Freeriders and Underdogs: Participation in Corporate Voting*

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Abstract

We document nontrivial discretionary participation in corporate voting: 24% in the US and 7% internationally. Lower participation by freeriders (majority-preference shareholders) than underdogs (minority-preference shareholders) can distort outcomes. A rational choice model, where participation depends on pivotal probabilities, costs, and benefits, illustrates these effects. Using structural estimation, we recover unobservable shareholder preferences and find that 16% of US and 11% of international nonroutine proposals do not represent the majority preference. Reducing voting costs does not always improve representativeness. Counterfactual analysis illustrates this nonmonotonic relationship: lowering costs decreases misrepresentation in board proposals but increases it in takeover defense proposals.

Keywords: voting participation; corporate governance; shareholder proposals; shareholder democracy; structural estimation

1 Introduction

Corporate voting helps uphold managerial accountability, aligns governance with shareholder interests and influences firm actions.[1] Shareholder voting support lends legitimacy to proposals in firms (Fisch, 2017) but also informs regulators and the greater public about shareholder preferences (Grewal et al., 2016; Hwang and Nili, 2020). This applies especially to controversial and partisan issues in today's increasingly polarized political landscape (Wu and Zechner, 2024;

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[1]See evidence in Cuñat et al. (2012); Thomas and Cotter (2007); Del Guercio et al. (2008); Cai et al. (2009); Levit and Malenko (2011a); Becht et al. (2016).

Ravina and Persico, 2025; Kempf and Tsoutsoura, 2024). For aggregating preferences in voting, majority rule is widely considered the most efficient method.[2] However, only in full participation does voting support equal the underlying shareholder preferences. This is not the case in reality, where not all shareholders vote, with average participation rates in the United States of 73% among all shareholders and, among those that are not insiders or mutual funds, only 24%.

Low participation rates raise concerns about the representativeness of corporate governance decisions and its broader implications for capital markets, including investor confidence and firms' ability to raise capital. Because shareholders' participation decisions depend on the perceived costs and benefits of voting (Brav et al., 2022; Hafeez et al., 2022), regulators have sought to overcome the low participation rates by lowering voting costs or increasing voting benefits. Historical examples for policy attempts are one-person-one-vote systems, voting caps, and policies favoring smaller shareholders (Dunlavy, 2006; Hirst, 2017; Haan, 2022). More recent efforts and suggestions have focused on improving accessibility, for example via electronic voting (Fisch, 2017). While insights from political elections suggest that reducing voting costs can increase turnout (Merlo and Palfrey, 2018), participation rates among certain shareholder groups remain persistently low. This raises doubts about the effectiveness of these policies and highlights gaps in the understanding of corporate voter turnout (Nili and Kastiel, 2016; Hirst, 2017; Griffin, 2019; Ricci and Sautter, 2022). Indeed, studying voting representation and assessing the impact of interventions is inherently challenging, as shareholder preferences are unobservable.

We structurally estimate a model of corporate voting to study how heterogeneity in shareholder preferences and strategic selection into voting affects voting outcomes relative to a full participation benchmark. A key feature of corporate voting is ownership heterogeneity, where voting power varies across shareholders: large shareholders hold more significant stakes than do dispersed shareholders. We incorporate this feature by categorizing shareholders into *regular* and *discretionary* voters. Regular voters include those with high incentives to vote (insiders and blockholders), and those that are required to vote (family voting trusts and investment funds). Discretionary voters, on the other hand, include hedge funds, private wealth managers, and retail investors, who choose whether to vote. Unlike regular voters, their preferences are unobservable, no polls akin to those used in political elections produce estimates, and trading can introduce additional noise.[3] Our model captures both ownership heterogeneity and the observability of regular voter preferences, enabling the analysis of corporate voting outcomes.[4]

The model highlights two opposing forces in corporate voting: the *freerider effect*, where discretionary voters supporting the majority abstain, relying on regular voters to secure their preferred outcome, and the *underdog effect*, where minority-side discretionary voters participate more actively to compensate for their disadvantage. These effects arise due to ownership heterogeneity and have implications for representativeness.[5] Our main theoretical result is that

[2]Wu and Zechner (2024) show that the ownership-weighted average shareholder preference maximizes aggregate welfare in the presence of activists. See also Zytnick (2022) for a discussion.

[3]We provide a microfoundation in the Online Appendix, where opportunistic and noise trading prior to the record date generates dispersion in the record-date composition of discretionary holders. For broader analyses of the interaction between trading and voting, see Levit et al. (2024); Meirowitz and Pi (2022).

[4]Jackson and Zytnick (2022) study individual investor preferences and show that ideological preferences and not proposal characteristics mostly determine the individual investor's vote.

[5]In political elections, where all voters are discretionary, the underdog effect describes the higher participation rate of supporters of the less popular option, while the freerider effect captures the tendency of voters supporting

reducing the cost of voting has a nonmonotonic effect on representativeness. When costs are high (or the benefit-to-cost ratio is low), majority-side discretionary voters (*freeriders*) are more likely to abstain than minority-side discretionary voters (*underdogs*), leading to systematic misrepresentation. Initially, lowering voting costs worsens this distortion by disproportionately mobilizing underdogs, pushing outcomes further from the full participation benchmark. However, once costs fall below a critical threshold, freeriders also begin to vote, restoring alignment between voting outcomes and the preferences of the overall shareholder base.

The model builds on the rational-choice framework (Riker and Ordeshook, 1968), where participation depends on the probability of being pivotal, the cost of voting, and the benefit of a preferred outcome. Specifically, we extend the *political elections model* of Myatt (2015), which derives closed-form pivotal probabilities under heterogeneous preferences, by incorporating regular voters—a central feature of corporate settings. We show that full freeriding occurs only in the presence of regular voters. This is precisely where misrepresentation, in the corporate setting, does not monotonically decrease with the benefit-to-cost ratio. These insights guide our empirical approach, where we test the model's predictions using shareholder voting data across different proposal types, industries, and countries.

We solve the model with the aim of estimating the preferences of discretionary voters—both freeriders and underdogs—using data on shareholder proposals. The participation rates and equilibrium regions are available in closed form, ensuring that estimation remains computationally robust. In our empirical analysis, we use voting data from ISS Voting Results. Our primary dataset consists of 66,300 proposals voted on in US firms from 2003 to 2020, complemented by 189,455 proposals from non-US firms from 2013 to 2020. Average voting participation in our sample was 73% for US proposals (similar to Brav et al. (2022) and Nili and Kastiel (2016)) and 40% outside the United States. To proxy for regular voters, we use insiders as well as investment funds, which in most jurisdictions have a fiduciary duty to vote. Excluding regular voters, discretionary participation rates were 24% in the United States and 7% internationally.

We then use the model to assess whether voting outcomes reflect the majority preference of the shareholder base. Specifically, we introduce the likelihood of a minority win as an empirical measure of *representativeness*, capturing how both regular and discretionary shareholders can influence corporate decisions through voting.[6] In our U.S. sample, we find that strategic selection into voting results in a 16% probability that the final outcome reflects the minority preference. This probability varies for subsamples: it is highest for management proposals related to governance (25%) and lowest for shareholder proposals on takeover defenses (3%). Outside the United States, representativeness issues are most pronounced in Japan, where the probability of a minority win reaches 27% and least in Brazil, Russia, and China (4%). On average, the likelihood of an unrepresentative outcome is 11% outside the United States. These findings highlight the utility of our model, which allows us to disentangle whether disparities arise from differences in voting costs and benefits or from the composition of the shareholder base. Low participation rates outside the United States are largely driven by *full freeriding*,

the favorite to abstain (see Palfrey and Rosenthal (1983)).

[6]Our empirical setup allows for more nuanced measures of the tension between insiders and dispersed shareholders, such as the difference in preferences between these groups or the likelihood that the majority of dispersed shareholders prevails. However, here, we focus on the broader measure of whether the outcome reflects the majority of the entire shareholder base.

where majority-side discretionary voters abstain, occurring in 72% of sample proposals. Even within the United States, full freeriding is prevalent, appearing in 83% of proposals. In this equilibrium, discretionary majority voters (*freeriders*) abstain, and increasing the benefits (or decreasing the costs) of voting paradoxically reduces representativeness by further mobilizing minority voters without inducing freeriders to participate. Conversely, in 11% of the sample, discretionary minority voters (*underdogs*) turn out fully, even though their probability of swinging the vote remains low. This represents a full underdog effect, where these voters participate disproportionately relative to their chances of influencing the outcome. Notably, the standard equilibrium observed in political elections—where both sides participate partially—occurs in only 7% of U.S. proposals, underscoring the distinctiveness of corporate voting.

In the corporate setting, our findings suggest that reducing voting costs may increase turnout but does not necessarily enhance representativeness. Identifying the equilibrium is therefore crucial for predicting outcomes and evaluating interventions—whether campaigns to reduce uncertainty about discretionary voter preferences (Lee and Souther, 2020), platforms like Moxy that lower voting costs, or regulatory reforms affecting voting costs and benefits (e.g., the EU Shareholder Rights Directive Pinto (2009)).

To inform such decisions, we conduct a counterfactual analysis by varying the benefit-to-cost ratio per share and calculating the resulting probability of misrepresentation. Consistent with our model, we find a nonmonotonic relationship: for instance, increasing the benefit (or reducing the cost) decreases misrepresentation in shareholder board proposals but increases it in shareholder takeover defense proposals. Internationally, we show that lowering voting costs (or increasing benefits) in the European Union can reduce misrepresentation—a stated aim of the Shareholder Rights Directive II—whereas in the United States, changes in voting costs or benefits have minimal impact on representativeness.

Our structural estimation has both advantages and limitations. The primary advantage is that it provides a framework to interpret corporate voting outcomes by disentangling the role of preferences (popularity of a proposal) from perceived benefits (importance of winning). This distinction allows the model to explain cases where an important proposal receives low turnout due to consensus, while a low-stakes proposal may attract high participation due to disagreement. Using this framework, we estimate both unobservable preferences and the benefit-to-cost ratio per share, offering insight into voting patterns.

The key limitation of our approach is its reliance on a specific theoretical model, with its attendant assumptions. Two are particularly important. First, we assume preference heterogeneity (private values) drives participation rather than information heterogeneity (private information). If voting aggregates dispersed private information (Feddersen and Pesendorfer, 1996), our model may not fully capture that. However, we show that our results hold across subsamples where information aggregation is more likely to matter, such as early versus late-season meetings.

Second, beyond specific functional choices, we test the robustness of our theoretical and empirical results to broader variations in the model's key primitives—the cost (assumed to be the same for all), valuation (assumed to be symmetric across freeriders and underdogs), and distribution of aggregate uncertainty (assumed to be uniform in the baseline). In the Online Appendix, we develop three extensions that systematically relax each of these assumptions:

idiosyncratic participation costs, a flexible (Beta) distribution for the latent support rate, and asymmetric private valuations across sides. Together, these extensions encompass heterogeneity in behavior, beliefs, and incentives, showing that our central qualitative prediction—the non-monotonic relation between the probability of misrepresentation and the benefit–cost ratio—persists across these richer environments. Empirically, we estimate the uniform-cost case (a tractable benchmark from the first extension) to gauge how heterogeneity affects estimation. Despite its simplicity, the baseline model matches the data remarkably well, outperforming both reduced-form OLS regressions and the aforementioned more complex variants that require numerical computation and therefore introduce estimation noise and additional computational cost.

Finally, while our model considers a corporate setting, it can also extend to political elections, particularly in contexts with regular voters—those with negative voting costs due to factors like civic duty (Manning and Edwards, 2014; Coate and Conlin, 2004; Feddersen and Sandroni, 2006) or mandate-driven behavior (Faravelli et al., 2015). These voters participate regardless of their pivotal probability or the benefits of their preferred outcome winning. Moreover, their preferences can often be inferred from external sources, such as party donations or electoral registers in the United States. Exploring how our framework can apply to political elections, where regular voters with identifiable preferences play a crucial role, is a promising direction for future research.

Literature. Our paper contributes to several strands of literature. First, we contribute to the theoretical literature on corporate voting (for an overview, see Malenko (2023)). Most of this literature focuses on the aggregation of dispersed private information in a framework with a common value (i.e., heterogeneous information and homogeneous preferences) and no differences in the ownership structure (Maug and Yilmaz, 2002; Maug and Rydqvist, 2009; Bond and Eraslan, 2010; Levit and Malenko, 2011a). The aforementioned papers do not allow for voter abstentions and hence are not apt for a study of voting participation.

Two theoretical papers relate closely to ours. Bar-Isaac and Shapiro (2020) assume heterogeneous information and homogeneous preferences but also consider differences in the ownership structure (i.e., blockholders and dispersed shareholders) and allow for abstention. They show that blockholders may abstain to assist the aggregation of information. Hence, their paper has normative implications for current US regulation, which requires blockholders (i.e., regular voters in our terminology) to vote all their shares.

In a departure from the literature, Levit et al. (2024) introduce heterogeneous preferences to study the interplay between trading and voting, showing that then, the firm's stock price may not fully reflect shareholders' welfare. An important result is that heterogeneous or private values can influence stock prices, depending on the preferences of the marginal post-voting trader relative to those of the marginal voter (Levit et al., 2024, Section 7.3). However, their model neither incorporates abstention in the voting game nor accounts for differences in ownership structure. Our contribution is to develop a model where both heterogeneous preferences and the ownership structure shape voting. While we do not explicitly model trading after the vote, we assume that its expected impact is common knowledge and affects the participation decision. Similarly, trading before the vote are incorporated if observable (e.g., via 13D or 13F filings).

Any unobservable trading adds to the aggregate uncertainty over the preferences of discretionary voters, further justifying its role in our framework.[7,8,9]

Second, we contribute to the literature on voting participation in political elections. Theoretical models of voter turnout in these elections (Downs, 1957; Riker and Ordeshook, 1968) conceptualize voting as a cost-benefit decision. One of the main paradigms is pivotal voting models (Palfrey and Rosenthal, 1983, 1985). These models explain turnout through the probability of being decisive but must confront the turnout paradox, as costly participation and low pivotal probabilities predict near zero turnout in large electorates. Introducing aggregate uncertainty about aggregate representation(Krishna and Morgan, 2012; Evren, 2012; Myatt, 2015) resolves. Our work is primarily based on the framework in Myatt (2015), extending it to the corporate setting by incorporating ownership heterogeneity, which leads to richer and more realistic equilibria. This shows that aggregate uncertainty sustains turnout while accounting for institutional differences between firms.[10]

Empirical studies of voter turnout models show pivotal voter models predict turnout well but overestimate electoral closeness due to aggregate uncertainty (Coate and Conlin, 2004; Coate et al., 2008).[11] Merlo and Palfrey (2018) validate pivotal-voter models through structural estimation, demonstrating effective parameter recovery.[12] They also examine how reducing voting costs influences turnout, akin to our study of cost reductions and representativeness. Kawai and Watanabe (2013) highlight how turnout skews preference aggregation, as high-cost or low-intensity voters are underrepresented. Bursztyn et al. (2024) provide causal evidence from Swiss referenda showing that closer elections increase participation. We contribute by examining how aggregate uncertainty and heterogeneity affect turnout and representativeness in corporate voting, where increased participation does not always enhance representation.

Third, we contribute to the empirical literature on corporate voting participation. Nili and Kastiel (2016) document declining participation in US corporate elections, despite technological advances and increasingly high-stakes proposals. Van der Elst (2011) finds that block ownership predicts participation, while Brav et al. (2022) show that retail investors vote more in smaller firms, when their stake is larger, and when they receive more agenda-related information.[13]

[7]Levit et al. (2023) study blockholders and the voting premium in a related framework to Levit et al. (2024). They show that minority blockholders can endogenously generate a voting premium (or even a negative one) by trading to influence the shareholder base, as dispersed shareholders free-ride on the blockholder's trades and the resulting liquidity of voting shares.

[8]For related instances of the voting and trading literature, see studies on the separation of cash flow and voting rights (Brav and Mathews, 2011; Zachariadis and Olaru, 2017), on how uninformed shareholders prefer their votes to be exercised by informed ones (Esô et al., 2014), and on how the ability of shareholders to trade after voting influences information aggregation (Meirowitz and Pi, 2022) as well as the literature on vote trading in political election, where agents with higher preference intensity buy votes from those less invested (Casella et al., 2012).

[9]Trading in our setup is analyzed in the Online Appendix. There we show that pre–record-date trading and share lending (OA.3.1) jointly shape the aggregate uncertainty in ownership composition, while post–record-date trading (OA.3.2) is neutral for equilibrium outcomes under our timing assumptions. These mechanisms help interpret the role of ownership dynamics around the record date, but trading is not the focus of our analysis.

[10]Other turnout theories include altruistic models that account for spillover benefits (Palfrey and Prisbrey, 1997), expressive voting models that emphasize intrinsic satisfaction (Brennan and Lomasky, 1993; Riker and Ordeshook, 1968), and ethical voting models that highlight civic duty and collective welfare (Coate and Conlin, 2004; Feddersen and Sandroni, 2006). For overviews, see Feddersen (2004); Geys (2006); Merlo (2006).

[11]Our model mitigates this by incorporating regular voters, leading to equilibria where outcomes are not ties.

[12]Altruistic models perform similarly, ethical voting the worst, and expressive voting falls in between.

[13]Turnout determinants in political elections exhibit similar patterns, varying with institutional settings (Blais, 2006), voter characteristics (Wolfinger and Rosenstone, 1980), altruism (Blais, 2000), and education or candidate

We extend this literature by structurally estimating a model that identifies how ownership, preferences, and proposal characteristics shape voting outcomes.

Fourth, we contribute to the literature that estimates the benefits of voting. This literature has so far focused on the stock market reaction to the passage of proposals (Cuñat et al., 2012; Cuñat et al., 2020; Bach and Metzger, 2018; Gantchev and Giannetti, 2020), the market for votes in the equity loan market (Christoffersen et al., 2007; Aggarwal et al., 2015), the discrepancy between stock and option prices around voting (Kalay et al., 2014), and trading patterns following voting outcomes (Li et al., 2021). We develop an alternative method to estimate benefits that relies on the voting participation decision.

Fifth, our study relates to papers that structurally estimate voting parameters in the corporate governance setting. Matvos and Ostrovsky (2010), in the context of director elections, show that (in the terminology of our paper) regular voters vote according to peer effects and heterogeneity in their management friendliness. Barry (2023) estimates the value of non-binding Say-on-Pay votes on CEO compensation, following the structural strand of the literature on CEO compensation (Taylor, 2010, 2013). Blonien et al. (2024) estimate errors in shareholder voting, and Pinnington (2023) studies the voting behavior of passive funds. We contribute by focusing on the participation decision and preferences of discretionary voters.

2 Voting in the United States

2.1 Data

Our voting outcome data is from ISS Voting Results, which covers Russell 3000 firms from 2003–2020. Following the literature on shareholder voting (Gordon and Pound, 1993; Cuñat et al., 2012; Bach and Metzger, 2018), we exclude routine proposals and director elections because they do not provide "no" votes and hence abstentions have another interpretation (Matvos and Ostrovsky, 2010; Cai et al., 2013). In tune with our theoretical analysis, we focus on simple-majority elections. The source of individual investment fund votes is ISS Mutual Fund Voting, which draws its data from mandatory N-PX filings. We aggregate fund-level voting data at the corresponding fund-family level, following evidence that funds rarely vote differently than their family (Fichtner et al., 2017).

We obtain data on institutional ownership from Factset, which sources its information from 13F and other filings. Institutions that report 13F filings include investment funds, which also disclose their votes on the N-PX forms, as well as hedge funds and other asset managers. We complement this data with the ownership fraction of insiders from Worldscope, as used by Iliev et al. (2015). This measure tabulates shares held by insiders and specifically includes (i) shares held by officers, directors, and their immediate families; (ii) shares held in trust; (iii) shares of the company held by any other corporation (except shares held in a fiduciary capacity by banks or other financial institutions); (iv) shares held by pension or benefit plans; and (v) shares held by individuals who own 5% or more of the outstanding shares. Importantly for our analysis, the measure explicitly excludes shares held in a fiduciary capacity by financial institutions that report their votes via N-PX forms.

trains (Abramson et al., 1992).

Our sample includes 39,274 meetings. There are, on average, 1.7 proposals per meeting and 66,300 proposals in total. (For a breakdown by year, see Table OA.1 in the online appendix.) Table 2 presents summary statistics——in Section A of the appendix with all main figures and tables. Our firms are comparable to those in the samples used by other papers on shareholder meetings (e.g., Cvijanović et al. (2016)), with an average market capitalization of $8.7 billion, leverage of 38%, and a market-to-book ratio of 3.7. The sample firms had, on average, 187 million outstanding shares, of which institutional investors owned 69%. Among these, 30% reported their votes (N-PX shares). Another 13% was owned by insiders. There were on average 286 distinct institutional shareholders per voting event.

2.2 Mandatory voting

In the United States, during our sample period 2003–2020, investment funds had a fiduciary duty to vote on behalf of their clients (SEC Final Rule IA-2106). This duty was enforceable for mutual funds and other registered investment management companies, which had to disclose their votes on the N-PX forms.[14] Using voting data from Broadridge, Zytnick (2022) shows that mutual funds indeed have a turnout of virtually 100%. We therefore assume that these funds always vote, and account for them as our regular voters, together with the fraction of insider holdings from Worldscope. Note that these contain both voting trusts, that must vote, and other insiders that are not legally obliged to vote. We believe that the assumption that insiders vote is reasonable as they are likely to have strong incentives to do so (for a stark example see Fos and Jiang (2015), who document cases in which CEOs exercise underwater options to obtain more votes).

To calculate the number of votes by discretionary voters, we subtract the votes of N-PX voters and votes of insiders–which we assume will support management–from the aggregates in each category (for, against, and abstentions). These other votes can come from other institutional investors such as hedge funds as well as from individuals such as retail shareholders. We then calculate discretionary participation as the number of these discretionary votes out of the total number of non-regular shares. Total participation is the number of all votes cast as a percentage of the shares outstanding or the sum of the discretionary and regular voting participation (100% by definition).

Regular voters can also formally abstain. The number of these abstentions is very small, with 1%. We include the official abstention votes to the sum of total votes cast when calculating participation rates.

2.3 Voting Participation: Stylized Facts

To set the stage for our analysis, we show basic summary statistics for voting support (as a fraction of the valid base) and participation in Table 2. The valid base can be either the number of shares outstanding or the number of voting shares and depends on state laws and company charters (Bach and Metzger, 2018).

[14]Disclosure of Proxy Voting Policies and Proxy Voting Records by Registered Management Investment Companies, Securities Act Release No. 8188, 68 Fed. Reg. 6564, 6565 (Feb. 7, 2003) (codified at 17 C.F.R. pts. 239, 249, 270, 274)

Average voting participation is nontrivial but not full. Total participation averages 73%, and discretionary participation averages 24%.[15] Management proposals receive on average more support than shareholder ones, with the highest support (91%) for Say-on-Pay proposals (See Section OA.5 for definitions). Shareholder proposals on corporate social responsibility (CSR) receive the least support (18%), consistent with the results in He et al. (2019). Discretionary voting participation does not align with support. It ranges from 14% in business-related management proposals to 48% in the very rare (6) CSR-related management proposals.

3 Model

In this section, we analyze the voting participation decision using a rational-choice framework that links participation to a voter's pivotal probability, the cost of participation, and the benefit derived from the preferred outcome's success (Riker and Ordeshook, 1968). Building on the political elections model of Myatt (2015), which provides closed-form calculations of pivotal probabilities, we extend the framework to incorporate differences in ownership structure—a defining feature of corporate elections, in contrast to the one-voter-one-vote assumption of Myatt (2015). Our model captures the interaction between the underdog effect, where voters supporting the less popular alternative are more likely to participate, and the freerider effect, where supporters of the leading option are less inclined to vote.

While the model is of independent interest for its insights into how voting costs and benefits shape representativeness, it is also designed to support empirical estimation in later sections. Specifically, the model provides closed-form expressions that link observable variables, such as the ownership structure, to unobservable primitives like the benefit-to-cost ratio. These relationships offer a foundation for identifying model parameters and estimating the probability that outcomes under discretionary participation differ from those under full participation. This strong selection effect, which alters outcomes rather than merely redistributing support, is a central focus of our analysis. The estimation framework and results, which leverage U.S. and international voting data, are presented in subsequent sections.

3.1 Setup

Consider a corporate proposal up for a vote with two options, R and L. Shareholders have heterogeneous preferences over the proposal: *if they vote*, they do so according to a pre-determined preference type, R or L. In total, shareholders own $n+1$ voting shares, which are split between two groups.[16] A fraction $\gamma \in [0,1)$ of the n shares belongs to *regular voters*, while the remaining $(1-\gamma)n+1$ belong to *discretionary* voters, with a single share and a single vote each.

Regular voters always vote. We capture their voting preference by a constant q, which is the fraction of this group who vote for R and is common knowledge at the voting stage. We assume without loss of generality:

[15] Broker nonvotes, where shareholders give no voting instructions to their broker, typically count toward quorum. In our sample, 97% of proposals meet quorum with broker nonvotes, and 90% would without them. However, broker nonvotes do not affect nonroutine proposal outcomes.

[16] The number of voting shares n approximates the firm's market capitalization divided by average holdings. For example, a \$10 million market cap and \$10,000 average holdings yield $n = 1,000$. In our US sample, the average number of institutional owners at a voting event is 256 (Table 2 Panel A).

A0: $q \in (1/2, 1)$.

So R is the *favorite* among regular voters, and L is the *underdog*. We can think of regular voters as either i) two subgroups (blockholders) with sizes q and $1-q$ supporting R and L, respectively; or, alternatively, ii) coalitions of dispersed shareholders, who always participate and vote in proportion q and $1-q$ for R and L.

Discretionary voters choose whether to vote. Their choice depends on a cost $c > 0$, which they face when they vote, regardless of the outcome, and an incremental benefit $v > 0$, which represents the difference in \$/share accruing to a discretionary voter when her pre-determined type wins. For our main analysis, we assume all discretionary voters are risk neutral and share the same c and v, regardless of their preference type R or L.[17] Among discretionary voters, option R has popularity $p \in (0, 1)$. The crux of the model is that there is aggregate uncertainty: p is unknown with strictly positive continuous density f in $(l, h) \subseteq (0, 1)$, and F the corresponding distribution, where $\overline{p} \equiv \mathbb{E}[p]$.[18] Finally, we assume that discretionary voters face an availability shock: even if they decide to vote, they will ultimately cast a vote with probability $\overline{a}$. This availability shock is in the context of voting what a liquidity shock is in the context of trading.[19]

Upon voting, the outcome is the simple majority of the votes cast, and in the case of a tie, a fair coin toss is the tie-breaker. The aforementioned structure is common knowledge. The only choice variable (strategy) is whether a discretionary voter votes. We look for symmetric strategies across preference types R or L of discretionary voters, and the solution concept is a Bayesian Nash Equilibrium. The model coincides with Myatt (2015) for $\gamma = 0$ (i.e., no regular voters), and hence we extend his for general $\gamma > 0$. Below we discuss the cost c and benefit v.

The cost c captures i) administrative costs such as time and effort (Brav et al., 2022) for the US and international examples, such as pre-registration requirements in Switzerland, power of attorney in Sweden, and the lack of electronic voting in some countries (Eckbo et al., 2010, 2011; Council of Institutional Investors, 2011)), and ii) the opportunity cost to keeping one's shares and voting. A significant example of the latter is loan fees earned in the share lending market. As Porras Prado et al. (2016, pp. 3212-3213) mention "Investors may trade off the income from lending with the potential risks of losing monitoring control through transferring shares to the equity lending market." Indeed, loan fees are higher around voting record dates (Aggarwal et al., 2015, Figure 1).

The benefit v is a private value to a discretionary voter. Possible sources of differences in private values among investors are i) incentives (Chevalier and Ellison, 1999; Cvijanović et al., 2022), ii) portfolio concerns (Matvos and Ostrovsky, 2010; Cohen and Schmidt, 2009; Bodnaruk and Rossi, 2016; He et al., 2019; Griffith and Lund, 2019), iii) preferences (Bolton et al., 2020; Zytnick, 2022; Dikolli et al., 2022; Jin and Noe, 2024), iv) conflicts of interest (Davis and Kim, 2007; Ashraf et al., 2012; Butler and Gurun, 2012; Cvijanović et al., 2016; Griffith and Lund,

[17] In Section OA.4 we present several model extensions, which generalize the baseline framework to accommodate heterogeneity in costs, non-uniform aggregate uncertainty, and valuation asymmetries—including the uniform (idiosyncratic) cost special case.

[18] Aggregate uncertainty over p is the key factor that overcomes the impossibility result of Palfrey and Rosenthal (1985), where costly voting leads to near-zero participation in large elections due to infinitesimal pivotal probabilities when p is a known constant.

[19] The availability shock aids in identifying unique equilibria. Following Myatt (2015), we initially assume a is a random probability within an interval and then take the limit as the interval collapses to a single point, yielding the degenerate case where $a = \overline{a}$.

2019; Michaely et al., 2023), v) opinions (Harrison and Kreps, 1978; Li et al., 2021; Kakhbod et al., 2022), vi) extreme information/no learning (Feddersen and Pesendorfer, 1997; Yilmaz, 2000), vii) governance philosophy (Couvert, 2021; Bubb and Catan, 2022), viii) separation of cash flow and voting rights (Burkart and Lee, 2008; Adams and Ferreira, 2008; Brav and Mathews, 2011; Zachariadis and Olaru, 2017), ix) time horizon (Bushee, 1998; Gaspar et al., 2005), and x) tax clienteles (Desai and Jin, 2011).

Our model abstracts from how precisely these voting benefits materialize. While not all proposals are binding, empirical evidence suggests that firms often face significant consequences for failing to abide by voting outcomes (Ferri, 2012; Levit and Malenko, 2011a; Malenko, 2023). For shareholder proposals, the implementation rate of majority decisions is approximately 70%, and close votes are associated with positive abnormal returns (Ertimur et al., 2013; Denis et al., 2019; Larcker et al., 2015). In director elections, a failure to abide by shareholder votes can lead to decreased CEO pay, increased CEO turnover, and adverse labor market outcomes (Del Guercio et al., 2008; Cai et al., 2009; Ertimur et al., 2018; Fos et al., 2017; Aggarwal et al., 2019). Moreover, a majority of firms adjust compensation following say-on-pay votes, with notable spillovers observed among related firms (e.g., with overlapping directors). For a further discussion of the model's assumptions, particularly, in relation to our estimation, see Section 7.

We next proceed to the model's primitives. We present the turnout rates and our main focus, the misrepresentation probability.

3.2 Primitives

Let us consider a discretionary voter (shareholder) of either type $i \in \{R, L\}$. According to the rational choice theory, she votes, abstains, or is indifferent, depending on whether the difference of expected benefit to cost $v \Pr[\text{Pivotal}|i] - c$ is positive, negative, or zero, respectively. The expression $\Pr[\text{Pivotal}|i]$ is her probability of being pivotal, that is, of affecting the outcome. We assume a symmetric participation strategy within each type $i \in \{R, L\}$ so that if we let t_i the turnout rate for i we have:

$$t_i = \begin{cases} 1, & \text{if } \Pr[\text{pivotal}|i] > c/v, \\ \in (0,1), & \text{if } \Pr[\text{pivotal}|i] = c/v, \\ 0, & \text{if } \Pr[\text{pivotal}|i] < c/v, \end{cases} \tag{1}$$

where, in turn, the three cases correspond to full, partial, and no participation, respectively. The following assumptions on model parameters ensure that discretionary participation is possible.
A1: $\gamma < 1/(2q)$.
A2: $v \geq c$.
Assumption 1 rules out the case in which either type of regular voter can decide the outcome unilaterally. (It is sufficient, given A0; i..e., $q > 1/2$). In OA.6, we discuss how Assumption 1 affects our estimation sample. Assumption 2 ensures that the cost should not exceed the benefit, and hence the inequalities in (1) are meaningful.

Now, given any turnout rate t_i and a realization of support of R among discretionary voters

p drawn from density f, the realized support for R is

$$\overbrace{\overline{a}(1-\gamma)pt_R}^{\text{discretionary support for } R} + \overbrace{\gamma q}^{\text{regular support for } R}, \tag{2}$$

where we indicate the corresponding discretionary and regular (voter) support, in the sum. Similarly, discretionary and regular support for L is $\overline{a}(1-\gamma)(1-p)t_L + \gamma(1-q)$. Hence, the outcome is determined by the sign of

$$O_{\text{disc}}(p; t_L, t_R) \equiv \overbrace{\gamma q + \overline{a}(1-\gamma)pt_R}^{\text{total support for } R} - [\overbrace{\gamma(1-q) + \overline{a}(1-\gamma)(1-p)t_L}^{\text{total support for } L}]. \tag{3}$$

If the difference is positive, negative, and zero, R wins, L wins, or they tie, respectively. Define p^* as the probability for which the outcome is a tie; i.e., $O_{\text{disc}}(p^*; t_L, t_R) = 0$, such that from (3):

$$p^* \equiv \frac{t_L}{t_R + t_L} - \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}(t_R + t_L)}. \tag{4}$$

Then, since $O_{\text{disc}}(p; t_L, t_R)$ is increasing in p, R wins if $p > p^*$ and L if $p < p^*$.

The probability p^* plays an important role in the calculation of the pivotal probabilities in the next subsection, and we will also use it to express the probability of misrepresentation. To this end, let $p^{\text{full}} \in (0, 1/2)$ be the probability of a tie under full participation. We calculate it by substituting for $t_L = t_R = 1$ in (4). There is no definitive ranking between p^* and p^{full} for generic turnout rates. Consider first the case, when $p^{\text{full}} < p^*$: i) for a realized p with $p^{\text{full}} < p < p^*$, L wins under discretionary participation but loses under full participation; ii) for $p < p^{\text{full}}$, L wins under both; iii) while for $p > p^*$, R wins under both. The outcome is similar in the case where $p^* < p^{\text{full}}$. Therefore, the voting outcome differs between discretionary and full participation; that is, we have *misrepresentation*, when $p \in \big(\min(p^*, p^{\text{full}}), \max(p^*, p^{\text{full}})\big)$. This allows us to express the probability of misrepresenation as follows:[20]

$$P^{\text{misrep}} \equiv \Pr\Big[p \in \Big(\min(p^*, p^{\text{full}}), \max(p^*, p^{\text{full}})\Big)\Big] = \int_{\min(p^*, p^{\text{full}})}^{\max(p^*, p^{\text{full}})} f(p)dp. \tag{5}$$

In the next subsection, we derive a closed form expression for $\Pr[\text{Pivotal}|i]$ in terms of model fundamentals. This will allow us to characterize parameter regions of different degrees of participation of either voter type i according to (1). For each pair of participation rates, t_L, t_R, we also then have corresponding expressions for $p^\star$ and P^{misrep} (see (4) and (5)).

3.3 Pivotal Probabilities in Large Elections

To calculate the pivotal probabilities, let us first consider how a discretionary voter can affect the outcome. She can be pivotal, if either i) her type is losing by one vote, she pushes the score to a tie, and the coin toss is favorable (with a probability of 1/2); or ii) there is already a tie, the coin toss is against her type and with her vote, and she gives a clear majority to her type.

[20] We assume $f(p) > 0$ for $p \in (l, h)$, so in equilibrium $p^* \in (l, h)$; now, if $p^{\text{full}} < l$ then L wins for sure under full participation and $P^{\text{misrep}} = F(p^*)$; similarly, if $p^{\text{full}} > h$ then R wins for sure and $P^{\text{misrep}} = 1 - F(p^*)$.

The expressions of the pivotal probabilities for an arbitrary number of voting shares n are cumbersome,[21] hindering intuition and the efficiency of our estimation. To simplify, we consider large n elections, adapting the methodology of Myatt (2015) to our $\gamma > 0$ setting. Below, we present the pivotal probabilities as approximated for n large. In this sense, we are looking at *approximate equilibria*, as defined by Myatt (2015, p. 10).

Lemma 1 (**Pivotal Probabilities**). *Assume A1. Then the pivotal probabilities for L and R in large elections are approximately:*

$$\Pr[\text{pivotal}|L] \approx \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})(t_R+t_L)} f\left(p^*\right)(1-p^*), \tag{6}$$

$$\Pr[\text{pivotal}|R] \approx \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}\overline{p}(t_R+t_L)} f\left(p^*\right)p^*, \tag{7}$$

where p^* is defined in (4).

The proofs of this lemma and all other main results are in Section B of the appendix.[22] The equations for the pivotal probabilities exhibit intuitive comparative statics. Pivotality—for either voter type i—increases if we have i) fewer total voters n, ii) a smaller fraction of discretionary voters $1-\gamma$, iii) a lower availability $\overline{a}$, iv) a lower sum of participation rates t_R+t_L, and v) lower support within a type, so $(1-\overline{p})$ for L and $\overline{p}$ for R. Moreover, the pivotal probabilities increase the closer p^* is to the mode of f, where the density is maximized, since this makes it more likely that having one extra vote for a type (L or R) may flip the voting outcome to that type's favor. Even though (6) and (7) are approximations, we subsequently treat them as equalities.

