Privilege Lost?

The Rise and Fall of a Dominant Global Currency

Kai Arvai* Nuno Coimbra†

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Abstract

Economic size matters for three key aspects of a dominant currency: safety, liquidity, and insurance. First, larger countries tend to be more diversified, increasing their debt capacity and safety. Secondly, the larger the share of a country in the supply of safe assets, the more liquid and attractive its bonds are. Finally, the larger a safe country's share in the world economy, the more its bonds appreciate in downturns, providing insurance for investors. We argue that these three aspects reinforce each other and show how they erode if the dominant country grows less than the rest of the world.

Keywords: Dominant Currency, Safe Assets, US Dollar, Default

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*Banque de France. E-mail address: kai.arvai@banque-france.fr

†Banque de France and CEPR. E-mail address: nuno.coimbra@banque-france.fr

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1 Introduction

"The US Treasury market is [...] a really important feature to promoting the dollar's continued leadership around the globe. [..] Having that reliable, safe and readily accessible and tradeable asset is critical. It was critical to the British in their time as the leaders in currency. It was critical to the Dutch before that."

- Gary Gensler, chair of the SEC, FT March 2024.

The United States and the US Dollar play a crucial role in the international financial system. Dollar-denominated US government bonds are used extensively as safe assets by investors around the world. This special demand can give rise to yield differentials, lowering the required interest rates the US needs to pay when compared to other countries. This became known as the US' *exorbitant privilege*, a term coined in the 1960s by Valéry Giscard d'Estaing, at the time the French Minister of Finance, when discussing the position of the USD in the Bretton Woods system.

More recently, this expression has often been applied to return differentials between the US investment position compared to the rest of the world. Gourinchas and Rey (2007), Maggiori (2017) and Gourinchas and Rey (2022) describe how the US ends up holding riskier foreign assets than the rest of the world, who in turn holds a larger proportion of safe assets from the US. This implies that the US earns a higher return in normal times ('exorbitant privilege') while suffering losses in crisis (exorbitant duty').

Some recent papers such as Jiang et al. (2022) and Atkeson et al. (2022) argue that the privilege has waned or even disappeared since the Great Financial Crisis. The debate is still open whether this was due to a shift in expected return differentials or a higher than expected frequency of crisis. This paper goes back to the classic meaning of the exorbitant privilege and focuses on the cost of debt and demand for liquid safe assets.

Motivated by that, we ask how a country obtains the position of a global safe haven in the first place, how it might eventually lose that position and how safety is linked to convenience yields. We argue that both safety and size are necessary conditions for becoming a dominant currency, but also that these two properties are inherently connected. The larger the economy, the more diversified it will tend to be and so, *ceteris paribus*, the degree of safety will rise with size. This safety leads to lower interest rates and also a larger debt capacity, reflected in a higher debt-to-GDP ratio. At the same time, safety and size alone are not sufficient for a country to earn the exorbitant privilege.

We identify two key channels, both linked to country size, through which convenience yields emerge. First, the larger the country, the deeper the market for its bonds, which increases the probability that investors match with lenders in order to finance their investment opportunities. This generates a *liquidity* premium for smaller countries. Secondly, countries with a larger share in the world economy appreciate during downturns, generating an *insurance* premium. We demonstrate how both of these channels require the safety status of the dominant country to generate convenience yields. Finally, we show how rate differentials between two countries decline as relative sizes converge. This eventually leads to a loss of the convenience yield privilege when both countries are equally large and safe, as the liquidity and insurance status erode.

Interest rate differentials in our model are composed of a risk, a liquidity and an insurance premium, with these three aspects reinforcing each other. While an abundant supply of bonds is good from the perspective of liquidity provision, there is a trade-off as the country's risk premium increases at the same time. As in the Triffin dilemma (Triffin, 1961), a country that acts as the global liquidity provider needs to be prudent not to overextend itself. Furthermore a loss of the exorbitant privilege also implies an erosion in debt capacity. Safety and insurance are also linked to each other, as the large safe heaven status relies on the dominant country remaining safe during crises. Finally, the insurance and liquidity channels also reinforce each other. Payoffs of investment opportunities in large and liquid country are more valuable in bad states than those from a small country. Moreover, increased demand due to the insurance channel increases debt capacity, ultimately leading to a larger bond supply and enhanced liquidity.

We develop a model in which a government strategically defaults on its debt obligations when repayment is too costly relative to default. Investors anticipate this possibility and demand higher or lower interest rates depending on the distribution of the underlying shocks. In larger economies, idiosyncratic shocks originate from more and more sources and therefore become less granular and less important, effectively diversifying fundamental risk.

We discuss which features in an economy determine whether there is growth with or without diversification. The first feature is the correlation of shocks. A large economy with a low shock correlation across units enjoys a lower aggregate shock variation. This decreases the likelihood of an aggregate tail-event that could lead to a default. Unlike optimal currency areas, an optimal debt area requires a low correlation of shocks. Furthermore, we also formalize how these idiosyncratic shocks are aggregated. This ag-

gregation reflects how shocks propagate through the economy, but also the government's institutional ability to perform transfers across units[1]. Growth, together with low shock correlation and such institutional features, decrease the risk premia.

We embed this framework in a two-country open economy model in which investors can save in bonds of two countries and governments behave strategically. We show that the relative size of both countries matters for interest rate spreads. As the small country grows and diversifies, default becomes less likely, its interest rate falls and debt-to-GDP rises, while the larger country's interest rate increases slightly. We also provide stylized facts showing that economic size is correlated with safety and a larger debt capacity.

To generate liquidity premia, we also include investment opportunities that are easier to finance when bonds are safe and abundant. The larger the country's supply of assets relative to the rest of the world, the more liquid they will be. Liquidity is modeled similar to Chahrour and Valchev (2022). However, if a country's share of the world economy falls, it will see its liquidity premium erode. In this environment, we discuss the role of financial development for liquidity yields. If one country is financially more developed, interest rate spreads can persist even with equally large and safe economies.

Insurance yields arise in our setup because investors want to hedge against shocks to their consumption. When a country is in a recession, its good will be scarce and its price high. But the larger the country, the larger the impact in the marginal utility of global investors, due to the good's share in their consumption basket. The larger this positive covariance between the price of the underlying good and marginal utility, the stronger is the insurance value of the bond. Bonds of (safe) larger countries will therefore be more expensive than bonds of a smaller country whose shocks affect the stochastic discount factor of global investors less, as in Hassan (2013). We highlight an important connection between insurance and safety: only if there is negligible default risk can this channel capitalize on the size of the country and generate *insurance* convenience yields. We also demonstrate how risk aversion of investors can drive up the insurance premium.

Literature The literature that discusses the US' exorbitant privilege has mostly focused on return differentials between the international investment position of the US, when compared to the rest of the world vis-a-vis the US. Gourinchas and Rey (2007) highlighted that the net external position to the US was akin to being the "world banker", borrowing short and lending long. In Gourinchas and Rey (2022), the same authors describe how

[1]This can also be interpreted as a proxy for institutional quality and its ability to provide inter-regional or sectoral smoothing.

the US position also has an element of an insurer for the rest of the world, as it provides safe assets while holding risky assets. The return differentials then arise because of risk premia and the US provides wealth transfers to the rest of the world during crises (the 'exorbitant duty'). The exorbitant privilege is therefore akin to an insurance premium during normal times. A similar view is taken by Maggiori (2017) who shows that the more financially developed country will take the riskier investment positions and so should be short in debt and long in equities. We argue that the larger country is also safer because it is more diversified, increasing its debt capacity as it will have a lower probability of default for a given debt-to-GDP ratio. We emphasize that the dominant country also needs this safety status to act as a global insurer. Moreover, its liquid safe assets can also easily be sold or used as collateral when investment opportunities arise, which can additionally increase spreads due to convenience yields.

What makes government debt safe? He et al. (2019) highlight strategic interactions between investors and the size of a country's debt market. Our paper focuses on the determination of a country's fundamentals as its size grows, and on strategic competition between governments' bond supply. We also allow for an endogenously changing aggregate amount of bonds, with governments taking into account both the impact on safety and liquidity.

Jiang et al. (2021) develop a model of the global financial cycle in which the US dollar plays a special role as a provider of safe assets. They assume that US bonds are safe and that convenience yields arise via money-in-utility. Our paper asks how such safety is generated in the first place and how this safety links to the emergence of convenience yields when compared to other foreign bonds. We therefore put the focus on the question why the US became the safe haven and overtook other countries. On China, Clayton et al. (2025) discuss how its slow entry into the international financial market can be rationalized via reputation building to gain credibility as a safe currency issuer. Our paper focuses more on the macroeconomic fundamentals as a determinant for safety. Brunnermeier et al. (2024) discuss what determines the fundamental price of such safe assets. They emphasize that the price also includes –next to the pure cash flow value– service flows via insurance. Our model emphasizes service values such as liquidity and insurance as well, but highlights how a reduction in safety can compromise the liquidity and insurance status.

Some recent papers argue that the privilege has waned or even disappeared in the years since the Great Financial Crisis. Jiang et al. (2022) show that the ex-post return

differential was no longer positive for the US from 2010 to 2019, and similarly Atkeson et al. (2022) demonstrate that the US' NFA position has substantially deteriorated. Our paper contributes to this strand of literature by explicitly modeling why US (government) assets are considered to be more safe and how the US obtains (and might lose) this position. Vicquéry (2022) provides systematic accounts for the historical structure of the international financial system, describing how the British Empire was at the center of the international monetary system, but was then overtaken by the US. Eichengreen et al. (2017) document in a similar way the evolution of the international financial system, noting how several currencies can be at the same time relevant for the global economy during a transition phase.

We are not the first who argue that the size of an economy and its global trade share are important for currency returns. Hassan (2013) shows that bonds of larger countries feature lower interest rates, as they insure against shocks that affect a larger part of the world economy. Therefore, larger countries would also be 'privileged' in the sense of low cost of debt. Our model includes this *insurance* channel explicitly by considering imperfect substitution across home and foreign goods, and by highlighting the safety status of the country as a necessary condition for insurance. We also discuss how investors' risk aversion can drive up the insurance premium. In a similar way, Richmond (2019) highlights the role of a country's trade centrality and provides evidence that a larger world GDP share of a country reduces its interest rates.

Choi et al. (2022) note that the US is the most important provider of safe assets and could exert market power over its asset supply. This results in under provision of US bonds and a markup with lower interest rates for the United States reflecting the exorbitant privilege. In our paper, we analyze changes in relative country size and the impact of supply on market depth and liquidity. We also consider governments as price makers for the provision of safe assets. Bianchi et al. (2023) provide a theory why convenience yields arise. They argue that financial institutions have precautionary demand for (safe) US assets as they are subject to volatile financial flows. This demand gives rise to convenience yields. We model the additional liquidity element of safe bonds as the ability to liquidate assets to finance investment opportunities, which in turn drives up demand for the safest bond and consequently lowers its interest rates. We also highlight the role of a country's financial development in explaining the magnitude of such liquidity yields.

Farhi and Maggiori (2018) develop a model of the international monetary system, in which a hegemon issues international reserves to the rest of the world. The hegemon has

–as in our model– a lack of commitment and can implicitly default on its reserves via a depreciation of its currency. The stability of the monetary system is determined by how many reserves are issued by the hegemon, which ultimately determines the probability of a default. They also discuss how the system changes if competition arises by other reserve issuers. Our approach differs as it explicitly considers the size of the country as an important determinant for its debt capacity, the insurance value of its bonds and their liquidity. Not only does the amount of debt relative to GDP matter for the default probability, but also the total amount of GDP and market size. Furthermore, we emphasize that governments consider the trade-off between safety on the one hand, as well as having large and liquid markets on the other hand.

Coppola et al. (2023) establish a liquidity-based theory in which economic agents decide to denominate their debt in the most liquid market. The government of the most liquid market has then the strongest incentive to invest to make its market even more liquid, reinforcing its status as a dominant currency. Our paper emphasizes the role of liquidity as well, and highlights its interaction with the economic size of the corresponding country, as well as the bonds' safety as a necessary condition. Importantly, a too large bond market could endanger the safety status of the dominant country. Our way to model the emergence of liquidity yields is very close to Chahrour and Valchev (2022) who emphasize the role of the overall size of an asset market. The larger the share of the bond in world markets, the easier it is to facilitate transactions. This is similar to Doepke and Schneider (2017) who show how agents coordinate on the usage of one dominant currency to reduce exposure to relative prcie risk and increase matching efficiency.

