Why HANK Matters for Stabilization Policy

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Abstract

When do optimal inflation and fiscal policy differ significantly between Heterogeneous-Agent (HA) and Representative-Agent (RA) models, and what are the underlying mechanisms? To answer this question, we derive jointly the optimal fiscal-monetary Ramsey policy in HA and RA models that incorporate both price and wage stickiness. We examine different sets of fiscal tools and analyze both supply and demand shocks. HA economies diverge significantly from RA economies when the severity of credit constraints varies over time, which is the case for demand shocks but less so for supply shocks. We introduce a new statistics, that we call the Marginal Value of Credit Constraint, to quantify this mechanism. Furthermore, inflation dynamics differ between HA and RA economies in response to demand shocks, particularly when fiscal policy is not employed as a stabilization tool over the business cycle. We identify the relevant fiscal tools to reduce inflation volatility over the business cycle.

Keywords: Heterogeneous agents, wage-price spiral, inflation, monetary policy, fiscal policy.

JEL codes: D31, E52, D52, E21.

1 Introduction

Heterogeneous-Agent (HA) models describe economies where agents face incomplete insurance markets for the idiosyncratic risk and credit constraints. These models with nominal frictions (HANK models) are now widely used to identify new transmission channels after various shocks or change in policy. The implications of these models for optimal stabilization policy are however unclear. Do we uncover new mechanisms about optimal fiscal and monetary stabilization policy compared to simpler complete-market (CM) models featuring a representative agent, or do they

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only generate small quantitative differences? For instance, some recent papers conclude that the allocation, and sometimes the optimal dynamics of inflation, are close in HA and CM models (LeGrand et al. (2022), Challe (2020)). We show that this not the case in general settings. HA models generate new mechanisms to think about monetary and fiscal stabilization policy. In a nutshell, these mechanisms rely on the time-varying demand for liquidity, which can be captured by simple statistics, but which crucially depend on the nature of the shock hitting the economy.

We show this property analyzing an heterogeneous agents models with both sticky prices and sticky wages. In this environment we solve jointly for optimal fiscal and monetary policy after a three types of shock: A TFP shock, a discount factor shock and a public spending shock. We consider a rich set of fiscal tools, as explained below. In each case, we solve for the optimal policy (using the same set of instruments) in the corresponding complete-market economy to identify the difference in allocation and instruments between the HA and CM economy.

In a first step, to provide a theoretical explanation of the difference between HA and CM models we focus on a simple environment which allows for analytical characterization of the difference between the allocation of HA and CM models. We consider a model with deterministic income fluctuations and credit constraints, as in Woodford (1990) and LeGrand and Ragot (2025). We solve for optimal policy in a flexible price environment where the tools are capital and labor tax and public debt, after two types of shock: a TFP shock and a discount factor shock. Optimal policy generates a dynamics of consumption and labor, which are exactly the same between CM and HA models after a TFP shock, but this dynamics is different after a discount factor shock. We prove that the difference in result comes from the fact that the normalized Lagrange multiplier on the credit constraints of agents is not time-varying after TFP shock, but it is time-varying after discount-factor shock. We called this normalized value the Marginal Value of Credit Constraint (MVCC) as it is the price a credit constrained agent would pay to relax the credit constraint by one unit. We show then show that a time-varying MVCC is a necessary condition for the allocation to be different in HA and CM economies. In other words, it isn't the presence of occasionally binding credit constraint (which may be affected by time-varying risk) which is driving a difference between CM and HA economies, but the fact that the intensity of the credit constraint is time-varying, what generates a time-varying demand for liquidity. This property happens to be shock specific. It is the case for discount factor shocks, but much less so for TFP shock. We show that the MVCC is related to the discount factor wedge, which is used by Nakajima (2005), Werning (2015), Acharya and Dogra (2021) and Berger et al. (2023) to compare aggregate allocation between HA and CM economies.

In a second step, we solve for optimal monetary and fiscal policy in the general incomplete-market model, with both sticky prices and sticky wages, and with a standard separable utility function. First, we present a called complete fiscal system, composed of three labor taxes, a capital tax and public debt. We theoretically show that when these tools are optimally time-varying optimal price and wage inflation is 0 after the three shocks (TFP, discount factor and public

spending). This results extend equivalence result of Correia et al. 2008 or LeGrand et al., 2022 to an environment of both sticky prices and wages. From this theoretical benchmark, we then remove some fiscal instruments, considering a limited set of time-varying instruments, and we solve for optimal policy in the CM and HA economy for each aggregate shock. This strategy allows identifying the missing tools generating the highest welfare loss or a significant deviation from price and wage stability. This strategy generates two sets of results.

First, whether RA and HA models differ for normative questions depends jointly on the nature of the shock and on the instruments available to the planner. The key mechanism is whether the precautionary saving motive varies along the business cycle, which is summarized by the MVCC: when the MVCC is constant, RA and HA allocations remain close, while when the MVCC is time-varying, the two diverge. A sufficient statistic is therefore the cyclicality of the MVCC, even if in practice the DFW provides an observationally equivalent measure under our benchmark calibration. We illustrate this mechanism with two contrasting cases. After a TFP shock, the MVCC is constant, so RA and HA models deliver very similar prescriptions, unless the fiscal system is severely restricted. By contrast, after a discount-factor shock the MVCC becomes time-varying, and RA and HA models differ sharply in their optimal allocations and policies, even when the fiscal system is relatively complete.

Second, the available set of fiscal instruments determines the extent to which the planner can stabilize inflation and real allocations. Income taxes and labor taxes are shown to be particularly efficient instruments: as soon as at least one of them is available, the HA and RA models remain close in terms of aggregate consumption and labor, and deviations from price stability are limited under the benchmark calibration. These two instruments also act as partial substitutes, so the quantitative difference from having one or the other is not very large. By contrast, when the employer social contribution is the only available tool, the planner cannot offset the distortions introduced by shocks. In this case, the economy exhibits pronounced deviations from both price and wage stability, and the divergence between RA and HA allocations becomes significant, regardless of whether the shock is supply-driven (TFP) or demand-driven (discount factor). Thus, the effectiveness of stabilization policy depends not only on the type of shock but also on the composition of the fiscal toolkit available to the planner.

Related literature. This paper belongs to the literature on optimal policy in heterogeneous agent model on one side, and on wage and price inflation dynamics on the other side. Deriving optimal policy in heterogeneous-agent models with aggregate shocks is a difficult theoretical and computational task. Some papers consider numerical methods to solve for optimal path of the instruments (Dyrda and Pedroni, 2022). Other papers rely on continuous-time techniques for the theoretical derivation of the first-order conditions of the planner (Nuño and Thomas, 2022 among others). Acharya et al. (2022) solve for optimal monetary policy using the tractability of the CARA-normal environment without capital. Bhandari et al. (2021) quantitatively solve for

optimal policies in a new-Keynesian model with aggregate shocks. Yang (2022) solves for the optimal monetary policy by optimizing on the coefficients of a Taylor rule. McKay and Wolf (2022) derive a general quadratic-linear formulation to solve for optimal policy rules. McKay and Reis (2021) study optimal automatic stabilizers in the context of the optimal replacement rate. Their main focus is on the trade-off between insurance and incentives in the presence of an aggregate demand effect. The mechanism we identify is different in that it directly affects the gap between the real wage and the marginal productivity of labor.In this paper, we use the tools of LeGrand and Ragot (2022a) and the improvements of LeGrand and Ragot (2022b) to solve for optimal fiscal and monetary policy with aggregate shocks. The gain of this approach is to allows to easily solve for optimal policy with many tools and with various nominal frictions. On the theoretical side, the Lagrangian approach pioneered in Marcet and Marimon (2019) enables us to derive the first-order conditions of the Ramsey planner in an environment with both wage and price rigidities.

Regarding the literature on wage and price inflation, models including both price and wage stickiness have been studied in RA economies (Blanchard, 1986, Galí, 2015, chapter 6, or Blanchard and Gali, 2007 among others). Erceg et al. (2000) study optimal monetary policy in this environment. Chugh (2006) study both optimal monetary policy and an optimal labor tax. Recently, Lorenzoni and Werning (2023) analyze more deeply optimal policy and the real wage dynamics in this environment.

2 Understanding the difference between RA and HA economies in a simple environment

In this section, we consider a tractable model, where the difference between the optimal policy and allocations between HA and RA economies can be analytically characterized. The main simplifying assumption is to consider deterministic productivity fluctuations, as in Woodford (1990) and in LeGrand and Ragot (2025) among others. We focus on the flexible price economy to simplify the exposition.

Production. The production function is $Y_t = Z_t L_t$. As prices are flexible, the real wage is $\tilde{w}_t = Z_t$.

The agents. The economy is populated two types of agents, denoted by A and B. A unit mass of agent A has a productivity 1 every odd period and a productivity 0 every even periods. A unit mass of agent B has a productivity 1 every even period and a productivity 0 every off periods. In each period, there is thus a unit mass of agents with productivity 1, and income fluctuations are deterministic. The agents having a positive productivity are called "employed", and are identified

with subscript e and agents with a 0 productivity are called called "unemployed", identified with subscript u.

The utility of agents is of the GHH type $U(c,l) = \log(c - \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}})$, where c and l are individual consumption and labor supply respectively, and $\varphi > 0$ is the Frisch elasticity of labor supply. We assume that all agents discount period $t+1$ utility at period t with a discount factor β_t, which can be time varying. As is standard, we call a change in β_t a preference shock (Galí, 2015). We only impose $0 < \beta_t < 1$ at each period.

In period 0, the discount factor of period t utility is:

$$\Theta_t = \Pi_{k=0}^{t} \beta_t, \tag{1}$$

which reduces to β^t if βs are constant.

The only friction in this economy is a credit constraint: Agents cannot borrow. We denote as $c_{e,t}, a_{e,t}, c_{u,t}, a_{u,t} \geq 0$ the consumption and saving levels of employed and unemployed agents in period t. We denote as R_t and w_t the gross real post-tax interest rate and wage rate. the two budget constraints of e and u agents can be written as:

$$c_{e,t} + a_{e,t} = R_t a_{u,t-1} + w_t l_{e,t}, \tag{2}$$

$$c_{u,t} + a_{u,t} = R_t a_{e,t-1}. \tag{3}$$

To simplify the exposition we assume that the initial amount of wealth is 0, $a_{e,-1} = a_{u,-1} = 0$. The two Euler equations are:

$$\left(c_{e,t} - \frac{l_{e,t}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{-1} \geq \beta R_{t+1} c_{u,t+1}^{-1}, \tag{4}$$

$$c_{u,t+1}^{-1} \geq \beta R_{t+1} \left(c_{e,t+1} - \frac{l_{e,t+1}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{-1}, \text{ with equality if } a_{u,t} > 0. \tag{5}$$

Because they have a null productivity, unemployed agents do not work: $l_{u,t}$. Due to the GHH utility function, the labor supply of unemployed employed agents is pinned down by the real wage—as aggregate labor supply L_t:

$$L_t = l_{e,t} = w_t^{\varphi}. \tag{6}$$

Government. The government issue a quantity of debt B_t. The financial market equilibrium is:

$$a_{e,t} + a_{u,t} = B_t. \tag{7}$$

The government raises linear capital tax τ_t^K and labor tax τ_t^L to finance interest payment on public debt. If $\tilde{r}_t$ denotes the net pre-tax interest rate, the gross post-tax interest rate is $R_t = 1 + (1 - \tau_t^K)\tilde{r}_t$. Similarly, the post-tax wage rate is $w_t = (1 - \tau_t^L)\tilde{w}_t$. The government budget constraint writes as $(1 + \tilde{r}_t)B_{t-1} \leq B_t + \tau_t^K \tilde{r}_t(a_{e,t-1} + a_{u,t-1}) + \tau_t^L \tilde{w}_t L_t$, which with $\tau_t^L \tilde{w}_t = \tilde{w}_t - w_t = Z_t - w_t$ becomes:

$$R_t B_{t-1} = (Z_t - w_t)L_t + B_t. \tag{8}$$

The Ramsey allocation. For a given sequence of MIT shocks, known at date 0, $\{\Theta_t, Z_t\}_{t\geq 0}$, the Ramsey program consists in the choice of the path of instruments $\{\tau_t^K, \tau_t^L, B_t\}_{t\geq 0}$ that implements a competitive equilibrium maximizing an aggregate welfare criterion, for given initial conditions. The criterion we consider is the standard Utilitarian objective, in which both agents are equally weighted, but where the discount factors are the same as those of agents and thus possibly time-varying. The Ramsey program can be written in post-tax terms as follows:

$$\max_{(c_{e,t},c_{u,t},a_{e,t},a_{u,t},l_{e,t}B_t,A_t,R_t,w_t)} \sum_{t=0}^{\infty} \Theta_t \left(\log \left(c_{e,t} - \frac{l_{e,t}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right) + \log(c_{u,t}) \right) \tag{9}$$

$$c_{e,t}, c_{u,t}, a_{e,t}, a_{u,t}, l_{e,t}, l_{u,t} \geq 0, \tag{10}$$

$$a_{e,-1} = a_{u,-1} = 0, \tag{11}$$

and subject to: the constraints (2)–(8) guaranteeing the optimality of individual choices (budget constraints, Euler equations and labor FOC with GHH utility function, respectively); the resource constraint (8); and the financial market clearing condition (7).

2.1 Two benchmark economies: The representative-agent and no-policy economies

Before characterizing the allocation for different shocks, we present two economies that we will use as benchmarks for our baseline HA economy.

The representative-agent economy. The representative-agent (RA) economy is an economy populated by a unique agent, who is endowed with the same GHH utility function TIn this case, the Ramsey program thus consists in maximizing the intertemporal welfare of this agent subject to the Euler equation and the resource constraint. It turns out that the optimal policy is straightforward to characterize in this case, as the first-best allocation can be achieved. Indeed, the planner simply implements $\tau_t^K = \tau_t^L = B_t = 0$. The labor supply is $L_t^{RA} = Z_t^{\varphi}$, while optimal consumption is $C_t^{RA} = Z_t^{1+\varphi}$.

The no-policy economy. In the no-policy (NP) economy, the planner does not implement any policy in the economy with credit constraints. The laissez-faire policy corresponds to $B_t = \tau_t^L = \tau_t^K = 0$. The allocation is thus straightforward to derive. The wage is equal to the productivity of labor and the employed agent supplies labor such that $l_{e,t} = L_t = Z_t^{\varphi}$. In the absence of storage of value, the employed agent consumes their income: $c_{e,t} = Z_t^{1+\varphi}$, while the unemployed agent consumes nothing: $c_{u,t} = 0$. There is no consumption smoothing and the aggregate consumption, denoted as C_t^{NP} in this economy, verifies $C_t^{NP} = C_t^{RA}$.

As a consequence, these two economies generate the same aggregate consumption path after both supply shocks $(Z_t)_t$ and demand shocks $(\beta_t)_t$.

2.2 The incomplete-market economy with optimal policy

The Ramsey program is expressed in equations (9)–(11). As a preliminary remark, we prove that the planner does not increase public debt up to the level for which credit constraints do not bind for unemployed agents. Indeed, this would imply a too high distorting labor tax. The credit constraint is thus assumed to be binding for unemployed agents. Then, using the labor supply (6), the program expression can be considerably simplified. All technical details can be found in Appendix A, and provide only the main results in this section.

We denote the aggregate consumption in this economy by $C_t^{HA} = c_{u,t} + a_{u,t}$, where HA stands for Heterogeneous-Agent with optimal policy. Our first result, stated in the following proposition, summarizes how the optimal allocations in HA and RA economies differ following discount factor and TFP shocks.

Proposition 1 *For any path of $(Z_t)_t$ and $(\beta_t)_t$, the optimal allocations in the HA and RA economies are connected via the following relationship:*

$$\frac{C_t^{HA}}{C_t^{RA}} = \left(\frac{1}{2} + \varphi\right)^{\varphi} \left(\varphi + \frac{1}{1+\beta_t}\right)^{-\varphi}.$$

Proposition 1, whose proof can be found in Appendix A tells two stories regarding the role of aggregate shocks.

Supply shocks Z_t only. When the economies are hit by TFP shocks only, then discount factors are not time-varying and $\beta_t = \beta$ for all dates. Proposition 1 then tells that the ratio $\frac{C_t^{HA}}{C_t^{RA}}$ is constant. This implies that $\hat{C}_t^{HA} = \hat{C}_t^{RA}$, where we denote by $\hat{x}_t$ the proportional deviation of variable x_t from its steady state value. Although RA and HA economies have different steady states, their aggregate response to TFP shocks is identical.

