Ideas and Firm Dynamics

When It Takes Two to Tango

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August 31, 2025

Abstract

Idea production is increasingly performed by teams of inventors rather than individuals. We study two-way interaction between idea team production and firm dynamics, examining how firms shape teams and how the rise of teams impacts firm dynamics. Using U.S. patent data in 1976-2020, we find the increasing trend of team innovation, 90% of which is driven within firms; team patent and quality rising in the number of inventors; and team characteristics varying with firm size, productivity, and age. Motivated by these facts, we develop a tractable framework microfounding inventor team formation within firms and show that team-based production can increase returns to scale in R&D labor, raising the optimal size of innovative firms. Embedding this framework into a canonical model of frictional labor market, we find that team invention increases tightness in the inventor labor market, increases R&D labor concentration in large firms, lowers the R&D employment share of small firms, and raises large firm wage premiums for knowledge workers, consistent with the data.

Keywords: innovation, R&D team, inventor allocation, firm dynamics, economic growth

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1 Introduction

The development of new ideas is a key driver of economic growth. Ideas can come either from individual inventors or from collaborations among teams of inventors. Teams have become increasingly important in the production of new ideas (Jones 2021; Pearce 2023), and firms play a critical role in determining collaborations between inventors. However, little is known about the process of inventor team formation within firms nor about its implications for firm dynamics, worker allocation, and macroeconomic outcomes. In this paper, we aim to fill this gap by studying the two-way interaction between inventor team formation and firm heterogeneity and dynamics.

We begin by using the USPTO patent database linked to Compustat and documenting stylized facts about inventor teams. First, patents developed by teams now account for 80% of all private patents in the U.S., compared to just 50% in 1980. Second, 90% of team collaborations are by inventors at the same firm. Thus, understanding the role of firms is critical to understanding team idea production. We find that patenting output per worker, measured by citation-weighted patents, rises with the number of inventors within a firm. This motivates our approach to modeling team idea production as increasing returns to scale in R&D labor. Additionally, we observe that team characteristics vary across firm size, productivity, and age distributions: small, less productive, or young firms are more likely to patent through solo inventors or small teams, and team size rises with firm size and age.

We then propose a novel and tractable way of modeling team invention within firms. Each worker draws a bilateral productivity from potential collaborations with other workers at the firm, and an individual's realized productivity is the maximum of these draws. Consequently, workers in large firms have higher expected productivity, as they have more chances to find a good match (e.g., a colleague with a complementary set of skills). With particular functional form assumptions, expected worker productivity grows in the number of employees at a diminishing rate and can easily be incorporated into a standard production function to resemble less-decreasing returns to scale in the

number of R&D workers.

Building on this microfoundation of team formation, we analyze how the shift from solo to team-based invention influences firm-level decisions, the aggregate allocation of inventors across firms, and firm distribution. We find that team-based invention increases the optimal scale of innovative firms, resulting in a higher concentration of R&D workers in large firms and a greater share of large firms. These findings are consistent with recent empirical evidence (Akcigit and Ates 2019; Akcigit and Goldschlag 2023a).

However, given another data evidence of wage differentials of inventors across firms and declining inventor mobility, inventor labor markets likely exhibit a certain degree of frictions.[1] Taking this view seriously, we embed our simple framework into a canonical model of a frictional labor market (Elsby and Michaels 2013). In this full-fledged model, a shift from solo to team-based invention generates the same results as in the frictionless model, with the exception of its effects on labor market tightness, wage differentials across firms, and inventor flows. Specifically, the transition to team production increases labor market tightness in the inventor market, raises the employment share of large firms and the wage premium paid at larger firms. This makes it harder for productive young firms to find workers and grow, with their growth rate declining from 2.74% to 2.26%. Consequently, the separation rate falls, and the hiring inaction region widens, reducing inventor mobility.

Related Literature A growing body of literature examines the match between individual inventors and firms (Akcigit and Goldschlag 2023a; Akcigit and Goldschlag 2023c; Babalievsky 2023; Manera 2022; Liu 2023). A separate strand of literature models firms as collections of coworkers or co-inventors, without incorporating additional dimensions of firm heterogeneity (Herkenhoff et al. 2024; Pearce 2023; Freund 2024). This paper bridges the gap between these two approaches by exploring how inventor team formation varies with firm-level characteristics and connects to firm dy-

[1] Akcigit and Goldschlag (2023b) document significant wage dispersion among inventors, with otherwise similar individuals earning substantially higher wages when they join large firms. They also show that inventor mobility has declined since 2000.

namics and inventor allocation aross firms.[2]

Our paper is also related to the literature on how interactions between individuals shape economic growth (Lucas and Moll 2014; Akcigit et al. 2018; Prato 2022). Relative to these papers, we abstract from the dynamics of individual-level knowledge accumulation and instead focus on the organizational features that may determine different forms of knowledge production at the firm level (Garicano and Rossi-Hansberg 2015). Our model predicts that R&D workers will increasingly concentrate at large firms, which highlights the importance of understanding how human capital accumulation among R&D workers occurs. If variety in employers and team members is a key source of knowledge accumulation, the trend toward greater worker concentration at large firms could pose challenges. On the other hand, the shift from solo invention to more collaborative efforts may accelerate on-the-job human capital accumulation for knowledge workers as they learn from each other through collaboration.

Several empirical papers highlight the importance of inventor teams within firms. Jaravel, Petkova, and Bell (2018) find that the death of a co-inventor negatively impacts subsequent inventor productivity. Bhaskarabhatla et al. (2021) show that a significant share of firm-level heterogeneity in research productivity is driven by differences in inventor quality. Kline et al. (2019) demonstrate that firms share around 30% of the rents from patenting with incumbent and high-paid employees, implying that a large share of the rents from patenting are retained by firm owners. Our paper builds on this literature by linking team innovation to firm dynamics, examining how firms shape inventor teams, and quantifying the impact of the rise of teams on firm-level as well as aggregate outcomes.

Finally, our findings contribute to understanding trends in productivity differences between large and small firms, particularly in the context of research productivity (Andrews, Criscuolo, and Gal 2016; Bessen et al. 2020; Pugsley, Sedlacek, and Sterk 2021; Cavenaile, Celik, and Tian 2020; Olmstead-Rumsey 2022; Akcigit and Ates 2023). Also

[2] Jarosch, Oberfield, and Rossi-Hansberg (2021) and Boerma, Tsyvinski, and Zimin (2023) study models of sorting into teams within firms, but not specifically for inventors.

our findings are related to the recent trends of declining business dynamism and high-growth young firm activities (Decker et al. 2014; Decker et al. 2016; Sterk, Sedláček, and Pugsley 2021; Akcigit and Ates 2023).

2 Empirics

2.1 Data

Our primary data sources are the USPTO PatentsView and S&P's Compustat. The USPTO PatentsView tracks all patents granted by the U.S. Patent and Trademark Office (USPTO) from 1976 onward. This database provides detailed information, including application and grant dates, technology class classifications, patent inventors, citation, and the names and addresses of patent assignees. Compustat is a comprehensive database maintained by S&P Global and provides detailed firm-level data for publicly traded companies. It includes information such as firm size, industry scope, balance sheets, income and cash flow statements, and stock prices.

We collect utility patents filed between 1976 and 2018, granted by the USPTO. To identify the patents assigned to US public firms and track them by different firm characteristics, we link the USPTO assignee identifier to the Compustat GVKEY using the bridge provided by Ding, Jo, and Kim (2022) and Braguinsky et al. (2023).[3] When a patent is assigned to multiple entities, we only keep the first assignee following the sequence order recorded by USPTO.

To measure the importance of a patent, we calculate five-year forward citations with vintage fixed effects removed, and obtain real patent values from Kogan et al. (2017) (KPSS values, hereafter). To obtain an accurate estimate of firm age, we also link PatentsView with the dataset provided by Ewens and Marx (2024) to obtain the firm's

[3]This bridge is constructed using the standard name and address matching process, complemented by an internet search-aided algorithm following Autor et al. (2020) as well as additional manual matching. The detailed methodology is described in Ding, Jo, and Kim (2022) and Braguinsky et al. (2023).

founding year.[4]

2.2 Stylized Facts

With this dataset, we document the following facts on team patenting, its properties, and its relationship to firms.

Fact 1: Team patents have increased from 50% in 1976 to 80% in 2018

Teams have played an increasingly dominant role in innovations over the past five decades. A patent is identified as a team patent if it has multiple inventors, and the group of inventors filing the same patent is identified as a team. Among all patents in the dataset, the share of team patents has risen from below 50% in 1976 to above 80% in 2018. This trend holds either measured in patent counts, KPSS values, or five-year forward citations, as shown in Figure 1.[5]

Fact 2: Firms shape teams; 90% of team patents are by inventors at the same firm

At the same time, most team collaboration in innovation occurs within firms rather than between them, highlighting the role of firms as a critical platform for team collaboration. We identify an inventor's affiliation in a given year as the firm with the largest number of patents filed by that inventor during that year. In the event of a tie, we assign the affiliation to the firm associated with the patent having the latest filing date. In the dataset, 89.5% of the team patents are filed by inventors affiliated with the same firm.

[4]Eventually, we aim to incorporate administrative data from the U.S. Census Bureau to extend our analysis, which is currently in progress.

[5]In this figure, we restrict our sample to Compustat firms with patent filings due to the use of KPSS values and to maintain consistency of the sample across series. The result is robust when using the full set of USPTO assignees.

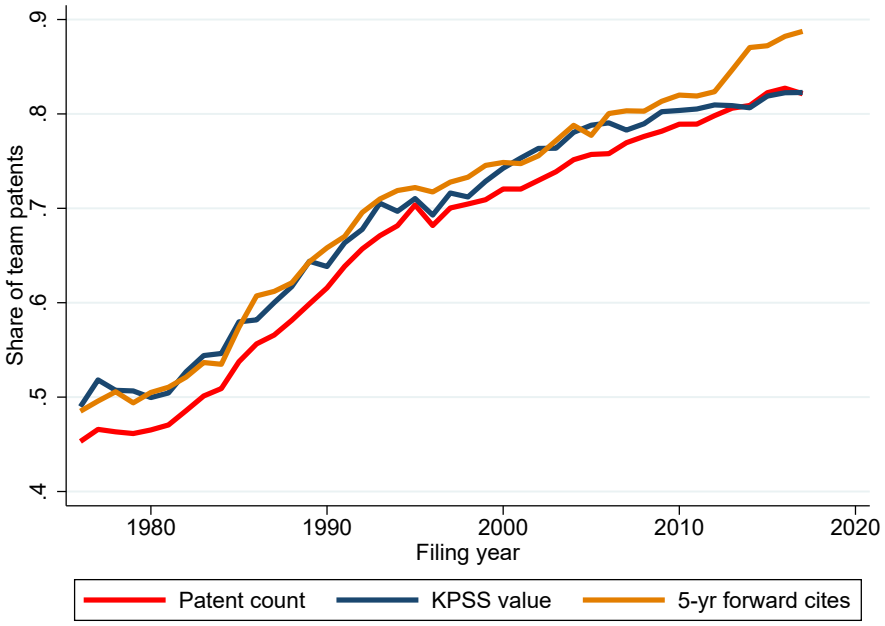


Figure 1: Team Patents Shares

Note: The figure represents the share of team patents at Compustat firms from 1976 to 2018. A team patent is defined as one assigned by multiple inventors. The number of patents is counted unweighted (red line), weighted in KPSS values (navy line) extracted from Kogan et al. (2017), and weighted in five-year forward citations with vintage fixed effects removed (orange line).

