable captures how attractive is to live in location i and commute to location j from the view-point of leisure time. Additionally, the variable v_{ij} captures how attractive is the i to j commute from a monetary point of view. It is important to emphasize that these two variables are not independent since the accessibility to leisure activities is indeed a function of the net hourly wage.

2.2 Residence and workplace location choices

Residents choose the residence and workplace location pair that offers them the highest utility. As describe in the previous section, indirect utility (12) has a Fréchet distributed random component. This means means that indirect utility is itself a Fréchet distributed random variable whose cumulative density function is

$$G_{ij}(V) = e^{-\Psi_{ij} V^{-\theta}} \quad \text{where} \quad \Psi_{ij} = \lambda_i \lambda_j r_i^{\beta(\gamma-1)} A_{ij}^{\frac{(1-\beta)(1-\rho)}{\rho}} v_{ij}^{\beta+(1-\alpha)(1-\beta)} \tag{13}$$

To find an expression for the residence and workplace choices, we must find the share of residents for which a particular ij offers the highest utility among all alternatives. Using the properties of the Fréchet distribution, it can be shown that

$$\begin{aligned} \pi_{ij} &= P[V_{ij} \geq \max_{rs} V_{rs}, \quad \forall r \neq i \; \forall s \neq j] \\ &= \frac{\lambda_i \lambda_j r_i^{\theta\beta(\gamma-1)} A_{ij}^{\frac{\theta(1-\beta)(1-\rho)}{\rho}} v_{ij}^{\theta(\beta+(1-\alpha)(1-\beta))}}{\sum_r \sum_s \lambda_r \lambda_s r_r^{\theta\beta(\gamma-1)} A_{rs}^{\frac{\theta(1-\beta)(1-\rho)}{\rho}} v_{rs}^{\theta(\beta+(1-\alpha)(1-\beta))}} \end{aligned} \tag{14}$$

The previous equation states that residents of the city are more likely to choose origin and destination pairs that have a high level of amenities (λ_i and λ_j), low rents (r_i), higher accessibility to leisure activities (A_{ij})and higher hourly wages (v_{ij}).

The share of residents living in location i can be obtained from (14) as the marginal probability of the origins

$$\pi_i = \sum_j \pi_{ij} \tag{15}$$

while the share of residents living in j is simply the marginal probability of the destinations

$$\pi_j = \sum_i \pi_{ij} \tag{16}$$

Additionally, it is also possible to find the share of commutes to j for residents living in i using the definition of conditional probability

$$\begin{aligned} \pi_{j|i} &= \frac{\pi_{ij}}{\pi_i} \\ &= \frac{\lambda_j A_{ij}^{\theta(1-\beta)(1-\rho)/\rho} v_{ij}^{\theta(\beta+(1-\alpha)(1-\beta))}}{\sum_s \lambda_s A_{is}^{\theta(1-\beta)(1-\rho)/\rho} v_{is}^{\theta(\beta+(1-\alpha)(1-\beta))}} \end{aligned} \tag{17}$$

2.3 Production

In every location of the city, there is a representative firm that produces the *numéirare* consumption good under conditions of perfect competition. To focus on the inclusion of non-commuting trips, we will assume that firms only use labor as their factor of production but this assumption can be easily relaxed. The production function of the representative firm in location j is

$$Y_j = A_j L_j^{\mu}, \qquad 0 < \mu < 1 \tag{18}$$

where A_j is the productivity of firms located in location j and L_j is total labor demand measured in hours. Profit maximization yields a labor demand equation in every location j of the city

$$L_j = \left(\frac{\mu A_j}{w_j} \right)^{\frac{1}{1-\mu}} \tag{19}$$

where the equilibrium wage is determined by the requirement that total hours of labor demand equals the total hours of labor supply.