Demand shock β_t only. When the economies are hit by demand shocks, the ratio $\frac{C_t^{HA}}{C_t^{RA}}$ is time-varying.[1] RA and HA economies then feature heterogeneous aggregate responses since $\hat{C}_t^{HA} \neq \hat{C}_t^{RA}$.

As a consequence, only demand shocks generate a difference in the aggregate response of HA and RA economies. However, for both supply and demand shocks, the paths of the instruments differ between RA and HA economies. Indeed, in the RA economy, setting $B_t = \tau_t^L = 0$ implements the first-best allocation in each period, and the path of all instruments is moving for both demand and supply shocks. The difference in the allocation response between HA and RA economies is our object of interest as it captures the difference between HA and RA for the two types of shocks.

2.3 Interpreting the results: Two statistics

We introduce two statistics to understand the differences between optimal allocations in RA and HA economies.

The Marginal Value of the Credit Constraints (MVCC). We define as ν_t the Lagrange multiplier of the credit constraint of u-agent. It is equal to $\nu_t := U_c(c_{u,t}, 0) - \beta R_{t+1} U_c(c_{e,t+1}, l_{e,t+1})$. In words, it is equal to the gap between the current and future discounted marginal utilities of agents u and therefore reflects how slack the Euler equation (5) of unemployed agents is. We then define the Marginal Value of Credit Constraints (MVCC) as the Lagrange multiplier ν_t normalized by the current marginal utility of agents u:

$$MVCC_t := \frac{\nu_t}{U_c(c_{u,t}, 0)}.$$

This unitless quantity has n simple microeconomic interpretation: It is the maximal fee that agents u would accept to pay to be able to borrow one extra unit of resources. If credit constraints do not bind, the MVCC will amount to zero as saving is optimal by construction (and hence no extra saving is needed). Using the definition of ν_t, the Euler equation of unemployed agents can be written as:

$$U_c(c_{u,t}, 0) = \beta_t \frac{R_{t+1}}{1 - MVCC_t} U_c(c_{e,t+1}, l_{e,t+1}),$$

which states that $\frac{R_{t+1}}{1-MVCC_t}$ is the gross interest rate for which zero savings is optimal for unemployed agents. When the $MVCC$ is constant, then the interest rate R_{t+1} is a sufficient statistics for the growth rate of the marginal utility of unemployed agents—and hence of the growth rate of their consumption. The same holds for employed agents, as their Euler equation holds with equality. This means that, when the MVCC is constant, the consumption dynamics

[1] Following the literature, we call discount factor shocks *demand shocks*, as they do not affect the production frontier.

of both employed and unemployed agents only depend on the real interest rate, and their consumption growth rates is the same and thus the same as in the RA economy. Therefore, only the dynamic properties of the MVCC – and not its steady-satte value – matter to determine to which extent the dynamic responses of RA and HA differ after an aggregate shocks.

The MVCC can be explicitly computed in this economy. As shown in Appendix A.2, its expression is:

$$MVCC_t = 1 - \frac{(1+\varphi(1+\beta_{t+1}))\,(1+\varphi(1+\beta_t))}{(1+2\varphi)^2},$$

which depends on the discount factor β_t but not on the TFP Z_t. In particular, the MVCC is constant when β is constant, regardless of the TFP shock. Therefore, this explains why the dynamics of aggregate consumption in RA and HA economies is the same after a TFP shock, but differ after a discount factor shock.

The Discount Factor Wedge (DFW). Following Nakajima (2005), Werning (2015), Acharya and Dogra (2021) and Berger et al. (2023), it is known that the aggregate allocation of HA models can be seen as the outcome of a RA model, in which some proper wedges have been introduced. In particular, the *discount factor wedge* (DFW) is the wedge such that the allocation and interest rate in the HA economies are an equilibrium outcome of an RA economy, in which the discount factor is multiplied by this wedge—while other elements are the same in the HA and RA economies. The DFW thus allows to reproduce the allocation and prices of the HA economy as an equilibrium outcome of a slightly modified RA economy, and also to quantify the distortions in the saving incentives due to market incompleteness. More formally, the DWF, denoted $\beta_t^{wedge} - 1$, is defined such that the Euler equation of a representative agent, endowed with the allocation of the HA economy and a transformed discount factor, holds with equality. Formally:

$$\left(C_t^{HA} - \frac{l_{HA,t}^{1+1/\varphi}}{1+1/\varphi}\right)^{-1} = \beta_t^{wedge}\beta_t R_{t+1}^{HA}\left(C_{t+1}^{HA} - \frac{l_{HA,t+1}^{1+1/\varphi}}{1+1/\varphi}\right)^{-1}$$

After some algebra, derived in Appendix A.3, we obtain the following expression:

$$DFW_t = \beta_t^{wedge} - 1 = \frac{(1+1/\varphi)\,(1+\varphi\,(1+\beta_{t+1})) - (1/2+\varphi)\,(1+\beta_{t+1})}{(1+1/\varphi)\,(1+\varphi\,(1+\beta_t)) - (1/2+\varphi)\,(1+\beta_t)} \times \frac{1+2\varphi}{1+\varphi\,(1+\beta_{t+1})} - 1.$$

The wedge DFW_t concerns the aggregate consumption of all agents (constrained and unconstrained). However, as the Euler equation of unconstrained agents holds with equality, movements of the DFW is mainly driven by unconstrained agents. In the quantitative model below, we report both the MVCC and the DFW and we check that they behave the same way and are observationally equiavlent. If the steady-state $MVCC$ is close to 0, one can prove that the deviations of DFW_t and of $MVCC_t$ are proportional, $\widehat{DFW}_t \propto \widehat{MVCC}_t$. As a consequence, both statistics provide the same information. The advantage of the MVCC is to provide a

micro-foundation for this information. The connection to the underlying constraint si also straightforward.

In the more general heterogeneous-agent model studied below, the MVCC will be different from 0 only for credit-constrained agents. In this case, the average value of the MVCC will behave as the discount factor wedge as we show in Section A.2.

Sufficient statistics and optimal policies. The simple model provides three results, which will help understanding the general model. First, it shows that some statistics can be used to understand to which extent the dynamics of HA and RA allocations differ. This statistic, that we call MVCC, captures the intensity of the credit constraint, which is the key imperfection for HA and RA to differ. Second, the optimal allocation of the HA economy can be shown to have the same dynamic properties as the one in the RA economy. This is the case, when the optimal instrument path is able to undo the effect of the shocks on the credit constraint, such that the MVCC stays constant. Third, the MVCC (or the DFW) are not sufficient statistics to assess the optimality of the observed policy. For instance, implementing a constant MVCC after a discount factor shock would be suboptimal, whereas this is the case for supply shocks.

3 The general model

In the general model, we relax many of the simplifying assumptions of the previous Section, and we introduce both sticky prices and sticky wages, together with a rich fiscal structure.

We consider a discrete-time economy populated by a continuum of size one of ex-ante identical agents. These agents are assumed to be distributed along a set J, with the non-atomic measure ℓ: $\ell(J) = 1$.[2]

3.1 Risk

Aggregate risks. Agents face an aggregate shock $(S_t)_t$. The shock is persistent but known at date 0 and should hence be considered as MIT shock. The aggregate shock can affect the economy through different channels: the TFP, Z_t, or the agents' discount factors, β_t, the government public spending, G_t, or the individual productivity levels, y_{it}. In the main text, we will mostly focus on two channels: the TFP and the discount factors, as in the theoretical section. Indeed, these two cases are sufficient to contrast Ramsey policies in HA and RA economies. We will report other cases in the appendix.

Idiosyncratic risk. We assume that the agents face an idiosyncratic productivity risk. The productivity process, denoted y, is assumed to follow a first-order Markov chain with transition

[2]We follow Green (1994) and assume that the law of large numbers holds.

matrix $\boldsymbol{\pi} = (\pi_{yy'})_{y,y'}$ and to take value in a finite set $\mathcal{Y}_t$, which possibly depends on the aggregate shock S_t. With wage w and labor supply l, an agent with productivity y earns the pre-tax labor income wyl. In each period, the fraction of agents with productivity y is constant and denoted by n_y. We normalize average productivity to 1, i.e., such that $\sum_y n_y y = 1$. The history of idiosyncratic productivity shocks up to date t for an agent i is denoted by $y_i^t = \{y_{i,0}, \ldots, y_{i,t}\}$, where $y_{i,\tau}$ is the date-τ productivity. The measure of idiosyncratic histories up-to-date t, denoted by θ_t, can be computed using the initial distribution and the transition matrix.

3.2 Preferences

Households are expected-utility maximizers endowed with time-separable preferences and a possibly time-varying discount factors. We will denote by $\beta_{t-1} \in (0,1)$ the discount factor between dates $t-1$ and t and by $\Theta_t = \Pi_{s=0}^{t-1}\beta_s$ the compounded discount factor between dates 0 and t. In each period, households enjoy utility $U(c,l)$ from the consumption c of the unique consumption good of the economy and suffer from the disutility of providing the labor supply l. We further assume that in each period, the instantaneous utility is separable in consumption and labor: $U(c,l) = u(c) - v(l)$, where $u, v : \mathbb{R}_+ \to \mathbb{R}$ are twice continuously differentiable and increasing. Furthermore, u is concave, with $u'(0) = \infty$, and v is convex.

3.3 Labor taxes

For the sake of generality, and for theoretical reasons which we develop in Section 3.8 below, we introduce a rich set of four linear taxes. We here present the two labor taxes, and introduce income and capital tax below.

First, we assume that unions bargain over the nominal wage rate, denoted by $\hat{W}_t$. Workers pay a linear labor tax τ_t^L on this income such that their post-tax nominal wage is $(1-\tau_t^L)\hat{W}_t$. Second, firms pay an additional labor tax, τ_t^S, which implies a wedge between the labor cost per efficient unit of labor, $\tilde{W}_t$, paid by firms and the wage $\hat{W}_t$ bargained by workers. This additional tax can be thought of as an employer social contribution that does not appear on the payroll of workers. Formally, the labor cost $\tilde{W}_t$, the bargained wage $\hat{W}_t$ and the tax τ^S verify the following relationship: $\hat{W}_t = (1-\tau_t^S)\tilde{W}_t$. The tax τ_t^S will have an effect on labor demand that will be internalized by unions in their bargaining strategy. Similarly, the tax τ_t^L will have an effect on labor income that will also be internalized. The difference between the two taxes is that τ_t^S has a direct effect on employment for a given bargained wage $\hat{W}_t$ but not on the wage W_t, whereas τ_t^L has a direct effect on the wage W_t for a given bargained wage $\hat{W}_t$, but no direct effect on employment.[3]

[3]We call direct effect the partial equilibrium effect of each variable. In general equilibrium (with endogenous income), these taxes obviously affect all variables through price variations.

3.4 Production

The specification of the production sector follows the New-Keynesian literature on price stickiness, adapted to the previous tax structure. The consumption good Y_t is produced by a unique profit-maximizing representative firm that combines intermediate goods $(y^f_{j,t})_j$ from different sectors indexed by $j \in [0,1]$ using a standard Dixit-Stiglitz aggregator with an elasticity of substitution, denoted ε_P:

$$Y_t = \left[\int_0^1 y^f_{j,t}{}^{\frac{\varepsilon_P - 1}{\varepsilon_P}} dj\right]^{\frac{\varepsilon_P}{\varepsilon_P - 1}}.$$

For any intermediate good $j \in [0,1]$, the production $y^f_{j,t}$ is realized by a monopolistic firm and sold at price $p_{j,t}$. Z_t is aggregate labor productivity. It is affected at period 0 by a shock ϵ^Z_0 and it follows a AR(1) process. $Z_t = e^{z_t}$,with

$$z_0 = 1 + \epsilon^Z_0 \text{ and } z_t = \rho^Z z_{t-1} \text{ for } t \geq 1, \rho^Z < 1.$$

Intermediate firms face a quadratic price adjustment cost à la Rotemberg when setting their price. Following the literature, the price adjustment cost is proportional to the magnitude of the firm's relative price change and equal to $\frac{\psi_p}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - 1\right)^2$. Denoting the price inflation rate as $\pi^P_t = \frac{P_t}{P_{t-1}} - 1$ where P_t is the implied price index, we obtain the standard equation characterizing the Phillips curve after standard derivation relegated in Appendix

$$\pi^P_t(1 + \pi^P_t) = \frac{\varepsilon_P - 1}{\psi_P}(m_t - 1) + \beta_t \mathbb{E}_t\left[\pi^P_{t+1}(1 + \pi^P_{t+1})\frac{Y_{t+1}}{Y_t}\right], \tag{12}$$

where $Y_t = Z_t L_t$.

3.5 Labor market: Labor supply and Union wage decision

Following the New Keynesian sticky-wage literature, labor hours are supplied monopolistically by unions (Erceg et al. (2000)Chugh (2006) Hagedorn et al. (2019) or Auclert et al., 2022 among others). There is a continuum of unions of size 1 indexed by k and each union k supplies L_{kt} hours of labor at date t with nominal wage $\hat{W}_{kt}$. Each union k sets its wage $\hat{W}_{kt}$ so as to maximize the intertemporal welfare of its members subject internalizing the demand by firms. We assume the presence of quadratic utility costs related to the adjustment of the nominal wage and equal to $\frac{\psi_W}{2}(\hat{W}_{kt}/\hat{W}_{kt-1} - 1)^2 dk$. The objective of union k is thus:

$$\max_{(\hat{W}_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \Theta_s \int_i \left(U(c_{i,s}, l_{i,s}) - \frac{\psi_W}{2}\left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1\right)^2\right)\ell(di),$$

This maximization will provide the New-Keynesian wage-Phillips curve:

$$\pi_t^W(\pi_t^W+1)=\frac{\varepsilon_W}{\psi_W}\underbrace{\left(v'(L_t)-\frac{\varepsilon_W-1}{\varepsilon_W}(1-\tau_t^L)\hat{w}_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)}_{\text{labor gap}}L_t+\beta_t\mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W+1)\right], \tag{13}$$

where $\pi_t^W=\frac{\hat{W}_t-\hat{W}_{t-1}}{\hat{W}_{t-1}}$ is the wage inflation rate of the wage bargained by unions, and $\hat{w}_t=\hat{W}_t/P_t$ is the real pre-tax wage, as $\hat{W}_t$ is the bargained nominal wage.

3.6 Assets

The only asset is nominal public debt, whose supply size is denoted by B_t at date t, and which pays off the pre-determined before-tax nominal interest rate i_{t-1}. Public debt is issued by the government and assumed to be default free. The financial market clearing implies that the net total savings of households, denoted A_t, must equal public debt:

$$A_t=B_t. \tag{14}$$

The corresponding real before-tax (net) interest rate for public debt, denoted by $\tilde{r}_t$, is defined by:

$$\tilde{r}_t=\frac{1+i_{t-1}}{1+\pi_t^P}-1. \tag{15}$$

3.7 Agents' program

Each agent enters the economy with an initial endowment of public debt $a_{i,-1}$ and an initial productivity level $y_{i,0}$. The joint initial distribution over public debt and productivity levels is denoted Λ_0. In later periods, each agent learns her productivity level $y_{i,t}$, supplies labor, and earns her savings payoffs. Since the labor supply L_t is chosen by unions, the labor income is $(1-\tau_t^L)\hat{w}_t y_{i,t}L_t$. The before-tax real financial payoff amounts to $\tilde{r}_t a_{i,t-1}$.

We assume that agents pay two other taxes. First, a capital tax $\hat{\tau}_t^K$ is levied on interest payment and implies a net asset payoff $(1-\tau_t^K)\tilde{r}_t a_{i,t-1}$. Second, an income tax τ_t^E is levied on total labor income, $\tau_t^E(1-\tau_t^L)\hat{w}_t y_{i,t}L_t$. The latter income tax τ_t^E is not internalized by the unions as each union as a marginal contribution to total income. As a consequence, the post-tax total income is equal to $(1-\tau_t^E)(1-\tau_t^L)\hat{w}_t y_{i,t}L_t$.