Fact 3: Patent output increases with the number of potential collaborators in a firm

For each assignee in the USPTO, we count the total number of inventors each year as the number of potential collaborators in a firm. We see how the number and output of patents in a firm depend on the number of collaborators in the firm.

First, for each assignee in a given year, we calculate patents per potential collaborator and then compute the average across pairs for each value of potential collaborators. Figure 2 presents a scatterplot of the average patent count against the number of potential collaborators. The figure shows that firms with a larger pool of potential collaborators tend to produce more patents, even after normalizing by the number of potential workers.

Next, we compute citations per potential worker for each firm in a given year and

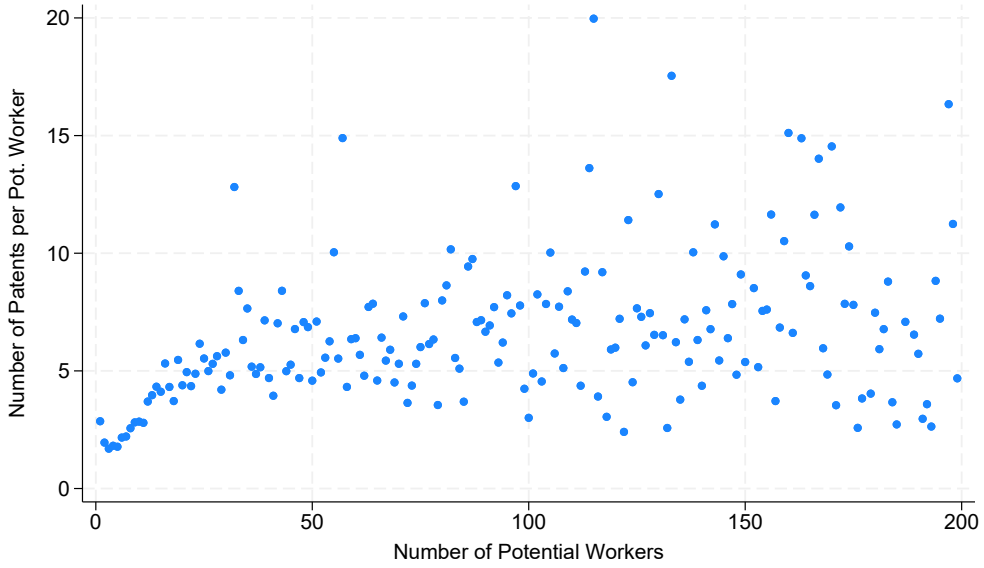


Figure 2: Patent Numbers and Potential Collaborators

Note: The figure shows a scatterplot of the number of potential workers (x-axis) against the average number of patents per potential worker (y-axis).

then take the average across firm-year pairs for each number of potential collaborators. Figure 3 presents the corresponding scatterplot, showing that firms with more potential collaborators also receive more citations per collaborator. This pattern remains robust with using adjusted citations—filtered by CPC technology class-year fixed effects—as shown in Figure 4.[6]

Fact 4: Larger firms have a larger share of team patents

The formation of teams varies systematically with firm size. Defining firm size by the number of inventors, we estimate the following regression to examine the relationship between firm size and the propensity to engage in team patenting:

$$TeamPatShare_{ft} = \eta_f + \gamma_t + \log(Num.Inventors)_{ft} + \epsilon_{ft} \tag{2.1}$$

[6]The adjustment accounts for potential effects driven by time trends or shifts in technology composition, which may arise from differences in citation distributions across technology classes or over time.

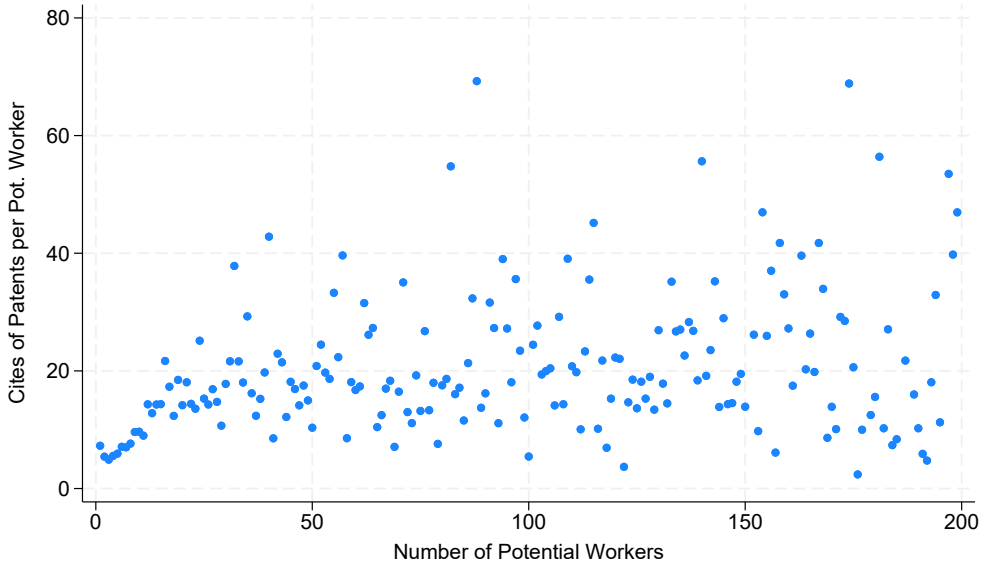


Figure 3: Patent Citations and Potential Collaborators

Note: The figure shows a scatterplot of the number of potential workers (x-axis) against the average citations per potential worker (y-axis).

where $TeamPatShare_{ft}$ is the share of team patents out of total patents assigned by firm f in year t; $\log(Num.Inventors)_{ft}$ is the log number of inventors within firm f in t; and η_f and γ_t are firm and year fixed effects, respectively.

Table 1 presents the results, where the first column uses all USPTO assignees, and the second column restricts the sample to Compustat firms. Across both samples, we find that larger firms exhibit a higher share of team patents than smaller firms. The effect is economically meaningful: a 1% increase in the number of inventors within a firm is associated with a 20.4 pp increase in the share of team patents for USPTO firms, and a 12 pp for Compustat firms.

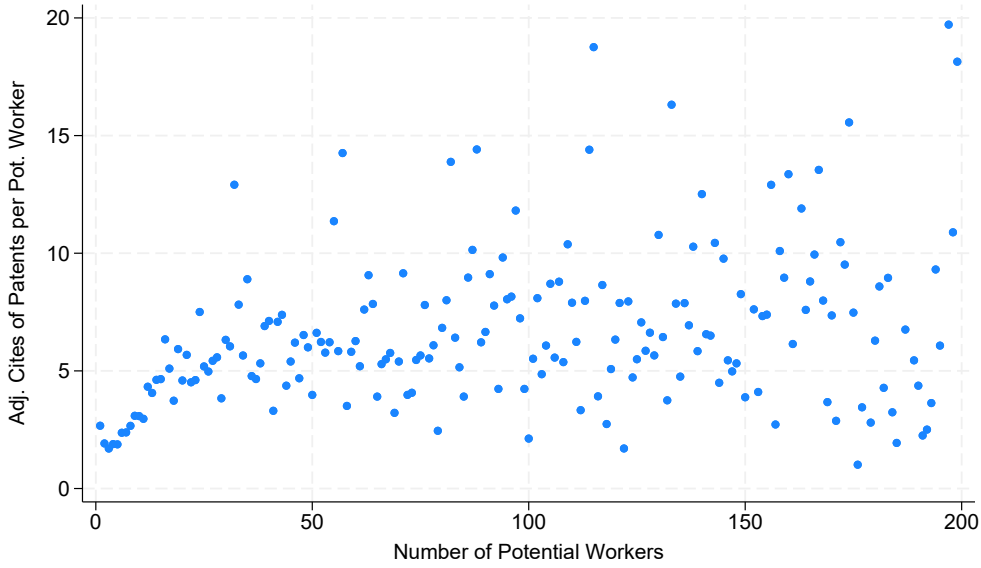


Figure 4: Patent Citations (Adjusted) and Potential Collaborators

Note: The figure shows a scatterplot of the number of potential workers (x-axis) against the average adjusted citations per potential worker (y-axis). The adjusted citations are calculated by filtering out technology class-year fixed effects.

Table 1: Team Patent Share and Firm Size

	All Firms	Public Firms
Log num. inventors	0.204^{***}	0.122^{***}
	(282.52)	(22.46)
N of Obs.	717,111	74,501
Firm FE	Yes	Yes
Year FE	Yes	Yes
Adj. R^2	0.491	0.375

Note: The table presents the regression result from (2.1), where the dependent variable is the share of team patents within a firm. The coefficient of interest is on the log number of inventors in the firm. Firm and year fixed effects are included, respectively. The first column covers all USPTO assignees, and the second column includes Compustat firms only. *** p<0.01, ** p<0.05, * p<0.1

Fact 5: Patent output in a firm increases in team patenting

We further examine the quality of team innovation using the citation-weighted number of patents at the firm level. To this end, we estimate the following regression:

$$\log(\text{CitationWgtPatents})_{ft} = \eta_f + \gamma_t + \text{TeamPatShare}_{ft} + X_{ft} + \epsilon_{ft}, \tag{2.2}$$

where $\log(\text{CitationWgtPatents})ft$ is the log of the citation-weighted number of patents of firm f in year t, and $\text{TeamPatShare}ft$ is the share of team patents. The vector X_{ft} includes firm-level controls such as sales, employment, and R&D expenses, whenever available. We estimate this regression for both the full sample of USPTO assignees and the subsample of Compustat firms.

