# THE ROLES OF BORROWER PRIVATE INFORMATION AND MORTGAGE RELIEF DESIGN IN FORECLOSURE PREVENTION

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*Working draft, Please do not distribute*

August 26, 2025

I study frictions that prevent financial institutions from granting debt relief to struggling borrowers in the U.S. residential mortgage market. I contribute to a growing literature on selection markets in industrial organization by exploring how asymmetric information, transaction costs, and aid generosity associated with granting debt relief affect mortgage foreclosure outcomes. To disentangle these mechanisms, I introduce a structural model in which banks decide whether to offer debt relief to potentially distressed borrowers when processing relief is costly and borrowers hold private information about their financial well-being. I use my model to estimate the impact of the Federal Home Affordable Modification Program (HAMP) on mortgages originated prior to 2008. I find that HAMP incentive payments decreased foreclosures by 3%, but that information frictions increased total foreclosures by 17% relative to full information. I also find that the level of borrower relief under HAMP was insufficient for preventing 93% of foreclosures.

*Keywords:* Game theoretic modeling, asymmetric information, foreclosure, mortgage, loan forgiveness

## I. INTRODUCTION

Mortgage foreclosure has broad social costs that impact all participants in housing markets. Foreclosure involves significant financial and non-financial costs to a homeowner, and can affect the neighborhood of the property by depressing home values and increasing the probability of future foreclosures (Congressional Oversight Panel, 2009a; Campbell et al., 2011; Anenberg and Kung, 2014; Gerardi et al., 2015; Mian et al., 2015). Past work also finds that foreclosure causes housing instability, reduced home-ownership, and increased rates of depression and divorce for property owners (Tsai, 2015; Diamond et al., 2020). Debt holders typically also take significant losses when a borrower's home is repossessed and sold through foreclosure; White (2009) estimates that the average foreclosure sale in November 2008 lost a debt holder 57% of the outstanding mortgage value. In a less dire economic period before the 2008 financial crisis, Campbell et al. (2011) estimate that foreclosure sales still lost 27% of the market value of the underlying home. Despite these direct and indirect costs of foreclosure, United States loan servicers foreclosed 7.8 million homes in the decade between 2007 and 2016 (CoreLogic, 2017).

Due to the broad social costs of foreclosure and the reluctance of mortgage servicers to renegotiate with borrowers, federal and local governments in the U.S. often incentivize servicers to grant debt forgiveness to financially distressed borrowers, instead of foreclosing on their homes. Servicers may be different from the original lender, and are responsible for interacting with borrowers, revising debt terms, and initiating foreclosure proceedings. This paper focuses on debt forgiveness granted through loan modifications that reduce a borrower's monthly payments and increase liquidity, which Ganong and Noel (2020a) and Scharlemann and Shore (2022) find meaningfully reduce borrower default. This forgiveness is predominately achieved through a mixture of maturity term extensions and interest rate reductions. The logic for encouraging loan modifications is straightforward: if a servicer can reduce the cost of a mortgage sufficiently, a borrower should be able to continue making payments. Although the loan can become less valuable to the servicer through modification, if reducing the debt produces a smaller loss than the debt holder would experience through foreclosure, relief should be beneficial for all parties.[1]

I study three mechanisms that can inhibit mortgage servicers from granting debt relief to borrowers: 1) asymmetric information, 2) transaction costs, and 3) the level of relief necessary. I look at these three mechanisms jointly because their interaction determines foreclosure outcomes and the effectiveness of public policy. First, the asymmetric information challenge arises from the fact that borrowers know more about their ability to make loan payments than their servicers. For example, an individual's impending divorce might eventually drive him into bankruptcy, whereas support from wealthy relatives could help keep him current on mortgage payments. Since servicers primarily make mortgage decisions based on a limited number of standard indicators, such as a borrower's credit score, monthly income and property value, they may miss important determinants of financial health. Under this information asymmetry, servicers will also worry that a reputation for generous debt relief will lead individuals to pretend to be financially distressed in order to gain loan forgiveness. To deter this sort of strategic behavior, servicers offer less relief to borrowers relative to full information.[2]

Second, even if servicers have enough information to allocate relief correctly, the transaction costs of screening borrowers and modifying contractual terms can dampen incentives to award relief. Granting debt relief is a bespoke process that can create significant costs for servicers. While mortgage agreements award loan servicers direct reimbursement for foreclosure procedures, servicers are not always similarly compensated for awarding borrowers with debt relief intended to reduce losses (Barclays, 2008). Past research has suggested that many servicing units lacked the resources to process the inflow of borrower defaults during the 2008 financial crisis, making it prohibitively costly to process additional debt relief to borrowers (Cordell et al., 2010; Maturana, 2017). Other research has also highlighted the role of restrictive servicing

[1] I assume that the servicer's objectives are aligned with the holder of the outstanding mortgage debt and I use the term "loan servicer" in what follows. If the language is more familiar to the reader, this combined entity could also be relabeled as "creditor", "bank" or "financial institution".