Agents earn this net total income and use it together with their past savings to consume $c_{i,t}$ and save $a_{i,t}$. Their budget constraint can be expressed as follows:

$$c_{i,t}+a_{i,t}=a_{i,t-1}+(1-\tau_t^K)\tilde{r}_t a_{i,t-1}+(1-\tau_t^E)\big((1-\tau_t^L)\hat{w}_t y_{i,t}L_t\big). \tag{16}$$

To simplify the previous notation, we define the post-tax real interest and wage rates as:

$$r_t = (1 - \tau_t^K)\tilde{r}_t, \tag{17}$$

$$w_t = (1 - \tau_t^E)(1 - \tau_t^L)\hat{w}_t = (1 - \tau_t^E)(1 - \tau_t^L)(1 - \tau_t^S)\tilde{w}_t. \tag{18}$$

As $\hat{W}_t/P_t = w_t/(1 - \tau_t^E)(1 - \tau_t^L)$, we have the law of motion of the post-tax real wage as a function of inflation and taxes

$$(1 + \pi_t^W)\frac{w_{t-1}}{(1 - \tau_{t-1}^L)(1 - \tau_{t-1}^E)} = \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^E)}(1 + \pi_t^P). \tag{19}$$

The agent's program can be finally be written as:

$$\max_{\{c_{i,t},a_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Theta_t \left(U(c_{i,t}, L_t)\right), \tag{20}$$

$$c_{i,t} + a_{i,t} = (1 + r_t)a_{i,t-1} + w_t y_{i,t} L_t, a_{i,t}, \tag{21}$$

and subject to the credit constraint $a_{i,t} \geq -\underline{a}$, and the consumption positivity constraint $c_{i,t} > 0$. The notation $\mathbb{E}_0$ is an expectation operator over both idiosyncratic and aggregate risks. The solution of the agent's program is a sequence of functions, defined over $([-\bar{a}; +\infty) \times \mathcal{Y}) \times \mathcal{Y}^t \times \mathbb{R}^t$ and denoted by $(c_t, a_t)_{t\geq 0}$, such that:[4]

$$c_{i,t} = c_t((a_{i,-1}, y_{i,0}), y_i^t, z^t),\ a_{i,t} = a_t((a_{i,-1}, y_{i,0}), y_i^t, z^t). \tag{22}$$

For the sake of simplicity, we will keep using the notation with the i-index. Denoting by $\nu_{i,t}$ the discounted Lagrange multipliers of the credit constraint, the Euler equation corresponding to the agent's program (20) is:

$$u'(c_{i,t}) = \beta_t \mathbb{E}_t \Big[(1 + r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t}. \tag{23}$$

with the complementary slackness condition:

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0,\ \nu_{i,t} \geq 0 \tag{24}$$

The MVCC

As in Section 2, one can easily introduce the MVCC. For agents i, it is $MVCC_{i,t} = \nu_{i,t}/u'(c_{i,t})$. Such that, for all agents:

$$u'(c_{i,t}) = \beta_t \mathbb{E}_t \Big[\frac{1 + r_{t+1}}{1 - MVCC_{i,t}} u'(c_{i,t+1})\Big].$$

[4]See e.g. Miao (2006), Cheridito and Sagredo (2016), and Açikgöz (2018) for a proof of the existence of such functions.

In the HA economy, there is a distribution of $MVCC$. When agents are not constrained $MVCC_{i,t} = 0$, and $0 < MVCC_{i,t} \leq 1$ otherwise. In quantitative Section, we will comment on the average $MVCC$ over agents.

3.8 Government and market clearing conditions

The government has to finance an exogenous public good expenditure G_t, by raising a quite large number of taxes and by issuing one-period riskless public debt. The government raises four linear taxes: (i) a tax τ_t^S based on labor cost $\tilde{w}_t$ and paid by employers, (ii) a tax τ_t^L based on bargained wage $\hat{w}_t$ and paid by workers, and finally (iii) a tax τ_t^E based on total income and paid by workers (iv) a capital tax τ_t^K. Importantly, the three labor instruments (τ_t^L, τ_t^S and τ_t^E) are independent and not redundant, on the one hand, τ_t^S creates a wedge between the labor cost and the bargained wage, while τ_t^L and τ_t^E create wedges between the bargained wedge and the net wage. On the other hand, τ_t^L is internalized by unions, while τ_t^E is not. These three taxes will play on different margins and will allow us derive our equivalence result below. Hence, they should be understood as theoretical tools needed to generate price and wage stability. Each tax will be removed in turn to consider more realistic fiscal settings and to assess how each fiscal instrument contributes to inflation volatility. In addition to capital and labor taxes and to public debt, the government also fully taxes the firms' profits, Ω_t, which limits the distortions implied by profit distribution.

We can now express the government budget constraint, as follows:

$$G_t + \frac{1+i_{t-1}}{1+\pi_t^P} B_{t-1} \leq \Omega_t + B_t + \tau_t^E (1-\tau_t^L) \hat{w}_t L_t + \tau_t^K \tilde{r}_t \int_i a_{i,t-1} \ell(di) + \tau_t^L \hat{w}_t L_t + \tau_t^S \tilde{w}_t L_t.$$

We can simplify the previous government budget constraint using the financial market clearing (14), the post-tax interest rate $\tilde{r}_t$ (15) and the post-tax rate definitions (17) as:

$$G_t + r_t B_{t-1} + w_t L_t \leq \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Y_t + B_t - B_{t-1}, \tag{25}$$

We finally express the financial market clearing condition and the economy resource constraints:

$$\int_i a_{i,t} \ell(di) = B_t, \tag{26}$$

$$\int_i c_{i,t} \ell(di) + G_t = \left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right) Z_t L_t. \tag{27}$$

Equilibrium definition. We can finally formulate our definition of competitive equilibrium.

Definition 1 (Sequential equilibrium) *For any exogenous paths of aggregate shocks (S_t), characterizing TFP $(Z_t)_t$, public spending $(G_t)_t$, discount factors $(\beta_t)_t$, and productivity levels,*

$(\mathcal{Y}_t)_t$, *a sequential competitive equilibrium is a collection of individual allocations* $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$, *of aggregate quantities* $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, *of price processes* $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$, *of monetary policy* $(i_t)_{t\geq 0}$, *fiscal policies* $(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$, *and inflation dynamics* $(\pi_t^W, \pi_t^P)_{t\geq 0}$ *such that, for an initial wealth and productivity distribution* $(a_{i,-1}, y_{i,0})_{i\in\mathcal{I}}$, *and for an initial value of public debt verifying* $B_{-1} = \int_i a_{i,-1}\ell(di)$, *we have:*

1. *given prices, the allocations* $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$ *solve the agent's optimization program (20)–(21);*
2. *financial, and goods markets clear at all dates: for any* $t \geq 0$, *equations (26) and (27) hold;*
3. *the government budget is balanced at all dates: equation (25) holds for all* $t \geq 0$*;*
4. *firms' profits* Ω_t *and the mark-up* m_t *are consistent with firms profit maximization.*
5. *the price inflation path* $(\pi_t^P)_{t\geq 0}$ *is consistent with the price Phillips curve (12), while the wage inflation path* $(\pi_t^W)_{t\geq 0}$ *is consistent with the wage Phillips curve (13);*
6. *the real and nominal rates* $(\tilde{r}_t, i_t)_{t\geq 0}$ *verify (15);*
7. *post tax rates* $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$ *are defined in equations (17)–(18).*

Social Welfare Function. Following LeGrand et al. (2022), we assume that the planner maximizes a generalized social welfare function, where the weights on each period utility can depend on the current productivity of the agent. The objective of the planner is thus:

$$W_0 = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\Theta_t\int_i \omega(y_t^i)\left(U(c_t^i, l_t^i)\right)\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right]. \tag{28}$$

This expression embeds the utilitarian case, where $\omega(y) = 1$ for all y. This generalization of the standard Social Welfare Function is now used either in quantitative work, such as LeGrand et al. (2022) and McKay and Wolf (2022), or in more theoretical investigations, as a deviation from the utilitarian case (see Dávila and Schaab, 2022). It will be used to ease the simulations and comparisons of economies in Section 5.

We assume that the economy starts from the steady-state situation where the fiscal system is optimally determined. Then in period 0, the economy is hit either by a demand shock or a supply shock. The whole paths of these shocks is known at period 0, and the planner sets optimally its available instruments under commitment.

We have introduced five fiscal instruments $(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$. This rich fiscal system is also a theoretical device to understand distortions in the HA economy with both price and wage stickiness. As will be clear below, this fiscal system is the minimal one such that there is no deviation from price stability in all cases. In what follows, we consider different fiscal systems,

where only some fiscal instruments can be optimally time-varying to smooth the effect of the shock, but not necessarily all of them. More precisely, we solve for optimal monetary policy considering a set $\mathcal{I} \subset \{\tau^L, \tau^S, \tau^E, \tau^K\}$ of available fiscal instruments. We will consider that public debt is always optimally set, which seems theoretically and empirically relevant in this environment. The set $\mathcal{I}$ is set once for all and does not change across periods. With a slight abuse of notation, we denote by $(\mathcal{I}_t)_{t\geq 0}$ the path of these available fiscal instruments. For all other instruments $I \in \{\tau^L, \tau^S, \tau^E, \tau^K, B\} \setminus \mathcal{I}$, we assume that the instrument is constant and set to its steady-state value: $I_t = I_{ss}$ at all dates.

We can define the notion of Ramsey equilibrium using this notion of social welfare function.

Definition 2 (Ramsey equilibrium) *For a given path of aggregate shocks $(S_t)_{t\geq 0}$, a Ramsey equilibrium with all instruments is the path of monetary policy $(i_t)_{t\geq 0}$, fiscal instruments $(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$, which selects a sequential equilibrium defined in 1, which is maximizing the social welfare function (28).*

A steady-state Ramsey equilibrium is a Ramsey equilibrium where aggregate real variables $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, prices $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$, monetary policy $(i_t)_{t\geq 0}$, fiscal policies $(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$, and inflation dynamics $(\pi_t^W, \pi_t^P)_{t\geq 0}$ are constant.

Definition 3 (Ramsey steady state) *A steady-state Ramsey equilibrium is a Ramsey equilibrium where aggregate real variables $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, prices $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$, monetary policy $(i_t)_{t\geq 0}$, fiscal policies $(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$, and inflation dynamics $(\pi_t^W, \pi_t^P)_{t\geq 0}$ are constant. The value of the instruments are denoted as $(\tau_{ss}^L, \tau_{ss}^S, \tau_{ss}^E, \tau_{ss}^K, B_{ss})$.*

Finally, we can define our optimal policy with a limited set of instruments.

Definition 4 (Ramsey equilibrium with limited instruments) *For a given path of aggregate shocks $(S_t)_{t\geq 0}$ and a given set of available instruments $\mathcal{I} \subset \{\tau^L, \tau^S, \tau^E, \tau^K\}$, a Ramsey equilibrium with a limited number of instruments is the path of monetary policy $(i_t)_{t\geq 0}$, a path of public debt $(B_t)_{t\geq 0}$, a path of fiscal instruments $(\mathcal{I}_t)_{t\geq 0}$, which selects a competitive equilibrium, which is maximizing the social welfare function (28) given that the unavailable instruments are set to their steady-state values: $I_t = I_{ss}$ for all $I \in \{\tau^L, \tau^S, \tau^E, \tau^K\} \setminus \mathcal{I}$.*

We first solve the Ramsey model without aggregate shock to compute the steady-state values of instruments, $(\tau_{ss}^L, \tau_{ss}^S, \tau_{ss}^E, \tau_{ss}^K)$, and then we solve for the optimal dynamics of the available instruments. For the simulation of the dynamics for a given set of available instruments $\mathcal{I}$, observe that non-available isntruments are set to their steady-state value. Therefore, no matter the choice of $\mathcal{I}$, the Ramsey equilibrium will feature the same steady-state allocation.

3.9 The complete-market economy

The complete market economy is very standard. A representative agent maximizes utility in an economy without financial constraints. The planner has distorting tools and public debt to finance the public spending. The provide the algebra in Appendix C.

4 Optimal policies with heterogeneous agents

4.1 Characterizing the Ramsey allocation

We now consider heterogeneous-agent economies and derive optimal policies for both supply and demand shocks, for a given set of available fiscal instruments $\mathcal{I}$. The Ramsey planner's program is

$$\max_{\left(\mathcal{I}_t,w_t,r_t,L_t,B_t,\pi_t^P,\pi_t^W,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Theta_t \left[\int_i \omega(y_t^i)\left(u(c_t^i)-v(L)\right)\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right], \quad (29)$$

subject to the budget of the State (25), the individual budget constraints (21), the individual Euler equation (23) and the slackness condition (24), the price Phillips curve (12), the Wage Phillips curve (13), the real wage dynamics (19), positivity constraints $c_t^i, l_t^i \geq 0$, given initial conditions a_{-1}^i, and finally the constraint on the instruments $I_t = I_{ss}$ for all unavailable instruments $I \notin \mathcal{I}$. We provide the full program in Appendix D, where we also derive the first-order conditions of the planner. As in LeGrand and Ragot (2025), we use some aspect of Marcet and Marimon (2019) to factorize the Lagrangian. On a technical note, the factorization of the price and wage Phillips curve is easy, as they can both be viewed as Euler equations, for firms and unions, respectively.

This economy faces different frictions, which are worth summarizing. The monetary economy features two sets of market imperfections. The first set is related to the goods market. Intermediary firms enjoy a monopoly power, which implies a price markup m_t that can differ from one. There is also a Rotemberg cost for price adjustment, which prevents firms from freely setting their price. Note that the good market imperfections are complementary: one vanishes when the other is absent, as can be seen from the price Phillips curve (12). The second set of imperfections is related to the labor market. The union implies that the labor supply of agents is not set optimally, while the Rotemberg cost for wages prevents unions from freely setting wages. Note that in the absence of Rotemberg cost, the labor supply still remains sub-optimal, as it remains set at the union level and is characterized by: $v'(L_t) = w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)$. The labor supply would be common to all agents, while it would be individual-specific and equal to $v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t})$, if agents were able to choose their own labor supply $l_{i,t}$. This sub-optimal common labor choice will play a major role in our equivalence results below.

To get further insights about the optimal allocation in the HA economy, it is useful to introduce the notion of *social valuation of liquidity (SVL) for agent i,* which is the value for

the planner to transfer one extra unit of consumption good to agent i in period t. In LeGrand and Ragot (2025), we show that this statistics simplifies the derivation of the first order of the planner, and that it is the same statistics as the *Generalized Social Marginal Welfare Weights* (GSMWW) introduced by Saez and Stantcheva (2016).

More precisely, we denote by $\Theta_t \lambda_{i,t}$ the Lagrange multipliers of the Euler equations (23 of agent i at date t. The Lagrange multiplier of the government budget constraint is $\Theta_t \mu_t$ (25). The Lagrange multipliers on the price and wage Phillips curves (12) and (13) are denoted $\Theta_t \gamma_{P,t}$ and $\Theta_t \gamma_{W,t}$, respectively. We can then express the Lagrangian of the program, that we denote by $\mathcal{L}$. From this Lagrangian, we can define the (SVL) $\psi_{i,t}$ as:

$$\psi_{i,t} := \frac{\partial \mathcal{L}}{\partial c_{i,t}},$$

which is the value for the planner to transfer one extra unit of consumption good to agent i in period t. The expression of $\psi_{i,t}$ can also be written as:

$$\psi_{i,t} := \underbrace{\omega_t^i u'(c_{i,t})}_{\text{direct effet}} - \underbrace{\left(\lambda_{i,t} - (1+r_t)\lambda_{i,t-1}\right) u''(c_{i,t})}_{\text{effect on savings}} - \underbrace{\frac{\varepsilon_W - 1}{\psi_W} \gamma_{W,t} \frac{w_t y_{i,t} L_t}{1-\tau_t^E} u''(c_{i,t})}_{\text{effect on the bargained wage}} \tag{30}$$

As can be seen in equation (30), this valuation consists of three terms. The first is the marginal utility of consumption $\omega_t^i u'(c_{i,t})$, which is the private valuation of liquidity for agent i multiplied by the current weight of agent i. The second term in (30) takes into consideration the impact of the extra consumption unit on saving incentives from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between periods t and $t+1$, and thus makes her Euler equation (either nominal or real) more "binding". This more "binding" constraint reduces the utility by the algebraic quantity $u''(c_{i,t})\lambda_{i,t}$. The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ and t and therefore "relaxes" the constraint of date $t-1$. This is reflected in the quantity $\lambda_{i,t-1}$. The third term is the effect of a transfer to households on the marginal incentives to bargain the wage by the union and is hence proportional to the Lagrange multiplier γ_W on the wage Phillips curve.

This expression is common to all HA economies we consider, whatever the set of tools. The expressions of the first-order conditions of the planner will obviously depend on the set of tools. The provide all the derivations in Appendix D.

The corresponding complete-market economy. We solve the same problem with a representative agent instead of the heterogeneous-agent structure. We provide the problem and the first-order conditions in Appendix C, to save some space, as the techniques to solve the model are more standard (although the problem as not been solved with a rich fiscal structure, to the best of our knowledge).