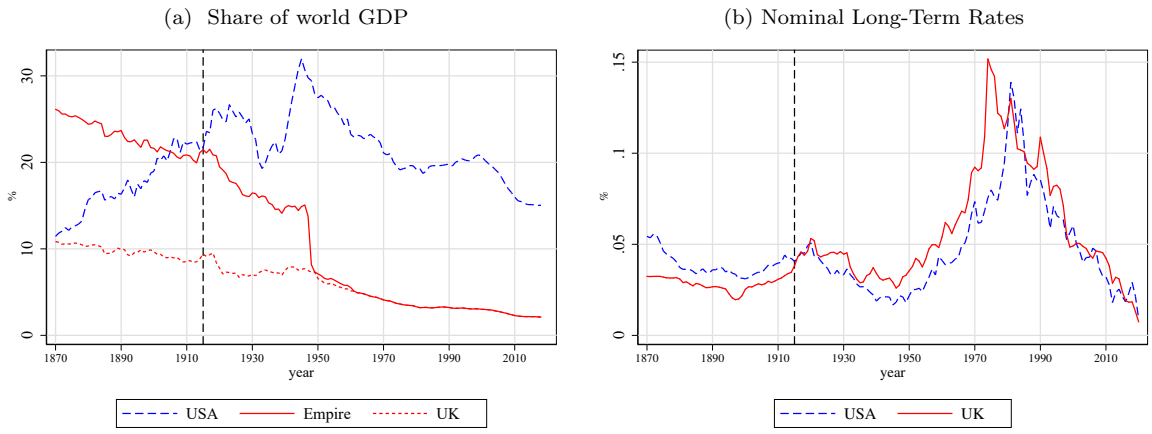
Last, there is a literature that measures the role of convenience or liquidity yields for US government bonds. Krishnamurthy and Vissing-Jorgensen (2012) analyze the aggregate demand for treasury debt and find that the special status of US treasury in terms of liquidity and safety drive down its interest rate by on average 73 basis points between 1926 and 2008 with 46bp attributed to liquidity and 27bp to safety. This special status of US debt saves the government interest rate costs of about 0.25 % of its GDP, a fact that is also present in our analysis as higher diversification and liquidity allow for a larger debt capacity of the dominant global currency country. More recently, Du et al. (2018) document a clear trend in the decline of US convenience yields, in particular for medium and long-run maturities. Our model also yields a decomposition of interest rate spreads into a credit risk, liquidity and insurance components. We show that convenience yields are a function of relative country size and that a decline in the hegemon's share in global activity reduces convenience yields.

2 Historical and stylized facts

There are not many instances where a country lost its position as the global dominant currency issuer, but one well-known example is the USD taking the spot of the British Sterling, as the British Empire waned. In Figure 1 (a), we highlight the historical evolution of the share of world GDP of the United States and the British Empire, consisting out of the United Kingdom and all of its colonies. Towards the end of the 19th century and the beginning of the 20th century, the US share of world GDP grew substantially, eventually overtaking the British Empire and becoming the world's largest economy at around 1915 (vertical dashed line).

Figure 1 (b) documents that this change in the position as the world's largest economy also coincided with a systematic change in interest rates that both central governments in Washington and London had to pay. As the right panel shows, the long-term interest rates of 10 year government bonds were higher for the United States throughout the period before 1915 [2]. After World War 1, the role reversed and the US was the country that paid lower interest rates. This might not only reflect the loss of the status as the entity with the greatest GDP, but also the loss of the safety status.

Figure 1: World GDP and interest rates



Note: World GDP share of the United States, the UK and the British Empire. The British Empire includes all countries and colonies for which the Empire has established control in a given year. GDP is in real 2011 USD, adjusted for relative price differences. *Source:* Bolt et al. (2018) and own calculations. Nominal long-term interest rates (10 year) of the US and the UK. *Source*: Jordà et al. (2019) and own calculations.

We furthermore examine different measures of interest rate spreads in Table A2 and Figure AA1, which take into account inflation and exchange rate movements. The evidence for an important shift after 1915 is present in all these measures, as is the waning of

[2] Chen et al. (2022) and Pflueger and Yared (2024) make a similar point about historical interest rate differentials between the dominant currency providers and other countries.

the privilege after the Great Financial Crisis. This is indicative of the privileged position the United States occupied for almost a century, as the global provider of safe assets and the USD as the dominant global currency.

More recently, the difference between both countries eroded substantially. Since 2009 spreads are very close to zero across the board and no longer even positive when looking at implied yields rather than bond returns, reflecting the possibility that the US could be losing its exorbitant privilege, as argued by Atkeson et al. (2022) and Jiang et al. (2022). As we will argue in this paper, this is also related to the accelerated fall in the share of the United States in global GDP since the 2000s.

2.1 Size, diversification and default

At the heart of our argument is that size combined with a sufficient degree of diversification allows a government to create safe, liquid bonds that can be used to insure global investors. Such governments have a higher debt capacity and can issue large amounts of bonds without increasing default risk too much.[3] A sufficiently large debt market then allows for the emergence of additional returns (convenience yields), as larger markets are more liquid and it becomes easier to facilitate transactions with that bond. In this section, we provide some simple stylized facts highlighting the correlations between size, volatility, and default.

Size and Volatility Larger countries tend to be less volatile than smaller ones, as can be seen in Figure 2 (a). This chart plots the standard deviation of GDP growth rates against the country's average log share of world GDP (PPP, 1980 to 2023). Moreover, countries with exceptionally large volatility are small, or –if they are larger– have a low degree of diversification in the economy, as they are large commodity producers, highlighted in red in Figure 2 (a). This correlation is in line with Di Giovanni and Levchenko (2012) who document a similar correlation for the period 1970-2006. Depending on the correlation of shocks and institutional features, our model is able to generate both large countries with low aggregate volatility and large countries with high volatility.

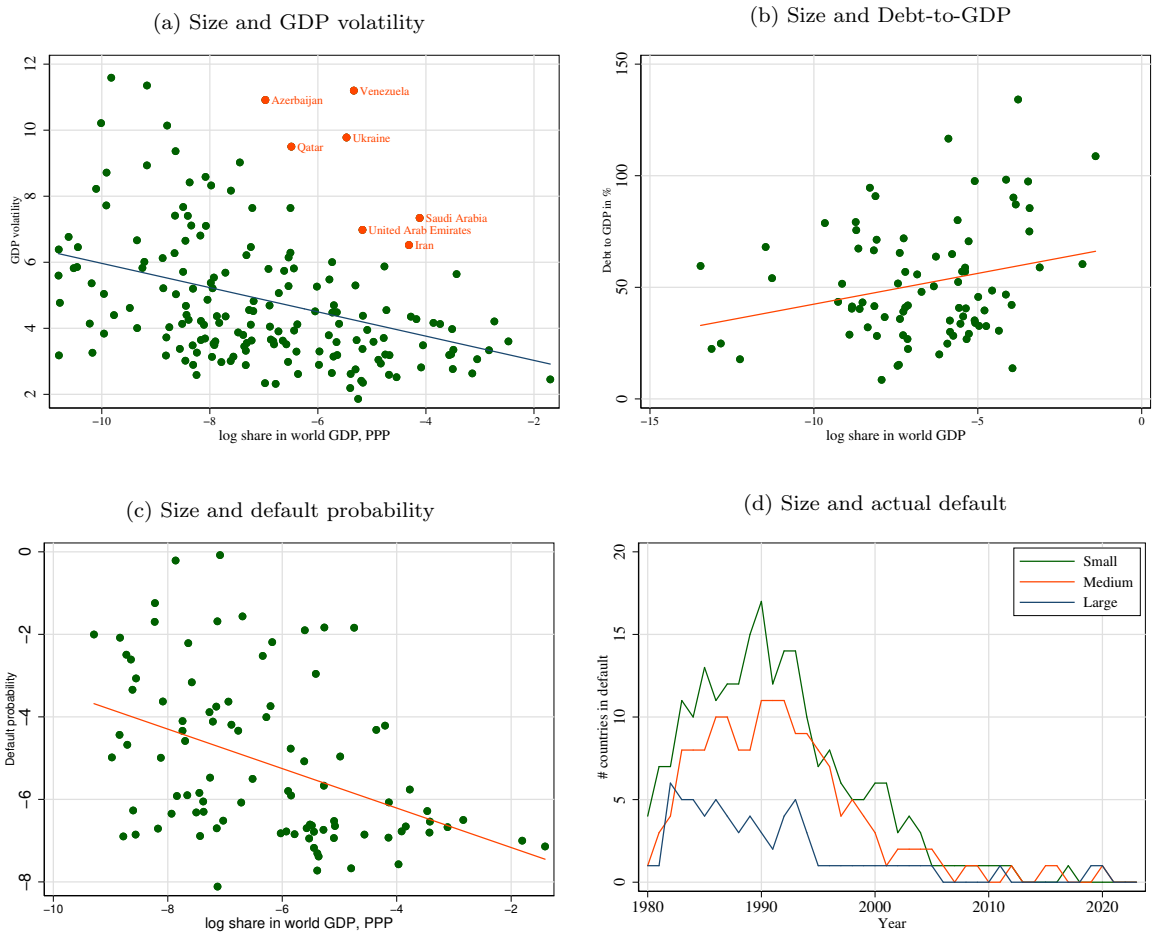
Size and Safety Using estimates of a government's default probability provided by Bloomberg[4] we show in Figure 2 (c) that there is a significant and negative correlation

[3] Imbs and Wacziarg (2003) noted that there's a U-shape in diversification as a country develops. That result is expressed relative to per capita income, as it concerns itself with development stages rather total size of the economy.

[4] Bloomberg uses its own SRSK model to estimate these probabilities of default based on several factors such as real GDP growth, foreign reserve to debt ratio, bank non-performing loan ratio, among others.

between size and safety. Large countries tend to have lower probabilities of default. Using the dataset of Asonuma and Trebesch (2016) for actual default events, we furthermore document in Figure 2 (d) that large countries have historically defaulted less than medium sized countries, while medium-sized countries have defaulted less than small countries. This empirical pattern supports a key feature of the model in which large countries tend to be safer. This correlation is crucial for the global insurance status.

Figure 2: Size and Safety



Note: Panel (a) shows the log GDP share and GDP volatility of 176 countries over time. Mean between 1980-2023, GDP in PPP with IMF data. Panel (b) shows the log GDP share and debt-to-GDP ratio of 84 countries in 2019 with IMF data. Extreme outliers in terms of debt-to-GDP are excluded. Those are Greece, Japan and Venezuela which are all relatively large countries. Panel (c) shows the probability of default of 99 countries for one year government bonds according to Bloomberg CDS data correlated with the corresponding share in world GDP, all data in log. Panel (d) shows the number of countries in a default state, sorted into three groups of small, medium and large countries. Each group has the same number of countries. *Source: IMF, Bloomberg, Asonuma and Trebesch (2016)*

Size and Debt-to-GDP An important implication of our model is that countries with better fundamentals have higher debt capacity. We should then observe higher debt-to-GDP ratios given the correlations described above. This can also be observed in the data. In Figure 2 (b), a positive correlation between size and debt-to-GDP ratios around the

world is apparent. The correlation of size and debt capacity is an important requirement in our model, as the ability to supply safe bonds abundantly is a key factor in determining liquidity premia.

3 Baseline model

We consider an economy with 2 countries, Home (H) and Foreign (F). Each country is populated by one central government and investors. As standard in the endogenous default literature, we assume the governments are impatient relative to investors, providing a motive to borrow. Due to limited commitment, the government has the option to (costly) default in period 2.

The two countries can be asymmetric in terms of their size $N \in \left\{N^H, N^F\right\}$. Size is defined as the number of economic units n that the country consists of. For now, units are ex-ante identical within and across countries and later we will explore heterogeneity in unit size across countries. We use the concept of unit as an umbrella term, which encompasses very different forms of growth. It could be interpreted as colonies or regions during the days of the British Empire, or as sectors, of an economy growing in complexity. The fundamental aspect that defines a unit in our framework is that each is a source of production, subject to idiosyncratic shocks. If economic growth occurs via an increase in the number of units, then it leads to diversification, as we will detail later. However, if it occurs by a proportional increase in the size of each unit, this will be a form of growth without diversification. The model is therefore able to distinguish the different impact of growing with and without diversification. For simplicity, we assume that units are ex-ante symmetric and the total mass of investors (and therefore their wealth) is proportional to country size N.

Section 3.1 details the government problem and output properties as a function of size, shock correlation and institutional characteristics. Section 3.2 details the investor's behavior in which the insurance motive becomes clear, while section 3.3 explains the setup for the investment opportunities to give rise to liquidity premia.

3.1 Government

We assume that the central government receives utility from a nation-wide aggregate of government consumption G_t:

$$\mathbb{E}\left[\log(G_1) + \delta \log(G_2)\right]. \tag{1}$$

δ is the time discount factor and log preferences imply some degree of (constant relative) risk aversion and willingness for intertemporal consumption smoothing. Government collects tax receipts by taxing the domestic good Y_t, which is an aggregator of N unit-specific endowments $y_{j,t}$:

$$Y_t = \left(\sum_{n=1}^{N} \omega_n^{\frac{1}{\theta}} y_{n,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}.$$

ω_n is a measure of unit size, and $y_{n,t}$ indicates unit n endowment in period t. We assume that the endowment is known and identical in the first period, while it is unit-specific and stochastic in the second. The shock process is the same for each unit with a cross-unit correlation ρ[5], and θ is the elasticity of substitution of endowments across units. When $\theta \to \infty$, output is simply the (weighted) sum of endowments across all units, as if endowment in one unit is perfectly substitutable with endowment from other units. For lower values, endowments become more complementary, implying that endowment dispersion across units in the second period will reduce the overall aggregator. $\theta = 1$ corresponds to a Cobb Douglas aggregator while the extreme case of $\theta \to 0$ gives the Leontief case, in which the amount of domestic goods Y_t is equal to the lowest endowment realization across all units.

In period 1, the government issues bonds B to Home and Foreign investors at price Q. The government budget constraint is then

$$G_1 = QB_1 + \tau Y_1 \tag{2}$$

where τ denotes the tax rate which is assumed to be fixed for simplicity.