2.4 Equilibrium

Given a set of exogenous parameters $\{\alpha, \beta, \gamma, \theta, \rho, \mu\}$, total population of the city R and vectors of location characteristics $\{\boldsymbol{a}, \boldsymbol{A}, \boldsymbol{\lambda^O}, \boldsymbol{\lambda^D}, \boldsymbol{H}\}$, the general equilibrium of the model is defined by the labor market clearing condition

$$\left(\frac{\mu A_j}{w_j} \right)^{\frac{1}{1-\mu}} = \sum_i \pi_{j|i} R_i \tag{20}$$

and the house market clearing condition

$$H_i = \frac{(1-\gamma)\beta T \sum_j v_{ij} \pi_{j|i} R_i}{r_i} \tag{21}$$

which define equilibrium conditions for the set of vectors $\{\boldsymbol{w}, \boldsymbol{q}\}$.

3 Estimation

From equation (39), the model predicts a semi-log gravity equation for non-commuting trips to destination k for the i to j commuter

$$\ln n_{k|ij} = \vartheta_k + \varrho_{ij} - \delta \tau_{k|ij} \tag{22}$$

where ϑ_k is a destination fixed effect that captures how attractive a location is, ϱ_{ij} is a fixed effect that captures the accessibility to leisure for the i to j commuter and $\tau_{k|ij}$ is the generalized trip cost to location k. The parameter δ is the semi-elasticity of non-commuting trips with respect to the generalized trip cost and it is a combination of several parameters.

Using our large-scale GPS data for the city of Budapest, we can identify for over 100,000 inhabitants both their commute as well as their visits to other locations of the city for leisure purpose. To measure the generalized cost $\tau_{k|ij}$, we need to make assumptions regarding the origin of the leisure trips. The easiest assumption is to consider that residents always choose the option that minimizes the generalized cost taking

$$\tau_{k|ij} = \min(c_{ik} + v_{ij}t_{ik}, c_{jk} + v_{ij}t_{jk}) \tag{23}$$

Following the argument in (), we augment the gravity equation derived from the model with a random component that captures deviation from the conditional expected value

$$\ln n_{k|ij} = \vartheta_k + \varrho_{ij} - \delta\tau_{k|ij} + \varepsilon_{ijk} \tag{24}$$

and we estimate the parameter δ with the PPML estimator. This estimation is still work in progress.

From equation (14), the model predicts a semi-log gravity equation for commuting trips from i to j

$$\ln \pi_{ij} = \varphi_i + \eta_j + \nu \ln A_{ij} + \kappa v_{ij} \tag{25}$$

where φ_i is an origin fixed effect, η_j is a destination fixed effect, A_{ij} is the accessibility to leisure and v_{ij} is the net hourly wage or value of time for the i to j commuter. The parameters κ and ν are composite parameters that capture how changes in the explanatory variables translate to changes in the commuting flows.

Similarly as before, we can rely on a PPML estimator to estimate both ν and κ provided that we can observe the accessibility to leisure A_{ij} and the net of commuting costs hourly wagev_{ij}. While the former is easy to observe, the latter is more challenging since its value depends on the values for the attractiveness of each location a_k.

To solve this problem, we show how the model can be used to solve for the value of a_k in every location provided that we observe both $n_{k|ij}$ and $\tau_{k|ij}$. Define the following excess demand function for each location k in the city

$$\mathbb{D}_k(\mathbf{a}) = \sum_i \sum_j n_{ijk}\tau_{ijk} - \sum_i \sum_j \frac{a_k \tau_{ijk}^{-\delta}}{\sum_t a_t \tau_{ijt}^{-(\delta-1)}} \sum_t n_{ijt}\tau_{ijt} \tag{26}$$

Our objective is to prove that there exists a unique $\boldsymbol{a}$ such that $\mathbb{D}_k(\boldsymbol{a}) = 0$ for all locations.

Following Monte, Redding, and Rossi-Hansberg 2018, we will divide the proof in three steps. In a first step, we will construct a derived excess demand function that is sufficiently well-behaved to apply Brouwer's fixed point theorem. Next, we will show that if the derived excess demand function has at least a solution, our original excess demand function must also have at least a solution. Finally, we will show that the original excess demand function also exhibits gross substitution, which guarantees that there is at most a solution. These three results together imply that there is a unique solution.

