Who Carries?*

Alex Ferreira† Giuliano Ferreira‡ Miguel León-Ledesma§
Rory Mullen¶

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Abstract

In representative agent models, consumption growth risk explains currency carry trade returns only when risk aversion is high. Yet, a highly risk-averse agent would not hold a carry trade portfolio. Heterogeneity helps by allowing a risk-tolerant minority of agents to hold carry trade portfolios while a risk-intolerant majority does not. We show that with heterogeneous risk aversion, standard models of international macroeconomics produce carry traders endogenously in economies with domestic bias in aggregate portfolios and low aggregate portfolio returns, as observed in Germany, Japan, and the United States, together holding half of global debt.

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†Universidade de São Paulo, Brazil. Email: alexferreira@usp.br
‡Universidade de São Paulo, Brazil. Email: giudqf@usp.br
§University of Exeter, United Kingdom and CEPR. Email: m.leon-ledesma@exeter.ac.uk
¶University of Warwick, United Kingdom. Email: rory.mullen@wbs.ac.uk

1 Introduction

The carry trade has two parts: the return and the portfolio position. Most research on the carry trade in foreign exchange markets seeks to rationalize the return but not the position. A satisfactory theory should rationalize both. In particular, a theory of the carry trade must grapple with the empirical observation that aggregate portfolio positions do not resemble aggregate carry trades in many countries where portfolios are heavily biased towards domestic assets.

Most existing research does not pin down portfolio positions. Consider the consumption-based asset pricing model of Lustig and Verdelhan (2007), where carry trade returns compensate an investor for holding high interest-rate currencies that depreciate when consumption growth is low. To match carry trade returns for the United States, Lustig and Verdelhan must assume a high degree of risk aversion. Standard extensions of their model would imply domestic portfolio bias for the highly risk-averse investor. Domestic portfolio bias would fit the data for the United States, but would look nothing like a carry trade, because interest rates in the United States have been low in recent decades. The same is true of Germany and Japan, as we report in Table 1. For these three economies, which together hold half of global debt, and others like them, we ask: who carries?

We build on the foundation laid by consumption-based asset pricing models of carry trade returns to show that heterogeneity offers a compelling answer. By fixing aggregate risk aversion at a high level while allowing for within-country variation across agents, it should be possible to generate plausible carry trade returns on portfolios held by a risk-tolerant minority of agents in an economy with domestic portfolio bias and low portfolio returns in the aggregate. Indeed, we prove sufficient conditions for exactly this result in a standard two-country dynamic stochastic general equilibrium model with incomplete markets that we augment with heterogeneity in risk aversion. The benefits of this approach are substantial.

First, heterogeneity bridges the gap between the finance literature on currency carry trades and the international macroeconomics literature on domestic bias in aggregate portfolio holdings. The finance literature typically employs representative-agent models to study carry trade returns. However, as discussed, a representative agent cannot be a carry trader in countries earning low returns on portfolios with domestic bias in aggregate—an empirically important case. Conversely, the international macroeconomics literature typically employs representative-agent models to study domestic bias in aggregate portfolio holdings, placing little emphasis on relative returns across currencies. Our heterogeneous-agent framework can simultaneously capture *individual carry trade positions* and *domestic bias in aggregate positions* in economies with low aggregate portfolio returns.

Second, the heterogeneity that drives the carry trade in our model also drives a wedge between the aggregate coefficient of risk aversion and the aggregate intertemporal elasticity

Table 1 – Domestic Currency Bias and Returns on Aggregate Debt Portfolios

	Group Share of Global Holdings	Country-Level Weighted Average (in %)		
		Domestic Currency Bias	Nominal Portfolio Return	Real Portfolio Return
Germany, Japan, United States	54.0	91.4	1.2	−0.3
All Other Countries	46.0	73.0	3.2	0.7

Notes. The table reports estimated domestic currency bias and returns on debt portfolios held by Germany, Japan, and the United States versus 181 other countries. Portfolio returns and domestic currency bias are estimated from OECD and IMF interest rates, exchange rates, and CPI inflation, combined with portfolio weights estimated from IMF and BIS debt holdings and issuance data over the period 2001–2021. Shares of global holdings are calculated for each group then averaged over years. All other values are calculated for each country then averaged over countries and years, using each country's debt holdings as weights. We exclude outlier interest rates and exchange rate returns exceeding three standard deviations from annual means. Appendices A.1 and A.2 describe our methodology, with robustness checks in Table A3.

of substitution. This aggregation wedge arises even for isoelastic utility functions with constant relative risk aversion. It transforms these preferences into a parsimonious and tractable alternative to the recursive preferences traditionally employed to separate risk aversion from intertemporal substitution. The aggregation wedge affects the responsiveness of macroeconomic variables to shocks, so the same heterogeneity that gives rise to individual carry traders also affects macroeconomic dynamics.

Third, the domestic bias in aggregate portfolios that our model generates raises the responsiveness of carry trade returns to shocks. In the model, a portfolio with domestic bias amounts to a hedge against low consumption growth. If consumption growth falls, hedged portfolio values rise. When a majority of agents are hedged, this initial rise produces a positive feedback effect that raises hedged portfolio values further. The feedback effect improves the hedging properties of portfolios with domestic bias, but worsens the anti-hedging properties of portfolios with international bias, which represent carry trade portfolios in the model. Thus, domestic bias in aggregate portfolios amplifies the risk of the carry trade, and carry traders earn higher returns.

We demonstrate these benefits and rationalize the coexistence of domestic portfolio bias and carry trade activity in low interest-rate economies using a model with an infinite horizon, incomplete markets, and two countries, each populated by a continuum of agents with heterogeneous risk aversion. In the model, individual coefficients of relative risk aversion equal the inverse of individual elasticities of intertemporal substitution, but this inverse relationship breaks when the individual parameters are aggregated. If aggregate risk aversion exceeds a low threshold, the aggregate portfolio displays domestic bias. While the majority of individual agents endogenously hold portfolios with domestic bias, a minority of agents hold carry trade portfolios, consistent with the evidence. Unlike

existing models of segmented markets that assume exogenous market participation, our agents endogenously sort into different portfolio strategies based on their risk preferences.

Importantly, we present a novel method to solve for both individual and aggregate portfolio holdings, extending existing methods in finance and international macroeconomics to a new class of heterogeneous-agent models. Our model admits approximate closed-form solutions for individual and aggregate consumption, real wealth, and real portfolio holdings, as well as for the real exchange rate and the carry trade real return. Conveniently, heterogeneity vanishes in the model's determinate non-stochastic steady state. Around this steady state, the model generates stationary, non-degenerate cross-sectional distributions of real wealth and consumption. Our method is powerful, but requires solving the model twice—once at the aggregate level and once at the individual level—doubling the technical complexity.

Given this complexity, we prioritize simplicity and do not attempt a full quantitative calibration. Instead, we design the model with a small number of parameters and deliberate abstractions to highlight the core theoretical mechanisms and demonstrate the solution method rather than directly fit the data. In this sense, we follow in the tradition of Devereux and Sutherland (2011), who also focus on the solution method rather than quantitative analysis. Nevertheless, our method extends directly to richer quantitative models with additional assets, shocks, and frictions.

While we use the carry trade as our framing device, we emphasize that our method has broader appeal. For example, the method can be used to endogenize the existence of hedgers and speculators, to study private versus social gains from international risk sharing, or to analyze the distributional consequences of macroeconomic policies in open economies. By solving for the complete cross-sectional distribution of portfolios in a workhorse international macro model, our approach opens new avenues for understanding how heterogeneity and financial markets interact.

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The remainder of the paper is organized as follows. In Section 2, we discuss related literature. In Section 3, we construct our two-country dynamic stochastic general equilibrium model with heterogeneity in risk aversion. Section 4 concludes.

2 Related Literature

In foreign exchange markets, carry trades exploit interest rate differentials across currencies. These trades violate uncovered interest parity in predictable ways, as documented by Hansen and Hodrick (1980), Fama (1984), and Engel (1984). Most explanations for these violations fall into one of three categories: financial frictions (Burnside et al., 2006; Brunnermeier et al., 2009; Gabaix and Maggiori, 2015), rare disasters (Barro,

2006; Burnside et al., 2011; Farhi and Gabaix, 2016), and risk-based factors (Lustig and Verdelhan, 2007; Lustig et al., 2011; Menkhoff et al., 2012; Lettau et al., 2014).

In the consumption-based capital asset pricing model, adapted to carry trades by Lustig and Verdelhan (2007), carry trade returns compensate investors for exposure to consumption growth risk. Despite some debate (Burnside, 2011; Lustig and Verdelhan, 2011), this framework remains influential in risk-based explanations.

Within this tradition, we are closest to Jylhä and Suominen (2011) and Filipe, Nissinen, and Suominen (2023), who develop a two-country model with endogenous carry traders using two types of myopic agent—one restricted to domestic markets, the other unrestricted. Richers (2019) develops a two-period model with a global investor and local agents, where agency frictions lead to persistent cross-country return differentials. Park (2022) incorporates heterogeneous beliefs in a two-country, two-period model with three types of agent, showing how pessimistic beliefs increase carry trade returns. Like these papers, we model carry traders in a consumption-based economy with heterogeneous agents. Unlike these papers, we model a continuum of unrestricted agents with differing risk preferences who optimize over an infinite horizon. Our framework incorporates richer heterogeneity into a standard international macroeconomic model and uniquely endogenizes carry traders in economies where most agents prefer low-return portfolios with domestic bias.

Our paper also relates to the literature on segmented international financial markets, in which a small set of intermediaries bears currency risk. A prominent example is Gabaix and Maggiori (2015), who exogenously restrict households to domestic assets and exogenously assign one household each period to serve as a global financier bearing currency risk. Our framework features no global financier. Instead, heterogeneity in risk preferences leads a minority of relatively risk-tolerant agents to voluntarily bear currency risk, while the majority of agents choose to avoid it. Thus, our model endogenously produces financier-like agents who optimally choose to enter the role, rather than being randomly assigned.

The profitability of the carry trade has been extensively studied, but evidence on actual carry trade positions is more limited. Galati et al. (2007) and Gagnon and Chaboud (2007) find modest evidence of carry trades primarily in FX derivatives markets. Curcuru et al. (2011) and Fong (2013) also document limited evidence of carry trades, even during times of reportedly high activity. Recent BIS evidence identifies hedge funds and global macro investors as main participants in carry trade strategies, primarily in FX derivatives markets (Bank for International Settlements, 2024). Using UK transaction-level data, Hacioğlu-Hoke et al. (2024) confirm hedge funds as important carry traders, but their positions represent only about 3% of net FX derivatives exposures. Specific carry trade channels have been observed in particular contexts—household foreign currency borrowing in Austria (Beer et al., 2010), European bank behavior during the sovereign debt crisis (Acharya and Steffen, 2015), and emerging market firms issuing USD bonds (Bruno and

Shin, 2017)—but these specialized cases do not suggest widespread dominance of carry trade strategies.

In contrast, a growing literature finds that hedging, not carry trade, drives much of the currency derivatives market. Allayannis and Ofek (2001) and Allayannis et al. (2001) find that large U.S. firms use currency derivatives to reduce exchange rate exposure, enhancing firm value. Bartram et al. (2010) and Bartram (2019) find that non-financial firms globally use currency derivatives to reduce exchange rate exposure. Bräuer and Hau (2023) show that institutional investor hedging explains significant exchange rate variation. Sialm and Zhu (2024) find that most U.S. international fixed income mutual funds hedge with currency forwards. Hacioğlu-Hoke et al. (2024) show that pension and investment funds, insurers, non-financial corporations, and non-dealer banks use currency derivatives primarily for hedging. Alfaro et al. (2024) survey evidence of hedging in transaction-level currency derivatives datasets. Liao and Zhang (2025) find that hedging has greater exchange rate effects in periods of financial distress.

In bond markets, domestic bias dominates. Lane (2005) finds strong regional bias in Euro-area bonds after the Euro's introduction, anticipating a later literature on domestic-*currency* bias. Fidora et al. (2007) and Coeurdacier and Rey (2013) find that domestic bias persisted more strongly in bonds than in equities over decades of financial globalization. Coeurdacier and Gourinchas (2016) show that bonds help hedge real exchange rate risk in portfolios with equities. Burger et al. (2018) find that investors strongly prefer domestic-currency bonds, with U.S. investors over-weighting USD bonds from foreign issuers. Recent work by Maggiori et al. (2020) and Florez-Orrego et al. (2024) using micro data show that global investors prefer domestic-currency bonds, with USD bonds as an exception. Faia et al. (2022) find domestic bias in Euro-area corporate bond holdings for insurers and pension funds but not mutual funds. Burietz and Ureche-Rangau (2020) find domestic bias in Euro-area syndicated lending.

These studies collectively suggest that while carry traders exist in both currency derivatives markets and specialized debt markets, they are not the dominant market participants. Hedging appears to be more important than speculation for many non-financial and financial firms. These findings reinforce our thesis that carry traders, while present, are outweighed by investors with non-speculative motives. We reconcile these findings with the carry trade literature using a heterogeneous-agent framework.

Our framework builds on methodological advances in solving for international portfolios in open-economy models with incomplete markets. Guu and Judd (2001), Devereux and Sutherland (2010, 2011), Tille and van Wincoop (2010), and Evans and Hnatkovska (2012) develop perturbation methods to solve open-economy models with portfolio choice under incomplete markets. More recently, Saucet (2025) develops a global solution for portfolio holdings in an open economy with heterogeneity across but not within countries. Our technical contribution extends the Devereux and Sutherland (2011) perturbation method

to allow for within-country heterogeneity in risk preferences for a continuum of agents (in addition to heterogeneity across countries). We provide approximate closed-form solutions for the distinct portfolio of each individual agent, and for the aggregate economy. While we solve only for steady-state holdings, this limitation can be addressed in future work using higher-order approximations.

Our work relates to a macroeconomics literature on preference heterogeneity that until recently followed the Bewley-Ayagari-Hugget tradition, where agents differ only in their history of idiosyncratic shocks. This was true of earlier HANK models (see Kaplan et al. 2018), where heterogeneity in the marginal propensity to consume (MPC) was shown to affect monetary policy transmission. Recently, Aguiar et al. (2020), Calvet et al. (2021) and Gelman (2021) have shown that preference heterogeneity rather than shocks can explain much of the observed heterogeneity in MPCs. Kekre and Lenel build New-Keynesian models with two-type heterogeneity in risk aversion: among households in a closed economy (Kekre and Lenel, 2022) and across countries in an open economy (Kekre and Lenel, 2024). Davis and van Wincoop (2024) develop an open-economy model featuring within-country heterogeneity in risk aversion and many types; unlike us, they focus on capital flow retrenchment and model portfolio choice over a two-period horizon.

Our work also relates to a finance literature on preference heterogeneity. Kogan and Uppal (2001) study portfolio holdings and asset pricing in a closed economy with heterogeneous risk aversion and incomplete markets, but unlike us they perturb parameters of the utility function around the logarithmic utility case. Guvenen (2009) studies asset prices in a two-agent closed-economy macro model with limited stock market participation and heterogeneity in the intertemporal elasticity of substitution (IES). Garleanu and Panageas (2015) study asset pricing in a closed-economy OLG model with two agents distinguished by their IES and risk aversion, and Panageas (2020) surveys the literature on closed-economy asset pricing with heterogeneity, providing a unifying framework.

This paper offers a unique combination of features that distinguishes it from prior research on preference heterogeneity. While existing papers share individual elements with our approach, none combines: (1) a continuum of agents with heterogeneous risk aversion rather than a limited number of types; (2) agents optimizing over an infinite horizon rather than myopic agents or two-period frameworks; (3) an open-economy setting that captures both within and across-country heterogeneity; and (4) unrestricted agents who can freely participate in both domestic and international markets. This combination allows us to study the effects of heterogeneity on individual and aggregate variables in a benchmark open-economy model, while explaining how domestically biased investors, hand-to-mouth consumers, and carry traders emerge endogenously from the same underlying preference distribution.

3 Theoretical Model

We now develop our heterogeneous-agent model and derive a series of propositions that characterize equilibrium portfolios and returns for individual agents and the aggregate economy. Our approach resolves the tension we highlighted in the introduction between the finance literature on carry trades and the international macroeconomics literature on domestic bias in aggregate portfolios, which representative-agent models cannot address.

The model admits exact aggregation across agents in the non-stochastic steady state and approximate aggregation locally around the steady state. These features allow us to derive approximate closed-form solutions for both individual and aggregate consumption, real wealth, and portfolio holdings, in addition to the real exchange rate and the carry trade real return. We deliberately specify relatively simple model primitives, in order to present solutions for the aggregate economy and individual agents as transparently as possible.

The mechanism at the core of the model is straightforward: agents endogenously choose their portfolio positions to achieve consumption volatility consistent with their risk preferences. Risk-tolerant agents choose higher expected consumption at the cost of higher consumption volatility, while risk-averse agents choose the opposite. The carry trade, which produces returns that rise and fall with consumption growth, appeals only to a risk-tolerant minority.

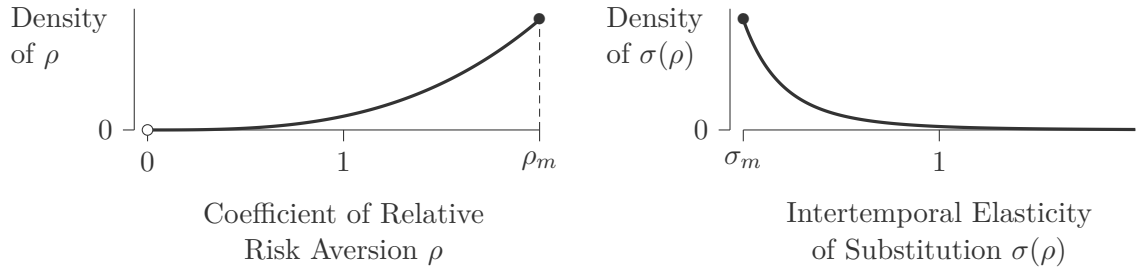
3.1 Model Primitives

There are two countries, home H and foreign F. To avoid repeating home and foreign versions of equations throughout the text, we use $i, j, k \in \{H, F\}$ to index countries and currencies and write all equations in terms of these indices. Equations with multiple indices should be understood to hold for any combination of index values unless we explicitly indicate otherwise.

A continuum of agents populates each country. We denote the set of agents $\mathcal{H}_i$ in country i and we assume a measure of one for this set. Each agent is uniquely identified within country by its coefficient of relative risk aversion, denoted ρ, so we use ρ as the index for agents. Each country has an identical distribution of risk aversion across agents, so each agent has a counterpart abroad with an identical coefficient of relative risk aversion.

We define an inverse Pareto density function to characterize the distribution of risk

Figure 1 – Distributions of Risk Aversion and Intertemporal Substitution



Notes. The figures plot the distribution of the coefficient of risk aversion ρ across individual agents (left) and the corresponding distribution of the intertemporal elasticity of substitution $\sigma(\rho)$ across individual agents (right) for an illustrative calibration of $\kappa = 4$, $\rho_m = 2$, and $\sigma_m = 1/2$. The two density functions plotted here represent equivalent heterogeneity across agents.

aversion across agents.[1] The density function, denoted $f(\rho)$ and given by

$$f(\rho) = \frac{\kappa}{\rho}\left(\frac{\rho}{\rho_m}\right)^{\kappa}, \tag{1}$$

has positive support over $\mathcal{R} = (0, \rho_m]$, where $\kappa > 1$ and $\rho_m > (\kappa^2 + \kappa)/(\kappa^2 - 1)$ are shape and scale parameters, respectively.[2] These parameter restrictions ensure that aggregate risk aversion is well-defined and sufficiently high. We illustrate the density function in the left panel of Figure 1. The density falls as ρ approaches zero from above, forming a left tail of agents with decreasing coefficients of risk aversion. Agents near the tip of the tail are nearly risk neutral; these will be the carry traders in our model.

Agent ρ in country i maximizes the expected present value of lifetime utility over consumption and wealth. Let $U_{it}(\rho)$ denote the agent's expected present value of lifetime utility,

$$U_{it}(\rho) = \mathrm{E}_t\left[\sum_{s=t}^{\infty} \beta^{s-t}\Big(U_{Cis}(\rho) + U_{Wis}(\rho)\Big)\right], \tag{2}$$

where $\beta \in (0.5, 1)$ denotes the subjective discount factor, and where $U_{Cit}(\rho)$ and $U_{Wit}(\rho)$ denote the agent's period utility from consumption and real wealth, respectively. We assume constant relative risk aversion for $U_{Cit}(\rho)$ and constant absolute risk aversion for

[1] Our results do not depend on the specific distribution that we assume. For example, uniform, triangular, normal, or log-normal distributions also work. We only require the distribution to support a small measure of sufficiently risk-tolerant agents and to imply a sufficient level of aggregate risk aversion. The inverse Pareto distribution does, however, have some empirical support: Kimball et al. (2008) estimate a qualitatively similar empirical distribution from a number of large-scale surveys conducted in the United States and internationally, where respondents state their preferences over risky gambles related to employment and lifetime earnings. On the other hand, Calvet et al. (2021) estimate a more symmetric empirical distribution of risk aversion using Swedish administrative data and assuming Epstein-Zin preferences.

[2] The agent with exactly $\rho = 1$ is measure zero, and assumed to have log utility.

$U_{Wit}(\rho)$,

$$U_{Cit}(\rho) = \frac{1}{1-\rho}\left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{1-\rho} \quad \text{and} \quad U_{Wit}(\rho) = -\frac{\theta}{\rho}\left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}}\right)^{-\rho}, \tag{3}$$

where $C_{it}(\rho)$ denotes the agent's consumption basket, with subscript i denoting the agent's country of residence, and $W^i_{it}(\rho)$ denotes the agent's real wealth, with subscript i denoting the agent's country of residence and superscript i denoting the numéraire basket. Agents have internal habits set at the non-stochastic steady-state levels of consumption and real wealth, where the absence of time subscripts on these variables indicates their steady state levels.[3]

The parameter θ governs the importance of real wealth as a direct source of utility. We introduce wealth into the utility function for practical reasons: it ensures stationarity in the model even for arbitrarily small values of θ that have no significant impact on equilibrium outcomes. Throughout the paper, we assume $\theta \in [0, \epsilon)$, where ϵ is a small positive number. Nevertheless, wealth-in-utility can be economically justified by factors like social status and is used as a modeling device in recent studies by Michaillat and Saez (2021), He et al. (2023), Michau et al. (2023), and Zhao (2023). Unlike these papers, we show that wealth-in-utility achieves stationarity while preserving aggregability in the presence of heterogeneity.

The consumption utility function in (3) directly links the coefficient of relative risk aversion to the intertemporal elasticity of substitution for individual agents, meaning that heterogeneity in the former implies heterogeneity in the latter. Letting $\sigma(\rho)$ denote the intertemporal elasticity of substitution with respect to consumption for agent ρ, the utility function in (3) implies

$$\sigma(\rho) = 1/\rho\,. \tag{4}$$

The inverse Pareto density function that we assume for ρ in (1) implies a Pareto density function for $\sigma(\rho)$ with shape parameter κ and scale parameter $\sigma_m = 1/\rho_m$. The Pareto density function falls in $\sigma(\rho)$, forming a right tail of agents with increasing intertemporal elasticities of substitution. We illustrate the density function for $\sigma(\rho)$ in the right panel of Figure 1.

Each agent's budget constraint ensures that real expenditure on consumption and bond holdings each period does not exceed the agent's real endowment income plus the gross real return on bond holdings from the previous period. We write the constraint as

[3]We choose these functional forms for tractability. Internal habits simplify the non-stochastic steady state and the aggregation of approximate equilibrium conditions. CARA utility over real wealth simplifies our specification of bond markets, because it admits the zero and negative values of real wealth that arise when bonds are in zero net supply. The model remains tractable if we instead specify CRRA utility over real wealth and bonds in positive net supply.

an equality,

$$C_{it}(\rho) + B^i_{iit}(\rho) + B^i_{ijt}(\rho) = \frac{P^i_{Cit}}{P^i_{it}} Y_{it}(\rho) + R^i_{it} B^i_{iit-1}(\rho) + R^i_{jt} B^i_{ijt-1}(\rho)\,, \quad i \neq j\,, \tag{5}$$

where $B^i_{ijt}(\rho)$ denotes the real value of bond holdings for agent ρ, with subscripts i and j denoting the agent's country of residence and the bond's country of issuance, respectively, and superscript i denoting the numéraire basket; P^i_{Cit} and P^i_{it} denote consumption good and basket prices, respectively, with subscript i denoting the country and superscript i denoting the numéraire currency; $Y_{it}(\rho)$ denotes agent ρ's real endowment, with subscript i denoting the agent's country of residence; and R^i_{jt} denotes the gross real return, with subscript j denoting the bond's country of issuance and superscript i denoting the numéraire basket. We state the equivalent nominal budget constraint in Appendix B.2.

We define agents' real wealth as the real value of their domestic and international nominal bond holdings, expressed in a common numéraire basket. That is,

$$W^i_{it}(\rho) = B^i_{iit}(\rho) + B^i_{ijt}(\rho)\,, \quad i \neq j\,. \tag{6}$$

Nominal bonds are single-period assets that pay one unit of the issuer's currency at maturity with certainty. We define agents' real bond holdings and the gross real return as

$$B^i_{ijt}(\rho) = \frac{A_{ijt}(\rho) P^i_{Bjt}}{P^i_{it}} \quad \text{and} \quad R^i_{jt} = \frac{S_{ijt}/P^i_{Bjt-1}}{P^i_{it}/P^i_{it-1}}\,, \tag{7}$$

respectively, where $A_{ijt}(\rho)$ denotes the quantity of bonds held by agent ρ, with subscripts i and j denoting the bond holder's country and the bond's country of issuance, respectively; P^i_{Bjt} denotes the nominal bond price, with subscript j denoting the bond's country of issuance and superscript i denoting the numéraire currency; and S_{ijt} denotes the nominal exchange rate in units of currency i per unit of currency j, with $S_{iit} = 1$ by definition.

Each period, all agents within a country receive an identical endowment, which is a positive random variable. Since the measure of agents in each country equals one, each agent's endowment equals the aggregate endowment,

$$Y_{it}(\rho) = Y_{it}\,. \tag{8}$$

We normalize the steady-state endowment to one, $Y_i = 1$, where the absence of a time subscript indicates the steady state.

Endowment goods trade internationally, and agents choose a quantity of each country's endowment good to consume. Agents differentiate between domestic and international endowment goods, and prefer variety in their consumption. We define each agent's

consumption basket as

$$C_{it}(\rho) = \gamma C_{iit}(\rho)^{\alpha_{ii}} C_{ijt}(\rho)^{\alpha_{ij}}\,, \quad i \neq j\,, \tag{9}$$

where $C_{ijt}(\rho)$ denotes the quantity of good j that agent ρ in country i consumes, and where the parameter α_{ij} denotes country i's expenditure share on the endowment good of country j. Expenditure shares sum to one, $\alpha_{ii}+\alpha_{ij} = 1$, and we assume that the shares exogenously embed domestic bias in agents' consumption preferences, $\alpha_{ii} \in (1/2, 1)$. Agents decide the composition of their consumption baskets after first deciding on their total consumption expenditure in a two-stage budgeting procedure. Their total consumption expenditure therefore constrains their expenditure on domestic and international goods,

$$P^i_{it} C_{it}(\rho) = P^i_{Cit} C_{iit}(\rho) + P^i_{Cjt} C_{ijt}(\rho)\,, \quad i \neq j\,, \tag{10}$$

where P^j_{Cit} denotes the price of good i in numéraire currency j.

Because goods and nominal bonds are differentiated across countries, each trades in a separate market with a separate clearing condition. Market clearing requires that aggregate demand across countries equals aggregate supply,

$$C_{iit} + C_{jit} = Y_{it} \quad \text{and} \quad B^i_{iit} + B^i_{jit} = 0\,, \quad i \neq j\,, \tag{11}$$

where we assume zero net supply of each country's nominal bonds.

The law of one price holds for goods and nominal bonds, so goods prices across countries and bond prices across countries must be equalized when expressed in a common numéraire currency. That is,

$$P^i_{Cit} = S_{ijt} P^j_{Cit} \quad \text{and} \quad P^i_{Bit} = S_{ijt} P^j_{Bit}\,. \tag{12}$$

Because consumption baskets differ across countries, consumption basket prices also differ across countries and we define the real exchange rate as the ratio of consumption basket prices expressed in a common numéraire currency,

$$Q_{ijt} = S_{ijt} P^j_{jt} \Big/ P^i_{it}\,, \tag{13}$$

where Q_{ijt} denotes the real exchange rate in units of consumption basket i per unit of consumption basket j, with $Q_{iit} = 1$ by definition.

A quantity equation determines the price level in each country, and we assume that each country's money supply is a positive random variable. We assume symmetric money supplies across countries in the non-stochastic steady state, when expressed in a common

numéraire currency,

$$M_{it}^{i} = Y_{it} P_{it}^{i} \quad \text{and} \quad M_{i}^{i} = S_{ij} M_{j}^{j} \,, \tag{14}$$

where M_{it}^{j} is the supply of money in country i in numéraire currency j, and where the absence of a time subscript indicates the steady state.

The model features four sources of uncertainty—home and foreign goods endowments and home and foreign money supplies—and only two nominal bonds, making financial markets incomplete. The logarithms of endowments and money supplies have positive and finite means, and each follows an independent and identically distributed process over time. We assume a block-symmetric variance-covariance matrix with positive and finite elements, where domestic covariances exceed cross-country covariances. This structure is detailed in Appendix B.1 and helps to ensure that portfolios with domestic bias earn lower expected real returns than portfolios with international bias.[4]

3.2 Model Propositions

We solve the model in two stages: first for the aggregate economy and then for individual agents, distinguishing between non-portfolio and portfolio problems at each stage. The model is tractable precisely because it separates into agent and aggregate, as well as non-portfolio and portfolio problems.

The propositions that follow provide a roadmap to our model. In Section 3.2.1 we state first-order optimality conditions. In Section 3.2.2, Propositions 1 and 2 characterize the non-stochastic steady state, where heterogeneity vanishes, contributing to the model's tractability. In Section 3.2.3, Propositions 3 and 4 introduce the aggregation wedge and demonstrate how agent-specific equilibrium conditions can be approximated and aggregated. In Section 3.2.4, Proposition 5 establishes stationarity for the aggregate economy, and we solve for aggregate consumption, real wealth, and the real exchange rate. In Section 3.2.5, Propositions 6 through 9 introduce the portfolio valuation multiplier and establish sufficient conditions for domestic bias in aggregate portfolios earning low real returns. In Section 3.2.6, we solve for agent-specific consumption and real wealth. In Section 3.2.7, Proposition 10 states the culminating result of our model: sufficient conditions for the existence of agents with carry trade portfolios that earn high returns in economies with domestically biased aggregate portfolios that earn low returns.

[4]Our core results extend to more general variance-covariance structures, described in Appendix B.1, that can produce persistent differences in interest rates across countries. We adopt the block-symmetric structure to simplify exposition in order to focus on within-country heterogeneity in portfolio holdings. See Hassan and Zhang (2021) for a review of cross-country interest rate differentials.

3.2.1 Utility Maximization.

Agents solve a two-stage utility maximization problem. In the first stage, agents maximize expected discounted lifetime utility in (2) by choosing quantities of the consumption basket to consume, and by constructing a portfolio of home and foreign nominal bonds. For agent ρ in country i, the first stage of the utility maximization problem yields the Euler equations

$$\beta \, \mathrm{E}_t \left[\left(\frac{C_{it+1}(\rho)}{C_i(\rho)} \right)^{-\rho} \frac{1}{C_i(\rho)} R^i_{jt+1} \right] = \left(\frac{C_{it}(\rho)}{C_i(\rho)} \right)^{-\rho} \frac{1}{C_i(\rho)} - \theta \left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}} \right)^{-\rho} . \tag{15}$$

If the wealth-in-utility parameter θ is positive, real wealth directly raises utility, in addition to its traditional indirect effect through consumption. A rise in present consumption that lowers real wealth will simultaneously lower utility from real wealth, and agents forming optimal consumption plans will account for this effect by subtracting the marginal utility of real wealth from the marginal utility of consumption in their Euler equations. This trade-off determines an optimal level of real wealth and ensures stationarity.