[2] The information issue in this paper is primarily driven by adverse selection, whereby borrowers act on private information that is not observed by the servicer. I rule out the moral hazard problem that arises from defaulting borrowers that expect to receive financial assistance by focusing on a sample of mortgages originated prior to the 2008 financial crisis – before the crisis there was almost no precedent of debt-burden reducing loan modifications.

agreements that prevented mortgage servicers from granting debt relief through loan modifications (Piskorski et al., 2010; Agarwal et al., 2012; Kruger, 2018).

Third, the level of relief necessary to prevent a foreclosure may lead to a greater loss for the debt holder than foreclosure itself. The mortgage servicer would then prefer the loss associated with foreclosure to the loss from an overly generous loan reduction. The social costs of foreclosure will still exceed the private costs faced by the mortgage servicer, creating scope for policy intervention.

I create an empirical model to quantify how the interrelated mechanisms affect mortgage debt relief and foreclosure outcomes. My approach formalizes the intuition of Adelino et al. (2013) around how information asymmetry undermines debt relief schemes in the interaction between borrowers and their servicers.[3] I model the strategic interaction between borrowers and loan servicers as a game of debt relief that ends in either repayment or foreclosure. I allow borrowers to hold private information about both their willingness to temporarily miss debt obligations (become *delinquent* on their loan) and willingness to stop paying altogether and enter foreclosure (*default* on their loan). The timing is as follows. A loan servicer first observes borrower characteristics and the distribution of borrower private information, and then commits to a conditional probability of awarding debt relief through a loan modification. A borrower then observes his relief probability and decides whether to go delinquent on his mortgage. Borrowers that go delinquent incur a cost of delinquency, but gain the possibility of earning a loan modification that reduces their debt burden. Borrowers that choose to go delinquent then realize a modification outcome based on the servicer's policy, and decide whether to resume repaying their loan or to default and enter foreclosure. Financially distressed borrowers do not necessarily make a choice in this game, but instead always go delinquent and then default. The servicer finds it beneficial to commit to the modification probability ex-ante, because this deters strategic behavior by borrowers that do not require relief to avoid foreclosure.

I estimate my model using a sample of around 162,000 Fannie Mae-guaranteed loans originated in California between 2004 and 2007, which I track until the end of 2019. By using California, the state with the largest number of home mortgages, I can focus on the mechanisms of interest by avoiding differences in state housing regulations, foreclosure practices, and banking market structure (Ghent and Kudlyak, 2011). I also split my sample by origination cohort because of well-documented evidence of declining lending standards leading into 2008 (Bajari et al., 2008; Demyanyk and Van Hemert, 2009; Palmer, 2023). Because I can observe the final payment or foreclosure outcomes for essentially all loans in my sample, my dataset avoids the significant outcome censoring that exists in earlier papers studying the 2008 financial crisis. Another strength of this dataset is that I can observe information on all proceeds and costs associated with the foreclosure process. Improved visibility on loan outcomes is critical for studying past foreclosure: the long timelines involved with resolving borrower default meant that U.S. foreclosure rates after 2008 remained elevated well into the mid-2010s. Central to the identification of the model, my data allows me to observe servicer identities and the loan modification outcomes of borrowers. My identifying variation comes from

[3] Adelino et al. (2013) discuss the role of information asymmetry and propose a theoretical model that suggests that lenders will be less likely to modify distressed loans if the probability that the borrower will start repaying without assistance is high (*self-cure*) or the probability of repayment with assistance is low (*re-default*). This dynamic also exists in my work.

differences in the rates of loan modification across servicers that are unrelated to borrower characteristics. The mortgage literature clearly documents heterogeneity in servicer modification rates that cannot be explained by borrower characteristics (Agarwal et al., 2011, 2017; Korgaonkar, 2020). In recent work, Kim et al. (2022) similarly leverage servicer-level variation to identify the effect of mortgage payment forbearance on borrower outcomes during the COVID-19 pandemic.

Consistent with work by Diamond et al. (2020), my results suggest that historical estimates for the social costs of foreclosure are too low, while also highlighting how information asymmetry can lead to the misallocation of debt relief. I find that there are significant benefits in correctly targeted relief for both borrowers and their servicers: on average a borrower that would enter foreclosure in absence of a loan modification places about $23,000 value on foreclosure avoidance. The value of foreclosure avoidance for the borrowers most at risk of foreclosure represents a lower bound on the overall value of foreclosure avoidance in the broader borrower population. Servicers gain an average of $73,000 per loan modification that prevents a foreclosure, due to the large expected losses from a foreclosure sale. For context, a comprehensive study by the U.S. Department of Housing and Urban Development (2010) found that the social cost of foreclosure was only $51,061 and that foreclosed homeowners only bear costs of $10,300 from this process, half the amount estimated in this paper. My ability to observe realized foreclosure outcomes through the entire 2008 financial crisis likely drives this gap. Unlike past work, my model also allows me to consider how information asymmetry forces the servicer to consider the costs of incorrectly awarded debt relief. Servicers lose over $41,000 on average per borrower who could have avoided foreclosure without assistance. From a market-level perspective, I find that the social loss from failing to grant a modification that would prevent a foreclosure is 36% larger than the private loss for a loan servicer. Conversely, the social loss from incorrectly granting a modification to a borrower that did not need it is 80% smaller than the private loss faced by the servicer. The relationship between private and social losses of modification highlight how servicer incentives can prevent socially beneficial debt relief to borrowers. My results suggest that servicers prevent 76% of foreclosures that could have been stopped by granting debt relief because they cannot perfectly screen borrowers, and face losses from making mistakes.