As in Hassan (2013), we assume that each unit of bonds pays out $\hat{P}_2$ units of Home goods, where $\hat{P}_2$ is the price, expressed in home goods, of the investor's consumption

[5]Later, we will calibrate the marginal probability distribution to be uniformly distributed between $\underline{u}$ and an upper limit $\bar{u}$, with the cross-unit correlation given by a copula, but for now the results are more general. We also consider equally sized units.

basket in period one.[6] More details on the payouts are provided in the investor's section 3.2. The government either pays back all the debt B to investors, or it partly defaults. In case of default, it only pays back a share $\kappa < 1$ of the original debt repayment to investors and it suffers a dead-weight loss of ψ of output Y_2 that cannot be recovered by the government or investors. Government consumption is then given by

$$G_2 = \begin{cases} \tau Y_2 - B\hat{P}_2 & \text{if no default} \\ (1-\psi)\tau Y_2 - \kappa B\hat{P}_2 & \text{if default} \end{cases}$$

The government defaults if country wide consumption G_2 is bigger under default than under repayment, that is if default costs are smaller than the debt haircut:

$$\psi\tau Y_2 < (1-\kappa)B\hat{P}_2 \tag{3}$$

Default therefore happens in recessions, when aggregate output is very low. Given the stochastic process for Y_2, this inequality is satisfied with probability p. The (ex-ante) default probability is therefore a function of debt, bond prices, model parameters and country size: $p = p(B, Q, \theta, \kappa, \psi, \rho, N)$. The impact of having more units on the default probability crucially depends on the shock correlation and the institutional features as we detail below.

3.1.1 Elasticity of substitution θ

To highlight the role of θ we consider a polar case of perfect substitutability and discuss features in intermediate cases that include Cobb Douglas and Leontief.

$\theta \to \infty$ (perfect substitutability) For $\theta \to \infty$, the CES aggregator sums up each unit. The government then defaults if the (weighted) average endowment $\bar{y}_2$ is lower than the haircut $1-\kappa$ on average repayment costs per unit, that is if

$$\psi\tau\bar{y}_2 < (1-\kappa)\frac{B\hat{P}_2}{N}.$$

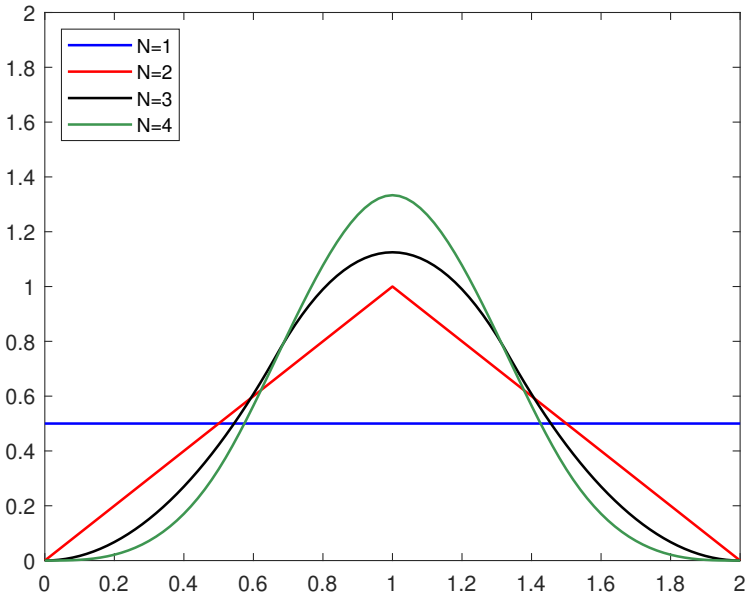
where $\bar{y}_2 = \sum_{n=1}^{N} \omega_n y_{2n}$ is the weighted average nationwide output. $\bar{y}_2$ has a probability distribution that is a function of N, and the inequality is satisfied with a certain probability (the default probability p). Under equal weights, if y_{2n} is uniform and IID, the

[6] Having the bonds payout $\hat{P}_2$ instead of is not strictly necessary in our model, but we keep the setup as close to Hassan (2013) as possible.

relevant distribution of the sum is an Irwin-Hall distribution, see Appendix B.1.

As long as the unit processes are not perfectly correlated ($\rho \neq 1$), considering more units makes the overall variability of the left hand side smaller for $\theta \to \infty$. Therefore, the inequality is less likely to be satisfied with larger N.[7]

Figure 3: PDF of aggregate output for $\theta \to \infty$ and $\rho = 0$



Note: Figure shows the PDF of $\bar{y_2}$ ($\theta \to \infty$). It is plotted for $N = \{1, 2, 3, 4\}$ where $y_{2n} \sim Uni(0, 2)$ and iid.

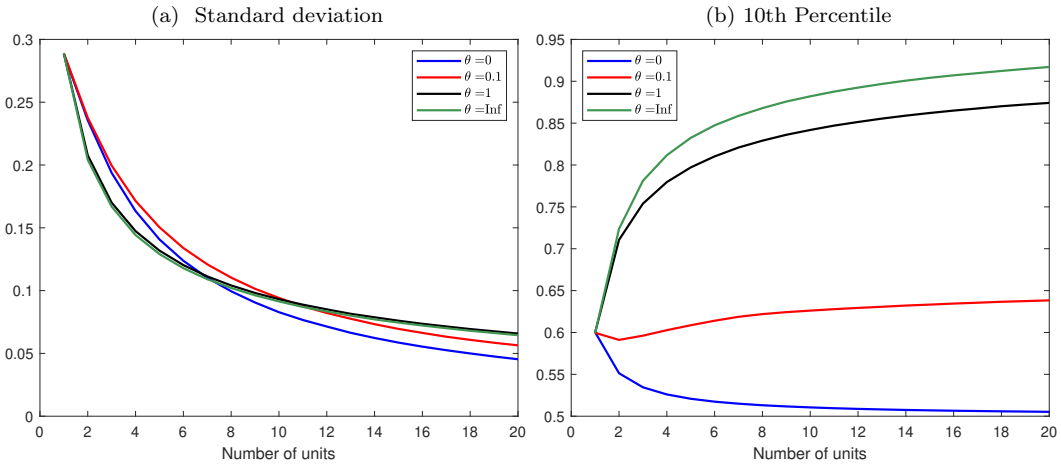
Figure 3 displays the probability distribution function of average national output for the polar case and varies the number of units between $N = 1$ (blue line) and $N = 4$ (green line). In the case of perfect substitutability ($\theta \to \infty$), the probability mass of the sum of uniforms increasingly centers around the mean while the tail mass shrinks substantially. The more units you have, the more likely it is that the average output is close to the mean. If mean output is large enough, this makes default less likely the more units there are.

$0 \leq \theta < \infty$ **(Cobb Douglas and intermediate)** For intermediate values, we can compute the probability mass of the tails to assess the extent countries are able to diversify. Lower values of θ indicate some degree of complementarity between units. This implies that the mean output decreases with smaller θ and the standard deviation of output decreases with a rising number of units. Overall, there is a diversification effect of having more units, even for small θ as there is less and less mass at the tail, as indicated in Figure 4. This is true for the case of Cobb Douglas ($\theta = 1$), as well as for small values such as

[7] As long as the probability of default is lower than 50%.

$\theta = 0.1$. Only if production is very complementary, such as the Leontief case with $\theta = 0$, the mass concentrates increasingly on the left which indicates rising tail risks.

Figure 4: Average output properties as a function of size



Note: Graph (a) shows the standard deviation, while (b) computes the 10th percentile of average output as a function of the number of units, for different values of θ. The shock process is Uni(0.5,1.5).

3.1.2 Cross-sectional output correlation ρ

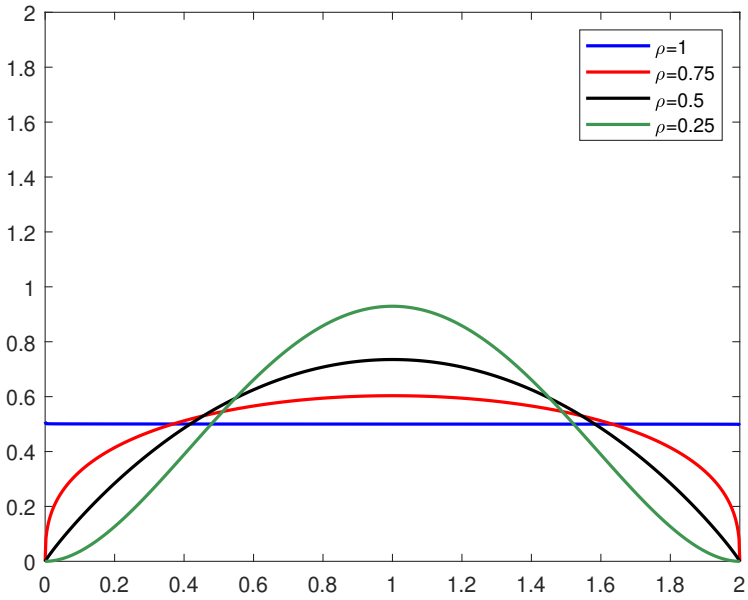
We also consider different values for the correlation of shocks across units to highlight the importance of diversification. Correlation affects the impact of size on diversification but also introduces limits to the degree of diversification that size can achieve. With positive correlation, economies with infinite units will still feature aggregate risk. This correlation could be interpreted as a measure of production complementarities, common components or simply reflect a measure for the diversity across economic units in a particular country.

We introduce correlation across units by making use of Gaussian copulas. A Gaussian copula is a joint probability distribution function over the hypercube $[0, 1]^N$ which can be mapped to the joint distribution of y_{2n} for $n = 1, .., N$. One particular feature of Gaussian copulas is that marginal probability distributions remain uniform and it can be fully specified by a simple correlation matrix Ω. For simplicity, we set the non-diagonal elements of Ω to be all equal to ρ, so we can characterize the copula with a single parameter.[8] Since marginal distributions are uniform, when $\rho = 0$ we have the same distribution as in the previous section, with each y_{2n} being drawn from a iid uniform distribution.

The higher the correlation across units, the closer the distribution gets to the original uniform distribution. In the extreme case of perfect correlation (the horizontal blue line),

[8]Since Ω is a correlation matrix, diagonal elements are all equal to 1, and we set all other elements to ρ. Note that ρ will generally be related, but not exactly equal, to the cross-sectional correlation, except in the cases of perfect ($\rho = 1$) or no ($\rho = 0$) correlation where the two coincide.

Figure 5: PDF of average aggregate output, N=4



Note: Figure shows the PDF of $\bar{Y}_2$ ($\theta \to \infty$). It is plotted for $N = 4$ where $y_{2j} \sim Uni(0, 2)$ withcorrelation ρ across units.

adding more units does not alter the aggregate distribution, as all units draw the same shock. As a consequence, the lower the correlation across units, the more diversified an economy gets and the less likely will be a default of the government. In a sense, an *optimal debt area* requires low (or even negative) correlation of shocks across units, as large nationwide recessions become less likely the more diversified the country is. Indeed, the literature has emphasized the importance of tail risks, by observing that interest rate spikes and sovereign default usually coincide with heavy recessions, see for example Arellano (2008), Neumeyer and Perri (2005) and Uribe and Yue (2006). In our setup more units with low shock correlation create safer and less variable fundamentals for the nationwide government, increasing the debt capacity of the country.

3.1.3 Nationwide and local shocks

We also consider a country-specific TFP component Z that is stochastic and scales output across all regions, providing an additional limit to diversification. Output in each region is then given by

$$y_{2n} = Z \cdot \epsilon_n \tag{4}$$

where $\epsilon_{n,t}$ is a unit-specific shock at time t, following the stochastic endowment process described in the previous sections. The probability of default still follows inequality (3). The nationwide component is independent of the number of regions, so it does not feature

a diversification effect. Aggregate output will be a combination of the country-specific shock Z and the un-diversified component of the unit-specific shocks.

3.1.4 The bond supply schedule

The government maximizes (1) subject to the aggregator and the payouts in each period and state. In our baseline setup, governments act as price-makers, behaving as monopolistic suppliers of their own bonds. They also engage in Cournot-style competition for funds in the international market. The problem of the Foreign government is analogous, with all variables being denoted by a star * and with the distinction that endowments are in country-F specific goods.

Under Cournot competition, interest rates and default probabilities are endogenous objects that depend not only on each country's choice of bond supply, but also the other country's. We therefore solve the two period-model via an iterative value function algorithm. Details of the government optimization problem are in Appendix B.2.

3.2 Investors

There is a mass of global investors proportional to the number of units in both countries $N + N^*$. Investors are ex-ante homogeneous and have CRRA preferences

$$U = C_1^{1-\sigma} + \beta E\left(C_2^{1-\sigma}\right) \tag{5}$$

where $\beta < 1$ is the time discount factor and σ the relative risk aversion coefficient. The aggregate consumption good is also a CES composite of home and foreign goods:

$$C_t = \left(\alpha^{1/\eta}\, {C_{H,t}}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{1/\eta}\, {C_{F,t}}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

$C_{j,t}$ indicates consumption of country j goods. The weights are allowed to vary according to the relative size of countries.

We normalize the price of the home good to 1 and assume that there are no trade frictions, so the law of one price always holds for both goods. The CES aggregator implies that the price index of the consumption basket (in terms of home goods) is $P_t = \left(\alpha + (1-\alpha)\, {P_{F,t}}^{1-\eta}\right)^{\frac{1}{1-\eta}}$. We also assume weights are proportional to the share of each country in the world economy such that $\alpha = \frac{E(Y_H)}{E(Y_H)+E(Y_F)}$. Since units are equal sized in the baseline model, this becomes (for now) equivalent to $N/(N+N^*)$.