3.4 Equilibrium Participation Rates and Regions

We now move on to characterizing the equilibrium participation rates, using the closed-form expressions for pivotal probabilities defined in terms of the model fundamentals in (1). The values of t_L and t_R can range from zero to one or lie anywhere in between, indicating a mixed strategy. However, since the probability p^* is nonnegative (see (4)), we can rule out equilibria without discretionary participation for the underdog L.

Lemma 2. *There is no equilibrium where discretionary voters of type L do not vote, so in any equilibrium $t_L \neq 0$.*

Intuitively, since regular voters on average favor R (see A0), there has to be some discretionary support for L, so that there is any possibility (no matter how small) for that side to win. Given Lemma 2, there are six possible equilibria to consider in terms of pairs of participation rates (t_L, t_R), where $t_L \in \{(0,1),1\}$, and $t_R \in \{0,(0,1),1\}$.[23] In the case of mixed strategies by either type i, we further assume that f (the density of p) is uniform to be able to solve the equation $\Pr[\text{pivotal}|i] = c/v$ for t_i, see (1):[24]

[21] They involve the Γ function and combinatorial expressions, see Myatt (2015, Eqs (4) & (5))

[22] Lemma 1 is the counterpart of Myatt (2015, Lemma 3) for $\gamma > 0$. The differences are i) the effective electorate size is here $(1-\gamma)n$, i.e., only the discretionary voters, and ii) p^* (i.e., the probability p for which we have a tie) now takes into account the preferences of regular voters (see (4)), who are absent in Myatt (2015).

[23] As we show below, the equilibria where either or both t_L and t_R lie in (0,1) are unique.

[24] This is because in (6), the rates appear in both the denominator and the numerator through $f(p^\star)$, see (4).

A3: $p \sim \mathcal{U}[l, h]$.

For each candidate equilibrium (t_L, t_R), we identify the regions of model parameters that satisfy the conditions imposed by (1) and align with the definitions of rates and probabilities. Consider, for instance, the equilibrium where $t_L = 1$ and $t_R \in (0, 1)$. The equilibrium conditions are $\Pr[\text{pivotal}|L] > c/v$ and $\Pr[\text{pivotal}|R] = c/v$, with pivotal probabilities specified in (6) and (7). The first condition imposes a parameter restriction, while the second provides an expression for t_R, subject to the requirement that the derived rate falls within $(0, 1)$. Using these rates, we calculate $p^\star$ and further refine the parameter regions to ensure $p^\star \in (l, h)$, the support of p. Each equilibrium is ultimately defined by inequalities involving n (the number of shares/votes), γ (the regular voters' block size), and $v/(cn)$, the benefit-to-cost ratio per share—a key parameter in our model and estimation.[25] Notably, these parameter regions are *non-overlapping*, ensuring that each equilibrium corresponds uniquely to a specific set of model fundamentals.

Table 1 summarizes the equilibria while omitting references to the region of n for brevity. The table identifies equilibria based on the participation rates of L and R and the regions of γ and $v/(cn)$ in which they exist. Propositions proving equilibrium existence are in Sections OA.1-OA.2. To streamline our discussion, the last column of Table 1 groups equilibria into three categories based on the extent of freeriding among discretionary supporters of R: (i) full freeriding, (ii) partial, and (iii) none.

3.5 Discussion of Equilibria

We begin our discussion of equilibria with our main theoretical result, which is the key departure from the case with no regular voters.

Proposition 1. *At least one full freeriding equilibrium, that is, m*0 *or* 10*, exists if only if* $\gamma > 0$.

Hence, without regular voters (i.e., $\gamma = 0$, the model of Myatt (2015)), there are no full freeriding equilibria. This statement is true irrespective of A3, the uniformity of p. The intuition is as follows: With only discretionary voters, they are all equally small. With aggregate uncertainty, this implies a nontrivial probability of affecting the outcome and makes voting worth the cost (for a given benefit) for supporters of both sides. In contrast, for $\gamma > 0$, given A0 (i.e., $q > 1/2$), the freeriders are on average at an advantage since regular voters support their side. This raises the bar for the benefit-to-cost ratio for which freeriders will be willing to participate, or conversely, for any $\gamma > 0$, there are values of $v/(cn)$ for which discretionary supporters of R do not vote; i.e., full freeriding.

In Figure 1 (a) and (c), we depict the equilibria of Table 1 in the space $\gamma \times v/(cn)$. As we see, regions $\Gamma \times V$ are non-overlapping between equilibria. Moreover, outside of the parameter regions, there are no equilibria in which the two types use symmetric strategies (pure or mixed). In the figures, we pick parameters, primarily, motivated from our US data: $h - l = 0.5$, $q = 0.8$, and availability $\overline{a} = 1$. We choose two values for $\overline{p}$, 0.7 and 0.3; bearing in mind that $q > 1/2$ from A0, the first choice corresponds to *Agreement* between the (majority of) regular and the

[25] Neither v nor c, independently, influences the regions or rates; see (1), (4), (6), and (7).

(majority of) discretionary voters, and the second to *Disagreement*.[26]

First, aggregate uncertainty regarding p is essential for sustaining equilibria with positive participation; that is, $l < h$ is necessary for the regions $\Gamma \times V$ in Figures 1 (a) and (c) to be non-empty. Intuitively, if voters were certain of others' preferences, then their perceived pivotal probability would be almost zero, and hence they would lose their incentives to participate.

Second, including regular voters (i.e., $\gamma > 0$) enhances the space where equilibria exist and results in a richer set of strategies (e.g., equilibrium $m0$) relative to Myatt (2015, Proposition 2) —i.e., the instance of our model for $\gamma = 0$. As proved in Proposition 1, in Myatt (2015), there are only equilibria mm, $1m$ (or $m1$ depending on whether $\overline{p}$ is larger or smaller than 0.5), and 11. This is also depicted in Figure 1 (a) and (c).

Third, strategic selection into voting, manifesting via the freerider and underdog effects, affects both rates and regions. Focusing on the rates, discretionary support for L (t_L) (weakly) increases with q (support for R among regular voters), while t_R (weakly) decreases. This reflects an intergroup underdog effect: discretionary voters for the underdog among regular voters turn out more to compensate, while those for R freeride and turn out less. Additionally, there is the intragroup underdog effect, as seen in Levine and Palfrey (2007) and Myatt (2015), where t_L (weakly) increases with $\overline{p}$ (average support for R among discretionary voters), while t_R decreases. Both rates also (weakly) increase with the benefit-to-cost ratio per share $v/(cn)$ and decrease with the number of voters n, availability $\overline{a}$, and dispersion $h - l$.

Fourth, selection effects also influence the parameter regions where specific equilibria exist, as shown by $\Gamma \times V$ in Figure 1 (a) and (c). Even when $l < h$, not all equilibria necessarily exist (see Propositions OA.1 and OA.3–OA.7): i) if $l > 1/2$ (strong Agreement, omitted from Figure 1) (a) and (c), equilibria 11 and $m1$, where R supporters fully participate, do not exist because R is the clear majority choice, reducing discretionary turnout for R; ii) if $l < 1/2$ and $\overline{p} > 1/2$ (Agreement), equilibrium $m1$ is absent; iii) if $\overline{p} < 1/2$ (Disagreement), all equilibria exist. Intuitively, for R supporters to fully participate ($t_R = 1$), some discretionary voters must prefer L. Greater disagreement ($\overline{p} < 1/2$) reduces the intergroup freeriding effect, shrinking the region for equilibria like $1m$ (see Figure 1 (a) and (c) and (OA.20)).

Finally, in Figure 1 (b) and (d), we reproduce Figure 1 (a) and (c) with equilibria re-labeled based on the extent of freeriding by R supporters. Notably, for a fixed γ, increasing $v/(cn)$ results in a progression from full to partial to no freeriding. This transition affects outcomes, shifting from R under full freeriding, to a Tie or R under partial freeriding, and eventually to a Tie, R, or even L with no freeriding. This observation, along with Proposition 1, will be crucial in the next subsection, where we analyze the misrepresentation probability as it relates to the benefit-to-cost ratio per share—a key parameter we aim to identify in the data using our model.

3.6 Misrepresentation Probability versus Benefit-to-Cost Ratio

In political elections, a higher benefit-to-cost ratio of voting leads to higher participation and reduced misrepresentation (e.g., Myatt (2015)). This principle underlies many policy interventions aimed at boosting turnout (Nili and Kastiel, 2016). However, in corporate voting, the

[26] In both graphs, the maximum value of γ is $1/(2q)$ (see A1) and we set the maximum of $v/(cn)$, arbitrarily, to 2.5 (i.e., the blue region theoretically extends to plus infinity.).

relationship between the cost of voting and representativeness is more complex due to ownership heterogeneity and strategic participation. In this section, we formally analyze how the probability of misrepresentation, P^{misrep}, depends on the benefit-to-cost ratio per share, $v/(cn)$. We demonstrate that the relationship is nonmonotonic—an effect unique to the corporate setting.

To establish this result, we prove that increasing $v/(cn)$ can first increase and then decrease misrepresentation, a pattern that does not arise in political elections. The key theoretical finding is stated in the following proposition:

Proposition 2. *If and only if $\gamma > 0$, there exists a set of parameters $\gamma, q, l, h, \overline{a}$ and an interval of $\{v/(cn)\}$ in which $\partial P^{misrep}/\partial\left(v/(cn)\right)$ changes sign from positive to negative.*

This result implies that a reduction in voting costs does not always improve representativeness. The effect is driven by ownership heterogeneity, which creates strategic incentives that can lead to equilibria where misrepresentation first rises before eventually falling. The presence of regular voters is critical for this dynamic, as highlighted in the following corollary:

Corollary 1. *If there are no regular voters ($\gamma = 0$), increasing the benefit-to-cost ratio per voter can only decrease the probability of misrepresentation.*

This distinction between corporate and political elections is central to our framework. The proof of Proposition 2 shows that the shift from a full freeriding equilibrium ($m0$ or 10) to a partial participation one ($1m$) as $v/(cn)$ causes $\partial P^{\text{misrep}}/\partial(v/(cn))$ to change sign. Intuitively, an increase in $v/(cn)$ in the full freeriding equilibria raises turnout by underdogs while leaving freeriders unaffected, thereby increasing misrepresentation. However, once underdogs fully participate and freeriders start voting (as in $1m$), misrepresentation declines with rising $v/(cn)$.

To analyze this result step by step, note that under A3, (5) simplifies to:

$$P^{\text{misrep}} = \begin{cases} \left(p^* - l\right)/\left(h - l\right), & \text{if } p^{\text{full}} < l, \\ \left|p^* - p^{\text{full}}\right|/\left(h - l\right), & \text{if } l \leq p^{\text{full}} \leq h, \\ \left(h - p^*\right)/\left(h - l\right), & \text{if } p^{\text{full}} > h. \end{cases} \tag{8}$$

Since p^{full} is constant and independent of c, the effect of $v/(cn)$ on P^{misrep} depends entirely on how p^* evolves in (4).[27] Below we summarize the probability p^* vs $v/(cn)$ across equilibria.

Eqm.:	mm	$1m$	10	11	$m1$	$m0$
$\partial p^*/\partial\left(v/(cn)\right)$	$=0$	<0	$=0$	$=0$	>0	>0

(9)

The effect of $v/(cn)$ varies across equilibria and determines the parameter regions where each equilibrium exists (Figure 1). In equilibrium mm, the outcome is always a tie, ensuring that total support for R matches that for L, leaving $p^* = \overline{p}$ and P^{misrep} unchanged as $v/(cn)$ varies. Similarly, in pure-strategy equilibria (10, 01, and 11), where participation rates remain fixed at corner values, p^* is unaffected. However, in mixed-strategy equilibria, $v/(cn)$ influences p^*. For

[27] The probability p^{full}, corresponding to (4) when $t_R = t_L = 1$, depends only on the ratio of regular to discretionary voters, $\gamma/(1-\gamma)$, and the degree of support for R among regular voters, $2q - 1$. It decreases with both factors and is unaffected by $v/(cn)$.

instance, in $1m$, an increase in $v/(cn)$ leads to greater participation among R, lowering their average discretionary support and thus decreasing p^*. Similar patterns hold in $m1$ and $m0$. The equilibrium transition path also depends on $v/(cn)$. Under Agreement ($\overline{p} = 0.7 > 0.5$) with $\gamma = 0.1$, increasing $v/(cn)$ shifts equilibria from $m0$ to mm, then to $1m$, and finally to 11. This sequence reflects a progression where L initially participates while R freerides, then both types partially participate, followed by full participation by L, and eventually full participation by both groups at sufficiently high $v/(cn)$.

Figures 2 illustrate these results. The left panels indicate equilibria, and the right panels the extent of freeriding. Panels (a)–(b) plot p^* against $v/(cn)$ for the Agreement case ($\overline{p} = 0.7$), assuming $\gamma = 0.1$ and $p^{\text{full}} \in (l, h) = (0.45, 0.95)$. As $v/(cn)$ increases, p^* first rises in equilibrium $m0$, stays constant at $\overline{p}$ in mm, decreases in $1m$, and stabilizes at p^{full} in 11. The corresponding plots for P^{misrep} in Panels (c)–(d) reveal an inverted-U shape, with misrepresentation peaking in mm where the outcome is always a tie. Panels (e)–(f) present the same analysis for the Disagreement case ($\overline{p} = 0.3$), again with $\gamma = 0.1$ and $p^{\text{full}} \in (0.05, 0.55)$. Under Disagreement, $p^* \leq p^{\text{full}}$, meaning that misrepresentation is highest in $m0$ and decreases as $v/(cn)$ rises. While monotonic in this case, nonmonotonicity can emerge for higher γ, as freeriders begin to participate in $1m$. Note that these graphs are illustrative but not general. For instance, at higher values of γ, nonmonotonicity can also occur under Disagreement.

These findings have critical implications for regulatory interventions to improve representativeness. Consider a regulator attempting to reduce misrepresentation under Agreement ($\overline{p} = 0.7$) when $v/(cn) = 0.25$. Figure 2(c) suggests that halving c to shift $v/(cn)$ to 0.5 would be ineffective, as the misrepresentation probability remains unchanged. Effective interventions must distinguish between Agreement and Disagreement cases, requiring knowledge of unobserved quantities, such as $\overline{p}$, $h - l$ (or std(p)), and the prevailing $v/(cn)$.

4 Empirics

The model allows us to infer unobservable voter preferences from observed voting data. Specifically, we use information on the (i) ownership of regular voters (γ) and number of shares (n), (ii) fraction of regular majority voters (q), and (iii) observed support for each option among discretionary voters. These inputs enable us to estimate the (i) mean and standard deviation of the fraction of discretionary majority voters (p) and (ii) benefit-to-cost ratio per share ($v/(cn)$). We estimate the model using a two-step generalized method of moments (GMM) estimator in an exactly identified system of equations.

4.1 Identification

In this section, we describe how variation in the discretionary support for the favorite , p, allows us to identify unobserved parameters. Let $dSuL$ and $dSuR$ denote the discretionary support for the underdog and favorite, respectively, both as a fraction of the total discretionary voter base, $1 - \gamma$. Given equilibrium participation rates and a realization of p, we have (see 2):

$$dSuL = t_L(1 - p), \quad dSuR = t_R p. \tag{10}$$

The observed discretionary votes are the product of the participation rates and the support for each alternative among discretionary voters.

Participation rates are *onto* (but not one-to-one) functions of the model parameters.[28] The participation rates (t_L, t_R) are uniquely determined for a given set of parameters. However, the mapping is not invertible, as different parameter values can lead to the same observed rates. Consider two distinct benefit-to-cost ratios, $v/(cn)$ and $v'/(cn)$, such that both $\gamma \times v/(cn)$ and $\gamma \times v'/(cn)$ lie within $\Gamma_{10} \times V_{10}$. In this case, both values result in equilibrium 10, yielding identical participation rates $t_L = 1$, $t_R = 0$, despite $v/(cn) \neq v'/(cn)$. Consequently, even if participation rates and exogenous parameters (γ, q) were directly observed, we would only recover the *interval bounds* of V_{10}. This same identification structure applies to equilibria 01 and 11. In contrast, for equilibria $1m$, $m1$, mm, and $m0$, observation of participation rates allows for a *point identification* of $v/(cn)$.

Hence, defining the observable exogenous parameters as $\boldsymbol{x} \equiv (\gamma, q)$,[29] the vector of parameters to estimate is: $\boldsymbol{\theta} \equiv (v/(cn)_{\text{lower}}, v/(cn)_{\text{upper}}, \overline{p}, \text{std}(p))$, where $v/(cn)_{\text{lower}}, v/(cn)_{\text{upper}}$ define the endpoints of the intervals for $v/(cn)$, explicitly allowing the benefit-to-cost ratio per share to be in a (possibly degenerate) interval. Thus, in our estimation, $v/(cn)$ may be *set identified* (see Molinari (2020)). A point estimate for $v/(cn)$ is obtained when $v/(cn)_{\text{lower}} = v/(cn)_{\text{upper}}$.

Now, returning to the observed discretionary support in (10), given $(\boldsymbol{x}, \boldsymbol{\theta})$, the equilibrium participation rates (t_L, t_R) are uniquely determined. Consequently, variation in $dSuL$ and $dSuR$ stems solely from the randomness in p. We exploit this randomness to identify the latent variables in $\boldsymbol{\theta}$.

4.2 The estimation problem

We now describe how moment conditions on the observed support for each voting option allow us to estimate the unobserved parameters. The goal is to estimate $\boldsymbol{\theta}$ by GMM using the following moment conditions, where the expectations are with respect to the density of p:

$$\begin{aligned}
\mathbb{E}[dSuL] &= t_L(\boldsymbol{x}, \boldsymbol{\theta})(1 - \overline{p}),\ \mathbb{E}[dSuR] = t_R(\boldsymbol{x}, \boldsymbol{\theta})\,\overline{p}, \\
\mathbb{E}[dSuL^2] &= t_L^2(\boldsymbol{x}, \boldsymbol{\theta})\,\text{std}(p)^2 + t_L^2(\boldsymbol{x}, \boldsymbol{\theta})(1 - \overline{p})^2,\ \mathbb{E}[dSuR^2] = t_R^2(\boldsymbol{x}, \boldsymbol{\theta})\,\text{std}(p)^2 + t_R^2(\boldsymbol{x}, \boldsymbol{\theta})\,\overline{p}^2.
\end{aligned}$$

Let $\boldsymbol{m} \equiv (dSuL, dSuR, dSuL^2, dSuR^2)$ and

$$\boldsymbol{g}(\boldsymbol{m}, \boldsymbol{\theta}) \equiv \begin{pmatrix} dSuL - t_L(\boldsymbol{x}, \boldsymbol{\theta})(1 - \overline{p}) \\ dSuR - t_R(\boldsymbol{x}, \boldsymbol{\theta})\,\overline{p} \\ dSuL^2 - t_L^2(\boldsymbol{x}, \boldsymbol{\theta})\,\text{std}(p)^2 - t_L^2(\boldsymbol{x}, \boldsymbol{\theta})(1 - \overline{p})^2 \\ dSuR^2 - t_R^2(\boldsymbol{x}, \boldsymbol{\theta})\,\text{std}(p)^2 - t_R^2(\boldsymbol{x}, \boldsymbol{\theta})\,\overline{p}^2 \end{pmatrix}.$$

Then $\mathbb{E}[\boldsymbol{g}(\boldsymbol{m}, \boldsymbol{\theta})] = 0$ is the moment condition used for defininining the GMM estimator for $\boldsymbol{\theta}$. Here, our nonlinear system is exactly identified, with four moment conditions to estimate four parameters. Per standard practice, for a sample $(p_1, \ldots, p_s)$ of iid realizations of discretionary support for the favorite p, we take averages $\boldsymbol{g}_s(\boldsymbol{\theta}) = \sum_{i=1}^{s} \boldsymbol{g}(\boldsymbol{m}_i, \boldsymbol{\theta})/s$, where we explicitly denote

[28] Participation rates are continuous, differentiable (almost everywhere), and nonlinear, as shown in Table 1.

[29] We do not explicitly use parameter n in the identification, as it is absorbed in the ratio $v/(cn)$.

the moments' dependence on the realization of p. Then our estimation problem is

$$\min_{\boldsymbol{\theta}} \boldsymbol{J}(\boldsymbol{\theta}) = s g_s(\boldsymbol{\theta})' \boldsymbol{W} g_s(\boldsymbol{\theta}), \tag{11}$$

where $\boldsymbol{W}$ is 4×4, positive-definite, and a symmetric weight matrix. In our estimation, we perform a two-step GMM (Hansen (1982); Hansen and Singleton (1982)). In the first step, we take $\boldsymbol{W} = \boldsymbol{I}$, the identity matrix, and obtain estimates $\hat{\boldsymbol{\theta}}$; in the second, we use $\hat{\boldsymbol{W}} = \left(\sum_{i=1}^{s} \boldsymbol{g}(\boldsymbol{m}_i, \hat{\boldsymbol{\theta}}) \boldsymbol{g}(\boldsymbol{m}_i, \hat{\boldsymbol{\theta}})' / s\right)^{-1}$. Standard errors are computed using the delta method (Wooldridge (2010, pp. 44-45)).[30] Next, we describe how we implement the estimation of $\boldsymbol{\theta}$.

4.3 Implementation

We arrange our data into bins to have a meaningful unit of estimation. Each bin quadruple sorts our data on i) the fraction of regular voters, γ; ii) numbers of shares, n; iii) proposal type; and iv) the fraction of regular voters with preference for R (the favorite), q (see Table OA.3 Panel B for a description of the variables). There are 23 proposal types, and for each of γ, q, and n, we split the data into terciles; so in total we have $23 \times 3 \times 3 \times 3 = 621$ bins.

Our *identifying assumption* is that, within each bin, the estimated parameters $\boldsymbol{\theta}$ are constant and that the bin specific averages of the exogenous parameters $\overline{\boldsymbol{x}} = (\overline{\gamma}, \overline{q})$ are representative. In other words, we postulate that variation in discretionary preference p across proposals is the only variation needed to identify the bin-specific parameters. We are faced with the common trade-off: more observations within a bin reduce our estimation error but also decrease the representativeness of the estimates. We show, however, that our results do not depend qualitatively on the size of the bins; see Table OA.6 Panel A.

For each bin, we compute the sample averages $\boldsymbol{g}_s(\boldsymbol{\theta})$. To solve (11), we perform an exhaustive search in a dense grid of points in the permissible space of the unobservable parameters $\boldsymbol{\theta}$. For each point, we compute the predicted discretionary support per preference type and for each equilibrium. Within each bin, we then pick the point in the grid of $\boldsymbol{\theta}$ with the lowest estimation error value $\boldsymbol{J}$. Recall that, for fixed parameters $\boldsymbol{x}$ and $\boldsymbol{\theta}$, the model predicts a unique equilibrium. In addition, the algorithm picks the parameter values that minimize the estimation error using an exhaustive search. Hence, we can be certain that no other parameter values (in the grid) and equilibrium would result in a lower estimation error, given the data.

Finally, we estimate P^{misrep} from the closed form expression in (8), using the estimated parameters $\boldsymbol{\theta}_{\text{est}}$ and observed sample averages $\overline{\boldsymbol{x}}$ for each bin.

4.4 Estimates

In Table 3, we report our parameter estimates. We report averages weighted by the number of observations, i.e., actual proposals voted upon, in each bin (our estimation unit). In Panel A, we show parameter estimates for all equilibria, whereas in Panel B, we split them by equilibrium.

Misrepresentation. We estimate a nontrivial probability P^{misrep} of a minority win of 15.6%, with a narrow 95% confidence interval between 14.7% and 16.3%. This average masks

[30] In particular, let $\boldsymbol{\Delta_i} \equiv [\partial\theta_i/\partial\overline{\mathbf{m}}_1, \partial\theta_i/\partial\overline{\mathbf{m}}_2, \partial\theta_i/\partial\overline{\mathbf{m}}_3, \partial\theta_i/\partial\overline{\mathbf{m}}_4]$, the sensitivity of parameters $\{\theta_i\}_{i=1}^4$ to sample averages of moments $\overline{\mathbf{m}}$. And let $\boldsymbol{S}$ the variance-covariance matrix of $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\theta})$, where θ is the second-step estimate. Then, the variance of our error in estimating parameter θ_i is given by $\boldsymbol{\Delta}_i \times \boldsymbol{S} \times \boldsymbol{\Delta}_i'$, for $i = \{1, 2, 3, 4\}$.

substantive heterogeneity (see Figure OA.7). In terms of proposal types, the minority is least likely to win in shareholder proposals on takeover defense (3%) and most likely in governance-related management proposals (25%).

Benefits and cost of voting. The average benefit-to-cost ratio per share, $v/(cn)$, is 0.37 with a 95% confidence interval of 0.26 to 0.48. Under rather primitive assumptions, we show, in Section OA.8, that these number are roughly comparable to the literature. In Figure OA.7, we report the estimates for different subsamples. Shareholder proposals have greater $v/(cn)$ than management proposals (Panel A). The proposals with the highest $v/(cn)$ are shareholder proposals related to takeover defense (Panel C); the lowest are management proposals related to board matters (Panel B).

Free-riding and Underdog effects. Our estimate for participation rates (Table 3, Panel A) is 58% for L, which is by magnitudes larger than the 3% for R, the side aligned with the majority of regular voters. Indeed, according to our estimates, L is not only the minority among regular voters but on average also the minority preference among discretionary voters: we estimate a 0.55 mean and a 0.26 standard deviation for the distribution of p, the fraction of discretionary voters for R. With an estimate of $\overline{p}$ higher than 0.5, the average case is one of Agreement (as in Figures 1– 2). The low participation by the majority corresponds to the prevalence (83%) of full freeriding equilibria ($m0$ and 10), with no participation by R (Panel B). Further, 17% are in an equilibrium with partial freeriding (mm and $1m$), and almost none (0.3%) in one with zero participation ($m1$ and 11). This ranking contrasts with political elections, where the mostly observed equilibrium is mm with partial participation from both sides.

Selection by equilibrium. The selection effects differ between equilibria (Panel B). While, by definition, P^{misrep} is zero under full participation from both sides (equilibrium 11), it is 5% in mm, where both sides partially participate, and 18% in $m0$, the most frequent equilibria with full freeriding. The $v/(cn)$, as predicted by our theoretical model, is lower in equilibria with more freeriding and the highest in the equilibrium with no freeriding. The corner solutions with full or no participation drive the overall participation rates. The low average turnout by R reflects the prevalence of freeriding. Even in the partial freeriding equilibria, turnout by R is as low as 17% –these are more comparable to equilibria with full freeriding than those with no freeriding (or full participation from R). In contrast, the minority side L participates 51% in $m0$, the equilibrium where the R side freerides fully. In the mixed equilibrium mm, the L side turns out 70%. The large difference in participation between discretionary voters of L and R implies that selection moves the voting outcome away from the population's preference.

5 Cross-country evidence

Administrative costs of voting are likely to be small in the United States, as most firms in the US support electronic voting and do not require cumbersome paperwork to prove ownership or pre-register for voting. To explore selection effects for different levels of the voting costs, we extend our sample to firms outside the United States in this section and link misrepresentation probability estimations to institutional determinants of the costs and benefits of voting.

Data. We obtain data for international firms from 2013-2020 on meetings and proposals from

ISS and data on votes per shareholder from Diligent.[31] As for the US sample, we exclude routine proposals, director elections, and say-on-pay frequency proposals (147,472 proposals). Our data on ownership comes from Factset.[32] Excluding proposals that violate A1 (the assumption that regular voters do not unilaterally decide the voting outcome),[33] our estimation sample includes 189,455 proposals from 59,693 meetings of 12,544 firms incorporated in 92 countries with data on proposals, meetings, and ownership. The countries with the most proposals in our sample are China, Japan, and the United Kingdom. The average firm is smaller than in the United States, with a market capitalization of $1.37 billion (see Table OA.4).

Sample. Ownership by N-PX firms and other mutual funds is lower outside the United States (Panel A of Table 4), with 1% (12% in the United States). In contrast, insiders hold 43% of the shares (9% in the United States). Participation is lower by a magnitude, with 40% compared to 72% in the United States. Discretionary participation is 7% (24% in the United States). Proposals mostly pass (with 96% votes for), with governance-related management proposals receiving the highest votes (98%) and defense-related ones the lowest (81%). Discretionary participation is highest for CSR proposals (20% for both shareholder and management proposals) and lowest for payout-related shareholder proposals (1%).

Estimates. Despite the vastly smaller participation rates, the distribution of equilibria and resulting representativeness resemble those in the United States (Figure OA.7 Panel F and Table 4 Panel B). As in the United States, the majority of international proposals (72% compared to 57% in the United States) exhibit full freeriding, followed by partial freeriding equilibria (28% compared to 37% in the United States). No-freeriding equilibria occur very rarely, as in the United States. This distribution results in participation of 46% by the underdog (L, compared to 43% in the United States) and 2% by the majority (R, compared to 4% in the United States). The likelihood of a minority win is 11%, very similar to 10% in the United States.

However, representativeness differs vastly across regions.[34] From Table 4 Panel B and Figure OA.7 Panel F, we see that Japan has the highest estimated P^{misrep} with 27%. This also relates to the lowest $v/(cn)$ for Japan 0.02, where meetings tend to be clustered. Indeed, participation is as low as 4% for L and zero for R. At the other end of the spectrum, Brazil, Russia, and China (BRC) have the lowest P^{misrep} (4%) and the highest $v/(cn)$ (0.22). There, participation rates are 63% on the L and 12% on the R.

[31] Formerly Insightia and ProxyInsight. Diligent collects information on votes from public sources, including NP-X files and mutual fund websites. See Michaely et al. (2023) for a more detailed description of its US data.

[32] Ultimately, we cover 335,917 proposals from 59,693 meetings of 17,693 firms incorporated in 96 countries with data on proposals, meetings, and ownership.

[33] Assumption A1 is violated much more often outside the United States, leading us to exclude 146,322 proposals in 60,135 firms (see OA.4). The excluded firms are significantly smaller, with an average market cap of $1.35 billion, compared to $1.37 billion in our sample firms. They have fewer N-PX ownership, with 0.96% compared to 1.08% in sample firms, but much higher insider ownership, with an average of 65%, compared to 25% in our sample. The average participation rate in excluded proposals is 46%, versus 35% in-sample, and discretionary participation 2% versus 8% in-sample. Management proposals comprise 52% of the excluded sample versus 47% in-sample.

[34] To account for differences in the institutional environment, we group countries for the estimation. We single out Japan and the United Kingdom, as they constitute large parts of our sample and have distinctive governance characteristics. We then group offshore financial centers and Commonwealth countries as a separate group, as their legal environments resemble each other. As India is part of the Commonwealth category, we group the remaining BRIC countries—Brazil, Russia, and China—in a BRC category. Because the distribution of q is very concentrated in the international sample, we do not use it to construct estimation bins. As a result, our estimation bin for the international sample is proposal type $\times$ country group $\times$ terciles of γ $\times$ terciles of n.

6 Counterfactuals

To guide regulatory or corporate policy to improve representativeness, we report the sign of the average effect of changes in the benefits or costs of voting on representativeness, in Table 8.

Countries. Our estimates suggest that reducing the cost of voting will not automatically make voting results more representative of the shareholder population. Indeed, this is only the case for Japan, the European Union, and offshore countries (Figure OA.9). For the United Kingdom, reducing the cost of voting will initially make results more representative, but only to a limited extend. For other Commonwealth countries and Brazil, Russia, and China, reducing the cost of voting is also likely to make voting less, not more representative. Note that our sample stops before the Shareholder Rights Directive II was implemented throughout the European Union. Our results suggest that the stated aim to decrease c was warranted. The United States is at a point where neither increasing nor decreasing the cost of voting will matter much to representativeness.

Proposals. In Figure OA.11, we depict the relationship between implied P^{misrep} and counterfactual $v/(cn)$ for each proposal type in turn for US proposals. An increase in $v/(cn)$ will reduce P^{misrep} for management proposals on restructuring, voting, board composition, and business matters, and shareholder proposals on CSR, governance, and voting. In contrast, it will increase P^{misrep} (and hence will harm representativeness) for management proposals on say-on-pay, takeover defense, and governance as well as shareholder proposals on compensation and takeover defense.

Trends. We repeat the analysis for US proposals across years in Figure OA.12. Now the range is the minimum and maximum $v/(cn)$ across years, so for 2020 and 2003, and the red cross signifies the estimated $v/(cn)$. The derivatives are positive in most of the first half of the sample years: then, increasing the benefits of voting or reducing its cost would have resulted in more misrepresentative results, not less. From 2012 onwards, the derivatives have been negative but turning positive: decreasing the cost of voting would only improve representativeness if the decrease is small and harm representative if the decrease is large.

Industries. In Figure OA.13, we show counterfactuals across industries for US proposals. Decreasing the cost of voting (and thus increasing the benefit-to-cost ratio) will benefit the representativeness of proposals in manufacturing, services, transportation, and wholesale but harm it in agriculture, finance, mining, and retail.

7 Discussion and Robustness

To facilitate the interpretation of the estimations, we now point out the limitations imposed by the model's assumptions. Our model focuses on the strategic decision of whether to participate in voting while taking other potential decisions as given. In particular, the model abstracts from why shareholders make certain proposals as well as how they obtain their shares, choose their preference type, and arrive at their common knowledge of the model's parameters. In reality, however, these decisions are likely to be endogenous, and hence the participation decision may depend on factors outside our model.

Of particular interest is how shareholders obtain decision-relevant information. In the model,

discretionary voters observe the parameters: $\{\gamma, q, \overline{p}, \text{std}(p), v, c, n, \overline{a}\}$.[35] Much of the information is easy to access due to disclosure regulations: the ownership structure (γ, n), which is disclosed in the invitation to vote; voting manifestos by institutional investors (q); and recommendations of proxy advisors, such as ISS and Glass Lewis (Iliev and Lowry (2014); Malenko and Shen (2016)), which can proxy for average investor preferences $\overline{p}$. Also, it is reasonable to assume that voters know their cost of voting c.

Other information can still be costly to acquire. For example, subscription services like Diligent provide aggregate voting predictions using the past voting of institutional investors. Here, shareholders can acquire costly information about the benefits of the proposals (i.e., v), the (dispersion of) preferences of the other shareholders (i.e., std(p)), or both. From the perspective of our model, any information acquisition cost is sunk at the stage of the participation decision. Moreover, our key assumption is not that discretionary voters have perfect information but that they are homogeneous in their amount of information. Absent heterogeneity in information, the channel we shut down is information aggregation. To evaluate the robustness of our results to this channel, we perform our estimations separately in more and less information-demanding proposals: proposals in meetings with only one nonroutine proposal and those in meetings with more proposals as well as meetings early in the season compared to those later in the season, when new trends in proposal have already been discussed; (see OA.9.3). We show that our qualitative results remain similar in each of these subsamples.

The primitives c, v, and the distribution of p are central to the model's mechanics: they jointly determine the costs, benefits, and uncertainty that drive turnout and misrepresentation. To assess the robustness of our theoretical results, we extend the model in the Online Appendix to relax each of these assumptions in turn. Specifically, we allow for heterogeneity in participation costs (§ OA.4.1), a flexible Beta distribution for aggregate uncertainty in the support for R (§ OA.4.2), and asymmetric private valuations (§ OA.4.3). Together, these extensions span heterogeneity in behavior, beliefs, and incentives, and demonstrate that our main qualitative result—the non-monotonic relation between the probability of misrepresentation and the benefit–cost ratio—remains robust across these generalizations.

Empirically, we estimate the *uniform-cost* specification derived from the first extension as a tractable benchmark. This version maintains all other baseline assumptions but replaces the constant cost with a uniformly distributed cost of voting,[36] allowing for investor heterogeneity (e.g., between hedge funds and retail groups) including effectively costless voters. Despite this additional flexibility, estimation errors are higher than under the homogeneous-cost baseline, which attests to the empirical discipline and strong fit of the simpler closed-form model—whose analytical tractability avoids the numerical approximation and computational noise required by the more general variants.

Equity lending is another practical concern. Aggarwal et al. (2015) report a 1.9 ppt rise in recalls by institutions around record dates, leading to a reduced supply and higher fees. In the presence of such a market, discretionary voters may decide to lend their shares instead of voting,

[35] Our estimates already take into account any actions by major stakeholders to affect those parameters. For example, investors buying shares to vote or managerial actions to affect the outcome (including campaigning or picking the location of the meeting). In essence, our identifying assumption requires that these parameters are fixed until voting is over.

[36] In the notation of OA.4, this corresponds to parameters $\delta = 1, \kappa = 1, p \sim \mathcal{U}[l, h], v_R = v_L = v$.

effectively increasing the opportunity cost component of c. If voting constitutes a significant factor in the equity lending market, the demand, supply, and interest should be endogenous to shareholder preferences and the voting decision. Given that the expected benefits of voting are greater for shareholders with a preference for the underdog, they should be willing to pay more to borrow shares. This will allow them to acquire more votes, which will lead to closer votes than would occur in a world without equity lending. Closer votes can lead to a higher incidence of the incomplete participation equilibrium mm. However, the additional borrowing demand around the record date is small, with estimates around 0.2% (Christoffersen et al. (2007)) to 0.3% (Aggarwal et al. (2015)). Consistent with these magnitudes, our estimates do not change qualitatively even in the top quintiles of equity demand and supply (see Section OA.9.3).