I leverage my results to study the effectiveness of the Federal Government's Home Affordable Modification Program (HAMP), which was launched in 2009 in response to mounting U.S. foreclosures. My structural model allows me to identify the drivers for foreclosure under HAMP. I find that the HAMP incentive payments made to servicers sufficiently offset bank costs to increase relief disbursement and to decrease realized foreclosures by as much as 3%, or 200,000 when scaled to the national level, over the decade from 2007 to 2016. Despite this, information asymmetry increased total foreclosures by up to 17% or the equivalent of 1.1 million properties and $110 billion of lost value over the same time period. Finally, I find that the level of borrower relief prescribed under HAMP was insufficient for preventing as many as 93% of foreclosures, highlighting the extent of borrower distress coming out of the 2008 financial crisis.

Finally, I assess the optimality of loan modification subsides offered through the HAMP program under different assumptions of the social costs of government expenditure and foreclosure externalities. I

consistently find that some subsidy spending was worthwhile from a social welfare perspective, but that the average level of subsidy exceeds the socially optimal level for the samples of Fannie Mae mortgages studied in this paper. Nonetheless, I find that overall welfare per loan is not sensitive to the level of subsidy given to mortgage servicers and that certain borrowers stood to significantly benefit from loan modification subsidies. For the borrowers most at risk of default, larger loan subsides could have increased average welfare by upwards of 30%. Given large potential effects for this vulnerable sub-group of borrowers and relatively small effects on overall welfare, I find that loan modification policy was generally worth pursuing.

These results augment past work on debt relief policy, and specific studies of the Federal HAMP, by jointly quantifying the mechanisms that affect foreclosure prevention. My headline findings are qualitatively consistent with previous findings. Agarwal et al. (2017) leverage program eligibility cut-offs to study the causal effect of HAMP on foreclosure outcomes and find that the program prevented 600,000 foreclosures nationally, but only reached one third of the intended population of households. Hembre (2018) leverages a more structural approach and finds that HAMP prevented slightly over 500,000 foreclosures nationally but that program costs were too large to be socially beneficial. My headline numbers are not directly comparable to these two papers because of my focus on a Fannie Mae-associated, Californian sample, but I similarly find that only one third of delinquent borrowers were awarded with loan modifications, program subsidies may have exceeded social welfare maximizing levels, and that a more conservative 200,000 foreclosures nationally were prevented due to the Federal Government's incentive payments made to loan servicers.[4]

By structurally modeling information asymmetry in mortgage debt relief, I contribute to a growing literature that leverages discrete choice methods from industrial organization to study selection markets. Einav et al. (2021) provide a detailed description of the history of this literature and outline the key empirical models used in past work. As emphasized in their paper, selection markets are ones in which the identity of the consumer affects the costs faced by the supplier. One of my main contributions is to illustrate the importance of this selection in the context of mortgage foreclosure policy, where underlying borrower types will create different costs and benefits for the debt holder. The topic of my work closely relates to Hendren (2013), Hendren (2017), and Herbst and Hendren (2021), studies that demonstrate how private information can lead to market unravelling in health insurance, private provision of job loss insurance and student loan markets respectively. Information asymmetry in healthcare markets has also received significant academic attention (Cardon and Hendel, 2001; Bundorf et al., 2012; Einav et al., 2013; Cabral and Mahoney, 2019; Marone and Sabety, 2022). Within household finance markets, structural approaches have also explored asymmetric information in annuities markets (Einav et al., 2007), small business credit markets (Crawford et al., 2018), online credit markets (Xin, 2023), and credit card markets (Nelson, 2022; Matcham, 2023). I add to the literature by providing a bespoke model for studying the effects of information in the context of mortgage debt relief and foreclosure prevention. My study of how these mechanisms can deter mortgage debt relief has not been explored with past structural methods, even though this approach is fundamental

[4]My estimate of national prevented foreclosures is computed based on my California sample and then scaled to the national level. California experienced a more acute housing crash after 2008 than the national average, a crash that was also accompanied with more foreclosures.

to disentangling their effects on equilibrium outcomes. Analogous to the previous literature, my paper demonstrates the potential for an "unravelling" in mortgage debt relief, in the sense that no relief is granted if households hold a sufficient amount of private information that loan servicers cannot observe.