Following Hassan (2013), the H bond pays out $\hat{P}_2 = P_1$ at $t = 2$, reflecting that bonds pay the exact number of H goods required to buy one unit of H's consumption bundle fixed at first-period prices. The F bond is similar, just that the value of the payment is fixed in terms of F goods. When expressed in terms of the numeraire (home goods) we have $\hat{P}_2^* = P_1^* \frac{P_{F,2}}{P_{F,1}}$.

If H goods are scarce in period 2, P_2 is low and the H bond provides a larger real return (in terms of the consumption basket). The reverse effect is true if F is scarce, $P_{F,2}$ is high relative to $P_{F,1}$, so the F bond pays higher real returns than expected. The main difference is that the larger the country, the larger the comovement of prices with marginal utility due to its size in the consumption basket. For example, an atomistic country's endowment being low still generates increases in its price, but this will not correlate with movements in the marginal utility of global investors. Therefore, the larger the share of a country's good in the consumption basket, the higher the insurance value of its bond. This is akin to Hassan (2013), although the mechanism there relied on tradable vs non-tradable goods.

Aggregate consumption by investors in period 1 is given by

$$C_{H,1} + P_{F,1}C_{F,1} \equiv P_1 C_1 = (1-\tau)(Y_{H,1} + P_{F,1}Y_{F,1}) - QB_H - Q^* P_{F,1} B_F \tag{6}$$

where B_j denotes $j \in \{H, F\}$ bonds held by investors. Investors can freely trade goods with each other, before buying bonds from both governments using the corresponding good.

In period 2 governments pay back their debt or choose to default. The investors budget constraint is:

$$C_{H,2} + P_{F,2}(s)C_{F,2} = (1-\tau)(Y_{H,2}(s) + P_{F,2}(s)Y_{F,2}(s)) + R(s)B_H + R^*(s)P_{F,2}(s)B_F$$

where

$$R(s) = \begin{cases} \hat{P}_2 + M & \text{if no H default} \\ \kappa \hat{P}_2 & \text{if H defaults} \end{cases} \quad , \quad R^*(s) = \begin{cases} \hat{P}_2^* + M^* & \text{if no F default} \\ \kappa \hat{P}_2^* & \text{if F defaults} \end{cases}$$

$$M = f(B_H + B_H^*, s, \Theta) \quad , \quad M^* = f(B_F + B_F^*, s, \Theta)$$

where M and M^* indicate additional returns from investment opportunities that are further detailed in section 3.3. The full investor's optimization problem is provided in

Appendix B.3.

The optimal intra-temporal decision for each household at time t is characterized by

$$P_{F,t} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\eta}} \left(\frac{C_{F,t}}{C_{H,t}}\right)^{-\frac{1}{\eta}}$$

To illustrate how the insurance aspect lowers the price for assets of a big country, consider the two Euler equations. The difference gives an expression for the safe asset discount:

$$0 = \mathbb{E}\left[\left(C_{H,2}(s)\right)^{-\frac{1}{\eta}} C_2(s)^{\frac{1}{\eta}-\sigma} \left(\frac{R(s)}{Q} - \frac{P_{F,2}R^*(s)}{P_{F,1}Q^*}\right)\right]$$

If there is a shortage of home goods, then P_{F2} decreases and conversely a shortage of foreign goods leads to higher $P_{F,2}$. In that sense, it is generally desirable to hold the bonds of the good for which there is a shortage of. Moreover, the derivative of marginal utility is directly related to the good's share within the consumption basket. This means that holding bonds of the larger country is particularly valuable since the covariance between marginal utility and its return will be larger than for the smaller country. However, if goods are perfectly substitutable there are no price movements to allow for a distinction in bond returns. And if agents are risk neutral, there is no covariance of marginal utilities with prices either. For realistic calibrations of η under log utility, this effect generates an insurance discount for the larger country as in Hassan (2013).[9]

3.3 Investment opportunities

We also assume that in the beginning of the second period, with probability χ, an investor i gains access to an investment opportunity in country j. This investment can be financed with country-j specific bonds *if and only if* country j is not in a default state. In that case, those with investment opportunities have the possibility to receive an additional return in terms of country j goods of R^+ on their (scalable) investment. The other mass of investors $(1-\chi)$ do not have direct access to these country-specific investment opportunities. However, they can lend their net worth to those that do. This lending procedure is subject to search and matching frictions. Financial intermediaries specialized in country-j bonds collect $\mathcal{B}_j = (1-\chi)B_j$ bonds and match it to the mass of investors

[9] If goods are very complementary, this intuition breaks down and can even reverse. In the case $\eta = \sigma = 1$, the two effects exactly cancel out as in Cole and Obstfeld (1991), so for $\sigma = 1 > \eta$ it would actually be the smaller country who would earn that premium.

with investment opportunities $X = \chi(N + N^*)$. Following Chahrour and Valchev (2022), we assume the following matching function :

$$M_j = m\frac{\mathcal{B}_j X}{\mathcal{B}_j + X}$$

where the parameter m represents velocity of asset holdings, reflecting the number of transactions a certain amount of bond holdings can sustain. It therefore also reflects the degree of financial development. The probability of a successful matching is then given by

$$\mu_j = \frac{M_j}{X} = m\frac{\mathcal{B}_j}{\mathcal{B}_j + X}. \tag{7}$$

The match surplus R^+ is distributed via Nash bargaining, so a fraction ν goes to the borrower with investment opportunities while $1-\nu$ is allocated to lenders. The intermediaries are perfectly competitive and receive zero profits. Finally, investors have the possibility of absconding with a fraction of the borrowed funds. As in Kiyotaki and Moore (1997), the participation constraint introduces a borrowing limit. Given the standard nature of the friction, we do not model the full agent-principal problem and summarize it through a reduced form borrowing limit, which is a fraction ζ of the borrower's own funds b. The expected additional return R_j^I due to the existence of investment opportunities is

$$R_j^I = (1 - p_j)\left(\chi R^+ + \chi\mu_j\zeta\nu R^+ + (1-\chi)R_j^D\right) \tag{8}$$

where R^D is the return earned from depositing funds with intermediaries.[10]. In this expression, the first term is the expected gain from getting an investment opportunity and investing his own funds. The second term reflects the gain from an investor with an investment opportunity if it matched with a lender. The last is the return that those without investment opportunities get from depositing their funds with financial intermediaries.

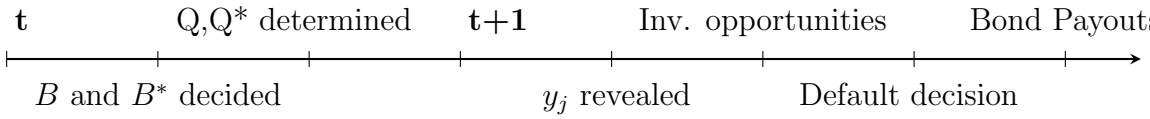
The main difference between additional returns of home and foreign bonds (per unit) lies in the default probability p_j, the matching probability μ_j and the good delivered by them (H vs F). The matching probability depends on the financial depth of the bonds market $\mathcal{B}_j$, which is a function of the current market value of the respective bond. As in Chahrour and Valchev (2022), the larger the size of the government bond market, the tighter the matching function and therefore more likely a borrower or a lender will

[10]Given intermediaries are perfectly competitive, this is simply the ratio of total fund earnings divided by the total deposited funds: $R_j^D = \frac{\mu_j\chi(1-\nu)\zeta R^+\mathcal{B}_j}{(1-\chi)\mathcal{B}_j}$

find a match. However, safety also plays an important role. Only bonds that are not in a default state can be used to finance investment opportunities. Large and safer bond markets will therefore be privileged from the perspective that matching between borrowers and lenders is easier. This in turn generates larger expected returns for the corresponding asset. Lowering credit risk not only lowers risk premia, but also increases convenience yields.

3.4 Timing and equilibrium

Figure 6: Timeline of baseline model



The timing of the events is the following: The government chooses the supply of debt B, B^*. Investors then decide whether to invest in bonds or consume, with prices Q, Q^* clearing the bond market. Once the next period begins, unit-specific shocks y_j are revealed and it becomes public knowledge which government will default on their debt. After that each investors can have an investment opportunity with probability $1-\chi$ while the intermediation between those with and without investment opportunities takes place. The bonds are paid out (or defaulted upon) and finally endowments are received and agents consume their remaining wealth. An illustration of the timeline is shown in Figure 6.

The investors optimize their portfolio under this timeline, giving rise to first order conditions that shape their demand curve for bonds. The full set of optimization problems, together with the budget constraint and the first order conditions, are described in more detail in Appendix B.3.

The equilibrium requires that the first order conditions of all investors and all governments are satisfied for a set of variables $\{C_H, C_F, P_F, B, B^*, Q, Q^*\}$. Bond markets clear, as well as goods markets according to the following conditions:

$$\mathcal{Y}_{Ht} = C_{Ht} + C^*_{Ht} + G \quad \text{and} \quad \mathcal{Y}_{Ft} = C_{Ft} + C^*_{Ft} + G^*$$

where $\mathcal{Y}_i$ is the stochastic output that includes potential gains from investment opportunities and losses from default. Furthermore, the budget constraints hold in all states.

3.5 Calibration

This section provides a calibration for key parameters of the model. Table 1 shows the parameters, the corresponding calibrated values and their description.

Table 1: Calibration

	Value	Description
Economy		
Y_1	1	Endowment in period 1
Y_2	$U[1-\frac{1}{3}, 1+\frac{1}{3}]$	Endowment process in period 2
Z	1/3	Aggregate TFP component
ρ	$\in [0,1]$	Unit shock correlation, baseline =0
N	50	Size of Home (dominant) country
N^*	$\in [1,...,50]$	Size of Foreign country
κ	0.46	Average haircut of 46%
ψ	0.085	Output loss of 8.5% if default
Investors		
σ	$\in \{0,1,1.5\}$	Risk aversion, baseline risk neutral
η	$\in \{3,\infty\}$	Trade elasticity, baseline =3
α	$\frac{E(Y_H)}{E(Y_H)+E(Y_F)}$	Consumption weights reflect relative size
β	0.96	Time discount factor
χ	0.33	Share investors with inv. opportunity
ν	0.8	Share investors in Nash bargaining
m	$\in \{2,8\}$	Matching efficiency, baseline =8 as in Chahrour and Valchev (2022)
ζ	0.5	Borrowing limit
R^+	0.05	Investment opportunity, additional 5% return
Government		
δ	0.5	Time discount factor, impatient government
τ	0.4	endowment taxation rate
θ	$\in [0,\infty)$	Unit smoothing parameter, baseline =0

The upper part of the table calibrates the properties of the economy. We calibrate the bounds of TFP and the unit endowment process such that for $N = 50$ and $\rho = 0$, the largest country has an output volatility of 0.03 (target USA), while the smallest country N=2 has an output volatility of 0.12, approximately the largest volatility observed in Fig 2, panel a). Furthermore, we calibrate the output loss in the case of default to 8.5% of GDP. This number is motivated by the literature, that finds an average output loss of 1.099% per year (Trebesch and Zabel, 2017) and an average duration of 8 years (Uribe

and Schmitt-Grohé, 2017). Together, these two numbers imply a net present value of the output losses (discounted at β) equal to around 8.5%. The average haircut for investors in the event of default is set to $\kappa = 0.46$, in line with the estimates of Graf von Luckner et al. (2025).

Next, we calibrate parameters that govern the investors' behavior. For the baseline, we calibrate investors to be risk neutral. In Section 4.2, we relax the assumption of risk neutrality and consider investors with $\sigma > 0$. The trade elasticity in the baseline is set to 3, corresponding to some degree of substitutability between Home and Foreign goods. This is in line with empirical estimates by Feenstra et al. (2018) who find estimates between 2 and 4. We also consider the case with perfect substitution ($\eta = \infty$) that nests the one good setup. We make sure that the share parameter α in the consumption aggregator is in line with the share of the country in the global economy at the deterministic steady. The time discount factor for investors is set to $\beta = 0.96$, implying yearly real rates of 4% for safe assets without convenience yields. The government discount factor is set to $\delta = 0.5$, meaning it is very impatient and has a strong incentive to issue debt in the first period. We set $\tau = 0.4$, implying governments retain around 40% of the economy's endowment. We focus on the case with $\theta \to \infty$ and $\rho = 0$ which implies relatively large diversification effects of growing economies.

Regarding investment opportunities, we choose a value of χ=0.33, indicating that about one third of investors will have a direct investment opportunity. R^+ is calibrated such that investment opportunities have a return of 5%. At the same time, we calibrate the Nash bargaining parameter ν to 80% indicating that investors with opportunities retain the majority of additional surplus, while intermediaries receive only a small share. The parameter m which proxies financial market development or the average credit duration is set to 8, as in Chahrour and Valchev (2022). Together with χ at 33%, a zero default probability of bonds, and the extra lending via financial markets investors expect to have close to 2% more returns on their bonds. With these values, interest rate spreads are at 35bp between countries with similar safety but a 1:10 size differential, as in the risk averse case in Figure 11, panel b . This is in line with Diamond and Van Tassel (2021) who find an average convenience yield of 35 bp [11].

[11] Other literature use different ways to measure convenience yields, resulting in different estimates. For example Krishnamurthy and Vissing-Jorgensen (2012) find slightly higher average US convenience yields for liquidity of 46 basis points. Our calibration is therefore on the conservative side, but we note our model can create a large set of yield differentials depending on safety difference, risk aversion, trade elasticities and liquidity yields.

4 Results

In this section, we highlight how key economic variables evolve as a function of Foreign country size N^*, while the original dominant currency country (Home) remains at constant N. The first subsection demonstrates how the Home country loses its privilege as the Foreign country becomes as big and safe as the Home country. We then disentangle the importance of the liquidity and insurance channels, and their connection to each other.