Before proceeding, it is important to notice that the excess demand function has the following properties:

- $\mathbb{D}_k(\boldsymbol{a})$ is continuous.
- $\mathbb{D}_k(\boldsymbol{a})$ is homogeneous of degree zero.
- $\sum_k \mathbb{D}_k(\boldsymbol{a}) = 0$.

To show that the excess demand system has at least a solution, I will first construct a derived system where the assumptions of Brouwer's fixed point theorem immediately apply. Next, I will show that if the derived excess demand system has at least a solution, then the original excess demand system must also a solution due to the third property outlined above.

Consider the following derived excess demand system

$$f_k(\tilde{\boldsymbol{a}}) = \frac{\tilde{a}_k + \max(\mathbb{D}_k(\tilde{\boldsymbol{a}}), 0)}{\sum_k \tilde{a}_k + \max(\mathbb{D}_k(\tilde{\boldsymbol{a}}), 0)} \tag{27}$$

where $\tilde{\boldsymbol{a}}$ belongs to the unit simplex. Notice that the previous derived excess is defined from the simplex to itself. Additionally, it is a continuous function since it is composed of continuous functions. Brouwer's fixed point theorem guarantees that this excess demand system has at least a solution.

Since $\sum_k \mathbb{D}_k(\boldsymbol{a}) = 0$, it cannot be that $\mathbb{D}_k > 0$ or that $\mathbb{D}_k < 0$ for all k. Additionally, if $\mathbb{D}_k > 0$ for some k, then $f_k(\boldsymbol{a}) \neq a_k$, which contradicts the fact that $\boldsymbol{a}$ is a fixed point. Therefore, it must be that $\mathbb{D}_k(\boldsymbol{a}) = 0$ for all k. This shows that there is at least a solution.

Next, notice that

$$\frac{\partial \mathbb{D}_k(\boldsymbol{a})}{\partial a_{k'}} < 0 \qquad \forall k, k' \neq k \tag{28}$$

which means that increasing the attractiveness of any location of the city other than itself decreases the excess demand of non-commuting trips to location k.

Since homogeneity of degree 0 implies that

$$\nabla \mathbb{D}(\boldsymbol{a})\boldsymbol{a} = 0 \tag{29}$$

it must be that

$$\frac{\partial \mathbb{D}_k(\boldsymbol{a})}{\partial a_k} > 0 \tag{30}$$

which shows that the excess demand system exhibits gross substitution.

Finally, since any excess demand system that exhibits gross substitution has at most one equation and we have previously proved that our demand system has at least one solution, it must be that our excess demand system has exactly one solution.

4 Simulations

In this section, we randomly generate data in a grid to show how we can solve for the attractiveness parameter a_k in every location of the grid. Along the section, we will assume that we know the value for δ, the semi-elasticity of the generalized trip cost to the number of non-commuting trips. In the previous step, we discussed how this parameter can be estimated.

We generate a grid with 64 different locations whose cells are $1km^2$ squares. To compute the travel times and costs distribution, we simply assume an homogeneous speed across the grid of 30 km/h and a monetary cost of 0.25 €/km.

Next, we randomly generate a distribution for population, employment, wages and rents. As it can be seen in figure 1, the distribution of the data across the grid seeks to replicate a monocentric city since the values of all four variables tend to peak in the central cell. Despite this, each variable has a different gradient: while the employment distribution is clearly monocentric, the population distribution tends to be more scattered and both the wage and rents are somewhat in between these two extremes.

The next step consists in generating the distribution for non-commuting trips. This time, we decided to generate a monocentric distribution but shifted towards the right-hand side of the grid. This seeks to capture the fact that the employment centre and the leisure centre may be located in different parts of the city. Figure 2 shows the results.