In the second stage, agents choose quantities of home and foreign goods to include in their consumption baskets, subject to the plans they formed in the first stage of the problem. The second-stage problem yields demand functions for home and foreign goods,

$$C_{ijt}(\rho) = \alpha_{ij} \frac{P^i_{it}}{P^i_{Cjt}} C_{it}(\rho) \, . \tag{16}$$

The demand functions in (16) can be rearranged to equate α_{ij} with the expenditure share on the good from country j for agents in country i.

Finally, the expenditure constraints in (10) and goods demands in (16) from agents' second-stage utility maximization problems together with the definition of the consumption basket in (9) imply an optimal price index. For the consumption basket in country i, the price index in numéraire currency k is

$$P^k_{it} = \left(P^k_{Cit} \right)^{\alpha_{ii}} \left(P^k_{Cjt} \right)^{\alpha_{ij}} , \quad i \neq j \, . \tag{17}$$

We derive the Euler equation in (15), the goods demands in (16), and the price index in (17) in Appendix B.3.

3.2.2 Non-Stochastic Steady State.

We now derive the non-stochastic steady-state equilibrium of the model, where all variables are constant and equal to their expected values. In the steady state, the model simplifies considerably because all heterogeneity vanishes.

Evaluating the agent-specific Euler equations in (15) at the non-stochastic steady state, we obtain

$$\theta C_i(\rho) = 1 - \beta R_j^i \,, \tag{18}$$

where the absence of time subscripts indicates the steady state. The left side of (18) is independent of j, so the steady-state real return on the home bond equals the steady-state real return on the foreign bond when returns are expressed in a common numéraire, $R_i^i = R_j^i$.

If we set $\theta = 0$, real wealth drops out of the utility function. The agent-specific Euler equations in (18) simplify to $R_i^i = R_j^i = 1/\beta$ and impose no optimality conditions on steady-state consumption for individual agents. Without such conditions, consumption is only constrained by agents' budgets, but budget constraints alone cannot determine both consumption and real wealth, so the steady state is indeterminate. Assuming an exogenous steady-state distribution of real wealth across agents would not fully repair the model because the model is also non-stationary when we set $\theta = 0$, as we later show. For these reasons, we focus on positive values of θ in the neighborhood of zero, $\theta \in [0, \epsilon)$.

Numerous alternatives to wealth-in-utility have been proposed to induce determinacy and stationarity in representative-agent models. Common alternatives include endogenous subjective discount factors, portfolio holding or adjustment costs, and overlapping generations, as discussed by Schmitt-Grohé and Uribe (2003) and Ghironi (2006). We find wealth in the utility function to be the simplest and most tractable device in our setting with heterogeneity in risk aversion, because other devices either hinder aggregation or introduce an incidental second dimension of heterogeneity into the model.

Because agents differ only in their aversion to risk and because risk is absent from the non-stochastic steady state, the steady-state distribution of consumption and real wealth across agents is degenerate, in the sense that all agents consume equal quantities and hold zero real wealth. This fact leads to a particularly simple steady-state solution for individual and aggregate non-portfolio variables and allows us to state the following proposition, which we prove in Appendix B.4.1.

Proposition 1 (Non-Stochastic Steady-State Non-Portfolio Equilibrium)**.** *Heterogeneity in risk aversion vanishes from the non-portfolio equilibrium conditions of the model in the non-stochastic steady state. The distribution of non-portfolio variables across agents is endogenously degenerate.*

Agent-specific and aggregate steady-state consumption baskets and goods demands are given by

$$C_i(\rho) = C_i = 1 \quad \textit{and} \quad C_{ij}(\rho) = C_{ij} = \alpha_{ij} \,, \tag{19}$$

steady-state price indices for consumption baskets, prices for consumption goods, and the

real exchange rate are given by

$$P_i^j = P_{Ci}^j = M_i^j \quad \textit{and} \quad Q_{ij} = 1\,, \tag{20}$$

and agent-specific and aggregate steady-state real wealth and real returns on nominal bonds are given by

$$W_i^j(\rho) = W_i^j = 0 \quad \textit{and} \quad R_i^j = (1-\theta)/\beta\,. \tag{21}$$

Because real returns on domestic and international nominal bonds are equal in the steady state, we drop country and currency superscripts and henceforth write R.

We now turn to steady-state portfolios. In contrast to the non-portfolio steady state, the portfolio steady state does depend on individual and aggregate risk aversion. We derive steady-state portfolio solutions by extending a method developed by Samuelson (1970) and Devereux and Sutherland (2011) to our heterogeneous-agent setting. Using the method, we derive order-zero approximate individual and aggregate portfolio holdings from second-order approximate equilibrium conditions. The latter conditions involve variance and covariance terms, which each agent evaluates differently according to their individual risk preferences. This fact produces the non-degenerate distribution of portfolio holdings across agents that we later characterize analytically. For now, we provide a limited characterization in the following proposition, which we prove in Appendix B.4.2.

Proposition 2 (Non-Stochastic Steady-State Portfolio Equilibrium)**.** *Agent-specific and aggregate steady-state portfolio holdings satisfy*

$$B_{ii}^i(\rho) + B_{ij}^i(\rho) = B_{ii}^i + B_{ij}^i = 0\,, \quad i \neq j\,. \tag{22}$$

Additionally, aggregate steady-state portfolio holdings satisfy

$$B_{ii}^i = B_{jj}^i\,, \tag{23}$$

while no such condition holds for agents, because bond markets must only clear in aggregate.

We later show that heterogeneity in agents' steady-state portfolios contributes to heterogeneity in consumption and real wealth dynamics through portfolio valuation effects. At the same time, the absence of heterogeneity in the non-portfolio steady state greatly simplifies aggregation, both in and around the steady state. We exploit this aggregability extensively in the following sections.

3.2.3 Model Aggregability

We aggregate variables across agents by integrating the product of individual agents' variables and the density function $f(\rho)$,

$$\begin{gathered} Y_{it} = \int_{\mathcal{R}} Y_{it}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad C_{it} = \int_{\mathcal{R}} C_{it}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad C_{ijt} = \int_{\mathcal{R}} C_{ijt}(\rho) f(\rho)\,\mathrm{d}\rho\,, \\ W_{it}^{j} = \int_{\mathcal{R}} W_{it}^{j}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad \text{and} \quad B_{ijt}^{k} = \int_{\mathcal{R}} B_{ijt}^{k}(\rho) f(\rho)\,\mathrm{d}\rho\,, \end{gathered} \tag{24}$$

where the absence of an agent index indicates an aggregate variable. The set of agents in each country has a measure of one, so our model makes no distinction between aggregates and averages.

We apply the same procedure to agents' coefficients of risk aversion and intertemporal elasticities of substitution to obtain aggregates of these parameters. The simple inverse relationship between the agent-specific parameters in (4) fails in aggregate, due to a wedge produced by the heterogeneity across agents, as we emphasize in the following proposition and prove in Appendix B.5.2.

Proposition 3 (Aggregate Preference Parameters)**.** *We define the aggregate coefficient of risk aversion $\bar{\rho}$ and intertemporal elasticity of substitution $\bar{\sigma}$ as*

$$\bar{\rho} = \int_{\mathcal{R}} \rho f(\rho)\,\mathrm{d}\rho \quad \textit{and} \quad \bar{\sigma} = \int_{\mathcal{R}} \sigma(\rho) f(\rho)\,\mathrm{d}\rho\,, \tag{25}$$

respectively. Given the density function in (1)*, it follows that*

$$\bar{\sigma} = \omega/\bar{\rho}\,, \quad \textit{with} \quad \omega = \kappa^2/(\kappa^2 - 1)\,, \tag{26}$$

where ω denotes the aggregation wedge. The wedge exceeds one for finite values of the shape parameter κ, and approaches one in the limit as κ approaches infinity. As κ rises, the cross-sectional distribution of risk aversion collapses around the scale parameter ρ_m.

The aggregation wedge ω decouples aggregate risk preferences from aggregate intertemporal substitutability, offering an intermediate degree of flexibility between time-separable preferences with constant relative risk aversion and the recursive preferences of Epstein and Zin (1989).

Aggregating variables and parameters over agents is straightforward, but aggregating equilibrium conditions over agents is not. Complications arise when an agent variable cannot be additively separated from the agent-specific coefficient of relative risk aversion. In particular, the agent Euler equations in (15) are not easily aggregable in their exact form. However, in the non-stochastic steady state, heterogeneity vanishes and exact aggregation over agents becomes trivial. The absence of heterogeneity in the steady state also makes approximations around the steady state particularly tractable.

We use Taylor series expansions to approximate the model. For positive variables like consumption baskets, goods demands, endowments, money supplies, prices, and exchange rates, hats denote logarithmic deviations from the steady state. Since portfolio holdings and real wealth are not always positive, we expand these variables in levels around their steady-state values. For real wealth, hats denote deviations relative to the steady-state endowment, $\hat{W}^j_{it}(\rho) = \Big(W^j_{it}(\rho) - W^j_i(\rho)\Big)/Y_j$, because steady-state real wealth is zero. For real returns, we also expand in levels, using hats to denote deviations relative to the steady-state level, $\hat{R}^j_{it} = \Big(R^j_{it} - R\Big)/R$.

Approximations of the agent Euler equations in (15) are simpler to aggregate than the exact equations because, with internal habits, the agent-specific coefficient of relative risk aversion is additively separable from agent variables in the approximations. We highlight this convenient properly in the following proposition, which we prove in Appendix B.5.3.

Proposition 4 (First-Order Aggregate Euler Equations)**.** *First-order Taylor expansions of agent Euler equations in* (15) *around the non-stochastic steady state are given by*

$$\hat{C}_{it}(\rho) = \theta \hat{W}^i_{it}(\rho) + \beta R \, \mathrm{E}_t \left[\hat{C}_{it+1}(\rho) - \frac{1}{\rho} \hat{R}^i_{jt+1} \right] + O(\epsilon^2) \,, \tag{27}$$

where the inverse of the agent-specific coefficient of relative risk aversion multiples the real return, an aggregate variable. Multiplying the agent-specific Euler equations by the density function in (1) *and integrating yields the aggregate Euler equations*

$$\hat{C}_{it} = \theta \hat{W}^i_{it} + \beta R \, \mathrm{E}_t \left[\hat{C}_{it+1} - \frac{\omega}{\rho} \hat{R}^i_{jt+1} \right] + O(\epsilon^2) \,, \tag{28}$$

where the aggregation wedge ω defined in (26) *summarizes the first-order effect of heterogeneity on the aggregate economy.*

The agent-specific coefficient of relative risk aversion does not appear directly in the remaining agent-specific equilibrium conditions, and first-order Taylor expansions of these conditions allow for straightforward first-order aggregation. The first-order Taylor expansion of the agent-specific budget constraints in (5) and the corresponding aggregate budget constraints are given by

$$\begin{aligned} \hat{W}^i_{it}(\rho) + \hat{C}_{it}(\rho) &= R \hat{W}^i_{it-1}(\rho) + R B^i_{ii}(\rho) \Big(\hat{R}^i_{it} - \hat{R}^i_{jt} \Big) \\ &\quad + \hat{P}^i_{Cit} - \hat{P}^i_{it} + \hat{Y}_{it}(\rho) + O(\epsilon^2) \quad \text{and} \\ \hat{W}^i_{it} + \hat{C}_{it} &= R \hat{W}^i_{it-1} + R B^i_{ii} \Big(\hat{R}^i_{it} - \hat{R}^i_{jt} \Big) \\ &\quad + \hat{P}^i_{Cit} - \hat{P}^i_{it} + \hat{Y}_{it} + O(\epsilon^2) \,, \quad i \neq j \,. \end{aligned} \tag{29}$$

The first-order Taylor expansion of agent-specific goods demands in (16) and the corre-

sponding aggregate goods demands are given by

$$\hat{C}_{ijt}(\rho) = \hat{P}^j_{it} - \hat{P}^j_{Cit} + \hat{C}_{it}(\rho) + O(\epsilon^2) \quad \text{and} \quad \hat{C}_{ijt} = \hat{P}^j_{it} - \hat{P}^j_{Cit} + \hat{C}_{it} + O(\epsilon^2)\,. \tag{30}$$

The remaining equilibrium conditions of the model depend solely on aggregate variables and require no aggregation. Their first-order Taylor expansions are standard and we provide them in Appendix B.6. In the following two sections, we use the aggregate equilibrium conditions to solve for aggregate consumption and real wealth, and then for aggregate portfolio holdings and returns. When the aggregate coefficient of relative risk aversion exceeds the aggregation wedge ω (a mild condition), we show that the aggregate portfolio displays domestic bias (Proposition 8) and earns low expected returns (Proposition 9), consistent with the patterns we document in Table 1 for Germany, Japan, and the United States.

3.2.4 The Aggregate Non-Portfolio Problem

We find it convenient to solve the model first for cross-country differences in consumption and real wealth, and for the real exchange rate. Using these solutions and market clearing conditions, we then derive country-specific solutions.

The aggregate equilibrium conditions derived from first-order aggregation of agent-specific equilibrium conditions, together with first-order approximations of the remaining of equilibrium conditions of the model, produce a system of aggregate equations with no agent-specific variables. This aggregate system can be solved independently from the agent-specific equilibrium conditions of the model.

We solve the aggregate system first for cross-country differenced aggregate consumption and real wealth. These solutions then determine the real exchange rate. To see that cross-country differences suffice, combine goods market clearing conditions in (11) with aggregate goods demand equations in (30) to obtain

$$\phi\hat{Y}_{i-jt} = \left(1-\phi^2\right)\hat{Q}_{ijt} + \phi^2\hat{C}_{i-jt} + O(\epsilon^2) \quad \text{with} \quad \phi \equiv \alpha_{ii} - \alpha_{ij}\,, \tag{31}$$

where the subscript $i-j$ denotes a cross-country difference. The parameter ϕ measures the strength of preferences for the domestic good, is symmetric across countries, and lies between zero and one, $\phi \in (0,1)$. Equation (31) states that the real exchange rate and cross-country difference in consumption adjust to clear international goods markets. Where the adjustment occurs—more through the real exchange rate or through consumption—depends on the strength of preferences for domestic goods and the tolerance of agents for consumption fluctuations.

The market clearing condition in (31) is one of three equations forming the cross-country differenced aggregate system. The other two are the cross-country differenced

aggregate Euler equation and the cross-country differenced aggregate budget constraint. These equations depend on deflated nominal variables—specifically, aggregate real wealth and the real return on nominal bonds—and it is useful to express these in a common numéraire. Using definitions (6) and (7), and noting that steady-state aggregate real wealth equals zero (Proposition 2), we obtain

$$\begin{aligned}\hat{W}^i_{i-jt} &= \hat{W}^i_{it} - \hat{W}^i_{jt} = \hat{W}^i_{it} - \hat{W}^j_{jt} + O(\epsilon^2) \quad \text{and} \\ \hat{R}^i_{i-jt} &= \hat{R}^i_{it} - \hat{R}^i_{jt} = \hat{R}^i_{it} - \hat{R}^j_{jt} - \left(\hat{Q}_{ijt} - \hat{Q}_{ijt-1}\right) + O(\epsilon^2)\,,\end{aligned} \tag{32}$$

where $\hat{W}^k_{i-jt}$ denotes the cross-country difference in aggregate real wealth and $\hat{R}^k_{i-jt}$ denotes the cross-country difference in real returns on nominal bonds, both in numéraire basket k.

With this notation in place, we write the cross-country differenced aggregate budget constraint and Euler equation as

$$\begin{aligned}\hat{W}^i_{i-jt} + \hat{C}_{i-jt} &= R\hat{W}^i_{i-jt-1} - \frac{1-\phi}{\phi}\hat{Q}_{ijt} + \hat{Y}_{i-jt} + R\hat{V}^i_{i-jt} + O(\epsilon^2) \quad \text{and} \\ \hat{C}_{i-jt} - \theta\hat{W}^i_{i-jt} &= \beta R\,\mathrm{E}_t\left[\hat{C}_{i-jt+1} - \frac{\omega}{\bar{\rho}}\left(\hat{R}^i_{i-jt+1} + \hat{Q}_{ijt+1} - \hat{Q}_{ijt}\right)\right] + O(\epsilon^2)\,,\end{aligned} \tag{33}$$

respectively, where we introduce the cross-country differenced aggregate portfolio valuation effect $\hat{V}^i_{i-jt}$, defined as

$$\hat{V}^i_{i-jt} \equiv B^i_{ii-ji}\hat{R}^i_{i-jt} + O(\epsilon^2) \quad \text{with} \quad B^i_{ii-ji} \equiv B^i_{ii} - B^i_{ji}\,. \tag{34}$$

The portfolio valuation effect is endogenous, but behaves as an independent and identically distributed random variable, as shown by Devereux and Sutherland (2011). Consequently, we can temporarily treat the effect as if it were exogenous and derive intermediate solutions in terms of it. Later, we can use these intermediate solutions together with second-order Taylor expansions of the Euler equations in (15) to derive final solutions.

Using the market clearing condition in (31) to eliminate the real exchange from the cross-country differenced aggregate Euler equation and budget constraint in (33), we obtain a system of two first-order approximate difference equations,

$$\mathrm{E}_t\left[\hat{\boldsymbol{Z}}_{i-jt+1}\right] = \boldsymbol{\mathcal{E}}^{(-)}_{ZZ}\hat{\boldsymbol{Z}}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_{ZY}\hat{Y}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_{ZV}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{35}$$

where $\hat{\boldsymbol{Z}}_{i-jt} = \begin{bmatrix}\hat{W}^i_{i-jt-1} & \hat{C}_{i-jt}\end{bmatrix}'$ denotes the (2×1) vector of cross-country differences in aggregate real wealth and aggregate consumption and where $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$, $\boldsymbol{\mathcal{E}}^{(-)}_{ZV}$, and $\boldsymbol{\mathcal{E}}^{(-)}_{ZY}$ denote matrices of partial elasticities. The matrix $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$, for example, is (2×2) with elements $\mathcal{E}^{(-)}_{mn}$, where $m, n \in \{W, C\}$. We provide a derivation of (35) in Appendix B.7, along with expressions for the full set of partial elasticities.

The cross-country differenced aggregate system in (35) has one non-predetermined

variable and admits a unique non-explosive solution if the matrix of partial elasticities $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$ has exactly one eigenvalue that lies strictly outside the unit circle, satisfying the conditions of Blanchard and Kahn (1980). If the second eigenvalue lies strictly inside the unit circle, the system is stationary.

Setting $\theta = 0$ removes wealth from the utility function and the aggregate system joins the class of non-stationary incomplete markets models discussed by Schmitt-Grohé and Uribe (2003) and Ghironi (2006). This knife-edge non-stationarity serves as a benchmark: by considering the neighborhood around $\theta = 0$, we establish a condition for stationarity, stated in the following proposition and proved in Appendix B.7.2.

Proposition 5 (Stationarity of the Cross-Country Differenced Aggregate System)**.** *Let $\theta \in [0, \epsilon)$, where ϵ is a small positive number. Then the matrix $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$ has two distinct real-valued eigenvalues, $\lambda^{(-)}_{ZW}$ and $\lambda^{(-)}_{ZC}$. Furthermore, $\lambda^{(-)}_{ZW}\big|_{\theta=0} = 1/\beta$ and $\lambda^{(-)}_{ZC}\big|_{\theta=0} = 1$, and*

$$\partial\lambda^{(-)}_{ZW}/\partial\theta\,\big|_{\theta=0} > 0 \quad \textit{and} \quad \partial\lambda^{(-)}_{ZC}/\partial\theta\,\big|_{\theta=0} < 0\,. \tag{36}$$

Hence, there exists a $\theta > 0$ that yields a unique and stationary rational expectations solution to the cross-country differenced aggregate system in (35)*.*

The aggregate system admits a unique and stationary solution when the wealth-in-utility parameter θ takes a small positive value, and we proceed under this assumption.[5]

To derive intermediate solutions to the aggregate system, we first factorize the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$ into a diagonal matrix of eigenvalues and a matrix of eigenvectors, decoupling the two difference equations in the system. We then impose the condition that the endogenous variables in the decoupled equations do not "explode too fast," as in Blanchard and Kahn (1980). This procedure yields intermediate solutions in terms of lagged aggregate real wealth, aggregate real portfolio valuation effects, and aggregate endowments in cross-country differences. Coefficients in the intermediate solution represent semi-partial elasticity—semi-partial because they fail to capture the effects of exogenous variables that act on aggregate real wealth and aggregate consumption indirectly through portfolio valuation effects.

Our intermediate solution to the cross-country differenced aggregate system in (35) is given by

$$\begin{bmatrix} \hat{W}^i_{i-jt} \\ \hat{C}_{i-jt} \end{bmatrix} = \boldsymbol{\eta}^{(-)}_{ZW}\hat{W}^i_{i-jt-1} + \boldsymbol{\eta}^{(-)}_{ZY}\hat{Y}_{i-jt} + \boldsymbol{\eta}^{(-)}_{ZV}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{37}$$

where $\boldsymbol{\eta}^{(-)}_{ZW}$, $\boldsymbol{\eta}^{(-)}_{ZY}$, and $\boldsymbol{\eta}^{(-)}_{ZV}$ denote matrices of semi-partial elasticities that depend only on the parameters of the model.[6]

[5]We remark that the aggregate cross-country differenced system also admits a unique and stationary solution when θ is negative and real wealth has a direct negative effect on utility.

[6]We reiterate the distinction between partial and semi-partial elasticities using the example of $\mathcal{E}^{(-)}_{WW}$ and $\eta^{(-)}_{WW}$. The semi-partial elasticity $\eta^{(-)}_{WW}$ captures the indirect effect of past wealth on current wealth

To obtain intermediate solutions for country-specific aggregate real wealth and consumption, we use the intermediate solutions for cross-country differences in (37) together with first-order Taylor expansions of the market clearing conditions in (11),

$$\hat{C}_{i+jt} = \hat{Y}_{i+jt} + O(\epsilon^2) \quad \text{and} \quad \hat{W}^i_{i+jt} = 0 + O(\epsilon^2)\,, \tag{38}$$

where the subscript $i+j$ denotes a cross-country sum. Intermediate solutions for country-specific aggregate consumption and real wealth are then obtained as

$$\hat{C}_{it} = \frac{1}{2}\big(\hat{C}_{i+jt} + \hat{C}_{i-jt}\big) + O(\epsilon^2) \quad \text{and} \quad \hat{W}^i_{it} = \frac{1}{2}\hat{W}^i_{i-jt} + O(\epsilon^2)\,, \tag{39}$$

using the market clearing conditions in (38) and the intermediate solutions for cross-country differences in (37).

We obtain an intermediate solution for the real exchange rate using the goods market clearing condition in (31) together with the intermediate solutions for cross-country differenced aggregate real wealth and aggregate consumption in (37),

$$\hat{Q}_{ijt} = \eta^{(-)}_{QW}\hat{W}^i_{i-jt-1} + \eta^{(-)}_{QY}\hat{Y}_{i-jt} + \eta^{(-)}_{QV}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{40}$$

where the semi-partial elasticities $\eta^{(-)}_{QW}$, $\eta^{(-)}_{QY}$, and $\eta^{(-)}_{QV}$ depend on parameters of the model.

We provide derivations of the intermediate solutions for cross-country differenced aggregate real wealth and consumption and for the real exchange rate in Appendix B.7.3, along with expressions for the full set of semi-partial elasticities. The solutions are intermediate because they depend on the endogenous cross-country differenced portfolio valuation effect $\hat{V}^i_{i-jt}$ defined in (34). The portfolio valuation effect has two components: cross-country differenced aggregate portfolio holdings and cross-country differenced real returns on nominal bonds.

In the following section, we turn to the aggregate portfolio problem and derive solutions for these two components. In the process, we introduce a portfolio valuation multiplier that amplifies carry trade returns when aggregate portfolios display domestic bias. We then establish sufficient conditions for this domestic bias to arise and show that these aggregate portfolios earn low returns—setting the stage for carry traders to emerge as an endogenous minority in these economies.

3.2.5 The Aggregate Portfolio Problem

We begin with the cross-country differenced real return on nominal bonds. Using first-order Taylor expansions of the real returns defined in (7) and the quantity equations defined in (14) together with our intermediate solutions for cross-country differenced aggregate real

via consumption, but not via portfolio valuation effects. In contrast, the partial elasticity $\mathcal{E}^{(-)}_{WW}$ captures for neither indirect effect. Later, we will solve for general elasticities that capture both indirect effects.

wealth and consumption in (37), we obtain

$$\hat{R}^i_{i-jt} = \eta^{(-)}_{RY}\hat{Y}_{i-jt} + \eta^{(-)}_{RM}\hat{M}^{i-j}_{i-jt} + \eta^{(-)}_{RV}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{41}$$

where $\hat{M}^{i-j}_{i-jt} \equiv \hat{M}^i_{it} - \hat{M}^j_{jt}$, and where the semi-partial elasticities $\eta^{(-)}_{RY}$, $\eta^{(-)}_{RM}$, and $\eta^{(-)}_{RV}$ depend only on the parameters of the model.

The valuation effect $\hat{V}^i_{i-jt}$ in (41) itself depends on the cross-country differenced real return. The expression can, therefore, be rearranged to isolate the cross-country differenced real return on the left-hand side,

$$\hat{R}^i_{i-jt} = \gamma^{(-)}_{RY}\hat{Y}_{i-jt} + \gamma^{(-)}_{RM}\hat{M}^{i-j}_{i-jt} + O(\epsilon^2)\,, \tag{42}$$

where $\gamma^{(-)}_{RY}$ and $\gamma^{(-)}_{RM}$ denote general elasticities, in that they account for general equilibrium effects of the exogenous domestic and international endowments and money supplies on real returns through all endogenous variables in the model, including portfolio valuation effects. These general elasticities are given by

$$\gamma^{(-)}_{RY} = \mu^{(-)}_{ii-ji}\eta^{(-)}_{RY} \quad \text{and} \quad \gamma^{(-)}_{RM} = \mu^{(-)}_{ii-ji}\eta^{(-)}_{RM} \quad \text{with} \quad \mu^{(-)}_{ii-ji} \equiv \frac{1}{1-\eta^{(-)}_{RV}B^i_{ii-ji}}\,, \tag{43}$$

where $\mu^{(-)}_{ii-ji}$ denotes the cross-country differenced portfolio valuation multiplier. The multiplier plays an important role in shaping the risk facing agents who trade in bonds markets. In particular, domestic bias in aggregate portfolio holdings amplifies risk while international bias attenuates it. The following proposition, which we prove in Appendix B.8.1, emphasizes this point.

Proposition 6 (The Portfolio Valuation Multiplier)**.** *The semi-partial elasticity $\eta^{(-)}_{RV}$ is positive. Bounded domestic bias in aggregate steady-state portfolio holdings implies*

$$1 < \mu^{(-)}_{ii-ji} \quad \forall\, 0 < B^i_{ii-ji} < \frac{2}{\eta^{(-)}_{RV}}. \tag{44}$$

In this case, the multiplier amplifies the response of the cross-country differenced real return to fluctuations in endowments and money supplies. In contrast, international bias in aggregate steady-state portfolio holdings implies

$$0 \leq \mu^{(-)}_{ii-ji} < 1 \quad \forall\, B^i_{ii-ji} < 0. \tag{45}$$

In this case, the multiplier attenuates the response of the cross-country differenced real return to fluctuations in endowments and money supplies.

The portfolio valuation multiplier in (43) depends on cross-country differenced steady-state aggregate portfolio holdings B^i_{ii-ji}, which the first-order Taylor expansions used thus

far leave aggregate portfolio holdings indeterminate. This indeterminacy arises because, to a first-order approximation, the bonds in our model are perfect substitutes, yielding identical expected returns. The Euler equations in (27) directly show this substitutability. Differencing across bonds, we have

$$\mathrm{E}_t\Big[R^i_{it+1}\Big] = \mathrm{E}_t\Big[R^i_{jt+1}\Big] + O(\epsilon^2)\,. \tag{46}$$

The first-order indeterminacy of portfolio holdings is a common feature of macroeconomic models with non-trivial portfolio choice, as Coeurdacier and Rey (2013) observe.

Higher-order approximation methods resolve this indeterminacy. Samuelson (1970) first developed these methods in partial equilibrium, assuming "riskiness" was "limited" and deriving order-n approximate local solutions for portfolio holdings using order-$(n+2)$ approximate equilibrium conditions. Devereux and Sutherland (2011) later extended the method to general equilibrium, recognizing that Samuelson's assumption of limited risk aligns with perturbation methods used to solve dynamic stochastic general equilibrium models. We use the Samuelson-Devereux-Sutherland method to derive aggregate portfolio holdings and later extend it to derive agent-specific portfolio holdings.

To derive order-zero approximate aggregate portfolio holdings, we require second-order approximate agent-specific Euler equations, which we then aggregate. Aggregation can be more challenging in second-order approximations because of quadratic terms, but these terms drop out when we difference Euler equations across domestic and international bonds, and the remaining terms aggregate straightforwardly. This fact allows us to state the following proposition, which we prove in Appendix B.8.2.

Proposition 7 (Second-Order Individual and Aggregate Euler Equations)**.** *Differencing second-order Taylor expansions of the agent-specific Euler equations in* (15) *across bonds, and then differencing and summing across countries, yields, respectively,*

$$\begin{aligned} 0 &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i-jt+1}(\rho) - \frac{1}{\rho}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3) \quad \textit{and} \\ \frac{2}{\rho}\,\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\Big] &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i+jt+1}(\rho) + \frac{1}{\rho}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3)\,, \end{aligned} \tag{47}$$

where the inverse of the agent-specific coefficient of risk aversion multiples only aggregate variables. Multiplying by the density function in (1) *and integrating yields second-order*

approximate differenced and summed aggregate Euler equations, respectively,

$$\begin{aligned} 0 &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i-jt+1} - \frac{\omega}{\bar{\rho}}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3) \quad \textit{and} \\ \omega\frac{2}{\bar{\rho}}\,\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\right] &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i+jt+1} + \frac{\omega}{\bar{\rho}}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3)\,. \end{aligned} \tag{48}$$

The second-order approximate cross-country differenced aggregate Euler equation in (48), together with our first-order approximate solutions, suffice to determine order-zero approximate aggregate portfolio holdings. The cross-country summed version of the same, together with our first-order approximate solutions, suffice to determine second-order approximate expected cross-country differenced real returns. We now solve for these two portfolio variables.