4.1 Privilege Lost: Challenger size and vanishing bond spreads

We set initially $N^* = 2$, and $N = 50$, so that there is an important size difference between the two countries, and then explore what would happen if F was closer or equal in size to the H country. We first highlight the case of risk neutral investors and later relax that assumption. Figure 7, displays the values for default probability, interest rates[12], Debt-to-GDP ratio and liquidity yields as a function of F's size N^* under perfect substitution across economic units ($\theta = \infty$).

In panel (a), we see that probability of default falls for F as N^* grows, while the debt-to-GDP ratio rises substantially in (c). As explained before, when country F grows, its debt capacity increases as well and governments use this opportunity to raise additional funds. F's probability of default falls as it is optimal for the government to benefit both in terms of quantities and prices, from the higher debt capacity. Eventually, when countries are exactly identical, the privilege is no longer there, and all variables equalize.

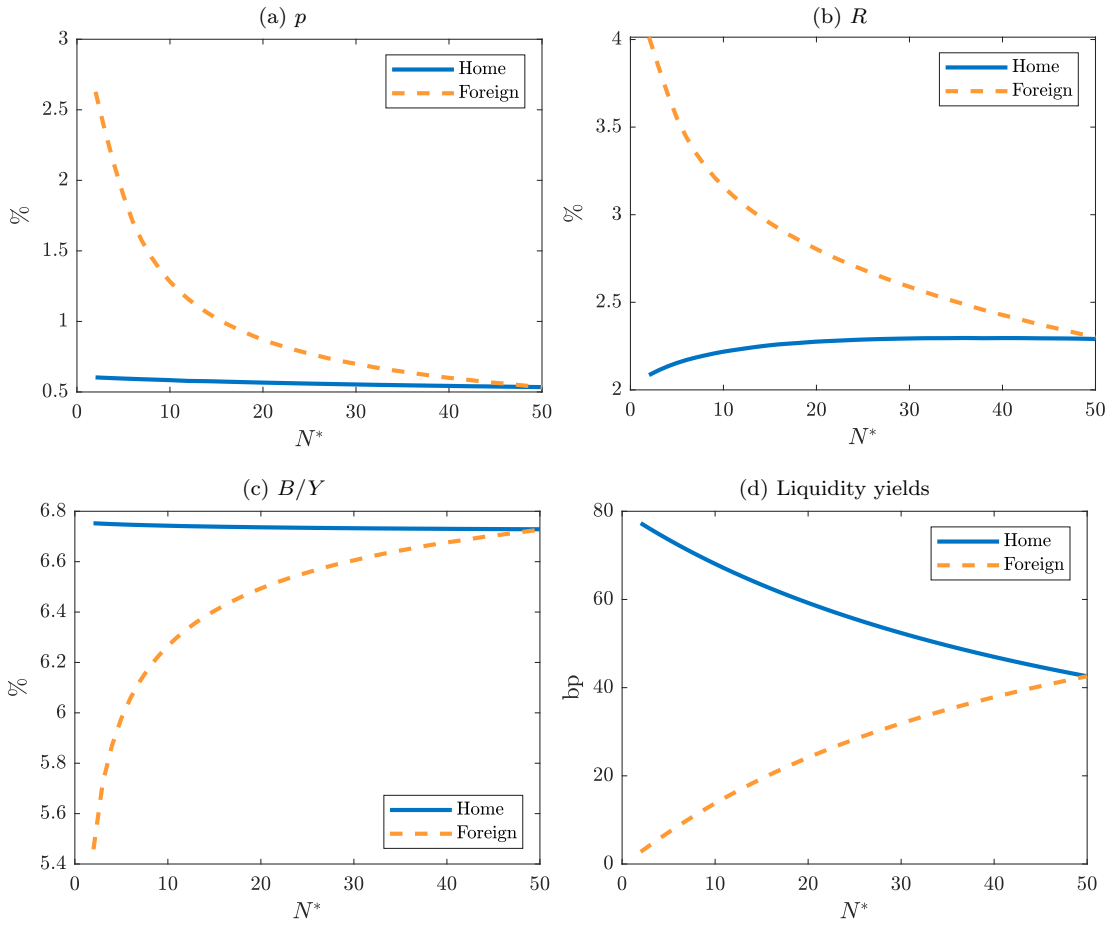
For the original dominant country, it is important to understand what happens to liquidity yields. We depict liquidity yields as a function of N^* in Figure 7 (d). These yields are defined as the expected gain from the ability to fund investment opportunities via the search and matching market:

$$R_j^{liq} = (1 - p_j) \left(\chi \mu_j \zeta \nu R^+ + (1 - \chi) \frac{\mu_j \chi (1 - \nu) \zeta R^+ \mathcal{B}_j}{(1 - \chi) \mathcal{B}_j} \right) \tag{9}$$

Those include elements from equation (8) without the direct gains from investment opportunities. As discussed in the previous section, this expression is a function of liquidity (the matching probability μ_j) and the probability of default p_j. As F becomes safer, the fall in the probability of default improves convenience yields, but also liquidity improves

[12]For exposition purposes, we present the results in terms of yields/interest rates expressed in terms of the Home good $R = 1/Q - 1$ and $R^* = 1/(P_{F1}Q^*/\mathbb{E}P_{F2}) - 1$.

Figure 7: Main variables under perfect substitution



Note: Values of key variables for Home (solid blue) and Foreign (dashed orange) as a function of N^*, with risk neutral investors, $\eta = 3$ and $R^+ = 5\%$.

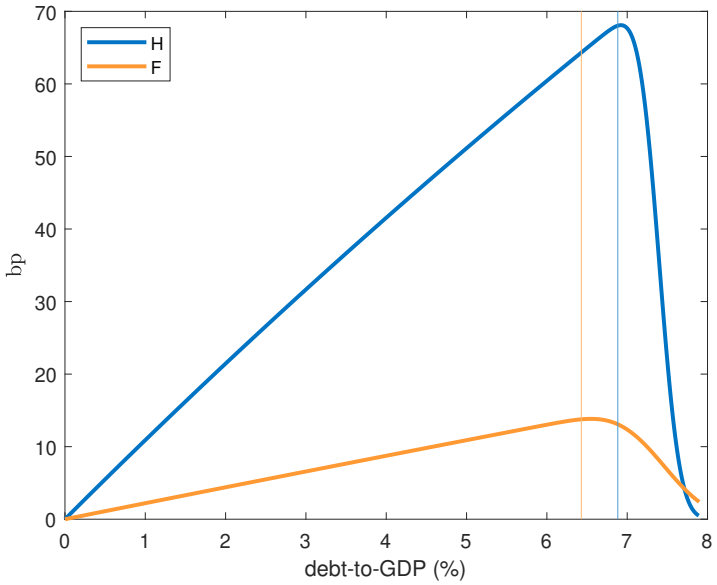
as the country (and its debt market depth) grows. At the same time, the dominant country share of the world economy is falling, not only is it eroding its *relative* advantage with respect to F but also reducing the *absolute* value of liquidity yields.

The dominant currency country therefore faces a dilemma. On the one hand, the more of its bonds are held and traded across the world, the better its liquidity properties (absent default), but on the other hand it needs to be prudent in bond emissions so that its debt remains safe and therefore useful as a financing currency[13]. For the smaller country, relative growth improves both dimensions. It increases debt capacity, and with it better safety and liquidity services. Figure 8 illustrates the liquidity *Laffer*-style curve that governments face for the case with $N = 50$ and $N^* = 10$. We plot expected liquidity returns from equation (9), as a function of each country's possible choices of debt-to-GDP, assuming the other country remains at their optimal choice. Liquidity returns increase for both countries initially, but much more so for H. This reflects not only the larger debt

[13]This reflects elements of the traditional Triffin dilemma, Triffin (1961).

capacity but also the relative differences in total debt supply as a proportion of the world economy. Liquidity gains are also quickly eroded once probabilities of default begin to lift-off, leading to a sharp drop after the peak. Both countries are on the left side of the graph, while the bigger country is closer to the peak, reflecting the higher risk-aversion of governments relative to investors.

Figure 8: Debt and liquidity



Note: Liquidity yields from equation (9) as a function of debt to GDP. H has size $N = 50$, F has $N^* = 10$. Vertical lines indicate optimal debt-to-GDP in equilibrium. $R^+ = 5\%, \eta = 3$.

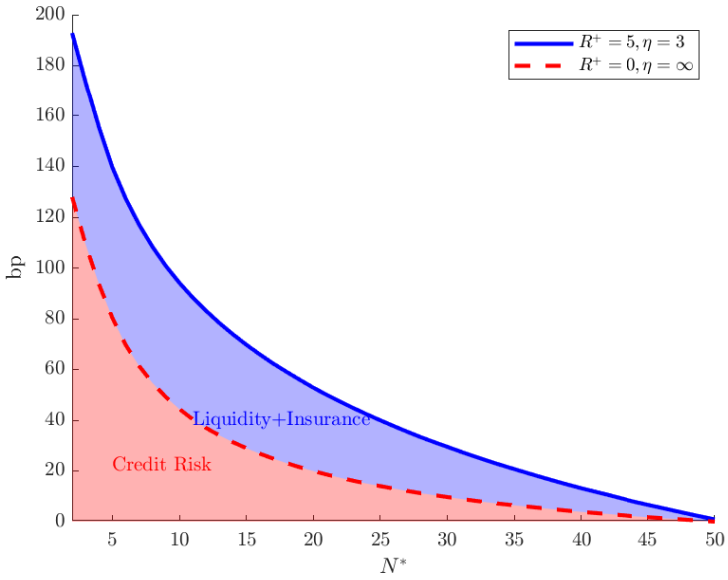
For the original anchor, convenience yields reduce when the other country grows, which forces a reduction in the debt-to-GDP ratio. It is optimal for H to make its debt safer as its dominant status erodes, while interest rates increase nevertheless due to lower convenience yields.

As a next step, we disentangle the spreads between the Home and Foreign country. Given the model outlined above, there are three sources of spreads: First, credit risk in the form of default probability. If one country is more likely to default than the other, investors want higher interest rates. Second, there is the liquidity component arising from the ability to finance investment opportunities, as explained above. The last source of spreads is the insurance aspect. As bonds pay out in country-specific goods, and the price of these goods fluctuates with the global state of the economy, the value of each bond depends on the state of the world. Negative shocks in a country make its good more valuable, ceteris paribus. Provided there is no default, these payouts are more valuable as well, since it is also a state of low aggregate consumption C_2. The larger the country, the larger the drop in aggregate consumption due to its weight in the consumption basket. Larger countries will therefore have a larger insurance value, since (on average) their real

return is higher in states of low aggregate consumption.[14]

There are two parameters that control the liquidity and insurance channels. R^+ determines the additional returns on investment opportunities. If set to zero, the liquidity channel is completely eliminated. For insurance, $\eta = \infty$ implies that both goods are perfectly substitutable, this means that the price of both goods is the same and the unit value of bond payouts in full repayment states is the same. Figure 9 shows the spreads between both countries in our baseline and in a model without the liquidity and insurance channels.

Figure 9: Spreads



Note: Spreads $(R^* - R)$ between Foreign and Home country. The solid blue line indicates the case for, $\eta = 3$, $R^+ = 5\%$, while the dashed red line shows the case without liquidity and insurance, $\eta = \infty$, $R^+ = 0$.

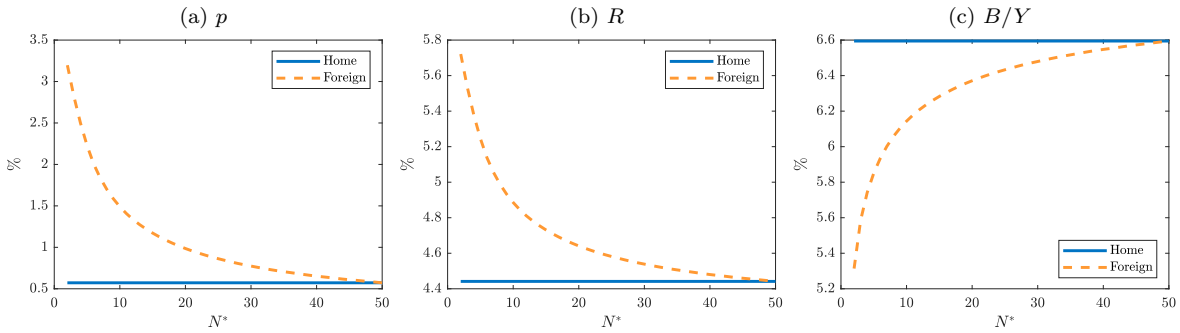
The initial spreads in the baseline are at around 190 basis points, with credit risk being the largest component. As the small country grows, it diversifies its economy and the increase in debt capacity reduces credit risk rapidly, as can be seen by the convex relationship between the probability of default and N^* (Fig 9). This is despite the rise in debt-to-GDP (Fig 7, panel c), as F takes advantage of its higher debt capacity. Liquidity yields reduce more progressively, with the convergence being still convex but much closer to linear in N^* (Fig 7, panel d). As N^* approaches N, the spread becomes mostly driven by the liquidity and insurance components.

We zoom in on the case where only credit risk matters, with $R^+ = 0$ and $\eta = 3$ in Figure 10. H is relatively insulated with respect to N^*. With risk-neutral investors, the

[14]It is more intuitive to think about an infinitesimally small country. A strong recession in that country will raise the price of its own good, but it will not move the aggregate consumption basket of investors and therefore has little insurance value for them.