Finally, we solve for the value of a_k in every location as explained in the previous section. The intuition is that we must be able to rationalize the observed non-commuting trips distribution given the generalized cost matrix. Locations that

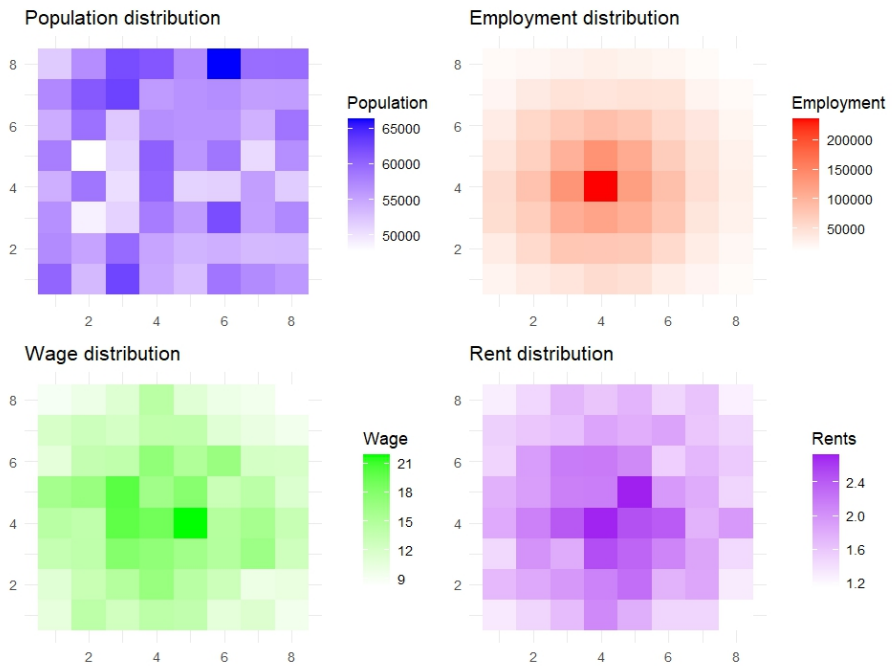


Figure 1: Distribution of population, employment, wages and rents.

receive a high (low) number of non-commuting trips despite being difficult (easy) to access should have a high (low) attractiveness parameter.

The results of the inversion can be seen in figure 3. The first thing to notice is that the cell with the highest number of non-commuting trips is also the one with the highest value for the attractiveness parameter a_k. This makes sense since this cell receives a high number of non-commuting trips despite not having the greatest accessibility.

Next, it is important to highlight the difference in the attractiveness parameter a_k between the left-hand side cells and the right-hand side cells. Due to the symmetry of the grid, these set of cells have similar accessibility. However, the right-hand side cells tend to receive more non-commuting trips which translates to a higher attractiveness parameter. Notice that this difference does not appear between the upper side cells and the downer side cells. In this case, this is because both set of cells have similar accessibility but they tend to receive the same number of non-commuting trips.

Finally, it is worth mentioning that the distribution of the attractiveness parameter a_k closely resembles the distribution of non-commuting trips $\sum_{i,j} n_{k|ij}$, something that suggests that the generalized trip costs are somewhat less important to determine non-commuting trips compared to the attractiveness param-