For portfolio holdings, we substitute the first-order approximate solutions (37) for $\hat{C}_{i-jt}$, (40) for $\hat{Q}_{ijt}$, and (42) for $\hat{R}^i_{i-jt+1}$ into the second-order approximate differenced Euler equation in (48). The final solution for B^i_{ii-ji} that obtains after substitution is somewhat unwieldy; we provide it in Appendix B.8.3 and focus here on an intermediate solution in terms of cross-country differenced real returns,

$$B^i_{ii-ji} = \zeta_B^{(-)}\frac{\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\right]}{\mathrm{E}_t\left[\left(\hat{R}^i_{i-jt+1}\right)^2\right]} + O(\epsilon)\,, \tag{49}$$

where $\zeta_B^{(-)}$ is a collection of model parameters, and where the conditional variance of the cross-country differenced real return and its conditional covariance with the cross-country differenced aggregate endowment are constant over time. The collection of parameters $\zeta_B^{(-)}$ is given by

$$\zeta_B^{(-)} = -\frac{\bar{\rho}\eta^{(-)}_{CY} - \omega\eta^{(-)}_{QY}}{\bar{\rho}\eta^{(-)}_{CV} - \omega\eta^{(-)}_{QV}}\,. \tag{50}$$

We use the bond market clearing condition in (31) together with the solution for cross-country differenced real bond holdings in (49) to obtain country-specific solutions,

$$B^i_{ii} = -B^i_{ij} = \frac{1}{2}B^i_{ii-ji}\,. \tag{51}$$

In Table 1, we documented strong domestic bias in the aggregate portfolio holdings of three large economies with low interest rates: Germany, Japan, and the United States. Our theoretical model endogenously produces domestic bias in aggregate portfolios when agents are sufficiently risk-averse on average. The intuition for this result is straightforward: risk averse agents prefer to hedge their exposure to endowment fluctuations; because the cross-country differenced real return $\hat{R}^i_{i-jt+1}$ covaries negatively with the cross-country

differenced aggregate endowment $\hat{Y}_{i-jt+1}$, a long position in the domestic asset funded by a short position in the international asset produces the desired hedge. We state the sufficient condition for domestic bias in aggregate portfolio holdings in the following proposition, which we prove in Appendix B.8.3.

Proposition 8 (Domestic Bias in Aggregate Portfolio Holdings)**.** *If $\bar{\rho} > \omega$, then steady-state aggregate portfolio holdings satisfy the order-zero approximate inequalities*

$$B^i_{ij} < 0 + O(\epsilon) < B^i_{ii}\,. \tag{52}$$

For finite values of the shape parameter κ from the distribution of risk aversion, the aggregation wedge ω exceeds one. As the shape parameter approaches infinity, heterogeneity in risk aversion vanishes and the aggregation wedge ω approaches one.

The condition that $\bar{\rho}$ exceeds ω is relatively mild, and is satisfied under our assumptions on ρ_m and κ. For example, the inverse Pareto distribution shown in Figure 1 with $\kappa = 4$ implies $\omega \approx 1.33$, while Lustig and Verdelhan (2007) assume a coefficient of risk aversion that exceeds 100, and Elminejad et al. (2022) estimate a median of 3.77 and a mean of 23.36 in a 92-study meta-analysis.

We provide a final solution for aggregate portfolio holdings in Appendix B.10.2, where we also relate our result to a classic finding of Cole and Obstfeld (1991). In particular, when $\bar{\rho} = \omega$ and $\theta = 0$, equation (50) implies $\zeta^{(-)}_B = 0$, yielding zero aggregate portfolio holdings. This special case parallels the Cole-Obstfeld result in representative agent models where portfolios are zero when the coefficient of relative risk aversion equals one. As we later show, individual agents in our model still hold non-zero portfolios in this special case, suggesting positive individual gains from international risk sharing.

Having established sufficient conditions for domestic bias in aggregate portfolio holdings, we now turn to expected returns. We obtain a second-order approximation of expected cross-country differenced real returns by substituting the first-order approximate intermediate solutions in (37) for $\hat{C}_{i-jt}$ and (40) for $\hat{Q}_{ijt}$ into the second-order approximate summed Euler equation in (48). The resulting intermediate solution,

$$\begin{aligned}
\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\Big] = {} & \frac{\omega/\bar{\rho}}{2}\,\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\hat{Y}_{i+jt+1}\Big] \\
& + \frac{\eta^{(-)}_{QY}}{2}\,\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\Big] \\
& + \frac{\eta^{(-)}_{QV}}{2}\,\mathrm{E}_t\Big[\Big(\hat{R}^i_{i-jt+1}\Big)^2\Big] + O(\epsilon^3)\,,
\end{aligned} \tag{53}$$

can be reduced to a final solution using first-order approximate realized real returns in (41) and the intermediate solution for aggregate cross-country differenced portfolio holdings in (49). In the following proposition, which we prove in Appendix B.8.4, we establish

sufficient conditions for the cross-country differenced expected real return to be negative.

Proposition 9 (Negative Expected Cross-Country Differenced Real Returns)**.** *If $\bar{\rho} > \omega$, then the cross-country differenced expected real return satisfies the inequality*

$$\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\right] < 0 + O(\epsilon^3) \tag{54}$$

and, to a second-order approximation, the domestic bond in country i offers domestic agents a lower expected real return than the international bond after currency conversion.

Intuitively, under Proposition 9, a relative rise in the domestic endowment lowers the domestic price level and raises the real return on the domestic nominal bond. At the same time, domestic agents with greater endowments import more, causing the real exchange rate to depreciate, which increases the real return on the international bond in terms of the domestic numéraire. The cross-country *differenced* real return will be negative when the depreciation effect dominates, which occurs when the preference for domestic goods is strong ($\phi > 0$) and aversion to consumption fluctuations is high ($\bar{\rho} > \omega$). Much of this intuition can be seen directly in the market clearing condition (31).

Under Propositions 8 and 9, country i holds an aggregate long position in the domestic bond and an aggregate short position in the international bond, leading to an expected negative real return. Thus, aggregate portfolios exhibit domestic bias despite earning low returns, as we observe for Germany, Japan, and the United States in Table 1.

Next, we examine individual agents to identify the carry traders in such economies. Following our aggregate approach, we first solve the non-portfolio problem in Section 3.2.6—largely procedural—before turning to the portfolio problem in Section 3.2.7, where we establish our central result in Proposition 10: agents with sufficiently low individual risk aversion optimally choose carry trade portfolios, forming an endogenous minority that profits by trading against the domestic bias of the majority.

3.2.6 The Agent's Non-Portfolio Problem

Agent-specific and aggregate equilibrium conditions are structurally similar, so we solve for agent-specific variables in deviations from aggregate variables rather than from the steady state. This approach not only provides analytical convenience but also strengthens the link between individual and aggregate solutions.

Our solution procedure follows the one used for aggregate variables, except that for individual agents we must solve systems of equations in both cross-country differences and sums, because we cannot rely on market clearing conditions as we did in the aggregate problem. Once we have solutions for differences and sums, we can derive country-specific solutions for the consumption, real wealth, and portfolio holdings of individual agents.

Our solutions depend on two state variables: agent-specific and aggregate real wealth. Since each agent is measure zero, these variables evolve independently. The presence of aggregate real wealth as a second state variable does complicate the problem, but only slightly because agents treat it as exogenous in their decision-making.

We begin with the cross-country differenced system for individual agents. Because the steps to solve the cross-country summed system follow the same logic, we present them in Appendix B.9.4 to save space.

Cross-Country Differences. We derive the cross-country differenced system for an individual agent from the first-order approximate individual and aggregate Euler equations in (27) and (28) and budget constraints in (29). We first take cross-country differences for the individual and aggregate equilibrium conditions separately, then take the difference between the individual and aggregate cross-country differences.

We obtain the first-order approximate double-differenced agent-specific Euler equation as

$$\begin{aligned}\Big(\hat{C}_{i-jt}(\rho) - \hat{C}_{i-jt}\Big) - \theta\Big(\hat{W}^i_{i-jt}(\rho) - \hat{W}^i_{i-jt}\Big) \\ = \beta R\,\mathrm{E}_t\left[\Big(\hat{C}_{i-jt+1}(\rho) - \hat{C}_{i-jt+1}\Big) - \left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\Big(\hat{Q}_{ijt+1} - \hat{Q}_{ijt}\Big)\right] + O(\epsilon^2)\,,\end{aligned} \tag{55}$$

where the subscript $i-j$ on agent-specific variables denotes cross-country differences between agents with identical coefficients of relative risk aversion, and where the intermediate solution for the real exchange rate in (40) can be used to eliminate the real exchange rate. Similarly, we obtain the first-order approximate double-differenced agent-specific budget constraint as

$$\begin{aligned}\Big(\hat{W}^i_{i-jt}(\rho) - \hat{W}^i_{i-jt}\Big) = R\Big(\hat{W}^i_{i-jt-1}(\rho) - \hat{W}^i_{i-jt-1}\Big) \\ - \Big(\hat{C}_{i-jt}(\rho) - \hat{C}_{i-jt}\Big) + R\Big(\hat{V}^i_{i-jt}(\rho) - \hat{V}^i_{i-jt}\Big) + O(\epsilon^2)\,,\end{aligned} \tag{56}$$

where we introduce the cross-country differenced agent portfolio valuation effect $\hat{V}^i_{i-jt}(\rho)$, defined as

$$\hat{V}^i_{i-jt}(\rho) \equiv B^i_{ii-ji}(\rho)\hat{R}^i_{i-jt} \quad \text{with} \quad B^i_{ii-ji}(\rho) \equiv B^i_{ii}(\rho) - B^i_{ji}(\rho)\,. \tag{57}$$

In the Euler equation (55) and the budget constraint (56), the agent's consumption always appears as a deviation from aggregate consumption, while aggregate consumption never enters these equations independently. Therefore, we can treat the deviation of an individual's consumption from the aggregate as a single variable. While the agent's real wealth appears as a deviation from aggregate real wealth, the latter also affects the equations separately through the real exchange rate in (40). As a result, we must treat individual and aggregate real wealth as separate state variables and include the law of

motion for aggregate real wealth (37) in the agent's cross-country differenced system.

The system is then given by three simultaneous difference equations: the Euler equation in (55), the budget constraint in (56), and the law of motion for aggregate real wealth that we solved in (37). After eliminating the real exchange rate using the intermediate solution in (40), we write the system in matrix form as

$$\begin{aligned}\mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}(\rho)\Big] = \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\hat{\boldsymbol{Z}}_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y}\hat{Y}_{i-jt} \\ + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)}\hat{V}^i_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V}\hat{V}^i_{i-jt} + O(\epsilon^2)\,,\end{aligned} \tag{58}$$

where $\hat{\boldsymbol{Z}}_{i-jt}(\rho) = \Big[\hat{W}^i_{i-jt-1}(\rho) \quad \hat{C}_{i-jt}(\rho) - \hat{C}_{i-jt} \quad \hat{W}^i_{i-jt-1}\Big]'$ denotes the (3×1) vector of the agent's cross-country differenced real wealth, cross-country differenced consumption in deviation from aggregate, and cross-country differenced aggregate real wealth, and where $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$, $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y}$, $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)}$, and $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V}$ denote matrices of partial elasticities. The matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$, for example, is (3×3) with elements $\mathcal{E}^{(-)}_{mn}$, $m, n \in \{W(\rho), C(\rho), W\}$. We provide a derivation of the agent's cross-country differenced system (58) in Appendix B.9.1, along with expressions for the full set of partial elasticities.

In Proposition B1 in Appendix B.9.2, we establish uniqueness and stationarity for the agent's cross-country differenced system. There, we follow the same strategy as we did with the aggregate cross-country differenced system, considering a neighborhood of small positive values for θ near the knife-edge non-stationary value of $\theta = 0$ to establish the result.

As we did for the aggregate system, we proceed under the assumption that θ takes a small positive value, and derive an intermediate solution to the agent's system in (58) by factorizing the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ into a diagonal matrix of eigenvalues and a matrix of eigenvectors and by imposing the condition that the endogenous variables in the system do not "explode too fast," again as in Blanchard and Kahn (1980). Our intermediate solutions to the agent's non-portfolio problem depend on the agent's lagged real wealth in deviation from aggregate, lagged aggregate real wealth independently, endowments, and portfolio valuation effects. Later, once the agent's portfolio problem has been solved, these intermediate non-portfolio solutions will combine with portfolio solutions to yield general solutions.

We introduce the "vanishing integral property" for deviations of agent variables from aggregate variables, denoting such deviations with tildes. Aggregating these deviations across agents results in zero as the deviations average out. Variables with this property remain specific to individual agents and retain the index ρ. For real wealth, consumption, and portfolio valuation effects, the deviations are written

$$\begin{gathered}\tilde{W}^i_{i-jt}(\rho) = \hat{W}^i_{i-jt}(\rho) - \hat{W}^i_{i-jt}\,, \quad \tilde{C}_{i-jt}(\rho) = \hat{C}_{i-jt}(\rho) - \hat{C}_{i-jt}\,, \\ \text{and} \quad \tilde{V}^i_{i-jt}(\rho) = \hat{V}^i_{i-jt}(\rho) - \hat{V}^i_{i-jt}\,.\end{gathered} \tag{59}$$

Some semi-partial elasticities in our intermediate solutions below will also have this property, so we also denote these with tildes. All semi-partial elasticities without tildes are independent of agent-specific coefficients of relative risk aversion and therefore common to all agents.

With this notation in place, we write our intermediate solution for the agent's double-differenced system as

$$\begin{aligned}\begin{bmatrix}\tilde{W}^i_{i-jt}(\rho)\\ \tilde{C}_{i-jt}(\rho)\end{bmatrix} &= \boldsymbol{\eta}^{(-)}_{Z(\rho)W(\rho)}\tilde{W}^i_{i-jt-1}(\rho) + \boldsymbol{\eta}^{(-)}_{Z(\rho)V(\rho)}\tilde{V}^i_{i-jt}(\rho)\\ &\quad + \tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)V}\hat{V}^i_{i-jt} + \tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)Y}\hat{Y}_{i-jt} + O(\epsilon^2)\,,\end{aligned} \tag{60}$$

where $\boldsymbol{\eta}^{(-)}_{Z(\rho)W(\rho)}$, $\boldsymbol{\eta}^{(-)}_{Z(\rho)V(\rho)}$, $\tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)V}$, and $\tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)Y}$ denote matrices of semi-partial elasticities that depend only on the parameters of the model. The tilde notation emphasizes the consistency between agent and aggregate problems, as aggregating (60) across agents results in zero. We derive this intermediate solution and provide expressions for the full set of semi-partial elasticities in Appendix B.9.3.

Cross-Country Sums. We solve the cross-country summed system using the same method as for the cross-country differenced system. The summed system is simpler because it does not involve the real exchange rate and has only one predetermined state variable, agent-specific real wealth.

For brevity, we present the solution in Appendix B.9.6. There, we establish stationarity and uniqueness in Proposition B2, before solving for cross-country summed agent consumption and real wealth in deviations from aggregates. The solution depends on lagged real wealth, lagged endowments, and portfolio valuation effects. Together, the cross-country differenced and summed solutions yield country-specific solutions for agent consumption and real wealth.

3.2.7 The Agent's Portfolio Problem

With the non-portfolio solutions in hand, we can now identify the carry traders in our model. Proposition 10 establishes our central result: while the majority of agents hold domestically-biased portfolios that hedge consumption risk, a risk-tolerant minority of agents optimally choose carry trade portfolios that profit from bearing this risk.

We establish this result by extending the Samuelson-Devereux-Sutherland method of higher-order approximation to our heterogeneous-agent setting. This extension allows us to derive closed-form solutions for the portfolio positions of individual agents in a general equilibrium continuum economy. Intuitively, agents with higher risk aversion prefer portfolios with less volatility and lower average returns, leading to consumption that is less volatile but lower on average. In contrast, agents with lower risk aversion

choose portfolios with more volatility and higher average returns, leading to consumption that is more volatile but higher on average. Given the range of different risk preferences, this model generates rich heterogeneity in investment and consumption patterns across agents within each country.

The portfolio problem of individual agents differs from the aggregate portfolio problem because there are no bond market clearing conditions for individual agents. Instead, we use the second-order approximate cross-country differenced and summed agent Euler equations in (47) to derive order-zero approximate cross-country differenced and summed agent portfolio holdings, respectively. We then combine these results to obtain country-specific solutions.

By differencing the second-order approximate cross-country differenced agent and aggregate Euler equations in (47) and (48), we obtain a second-order double-differenced Euler equation,

$$0 = \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\tilde{C}_{i-jt+1}(\rho) - \left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3)\,. \tag{61}$$

Combining (61) with our intermediate solution in (60) and the intermediate solution for the real exchange rate in (40), we obtain an intermediate order-zero approximate solution for agent-specific double-differenced domestic bond holdings,

$$\tilde{B}^i_{ii-ji}(\rho) = \zeta^{(-)}_{B(\rho)}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\frac{\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\right]}{\mathrm{E}_t\left[\left(\hat{R}^i_{i-jt+1}\right)^2\right]} + O(\epsilon)\,, \tag{62}$$

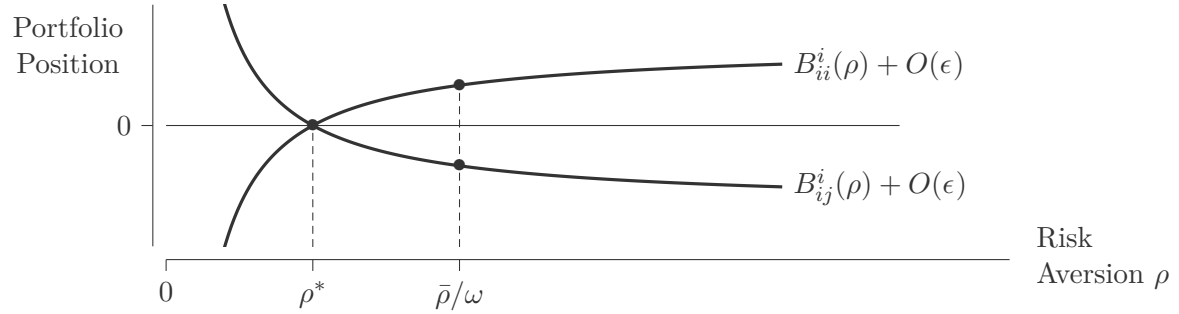
where $\tilde{B}^i_{ii-ji}(\rho) \equiv B^i_{ii-ji}(\rho) - B^i_{ii-ji}$, and where the variance of the cross-country differenced real return and its covariance with the cross-country differenced endowment are constant. Here, $\zeta^{(-)}_{B(\rho)}$ is a collection of model parameters independent of any individual agent's coefficient of relative risk aversion. We provide an expression for $\zeta^{(-)}_{B(\rho)}$ in Appendix B.10.

By differencing the second-order approximate cross-country summed agent and aggregate Euler equations (47) and (48), we obtain an Euler equation with cross-country summed agent-specific consumption expressed in deviation from aggregate,

$$0 = \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\tilde{C}_{i+jt+1}(\rho) - \frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\hat{Y}_{i+jt+1}\right)\right] + O(\epsilon^3)\,. \tag{63}$$

Combining (63) with the intermediate solution for cross-country summed agent-specific consumption in appendix equation (B111), we obtain an intermediate order-zero approximate solution for agent-specific cross-country summed bond holdings in deviation from

Figure 2 – The Cross-Section of Agents' Portfolio Holdings



Notes. The figure shows portfolio holdings of the domestic bond $B^i_{ii}(\rho)$ and international bond $B^i_{ij}(\rho)$ for agents in country i. The horizontal axis shows agents' coefficients of relative risk aversion and the vertical axis shows agents' portfolio positions. Agents' domestic bond positions rise in risk aversion, while their international bond positions fall. The agent with $\rho = \rho^*$ is hand-to-mouth. Agents with $\rho < \rho^*$ are carry traders with short domestic bond positions and long international bond positions. The agent with $\rho = \bar{\rho}/\omega$ holds a portfolio with domestic bias that matches the aggregate portfolio.

aggregate,

$$\tilde{B}^i_{ii+ji}(\rho) = \zeta^{(+)}_{B(\rho)} \left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right) \frac{\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i+jt+1}\right]}{\mathrm{E}_t\left[\left(\hat{R}^i_{i-jt+1}\right)^2\right]} + O(\epsilon)\,, \tag{64}$$

where $\tilde{B}^i_{ii+ji}(\rho) \equiv B^i_{ii+ji}(\rho) - B^i_{ii+ji}$, and where the variance of the cross-country differenced real return and its covariance with the cross-country summed endowment are constant. Here, $\zeta^{(+)}_{B(\rho)}$ is a collection of model parameters independent of agent-specific coefficients of relative risk aversion. We provide an expression for $\zeta^{(+)}_{B(\rho)}$ in Appendix B.10.

Combining agent-specific cross-country differenced and summed domestic bond holdings, we obtain an intermediate country-specific solution,

$$\tilde{B}^i_{ii}(\rho) = \frac{1}{2}\left(\tilde{B}^i_{ii-ji}(\rho) + \tilde{B}^i_{ii+ji}(\rho)\right) + O(\epsilon)\,, \tag{65}$$

where $\tilde{B}^i_{ii}(\rho) \equiv B^i_{ii}(\rho) - B^i_{ii}$, and where $B^i_{ii}(\rho) = -B^i_{ij}(\rho)$ from Proposition 2. A final solution for $\tilde{B}^i_{ii}(\rho)$ can be obtained directly from (65) using (41), (62), and (64), as we show in Appendix B.10.2.

We argued in the introduction that economies with domestically-biased aggregate portfolios earning low returns—like Germany, Japan, and the United States—are not engaged in aggregate carry trades. To model carry trades in such economies, individual carry traders must be outweighed by agents who favor domestic assets. Our heterogeneous-agent model captures this cross-sectional pattern of investment, as we establish in the following culminating proposition, which we prove in Appendix B.10.1.

Proposition 10 (International Bias in Agent-Specific Portfolio Holdings)**.** *If $\bar{\rho} > \omega$, then*

a threshold level of risk aversion $\rho^* \in (0, \bar{\rho}/\omega)$ *exists such that*

$$\begin{aligned} B^i_{ij} < 0 + O(\epsilon) < B^i_{ii} \quad \textit{while} \\ B^i_{ii}(\rho) < 0 + O(\epsilon) < B^i_{ij}(\rho) \quad \forall \rho < \rho^* \,. \end{aligned} \tag{66}$$

To an order-zero approximation, the aggregate portfolio position in country i *exhibits domestic bias while individual portfolio positions exhibit international bias for agents with coefficients of relative risk aversion below the threshold value* ρ^**. Portfolios with domestic bias earn negative expected returns, while those with international bias earn positive expected returns.*

Figure 2 illustrates order-zero approximate holdings of domestic and international bonds for the cross-section of individual agents in country i. In the illustration, country i's aggregate portfolio is biased towards the domestic asset. The agent with $\rho = \rho^*$ is hand-to-mouth, agents with $\rho < \rho^*$ hold carry trade portfolios earning positive expected returns, and agents with $\rho > \rho^*$ hold portfolios with domestic bias earning negative expected returns. The agent with $\rho = \bar{\rho}/\omega$ holds exactly the aggregate portfolio. In this way, the model endogenizes a continuum of investor types within a tractable, two-country, dynamic stochastic general equilibrium framework.

We provide a final solution for the portfolio holdings of individual agents in Appendix B.10.2, and also consider a special case related to Cole and Obstfeld (1991), where $\bar{\rho} = \omega$ and $\theta = 0$ imply zero aggregate portfolio holdings. In this special case, the closed-form solution for the portfolio holdings of individual agents simplifies dramatically. Individual agents hold non-zero positions that either hedge ($\rho > 1$) or speculate ($\rho < 1$) against consumption growth risk, but these positions exactly offset each other in the aggregate.

4 Conclusions

The currency carry trade literature has focused on rationalizing returns, but to realize these returns, investors must hold carry trade portfolios, which have been largely overlooked. A substantial literature documents strong domestic bias in aggregate portfolio holdings for low-interest-rate economies like Germany, Japan, and the United States, where domestic bias does not resemble an aggregate carry trade. In these countries, individual carry traders must therefore be outweighed by agents favoring domestic-currency assets. A complete theory of the carry trade should explain both carry trade returns and positions, while acknowledging that most agents are not carry traders—a fact representative-agent models cannot accommodate.

We resolve this tension between the finance literature on the carry trade and the international macroeconomics literature on domestic bias in aggregate portfolios by developing a theoretical model that features heterogeneity in risk aversion and captures a

full range of individual portfolio positions. To solve the model, we extend the Samuelson-Devereux-Sutherland solution method of higher-order approximations to heterogeneous agents. The model is determinate, stationary, and highly tractable, admitting closed-form solutions for both individual and aggregate variables in a familiar discrete-time setting, producing rich heterogeneity in investment and consumption behavior. Individual portfolio positions in the model endogenously range from speculative carry trades to hand-to-mouth positions to domestically-biased hedges, while the aggregate portfolio displays domestic bias. The model reconciles the existence of individual carry traders with the empirical observation of domestic bias in aggregate portfolios that earn low returns.

At the core of the model is a trade-off between the hedge that portfolios with domestic bias provide and the lower returns they offer. When the home endowment rises relative to foreign, the price of the home good falls and the real return on the home nominal bond rises. Home agents consume more of both home and foreign goods, boosting imports and depreciating the home currency. Consequently, the real return on the foreign nominal bond also rises for home agents. The *difference* in real returns—home minus foreign—depends on the responsiveness of the real exchange rate, which is greater when agents have higher average risk aversion and a stronger preference for the domestic good. The real exchange rate response dominates in our model, and the foreign real return rises above home, rendering long positions in the home bond combined with short positions in the foreign bond an effective hedge against consumption growth risk. Agents with higher risk aversion take this hedge, accepting a negative difference in returns, while agents with lower risk aversion take carry trades to exploit a positive difference in returns.

Though we frame our analysis around currency carry trades and develop our solution method in a stylized model without quantitative calibration, the method readily extends to more complex quantitative frameworks incorporating multiple assets, additional shocks, and richer frictions. The approach has wide-ranging applications: it can be used to model how financial market participation emerges endogenously, to examine distributional effects of international economic policies, to study wealth inequality's role in global imbalances, or to investigate any environment where heterogeneous preferences within countries determine both individual portfolio choices and macroeconomic aggregates. By deriving the full distribution of portfolio holdings in a standard open-economy framework, our method provides new tools for analyzing the interplay between heterogeneity and international financial markets.

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ONLINE APPENDIX

A Empirical Appendix

This appendix details our empirical methodology for estimating the country-level debt holdings that underlie Table 1. Our primary data sources are the IMF Coordinated Portfolio Investment Survey (CPIS), Securities Held as Foreign Exchange Reserves (SEFER), and International Financial Statistics (IFS) datasets, the BIS Debt Securities Statistics (DDS) and International Debt Securities (IDS) datasets, the OECD Main Economic Indicators (MEI) dataset, and the CEPII Gravity dataset.

While researchers including Fidora, Fratzscher, and Thimann (2007), Lane and Shambaugh (2010), Coppola, Maggiori, Neiman, and Schreger (2021), and Allen, Gautam, and Juvenal (2023) have produced estimates of aggregate portfolio holdings, the estimates available publicly lack one of two features we need to compute currency-adjusted returns on country-level debt portfolios: (1) estimates of both international *and* domestic debt holdings, rather than international *or* domestic holdings alone; and (2) decomposition of holdings by both issuer country *and* issuance currency, rather than by issuer *or* currency alone. Our estimates have these two features, and rely exclusively on publicly-available data sources.

The appendix is organized as follows. Section A.1 describes our procedure for estimating bilateral debt holdings by issuer and currency, while Section A.2 explains our computation of domestic currency bias and returns on aggregate debt portfolios and provides robustness checks.

A.1 Estimating Debt Holdings by Issuer and Currency

This section describes how we estimate country-level debt holdings by currency, which we use in Table 1. Our approach combines the procedures in Fidora et al. (2007) for estimating investor-country holdings of domestic debt and in Lane and Shambaugh (2010) for estimating the currency composition of investor-country holdings of international debt.[7]

Throughout this section, *debt holdings* refer to country-level aggregate net debt holdings in nominal terms in numéraire currency USD, where aggregate net means the sum of long positions (with positive sign) and short positions (with negative sign) over all agents (households, firms, government) residing in a given country. Abusing the notation of

[7] Allen et al. (2023) update the work of Lane and Shambaugh (2010) using confidential IMF survey data. However, their estimates exclude domestic holdings and are available by currency only, rather than by currency and issuer, making them less suitable for our study.

our theoretical model, we denote these holdings B_{ijt}, although they are nominal here. Domestic debt holdings refer to debt held by residents of a given country that was issued by residents of the same country. International debt holdings refer to debt held by residents of a given country that was issued by residents of another country. Debt may be issued in domestic or international markets (as defined by the BIS), and the issuance market, as a concept, is distinct from the residence of the issuer or holder of debt.[8]

In subsection A.1.1, we use gravity models to estimate the domestic and international debt holdings of investor countries by issuer country. We first estimate a model for international debt holdings, then a second model for both domestic and international debt holdings. In subsection A.1.2, we estimate currency weights, which allow us to compute domestic and international debt holdings of investor countries, by issuer country *and* issuance currency. The initial weights are based on the reported currency composition of debt issued in international markets, but we adjust these weights to account for investor-country holdings of international debt issued in the domestic market of the issuer. Finally, we use a biproportional fitting procedure to reconcile the estimated debt holdings of investor country by issuer country and issuance currency with debt holdings reported to the IMF by issuance currency alone.

A.1.1 Estimating Debt Holdings by Issuer Country

To estimate portfolio holdings by issuer country, we proceed in three steps. In step one, we fit a first-stage gravity model to CPIS-reported international debt holdings, and use the fitted first-stage model to obtain first-stage predictions of international debt holdings for non-reporting countries. In step two, we estimate domestic debt holdings as the residual difference between total debt issuance and rest-of-world international debt holdings for each issuer country. In step three, we fit a second-stage gravity model to domestic and international debt holdings, and use the fitted second-stage model to obtain final predictions of domestic and international debt holdings for non-reporting countries.

Step One. We fit a first-stage gravity model to CPIS-reported international debt holdings using pseudo-Poisson maximum likelihood (PPML), which performs well in the presence of heteroskedasticity (Silva and Tenreyro, 2006; Santos Silva and Tenreyro, 2022). Our unbalanced panel contains 177 981 investor-issuer-year observations spanning 2001–2021 and covering 83 investor countries and 196 issuer countries. Because we use the model to predict missing debt holdings for non-reporting countries, we limit regressors to those with good coverage across countries to minimize data loss.

[8]For example, a German resident might issue German debt in German debt markets under German law, that is held by a US resident. From the US perspective, this represents holdings of international debt issued in the German domestic market. Alternatively, a German resident might issue German debt in a US market under US law, that is held by a German resident. From the German perspective, this represents holdings of domestic debt issued in the German international market.

The gravity variables we select, obtained from Conte et al. (2022), include geographic distance, a contiguity indicator, a common language indicator, GDP, GDP per capita, an EU membership indicator, and a WTO membership indicator. Additionally, we construct a common-currency indicator, a tax haven indicator based on the list in Coppola et al. (2021) (used for interaction terms), and a crisis indicator for both investor and issuer countries from the banking, debt, and currency crises identified by Nguyen et al. (2022). We also include interactions for investor and issuer EU membership, WTO membership, GDP, and GDP per capita. Finally, we include investor, issuer, and year fixed effects. Our specification is given by

$$\mathrm{E}\Big[B_{ijt} \mid \boldsymbol{X}_{ijt},\, \boldsymbol{IT}_{ijt},\, \boldsymbol{FE}_{ijt}\Big] = \exp\Big(\boldsymbol{\beta}_X' \boldsymbol{X}_{ijt} + \boldsymbol{\beta}_{IT}' \boldsymbol{IT}_{ijt} + \boldsymbol{\beta}_{FE}' \boldsymbol{FE}_{ijt}\Big)\,, \quad i \neq j\,, \tag{A1}$$

where B_{ijt} denotes investor country i's holding of debt issued by country j in year t. The vector $\boldsymbol{X}_{ijt}$ collects a constant and the gravity and indicator variables. The vector $\boldsymbol{IT}_{ijt}$ collects interaction terms, and $\boldsymbol{FE}_{ijt}$ collects fixed effects.

After dropping 5 multicollinear fixed effects identified using QR decomposition, we estimate the first-stage model with 302 regressors and 171 789 observations.[9] We estimate the model both with and without L2 regularization ($\alpha = 0.0001$). Our preferred estimates, used in Table 1, include regularization. First-stage results for a subset of regressors are reported in columns two and four of Table A1. The correlation between predicted and observed values is 93.0 without regularization and 92.88 with regularization.