We also make procedural assumptions that are standard in the literature. (See Heard and Sherman (1987); McGurn (1989); Monks and Minow (2003) for further details about the mechanics of proxy voting.) First, we assume that voting occurs simultaneously. In practice, there might be instances of early access to tabulations (Bach and Metzger (2018)). However, most voters and brokers submit their votes at the deadline to prevent access to such information and to avoid having to change their votes should they change their opinion. Second, we assume that regular voters never abstain from voting, which is consistent with the data with fewer than 1% of empty votes cast within our sample. Third, our model assumes that the vote is for a single issue/proposal—i.e., it abstracts from bundling, which occurs in reality. To address this, in Section OA.9.3, we show that our results hold qualitatively in meetings with more proposals. Section OA.9.2 discusses robustness to alternative estimation choices.

Finally, not all proposals are binding. Of exception (in the US) are management proposals on restructuring-related issues such as equity issuance and acquisitions financed with stock Tallarita (2022). We do not model explicitly whether proposals are binding. Levit and Malenko (2011b) show that non-binding (so-called precatory) proposals can still be effective at influencing management and Cai and Walkling (2011); Correa and Lel (2016); Cuñat et al. (2015) document empirically effects of non-binding Say-on-Pay votes. In our data, if non-binding proposals were ignored by management altogether, we should observe a low estimated benefit-to-cost ratio for those. Empirically, this is not the case: restructuring-related management proposals have no higher benefit-to-cost estimate (0.16, compared to an average of 0.32 for management proposals) than other proposals. One potential explanation for the higher benefit-to-cost estimate for non-binding proposals is that only the most important issues are selected to be voted upon (as in the argument by Barry (2023)), while restructuring rules mean that all related matters must be voted on.

8 Conclusion

We analyze voting participation as a trade-off between the costs and benefits of voting, focusing on how shareholders' votes are influenced by the actions of others. In our model, we demonstrate that shareholders with popular preferences tend to participate less due to a freerider effect, while those with unpopular preferences participate more, resulting in the underdog effect. Consequently, the less popular side is over-represented in voting outcomes, which can lead to unrepresentative results. Using US and international data, we find a strong underdog effect

alongside a significant freerider effect, yielding a 10% probability of unrepresentative outcomes in the United States and 11% internationally. Our estimation algorithm can aid firms and regulators in identifying representative voting outcomes and crucial proposals, outperforming reduced form models by significantly reducing estimation errors. By accounting for the competing effects of freeriders and underdogs, we enhance prediction accuracy regarding voting support.

Moreover, in counterfactuals, we assess how reducing voting costs can influence representativeness. Our findings indicate that the likelihood of misrepresentation depends non-monotonically on the voting benefit-to-cost ratio and unobservable parameters that our model estimates. We demonstrate the model's versatility through counterfactual analyses across various proposal types, years, industries, and countries. This study is the first to infer voter preferences from participation rates in a corporate setting, providing a framework for interpreting voting support. We hope it lays the foundation for future research on these effects.

APPENDIX

A. Figures and Tables

Figure 1: **Equilibrium regions** under Agreement and Disagreement between regular and discretionary voters ($\overline{a} = 1$, $h - l = 0.5$, $q = 0.8$, $p \sim \mathcal{U}[l, h]$). (a) and (c) splits the regions based on the specific equilibrium, while (b) and (d) groups them based on the extent of freeriding.

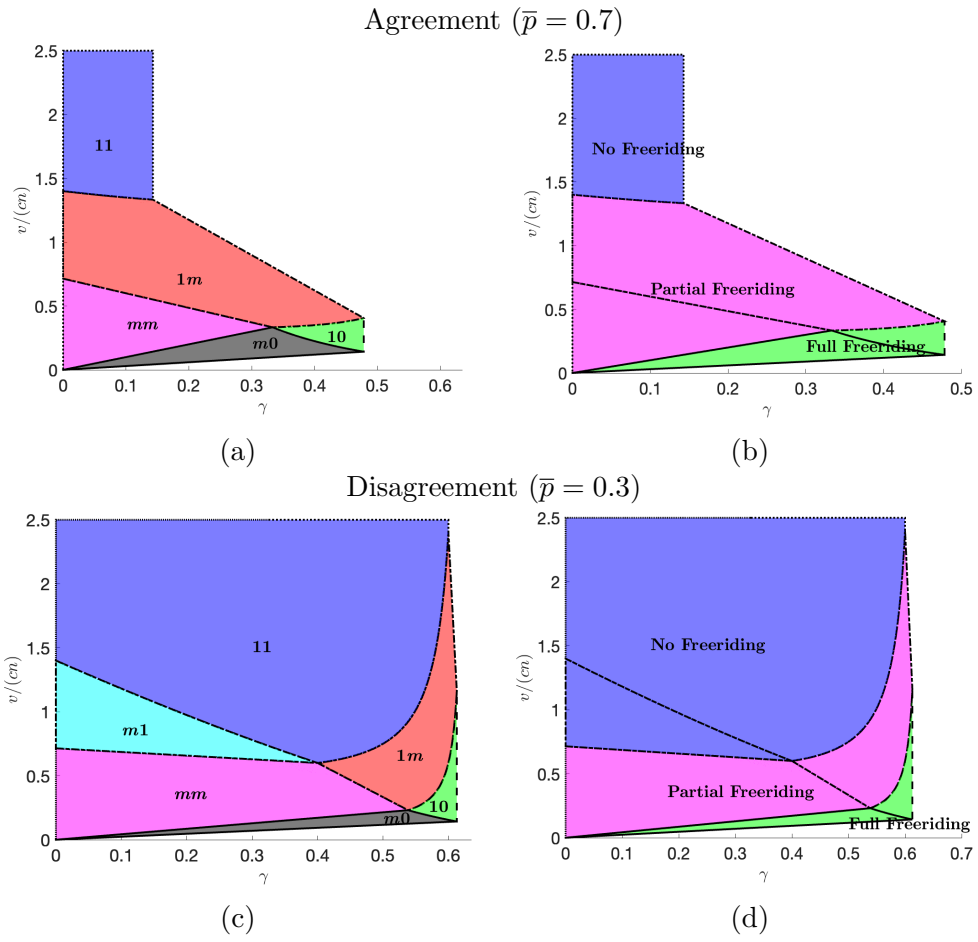


Figure 2: **Probabilities** p^* (top panels (a)–(b) & (e)–(f)) and P^{misrep} (bottom panels (c)–(d) & (g)–(h)) vs $v/(cn)$ under Agreement and Disagreement between the two groups of voters ($\gamma = 0.1$, $\overline{a} = 1$, $h - l = 0.5$, $q = 0.8$, $p \sim \mathcal{U}[l, h]$). In the top panels the black dashed line is $p^{\text{full}} \approx 0.47$ and the red dashed line is $\overline{p}$. The corresponding equilibria ((a), (c), (e), (g)) or the extent of freeriding ((b), (d), (f), (h)) are also mentioned on the graphs.

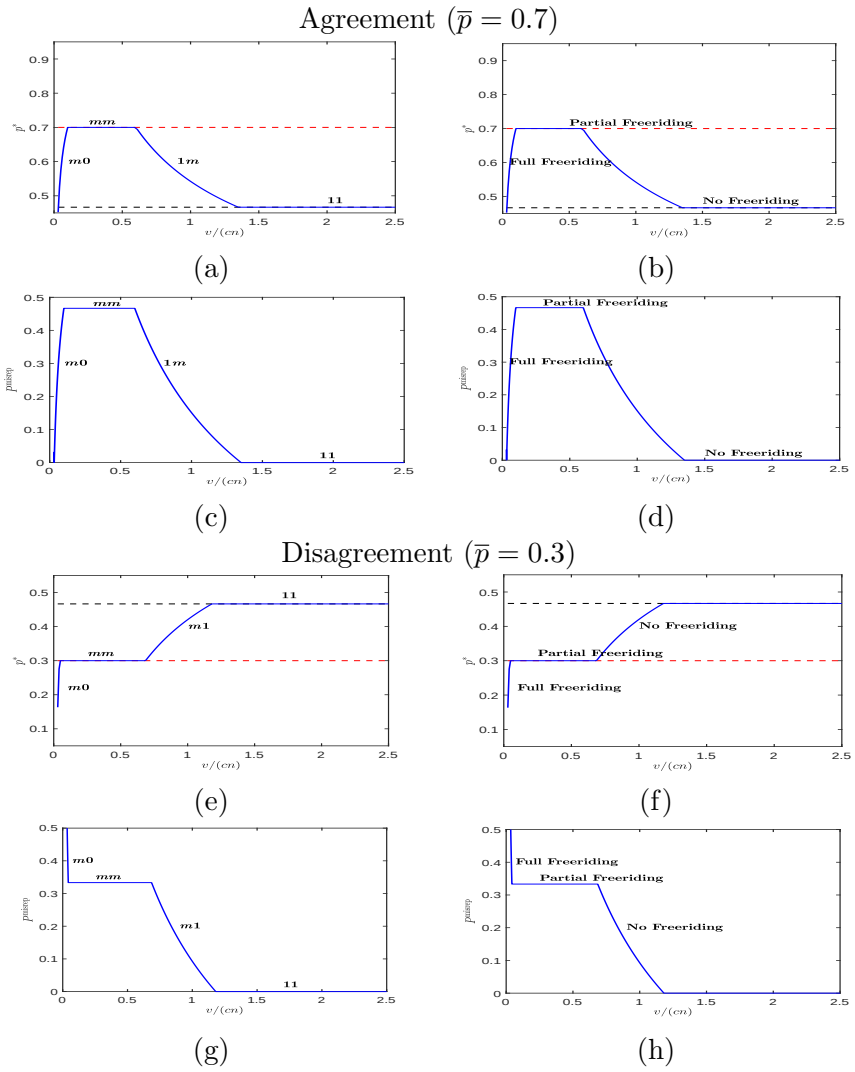


Table 1: **All possible equilibria.** Column 1 identifies each equilibrium, with notation indicating L's and R's participation levels (m for mixed, 1 for full, 0 for none). Column 2 cites the relevant proposition, while columns 3 and 4 provide t_L and t_R, referencing explicit formulas where applicable. Columns 5 and 6 outline the parameter regions for γ, Γ and $v/(cn), V$. Column 7 reports the average outcome based on $\overline{p} - p^*$. Column 8 classifies equilibria by the extent of freeriding among R-supporters: full $(m0, 10)$, partial $(mm, 1m)$, and none $(m1, 11)$.

(1) Eqm.	(2) Prop.	(3) t_L	(4) t_R	(5) $\gamma \in$	(6) $v/(cn) \in$	(7) Avg. outcome	(8) Freeriding
$m0$	OA.7	$\in (0,1)$, (OA.36)	$=0$	Γ_{m0}, (OA.34)	V_{m0}, (OA.35)	Right	Full
10	OA.4	$=1$	$=0$	Γ_{10}, (OA.24)	V_{10}, (OA.25)	Right	Full
mm	OA.1	$\in (0,1)$, (OA.4)	$\in (0,1)$, (OA.5)	Γ_{mm}, (OA.2)	V_{mm}, (OA.3)	Tie	Partial
$1m$	OA.3	$=1$	$\in (0,1)$, (OA.22)	Γ_{1m}, (OA.20)	V_{1m}, (OA.21)	Tie/Right	Partial
$m1$	OA.6	$\in (0,1)$, (OA.32)	$=1$	Γ_{m1}, (OA.30)	V_{m1}, (OA.31)	Left	No
11	OA.5	$=1$	$=1$	Γ_{11}, (OA.27)	V_{11}, (OA.28)	Left/Tie/Right	No

Table 2: **Univariate Statistics** for a sample of proposals voted upon in US firms 2003–2020. Panel A shows firm characteristics (at the firm-year level), statistics for ownership of voting shares (at the meeting level), and for participation (at the proposal level). Panel B table shows univariate statistics by proposal type.

Panel A: Summary Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Market capitalization	34133	8670	36300	-849	1320000
Leverage	26901	0.38	0.48	-0.8	17.31
Market/Book	34746	3.66	159.59	-4146.31	28657.47
% institutional ownership	35638	0.69	0.28	0	1
of which: N-PX	39193	0.3	0.22	0	1
closely held	33601	12.91	16.84	0	99.14
Shares outstanding (M)	39274	187	608	0.099665	29100
n	39274	286	379	0	4488
Participation (all)	66300	0.73	0.18	0	1
Participation (voluntary)	66300	0.24	0.29	0	1

Panel B: Participation (%)

	Shareholder Proposals				**Management Proposals**			
Proposal type	Frequency	Support (%)	Total	Discretionary	Frequency	Support (%)	Total	Discretionary
Board	1012	0.29	0.72	0.43	606	0.9	0.79	0.15
Business	82	0.2	0.62	0.34	436	0.89	0.64	0.14
CSR	2996	0.18	0.66	0.37	6	0.82	0.83	0.48
Compensation	1310	0.3	0.7	0.39	21892	0.87	0.73	0.22
Defense	144	0.57	0.71	0.43	293	0.8	0.68	0.26
Governance	254	0.36	0.63	0.32	1085	0.84	0.75	0.23
Restructuring	286	0.29	0.75	0.47	6984	0.84	0.71	0.28
SOP	0				25219	0.91	0.74	0.19
Voting	2567	0.51	0.73	0.47	1128	0.89	0.79	0.21
Total	8651	0.32	0.7	0.41	57649	0.88	0.74	0.22

Table 3: **Selection.** This table shows estimates for proposals voted upon in US firms 2003–2020 excluding observations that do not satisfy A1 (see table OA.3. Panel A includes the mean, confidence interval, standard deviation, minimum, and maximum for the whole sample. Panel B shows the mean for each equilibrium.

Panel A: Parameter estimates

Estimate	N	Mean	Confidence Interval	Std.	Min	Max
P^{misrep}	33939	0.156	[0.147;0.163]	0.112	0	0.506s.e.¿s.e.
		[0.004;0.005]				
$v/(cn)$ Benefit-cost ratio	33939	0.369	[0.259;0.48]	0.566	0.023	4.56s.e.¿s.e.
		[0.057;0.056]				
tL Participation rate L	33939	0.581	[0.573;0.593]	0.225	0.069	1s.e.¿s.e.
-		[0.006;0.004]				
tR Participation rate R	33939	0.031	[0.027;0.037]	0.102	0	1s.e.¿s.e.
-		[0.003;0.002]				
$pbar$ Mean	33939	0.552	[0.512;0.583]	0.047	0.203	0.789s.e.¿s.e.
-		[0.016;0.02]				
$std(p)$	33939	0.257	[0.245;0.268]	0.027	0.1	0.288s.e.¿s.e.
-		[0.006;0.006]				

Panel B: By equilibrium

	mm	$1m$	10	11	$m1$	$m0$	Full Freeriding	Partial Freeriding	No Freeriding
N	2250	3673	323	11	0	27682	28005	5923	11
(as a fraction of N)	7%	11%	1%	0%	0%	82%	83%	17%	0%
P^{misrep}	0.11	0.05	0	0	0	0.18	0.17	0.07	0
$v/(cn)$	0.72	1.79	0.16	2.05	0	0.15	0.15	1.38	2.05
tL Participation rate L	0.7	1	1	1	0	0.51	0.52	0.89	1
tR Participation rate R	0.29	0.1	0	1	0	0	0	0.17	1

Table 4: **International.** This table shows in Panel A univeriate statistics for a sample of non-US proposals from 2013–2020. Panel B shows, for those proposals that do not violate assumption 1, the fraction of proposals per equilibrium.

Panel A: Summary Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Market capitalization	74602	1360	1610	2.16	4710
N-PX shares	102262	0.01	0.05	0	0.99
Closely held	94096	0.43	0.24	0	1
Participation (all)	339712	0.4	0.33	0	1
Participation (voluntary)	349238	0.07	0.2	0	1

Panel B: Shareholder preference by country

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Total	Equilibrium (as a fraction of N)			Estimates			
All	N	Full Freeriding	Partial Freeriding	No Freeriding	$v/(cn)$	t_L	t_R	P^{misrep}
Brazil, Russia, China	16415	0.54	0.46	0	0.22	0.63	0.12	0.04
Commonwealth	29338	0.63	0.37	0	0.12	0.47	0.01	0.09
EU	28431	0.65	0.35	0	0.14	0.43	0.02	0.12
Japan	1347	1	0	0	0.02	0.04	0	0.27
Offshore	1371	0.54	0.46	0	0.12	0.24	0.04	0.15
Rest of World	40355	0.82	0.18	0	0.12	0.5	0	0.12
UK	17210	0.94	0.06	0	0.08	0.3	0	0.16
Total Non-US	134467	0.72	0.28	0	0.13	0.46	0.02	0.11

Table 5: **Counterfactuals** – change in misrepresentation given a change in the benefits or costs of voting. This table shows the sign of the derivatives of P^{misrep} by increasing $v/(cn)$ in Panel A and by P^{misrep} by decreasing $v/(cn)$ in Panel B. This has the maximum $v/(cn)$ so the entry refers to the sign as we approach the current level of the benefit cost ratio per share.

Panel A: increasing $v/(cn)$

Subsample	-	0	+	-, turning 0	-, turning +	0, turning -	+, turning 0	+, turning -
Countries	Japan EU Offshore	US rest of world	Commonwealth Brazil,Russia.China	UK				
Management Proposals	Board Restructuring Voting	Compensation	Defense Governance Say-On-Pay		Business			
Shareholder Proposals	Board Business CSR Governance Voting	Restructuring	Compensation Defense					
Years			2003-5;2009-2011		2006-8;2012-2020			
Industry	Manufacturing Services Transportation Wholesale	Construction	Agriculture Finance Mining Retail					

Panel B: decreasing $v/(cn)$

Subsample	-	0	+	-, turning 0	-, turning +	0, turning -	+, turning 0	+, turning -
Countries	Brazil,Russia.China EU rest of world	US	Commonwealth Japan Offshore					UK
Management Proposals	Say-On-Pay Restructuring Voting		Board Compensation Defense Governance					
Shareholder Proposals	Board Compensation Governance	Defense	Business CSR Restructuring Voting					
Years			2003-5;2018-20		2006-8			2012-7
Industry	Agriculture Construction Manufacturing Retail Services Transportation		Finance Mining Wholesale					

B. Proofs of Main Results

Proof of Lemma 1. Define $u_R \equiv apt_R$ and $u_L \equiv a(1-p)t_L$, the actual voting probabilities for R and L among discretionary voters, respectively, and the probability of absentee votes $u_0 \equiv 1 - u_R - u_L$. Vector $u \equiv (u_R, u_L, u_0)$ lives in the two-dimensional unit simplex Λ. Beliefs regarding u are represented by continuous and bounded density $h(\cdot|i)$, for $i \in \{R, L\}$. Then the number of discretionary votes b_R, b_L, and $(1-\gamma)n - b_R - b_L$ follow a multinomial distribution, with probabilities $u \in \Lambda$. Hence, by adapting Myatt (2015, Eqs (4) & (5)) for our purposes, we calculate the probability of a tie with x discretionary votes for R by taking the following expectation over u:

$$\begin{aligned}
\Pr\left[b_R = x, b_L = x + (2q-1)\gamma n \middle| \, h(\cdot|i)\right] &= \\
\int_\Lambda \frac{((1-\gamma)n)!\, {u_R}^x {u_L}^{x+(2q-1)\gamma n} {u_0}^{(1-2q\gamma)n-2x}}{x!(x+(2q-1)\gamma n)!((1-2q\gamma)n-2x)!} h(u|i)du &\approx \\
h\left(\frac{x}{(1-\gamma)n}, \frac{x}{(1-\gamma)n} + \frac{(2q-1)\gamma}{(1-\gamma)}, \frac{1-2q\gamma}{1-\gamma} - 2\frac{x}{(1-\gamma)n}\middle| i\right) \frac{\Gamma\left((1-\gamma)n+1\right)}{\Gamma\left((1-\gamma)n+3\right)} &\approx \\
\frac{1}{(1-\gamma)n} \frac{h\left(\frac{x}{(1-\gamma)n}, \frac{x}{(1-\gamma)n} + \frac{(2q-1)\gamma}{(1-\gamma)}, \frac{1-2q\gamma}{1-\gamma} - 2\frac{x}{(1-\gamma)n}\middle| i\right)}{(1-\gamma)n},&
\end{aligned}$$

for $i \in \{R, L\}$, where the first equality follows from the multinomial distribution. The first approximation relies on the observation that, for n large, most of the distribution will be concentrated at its mode and uses properties of the Dirichlet density. The second approximation follows from the definition of $\Gamma(y+1) = y!$, for $y \in \mathbb{N}$, and that, for n large, $(1-\gamma)n+1 \approx (1-\gamma)n$, $(1-\gamma)n + 2 \approx (1-\gamma)n$. Hence, summing over all possible x, the overall probability of a tie is:

$$\sum_{x=0}^{n/2-q\gamma n} \Pr\left[b_R = x, b_L = x + (2q-1)\gamma n \middle| \, h(\cdot|i)\right],$$

where we assume for simplicity, in the context of this proof, that both $n/2$ and γqn are integers. Given the above approximations, for n large, the sum can be approximated by the integral

$$\frac{1}{(1-\gamma)n} \int_0^{\frac{1/2-q\gamma}{1-\gamma}} h(y, y + (2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma) - 2y|i)dy.$$

Therefore, by employing Myatt (2015, Lemma 2), the probability of a tie and a near tie are equal for a large n and hence for $i \in \{R, L\}$:

$$\Pr[\text{Pivotal}|i] \approx \frac{1}{(1-\gamma)n} \int_0^{\frac{1/2-q\gamma}{1-\gamma}} h(y, y + (2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma) - 2y|i)dy. \tag{12}$$

Below, we revert the above expressions from vector u to vector (p, a) so that we can transition from density h to densities f and g. Recall that $u_R = apt_R$, and $u_L = a(1-p)t_L$; hence, the Jacobian $\partial(u_R, u_L)/\partial(p, a)$ has a determinant equal to a_L. Moreover, note that each shareholder

updates her beliefs based on her own availability, and so

$$h(x,y,1-x-y|i)=\frac{f\left(p(x,y)|i\right)g\left(a(x,y)|\text{available}\right)}{a(x,y)t_Lt_R},$$

for any $x,y\in(0,1)$ and $i\in\{R,L\}$, where

$$g\left(a|\text{available}\right)=\frac{g(a)a}{\overline{a}},\ f(p|L)=f(p)\frac{1-p}{1-\overline{p}},\ f(p|R)=f(p)\frac{p}{\overline{p}}.$$

Hence, for $u_R=y$ and $u_L=y+(2q-1)\gamma/(1-\gamma)$ from (12), after some simple algebra, we define

$$\begin{aligned}p(a) &\equiv \frac{t_L}{t_R+t_L}-\frac{(2q-1)\gamma}{1-\gamma}\frac{1}{a(t_R+t_L)},\\ a(y) &\equiv \frac{y(t_R+t_L)}{t_Rt_L}+\frac{(2q-1)\gamma}{1-\gamma}\frac{1}{t_L},\end{aligned}$$

so that $p(\overline{a})=p^*$ (see (4)). By substituting all the above in the integrand of (12), we have

$$\begin{aligned}&h(y,y+(2q-1)\gamma/(1-\gamma),(1-2q\gamma)/(1-\gamma)-2y|R)\\ =&\frac{f\left(p(a(y))\,g\left(a(y)\right)a(y)p\left(a(y)\right)\right)}{a(y)t_Rt_L\overline{a}\overline{p}}=p\left(a(y)\right)f\left(p(a(y))\,g\left(a(y)\right)\frac{1}{t_Rt_L\overline{a}\overline{p}},\ \text{and}\end{aligned}\tag{13}$$

$$\begin{aligned}&h(y,y+(2q-1)\gamma/(1-\gamma),(1-2q\gamma)/(1-\gamma)-2y|L)\\ &=\left(1-p\left(a(y)\right)\right)f\left(p(a(y))\,g\left(a(y)\right)\frac{1}{t_Rt_L\overline{a}(1-\overline{p})}.\end{aligned}\tag{14}$$

Now, to calculate the integral in (12), we change the variable from y to $a=a(y)$. We have $da=dy(t_R+t_L)/(_L)$ and

$$\begin{aligned}a(0) &= \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{t_L},\\ a\left((1/2-q\gamma)/(1-\gamma)\right) &= \frac{t_R\left(1/2-\gamma(1-q)\right)+t_L\left(1/2-\gamma q\right)}{(1-\gamma)t_Rt_L}.\end{aligned}$$

Then we have:

$$\begin{aligned}\Pr[\text{Pivotal}|R] &\approx \frac{1}{(1-\gamma)n}\int_0^{\frac{1/2-q\gamma}{1-\gamma}}h(y,y+(2q-1)\gamma/(1-\gamma),(1-2q\gamma)/(1-\gamma)-2y|R)dy.dy\\ &= \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}\overline{p}(t_R+t_L)}\int_{a(0)}^{a((1/2-q\gamma)/(1-\gamma))}f(p(a))p(a)g(a)da,\end{aligned}\tag{15}$$

where, in the second line, we substituted from (13) as well as using (14)

$$\Pr[\text{Pivotal}|L] = \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})(t_R+t_L)}\int_{a(0)}^{a((1/2-q\gamma)/(1-\gamma))}f(p(a))(1-p(a))g(a)da.\tag{16}$$

To develop a simple formula for estimation purposes, we assume that a follows a degenerate distribution around its mean; that is, $g(a)=\delta(a-\overline{a})$, where δ is the Dirac function. Then, to

have a strictly positive probability of being pivotal, we need:

$$a(0) < \overline{a} < a\left((1/2 - q\gamma)/(1-\gamma)\right) \iff \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{t_L} < \overline{a} < \frac{t_R\left(1/2 - \gamma(1-q)\right) + t_L\left(1/2 - \gamma q\right)}{(1-\gamma)_L}. \tag{17}$$

The above is a restriction on equilibrium t_R, t_L. Let us start with the lower bound on $\overline{a}$ in (17). This restriction can be equivalently written as:

$$\overline{a}t_L(1-\gamma) + (1 - q\gamma) > q\gamma.$$

In words, this says that, if all expected support for R is zero (i.e., $\overline{p} = 0$), then there is t_L so that L wins. This has to be true in any equilibrium for discretionary supporters of L to participate. Now the restriction imposed by the upper bound in (17) can be equivalently written as:

$$t_R\left(1/2 - \gamma(1-q)\right)\left(\overline{a}t_L - 1\right) + t_L\left(1/2 - \gamma q\right)\left(\overline{a}t_R - 1\right) < 0,$$

which is always true as (from $q > 1/2$ and A1): $\gamma(1-q) < \gamma q < 1/2$; $\overline{a} \leq 1$; and $t_{R,L} \leq 1$. Hence, (17) will always be satisfied in equilibrium. Also, note that, for (15) and (16) not to be zero, we need $p(\overline{a}) = p^* \in (l, h)$, which we will check for each equilibrium separately. Therefore, given $g(a) = \delta(a - \overline{a})$ and A1, we have that (16) leads to (6), and, similarly, (15) becomes (7), where p^* is given by (4). ■

Proof of Proposition 1. This follows directly from equations (OA.34–OA.35) and (OA.24–OA.25) that define the permissible $\gamma \times v/(cn)$ for which equilibria, $m0$ and 10 exist, i.e., $\Gamma_{m0} \times V_{m0}$ and $\Gamma_{10} \times V_{10}$, respectively. In particular:

Equilibrium $m0$**.** Although $\{0\} \in \Gamma_{m0}$, when $\gamma = 0$, we have $V_{m0} = \{0\}$ (see (OA.35)), meaning that equilibrium 10 exists only for a measure zero set of $v/(cn)$ in the absence of regular voters. On the other hand, again from (OA.34–OA.35), for any $\gamma > 0$, we can find a positive measure of $v/(cn) \in V_{m0}$ so that equilibrium $m0$ exists.

Equilibrium 10**.** We have that $\{0\} \notin \Gamma_{10}$, so, for $\gamma = 0$, there is no equilibrium 10. While, from (OA.24) for $\gamma > (1-\overline{p})\overline{a}/\left(2q - 1 + (1-\overline{p})\overline{a}\right)$, there is also a range of $v/(cn) \in V_{m0}$ (see (OA.25)) for which equilibrium 10 exists. ■

Proof of Proposition 2. For any fixed parameters $\gamma, q, l, h, \overline{\alpha}$, we follow $v/(cn)$ on the interval from $(0, +\infty)$. As the benefit-to-cost ratio per share increases, different equilibria exist (see Table 1 and Figures 1).We can identify eight possible equilibrium paths occurring in non-overlapping but adjacent intervals of $v/(cn)$:

1. $m0 \to mm \to 1m \to 11$,
2. $m0 \to mm \to m1 \to 11$,
3. $m0 \to 10 \to 1m \to 11$,
4. $mm \to 1m \to 11$,
5. $mm \to m1 \to 11$,
6. $m0 \to mm \to 1m$,
7. $m0 \to 10 \to 1m$,
8. $m0$.

Which one occurs depends on the parameters $\gamma, q, l, h, \overline{a}$. In particular, given Proposition 1, that no full free-riding equilibria exist for $\gamma = 0$, only Paths 4 and 5 are pertinent for the Myatt (2015) model. Moreover, equilibrium $m1$ is only possible for Disagreement; i.e., $\overline{p} < 1/2$, while mm requires sufficiently low γ, and 10 sufficiently high γ. Finally, for any fixed parameter, there

exist $\{v/(cn)\}$ from zero to an upper value, for which no equilibrium exists; in the case where this upper value is finite and an equilibrium exists once we exceed that upper value, then there are no further gaps *between* equilibrium regions.

To determine the sign of $\partial P^{\text{misrep}}/\partial(v/(cn))$ across each path, we rely on (8), which relates P^{misrep} with $p^\star$ (see 4) and p^{full} (equal to $1/2-(2q-1)\gamma/(2\overline{a}(1-\gamma))$ by setting $t_L=t_R=1$ in (4)). We also rely on the expressions in (9) for $\partial p^\star/\partial(v/(cn))$. Note that, for $v/(cn)\to \min V_{m0}$, we have $p^\star\to l$ (see (OA.96) (i.e., the lower end of the distribution of p). Also, in equilibrium mm by definition $p^\star=\overline{p}$; now, let p_{10} the value of $p^\star$ for equilibrium 10 (equal to $1-(2q-1)\gamma/(\overline{a}(1-\gamma))$ by setting $t_L=1$, $t_R=0$ in (4); it is easy to see that $p_{10}>p^{\text{full}}$.

Given this we have, in turn, for each of the paths above:

Path 1

Eqm.:	$m0$	mm	$1m$	11
$\partial p^\star/\partial(v/(cn))$	>0	$=0$	<0	$=0$
$p^\star$	$l\nearrow\overline{p}$	$\overline{p}$	$\overline{p}\searrow p^{\text{full}}$	p^{full}
$\partial P^{\text{misrep}}/\partial(v/(cn))$	<0, for $l<p^\star<p^{\text{full}}$, >0, for $p^{\text{full}}<p^\star<\overline{p}$	$=0$	<0	$=0$.

In the table above, the first row are the equilibria in the corresponding path. The second is the derivative of $p^\star$ with respect to the benefit-to-cost ratio per share, from the expressions in (9). The third row is the corresponding interval of values of $p^\star$ (or the single value when constant), where we also incorporate with arrows the sign of the derivative; for example, in $m0$, $p^\star$ increases from l to $\overline{p}$, and we denote this as $l\nearrow\overline{p}$. The fourth and last row contains the derivative of P^{misrep} with respect to the benefit-to-cost ratio per share; to understand why these are the signs, note that for Path 1 and in equilibrium $m0$, $l<p^\star<p^{\text{full}}$ for low $v/(cn)\in V_{m0}$ and $p^{\text{full}}<p^\star<\overline{p}$ for higher $v/(cn)\in V_{m0}$; hence, from (8), we have $P^{\text{misrep}}=-(p^\star-p^{\text{full}})/(h-l)$ in the former and $P^{\text{misrep}}=+(p^\star-p^{\text{full}})/(h-l)$, in the latter; the corresponding derivative then follows from that of $p^\star$ in $m0$. Likewise, for the other equilibria in this path.

Therefore, in equilibrium Path 1, the derivative of $\partial P^{\text{misrep}}/\partial(v/(cn))$ changes sign twice from negative to positive and then again to negative. Moreover, this path exists for both Agreement and Disagreement (see Figure 1).

The corresponding tables for the other paths appear below. Those tables provide a complete characterization of the derivative of P^{misrep} with respect to $v/(cn)$ for *every* possible equilibrium path. For the result stated in the proposition and the ensuing corollary, Path 1 is sufficient together with the observation that, in Paths 4 and 5, which are pertinent in the $\gamma=0$ case, the derivative $\partial P^{\text{misrep}}/\partial(v/(cn))$ is always nonpositive. ■

Path 2

Eqm.:	$m0$	mm	$m1$	11
$\partial p^\star/\partial(v/(cn))$	>0	$=0$	>0	$=0$
$p^\star$	$l\nearrow\overline{p}$	$\overline{p}$	$\overline{p}\nearrow p^{\text{full}}$	p^{full}
$\partial P^{\text{misrep}}/\partial(v/(cn))$	<0	$=0$	<0	$=0$.

Path 3

Eqm.:	$m0$	10	$1m$	11
$\partial p^\star/\partial(v/(cn))$	>0	$=0$	<0	$=0$
$p^\star$	$l\nearrow p_{10}$	p_{10}	$p_{10}\searrow p^{\text{full}}$	p^{full}
$\partial P^{\text{misrep}}/\partial(v/(cn))$	<0, for $l<p^\star<p^{\text{full}}$, >0, for $p^{\text{full}}<p^\star<p_{10}$	$=0$	<0	$=0$.

Path 4

Eqm.:	mm	$1m$	11
$\partial p^\star / \partial \left(v/\left(cn \right) \right)$	$=0$	<0	$=0$
$p^\star$	$\overline{p}$	$\overline{p} \searrow p^{\text{full}}$	p^{full}
$\partial P^{\text{misrep}} / \partial \left(v/\left(cn \right) \right)$	$=0$	<0	$=0.$

Path 5

Eqm.:	mm	$m1$	11
$\partial p^\star / \partial \left(v/\left(cn \right) \right)$	$=0$	>0	$=0$
$p^\star$	$\overline{p}$	$\overline{p} \nearrow p^{\text{full}}$	p^{full}
$\partial P^{\text{misrep}} / \partial \left(v/\left(cn \right) \right)$	$=0$	<0	$=0.$

Path 6

Eqm.:	$m0$	mm	$1m$
$\partial p^\star / \partial \left(v/\left(cn \right) \right)$	>0	$=0$	<0
$p^\star$	$l \nearrow \overline{p}$	$\overline{p}$	$\overline{p} \searrow p^{\text{full}}$
$\partial P^{\text{misrep}} / \partial \left(v/\left(cn \right) \right)$	<0, for $l < p^\star < p^{\text{full}}$, >0, for $p^{\text{full}} < p^\star < \overline{p}$	$=0$	$<0.$

Path 7

Eqm.:	$m0$	10	$1m$
$\partial p^\star / \partial \left(v/\left(cn \right) \right)$	>0	$=0$	<0
$p^\star$	$l \nearrow p_{10}$	p_{10}	$p_{10} \searrow p^{\text{full}}$
$\partial P^{\text{misrep}} / \partial \left(v/\left(cn \right) \right)$	<0, for $l < p^\star < p^{\text{full}}$, >0, for $p^{\text{full}} < p^\star < p_{10}$	$=0$	$<0.$

Path 8

Eqm.:	$m0$
$\partial p^\star / \partial \left(v/\left(cn \right) \right)$	>0
$p^\star$	$l \nearrow p_{10}$
$\partial P^{\text{misrep}} / \partial \left(v/\left(cn \right) \right)$	<0, for $l < p^\star < p^{\text{full}}$, >0, for $p^{\text{full}} < p^\star < p_{10}$

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Online Appendix

"Freeriders and Underdogs: Participation in Corporate Voting"

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The online appendix provides supplementary theoretical and empirical material supporting the main text. Sections OA.1 and OA.2 contain the results mentioned in 1. In particular, Section OA.1 establishes the equilibrium conditions for incomplete participation from both sides mm, while Section OA.2 extends this analysis to equilibria where at least one side fully participates or abstains $(1m, 10, 11, m1, m0)$. Throughout we assume $p \in (l, h) \subseteq (0, 1)$, $\overline{a} \in (0, 1]$, g is degenerate, so that $a = \overline{a}$, and maintain A0–A2 as the foundational assumptions for the theoretical results; let also $d \equiv h - l$.

Then Section OA.3 analyzes ex-ante and ex-post trading around the record date, showing how pre-record trading micro-foundations generate aggregate uncertainty and why post-record trading is neutral under our timing assumptions. Section OA.4 develops three extensions of the model—heterogeneous costs (OA.4.1), non-uniform aggregate uncertainty (OA.4.2), and asymmetric valuations (OA.4.3)—to test the robustness and empirical implications of the main results.