Table 2: Team Patent Share and Citation-weighted Patents

Variable	All Firms	Public Firms
TeamPatShare	0.074^{***}	0.113^{***}
	(20.49)	(7.85)
Log past pats.	0.140^{***}	0.308^{***}
	(74.66)	(20.25)
Log sales	—	-0.019
		(-1.09)
Log emp.	—	0.256^{***}
		(6.64)
Log R&D	—	0.120^{***}
		(7.05)
N of Obs.	717,111	69,193
Firm FE	Yes	Yes
Year FE	Yes	Yes
Adj. R^2	0.535	0.665

Note: The table presents the regression result from (2.2), where the dependent variable is the log of citation-weighted patent counts within a firm. The coefficient of interest is on the share of team patents in the firm. Firm and year fixed effects are included, respectively. The first column covers all USPTO assignees, and the second column includes Compustat firms only. For Compustat firms, we include firm sales, employment size, and R&D expenditure as additional controls. *** p<0.01, ** p<0.05, * p<0.1

The results, reported in Table 2, show a positive association between the share of team patents and the citation-weighted number of patents. Specifically, the estimates suggest that if a firm were to shift from producing all patents solo to producing them entirely in teams, its citation-weighted patent output would increase by 7.4% among USPTO assignees and by 11.3% among Compustat firms.

Fact 6: Team size rises with firm size

Furthermore, there is heterogeneity in team size across firms of different size. To analyze this, we classify firms into three groups the number of inventors in a year as a proxy for firm size in the USPTO. We define firms as small if they have fewer than five inventors, medium-sized if they have between five and ten inventors, and large if they have more than ten inventors.

For each year, we calculate the average team sizes for these three groups and examine their variations over time, as shown in Figures 5. The pattern indicates that medium-sized and large firms consistently have larger teams than small firms.

Furthermore, this pattern also holds among Compustat firms, where more precise firm size information is available based on the number of employees. Figure 6 shows this pattern across firms of different employment size. The results suggest that larger firms have larger teams than smaller firms in Compustat.

Furthermore, we also investigate different firm characteristics, such as productivity and age. We find consistent results where more productive firms (with productivity defined as the number of patents in USPTO or revenue productivity in Compustat) tend to have larger teams compared to less productive firms. Also, we find a similar trend with respect to firm age, where younger firms tend to have smaller firm sizes compared to older firms. See more details in Appendix A.

Fact 7: Teams are short-lived

We also find the duration over which a team remains active in patenting. We find that most teams are short-lived: 80% of teams patent together once in our sample. This implies that serial teams are unlikely. Also, taken together with Fact 1, this finding suggests that there may be multiple variations or turnover of teams within firms.

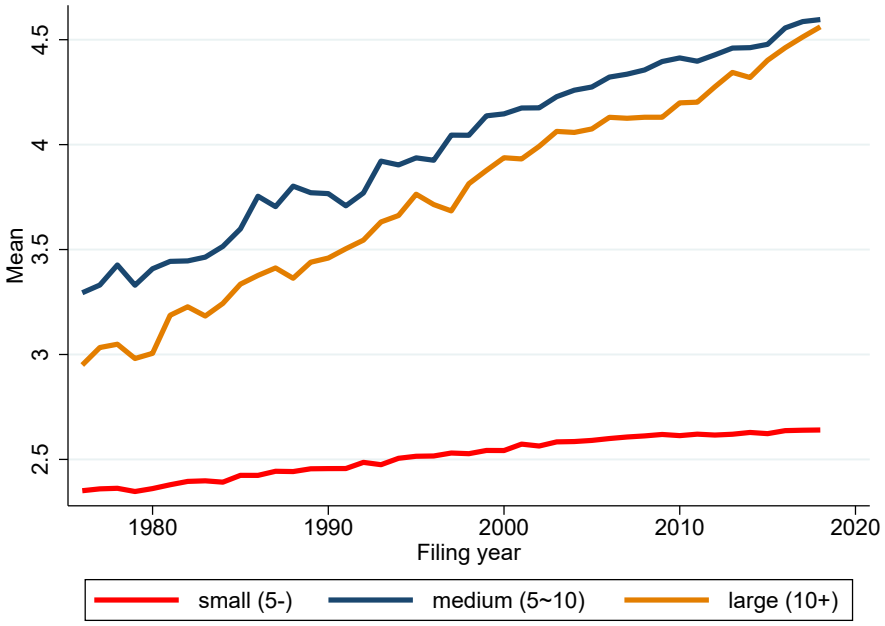


Figure 5: Average Team Sizes in a Firm (By Number of Inventors)

Note: The figure illustrates the average team size in firms, categorized by firm size groups based on the number of inventors. Small firms are those with five or fewer inventors (red line), medium-sized firms are those filing between five and ten inventors (navy line), and large firms are those with more than ten inventors (orange line) in the USPTO data.

Fact 8: Firms are less likely to be single-inventor firms now than in the past

We explore how the fraction of patenting firms with a single inventor in a given year has changed over time. Figure 7 illustrates the decreasing trend of this ratio over time. In addition, we find that this trend also holds amongst young firms given the fact that young firms are likely to be single-inventor firms. See more details in Appendix B.

On top of these findings, we also document the rising trend of team diversity and complementarities within teams in the Appendix. See Appendix C and D for details.

3 Simple Model

In this section, we construct a simple static model that provides a micro-foundation for team production and explains the role of firms in shaping it. The model also helps ac-

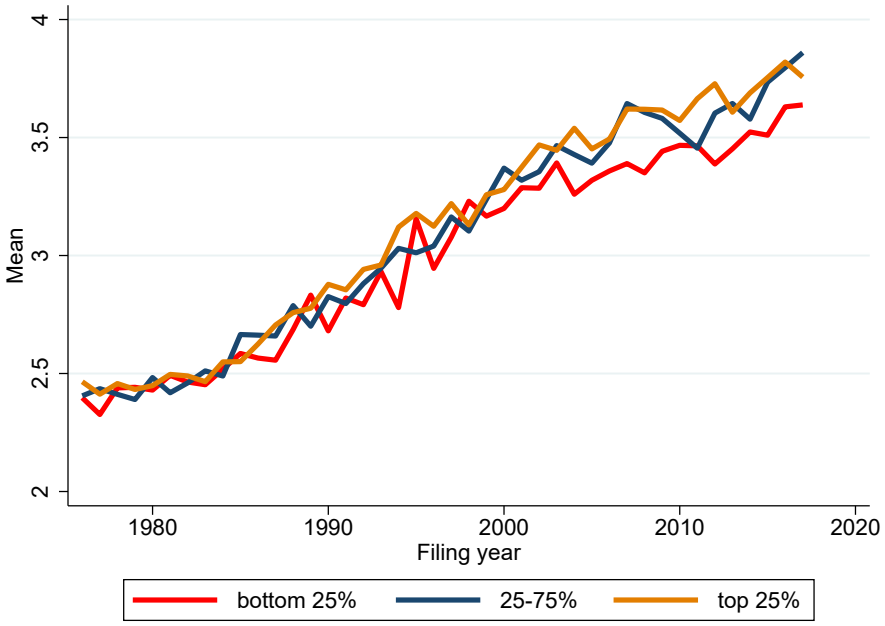


Figure 6: Average Team Sizes in a Firm over Years (By Firm Employment)

Note: The figure illustrates the average team size in firms, categorized by firm employment size. Firms are classified by their percentile of the cross-sectional size distribution within a pair of industry (NAICS4) and year. Top 25% (red line) indicates the top 25 percentile firms, 25-75% (navy line) indicates firms between 25 and 75 percentiles, and bottom 25% (orange line) indicates those in the bottom 25 percentile in employment size distribution. The solid lines represent the mean team size across firms, while the short-dashed lines indicate the median team size for firms in each group in a given year.

count for the stylized facts observed in the data. In this version, we prioritize parsimony to enhance expositional clarity.

3.1 Team Production

Suppose worker i has the following productivity:

$$\begin{cases} z_{iit} = \bar{z} & \text{if solo} \\ z_{ijt} \sim P(x_{min}^{team}, \gamma^{team}) & \text{if working with another worker } j, \end{cases} \tag{3.3}$$

where a worker can either operate independently or collaborate with another worker within a firm. When working alone, productivity is constant at $\bar{z}$. When collaborating,

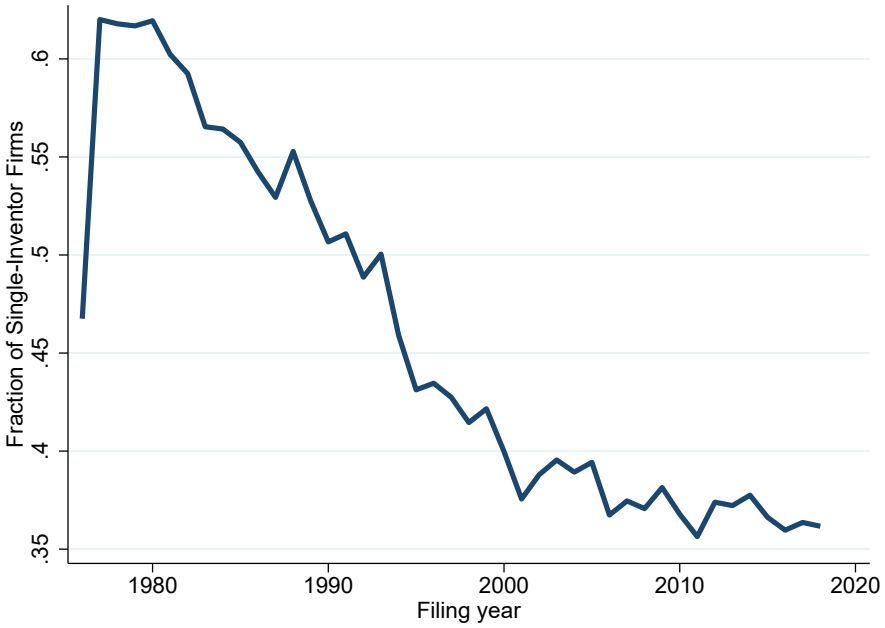


Figure 7: Fraction of Single-Inventor Firms

Note: The figure shows the fraction of patenting firms with a single inventor filing patents in a given year.

the workers draws a symmetric, pairwise productivity z_{ijt} from a Pareto distribution $P(x_{min}^{team}, \gamma^{team})$.[7]

Now assume that firms can observe all potential collaborations among their workers and assign each worker to the co-worker who yields the highest pairwise productivity. This defines team productivity z_{it}^{team} for worker i within a firm, which is the highest productivity the worker obtains amongst possible collaborations he can form within a firm. The team productivity for worker i in a firm of size n is given by

$$z_{it}^{team} = \max\{z_{i1t}, z_{i2t}, ..., z_{int}\}. \tag{3.4}$$

Based on the Pareto distribution, the expected team productivity of worker i in a firm

[7]For simplicity, we assume each potential collaboration consists of only two workers. This is consistent with empirical evidence showing that most team inventions are done by small teams.

of size n can be derived as

$$\mathbb{E}(z_{it}^{team}) = x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})n^{\frac{1}{\gamma^{team}}}. \tag{3.5}$$

See Appendix E.1 for derivation. Note that this property also holds with a continuum of workers by interpreting n as the mass of workers rather than a discrete count. See details in Appendix E.2. This provides a tighter connection to our model, which assumes a continuum of workers.