The MVPF. In both RA and HA economies, the MVPF is the Lagrange multiplier on the budget of the State. Although the steady-state value of the Lagrange multiplier will be the same in all HA economy (as the steady states are the same), its dynamics will depend on the state of tools S. As mentioned above, the $MVPF$ is now a very complex function of the allocation.

4.2 The equivalence result

We can state our main equivalence result.

Proposition 2 (An equivalence result) *In the HA economy, when all instruments* $(\tau^E, \tau^S, \tau^L, \tau^K)$ *are optimally chosen, the planner exactly implements* $\pi_t^P = 0$ *and* $\pi_t^W = 0$*, for every path of aggregate shocks* $(S_t)_{t\geq 0}$.

Proposition 2 generalizes the equivalence result of Correia et al. (2008) and Correia et al. (2013) for representative agent economies and LeGrand et al. (2022) for heterogeneous-agent economy to the case where there are both sticky prices and sticky wages. Interestingly, compared to LeGrand et al. (2022), we need two additional instruments (τ_t^E, τ_t^S), whereas we introduce one additional nominal constraint. Indeed, we need one instrument to prevent wage inflation (which destroys resources) and another one to reproduce the flexible price labor supply and neutralize the market power of unions. In the presence of a sufficiently large fiscal system, monetary policy has no role but price stability. This holds for each possible path of aggregate shocks, regardless of their impacted: TFP, public spending, discount factors, or productivity levels. Importantly, the result requires the presence of two labor taxes. The first labor tax τ^S (internalized by the planner) enables the planner to "isolate" the pre-tax rate $\tilde{w}_t$ that is determined by the allocation (with a zero price inflation) from the union wage $\hat{w}_t$ that is determined by the inflation path $(\pi_t^W)_t$. Removing τ^S as an independent instrument imposes a constraint between the factor price $\tilde{w}_t$ and the wage inflation path. In other words, the planner would have to balance the effects of price inflation (determining $\tilde{w}_t$) and of wage inflation (determining $\hat{w}_t$). The second labor tax τ_t^E enables the planner to simultaneously set the labor supply optimally and close the wage gap in the wage Phillips curve. Removing τ_t^E would imply that the planner would need to tradeoff two inefficiencies: (i) the sub-optimal labor supply due the market power of unions and (ii) the cost of wage inflation. Should one of these two instruments be removed, Proposition 2 would not hold anymore and the economy would feature positive inflation on wages or on prices.

Overall, the first item of Proposition 2 rationalizes our tax system, which is the minimal tax system for which price stability is optimal.[5]

To assess the deviation of the allocations and the quantitative deviation from price stability, we now provide a quantitative investigation of economies in which we vary the set of available

[5]More precisely, other tax systems could correspond to price and wage stability. For instance, it could be possible to consider time-varying consumption tax as in Correia et al. (2008). However, the number of independent instruments would not be smaller. We consider our tax system to be not unrealistic.

fiscal instruments.

5 Quantitative analysis of optimal policies

We now focus on two pairs of economies to derive the main lessons for the differences between the HA and the RA allocations on the one hand, and for the deviation form price and wage stability on the other hand. In each case, the economy differs according to the set of taxes that are available to the planner (others are kept constant and set to their steady-state value). As we consider four taxes $(\tau^E, \tau^S, \tau^L, \tau^K)$, we can thus analyze 15 economies (since we need at least one tax to adjust). In addition, we can compare optimal policy in each economy after 2 types of aggregate shocks: TFP and discount factor shocks. We can thus analyze 60 economies when we count both RA and HA environments. We focus in the main text on this selection of four economies to highlight the main mechanisms and results. We relegate to the appendix the other cases that we do not discuss here.

Each pair of economies we consider contrasts the implications of a TFP shock to those of a discount factor shock. This is the consistent extension of our theoretical investigation of Section 2. These two shocks indeed offer very different implications for optimal policy. We consider first the economies when all fiscal instruments are available: $\mathcal{I}^{(1)} = (\tau^K, \tau^L, \tau^S, \tau^E)$ and then a case when only the capital tax τ^K and the employer social contribution τ^S are available: $\mathcal{I}^{(2)} = (\tau^K, \tau^S)$. We have made the choice to keep the capital tax τ^K, as the capital tax has a role in period 0 only because the nominal interest rate is pre-determined. To avoid possible confusing results, we thus chose not to remove it.

In order to provide these simulations we first calibrate the model in Section 5.1. We explain how to compute optimal policies in HA economies in Section 5.2.

5.1 The calibration and steady-state distribution

The period is a quarter.

Aggregate shock. The aggregate shock (S_t) follows an AR(1) process, such that $S_t = \rho S_{t-1}$, where we set $\rho = 0.95$ to consider the same persistence for all channels of the aggregate shock. We also set $S_0 = 1$ but will consider other initial adjustments that will be channel-specific.

Technology and TFP shock. The production function is: $Y = ZL$. The TFP process is a standard AR(1) process and verifies $Z_t = \exp(z_0 S_t)$, where $z_t < 0$ is the period 0 negative TFP shock.

Preferences. The steady-state discount factor is $\beta = 0.99$, and the period utility function is: $\frac{c^{1-\sigma}-1}{1-\sigma} - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$. The Frisch elasticity of labor supply is set to $\varphi = 0.5$, which is the

value recommended by Chetty et al. (2011) for the intensive margin in HA models. The scaling parameter is $\chi = 0.01$, which implies an aggregate labor supply of roughly 1/3.

The process for β_t verifies $\beta_t = \beta \times \exp(b_0 S_t)$, where $b_0 > 0$ is a period-0 positive discount factor shock.

Idiosyncratic risk. We use a standard productivity process: $\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y$, with $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. We calibrate a persistence of the productivity process $\rho_y = 0.994$ and a standard deviation of $\sigma_y = 0.06$. These values are consistent with empirical estimates (Krueger et al., 2018), and generates a steady-state Gini of wealth of 0.78, which is again in line with the data.[6] Finally, we use the Rouwenhorst (1995) procedure to discretize the productivity process into 10 idiosyncratic states $\{y_1, \ldots, y_{10}\}$ with a constant transition matrix. To consider an increase in the variance of the productivity process, we define $y_{i,t} := y_i + (-1)^{1_{i \leq 5}} n_i^{-1} u_t \delta_0$, where $\delta_0 > 0$ is the initial variance shock, n_i is the share of agents with productivity y_i and $(-1)^{1_{i \leq 5}} = -1$ if $i \leq 5$ and 1 otherwise. The average productivity level is unchanged but the variance gets higher. Low productivity levels agent receive lower levels and high ones get higher levels.

Steady state taxes and public debt, and public spending shock. We first solve the model with constant exogenous taxes and explain below the choice of the Social Welfare Function (SWF). We first assume that employer social contributions and capital taxes are 0, $\tau^S = \tau^K = 0$. The income tax τ^E is set to $\frac{1}{\varepsilon_W}$ to undo distortions on the labor market due to the monopoly power of unions. We consider that $\tau^L = 16\%$. This last value (together with the value of public debt explained below) implies that public spending over GDP is 15, which is close to the US value in 2007. The amount of public debt (which is the only asset here) is set to the annual value of 128% of GDP. As public debt is the only asset in our economy, we target this amount to obtain an average Marginal Propensity to Consume (MPC) of 0.3.[7]

The process for G_t is a AR(1) process, with $G_t = G \exp(g_0 S_t)$ where $g_0 > 0$ is the period-0 positive public spending shock.

Monetary parameters. Following the literature and in particular Schmitt-Grohé and Uribe (2005), we assume that the elasticity of substitution is $\varepsilon_P = 6$ across goods and $\varepsilon_W = 21$ across labor types. The price adjustment cost is set to $\psi_P = 100$, such that the slope of the price Phillips curve is $\frac{\varepsilon_P - 1}{\psi_P} = 5\%$ (see Bilbiie and Ragot, 2021, for a discussion and references). The wage adjustment cost is set to $\psi_W = 2100$, such that the slope of the wage Phillips curve is 1%, assuming wages to be stickier than prices.[8] Finally, as there is no inflation on prices or wages at

[6] The Gini of wealth is 0.78 using the SCF data in 2007, before the 2008 Great Recession.

[7] We thus adopt a liquid one-asset liquid wealth calibration to match a realistic MPC (Kaplan and Violante, 2022).

[8] We have performed sensitivity analysis regarding these coefficients. Our qualitative results appear not to be sensitive to these values, even if inflation and wage volatility increases with the slopes of Phillips curves.

the steady state: $\pi^P = \pi^W = 0$, these coefficients only matter in the dynamics.

Table 1 provides a summary of the model parameters.

Parameter	Description	Value	Target
	Preference and technology		
β	Discount factor	0.99	Quarterly calibration
σ	Curvature utility	2	
$\bar{a}$	Credit limit	0	
χ	Scaling param. labor supply	0.01	$L = 1/3$
φ	Frisch elasticity labor supply	0.5	Chetty et al. (2011)
	Shock process		
ρ_y	Autocorrelation idio. income	0.993	Krueger et al. (2018)
σ_y	Standard dev. idio. income	6%	$Gini = 0.78$
ρ_z	Autocorrelation TFP shock	0.95	
	Tax system		
τ^L	Labor tax	16%	$G/Y = 15\%$
τ^E	Other tax	4.74%	$1/\varepsilon_w$
τ^S,τ^E,τ^K	Other tax	0%	
B/Y	Public debt over yearly GDP	128%	$MPC = 0.3$
G/Y	Public spending over yearly GDP	15%	Targeted
	Monetary parameters		
ε_p	Elasticity of sub. between goods	6	Schmitt-Grohé and Uribe (2005)
ψ_p	Price adjustment cost	100	Price PC 5%
ε_w	Elasticity of sub. labor inputs	21	Schmitt-Grohé and Uribe (2005)
ψ_w	Wage adjustment cost	2100	Wage PC 1%

Table 1: Parameter values in the baseline calibration. See text for descriptions and targets.

Calibration of the complete-market economy. The calibration of the CM economy considers the same preference parameters as in the HA economy. We denote with upper-script CM (HA) the allocation in the CM (HA) economy. In the CM economy, the first-best can be achieved at the steady-state. The steady-state labor supply L^{CM} (with $\pi^W = 0$) is determined by $v'(L^{CM}) = u'(c^{CM})$. We set public spending (G^{CM}), in the CM economy such that the public-spending-to-GDP ratios are identical in the two economies: $G^{CM}/Y^{CM} = G^{HA}/Y^{HA}$.

5.2 Simulating optimal policies in the HA economies

To investigate the optimal dynamics of the model, we perform the following experiment – which is standard in the New Keynesian RA literature, but which must be adapted to the HA case.

We first solve for the optimal policy for a given set of instruments and consider the steady-state allocation – which is the long run allocation in the absence of any aggregate shock. We then assume that the economy starts from this Ramsey steady-state and then implement a period-0 MIT shock, which is either supply or demand driven. This procedure allows us to quantify to which extent the economy is perturbated from that the steady state before converging back to the latter. The magnitude of the economy response to the shock will depend both on the nature of the shock and on the set of instruments that are available to the planner.

The steady state crucially depends on the Social Welfare Function used in the Ramsey program, as well as on the tools that the planner has access to. To overcome this difficulty and to start from the same steady state in all cases, we use the inverse optimal taxation approach, as in Heathcote and Tsujiyama (2021) and LeGrand and Ragot (2025). More precisely, we consider the same steady-state fiscal instruments, defined by $\tau^S = \tau^K = 0$, $\tau^E = 1/\varepsilon_w$ and $\tau^L > 0$, and estimate the weights of the SWF for each set of fiscal tools to ensure that this steady state is optimal. Indeed, each instrument of the planner generates a first-order condition, which imposes one restriction on the SWF.[9]However, these constraints are not sufficient to uniquely pin down the SWF. We thus choose the closest SWF to the utilitarian SWF (where all weights are equal) that satisfies these restrictions, which is the closest one to the utilitarian SWF (where all weights are equal). We also verify that the SWF does not quantitatively affect the dynamics of the allocation at the first order.

The Ramsey problem in HA models cannot be solved with usual simulation techniques. Indeed, the Ramsey equilibrium involves a joint distribution across wealth and Lagrange multipliers, which is a high-dimensional object. While the steady-state values of Lagrange multipliers are already difficult to compute, the Ramsey solution actually requires solving the dynamics of this joint distribution. For this reason, we use the truncation method of LeGrand and Ragot (2022a) to determine the joint distribution of individual wealth and Lagrange multipliers.[10] The accuracy of optimal policies has been analyzed in LeGrand and Ragot (2023) for both the steady state and the dynamics. In addition, an improvement to efficiently reduce the state space is provided in LeGrand and Ragot (2022b). We detail the calculations in Appendix, and refer to these papers for details about the method.

To find the steady-state values of the Lagrange multipliers and SWF for a given fiscal policy, we use the following algorithm:

1. Set a truncation structure (a maximum truncation length N) and set the desired values for

[9]This strategy ensures that a consistent steady-state exists. An alternative would be to consider a given SWF function and solve for the optimal Ramsey steady state. In this case, the steady-state allocation can be unrealistic (Auclert et al., 2024 or LeGrand and Ragot, 2025 for a discussion). As in standard New Keynesian models, optimal steady-state price and wage inflation rates are null, regardless of the social welfare function. As a consequence, steady-state price stability does not impose any restriction on the SWF.

[10]Optimizing on simple rules in the spirit of Krusell and Smith (1998) is also hard to implement as their are many independent instruments.

the various instruments.

2. Solve the steady-state allocation of the full-fledged Bewley model with the previous instrument values, using standard techniques.

3. Compute the truncated representation of the economy, i.e., aggregate over truncated histories.

4. Compute the steady-state Ramsey solution in truncated economy with the following steps:

   (a) Derive first-order conditions of the planner for each instrument in the truncated representation.

   (b) Compute the SWF weights that are the closest to 1, for which all the planner's FOCs hold.

   (c) Compute associated Lagrange multipliers.

   (d) The truncated representation, together with the fiscal instruments, the estimated SWF, and Lagrange multipliers is a steady-state optimal Ramsey allocation for the truncated representation.

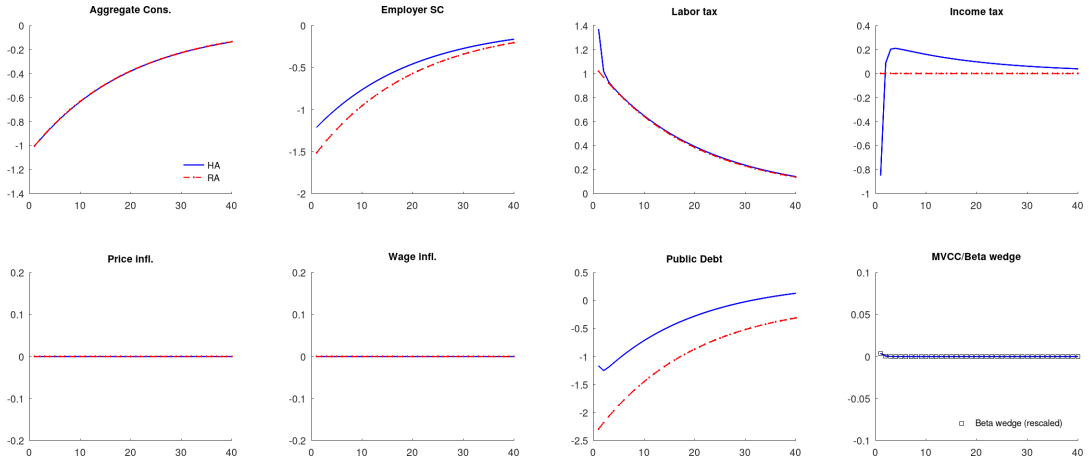
5. Compute the optimal dynamics of instruments and allocation in the truncated economy using the FOCs of the planner – as is standard in any finite state space model.

We use the refined truncation approach, with a number of length for the refinement equals to $N = 8$. We check that the results do not depend on the choice of the truncation length. As in LeGrand and Ragot (2022a), the truncation method provides accurate results, thanks to the introduction of the ξs parameters. We check that the dynamics does not depend on the truncation length.