Section OA.5 categorizes and describes the proposals, while Section OA.6 describes the estimation sample satisfying our assumptions. Section OA.7 evaluates the model's fit by benchmarking its predictions against reduced-form estimates. Section OA.8 interprets the estimated benefit-to-cost ratios in terms of return on investment from a successful vote. Section OA.9 presents robustness checks, including alternative estimation assumptions, the role of equity lending, and variation in the availability of shares for voting. Additional tables and figures supporting the empirical results appear at the end of the appendix.

OA.1 Equilibrium with Internal Participation

Proposition OA.2 contains the necessary and sufficient conditions for the existence of equilibrium mm for an arbitrary number of voters n. Proposition OA.1, which we present first, is the restriction of Proposition OA.2 to large n (case (b), which is the only one that survives as $n \to \infty$).

Proposition OA.1 (**Equilibrium** mm)**.** *If*

$$n \in N_{mm} \equiv \left(\frac{f(\overline{p})\left(\overline{a}(1-\overline{p}) + 2q - 1\right)}{\overline{a}(2q-1)}, \infty \right), \tag{OA.1}$$

$$\gamma \in \Gamma_{mm} \equiv \left(\frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}, \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p}) + 2q - 1} \right), \tag{OA.2}$$

$$\frac{v}{cn} \in V_{mm} \equiv \left(\frac{\gamma(2q-1)}{f(\overline{p})(1-\overline{p})}, \min\left\{ \frac{\left(\overline{a} - \gamma\left(\overline{a} - 1 + 2q\right)\right)}{f(\overline{p})\overline{p}}, \frac{\left(\overline{a} - \gamma\left(\overline{a} + 1 - 2q\right)\right)}{f(\overline{p})(1-\overline{p})} \right\} \right) \tag{OA.3}$$

then there exists an equilibrium with incomplete participation by both types; that is, t_L, $t_R \in (0, 1)$

given by

$$t_L = \frac{f(\overline{p})\overline{p}}{(1-\gamma)\overline{a}}\frac{v}{cn} + \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}, \tag{OA.4}$$

$$t_R = \frac{f(\overline{p})(1-\overline{p})}{(1-\gamma)\overline{a}}\frac{v}{cn} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}. \tag{OA.5}$$

Furthermore, in equilibrium, the probability that either L or R is pivotal is equal to the common cost-to-benefit ratio c/v (1), and the total expected votes for L and R are equal

$$\overline{a}(1-\gamma)p^* t_R + \gamma q = \overline{a}(1-\gamma)(1-p^*)t_L + \gamma(1-q), \tag{OA.6}$$

i.e., $p^* = \overline{p}$, so that the expected outcome is a tie.

Proposition OA.2 (**Equilibrium** *mm* **for any** n)**.** *Assume that*

$$\frac{v}{c} < \min\left\{\frac{n\left(\overline{a}-\gamma\left(\overline{a}-1+2q\right)\right)}{f(\overline{p})\overline{p}}, \frac{n\left(\overline{a}-\gamma\left(\overline{a}+1-2q\right)\right)}{f(\overline{p})(1-\overline{p})}\right\}.$$

In addition, consider the following disjoint parameter regions:

(a) Small regular block size:

$$\frac{v}{c} \geq 1, \quad n > \frac{f(\overline{p})\overline{p}}{\overline{a}}, \text{ and}$$

(i) Many voters, high availability:

$$n > \frac{f(\overline{p})(1-\overline{p})}{\overline{a}}, \overline{a} > 2q-1, \text{and } 0 \leq \gamma < \min\left\{\frac{\overline{a}n - f(\overline{p})(1-\overline{p})}{\left(\overline{a}+1-2q\right)n}, \frac{\overline{a}n - f(\overline{p})\overline{p}}{\left(\overline{a}-1+2q\right)n}, \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}\right\}, \text{or}$$

(ii) Many voters, low availability:

$$n > \frac{f(\overline{p})(1-\overline{p})}{\overline{a}}, \overline{a} < 2q-1, \text{and } 0 \leq \gamma < \min\left\{\frac{\overline{a}n - f(\overline{p})\overline{p}}{\left(\overline{a}-1+2q\right)n}, \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}\right\}, \text{ or}$$

(iii) Few voters, low availability:

$$\frac{f(\overline{p})\left(\overline{a}(1-\overline{p})+2q-1\right)}{\overline{a}(2q-1)} < n < \frac{f(\overline{p})(1-\overline{p})}{\overline{a}}, \overline{a} < 2q-1, \overline{p} < \frac{1}{2}, \quad \text{and}$$
$$\frac{\overline{a}n - f(\overline{p})(1-\overline{p})}{\left(\overline{a}+1-2q\right)n} < \gamma < \frac{\overline{a}n - f(\overline{p})\overline{p}}{\left(\overline{a}-1+2q\right)n}.$$

(b) Large regular block size (Many voters, any availability):

$$\frac{v}{c} > \frac{n\gamma(2q-1)}{f(\overline{p})(1-\overline{p})}, \quad n > \frac{f(\overline{p})\left(\overline{a}(1-\overline{p})+2q-1\right)}{\overline{a}(2q-1)}, \text{ and}$$
$$\frac{f(\overline{p})(1-\overline{p})}{n(2q-1)} < \gamma < \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p})+2q-1}.$$

These conditions are necessary and sufficient for the existence of an incomplete participation equilibrium by both types—that is, $t_L, t_R \in (0,1)$, which are given by equations (OA.4) and

(OA.5). Finally, in such an equilibrium, the probability of being pivotal for either R of L is equal to the common cost-to-benefit ratio c/v (1), and the expected votes for R and L are equal (OA.6); i.e., $p^* = \overline{p}$.

Proof. We proceed in two steps. In the first, we derive the expressions for the rates, and in the second, we determine the feasible parameter regions.

Step 1: Derivation of Rates Given the expressions for the pivotal probabilities, we now seek to determine whether an equilibrium exists with incomplete participation for both L and R (i.e., t_L, $t_R \in (0,1)$). From (1), we know that, since the cost-to-benefit ratio is the same for both types, the pivotal probabilities should also be the same for both. Hence, using (7) and (6), in equilibrium, we must have:

$$p^* \quad = \quad \overline{p}, \tag{OA.7}$$

which, given (OA.6), means that at equilibrium the total average supports for L and R are equal. Hence, the expected outcome is a tie. From (OA.7), the pivotal probabilities in equilibrium are:

$$\Pr[\text{Pivotal}|R] = \Pr[\text{Pivotal}|L] \quad = \quad \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(t_R+t_L)} f\left(p^*\right).$$

Moreover, the pivotal probability for type R is equal to her cost-to-benefit ratio (1) in the equilibrium with incomplete participation; hence,

$$t_R + t_L \quad = \quad \frac{1}{(1-\gamma)n\overline{a}} f(\overline{p}) \frac{v}{c}. \tag{OA.8}$$

Furthermore, according to the definition of p^* (4) and the fact that it is equal to $\overline{p}$ (OA.7), after some simple algebra, we have

$$t_L \quad = \quad (t_R + t_L)\overline{p} + \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}}. \tag{OA.9}$$

Using (OA.8) with (OA.9), we derive the equilibrium t_L and t_R, as given in (OA.4) and (OA.5).

Step 2: Bounds on the Parameters Incomplete participation means that $(t_L, t_R) \in (0,1)$. To ensure this, we now derive the restrictions on the parameters of the model—in particular, n, γ and v/c or equivalently $v/(cn)$. By definition, what we need for incomplete participation is $(t_L, t_R) \in (0,1)$. According to (OA.4), it is evident that $t_L > 0$ for all parameter values. The condition $t_L < 1$ is equivalent to:

$$\gamma \quad < \quad \frac{\overline{a}}{2q-1+\overline{a}}, \text{ and} \tag{OA.10}$$

$$\frac{v}{c} \quad < \quad \frac{n\left(\overline{a} - \gamma\left(\overline{a} - 1 + 2q\right)\right)}{f(\overline{p})\overline{p}}. \tag{OA.11}$$

Given our assumption that $q > 1/2$, the upper bound on γ in (OA.10) takes precedence over A1 (i.e., $\gamma < 1/(2q)$). Now, the condition $t_R > 0$ is equivalent to:

$$\frac{v}{c} > \frac{n\gamma(2q-1)}{f(\overline{p})(1-\overline{p})}. \tag{OA.12}$$

For (OA.12) to define the relevant lower bound on v/c given A2 (i.e., $v/c \geq 1$), we need a lower bound on the regular block size and the number of voting shares,

$$\gamma > \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}, \text{ and} \tag{OA.13}$$

$$n > \frac{2qf(\overline{p})(1-\overline{p})}{2q-1}; \tag{OA.14}$$

otherwise, we need to impose $v/c \geq 1$ from A2. Finally, the condition $t_R < 1$ is equivalent to:

$$\frac{v}{c} < \frac{n\left(\overline{a} - \gamma\left(\overline{a} + 1 - 2q\right)\right)}{f(\overline{p})(1-\overline{p})}. \tag{OA.15}$$

Hence, for the benefit-to-cost ratio v/c, we have two possible upper bounds: (OA.11) and (OA.15). Either can be relevant, depending on the parameter values. Therefore, the upper bound on v/c is

$$\frac{v}{c} < \min\left\{\frac{n\left(\overline{a} - \gamma\left(\overline{a} - 1 + 2q\right)\right)}{f(\overline{p})\overline{p}}, \frac{n\left(\overline{a} - \gamma\left(\overline{a} + 1 - 2q\right)\right)}{f(\overline{p})(1-\overline{p})}\right\}. \tag{OA.16}$$

For v/c, we also have a lower bound, which is either (OA.12), if γ satisfies (OA.13), or one. In either case, we need to ensure that the lower bound is smaller than the upper bound on v/c in (OA.16). Let us look at each case in turn.

Case A: If (OA.13) holds, then to ensure that the upper bounds on v/c in (OA.16) are larger than the lower bound in (OA.12), we have another restriction on γ,

$$\gamma < \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p}) + 2q - 1}. \tag{OA.17}$$

From the two possible upper bounds of γ in (OA.10) and (OA.17), we can show that, for $q > 1/2$, the relevant bound is condition (OA.17). Finally, we need to ensure that the lower bound of γ in (OA.13) is lower than the upper bound in (OA.17). This puts a lower bound on the number of voting shares:

$$n > \frac{f(\overline{p})\left(\overline{a}(1-\overline{p}) + 2q - 1\right)}{\overline{a}(2q-1)},$$

which supersedes the other lower bound of (OA.14).

Case B: If (OA.13) does not hold, then to ensure that the upper bound on v/c in (OA.16) exceeds the lower bound imposed by A2, we add the following two restrictions on γ,

$$n(\overline{a} - (2q-1))\gamma < \overline{a}n - f(\overline{p})(1-\overline{p}), \text{ and } n(\overline{a} + (2q-1))\gamma < \overline{a}n - f(\overline{p})\overline{p}. \tag{OA.18}$$

For these to be satisfied, we need $n > f(\overline{p})\overline{p}/\overline{a}$. Observe also that the second restriction in (OA.18) implies the upper bound in (OA.10), so this latter bound can be ignored in what follows. Then we have the following subcases:
i) If $n > f(\overline{p})(1-\overline{p})/\overline{a}$ and $\overline{a} < 2q-1$, then the only other restriction on γ (i.e., other than $\gamma < f(\overline{p})(1-\overline{p})/(n(2q-1))$) can be written as $\gamma < (\overline{a}n - f(\overline{p})\overline{p}) / ((\overline{a}-1+2q)\, n)$.
ii) If $n > f(\overline{p})(1-\overline{p})/\overline{a}$ and $\overline{a} > 2q-1$, then the added restriction on γ can be written as

$$\gamma < \min\left\{ \frac{\overline{a}n - f(\overline{p})(1-\overline{p})}{(\overline{a}+1-2q)\, n}, \frac{\overline{a}n - f(\overline{p})\overline{p}}{(\overline{a}-1+2q)\, n} \right\}.$$

iii) If $n < f(\overline{p})(1-\overline{p})/\overline{a}$ and $\overline{a} > 2q-1$, then there is no equilibrium with incomplete participation.
iv) If $n < f(\overline{p})(1-\overline{p})/\overline{a}$, $\overline{a} < 2q-1$, and $\overline{p} < 1/2$, then we need

$$\frac{\overline{a}n - f(\overline{p})(1-\overline{p})}{(\overline{a}+1-2q)\, n} < \quad \gamma \quad < \frac{\overline{a}n - f(\overline{p})\overline{p}}{(\overline{a}-1+2q)\, n}, \text{ for } \frac{f(\overline{p})\,(\overline{a}(1-\overline{p})+2q-1)}{\overline{a}(2q-1)} < n < \frac{f(\overline{p})(1-\overline{p})}{\overline{a}}.$$

We summarize all the above in the statement of Proposition OA.2. ■

OA.2 Equilibria with Corner Participation

In this section, we present the propositions and corresponding proofs for the equilibria where at least one type of discretionary voter has a corner participation rate.

Proposition OA.3 (Equilibrium $1m$)**.** *Assume that $p \sim \mathcal{U}[l,h]$. If and only if*

$$n \in N_{1m} \equiv \left(\max\left\{ \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{4(1-\gamma)^2\overline{a}^2\overline{p}d}, \frac{1}{\overline{p}d}\frac{l^2}{(1-\gamma)\overline{a} - (2q-1)\gamma} \right\}, \infty \right), \tag{OA.19}$$

$$\gamma \in \Gamma_{1m} \equiv \left(\frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p}) + 2q - 1}\mathbb{I}\left(\overline{p} < \frac{1}{2}\right), \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)} \right), \tag{OA.20}$$

$$\frac{v}{cn} \in V_{1m} \equiv \left(\max\left\{ \frac{(1-\gamma)^2\overline{a}^2\overline{p}d}{(1-\gamma)\overline{a} - (2q-1)\gamma}, \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{\overline{p}}d, \frac{1}{n} \right\}, \right.$$
$$\left. \min\left\{ \frac{4(1-\gamma)^2\overline{a}^2\overline{p}d}{(1-\gamma)\overline{a} - (2q-1)\gamma}, \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{l^2}\overline{p}d \right\} \right), \tag{OA.21}$$

where $\mathbb{I}$ is the indicator function, there exists an equilibrium with $t_L = 1$ and $t_R \in (0,1)$ given by

$$t_R = \frac{1}{(1-\gamma)\overline{a}}\sqrt{\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{d\overline{p}}\frac{v}{cn}} - 1. \tag{OA.22}$$

Proposition OA.4 (Equilibrium 10)**.** *If and only if*

$$n \in N_{10} \equiv \left(\frac{f(p^*)\, p^*}{(1-\gamma)\overline{a}\overline{p}}, \infty \right), \tag{OA.23}$$

$$\gamma \in \Gamma_{10} \equiv \left(\frac{(1-\overline{p})\overline{a}}{2q-1+(1-\overline{p})\overline{a}}, \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)} \right), \tag{OA.24}$$

$$\frac{v}{cn} \in V_{10} \equiv \left(\max\left\{ \frac{1}{n}, \frac{(1-\gamma)\overline{a}(1-\overline{p})}{f(p^*)\,(1-p^*)} \right\}, \frac{(1-\gamma)\overline{a}\overline{p}}{f(p^*)\, p^*} \right), \tag{OA.25}$$

there exists an equilibrium with $t_L = 1$ and $t_R = 0$, where from (4) $p^* = 1-(2q-1)\gamma/((1-\gamma)\overline{a})$.

Proposition OA.5 (Equilibrium 11)**.** *If and only if* $l < 1/2$ *and*

$$n \in N_{11} \equiv (0,\infty), \tag{OA.26}$$

$$\gamma \in \Gamma_{11} \equiv \left(\max\left\{0, \frac{\overline{a}(1-2h)}{\overline{a}(1-2h)+2q-1}\mathbb{I}\left(h<\frac{1}{2}\right)\right\}, \frac{\overline{a}(1-2l)}{\overline{a}(1-2l)+2q-1}\right), \tag{OA.27}$$

$$\frac{v}{cn} \in V_{11} \equiv \left(\max\left\{\frac{(1-\gamma)\overline{a}\overline{p}}{f\left(p^*\right)\frac{p^*}{2}}, \frac{(1-\gamma)\overline{a}(1-\overline{p})}{f\left(p^*\right)\frac{1-p^*}{2}}, \frac{1}{n}\right\}, \infty\right), \tag{OA.28}$$

where $\mathbb{I}$ *is the indicator function, there exists an equilibrium with* $t_L = 1$ *and* $t_R = 1$*, where from (4)* $p^* = 1/2 - (2q-1)\gamma/\left(2(1-\gamma)\overline{a}\right)$*.*

Proposition OA.6 (Equilibrium $m1$)**.** *Assume that* $p \sim \mathcal{U}[l,h]$*. If and only if* $\overline{p} < 1/2$ *and*

$$n \in N_{m1} \equiv \left(\max\left\{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{4d(1-\gamma)^2\overline{a}^2(1-\overline{p})}, \frac{(1-h)^2}{d(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right)}\right\}, \infty\right), \tag{OA.29}$$

$$\gamma \in \Gamma_{m1} \equiv \left(0, \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1}\right), \tag{OA.30}$$

$$\frac{v}{cn} \in V_{m1} \equiv \left(\max\left\{\frac{1}{n}, d\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{(1-\overline{p})}\right\}, \min\left\{d\frac{4(1-\gamma)^2\overline{a}^2(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}, d\frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2}\right\}\right), \tag{OA.31}$$

there exists an equilibrium with $t_R = 1$ *and* $t_L \in (0,1)$ *given by*

$$t_L = \frac{1}{(1-\gamma)\overline{a}}\sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{d(1-\overline{p})}\frac{v}{cn}} - 1. \tag{OA.32}$$

Proposition OA.7 (Equilibrium m0)**.** *Assume that* $p \sim \mathcal{U}[l,h]$*. If and only if*

$$n \in N_{m0} \equiv \left(\max\left\{\frac{(2q-1)\gamma}{d(1-\gamma)^2\overline{a}^2(1-\overline{p})}, \frac{1-\overline{p}}{d(2q-1)\gamma}\right\}, \infty\right), \tag{OA.33}$$

$$\gamma \in \Gamma_{m0} \equiv \left(0, \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)}\right), \tag{OA.34}$$

$$\frac{v}{cn} \in V_{m0} \equiv \left(\max\left\{\frac{1}{n}, \frac{d(2q-1)\gamma(1-\overline{p})}{(1-l)^2}\right\}, \min\left\{d\frac{(1-\gamma)^2\overline{a}^2(1-p)}{(2q-1)\gamma}, \frac{d(2q-1)\gamma}{1-\overline{p}}\right\}\right), \tag{OA.35}$$

there exists an equilibrium with $t_R = 0$ *and* $t_L \in (0,1)$ *given by*

$$t_L = \frac{1}{(1-\gamma)\overline{a}}\sqrt{\frac{(2q-1)\gamma}{d(1-\overline{p})}\frac{v}{cn}}. \tag{OA.36}$$

OA.2.1 Proofs of Propositions OA.3–OA.7

Given Lemma (2) that t_L cannot be zero in any equilibrium, the equilibria we need to inquire about—other than mm in Proposition OA.1, presented in the previous section—are as follows:

$$t_L = 1, t_R \in (0,1), \tag{1m}$$
$$t_L = 1, t_R = 0, \tag{10}$$
$$t_L = 1, t_R = 1, \tag{11}$$
$$t_L \in (0,1), t_R = 1, \tag{m1}$$
$$t_L \in (0,1), t_R = 0. \tag{m0}$$

We begin with equilibria $1m$, 10, and 11 where $t_L = 1$. Let:

$$K \equiv 1 - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}, \tag{OA.37}$$

so from (4), in those equilibria, $p^* = K/(1+t_R)$. Given this, we need $K > 0$, or else $p^* < 0$; i.e.,

$$1 > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \iff \overline{a} - \overline{a}\gamma > (2q-1)\gamma \iff \overline{a} > (2q-1+\overline{a})\gamma \iff \gamma < \frac{\overline{a}}{2q-1+\overline{a}}. \tag{OA.38}$$

This upper bound on γ takes precedence over A1 (i.e., $\gamma < 1/(2q)$), since $\overline{a} < 1$.

We also need $p^* < 1$; i.e.,

$$K\frac{1}{1+t_R} < 1 \iff t_R > K - 1,$$

which is always true since $K < 1$ (see (OA.37)).

Moreover, for $t_L = 1$, we must have:

$$\begin{aligned}\mathbb{P}\left[\text{pivotal}|L\right] > \frac{c}{v} &\iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})}\frac{f(p^*)(1-p^*)}{1+t_R} > \frac{c}{v} \iff \frac{f(p^*)(1-p^*)}{1+t_R} > \frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \\ &\iff f\left(K\frac{1}{1+t_R}\right)\frac{1+t_R-K}{(1+t_R)^2} > \frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}),\end{aligned} \tag{OA.39}$$

where in the second inequality we substituted for the pivotal probability from (6).

Now we focus on each of the equilibria with $t_L = 1$ in turn. The aim of the proofs is to derive the regions of n, γ, and $v/(cn)$ so that, given $t_L = 1$, we have that $t_R \in (0,1)$ and probability $p^* \in (l,h)$ (while also respecting assumptions A1 and A2).

Equilibrium $1m$: Deriving the expression for t_R: For $t_R \in (0,1)$, we must have:

$$\mathbb{P}\left[\text{pivotal}|R\right] = \frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}\overline{p}}\frac{f(p^*)p^*}{1+t_R} = \frac{c}{v} \iff f\left(K\frac{1}{1+t_R}\right)\frac{K}{(1+t_R)^2} = \frac{c}{v}(1-\gamma)n\overline{a}\overline{p}, \tag{OA.40}$$

where, in the second equality, we substituted for the pivotal probability from (7) and, in the third, for $p^* = K/(1+t_R)$. Let us assume that $p \sim \mathcal{U}[l,h], 0 \le l < h \le 1$, and $p^* \in (l,h)$, which we check below; then $\overline{p} = (h+l)/2$, $f(p^*) = 1/(h-l) = 1/d$, and (OA.40) becomes:

$$\frac{1}{d}\frac{K}{(1+t_R)^2} = \frac{c}{v}(1-\gamma)n\overline{a}\overline{p} \iff (1+t_R)^2 = \frac{K}{\frac{dc}{v}(1-\gamma)n\overline{a}\overline{p}} = \frac{1-\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}}{\frac{dc}{v}(1-\gamma)n\overline{a}\overline{p}},$$

which, if we solve, yields (OA.22), where in the last equality we substituted for K from (OA.37). Checking that $t_R \in (0,1)$: We need to ensure that

$$t_R < 1 \iff \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\frac{dc}{v}n\overline{p}}\frac{1}{(1-\gamma)^2\overline{a}^2} < 4 \iff \frac{v}{c} < nd\frac{4(1-\gamma)^2\overline{a}^2\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}, \quad \text{(OA.41)}$$

where, in the second inequality, we substituted from (OA.22); and, similarly,

$$t_R > 0 \iff \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\frac{dc}{v}n\overline{p}} > (1-\gamma)^2\overline{a}^2 \iff \frac{v}{c} > nd\frac{(1-\gamma)^2\overline{a}^2\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}. \quad \text{(OA.42)}$$

In addition, from (OA.39), for $p \sim \mathcal{U}[l,h]$ we have:

$$\begin{aligned}
&\frac{1}{d}\frac{1+t_R-K}{(1+t_R)^2} > \frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \iff \frac{1}{1+t_R} - \frac{K}{(1+t_R)^2} > \frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \\
\iff &\sqrt{\frac{\frac{dc}{v}n\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}}(1-\gamma)\overline{a} - \frac{dc}{v}(1-\gamma)n\overline{a}\overline{p} > \frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \\
\iff &\sqrt{\frac{v}{dcn}\frac{\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}} > \overline{p}+1-\overline{p} \iff \frac{v}{c} > nd\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\overline{p}}, \quad \text{(OA.43)}
\end{aligned}$$

where, in the third inequality, we substituted for t_R from (OA.22).

Now we compare the two lower bounds on v/c (OA.42), and (OA.43), to see whether they either of them can be relevant:

$$(1-\gamma)^2\overline{a}\overline{p}^2 > [(1-\gamma)\overline{a}-(2q-1)\gamma]^2 \iff \overline{a}(1-\overline{p}) < \gamma\left[\overline{a}(1-\overline{p})+2q-1\right] \iff \gamma > \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p})+2q-1}.$$

Is this last requirement consistent with the current lower bound on γ (OA.38)?

$$\frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p})+2q-1} < \frac{\overline{a}}{2q-1+\overline{a}} \iff (1-\overline{p})(2q-1)+(1-\overline{p})\overline{a} < \overline{a}(1-\overline{p})+2q-1 \iff \overline{p} > 0,$$

which is always true; hence, either of (OA.42) and (OA.43) can be relevant.

Moreoever, for existence of equilibrium $1m$, we need to check that the upper bound on v/c in (OA.41) is larger than both the two possible lower bounds in (OA.42) and (OA.43). First, from (OA.41) and (OA.43), we have existence if and only if

$$\begin{aligned}
&4(1-\gamma)^2\overline{a}^2\overline{p}^2 > [(1-\gamma)\overline{a}-(2q-1)\gamma]^2 \iff 2(1-\gamma)\overline{a}\overline{p} > (1-\gamma)\overline{a}-(2q-1)\gamma \\
\iff &(1-\gamma)\overline{a}(1-2\overline{p})-(2q-1)\gamma < 0 \iff \overline{a}(1-2\overline{p}) < \gamma\left[\overline{a}(1-2\overline{p})+2q-1\right]. \quad \text{(OA.44)}
\end{aligned}$$

We inquire on the validity of (OA.44) by considering the following cases:
Case A: If $1-2\overline{p}>0 \iff \overline{p}<1/2$, then from (OA.44) we need:

$$\gamma > \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1}. \tag{OA.45}$$

For this to be consistent with the current upper bound on γ (OA.38), we further need:

$$\frac{\overline{a}}{2q-1+\overline{a}} > \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1} \iff \overline{a}(1-2\overline{p})+2q-1 > (1-2\overline{p})(2q-1)+\overline{a}(1-2\overline{p}) \iff \overline{p}>0,$$

which is always true.
Case B: If $1-2\overline{p}<0 \iff \overline{p}>1/2$ and $\overline{a}(1-2\overline{p})+2q-1>0 \iff \overline{p}<(2q-1)/(2\overline{a})+1/2$; i.e., if $1/2<\overline{p}<(2q-1)/(2\overline{a})+1/2$, then (OA.44) is true without any further restrictions.
Case C: If $\overline{p}>(2q-1)/(2\overline{a})+1/2$ for (OA.44) to be true we need:

$$\overline{a}(2\overline{p}-1) > \gamma\left[\overline{a}(2\overline{p}-1)-(2q-1)\right] \iff \gamma < \frac{\overline{a}(2\overline{p}-1)}{\overline{a}(2\overline{p}-1)-(2q-1)}. \tag{OA.46}$$

We need to check how the above compares with the current upper bound on γ (OA.38):

$$\frac{2\overline{p}-1}{\overline{a}(2\overline{p}-1)-(2q-1)} < \frac{1}{2q-1+\overline{a}} \iff (2\overline{p}-1)(2q-1)+(2\overline{p}-1)\overline{a} < (2\overline{p}-1)\overline{a}-(2q-1)$$
$$\iff 2\overline{p}-1<-1 \iff \overline{p}<0,$$

which is false, so (OA.46) is weaker than (OA.38) and does not take precedence.

Second, the lower bound on v/c in (OA.42) is smaller than the upper bound in (OA.41).

Let us summarize our findings up to this point: An equilibrium with $t_L=1, t_R\in(0,1)$ given by (OA.22) exists if:

$$\max\left\{\frac{(1-\gamma)^2\overline{a}^2 n\overline{p}d}{(1-\gamma)\overline{a}-(2q-1)\gamma}, \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\overline{p}}nd\right\} \tag{OA.47}$$

$$< \frac{v}{c} <$$

$$nd\frac{4(1-\gamma)^2\overline{a}^2\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}, \tag{OA.48}$$

$$\text{and} \qquad \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1}\mathbb{I}(\overline{p}<1/2) < \gamma < \frac{\overline{a}}{2q-1+\overline{a}}, \tag{OA.49}$$

where $\mathbb{I}$ is the indicator function.
Checking assumptions A1 and A2: As mentioned, for all equilibria with $t_L=1$, the upper bound in (OA.38) takes precedence over the one in A1. We continue with the requirement of A2 that $v/c\geq 1$. Hence, for an equilibrium to exist, we need that expression (OA.48) (the current upper

bound) exceeds one:

$$d4(1-\gamma)^2\overline{a}n\overline{p} > (1-\gamma)\overline{a} - (2q-1)\gamma \iff n > \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{4(1-\gamma)^2\overline{a}^2\overline{p}d}. \tag{OA.50}$$

This implies a lower bound on the number of voters n. Next we compare the lower bound imposed by A2 with the current ones in (OA.47). In turn we have:
i) For the lower bound (OA.42) to be the relevant one, we need:

$$\begin{aligned} \text{relative to '1'} \rightarrow \quad & n > \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{(1-\gamma)^2\overline{a}^2\overline{p}d}, \\ \text{and relative to (OA.43)} \rightarrow \quad & \gamma > \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p}) + 2q - 1}. \end{aligned}$$

ii) For the lower bound (OA.43) to be the relevant one, we need:

$$\begin{aligned} \text{relative to '1'} \rightarrow \quad & n > \frac{\overline{p}}{d\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]}, \\ \text{and relative to (OA.42)} \rightarrow \quad & \gamma < \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p}) + 2q - 1}. \end{aligned}$$

iii) For the lower bound in A2 (i.e., 1) to be the relevant one, we need:

$$\begin{aligned} \text{relative to (OA.42)} \rightarrow \quad & n < \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{(1-\gamma)^2\overline{a}^2\overline{p}d}, \\ \text{and relative to (OA.43)} \rightarrow \quad & n < \frac{\overline{p}}{\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]d}. \end{aligned}$$

Hence, depending on the parameters, any of the three lower bounds on v/c can be relevant.
Checking that $p^* \in (l,h)$: Since $p \in [l,h]$, we also need to check that $l < p^* < h$. Recall that, for the equilibrium under consideration (i.e., $1m$), $p^* = K/(1+t_R)$, where K is given by (OA.37) and t_R by (OA.22). Hence, after some algebra, we write the requirement on p^* as:

$$l < p^* < h \iff \frac{1}{h^2} < \frac{1}{p^{*2}} < \frac{1}{l^2} \iff \frac{1}{h^2} < \frac{v}{nc}\frac{1}{d\overline{p}}\frac{1}{\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]} < \frac{1}{l^2}$$

$$\iff \frac{nd\overline{p}[(1-\gamma)\overline{a} - (2q-1)\gamma]}{h^2} \tag{OA.51}$$

$$< \frac{v}{c} <$$

$$\frac{nd\overline{p}[(1-\gamma)\overline{a} - (2q-1)\gamma]}{l^2}. \tag{OA.52}$$

How do these reconcile with the existing bounds (OA.47) and (OA.48)? In turn, we have:
i) (OA.51) versus the second part of the maximum in the lower bound (OA.47) (i.e., the right-hand-side (RHS) of (OA.43)):

$$\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{\overline{p}}d > d\overline{p}\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{h^2} \iff \frac{1}{\overline{p}} > \frac{\overline{p}}{h^2} \iff \overline{p} < h,$$

which is always true; hence, (OA.51) is never the relevant lower bound.
ii) (OA.52) versus the upper bound in (OA.48):

$$\frac{d\overline{p}\,[(1-\gamma)\overline{a}-(2q-1)\gamma]}{l^2}>\frac{4(1-\gamma)^2\overline{a}^2\overline{p}d}{(1-\gamma)\overline{a}-(2q-1)\gamma} \iff [(1-\gamma)\overline{a}-(2q-1)\gamma]^2>4(1-\gamma)^2\overline{a}^2l^2$$
$$\iff (1-\gamma)\overline{a}-(2q-1)\gamma>2(1-\gamma)\overline{a}l \iff (1-2l)\overline{a}>(2q-1+\overline{a}(1-2l))\gamma. \quad \text{(OA.53)}$$

To inquire on the validity of (OA.53), we need to consider the following cases:
Case A: If $1-2l>0 \iff l<1/2$, then we need $\gamma<(1-2l)\overline{a}(2q-1+\overline{a}(1-2l))$ for the current upper bound on v/c in (OA.48) to be the relevant one. Otherwise, (OA.52) takes precedence. Note that the above upper bound on γ is lower than our current one in (OA.49) and so is in the feasible region.
Case B: If $1-2l<0 \iff l>1/2$, then:
B-1) If $2q-1+\overline{a}(1-2l)>0 \iff \overline{a}<(2q-1)/(2l-1)$, (OA.52) is the relevant upper bound.
B-2) If $2q-1+\overline{a}(1-2l)<0 \iff \overline{a}>(2q-1)/(2l-1)$, then we need $\gamma>(2l-1)\overline{a}/(1-2q+\overline{a}(2l-1))$ for the current upper bound on v/c in (OA.48) to be the relevant one. Otherwise, (OA.52) takes precedence. Note that again the above upper bound on γ is lower than our current one in (OA.49) and so is in the feasible region.

Therefore, (OA.52) can be the relevant upper bound, and hence we need to ensure that it is larger than the existing possible lower bounds on v/c in (OA.47) and A2. We have, in turn:
i) Comparing (OA.52) versus 1 (in A2), leads to

$$\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{l^2}n\overline{p}d>1 \iff n>\frac{1}{\overline{p}d}\frac{l^2}{(1-\gamma)\overline{a}-(2q-1)\gamma}. \quad \text{(OA.54)}$$

How does (OA.54) compare with the existing lower bound on n in (OA.50)?

$$\frac{1}{\overline{p}d}\frac{l^2}{(1-\gamma)\overline{a}-(2q-1)\gamma}>\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{4(1-\gamma)^2\overline{a}^2\overline{p}d} \iff 2l(1-\gamma)\overline{a}>(1-\gamma)\overline{a}-(2q-1)\gamma$$
$$\iff (1-\gamma)\overline{a}(1-2l)<(2q-1)\gamma.$$

Hence, if $1-2l<0 \iff l>1/2$, then the above is always true, and (OA.54) takes precedence. While if $1-2l>0 \iff l<1/2$, then we need: $\gamma>\overline{a}(1-2l)/(2q-1+\overline{a}(1-2l))$, which is within the feasible interval for γ, since $l>0$. Therefore, for $l<1/2$, either of (OA.54) and (OA.50) can be the relevant lower bound on n. This results in region N_{1m} defined in (OA.19).
ii) Comparing (OA.52) to the first term in the maximum for the lower bound in (OA.47), implies

$$\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{l^2}n\overline{p}d>\frac{(1-\gamma)^2\overline{a}^2n\overline{p}d}{(1-\gamma)\overline{a}-(2q-1)\gamma} \iff (1-\gamma)\overline{a}-(2q-1)\gamma>(1-\gamma)\overline{a}l$$
$$\iff (1-\gamma)\overline{a}(1-l)>(2q-1)\gamma \iff \overline{a}(1-l)>[2q-1+\overline{a}(1-l)]\,\gamma \iff \gamma<\frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)}. \quad \text{(OA.55)}$$

Now, the upper bound in (OA.55) is lower than the one in (OA.49), so it takes precedence (and it is also larger than the current lower bound in (OA.49), since $\overline{p}>l$). This together with the lower bound in (OA.49), results in region Γ_{1m} defined in (OA.20).

iii) Comparing (OA.52) versus the second term in the maximum for the lower bound in (OA.48):

$$\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{l^2}n\overline{p}d > \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\overline{p}}nd \iff \overline{p} > l,$$

which is always true. Hence, the upper bound in (OA.48) needs to be amended to include (OA.52). While, as mentioned before, the lower bounds in (OA.47) need to be amended to include the bound imposed by A2. These result in region V_{1m} defined in (OA.21) (where we further divide all expressions by n). This concludes the proof of Proposition OA.3. ■

Equilibrium 10. Drawing inferences from the values of t_L, t_R: For $t_L = 1$ and $t_R = 0$, we have that $p^* = K/(1+t_R) = K$, where K is defined in (OA.37). Moreover, no discretionary participation of R means that (adapting (OA.40)) we have:

$$\mathbb{P}\left[\text{pivotal}|R\right] < \frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}}\frac{f(p^*)p^*}{1+0} < \frac{c}{v} \iff \frac{v}{c} < \frac{(1-\gamma)n\overline{ap}}{f(K)K}. \tag{OA.56}$$

While for full discretionary participation of L, we need from (OA.39):

$$\frac{v}{c} > \frac{(1-\gamma)n\overline{a}(1-\overline{p})}{f(K)(1-K)}. \tag{OA.57}$$

For existence of equilibrium, we need the lower bound on v/c in (OA.57) to be smaller than the upper bound implied by (OA.56). Hence,

$$\frac{(1-\gamma)n\overline{a}(1-\overline{p})}{(1-\gamma)n\overline{ap}} < \frac{f(K)(1-K)}{f(K)K} \iff 1-\overline{p}-\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} < 0 \iff \gamma > \frac{(1-\overline{p})\overline{a}}{2q-1+(1-\overline{p})\overline{a}}, \tag{OA.58}$$

where, in the second inequality, we substituted for K from (OA.37) and canceled common terms. Is this new lower bound smaller than the current upper bound in (OA.38)? Existence requires:

$$\frac{(1-\overline{p})\overline{a}}{2q-1+(1-\overline{p})\overline{a}} < \frac{\overline{a}}{2q-1+\overline{a}} \iff (1-\overline{p})\overline{a}\left[2q-1+\overline{a}\right] < (2q-1+(1-\overline{p})\overline{a})\overline{a}$$
$$\iff (1-\overline{p}-1)\overline{a}(2q-1) < 0 \iff -\overline{pa}(2q-1) < 0,$$

which is always true, so the lower bound in (OA.58) is in the feasible region of γ without further parameter restrictions.