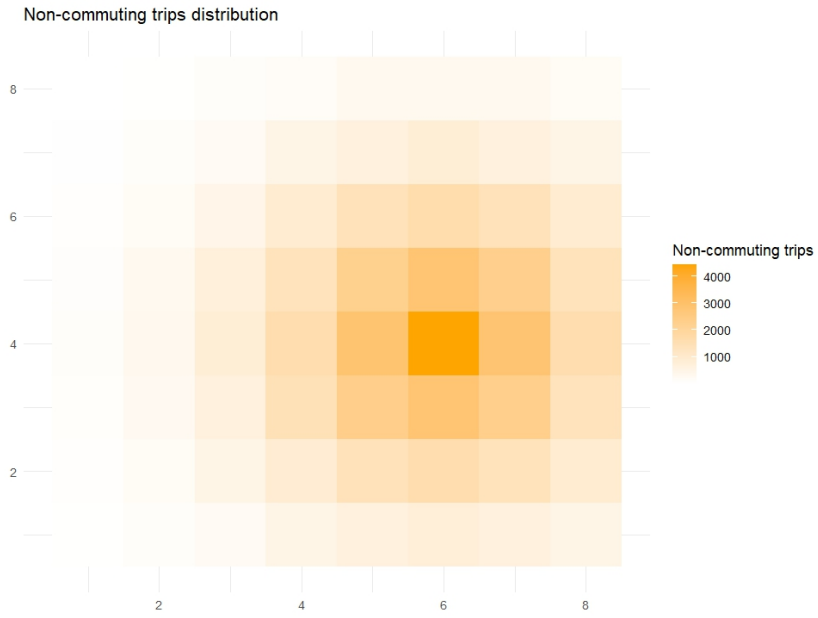


Figure 2: Distribution of non-commuting trips

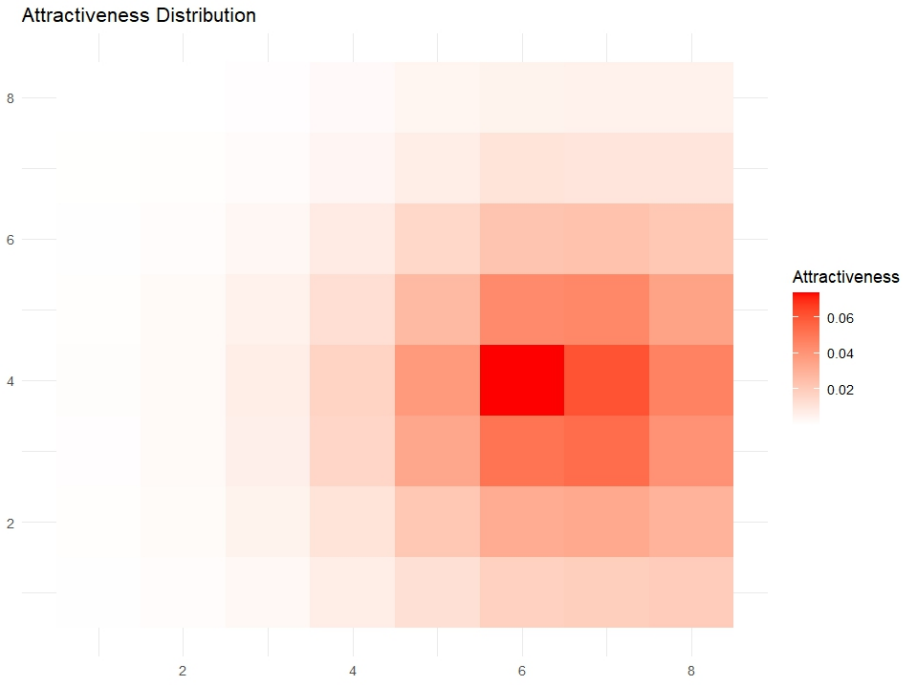


Figure 3: Attractiveness a_k distribution

eter a_k. We believe this is a feature of this particular grid since we assume an homogeneous speed and cost. In more real-life example, we trust the inversion will be available to highlight the importance of both variables.

5 Conclusions

This paper proposes a new quantitative spatial equilibrium model that features non-commuting trips to leisure activities. To model non-commuting trips, we assume that the utility function of residents exhibits love for variety in the total time spent in leisure activities across different locations. While residents enjoy visiting any location of the city, their temporal constraint forces them to prioritize some locations over others. Additionally, the assumption that the utility per visit exhibits marginal decreasing utility guarantees that some locations are worth visiting more than once even though residents must pay a fix monetary cost in every visit.

The theoretical model predicts a gravity structural equation for the number of non-commuting trips that depends on both the attractiveness and accessibility of the destination relative to any other location of the city. We empirically estimate the parameters governing this gravity equation using large-scale GPS data for the city of Budapest.

Our theoretical framework suggests that the accessibility to leisure activities is also a determining variable for the residential and labor choices of residents, aside from other variables that the literature in quantitative spatial economics has taken into account. To quantify its importance, we estimate a gravity equation that contains the accessibility to leisure as an explanatory variable. To measure it, we show how our model can be used to solve for its value provided that the distribution of non-commuting trips as well as the generalized trip cost matrix is observed.