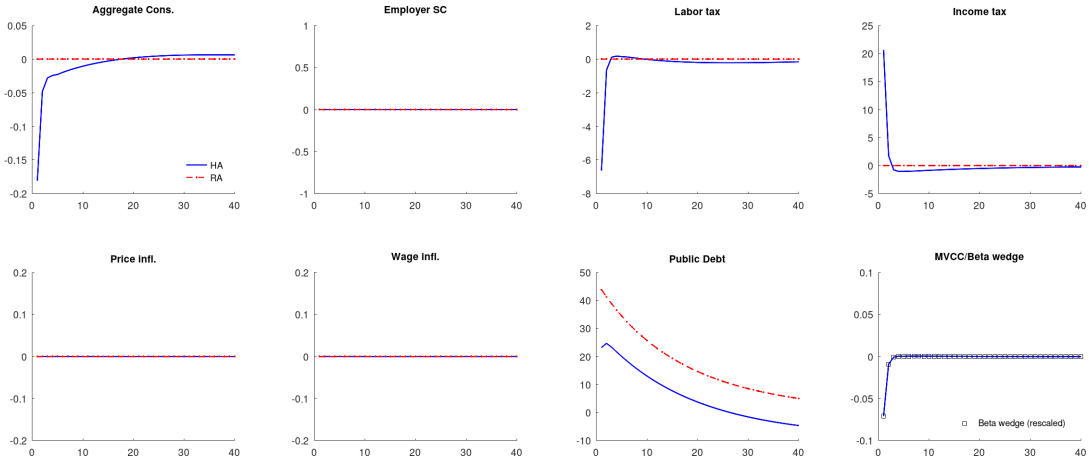
5.3 The economies with all instruments

We consider an economy where the planner has access to all fiscal tools, with no restriction. These economies are subject either to a TFP shock or to a discount factor shock. We plot the Impulse Response Functions (IRFs) of allocations and planner's instruments after either transitory negative technology shock or a positive discount factor shock. The blue plain line corresponds to the HA economy and the red dashed one to the CM economy. We report the IRFs for eight key variables over 40 periods: aggregate consumption (panel 1) in percentage proportional deviation from the steady state; the three labor taxes, all of them in level deviations: employer social contribution τ_t^S (panel 2), the labor tax τ_t^L (panel 3), and the income tax τ_t^E (panel 4); the two inflation rates, for prices (π^P, panel 5) and wages (π^W, panel 6), both also in level deviations; the public debt (B_t, panel 7); the MVCC and the discount factor wedge (panel 8). In the latter panel, we only report the MVCC for the HA economy in blue plain line—as it is

meaningless for the CM economy. We also report the discount factor wedge with black squares. We rescale the series for the discount factor wedge by the same constant (approximately equal to 2), that only depends on steady-state parameters and is the same for across economies. As will be visible the (rescaled) series of discount factor wedge and the one of MVCC are nearly—even though not exactly—identical across all simulations.



(a) TFP shock.



(b) Discount factor shock.

Figure 1: Dynamics of the economy when all instruments are available, after a TFP and discount factor shock. The Heterogeneous-Agent economy (HA) is in blue and the Complete-Market (CM) is in red. Variables are in percentage proportional change, except tax rates and inflation rates which are in percentage level change.

First, both price and wage inflation rates are null along the dynamics as expected. This price and wage stability is independent of the nature of the shock and holds in Figures 1a and 1b. Second, the response of aggregate allocation markedly differ depending on the shock. In the case of the TFP shock, the aggregate consumption response is virtually identical between the HA and the CM economies. Oppositely, in the case of the discount factor shock, aggregate

consumption does not react in the CM economy but drops at impact before converging back rapidly to zero in the HA economy. The CM economy implements the first-best allocation that is independent of agents' discount factor. Only two quantities react to the discount factor shock in the CM economy: the interest rate and the public debt. The interest rate does not play any role in the allocation and is actually a residual of the Euler equation. The public debt is computed from the government's budget constraints and increases reflecting the changes in the interest rate. In the HA economy, the aggregate allocation is sensitive to a discount factor shock. Agents become more patient, which makes them save more. Because of credit constraints, the economy is non-Ricardian and these larger individual savings translates into lower aggregate consumption. This explains the drop at impact of aggregate consumption on Panel 1. Similarly, the larger individual savings implies that public debt must increase to accommodate this extra saving demand coming from more patient agents. As explained in the theoretical model of Section 2, the MVPF (panel 8) is a very good indicator of whether HA and CM economies do differ. Indeed, as we explained, the MVCC measures to which extent the credit constraints are binding in the economy and hence this measures to which extent the economy is non-Ricardian. For the TFP shock of Figure 1a, the MVCC barely changes, indicating that HA and CM allocations do not differ much. For the discount factor shock of Figure 1b, the response of the MVCC is very strong at impact, indicating that the non-Ricardian dimension quantitatively matters in this case and that the response of aggregate consumption is different in HA and CM economies. As mentioned, the responses of MVCC and discount factor wedge are observationally equivalent.

Finally, the fiscal tools reaction is different across economies and across shocks. After the TFP shock, the planner substantially decreases social the employer contribution (τ^S). This allows the cost and the marginal productivity of labor to get closer. Subsidizing labor demand helps limit the negative impact of the contraction. Oppositely, the planner raises the labor tax (τ^L) by a comparable amount to avoid decreasing too much the wage targeted by unions and to distort too heavily the household supply of labor. These two sizable movements hold both in the HA and RA cases. Finally, regarding the income tax (τ^E), it is not affected in the RA economy as this is a redundant instrument that is of no help in the RA economy.[11] This is not the case in the HA economy, where it is slightly reduced at impact to relax the households' budget constraints—which explains increase in τ^L at impact that is slightly larger in HA than in RA.

5.4 The economies when τ^L is not available

Figure 2 plots the response of the HA and RA economies after a TFP and discount factor shocks, when both the income tax τ^E and the employer social contribution, τ^S are available for the

[11] Individual and government budget constraints can be combined in the resource constraint, where the wage and the interest rates play no role. The wage rate w thus influence the Phillips curves (12)–(13) and the consistency equation (19) between inflation rates. This is the exact same role as $w/(1-\tau^E)$ making both w and τ^E substitute.

planner. For both shocks, the labor tax does not move by construction.

The outcomes differ for the two shocks. First for the TFP shock in Figure 2a, the real effect of the shock is similar in the HA and RA economies, despite the absence of τ^L. This is corroborated by MVCC and β-wedges, which barely move. Compared to the case with all instruments, there is a (small) inflation response, both for prices and wages, but it is similar in HA and RA economies. Unlike in the full instrument case, the income tax sharply increase in the HA economy at impact, while it was slightly decreasing when all instruments were available. The reason is that this increase substitutes for the increase in the labor tax of the full instrument case, as the labor tax is here constrained to remain constant. The fact that the full instrument case and this one are overall quite close from each other shows that the income tax τ^E can act as a decent substitute for the absence of τ^L.

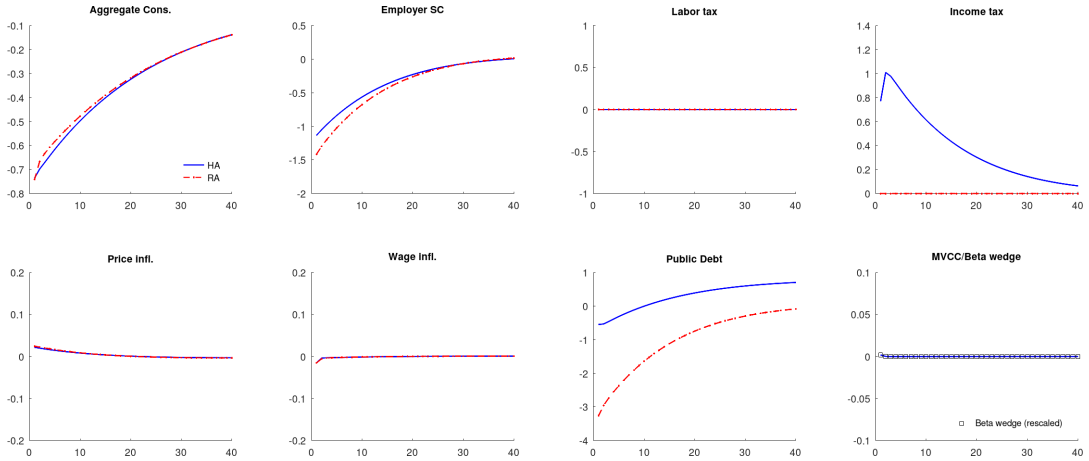
Second in the case of the discount factor in Figure 2b, the picture is slightly different. As in the all instrument case, the real response differs in the HA and RA economies—as there a β-shock still has no impact in the RA economy. This is reflected in the MVCC and β-wedges. However, the drop in consumption is much larger when τ^L is not available, which reflects the absence of the labor tax. While in the full instrument case the labor tax was sharply decreasing and the employer social contribution was kept constant, the absence of a time-varying labor tax implies that the employer social contribution has to act as a substitute—since the income tax increases at impact, cannot play this role. This explains why the social contribution now decreases at impact. However, the employer social contribution is a poor substitute for the labor tax. In addition to the larger real effect that we mentioned, the planner must also deviate from price and wage stability to account for the lack of availability of the labor tax.

For the sake of conciseness, we do not report here the cases where only τ^E or τ^S are not available. In the absence of the income tax τ^E, the picture is overall quite similar to the one in the absence of τ^L that we document here. This again shows that these two instruments are reasonably good substitutes. When τ^S is missing the picture is very similar to the full instrument case showing that this role of this instrument is not as effective as the other two. This is confirmed in the following section.

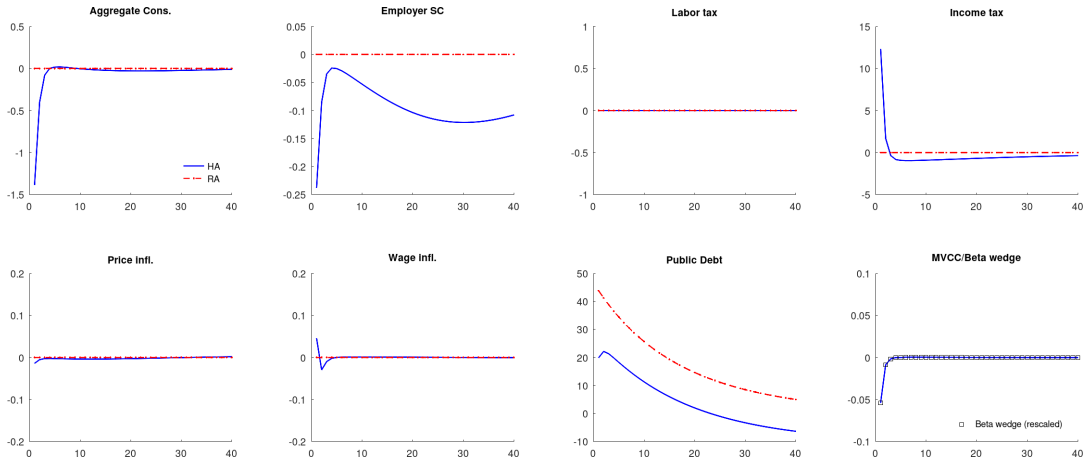
5.5 The economies with τ^S as a unique fiscal instrument

We plot in Figure 3 the response of the HA and RA economies after a TFP and discount factor shocks, when the employer social contribution, τ^S, is the sole fiscal instrument available on labor for the planner. The capital tax is also still present. For both shocks, the labor and income taxes do not move by construction.

Overall, the picture is markedly different from the all instrument case of Figure 1, and even from the no-τ^L case of Figure 2. For both shocks, the real response is different in the HA and RA economies when only τ^S is available. This is again reflected in the MVCC and β-wedges.



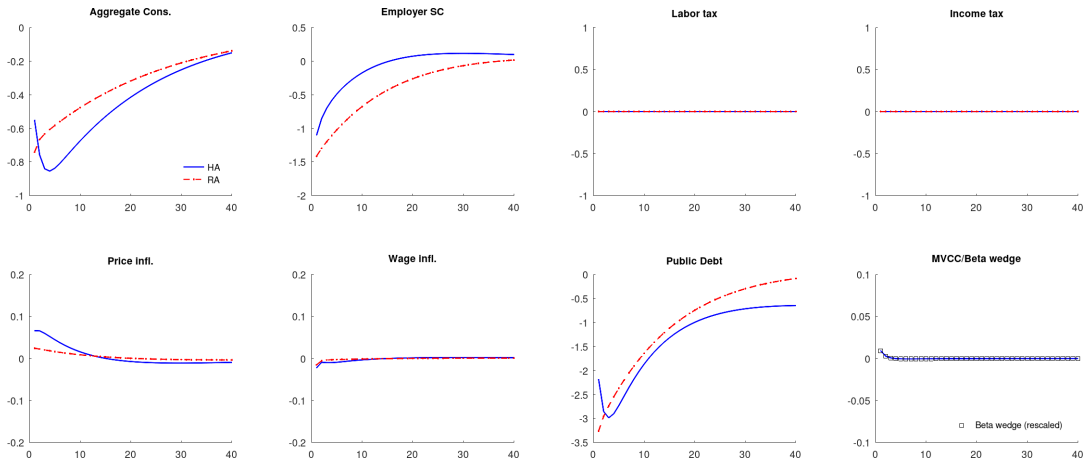
(a) TFP shock.



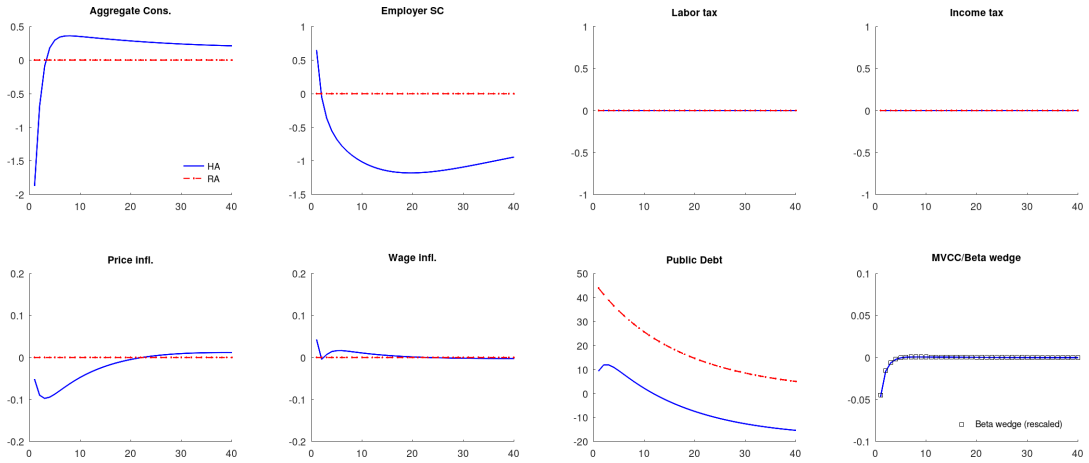
(b) Discount factor shock.

Figure 2: Dynamics of the economy when τ^S is the only available labor fiscal instrument, after a TFP and discount factor shock. The Heterogeneous-Agent economy (HA) is in blue and the Complete-Market (CM) is in red. Variables are in percentage proportional change, except tax rates and inflation rates which are in percentage level change.

Furthermore, the planner deviates from price and wage stability—for both shocks in the HA economy and for the TFP shock only in the RA economy. This further reinforces the idea that τ^S is not a very effective instrument for the planner,



(a) TFP shock.



(b) Discount factor shock.

Figure 3: Dynamics of the economy when τ^S is the only available labor fiscal instrument, after a TFP and discount factor shock. The Heterogeneous-Agent economy (HA) is in blue and the Complete-Market (CM) is in red. Variables are in percentage proportional change, except tax rates and inflation rates which are in percentage level change.

5.6 The effect of missing instruments

We now provide the result with additional simulations. To save some space, we summarize results and refer to the IRFs in Appendix.

First, we considered the economy keeping the capital tax constant, and considering $\left\{\tau_t^S, \tau_t^L, \tau_t^E\right\}$. For the given calibration, inflation barely moves on impact. When we decrease the coefficient on

price stickiness to a low value of $\psi_p = 10$ instead of 100, then inflation increases significantly on impact, for one period. This spike in inflation is a substitute for the missing capital tax. It indeed decreases the real interest rate for one period due to the Fisher effect, as it is unexpected. As a consequence, inflation is a substitute for missing capital tax, when price are flexible enough, this result is consistent with LeGrand et al. (2022).

Second, we find that the allocations in HA and CM models after positive public spending shocks are similar to negative TFP shocks, as mentioned above. The difference between HA and CM allocation and price increases when there the number of instruments decreases. Third, the discount factor shock is generating the biggest difference between HA and CM economies, as the MVCC varies significantly in HA economies. Fourth, for public spending shock and discount factor shock, the use of time-varying employers' social contribution (which is a time-varying labor subsidy or taxes not directly affecting the bargaining of wages) is sufficient tool to quantitatively reduce optimal inflation variability. Finally, the path of public debt varies significantly for HA and CM economies, even when allocations are close. Indeed, in HA economy, the planner adjust to the demand for liquidity for agents to self-insure.