Checking assumptions A1 and A2: As mentioned for all the equilibria with $t_L = 1$, the upper bound in (OA.38) takes precedence over the one in A1. What about A2 that $v/c \geq 1$? For existence, we need 1 to be smaller than the current upper bound in (OA.56):

$$1 \leq \frac{(1-\gamma)n\overline{ap}}{f(K)K} \iff n \geq \frac{f(K)K}{(1-\gamma)\overline{ap}}, \tag{OA.59}$$

which imposes a lower bound on the number of voters. This bound defines N_{10} in (OA.23). Then the lower bound of v/c in (OA.57) together with 1 and the upper bound in (OA.56) define V_{10} in (OA.25) (where we further divide all expressions by n).

Checking that $p^* \in (l,h)$: Since $p \in [l,h]$, we also need to check that $l < p^* < h$ or, for equilibrium 10, $l < K < h$. Substituting for K from (OA.37) and after a bit of algebra, we arrive at:

$$\frac{\overline{a}(1-h)}{2q-1+\overline{a}(1-h)} \tag{OA.60}$$

$$< \gamma <$$

$$\frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)}. \tag{OA.61}$$

How does (OA.60) compare with the current lower bound in (OA.58)?

$$\frac{\overline{a}(1-h)}{2q-1+\overline{a}(1-h)} < \frac{\overline{a}(1-\overline{p})}{2q-1+\overline{a}(1-\overline{p})} \iff \frac{2q-1}{\overline{a}(1-h)} > \frac{2q-1}{\overline{a}(1-\overline{p})} \iff \overline{p} < h,$$

which is always true, and therefore the current lower bound stands.
How does (OA.61) compare with the current upper bound in (OA.38)?

$$\frac{\overline{a}}{2q-1+\overline{a}} < \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)} \iff \frac{2q-1}{\overline{a}} > \frac{2q-1}{\overline{a}(1-l)} \iff l < 0,$$

which is never true, and so (OA.61) takes precedence. So these upper and lower bounds in (OA.58) define region Γ_{10} in (OA.24), concluding the proof of Proposition OA.4. ■

Equilibrium 11. Drawing inferences from the values of t_L, t_R: For $t_R = 1$ we have $p^* = K/2$, and we need (adapting (OA.40)):

$$\mathbb{P}\left[\text{pivotal}|R\right] > \frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}\overline{p}}\frac{f(p^*)p^*}{2} > \frac{c}{v}, \iff \frac{v}{c} > \frac{(1-\gamma)n\overline{a}\overline{p}}{f\left(\frac{K}{2}\right)\frac{K}{4}}. \tag{OA.62}$$

For $t_L = 1$, we have from (OA.39):

$$f\left(\frac{K}{2}\right)\frac{2-K}{4} > \frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \iff \frac{v}{c} > \frac{(1-\gamma)n\overline{a}(1-\overline{p})}{f\left(\frac{K}{2}\right)\frac{2-K}{4}}. \tag{OA.63}$$

Depending on parameter values, either of (OA.62) or (OA.63) can be the relevant lower bound.
Checking assumptions A1 and A2: As mentioned for all the equilibria with $t_L = 1$, the upper bound in (OA.38) takes precedence over the one in A1. Now lower bounds (OA.62) and (OA.63), together with 1 from A2 define region V_{11} in (OA.28) (where we divide all expressions by n). Note that there is no upper bound on v/c in equilibrium 11.
Checking that $p^* \in (l,h)$: Since $p \in [l,h]$, we also need to check that $l < p^* < h$. Using that now $p^* = K/2$ and (OA.37) for K we arrive at the

$$1 - 2h \leq \frac{2q-1}{\overline{a}}\frac{\gamma}{1-\gamma} \leq 1 - 2l. \tag{OA.64}$$

Therefore, for an equilibrium to exist, we need $1 - 2l \geq 0 \iff l \leq 1/2$, and this is part of the requirements in the statement of Proposition OA.5. Given this from the second inequality in

(OA.64), we have the requirement:

$$\frac{2q-1}{\overline{a}}\frac{\gamma}{1-\gamma} \leq 1-2l \iff \gamma \leq \frac{(1-2l)\overline{a}}{(1-2l)\overline{a}+2q-1}, \tag{OA.65}$$

which as an upper bound takes precedence over the current one in (OA.38). Now, for $1-2h$, we consider two cases:

Case A: If $1-2h<0 \iff h>1/2$, then the first inequality in (OA.64) is trivially true without further restrictions on parameters.

Case B: If $1-2h>0 \iff h<1/2$, then we also need:

$$\gamma \geq \frac{(1-2h)\overline{a}}{(1-2h)\overline{a}+2q-1}. \tag{OA.66}$$

The lower bound on γ in (OA.66) is always smaller than the current upper bound in (OA.65); putting them together leads to region Γ_{11} is (OA.27). Note that we do not have any restriction on the number of voters n, and hence the region N_{11} in (OA.26) is the entire positive half-line. This concludes the proof of Proposition OA.5. ■

Below we move on to the proofs pertaining to equilibria where $t_L<1$, i.e., $m1$ and $m0$. Again, the main goal is to derive the regions of n, γ, and $v/(cn)$ so that, given $t_R=1$ or 0, we have that $t_L \in (0,1)$ and probability $p^* \in (l,h)$ (while still requiring A1 and A2).

Equilibrium *m1*. Deriving p^*: Here we inquire whether $t_L \in (0,1)$, $t_R=1$ can be an equilibrium. First, from (4), we have:

$$p^* = \frac{t_L}{1+t_L} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\frac{1}{1+t_L} \iff 1-p^* = \frac{1}{1+t_L}\left[1+\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\right]. \tag{OA.67}$$

The term in the square brackets is positive so $p^*<1$ but we further need that

$$p^*>0 \iff 1-p^*<1 \iff 1+t_L>1+\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \iff t_L>\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}. \tag{OA.68}$$

So, for an equilibrium where L play a mixed strategy to exist we must have:

$$\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}<1 \iff (2q-1+\overline{a})\gamma<\overline{a} \iff \gamma<\frac{\overline{a}}{2q-1+\overline{a}}, \tag{OA.69}$$

which is the same as (OA.38), and so this new upper bound on γ takes precedence over the one required by A1 (i.e., $\gamma<1/(2q)$).

Deriving the expression for t_L: For $t_L \in (0,1)$, $t_R=1$ to be an equilibrium, we must have that the pivotal probabilities in (6) and (7), respectively, satisfy:

$$\mathbb{P}\left[\text{pivotal}|L\right] = \frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})(1+t_L)}f(p^*)(1-p^*) = \frac{c}{v}, \tag{OA.70}$$

$$\mathbb{P}\left[\text{pivotal}|R\right] > \frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}\overline{p}(1+t_L)}f(p^*)p^* > \frac{c}{v}. \tag{OA.71}$$

Assume that $p \sim \mathcal{U}[l,h]$, so that, provided that $p^* \in (l,h)$ (which we check below), we have

$f(p^*) = 1/d$, where $d = h - l$; then (OA.70) becomes:

$$\frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})}\frac{1}{(1+t_L)}\frac{1}{d}\frac{1+\dfrac{(2q-1)\gamma}{(1-\gamma)\overline{a}}}{1+t_L} = \frac{c}{v} \iff (1+t_L)^2 = \frac{1+\dfrac{(2q-1)\gamma}{(1-\gamma)\overline{a}}}{\dfrac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p})},$$

which, if we solve, yields (OA.32), where, in the first equality, we also substituted for $1 - p^*$ from (OA.67).

Checking that $t_L \in (0,1)$: We need to ensure that, given (OA.32):

$$t_L < 1 \iff \sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}}\frac{1}{(1-\gamma)\overline{a}} < 2 \iff \frac{v}{c} < d\frac{4(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}, \quad \text{(OA.72)}$$

$$t_L > 0 \iff \frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})} > (1-\gamma)^2\overline{a}^2 \iff \frac{v}{c} > d\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}. \quad \text{(OA.73)}$$

Clearly, the upper bound on v/c imposed by (OA.72) is larger than the lower bound in (OA.73).

Drawing inferences from the values of t_L, t_R: In addition, from (OA.71), we have for $p \sim \mathcal{U}[l,h]$:

$$\begin{aligned}
&\frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}(1+t_L)^2}\frac{1}{d}\left(t_L - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\right) > \frac{c}{v} \iff \left(\frac{1}{1+t_L} - \frac{L}{(1+t_L)^2}\right) > \frac{dc}{v}(1-\gamma)n\overline{ap} \\
\iff &\sqrt{\frac{\frac{dc}{v}n(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}}(1-\gamma)\overline{a} - \frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p}) > \frac{dc}{v}(1-\gamma)n\overline{ap} \\
\iff &\sqrt{\frac{\frac{v}{cdn}(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}} > 1 \iff \frac{v}{c} > nd\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{1-\overline{p}}, \qquad \text{(OA.74)}
\end{aligned}$$

where, in the second line, we substituted for the computed t_L from (OA.32). Let us also see how (OA.68) changes in terms of t_L. Substituting we have:

$$\sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}}\frac{1}{(1-\gamma)\overline{a}} - 1 > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \iff \frac{v}{c} > nd(1-\overline{p})\left[(2q-1)\gamma + (1-\gamma)\overline{a}\right]. \quad \text{(OA.75)}$$

Note that, since $1/(1-\overline{p}) > 1-\overline{p}$, (OA.74) always supersedes (OA.75) as a lower bound on v/c. How about ranking the lower bounds on v/c imposed by (OA.73) and (OA.74)? We have:

$$\begin{aligned}
&(1-\gamma)^2\overline{a}^2(1-\overline{p})^2 > \left[(1-\gamma)\overline{a}+(2q-1)\gamma\right]^2 \iff (1-\gamma)\overline{a}(1-\overline{p}) > (1-\gamma)\overline{a}+(2q-1)\gamma \\
\iff &(1-\gamma)\overline{a}(1-\overline{p}-1) > (2q-1)\gamma \iff -(1-\gamma)\overline{ap} > (2q-1)\gamma,
\end{aligned}$$

which is not true for any parameter value; therefore, (OA.74) supersedes (OA.73) and is the (up to now) relevant lower bound on v/c.

Now we need to check whether the upper bound on v/c in (OA.72) exceeds the lower bound

in (OA.74); otherwise equilibrium $m1$ does not exist. We have:

$$4(1-\gamma)^2\overline{a}^2(1-\overline{p})^2 > [(1-\gamma)\overline{a}+(2q-1)\gamma]^2 \iff 2(1-\gamma)\overline{a}(1-\overline{p}) > (1-\gamma)\overline{a}+(2q-1)\gamma$$
$$\iff (1-\gamma)\overline{a}(2-2\overline{p}-1) > (2q-1)\gamma \iff \overline{a}(1-2\overline{p}) > \gamma\left[\overline{a}(1-2\overline{p})+2q-1\right]. \tag{OA.76}$$

We consider the following cases to inquire on the validity of (OA.76).
Case A: If $1-2\overline{p}>0 \iff \overline{p}<1/2$; then, we need:

$$\gamma < \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1}. \tag{OA.77}$$

How does this compare with the current upper bound on γ in (OA.69)?

$$\frac{\overline{a}}{2q-1+\overline{a}} > \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1} \iff \overline{p}>0,$$

which is always true; therefore, (OA.77) supersedes (OA.69) in this case. Hence, for $\overline{p}<1/2$ and for γ less than the bound in (OA.77), inequality (OA.76) holds and equilibrium exists.
Case B: If $1-2\overline{p}<0 \iff \overline{p}>1/2$ and $\overline{a}(1-2\overline{p})+2q-1>0 \iff \overline{p}<1/2+(2q-1)/(2\overline{a})$, then inequality (OA.76) does not hold for any $\gamma>0$. Hence, equilibrium does not exist.
Case C: If $1-2\overline{p}<0 \iff \overline{p}>1/2$ and $\overline{a}(1-2\overline{p})+2q-1<0 \iff \overline{p}>1/2+(2q-1)/(2\overline{a})$, then, we need

$$\overline{a}(2\overline{p}-1) < \gamma\left[\overline{a}(2\overline{p}-1)-(2q-1)\right] \iff \gamma > \frac{\overline{a}(2\overline{p}-1)}{\overline{a}(2\overline{p}-1)-(2q-1)}.$$

For an equilibrium to exist, we need that this lower bound be smaller than the current upper bound of γ in (OA.69); that is,

$$\frac{\overline{a}(2\overline{p}-1)}{\overline{a}(2\overline{p}-1)-(2q-1)} < \frac{\overline{a}}{2q-1+\overline{a}} \iff (2q-1)2\overline{p}<0, \tag{OA.78}$$

which is never true, so also in this case inequality (OA.76) does not hold, and this equilibrium does not exist.
Therefore, equilibrium $m1$ only exists for $\overline{p}<1/2$, and so we add this restriction in the statement of Proposition OA.6. Additionally, we need the upper bound on γ in (OA.77) that hence defines region Γ_{m1} in (OA.30).
Checking assumptions A1 and A2: As mentioned, the upper bound on γ in (OA.77) supersedes the one in (OA.69), which, in turn, supersedes the bound imposed by A1 (i.e., $\gamma<1/(2q)$). Therefore, for $\overline{p}<1/2$, where an equilibrium exists, (OA.77) is the relevant bound. How about A2 that $v/c\geq 1$? We check with the current upper bound in (OA.72) for existence:

$$1 < d\frac{4(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma} \iff n > \frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{4d(1-\gamma)^2\overline{a}^2(1-\overline{p})}. \tag{OA.79}$$

So this imposes a lower bound on the number of voters n. How does 1 compare with the lower

bound in (OA.74)? If

$$n > \frac{(1-\overline{p})}{d\left[(1-\gamma)\overline{a}+(2q-1)\gamma\right]},$$

(OA.74) is relevant; otherwise, 1 is relevant.

Checking that $p^* \in (l,h)$: Since $p \in [l,h]$, we also need to check that $l < p^* < h$. For the case $t_L \in (0,1), t_R = 1$, we have from (4):

$$p^* = \frac{t_L}{1+t_L} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\frac{1}{1+t_L} \iff 1-p^* = \frac{1}{1+t_L} + \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\frac{1}{1+t_L}$$

$$\iff 1-p^* = \sqrt{\frac{dcn}{v}(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right)}, \tag{OA.80}$$

where, in the last equality, we used expression (OA.32) for t_L and did a bit of algebra. Using (OA.80) the requirement that $l < p^* < h$, becomes:

$$1-h < 1-p^* < 1-l \iff (1-h)^2 < \frac{dcn}{v}(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right) < (1-l)^2$$

$$\iff \frac{nd(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right)}{(1-l)^2} \tag{OA.81}$$

$$< \frac{v}{c} <$$

$$\frac{nd(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right)}{(1-h)^2}. \tag{OA.82}$$

We now compare the lower bound in (OA.81) with the current one in (OA.74):

$$\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{1-\overline{p}} > \frac{(1-\overline{p})\left[(1-\gamma)\overline{a}+(2q-1)\gamma\right]}{(1-l)^2} \iff (1-l)^2 > (1-\overline{p})^2 \iff l < \overline{p},$$

which is always true; therefore, (OA.74) supersedes the bound in (OA.81) and remains the relevant lower bound on v/c.

How about (OA.82) versus the current upper bound on v/c in (OA.72)? We have:

$$\frac{4(1-\gamma)^2\overline{a}^2(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma} < \frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2} \iff 2(1-\gamma)\overline{a}(1-h) < (1-\gamma)\overline{a}+(2q-1)\gamma$$

$$\iff (1-\gamma)\overline{a}\left[2(1-h)-1\right] < (2q-1)\gamma \iff \left[2(1-h)-1\right]\overline{a} < \left[2q-1+(2(1-h)-1)\,\overline{a}\right]\gamma.$$

The sign of the above inequality depends on whether: $2(1-h)-1 > 0 \iff h < 1/2$ or not. Hence, either of (OA.82) or (OA.72) can be the relevant bound, depending on the parameters.

We need to ensure that the new possible upper bound in (OA.82) exceeds the existing lower bounds: the one in A2 (i.e., $v/c \geq 1$) and the one in (OA.74). We have in turn:

i) Comparing (OA.82) vs 1, leads to:

$$\frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2}nd > 1 \iff n > \frac{(1-h)^2}{d(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}. \tag{OA.83}$$

How does (OA.83) compare with the existing lower bound on n in (OA.79)? We have:

$$\frac{(1-h)^2}{d(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)} > \frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{4d(1-\gamma)^2\overline{a}^2(1-\overline{p})} \iff (1-\gamma)\overline{a}(1-2h) > (2q-1)\gamma.$$

Hence, if $1-2h<0 \iff h>1/2$, then the current bound (OA.79) is the relevant one.
While, if $1-2h>0 \iff h<1/2$, then the current bound holds if and only if:

$$(1-\gamma)\overline{a}(1-2h) < (2q-1)\gamma \iff \gamma > \frac{\overline{a}(1-2h)}{2q-1+\overline{a}(1-2h)}.$$

How does this lower bound on γ compare with our current one in (OA.77)?

$$\frac{\overline{a}(1-2h)}{2q-1+\overline{a}(1-2h)} < \frac{\overline{a}(1-2\overline{p})}{2q-1+\overline{a}(1-2\overline{p})} \iff h>\overline{p},$$

which is always true. Therefore, either of the lower bounds on n in (OA.83) and (OA.79) can be relevant for $h<1/2$. And they are necessary for an equilibrium to exist. These two lower bounds together define interval N_{m1} in (OA.29).
ii) Comparing (OA.82) vs (OA.74), leads to:

$$\frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2}nd > nd\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{(1-\overline{p})} \iff \overline{p}<h,$$

which is again always true, so (OA.82) exceeds (OA.74) without any further restrictions.
So either of (OA.82) and (OA.72) can be the relevant upper bound on v/c, consistent with the possible lower bounds from A2 (i.e., 1) and in (OA.74). These bounds define interval V_{m1} in (OA.21). This concludes the proof of Proposition OA.6. ■

Equilibrium *m0*. Deriving p^*: Now we inquire about the existence of equilibrium with $t_L \in (0,1)$ and $t_R=0$. Then p^* in (4) becomes:

$$p^* = 1-\frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}t_L} \iff 1-p^* = \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}t_L}. \tag{OA.84}$$

We have $1-p^*>0$ but also need:

$$1-p^*<1 \iff (2q-1)\gamma < (1-\gamma)\overline{a}t_L \iff t_L > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}, \tag{OA.85}$$

and, for equilibrium $m0$ to exist, the right hand side in (OA.85) cannot exceed one, which yields:

$$\gamma < \frac{\overline{a}}{2q-1+\overline{a}}, \tag{OA.86}$$

which is the same as (OA.69) and so, like there, this new upper bound on γ takes precedence over the one required by A1.
Deriving the expression for t_L: For $t_L \in (0,1)$, $t_R=0$ to be an equilibrium we must have that

the pivotal probabilities in (6) and (7), respectively, satisfy:

$$\mathbb{P}\left[\text{pivotal}|R\right] < \frac{c}{v} \iff \frac{1}{(1-\gamma)n\overline{a}\overline{p}}\frac{1}{t_{L}}f(p^{*})p^{*} < \frac{c}{v}, \tag{OA.87}$$

$$\mathbb{P}\left[\text{pivotal}|L\right] = \frac{c}{v} \iff \frac{1}{(1-\gamma)n\overline{a}(1-\overline{p})}\frac{1}{t_{L}}f(p^{*})(1-p^{*}) = \frac{c}{v}. \tag{OA.88}$$

Assume that $p \sim \mathcal{U}[l,h]$, so that if $p^{*} \in (l,h)$ (which we check below), then $f(p^{*}) = 1/d$, where $d = h - l$; then (OA.88) becomes:

$$\frac{1}{(1-\gamma)^{2}n\overline{a}^{2}(1-\overline{p})}\frac{1}{t_{L}^{2}}\frac{1}{d}(2q-1)\gamma = \frac{c}{v} \iff t_{L} = \sqrt{\frac{(2q-1)\gamma}{n(1-\overline{p})d\frac{c}{v}}}\frac{1}{(1-\gamma)\overline{a}},$$

which coincides with (OA.36).

Checking that $t_{L} \in (0,1)$: We need to ensure that, given (OA.36):

$$t_{L} < 1 \iff \frac{(2q-1)\gamma}{n(1-\overline{p})d\frac{c}{v}} < (1-\gamma)^{2}\overline{a}^{2} \iff \frac{v}{c} < d\frac{(1-\gamma)^{2}\overline{a}^{2}n(1-\overline{p})}{(2q-1)\gamma}, \tag{OA.89}$$

and $t_{L} > 0$, which, in this case, is always true.

Drawing inferences from the values of t_{L}, t_{R}: Additionally, from (OA.87), we have for $p \sim \mathcal{U}[l,h]$:

$$\begin{aligned}
&\frac{1}{(1-\gamma)n\overline{a}\overline{p}}\frac{1}{d}\frac{1}{t_{L}}p^{*} < \frac{c}{v} \iff \frac{1}{(1-\gamma)n\overline{a}\overline{p}}\frac{1}{d}\frac{1}{t_{L}}\left(1 - \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}t_{L}}\right) < \frac{c}{v} \\
&\iff \sqrt{\frac{n(1-\overline{p})d\frac{c}{v}}{(2q-1)\gamma}}(1-\gamma)\overline{a} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\frac{(1-\gamma)^{2}\overline{a}^{2}n(1-\overline{p})d\frac{c}{v}}{(2q-1)\gamma} < d\frac{c}{v}(1-\gamma)n\overline{a}\overline{p} \iff \frac{v}{c} < d\frac{n(2q-1)\gamma}{1-\overline{p}},
\end{aligned} \tag{OA.90}$$

where, in the first equality, we used $f(p^{*}) = 1/d$, in the second inequality, we substituted for p^{*} from (OA.84), and, in the third inequality, we substituted for t_{L} from (OA.36). Between (OA.89) and (OA.90), which is the relevant upper bound for v/c?

$$d\frac{n(2q-1)\gamma}{1-\overline{p}} < d\frac{(1-\gamma)^{2}\overline{a}^{2}n(1-\overline{p})}{(2q-1)\gamma} \iff (2q-1)\gamma < (1-\gamma)\overline{a}(1-\overline{p}) \iff \gamma < \frac{\overline{a}(1-\overline{p})}{2q-1+\overline{a}(1-\overline{p})} \tag{OA.91}$$

Hence, if (OA.91) is true, then (OA.90) is relevant; otherwise (OA.89) is relevant. Note that (OA.91) is smaller than (OA.86), and hence in the feasible (up until now) region for γ.

Moreover, let us see how (OA.85) changes in terms of the computed t_{L} in (OA.36). We have:

$$\sqrt{\frac{(2q-1)\gamma}{n(1-\overline{p})d\frac{c}{v}}}\frac{1}{(1-\gamma)\overline{a}} > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \iff \sqrt{\frac{1}{n(1-\overline{p})d\frac{c}{v}}} > \sqrt{(2q-1)\gamma} \iff \frac{v}{c} > dn(1-\overline{p})(2q-1)\gamma. \tag{OA.92}$$

For the existence of equilibrium $m0$, we need both possible upper bounds on v/c (OA.89) and (OA.90) to ex eed the lower bound in (OA.92). We have in turn:

i) Comparing (OA.89) with (OA.92):

$$d\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(2q-1)\gamma} > dn(1-\overline{p})(2q-1)\gamma \iff (1-\gamma)^2\overline{a}^2 > (2q-1)^2\gamma^2 \iff \gamma < \frac{\overline{a}}{2q-1+\overline{a}},$$

which is identical to the current upper bound on γ in (OA.86) and hence true in the region we are considering.

ii) Comparing (OA.90) with (OA.92):

$$d\frac{n(2q-1)\gamma}{1-\overline{p}} > dn(1-\overline{p})(2q-1)\gamma \iff \overline{p} > 0,$$

which is also always true.

Checking assumptions A1 and A2: As mentioned, the upper bound on γ in (OA.86) supersedes the bound imposed by A1 (i.e., $\gamma < 1/(2q)$). This upper bound and the generic lower bound of 0 define are the current candidates for Γ_{m0}. Now we check the consistency of the upper bounds for v/c in (OA.89) and (OA.90) with respect to A2 (i.e., $v/c \geq 1$):

i) Comparing 1 with the bound in (OA.89):

$$n > \frac{(2q-1)\gamma}{d(1-\gamma)^2\overline{a}^2(1-\overline{p})}. \tag{OA.93}$$

ii) Comparing 1 with the bound in (OA.90):

$$n > \frac{1-\overline{p}}{d(2q-1)\gamma}. \tag{OA.94}$$

Expressions (OA.93) and (OA.94) impose lower bounds on n. Either can be relevant, depending on parameter values. Together they define interval N_{m0} in (OA.33).

Checking that $p^* \in (l,h)$: Given that $p \in [l,h]$, we need to also ensure that $l < p^* < h$. From the expression for p^* in (OA.84) and t_L given by (OA.36), we have: $1-p^* = \sqrt{(2q-1)\gamma\frac{dcn}{v}(1-\overline{p})}$ so

$$\begin{aligned} & l < p^* < h \iff 1-h < 1-p^* < 1-l \iff (1-h)^2 < (1-p^*)^2 < (1-l)^2 \\ \iff & \frac{nd(2q-1)\gamma(1-\overline{p})}{(1-l)^2} && \text{(OA.95)} \\ & < \frac{v}{c} < \\ & \frac{nd(2q-1)\gamma(1-\overline{p})}{(1-h)^2}. && \text{(OA.96)} \end{aligned}$$

How does the lower bound in (OA.95) compare with the existing one in (OA.92)?

$$\frac{nd(2q-1)\gamma(1-\overline{p})}{(1-l)^2} > dn(1-\overline{p})(2q-1)\gamma \iff (1-l)^2 < 1,$$

which is always true, and hence (OA.95) takes precedence over (OA.92).

Now how does the upper bound (OA.96) compare with the existing ones in (OA.89) and (OA.90)? We have in turn:

i) Comparing (OA.96) with the bound in (OA.89):

$$\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(2q-1)\gamma} < \frac{nd(2q-1)\gamma(1-\overline{p})}{(1-h)^2} \iff \gamma > \frac{\overline{a}(1-h)}{2q-1+\overline{a}(1-h)}. \tag{OA.97}$$

Hence, we need to inquire further. Note that (OA.97) is smaller than (OA.86) and so in the feasible region of γ. Hence, either of (OA.89) and (OA.96) can be the relevant v/c upper bound.
i) Comparing (OA.96) with the bound in (OA.90):

$$\frac{d(2q-1)\gamma n}{1-\overline{p}} < \frac{nd(2q-1)\gamma(1-\overline{p})}{(1-h)^2} \iff \overline{p} < h,$$

which is always true, and hence (OA.96) is never the relevant upper bound on v/c.
Combining the lower bound (OA.95) with the one implied by A2 (i.e., $v/c \geq 1$) and the upper bounds in (OA.89) and (OA.90) defines interval V_{m0} in (OA.35) (where we also divide all parameters by n).

Finally we need to check whether we need to update the bounds on γ to make sure that the lower bound on v/c is indeed lower than the upper. After some algebra we coclude that we need to tighten the upper bound on γ (lower bounds remains at zero) to $\bar{a}(1-l)/\left((2q-1)+\bar{a}(1-l)\right)$, which results to Γ_{m0} in (OA.34). This concludes the proof of Proposition OA.7. ■

OA.3 Trading and the Timing of Ownership

The first part of this section clarifies how trading and share lending around the record date relate to the aggregate uncertainty $f(p)$ in our model. We show that *ex-ante trading* provides a natural micro-foundation for that uncertainty. Then, in the brief second part, we discuss how *ex-post trading* is, under our timing assumptions, neutral for equilibrium outcomes.

OA.3.1 Ex-Ante Trading as a Micro-Foundation for Aggregate Uncertainty

Let n denote total shares and $\gamma \in [0,1)$ the fraction held by regular (non-discretionary) voters. Between an observation date t_0 and the record date T, discretionary shares may be traded or lent. Following Kyle (1985), aggregate order flow is

$$y = x_I + u,$$

where x_I represents informed trading and $u \sim \mathcal{N}(0, \sigma_u^2)$ is noise trading. Let $\phi = \Phi(|y|) \in [0,1]$ be the (random) fraction of discretionary shares whose ownership changes before T, with $\Phi' > 0$. If a share turns over, its new owner's vote is drawn i.i.d. from the stationary cross-sectional distribution (probability $\tilde{p}$ of supporting R). If p_0 is the discretionary support for R at t_0, then

$$p_T \;=\; (1-\phi)p_0 + \phi\,\tilde{p}, \qquad \tilde{p} \sim \tfrac{1}{(1-\gamma)n}\,\text{Binomial}\big((1-\gamma)n, \bar{p}\big), \tag{OA.98}$$

where $\bar{p}$ reflects the stationary composition of discretionary types.

Lemma 3 (Distribution of the record-date composition under turnover)**.** *Let ϕ have a continuous density g_ϕ on $[0,1]$ induced by the noise u. Then:*

(i) ***Conditional moments.***

$$\mathbb{E}[p_T \mid p_0, \phi] = (1-\phi)p_0 + \phi\bar{p}, \quad \mathrm{Var}(p_T \mid p_0, \phi) = \phi^2 \frac{\bar{p}(1-\bar{p})}{(1-\gamma)n}.$$

(ii) ***Mixture density.*** *The unconditional density is a continuous mixture*

$$f_{p_T}(p) = \int_0^1 f_{p_T|\phi}(p \mid \varphi)\, g_\phi(\varphi)\, d\varphi.$$

If g_ϕ has non-degenerate support and $(1-\gamma)n < \infty$, then $f_{p_T}(p)$ is continuous and strictly positive on an interval $[l, h] \subset (0, 1)$.

Proof. (i) follows from (OA.98) and the Binomial moments. (ii) Conditioning on $\phi = \varphi$, p_T lies on the grid $\{(1-\varphi)p_0 + \varphi\, k/M : k = 0, \ldots, M\}$ with $M = (1-\gamma)n$. Integrating over a continuous g_ϕ smears the grid into a continuous law on $[l, h]$ with a strictly positive density on its interior. □

Proposition OA.8 (Micro-foundation of aggregate uncertainty)**.** *Under Lemma 3, the model's random variable p can be interpreted as the realization of p_T in* (OA.98)*. Hence, the density $f(p)$ used in all equilibrium expressions is the record-date distribution generated by ex-ante trading and lending.*

Proof. Direct from Lemma 3: p_T has continuous density on $[l, h]$ with mean between p_0 and $\bar{p}$ and non-zero variance whenever $\phi > 0$ with positive probability. Substituting $f(p_T)$ for $f(p)$ in the pivotal-probability expressions leaves all equilibrium relations unchanged. □

Interpretation and empirical link. Equation (OA.98) makes explicit how order-flow volatility σ_u and turnover ϕ map into dispersion of p_T. Higher σ_u or lending intensity increases $\mathrm{Var}(p_T)$, i.e. a wider $[l, h]$ and lower local density $f(p^*)$. Record-date turnover or loan-fee spikes can serve as empirical proxies for this dispersion.

Asymmetric turnover. If R- and L-supporters differ in turnover, with $\phi_R, \phi_L \in [0, 1]$, then

$$\mathbb{E}[p_T \mid p_0] = (1-\phi_R)p_0 + (\phi_R p_0 + \phi_L(1-p_0))\bar{p}, \tag{OA.99}$$

so if $\phi_R > \phi_L$, the expected composition moves closer to $\bar{p}$, biasing the observed mean toward the freeriders' side.

OA.3.2 Ex-Post Trading and Timing

After the record date and the vote, shares may be re-traded, but such transactions do not affect pivotality or turnout under our static timing. At stage (i) agents form beliefs and choose t_L, t_R; (ii) the outcome is realized and payoffs accrue; (iii) only then does secondary-market trading occur.

Neutrality result. If v (or v_R, v_L in the asymmetric model Extension III below) already incorporates the expected resale value $\mathbb{E}[\Delta V]$ at the record date, then ex-post trading after ΔV is realized leaves the first-order condition $\Pr[\text{pivotal} \mid i] = C(t_i)/v_i \; i \in \{L, R\}$—or the corresponding corner conditions—unchanged, where $C(t_i)$ is the inverse CDF (quantile function) of voting cost (anticipating our extensions in the subsequent sections; $C(t_i) = c$ in our baseline model); hence

$$\frac{\partial t_i}{\partial(\text{ex-post trades})} = \frac{\partial p^*}{\partial(\text{ex-post trades})} = \frac{\partial P^{\text{misrep}}}{\partial(\text{ex-post trades})} = 0.$$

Within this static framework, ex-post trading is neutral.[1] In richer intertemporal models, anticipated resale could feed back into both trading ex-ante and voting incentives by making v endogenous to expected prices (see Meirowitz and Pi, 2022; Levit et al., 2024); those dynamics are beyond the scope of our paper, which focuses on voting participation.

OA.4 Model Extensions and Robustness

We develop three extensions to assess robustness and clarify scope:[2]

1. a generalized distribution of individual voting costs, $C(t) = \bar{c}\, t^{\delta}$;
2. a flexible (Beta) distribution for aggregate uncertainty in p;
3. asymmetric private valuations, $v_L \neq v_R$.

Across these, we track what changes and what remains intact. Our core insight—the non–monotonic relation between the misrepresentation probability P^{misrep} and the benefit–cost ratio per share when $\gamma > 0$—is robust: it holds exactly under constant costs (where full freeriding can occur) and persists once cost heterogeneity rules out corner abstention.

Throughout, define

$$B \;\equiv\; \frac{(2q-1)\gamma}{(1-\gamma)\bar{a}}, \text{ with } p^* \;=\; \frac{t_L - B}{t_R + t_L}, \text{ from (4)}$$

and, again, let $d = h - l$ denote the width of the support of p. Recall, from Lemma 1, we have that for large n, the pivotal probabilities for discretionary L and R voters are given by (6) and (7), reproduced here for convenience:

$$\Pr[\text{pivotal} \mid L] = \frac{f(p^*)(1-p^*)}{(1-\gamma)n\,\bar{a}\,(1-\bar{p})\,(t_R + t_L)}, \quad \Pr[\text{pivotal} \mid R] = \frac{f(p^*)p^*}{(1-\gamma)n\,\bar{a}\,\bar{p}\,(t_R + t_L)}.$$

[1] Neutrality hinges on the timing (record-date payoffs priced in) and on no additional constraints (e.g., borrowing limits, strategic lending) that would re-enter the first order conditions (FOCs).

[2] These extensions are qualified in the political election context in Myatt (2015, secs. 4–6).

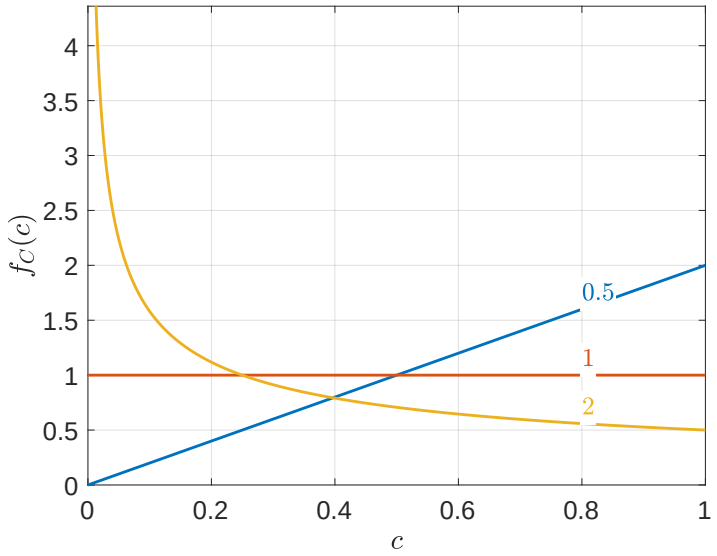


Figure OA.1: Cost density $f_c(c) = (c/\bar{c})^{1/\delta-1} / (\delta\bar{c})$ for different curvature parameters $\delta = \{0.5, 1, 2\}$ and $\bar{c} = 1$.