My findings have clear policy relevance and no previous paper, to my knowledge, formally quantifies how information, transaction costs and relief sufficiency jointly influence loan modification and foreclosure outcomes. While significant scrutiny has been correctly applied to mortgage origination practices in the lead up to the 2008 financial crisis, this paper demonstrates that the severity of U.S. foreclosures after the crisis can also be attributed to the design of debt relief programs and the behavior of financial intermediaries that allocated aid. I show that debt relief can be a powerful approach for preventing mortgage foreclosure, but that its effectiveness depends on a number of factors. Though I find that information asymmetry between borrowers and their servicers is a primary cause for inefficient allocation of relief, overcoming the information issue can be challenging. Governments can directly reduce the transaction costs faced by mortgage servicers through incentive payments or subsidies, but the the social benefit and the effect on foreclosure outcomes will depend on how well-informed servicers are about borrower default probabilities. Subsidies and financial incentives can certainly help alter servicer decisions, but without careful consideration, policymakers risk unintentionally subsidizing relief that would have occurred in the absence of incentives, or spending to the point that the cost of subsidies outweigh their benefits. Finally, policymakers designing debt relief programs must consider carefully the level of relief required to help borrowers avoid foreclosure. Insufficient relief will have no effect on foreclosure outcomes, while overly generous relief will make servicers reluctant to voluntarily participate in the program because of the losses associated with debt forgiveness.

The paper proceeds as follows. Section II. provides additional background on foreclosures in the United States and the government's response to them. Section III. presents my theoretical model of foreclosure prevention policy. Section IV. describes my data and presents descriptive evidence to support my identifying assumptions. Section V. outlines the empirical implementation of my model and the identification strategy. Section VI. presents model estimates and main results. Section VII. leverages the results to evaluate foreclosures outcomes and social welfare under alternative information levels and government subsidies. Finally, Section VIII. concludes and discusses future research directions.

## II. BACKGROUND

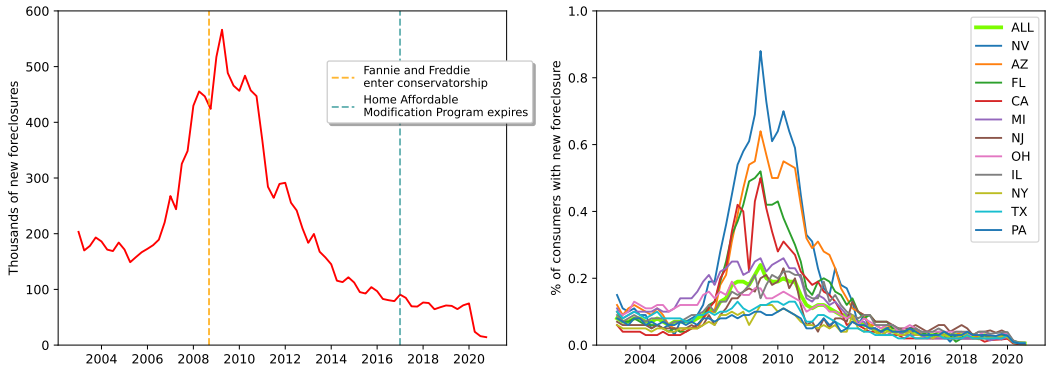
The 2008 financial crisis and associated collapse in the United States residential housing market led to historic levels of residential mortgage foreclosure that persisted half a decade after the start of the downturn. Figure I above depicts the trends in national foreclosures (left) and the share of consumers entering foreclosure by state (right) between 2003 and the start of 2020. At its worst, the number of new quarterly foreclosures in the U.S. rose from around 170,000 in the fourth quarter of 2005 to a peak of 570,000 in the third quarter of 2009, a more than three-fold increase. The foreclosure crisis was particularly acute in a handful of states. Arizona, California, Florida and Nevada each experienced rates of quarterly foreclosure

more than double the national average.

FIGURE I
TRENDS IN UNITED STATES FORECLOSURES

FIGURE I
New quarterly foreclosures

FIGURE I
% of consumers with new foreclosures, by state




*Notes:* Data from New York Fed Consumer Credit Panel/Equifax

Economic distress and the dramatic rise in mortgage foreclosures quickly attracted policy interest, due to the risk of further market deterioration and the housing market's important role in financial markets. The U.S. Treasury publicly announced that Fannie Mae and Freddie Mac were going to be taken into Federal conservatorship on September 7, 2008 due to mounting financial losses. The following month, Congress authorized $700 billion for the Troubled Asset Relief Program ("TARP") which established several wide-reaching initiatives relating to the U.S. financial system, economic growth and the housing market. As a part of TARP, the Federal Government launched the Making Home Affordable program in February 2009, with the explicit goal of assisting struggling homeowners and preventing foreclosure. Of these homeowner-facing initiatives, the single largest program was the Home Affordable Modification Program (HAMP) which was allocated $75 billion. The objective of HAMP was to encourage financial institutions to grant loan modifications to help reduce the debt burden for struggling borrowers.