The expected team productivity in (3.5) increases with firm size n, the scale parameter x_{min}^{team}, and the inverse of the tail parameter γ^{team} of the Pareto distribution. Figure 8 presents this feature. At the worker level, team productivity exceeds solo productivity when the worker belongs to a sufficiently large firm (with size n). Figure 9 illustrates this relationship, showing a firm-size cutoff above which workers attain higher productivity in teams than when working alone. Therefore, workers in larger firms benefit more from collaborating in teams than from working alone. This provides a potential mechanism supporting the empirical evidence of increasing inventor teams at larger firms.

3.2 Firms

Given this structure, we can now characterize the firm's problem. Let firm productivity z_{ft} be determined by the average worker productivity within the firm, combined with an idiosyncratic firm-level shock ε_{ft}. Since workers are homogeneous, the average productivity of workers in a firm coincides with the productivity of any individual worker within that firm. Equation (4.9) characterizes the firm productivity.

$$z_{ft} = \begin{cases} \bar{z}\varepsilon_{ft} & \text{w/ solo production} \\ \underbrace{x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})n_{ft}^{\frac{1}{\gamma^{team}}}}_{=\mathbb{E}(z_{it}^{team})}\varepsilon_{ft} & \text{w/ team production.} \end{cases} \tag{3.6}$$



Figure 8: Expected Team Productivity $\mathbb{E}(z_{it}^{team})$

Note: The figure illustrates the expected team productivity by firm size (top) and by different values of scale (middle) and tail (bottom) parameters from the Pareto distribution.

Firms (f) maximize profits and decide optimal size by taking wages as given.

$$\max_{n_{ft}} z_{ft} n_{ft}^{\alpha} - w_t n_{ft},$$

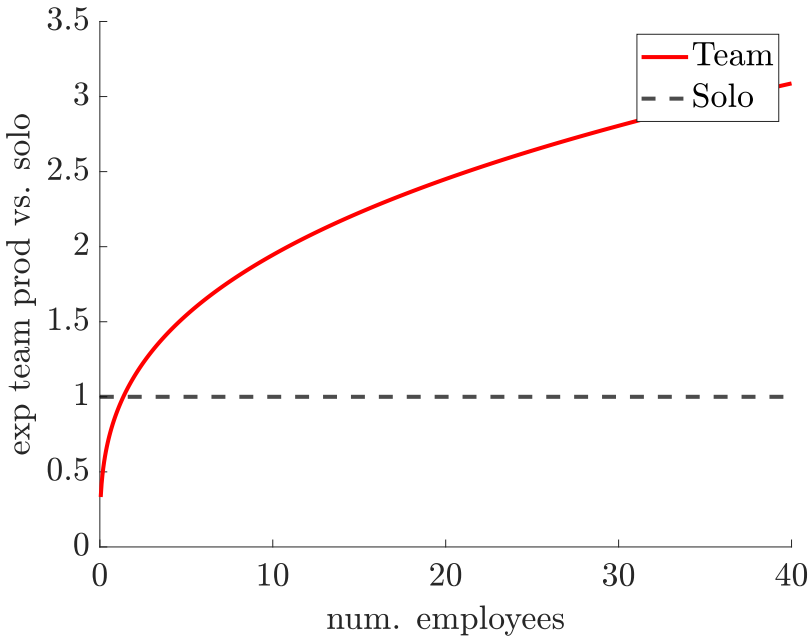


Figure 9: Team vs Solo Productivity

Note: The figure compares worker-level productivity under team and solo production. The red solid line indicates team production, while the black dashed line shows solo production.

where n_{ft} denotes firm size and w_t the wage. Note that, under team production, firm productivity increases with firm size, as shown in (4.9). Consequently, switching from solo to team production generates higher returns to scale for firms. This is because the probability of drawing better productivity for forming a team increases with the number of workers within a firm, which provides more potential collaborations.

The remaining steps are standard. The optimal size of firms under solo and team production can be derived, respectively, as follows:

$$n_{ft}^* = \begin{cases} \left(\frac{\alpha z_{ft}\bar{z}}{w_t}\right)^{\frac{1}{1-\alpha}} & \text{w/ solo production} \\ \left(\frac{(\frac{1}{\gamma^{team}}+\alpha) z_{ft} x_{min}^{team} \Gamma(1-\frac{1}{\gamma^{team}})}{w}\right)^{\frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)}} & \text{w/ team production.} \end{cases} \tag{3.7}$$

3.3 Equilibrium Wage

For now, we assume a frictionless labor market with a fixed labor supply $\bar{N}$ to highlight the fundamental differences between solo and team production and their effects on

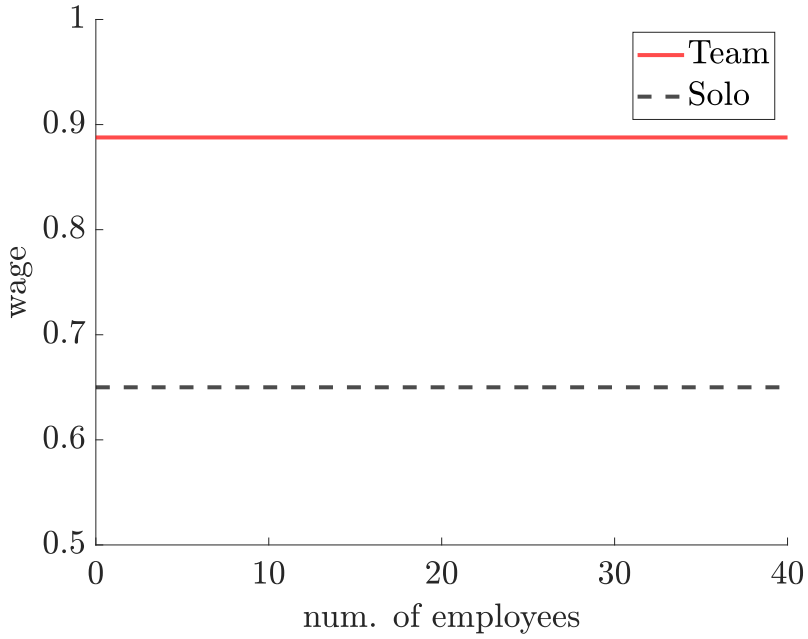


Figure 10: Equilibrium Wage under Team vs Solo Production

Note: The figure shows the equilibrium wages under team and solo production. The red solid line indicates team production, while the black dashed line shows solo production.

firm labor demand and labor reallocation.[8] Under this assumption, equilibrium wages are determined by the labor market clearing condition:

$$\int n^*(z_{ft}, \bar{z}; w) dg(z_{ft}) = \bar{N},$$

where $n^*(z_{ft}, \bar{z}; w) \equiv n^*_{ft}$ in (3.7).

3.4 Model Implications

Suppose there are two types of economies: one with only solo production and the other with team production. We derive the equilibrium outcomes under each type of economy.

[8] In the next section, we extend the analysis to a frictional labor market.

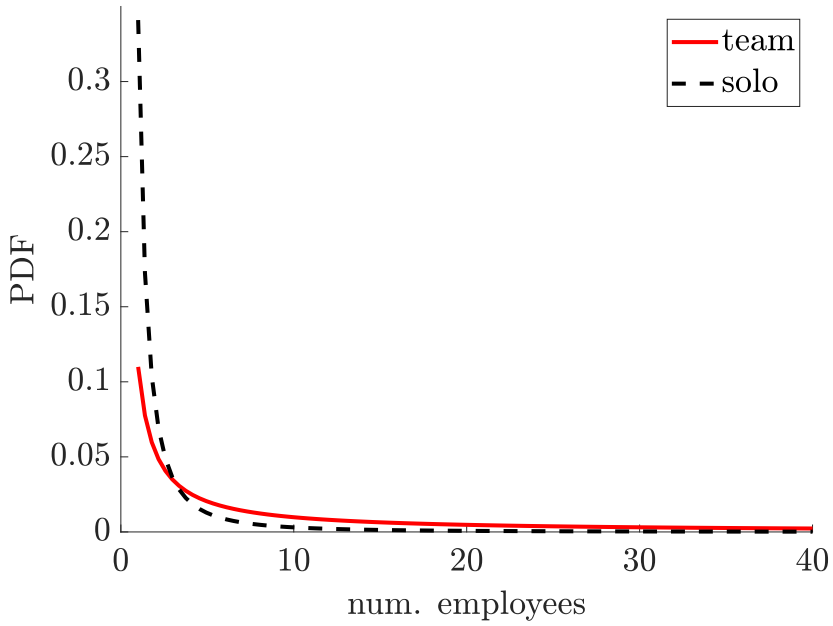


Figure 11: Size Distribution under Team vs Solo Production

Note: The figure shows the firm size distribution under team and solo production. The red solid line indicates team production, while the black dashed line shows solo production.

Figure 10 presents the equilibrium wages, and Figure 11 shows the firm size distribution, under solo and team production. The red solid line indicates for team production, and the black First, the equilibrium wage rises with the transition from solo to team production. This is due to having increased returns to scale with team production, which increases labor demand across firms and the equilibrium level of the wage. Second, under team production, the firm size distribution is more skewed toward larger firms. This is also intuitive, as the increased returns to scale are more pronounced for larger firms, which are better able to achieve higher team productivity with a larger set of workers. Although higher wages tend to reduce labor demand across firms, this effect is offset by the increased returns to scale from team production, particularly for larger firms. As a result, switching from solo to team production reallocates labor toward larger firms, consistent with the observed increase in inventor concentration at larger firms in the data.

4 Full-fledged Model

We extend the simple model to account for the frictional nature of the inventor labor market (Akcigit and Goldschlag 2023c).[9] To do so, we extend Elsby and Michaels (2013) to include the team production framework in (4.9).

4.1 Set-up

The main setup of the model follows Elsby and Michaels (2013). The model is in discrete time and has an infinite horizon, in which firms use labor to produce (innovative) output. We focus on the steady-state equilibrium of the model. To simplify notation, we omit the time subscript whenever possible, while using the superscript $'$ to denote variables in the next period $(t+1)$, and the subscript -1 to denote variables from the previous period $(t-1)$.

Workers are homogeneous and can only search from the unemployment pool. A mass of labor force is fixed to $\bar{N}$. Firms are multi-worker and face search frictions. Each firm must pay a constant vacancy cost c. A mass of firms is normalized to one, and there is no firm entry and exit.

Let U denote the total number of unemployed workers and V the total number of vacancies in the economy. Search is assumed to be random for both workers and firms. Labor market frictions are captured by a matching function, which determines the number of successful matches,

$$M(U,V) = \mu U^{\phi} V^{1-\phi}. \tag{4.8}$$

Since not all searching workers find jobs and not all vacancies are filled, the matching function implies a degree of frictions in the labor market. It also characterizes job finding rate for workers, $f(\theta) = \frac{M}{U}$ and job filling rate for firms, $q(\theta) = \frac{M}{V}$, where labor market tightness is defined as $\theta = \frac{V}{U}$. Thus, the total number of hires in the economy

[9] Akcigit and Goldschlag (2023c) find that large firms pay 12% higher wages to a given inventor, and inventor mobility across firms has declined since 2000.

is constrained by the matching function.