OA.4.1 Extension I: Generalized Idiosyncratic Cost Distribution

This extension introduces heterogeneity in participation frictions by allowing each discretionary shareholder to have idiosyncratic cost $c \in [0, \bar{c}]$ with flexible *inverse CDF* (quantile)

$$C(t) = \bar{c}\, t^{\delta}, \qquad \delta \geq 0,\ \bar{c} > 0, \tag{OA.100}$$

nesting the baseline $\delta = 0$ (constant cost) and $\delta = 1$ (uniform idiosyncratic costs). For $\delta > 0$, the CDF/density of c are

$$F_c(c) = \left(\tfrac{c}{\bar{c}}\right)^{1/\delta}, \qquad f_c(c) = \frac{1}{\delta\bar{c}}\left(\tfrac{c}{\bar{c}}\right)^{1/\delta-1}.$$

The mean is $\mathbb{E}[c] = \bar{c}/(1+\delta)$. As $\delta \to 0$, costs concentrate at $\bar{c}$; as $\delta \to \infty$, mass piles near zero. Thus variance is 0 at both extremes and attains an interior maximum for some finite $\delta^\star > 0$. The aim is to test whether our main comparative statics and the piecewise non-monotonic pattern of P^{misrep} survive when a thin tail of very low-cost ("always-voting") types coexists with higher-cost voters. Figure OA.1 depicts the density $f_c(c)$ for $\delta = 0.5, 1, 2$ and $\bar{c} = 1$. Below we proceed in several steps.

Step 1: Corner equilibria are ruled out for $\delta > 0$.

Lemma 4 (No full freeriding when $\delta > 0$)**.** *If $\delta > 0$, equilibria $m0$ and 10 do not exist.*

Proof. Because $C(0) = 0$, any type with sufficiently low cost has positive net gain from voting whenever $\Pr[\text{pivotal} \mid i] > 0$. If the other side participates (interior or partial), $\Pr[\text{pivotal} \mid i] > 0$ by Lemma 1. If both sides abstain, a measure of zero-cost types would profitably deviate to vote. Hence no corner with one side at zero turnout can be an equilibrium. □

Step 2: The mm equilibrium ($t_L,\, t_R \in (0,1)$.) Let $u_{cn} \equiv v/(\bar{c}n)$. The interior FOCs are

$$\Pr[\text{pivotal} \mid i] = \frac{C(t_i)}{v} = \frac{t_i^\delta}{n u_{cn}}, \qquad i \in \{L, R\}.$$

Using Lemma 1 for the pivotal probabilities, dividing and summing the FOCs yields:

$$\left(\tfrac{t_R}{t_L}\right)^\delta = \frac{t_L - B}{t_R + B} \cdot \frac{1-\bar{p}}{\bar{p}}, \tag{OA.101}$$

$$\frac{1}{(1-\gamma)\bar{a}\, d} \, \frac{\frac{t_L}{\bar{p}} + \frac{t_R}{1-\bar{p}} + B(\frac{1}{1-\bar{p}} - \frac{1}{\bar{p}})}{(t_R + t_L)^2} = \frac{t_L^\delta + t_R^\delta}{u_{cn}}, \tag{OA.102}$$

with $f = 1/d$ under A3 (uniform $p \sim \mathcal{U}[l,h]$). Note that (OA.101) implies that we need to have $t_L > B$ in any equilibrium.

The interior case requires solving the coupled system (OA.101)–(OA.102) for (t_L, t_R) jointly. Unlike the $1m$ case, below, where one equation in one unknown yields global monotonicity and clean comparative statics, the mm system is two-dimensional and nonlinearly interdependent through both p^* and (t_L, t_R). This coupling makes a general closed-form or global existence result intractable: even under a uniform $f(p) = 1/d$, the nonlinear interdependence of the two FOCs in (t_L, t_R) means that the Jacobian can change sign depending on $(\bar{p}, \gamma, \delta, B)$, precluding global monotonicity and closed-form solutions. When $f(p)$ is later allowed to vary (as in Extension II), such curvature further complicates existence and stability. Nevertheless, we can show the following benchmark result.

Lemma 5 (Existence under symmetry)**.** *If $\bar{p} = \frac{1}{2}$ and $B = 0$ and $f(p) > 0$ for all $p \in (l,h)$, then a symmetric interior equilibrium exists with $t_L = t_R \in (0,1)$ satisfying*

$$t^{\delta+1} = \frac{u_{cn}}{2(1-\gamma)\bar{a}\, d}.$$

This solution is locally unique whenever $\delta > 0$.

Proof. Substituting $t_L = t_R = t$ into (OA.101)–(OA.102) eliminates p^* and yields the scalar equation above. Since $t^{\delta+1}$ is strictly increasing on $[0,1]$, there is a unique solution. Local uniqueness for small deviations from symmetry follows from the implicit function theorem. □

By the implicit function theorem, the symmetric interior solution persists under small perturbations $(B, \bar{p} - 1/2)$ provided the Jacobian at the symmetric point is nonsingular and the density $f(p^*)$ is strictly positive at that point (which holds under A3). These conditions are verified explicitly in the subsection OA4.4 "Jacobian at Symmetry and Local Monotonicity of S in u_{cn}." Hence, for B sufficiently small and $\bar{p}$ sufficiently close to $1/2$, a unique interior mm equilibrium exists by continuity. Global existence for arbitrary parameters is numerical, but the system is well behaved in a neighborhood of symmetry.

Local comparative statics in mm. Write $S \equiv t_L + t_R$ and $r \equiv t_R/t_L$, so that

$$p^* = \frac{t_L - B}{t_L + t_R} = \frac{1}{1+r} - \frac{B}{S}.$$

The ratio condition (OA.101) pins down r as a function of $(p^*, B, \bar{p}, \delta)$, while the scale condition (OA.102) pins down S given (r, p^*, u_{cn}).

Lemma 6 (Turnout mass increases with incentives in *mm*)**.** *Under A3 (uniform p) and for any $\delta \geq 0$, the total turnout mass $S = t_L + t_R$ is strictly increasing in u_{cn} along the mm equilibrium locus:*

$$\frac{\partial S}{\partial u_{cn}} > 0.$$

Proof. Define $S = t_L + t_R$ and $r = t_R/t_L$. The *mm* equilibrium is the solution to the system $G_1(r, S) = 0$ and $G_2(r, S, u_{cn}) = 0$, where G_1 is the ratio condition (OA.101) (which does not depend on u_{cn}) and G_2 is the scale condition (OA.102), written as $G_2 = \text{LHS} - \text{RHS} = 0$. By the implicit function theorem, along the *mm* locus,

$$\begin{pmatrix} \partial r/\partial u_{cn} \\ \partial S/\partial u_{cn} \end{pmatrix} = -J^{-1} \begin{pmatrix} \partial G_1/\partial u_{cn} \\ \partial G_2/\partial u_{cn} \end{pmatrix}, \qquad J = \begin{pmatrix} \partial G_1/\partial r & \partial G_1/\partial S \\ \partial G_2/\partial r & \partial G_2/\partial S \end{pmatrix}.$$

Since G_1 does not depend on u_{cn}, $\partial G_1/\partial u_{cn} = 0$. The derivative of G_2 is $\partial G_2/\partial u_{cn} = \partial/\partial u_{cn}[-(t_L^\delta + t_R^\delta)/u_{cn}] = (t_L^\delta + t_R^\delta)/u_{cn}^2 > 0$. Cramer's rule yields

$$\frac{\partial S}{\partial u_{cn}} = -\frac{\det\begin{pmatrix} \partial G_1/\partial r & 0 \\ \partial G_2/\partial r & \partial G_2/\partial u_{cn} \end{pmatrix}}{\det(J)} = -\frac{(\partial G_1/\partial r)\,(\partial G_2/\partial u_{cn})}{\det(J)}.$$

As shown in OA4.4 calculation for the symmetric benchmark, $\partial G_1/\partial r > 0$ and $\det(J) < 0$. Since $\partial G_2/\partial u_{cn} > 0$, the full expression is

$$\frac{\partial S}{\partial u_{cn}} = -\frac{(+)(+)}{(-)} > 0.$$

This positive sign holds by continuity in a neighbourhood of the symmetric point. □

Note, that the same argument applies for small perturbations away from symmetry, where B is small and $\bar{p}$ is near $1/2$ (See Appendix OA.4.4 for a Jacobian calculation establishing local nonsingularity and $\partial S/\partial u_{cn} > 0$ at symmetry.)

Proposition OA.9 (Sign of $\partial p^*/\partial u_{cn}$ in *mm* for $B > 0$)**.** *Along the mm equilibrium locus,*

$$\frac{\partial p^*}{\partial u_{cn}} \;=\; -\frac{1}{(1+r)^2}\,\frac{\partial r}{\partial u_{cn}} \;+\; \frac{B}{S^2}\,\frac{\partial S}{\partial u_{cn}}.$$

Hence:

1. *If the turnout ratio r is locally insensitive to u_{cn} (e.g., $\bar{p}$ near $1/2$ and B small), then $\partial p^*/\partial u_{cn} > 0$ since $B > 0$ and $\partial S/\partial u_{cn} > 0$ by Lemma 6.*

2. *More generally, a sufficient condition for $\partial p^*/\partial u_{cn} > 0$ is*

$$\left|\frac{\partial r}{\partial u_{cn}}\right| \;\leq\; \frac{B(1+r)^2}{S^2}\,\frac{\partial S}{\partial u_{cn}}.$$

3. If $\partial r/\partial u_{cn} > 0$ is large enough to overturn the B-term, then $\partial p^*/\partial u_{cn} < 0$.

Proof. Differentiate $p^* = 1/(1+r) - B/S$ and use Lemma 6. The decomposition is algebraic; the sign statements follow immediately. □

Corollary 2 (Implications for P^{misrep} inside mm)**.** *Because $P^{\text{misrep}} = |p^* - p^{\text{full}}|/d$ (for $p^{\text{full}} \in (l,h)$), the within-mm slope with respect to u_{cn} is governed by the sign of $\partial p^*/\partial u_{cn}$. In particular, when B is moderate and r is locally flat in u_{cn},* P^{misrep} increases *with u_{cn} inside mm. This delivers the upward limb that (when spliced with the strictly decreasing $1m$ limb below) generates the global hump in $P^{\text{misrep}}(u_{cn})$ for $\delta > 0$.*

Step 3: The $1m$ equilibrium ($t_L = 1, t_R \in (0,1)$). Using $p^* = \frac{1-B}{1+t_R}$, the R-side FOC becomes

$$t_R^{\delta}(1+t_R)^2 = \frac{(1-B)\,u_{cn}}{(1-\gamma)\bar{a}\,\bar{p}\,d}. \tag{OA.103}$$

Proposition OA.10 ($1m$ existence and uniqueness)**.** *Let $h_\delta(t) = t^\delta(1+t)^2$ on $[0,1]$. Then h_δ is strictly increasing, $h_\delta(0) = 0$, $h_\delta(1) = 4$. Hence* (OA.103) *has a unique $t_R \in (0,1]$ iff*

$$K \equiv \frac{(1-B)\,u_{cn}}{(1-\gamma)\bar{a}\,\bar{p}\,d} \in (0,4].$$

At $K = 4$ the equilibrium coincides with 11 ($t_R = 1$).

Proof. $h'_\delta(t) = t^{\delta-1}(1+t)[\delta(1+t)+2t] > 0$ on $(0,1]$; thus h_δ bijects $[0,1]$ to $[0,4]$. □

Proposition OA.11 (Comparative statics in $1m$)**.** *Let t_R solve* (OA.103)*. Then*

$$\frac{\partial t_R}{\partial u_{cn}} > 0, \quad \frac{\partial t_R}{\partial B} < 0, \quad \frac{\partial t_R}{\partial \bar{p}} < 0, \quad \frac{\partial t_R}{\partial d} < 0,$$

and $p^* = (1-B)/(1+t_R)$ decreases *in u_{cn},* increases *in $\bar{p}$ and d, and its comparative static with respect to B is, in general,* ambiguous*.*

Proof. Differentiate $h_\delta(t_R) = K$ with $K = \frac{(1-B)u_{cn}}{(1-\gamma)\bar{a}\,\bar{p}\,d}$ and use $\partial t_R/\partial K = 1/h'_\delta(t_R) > 0$; apply the chain rule. □

Lemma 7 (Turnout elasticity in $1m$)**.** *For $t_R \in (0,1)$,*

$$\frac{1}{\delta+1} \le \frac{\partial \log t_R}{\partial \log u_{cn}} \le \frac{1}{\delta},$$

tight as $t_R \uparrow 1$ and $t_R \downarrow 0$, respectively.

Proof. Differentiate $\log h_\delta(t_R) = \log K$. Since $\partial \log h_\delta/\partial \log t_R = \delta + 2t_R/(1+t_R) \in [\delta, \delta+1]$, take reciprocals. □

Step 4: Equilibria $m1$ and 11. As in the baseline, $m1$ is symmetric to $1m$. And 11 occurs when both FOCs bind at $t_L = t_R = 1$, i.e., $K \ge 4$, yielding $p^* = p^{\text{full}}$ and $P^{\text{misrep}} = 0$. We now present the main result of this extension:

Proposition OA.12 (Within-regime monotonicity; possible upward slope in mm)**.** *With uniform p, $\delta > 0$, $\gamma > 0$:*

i) In mm, there exist parameter values for which p^* increases with u_{cn}, so that most importantly P^{misrep} slopes upward within mm.

ii) In $1m$, $p^* = (1-B)/(1+t_R)$ is strictly decreasing in u_{cn}; hence P^{misrep} is decreasing in u_{cn} as long as p^* does not cross p^{full} within the $1m$ region.

Proof. (i) Write $S = t_L + t_R$, $r = t_R/t_L$, so $p^* = \frac{1}{1+r} - \frac{B}{S}$. From Lemma 6, $\partial S/\partial u_{cn} > 0$. Differentiating p^* gives

$$\frac{\partial p^*}{\partial u_{cn}} = \underbrace{-\frac{1}{(1+r)^2}\frac{\partial r}{\partial u_{cn}}}_{\text{Ratio Effect}} + \underbrace{\frac{B}{S^2}\frac{\partial S}{\partial u_{cn}}}_{\text{Scale Effect}}.$$

The sign depends on the relative magnitudes of these two terms. The Scale Effect is unambiguously positive (for $B > 0$). Therefore, $\partial p^*/\partial u_{cn} > 0$ if and only if the positive Scale Effect dominates the Ratio Effect, which occurs if

$$\frac{B}{S^2}\frac{\partial S}{\partial u_{cn}} > \frac{1}{(1+r)^2}\frac{\partial r}{\partial u_{cn}}.$$

This condition is met for parameters where the equilibrium r is locally insensitive to u_{cn}, allowing the positive Scale Effect to dominate. The numerical results in Figure OA.2 (b-d) illustrate the existence of such an upward-sloping region.

(ii) From (OA.103), $\partial t_R/\partial u_{cn} > 0$; thus $\partial p^*/\partial u_{cn} = -(1-B)(1+t_R)^{-2}\partial t_R/\partial u_{cn} < 0$. □

Step 5: Implications for misrepresentation. In mm, $P^{\text{misrep}}(u_{cn})$ can slope up or down. This creates a global hump as we transition from mm to $1m$; this is because in $1m$, Proposition OA.11 implies p^* decreases monotonically with u_{cn}, and

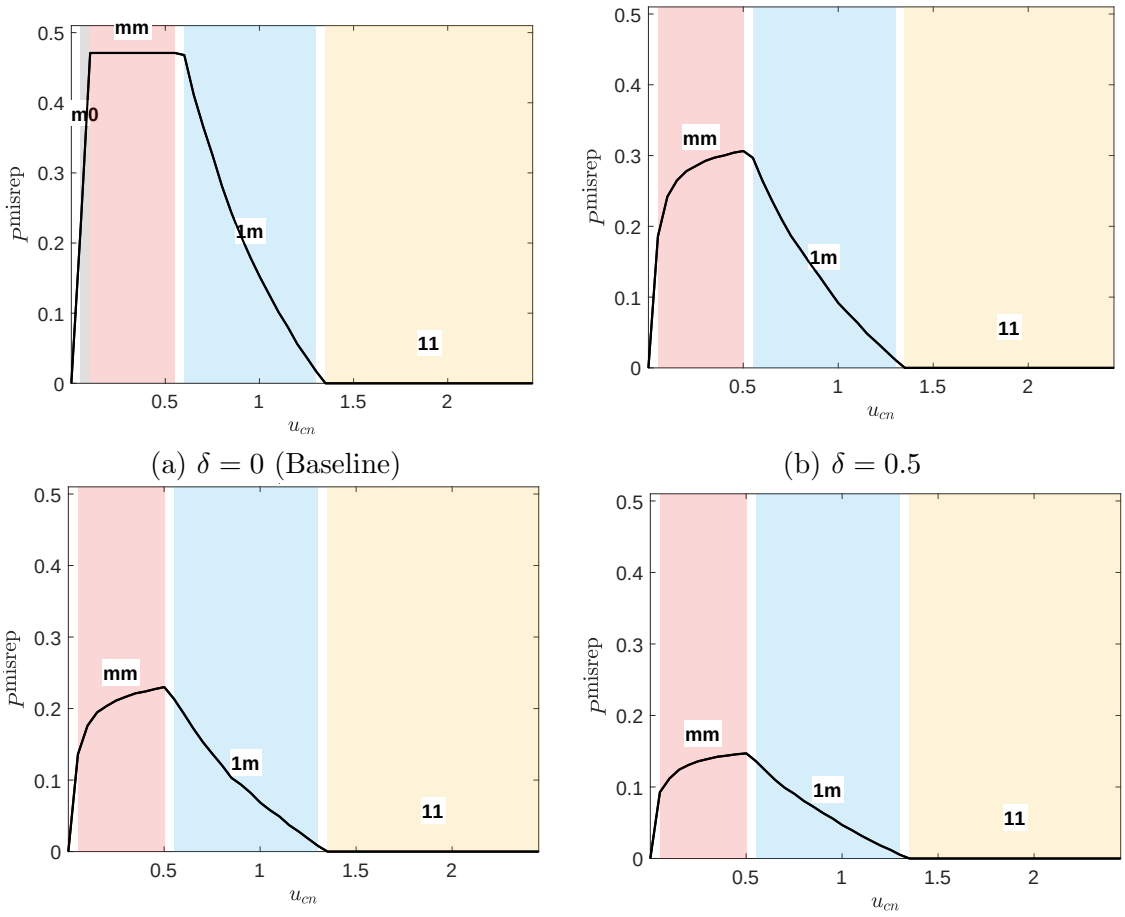
$$P^{\text{misrep}} = \frac{|p^* - p^{\text{full}}|}{d}, \text{ for } p^{\text{full}} \in (l, h)$$

is strictly decreasing in u_{cn} until vanishing at $1m \to 11$.

Proposition OA.13 (Non-monotonicity of P^{misrep})**.** *For $\gamma > 0$, $\delta > 0$, if u_{cn} varies so the equilibrium sequence is $mm \to 1m \to 11$, then $P^{\text{misrep}}(u_{cn})$ is monotone within each region and has at most one interior maximum across transitions.*

Proof. By Lemma 4, corners $m0/10$ are ruled out. From Proposition OA.12 part (i), inside mm, it may increase. Inside $1m$, P^{misrep} it is decreasing in u_{cn} (part (ii)). A function that is monotone on each subinterval and changes slope sign once across the boundary has at most one interior maximum, realized at a regime transition. □

In Figure OA.2 we depict the probability of misrepresentation vs the benefit to (max) cost ratio per voter, for different $\delta = \{0, 0.5, 1, 2\}$. As we can see for higher δ, since there are probabilistically more "always voters," P^{misrep} falls globally; importantly, however, there is still an inverse U-shape relationship vs u_{cn}, which is our main qualitative result.



(a) $\delta = 0$ (Baseline)

(b) $\delta = 0.5$

(c) $\delta = 1$ (Uniform Cost)

(d) $\delta = 2$

Figure OA.2: **Misrepresentation and cost curvature.** P^{misrep} vs. $u_{cn} = v/(\bar{c}n)$ for $\delta = \{0, 0.5, 1, 2\}$. Each curve splices interior mm and $1m$ equilibria. Parameters: $\gamma = 0.1$, $\bar{a} = 1$, $q = 0.8$, $\bar{p} = 0.6$, and $p \in [l, h] = [0.45, 0.95]$; here $u_{cn} = v/(\bar{c}n)$. Even with $\delta > 0$ (no corners), a single interior hump arises from the $mm \to 1m$ transition.

OA.4.2 Extension II: Beta–Distributed Aggregate Uncertainty

This extension relaxes the assumption that aggregate uncertainty over the latent support rate p is uniform. Instead, we let $p \in (l, h)$ follow a scaled Beta(α, β), a flexible two–parameter family that can generate symmetry, skewness, and varying tail thickness. This allows us to test whether our main results—particularly the shape of the misrepresentation probability $P^{\text{misrep}}(u_{cn})$—depend on the shape of the underlying uncertainty distribution. The Beta specification can be viewed as a microfoundation for asymmetric expectations across investors, while maintaining tractability of pivotal probabilities.

Beta density and notation. With $d = h - l$, define the scaled Beta density and CDF:

$$f^{(l,h)}_{\alpha,\beta}(p) = \frac{1}{d}\frac{\left(\frac{p-l}{d}\right)^{\alpha-1}\left(1-\frac{p-l}{d}\right)^{\beta-1}}{\mathcal{B}(\alpha,\beta)}, \qquad F^{(l,h)}_{\alpha,\beta}(p) = \mathcal{I}_x(\alpha,\beta), \quad x = \tfrac{p-l}{d}, \tag{OA.104}$$

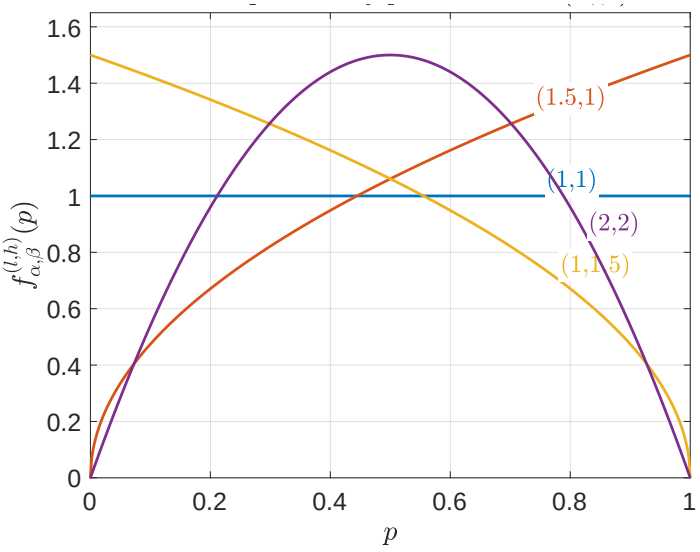


Figure OA.3: **Beta distributions of aggregate uncertainty.** The scaled Beta(α, β) nests the uniform ($\alpha = \beta = 1$), allows skewness ($\alpha \neq \beta$), and captures varying tail thickness. The figure depicts the density for $(\alpha, \beta) = \{(1,1), (1.5,1), (1,1.5), (2,2)\}$, and parameters $l = 0.45$, $h = 0.85$.

where $\mathcal{B}(\alpha, \beta)$ is the Euler Beta function and $\mathcal{I}_x(\alpha, \beta)$ is the regularized incomplete Beta function (the CDF). The mean and variance are

$$\bar{p} = l + d\,\frac{\alpha}{\alpha+\beta}, \qquad \mathrm{Var}(p) = d^2\,\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}.$$

We depict the beta density $f^{(l,h)}_{\alpha,\beta}$ for different $(\alpha, \beta) = \{(1,1), (1.5,1), (1,1.5), (2,2)\}$, and parameters $l = 0.45$, $h = 0.85$ in Figure OA.3.

Step 1: Interior (mm) equilibrium. The ratio equation is unaffected by the change in f:

$$\left(\tfrac{t_R}{t_L}\right)^{\delta} = \frac{t_L - B}{t_R + B} \cdot \frac{1-\bar{p}}{\bar{p}}. \tag{OA.105}$$

The scale condition now weights pivotality by the local density of p:

$$\frac{f^{(l,h)}_{\alpha,\beta}(p^*)}{(1-\gamma)\bar{a}\,(t_R + t_L)}\left(\frac{p^*}{\bar{p}} + \frac{1-p^*}{1-\bar{p}}\right) = \frac{t_L^{\delta} + t_R^{\delta}}{u_{cn}}. \tag{OA.106}$$

Existence and comparative statics in mm. The system (OA.105)—(OA.106) is analogous to that in Extension I, but its curvature now depends on the local slope of $f_{\alpha,\beta}$. General existence is again numerical, but the comparative statics can be characterized qualitatively.

Lemma 8 (Turnout mass increases with incentives under Beta)**.** *For any $\delta \geq 0$ and any Beta density $f^{(l,h)}_{\alpha,\beta}$ with $f(p) > 0$ on its support, total turnout $S = t_L + t_R$ is strictly increasing in u_{cn} along the mm locus: $\partial S/\partial u_{cn} > 0$.*

Proof. As in Lemma 6, write the mm system as $G_1(r, S) = 0$ (ratio) and $G_2(r, S, u_{cn}) = 0$ (scale), now with the Beta density in (OA.106) and $G_2 = \text{LHS} - \text{RHS}$. Again $\partial G_1/\partial u_{cn} = 0$

and $\partial G_2/\partial u_{cn} = (t_L^\delta + t_R^\delta)/u_{cn}^2 > 0$. The Jacobian J has the same structure as in Lemma 4. By Cramer's rule:

$$\frac{\partial S}{\partial u_{cn}} = -\frac{(\partial G_1/\partial r)\,(\partial G_2/\partial u_{cn})}{\det(J)}.$$

As established in OA4.4 for the symmetric benchmark (which holds for a Beta density as well, as noted there), $\partial G_1/\partial r > 0$ and $\det(J) < 0$. Since $\partial G_2/\partial u_{cn} > 0$, we have

$$\frac{\partial S}{\partial u_{cn}} = -\frac{(+)(+)}{(-)} > 0.$$

□

Proposition OA.14 (mm comparative statics under Beta)**.** *The change in the pivotal point p^* with respect to incentives u_{cn} is given by the decomposition:*

$$\frac{\partial p^*}{\partial u_{cn}} \;=\; -\frac{1}{(1+r)^2}\,\frac{\partial r}{\partial u_{cn}} \;+\; \frac{B}{S^2}\,\frac{\partial S}{\partial u_{cn}}.$$

where $S = t_L + t_R$ and $r = t_R/t_L$. The introduction of $f(p^*)$ in the scale equation (OA.106) *primarily affects the equilibrium magnitudes and the sensitivity of r to u_{cn}.*

1. *By Lemma 8, $\partial S/\partial u_{cn} > 0$, meaning the second term (the "Scale Effect") is positive (as $B > 0$).*
2. *The sign of $\partial p^*/\partial u_{cn}$ depends on the sign and magnitude of the first term (the "Ratio Effect").*
3. *If the Ratio Effect is small (e.g., r is locally insensitive to u_{cn}), the positive Scale Effect dominates and $\partial p^*/\partial u_{cn} > 0$. The slope of the density, $f'(p^*)$, influences the magnitude of $\partial S/\partial u_{cn}$ and $\partial r/\partial u_{cn}$ but does not solely determine their signs.*

Proof. The decomposition is an algebraic identity from differentiating $p^* = 1/(1+r) - B/S$. Part (1) follows from Lemma 8 and the assumption $B > 0$. Parts (2) and (3) follow from analyzing the decomposition. The partial derivatives $\partial S/\partial u_{cn}$ and $\partial r/\partial u_{cn}$ are the solutions to the system of FOCs found via the implicit function theorem. The presence of $f(p^*)$ in (OA.106) adds a term $\propto f'(p^*)$ to the Jacobian of the system, influencing the final magnitudes of these derivatives. For example, when p^* increases (as u_{cn} rises), if $f'(p^*) < 0$, the falling density $f(p^*)$ helps the LHS of (OA.106) to decrease, reinforcing the equilibrium adjustment. □

Step 2: The $1m$ equilibrium ($t_L = 1, t_R \in (0,1)$). Substituting $p^* = (1-B)/(1+t_R)$ gives:

$$t_R^\delta(1+t_R)^2 = \frac{(1-B)\,u_{cn}}{(1-\gamma)\bar{a}\,\bar{p}}\,f_{\alpha,\beta}^{(l,h)}\Big(\frac{1-B}{1+t_R}\Big). \tag{OA.107}$$

Proposition OA.15 (Existence and comparative statics in $1m$ under Beta)**.** *If $f_{\alpha,\beta}^{(l,h)}(p^*)$ is monotone on the relevant range of $p^* = (1-B)/(1+t_R)$, then:*

1. *a unique $t_R \in (0,1]$ solves* (OA.107)*;*
2. *$\partial t_R/\partial u_{cn} > 0$ and $\partial p^*/\partial u_{cn} < 0$.*

Proof. The LHS of (OA.107) $L(t) = t^\delta(1+t)^2$ is strictly increasing; The RHS $R(t) \propto f(p^*(t))$ is strictly monotone in the opposite direction if f is monotone and p^* decreases in t. Implicit differentiation of the equality $L(t_R) = R(t_R, u_{cn})$ shows the stated signs. □

Step 3: Equilibria $m1$ and 11. As before, $m1$ mirrors $1m$ by symmetry; 11 arises when both sides participate fully ($t_L = t_R = 1$) with $K \geq 4$. The Beta density affects only the slope of the scale condition, not the regime boundaries themselves.

Step 4: Implications for misrepresentation. The misrepresentation probability becomes

$$P^{\text{misrep}} = \left|F^{(l,h)}_{\alpha,\beta}(p^*) - F^{(l,h)}_{\alpha,\beta}(p^{\text{full}})\right| = \left|\mathcal{I}_{x^*}(\alpha,\beta) - \mathcal{I}_{x^{\text{full}}}(\alpha,\beta)\right|.$$

Within each equilibrium region, p^* varies monotonically with u_{cn}; across transitions $mm \to 1m \to 11$, the global pattern remains piecewise non-monotone with at most one interior hump (as in Proposition OA.13). Skewness and tail thickness (via α, β) primarily shift the *location* and *scale* of the hump but not its qualitative existence.

Figure OA.4 illustrates these effects for several $(\alpha, \beta) = \{(1.5, 1), (1, 1), (1, 1.5), (2, 2)\}$, holding other primitives fixed: when f is skewed, the hump shifts horizontally (toward the side with more mass); when f is more peaked (large $\alpha + \beta$), the curve flattens due to higher pivotality near p^*.[3]

OA.4.3 Extension III: Asymmetric Private Valuations

What this captures. Different investor groups may value outcomes asymmetrically (e.g., due to portfolio concentration, hedging, or preferences). We allow $v_L \neq v_R$ to test whether turnout tilts, thresholds, and the global $P^{\text{misrep}}(u_{cn})$ pattern persist.

Primitives and notation. Let $v_L, v_R > 0$ and $\kappa \equiv v_R/v_L$. Now, (slightly abusing our notation from Extensions I and II) let $u_{cn} \equiv v_L/(\bar{c}n)$ (we normalize by v_L; results are symmetric if we normalize by v_R). The interior FOCs become

$$\Pr[\text{piv} \mid L] = \frac{\bar{c}\, t_L^\delta}{v_L}, \qquad \Pr[\text{piv} \mid R] = \frac{\bar{c}\, t_R^\delta}{v_R}. \tag{OA.108}$$

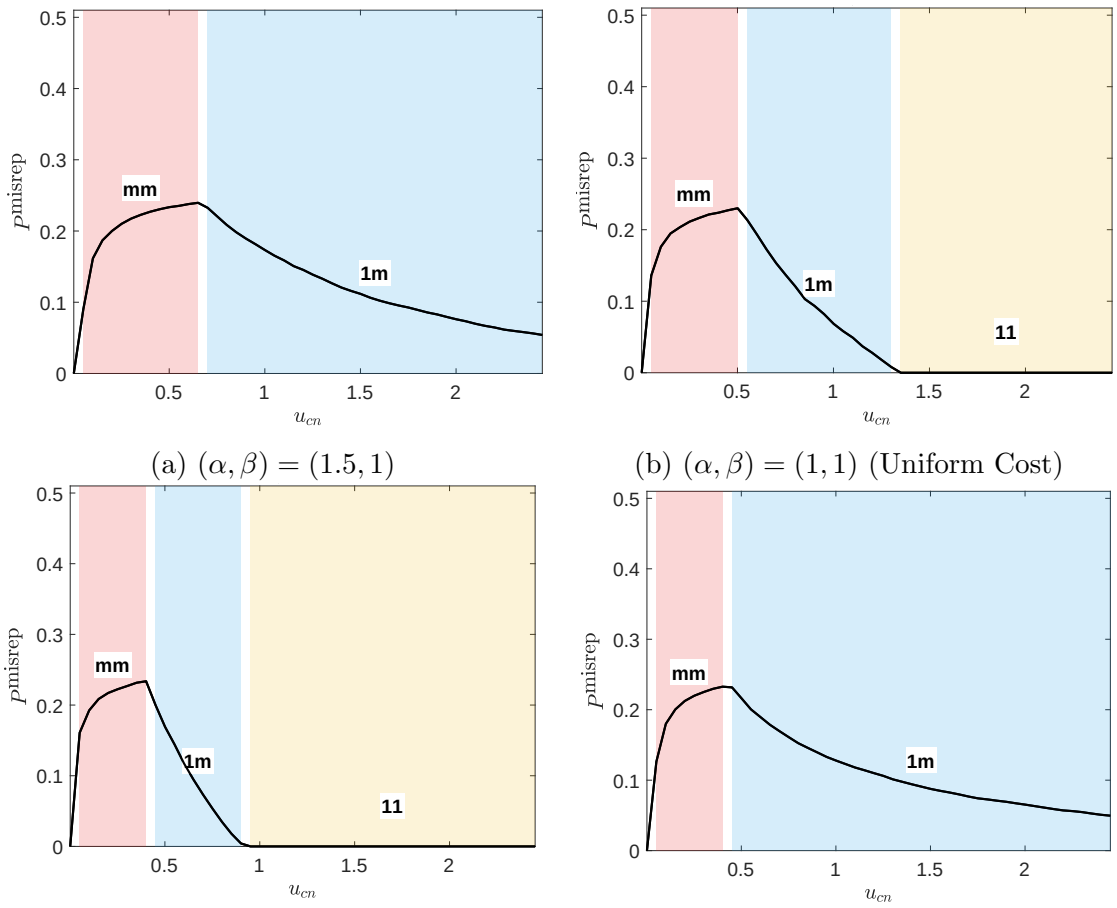
Step 1: mm equilibrium (both mixed). Using Lemma 1 and dividing (OA.108) gives the *asymmetric ratio*:

$$\left(\tfrac{t_R}{t_L}\right)^\delta = \kappa \frac{p^*(1-\bar{p})}{(1-p^*)\bar{p}}, \qquad p^* = \frac{t_L - B}{t_R + t_L}. \tag{OA.109}$$

Summing the FOCs yields the *asymmetric scale*:

$$\frac{f(p^*)}{(1-\gamma)\bar{a}\,(t_R + t_L)}\left(\tfrac{p^*}{\bar{p}} + \tfrac{1-p^*}{1-\bar{p}}\right) = \frac{t_L^\delta + t_R^\delta/\kappa}{u_{cn}}. \tag{OA.110}$$

[3] The results apply on either side of the mode $(\alpha - 1)/(\alpha + \beta - 2)$ (scaled to $[l, h]$). If p^* crosses the mode as u_{cn} varies, local slopes can flip.



(a) $(\alpha, \beta) = (1.5, 1)$ (b) $(\alpha, \beta) = (1, 1)$ (Uniform Cost)

(c) $(\alpha, \beta) = (1, 1.5)$ (d) $(\alpha, \beta) = (2, 2)$

Figure OA.4: **Misrepresentation under Beta uncertainty.** $P^{\text{misrep}}(u_{cn})$ for selected Beta(α, β) parameters $\{(1.5, 1), (1, 1), (1, 1.5), (2, 2)\}$. Skewness shifts the hump horizontally; the regime-transition mechanism persists. Parameters: $\gamma = 0.1$, $\bar{a} = 1$, $q = 0.8$, $d = 0.5$, $\delta = 1$, $l = 0.45$, and $h = 0.95$; here $u_{cn} = v/(\bar{c}n)$.

Existence/tractability. As in the symmetric case, general closed forms are unavailable. Under symmetry of f around $\bar{p} = \frac{1}{2}$ and $B = 0$, small deviations $\kappa \approx 1$ generate a unique interior solution by the implicit function theorem; numerically the system is well behaved.