Step Two. We compute domestic holdings of domestic debt as the residual between total reported issuance amounts outstanding and the sum of reported plus gravity-estimated rest-of-world holdings for issuer countries with available data.

Total issuance amounts outstanding are available for 38 countries in the BIS DSS dataset with good coverage over our sample period. For 20 additional countries with poor or no DSS coverage, we approximate total issuance by summing domestic and international market issuance amounts from the BIS IDS dataset.[10]

To measure rest-of-world holdings of issuer-country debt, we aggregate CPIS-reported holdings for reporting countries and first-stage gravity-estimated holdings for non-reporting countries. This aggregate covers all economic sectors (including government) except for central banks. Central Bank holdings are reported separately in the IMF SEFER dataset. We add SEFER-reported holdings to the CPIS aggregate to obtain an estimate of total rest-of-world holdings.

[9] Our implementation of QR decomposition removes the fixed effects with the lowest-valued diagonal elements of the R matrix until the condition number equals approximately 50.

[10] As Gruić and Wooldridge (2012) note, summing domestic and international issuance may involve double-counting. However, in cases where both DSS and IDS total issuance are available, we find the totals to be very similar. DSS data represent 87.13% of total reported issuance in an average year in our sample, and IDS data represent 12.87%.

Table A1 – First- and Second-Stage PPML Gravity Model Estimation

Independent Variable	No Regularization		L2 Regularization	
	Stage One	Stage Two	Stage One	Stage Two
Constant	−17.91	−11.95	−7.07	−7.62
	(0.00)	(0.00)		
Distance	−0.39	−0.48	−0.38	−0.48
	(0.00)	(0.00)		
Common Language	0.23	0.27	0.23	0.27
	(0.00)	(0.00)		
Common Currency	0.68	0.93	0.68	0.93
	(0.00)	(0.00)		
Investor GDP	3.50	1.75	1.22	1.23
	(0.00)	(0.00)		
Issuer GDP	4.71	1.15	1.86	1.76
	(0.00)	(0.00)		
Investor GDP/Cap	−0.68	1.26	1.31	1.67
	(0.00)	(0.00)		
Issuer GDP/Cap	−1.81	1.76	0.67	1.27
	(0.00)	(0.00)		
Domestic Debt		6.19		6.14
		(0.00)		
Domestic Debt × Investor GDP		−0.49		−0.48
		(0.00)		
Domestic Debt × Investor GDP/Cap		−2.69		−2.67
		(0.00)		
Domestic Debt × Investor Tax Haven		−1.42		−1.40
		(0.00)		
Year FE	Yes	Yes	Yes	Yes
Investor and Issuer FE	Yes	Yes	Yes	Yes
Observations	171 789	172 811	171 789	172 811
Correlation	93.00	99.55	92.88	99.55

Notes. The table reports PPML estimates from the first- and second-stage gravity models described in Section A.1.1. The first stage uses CPIS-reported international holdings; the second stage incorporates domestic holdings computed as the residual between total issuance and rest-of-world holdings. P-values are shown below coefficients. Observation counts and the correlation between observed and predicted holdings appear at the bottom. Columns labeled "L2 Regularization" apply a penalty of $\alpha = 0.0001$, and p-values are not available for these specifications. Fixed effects and less economically significant covariates are omitted.

Following Fidora et al. (2007), we define domestic holdings of domestic debt as the difference between total issuance amounts outstanding and total rest-of-world holdings:

$$B_{iit} = I_{it} - B_{-iit}\,, \quad \text{where } B_{-iit} \equiv \sum_{j \neq i} B_{jit} + B_{CBit}\,, \tag{A2}$$

where I_{it} denotes issuer-country i's total debt issuance amounts outstanding, B_{CBit} represents SEFER-reported central bank holdings of country i's debt, and B_{-iit} denotes rest-of-world holdings of country i's debt.

For six countries (Estonia, India, Ireland, Lebanon, Luxembourg, and Taiwan), CPIS-reported rest-of-world holdings alone, before adding gravity-estimated holdings, exceed total reported outstanding issuance. In these cases, which together account for 2.27% of total reported outstanding issuance in an average year, the residual-based estimates of domestic debt holdings become negative and we exclude them, instead relying on second-stage gravity estimates, described next.

Step Three. We fit a second-stage gravity model that incorporates both the CPIS-reported international debt holdings and the residual-based domestic debt holdings from the previous steps. We continue to use the PPML estimator and the same baseline regressors, but we now add a domestic debt indicator (equaling one if the investor and issuer countries coincide) and interact this indicator with GDP, GDP per capita, and the tax haven indicator. All continuous variables remain logged and standardized. Our specification is given by

$$\begin{aligned} &\mathrm{E}\Big[B_{ijt} \mid \boldsymbol{X}_{ijt},\, \boldsymbol{IT}_{ijt},\, \boldsymbol{FE}_{ijt},\, \boldsymbol{DOM}_{ijt}\Big] \\ &\quad = \exp\Big(\boldsymbol{\beta}_X^\top \boldsymbol{X}_{ijt} + \boldsymbol{\beta}_{IT}^\top \boldsymbol{IT}_{ijt} + \boldsymbol{\beta}_{FE}^\top \boldsymbol{FE}_{ijt} + \boldsymbol{\beta}_{DOM}^\top \boldsymbol{DOM}_{ijt}\Big)\,, \end{aligned} \tag{A3}$$

where B_{ijt} now encompasses both reported international and residual-based domestic debt holdings. The vector $\boldsymbol{DOM}_{ijt}$ contains a domestic debt indicator that equals one for $i = j$ and zero otherwise, and its interactions.

Columns three and five of Table A1 present the second-stage results. The coefficient on the domestic debt indicator is positive and both economically and statistically significant, reflecting a strong domestic bias. By contrast, the interaction terms with GDP per capita and the tax haven indicator are significantly negative, suggesting that wealthier countries and tax havens exhibit a smaller *incremental* domestic bias once we control for other factors. This does not imply that they hold less domestic debt in absolute terms or as a share of total debt; rather, it implies only that the domestic debt indicator effect is attenuated.

After fitting the second-stage model, we recalculate rest-of-world holdings by combining CPIS-reported debt holdings with second-stage gravity-estimated debt holdings for non-

reporting countries, then update our residual-based estimates of domestic debt holdings accordingly. Our final data set of bilateral debt holdings thus includes both reported and estimated holdings of international and domestic debt for all countries in our sample.

A.1.2 Estimating Weights by Issuer Country and Issuance Currency

To obtain estimates of investor-country debt holdings by issuer *and* currency, we proceed in five steps. First, we compute *holding weights* to capture each issuer's share in the total holdings of each investor country. Second, we compute international-market *issuance weights* to capture the currency denomination of debt issued by each investor country in international markets. Third, we compute *access weights* to capture the extent to which domestic and international investors access the issuer's domestic and international markets. Fourth, we combine these issuance and access weights to obtain initial *portfolio weights*. Fifth, we adjust the initial portfolio weights using a biproportional fitting algorithm to match the currency composition of total debt holdings reported by investor countries in CPIS. The final portfolio weights decompose each investor country's total debt holdings into holdings by issuer country and issuance currency.

Step One: Holding Weights. We compute holding weights as issuer country shares in the total debt holdings of investor countries. Holding weights for investor-country i satisfy following identity,

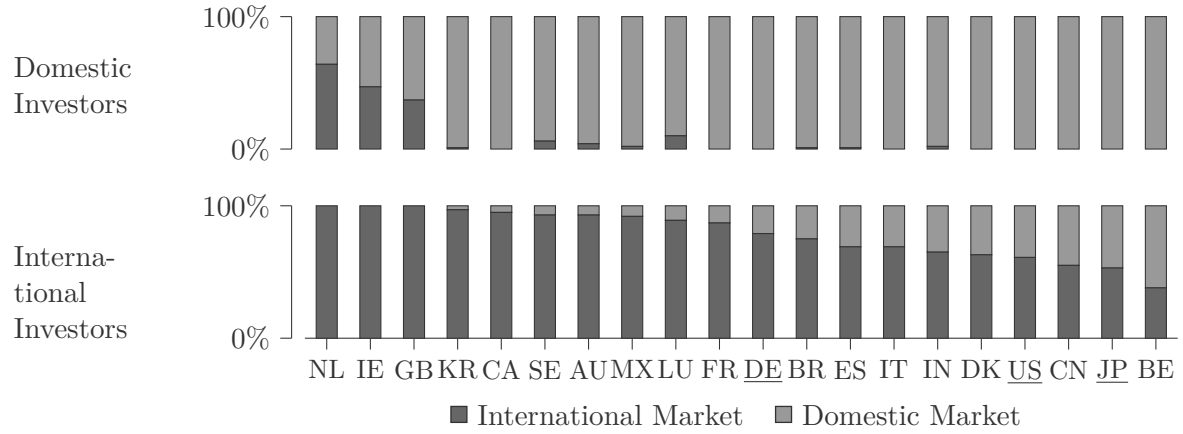
$$B_{it} = \sum_j B_{ijt} \quad \Leftrightarrow \quad 1 = \sum_j b_{ijt}\,, \tag{A4}$$

where B_{it} denotes the total debt holdings of country i and $b_{ijt} \equiv B_{ijt}/B_{it}$ denotes the share of debt issued by country j in the total debt holdings of country i.

Step Two: Access Weights. We compute access weights based on the difference between total rest-of-world holdings and total international-market debt issuance for each issuer country. If the difference is positive, with rest-of-world holdings exceeding international-market issuance, we infer that international investors hold debt from the issuer's domestic market. If the difference is negative, with international-market debt issuance exceeding rest-of-world holdings, we infer that domestic investors hold debt from the issuer's international market.[11]

[11]The BIS distinguishes between debt issued in domestic and international markets. In domestic markets, issuers predominantly issue debt in their domestic currency. Although international-market debt also is increasingly denominated in the issuer's domestic currency (see Gruić and Wooldridge 2012, Du and Schreger 2016, and Burger et al. 2018), a substantial share remains in USD or other international currencies. Because the currency composition differs by market, the market accessed by investors matters for our estimates. Gruić and Wooldridge (2012) document a divergence between international bond issues (from BIS IDS data) and cross-border bond holdings (from IMF CPIS data). This discrepancy suggests that international investors sometimes buy debt in the issuer's domestic market, while domestic investors

Figure A1 – Market Access Weights for Top-Twenty Issuer Countries



Notes. The figure shows access weights for domestic and international investors in the top 20 issuer countries by total issuance, averaged over the period 2001–2021. The upper panel shows weights for domestic investors, who generally buy domestic debt from domestic markets. The lower panel shows weights for international investors, who generally buy international debt from international markets. In 5.87% of country-year observations, weights fall outside the interval $[0, 1]$ and are truncated to zero or one.

We compute separate access weights for domestic and international investors. For domestic investors residing in issuer country i, access weights satisfy the following identity,

$$\begin{aligned} B_{iit} &= \max\Big(I_{it}^{(IM)} - B_{-iit}, 0\Big) + \Big(B_{iit} - \max\Big(I_{it}^{(IM)} - B_{-iit}, 0\Big)\Big) \\ \Leftrightarrow \quad 1 &= a_{iit}^{(IM)} + a_{iit}^{(DM)}\,, \end{aligned} \tag{A5}$$

where $a_{iit}^{(IM)} \equiv \max(I_{it}^{(IM)} - B_{-iit}, 0)/B_{iit}$ denotes the domestic-investor access weight to issuer i's international market, and $a_{iit}^{(DM)} = 1 - a_{iit}^{(IM)}$ denotes the domestic-investor access weight to issuer i's domestic market. For international investors outside issuer country i, access weights satisfy a similar identity,

$$\begin{aligned} B_{-iit} &= \max\Big(B_{-iit} - I_{it}^{(IM)}, 0\Big) + \Big(B_{-iit} - \max\Big(B_{-iit} - I_{it}^{(IM)}, 0\Big)\Big) \\ \Leftrightarrow \quad 1 &= a_{-iit}^{(DM)} + a_{-iit}^{(IM)}\,, \end{aligned} \tag{A6}$$

where $a_{-iit}^{(DM)} \equiv \max(B_{-iit} - I_{it}^{(IM)}, 0)/B_{-iit}$ denotes the international-investor access weight to issuer i's domestic market, and $a_{-iit}^{(IM)} = 1 - a_{-iit}^{(DM)}$ denotes the international-investor access weight to issuer i's international market.

Figure A1 illustrates access weights averaged over 2001–2021 for the top 20 issuer countries, separately for domestic and international investors. The upper panel shows that domestic investors generally buy domestic debt in domestic markets, although in

sometimes buy debt in the international market of their own country. Burger and Warnock (2007) study this phenomenon in the bond markets of 40 countries.

Great Britain, the Netherlands, and Ireland they buy significant amounts of domestic debt issued in international markets. The lower panel shows that international investors generally buy international debt in international markets, although for the United States, China, Japan, and Belgium, they purchase significant amounts of debt issued in domestic markets. In 5.87% of country-year observations, weights fall outside the interval $[0, 1]$ and are truncated to zero or one.

Step Three: Issuance Weights. We compute issuance weights for 142 issuer countries that report international-market issuance in total and by currency for USD, EUR, and the issuer's domestic currency in the BIS IDS data. When values are present for at least one of these currencies, we set any remaining missing values to zero. We then define issuance in "other" currencies (neither USD, EUR, nor the issuer's domestic currency) as the remainder after subtracting these three currency totals from the overall total international-market issuance.[12]

Issuance weights for issuer-country i satisfy the following identity,

$$\begin{aligned} I_{it}^{(IM)} &= I_{iit}^{(IM)} + I_{iUSDt}^{(IM)} + I_{iEURt}^{(IM)} + I_{iOTHt}^{(IM)} \\ \Leftrightarrow \quad 1 &= v_{iit}^{(IM)} + v_{iUSDt}^{(IM)} + v_{iEURt}^{(IM)} + v_{iOTHt}^{(IM)}, \end{aligned} \tag{A7}$$

where $I_{it}^{(IM)}$ denotes total international-market issuance, $I_{ijt}^{(IM)}$ denotes issuance in currency j. We define the issuance weight for issuer country i and issuance currency j as $v_{ijt}^{(IM)} \equiv I_{ijt}^{(IM)}/I_{it}^{(IM)}$. In particular, $I_{iit}^{(IM)}$ and $v_{iit}^{(IM)}$ denote the issuance amount and weight for issuer country i's international-market issuance in its domestic currency. For domestic-market issuance, we assume that all debt is issued in the issuer's domestic currency, so that $v_{iit}^{(DM)} \equiv 1$.[13]

Step Four: Portfolio Weights. We compute portfolio weights for investor-country debt holdings by combining holding weights with issuer weights and access weights. Portfolio weights allow us to decompose each investor country's total holdings into holdings by issuer country *and* issuance currency. These weights play an important role in the calculation of portfolio returns on investor-country debt holdings in Section A.2.

We compute separate portfolio weights for domestic and international debt holdings.

[12] A large share of country-year observations (72.15%) have missing values for USD, EUR, or domestic currency issuance in the IDS data. However, after setting these missing values to zero, the average share of international market issuance in the other-currency category remains low at 6.23%, compared with 8.64% for observations with no missing values. This suggests that the zero-fill assumption does not artificially inflate the other-currency category.

[13] This assumption appears reasonable for most countries. Gruić and Wooldridge (2012) observe that "bonds issued in the local market are typically issued in local currency under local law," but note international financial centers and dollarised or euroised economies as possible exceptions.

For domestic debt holdings, portfolio weights are given by

$$w_{iikt} \equiv \begin{cases} b_{iit} a_{iit}^{(IM)} v_{ikt}^{(IM)}\,, & i \neq k\,, \\ b_{iit}\left(a_{iit}^{(IM)} v_{iit}^{(IM)} + a_{iit}^{(DM)}\right), & i = k\,, \end{cases} \tag{A8}$$

where w_{iikt} denotes the portfolio weight for investor country i's holdings of domestic debt issued in currency k. For international debt holdings, portfolio weights are given by

$$w_{ijkt} \equiv \begin{cases} b_{ijt} a_{-jjt}^{(IM)} v_{jkt}^{(IM)}\,, & i \neq j \neq k\,, \\ b_{ijt}\left(a_{-jjt}^{(IM)} v_{jjt}^{(IM)} + a_{-jjt}^{(DM)}\right), & i \neq j = k\,, \end{cases} \tag{A9}$$

where w_{ijkt} denotes the portfolio weight for investor country i's holdings of debt issued by country j in currency k, where $i \neq j$. For country-year observations where we lack IDS data and cannot compute access or issuance weights, we use the global average of non-missing access and issuance weights to compute portfolio weights.[14]

Step Five: Biproportional Fitting. Using the initial portfolio weights from step four, we can derive total international debt holdings by currency for each investor country. For a subset of 66 countries holding 80.20% of global debt in an average year, we can compare these derived totals with a CPIS-reported totals by currency, and thereby obtain an indirect measure of accuracy for the underlying portfolio weights. To obtain these derived totals, we multiply total international debt holdings by portfolio weights and aggregate over issuers.

The upper panel of Figure A2 shows the discrepancy between our derived totals and the CPIS-reported totals by currency. With accurate portfolio weights, observations would fall on the 45-degree line. Instead, our portfolio weights overestimate international debt holdings in EUR and USD when the true values are low and underestimate when the true values are high. The opposite is true of CHF, GBP, and JPY, but these currencies represent a much smaller fraction of international debt.

To improve the estimates, we adjust the initial portfolio weights using a biproportional fitting algorithm. We introduce the following notation. Let B_{ijkt} denote country i's holding of debt issued by country j in currency k, computed as a portfolio weight multiplied by country i's total holdings, and let $B_{ij\bullet t}$ and $B_{i\bullet kt}$ denote total debt holdings by issuer and

[14] Many of the smallest issuer countries lack IDS data. While these countries are numerous (causing 42.29% of portfolio weights to be computed with global averages), the debt they issue accounts for a negligible 0.29% of global holdings in an average year.

by currency, respectively. That is, let

$$B_{ijkt} = w_{ijkt} B_{it}\,, \quad B_{ij\bullet t} = \sum_k B_{ijkt}\,, \quad \text{and} \quad B_{i\bullet kt} = \sum_j B_{ijkt}\,. \tag{A10}$$

Note that $B_{ij\bullet t}$ has the same meaning as B_{ijt} used elsewhere; we adopt the "dot" notation here to avoid confusion with $B_{i\bullet kt}$. Finally, let $B^{\dagger}_{i\bullet kt}$ denote CPIS-reported (as opposed to derived) total holdings by currency.

For each investor–year combination, we arrange initial estimates of holdings by issuer and currency into a matrix, with issuers j in rows and currencies k in columns. We iteratively adjust rows and columns to obtain a set of holdings $B^{\dagger}_{ijkt}$ such that

$$B_{ij\bullet t} = \sum_k B^{\dagger}_{ijkt} \quad \text{and} \quad B^{\dagger}_{i\bullet kt} = \sum_j B^{\dagger}_{ijkt} \quad \forall j, k\,. \tag{A11}$$

The row target $B_{ij\bullet t}$ ensures that the issuer-level totals remain unchanged from our estimates in Section A.1.1, while the column target $B^{\dagger}_{i\bullet kt}$ forces the currency composition to align with the CPIS-reported values.

At iteration n, we first scale each row so that its sum equals the corresponding row target $B_{ij\bullet t}$. Specifically, for issuer (row) j, we update

$$B^{\left(n+\frac{1}{2}\right)}_{ijkt} = B^{(n)}_{ijkt} \frac{B_{ij\bullet t}}{\sum_k B^{(n)}_{ijkt}}, \quad \forall\, k\,. \tag{A12}$$

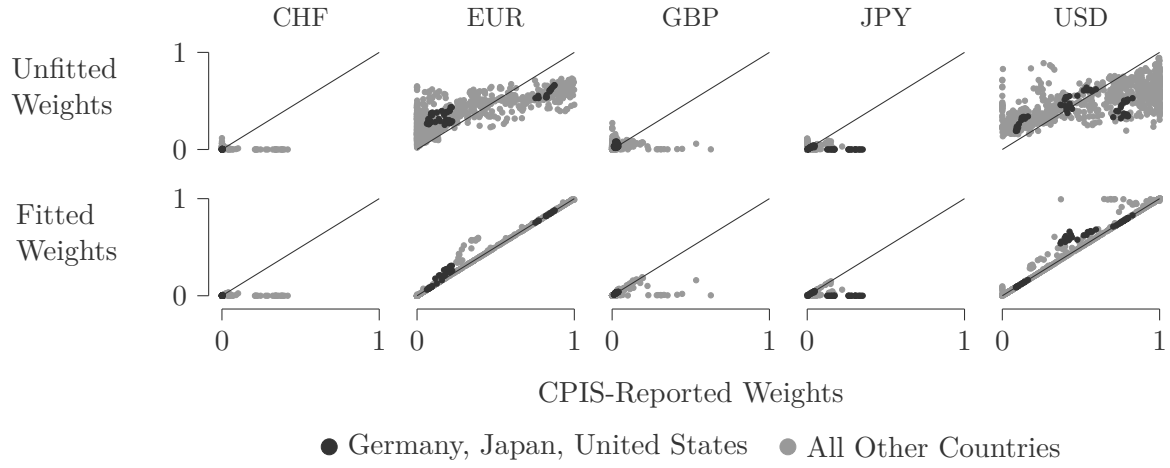
Next, we scale each column so that its sum equals the corresponding column target $B^{\dagger}_{i\bullet kt}$. Specifically, for currency (column) k, we update

$$B^{(n+1)}_{ijkt} = B^{\left(n+\frac{1}{2}\right)}_{ijkt} \frac{B^{\dagger}_{i\bullet kt}}{\sum_j B^{\left(n+\frac{1}{2}\right)}_{ijkt}}, \quad \forall\, j\,. \tag{A13}$$

These steps are repeated until the maximum deviation between the current row and column sums and their respective targets falls below a given tolerance. If convergence is not achieved within a maximum number of iterations, we apply a final re-scaling to enforce the row constraints. The final values $B^{\dagger}_{ijkt}$ are then used to update portfolio weights w_{ijkt}.

Convergence often fails because our initial estimates B_{ijkt} are derived from IDS data covering USD, EUR, and domestic currency, while our currency targets are derived from CPIS data covering USD, EUR, JPY, GBP, and CHF. In the IDS data, we construct a residual "other" category that includes JPY, GBP, and CHF when these are non-domestic currencies. As a result, the JPY, GBP, and CHF columns are sparsely populated, which prevents convergence to the CPIS-reported targets in these currencies. However, these currencies account for only 11.22% of total international debt holdings, and the algorithm is effective for USD and EUR. The lower panel of Figure A2 illustrates this effectiveness,

Figure A2 – Biproportional Fitting of Portfolio Weights to Currency Weights



Notes. The figure shows scatter plots of estimated and CPIS-reported currency weights for CHF, EUR, GBP, JPY, and USD from 2001 to 2021 before and after the biproportional fitting described in Section A.1.2, step five. Germany, Japan, and the United States are shown in black, and all other countries are shown in gray. The top row shows unfitted estimates, and the bottom row shows fitted estimates. Each plot includes a 45-degree reference line. After fitting, estimates for USD and EUR align closely with reported values. Estimates for GBP, CHF, and JPY remain less well aligned, but these currencies account for only 11.22% of international debt holdings.

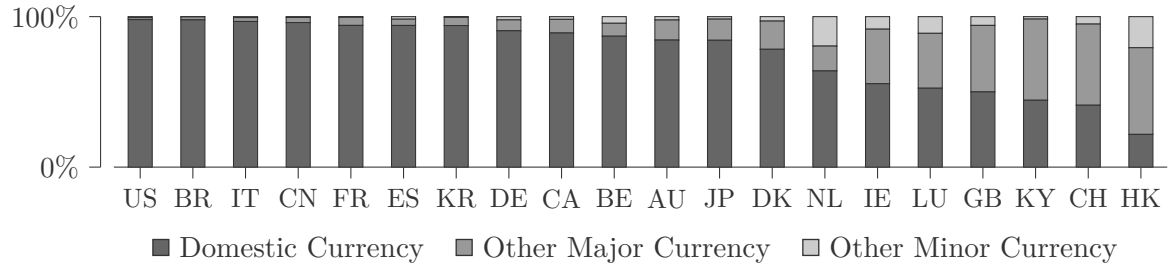
with fitted values clustering around the 45-degree line for USD and EUR.

Final Portfolio Weights by Issuance Currency Our final estimates of portfolio weights by issuance currency indicate a strong domestic-currency bias in aggregate net debt holdings. This finding aligns with an established literature documenting the tendency for investors to disproportionately hold domestic debt (Fidora et al., 2007; Coeurdacier and Rey, 2013) and domestic-currency debt (Burger et al., 2018; Maggiori et al., 2020). Strong domestic-currency bias in low interest-rate currencies is hard to reconcile with widespread carry trade activity in debt markets. Carry trading may be more prevalent in currency derivatives markets, but recent research using derivatives positions micro-data suggests that currency derivatives are predominantly used for hedging purposes, not for speculative carry trading (see the work of Hacioğlu-Hoke et al. 2024 and Alfaro et al. 2024, reviewed in Section 2).

Figure A3 summarizes our currency composition estimates for the top twenty investor countries by total holdings. The figure shows the share of each country's debt holdings in three currency categories: domestic currency, major non-domestic currencies (USD, EUR, JPY, GBP, CHF), and minor non-domestic currencies. In most countries, domestic-currency holdings dominate portfolios.[15] However, large international financial centers like

[15]We verify the high domestic currency share in the United States with a direct calculation. US debt issuance averaged 33 200 USD bn in BIS reported data during our sample period. Non-US investors held 4 863 USD bn (14.6% of total issuance) and global central banks held 2 087 USD bn (6.3% of total issuance), based on IMF data. This implies 26 250 USD bn in US holdings of domestic debt (assumed

Figure A3 – Currency Composition of Aggregate Net Debt Holdings



Notes. The figure shows the currency composition of debt holdings for the twenty investor countries with the highest average debt holdings, 2001–2021. Holdings are classified as domestic currency, other major currency (USD, EUR, JPY, GBP, CHF), or other minor currency, referring to any other currencies. Countries are ordered by domestic currency share. Currency weights are estimated using IMF CPIS and BIS DDS and IDS databases. Appendix A.1 details the methodology.

the United Kingdom, the Cayman Islands, Switzerland, and Hong Kong are exceptions where major non-domestic currencies play an important role. Notably, Australia (AU) and Japan (JP) allocate similarly large shares of holdings to domestic-currency debt. This finding is hard to reconcile with widespread carry trade activity in Japanese debt markets, where interest rates have been persistently low.

A.2 Domestic Currency Bias and Portfolio Returns

Table 1 reports average estimates of domestic currency bias and portfolio returns on debt holdings for Germany, Japan, and the United States as one group, and for 181 rest-of-world countries as another group. Aggregate debt portfolios in Germany, Japan, and the United States earn low returns and exhibit strong domestic-currency bias. In this appendix section, we describe the data and methodology used to produce these estimates.

Domestic Currency Bias. We compute domestic currency bias using the debt holdings and portfolio weights estimated in Section A.1. Following Fidora et al. (2007), we define

$$\begin{aligned} DCB_{it} &= 100\left(w^*_{-it} - w_{-it}\right)/w^*_{-it}\,, \quad \text{with} \\ w_{-it} &= \sum_j \sum_{k\neq i} w_{ijkt} \quad \text{and} \quad w^*_{-it} = \sum_j \sum_{k\neq i} w_{ijkt} B_{it} \Big/ \sum_i B_{it}\,, \end{aligned} \tag{A14}$$

where DCB_{it} denotes domestic currency bias for country i, w_{-it} denotes the share of international-currency debt in i's debt holdings, and w^*_{-it} denotes the share of international-currency debt in global debt holdings (with "international" defined relative to country i). Domestic currency bias is bounded above by 100 and unbounded below. It is negative

USD-denominated). US investors held 2 451 USD bn in foreign-issued debt, with an IMF-reported 78% denominated in USD. Total US holdings in USD were therefore 28 160 USD bn (26 250 bn domestic + 1 910 bn foreign-issued in USD), representing 98.1% of total US debt holdings (28 700 USD bn).

in only 2.38% of country-year observations, indicating that most countries overweight domestic currency in their debt holdings.

Portfolio Returns. We compute nominal portfolio returns as a weighted average of issuer-country nominal interest rates adjusted for currency returns, taken over all positions in an investor country's debt holdings. Real portfolio returns equal nominal portfolio returns adjusted for the investor country's domestic rate of CPI inflation. We obtain interest rates from the OECD MEI database and IMF IFS database. Exchange rates and rates of CPI inflation are also from IFS.

Short-term interest rates serve as indicative interest rates for our portfolio return calculations.[16] For each issuer country, we use OECD MEI short-term interest rates where available, and IMF IFS treasury bill rates where MEI rates are unavailable.[17] These two sources cover 2 069 country-year observations. For an additional 1 032 country-year observations, these sources are unavailable but IFS deposit rates are available. In these cases, we use a fitted model to predict short-term interest rates from deposit rates,

$$R^i_{Bit} = \alpha + \beta_D R^i_{Dit} + \boldsymbol{\beta}'_{FE} \boldsymbol{FE}_{it} + \epsilon_{it} \,, \tag{A15}$$

where R^i_{Bit} and R^i_{Dit} denote gross short-term and deposit rates, respectively, with subscript i denoting the issuer country and superscript i denoting the numéraire currency, set as the domestic currency of the issuing country, and $\boldsymbol{FE}_{it}$ denotes country and year fixed effects. The regression yields an $R^2 = 0.82$ and coefficient estimates $\hat{\beta}_D = 0.01$ and $\hat{\alpha} = 98.83$, both significant at the 1% level. In total, reported or predicted rates are available for 3 101 country-year observations.

End-of-year nominal exchange rates from IFS are available for 3 778 country-year observations, reported with respect to USD. With the nominal exchange rate between currency i and USD denoted $S_{i\text{USD}t}$, we define the gross nominal currency return for an arbitrary currency pair as

$$R_{Sijt} = R_{Si\text{USD}t} / R_{Sj\text{USD}t} \,, \quad \text{where} \quad R_{Si\text{USD}t} = S_{i\text{USD}t} / S_{i\text{USD}t-1} \quad \forall\, i \,. \tag{A16}$$

[16] Our estimates cover debt of all maturities. Ideally, we would match interest rate terms to debt maturities when computing returns, but short-term and long-term debt data are less complete in BIS and IMF sources. For robustness, Panel A3c of Table A3 reports portfolio returns for short-term debt using available data.

[17] MEI short-term rates are "based on three-month money market rates where available. Typical standardised names are 'money market rate' and 'treasury bill rate' " (OECD, OECD). IMF treasury bill rates are rates "at which short-term government debt securities are issued or traded in the market," with details varying slightly from country to country (International Monetary Fund, 2023a,b).

Table A2 – Summary of Domestic Currency Bias and Portfolio Returns

Variable		Summary Statistics (in %)			
	Count	Mean	SD	Median	IQR
Portfolio Return, Nominal	3 778	5.6	8.8	3.9	6.5
Portfolio Return, Real	3 459	0.5	7.1	0.4	5.0
Coverage Ratio	3 778	84.7	25.6	95.6	8.8
Domestic Currency Bias	3 778	62.9	29.0	73.1	42.0
Currency Return	3 778	2.3	11.8	0.0	7.8
CPI Inflation Rate	3 459	5.3	9.4	3.3	5.1

Notes. The table presents summary statistics for country-year estimates of nominal and real portfolio returns, coverage ratios, and domestic currency bias in the upper panel, and for currency returns and CPI rates of inflation in the lower panel. Statistics are computed for the period 2001–2021. The count column counts non-missing country-year observations. Mean, standard deviation (SD), median, and interquartile range (IQR) are presented in percentages.