4.2 Firms

Firms operate with the following production function:

$$y = \varphi z n^{\alpha}, \quad z = \begin{cases} \bar{z}\varepsilon & \text{w/ solo production} \\ x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})n^{\frac{1}{\gamma^{team}}}\varepsilon & \text{w/ team production} \end{cases}, \quad \varepsilon' \sim G(\varepsilon', \varepsilon), \tag{4.9}$$

where $\varphi \sim P(x_{min}^{\varphi}, \gamma^{\varphi})$ is a time-invariant productivity type, drawn from Pareto distribution, and ε' follows a persistent and mean-reverting process.[10,11] Specifically, we follow Elsby and Michaels (2013) and assume that the idiosyncratic shock ε evolves according to:

$$\varepsilon' = \begin{cases} \varepsilon & \text{with probability } 1-\lambda, \\ \varepsilon' \sim P(x_{min}^{\varepsilon}, \gamma^{\varepsilon}) & \text{with probability } \lambda, \end{cases} \tag{4.10}$$

which specifies the process $G(\varepsilon', \varepsilon)$, where $\lambda < 1$ captures the persistence of the shock, and innovations are drawn from a Pareto distribution with scale and tail parameters $(x_{min}^{\varepsilon}, \gamma^{\varepsilon})$. We also assume $\alpha < 1$, where the production function exhibits decreasing returns to scale.

For now, we assume that firms operate either with solo or team production, which is exogenously taken as given in the economy. Under each solo or team production, firms realize their productivity shock ε each period and make decision to employ workers. They can grow by hiring workers or contract by separating workers or stay inactive given the existence of constant vacancy cost. If firms want to hire workers, they need to post vacancies v by paying the per-vacancy cost c. The vacancies are matched with unemployed workers through the matching function (4.8). After the matching process,

[10] Having a permanent firm type allows us to match the firm size distribution in the data more accurately, without materially affecting the derivation of the optimal employment decision or the equilibrium outcomes.

[11] Here, we assume that firms make an ex-ante decision based on expected team productivity with their choice of n.

firms perform production and wage setting.

A firm's value function (given type φ) is characterized as follows:

$$\Pi(n_{-1}, \varepsilon, \varphi) = \max_{n,v} \left\{ y(n, \varepsilon, \varphi) - w(n, \varepsilon, \varphi)n - cv + \beta \int \Pi(n, \varepsilon', \varphi) dG(\varepsilon'|\varepsilon) \right\}, \quad (4.11)$$

where $y(n, \varepsilon, \varphi)$ denotes the production function given in (4.9), and $w(n, \varepsilon, \varphi)$ is the wage determined through bargaining for a firm of size n and productivity ε with type φ.[12] Note that, due to search frictions in the labor market, the hiring of workers is subject to the following constraint:

$$n = n_{-1} + vq(\theta), \quad \text{if } n > n_{-1}. \quad (4.12)$$

Rephrasing (4.11) with (4.12) gives

$$\Pi(n_{-1}, \varepsilon, \varphi) = \max_{n} \left\{ y(n, \varepsilon, \varphi) - w(n, \varepsilon, \varphi)n - c\frac{(n - n_{-1})}{q(\theta)}\mathbb{I}^{+} + \beta \int \Pi(n, \varepsilon', \varphi) dG(\varepsilon'|\varepsilon) \right\}, \quad (4.13)$$

where $\mathbb{I}^{+}$ indicates a dummy for hiring firms ($n > n_{-1}$). Note that the value function exhibits a kink at $n = n_{-1}$, which generates an inaction range in which firms optimally choose to remain inactive—neither hiring nor separating workers—for a certain range of productivity ε and initial size n_{-1}.

4.3 Workers

The value function of unemployed workers is as follows

$$\Gamma = b + \beta \mathbb{E}_{(n', \varepsilon', \varphi)}[(1 - f(\theta))\Gamma' + f(\theta)W(n', \varepsilon', \varphi)], \quad (4.14)$$

where b is unemployment benefits or leisure value to a worker, $W(n', \varepsilon', \varphi)$ is the value of being employed at a firm of size n' and productivity ε' with type φ, and $\mathbb{E}_{(n', \varepsilon', \varphi)}$ denotes

[12] The bargaining protocol will be specified in the next subsection.

the expectation over the distribution of $(n', \varepsilon', \varphi)$ among firms posting vacancies.

The value of employed workers at a firm of size n, productivity ε, and type φ is given by

$$W(n, \varepsilon, \varphi) = w(n, \varepsilon, \varphi) + \beta \mathbb{E}_{\varepsilon'|\varepsilon}[(1 - s)W(n', \varepsilon', \varphi) + s\Gamma'], \tag{4.15}$$

where $w(n, \varepsilon, \varphi)$ is the bargained wage at the firm, s is separation probability, which is endogenously determined by the firm's problem (4.13), and $\mathbb{E}_{\varepsilon'|\varepsilon}$ denotes the expectation over ε' given ε. Note that if the worker is laid off from the firm, she returns to the unemployment pool and receives the value Γ'.

4.4 Wage Setting

We follow Stole and Zwiebel (1996) and Elsby and Michaels (2013) to specify a wage bargaining protocol in the context of multi-worker firms with decreasing returns to scale. As emphasized by Elsby and Michaels (2013), the marginal surplus of a worker within a firm differs between the marginal worker and infra-marginal workers when production exhibits decreasing returns to scale. To account for this, we adopt the bargaining protocol from Stole and Zwiebel (1996), which implements Nash bargaining over the marginal surplus of a match.[13]

The marginal surplus of a worker to a firm (after the firm's employment decision has been made) is given by:

$$J(n, \varepsilon, \varphi) = y(n, \varepsilon, \varphi) - w(n, \varepsilon, \varphi) - w_n(n, \varepsilon, \varphi)n + \beta \int \frac{\partial \Pi(n, \varepsilon', \varphi)}{\partial n} dG(\varepsilon'|\varepsilon), \tag{4.16}$$

where the vacancy cost is already sunk at this point.

Similarly, the marginal surplus for a worker employed at a firm of size n and productivity ε is

$$W(n, \varepsilon, \varphi) - \Gamma,$$

[13] Conceptually, this corresponds to the firm negotiating with each worker in turn, where the breakdown of a negotiation with any individual worker triggers the renegotiation of wages with all remaining workers.

where $W(n, \varepsilon, \varphi)$ and Γ are defined in (4.15) and (4.14), respectively.

Wages are determined through the Nash bargaining between a firm and its workers over the marginal surplus as follows:

$$(1-\eta)(W(n,\varepsilon,\varphi)-\Gamma) = \eta J(n,\varepsilon,\varphi), \tag{4.17}$$

where η denotes the worker's bargaining power. Thus, the bargained wage between a firm of size n, productivity ε, and type φ and workers is the solution of the following differential equation:

$$w(n,\varepsilon,\varphi) = \eta\Big[y(n,\varepsilon,\varphi) - w_n(n,\varepsilon,\varphi)n + \beta f(\theta)\frac{c}{q(\theta)}\Big] + (1-\eta)b. \tag{4.18}$$

4.5 Optimal Employment Decision

To obtain the firm's optimal employment decision, we revisit (4.13) and get the first-order condition with respect to n. Combining it with the wage equation (4.18) yields the firm's optimal employment decision as follows:

$$n(n_{-1},\varepsilon,\varphi) = \begin{cases} R_v^{-1}(\varepsilon|\varphi), & \text{if } \varepsilon > R_v(n_{-1},\varphi) \\ n_{-1}, & \text{if } \varepsilon \in [R(n_{-1},\varphi), R_v(n_{-1},\varphi)] \\ R^{-1}(\varepsilon|\varphi), & \text{if } \varepsilon < R(n_{-1},\varphi), \end{cases} \tag{4.19}$$

where the functions $R_v(\cdot)$ and $R(\cdot)$ are determined by the followings; if firms operate in solo production,

$$(1-\eta)\Big[\frac{\varphi\bar{z}R_v(n,\varphi)\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\Big] - \eta\beta f(\theta)\frac{c}{q(\theta)} + \beta\int\frac{\partial\Pi(n,\varepsilon',\varphi)}{\partial n}dG(\varepsilon'|R_v(n,\varphi)) \equiv \frac{c}{q(\theta)}, \tag{4.20}$$

$$(1-\eta)\Big[\frac{\varphi\bar{z}R(n,\varphi)\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\Big] - \eta\beta f(\theta)\frac{c}{q(\theta)} + \beta\int\frac{\partial\Pi(n,\varepsilon',\varphi)}{\partial n}dG(\varepsilon'|R(n,\varphi)) \equiv 0; \tag{4.21}$$

and if firms work with team production,

$$(1-\eta)\Big[\frac{\varphi x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})R_v(n,\varphi)(\alpha+\frac{1}{\gamma^{team}})n^{\alpha+\frac{1}{\gamma^{team}}-1}}{1-\eta(1-\alpha-\frac{1}{\gamma^{team}})}-b\Big]-\eta\beta f(\theta)\frac{c}{q(\theta)}$$
$$+\beta\int\frac{\partial\Pi(n,\varepsilon',\varphi)}{\partial n}dG(\varepsilon'|R_v(n,\varphi))\equiv\frac{c}{q(\theta)}, \tag{4.22}$$

$$(1-\eta)\Big[\frac{\varphi x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})R(n,\varphi)(\alpha+\frac{1}{\gamma^{team}})n^{\alpha+\frac{1}{\gamma^{team}}-1}}{1-\eta(1-\alpha-\frac{1}{\gamma^{team}})}-b\Big]-\eta\beta f(\theta)\frac{c}{q(\theta)}$$
$$+\beta\int\frac{\partial\Pi(n,\varepsilon',\varphi)}{\partial n}dG(\varepsilon'|R(n,\varphi))\equiv 0. \tag{4.23}$$

There are several things to note from the decision rule (4.19). First, there exist two productivity cutoffs $R_v(n_{-1},\varphi)$ and $R(n_{-1},\varphi)$, given initial size n_{-1} and type φ. The first cutoff, $R_v(n_{-1},\varphi)$, marks the productivity level above which firms start posting vacancies and hiring workers. The second cutoff, $R(n_{-1},\varphi)$, identifies the productivity below which firms start separating workers to reduce their size. For productivity levels between these two cutoffs, firms stay inactive without hiring or separating workers.

4.6 Aggregation

We define aggregate steady state equilibrium and pin down aggregate variables such as aggregate unemployment U, vacancies V, and labor market tightness θ.