Proposition OA.16 (*mm* comparative statics with asymmetry)**.** *Along the mm locus, increasing κ (higher R valuation) raises the turnout ratio t_R/t_L via* (OA.109)*. Moreover, for fixed u_{cn}, p^* shifts* leftward *(toward L).*

Proof. From (OA.109), $\partial(t_R/t_L)/\partial\kappa > 0$. Write $p^* = 1/(1+r) - B/S$, $r \equiv t_R/t_L$. Holding S locally fixed, the partial derivative is $\partial p^*/\partial r = -1/(1+r)^2 < 0$. Since an increase in κ causes an increase in r, this dominant "Ratio Effect" *decreases* p^*. The change in S provides a second-order feedback, but the primary effect from the ratio adjustment is a leftward shift of the pivotal point. □

Step 2: $1m$ equilibrium ($t_L = 1$, $t_R \in (0,1)$). Substituting $p^* = (1-B)/(1+t_R)$ into the FOC for R (OA.108) yields

$$t_R^{\delta}(1+t_R)^2 = \frac{(1-B)\,u_{cn}}{(1-\gamma)\bar{a}\,\bar{p}}\kappa\, f(p^*), \qquad p^* = \frac{1-B}{1+t_R}. \tag{OA.111}$$

Proposition OA.17 ($1m$ existence and comparative statics)**.** *For any fixed $f(p^*)$ on a monotone branch and $\delta \geq 0$, there is a unique $t_R \in (0,1]$ solving* (OA.111)*. Moreover,*

$$\frac{\partial t_R}{\partial u_{cn}} > 0, \qquad \frac{\partial t_R}{\partial \kappa} > 0, \qquad \frac{\partial p^*}{\partial u_{cn}} < 0, \qquad \frac{\partial p^*}{\partial \kappa} < 0.$$

Proof. Monotonicity of $t^{\delta}(1+t)^2$ gives uniqueness. Differentiate (OA.111) noting that the RHS scales as κu_{cn}; use $h'_{\delta} > 0$. □

Step 3: $m1$ and 11. By symmetry, $m1$ obtains when $v_L > v_R$ ($\kappa < 1$), with analogous formulas. For sufficiently large incentives (large u_{cn}), both sides reach $t_L = t_R = 1$ (11), giving $p^* = p^{\text{full}}$ and $P^{\text{misrep}} = 0$.

Step 4: Implications for P^{misrep}. Within $1m$, p^* *decreases* with u_{cn} and also *decreases* with κ (by Proposition OA.17); hence, for fixed u_{cn}, a larger κ (higher R valuation) tilts representation toward R (higher t_R) but shifts the pivotal point p^* toward the lower-valuation side (L). Within mm, increasing κ also pushes p^* leftward (Proposition OA.16), which can either raise or lower P^{misrep} depending on p^{full} and f. Globally, across $mm \to 1m \to 11$, the curve $P^{\text{misrep}}(u_{cn})$ remains piecewise non–monotone with at most one interior hump, but the hump's horizontal position shifts with κ.

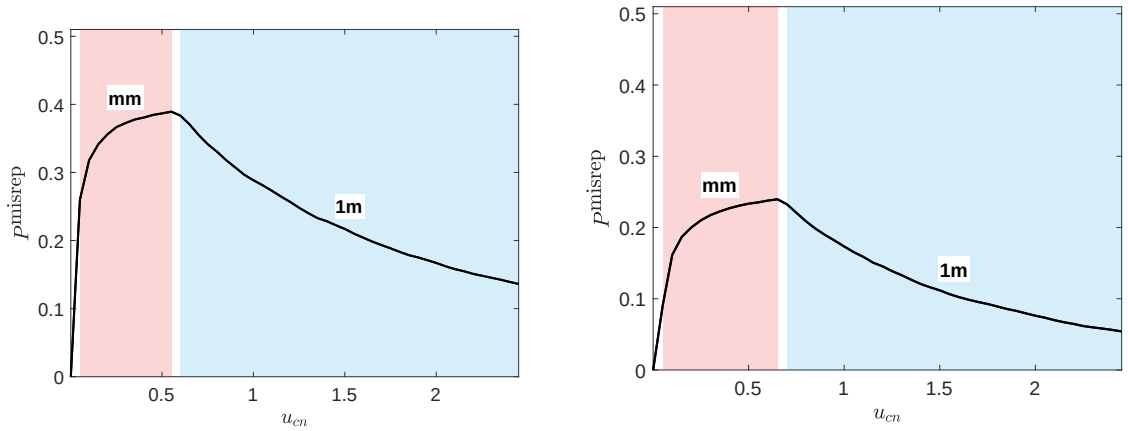
Figure OA.5 visualizes these effects by varying $\kappa = v_R/v_L = \{0.5, 1, 2\}$ while holding other primitives fixed; the peak shifts toward the higher valuation side and its height falls as valuation asymmetry grows.

Robustness Summary

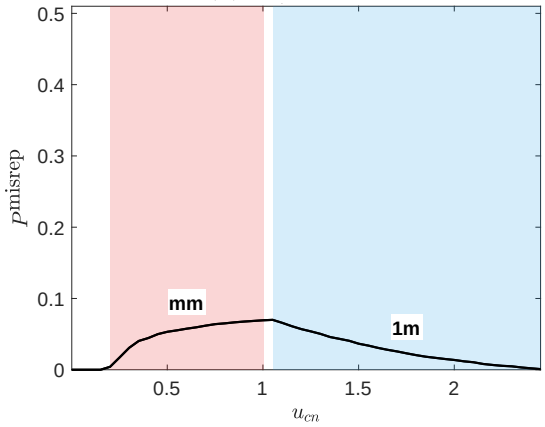
Theorem 1 (Robust non–monotonicity and empirical implications)**.** *Let $\gamma > 0$.*

1. *Under constant costs ($\delta = 0$), freeriding corners ($m0/10$) may exist; $P^{\text{misrep}}(u_{cn})$ is inverted–U.*
2. *For $\delta > 0$, $m0/10$ are eliminated; within mm or $1m$, P^{misrep} is monotone (this is also true for $\delta = 0$); across $mm \to 1m \to 11$, the overall curve is piecewise non–monotone with at most one interior maximum (Prop. OA.13).*
3. *Beta–distributed p rescales pivotality via $f(p^*)$ and can flip within–mm slopes (Prop. OA.14) without changing the regime–transition mechanism.*
4. *Asymmetric valuations ($\kappa \neq 1$) tilt turnout and shift the hump toward the* lower *valuation side; identification of κ requires exogenous variation that differentially affects valuations (Prop. OA.17).*[4]

[4] Analogous identification issues apply to α, β, and δ.



(a) $\kappa = 0.5$ (b) $\kappa = 1$



(c) $\kappa = 2$

Figure OA.5: **Misrepresentation under valuation asymmetry.** $P^{\text{misrep}}(u_{cn})$ for selected valuation ratios $\kappa = v_R/v_L = \{0.5, 1, 2\}$. As κ increases (R's valuation rises), the hump shifts toward the lower-valuation side (L) and flattens due to higher overall turnout. Parameters: $\gamma = 0.2$, $\bar{a} = 1$, $(\alpha, \beta) = (1.5, 1)$, $l = 0.45$, $h = 0.95$, $q = 0.8$, $\delta = 1$; here $u_{cn} = v_L/(\bar{c}n)$.

Proof. Follows directly from Propositions OA.13, OA.14, and OA.17. □

OA.4.4 Jacobian at Symmetry and Local Monotonicity of S in u_{cn}

This note verifies that, at the symmetric benchmark ($B = 0$, $\bar{p} = \frac{1}{2}$) in Extension I, the mm system is locally nonsingular and that the total turnout mass $S = t_L + t_R$ increases in u_{cn}.

Setup. Let $r \equiv t_R/t_L$ and $S \equiv t_L + t_R$. The mm equilibrium solves

$$G_1(r, S) = 0 \quad \text{(ratio)}, \qquad G_2(r, S, u_{cn}) = 0 \quad \text{(scale)}.$$

Under A3 (uniform $p \sim \mathcal{U}[l,h]$), the ratio and scale conditions (OA.101)–(OA.102) become, in (r,S) coordinates,

$$G_1(r,S) = r^\delta - \frac{t_L - B}{t_R + B} \cdot \frac{1-\bar{p}}{\bar{p}} = r^\delta - \frac{\frac{S}{1+r} - B}{\frac{rS}{1+r} + B} \cdot \frac{1-\bar{p}}{\bar{p}},$$

$$G_2(r,S,u_{cn}) = \frac{1}{(1-\gamma)\bar{a}\,d} \cdot \frac{\frac{t_L}{\bar{p}} + \frac{t_R}{1-\bar{p}} + B\left(\frac{1}{\bar{p}} - \frac{1}{1-\bar{p}}\right)}{S^2} \; - \; \frac{t_L^\delta + t_R^\delta}{u_{cn}},$$

with $t_L = S/(1+r)$ and $t_R = rS/(1+r)$.

Symmetric point ($B=0$, $\bar{p}=\frac{1}{2}$, $r=1$, $t_L = t_R = S/2$). At $B=0$ and $\bar{p}=\frac{1}{2}$, the ratio equation reduces to

$$G_1(r,S) = r^\delta - \frac{1}{r},$$

so the symmetric solution is $r=1$. The scale equation becomes

$$G_2(r,S,u_{cn}) = \frac{2}{(1-\gamma)\bar{a}d} \cdot \frac{1}{S} \; - \; \frac{2(S/2)^\delta}{u_{cn}} \; = \; \frac{2}{(1-\gamma)\bar{a}d} \cdot \frac{1}{S} \; - \; \frac{2^{1-\delta}S^\delta}{u_{cn}}.$$

Jacobian and its sign. Let

$$J \; \equiv \; \begin{pmatrix} \partial G_1/\partial r & \partial G_1/\partial S \\ \partial G_2/\partial r & \partial G_2/\partial S \end{pmatrix} \quad \text{evaluated at } (r,S) = (1,S).$$

Elementary differentiation gives:

$$\left.\frac{\partial G_1}{\partial r}\right|_{r=1} = \delta\, 1^{\delta-1} + \frac{1}{1^2} = \delta + 1 \; > \; 0, \qquad \left.\frac{\partial G_1}{\partial S}\right|_{B=0,\bar{p}=1/2} = 0 \quad (G_1 \text{ independent of } S \text{ at symmetry}).$$

For G_2,

$$\left.\frac{\partial G_2}{\partial r}\right|_{B=0,\bar{p}=1/2,r=1} = 0 \quad (\text{LHS independent of } r\text{; RHS even in } r \text{ at } r=1),$$

and

$$\left.\frac{\partial G_2}{\partial S}\right| = -\frac{2}{(1-\gamma)\bar{a}d} \cdot \frac{1}{S^2} \; - \; \frac{2^{1-\delta}\delta}{u_{cn}}\, S^{\delta-1} \; < \; 0.$$

Hence the Jacobian is diagonal at symmetry with entries $(\delta+1) > 0$ and a negative number, so

$$\det(J) = (\delta+1)\left(\frac{\partial G_2}{\partial S}\right) \; < \; 0,$$

and, in particular, $\det(J) \neq 0$ (nonsingularity).

Sign of $\partial S/\partial u_{cn}$. IFT gives

$$\begin{pmatrix} \partial r/\partial u_{cn} \\ \partial S/\partial u_{cn} \end{pmatrix} = -\, J^{-1} \begin{pmatrix} \partial G_1/\partial u_{cn} \\ \partial G_2/\partial u_{cn} \end{pmatrix}.$$

Here $\partial G_1/\partial u_{cn} = 0$ and

$$\frac{\partial G_2}{\partial u_{cn}} = \frac{\partial}{\partial u_{cn}}\left[-\frac{t_L^\delta + t_R^\delta}{u_{cn}}\right] = \frac{t_L^\delta + t_R^\delta}{u_{cn}^2} \;>\; 0.$$

By Cramer's rule,

$$\frac{\partial S}{\partial u_{cn}} = -\frac{\det\begin{pmatrix}\partial G_1/\partial r & 0\\ \partial G_2/\partial r & \partial G_2/\partial u_{cn}\end{pmatrix}}{\det(J)} = -\frac{(\partial G_1/\partial r)\,(\partial G_2/\partial u_{cn})}{\det(J)}.$$

At symmetry, $(\partial G_1/\partial r) > 0$, $(\partial G_2/\partial u_{cn}) > 0$, and $\det(J) < 0$, so

$$\frac{\partial S}{\partial u_{cn}} \;>\; 0.$$

This establishes local monotonicity of S in u_{cn} and confirms nonsingularity of the system at the symmetric point.

Remark (Beta case). With a Beta density $f_{\alpha,\beta}^{(l,h)}$ (Extension II), the scale equation's LHS near symmetry is still proportional to $f(p^*)/S$ and, at $B = 0$ and $\bar{p} = 1/2$, $p^* = 1/2$ is fixed at $r = 1$. Thus $\partial G_2/\partial r = 0$ and $\partial G_2/\partial S < 0$ remain valid at symmetry. The same Jacobian calculation yields $\det(J) < 0$ and $\partial S/\partial u_{cn} > 0$ locally.

OA.5 Proposal Classification

The ISS's functional classification of proposals into 257 types is fine enough to risk obscuring the economic meaning of each proposal type. For example, ISS assigns a different proposal type for Amend Articles/Bylaws/Charter to Remove Antitakeover Provisions (S0326), Approve/Amend Terms of Existing Poison Pill(S0322),' and Submit Shareholder Rights Plan (Poison Pill) to Shareholder Vote (S0302), even though these proposals address the same economic issue—takeover defense. For this reason, it is useful to work with a coarser and more economically meaningful classification. Our classification groups corporate governance proposals into four economically relevant types. We list these types along with their frequency in our sample in Table 2. The set of types is chosen to reflect the main issues that arise in the literature on voting and corporate governance (for example, Knoeber (1986); LaPorta et al. (1998); Grullon and Michaely (2002); Gompers et al. (2003); Bebchuk et al. (2009); Becht et al. (2009); Bebchuk and Fried (2009); Ferri and Maber (2012)). Once the set of types is chosen, the proposals are classified in a straightforward way, based on their description, as illustrated in the above example. In Table OA.2, we list the top three proposals per type.

OA.6 Estimation sample

In Table OA.3 Panel A, we describe our estimation sample–excluding observations that do not satisfy A1, that is, proposals in meetings where regular (mandatory) voters' ownership is too

large for discretionary voting to be consequential. In total, we exclude 20,550 proposals in 18,170 meetings. The excluded companies are significantly smaller ($5 versus $10 b in the sample firms) and less levered (32% versus 41% in the sample firms). They have more N-PX ownership (48% versus 21% in sample firms) and more insider ownership (21% versus 8% in sample firms). The total participation is 79% in the excluded firms (versus 70% in sample firms), but voluntary participation only 14%, compared to 29% in sample firms. Among the excluded proposals, 91% are management proposals, compared to 85% among our sample proposals. In Table OA.3 Panel B, we report univariate statistics for the input parameters for our estimation.

OA.7 Model fit

We compare the precision of out-of-sample predictions of our model with that of a reduced form model with a parsimonious set of fixed effects (firm, year, and proposal type) and explanatory variables based on the literature (Table 3 of Malenko and Shen (2016)). With both the reduced form model and our model, we calculate model parameters with all years up to 2015. For proposals from 2016–2020, we then use these parameters (ie, $v/(cn)$ and the distribution parameters of p) to predict total participation, $dSuL$, $dSuR$, and the outcome index $O_{\text{disc}}(p)$ in (3), where we drop for brevity the dependence on the participation rates. We provide the estimates in Table OA.5. Our model produces smaller mean squared errors (MSE) than the reduced-form model (Panel C in Table OA.3). The difference is significant, according to a Diebold and Mariano (1995) test; for this test, we treat the predictions as a time series with the meeting order as a time stamp.

OA.8 The magnitude of the benefits of voting

The algorithm estimates the benefit-to-cost ratio per share $v/(cn)$. In this section, we set these estimates into the context of the literature.

Since we are the first to estimate the benefit-to-cost ratio, we transform it to a more comparable measure, the return on investment, conditional on a win for one's preferred side. This transformation requires assumptions about two unknowns: the number of shares n and the cost of voting c. For this section, we will vary n and assume $c = 1$; please see Section 6 for an in-depth discussion of the cost of voting. Assuming a higher cost would linearly translate into lower returns. Under this assumption, one can interpret the ratio between the benefit-to-cost ratio per share $v/(cn)$ and the average ownership stake in dollars ($) as the average return to winning.

Figure OA.6 depicts the "return on winning" as a function of the ratio of market capitalization and n, that is, the average value of shares held prior to the vote. We exclude estimations in which the benefit-to-cost ratio falls below 1 (i.e., irrelevant proposals).

The graph shows that an average ownership stake over $300,000 can yield realistic estimates of the benefit-to-cost ratios (below 20%). For an average ownership stake of $1.5 million —that is, the average holding size of insiders convicted by the SEC (Ahern (2017))— and a cost of $1, the return is 0.6%. This result compares to an average return of 1.6% for the passing of

governance shareholder proposals by Cuñat et al. (2012). Hence, as in Ahern (2017), using the average ownership stake yields estimates that are roughly a third of estimates in the literature.

OA.9 Robustness

Now we conduct several robustness tests, which are reported in Table OA.6.

OA.9.1 Alternative model with uniform idiosyncratic costs

Our baseline estimates imply a predominance of the $m0$ equilibrium, with high turnout by the underdog side (t_L) despite an expected win by R. To test the robustness of this finding, we estimate a variant of the model that introduces idiosyncratic costs of voting—what we refer to as the *uniform-cost* case—derived from Extension I in Section OA.4 (OA.4.1). In the notation of that section, this corresponds to parameters $\delta = 1$, $\kappa = 1$, $p \sim \mathcal{U}[l, h]$, and $v_R = v_L = v$. This specification allows for heterogeneous participation costs across discretionary shareholders, while preserving symmetry in valuations and aggregate uncertainty.

Because closed-form solutions for turnout rates are unavailable in this case, numerical estimation is required and introduces additional computational noise. To evaluate the model's fit, we compare its estimation errors to those from our baseline constant-cost specification.

As reported in Table OA.6, the uniform-cost variant exhibits estimation errors approximately 4% higher than in the baseline. In this setting, almost all proposals are estimated to lie in the mm equilibrium with partial participation on both sides, implying a lower overall probability of misrepresentation (11%) relative to the baseline model.

OA.9.2 Alternative Estimation Methods

In this section, we show how robust our estimations are to alternative estimation choices. For all of the following variations, MSEs are significantly smaller than the comparison model.

Availability. Our baseline estimations assume that all shareholders are available to vote ($\overline{a} = 1$). However, Holderness and Pontiff (2016) document that not all shareholders participate in valuable rights offerings, suggesting that some miss even high-impact corporate events. Assuming an $\overline{a}$ of 0.9 or 0.8 instead more than doubles the estimation errors. With a 10% (20%) of shareholders not participating, participation rates are higher on the right by 11% (15%), and participation rates for L are lower by 9% for a 0.9 $\overline{a}$ and higher by 11% for 0.8. The probability of a misrepresentative outcome is lower, with 11% (8%), and the benefit-cost ratio almost unchanged for 0.9 $\overline{a}$ and 7% for 0.8 $\overline{a}$.

Alternative bins. Our baseline estimations use bins per proposal type$\times\gamma$ tercile$\times n$ tercile$\times q$ tercile. In Table OA.6 Panel A, we report the performance and equilibrium incidence for bins using quartiles of γ, q, and n instead. The estimates are similar to the baseline ones.

Size instead of the number of distinct investors. Our baseline estimations use the number of distinct institutional investors that do not submit N-PX forms as the measure for n. However,

this number does not take non-institutional investors into account. Another proxy for n, which we use here, is the size of the firm: Brav et al. (2022) shows it correlates highly with the number of actual shareholder accounts. In Table OA.6 Panel A, we report the performance and equilibrium incidence using market capitalization for n. Using capitalization necessarily shrinks our sample because we did not require price data for any of our other estimates. This results in smaller errors and reallocates the equilibrium distribution from $m1$ to mm, reducing the participation estimates on the right. It does not, however, change the estimate for the benefit-cost ratio or the probability of an unrepresentative result much.

OA.9.3 Sample Splits

In this section, we examine our model's performance and the robustness of the estimates for subsamples in which our assumptions are less likely to hold. All results are reported in Table OA.6 Panel B.

Equity lending. In the presence of an equity lending market, discretionary voters may decide to lend their shares instead of voting, effectively increasing the opportunity costs of voting. Currently, the US equity lending market operates over-the-counter, and data are available from Markit for the period 2001-2016.[5] The Markit database covers over 90% of that market and contains firm-quarter-level information on the supply of lendable shares for most stocks listed on public exchanges. Following Campello and Saffi (2015), we define equity lending supply as the difference between the value of a firm's lendable shares and the number of lendable shares currently on loan, divided by the firm's market capitalization. This calculation precisely measures the net lendable supply. We define equity lending demand as the value of shares actually borrowed divided by the firm's market capitalization. We then compare the firms above and below the median in terms of the demand and supply of equity lending and investigate the effect on our estimation results. Consistent with equity lending affecting the cost of voting, we estimate a higher benefit-to-cost ratio (a lower cost of voting) when price of lending is low (when demand is low or when supply is high). Estimates for the probability of unrepresentative outcomes are lower than in our baseline version, especially for the subsample with high equity lending demand. Estimation errors are similar to our baseline estimations in the subsamples of high demand and low supply, but larger than in our baseline estimations in the subsamples of low demand and high supply.

Number of proposals. Since our model focuses on one proposal, results for meetings with only one nonroutine proposal should be more representative. However, estimation errors are actually higher for single-proposal meetings than for multi-proposal ones. Both our estimates for the benefit-to-cost ratio and for the probability of an unrepresentative outcome are higher for meetings with multiple proposals.

Ownership concentration. Our model assumes equal holdings by dispersed shareholders. To see how relevant this assumption is, we split the sample into terciles of the Herfindahl

[5]We are most grateful to Pedro Saffi for providing us with part of this dataset.

concentration index of institutional ownership. We obtain a higher estimation error for the tercile with the higher concentration. Our estimate for the benefit-to-cost ratio is higher for the subsample with a lower ownership concentration, the probability of an unrepresentative outcome is higher for the subsample with a higher ownership concentration.

Importance of information. An important ingredient of the voting process missing from our model is information aggregation of dispersed private information. Hence, the algorithm should be more suitable when information aggregation matters less. This should be the case for meetings with only one proposal, firms with less concentrated shareholder bases (see the previous paragraphs), or for proposals later in the proxy season, after shareholders have already observed the voting preferences in many firms and guidance on the respective proposal types. We divide the sample by the date of the meeting within each year. The estimation errors are higher for later meetings although there information should matter less. Our estimate for the benefit-to-cost ratio is higher for late meetings; the probability of an unrepresentative outcome is lower for late meetings. Equilibrium incidences are similar across the subsamples and similar to the base algorithm.

Table OA.1: **Number of observations per year.** This table shows the number of proposals and meetings per year in our sample of US firms from 2003–2020.

Panel A: Number of observations per year

Year	Proposals	Meetings
2003	224	136
2004	2196	1282
2005	2297	1378
2006	2311	1401
2007	2184	1324
2008	2248	1375
2009	2760	1622
2010	2449	1455
2011	5109	3043
2012	4758	2785
2013	5470	3111
2014	5481	3223
2015	5314	3031
2016	5386	3053
2017	5450	3117
2018	4746	2929
2019	4318	2781
2020	3599	2228

Table OA.2: **Proposal type classification.** This table lists the most frequent ISS proposal descriptions by proposal type, for the sample of US proposals 2003-2020.

Proposal type	Most frequent ISS proposal descriptions
Board	Require Independent Board Chairman, Board Diversity, Establish Committee
Executive compensation	Advisory Vote to Ratify Named Executive Officers' Compensation, Stock Retention/Holding Period, Performance-Based and/or Time-Based Equity Awards
Takeover defense	Submit or Remove Shareholder Rights Plan (Poison Pill) Requirement
Voting	Declassify the Board of Directors, Require a Majority Vote for the Election of Directors, Proxy Access
Other governance	Amend Articles/Bylaws/Charter, Reincorporate in Another State

Table OA.3: **Estimation.** This table shows statistics on input data for the estimation and model fit for a sample of proposals voted upon in US firms from 2003–2020. Panel A compares the estimation sample to excluded observations. Panel B provides the univariate statistics for the input data. Panel C reports mean squared error (MSE) of our baseline estimations and the ones in Table OA.5, their difference, the test statistic and the p-value of the Diebold-Mariano test for equality of predictive accuracy.

Panel A: Estimation sample

Variable	Sample Mean	Excluded Mean	t	p
Meeting level				
Market capitalization	10371.33	5218.45	12.37	0
Leverage	0.41	0.32	14.7	0
Market/Book	3.97	3.11	0.48	0.63
% N-PX ownership	0.21	0.48	-139.32	0
closely held	8.49	20.61	-67.72	0
Number of shares	2130150.71	1340612.92	12.07	0
Proposal level				
Participation (all)	0.7	0.79	-60.49	0
Participation (voluntary)	0.29	0.14	58.98	0
Management	0.85	0.91	-18.73	0

Panel B: Input parameters of the algorithm

Variable	Name	N	Mean	Std. Dev.	Min	Max
Firm-time specific						
Institutional and insider ownership (%)	γ	33939	0.32	0.11	0	0.93
Number of voting shares approximated by the number of accounts	n	33939	456	599	0	4513
Proposal-type specific						
Average support for the most populous choice amongst institutional and insider ownership (%)	q	33939	0.97	0.06	0.68	1
Discretionary (non-institutional, non–insider) support for the minority preference (%)	$dSuL$	33939	0.13	0.09	0.02	0.53
... for the majority preference	$dSuR$	33939	0.11	0.06	0	0.59
...for the minority preference, squared (%)	$dSuL^2$	33939	0.06	0.05	0	0.33
...for the majority preference, squared (%)	$dSuR^2$	33939	0.06	0.03	0	0.39

Panel C: Comparison between reduced-form and estimation forecasts

	Participation	$dSuL$	$dSuR$	$O_{\text{disc}}(\overline{p})$
In-sample - 2003–2015				
MSE algorithm	0.07	0.06	0.04	0.06
MSE Malenko-Shen*	0.11	0.11	0.06	0.14
Difference	-0.03	-0.05	-0.02	-0.07
S (1)	-9.69	-12.63	-8.41	-15.68
p-value	<0.0000	<0.0000	<0.0000	<0.0000
Out-of-sample - 2016–2020				
MSE baseline estimation	0.07	0.05	0.03	0.06
MSE reduced form	0.08	0.08	0.05	0.11
Difference	-0.02	-0.03	-0.02	-0.05
S (1)	-5.89	-7.47	-7.93	-9.11
p-value	<0.0000	<0.0000	<0.0000	<0.0000

Table OA.4: **International estimation sample.** This table shows a comparison between sample proposals and those excluded from the sample because they violate assumption 1 for a sample of non-US proposals from 2013–2020.

Variable	Sample Mean	Excluded Mean	t	p
Meeting level				
Market capitalization	1371.8	1345.22	2.21	0.03
N-PX	0.0108	0.0096	3.98	0
closely held	0.25	0.65	-424.21	0
Proposal level				
Participation (all)	0.35	0.46	-95.57	0
Participation (voluntary)	0.08	0.02	101.08	0
Management	0.47	0.52	-24.91	0

Table OA.5: **Comparison of Estimation Methods.** This table compares the prediction accuracy of the algorithm to reduced-form models for each proposal. For the dependent variable and the relevant sample see the table caption. Independent variables are those from Table 3 of Malenko and Shen (2016), excluding the ISS recommendation. Standard errors are in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dependent variable	Total participation	$dSuL$	$dSuR$	$O_{\text{disc}}(p)$	Total participation	$dSuL$	$dSuR$	$O_{\text{disc}}(p)$	Total participation
Sample	All	All	All	All	Until 2015	Until 2015	Until 2015	Until 2015	All
Regression	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	Tobit
MaxTSR	0.000**	0	0	0.000*	0	0	0	0	0
	(2.30485)	(-0.314976)	(-1.50538)	(1.64742)	(0.703731)	(-0.0641738)	(1.05055)	(0.937787)	(0.937787)
BelowCutoffMaxTSR	-0.001	0.001	0.002**	-0.001	0.001	0.001	-0.001	-0.001	-0.001
	(-1.17056)	(1.11692)	(2.18469)	(-1.46012)	(0.751212)	(1.57884)	(-1.08535)	(-0.758522)	(-0.758522)
Total Compensation	0	0	0	-0.000***	0	0	-0.000***	0	0
	(-0.49759)	(-1.19921)	(0.865891)	(-2.89472)	(-0.590909)	(1.34024)	(-2.60533)	(-0.384204)	(-0.384204)
TDC1 Change	0	0	0	0	0	0	0	0	0
	(-0.149919)	(-1.04552)	(-0.489482)	(-0.79465)	(-1.29036)	(-0.556337)	(-1.02338)	(-0.368289)	(-0.368289)
Stock Compensation	0.001	-0.019	-0.001	-0.018	-0.001	0.033	-0.033	-0.028	-0.028
	(0.0278033)	(-0.699882)	(-0.031913)	(-0.94289)	(-0.0176108)	(1.02828)	(-1.40162)	(-0.783872)	(-0.783872)
Director holdings	-0.005*	-0.001	0.002	-0.004**	-0.005**	-0.001	-0.004**	0	0
	(-1.81919)	(-0.476174)	(1.02766)	(-2.09961)	(-2.00133)	(-0.561483)	(-1.99099)	(0.0372347)	(0.0372347)
Log Equity	0.037**	0.012	-0.016	0.028***	0.003	-0.01	0.013	0.043*	0.043*
	(2.46139)	(0.865865)	(-1.1617)	(2.83553)	(0.130112)	(-0.491534)	(0.836798)	(1.8714)	(1.8714)
ROA	0.543***	0.495***	-0.201	0.696***	0.601***	0.061	0.540***	0.255	0.255
	(3.5315)	(3.5268)	(-1.45104)	(6.99102)	(3.70058)	(0.383812)	(4.55812)	(1.45122)	(1.45122)
Market/book	0.001**	0	-0.000**	0.001***	-0.001	-0.001	0	0.004	0.004
	(2.48003)	(0.990964)	(-2.03167)	(4.22081)	(-0.550798)	(-0.434887)	(-0.172388)	(1.43457)	(1.43457)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Proposal type FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.35	0.468	0.367	0.539	0.477	0.372	0.536	0.367	0
$\#N$	2358	2358	2358	2358	1832	1832	1832	1832	0

Table OA.6: **Robustness.** This table reports alternative estimations. In Panel A we maintain the whole sample and change some attribute of our algorithm. The first column describes whether the algorithm assumes the same cost of voting across voters (base) or cost is idiosyncratic (uniform-cost model); whether availability is 1 (base) or rather 0.9, or 0.8; whether the algorithm matches the moments of terciles (base) or the moments of quartiles, and whether it uses size terciles instead of terciles of n (base). In Panel B we maintain the baseline algorithm and look into different (sub)samples. The first column reports whether the entire sample or a subsample was used, where "High/Low forecast standard deviation" refers to the highest/lowest tercile of analyst forecast standard deviation); "Early/Late meetings" refer to meetings held before/after the median proxy meeting date; "Early/Late meetings–ISS" refer to meetings held before/after ISS has issued recommendations for the respective proposal type in both directions; and "High/low equity lending supply/demand" refers to the quarterly highest/lowest quintile in equity lending supply/demand. In both panels we report: the number of observations; the MSE; the estimates for the benefit-cost ratio, participation on L and R, and the probability of an outcome non-representative of the population preference or the preference of regular voters; and the incidence of each equilibrium type for these estimations (columns (7)–(9)).

Panel A: Whole sample

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	All	Estimates					Equilibrium (as a fraction of N)		
Algorithm	N	MSE	$v/(cn)$	t_L	t_R	p^{misrep}	Full Freeriding	Partial Freeriding	No Freeriding
Base	33939	0.67	0.37	0.58	0.03	0.16	0.83	0.17	0
Idiosyncratic cost	33936	0.69	0.16	0.6	0.12	0.11	0	1	0
$\overline{a} = 0.9$	21952	2.27	0.36	0.5	0.14	0.11	0.51	0.49	0
$\overline{a} = 0.8$	21906	2.98	0.43	0.69	0.18	0.09	0.33	0.67	0
Quartiles	20915	0.67	0.25	0.66	0.03	0.11	0.87	0.13	0
Size instead of N	22101	0.56	0.32	0.45	0.11	0.14	0.49	0.51	0

Panel B: Baseline algorithm

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Sample	N	MSE	$v/(cn)$	t_L	t_R	P^{misrep}	Full Freeriding	Partial Freeriding	No Freeriding
Low equity lending demand	8243	1	0.33	0.6	0.06	0.14	0.8	0.2	0
High equity lending demand	8538	0.66	0.32	0.61	0.01	0.09	0.88	0.12	0
Low equity lending supply	5061	0.74	0.22	0.49	0.02	0.15	0.9	0.1	0
High equity lending supply	11176	2.3	0.35	0.65	0.04	0.12	0.83	0.17	0
Single proposal	10168	0.74	0.34	0.62	0	0.12	0.91	0.09	0
Bundle	11016	0.63	0.36	0.6	0.07	0.15	0.74	0.26	0
Low ownership concentration	11145	1.02	0.35	0.63	0.04	0.14	0.85	0.15	0
High ownership concentration	7559	1.63	0.27	0.48	0.02	0.19	0.86	0.14	0
Early meeting	12817	0.72	0.27	0.61	0.04	0.14	0.86	0.14	0
Late meeting	7914	16.12	0.34	0.57	0.02	0.13	0.82	0.18	0

Figure OA.6: **Benefit-to-cost ratio vs. average ownership stake.** This figure depicts $v/(cn)$ divided by the holding size, that is, the ratio of market capitalization and n, as a function of holding size, for a sample of US proposals between 2003–2020. We exclude estimations in which the benefit-to-cost ratio is lower than 1 (i.e., irrelevant proposals).

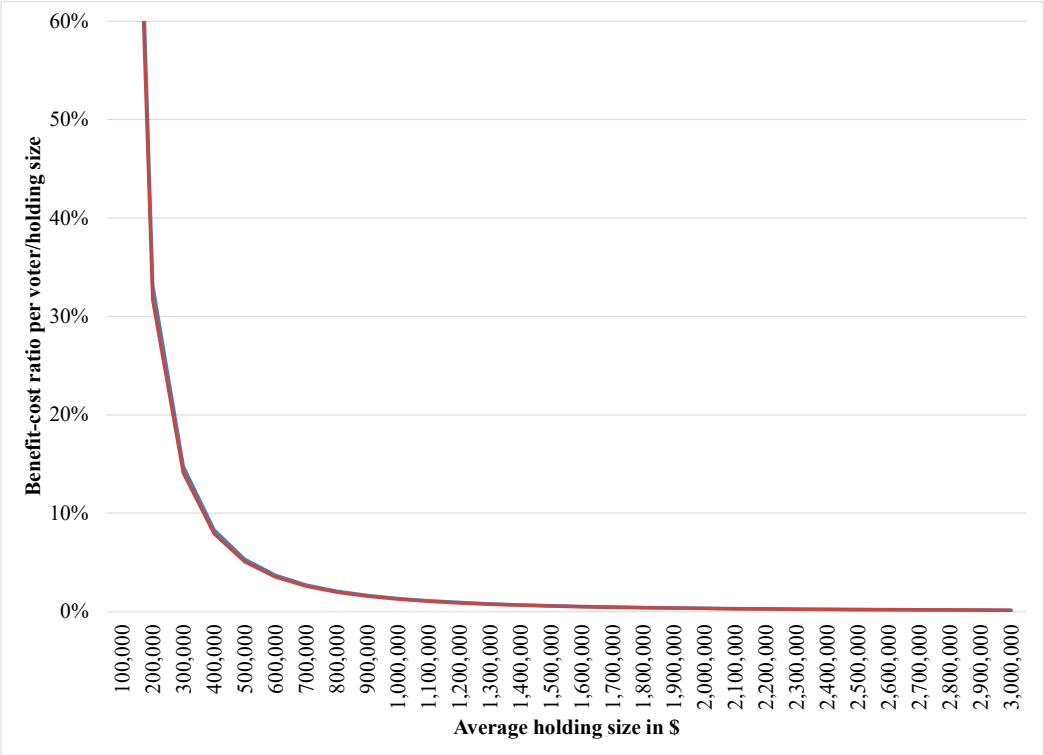
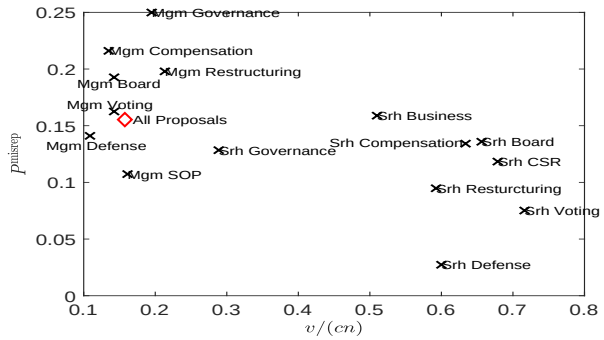
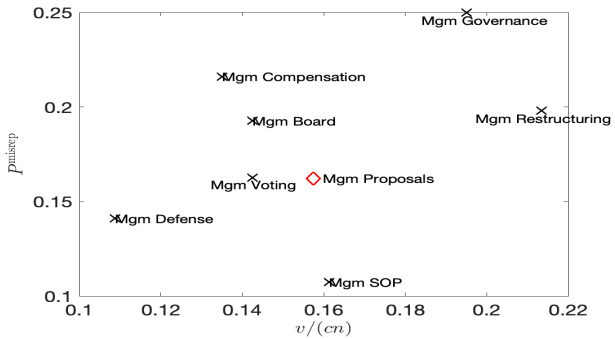


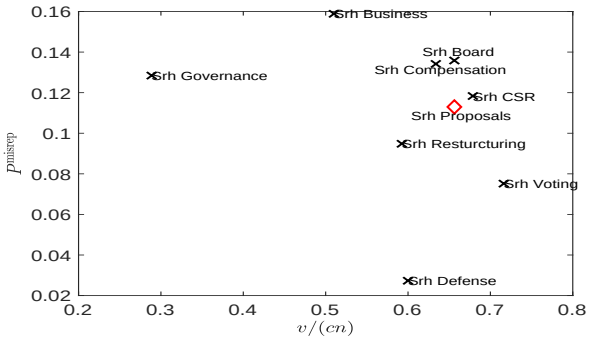
Figure OA.7: **Probability of misrepresentation $\mathbf{P^{misrep}}$ and benefit-cost ratios $\mathbf{v/(cn)}$.** These figures shows P^{misrep} as a function of $v/(cn)$, for a sample of US proposals between 2003–2020 in Panels (a)-(e) and in non-US proposals between 2013-2020 in Panel (f). In Panel (a)-(c), we report estimates by proposal type. In Panel (d)((e)), we sort the proposals in bins of q, γ, n, and year (industry) instead of proposal type for the estimation, and report the estimates by year (industry). In Panel (e), we sort the proposals in bins of γ, n, and country group, and report the estimates by country group.