We define the nominal portfolio return R^i_{NPit} for country i in numéraire currency i as

$$R^i_{NPit} = \sum_j \sum_k w_{ijkt} R_{Sikt} R^j_{Bjt}\,, \tag{A17}$$

where we apply the domestic short-term interest rate R^j_{Bjt} to debt issued by j in any currency.[18] We then use domestic rates of CPI inflation for the investor country to convert nominal portfolio returns into real portfolio returns, $R_{Pit} = R^i_{NPit}/\Pi^i_{it}$.

For some debt-issuing countries and years, short-term interest rates or currency returns are unavailable. For this reason, we compute portfolio returns for each investor country on the fraction of portfolio holdings where data are available, re-scaling portfolio weights by the coverage ratio, which we define as

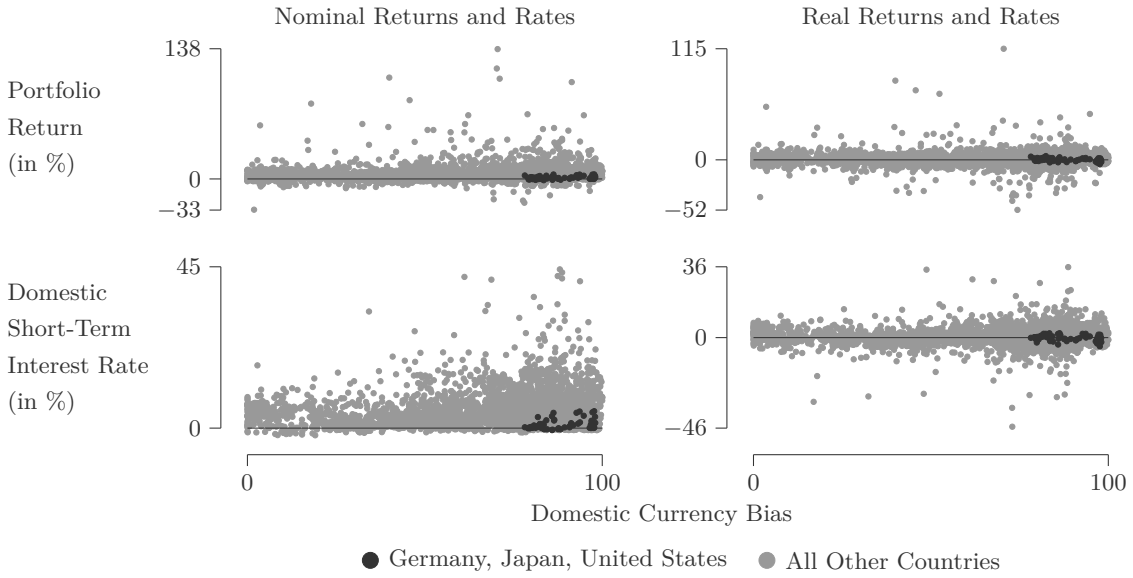
$$CR_{it} = \sum_j \sum_k w_{ijkt} \mathbb{1}_{ijk} \tag{A18}$$

where the indicator $\mathbb{1}_{ijk}$ equals zero when interest rates or exchange rates are missing for investor country i, issuer country j, or currency k. The median coverage ratio in our sample is 95.6%. Table A2 provides descriptive statistics for portfolio returns, coverage ratios, domestic currency bias, currency returns, and rates of inflation in our sample.

Figure A4 presents scatter plots of portfolio returns and domestic short-term interest

[18]The IMF and OECD have poor coverage of interest rates on debt issued in non-domestic currency. Empirically, interest rates on debt issued in domestic and international currencies do differ; for reference, the ECB reports a median international-minus-domestic currency rate spread of +0.8pp (advanced economies) and −1.6pp (emerging economies) (European Central Bank, 2014, Section C). While these spreads are large, they would be multiplied by small portfolio weights in our analysis, and would not significantly impact the main empirical findings we report in Table 1. Du and Schreger (2016, 2022) study the spread for sovereign debt in emerging markets, emphasizing currency risk, credit risk, and corporate balance sheet mismatches as important determinants, which our simplification does not capture.

Figure A4 – Domestic Currency Bias, Portfolio Returns, and Interest Rates



Notes. The figure shows domestic currency bias and both nominal and real portfolio returns and domestic short-term rates for Germany, Japan, and the United States versus 181 other countries, estimated annually. Germany, Japan, and the United States are shown in black; all other countries are shown in gray. Portfolio returns are calculated using short-term interest rates, exchange rates, and CPI inflation rates from the OECD and IMF, with portfolio weights estimated from IMF and BIS debt holdings and issuance data. We exclude 2.38% of estimates with negative domestic currency bias and country-year observations with interest or exchange rates more than three standard deviations above yearly mean.

rates against domestic currency bias. The horizontal axes show domestic currency bias. In the upper plots, the vertical axis shows portfolio returns, nominal on the left and real on the right. In the lower plots, the vertical axis shows domestic short-term interest rates, nominal on the left and real on the right. Germany, Japan, and the United States are shown in black, and their observations cluster near near 100 on the horizontal axis and near zero on the vertical axis. All other countries are shown in gray and exhibit more variation in domestic currency bias, portfolio returns, and domestic short-term interest rates.

Table A3 provides three robustness checks for the result presented in Table 1 that Germany, Japan, and the United States have higher domestic currency bias and lower returns on aggregate debt portfolios than other countries, on average. Panel A3a shows that the same pattern holds with unweighted averages. Panel A3b shows that the same pattern holds when we include interest rate and exchange rate return outliers. Panel A3c shows that the same pattern holds when we compute domestic currency bias and aggregate portfolio returns using short-term debt only. In all three robustness checks, the qualitative result of higher domestic currency bias and lower returns on aggregate debt portfolios for Germany, Japan, and the United States remains unchanged.

Table A3 – Domestic Currency Bias and Returns on Aggregate Debt Portfolios

(a) Weighted versus Unweighted Country Averages, Outliers Excluded, All Debt

	Weighted Average	Group Share of Global Holdings	Country-Level Average (in %)			
			Domestic Currency Bias	Nominal Portfolio Return	Real Portfolio Return	Portfolio Coverage Ratio
DE, JP, US	Yes	53.97	91.37	1.20	−0.27	97.89
All Other	Yes	46.03	72.96	3.16	0.73	94.02
DE, JP, US	No	53.97	88.60	1.21	−0.05	97.70
All Other	No	46.03	62.44	5.71	0.55	84.47

(b) Weighted Country Averages, Outliers Excluded versus Included, All Debt

	Outliers Excluded	Group Share of Global Holdings	Country-Level Weighted Average (in %)			
			Domestic Currency Bias	Nominal Portfolio Return	Real Portfolio Return	Portfolio Coverage Ratio
DE, JP, US	Yes	53.97	91.37	1.20	−0.27	97.89
All Other	Yes	46.03	72.96	3.16	0.73	94.02
DE, JP, US	No	53.94	91.37	1.21	−0.26	97.91
All Other	No	46.06	72.93	3.34	0.80	94.15

(c) Weighted Country Averages, Outliers Excluded, All Debt versus Short-Term Debt Only

	Short Term Only	Group Share of Global Holdings	Country-Level Weighted Average (in %)			
			Domestic Currency Bias	Nominal Portfolio Return	Real Portfolio Return	Portfolio Coverage Ratio
DE, JP, US	No	53.97	91.37	1.20	−0.27	97.89
All Other	No	46.03	72.96	3.16	0.73	94.02
DE, JP, US	Yes	52.27	99.52	1.20	−0.68	99.92
All Other	Yes	47.73	95.29	3.15	0.14	92.06

Notes. The table shows three robustness checks for Table 1, which reports domestic currency bias and returns on debt portfolios for Germany, Japan, and the United States versus 181 other countries. Panel A3a compares weighted and unweighted country averages. Panel A3b compares results with and without outliers (interest rates and exchange rate returns exceeding three standard deviations from annual means). Panel A3c compares all maturities to short-term debt only. All estimates use OECD and IMF data for interest rates, exchange rates, and CPI inflation, with portfolio weights from IMF and BIS holdings and issuance data (2001–2021). Global holdings shares are averaged by group across years; other values are averaged across countries and years, using debt holdings as weights where applicable.

B Theoretical Appendix

Appendix B provides derivations and proofs for the model in Section 3. It covers parameter restrictions (B.1), real and nominal budget constraints (B.2), utility maximization (B.3), the non-stochastic steady state (B.4), first-order Taylor expansions of equilibrium conditions (B.6), model aggregability (B.5), the aggregate non-portfolio problem (B.7) and portfolio problem (B.8), and the individual non-portfolio problem (B.9) and portfolio problem (B.10).

B.1 Parameter Restrictions

In this section, we collect our assumptions on parameter restrictions for easier reference throughout the appendix.

Assumption 1 (Preference Parameters)**.** *The preference parameters unrelated to risk aversion satisfy*

$$\theta \in [0, \varepsilon), \quad \beta \in (0.5, 1), \quad \textit{and} \quad \alpha_{ii} \in (0.5, 1), \tag{B1}$$

where $\alpha_{ii} + \alpha_{ij} = 1$ *so that domestic bias in consumption* $\phi = \alpha_{ii} - \alpha_{ij}$ *lies in* $(0, 1)$*.*

These restrictions ensure that individual preferences yield a well-defined steady state with domestic portfolio bias in the model. Agents favor consumption of the domestic good when $\alpha_{ii} \in (0.5, 1)$, which strengthens the incentive to hold domestic bonds and helps to produce domestic bias in aggregate portfolio holdings.

Assumption 2 (Distribution of Risk Aversion)**.** *We assume risk aversion* ρ *lies in the interval* $(0, \rho_m]$*, and that the parameters of the cross-sectional distribution of risk aversion satisfy*

$$\kappa > 1 \quad \textit{and} \quad \rho_m > (\kappa^2 + \kappa)/(\kappa^2 - 1), \tag{B2}$$

ensuring that the distribution has finite mean and variance, and that aggregate risk aversion is sufficiently high ($\bar{\rho} > \omega$*).*

Sufficiently high risk aversion is needed for Propositions 8, 9, and 10 establishing domestic bias in aggregate portfolios, low expected returns on aggregate portfolios, and the existence of carry traders. With moderate heterogeneity this assumption is mild, and, in the limiting representative-agent case ($\kappa \to \infty$), amounts to $\rho_m > 1$.

Assumption 3 (Endowments and Money Supplies)**.** *The logarithmic deviations of endowments and money supplies from their non-stochastic steady-state values have zero means, positive and finite variances and covariances, and follow independent, identically*

distributed processes over time. With $\hat{\boldsymbol{\xi}}_t \equiv [\hat{Y}_{it},\, \hat{Y}_{jt},\, \hat{M}^i_{it},\, \hat{M}^j_{jt}]'$, $i \neq j$, *we assume*

$$\mathrm{E}_{t-1}\left[\hat{\boldsymbol{\xi}}_t \hat{\boldsymbol{\xi}}_t'\right] = \begin{bmatrix} \sigma_Y^2 & \sigma_{YY} & \sigma_{YM} & \sigma^*_{YM} \\ \sigma_{YY} & \sigma_Y^2 & \sigma^*_{YM} & \sigma_{YM} \\ \sigma_{YM} & \sigma^*_{YM} & \sigma_M^2 & \sigma_{MM} \\ \sigma^*_{YM} & \sigma_{YM} & \sigma_{MM} & \sigma_M^2 \end{bmatrix}, \tag{B3}$$

where $\sigma_{YM} > \sigma^*_{YM}$, $\sigma_Y^2 > \sigma_{YY}$, *and* $\sigma_M^2 > \sigma_{MM}$, *ensuring that domestic covariances exceed cross-country covariances and that neither endowment shocks nor money supplies are perfectly correlated across countries.*

Assumption 3, which is stated verbally in section 3.1, preserves cross-country symmetry, and helps to produce low expected returns on portfolios with domestic bias. However, our results also hold under less restrictive assumptions. Specifically, all propositions, including Propositions 8, 9, and 10, hold if log endowments and money supplies have non-zero variances, are imperfectly correlated across countries, and satisfy

$$\mathrm{E}_t\left[\hat{Y}_{i-jt+1}\hat{Y}_{i+jt+1}\right] \geq 0\,, \quad \mathrm{E}_t\left[\hat{M}^{i-j}_{i-jt+1}\hat{Y}_{i-jt+1}\right] \geq 0\,,$$
$$\text{and} \quad \mathrm{E}_t\left[\hat{M}^{i-j}_{i-jt+1}\hat{Y}_{i+jt+1}\right] \geq 0\,, \quad i \neq j\,.$$

Though not our focus, the less-restrictive conditions do admit asymmetric variance-covariance structures that can generate persistent differences in returns across countries.

B.2 Nominal and Real Budget Constraints

In this section, we relate the nominal budget constraint to the real budget constraint in (5), and the nominal bond return to the real bond return in (7). Nominal and real budget constraints are mathematically equivalent and lead to identical first-order conditions.

Substituting the real return and real portfolio holdings into the real budget constraint in (5) using the definitions in (7), we obtain

$$\begin{aligned} &P^i_{it}\frac{A_{iit}(\rho)P^i_{Bit}}{P^i_{it}} + P^i_{it}\frac{A_{ijt}(\rho)P^i_{Bjt}}{P^i_{it}} + P^i_{it}C_{it}(\rho) \\ &= P^i_{it}\frac{S_{iit}/P^i_{it}}{P^i_{Bit-1}/P^i_{it-1}}\frac{A_{iit-1}(\rho)P^i_{Bit-1}}{P^i_{it-1}} + P^i_{it}\frac{S_{ijt}/P^i_{it}}{P^i_{Bjt-1}/P^i_{it-1}}\frac{A_{ijt-1}(\rho)P^i_{Bjt-1}}{P^i_{it-1}} + P^i_{Cit}Y_{it}(\rho)\,, \quad i \neq j\,, \end{aligned}$$

which simplifies to the agent's nominal budget constraint,

$$A_{iit}(\rho)P^i_{Bit} + A_{ijt}(\rho)P^i_{Bjt} + P^i_{it}C_{it}(\rho) = A_{iit-1}(\rho) + S_{ijt}A_{ijt-1}(\rho) + P^i_{Cit}Y_{it}(\rho)\,, \quad i \neq j\,.$$

The Euler equations in (27) would remain unchanged if we were to derive them from nominal instead of real budget constraints, maximizing with respect bond quantities rather

than real values of bond holdings.

The Euler equations in (27) can be written in terms of nominal instead of real returns using the following expression,

$$\begin{aligned} R^j_{it} &= \frac{S_{jit}/P^j_{jt}}{P^j_{Bit-1}/P^j_{jt-1}} \\ &= \frac{S_{jit}/P^j_{Bit-1}}{P^j_{jt}/P^j_{jt-1}} = R^j_{Bit}/\Pi^j_{it}\,, \end{aligned} \tag{B4}$$

where R^j_{Bit} denotes the gross nominal return and Π^j_{it} denotes the gross rate of inflation. A version of the Fisher equation obtains as a first-order Taylor expansion of (B4),

$$\hat{R}^j_{Bit} = \hat{R}^j_{it} + \hat{\Pi}^j_t + O(\epsilon^2)\,. \tag{B5}$$

B.3 Utility Maximization

For the first-stage utility maximization problem, agent ρ in country i solves

$$\begin{aligned} \max\ \mathcal{L}^{(1)}_{it}(\rho) = &\sum_{s=t}^{\infty} \beta^{s-t}\,\mathrm{E}_t\left[\frac{1}{1-\rho}\left(\frac{C_{is}(\rho)}{C_i(\rho)}\right)^{1-\rho} + \frac{\theta}{1-\rho}\left(\frac{e^{W^i_{is}(\rho)}}{e^{W^i_i(\rho)}}\right)^{1-\rho}\right] \\ &+ \sum_{s=t}^{\infty} \beta^{s-t}\,\mathrm{E}_t\Big[\mu^{(1)}_{is}(\rho)\Big\{R^i_{is}B^i_{iis-1}(\rho) + R^i_{js}B^i_{ijs-1}(\rho) + W^i_{is}(\rho)\Big\}\Big] \\ &- \sum_{s=t}^{\infty} \beta^{s-t}\,\mathrm{E}_t\Big[\mu^{(1)}_{is}(\rho)\Big\{B^i_{iis}(\rho) + B^i_{ijs}(\rho) + C_{is}(\rho)\Big\}\Big]\,, \quad i \neq j \end{aligned} \tag{B6}$$

by choice of $\Big\{C_s(\rho), B^i_{iis}(\rho), B^i_{ijs}(\rho)\Big\}$ for $i \neq j$ and for $s \geq t$, where $\mathcal{L}^{(1)}_{it}(\rho)$ denotes the agent's Lagrangian for the first-stage problem and $\mu^{(1)}_{it}(\rho)$ denotes the Lagrangian multiplier on the agent's budget constraint. First-order conditions with respect to consumption and real bond holdings are given by

$$\begin{aligned} \mu^{(1)}_{it}(\rho) &= \left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{-\rho}\frac{1}{C_i(\rho)} \quad \text{and} \\ \mu^{(1)}_{it}(\rho) &= \theta\left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}}\right)^{-\rho} + \beta\,\mathrm{E}_t\Big[\mu^{(1)}_{it+1}(\rho)R^i_{jt+1}\Big]\,. \end{aligned} \tag{B7}$$

Combining the first-order conditions in (B7) yields the Euler equations in (15).

For the second-stage utility maximization problem, agent ρ in country i solves

$$\begin{aligned} \max\ \mathcal{L}^{(2)}_{it}(\rho) = &\gamma C_{iit}(\rho)^{\alpha_{ii}} C_{ijt}(\rho)^{\alpha_{ij}} \\ &+ \mu^{(2)}_{it}(\rho)\Big(P^j_{it}C_{it}(\rho) - P^i_{Cit}C_{iit}(\rho) - P^i_{Cjt}C_{ijt}(\rho)\Big)\,, \quad i \neq j \end{aligned} \tag{B8}$$

by choice of $C_{iit}(\rho)$ and $C_{ijt}(\rho)$ for $i \neq j$, where $\mathcal{L}_{it}^{(2)}(\rho)$ denotes the agent's Lagrangian for the second-stage problem and $\mu_{it}^{(2)}(\rho)$ denotes the Lagrangian multiplier on the agent's expenditure constraint in (10). First-order conditions with respect to domestic and international consumption goods are given by

$$\mu_{it}^{(2)}(\rho)P_{Cit}^{i} = \gamma\alpha_{ii}\left(\frac{C_{ijt}(\rho)}{C_{iit}(\rho)}\right)^{\alpha_{ij}} \quad \text{and} \quad \mu_{it}^{(2)}(\rho)P_{Cjt}^{i} = \gamma\alpha_{ij}\left(\frac{C_{iit}(\rho)}{C_{ijt}(\rho)}\right)^{\alpha_{ii}}, \quad i \neq j\,. \tag{B9}$$

Combining (B9) with the expenditure constraint in (10) yields the goods demands in (16). Combining (B9) with the definition of the consumption basket in (9) yields the price index in (17).

B.4 The Non-Stochastic Steady State

In this section, we prove Propositions 1 and 2 characterizing the non-stochastic steady-state equilibrium of the model.

B.4.1 Proof of Proposition 1

Proposition 1 characterizes the non-portfolio non-stochastic steady state. We begin by deriving the steady-state price indices, goods prices, and the real exchange rate. From the quantity equations and the equality of steady-state money supplies in (14),

$$M_i^i = P_{Ci}^i Y_i = S_{ij} P_{Cjt}^j Y_j = S_{ij} M_j^j = M_j^i\,. \tag{B10}$$

Using the normalization of steady-state real endowments in (8) together with the law of one price in (17), it follows that common-currency prices for the domestic and international goods are equal in the steady state,

$$P_{Ci}^i = P_{Cj}^i\,, \tag{B11}$$

and using the price indices in (17) and the common-currency equality of goods prices, it follows that price indices and goods prices are equal in the steady state,

$$P_i^j = P_{Ci}^j\,. \tag{B12}$$

Using the equality of prices derived above together with the definition of the real exchange rate in (13) evaluated at the steady state,

$$Q_{ij} = \frac{S_{ij}P_j^j}{P_i^i} = \frac{P_j^i}{P_i^i} = \frac{P_{Cj}^i}{P_{Ci}^i} = \frac{P_{Cj}^i}{P_{Cj}^i} = 1\,. \tag{B13}$$

We now derive steady-state consumption. The left-hand side of the steady-state Euler

equations in (18) is independent of j, so real returns on domestic and international nominal bonds are equal in the steady state,

$$R_i^i = R_j^i \,, \tag{B14}$$

and the right-hand side is independent of ρ, so individual and aggregate consumption baskets are equal in the steady state (given that the measure of agents equals one in each country),

$$C_i(\rho) = C_i \,. \tag{B15}$$

From the goods demands in (16), together with steady-state prices in (20),

$$C_{ij}(\rho) = C_{ij} = \alpha_{ij} C_i \,, \tag{B16}$$

and together with goods market clearing in (11), the equality of individual and aggregate endowments in (8), and the parameter restrictions in (1),

$$C_i(\rho) = C_i = Y_i = Y_i(\rho) = 1 \,. \tag{B17}$$

Since $C_i(\rho) = 1$, it follows from (18) that

$$R_i^j = \frac{1-\theta}{\beta} = R \,. \tag{B18}$$

We now derive steady-state real wealth. From the budget constraints in (5), using the definition of real wealth in (6) and the equality of steady-state real returns in (B14),

$$W_i^i(\rho) + C_i(\rho) = R W_i^i(\rho) + Y_i(\rho) \,. \tag{B19}$$

Using the normalized values of steady-state consumption and endowments, and rearranging, the budget constraint becomes

$$W_i^i(\rho)(1-R) = 0 \,. \tag{B20}$$

Given that the measure of agents equals one in each country, it follows that

$$W_i^i(\rho) = W_i^i = W_j^j = W_j^j(\rho) \,. \tag{B21}$$

From the bond market clearing conditions in (11), summing across bonds,

$$B_{ii}^i + B_{ji}^i + B_{jj}^i + B_{ij}^i = 0 \quad \Leftrightarrow \quad W_i^i + W_j^i = 0 \,, \quad i \neq j \,, \tag{B22}$$

using the definition of real wealth in (6). From the definitions of real wealth in (6) and

real bond holdings in (7), together with the steady-state real exchange rate in (20),

$$W_j^i = Q_{ij} W_j^j = W_j^j \,. \tag{B23}$$

Hence, we have

$$W_i^i = W_j^j \,, \quad W_i^i + W_j^i = 0 \,, \quad \text{and} \quad W_j^i = W_j^j \,, \tag{B24}$$

and therefore

$$W_i^j(\rho) = W_i^j = 0 \,. \tag{B25}$$

B.4.2 Proof of Proposition 2

Proposition 2 provides an initial characterization of the portfolio non-stochastic steady state. This initial characterization is limited, and derives from the solution for steady-state individual and aggregate real wealth and from market clearing. From steady-state real wealth in (21), using the definition of real wealth in (6), we have

$$B_{ii}^i(\rho) = -B_{ij}^i(\rho) \quad \text{and} \quad B_{ii}^i = -B_{ij}^i \,. \tag{B26}$$

Combining these results with market clearing in (11), we have

$$B_{ii}^i = B_{jj}^i \,. \tag{B27}$$

We derive more complete characterizations of steady-state individual and aggregate portfolio holdings (to order-zero approximations) in Proposition 10.

B.5 Model Aggregability

In this section, we derive expressions for the aggregate coefficient of relative risk aversion and the aggregate intertemporal elasticity of substitution, and prove Propositions 3 and 4.

B.5.1 Aggregating Preference Parameters

Given the CRRA consumption utility in (3), heterogeneity can be defined equivalently in terms of either the coefficient of relative risk aversion or the intertemporal elasticity of substitution, both with respect to consumption. Using the distribution in (1), the aggregate coefficient of relative risk aversion and intertemporal elasticity of substitution are given by

$$\bar{\rho} = \int_{\mathcal{R}} \rho f(\rho) \, \mathrm{d}\rho = \frac{\kappa \rho_m}{\kappa + 1} \quad \text{and} \quad \bar{\sigma} = \int_{\mathcal{R}} \sigma(\rho) f(\rho) \, \mathrm{d}\rho = \frac{\kappa}{(\kappa - 1)\rho_m} \,, \tag{B28}$$

where we have used equation (4) for $\sigma(\rho)$.

B.5.2 Proof of Proposition 3

Proposition 3 introduces the aggregation wedge ω. Using the expressions for $\bar{\rho}$ and $\bar{\sigma}$ in (B28) and the definition of ω in (26), we have

$$\bar{\sigma} = \frac{\omega}{\bar{\rho}} \quad \Leftrightarrow \quad \frac{\kappa}{(\kappa-1)\rho_m} = \omega \frac{\kappa+1}{\kappa \rho_m} \quad \Leftrightarrow \quad \omega = \frac{\kappa^2}{\kappa^2-1}\,.$$

By inspection, the wedge is greater than one and decreasing in κ for $\kappa > 1$.

B.5.3 Proof of Proposition 4

Proposition 4 states the first-order approximate individual and aggregate Euler equations. We begin with the individual Euler equation. Combining fist-order conditions in (B7), we obtain

$$\left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{-\rho} \frac{1}{C_i(\rho)} = \theta \left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}}\right)^{-\rho} + \beta\, \mathrm{E}_t\left[\left(\frac{C_{it+1}(\rho)}{C_i(\rho)}\right)^{-\rho} \frac{1}{C_i(\rho)} R^i_{jt+1}\right].$$

A standard first-order Taylor expansion of this expression around the non-stochastic steady state then yields (27).

The expression in (27) aggregates straightforwardly. The coefficient of relative risk aversion is the only source heterogeneity in the expression, and it multiplies an aggregate variable. Integrating (27) over agents,

$$\begin{aligned} \int_{\mathcal{R}} \hat{C}_{it}(\rho) f(\rho)\,\mathrm{d}\rho &= \theta \int_{\mathcal{R}} \hat{W}^i_{it}(\rho) f(\rho)\,\mathrm{d}\rho \\ &+ \beta R\, \mathrm{E}_t\left[\int_{\mathcal{R}} \hat{C}_{it+1}(\rho) f(\rho)\,\mathrm{d}\rho - \int_{\mathcal{R}} \frac{1}{\rho} f(\rho)\,\mathrm{d}\rho \hat{R}^i_{jt+1}\right] + O(\epsilon^2)\,, \end{aligned} \tag{B29}$$

and noting that (24) implies

$$\hat{C}_{it} = \int_{\mathcal{R}} \hat{C}_{it}(\rho) f(\rho)\,\mathrm{d}\rho + O(\epsilon^2) \quad \text{and} \quad \hat{W}^j_{it} = \int_{\mathcal{R}} \hat{W}^j_{it}(\rho) f(\rho)\,\mathrm{d}\rho + O(\epsilon^2)\,,$$

we obtain (28) from (B29) directly. Expressions in (4) and (25) are used for the coefficient of relative risk aversion.

B.6 First-Order Taylor Expansions

Several Taylor expansions already appear in the body of the paper; these include individual Euler equations in (27), budget constraints in (29), and goods demands in (30), as well as market clearing conditions for goods and bonds in (31). For the remaining equilibrium

conditions, Taylor expansions are given below:

$$
\begin{aligned}
&\text{Agent Real Wealth:} && \hat{W}^j_{it}(\rho) = \hat{B}^j_{iit}(\rho) + \hat{B}^j_{ijt}(\rho) + O(\epsilon^2)\,, \quad i \neq j\,,\\
&\text{Real Bond Returns:} && \hat{R}^j_{it} = \left(\hat{S}_{jit} - \hat{P}^j_{Bit-1}\right) - \left(\hat{P}^j_{it} - \hat{P}^j_{it-1}\right) + O(\epsilon^2)\,,\\
&\text{Price Indices:} && \hat{P}^k_{it} = \alpha_{ii}\hat{P}^k_{Cit} + \alpha_{ij}\hat{P}^k_{Cjt} + O(\epsilon^2)\,, \quad i \neq j\,,\\
&\text{Consumption LoP:} && \hat{P}^i_{Cit} = \hat{S}_{ijt} + \hat{P}^j_{Cit} + O(\epsilon^2)\,,\\
&\text{Bond LoP:} && \hat{P}^i_{Bit} = \hat{S}_{ijt} + \hat{P}^j_{Bit} + O(\epsilon^2)\,,\\
&\text{Real Exchange Rate:} && \hat{Q}_{ijt} = \hat{S}_{ijt} + \hat{P}^j_{jt} - \hat{P}^i_{it} + O(\epsilon^2)\,, \quad \text{and}\\
&\text{Quantity Equations:} && \hat{M}^j_{it} = \hat{Y}_{it} + \hat{P}^j_{it} + O(\epsilon^2)\,.
\end{aligned}
\tag{B30}
$$

B.7 The Aggregate Non-Portfolio Problem

In the aggregate non-portfolio problem we solve for aggregate consumption and real wealth and for the real exchange rate in terms of lagged state variables, exogenous variables, and parameters. We first derive cross-country differenced solutions, and then use market clearing conditions to derive country-specific solutions.

B.7.1 The Aggregate Non-Portfolio System

To derive the cross-country differenced aggregate non-portfolio system in (35), we combine the market clearing condition in (31) with the aggregate budget constraints in (29) differenced across countries and aggregate Euler equations in (28) differenced across countries. Using the market clearing condition in (31) to eliminate the real exchange rate from the cross-country differenced aggregate budget constraint in (33), we obtain

$$
\hat{W}^i_{i-jt} = \frac{1-\theta}{\beta}\hat{W}^i_{i-jt-1} - \frac{1-\phi}{1-\phi^2}\hat{C}_{i-jt} + \phi\frac{1-\phi}{1-\phi^2}\hat{Y}_{i-jt} + \frac{1-\theta}{\beta}\hat{V}^i_{i-jt} + O(\epsilon^2)\,. \tag{B31}
$$

Similarly, using the market clearing condition in (31) to eliminate the real exchange rate from the cross-country differenced aggregate Euler equation in (33), we obtain

$$
\begin{aligned}
&\theta\hat{W}^i_{i-jt} + (1-\theta)\left(1 + \frac{\omega}{\bar{\rho}}\frac{\phi^2}{1-\phi^2}\right)\mathrm{E}_t\left[\hat{C}_{i-jt+1}\right]\\
&\quad = \left(1 + (1-\theta)\frac{\omega}{\bar{\rho}}\frac{\phi^2}{1-\phi^2}\right)\hat{C}_{i-jt} + (1-\theta)\frac{\omega}{\bar{\rho}}\frac{\phi}{1-\phi^2}\hat{Y}_{i-jt} + O(\epsilon^2)\,.
\end{aligned}
\tag{B32}
$$

These two equations, (B31) and (B32), constitute the cross-country differenced aggregate system. The system depends on three endogenous variables: cross-country differenced aggregate consumption $\hat{C}_{i-jt}$, cross-country differenced aggregate real wealth $\hat{W}^i_{i-jt}$, and the cross-country differenced aggregate portfolio valuation effect $\hat{V}^i_{i-jt}$. We solve the system initially for $\hat{C}_{i-jt}$ and $\hat{W}^i_{i-jt}$ in terms of $\hat{V}^i_{i-jt}$, describing these solutions as *intermediate*

solutions. Solving for $\hat{V}^i_{i-jt}$ requires second-order Taylor expansions of Euler equations and a solution for B^i_{ii-ji}, which we derive in B.8.