First, based on the shock process in (4.10), we define the steady-state cumulative distribution of employment. Let $\tilde{G}(\varepsilon)$ denote the cumulative distribution function of the Pareto distribution of innovations in (4.10). In steady-state equilibrium, for a given level of employment n, the inflow into this state must equal the outflow from it. Denoting by $H(n,\varphi)$ the steady-state cumulative distribution function of employment conditional on type φ, this condition is expressed as

$$H(n,\varphi)\big(1-\tilde{G}(R_v(n,\varphi))\big)=\big(1-H(n,\varphi)\big)\tilde{G}(R(n,\varphi)), \tag{4.24}$$

where the left-hand side represents the inflow into size n, and the right-hand side represents the outflow from size n, given firm type φ. Rearranging (4.24), we can express the steady-state cumulative distribution function of employment as

$$H(n,\varphi) = \frac{\tilde{G}(R(n,\varphi))}{1-\tilde{G}(R_v(n,\varphi)) + \tilde{G}(R(n,\varphi))}. \tag{4.25}$$

Next, integrating (4.25) across φ, we obtain the steady-state unconditional cumulative distribution function of employment $\tilde{H}(n)$ as follows:

$$\tilde{H}(n) = \int H(n,\varphi) dF(\varphi), \tag{4.26}$$

where $F(\varphi)$ denotes the cumulative distribution function of the Pareto distribution of firm type.

Based on the steady-state distribution, we define the Job Creation condition as follows:

$$U(\theta) + \int n d\tilde{H}(n;\theta) = \bar{N}, \tag{4.27}$$

which ensures that the total number of unemployed and employed workers equals the total workforce $\bar{N}$.[14]

Furthermore, we can derive the Beveridge Curve from the evolution of unemployment over time, where the inflow into the unemployment pool (the total number of separations) subtracting the outflow from the unemployment pool (the total hires of unemployed workers) characterizes the change in the unemployment stock between t and $t+1$. This follows

$$U' - U = S(\theta) - f(\theta)U,$$

and $U' = U$ in the steady-state equilibrium. As a result the Beveridge Curve is given by

$$U(\theta) = \frac{S(\theta)}{f(\theta)}, \tag{4.28}$$

[14]This is the analog of the job creation condition in the standard Mortensen and Pissarides (1994) model.

where $S(\theta)$ is the total number of separations, which can be obtained by

$$S(\theta) = \lambda \int \int (1 - H(n, \varphi))\tilde{G}(R(n, \varphi))dndF(\varphi).$$

5 Quantitative Analysis

5.1 Calibration

We set the baseline economy to solo production and calibrate the model following Elsby and Michaels (2013). The time period is set to one week.

First, we set the matching elasticity, ϕ, to 0.6, following the estimates in Petrongolo and Pissarides (2001). The matching efficiency parameter, μ, is set to 0.129 by targeting a mean labor market tightness of $\theta = 0.72$, as reported in Pissarides (2009). We target a labor share of 0.72 to pin down α, based on Gomme and Rupert (2007), and use the quarterly interest rate to set β to 0.999. The leisure value, b, is chosen to match the weekly unemployment inflow rate of $s = 0.0078$, consistent with Shimer (2012).

Worker hiring costs are targeted to be 14% of the quarterly worker wage, following Silva and Toledo (2013), to calibrate the vacancy cost c. We use the elasticity of the wage with respect to output per worker from Pissarides (2009) to set $\eta = 0.443$. The size of the labor force, $\bar{N}$, is chosen to match a mean unemployment rate of 6.5%, which corresponds to a weekly job-finding rate of 0.1125. We set the arrival rate of innovations in productivity shock, λ, to match the share of inactive firms in the Longitudinal Business Database (LBD, henceforth). Inactive firms are defined as those whose log employment changes by less than 5% in a year.

Lastly, we set the lower bound $x^{\varepsilon}_{\min}$ of firm productivity, ε, to normalize the mean productivity to one, and the shape parameter, γ^{ε}, to match the dispersion in employment growth in LBD, following Elsby and Michaels (2013). Also, the minimum value $x^{\varphi}_{\min}$ of the firm time-invariant type, φ, is set to match the minimum establishment-level employment of one worker, and its shape parameter, γ^{φ}, is chosen to match a mean

Table 3: Parameterization

Parameter	Definition	Value	Source
ϕ	Matching elasticity	0.600	Petrongolo and Pissarides, 2009
μ	Matching efficiency	0.129	Pissarides, 2009
α	$F(n) = n^{\alpha}$	0.601	Labor share = 0.72
β	Discount factor	0.999	Quarterly interest rate = 0.012
b	Value of leisure	0.387	Mean inflow rate = 0.0078
c	Flow vacancy cost	0.133	Hiring cost = 14% quarterly wage
η	Worker bargaining power	0.443	Cyclicality of wage in baseline MP
$\bar{N}$	Labor force	18.65	Mean job-finding rate = 0.1125
λ	Arrival rate of x	0.043	LBD data: $\Pr(\Delta \ln n < \lvert 0.05 \rvert) = 0.415$
x^{ε}_{min}	Lower bound of ε	1.000	Normalization
γ^{ε}	Shape parameter of the dist. of ε	0.250	LBD data: $\sigma(\Delta \ln n) = 0.416$
x^{φ}_{min}	Lower bound of φ	1.218	Mean employment = 17.38
γ^{φ}	Shape parameter of the dist. of φ	1.009	Minimum employment = 1

establishment size of 17.38, based on Small Business Administration (SBA) data.

The parameterization is summarized in Table 3, and the resulting calibrated economy is presented in Table 4.

5.2 Counterfactual Exercise: from Solo to Team Production

In this section, we perform a counterfactual exercise in which the production function shifts from solo to team-based production. This exercise captures the rise in team patenting and the declining trend of solo innovation in recent periods. This may reflect a structural change in technology development, potentially arising from the increasing complexity of knowledge—making innovation more difficult for individuals to achieve—or from gains in team productivity. By shifting the production function from solo to team production, we model how these structural changes in technology influence firm behavior and aggregate outcomes in the economy.

The results are summarized in Table 5. The first column reports the baseline economy with solo production, while the second column presents the counterfactual economy with a transition to team production. The top panel displays the aggregate labor market outcomes. Moving from solo to team production raises total output and lowers

Table 4: Baseline Economy (under Solo Production)

Output	11.16
Tightness θ	0.70
Unemployment rate, %	6.55
Wage rate	0.46
Separation rate, weekly %	0.78
$\mathbb{E}(n)$	17.25
Emp. share, small firms, %	9.59
Emp. share, large firms, %	41.07
Growth rate, small prod. firms, %	2.74
Share of firms with 0 growth, %	39.02
Wage prem., large firms, %	0.88

Table 5: Counterfactual Exercise

	Solo Production (Baseline)	Team Production
Output	11.16	11.64
Tightness θ	0.70	0.89
Unemployment rate, %	6.55	5.81
Wage rate	0.46	0.48
Separation rate, weekly %	0.78	0.76
$\mathbb{E}(n)$	17.25	17.36
Emp. share, small firms, %	9.59	9.51
Emp. share, large firms, %	41.07	43.31
Growth rate, small prod. firms, %	2.74	2.26
Share of firms with 0 growth, %	39.02	41.35
Wage prem., large firms, %	0.88	0.90

the unemployment rate. Labor market tightness and the average wage also increase, reflecting the higher returns to scale under team production, which boosts labor demand and market tightness. The bottom panel reports the firm-level outcomes. In the counterfactual economy, average firm size rises, accompanied by a reallocation of labor across firms. In particular, team production leads to a higher concentration of inventors at larger firms. The employment share of large firms—defined as those with 1,000 or more employees—increases, while the share of small firms with 5 or fewer employees declines. Given the rise of labor market tightness, which offsets the effect of increased returns to scale under team production, the average growth rate at small

firms with high productivity declines as well. Furthermore, the rise in labor market tightness widens the inaction range, as reflected in the higher share of firms exhibiting zero employment growth. Lastly, the size–wage premium increases with team production. The higher returns to scale mitigate differences in the marginal product of labor across firms of different sizes, reducing the lower marginal product typically observed at larger firms. Also, the rise in returns to scale disproportionately increases labor demand at larger firms, which increases their willingness to share a larger portion of the marginal surplus from a match.

6 Conclusion

Understanding inventor teams has become critical for studying innovation due to the increasing share of ideas produced by teams rather than individuals. Inventors do not collaborate randomly with others, but are more likely to team up with colleagues within the same firm. We provide new evidence showing that larger, more productive, or older firms are more likely to form inventor teams, assemble larger teams, and create teams with more diverse skill sets.

In our new theoretical framework, this happens because inventors are more likely to find a productive collaboration when the set of coworkers is larger, causing team production to increase returns to scale. In the data, firms' innovative output per worker is increasing and concave in the number of incumbent inventors, consistent with our theory. In our first, frictionless model, this can already explain rising inventor concentration at large firms. In a frictional labor market the switch to team production squeezes small, productive firms looking for new workers, and the set of incumbent workers acts like an asset for large firms.

In future versions, we plan to calibrate the model to inventor labor market data and use the model as a laboratory to ask whether the rise in team patenting is driven by changes in underlying distribution of firm productivity, or whether changes in the expected productivity of solo vs. team invention have driven the changes in firm dy-

namics. A decline in solo productivity would have very different implications from a rise in the average productivity of inventor teams, for example. Another direction for future work is to ask how the changing nature of idea production in teams, and within specific firms, will affect the dynamics of knowledge accumulation for R&D workers.

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Appendix

A Team Size, Firm Productivity, and Age

We measure firm productivity by the number of patents in USPTO or revenue productivity (TFPR) in Compustat as proxy. Using each measure, we classify firms into three groups. For the number of patents, we categorize firms filing a single patent as small (or low-productivity) firms, those filing between two and five patents as moderately sized (or moderately productive) firms, and firms filing more than five patents as large (or highly productive) firms. With TFPR, firms grouped into three categories based on their percentiles in the within-industry (NAICS4) cross-sectional productivity distribution for each year. Figure A12 and A13 display that team size increases in more productive firms, in terms of both patent counts and TFPR.