Loan modification became the government's primary method of addressing the dramatic rise in residential foreclosures after 2008. Prior to the economic downturn, private loan modification was uncommon and lacked standardization across financial institutions. Bank modifications would frequently take the form of a "capitalization" where borrowers were brought current on their loans by simply adding missed payments to a borrower's outstanding mortgage balance, increasing monthly payments for borrowers already struggling to repay debt (Agarwal et al., 2017). The Government introduced loan modification guidelines and provided participating mortgage institutions incentives to grant modifications. Federal guidelines established eligibility criteria and income targeting rules for modification that would reduce an eligible borrower's combined housing expenses to a set share of their income, to improve affordability for borrowers.[5] HAMP

[5]The precise target share varied by Federal program. Under HAMP, the target income share was 38% whereas the Federal Deposit Insurance Corporation's loan modification formula used a 31% target income share.

loan modifications were only available for mortgages that were originated on or before January 1, 2009.

Incentive compensation under HAMP featured a mix of upfront and on-going payments to servicers and debt holders. Servicers were eligible to receive a one-off payment up to $1,500 for each completed loan modification. After modification, servicers were also eligible for three to five annual "pay for success" payments which were equal to the lesser of $1,000 or one-half of the reduction in the borrower's annualized monthly payment. Servicers became ineligible for the on-going payments if a modified loan ceased to be in good standing (Supplemental Directive 09-01, 2009).

Table I presents an example loan modification taken from my dataset, with specific numbers rounded for exposition. The three primary levers by which a lender can modify a mortgage loan are the 1) annual interest rate, 2) the term of loan, and 3) the outstanding principal balance. Monthly payments can be reduced using any combination of reducing the annual interest rate, extending the term of the loan or forgiving outstanding principal balance. The most common form of loan modification following the 2008 financial crisis involved the combination of a reduced interest rate and an extended term length, leading to potentially large decreases in the monthly payments of borrowers.[6] In the example, the monthly loan payment is reduced by $858 through a combination of cutting the annual interest rate in half and increasing the term length by 30 years. This decrease in monthly payments translates into a meaningful 33% loss on the mortgage value for the creditor relative to full repayment without a modification. Though this sort of modification presents a meaningful loss for the creditor, the relevant comparison was typically an even greater loss of 40-50% of the mortgage balance through foreclosure.

Loan modification is mechanically similar to refinancing, but the two mortgage changes differ in important ways. Delinquent borrowers are usually ineligible for refinancing because being current on one's existing mortgage is a common prerequisite. Modification terms also tend to be more generous and involve smaller transaction costs than a refinance, because they are used as a last resort to prevent foreclosure. Banks typically won't agree to a loan modification unless a borrower is at serious risk of foreclosure (Akin, 2023).

To protect debt-holder interests, given the losses associated with modification, government guidelines required mortgage servicers to conduct net present value (NPV) tests to assess whether granting a loan modification would reduce expected payouts (Congressional Oversight Panel, 2009b). Servicers were required to grant loan modification to borrowers whenever it yielded a positive net present value.

TABLE I
EXAMPLE OF A LOAN MODIFICATION

| | Annual interest | Term | Principal* | Monthly payments | NPV for creditor with full repayment** |
|---|---|---|---|---|---|
| Pre-modification | 6% | 10 years | $116k | $1,288 | $116k |
| Post-modification | 3% | 40 years | $120k | $430 | $78$k$ |
| | | | | *-$858 per month* | *33% loss of value to creditor* |

*Notes:* * Past missed payments generally added back to principal once a loan has been modified.
** It is standard to calculate lender losses by discounting the original interest rate. For an example, see Maturana (2017).

[6] In practice, lenders are hesitant to write-down principal so most modifications involved adjusting only the interest rate and term of the loan.

In general, borrowers needed to demonstrate financial hardship and a risk of "imminent default" to qualify for a loan modification.[7] Figure X in the Appendix displays the primary table from Fannie Mae and Freddie Mac's current Mortgage Assistance Application form. The table demonstrates the wide variety of potentially viable arguments for financial hardship and even leaves room for borrowers to claim an alternative type of hardship not listed.

Uncertainty for borrowers and mortgage servicers arises from vagueness in the definition of financial hardship and imminent default, differences in hardship documentation requirements and the borrower's ability support their claim of hardship with evidence. Borrowers were often uncertain about their eligibility for loan modification and struggled with the processes of earning a loan modification, even when they were eligible. Despite standardized modification criteria and program participation by major mortgage institutions, loan servicers also differed widely in their willingness to grant loan modifications and their propensity to process foreclosures, suggesting that they had some discretion over decision making. In my model I assume that borrowers faced randomness in their likelihood of earning a loan modification.