We write the cross-country differenced aggregate system in (B31) and (B32) as

$$\boldsymbol{\mathcal{E}}_0^{(-)}\,\mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}\Big] = \boldsymbol{\mathcal{E}}_1^{(-)}\hat{\boldsymbol{Z}}_{i-jt} + \boldsymbol{\mathcal{E}}_2^{(-)}\hat{Y}_{i-jt} + \boldsymbol{\mathcal{E}}_3^{(-)}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{B33}$$

where the coefficient matrices are given by

$$\begin{aligned}
\boldsymbol{\mathcal{E}}_0^{(-)} &= \begin{bmatrix} 1 & 0 \\ \theta & (1-\theta)\Big(1+\frac{\omega}{\bar{\rho}}\frac{\phi^2}{1-\phi^2}\Big) \end{bmatrix}, \quad \boldsymbol{\mathcal{E}}_1^{(-)} = \begin{bmatrix} \frac{1-\theta}{\beta} & -\frac{1-\phi}{1-\phi^2} \\ 0 & 1+(1-\theta)\frac{\omega}{\bar{\rho}}\frac{\phi^2}{1-\phi^2} \end{bmatrix}, \\
\boldsymbol{\mathcal{E}}_2^{(-)} &= \begin{bmatrix} \phi\frac{1-\phi}{1-\phi^2} \\ -(1-\theta)\frac{\omega}{\bar{\rho}}\frac{\phi}{1-\phi^2} \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\mathcal{E}}_3^{(-)} = \begin{bmatrix} \frac{1-\theta}{\beta} \\ 0 \end{bmatrix}.
\end{aligned} \tag{B34}$$

The matrix $\boldsymbol{\mathcal{E}}_0^{(-)}$ is invertible if its determinant is non-zero. Therefore, we require that

$$\det\Big(\boldsymbol{\mathcal{E}}_0^{(-)}\Big) = (1-\theta)\left(1+\frac{\omega}{\bar{\rho}}\frac{\phi^2}{1-\phi^2}\right) \neq 0\,, \tag{B35}$$

which is always satisfied under Assumptions 1 and 2. Inverting $\boldsymbol{\mathcal{E}}_0^{(-)}$, we rewrite the aggregate system in (B33) to obtain (35), with partial elasticity matrices given by

$$\begin{aligned}
\boldsymbol{\mathcal{E}}_{ZZ}^{(-)} &= \begin{bmatrix} \mathcal{E}_{WW}^{(-)} & \mathcal{E}_{WC}^{(-)} \\ \mathcal{E}_{CW}^{(-)} & \mathcal{E}_{CC}^{(-)} \end{bmatrix} = \begin{bmatrix} \frac{1-\theta}{\beta} & -\frac{1-\phi}{1-\phi^2} \\ -\frac{\theta}{\beta}\frac{1-\phi^2}{1-\Big(1-\frac{\omega}{\bar{\rho}}\Big)\phi^2} & \frac{1}{1-\theta}\frac{1+\theta(1-\phi)-(1-(1-\theta)\frac{\omega}{\bar{\rho}})\phi^2}{1-\Big(1-\frac{\omega}{\bar{\rho}}\Big)\phi^2} \end{bmatrix}, \\
\boldsymbol{\mathcal{E}}_{ZY}^{(-)} &= \begin{bmatrix} \mathcal{E}_{WY}^{(-)} \\ \mathcal{E}_{CY}^{(-)} \end{bmatrix} = \begin{bmatrix} \phi\frac{1-\phi}{1-\phi^2} \\ -\frac{\phi}{1-\Big(1-\frac{\omega}{\bar{\rho}}\Big)\phi^2}\Big(\frac{\omega}{\bar{\rho}}+\frac{\theta(1-\phi)}{1-\theta}\Big) \end{bmatrix}, \quad \text{and} \\
\boldsymbol{\mathcal{E}}_{ZV}^{(-)} &= \begin{bmatrix} \mathcal{E}_{WV}^{(-)} \\ \mathcal{E}_{CV}^{(-)} \end{bmatrix} = \begin{bmatrix} \frac{1-\theta}{\beta} \\ -\frac{\theta}{\beta}\frac{1-\phi^2}{1-\Big(1-\frac{\omega}{\bar{\rho}}\Big)\phi^2} \end{bmatrix}.
\end{aligned} \tag{B36}$$

Before solving the aggregate cross-country differenced system in (35), we establish the conditions for a unique and stationary rational expectations solution following Blanchard and Kahn (1980). The eigendecomposition of $\boldsymbol{\mathcal{E}}_{ZZ}^{(-)}$ is given by

$$\boldsymbol{\mathcal{E}}_{ZZ}^{(-)} = \boldsymbol{\mathcal{V}}_{ZZ}^{(-)}\boldsymbol{\Lambda}_{ZZ}^{(-)}\boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}} \tag{B37}$$

where $\boldsymbol{\mathcal{V}}_{ZZ}^{(-)}$ is a matrix of eigenvectors and $\boldsymbol{\Lambda}_{ZZ}^{(-)}$ a diagonal matrix of eigenvalues,

$$\begin{aligned}\boldsymbol{\mathcal{V}}_{ZZ}^{(-)} &= \begin{bmatrix}\nu_{WW}^{(-)} & \nu_{WC}^{(-)} \\ \nu_{CW}^{(-)} & \nu_{CC}^{(-)}\end{bmatrix} = \begin{bmatrix}\mathcal{E}_{WC}^{(-)} & \mathcal{E}_{WC}^{(-)} \\ \lambda_{ZW}^{(-)} - \mathcal{E}_{WW}^{(-)} & \lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\end{bmatrix}, \quad \text{and} \\ \boldsymbol{\Lambda}_{ZZ}^{(-)} &= \begin{bmatrix}\lambda_{ZW}^{(-)} & 0 \\ 0 & \lambda_{ZC}^{(-)}\end{bmatrix},\end{aligned} \tag{B38}$$

where

$$\lambda_{ZW}^{(-)} = \frac{1}{2}\Big(\mathcal{E}_{WW}^{(-)} + \mathcal{E}_{CC}^{(-)}\Big) + \sqrt{\frac{1}{4}\Big(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\Big)^2 + \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)}}\,, \quad \text{and} \tag{B39}$$

$$\lambda_{ZC}^{(-)} = \frac{1}{2}\Big(\mathcal{E}_{WW}^{(-)} + \mathcal{E}_{CC}^{(-)}\Big) - \sqrt{\frac{1}{4}\Big(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\Big)^2 + \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)}}\,. \tag{B40}$$

B.7.2 Proof of Proposition 5

Proposition 5 states that $\lambda_{ZW}^{(-)}$ and $\lambda_{ZC}^{(-)}$ are distinct and real-valued if $\theta \geq 0$. To prove this statement, it suffices to show that the discriminant in (B39) and (B40) is positive. The discriminant Δ is given by

$$\Delta = \frac{1}{4}\Big(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\Big)^2 + \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)}\,. \tag{B41}$$

Suppose $\theta > 0$. The squared term is always positive or zero, so we need to show that the second term is positive,

$$\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)} > 0 \quad \Leftrightarrow \quad \frac{\theta}{\beta}\frac{1-\phi}{1-\Big(1-\frac{\omega}{\bar{\rho}}\Big)\phi^2} > 0\,, \tag{B42}$$

where we have used the partial elasticities in (B36). Under Assumptions 1 and 2, the inequality in (B42) is always satisfied. Now suppose $\theta = 0$. The second term is zero, $\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)} = 0$, but the squared term is positive, since $\mathcal{E}_{WW}^{(-)}\Big|_{\theta=0} - \mathcal{E}_{CC}^{(-)}\Big|_{\theta=0} = 1/\beta - 1 > 0$ under Assumption 1, so the discriminant Δ is again positive. Thus, $\theta \geq 0$ is sufficient for distinct and real-valued eigenvalues.

The case where $\theta = 0$ serves as a helpful benchmark for proving the remaining statements in Proposition 5. Table B1 reports the partial elasticities in (35) and their derivatives with respect to θ, evaluated at $\theta = 0$.

Proposition 5 states that $\lambda_{ZW}^{(-)}$ evaluated at $\theta = 0$ is strictly greater than one and $\lambda_{ZC}^{(-)}$ evaluated at $\theta = 0$ is exactly one. Using Table B1 to evaluate (B39) at $\theta = 0$,

$$\lambda_{ZW}^{(-)}\Big|_{\theta=0} = \frac{1}{2}\left(\frac{1}{\beta} + 1\right) + \frac{1}{2}\left|\frac{1}{\beta} - 1\right| = \frac{1}{\beta} = \mathcal{E}_{WW}^{(-)}\Big|_{\theta=0} > 1\,, \tag{B43}$$

Table B1 – Aggregate Partial Elasticities and Their Derivatives at $\theta = 0$

Partial Elasticity	Value of Partial Elasticity Evaluated at $\theta = 0$		Derivative with Respect to θ Evaluated at $\theta = 0$	
	Expression	Sign	Expression	Sign
$\mathcal{E}^{(-)}_{WW}$	$\frac{1}{\beta}$	$+$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{WC}$	$-\frac{1-\phi}{1-\phi^2}$	$-$	0	$+$
$\mathcal{E}^{(-)}_{CW}$	0	$\cdot$	$-\frac{1}{\beta}\frac{1-\phi^2}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}$	$-$
$\mathcal{E}^{(-)}_{CC}$	1	$+$	$\frac{2-\phi-\phi^2}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}$	$+$
$\mathcal{E}^{(-)}_{WY}$	$\phi\frac{1-\phi}{1-\phi^2}$	$+$	0	$\cdot$
$\mathcal{E}^{(-)}_{CY}$	$-\frac{\phi\frac{\omega}{\bar{\rho}}}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}$	$-$	$-\frac{\phi(1-\phi)}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}$	$-$
$\mathcal{E}^{(-)}_{WV}$	$\frac{1}{\beta}$	$+$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{CV}$	0	$\cdot$	$-\frac{1}{\beta}\frac{1-\phi^2}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}$	$-$

Notes. Column one lists the partial elasticities that appear in the cross-country differenced aggregate system (35). Columns two and four, respectively, give expressions for the partial elasticities and their derivatives with respect to θ, both evaluated at $\theta = 0$. Columns three and five, respectively, give the signs of the partial elasticities and their derivatives, both evaluated at $\theta = 0$. Signs are established under Assumptions 1 and 2.

and (B40) at $\theta = 0$,

$$\lambda^{(-)}_{ZC}\big|_{\theta=0} = \frac{1}{2}\left(\frac{1}{\beta}+1\right) - \frac{1}{2}\left|\frac{1}{\beta}-1\right| = \mathcal{E}^{(-)}_{CC}\big|_{\theta=0} = 1\,. \tag{B44}$$

Proposition 5 also states that the partial derivative of $\lambda^{(-)}_{ZW}$ with respect to θ evaluated at $\theta = 0$ is positive, and the partial derivative of $\lambda^{(-)}_{ZC}$ with respect to θ evaluated at $\theta = 0$ is negative. To establish this result, we differentiate the eigenvalues with respect to θ,

$$\begin{aligned}\frac{\partial\lambda^{(-)}_{\pm}}{\partial\theta} = {} & \frac{1}{2}\left(\frac{\partial\mathcal{E}^{(-)}_{ZW}}{\partial\theta}+\frac{\partial\mathcal{E}^{(-)}_{ZC}}{\partial\theta}\right) \pm \frac{1}{2}\left[\frac{1}{4}\left(\mathcal{E}^{(-)}_{WW}-\mathcal{E}^{(-)}_{CC}\right)^2+\mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CW}\right]^{-\frac{1}{2}} \\ & \times\left[\frac{1}{2}\left(\mathcal{E}^{(-)}_{WW}-\mathcal{E}^{(-)}_{CC}\right)\left(\frac{\partial\mathcal{E}^{(-)}_{WW}}{\partial\theta}-\frac{\partial\mathcal{E}^{(-)}_{CC}}{\partial\theta}\right)+\frac{\partial\mathcal{E}^{(-)}_{WC}}{\partial\theta}\mathcal{E}^{(-)}_{CW}+\mathcal{E}^{(-)}_{WC}\frac{\partial\mathcal{E}^{(-)}_{CW}}{\partial\theta}\right],\end{aligned} \tag{B45}$$

where the "+" case corresponds to eigenvalue $\lambda^{(-)}_{ZW}$ and the "−" case corresponds to

eigenvalue $\lambda_{ZC}^{(-)}$. Using Table B1, we then evaluate (B45) at $\theta = 0$,

$$\begin{aligned}\left.\frac{\partial \lambda_{ZW}^{(-)}}{\partial \theta}\right|_{\theta=0} &= \left.\frac{\partial \mathcal{E}_{WW}^{(-)}}{\partial \theta}\right|_{\theta=0} + \frac{\beta}{1-\beta}\mathcal{E}_{WC}^{(-)}\big|_{\theta=0}\left.\frac{\partial \mathcal{E}_{CW}^{(-)}}{\partial \theta}\right|_{\theta=0} \\ &= \frac{1}{1-\beta}\frac{1-\phi}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2} - \frac{1}{\beta}\,,\end{aligned} \tag{B46}$$

and

$$\begin{aligned}\left.\frac{\partial \lambda_{ZC}^{(-)}}{\partial \theta}\right|_{\theta=0} &= \left.\frac{\partial \mathcal{E}_{CC}^{(-)}}{\partial \theta}\right|_{\theta=0} - \frac{\beta}{1-\beta}\mathcal{E}_{WC}^{(-)}\big|_{\theta=0}\left.\frac{\partial \mathcal{E}_{CW}^{(-)}}{\partial \theta}\right|_{\theta=0} \\ &= \frac{1}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}\left[2-\phi-\phi^2-\frac{1-\phi}{1-\beta}\right].\end{aligned} \tag{B47}$$

The right-hand side of (B46) is positive if

$$\beta > \frac{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}{2-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2-\phi}\,, \tag{B48}$$

and the right-hand side of (B47) is negative if

$$\beta > \frac{1-\phi^2}{2-\phi^2-\phi}\,. \tag{B49}$$

The inequality in (B49) is implied by the inequality in (B48). Hence, (B48) provides a sufficient condition for the derivatives of $\lambda_{ZW}^{(-)}$ and $\lambda_{ZC}^{(-)}$ with respect to θ to be positive and negative, respectively, when evaluated at $\theta = 0$.

Assumptions 1 and 2 ensure that (B48) is satisfied. By continuity, a small positive value for θ then pushes the eigenvalues $\lambda_{ZC}^{(-)}$ below one and $\lambda_{ZW}^{(-)}$ further above one, ensuring a unique and stationary rational expectations solution to (35).

B.7.3 Intermediate Aggregate Non-Portfolio Solution

We now derive intermediate solutions for aggregate real wealth, aggregate consumption, and the real exchange rate in terms lagged state variables, exogenous variables and parameters, and the endogenous aggregate portfolio valuation effect. Using the eigendecomposition in (B37), we rewrite the aggregate cross-country differenced system in (B33) as

$$\begin{aligned}&\boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}}\,\mathrm{E}_t\left[\hat{\boldsymbol{Z}}_{i-jt+1}\right] \\ &\quad= \boldsymbol{\Lambda}_{ZZ}^{(-)}\boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}}\hat{\boldsymbol{Z}}_{i-jt} + \boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}}\boldsymbol{\mathcal{E}}_{ZY}^{(-)}\hat{Y}_{i-jt} + \boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}}\boldsymbol{\mathcal{E}}_{ZV}^{(-)}\hat{V}_{i-jt}^{i} + O(\epsilon^2)\,,\end{aligned} \tag{B50}$$

and from (B50) we extract the equation associated with the unstable eigenvalue $\lambda_{ZW}^{(-)}$,

$$\begin{aligned}\mathrm{E}_t\Big[\hat{Z}_{i-jt+1}\Big] = \lambda_{ZW}^{(-)}\hat{Z}_{i-jt} + \Big[\Big(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WY}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CY}^{(-)}\Big]\hat{V}_{i-jt}^{i} \\ + \Big[\Big(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WV}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CV}^{(-)}\Big]\hat{Y}_{i-jt} + O(\epsilon^2)\,,\end{aligned} \tag{B51}$$

where $\hat{Z}_{i-jt+1} = \Big(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\Big)\hat{W}_{i-jt}^{i} - \mathcal{E}_{WC}^{(-)}\hat{C}_{i-jt+1}$.

Because $\lambda_{ZW}^{(-)}$ is greater than one, the left-hand side of (B51) must be zero to rule out explosive paths. Setting the left-hand side to zero, we obtain

$$\hat{C}_{i-jt} = \eta_{CW}^{(-)}\hat{W}_{i-jt-1}^{i} + \eta_{CY}^{(-)}\hat{Y}_{i-jt} + \eta_{CV}^{(-)}\hat{V}_{i-jt}^{i} + O(\epsilon^2)\,, \tag{B52}$$

and using (B52) to eliminate $\hat{C}_{i-jt}$ from (B31), we obtain

$$\hat{W}_{i-jt}^{i} = \eta_{WW}^{(-)}\hat{W}_{i-jt-1}^{i} + \eta_{WY}^{(-)}\hat{Y}_{i-jt} + \eta_{WV}^{(-)}\hat{V}_{i-jt}^{i} + O(\epsilon^2)\,. \tag{B53}$$

Writing the intermediate solutions in (B52) and (B53) in more compact matrix form, we obtain (37), where the semi-partial elasticity matrices are given by

$$\boldsymbol{\eta}_{ZW}^{(-)} = \begin{bmatrix}\eta_{WW}^{(-)} \\ \eta_{CW}^{(-)}\end{bmatrix} = \begin{bmatrix}\lambda_{ZC}^{(-)} \\ \frac{\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}}{\mathcal{E}_{WC}^{(-)}}\end{bmatrix},$$

$$\boldsymbol{\eta}_{ZY}^{(-)} = \begin{bmatrix}\eta_{WY}^{(-)} \\ \eta_{CY}^{(-)}\end{bmatrix} = \begin{bmatrix}\frac{\Big(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WY}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CY}^{(-)}}{\lambda_{ZW}^{(-)}} - \phi\mathcal{E}_{WC}^{(-)} \\ \frac{\Big(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WY}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CY}^{(-)}}{\lambda_{ZW}^{(-)}\mathcal{E}_{WC}^{(-)}}\end{bmatrix}, \quad \text{and} \tag{B54}$$

$$\boldsymbol{\eta}_{ZV}^{(-)} = \begin{bmatrix}\eta_{WV}^{(-)} \\ \eta_{CV}^{(-)}\end{bmatrix} = \begin{bmatrix}\frac{\Big(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WV}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CV}^{(-)}}{\lambda_{ZW}^{(-)}} + \mathcal{E}_{WW}^{(-)} \\ \frac{\Big(\lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WV}^{(-)} - \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CV}^{(-)}}{\lambda_{ZW}^{(-)}\mathcal{E}_{WC}^{(-)}}\end{bmatrix}.$$

For the real exchange rate, we combine the market clearing condition in (31) with our intermediate solution for cross-country differenced consumption in (B52) to obtain (41),

with semi-partial elasticities

$$\begin{aligned}
\eta^{(-)}_{QW} &= -\frac{\phi^2}{1-\phi^2}\frac{\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}}{\mathcal{E}^{(-)}_{WC}} \\
\eta^{(-)}_{QY} &= \frac{\phi}{1-\phi^2}\left(1 - \phi\frac{\left(\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}\right)\mathcal{E}^{(-)}_{WY} - \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CY}}{\lambda^{(-)}_{ZW}\mathcal{E}^{(-)}_{WC}}\right), \quad \text{and} \\
\eta^{(-)}_{QV} &= -\frac{\phi^2}{1-\phi^2}\frac{\left(\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}\right)\mathcal{E}^{(-)}_{WV} - \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CV}}{\lambda^{(-)}_{ZW}\mathcal{E}^{(-)}_{WC}}\,.
\end{aligned} \tag{B55}$$

The semi-partial elasticities in (B54) and (B55) can be written in terms of model parameters using the partial elasticities in (B36) and the eigenvalues in (B39) and (B40).

B.8 The Aggregate Portfolio Problem

In the aggregate portfolio problem we solve for cross-country differenced realized and expected real returns and cross-country differenced aggregate portfolio holdings. Beginning with realized real returns, we take the cross-country difference of realized real returns in (B30) to obtain

$$\hat{R}^i_{i-jt+1} = -\hat{S}_{jit+1} - \hat{P}^i_{Bi-jt} + O(\epsilon^2)\,, \tag{B56}$$

which we combine with the first-order expansion of expected real returns in (46) to obtain

$$\hat{R}^i_{i-jt+1} = -\Big(S_{ijt+1} - \mathrm{E}_t\Big[S_{ijt+1}\Big]\Big) + O(\epsilon^2)\,. \tag{B57}$$

Using first-order Taylor expansions of the real exchange rate, price indices, and quantity equations in (B30), we write the nominal exchange rate as

$$\hat{S}_{ijt} = \frac{1}{\phi}\hat{Q}_{ijt} + \hat{M}^{i-j}_{i-jt} - \hat{Y}_{i-jt} + O(\epsilon^2)\,, \tag{B58}$$

and combining with the intermediate solution for the real exchange rate in (41) and the expression for the cross-country differenced real return in (B57), we obtain an intermediate solution for realized real returns,

$$\hat{R}^i_{i-jt} = \eta^{(-)}_{RY}\hat{Y}_{i-jt} + \eta^{(-)}_{RM}\hat{M}^{i-j}_{i-jt} + \eta^{(-)}_{RV}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{B59}$$

Table B2 – Aggregate Semi-Partial Elasticities at $\theta = 0$

Semi-Partial Elasticity	Evaluation at $\theta = 0$	
	Expression	Sign
$\eta^{(-)}_{CY}$	$(1-\beta)\phi + \frac{\beta\phi\frac{\omega}{\bar{\rho}}}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}$	$+$
$\eta^{(-)}_{WY}$	$\beta\phi\frac{(1-\phi)\left(1-\frac{\omega}{\bar{\rho}}\right)}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}$	$+$
$\eta^{(-)}_{QY}$	$\phi + \beta\phi\frac{1-\frac{\omega}{\bar{\rho}}}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}$	$+$
$\eta^{(-)}_{RY}$	$-\beta\phi^2\frac{1-\frac{\omega}{\bar{\rho}}}{1-\left(1-\frac{\omega}{\bar{\rho}}\right)\phi^2}$	$-$
$\eta^{(-)}_{CV}$	$\frac{1-\beta}{\beta}\frac{1-\phi^2}{1-\phi}$	$+$
$\eta^{(-)}_{WV}$	$\frac{1-\phi^2-(1-\beta)(1-\phi)}{\beta\left(1-\phi^2\right)}$	$+$
$\eta^{(-)}_{QV}$	$-\frac{1-\beta}{\beta}\frac{\phi^2}{1-\phi}$	$-$
$\eta^{(-)}_{QW}$	$-\frac{1-\beta}{\beta}\frac{\phi^2}{1-\phi}$	$-$
$\eta^{(-)}_{RV}$	$\frac{1-\beta}{\beta}\frac{1-\phi^2}{1-\phi}$	$+$
$\eta^{(-)}_{RM}$	-1	$-$

Notes. The table shows semi-partial elasticities from the cross-country differenced aggregate non-portfolio and portfolio problems, evaluated at $\theta = 0$. Column two give an expression for each elasticity, and column three gives its sign under Assumptions 1 and 2.

where the real return semi-partial elasticities are given by

$$\begin{aligned}
\eta^{(-)}_{RY} &= \frac{\phi}{1-\phi^2}\left(\frac{\left(\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}\right)\mathcal{E}^{(-)}_{WY} - \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CY}}{\lambda^{(-)}_{ZW}\mathcal{E}^{(-)}_{WC}} - \phi\right), \\
\eta^{(-)}_{RM} &= -1\,, \text{ and} \\
\eta^{(-)}_{RV} &= \frac{\phi}{1-\phi^2}\frac{\left(\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}\right)\mathcal{E}^{(-)}_{WV} - \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CV}}{\lambda^{(-)}_{ZW}\mathcal{E}^{(-)}_{WC}}\,.
\end{aligned} \tag{B60}$$

We substitute the portfolio valuation effect (34) to into (B59) and rearrange to obtain the cross-country differenced real return in terms of general elasticities in (42), where the semi-partial elasticities $\eta^{(-)}_{RY}$, $\eta^{(-)}_{RM}$, and $\eta^{(-)}_{RV}$ can be written in terms of model parameters using the partial elasticities in (B36), but where B^i_{ii-ji} remains to be solved. For use later, we report the semi-partial elasticities from the aggregate portfolio problem in Table B2, evaluate at $\theta = 0$.

B.8.1 Proof of Proposition 6

The cross-country differenced aggregate portfolio valuation multiplier $\mu^{(-)}_{ii-ji}$ in (43) depends on the semi-partial elasticity $\eta^{(-)}_{RV}$ in (B60), and exceeds one in absolute value if

$$\frac{1}{\left|1-\eta^{(-)}_{RV}B^{i}_{ii-ji}\right|}>1\,. \tag{B61}$$

From Table B2, $\eta^{(-)}_{RV}>0$ at $\theta=0$. Because $\eta^{(-)}_{RV}$ is continuous in θ in the neighborhood of $\theta=0$, its sign is preserved for $\theta\in[0,\epsilon)$. We consider two cases. First, suppose $1-\eta^{(-)}_{RV}B^{i}_{ii-ji}>0$. In this case, $1>1-\eta^{(-)}_{RV}B^{i}_{ii-ji}$ is equivalent to $0<B^{i}_{ii-ji}$. Hence, $0<B^{i}_{ii-ji}<1/\eta^{(-)}_{RV}$ implies $\left|\mu^{(-)}_{ii-ji}\right|>1$. Second, suppose $1-\eta^{(-)}_{RV}B^{i}_{ii-ji}<0$. In this case, $1>\eta^{(-)}_{RV}B^{i}_{ii-ji}-1$ is equivalent to $B^{i}_{ii-ji}<2/\eta^{(-)}_{RV}$. Hence, $\frac{1}{\eta^{(-)}_{RV}}<B^{i}_{ii-ji}<\frac{2}{\eta^{(-)}_{RV}}$ implies $\left|\mu^{(-)}_{ii-ji}\right|>1$. Combining cases one and two,

$$0<B^{i}_{ii-ji}<\frac{2}{\eta^{(-)}_{RV}}\quad\Rightarrow\quad\left|\mu^{(-)}_{ii-ji}\right|>1\,. \tag{B62}$$

As an illustration, suppose $\theta=0$, $\phi=0.5$, and $\beta=0.95$. In this case, $2/\eta^{(-)}_{RV}=38$; domestic bias in aggregate portfolio holdings amplifies the response of the cross-country differenced real return to endowment and money supply shocks, provided that the cross-country differenced aggregate holdings of country i's bond are less than 38 times the size of the steady-state economy, which is normalized to one.

In contrast, $\mu^{(-)}_{ii-ji}$ lies between zero and one if

$$0\leq\frac{1}{1-\eta^{(-)}_{RV}B^{i}_{ii-ji}}<1\,. \tag{B63}$$

Given that $\eta^{(-)}_{RV}$ is positive, these inequalities are satisfied for any $B^{i}_{ii-ji}<0$. In other words, any amount of international bias in aggregate portfolio holdings attenuates rather than amplifies the response of the cross-country differenced real return to endowment and money supply shocks.

B.8.2 Proof of Proposition 7

Second-order Taylor expansions of the individual Euler equations in (15) are given by

$$\begin{aligned}&\hat{C}_{it}(\rho)-\frac{\rho}{2}\hat{C}_{it}(\rho)^{2}-\theta\hat{W}^{i}_{it}(\rho)+\frac{\theta\rho}{2}\hat{W}^{i}_{it}(\rho)\\&\quad=\beta R\,\mathrm{E}_t\left[\hat{C}_{it+1}(\rho)-\frac{\rho}{2}\hat{C}_{it+1}(\rho)^{2}-\frac{1}{\rho}\hat{R}^{i}_{jt+1}+\hat{C}_{it+1}(\rho)\hat{R}^{i}_{jt+1}\right]+O(\epsilon^{3})\,.\end{aligned} \tag{B64}$$

Differencing (B64) across bonds, i minus j, for agent ρ in country i, we obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\Big] = \rho\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho)\hat{R}^i_{i-jt+1}\Big] + O(\epsilon^3)\,. \tag{B65}$$

We use this second-order approximate individual Euler equation to obtain the cross-country differenced and cross-country summed Euler equations that appear in Proposition 7.

We begin by deriving the second-order approximate cross-country *differenced* individual Euler equations in (47). Differencing (B65) across countries for agents with identical coefficients of relative risk aversion, we obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1} - \hat{R}^j_{i-jt+1}\Big] = \rho\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho)\hat{R}^i_{i-jt+1} - \hat{C}_{jt+1}(\rho)\hat{R}^j_{i-jt+1}\Big] + O(\epsilon^3). \tag{B66}$$

The cross-country differenced real return in numéraire currency j, denoted $\hat{R}^j_{i-jt+1}$, now appears on both sides of the cross-country differenced Euler equations in (B66), which we convert to numéraire currency i before proceeding. To preserve the order of approximation in (B66), we need a second-order approximate conversion for the left-hand side of (B66) and a first-order approximate conversion for the right-hand side. Using the definition of the real return in (7) and the definition of the real exchange rate in (13), we obtain

$$\begin{aligned} &\hat{R}^j_{i-jt+1} = \hat{R}^i_{i-jt+1} + O(\epsilon^2) \quad \text{and} \\ &\hat{R}^j_{i-jt+1} = \hat{R}^i_{i-jt+1} - \hat{R}^i_{i-jt+1}\Big(\hat{Q}_{ijt+1} - \hat{Q}_{ijt}\Big) + O(\epsilon^3)\,. \end{aligned} \tag{B67}$$

Using (B67) to convert $\hat{R}^j_{i-jt+1}$ to numéraire currency i in (B66), we obtain the second-order approximate cross-country differenced individual Euler equation in (47).

We next derive the second-order approximate cross-country *summed* individual Euler equations in (47). Summing (B65) across countries for agents with identical coefficients of relative risk aversion ρ, we obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1} + \hat{R}^j_{i-jt+1}\Big] = \rho\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho)\hat{R}^i_{i-jt+1} + \hat{C}_{jt+1}(\rho)\hat{R}^j_{i-jt+1}\Big] + O(\epsilon^3)\,. \tag{B68}$$

Again using (B67) to convert $\hat{R}^j_{i-jt+1}$ to numéraire currency i, we obtain the second-order approximate cross-country summed individual Euler equation in (47).

To obtain aggregate second-order approximate cross-country differenced and summed Euler equations, we exploit the fact that ρ multiplies no individual agent's variables in (47) and integrate with respect to ρ, leading straightforwardly to the expressions in (48).

B.8.3 Proof of Proposition 8

To prove Proposition 8 on domestic bias in aggregate portfolio holdings, we first derive a final solution for cross-country differenced portfolio holdings, which we omitted from Section 3.2.5.

To obtain the intermediate solution in (49), we combine the aggregate cross-country differenced portfolio valuation effect in (34), cross-country differenced aggregate consumption in (37) and the real exchange rate in (40) with the cross-country differenced aggregate Euler equations (48). This solution is intermediate because it depends on $\hat{R}^i_{i-jt+1}$.