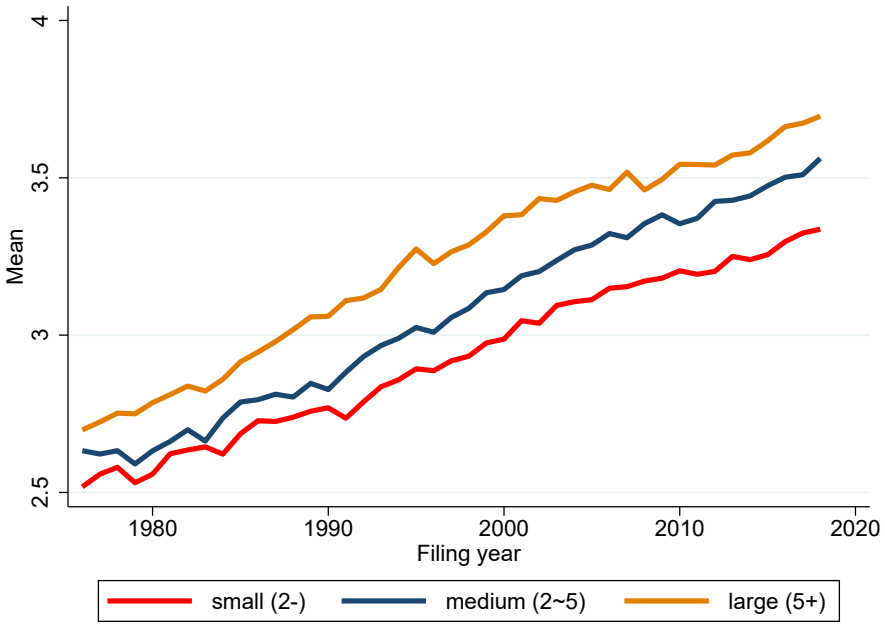


Figure A12: Average Team Sizes in a Firm (By Number of Patents)

Note: The figure illustrates the average team size in firms, categorized by firm size groups based on the number of patents filed. Small firms are those with one patent (red line), medium-sized firms are those filing between two and five patents (navy line), and large firms are those filing more than five patents (orange line) in the USPTO data.

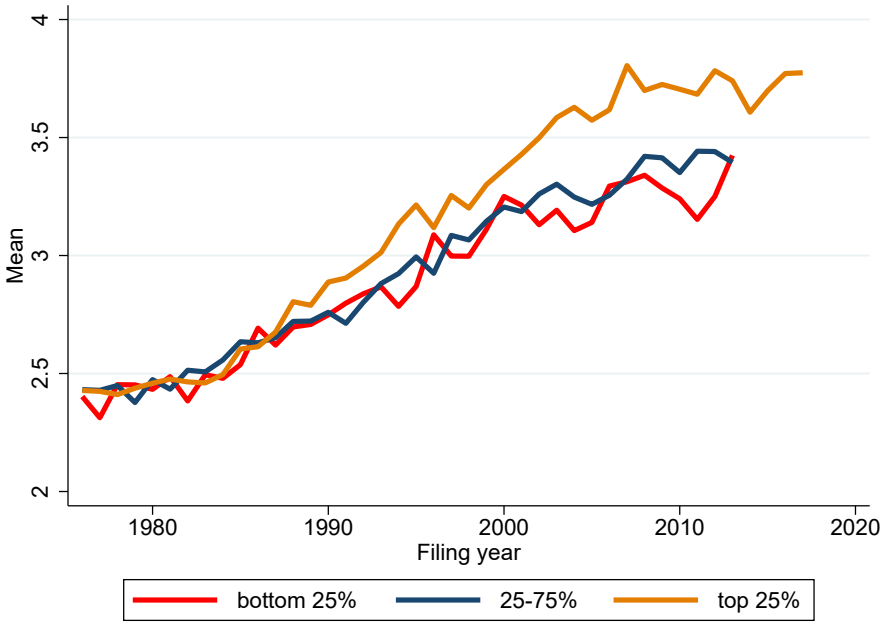


Figure A13: Average Team Sizes in a Firm over Years (By Firm TFPR)

Note: The figure illustrates the average team size in firms, categorized by firm productivity. Firms are classified by their percentile of the cross-sectional productivity distribution within a pair of industry (NAICS4) and year. Top 25% (red line) indicates the top 25 percentile firms, 25-75% (navy line) indicates firms between 25 and 75 percentiles, and bottom 25% (orange line) indicates those in the bottom 25 percentile in productivity distribution. Firm productivity is measured by the revenue productivity in Compustat.

In addition, firm age is calculated by subtracting the founding year from Ewens and Marx (2024) from the current year. With firm age, those younger than six years are considered young firms, those between six and twenty years are medium-aged firms, and those older than twenty years are categorized as old firms. A similar trend is observed with respect to firm age. The trends in team sizes across these age groups are plotted in Figure A14. After the year 2000, young firms tend to have smaller team sizes compared to older firms.

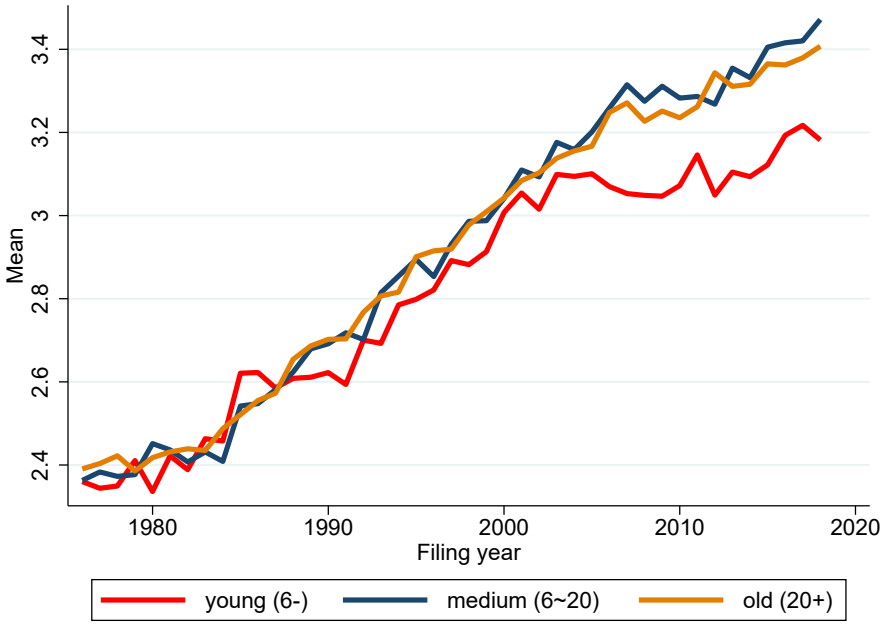


Figure A14: Average Team Sizes in a Firm over Years (By Firm Age)

Note: The figure illustrates the average team size in firms, categorized by firm age groups. Firm age data is sourced from Ewens and Marx (2024). Firms are classified as young if their age is five years or less (red line), medium-aged if their age is between six and twenty years (navy line), and old if their age is above twenty years (orange line).

B Single Inventor Share of Young Firms

Young firms are particularly likely to be single-inventor firms. To see the trend of the likelihood for young firms over time, we calculate the proportion of single-inventor firms among all young firms that have ever filed a patent in a given year. This is plotted in Figure A15, showing a declining trend of the fraction of single-inventor young firms over time.

C Team Diversity

Another trend over the past decades is rising knowledge diversity within teams. To measure knowledge diversity, we first identify the "core" CPC subgroup associated with

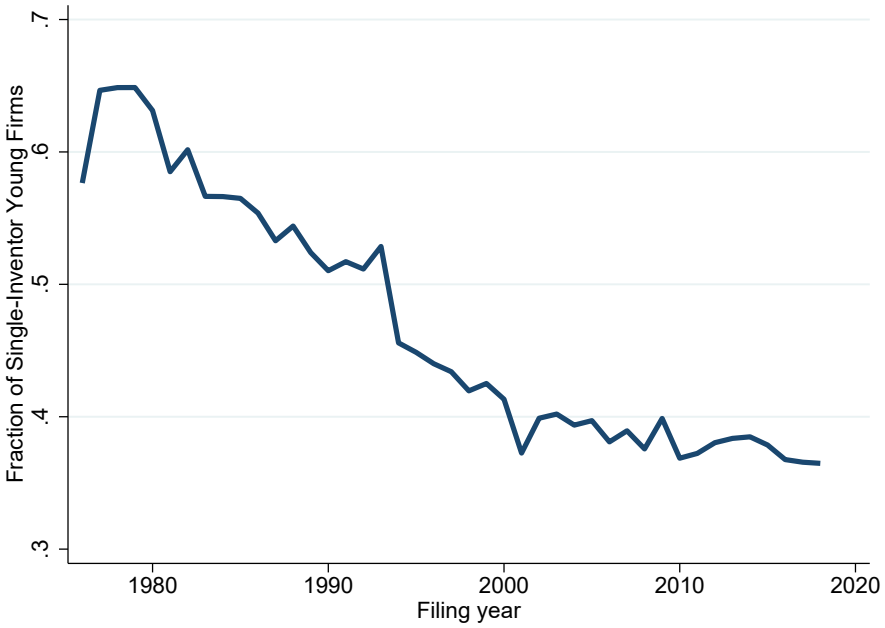


Figure A15: Fraction of Single-Inventor Young Firms

Note: The figure shows the fraction of young firms (aged five or less) with a single inventor in a given year. Firm age is sourced from Ewens and Marx (2024).

an inventor in a given year as a proxy for the inventor's type or area of expertise.[15,16] The core CPC subgroup of an inventor is defined as the subgroup with the highest number of patents filed by that inventor up to that year.[17] For each team, we calculate the number of core CPC subgroups associated with its inventors, which serves as a measure of the team's knowledge diversity in that year. We then average this knowledge diversity measure across all teams within a firm for that year. Finally, we compute the average of these firm-level measures across all firms in a given year.

The result is plotted in Figure A16, which shows an upward trend and an increasing

[15]For example, G06N 20/00 is "Machine learning" or A23B 11/00 is "Preservation of milk or dairy products."

[16]For a patent associated with multiple CPC subgroups, we pick the main CPC subgroups based on the sequence variable provided by the USPTO.

[17]Eventually, we aim to leverage the Longitudinal Employer-Household Dynamics (LEHD) dataset, which contains detailed demographic and educational information for workers covered by the U.S. Unemployment Insurance (UI) system. By linking it to the Longitudinal Business Database (LBD) and USPTO inventor data, we plan to track workers' patenting activities within and across firms. This is in progress.

likelihood of the average team working with inventors with different specialties.