Home country's debt, default probability and interest rates do not depend on N^*, since neither the insurance or the liquidity channel are present. At the same time, the larger size of F helps it diversify, reducing credit risk and interest rates, with spreads eventually becoming zero due to the countries being identical when $N^* = N$.

Figure 10: Main variables without liquidity and insurance specification



Note: Steady state values of key variables for Home (solid blue) and Foreign (dashed orange) as a function of N^*, with risk neutral investors, $\eta = \infty$ and $R^+ = 0\%$.

How important is liquidity relative to insurance? To answer this, we need a setup in which the insurance channel is properly isolated. There is however a crucial connection between safety and insurance, as only bonds that do not default in a bad state of the economy can fully reap the benefits from the insurance channel. At the same time it is true that default is especially likely in bad states. Moreover, changing the number of units alters the volatility of F. This implies that the share of F on global volatility does not necessarily increase as it grows.

Because safety and insurance are interconnected in an important way, we show another setup in the next subsection in which we abstract from the credit risk channel to better isolate the insurance channel.

4.2 Insurance and liquidity in small and safe countries

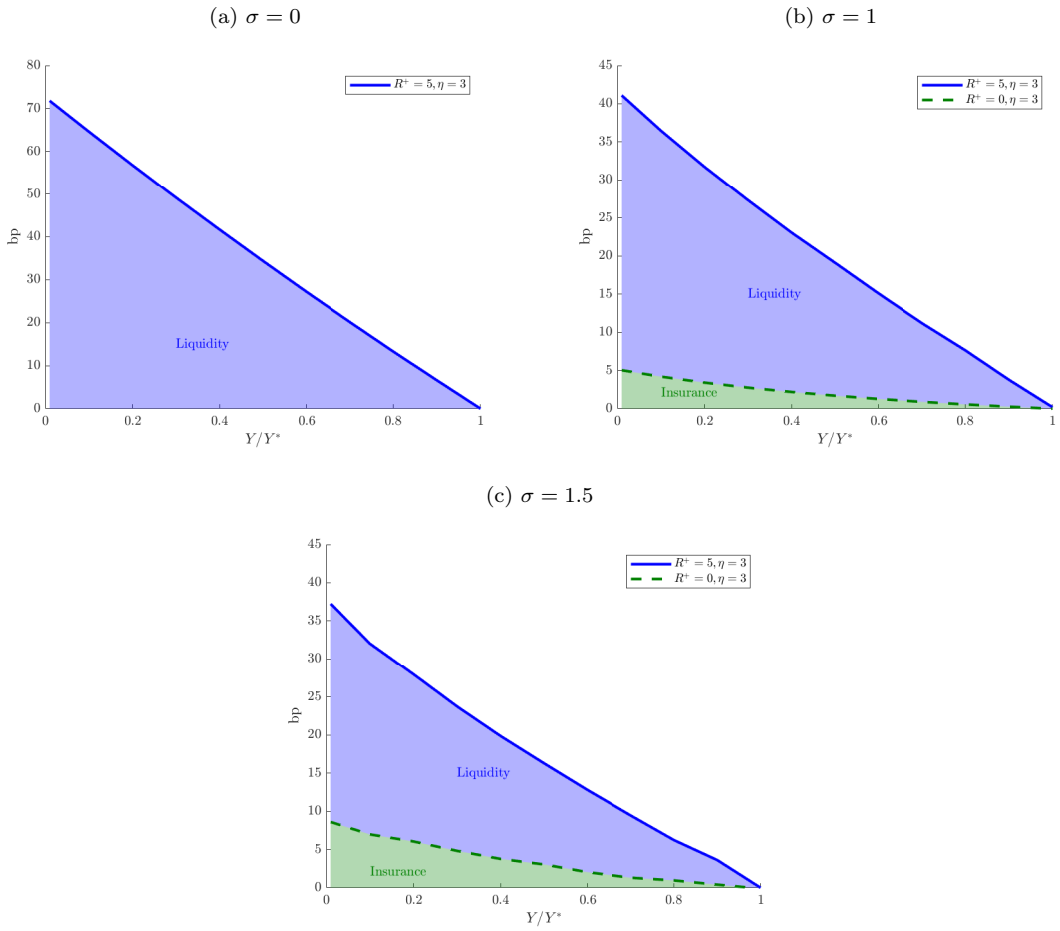
In this subsection, we turn off the credit risk channel by enabling countries to commit to repayment and consider countries that are small and diversified. We do so by setting the number of economic units in both countries to be equal, but allowing variation in the size of each unit. We also ensure that the consumption basket share α remains consistent with the country's global GDP share.

This implies that both countries have the same level of diversification and would choose the same debt-to-GDP if size was irrelevant. However, this will not be the case due to

liquidity differences in their bond supply and also due to the insurance effect described in the previous section.

We focus on the interest rate spread as we increase the risk aversion of investors. In all cases, we set the trade elasticity to $\eta = 3$ again. The first graph plots the spread with risk neutral ($\sigma = 0$) investors. We then move to more risk-averse investors with $\sigma = 1$ and $\sigma = 1.5$.

Figure 11: Spreads with risk averse investors



Note: Spreads ($R^* - R$) between Foreign and Home country. The solid blue line indicates the case for, $\eta = 3$, $R^+ = 5\%$, while the dashed green line shows the case without liquidity and with insurance only, $\eta = 3$, $R^+ = 0$. Investors are risk neutral in panel (a) risk averse with $\sigma = 1$ in panel (b), and even more risk averse with $\sigma = 1.5$ in panel (c). There are two output states Y_{low} and Y_{high} for both countries.

The entire spread differential can be explained by liquidity yields when investors are risk neutral, as there is no demand for insurance. With risk neutrality, globally bad states that originate from recessions in the large country are not weighted more in the Euler equation. The absence of this hedging motive with risk neutral investors implies no insurance discount when $\sigma = 0$. This changes as soon as there is some risk aversion, as high marginal utility states and price movements weigh payoffs from the larger country more. With $\sigma = 1$ insurance yields are 5 basis points and rise to almost 10 bp when $\sigma = 1.5$. Note

that $\sigma > 0$ also leads to non infinite IES, which makes investors less willing to tilt their consumption towards *t+1* too much. This reduces total debt supply, and explains the lower liquidity yields as σ grows. Additionally the uncertainty in idiosyncratic liquidity payoffs reduces the value of liquidity returns. The more risk averse investors are, the lower debt capacity and the more important is the insurance motive. So the share of convenience yields explained by the insurance channel grows with investor risk-aversion. Nevertheless, for a given investor risk-aversion, the insurance channel actually reinforces the liquidity advantage of the larger country. Since liquidity payouts of H are also in H goods, they are more valuable in the bad state, as they increase the availability of scarce goods for bond holders. The insurance status therefore reinforces the liquidity channel.

Gourinchas and Rey (2022) characterize the position of the US as an insurer for global investors: While the US enjoys in normal times an "exorbitant privilege" in form of higher returns on its asset position, this privilege turns into a duty in times of a global crisis, in which the US effectively transfers wealth towards the rest of the world. In our model, the dominant country also plays that role, through the insurance channel of Hassan (2013).

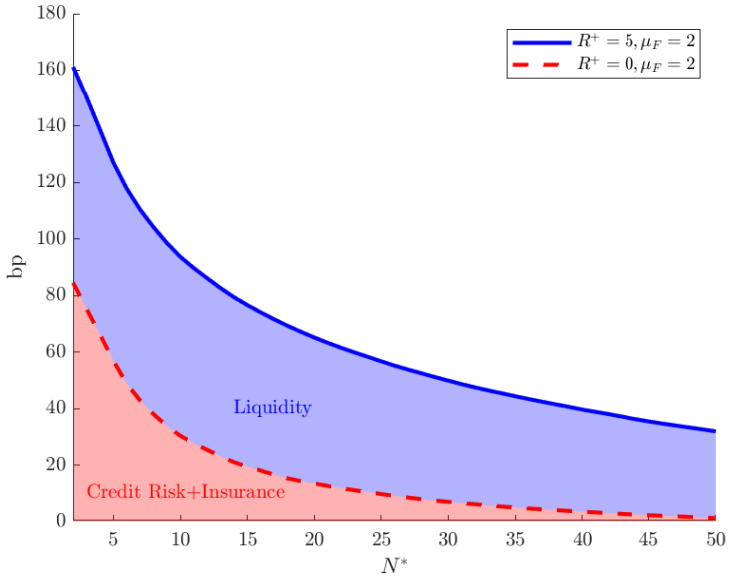
Although Gourinchas and Rey (2022) formulate the notion of privilege and duty through the heterogeneous portfolio composition of the US and the rest of the world, even without those elements the hegemon assumes the role of global insurer, since on average the real return on its bonds will be higher in negative global states due to currency appreciation. This leads ex-ante to a lower price of its bonds, but it is accompanied by a transfer of wealth to foreign bond holders during crises.

4.3 Size and financial development

Next we consider an asymmetric setup in which one country is less financially developed than another. We implement this by setting the asset velocity parameter in equation (7) to 2 instead of 8 for the Foreign country. This implies that the matching process in F is less efficient. For example, the matching probability for investors is 4 times higher in the large financially developed country, if both had the same size and debt-to-GDP. We go back to the setup of Section 4.1 with credit risk, risk-neutral investors, and varying numbers of units. Similarly to before, we look at the spread as F grows, turning the liquidity channel on in the baseline with $R^+ = 5\%$ and off with $R^+ = 0\%$.

In such a setup, spreads of 40 basis points persist even if one country overtakes the other in terms of GDP size. While the financially developed country enjoys additional

Figure 12: Spreads with heterogeneous financial development



Note: Spreads $(R^* - R)$ between Foreign and Home country. The solid blue line indicates the case for, $\eta = 3$, $R^+ = 5\%$, while the dashed red line shows the case without liquidity but with insurance, $\eta = 3$, $R^+ = 0$. The credit velocity parameter m is set to 2 for F and 8 for H.

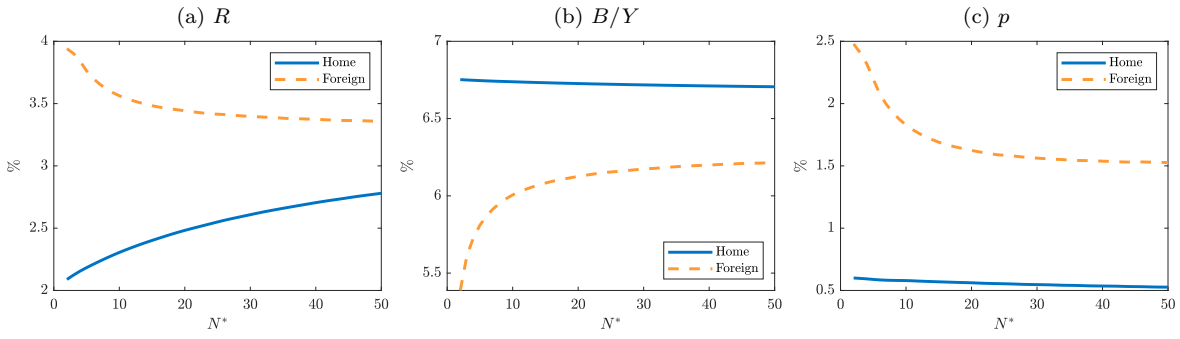
liquidity yields, its debt to GDP ratio is slightly higher. This means that the dominant country is more leveraged and slightly more risky than the financially less developed country. Spreads are almost entirely explained by liquidity yields, while the risk component plays a less meaningful role here. Additional figures for this case are in the Appendix Figure A2.

4.4 Lack of diversification

If one country has a higher correlation of shocks between units than another, it diversifies its output less when it grows. This is what we consider in the setup here, where H has a correlation of 0 as before, while F has a correlation of 0.1. A slightly higher shock correlation leads to a substantially less diversified tax base. This results in a default probability for the Foreign country of 1.5%, if it has the same size as the Home country which only has a 0.5% default probability. As a result, the debt capacity of the Home country is larger, while having lower interest rates at the same time.

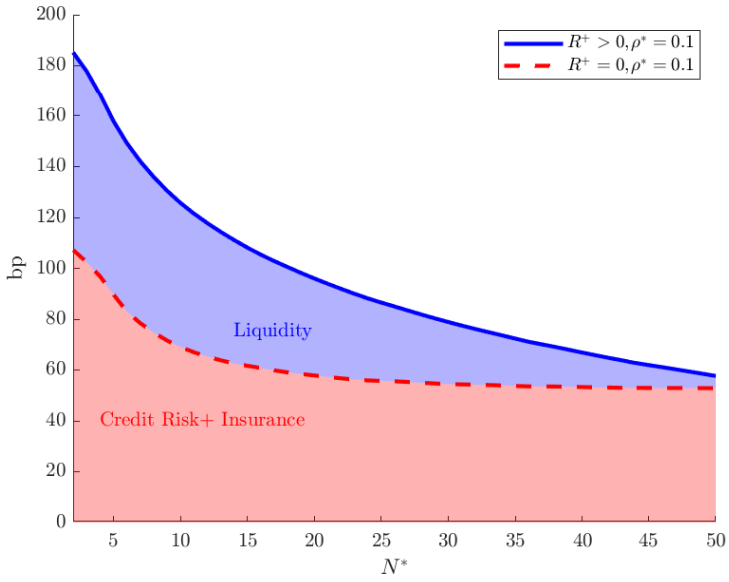
When considering the spread decomposition as before, the credit (and insurance) component are more prominent, determining the spreads even when they are equally large. The liquidity component is still responsible for a small amount of spreads, as H's debt market is larger.