Using the first-order approximate solution for $\hat{R}^i_{i-jt+1}$ in (41), and rearranging to isolate B^i_{ii-ji} on the left-hand side, we then obtain the final solution as

$$B^i_{ii-ji} = \frac{\mathrm{E}_{t-1}\left[\zeta^{(-)}_{B_1}\left(\hat{Y}_{i-jt}\right)^2 + \zeta^{(-)}_{B_2}\left(\hat{Y}_{i-jt}\hat{M}^{i-j}_{i-jt}\right)\right]}{\mathrm{E}_{t-1}\left[\zeta^{(-)}_{B_3}\left(\hat{Y}_{i-jt}\right)^2 + \zeta^{(-)}_{B_4}\left(\hat{Y}_{i-jt}\hat{M}^{i-j}_{i-jt}\right) + \zeta^{(-)}_{B_5}\left(\hat{M}^{i-j}_{i-jt}\right)^2\right]} + O(\epsilon)\,, \tag{B69}$$

where the coefficients are given by

$$\begin{aligned}
\zeta^{(-)}_{B_1} &= \eta^{(-)}_{RY}\left(\eta^{(-)}_{CY} - \frac{\omega}{\bar{\rho}}\eta^{(-)}_{QY}\right), \\
\zeta^{(-)}_{B_2} &= \eta^{(-)}_{RM}\left(\eta^{(-)}_{CY} - \frac{\omega}{\bar{\rho}}\eta^{(-)}_{QY}\right), \\
\zeta^{(-)}_{B_3} &= \eta^{(-)}_{RY}\eta^{(-)}_{RV}\left(\eta^{(-)}_{CY} - \frac{\omega}{\bar{\rho}}\eta^{(-)}_{QY}\right) - \left(\eta^{(-)}_{RY}\right)^2\left(\eta^{(-)}_{CV} - \frac{\omega}{\bar{\rho}}\eta^{(-)}_{QV}\right), \\
\zeta^{(-)}_{B_4} &= \eta^{(-)}_{RM}\eta^{(-)}_{RV}\left(\eta^{(-)}_{CY} - \frac{\omega}{\bar{\rho}}\eta^{(-)}_{QY}\right) - 2\eta^{(-)}_{RM}\eta^{(-)}_{RY}\left(\eta^{(-)}_{CV} - \frac{\omega}{\bar{\rho}}\eta^{(-)}_{QV}\right), \text{ and} \\
\zeta^{(-)}_{B_5} &= -\left(\eta^{(-)}_{RM}\right)^2\left(\eta^{(-)}_{CV} - \frac{\omega}{\bar{\rho}}\eta^{(-)}_{RV}\right),
\end{aligned} \tag{B70}$$

and where the semi-partial elasticities that appear in the coefficients are defined in equations (B54), (B55), and (B60). These coefficients are continuous in θ in the neighborhood of $\theta = 0$, so it will suffice for the proof to establish signs for the coefficients at $\theta = 0$, as these signs are preserved for θ sufficiently close to zero.

To establish signs for the coefficients in (B70), we must first establish signs for the semi-partial elasticities that appear in the coefficients. These semi-partial elasticities evaluated at $\theta = 0$ are given in Table B2. Using the table, we can further establish signs of two compound terms that appear repeatedly in the coefficients in (B70),

$$\left(\eta^{(-)}_{CY} - \frac{\omega}{\bar{\rho}}\eta^{(-)}_{QY}\right)\Bigg|_{\theta=0} > 0 \quad \text{and} \quad \left(\eta^{(-)}_{CV} - \frac{\omega}{\bar{\rho}}\eta^{(-)}_{RV}\right)\Bigg|_{\theta=0} > 0\,, \tag{B71}$$

under Assumptions 1 and 2.

Using these terms, Table B2, and (B71), we find that the coefficients in (B70) are

negative when evaluated at $\theta = 0$:

$$\begin{gathered}\zeta_{B_1}^{(-)}\Big|_{\theta=0} < 0\,, \quad \zeta_{B_2}^{(-)}\Big|_{\theta=0} < 0\,, \quad \zeta_{B_3}^{(-)}\Big|_{\theta=0} < 0 \\ \zeta_{B_4}^{(-)}\Big|_{\theta=0} < 0\,, \quad \text{and} \quad \zeta_{B_5}^{(-)}\Big|_{\theta=0} < 0\,.\end{gathered} \tag{B72}$$

Assumption 3, together with the results in (B69) and (B72), are sufficient to conclude that $B_{ii-ji}^{i} > 0$. From the definition of cross-country differenced steady-state bond holdings in (34) and the non-stochastic steady-state portfolio equilibrium in Proposition 2, it follows that $B_{ij}^{i} < 0 + O(\epsilon) < B_{ii}^{i}$ as stated in Proposition 8.

B.8.4 Proof of Proposition 9

Proposition 9 states that the cross-country differenced real return is negative. To establish this result, we begin from the intermediate solution for expected cross-country differenced real returns in (53). We need to establish the signs for the three terms under expectation operators on the right-hand side of this expression, as well as for the coefficients on these terms.

We begin with the coefficients. The coefficient $\omega\bar{\rho}/2$ is positive by inspection, under Assumption 2. The signs of the remaining two coefficients are determined by the signs of the semi-partial elasticities $\eta_{QY}^{(-)}$ and $\eta_{QV}^{(-)}$. From Table B2, these are positive and negative at $\theta = 0$, respectively, under Assumptions 1 and 2. By continuity, these signs are preserved for small positive values of θ.

It remains to establish the signs of the three terms under expectation operators on the right-hand side of (53). The first term is $\mathrm{E}_t\Big[\hat{R}_{i-jt+1}^{i}\hat{Y}_{i+jt+1}\Big]$. We use the cross-country differenced realized return in (42) to obtain

$$\mathrm{E}_t\Big[\hat{R}_{i-jt+1}^{i}\hat{Y}_{i+jt+1}\Big] = \gamma_{RY}^{(-)}\,\mathrm{E}_t\Big[\hat{Y}_{i-jt+1}\hat{Y}_{i+jt+1}\Big] + \gamma_{RM}^{(-)}\,\mathrm{E}_t\Big[\hat{M}_{i-jt+1}^{i-j}\hat{Y}_{i+jt+1}\Big] + O(\epsilon^3)\,,$$

Under Assumption 3, this term is negative if the general elasticities $\gamma_{RY}^{(-)}$ and $\gamma_{RM}^{(-)}$ are negative. These general elasticities are given in (43). Their numerators $\eta_{RY}^{(-)}$ and $\eta_{RM}^{(-)}$ are negative at $\theta = 0$, as shown in Table B2. Their denominator $1 - \eta_{RV}^{(-)}B_{ii-ji}^{i}$ is positive if

$$\eta_{RV}^{(-)}B_{ii-ji}^{i} < 1\,,$$

or, using the solution for B_{ii-ji}^{i} in (B69), if

$$\begin{aligned}&\left(\eta_{CV}^{(-)} - \frac{\omega}{\bar{\rho}}\eta_{QV}^{(-)}\right)\left(\eta_{RY}^{(-)}\right)^2\mathrm{E}_t\left[\left(\hat{Y}_{i-jt+1}\right)^2\right] \\ &\quad + 2\left(\eta_{CV}^{(-)} - \frac{\omega}{\bar{\rho}}\eta_{QV}^{(-)}\right)\eta_{RY}^{(-)}\eta_{RM}^{(-)}\,\mathrm{E}_t\Big[\hat{M}_{i-jt+1}^{i-j}\hat{Y}_{i-jt+1}\Big] > \zeta_{B_5}^{(-)}\,\mathrm{E}_t\left[\left(\hat{M}_{i-jt+1}^{i-j}\right)^2\right].\end{aligned} \tag{B73}$$

Evaluating this inequality at $\theta = 0$ using the semi-partial elasticity signs from Table B2, and using the signs for the cross-country differenced bond holding coefficients in (B72), the left-hand side of (B73) is positive while the right-hand side is negative under Assumptions 1–3. By continuity, the inequality is preserved for values of θ in the neighborhood of zero. Thus, $\gamma_{RY}^{(-)} < 0$ and $\gamma_{RM}^{(-)} < 0$, and therefore $\mathrm{E}_t\left[\hat{R}_{i-jt+1}^{i}\hat{Y}_{i+jt+1}\right] \leq 0$.

The second term is $\mathrm{E}_t\left[\hat{R}_{i-jt+1}^{i}\hat{Y}_{i-jt+1}\right]$. We again use the cross-country differenced real returns in (42) to obtain

$$\mathrm{E}_t\left[\hat{R}_{i-jt+1}^{i}\hat{Y}_{i-jt+1}\right] = \gamma_{RY}^{(-)}\,\mathrm{E}_t\left[\left(\hat{Y}_{i-jt+1}\right)^2\right] + \gamma_{RM}^{(-)}\,\mathrm{E}_t\left[\hat{M}_{i-jt+1}^{i-j}\hat{Y}_{i-jt+1}\right] + O(\epsilon^3)\,,$$

and using $\gamma_{RY}^{(-)} < 0$ and $\gamma_{RM}^{(-)} < 0$ from above, we immediately obtain $\mathrm{E}_t\left[\hat{R}_{i-jt+1}^{i}\hat{Y}_{i-jt+1}\right] < 0$ under Assumptions 1–3. The third term, $\mathrm{E}_t\left[\left(\hat{R}_{i-jt+1}^{i}\right)^2\right]$, is squared and therefore non-negative.

The signs of the three coefficients and three terms under expectation operators on the right-hand side of (53) under Assumptions 1–3 imply that $\mathrm{E}_t\left[\hat{R}_{i-jt+1}^{i}\right] < 0 + O(\epsilon^3)$, as stated in Proposition 9. Importantly, these assumptions were also sufficient for domestic bias in aggregate portfolio holdings, as shown in Proposition 8. Propositions 8 and 9 together capture an empirical pattern that we observe in Germany, Japan, and the United States and document in Table 1 and Figure A4: strong domestic bias in aggregate portfolio holdings despite relatively low returns.

B.9 The Individual Non-Portfolio Problem

In the individual non-portfolio problem, we solve for the individual consumption and real wealth of agents in terms of exogenous variables and parameters. Unlike the aggregate system, where we solved for cross-country differences and used market clearing conditions to derive country-specific solutions, the individual system requires us to solve for both cross-country differences and cross-country sums. We derive these solutions in the following two subsections.

B.9.1 The Individual Non-Portfolio System (−)

The individual Euler equations and budget constraints of agents were first-order approximated, cross-country differenced, and differenced with aggregate Euler and budget equations in (55) and (56). We begin from these equations.

Eliminating the real exchange rate from the cross-country differenced individual Euler

equation (55) using the intermediate solution for the real exchange rate in (40), we obtain

$$\begin{aligned}&\theta\hat{W}^i_{i-jt}(\rho) + \beta R\,\mathrm{E}_t\big[\tilde{C}_{i-jt+1}(\rho)\big] - \left(\theta + \beta R\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QV}\right)\hat{W}^i_{i-jt} \\ &\quad = \tilde{C}_{i-jt}(\rho) - \beta R\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\left(\eta^{(-)}_{QW}\hat{W}^i_{i-jt-1} + \eta^{(-)}_{QV}\hat{Y}_{i-jt} + \eta^{(-)}_{QV}\hat{V}^i_{i-jt}\right) \quad + O(\epsilon^2)\,,\end{aligned} \tag{B74}$$

where $\tilde{C}_{i-jt}(\rho) = \hat{C}_{i-jt}(\rho) - \hat{C}_{i-jt}$ denotes the difference between cross-country differenced agent and aggregate consumption, using the tilde notation to denote differences between agent and aggregate variables or parameters that integrate to zero.

Cross-country differenced aggregate real wealth $\hat{W}^i_{i-jt}$ now appears in (B74). A third equation, the law of motion for cross-country differenced aggregate real wealth in (37), must therefore join the Euler equation (B74) and budget constraint (56) to form the cross-country differenced individual system. We write the system in matrix form as

$$\begin{aligned}\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}\,\mathrm{E}_t\big[\hat{\boldsymbol{Z}}_{i-jt+1}(\rho)\big] &= \boldsymbol{\mathcal{E}}^{(-)}_{1(\rho)}\hat{\boldsymbol{Z}}_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{2(\rho)}\hat{Y}_{i-jt} \\ &\quad + \boldsymbol{\mathcal{E}}^{(-)}_{3(\rho)}\hat{V}^i_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{4(\rho)}\hat{V}^i_{i-jt} + O(\epsilon^2)\,,\end{aligned} \tag{B75}$$

where $\hat{\boldsymbol{Z}}_{i-jt}(\rho) = \begin{bmatrix}\hat{W}^i_{i-jt-1}(\rho) & \tilde{C}_{i-jt}(\rho) & \hat{W}^i_{i-jt-1}\end{bmatrix}'$ denotes the (3×1) vector of cross-country differenced individual real wealth, cross-country differenced individual consumption in deviations from aggregate, and cross-country differenced aggregate real wealth, and where

$$\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)} = \begin{bmatrix}1 & 0 & -1 \\ \theta & 1-\theta & -\left(\theta + (1-\theta)\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QW}\right) \\ 0 & 0 & 1\end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{1(\rho)} = \begin{bmatrix}\frac{1-\theta}{\beta} & -1 & -\frac{1-\theta}{\beta} \\ 0 & 1 & -(1-\theta)\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QW} \\ 0 & 0 & \eta^{(-)}_{WW}\end{bmatrix}, \quad \boldsymbol{\mathcal{E}}^{(-)}_{2(\rho)} = \begin{bmatrix}0 \\ -(1-\theta)\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QY} \\ \eta^{(-)}_{WY}\end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{3(\rho)} = \begin{bmatrix}\frac{1-\theta}{\beta} \\ 0 \\ 0\end{bmatrix}, \quad \text{and} \quad \boldsymbol{\mathcal{E}}^{(-)}_{4(\rho)} = \begin{bmatrix}0 \\ -(1-\theta)\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QV} \\ \eta^{(-)}_{WV}\end{bmatrix}.$$

The matrix $\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}$ is invertible if its determinant is non-zero. Therefore, we require that $\det\left(\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}\right) = 1 - \theta \neq 0$, which is always satisfied under Assumption 1.

Inverting $\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}$, we can rewrite (B75) to obtain the system as it appears in (58), where

matrices of partial elasticities are given by

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)} = \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)W(\rho)} & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} & \mathcal{E}^{(-)}_{W(\rho)W} \\ \mathcal{E}^{(-)}_{C(\rho)W(\rho)} & \mathcal{E}^{(-)}_{C(\rho)C(\rho)} & \tilde{\mathcal{E}}^{(-)}_{C(\rho)W} \\ \mathcal{E}^{(-)}_{WW(\rho)} & \mathcal{E}^{(-)}_{WC(\rho)} & \mathcal{E}^{(-)}_{WW} \end{bmatrix} = \begin{bmatrix} \frac{1-\theta}{\beta} & -1 & \eta^{(-)}_{WW} - \frac{1-\theta}{\beta} \\ -\frac{\theta}{\beta} & \frac{1+\theta}{1-\theta} & \frac{\theta}{\beta} - \eta^{(-)}_{QW}\left(1-\eta^{(-)}_{WW}\right)\left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right) \\ 0 & 0 & \eta^{(-)}_{WW} \end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y} = \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)Y} \\ \tilde{\mathcal{E}}^{(-)}_{C(\rho)Y} \\ \mathcal{E}^{(-)}_{WY} \end{bmatrix} = \begin{bmatrix} \eta^{(-)}_{WY} \\ \left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\left(\eta^{(-)}_{QW}\eta^{(-)}_{WY} - \eta^{(-)}_{QY}\right) \\ \eta^{(-)}_{WY} \end{bmatrix}, \tag{B76}$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)} = \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)V(\rho)} \\ \mathcal{E}^{(-)}_{C(\rho)V(\rho)} \\ \mathcal{E}^{(-)}_{WV(\rho)} \end{bmatrix} = \begin{bmatrix} \frac{1-\theta}{\beta} \\ -\frac{\theta}{\beta} \\ 0 \end{bmatrix}, \quad \text{and}$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V} = \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)V} \\ \tilde{\mathcal{E}}^{(-)}_{C(\rho)V} \\ \mathcal{E}^{(-)}_{WV} \end{bmatrix} = \begin{bmatrix} \eta^{(-)}_{WV} \\ \left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\left(\eta^{(-)}_{QW}\eta^{(-)}_{WV} - \eta^{(-)}_{QV}\right) \\ \eta^{(-)}_{WV} \end{bmatrix}.$$

Before solving the cross-country differenced individual system, we decompose the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ into eigenvalues and eigenvectors and establish the conditions for a unique and stationary rational expectations solution following Blanchard and Kahn (1980). The eigendecomposition of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ is given by

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)} = \boldsymbol{\mathcal{V}}^{(-)}_{Z(\rho)Z(\rho)} \boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)} \boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)} \tag{B77}$$

where $\boldsymbol{\mathcal{V}}^{(-)}_{Z(\rho)Z(\rho)}$ is a matrix of eigenvectors of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ and where $\boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)}$ is a diagonal matrix of eigenvalues of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$. We write the matrix of eigenvectors as

$$\begin{aligned} \boldsymbol{\mathcal{V}}^{(-)}_{Z(\rho)Z(\rho)} &= \begin{bmatrix} \nu^{(-)}_{W(\rho)W(\rho)} & \nu^{(-)}_{W(\rho)C(\rho)} & \nu^{(-)}_{W(\rho)W} \\ \nu^{(-)}_{C(\rho)W(\rho)} & \nu^{(-)}_{C(\rho)C(\rho)} & \nu^{(-)}_{C(\rho)W} \\ \nu^{(-)}_{WW(\rho)} & \nu^{(-)}_{WC(\rho)} & \nu^{(-)}_{WW} \end{bmatrix} \\ &= \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)C(\rho)} & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} \\ \lambda^{(-)}_{Z(\rho)W(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)} & \lambda^{(-)}_{ZC(\rho)} - \mathcal{E}^{(-)}_{WW} & \lambda^{(-)}_{ZW} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W} \\ 0 & 0 & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} \end{bmatrix}, \end{aligned} \tag{B78}$$

and the diagonal matrix of eigenvalues as

$$\boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)} = \begin{bmatrix} \lambda^{(-)}_{Z(\rho)W(\rho)} & 0 & 0 \\ 0 & \lambda^{(-)}_{Z(\rho)C(\rho)} & 0 \\ 0 & 0 & \lambda^{(-)}_{Z(\rho)W} \end{bmatrix}, \tag{B79}$$

where $\lambda^{(-)}_{Z(\rho)W} = \eta^{(-)}_{WW} = \lambda^{(-)}_{ZW}$, with $\lambda^{(-)}_{ZW}$ given in equation (B39), and where

$$\begin{aligned}\lambda^{(-)}_{Z(\rho)W(\rho)} = &\frac{1}{2}\Big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} + \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big) \\ &+ \sqrt{\frac{1}{4}\Big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}}\end{aligned} \tag{B80}$$

and

$$\begin{aligned}\lambda^{(-)}_{Z(\rho)C(\rho)} = &\frac{1}{2}\Big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} + \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big) \\ &- \sqrt{\frac{1}{4}\Big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}}.\end{aligned} \tag{B81}$$

B.9.2 Stationarity of the Individual System (−)

We now establish uniqueness and stationarity for the cross-country differenced individual system. Following the same strategy as we did with the cross-country differenced aggregate system, we consider the neighborhood of values for θ around the knife-edge non-stationary case of $\theta = 0$ to establish the result. We state the following proposition.

Proposition B1 (Stationarity of the Cross-Country Differenced Individual System)**.** *The matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ has two distinct real-valued eigenvalues, $\lambda^{(-)}_{Z(\rho)W(\rho)}$ and $\lambda^{(-)}_{Z(\rho)C(\rho)}$. Furthermore, $\lambda^{(-)}_{Z(\rho)W(\rho)}\Big|_{\theta=0} = 1/\beta$ and $\lambda^{(-)}_{Z(\rho)C(\rho)}\Big|_{\theta=0} = 1$, and*

$$\partial\lambda^{(-)}_{Z(\rho)W(\rho)}/\partial\theta\,\Big|_{\theta=0} > 0 \quad \textit{and} \quad \partial\lambda^{(-)}_{ZC(\rho)}/\partial\theta\,\Big|_{\theta=0} < 0\,. \tag{B82}$$

The third eigenvalue $\lambda^{(-)}_{Z(\rho)C}$ equals the aggregate eigenvalue $\lambda^{(-)}_{ZC}$, which lies between zero and one if θ takes a small positive value (Proposition 5). Hence, there exists a $\theta > 0$ that yields a unique and stationary rational expectations solution to the cross-country differenced individual system in (58)*. All three eigenvalues are independent of ρ.*

To prove this statement, it suffices to show that the discriminant in (B80) and (B81) is positive. The discriminant, denoted $\Delta^{(-)}_{\rho}$, is given by

$$\Delta^{(-)}_{\rho} = \frac{1}{4}\Big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}\,. \tag{B83}$$

Suppose $\theta > 0$. The squared term is always positive or zero, so it suffices to show that the second term is positive. We have

$$\mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)} = \frac{\theta}{\beta} > 0\,, \tag{B84}$$

where we have used the partial elasticities in (B76) under Assumptions 1 and 2. Suppose $\theta = 0$. The second term is zero, $\mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)} = 0$, but the squared term is positive, since $\mathcal{E}^{(-)}_{W(\rho)W(\rho)}\Big|_{\theta=0} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big|_{\theta=0} = 1/\beta - 1 > 0$ under Assumptions 1, so the discriminant

Table B3 – Individual Partial Elasticities and Their Derivatives at $\theta = 0$

Partial Elasticity	Value of Partial Elasticity Evaluated at $\theta = 0$		Derivative with Respect to θ Evaluated at $\theta = 0$	
	Expression	Sign	Expression	Sign
$\mathcal{E}^{(-)}_{W(\rho)W(\rho)}$	$\frac{1}{\beta}$	$+$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{W(\rho)C(\rho)}$	-1	$-$	0	$\cdot$
$\mathcal{E}^{(-)}_{C(\rho)W(\rho)}$	0	$\cdot$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{C(\rho)C(\rho)}$	1	$+$	$1+\frac{1}{\beta}$	$+$

Notes. Column one lists partial elasticities that appear in the cross-country differenced individual system (58). Columns two and four give expressions for the partial elasticities and their derivatives with respect to θ, respectively, both evaluated at $\theta = 0$. Columns three and five give the signs of the partial elasticities and their derivatives, respectively, both evaluated at $\theta = 0$. Signs are established under Assumptions 1 and 2.

$\Delta^{(-)}_{\rho}$ is again positive. Thus, the eigenvalues in (B80) and (B81) are distinct and real-valued under Assumptions 1–3.

As in the cross-country differenced aggregate problem, the case where $\theta = 0$ serves as a helpful benchmark here for proving the further statements in Proposition B1. In Table B3, we evaluate at $\theta = 0$ the partial elasticities in (B76) and their derivatives with respect to θ.

Proposition B1 states that $\lambda^{(-)}_{Z(\rho)W(\rho)}$ evaluated at $\theta = 0$ exceeds one and $\lambda^{(-)}_{Z(\rho)C(\rho)}$ evaluated at $\theta = 0$ equals one. Using Table B3 to evaluate (B80) and (B81) at $\theta = 0$,

$$\lambda^{(-)}_{Z(\rho)W(\rho)}\Big|_{\theta=0} = \mathcal{E}^{(-)}_{WW}\Big|_{\theta=0} > 1 \quad \text{and} \quad \lambda^{(-)}_{Z(\rho)C(\rho)}\Big|_{\theta=0} = \mathcal{E}^{(-)}_{CC}\Big|_{\theta=0} = 1\,. \tag{B85}$$

Proposition B1 further states that the partial derivative of $\lambda^{(-)}_{Z(\rho)W(\rho)}$ with respect to θ evaluated at $\theta = 0$ is positive, and the partial derivative of $\lambda^{(-)}_{Z(\rho)C(\rho)}$ with respect to θ evaluated at $\theta = 0$ is negative. To establish this result, we differentiate the eigenvalues with respect to θ,

$$\begin{aligned}\frac{\partial \lambda^{(-)}_{\pm}}{\partial \theta} = \frac{1}{2}&\left(\frac{\partial \mathcal{E}^{(-)}_{C(\rho)C(\rho)}}{\partial \theta} + \frac{\partial \mathcal{E}^{(-)}_{W(\rho)W(\rho)}}{\partial \theta}\right) \\ &\pm \frac{1}{2}\left[\frac{1}{4}\left(\mathcal{E}^{(-)}_{C(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\right)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}\right]^{-\frac{1}{2}} \\ &\times \left[\frac{1}{2}\left(\mathcal{E}^{(-)}_{C(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\right)\left(\frac{\partial \mathcal{E}^{(-)}_{C(\rho)C(\rho)}}{\partial \theta} - \frac{\partial \mathcal{E}^{(-)}_{W(\rho)W(\rho)}}{\partial \theta}\right)\right. \\ &\qquad \left. + \frac{\partial \mathcal{E}^{(-)}_{W(\rho)C(\rho)}}{\partial \theta}\mathcal{E}^{(-)}_{C(\rho)W(\rho)} + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\frac{\partial \mathcal{E}^{(-)}_{C(\rho)W(\rho)}}{\partial \theta}\right],\end{aligned} \tag{B86}$$

where the "+" case corresponds to $\lambda^{(-)}_{Z(\rho)W(\rho)}$ and the "−" case corresponds to $\lambda^{(-)}_{Z(\rho)C(\rho)}$. Using Table B3, we evaluate (B86) at $\theta = 0$ to obtain

$$\left.\frac{\partial \lambda^{(-)}_{Z(\rho)W(\rho)}}{\partial \theta}\right|_{\theta=0} = \frac{1}{1-\beta} - \frac{1}{\beta} \quad \text{and} \quad \left.\frac{\partial \lambda^{(-)}_{Z(\rho)C(\rho)}}{\partial \theta}\right|_{\theta=0} = \frac{1}{\beta} - \frac{1}{1-\beta}\,. \tag{B87}$$

The derivatives with respect to $\lambda^{(-)}_{Z(\rho)W(\rho)}$ and $\lambda^{(-)}_{Z(\rho)C(\rho)}$ are positive and negative, respectively, if $1/2 < \beta < 1$, which is satisfied under Assumption 1.

Finally, Proposition B1 states that the three eigenvalues of the matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ are independent of ρ. This result follows straightforwardly from the definitions of the eigenvalues in (B40), (B80), and (B81), together with the definitions of the partial elasticities in (B36) and (B76).

In view of the eigenvalues at $\theta = 0$ in (B44) and (B85), and the derivatives at $\theta = 0$ in (B49), (B86) and (B87) under Assumptions 1 and 2, a small positive value for θ pushes $\lambda^{(-)}_{Z(\rho)C(\rho)}$ and $\lambda^{(-)}_{Z(\rho)C}$ below one while pushing $\lambda^{(-)}_{Z(\rho)W(\rho)}$ further above one, thus satisfying the conditions in Blanchard and Kahn (1980) for a unique and stationary rational expectations solution to the cross-country differenced individual system in (58).

B.9.3 Intermediate Individual Non-Portfolio Solution (−)

We now derive intermediate solutions for the individual real wealth and consumption of agents in terms of lagged individual and aggregate state variables, exogenous variables and parameters, and the endogenous individual and aggregate portfolio valuation effects. Using the eigendecomposition in (58), we re-write the individual cross-country differenced system in (B75) as

$$\begin{aligned}
&\boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)}\,\mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}(\rho)\Big] = \boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)}\boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)}\hat{\boldsymbol{Z}}_{i-jt}(\rho) \\
&\quad + \boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)}\Big(\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y}\hat{Y}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)}\hat{V}^{i}_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V}\hat{V}^{i}_{i-jt}\Big) + O(\epsilon^2)\,,
\end{aligned} \tag{B88}$$

and from (B88) we extract the equation associated with the unstable eigenvalue $\lambda^{(-)}_{Z(\rho)W(\rho)}$,

$$\begin{aligned}
&\mathrm{E}_t\Big[\hat{Z}_{i-jt+1}(\rho)\Big] = \lambda^{(-)}_{Z(\rho)W(\rho)}\hat{Z}_{i-jt}(\rho) \\
&+ \Big[\Big(\lambda^{(-)}_{Z(\rho)W} - \mathcal{E}^{(-)}_{W(\rho)W}\Big)\eta^{(-)}_{WY} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\mathcal{E}^{(-)}_{W(\rho)Y} - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\tilde{\mathcal{E}}^{(-)}_{C(\rho)Y}\Big]\hat{Y}_{i-jt} \\
&+ \Big[\Big(\lambda^{(-)}_{Z(\rho)W} - \mathcal{E}^{(-)}_{W(\rho)W}\Big)\eta^{(-)}_{WV} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\mathcal{E}^{(-)}_{W(\rho)V} - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\tilde{\mathcal{E}}^{(-)}_{C(\rho)V}\Big]\hat{V}^{i}_{i-jt} \\
&+ \Big[\Big(\lambda^{(-)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\Big)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)V(\rho)}\Big]\tilde{V}^{i}_{i-jt}(\rho) + O(\epsilon^2)\,,
\end{aligned} \tag{B89}$$

where $\hat{Z}_{i-jt+1}(\rho) = \Big(\lambda^{(-)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\Big)\hat{W}^{i}_{i-jt}(\rho) - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\tilde{C}_{i-jt+1}(\rho)$.

Because $\lambda^{(-)}_{Z(\rho)W(\rho)}$ is greater than one, the left-hand side of (B51) must be zero to rule

out explosive paths. Setting the left-hand side to zero, we obtain

$$\begin{aligned}\tilde{C}_{i-jt}(\rho) = \eta^{(-)}_{C(\rho)W(\rho)}\tilde{W}^{i}_{i-jt-1}(\rho) + \eta^{(-)}_{C(\rho)C(\rho)}\tilde{V}^{i}_{i-jt}(\rho) \\ + \tilde{\eta}^{(-)}_{C(\rho)V}\hat{V}^{i}_{i-jt} + \tilde{\eta}^{(-)}_{C(\rho)Y}\hat{Y}_{i-jt} + O(\epsilon^2)\,,\end{aligned} \tag{B90}$$

and using (B90) to eliminate $\tilde{C}_{i-jt}(\rho)$ from the cross-country differenced individual budget constraint in (56), we obtain

$$\begin{aligned}\tilde{W}^{i}_{i-jt}(\rho) = \eta^{(-)}_{W(\rho)W(\rho)}\tilde{W}^{i}_{i-jt-1}(\rho) + \eta^{(-)}_{W(\rho)V(\rho)}\tilde{V}^{i}_{i-jt}(\rho) \\ + \tilde{\eta}^{(-)}_{W(\rho)V}\hat{V}^{i}_{i-jt} + \tilde{\eta}^{(-)}_{W(\rho)Y}\hat{Y}_{i-jt} + O(\epsilon^2)\,.\end{aligned} \tag{B91}$$

Writing the intermediate solutions (B90) and (B91) in more compact matrix form, we obtain (60), where the semi-partial elasticity matrices are given by

$$\begin{aligned}\boldsymbol{\eta}^{(-)}_{Z(\rho)W(\rho)} &= \begin{bmatrix}\eta^{(-)}_{W(\rho)W(\rho)} \\ \eta^{(-)}_{C(\rho)W(\rho)}\end{bmatrix} = \begin{bmatrix}\lambda^{(-)}_{Z(\rho)C(\rho)} \\ \mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \lambda^{(-)}_{Z(\rho)C(\rho)}\end{bmatrix}, \\ \boldsymbol{\eta}^{(-)}_{Z(\rho)V(\rho)} &= \begin{bmatrix}\eta^{(-)}_{W(\rho)V(\rho)} \\ \eta^{(-)}_{C(\rho)V(\rho)}\end{bmatrix} = \begin{bmatrix}\frac{1-\theta}{\beta} - \frac{\left(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \lambda^{(-)}_{Z(\rho)C(\rho)}\right)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \\ \frac{\left(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \lambda^{(-)}_{Z(\rho)C(\rho)}\right)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\end{bmatrix}, \\ \tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)V} &= \begin{bmatrix}\tilde{\eta}^{(-)}_{W(\rho)V} \\ \tilde{\eta}^{(-)}_{C(\rho)V}\end{bmatrix} = \begin{bmatrix}\left(\frac{\omega}{\bar{\rho}} - \frac{1}{\rho}\right)\frac{\eta^{(-)}_{QV} - \eta^{(-)}_{QW}\eta^{(-)}_{WV}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \\ \left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\frac{\eta^{(-)}_{QV} - \eta^{(-)}_{QW}\eta^{(-)}_{WV}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\end{bmatrix}, \quad \text{and} \\ \tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)Y} &= \begin{bmatrix}\tilde{\eta}^{(-)}_{W(\rho)Y} \\ \tilde{\eta}^{(-)}_{C(\rho)Y}\end{bmatrix} = \begin{bmatrix}\left(\frac{\omega}{\bar{\rho}} - \frac{1}{\rho}\right)\frac{\eta^{(-)}_{QY} - \eta^{(-)}_{QW}\eta^{(-)}_{WY}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \\ \left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\frac{\eta^{(-)}_{QY} - \eta^{(-)}_{QW}\eta^{(-)}_{WY}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\end{bmatrix}.\end{aligned} \tag{B92}$$

Next, we follow a similar procedure to derive an intermediate solution to the cross-country summed non-portfolio system of individual agents.