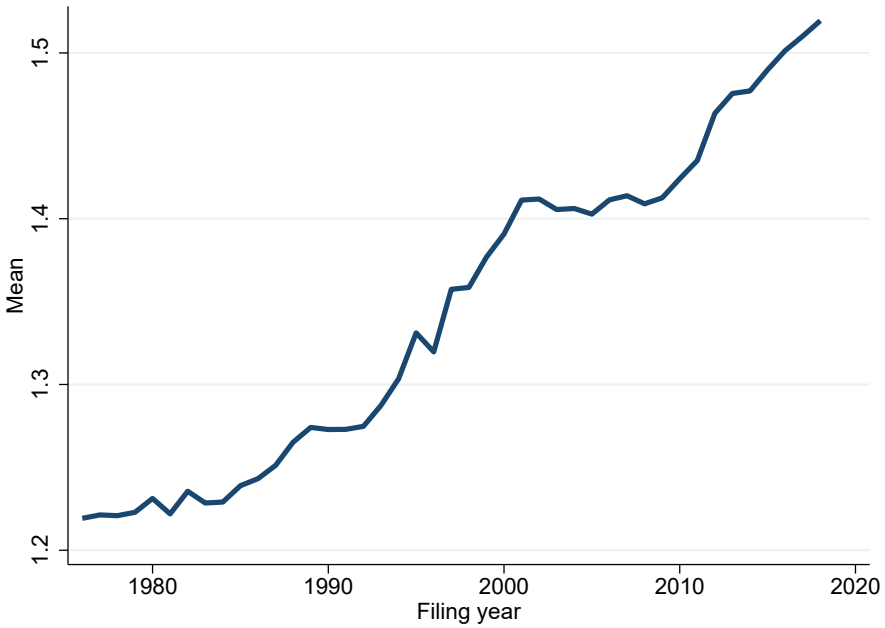


Figure A16: Average Number of Inventor CPC Subgroups in Teams in a Firm

Note: The figure illustrates the average number of core CPC subgroups associated with inventors on team patents in USPTO. A core CPC subgroup is defined as the modal CPC subgroup in an inventor's patenting history up to a given year. For each firm, we calculate the average number of core CPC subgroups for inventors on team patents and then compute the mean across all firms in a given year.

D Team Complementarities

We also aim to directly assess differences in complementarities among patenting team members across firms of varying sizes and ages, and to explore how these differences have evolved over time.

To do this, we estimate the linear model of team complementarities proposed by Bonhomme (2021):

$$Y_j = \lambda_n(\alpha_{i1(j)} + \cdots + \alpha_{in(j)}) + \epsilon_j,$$

where Y_{nj} is the 5-year forward citations of patents produced by team j of size n (including single inventor patents), α_i is individual i's fixed effect, and λ_n is the team-size

scaling factor for size n, which takes value one for single inventor teams.

The procedure of model estimation follows Section 3 in Bonhomme (2021). First, we use the identification condition to iteratively determine the subset of inventors for which the inventor fixed effects can be determined, and restrict the sample to this subset. Next, we estimate the complementarity coefficients from a linear moment condition. Last, the inventor fixed effects can be estimated in a standard OLS regression.

The interpretation is as follows. Output (citations) is a linear function of the team members' fixed effects. λ_n captures the effects of working in a team of size n. If $\lambda_n = 1$ for $n > 1$, the citations produced from this collaboration are the same as the sum of what the team members could produce individually. If $\lambda_n > 1$ it indicates that the team produces more citations than the members could generate on their own. An increase in λ_n over time suggests increasing complementarities for teams of size n. However, this model does not account for selection into teams or the impact of teams on the probability of securing a patent, among other factors.

As shown by the coefficients listed in Table A1, firms in the post-2000 period usually exhibit larger team complementarities than firms in the pre-2000 period, especially for teams of size two. Notably, complementarities for teams of two grew significantly for older firms after 2000.

This exercise also allows us to estimate inventor fixed effects that control for participation in teams of different sizes (say, selection into solo invention). Solo inventors, those with above median share (20%) of their patents filed alone, are slightly positively selected but this gaps has narrowed over time (Figure A17). Surprisingly, we find evidence of negative rather than positive assortative matching into teams of two based on the fixed effects we estimate (Figure A18). Formally, the exercise computes the a histogram and the mean of the absolute difference between team members' estimated fixed effects in the data versus 1000 random draws of two person teams the same size as the data with sampling weights for inventors the same in the bootstrap as that inventor's share of patents in the data. The mean expected difference in the fixed effects is 6% higher in the data, mean quality differences are larger in the data than patenting

teams drawn at random (negative assortative matching).

Table A1: Complementarity estimates for periods and subgroups of firms

Period	Group	λ_1	λ_2	λ_3	$\lambda_{\geq 4}$	# of Patents	# of Inventors
≤ 2000	All	1.0000	0.5674	0.4147	0.3116	1,022,209	206,849
≤ 2000	Small	1.0000	0.5699	0.4089	0.3209	48,392	10,697
≤ 2000	Large	1.0000	0.5627	0.4116	0.3088	885,695	181,694
≤ 2000	Young	1.0000	0.5919	0.4424	0.3581	12,982	3,234
≤ 2000	Old	1.0000	0.5587	0.4173	0.3149	323,371	65,928
> 2000	All	1.0000	0.6607	0.4803	0.3543	3,103,493	590,872
> 2000	Small	1.0000	0.6447	0.5027	0.3949	308,769	71,193
> 2000	Large	1.0000	0.6569	0.4750	0.3492	2,652,074	503,741
> 2000	Young	1.0000	0.5870	0.4526	0.3489	49,209	12,803
> 2000	Old	1.0000	0.6417	0.4631	0.3384	906,673	181,569

Note: This table lists the complementarity estimates for various periods and subgroups of firms. The dataset consists of all USPTO patents filed between 1976 and 2018. The sample of patents is divided into two periods based on their filing years: the first period includes patents filed no later than 2000, and the second period includes patents filed after 2000. Within each period, the sample is further divided by firm size and firm age. The group of small firms consists of those classified as small or micro entities in the USPTO dataset, while the group of large firms includes the remaining firms. The group of young firms includes those with an age of five years or less, where firm age is defined as the difference between the current year and the founding year from Ewens and Marx (2024). The group of old firms includes those with an age greater than five years.

E Derivation of Team Productivity

E.1 Discrete Firm Size

Recall that bilateral productivity is:

$$z_{ijt} \sim P(x_{min}^{team}, \gamma^{team}), \quad P(z \leq x) = 1 - (\frac{x_{min}^{team}}{x})^{\gamma^{team}} \quad \forall x \geq x_{min}^{team}$$

Let the max of n draws be

$$M_n \equiv \max\{z_{1jt}, z_{2jt}, .., z_{njt}\}$$

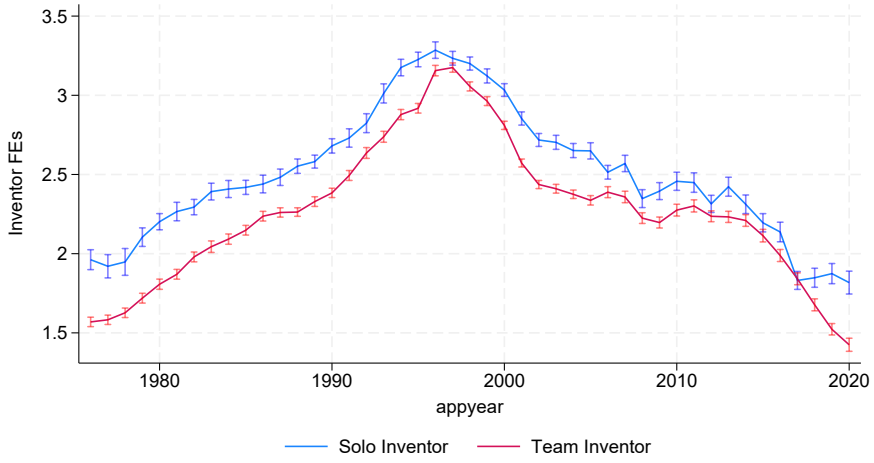


Figure A17: Mean estimated inventor fixed effect α_i. The median share of solo patents for inventors is 20%. Above = solo inventor, below = team inventor.

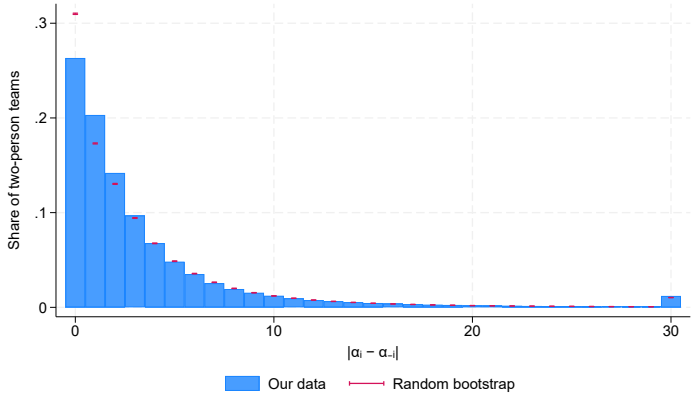


Figure A18: Histogram of the absolute difference in two-person team member fixed effects $|\alpha_i - \alpha_{-i}|$ in the data vs. 1000 random samples of two person teams with the same number of teams and sampling weights on inventors as in the data. Mean difference: 4.293 in the data and 4.041 in random bootstrap.

then

$$F_{M_n}(x) \equiv P(M_n \leq x) = \left[1 - (\frac{x_{min}^{team}}{x})^{\gamma^{team}}\right]^n \quad \forall x \geq x_{min}^{team}$$

Then, we can derive the pdf as follows:

$$f_{M_n}(x) = n\gamma^{team}(\frac{(x_{min}^{team})^{\gamma^{team}}}{x^{\gamma^{team}+1}})\left[1 - (\frac{x_{min}^{team}}{x})^{\gamma^{team}}\right]^{n-1} \quad \forall x \geq x_{min}^{team}.$$

Thus, the expected value is

$$\mathbb{E}(M_n) = \int_{x_{min}^{team}} x f_{M_n}(x)dx = x_{min}^{team} nB(n, 1 - \frac{1}{\gamma^{team}}),$$

where B is Beta function.[18]

Note that for large n, the Beta function asymptotically becomes:

$$B(n, 1 - \frac{1}{\gamma^{team}}) = \frac{\Gamma(n)\Gamma(1 - \frac{1}{\gamma^{team}})}{\Gamma(n + 1 - \frac{1}{\gamma^{team}})} \simeq \Gamma(1 - \frac{1}{\gamma^{team}})n^{-1+\frac{1}{\gamma^{team}}}.$$

Therefore, the expected productivity of working in a firm with n team members becomes:

$$\mathbb{E}(M_n) = x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})n^{\frac{1}{\gamma^{team}}} \tag{E.29}$$

which increases in n.

E.2 Continuous Firm Size

Assume that worker bilateral productivity is drawn from an inhomogeneous Poisson point process with intensity function within firm of size $n \in [0, \infty)$.

$$\lambda(x) = n\frac{f(x)}{F(x)}$$

[18] It can be derived with a transformation of variable with $t = 1 - (x_{min}^{team}/x)^{\gamma^{team}}$. So that it can be rephrased as $\mathbb{E}(M_n) = nx_{min}^{team} \int_0^1 t^{n-1}(1-t)^{-\frac{1}{\gamma^{team}}} dt$.

Then, the distribution function of the maximum talent is

$$\begin{aligned} F_{z_{it}^{team}}(x) &= \mathbb{P}[N(x, +\infty) = 0] \\ &= \exp\left(-\int_x^{+\infty} \lambda(u)du\right) \\ &= F_{z_{it}}(x)^n, \end{aligned}$$

and thus, everything follows the same as before.