Figure 13: Variables with different diversification



Note: Steady state values of key variables for Home (solid blue) and Foreign (dashed orange) as a function of size with risk neutral investors, $\sigma = 0$, $\eta = 3$, $R^+ = 5\%$ and $\rho_F = 0.1$.

Figure 14: Spreads with heterogeneous shock correlation



Note: Spreads ($R^* - R$) between Foreign and Home country. The solid blue line indicates the case for, $\eta = 3$, $R^+ = 5\%$, $\rho_F = 0.1, \rho_H = 0$ while the dashed red line shows the case without liquidity but with insurance and risk, $\eta = 3$, $R^+ = 0$.

5 Conclusion

We develop a model of the international financial system in which governments can default on their outstanding debt. In this paper, we focus on the role of economic size, and how it interacts with three key aspects of a dominant currency: safety, liquidity and insurance.

First, growth can be accompanied by diversification, which improves the fundamentals of the economy and increases its debt capacity. Secondly, large countries with high debt capacity are able to provide large amounts of safe assets which can be used as collateral or as stores of value. Having a large safe asset supply, relative to the world economy, underpins liquidity and facilitates the smooth functioning of international financial markets.

Finally, its goods being a large component of global consumption also increases the asset's role as a safe haven. During times of low global consumption, the denomination of the bond appreciates, making it a very good hedge compared to bonds of smaller countries.

However, its position can be challenged if other countries grow in size and their debt becomes safe as well. This leads to a gradual decline in interest rate spreads and eventually to a loss of the safe haven privilege as the other country's economy becomes larger (and potentially safer) than the incumbent safe haven country. Our model can explain how global interest rates on public debt declined while debt-to-GDP ratios were rising and US convenience yields were falling.

Finally, we show that the more efficient the country's financial markets are, the larger the convenience yields for a given bond supply. Large and safe countries might nevertheless find it hard to challenge the dominant currency if they do not match the financial development of the hegemon.

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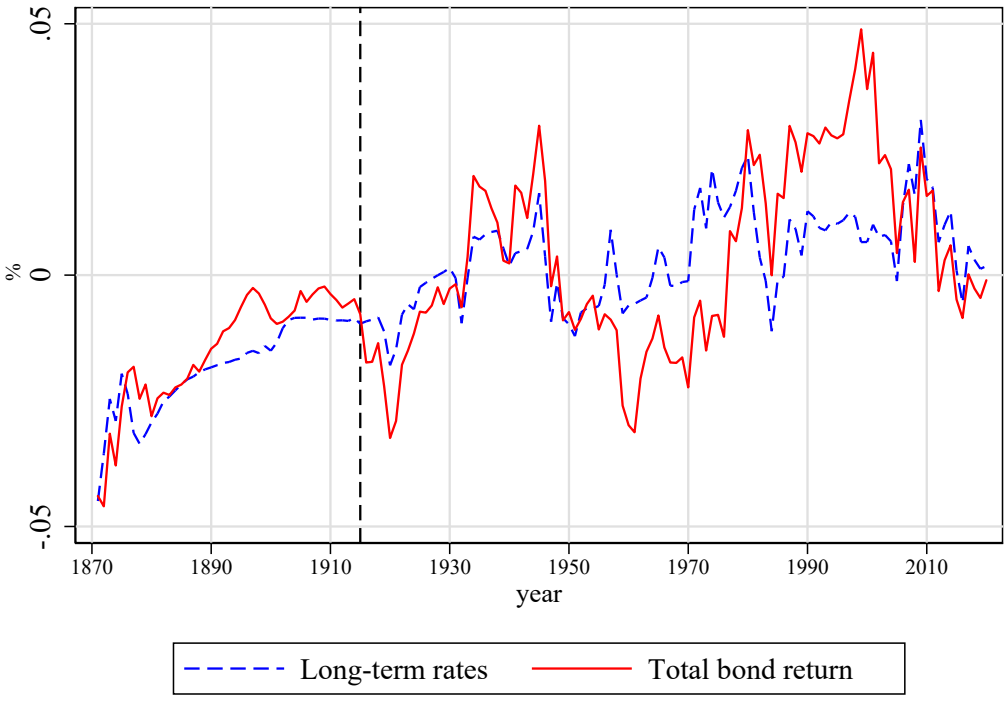
A Additional Tables and Figures

Table 2: UK-US long term interest spreads over time

Spread rate	1870-1914	1915-2008	2009-2020
Real long rate	-1.81%	0.13%	-0.24%
Real total bond return	-1.59%	0.17%	0.16%
USD long rate	-1.34%	1.29%	-0.11%
USD total bond returns	-1.13%	1.33%	0.07%
Real USD long rate	-1.36%	1.27%	-0.10%
Real USD total bond returns	-1.13%	1.32%	0.11%

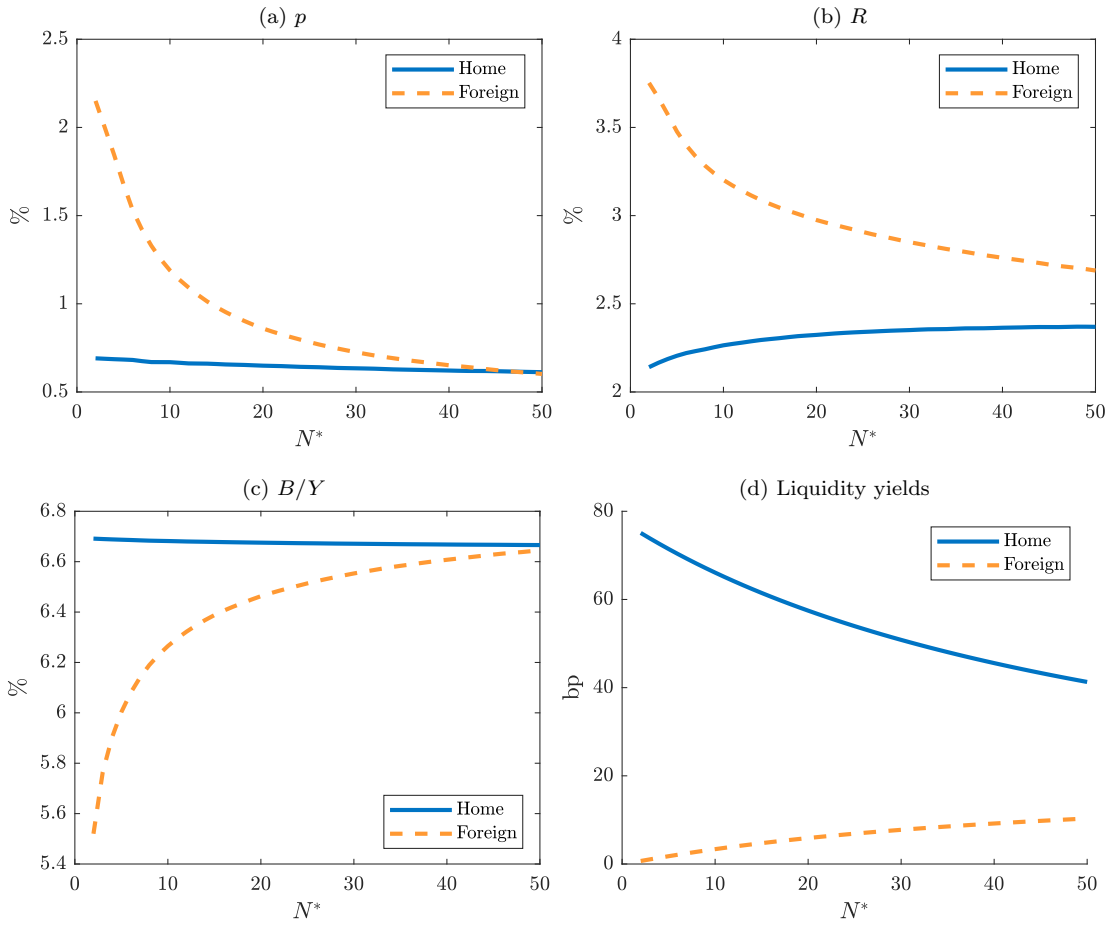
Note: Yearly average spreads of long term interest rates between the United Kingdom and the United States, excluding the World War periods of 1914-1919 and 1939-1947. *Source*: Jordà et al. (2019) and own calculations.

Figure A1: Interest Rate Spreads



Interest rates spreads in USD are calculated the following way: You invest one USD in UK bonds in Pounds to exchange rates in period t and receive the interest rate in period $t+1$ in Pound, then exchanging those back to USD the current exchange rate. This is compared to US long-term bond interest rate payments in US Dollar. Spreads are very volatile, we apply a quarter-centennial moving average to smooth out short-run fluctuations. *Source*: Jordà et al. (2019) and own calculations.

Figure A2: Main variables with heterogeneous financial development



Note: Values of key variables for Home (solid blue) and Foreign (dashed orange) as a function of N^*, with risk neutral investors, $\eta = 3$, $R^+ = 5\%$, $m = 8, m^* = 2$.

B Model and derivations

B.1 Combining Distributions

$\theta \to \infty$ In general the sum of two independently distributed random variables X, Y is given by:

$$Z = X + Y, \quad P(Z = z) = \sum_{k=-\infty}^{\infty} P(X = k)P(Y = z - k)$$

If we have continuous density functions f for X and g for Y, then this expression is

$$H(z) = \int_{\infty}^{\infty} F(z - t)g(t)dt$$

the convolution of the probability distribution is then

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)$$

The probability can therefore be computed for N units. For the Uniform a generalization to N yields the Irwin-Hall distribution which is given by the following cdf[15]:

$$F(x,N) = \frac{1}{2} + \frac{1}{2N!}\sum_{k=0}^{N}(-1)^k \binom{N}{k}(x-k)^N sgn(x-k)$$

with

$$p = F\left(\frac{BR - N\kappa}{1-\kappa}\right) \tag{10}$$

$\theta \to 1$ For N units we have

$$\prod_i^N \psi_i < \frac{(Y_2 - BR/N)^N}{((1-\kappa)Y_2)^N}$$

Let the product of N uniform distributions be Z, we then have

$$f_N(z) = \frac{(-\ln z)^{N-1}}{(N-1)!}$$

B.2 Government: Intertemporal problem and default

The government problem is characterized by

$$\begin{aligned}
\max_B \quad & \log(G_1) + \delta\left[(1-p)\mathbb{E}_{Y_2>\tilde{Y_2}}\log(G_2(rep)) + p\mathbb{E}_{Y_2<\tilde{Y_2}}\log G_2(def)\right] \\
\text{s.t.} \quad & G_1 = \tau Y_1 + QB \\
& G_2 = \begin{cases} \tau Z Y_2 - B\hat{P}_2 & \text{if repay} \\ (1-\psi)\tau Z Y_2 - \kappa B\hat{P}_2 & \text{if default} \end{cases} \\
& Y_t = \left(\sum_{j=1}^{J} \omega_j^{\frac{1}{\theta}} y_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}.
\end{aligned}$$

[15]The CDF of the sum of U[a,b] distributions can be found through standard scaling and shifting.

Z is aggregate TFP, the threshold $\tilde{Y_2}$ characterizes the default decision and determines the default probability p:

for $\theta \to \infty$ a government defaults if aggregate output is sufficiently small:

$$\bar{Y_2} \equiv \sum_{j=1}^{J} Y_{2j} < \frac{(1-\kappa)B\hat{P}_2}{\psi Z} \equiv \tilde{Y}_2^{\infty}.$$

for a given density function f, the probability of default is then a function of B, Q and other parameters, $p = f(\bar{Y_2} < \tilde{Y}_2^{\infty})$. If f in uniformly distributed as in the example of the main text, the sum of uniformly distributed variables follows an Irwin-Hall distribution.

for $\theta \to 1$

$$\tau Z \prod_{j=1}^{J} Y_{2j}^{\omega_j} - B\hat{P}_2 < \tau Z \prod_{j=1}^{J} (1-\psi) Y_{2j}^{\omega_j} - \kappa \hat{P}_2 B$$

implicit function which gives $\tilde{Y}_2^1$

for $\theta \to 0$ $G_t \to \min g_{it}$ default if

$$\min_j \{Y_{2j}\} < \frac{(1-\kappa)B\hat{P}_2}{\psi Z} \equiv \tilde{Y}_2^0.$$

for iid uniforms between $[a, b]$ the density function of the minimum value is

$$F(y) = \begin{cases} 1 - \left(1 - \frac{y}{b-a}\right)^n & : y \in (a,b) \\ 0 & : y < a \\ 1 & : y > b \end{cases} \qquad f(y) = \begin{cases} \frac{n}{b-a}\left(1 - \frac{y}{b-a}\right)^{n-1} & : y \in (a,b) \\ 0 & : \text{ otherwise} \end{cases}$$