Servicers faced uncertainty about the degree and accuracy of borrower financial distress. While the majority of borrowers were severely impacted by the rapid economic downturn, the uncertainty created room for strategic behavior from more financially stable borrowers. In a statement to the U.S. House of Representatives, the Executive Vice President of Wells Fargo's loan servicing division emphasized this issue by stating the importance of: *[...] striking the delicate balance between providing aggressive solutions for those in need and guarding against moral hazard*.[8] Policy documents also explicitly forbid federal program participants from "intentionally" defaulting on their outstanding debt, but it is unclear how feasible it was to enforce such rules against individual borrowers.[9] Past academic work has also provided evidence for strategic borrower behavior in the residential mortgage market (Guiso et al., 2013; Mayer et al., 2014; Gerardi et al., 2018).

Despite an acute foreclosure crisis and mixed results from loan modification in the aftermath of the 2008 financial crisis, loan modification continues as a policy tool for foreclosure prevention today. After the Federal HAMP expired in 2016, Fannie Mae and Freddie Mac continued to offer loan modifications through their "Flex Modification" programs. Job losses associated with the COVID-19 pandemic created concern about a new wave of foreclosures and renewed interest in loan modification programs. On July 23rd, 2021 the Biden administration announced a federal initiative that would again help make mortgage debt more affordable by reducing interest rates and extending mortgage terms (Ackerman, 2021). Given the ongoing relevance of loan modification policies, it is crucial to understand what can be improved from the past decade of foreclosure prevention efforts.

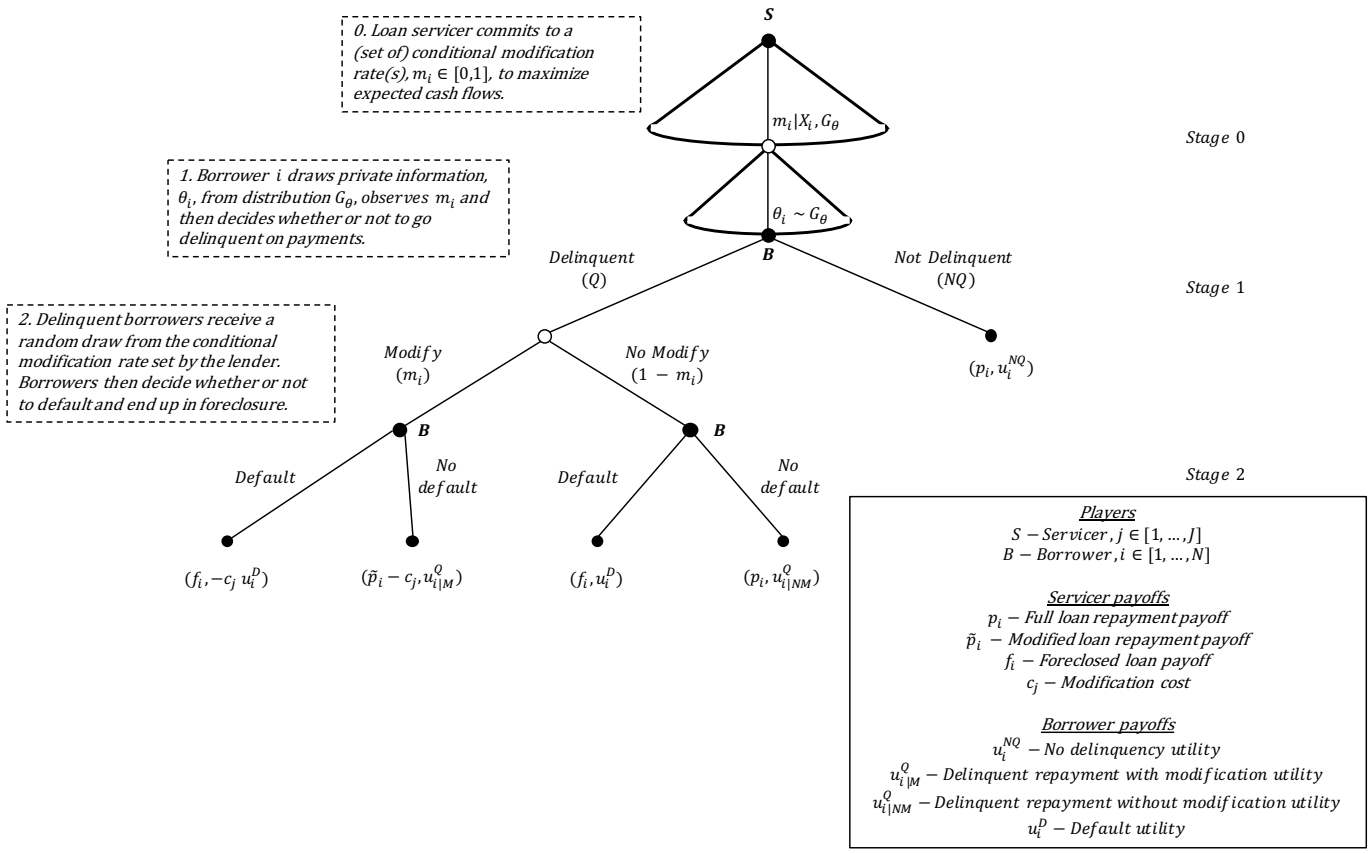
[7] For a detailed description of criteria under HAMP and the FDIC consider Agarwal et al. (2017) or Mulligan (2010).