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A Appendix

A.1 Solution for the residents maximization problem

The maximization problem for a resident who lives in i and commutes to j is

$$\max_{\boldsymbol{l_{ij}},\boldsymbol{n_{ij}},x_{ij},g_{ij},h_{ij}} \left(\sum_k (n_{k|ij} a_k l_{k|ij}^{\alpha})^{\rho}\right)^{\frac{1-\beta}{\rho}} g_{ij}^{\beta\gamma} h_{ij}^{\beta(1-\gamma)}$$

subject to

$$\sum_k n_{k|ij}(t_{k|ij} + l_{k|ij}) + x_{ij}(W + t_{ij}) = T$$

$$g_{ij} + r_i h_{ij} + \sum_k n_{k|ij} c_{k|ij} + x_{ij} c_{ij} = x_{ij} w_j W$$

The Lagrangian of the maximization problem is

$$\begin{aligned}\mathcal{L}(\boldsymbol{l_{ij}}, \boldsymbol{n_{ij}}, x_{ij}, g_{ij}, h_{ij}, \mu_T, \mu_M) = &\left(\sum_k (n_{k|ij} a_k l_{k|ij}^{\alpha})^{\rho}\right)^{\frac{1-\beta}{\rho}} g_{ij}^{\beta\gamma} h_{ij}^{\beta(1-\gamma)} \\ &- \mu_T \left(\sum_k n_{k|ij}(t_{k|ij} + l_{k|ij}) + x_{ij}(W + t_{ij}) - T\right) \\ &- \mu_M \left(g_{ij} + r_i h_{ij} + \sum_k n_{k|ij} c_{k|ij} + x_{ij} c_{ij} - x_{ij} w_j W\right)\end{aligned}$$

and its first-order conditions are

$$(1-\beta) L_{ij}^{(1-\beta-\rho)} C_{ij}^{\beta} n_{k|ij}^{-(1-\rho)} a_k^{\rho} l_{k|ij}^{-(1-\alpha\rho)} \alpha = \mu_T \quad \forall k \tag{31}$$

$$(1-\beta) L_{ij}^{(1-\beta-\rho)} C_{ij}^{\beta} n_{k|ij}^{-(1-\rho)} a_k^{\rho} l_{k|ij}^{\alpha\rho} = \mu_T (l_{k|ij} + t_{k|ij}) + \mu_M c_{k|ij} \quad \forall k \tag{32}$$

$$\mu_M (w_j W - c_{ij}) = \mu_T (W + t_{ij}) \tag{33}$$

$$L_{ij}^{1-\beta} \beta\gamma g_{ij}^{\beta\gamma-1} h_{ij}^{\beta(1-\gamma)} = \mu_M \tag{34}$$

$$L_{ij}^{1-\beta} g_{ij}^{\beta\gamma} \beta(1-\gamma) h_{ij}^{\beta(1-\gamma)-1} = \mu_M r_i \tag{35}$$

$$\sum_k n_{k|ij}(t_{k|ij} + l_{k|ij}) + x_{ij}(W + t_{ij}) = T \tag{36}$$

$$g_{ij} + r_i h_{ij} + \sum_k n_{k|ij} c_{k|ij} + x_{ij} c_{ij} = x_{ij} w_j W \tag{37}$$

The ratio of (31) and (32) gives an expression for the optimal time spent at a location k

$$l_{k|ij} = \frac{\alpha}{1-\alpha} \frac{\mu_M}{\mu_T} \left(\frac{\mu_T}{\mu_M} t_{k|ij} + c_{k|ij} \right)$$

which can be rewritten using equation (33) as

$$l_{k|ij} = \frac{\alpha}{1-\alpha} \frac{\tau_{k|ij}}{v_{ij}} \tag{38}$$

where $v_{ij} = \frac{w_j - c_{ij}}{W + t_{ij}}$ is the value of time for the i to j commuter and $\tau_{k|ij} = v_{ij} t_{k|ij} + c_{k|ij}$ is the generalized trip cost to location k for the i to j commuter. The ratio of (32) between two different locations k and t is