First order conditions:

for $\theta \to \infty$ (additive) the government's first order condition is

$$\frac{Q}{J\tau Y_1 + QB} - \delta \left[(1-p)\mathbb{E}_{\bar{Y}_2 > \tilde{Y}_2} \frac{\hat{P}_2}{\tau Z \sum_{j=1}^{J} Y_{2j} - B\hat{P}_2} + p\mathbb{E}_{\bar{Y}_2 < \tilde{Y}_2} \frac{\kappa \hat{P}_2}{(1-\psi)\tau Z \sum_j^J Y_{2j} - \kappa B \hat{P}_2} \right] = 0$$

which is

$$\frac{Q}{J\tau Y_1 + QB} - \delta \left[(1-p) \int_{\tilde{Y}_2}^{\bar{Y}} \frac{f(Y_2^{\Sigma})}{1 - F(\tilde{Y}_2)} \frac{\hat{P}_2}{\tau Z \sum_j^J Y_{2j} - B\hat{P}_2} dY_2^{\Sigma} + p \int_{\underline{Y}}^{\tilde{Y}_2} \frac{f(Y_2^{\Sigma})}{F(\tilde{Y}_2)} \frac{\kappa \hat{P}_2}{(1-\psi)\tau Z \sum_j^J Y_{2j} - \kappa B \hat{P}_2} dY_2^{\Sigma} \right]$$

for $\theta \to 0$ (Leontief) the government considers the worst off unit, therefore the foc is

$$\frac{Q/J}{\tau Y_1 + QB/J} - \delta \left[(1-p)\mathbb{E}_{Y_2 > \tilde{Y}_2} \frac{\hat{P}_2/J}{\tau Z \min\{Y_{2j}\} - B\hat{P}_2/J} + p\mathbb{E}_{Y_2 < \tilde{Y}_2} \frac{\kappa \hat{P}_2/J}{(1-\psi)\tau Z \min\{Y_{2j}\} - \kappa B\hat{P}_2/J} \right] = 0$$

B.3 Investor problem

Investors in H maximizes an Epstein Zin utility function, that nests the CRRA case for $\xi = 0$:

$$\max_{B_H, B_F, C_H, C_F, C} \qquad U = \left[(1-\beta) C_1^{1-\sigma} + \beta E_1 \left(C_2^{1-\xi} \right)^{\frac{1-\sigma}{1-\xi}} \right]^{\frac{1}{1-\sigma}} \tag{11}$$

$$\text{s.t.} \quad [\mu] \qquad C = \left(\alpha^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{12}$$

$$[\lambda_1] \qquad P_1 C_1 = (1-\tau) Y_1 - QB_H - Q^* P_{F1} B_F \tag{13}$$

$$[\lambda_2] \qquad P_2 C_2 = (1-\tau) Y_2 + RB_H + R^* P_{F,2} B_F \tag{14}$$

$$R = \begin{cases} \hat{P}_2 + M & \text{if no default} \\ \kappa \hat{P}_2 & \text{if default} \end{cases} \tag{15}$$

$$R^* = \begin{cases} \hat{P}_2^* + M^* & \text{if no default} \\ \kappa \hat{P}_2^* & \text{if default} \end{cases} \tag{16}$$

$$M = f(B_H + B_H^*, S, \Theta) \tag{17}$$

$$M^* = f(B_F + B_F^*, S, \Theta) \tag{18}$$

where R are the state contingent payouts that include direct payments $\hat{P}_2$ with or without default and investment opportunities M, and $Y_t = Y_{Ht} + P_{Ft} Y_{Ft}$.

We take the derivative with respect to C_{H1}, C_{H2}, B_H

$$\begin{aligned}
\frac{\partial U}{\partial C_{H1}}: &\quad (1-\beta) C_1^{-\sigma} \cdot \alpha^{\frac{1}{\eta}} \cdot (C_{H1})^{\frac{-1}{\eta}} \cdot C_1^{\frac{1}{\eta}-\sigma} U^{\sigma} = \lambda_1 \\
\frac{\partial U}{\partial C_{H2}}: &\quad \beta \pi(s) \alpha^{\frac{1}{\eta}} \left(C_{H2}(s) \right)^{-\frac{1}{\eta}} C_2(s)^{\frac{1}{\eta}-\xi} \mathbb{E} \left(C_2^{1-\xi} \right)^{\frac{\xi-\sigma}{1-\xi}-1} U^{\sigma} = \pi(s) \lambda_2(s) \\
\Rightarrow: &\quad \mathbb{E} \left(\beta C_2(s)^{-\xi} \alpha^{\frac{1}{\eta}} \cdot \left(C_{H2}(s) \right)^{-\frac{1}{\eta}} \cdot C_2(s)^{\frac{1}{\eta}-\xi} \mathbb{E} \left[C_2^{1-\xi} \right]^{\frac{1-\sigma}{1-\xi}-1} U^{\sigma} \right) = \mathbb{E}[\lambda_2] \\
\frac{\partial U}{\partial B_H}: &\quad \lambda_1 Q = \mathbb{E}[\lambda_2 R]
\end{aligned}$$

Plug in the two consumption first order conditions into the bond first order condition

to obtain the Euler equation

$$(1-\beta)\cdot\alpha^{\frac{1}{\eta}}\cdot(C_{H1})^{\frac{-1}{\eta}}\cdot C_1^{\frac{1}{\eta}-\sigma}U^{\sigma}Q=\sum_{s}^{S}\beta\pi(s)\alpha^{\frac{1}{\eta}}\left(C_{H2}(s)\right)^{-\frac{1}{\eta}}C_2(s)^{\frac{1}{\eta}-\xi}\mathbb{E}\left(C_2^{1-\xi}\right)^{\frac{\xi-\sigma}{1-\xi}}U^{\sigma}R(s)$$

where $s\in\{1,...,S\}$ denotes the set of possible states in the second period. If endowments are non-stochastic, then only the payouts of bonds change over states (default, non-default, investment opportunity, no investment opportunity etc.). Simplifying and doing the same for B_F we have:

$$(1-\beta)(C_{H1})^{\frac{-1}{\eta}}C_1^{\frac{1}{\eta}-\sigma}=\sum_{s}^{S}\beta\pi_s\left(C_{H2}(s)\right)^{-\frac{1}{\eta}}C_2(s)^{\frac{1}{\eta}-\xi}\mathbb{E}\left(C_2^{1-\xi}\right)^{\frac{\xi-\sigma}{1-\xi}}\frac{R_s}{Q} \tag{19}$$

$$(1-\beta)P_{F1}(C_{H1})^{\frac{-1}{\eta}}C_1^{\frac{1}{\eta}-\sigma}=\sum_{s}^{S}\beta\pi_s\left(C_{H2}(s)\right)^{-\frac{1}{\eta}}C_2(s)^{\frac{1}{\eta}-\xi}\mathbb{E}\left(C_2^{1-\xi}\right)^{\frac{\xi-\sigma}{1-\xi}}\frac{R_s^*P_{F,2}}{Q^*} \tag{20}$$

The Euler equation in the CRRA case is

$$C_{H1}^{-\frac{1}{\eta}}C_1^{\frac{1}{\eta}-\sigma}=\beta\mathbb{E}\left[C_{H2,s}^{-\frac{1}{\eta}}C_{2,s}^{\frac{1}{\eta}-\sigma}\frac{R_s}{Q}\right]$$

with Cobb Douglas trade consumption aggregation ($\eta=1$), we have

$$C_{H1}^{-1}C_1^{1-\sigma}=\beta\mathbb{E}\left[C_{H2,s}^{-1}C_{2,s}^{1-\sigma}\frac{R_s}{Q}\right]$$

The optimal intratemporal decision for each household in time t is characterized by

$$\frac{P_{Ht}}{P_{Ft}}=\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\eta}}\left(\frac{C_{Ht}}{C_{Ft}}\right)^{-\frac{1}{\eta}}$$

Normalizing $P_{Ht}=1$ as the numeraire we have:

$$P_{Ft}=\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\eta}}\left(\frac{C_{Ht}}{C_{Ft}}\right)^{\frac{1}{\eta}}$$
$$(P_{Ft})^{\eta}=\frac{1-\alpha}{\alpha}\frac{C_{Ht}}{C_{Ft}}$$

we then use the budget constraint in period 2, noting that $P_2^FC_2^F=P_2^FW_2^F$ to get an expression for C_{H2} as a function of endowments and state dependent payouts and plug this into the previous equation to get a price for traded goods.

Subtract the two Euler equations to get the safe asset discount:

$$0 = \mathbb{E}\left[\left(C_{H2}(s)\right)^{-\frac{1}{\eta}} C_2(s)^{\frac{1}{\eta}-\sigma}\left(\frac{R_s}{Q} - \frac{P_{F2}R_s^*}{P_{F,1}Q^*}\right)\right]$$

B.4 Period 1 prices

We can also compute the price of period 1 as a function of endowment and asset trading, using the following equations

$$\begin{aligned}
C_H + G = Y_H, \quad C_F + G^* = Y_F,\\
C_H = \alpha\left(\frac{P_H}{P}\right)^{-\eta} C, \quad C_F = (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C,\\
G = \tau Y_H + QB_H, \quad G^* = \tau Y_F + Q^* B_F\\
PC = (1-\tau)Y_H + (1-\tau)P_F Y_F - QB_H - Q^* P_F B_F
\end{aligned}$$

Plug in demand into the resource constraint

$$\alpha\left(\frac{P_H}{P}\right)^{-\eta} C + \tau Y_H + QB_H = Y_H,$$

Plug in investors budget constraint

$$\alpha\left(\frac{P_H}{P}\right)^{-\eta} \frac{(1-\tau)Y_H + (1-\tau)P_F Y_F - QB_H - Q^* P_F B_F}{P} + \tau Y_H + QB_H = Y_H,$$

$\eta = 1, P_H = 1$

$$\begin{aligned}
\alpha\left[(1-\tau)Y_H + (1-\tau)P_F Y_F - QB_H - Q^* P_F B_F\right] + \tau Y_H + QB_H = Y_H,\\
\alpha\left[(1-\tau)P_F Y_F - Q^* P_F B_F\right] = Y_H - \alpha(1-\tau)Y_H + \alpha QB_H - \tau Y_H - QB_H
\end{aligned}$$

Isolating P_F

$$P_F = \frac{Y_H - \alpha(1-\tau)Y_H + \alpha QB_H - \tau Y_H - QB_H}{\alpha\left[(1-\tau)Y_F - Q^* B_F\right]}$$

rewriting nicely

$$P_F = \frac{(1-\alpha)\left[(1-\tau)Y_H - QB_H\right]}{\alpha\left[(1-\tau)Y_F - Q^*B_F\right]}$$

$\eta > 1$

$$\alpha\left(\frac{P_H}{P}\right)^{-\eta}\frac{(1-\tau)Y_H + (1-\tau)P_FY_F - QB_H - Q^*P_FB_F}{P} + \tau Y_H + QB_H = Y_H,$$

For the normalization $P_H = 1$ and the price index from below we have

$$\alpha\left(\frac{1}{\left(\alpha+(1-\alpha)P_F^{1-\eta}\right)^{\frac{1}{1-\eta}}}\right)^{-\eta}\frac{(1-\tau)Y_H+(1-\tau)P_FY_F-QB_H-Q^*P_FB_F}{P}+\tau Y_H+QB_H = Y_H$$

which gives

$$\alpha\left(\left(\alpha+(1-\alpha)P_F^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{\eta-1}((1-\tau)Y_H+(1-\tau)P_FY_F-QB_H-Q^*P_FB_F)+\tau Y_H+QB_H = Y_H$$

simplify to

$$\alpha\frac{(1-\tau)Y_H + (1-\tau)P_FY_F - QB_H - Q^*P_FB_F}{\alpha+(1-\alpha)P_F^{1-\eta}} + \tau Y_H + QB_H = Y_H$$

Multiply

$$\alpha\left((1-\tau)Y_H + (1-\tau)P_FY_F - QB_H - Q^*P_FB_F\right) = (\alpha+(1-\alpha)P_F^{1-\eta})(Y_H - \tau Y_H - QB_H)$$

Isolating the P_F

$$\alpha\left((1-\tau)P_FY_F - Q^*P_FB_F\right) = (1-\alpha)P_F^{1-\eta}(Y_H - \tau Y_H - QB_H)$$

Further isolating

$$\alpha\left((1-\tau)Y_F - Q^*B_F\right) = (1-\alpha)P_F^{-\eta}(Y_H - \tau Y_H - QB_H)$$

which gives

$$\left(\frac{(1-\alpha)[(1-\tau)Y_H - QB_H]}{\alpha\left[(1-\tau)Y_F - Q^*B_F\right]}\right)^{\frac{1}{\eta}} = P_F$$

B.5 Price index

Consumption is CES

$$C = \left(\alpha^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

$$\max PC - P_H C_H - P_F C_F$$

The first order condition is given by

$$C_H = \alpha \left(\frac{P_H}{P} \right)^{-\eta} C, \quad C_F = (1-\alpha) \left(\frac{P_F}{P} \right)^{-\eta} C$$

Plug into budget constraint

$$PC = P_H^{1-\eta} P^{\eta} C\alpha + (1-\alpha) C P_F^{1-\eta} P^{\eta}$$

C cancels, solve for P

$$P = \left(\alpha P_H^{1-\eta} + (1-\alpha) P_F^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

B.6 Pseudo-algorithm

1. Discretize the governments choice space $\mathbf{B}$ and $\mathbf{B}^*$
2. Guess a solution for each government $\tilde{B}$ and $\tilde{B}^*$
3. For every point in the choice space $\mathbf{B}$, recover $(Q, Q^*, P_{F,1})$ from the investors optimality conditions and the resource constraint, assuming $B^* = \tilde{B}^*$.

   $\rightarrow$ Compute $V(\mathbf{B})$ and select the optimal choice $B \in \mathbf{B}$ that maximizes it
4. Repeat the same process analogously for every point in $\mathbf{B}^*$, recovering optimal B^*
5. Compare optimal B and B^* with guess and update $\tilde{B}$ and $\tilde{B}^*$ accordingly
6. Repeat steps 3 to 5 until convergence