6 Conclusion

We have solved for optimal fiscal and monetary policy in an economy plagued with both sticky prices and sticky wages, considering various fiscal instruments. We compare the outcome of a complete-market economy and an heterogeneous-agent economy for two shocks, a TFP shock and discount factor shock. As a benchmark, we first provide a complete fiscal system, which is defined as a fiscal system for which both price and wage inflation are zero after the three shocks. In other words, for the fiscal system any possibly useful effects of inflation are more efficiently captured by time-varying taxes, which avoid additional inflation costs.These complete fiscal system is a theoretical benchmark that the we use to consider both allocation and inflation when fixed some fiscal tools in the dynamics.

When all instruments are available the allocation of HA and CM economies are very close for both TFP (and public spending shock), but very different for discount factor shock. This result comes from the different effect of each shock on the average tightness of the credit constraint of agents, that we have labelled the Marginal Value of Credit Constraint (MVCC). This result helps rationalize different outcome found in the literature, which appears to be shock specific.

We then explore the effect of missing fiscal tools on optimal inflation. In particular, we identify the fiscal tools which are necessary to implement price stability. This allows to understand the wedges that inflation is affecting to improve welfare, and provide some insight for monetary policy with heterogeneous agents. We find that inflation is a substitute for missing capital taxes if prices are flexible enough. Long-lasting inflation is used to reduce the labor wedge, which is the difference between the real wage rate and the marginal productivity of labor, when some

labor taxes are not-time-varying.

Finally, we analyze the welfare effect of missing fiscal tools. This last experiment is interesting to identify the most welfare-enhancing fiscal tool after macroeconomic shocks. There is a consensus that monetary is the relevant instrument for business cycle stabilization, except at the zero lower bound. It is interesting to observe that it is not necessarily the case with nominal wage rigidity. Fiscal instrument can significantly improve welfare, but they are shock specific. In particular, time-varying labor subsidy is a useful tools to reduce the labor wedge after a TFP shock. It is noteworthy that these tools have been recently been used in Europe to stabilize employment. In Germany, the so-called *kurzarbeit* device played this role, while in France, the *activité partielle* policy was a wage subsidy to reduce layoffs during the Covid-19 crisis. These tools time-varying fiscal policy non-Keynesian stabilizers are their goal is not to simply manage aggregate demand and economic activity due to fiscal multipliers, but to directly affect the labor wedge.

From this analysis, we conclude that HA economies provide new insight about optimal stabilization policy, which appear to be shock specific.

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Appendix

A The simple model

The program of the planner in the economy with credit constraint (presented in Section 2) is:

$$\max_{(c_{e,t},c_{u,t},a_{e,t},a_{u,t},l_{e,t}B_t,A_t,R_t,w_t)} \sum_{t=0}^{\infty} \Theta_t \Big(U(c_{e,t}, l_{e,t}) + U(c_{u,t}, 0) \Big) \tag{31}$$

$$\text{s.t. } c_{e,t} + a_{e,t} = R_t a_{u,t-1} + w_t l_{e,t}, \tag{32}$$

$$c_{u,t} + a_{u,t} = R_t a_{e,t-1}, \tag{33}$$

$$U_c(c_{e,t}, l_{e,t}) = \beta_t R_{t+1} U_c(c_{u,t+1}, 0), \tag{34}$$

$$U_c(c_{u,t}, 0) \geq \beta_t R_{t+1} U_c(c_{e,t+1}, l_{e,t+1}), \text{ with equality if } a_{u,t} > 0, \tag{35}$$

$$-U_l(c_{e,t}, l_{e,t}) = w_t U_c(c_{e,t}, l_{e,t}), \tag{36}$$

$$R_t B_{t-1} + w_t l_{e,t} = Z_t l_{e,t} + B_t. \tag{37}$$

$$A_t = a_{e,t} + a_{u,t}, \tag{38}$$

$$A_t = B_t, \tag{39}$$

$$a_{e,t}, a_{u,t} \geq 0, \tag{40}$$

$$c_{e,t}, c_{u,t} > 0 \text{ and } l_{e,t}, l_{u,t} \geq 0. \tag{41}$$

We first assume that $a_u = 0$, and show below (in Section A.1) that it is the case.

If $a_{u,t} = 0$, using the GHH property, the first-order condition for labor supply is $l_t = (\chi w_t)^{\varphi}$. The Euler equation of employed agents is (using $c_{u,t+1} = R_{t+1} a_{e,t}$):

$$\left(w_t \left(\chi w_t \right)^{\varphi} - a_{e,t} - \frac{\left(\chi w_t \right)^{1+\varphi}}{1 + 1/\varphi} \right)^{-1} = \beta_t R_{t+1} \left(R_{t+1} a_{e,t} \right)^{-1},$$

One finds the saving function:

$$a_{e,t} = \frac{\beta_t}{1+\beta_t} \frac{1}{1+\varphi} w_t^{1+\varphi}$$

From that we get the relationships:

$$c_{e,t} = \left(1 - \frac{\beta_t}{1+\beta_t} \frac{1}{1+\varphi} \right) w_t^{1+\varphi} \tag{42}$$

$$c_{e,t} - \frac{l^{1+1/\varphi}}{1+1/\varphi} = \left(\frac{1}{1+\varphi} - \frac{\beta_t}{1+\beta_t} \frac{1}{1+\varphi} \right) w_t^{1+\varphi}$$

$$c_{u,t} = R_t \frac{\beta_{t-1}}{1+\beta_{t-1}} \frac{1}{1+\varphi} w_{t-1}^{1+\varphi} \tag{43}$$

We will use the change of variables:

$$\begin{aligned}\eta_t &:= \frac{1}{1+\varphi}\frac{\beta_t}{1+\beta_t} \\ x_t &:= w_t^{1+\varphi}\end{aligned} \tag{44}$$

and $x_t := w_t^{1+\varphi}$, we have $\frac{1}{1+\varphi}\frac{1}{1+\beta_t} = \frac{1}{1+\varphi} - \eta_t$.

Using $B_t = a_{e,t}$, the budget of the state can be written as**:**

$$G_t + R_t\eta_{t-1}x_{t-1} + = Z_t x_t^{\frac{\varphi}{1+\varphi}} - x_t + \eta_t x_t.$$

Using these relationships in the program of the planner, one finds

$$\begin{aligned}\max_{(R_t,w_t)} &\sum_{t=0}^{\infty}\Xi_t\Big(\log x_t + \log R_t + \log x_{t-1}\Big) \\ &- \sum_{t=0}^{\infty}\Xi_t\mu_t\left(\frac{G_t}{\chi^\varphi} + R_t\eta_{t-1}x_{t-1} - Z_t x_t^{\frac{\varphi}{1+\varphi}} + x_t - \eta_t x_t\right).\end{aligned}$$

The two first-order conditions and the budget constraint of the state form a system of 3 equations in the three unknowns R_t, μ_t, x_t

$$\begin{aligned}(1+\beta_{t+1})\frac{1}{x_t} &= \mu_t\left(-Z_t\frac{\varphi}{1+\varphi}x_t^{-\frac{1}{1+\varphi}} + 1 - \eta_t\right) + \beta_{t+1}\mu_{t+1}R_{t+1}\eta_t \\ 1 &= R_t\mu_t\eta_{t-1}x_{t-1} \\ R_t\eta_{t-1}x_{t-1} &= Z_t x_t^{\frac{\varphi}{1+\varphi}} - x_t + \eta_t x_t\end{aligned}$$

Substituting for R_t and μ_t, one finds $\left(\frac{1+\varphi}{1/2+\varphi}(1-\eta_t)\right)^{-(1+\varphi)} Z_t^{1+\varphi} = x_t$. Then:

$$R_t = \frac{x_t}{\eta_{t-1}x_{t-1}}\frac{1}{1+2\varphi}(1-\eta_t) \tag{45}$$

Then, using the expression of $c_{e,t}$ and $c_{u,t}$ one finds $\frac{c_{e,t}}{c_{u,t}} = 1+2\varphi$ and $c_{e,t} = (1-\eta_t)\,x_t$.

Then:

$$C_t^{HA} = c_{e,t} + c_{u,t} = \left(\frac{1+\varphi}{1/2+\varphi}\right)^{-\varphi}(1-\eta_t)^{-\varphi}\,Z_t^{1+\varphi}. \tag{46}$$

Steady-state interest rate

At the steady state, $Z_t = 1$ and $\eta = \frac{1}{1+\varphi}\frac{\beta}{1+\beta}$, thus:

$$R = \frac{1+\beta}{\beta}\frac{1+\varphi}{1+2\varphi} - \frac{1}{\beta}\frac{\beta}{1+2\varphi} = \frac{1+(1+\beta)\,\varphi}{1+2\varphi}\times\frac{1}{\beta}$$

As $\beta < 1$, $\frac{1+(1+\beta)\varphi}{1+2\varphi} < 1$ and $R < \frac{1}{\beta}$, and credit constraints are binding at the steady-state.

Comparison with RA economy

Using $C_t^{RA} = Z_t^{1+\varphi}$, and the expression of C_t^{HA} in (46), we deduce:

$$\frac{C_t^{HA}}{C_t^{RA}} = \left(\frac{1+\varphi}{1/2+\varphi}\right)^{-\varphi}\left(1 - \frac{1}{1+\varphi}\frac{\beta_t}{1+\beta_t}\right)^{-\varphi}$$

A.1 An existence proof

We now prove that $a_u = 0$, and that credit constraints are binding at the steady state. The proof is is related to the one presented in LeGrand and Ragot (2025), but it simpler, as we consider an economy without capital. We first prove that, when credit constraints are not binding, then the optimal level of public debt implements $a_u = 0$. Second, we show that when $a_u = 0$, the planner wants the credit constraint to bind and $R < \frac{1}{\beta}$.

First, in a Ramsey equilibrium where credit constraint doesn't bind, the economy is a simple two-agents economy with distorting taxes. The planner minimizes distortions (i.e labor taxes) to a level consistent with credit constraints not binding. This implies that B reached the minimum level for which $a_{u,t} \geq 0$,and $a_u = 0$in the Ramsey steady-state.

Second, the proof of the previous Section has assumed that $a_{u,t} = 0$ to conclude that in this case, the planner wants to credit constraints to bind. In other words, starting from $a_{u,t} = 0$, the previous proof shows that the optimal program implements $R < \frac{1}{\beta}$. The gain of a marginal increase in production efficiency by reducing labor taxes is higher that the marginal cost of reducing allocation efficiency, by decreasing public debt and the real interest rate.

A.2 Expression of the MVCC

One can compute the value of Lagrange multiplier on the credit constraint of unemployed agent.

$$\nu_t = c_{u,t}^{-1} - \beta R_{t+1}\left(c_{e,t} - \frac{\chi l_{e,t}^{1/\varphi+1}}{1+1/\varphi}\right)^{-1}$$

Using (42)-(43), one finds, after some algebra

$$MVCC_t = \frac{\nu_t}{u'\left(c_{u,t}\right)} = 1 - \frac{\left(1+\varphi\left(1+\beta_{t+1}\right)\right)\left(1+\varphi\left(1+\beta_t\right)\right)}{\left(1+2\varphi\right)^2}$$

Note that $MVCC_t > 0$ is equivalent to

$$\left(1+\varphi\left(1+\beta_{t+1}\right)\right)\left(1+\varphi\left(1+\beta_t\right)\right) < \left(1+2\varphi\right)^2$$

which is always true as $0 < \beta_{t+1} < 1$.

A.3 Expression of the DFW

First, the real interest rate in the HA economy is, from (45) and the expression of x_t and η_t:

$$R_t^{HA} = \frac{\left(1 - \frac{1}{1+\varphi}\frac{\beta_t}{1+\beta_t}\right)^{-(1+\varphi)}}{\frac{1}{1+\varphi}\frac{\beta_{t-1}}{1+\beta_{t-1}}\left(1 - \frac{1}{1+\varphi}\frac{\beta_{t-1}}{1+\beta_{t-1}}\right)^{-(1+\varphi)}} \left(\frac{Z_t}{Z_{t-1}}\right)^{1+\varphi} \frac{1}{1+2\varphi}\left(1 - \frac{1}{1+\varphi}\frac{\beta_t}{1+\beta_t}\right) \tag{47}$$

As as $l_{HA,t} = x_t^{\frac{\varphi}{1+\varphi}}$ the DFW β_t^{wedge} is defined by:

$$\frac{1}{R_{t+1}^{HA}} = \beta_t \beta_t^{wedge} \left(\frac{C_{t+1}^{HA} - \frac{l_{HA,t+1}^{1+1/\varphi}}{1+1/\varphi}}{C_t^{HA} - \frac{l_{HA,t+1}^{1+1/\varphi}}{1+1/\varphi}}\right)^{-1}$$

Substituting for $C_t^{HA}, C_t^{HA}, R_{t+1}^{HA}$ one finds after some algebra:

$$\beta_t^{wedge} = \frac{(1+1/\varphi)(1+\varphi(1+\beta_{t+1})) - (1/2+\varphi)(1+\beta_{t+1})}{(1+1/\varphi)(1+\varphi(1+\beta_t)) - (1/2+\varphi)(1+\beta_t)} \times \frac{1+2\varphi}{1+\varphi(1+\beta_{t+1})}$$

A.4 Relationship between MVCC and DFW

Using the expression of MVCC, one finds:

$$\beta_t^{wedge} = \frac{1}{1 - MVCC_t}\left(1 - \frac{1+2\varphi}{2(1+\varphi)+\varphi(1+\beta_t)} MVCC_t\right)$$

Assume $MVCC_t \simeq 0$, then:

$$\beta_t^{wedge} = 1 + \frac{1+\varphi(1+\beta_t)}{2(1+\varphi)+\varphi(1+\beta_t)} MVCC_t$$

Thus at the first order $\tilde{\beta}_t^{wedge} = \left(1 - \frac{1}{\beta^{wedge}}\right) M\tilde{V}CC_t$, where variables with tilda represent proportional deviations.