$$\frac{n_{k|ij}^{-(1-\rho)} a_k^{\rho} l_{k|ij}^{\alpha\rho}}{n_{t|ij}^{-(1-\rho)} a_t^{\rho} l_{t|ij}^{\alpha\rho}} = \frac{\mu_T(l_{k|ij} + t_{k|ij}) + \mu_M c_{k|ij}}{\mu_T(l_{t|ij} + t_{t|ij}) + \mu_M c_{t|ij}}$$

which can be further simplified using equation (38) to yield

$$\frac{n_{k|ij}^{-(1-\rho)} a_k^{\rho} \tau_{k|ij}^{\alpha\rho-1}}{n_{t|ij}^{-(1-\rho)} a_t^{\rho} \tau_{t|ij}^{\alpha\rho-1}} = 1$$

Rearranging, multiplying by $\tau_{t|ij}$ and summing across the t index leads to

$$\sum_t n_{t|ij} \tau_{t|ij} = n_{k|ij} a_k^{-\frac{\rho}{1-\rho}} \tau_{k|ij}^{\frac{1-\alpha\rho}{1-\rho}} \sum_t a_t^{\frac{\rho}{1-\rho}} \tau_{t|ij}^{-\frac{\rho(1-\alpha)}{1-\rho}}$$

which can be solved to find the relative number of trips to location k

$$n_{k|ij} = \frac{a_k^{\frac{\rho}{1-\rho}} \tau_{k|ij}^{-\frac{1-\alpha\rho}{1-\rho}}}{\sum_t a_t^{\frac{\rho}{1-\rho}} \tau_{k|ij}^{\frac{-\rho(1-\alpha)}{1-\rho}}} \sum_t n_{t|ij} \tau_{t|ij} \tag{39}$$

Similarly, the ratio of (34) and (35) gives an equation for the relative consumption expenditures

$$\frac{g_{ij}}{h_{ij}r_i} = \frac{\gamma}{1-\gamma} \tag{40}$$

The next step consists in finding the value for total expenditure. Multiplying equation (32) by $n_{k|ij}$ and summing across location, we find that

$$(1-\beta)L_{ij}^{1-\beta}C_{ij}^{\beta} = \mu_T \left(\sum_k n_{k|ij}(t_{k|ij} + l_{k|ij}) \right) + \mu_M \sum_k n_{k|ij}c_{k|ij}$$

Additionally, equations (34) and (35) can be combined to yield

$$\beta L_{ij}^{1-\beta}C_{ij}^{\beta} = \mu_M(g_{ij} + r_i h_{ij})$$

The ratio of these two last equations is

$$\frac{1-\beta}{\beta} = \frac{\mu_T \sum_k n_{k|ij}(t_{k|ij} + l_{k|ij}) + \mu_M \sum_k n_{k|ij}c_{k|ij}}{\mu_M(g_{ij} + r_i h_{ij})}$$

which can be simplified to

$$g_{ij} + r_i h_{ij} = \beta v_{ij} T \tag{41}$$

using equations (33), (36) and (37).

Combining the previous result with equation (41), we can find an equation for total housing consumption

$$h_{ij} = \frac{\beta(1-\gamma)v_{ij}T}{r_i} \tag{42}$$

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$$g_{ij} = \beta\gamma v_{ij} T \tag{43}$$

Next, combining equations (33), (36), (37) and (38), we can find an expression for the total generalized trip costs

$$\sum_k n_{k|ij}\tau_{k|ij} = (1-\alpha)(1-\beta)v_{ij}T \tag{44}$$

which allow us to pin down the number of non-commuting trips in (39). Finally, indirect utility is proportional to

$$V_{ij} \propto r_i^{\beta(\gamma-1)} A_{ij}^{\frac{(1-\rho)(1-\beta)}{\rho}} v_{ij}^{(1-\alpha)(1-\beta)+\beta} \tag{45}$$

where $A_{ij} = \left(\sum_k a_k^{\frac{\rho}{1-\rho}} \tau_{k|ij}^{-\frac{\rho(1-\alpha)}{1-\rho}} \right)$ and $v_{ij} = \frac{w_j W - c_{ij}}{W + t_{ij}}$