B.9.4 The Individual Non-Portfolio System (+)

We use the first-order approximate individual and aggregate Euler equations in (27) and (28) and the first-order approximate individual and aggregate budget constraints in (29) to derive the cross-country summed individual system in deviations from aggregates. This system is simpler than the cross-country differenced individual system, first because the real exchange rate drops out of the system and second because the system has only one predetermined state variable, the individual agent's real wealth. Aggregate real wealth does not enter because its cross-country sum is zero with assets in zero net supply.

Using first-order approximate individual and aggregate Euler equations, we obtain the first-order approximate cross-country summed individual Euler equation in deviations from aggregates,

$$\begin{aligned}\theta\Big(\hat{W}^i_{i+jt}(\rho) - \hat{W}^i_{i+jt}\Big) + \beta R\,\mathrm{E}_t\Big[\Big(\hat{C}_{i+jt+1}(\rho) - \hat{C}_{i+jt+1}\Big)\Big] \\ = \Big(\hat{C}_{i+jt}(\rho) - \hat{C}_{i+jt}\Big) - \frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\hat{Y}_{i+jt} + O(\epsilon^2)\,,\end{aligned} \tag{B93}$$

where the subscript $i+j$ on individual variables denotes cross-country sums between agents with identical coefficients of relative risk aversion. Similarly, using first-order approximate individual and aggregate budget constraints, we obtain the first-order approximate cross-country summed individual budget constraint in deviations from aggregates,

$$\begin{aligned}\Big(\hat{W}^i_{i+jt}(\rho) - \hat{W}^i_{i+jt}\Big) = R\Big(\hat{W}^i_{i+jt-1}(\rho) - \hat{W}^i_{i+jt-1}\Big) \\ - \Big(\hat{C}_{i+jt}(\rho) - \hat{C}_{i+jt}\Big) + R\Big(\hat{V}^i_{i+jt}(\rho) - \hat{V}^i_{i+jt}\Big) + O(\epsilon^2)\,,\end{aligned} \tag{B94}$$

where we have introduced a new variable $\hat{V}^i_{i+jt}(\rho)$ in the budget constraint, which we call the cross-country summed agent portfolio valuation effect and define as

$$\hat{V}^i_{i+jt}(\rho) = B^i_{ii+ji}(\rho)\hat{R}^i_{i-jt}\,, \quad \text{where} \quad B^i_{ii+ji}(\rho) = B^i_{ii}(\rho) + B^i_{ji}(\rho)\,. \tag{B95}$$

Unlike in the cross-country differenced agent problem, neither the cross-country summed Euler equation in (B93) nor the cross-country summed budget constraint in (B94) depends on the real exchange rate. The absence of the real exchange rate allows us to write the cross-country summed agent Euler equation and budget constraint directly in deviations from aggregates. Continuing to use tildes to indicate the vanishing integral property, we have

$$\begin{gathered}\tilde{W}^i_{i+jt}(\rho) = \hat{W}^i_{i+jt}(\rho) - \hat{W}^i_{i+jt}\,, \quad \tilde{C}_{i+jt}(\rho) = \hat{C}_{i+jt}(\rho) - \hat{C}_{i+jt}\,, \\ \text{and} \quad \tilde{V}^i_{i+jt}(\rho) = \hat{V}^i_{i+jt}(\rho) - \hat{V}^i_{i+jt}\,,\end{gathered} \tag{B96}$$

where, due to our assumption that bonds are in zero net supply, cross-country summed aggregate real wealth and portfolio valuation effects equal zero, $\hat{W}^i_{i+jt} = \hat{V}^i_{i+jt} = 0$. To maintain consistency in our notation, however, we still use the tilde to denote cross-country summed individual real wealth and portfolio valuation effects in deviations from aggregates.

The two equations (B93) and (B94) form the cross-country summed individual system, which we write in matrix form as

$$\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)}\,\mathrm{E}_t\Big[\tilde{\boldsymbol{Z}}_{i+jt+1}(\rho)\Big] = \boldsymbol{\mathcal{E}}^{(+)}_{1(\rho)}\tilde{\boldsymbol{Z}}_{i+jt}(\rho) + \tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{2(\rho)}\hat{Y}_{i+jt} + \boldsymbol{\mathcal{E}}^{(+)}_{3(\rho)}\tilde{V}^i_{i+jt}(\rho) + O(\epsilon^2)\,, \tag{B97}$$

where $\tilde{\boldsymbol{Z}}_{i+jt}(\rho) = \begin{bmatrix}\tilde{W}^i_{i+jt-1}(\rho) & \tilde{C}_{i+jt}(\rho)\end{bmatrix}'$ denotes the (2×1) vector of cross-country summed individual real wealth and consumption in deviation from aggregate, where

$$\begin{aligned}
\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)} &= \begin{bmatrix} 1 & 0 \\ \frac{\theta}{1-\theta} & 1 \end{bmatrix}, & \boldsymbol{\mathcal{E}}^{(+)}_{1(\rho)} &= \begin{bmatrix} \frac{1-\theta}{\beta} & -1 \\ 0 & \frac{1}{1-\theta} \end{bmatrix}, \\
\tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{2(\rho)} &= \begin{bmatrix} 0 \\ -\frac{1}{1-\theta}\frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\rho}\right) \end{bmatrix}, \quad \text{and} & \boldsymbol{\mathcal{E}}^{(+)}_{3(\rho)} &= \begin{bmatrix} \frac{1-\theta}{\beta} \\ 0 \end{bmatrix}.
\end{aligned} \tag{B98}$$

The matrix $\mathcal{E}^{(+)}_{0(\rho)}$ is invertible if its determinant is non-zero. We have

$$\det\left(\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)}\right) = 1 \neq 0\,, \tag{B99}$$

so $\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)}$ is invertible, and we rewrite the cross-country summed agent system as

$$\mathrm{E}_t\left[\tilde{\boldsymbol{Z}}_{i+jt+1}(\rho)\right] = \boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}\tilde{\boldsymbol{Z}}_{i+jt}(\rho) + \tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{Z(\rho)Y}\hat{Y}_{i+jt} + \boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)V(\rho)}\tilde{V}^i_{i+jt}(\rho) + O(\epsilon^2)\,, \tag{B100}$$

where

$$\begin{aligned}
\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)} &= \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)W(\rho)} & \mathcal{E}^{(+)}_{W(\rho)C(\rho)} \\ \mathcal{E}^{(+)}_{C(\rho)W(\rho)} & \mathcal{E}^{(+)}_{C(\rho)C(\rho)} \end{bmatrix} = \begin{bmatrix} \frac{1-\theta}{\beta} & -1 \\ -\frac{\theta}{\beta} & \frac{1+\theta}{1-\theta} \end{bmatrix}, \\
\tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{Z(\rho)Y} &= \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)Y} \\ \tilde{\mathcal{E}}^{(+)}_{C(\rho)Y} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{1-\theta}\frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\rho}\right) \end{bmatrix}, \quad \text{and} \\
\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)V(\rho)} &= \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)V(\rho)} \\ \mathcal{E}^{(+)}_{C(\rho)V(\rho)} \end{bmatrix} = \begin{bmatrix} \frac{1-\theta}{\beta} \\ -\frac{\theta}{\beta} \end{bmatrix}.
\end{aligned} \tag{B101}$$

Before solving the cross-country summed individual system, we decompose the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ into eigenvalues and eigenvectors and establish the conditions for a unique and stationary rational expectations solution following Blanchard and Kahn (1980). The eigendecomposition of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ is given by

$$\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)} = \boldsymbol{\mathcal{V}}^{(+)}_{Z(\rho)Z(\rho)}\boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)}\boldsymbol{\mathcal{V}}^{(+)-1}_{Z(\rho)Z(\rho)} \tag{B102}$$

where $\boldsymbol{\mathcal{V}}^{(+)}_{Z(\rho)Z(\rho)}$ is a matrix of eigenvectors of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ and where $\boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)}$ is a diagonal

matrix of eigenvalues of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$,

$$\begin{aligned}\boldsymbol{\nu}^{(+)}_{Z(\rho)Z(\rho)} &= \begin{bmatrix}\nu^{(+)}_{W(\rho)W(\rho)} & \nu^{(+)}_{W(\rho)C(\rho)} \\ \nu^{(+)}_{C(\rho)W(\rho)} & \nu^{(+)}_{C(\rho)C(\rho)}\end{bmatrix} \\ &= \begin{bmatrix}\mathcal{E}^{(+)}_{W(\rho)C(\rho)} & \mathcal{E}^{(+)}_{W(\rho)C(\rho)} \\ \lambda^{(+)}_{Z(\rho)W(\rho)} - \mathcal{E}^{(+)}_{W(\rho)W(\rho)} & \lambda^{(+)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(+)}_{W(\rho)W(\rho)}\end{bmatrix}, \quad \text{and} \\ \boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)} &= \begin{bmatrix}\lambda^{(+)}_{Z(\rho)W(\rho)} & 0 \\ 0 & \lambda^{(+)}_{Z(\rho)C(\rho)}\end{bmatrix},\end{aligned} \tag{B103}$$

where

$$\begin{aligned}\lambda^{(+)}_{Z(\rho)W(\rho)} = \frac{1}{2}\Big(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} + \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\Big) \\ + \sqrt{\frac{1}{4}\Big(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\Big)^2 + \mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)W(\rho)}}\end{aligned} \tag{B104}$$

and

$$\begin{aligned}\lambda^{(+)}_{Z(\rho)C(\rho)} = \frac{1}{2}\Big(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} + \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\Big) \\ - \sqrt{\frac{1}{4}\Big(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\Big)^2 + \mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)W(\rho)}}.\end{aligned} \tag{B105}$$

B.9.5 Stationarity of the Individual System (+)

We now establish uniqueness and stationarity for the cross-country summed individual system. Stationarity of the system follows straightforwardly from a comparison of the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ in (B101) with the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ in (B76). We state the following proposition.

Proposition B2 (Stationarity of the Cross-Country Summed Individual System)**.** *The eigenvalues of* $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ *are given by*

$$\lambda^{(+)}_{Z(\rho)W(\rho)} = \lambda^{(-)}_{Z(\rho)W(\rho)} \quad \textit{and} \quad \lambda^{(+)}_{Z(\rho)C(\rho)} = \lambda^{(-)}_{Z(\rho)C(\rho)}\,. \tag{B106}$$

From Proposition B1, there exists a $\theta > 0$ *that yields a unique and stationary rational expectations solution to the cross-country summed individual system in* (B100)*.*

Let $\Big[\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\Big]_{33}$ denote the (2×2) submatrix of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ formed by deleting the third row and third column from $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$. Then, by inspection, $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)} = \Big[\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\Big]_{33}$, and the eigenvalues of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ equal the eigenvalues of the submatrix $\Big[\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\Big]_{33}$. That is, $\lambda^{(+)}_{Z(\rho)W(\rho)} = \lambda^{(-)}_{Z(\rho)W(\rho)}$ and $\lambda^{(+)}_{Z(\rho)C(\rho)} = \lambda^{(-)}_{Z(\rho)C(\rho)}$. Thus, the stationarity and uniqueness conditions for the cross-country summed and differenced agent systems are the same, and satisfied under Assumptions 1 and 2 as shown in Proposition B1.

B.9.6 Intermediate Individual Non-Portfolio Solution (+)

We now derive intermediate solutions for cross-country summed individual real wealth and consumption in deviation from aggregate, written in terms of lagged individual state variables, exogenous variables and parameters, and the endogenous individual portfolio valuation effect. Using the eigendecomposition in (B102), we re-write the system in (B75) as

$$\begin{aligned}\boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)}\,\mathrm{E}_t\Big[\tilde{\boldsymbol{Z}}_{i+jt+1}(\rho)\Big] &= \boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)}\boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)}\tilde{\boldsymbol{Z}}_{i+jt}(\rho)\\ &+ \boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)}\Big(\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Y}\hat{Y}_{i+jt} + \boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)V(\rho)}\hat{V}^{i}_{i+jt}(\rho)\Big) + O(\epsilon^2)\,,\end{aligned} \tag{B107}$$

and from (B107) we extract the equation associated with the unstable eigenvalue $\lambda^{(+)}_{Z(\rho)W(\rho)}$,

$$\begin{aligned}\mathrm{E}_t\Big[\tilde{Z}_{i+jt+1}(\rho)\Big] &= \lambda^{(+)}_{Z(\rho)W(\rho)}\tilde{Z}_{i+jt}(\rho) - \mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)Y}\hat{Y}_{i+jt}\\ &+ \Big[\Big(\lambda^{(+)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(+)}_{W(\rho)W(\rho)}\Big)\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)V(\rho)}\Big]\tilde{V}^{i}_{i+jt}(\rho) + O(\epsilon^2)\,,\end{aligned} \tag{B108}$$

where $\tilde{Z}_{i+jt+1}(\rho) = \Big(\lambda^{(+)}_{Z(\rho)W(\rho)} - \mathcal{E}^{(+)}_{W(\rho)W(\rho)}\Big)\tilde{W}^{i}_{i+jt}(\rho) - \mathcal{E}^{(+)}_{W(\rho)W(\rho)}\tilde{C}_{i+jt+1}(\rho)$.

Because $\lambda^{(+)}_{Z(\rho)W(\rho)}$ is greater than one, the left-hand side of (B108) must be zero to rule out explosive paths. Setting the left-hand side to zero, we obtain

$$\tilde{C}_{i+jt}(\rho) = \eta^{(+)}_{C(\rho)W(\rho)}\tilde{W}^{i}_{i+jt-1}(\rho) + \tilde{\eta}^{(+)}_{C(\rho)Y}\hat{Y}_{i+jt} + \eta^{(+)}_{C(\rho)V(\rho)}\tilde{V}^{i}_{i+jt}(\rho) + O(\epsilon^2)\,, \tag{B109}$$

and using (B109) to eliminate $\tilde{C}_{i+jt}(\rho)$ from the cross-country summed individual budget constraint in (B94), we obtain

$$\tilde{W}^{i}_{i+jt}(\rho) = \eta^{(+)}_{W(\rho)W(\rho)}\tilde{W}^{i}_{i+jt-1}(\rho) + \tilde{\eta}^{(+)}_{C(\rho)Y}\hat{Y}_{i+jt} + \eta^{(+)}_{W(\rho)V(\rho)}\tilde{V}^{i}_{i+jt}(\rho) + O(\epsilon^2)\,. \tag{B110}$$

Writing the intermediate solutions in (B109) and (B110) in matrix form, we have

$$\begin{bmatrix}\tilde{W}^{i}_{i+jt}(\rho)\\ \tilde{C}_{i+jt}(\rho)\end{bmatrix} = \boldsymbol{\eta}^{(+)}_{Z(\rho)W(\rho)}\tilde{W}^{i}_{i+jt-1}(\rho) + \tilde{\boldsymbol{\eta}}^{(+)}_{Z(\rho)Y}\hat{Y}_{i+jt} + \boldsymbol{\eta}^{(+)}_{Z(\rho)V(\rho)}\tilde{V}^{i}_{i+jt}(\rho) + O(\epsilon^2)\,, \tag{B111}$$

where the cross-country summed agent wealth and consumption semi-partial elasticity matrices are given by

$$\boldsymbol{\eta}^{(+)}_{Z(\rho)W(\rho)} = \begin{bmatrix}\eta^{(+)}_{W(\rho)W(\rho)}\\ \eta^{(+)}_{C(\rho)W(\rho)}\end{bmatrix} = \begin{bmatrix}\lambda^{(+)}_{Z(\rho)C(\rho)}\\ \mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \lambda^{(+)}_{Z(\rho)C(\rho)}\end{bmatrix},$$

$$\tilde{\boldsymbol{\eta}}^{(+)}_{Z(\rho)Y} = \begin{bmatrix}\tilde{\eta}^{(+)}_{W(\rho)Y}\\ \tilde{\eta}^{(+)}_{C(\rho)Y}\end{bmatrix} = \begin{bmatrix}\dfrac{\tilde{\mathcal{E}}^{(+)}_{C(\rho)Y}}{\lambda^{(+)}_{Z(\rho)W(\rho)}}\\ -\dfrac{\tilde{\mathcal{E}}^{(+)}_{C(\rho)Y}}{\lambda^{(+)}_{Z(\rho)W(\rho)}}\end{bmatrix}, \quad \text{and} \tag{B112}$$

$$\boldsymbol{\eta}^{(+)}_{Z(\rho)V(\rho)} = \begin{bmatrix} \eta^{(+)}_{W(\rho)V(\rho)} \\ \eta^{(+)}_{C(\rho)V(\rho)} \end{bmatrix} = \begin{bmatrix} \frac{1-\theta}{\beta} - \frac{\left(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \lambda^{(+)}_{Z(\rho)C(\rho)}\right)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(+)}_{Z(\rho)W(\rho)}} \\ \frac{\left(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \lambda^{(+)}_{Z(\rho)C(\rho)}\right)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(+)}_{Z(\rho)W(\rho)}} \end{bmatrix}.$$

As with the cross-country differenced solution in (60), the cross-country summed solution in (B111) is intermediate in that it depends on the endogenous portfolio valuation effect $\tilde{V}^{i}_{i+jt}(\rho)$. This effect in turn depends on the endogenous steady-state cross-country sum of individual portfolio holdings, for which we solve next.

B.10 The Individual Portfolio Problem

We now solve for zero-order approximate cross-country differenced and summed agent portfolio holdings. These solutions then combine to yield country-specific solutions for each individual agent's holdings of domestic and international bonds.

To derive cross-country differenced individual portfolio holdings, we begin from equation (61). Using the real exchange rate in (40), the definition of the cross-country differenced individual portfolio valuation effect in (57), and the first-order approximate intermediate solution for cross-country differenced individual consumption in (60), we obtain the intermediate solution in (62). The collection of parameters $\zeta^{(-)}_{B(\rho)}$ in (62) is given by

$$\begin{aligned} \zeta^{(-)}_{B(\rho)} = {} & \frac{1}{\eta^{(-)}_{C(\rho)V(\rho)}} \left[\left(1 - \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \right) \eta^{(-)}_{QY} + \frac{\eta^{(-)}_{QW}\eta^{(-)}_{WY}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \right] \\ & - \frac{1}{\eta^{(-)}_{C(\rho)V(\rho)}} \frac{\bar{\rho}\eta^{(-)}_{CY} - \omega\eta^{(-)}_{QY}}{\bar{\rho}\eta^{(-)}_{CV} - \omega\eta^{(-)}_{QV}} \left[\left(1 - \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \right) \eta^{(-)}_{QV} + \frac{\eta^{(-)}_{QW}\eta^{(-)}_{WV}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \right]. \end{aligned} \tag{B113}$$

To derive cross-country summed individual portfolio holdings, we begin from equation (63). Using the real exchange rate in (40), the definition of the cross-country summed individual portfolio valuation effect in (B95), and the first-order approximate intermediate solutions for cross-country summed individual consumption in (B111), we obtain the intermediate solution in (64). The collection of parameters $\zeta^{(+)}_{B(\rho)}$ in (64) is given by

$$\zeta^{(+)}_{B(\rho)} = \frac{\bar{\rho}}{\omega} \frac{1}{\eta^{(+)}_{C(\rho)V(\rho)}} \left(1 - \frac{1}{(1-\theta)\lambda^{(-)}_{Z(\rho)W(\rho)}} \right). \tag{B114}$$

The solutions in equations (62) and (64) are intermediate, as the cross-country differenced real return appears on the right-hand side of each. Final solutions obtain straightforwardly by eliminating the cross-country differenced real return using the first-

order approximate general solution in (42). For brevity, we omit this derivation here, noting that (62) and (64) are sufficient to establish our main result for agent portfolio holdings in Proposition (10). We provide a final solution for individual portfolio holdings in Section B.10.2. For use later, we note that

$$\eta^{(-)}_{C(\rho)V(\rho)}\Big|_{\theta=0} = \eta^{(+)}_{C(\rho)V(\rho)}\Big|_{\theta=0} = \frac{1-\beta}{\beta} > 0\,. \tag{B115}$$

B.10.1 Proof of Proposition 10

Proposition 10 establishes sufficient conditions for international portfolio bias in agent portfolio holdings for a subset of risk-tolerant agents. These conditions simultaneously satisfy the conditions for domestic bias in aggregate portfolio holdings in Proposition 8 and for negative expected cross-country differenced real returns in Proposition 9.

We begin with equation (65) for agent holdings of the domestic bond in country i. Using the intermediate solutions for cross-country differenced and summed agent holdings of this bond in (62) and (64), respectively, we obtain

$$\tilde{B}^i_{ii}(\rho) = \frac{1}{2}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\frac{\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\zeta^{(-)}_{B(\rho)}\hat{Y}_{i-jt+1} + \zeta^{(+)}_{B(\rho)}\hat{Y}_{i+jt+1}\right)\right]}{\mathrm{E}_t\left[\left(\hat{R}^i_{i-jt+1}\right)^2\right]} + O(\epsilon)\,, \tag{B116}$$

In the proof of Proposition (9), we established that $\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\right] \leq 0$ and that $\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i+jt+1}\right] < 0$ under Assumptions 1 and 2. To establish the sign of $\tilde{B}^i_{ii}(\rho)$, it remains to establish the signs of $\zeta^{(-)}_{B(\rho)}$ and $\zeta^{(+)}_{B(\rho)}$.

We first establish the sign of $\zeta^{(-)}_{B(\rho)}$ assuming $\theta = 0$. Splitting the expression in (B113) into four terms, we have

$$\zeta^{(-)}_{B(\rho)} = \zeta^{(-)(T_1)}_{B(\rho)}\left(\zeta^{(-)(T_2)}_{B(\rho)} + \zeta^{(-)(T_3)}_{B(\rho)}\zeta^{(-)(T_4)}_{B(\rho)}\right), \tag{B117}$$

where

$$\begin{aligned}
\zeta^{(-)(T_1)}_{B(\rho)} &= \frac{1}{\eta^{(-)}_{C(\rho)V(\rho)}}\,,\\
\zeta^{(-)(T_2)}_{B(\rho)} &= \left(1 - \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\right)\eta^{(-)}_{QY} + \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\eta^{(-)}_{QW}\eta^{(-)}_{WY}\,,\\
\zeta^{(-)(T_3)}_{B(\rho)} &= -\frac{\bar{\rho}\eta^{(-)}_{CY} - \omega\eta^{(-)}_{QY}}{\bar{\rho}\eta^{(-)}_{CV} - \omega\eta^{(-)}_{QV}}\,, \quad \text{and}\\
\zeta^{(-)(T_4)}_{B(\rho)} &= \left(1 - \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\right)\eta^{(-)}_{QV} + \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\eta^{(-)}_{QW}\eta^{(-)}_{WV}\,.
\end{aligned} \tag{B118}$$

Using Table B2, the semi-partial elasticities in (B115), and the expression for $\lambda^{(-)}_{Z(\rho)W}\Big|_{W=0}$ in Proposition B1, we obtain

$$\begin{aligned}
\zeta^{(-)(T_1)}_{B(\rho)}\Big|_{\theta=0} &= \frac{\beta}{1-\beta}\Big|_{\theta=0} > 0\,,\\
\zeta^{(-)(T_2)}_{B(\rho)}\Big|_{\theta=0} &= (1-\beta)\left[1+\beta\frac{\left(1-\phi^2\right)\left(1-\frac{\omega}{\bar\rho}\right)}{1-\left(1-\frac{\omega}{\bar\rho}\right)\phi^2}\right] > 0\,,\\
\zeta^{(-)(T_3)}_{B(\rho)}\Big|_{\theta=0} &= -\phi\left(1-\frac{\omega}{\bar\rho}\right)\frac{(1-\beta)^2}{\beta}\frac{1-\left(1-\frac{\omega}{\bar\rho}\right)\phi^2}{1-\phi} < 0\,, \quad \text{and}\\
\zeta^{(-)(T_4)}_{B(\rho)}\Big|_{\theta=0} &= -\frac{(1-\beta)^2}{\beta}\frac{\phi^2}{1-\phi}\left(1+\frac{1-\phi^2-(1-\beta)(1-\phi)}{1-\phi^2}\right) < 0\,.
\end{aligned} \tag{B119}$$

Next, we establish the sign of $\zeta^{(+)}_{B(\rho)}$ assuming $\theta = 0$. Using Table B2 and the expression for the eigenvalue $\lambda^{(-)}_{Z(\rho)W}$ in Proposition B1, we obtain $\zeta^{(+)}_{B(\rho)}\Big|_{\theta=0} = \beta\bar\rho/\omega > 0$ under Assumptions 1 and 2. By continuity, the signs of $\zeta^{(-)}_{B(\rho)}$ and $\zeta^{(+)}_{B(\rho)}$ are preserved for small positive values of θ.

Returning to equation (B116), we conclude that $\tilde B^i_{ii}(\rho)$ is negative for any $\rho < \bar\rho/\omega$ under Assumptions 1–3, and that $\tilde B^i_{ii}(\rho)$ approaches negative infinity as ρ approaches zero from above,

$$\lim_{\rho\downarrow 0}\tilde B^i_{ii}(\rho) = -\infty\,. \tag{B120}$$

Thus, recalling that $\tilde B^i_{ii}(\rho) \equiv B^i_{ii}(\rho) - B^i_{ii}$, there exists a $\rho^* \in (0, \bar\rho/\omega)$ such that $B^i_{ii}(\rho) < 0$ for any $\rho < \rho^*$ and any finite value of B^i_{ii}.

B.10.2 Final Solution and Special Case of Cole and Obstfeld (1991)

Substituting cross-country differenced real returns (42) and general elasticities (43) into (B116), we obtain our final solution for individual portfolio holdings,

$$\begin{aligned}
B^i_{ii}(\rho) = B^i_{ii} &+ \left(\frac{1}{\rho}-\frac{\omega}{\bar\rho}\right)\left(\frac{1}{2}-\eta^{(-)}_{RV}B^i_{ii}\right)\\
&\times\frac{\mathrm{E}_{t-1}\left[\left(\eta^{(-)}_{RY}\hat Y_{i-jt}+\eta^{(-)}_{RM}\hat M^{i-j}_{i-jt}\right)\left(\zeta^{(-)}_{B(\rho)}\hat Y_{i-jt}+\zeta^{(+)}_{B(\rho)}\hat Y_{i+jt}\right)\right]}{\mathrm{E}_{t-1}\left[\left(\eta^{(-)}_{RY}\hat Y_{i-jt}+\eta^{(-)}_{RM}\hat M^{i-j}_{i-jt}\right)^2\right]} + O(\epsilon)\,,
\end{aligned} \tag{B121}$$

where from (51) and (B69), aggregate portfolio holdings are given by

$$B^i_{ii} = \frac{1}{2}\frac{\mathrm{E}_{t-1}\left[\zeta^{(-)}_{B_1}\left(\hat Y_{i-jt}\right)^2+\zeta^{(-)}_{B_2}\left(\hat Y_{i-jt}\hat M^{i-j}_{i-jt}\right)\right]}{\mathrm{E}_{t-1}\left[\zeta^{(-)}_{B_3}\left(\hat Y_{i-jt}\right)^2+\zeta^{(-)}_{B_4}\left(\hat Y_{i-jt}\hat M^{i-j}_{i-jt}\right)+\zeta^{(-)}_{B_5}\left(\hat M^{i-j}_{i-jt}\right)^2\right]} + O(\epsilon)\,. \tag{B122}$$

The semi-partial elasticities $\eta^{(-)}_{RV}$, $\eta^{(-)}_{RY}$, and $\eta^{(-)}_{RM}$ defined in (B60), the coefficients $\zeta^{(-)}_{B(\rho)}$ and $\zeta^{(+)}_{B(\rho)}$ defined in (B113) and (B114), and the coefficients $\zeta^{(-)}_{B_1}$, $\zeta^{(-)}_{B_2}$, $\zeta^{(-)}_{B_3}$, $\zeta^{(-)}_{B_4}$, and $\zeta^{(-)}_{B_5}$ defined (B70) depend on model parameters and are therefore exogenous, as are preference parameters ρ, $\bar{\rho}$, and ω, and variables $\hat{Y}_{i-jt}$, $\hat{Y}_{i+jt}$, and $\hat{M}^{i-j}_{i-jt}$.

A simpler solution for portfolio holdings obtains as a special case of (B121) and (B122), where $\bar{\rho} = \omega$ and $\theta = 0$. This special case, while producing non-stationary dynamics and no domestic portfolio bias in aggregate, is nevertheless instructive. Under the special assumptions on $\bar{\rho}$ and θ, semi-partial elasticities $\eta^{(-)}_{CY}$ and $\eta^{(-)}_{QY}$ equalize, so that aggregate portfolio holdings equal zero, by inspection of (50). This result parallels the classic finding in representative agent models where portfolios are zero when the coefficient of relative risk aversion equals one (Cole and Obstfeld, 1991).

The economic intuition is identical: when aggregate consumption and the real exchange rate respond with equal elasticity to endowment shocks, real exchange rate movements provide complete hedging for fluctuations in aggregate consumption. A negative domestic endowment shock reduces aggregate consumption but depreciates the real exchange rate by exactly the same proportion, leaving the value of consumption in foreign goods unchanged. Since the real exchange rate already insures against endowment risk, there is no additional hedging benefit from holding foreign assets at the aggregate level.

Since bonds are the only source of nominal rigidity, and aggregate portfolio holdings are zero, fluctuations in money supplies are irrelevant for the aggregate economy. Individual agents, on the other hand, still hold non-zero portfolios due to heterogeneity in risk aversion, and therefore face nominal rigidities and care about money supplies, even while their positions net out in the aggregate. This intuition is reflected in the simplified versions of (B121) and (B122) that obtain in the special case,

$$B^i_{ii}(\rho) = \beta\frac{\phi}{2}\left(1-\frac{1}{\rho}\right)\frac{\mathrm{E}_{t-1}\left[\hat{Y}_{i-jt}\hat{M}^{i-j}_{i-jt}\right]}{\mathrm{E}_{t-1}\left[\left(\hat{M}^{i-j}_{i-jt}\right)^2\right]} + O(\epsilon) \quad \text{and} \quad B^i_{ii} = 0 + O(\epsilon)\,. \tag{B123}$$

If $\mathrm{E}_{t-1}\left[\hat{Y}_{i-jt}\hat{M}^{i-j}_{i-jt}\right]$ is positive, the cross-country difference in money supplies will tend to rise with cross-country endowments—for example, as central banks conduct policy to stabilize price levels. Domestic bonds then serve as a hedge against consumption growth risk, and agents with $\rho > 1$ take the hedge while agents with $\rho < 1$ take the opposite positions. When aggregate risk aversion $\bar{\rho}$ equals exactly the aggregation wedge ω, carry traders and agents with domestic portfolio bias have equal weight in bond markets. When aggregate risk aversion rises above this knife-edge value, agents with domestic bias outweigh carry traders.