[8] Hearing before the subcommittee on Housing and Community Opportunity (February 2009), *Loan Modifications: Are Mortgage Servicers Assisting Borrowers with Unaffordable Mortgages?* Serial No. 111-6. p30.

[9] The Housing and Economic Recovery Act of 2008, Title IV-HOPE for Homeowners.

# III. EMPIRICAL MODEL

FIGURE II
LOAN MODIFICATION GAME TREE




I model the interaction between the servicer and a given borrower as a game of incomplete information with commitment using three decision stages. Figure II depicts the overall structure of this game. The timing is as follows:

**Stage 0: Servicer sets modification policy –** The game begins with the servicer, $S$, committing to a set of conditional probabilities of modification for each pre-existing loan $i \in [1, \ldots, N]$ that it faces. This probability, $m_i \in [0, 1]$, maximizes the expected future cash flows from loan $i$. These modification probabilities depend on a given borrower's observable characteristics, $X_i$, the amount of debt owed to the bank, $p_i$, debt owed after a loan modification, $\tilde{p}_i$, payoffs from foreclosure $f_i$, a joint distribution of unobservable characteristics, $G_\theta$, and the servicer's cost of processing a modification, $c_j$. Borrowers are only eligible for modification after entering delinquency, so the servicer internalizes the fact that it will only be facing borrowers willing to miss payments. Since $m_i$ reflects the probability that the borrower will receive a loan modification, if $m_i = 0$ then the borrower will never receive a modification and if $m_i = 1$ then the borrower is guaranteed one. A key underlying assumption is that $S$ has a way to commit to this policy and it is worthwhile to commit to establishing a reputation with borrowers.

**Stage 1: Borrower delinquency decision –** After the modification policy has been set, a given borrower $i$ decides whether to go delinquent on loan payments after observing his $m_i$, observables, $X_i$, and set of unobservables, $\theta_i$, drawn from the joint distribution $G_\theta$. By going delinquent on payments, the borrower demonstrates financial distress and may be awarded a loan modification. By choosing not to be delinquent on payments, the borrower avoids any costs associated with late payment but also forgoes the possibility of earning a loan modification. If a borrower has chosen to go delinquent, he receives a Bernoulli draw with probability $m_i$ and either realizes a loan modification or does not. If a borrower does not go delinquent on payments then the game ends with full repayment of the outstanding debt.

**Stage 2: Borrower default decision –** If a borrower has been delinquent on his payments and has received a modification realization, he must then make a final decision about whether or not to default. Servicer and borrower payoffs will vary based on this decision. In my model the final default decision is synonymous with foreclosure and the borrower will lose his collateral/home. Associating default with foreclosure in this way is relatively uncommon in the past mortgage literature, though it is the definition used in Ghent and Kudlyak (2011).

The model set-up simplifies an inherently dynamic process of mortgage debt relief in order to gain tractability in the interaction between a mortgage servicer and his borrowers. By compressing mortgage resolution after financial distress into my three stage model of commitment, I trade-off model richness for improved tractability for estimating the role of information asymmetry in the loan servicer's policy setting decision. Avoiding a fully dynamic model comes with certain costs, for example, my model set-up cannot provide deep analysis of borrower expectations and learning, or servicer dynamic reputational considerations. Bajari et al. (2013) and Fang et al. (2016) propose dynamic, single-agent models to study borrower default decisions and to learn about borrower discounting.

My set-up is also conditional on a set of loans that have already been made out to borrowers. This means that I focus on the effect of information on mortgage debt relief given a set of loans. Focusing on originated loans rules out a moral hazard effect where the occurrence of loan modification changes the overall distribution of borrower types in the population. I further limit the potential effect of this moral hazard by focusing on loans made before 2008 in estimation given the unprecedented decline in the housing market and Federal Government aid for loan modification that followed. It is unlikely that borrowers would have been anticipating such outcomes when taking out mortgages at the start of the decade.

## *III.A Borrower behavior*

Expected borrower behavior drives the loan servicer's cash flow maximization decision, so it is helpful to formally define the borrower's utility function and work backwards through the model stages before providing further detail on the servicer's problem.

### 1. Borrower utilities

An individual borrower $i$'s payoffs will depend on player actions and the modification realization. I define borrower $i$'s utility to depend on three key objects:

1. **Value of keeping home relative to defaulting and entering foreclosure ($H_i$):** $H_i$ captures the value derived from a property, the benefit of avoiding foreclosure and the opportunity cost associated with paying off a mortgage. The term captures a variety of effects, including the value of the mortgaged property, and the transaction costs of moving to a different home. The benefit of avoiding foreclosure will play an important role because foreclosure significantly damages a borrower's credit score and ability to engage with financial markets in the future. Opportunity costs of remaining in the property will also be picked up by this term: for a borrower facing a decision between affording food and making mortgage payments, $H_i$ will be low, all else equal. Alternatively, limited outside options for accommodations will drive $H_i$ up for a given borrower.