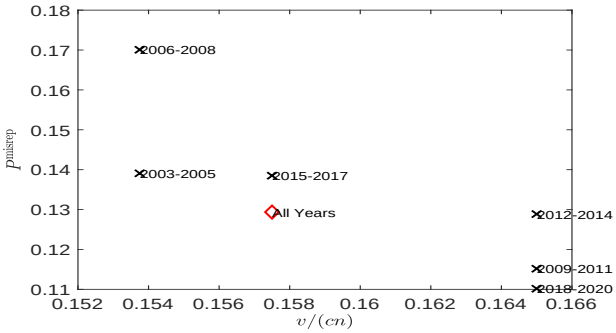
(a) By Proposal Type



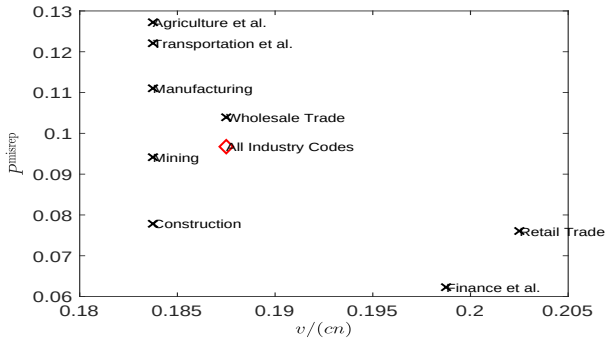
(b) By Proposal Type - Management Proposals



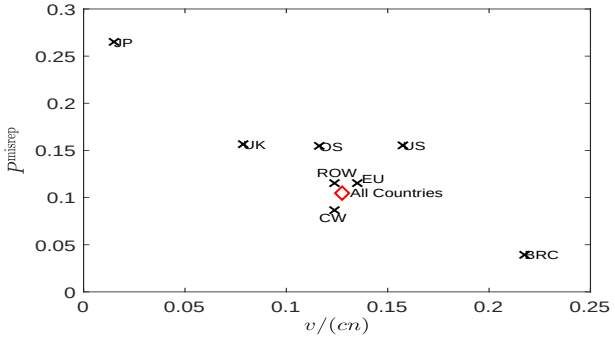
(c) By Proposal Type - Shareholder Proposals



(d) By Year

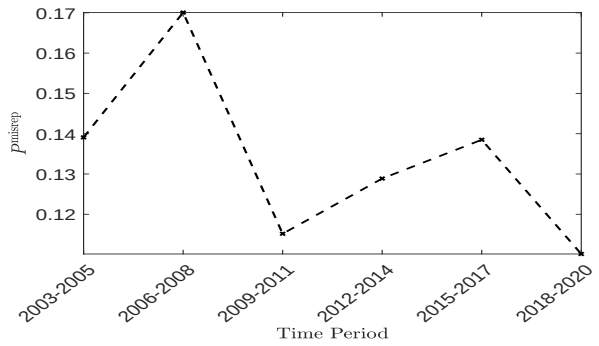


(e) By Industry

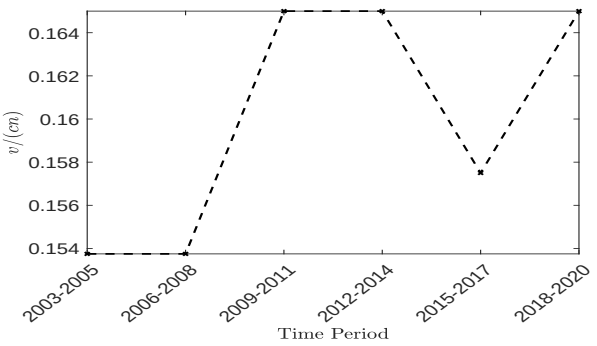


(f) By Country

Figure OA.8: **Trends in v/(cn) and $\mathbf{P^{misrep}}$.** These figures shows P^{misrep} (Panel (a)) and $v/(cn)$ (Panel (b)), for a sample of US proposals between 2003–2020. We sort the proposals in bins of q, γ, n, and year instead of proposal type for the estimation, and report the estimates by year.

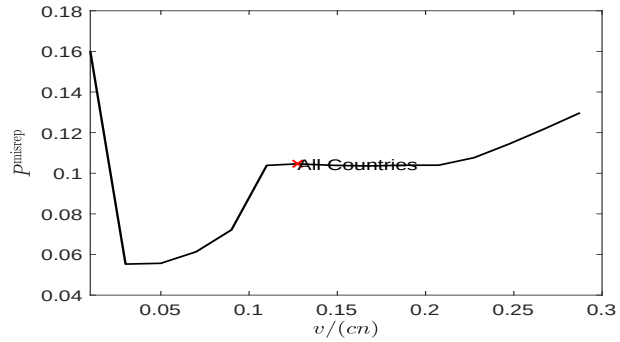


(a) Misrepresentation Probability

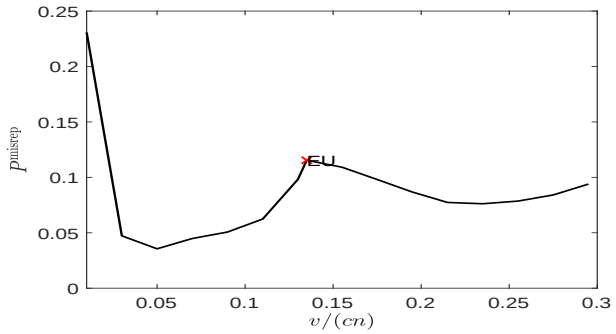


(b) Benefit-to-cost ratio per share

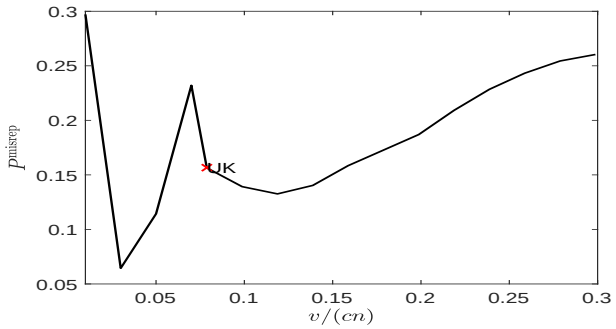
Figure OA.9: **Counterfactual probability of misrepresentation $\mathbf{P^{misrep}}$ and benefit-cost ratios v/(cn) by Country.** This figure shows the counterfactual P^{misrep} for the subsample given in the caption when we change $v/(cn)$ but hold all other parameters constant at the weighted sample median, for a sample of non-US proposals 2013–2020 in Panels (a)-(h) and US proposals 2003–2020 in Panel (i).



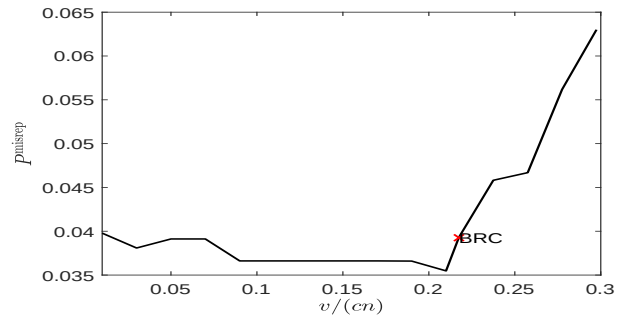
(a) All Countries



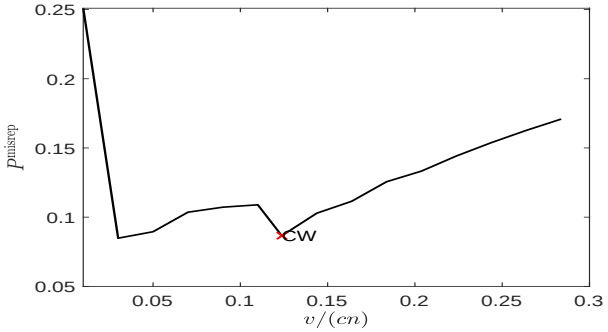
(b) Countries: European Union



(c) Countries: United Kingdom

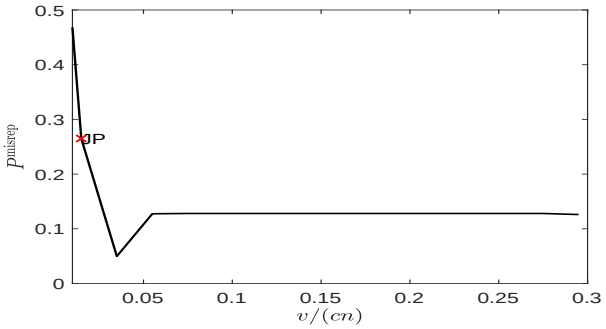


(d) Countries: Brazil, Russia, China

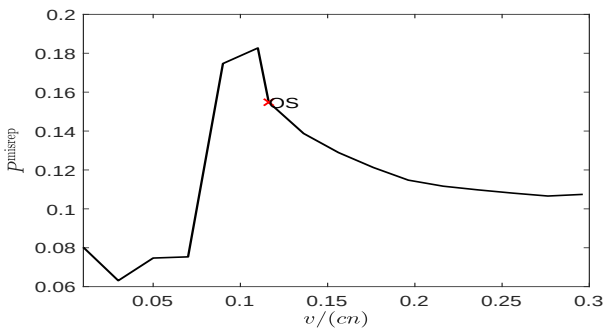


(e) Countries: Commonwealth

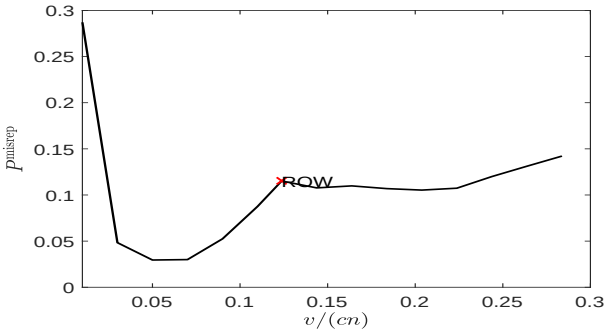
Counterfactual probability of misrepresentation P^{misrep} and benefit-cost ratios v/(cn) by Country (continued)



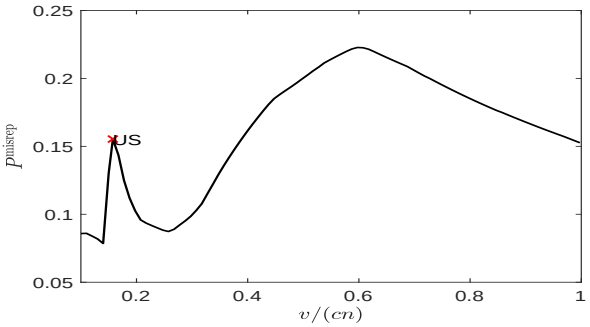
(f) Countries: Japan



(g) Countries: Offshore

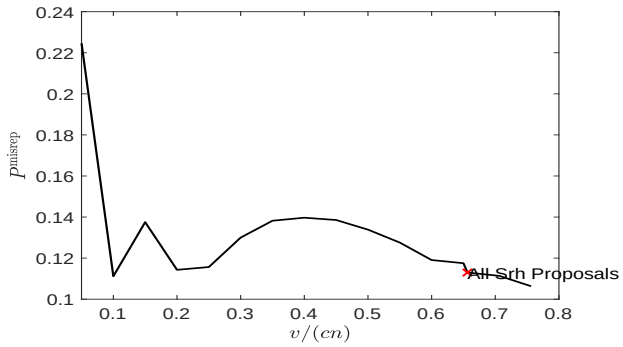


(h) Countries: Rest of World

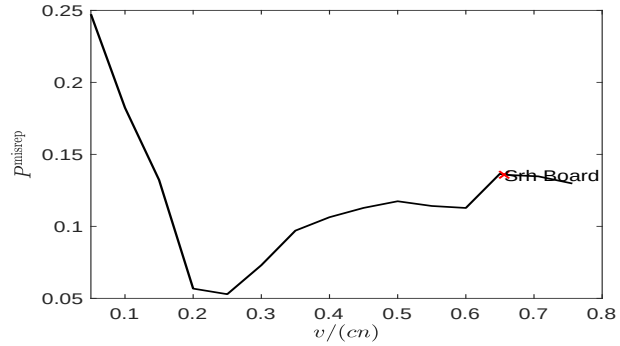


(i) Countries: United States

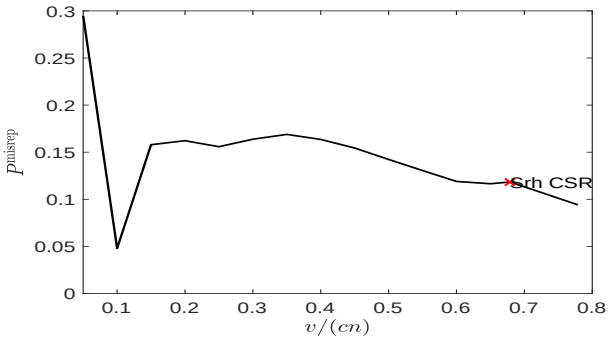
Figure OA.10: **Counterfactual probability of misrepresentation $\mathbf{P^{misrep}}$ and benefit-cost ratios $\mathbf{v/(cn)}$ by proposal type (shareholder proposals).** This figure shows the counterfactual P^{misrep} for the subsample given in the caption when we change $v/(cn)$ but hold all other parameters constant at the weighted sample median, for a sample of US shareholder proposals 2003–2020.



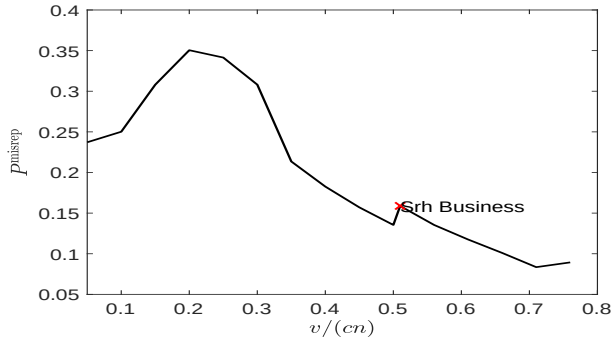
(a) All Shareholder Proposals



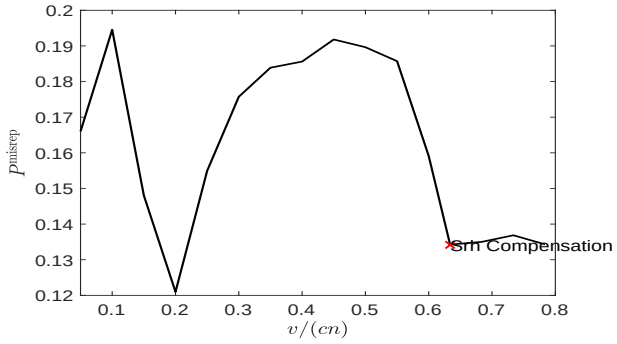
(b) Shareholder Board Proposals



(c) Shareholder CSR Proposals

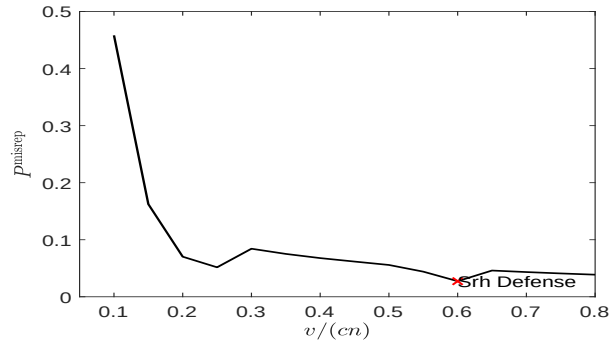


(d) Shareholder Business Proposals

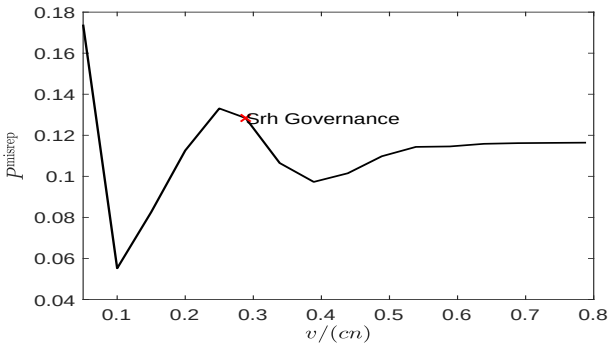


(e) Shareholder Compensation Proposals

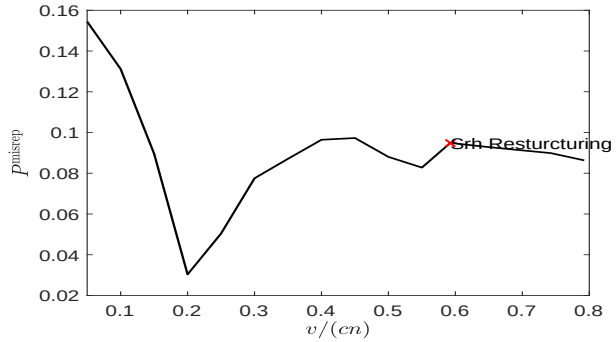
Counterfactual probability of misrepresentation P^{misrep} and benefit-cost ratios v/(cn) by proposal type (shareholder proposals) - continued



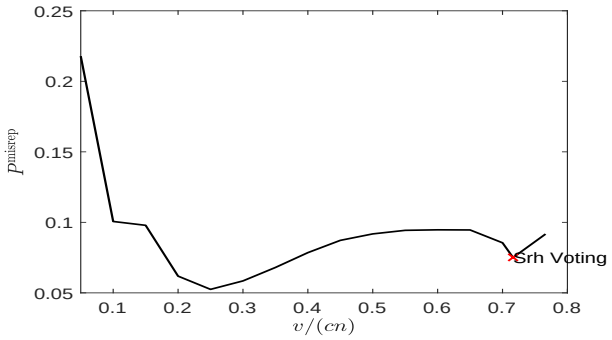
(f) Shareholder Defense Proposals



(g) Shareholder Governance Proposals

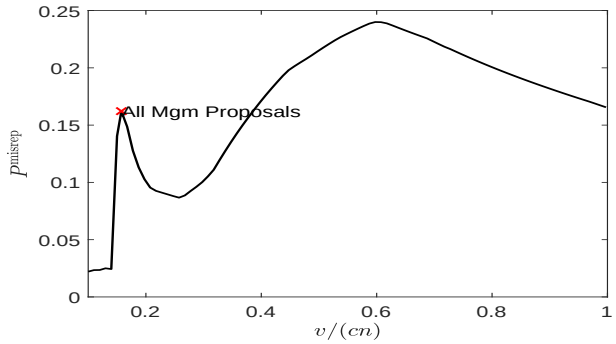


(h) Shareholder Restructuring Proposals

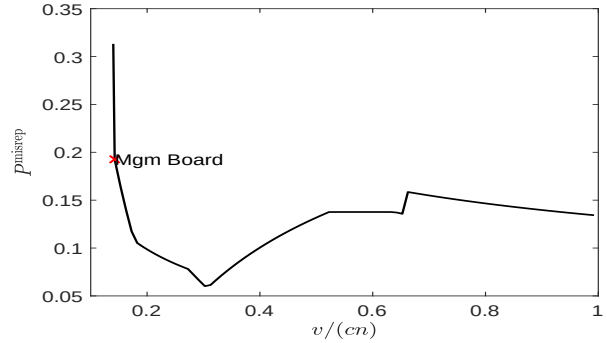


(i) Shareholder Voting Proposals

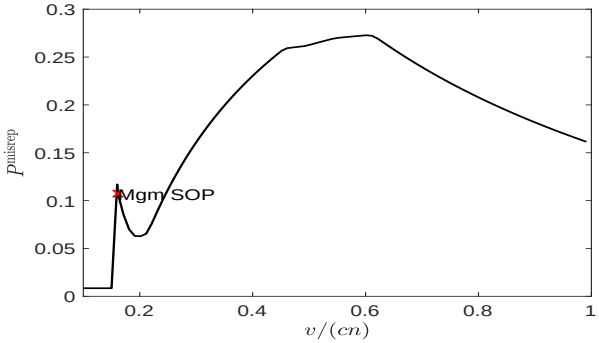
Figure OA.11: **Counterfactual probability of misrepresentation $\mathbf{P^{misrep}}$ and benefit-cost ratios v/(cn) by proposal type (management proposals).** This figure shows the counterfactual P^{misrep} for the subsample given in the caption when we change $v/(cn)$ but hold all other parameters constant at the weighted sample median, for a sample of US management proposals 2003–2020.



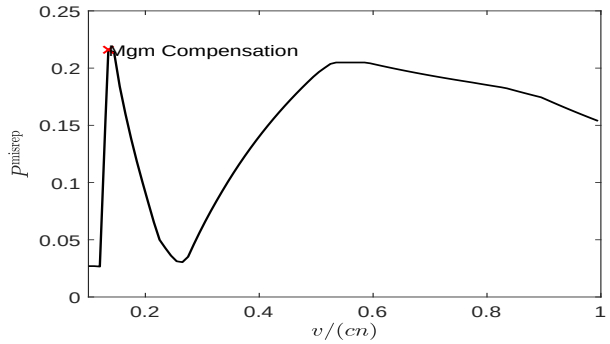
(a) All Management Proposals



(b) Management Board Proposals

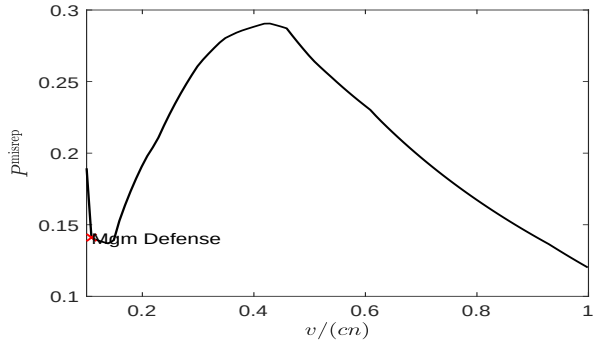


(c) Management SOP Proposals

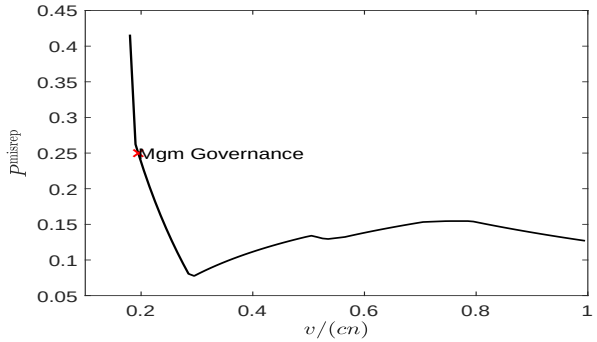


(d) Management Compensation Proposals

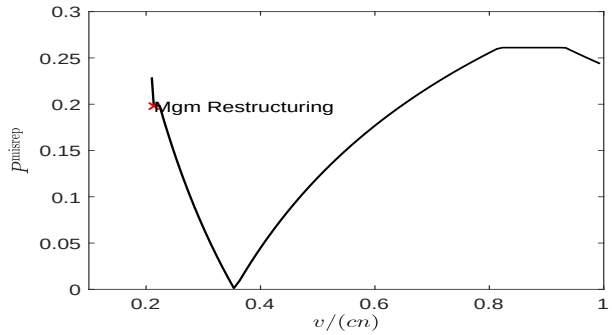
Counterfactual probability of misrepresentation P^{misrep} and benefit-cost ratios v/(cn) by proposal type (management proposals) - continued



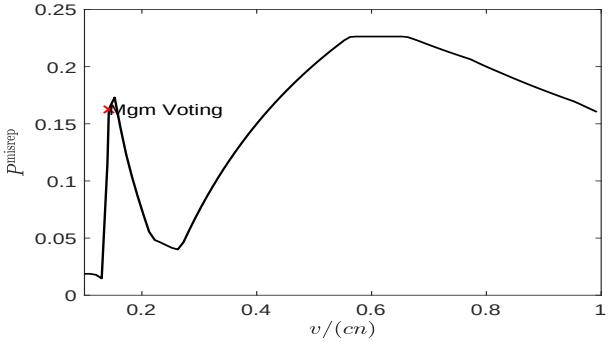
(e) Management Defense Proposals



(f) Management Governance Proposals

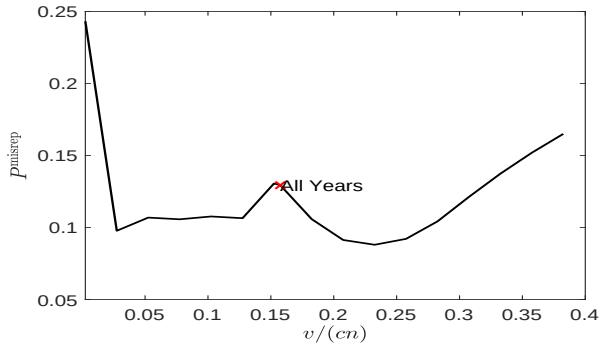


(g) Management Restructuring Proposals

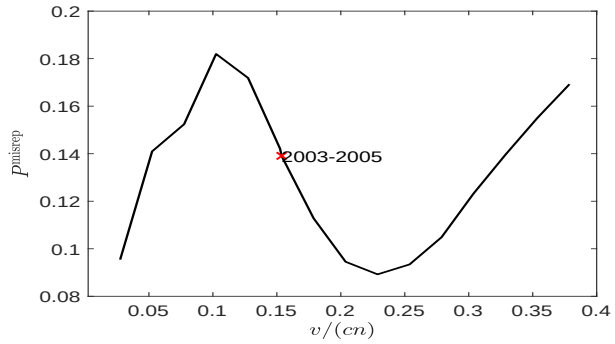


(h) Management Voting Proposals

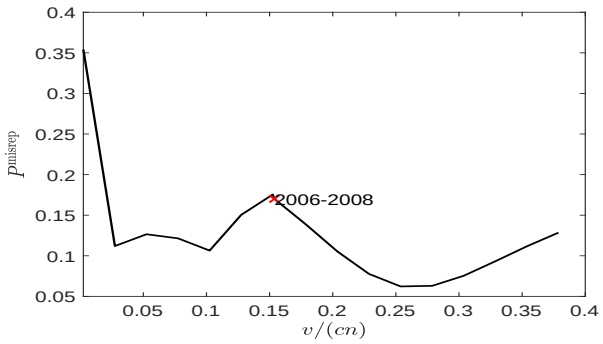
Figure OA.12: **Counterfactual probability of misrepresentation $\mathbf{P^{misrep}}$ and benefit-cost ratios $\mathbf{v/(cn)}$ by time period.** This figure shows the counterfactual P^{misrep} for the period given in the caption when we change $v/(cn)$ but hold all other parameters constant at the weighted sample median, for a sample of US shareholder proposals 2003–2020. We sort the proposals in bins of q, γ, n, and time period instead of proposal type for the estimation.



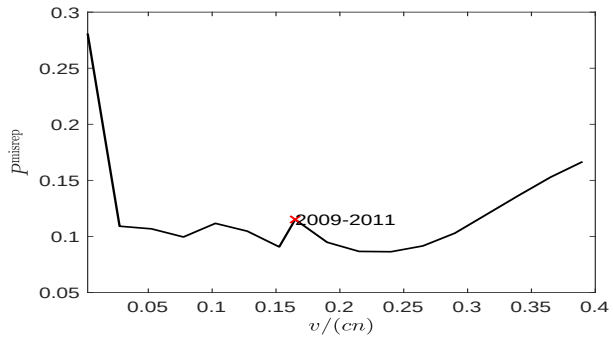
(a) All Years



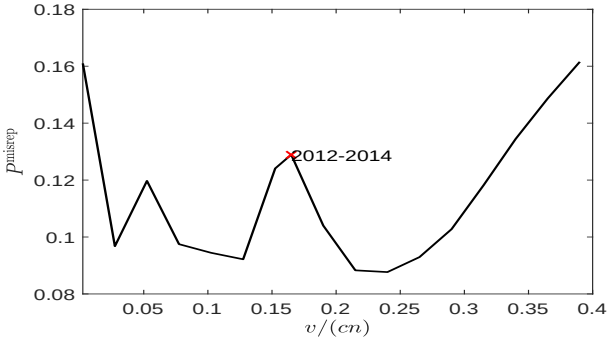
(b) Years 2003-05



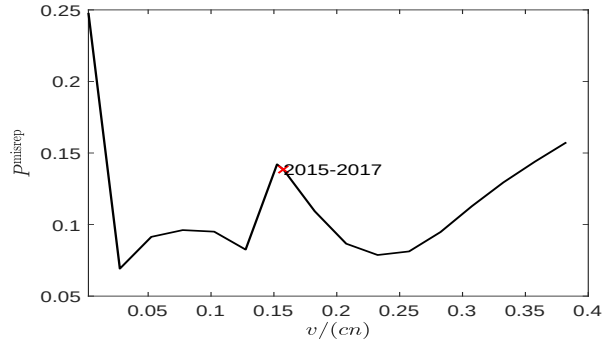
(c) Years 2006-08



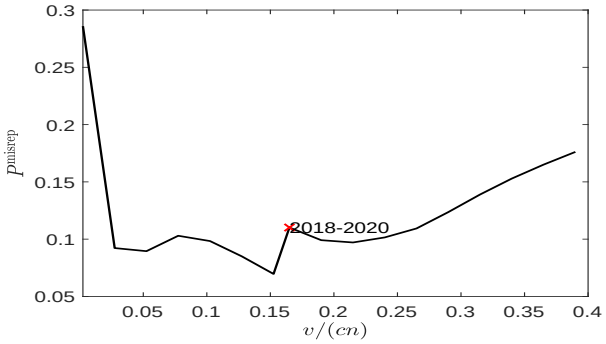
(d) Years 2009-11



(e) Years 2012-14

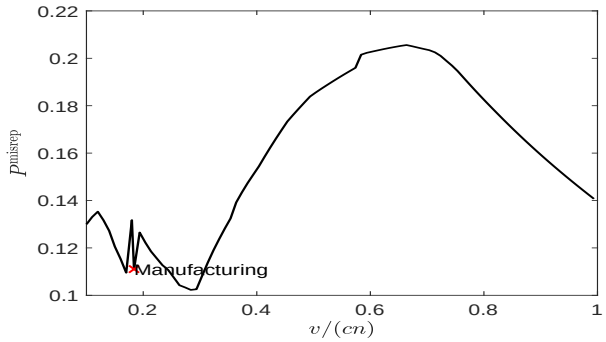


(f) Years 2015-17

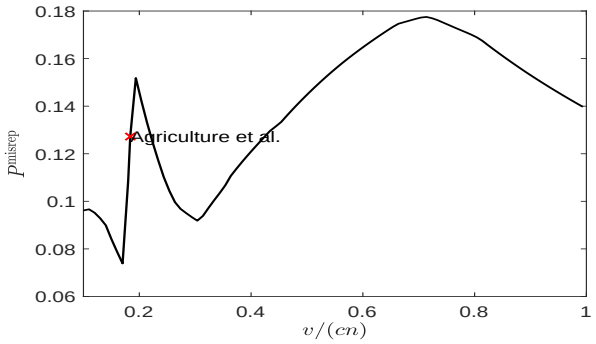


(g) Years 2018-20

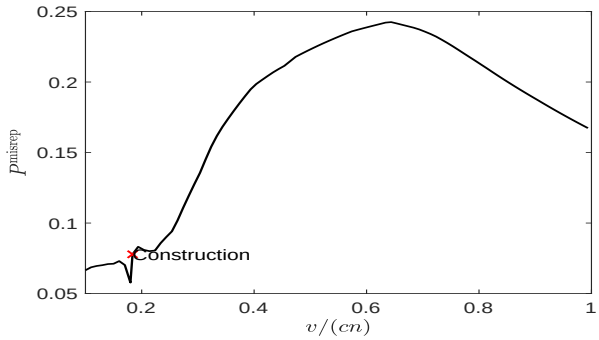
Figure OA.13: **Counterfactual probability of misrepresentation $\mathbf{P^{misrep}}$ and benefit-cost ratios $\mathbf{v/(cn)}$ by industry.** This figure shows the counterfactual P^{misrep} for the industry given in the caption when we change $v/(cn)$ but hold all other parameters constant at the weighted sample median, for a sample of US shareholder proposals 2003–2020. We sort the proposals in bins of q, γ, n, and industry instead of proposal type for the estimation.



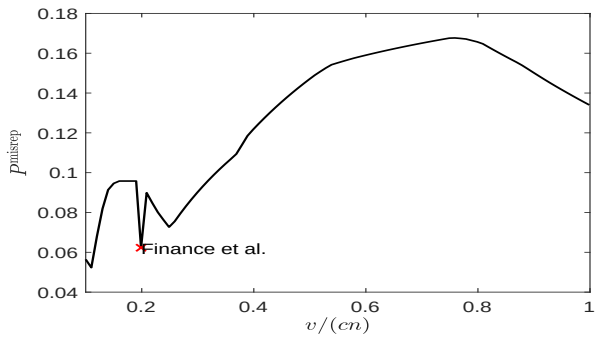
(a) Manufacturing



(b) Agriculture

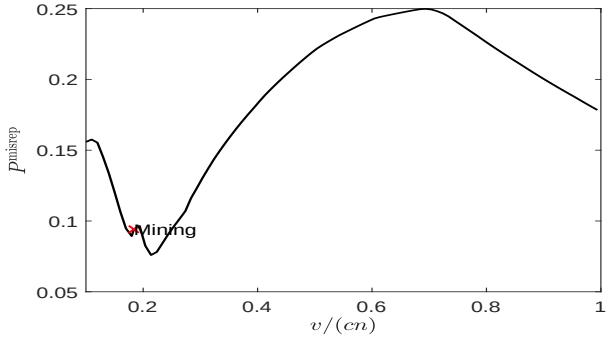


(c) Construction

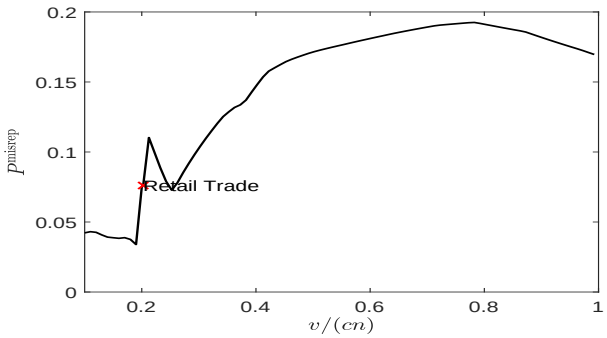


(d) Finance

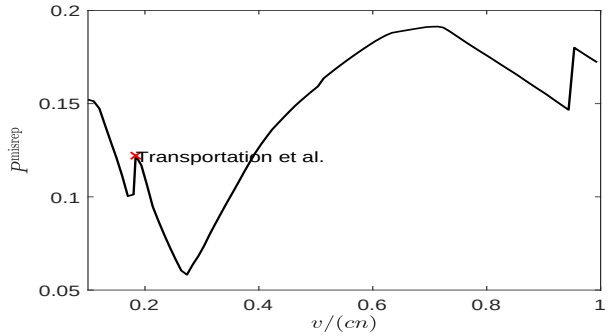
Counterfactual probability of misrepresentation P^{misrep} and benefit-cost ratios v/(cn) by industry - continued



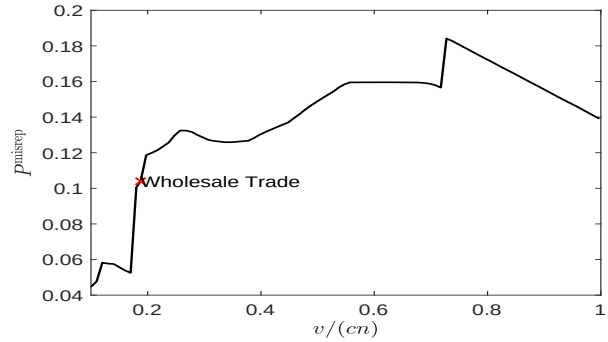
(e) Mining



(f) Retail



(g) Transportation



(h) Wholesale