B Derivation of the wage-Phillips curve

There is a continuum of unions of size 1 indexed by k and each union k supplies L_{kt} hours of labor at date t with nominal wage $\hat{W}_{kt}$. Union-specific labor supplies are then aggregated into aggregate labor supply by a competitive technology featuring a constant elasticity of substitution ε_W:

$$L_t = \left(\int_k L_{kt}^{\frac{\varepsilon_W - 1}{\varepsilon_W}} dk\right)^{\frac{\varepsilon_W}{\varepsilon_W - 1}}. \tag{48}$$

The competitive aggregator demands the union labor supplies $(L_{kt})_k$ that minimize the total labor cost $\int_k \hat{W}_{kt} L_{k,t} dk$ subject to the aggregation constraint (48), where $\hat{W}_{kt}$ is the bargained

nominal wage of the members of union k. The demand for labor of union k depends on the total labor cost paid by the firm $\tilde{W}_{kt}$: $L_{kt} = \left(\frac{\tilde{W}_{kt}}{\tilde{W}_t}\right)^{-\varepsilon_W}$, where $\tilde{W}_t = \left(\int_k \tilde{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{1}{1-\varepsilon_W}}$ is the total nominal wage index. As the labor demand depends on relative wages, and $\frac{\tilde{W}_{kt}}{\tilde{W}_t} = \frac{\hat{W}_{kt}}{\hat{W}_t}\frac{1-\tau_t^S}{1-\tau_t^S} = \frac{\hat{W}_{kt}}{\hat{W}_t}$, total labor demand can be written as:

$$L_{kt} = \left(\frac{\hat{W}_{kt}}{\hat{W}_t}\right)^{-\varepsilon_W} L_t, \tag{49}$$

where $\hat{W}_t = \left(\int_k \hat{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{1}{1-\varepsilon_W}}$ is the bargained nominal wage index. Each union k sets its wage $\hat{W}_{kt}$ so as to maximize the intertemporal welfare of its members subject to fulfilling the demand of equation (49). We assume the presence of quadratic utility costs related to the adjustment of the nominal wage and equal to $\frac{\psi_W}{2}(\hat{W}_{kt}/\hat{W}_{kt-1} - 1)^2 dk$. The objective of union k is thus:

$$\max_{(\hat{W}_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \int_i \left(u(c_{i,s}) - v(l_{i,s}) - \frac{\psi_W}{2}\left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1\right)^2 \right) \ell(di),$$

$$\max_{(\hat{W}_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \int_i \left(u(c_{i,s}) - v(l_{i,s}) - \frac{\psi_W}{2}\left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1\right)^2 \right) \ell(di),$$

subject to (49) and where $c_{i,t}$ and $l_{i,t}$ are the consumption and labor supply of agent i. The first-order condition with respect to W_{kt} thus writes as:

$$\pi_t^W(\pi_t^W + 1) = \frac{\hat{W}_{kt}}{\psi_W} \int_i \left(u'(c_{i,t}) \frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} - v'(l_{i,t}) \frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} \right) \ell(di) + \beta \mathbb{E}_t \left[\pi_{t+1}^W (\pi_{t+1}^W + 1) \right], \tag{50}$$

where the wage inflation rate is denoted by:

$$\pi_t^W = \frac{\hat{W}_{k,t}}{\hat{W}_{k,t-1}} - 1.$$

The labor supply l_{it} of agent i is the sum of her hours l_{ikt} supplied to union k, summed over all unions: $l_{it} = \int_k l_{ikt} dk$. Each union is assumed to request its members to supply an uniform number of hours, such that: $l_{ikt} = L_{kt}$. We thus deduce from (49):

$$\hat{W}_{kt} \frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} = \hat{W}_{kt} \frac{\partial (\int_k (\frac{\hat{W}_{kt}}{\hat{W}_t})^{-\varepsilon_W} L_t dk)}{\partial \hat{W}_{kt}} = -\varepsilon_W L_{kt}. \tag{51}$$

To compute the derivative of consumption $\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}}$, it should observed that it is equal to the derivative of its net total income. The net total income of agent i writes as $(1 - \tau_t^L)\hat{W}_{kt} y_{i,t} l_{i,t}/P_t$,

where τ_t^L is the labor tax. Formally:

$$\begin{aligned}\frac{1}{c_{i,t}}\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= \frac{1}{\hat{W}_{kt}} + \frac{1}{l_{i,t}}\frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} \\ &= \frac{1}{\hat{W}_{kt}} - \frac{\varepsilon_W}{\hat{W}_{kt}}\frac{L_{kt}}{l_{i,t}} \\ \hat{W}_{kt}\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= (1-\varepsilon_W)(1-\tau_t^L)\hat{W}_{kt}y_{i,t}l_{i,t}/P_t \end{aligned} \tag{52}$$

We focus on the symmetric equilibrium where all unions choose to set the same wage $\hat{W}_{kt} = \hat{W}_t$, hence all households work the same number of hours, equal to $l_{it} = L_t$. Combining (50) with the partial derivatives (51) and (52), we deduce the following Phillips curve for wage inflation:

$$\pi_t^W(\pi_t^W+1) = \frac{\varepsilon_W}{\psi_W}\underbrace{\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}(1-\tau_t^L)\hat{w}_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)}_{\text{labor gap}}L_t + \beta\mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W+1)\right], \tag{53}$$

where $\hat{w}_t = \hat{W}_t/P_t$ is the real pre-tax wage.

C Characterization of the Ramsey allocation in the RA case

We solve for the Ramsey allocation in the RA case, for both demand and supply shocks.

C.1 First-best allocation

In the first-best allocation, the resource constraint imposes that total consumption is financed out of production: $G_t + C_t = Z_t L_t$. The labor supply is thus determined by the solution to the following program: $\max_{L_t} u(Z_t L_t - G_t) - v(L_t)$. The first-order condition defines the first-best labor supply L_t^{FB} as the solution of:

$$Z_t u'(Z_t L_t^{FB} - G_t) \quad = \quad v'(L_t^{FB}), \tag{54}$$

which can be shown to admit a unique solution under standard assumption (u increasing concave with $u'(0) = \infty$ and $u'(\infty) = 0$ and v increasing convex).

Consider the following particular case. We set $G_t = 0$, $u'(c) = c^{-\gamma}$, and $v'(L) = \chi^{-1}L^{1/\phi}$ such that $\gamma > 0$ is the inverse of the IES and $\phi > 0$ is the Frisch elasticity of labor supply. We obtain: $L_t^{FB} = \chi^{\frac{1}{\frac{1}{\phi}+\gamma}} Z_t^{\frac{1-\gamma}{\frac{1}{\phi}+\gamma}}$.

C.2 Representative-agent model with a full set of instruments

We show that when the planner has access to the full set of instrument, the first-best allocation can be implemented for both demand and supply shocks. This requires $\pi^W = \pi^P = 0$, to avoid

price or wage adjustment costs. The equations defining the equilibrium allocation are

$$w_t = \left(1-\tau_t^E\right)\left(1-\tau_t^L\right)\left(1-\tau_t^S\right) Z_t,$$

$$v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{1-\tau_t^E}u'(C_t) = 0 \tag{55}$$

$$\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_t^S)(1-\tau_t^E)} = 1 \tag{56}$$

$$1 = \frac{w_t}{w_{t-1}}\frac{(1-\tau_{t-1}^L)}{(1-\tau_t^L)} \tag{57}$$

$$C_t = Z_t L_t - G_t$$

$$G_t + (1+r_t)B_{t-1} + (w_t - Z_t)L_t = B_t \tag{58}$$

$$u'(C_t) = \beta(1+r_{t+1})u'(C_{t+1}) \tag{59}$$

Note that the Euler equation (59) determines the real interest r_t^{FB} $t \geq 1$ from the first-best path of consumption C_t^{FB}. Importantly, this equations don't determine the period-0 interest rate r_0.

Equation (58à is the budget constraint of the government.

Equation (57) implies that there is a α such that $1-\tau_t^L := \alpha w_t$. For the allocation to be the first best, equations (55) and (54) implies that

$$1-\tau_t^E \quad := \quad \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{Z_t}$$

Then equation (56) implies

$$1-\tau_t^S := \frac{\varepsilon_W}{\varepsilon_W - 1}\frac{1}{\alpha w_t}$$

Then the budget of the government implies

$$w_t = Z_t - \frac{G_t + (1+r_t)B_{t-1} - B_t}{L_t} \tag{60}$$

Implementation results: For any path of G_t, Z_t and path of public debt B_t, for $t \geq 0$, the first best can be implemented.

The proof is direct. Consider a path G_t, Z_t, B_t and the first-best labor supply L_t^{FB}. It gives a path of consumption determining the real interest rate r_t, $t \geq 1$. For any r_0 (which is an additional free variable), the equation (60) determines a path for the real wage rate. Then for any α, $1-\tau_t^L = \alpha w_t$, $1-\tau_t^E = \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{Z_t}$, $1-\tau_t^S = \frac{\varepsilon_W}{\varepsilon_W - 1}\frac{1}{\alpha w_t}$ is a market equilibrium.

Note thus that public debt is not determined in this implementation.

Note that in a steady-state equilibrium (where $Z = 1$ and B, G, w are constant) , we have

$$B_{SS} = \frac{\beta}{1-\beta}\left((1-w)\, L_{SS}^{FB} - G_{SS}\right)$$

C.3 RA model : Representative agents without time-varying τ_t^E

We assume that the economy is in steady state, where public debt is B_{SS}. and hit by the shock at period 0. In the previous analysis, we can impose $\tau_t^E = \tau_{SS}^E$.

$$w_t = \frac{\varepsilon_W}{\varepsilon_W - 1}\left(1 - \tau_{ss}^E\right) Z_t$$

Then, $1 - \tau_t^L = \alpha \frac{\varepsilon_W}{\varepsilon_W - 1}\left(1 - \tau_{ss}^E\right) Z_t$, , $1 - \tau_t^S = \frac{1}{\alpha(1-\tau_{ss}^E) Z_t}$.

The budget of the state implies (for $t \geq 0$, with the notation $B_{-1} = B_{SS}$)

$$(1 + r_t)\, B_{t-1} - B_t = \Theta_t$$

with

$$\Theta_t := \left(1 - \frac{\varepsilon_W}{\varepsilon_W - 1}\left(1 - \tau_{ss}^E\right)\right) Z_t L_t^{FB} - G_t$$

The variable Θ_t is uniquely determined. This uniquely determines the path of public converging back to the steady state. To see that, first observe that the period-0 interest rate 0 r_0 is a free parameter determined by period-0 capital tax $\hat{\tau}_0^K$.

$$B_{-1}\,(1 + r_0) = \sum_{t=0} \frac{G_t}{R_{0,t}} + \lim_{T \to \infty} \frac{B_T}{R_{t,T}}$$

To have $\lim_{T\to\infty} \frac{B_T}{R_{t,T}} = 0$, we must choose the initial capital tax such that

$$(1 + r_0)\, B_{-1} = \sum_{k=t}^{\infty} \frac{\Theta_k}{\prod_{j=t+1}^{k}\left(1 + r_j^{FB}\right)} + \lim_{T \to \infty} \frac{B_T}{\prod_{j=t+1}^{T}\left(1 + r_j^{FB}\right)}$$

(with the notation $\prod_{j=t+1}^{t} = 1$). The term $\lim_{T\to\infty} \frac{B_T}{\prod_{j=t+1}^{T}(1+r_j)} = 0$ if the economy converges back to the steady state. The unique period 0 allowing the public debt to converge back to the steady state is

$$1 + r_0 = \frac{\sum_{k=t}^{\infty} \frac{\Theta_k}{\prod_{j=t+1}^{k}\left(1 + r_j^{FB}\right)}}{B_{SS}},$$

which is uniquely determined.

C.4 RA with Demand shock

We now show that whatever the fiscal system (economy 3 and 4), the first-best allocation can be implemented with demand shocks. The proof follows the consideration of the previous Section.

We now assume that $\tau^E = \tau^E_{SS}$ and $\tau^E = \tau^E_{SS}$. We now focus on the case where $1-\tau^E_{ss} = \frac{\varepsilon_W - 1}{\varepsilon_W}$, to determine uniquely the path of the instruments. Any other value would not quantitatively change the allocation, and qualitatively the path of the instruments.

In this case, we have $w = Z = 1$. Then $1 - \tau^L_t = \alpha$, and $1 - \tau^S = \frac{\varepsilon_W}{\varepsilon_W - 1}\frac{1}{\alpha}$

and

$$1 + r_0 = \frac{1-\beta}{\beta}\sum_{k=t}^{\infty}\frac{G_k/G_{SS}}{\prod_{j=t+1}^{k}\left(1+r^{FB}_j\right)},$$

implements the first-best allocation.

C.5 The RA economy without time-varying τ^E_t and τ^S_t, with optimal τ^L_t and supply shocks

The first-best cannot be implemented, and we must solve for the Ramsey allocation. We provide equations for both demand and supply shocks and then discuss each case in turn.

The program is:

$$\begin{aligned}
&\max_{\left(\tau^L_t,\tau^S_t,\tau^K_t,B_t,T_t,\pi^P_t,\pi^W_t,w_t,r_t,\Omega_t,\tilde{R}^N_t,L_t,c_t,a_t\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-v(L_t)\right) - \frac{\psi_W}{2}(\pi^W_t)^2\right],\\
&G_t + (1+r_t)B_{t-1} + w_t L_t \leq \left(1 - \frac{\psi_P}{2}(\pi^P_t)^2\right)Z_t L_t + B_t,\\
&c_t + a_t = (1+r_t)B_{t-1} + w_t L_t,\\
&u'(c_t) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{t+1})\Big],\\
&\pi^W_t(\pi^W_t+1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{1-\tau^E_{ss}}u'(c_t)\right)L_t + \beta\mathbb{E}_t\Big[\pi^W_{t+1}(\pi^W_{t+1}+1)\Big],\\
&\pi^P_t(1+\pi^P_t) = \frac{\varepsilon_P - 1}{\psi_P}\Big(\frac{1}{Z_t}\frac{w_t}{(1-\tau^L_t)(1-\tau^E_{ss})} - 1\Big) + \beta\mathbb{E}_t\Big(\pi^P_{t+1}(1+\pi^P_{t+1})\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\Big),\\
&(1+\pi^W_t)\frac{w_{t-1}}{1-\tau^L_{t-1}} = \frac{w_t}{1-\tau^L_t}(1+\pi^P_t),
\end{aligned}$$

Define

$$\begin{aligned}
T_t &= (1+r_t)B_{t-1} - B_t\\
x_t &= \frac{w_t}{1-\tau^L_t}
\end{aligned}$$

Then the program is (using $\frac{\varepsilon_W-1}{\varepsilon_W} = 1-\tau^E_{ss}$)

$$\max_{\left(x_t T_t, \pi^P_t, \pi^W_t, w_t, L_t, c_t\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-v(L_t)\right)-\frac{\psi_W}{2}(\pi^W_t)^2\right],$$

$$G_t + T_t + \left(1-\tau^L_t\right)x_t L_t \leq \left(1-\frac{\psi_P}{2}(\pi^P_t)^2\right)Z_t L_t,$$

$$c_t = T_t + \left(1-\tau^L_t\right)x_t L_t,$$

$$\pi^W_t(\pi^W_t+1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t)-\left(1-\tau^L_t\right)x_t u'(c_t)\right)L_t + \beta\mathbb{E}_t\left[\pi^W_{t+1}(\pi^W_{t+1}+1)\right],$$

$$\pi^P_t(1+\pi^P_t) = \frac{\varepsilon_P-1}{\psi_P}\left(\frac{\varepsilon_W}{\varepsilon_W-1}\frac{1}{Z_t}x_t - 1\right) + \beta\mathbb{E}_t\left(\pi^P_{t+1}(1+\pi^P_{t+1})\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\right),$$

$$(1+\pi^W_t)x_{t-1} = x_t(1+\pi^P_t),$$

while the corresponding Lagrangian becomes $c_t = T_t + \left(1-\tau^L_t\right)x_t L_t$

$$\begin{aligned}
\mathcal{L} &= \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-v(L_t)-\frac{\psi_W}{2}(\pi^W_t)^2\right)\\
&- \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_{W,t}-\gamma_{W,t-1})\pi^W_t(1+\pi^W_t)\\
&+ \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_{W,t}\left(v'(L_t)-\left(1-\tau^L_t\right)x_t u'(c_t)\right)L_t\\
&- \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_{P,t}-\gamma_{P,t-1})\pi^P_t(1+\pi^P_t)Z_t L_t + \frac{\varepsilon_P-1}{\psi_P}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W-1}x_t - Z_t\right)L_t\\
&+ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mu_t\left((1-\frac{\psi_P}{2}(\pi^P_t)^2)Z_t L_t - G_t - T_t - \left(1-\tau^L_t\right)x_t L_t\right)\\
&+ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Lambda_t\left((1+\pi^W_t)x_{t-1} - x_t(1+\pi^P_t)\right)
\end{aligned}$$

We now turn to the computation of the FOCs.

Consider

$$\psi_t := \frac{d\mathcal{L}}{dc} = u'(c_t) - \underbrace{\frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(1-\tau^L_t\right)x_t L_t u''(c_t)}_{\text{effect on wage inflation}}$$

FOC wrt π^W_t.

$$-\psi_W\pi^W_t - (\gamma_{W,t}-\gamma_{W,t-1})(2\pi^W_t+1) + \Lambda_t x_{t-1} = 0.$$

FOC wrt π^P_t.

$$-(\gamma_{P,t}-\gamma_{P,t-1})(2\pi^P_t+1) - \mu_t\psi_P\pi^P_t - \frac{\Lambda_t}{Z_t L_t}x_t = 0.$$

FOC wrt x_t.

$$0 = \left(1-\tau_t^L\right) L_t\psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(1-\tau_t^L\right)u'(c_t)L_t + \frac{\varepsilon_P-1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W-1}L_t$$
$$- \mu_t\left(1-\tau_t^L\right)L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$0 = \left(1-\tau_t^L\right)x_t\psi_t - v'(L_t) + \mu_t\left(\left(1-\frac{\psi_P}{2}(\pi_t^P)^2\right)Z_t - \left(1-\tau_t^L\right)x_t\right)$$
$$+ \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t) - \left(1-\tau_t^L\right)x_t u'(c_t)\right)$$
$$- (\gamma_{P,t}-\gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P-1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W-1}x_t - Z_t\right).$$

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt $1-\tau_t^L$.

$$0 = L_t x_t\psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}x_t u'(c_t)L_t - \mu_t x_t L_t.$$

Simplifying

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt $1-\tau_t^L$.

$$0 = \psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}u'(c_t) - \mu_t.$$
$$0 = \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}u'(c_t)\left(1 - \frac{-(c_t-T_t)\,u''(c_t)}{u'(c_t)}\right)$$

In this case, one can check that one has $\gamma_{W,t}=0$ (The wage Phillips curve is not a constraint)

FOC wrt π_t^W.

$$-\psi_W\pi_t^W + \Lambda_t x_{t-1} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t}-\gamma_{P,t-1})(2\pi_t^P+1) - \mu_t\psi_P\pi_t^P - \frac{\Lambda_t}{Z_t L_t}x_t = 0.$$

FOC wrt x_t.

$$0 = \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W - 1}L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

Using the FOC wrt to τ^L, we have:

FOC wrt L_t.

$$\begin{aligned} 0 = &-v'(L_t) + \mu_t\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t \\ &- (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W - 1}x_t - Z_t\right). \end{aligned}$$

Simplifying

$$\psi_W\pi_t^W = \Lambda_t x_{t-1}.$$

and

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) = \mu_t\psi_P\pi_t^P + \frac{\Lambda_t}{Z_t L_t}x_t.$$

FOC wrt x_t.

$$0 = \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W - 1}L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$\begin{aligned} v'(L_t) = &\mu_t\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t \\ &- (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W - 1}x_t - Z_t\right). \end{aligned}$$

Determining the path of public debt from the path of T_t

The dynamics of public debt is

$$B_t = (1+r_t)B_{t-1} + T_t$$

At the moment of the shock, at period 0, the planner can change capital tax.

$$1 + r_0 = \frac{-\sum_{k=t}^{\infty}\frac{T_k}{\prod_{j=t+1}^{k}(1+r_j^{FB})}}{B_{SS}},$$

C.6 RA analysis without (time-varying) τ_t^E, τ_t^L, with time-varying τ_t^S

With the same change of variable as in the previous case,

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^E,\tau_t^K,\pi_t^P,\pi_t^W,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-v(L_t)\right)\ell(di)-\frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t+T+w_tL_t\leq\Big(1-\frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_tL_t,$$

$$c_t=T_t+w_tL_t,$$

$$\pi_t^W(\pi_t^W+1)=\frac{\varepsilon_W}{\psi_W}\Big(v'(L_t)-w_tu'(c_t)\ell(di)\Big)L_t+\beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big],$$

$$\pi_t^P(1+\pi_t^P)=\frac{\varepsilon_P-1}{\psi_P}\Big(\frac{1}{Z_t}\frac{w_t}{(1-\tau_{ss}^L)(1-\tau_t^S)(1-\tau_{ss}^E)}-1\Big)+\beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_tL_t}\Big),$$

$$(1+\pi_t^W)w_{t-1}=w_t(1+\pi_t^P).$$

Define

$$z_t=\frac{1}{(1-\tau_{ss}^L)(1-\tau_t^S)(1-\tau_{ss}^E)}=\frac{1-\tau_{ss}^S}{1-\tau_t^S}$$

$$\begin{aligned}
\mathcal{L}=&\,\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(u(c_t)-v(L_t)-\frac{\psi_W}{2}(\pi_t^W)^2)\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_{W,t}-\gamma_{W,t-1})\pi_t^W(1+\pi_t^W)\\
&+\frac{\varepsilon_W}{\psi_W}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_{W,t}\left(v'(L_t)-w_tu'(c_t)\right)L_t\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_{P,t}-\gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_tL_t+\frac{\varepsilon_P-1}{\psi_P}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_{P,t}\left(wz_t-Z_t\right)L_t\\
&+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mu_t\Big((1-\frac{\psi_P}{2}(\pi_t^P)^2)Z_tL_t-G_t-T_t-w_tL_t\Big)\\
&+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Lambda_t\left((1+\pi_t^W)w_{t-1}-w_t(1+\pi_t^P)\right)
\end{aligned}$$

FOC wrt π_t^W.

$$-\psi_W\pi_t^W-(\gamma_{W,t}-\gamma_{W,t-1})(2\pi_t^W+1)+\Lambda_tw_{t-1}=0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t}-\gamma_{P,t-1})(2\pi_t^P+1)-\mu_t\psi_P\pi_t^P-\frac{\Lambda_t}{Z_tL_t}w_t=0.$$

FOC wrt w_t.

$$0 = L_t\psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}u'(c_t)L_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}L_t - \mu_t L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$\begin{aligned}0 = {} & w_t\psi_t - v'(L_t) + \mu_t\left(\left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right)Z_t - w_t\right) + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t) - w_t u'(c_t)\right) \\ & - (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(wz_t - Z_t\right).\end{aligned}$$

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt z_t.

$$0 = \gamma_{P,t}.$$

Simplifying

FOC wrt π_t^W.

$$\psi_W\pi_t^W = -(\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t w_{t-1}.$$

FOC wrt π_t^P.

$$-\mu_t\psi_P\pi_t^P = \frac{\Lambda_t}{Z_t L_t}w_{t-1}.$$

FOC wrt w_t.

$$0 = -\frac{\varepsilon_W}{\psi_W}\gamma_{W,t}u'(c_t)L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$v'(L_t) = +\mu_t\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t) - w_t u'(c_t)\right)$$

$$\mu_t = u'(c_t).$$

D Ramsey program for HA models

D.1 Flexible-price equilibrium

We here assume here that the planner must choose a common labor supply for all agents, in a flexible price economy: $\pi_t^P = \pi_t^W = 0$. The program is:

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^K,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i)-v(L_t)\right)\ell(di)\right],$$

$$G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq Z_t L_t + \int_i a_{i,t}\ell(di),$$

$$\text{for all } i\in\mathcal{I}\text{: } c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t} L_t,$$

$$a_{i,t}\geq -\overline{a}, \nu_{i,t}(a_{i,t}+\overline{a}) = 0,\ \nu_{i,t}\geq 0,$$

$$u'(c_{i,t}) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t}.$$

$$u'(c_{i,t}) = \frac{\beta}{1-MVCC_{i,t}}\mathbb{E}_t\Big[(1+r_{t+1})\,u'(c_{i,t+1})\Big]$$

The Lagrangian can be written as:

$$\mathcal{L} = \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i \omega_t^i(u(c_{i,t})-v(L_t))\ell(di) - \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i(\lambda_{i,c,t}-(1+r_t)\lambda_{i,c,t-1})\,u'(c_{i,t})\ell(di)$$
$$+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mu_t\left(Z_tL_t+\int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_tL_t - T_t\right).$$

We recall that $\psi_{i,t} = \omega_t^i u'(c_{i,t}) - (\lambda_{i,c,t}-(1+r_t)\lambda_{i,c,t-1})\,u''(c_{i,t})$. Compared to (30), we drop the FP subscript for the sake of simplicity. We compute the FOCs wrt four independent instruments: r_t, w_t, L_t and $(a_{i,t})_i$. The other instruments can be recovered from the constraints.

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1}u'(c_{i,t})\ell(di) = 0. \tag{61}$$

FOC wrt w_t.

$$\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) = 0.$$

FOC wrt L_t. Using the FOC on w_t:

$$\int_i \omega_{i,t}\ell(di)v'(L_t) = \mu_t Z_t = Z_t\int_i y_{i,t}\psi_{i,t}\ell(di).$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \right].$$

D.2 The HA economy with all instruments

The program is:

$$\max_{\left(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t \geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right],$$

$$G_t + (1 + r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t \leq \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2\right) Z_t L_t + \int_i a_{i,t} \ell(di),$$

$$\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + a_{i,t} = (1 + r_t) a_{i,t-1} + w_t y_{i,t} L_t,$$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0, \ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \left[(1 + r_{t+1}) u'(c_{i,t+1}) \right] + \nu_{i,t},$$

$$\pi_t^W (\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{1 - \tau_t^E} \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right) L_t + \beta \mathbb{E}_t \left[\pi_{t+1}^W (\pi_{t+1}^W + 1) \right],$$

$$\pi_t^P (1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P} \left(\frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^S)(1 - \tau_t^E)} - 1 \right) + \beta \mathbb{E}_t \left(\pi_{t+1}^P (1 + \pi_{t+1}^P) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} \right),$$

$$(1 + \pi_t^W) \frac{w_{t-1}}{1 - \tau_{t-1}^L} = \frac{w_t}{1 - \tau_t^L} (1 + \pi_t^P).$$

We can set:

- τ_t^S such that $1 - \tau_t^S = \frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^E)}$, hence $\frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^S)(1 - \tau_t^E)} - 1$ and $\pi_t^P = 0$.
- τ_t^E is a free parameter that can be deduced from π_t^W and the allocation. Hence, the wage Phillips curve is not a constraint.
- π_t^W only reduces utility and is an independent parameter that can be set through τ^L, hence $\pi_t^W = 0$

The program then reduces to the same one as in the flexible-price economy without union:

Recovering taxes from the allocation We then have

$$1 - \tau_t^E = \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \frac{\int_i y_{i,t} u'(c_{i,t}) \ell(di)}{v'(L_t)}$$

$$1 - \tau_t^L = \alpha w_t$$

$$1 - \tau_t^S = \frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^E)}$$

D.3 The HA economy without τ_t^E

We impose $\tau_t^E = 0$. The program is otherwise the same as in Section D.2. In particular, τ_t^S only appears in the price Phillips curve. As consequence, this equation is not a constraint and τ_t^S is set, such that $\pi_t^P = 0$. Inflation indeed only destroys resources here. We then obtain the following program:

$$\max_{\left(\tau_t^L, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t)\right) \ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq Z_t L_t + \int_i a_{i,t}\ell(di),$$

$$\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + y_{i,t} w_t L_t + T_t,$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t},$$

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t + \beta \mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big],$$

Because of $\tau_t^E = 0$, we cannot have simultaneously optimal labor supply and $\pi_t^W = 0$: the planner has to balance the relative costs of wage inflation with the suboptimal provision of labor supply. The Lagrangian is:

$$\begin{aligned}
\mathcal{L} = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1}\right) u'(c_{i,t})\ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W(1+\pi_t^W) \\
& + \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t - T_t\right).
\end{aligned}$$

We recall that in this economy, we have $\psi_{i,t} = \omega_t^i u'(c_{i,t}) - \left(\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1}\right) u''(c_{i,t}) - \frac{\varepsilon_W - 1}{\psi_W}\gamma_{W,t} w_t y_{i,t} u''(c_{i,t}) L_t$, where we also drop the superscript.

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t.

$$\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) = \gamma_{W,t}\frac{\varepsilon_W - 1}{\psi_W}\int_i y_{i,t}u'(c_{i,t})\ell(di).$$

FOC wrt L_t. Using the FOC wrt w_t:

$$-\int_i \omega_{i,t}\ell(di)v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) = 0.$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta\mathbb{E}_t\left[(1+r_{t+1})\hat{\psi}_{i,t+1}\right].$$

D.4 The HA economy without τ_t^E and τ_t^S with τ_t^L

In this case, there is no obvious simplification and the program is:

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^K,B_t,T_t,\pi_t^P,\pi_t^W,w_t,r_t,\Omega_t,\tilde{R}_t^N,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\Big(u(c_t^i) - v(L_t)\Big)\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t L_t + \int_i a_{i,t}\ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t}L_t$,

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0,\ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t},$$

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\Big(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}w_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\Big)L_t + \beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big],$$

$$\pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P}\Big(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)} - 1\Big) + \beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\Big),$$

$$(1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L} = \frac{w_t}{1-\tau_t^L}(1+\pi_t^P),$$

while the corresponding Lagrangian becomes:

$$\begin{aligned}
\mathcal{L} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W) \\
&+ \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{w_t}{(1-\tau_t^L)} - Z_t \right) L_t \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big((1 - \frac{\psi_P}{2}(\pi_t^P)^2) Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t) \int_i a_{i,t-1}\ell(di) - w_t L_t \Big) \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1+\pi_t^W) \frac{w_{t-1}}{1-\tau_{t-1}^L} - \frac{w_t}{1-\tau_t^L}(1+\pi_t^P) \right)
\end{aligned}$$

We now turn to the computation of the FOCs.

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t \frac{w_{t-1}}{1-\tau_{t-1}^L} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} \frac{w_t}{1-\tau_t^L} = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1} \hat{\psi}_{i,t} \ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t. Using the FOC wrt to τ^L, we have:

$$0 = \int_i y_{i,t} \hat{\psi}_{i,t} \ell(di) - \gamma_{W,t} \frac{\varepsilon_W - 1}{\psi_W} \int_i y_{i,t} u'(c_{i,t})\ell(di).$$

FOC wrt L_t. Using the FOC wrt w_t:

$$\begin{aligned}
0 = &- \int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Z_t + \frac{\varepsilon_W}{\psi_W} \gamma_{W,t} \left(v''(L_t) L_t + v'(L_t) \right) \\
&- (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t + \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} \left(\frac{w_t}{(1-\tau_t^L)} - Z_t \right).
\end{aligned}$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \right].$$

FOC wrt τ_t^L. We derive wrt $\frac{1}{1-\tau_t^L}$ and obtain:

$$0 = \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} L_t - \Lambda_t (1 + \pi_t^P) + \beta \mathbb{E}_t \left[\Lambda_{t+1} (1 + \pi_{t+1}^W) \right].$$

D.5 The HA economy without time-varying τ_t^E and τ_t^L with τ_t^S

I consider $\tau_{ss}^E = \left(\frac{\varepsilon_W - 1}{\varepsilon_W} \right)^{-1}$ and $\tau_t^L = \tau_{ss}^L$

In this case, there is no obvious simplification and the program is:

$$\max_{\left(\tau_t^S, \tau_t^K, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, \Omega_t, \tilde{R}_t^N, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t \geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \Big(u(c_t^i) - v(L_t) \Big) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right],$$

$$G_t + (1 + r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t + T_t \leq \Big(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \Big) Z_t L_t + \int_i a_{i,t} \ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1 + r_t) a_{i,t-1} + w_t y_{i,t} L_t,$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t} (a_{i,t} + \overline{a}) = 0, \ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1}) u'(c_{i,t+1}) \Big] + \nu_{i,t},$$

$$\pi_t^W (\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \Big(v'(L_t) - w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di) \Big) L_t + \beta \mathbb{E}_t \Big[\pi_{t+1}^W (\pi_{t+1}^W + 1) \Big],$$

$$\pi_t^P (1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P} \Big(\frac{1}{Z_t} \frac{w_t}{(1 - \tau^L)(1 - \tau_t^S)(1 - \tau_{ss}^E)} - 1 \Big) + \beta \mathbb{E}_t \Big(\pi_{t+1}^P (1 + \pi_{t+1}^P) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} \Big),$$

$$(1 + \pi_t^W) w_{t-1} = w_t (1 + \pi_t^P),$$

while the corresponding Lagrangian becomes:

$$
\begin{aligned}
\mathcal{L} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t)) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t}) \ell(di) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1}) \pi_t^W (1+\pi_t^W) \\
&+ \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right) L_t \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1}) \pi_t^P (1+\pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{w_t}{(1-\tau_{SS}^L)(1-\tau_{SS}^E)(1-\tau_t^S)} - Z_t \right) L_t \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big((1 - \frac{\psi_P}{2} (\pi_t^P)^2) Z_t L_t + \int_i a_{i,t} \ell(di) - G_t - (1+r_t) \int_i a_{i,t-1} \ell(di) - w_t L_t \Big) \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1+\pi_t^W) w_{t-1} - w_t (1+\pi_t^P) \right)
\end{aligned}
$$

We now turn to the computation of the FOCs.

FOC wrt τ_t^S. We derive wrt $\frac{1}{1-\tau_t^S}$ and obtain:

$$0 = \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} \frac{w_t}{(1-\tau_{SS}^L)(1-\tau_{SS}^E)} L_t$$

or

$$\gamma_{P,t} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} w_t = 0,$$

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t \frac{w_{t-1}}{1-\tau_{t-1}^L} = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1} \hat{\psi}_{i,t} \ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t}) \ell(di) = 0.$$

FOC wrt w_t.

$$0 = \int_i y_{i,t} \hat{\psi}_{i,t} \ell(di) - \gamma_{W,t} \frac{\varepsilon_W}{\psi_W} \int_i y_{i,t} u'(c_{i,t}) \ell(di).$$

FOC wrt L_t.

$$\begin{aligned}
\mathcal{L} = &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W) \\
&+ \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t \Big)
\end{aligned}$$

$$\begin{aligned}
0 = &- \int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) \\
&+ w_t \left(\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) - \gamma_{W,t}\frac{\varepsilon_W}{\psi_W}\int_i y_{i,t} u'(c_{i,t})\ell(di) \right)
\end{aligned}$$

Using the FOC wrt w_t:

$$0 = - \int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right)$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1+r_{t+1})\hat{\psi}_{i,t+1} \right].$$