2. **Delinquency cost of temporarily missing a mortgage payment ($Q_i$):** $Q_i$ represents the collective costs of temporary payment delinquency, which include a decline in a borrower's credit score, late interest fees, and the potential stress involved with interacting with a debt collector. This term will also capture the opportunity cost of on-time payment: a borrower facing a liquidity constraint will find it costly to remain current on payments, leading to a lower $Q_i$. The cost can also capture a borrower's potentially naive perception of delinquency costs.

3. **Payment disutility on the mortgage debt (unmodified $p_i$ or modified $\tilde{p}_i$):** The payment disutility term measures the discounted stream of payments that need to be made on a mortgage to pay off outstanding debt. I denote unmodified payment disutility as $p_i$ and modified payment disutility as $\tilde{p}_i$. It will always be the case that $p_i > \tilde{p}_i$. The payment disutility decreases the net utility value of remaining in a home since a loan balance must be paid off to retain the mortgaged property.

I allow both the $H_i$ and $Q_i$ terms to have heterogeneity across borrowers and to contain both observable and unobservable components, reflecting the fact that the borrower may hold some private information about these two objects. In addition to this, I allow the loan servicer to hold information about a borrower's probability of defaulting that is unavailable to the econometrician. Allowing the servicer to have additional information about the borrower reflects the fact that financial institutions request additional information from borrowers as a part of the loan modification process that is not observable in my dataset. The assumption of servicers with unobservable information about the borrower introduces an endogeneity problem that I need to address as a part of my identification argument. With a better-informed servicer, loans that receive modification will be unobservably different from loans that do not. Ignoring this aspect risks overstating the effectiveness of modifications in foreclosure prevention.

I specify the form of the borrower utility function for all potential outcomes of the game below.

**Repay without delinquency**

$$u_i^{NQ} = \underbrace{H_i(x_i'\beta, \xi_i, \varepsilon_i)}_{\text{Home utility relative to default } (H_i)} - \underbrace{p_i}_{\text{Payment disutility}}$$

**Repay with delinquency but without modification**

$$u_i^{Q}|NM = H_i(x_i'\beta, \xi_i, \varepsilon_i) - p_i - \underbrace{Q_i(w_i'\lambda, \eta_i)}_{\text{Delinquency cost } (Q_i)}$$

**Repay with delinquency and modification**

$$u_i^{Q}|M = H_i(x_i'\beta, \xi_i, \varepsilon_i) - \underbrace{\tilde{p}_i}_{\text{Modification disutility}} - Q_i(w_i'\lambda, \eta_i)$$

**Default with or without modification**

$$u_i^{D} = 0 - 0 - Q_i(w_i'\lambda, \eta_i)$$

■ Unknown to econometrician ■ Unknown to econometrician *and* loan servicer

- $x_i$ is the set of observable characteristics that explains the mean value of keeping a home relative to defaulting. It contains characteristics of the borrower and the property, such as the value of the home, its geographic location and the income of the borrower. $\beta$ is a vector of parameters for these observables.

- $w_i$ is the set of observable characteristics that explains the mean delinquency cost. It is allowed to contain a different set of observables than $x_i$, including observables such as a borrower's credit score. $\lambda$ is a vector of parameters for these observables.

- $\xi_i$ explains the component of the variance of $H_i$ that *is observed by the servicer* but is not known to the econometrician. This term is incorporated into the utility specification to reflect the fact that the loan servicer possesses additional information that a researcher doesn't have access to in conventional loan servicing data. $\xi_i$ includes any information that a borrower would report to his servicer in order to be eligible for a loan modification. Examples of this sort of information include job losses, recent divorces and occurrences of major injury or death.

- $\varepsilon_i$ is the private information of the borrower about $H_i$ that further explains the variance of value of keeping the home. $\varepsilon_i$ captures information that servicers cannot learn through typical means, including a borrower's emotional ties to a particular neighborhood or community, ability to access credit through informal markets and outside options for housing if they cannot remain at their property. The distribution of $\varepsilon_i$ is known to the loan servicer.

- $\eta_i$ is the private information of the borrower about their delinquency cost and it explains the variance in $Q_i$. Here I assume that the servicer does not observe individual heterogeneity beyond differences in $w_i$ values. The $\eta_i$ term captures factors such as an individual's financial sophistication, ability to negotiate with their bank and perception of being behind on debt.

- $\xi_i$, $\varepsilon_i$ and $\eta_i$ are drawn from a joint distribution $G_\theta$, with known parameters. These drive heterogeneity between borrowers in their values of $H_i$ and $Q_i$.
- $p_i$ and $\tilde{p}_i$ are the net present values of future mortgage payments for an unmodified and modified loan respectively. These are observable for the econometrician and market participants. The parameter on payment disutility has been normalized to 1 so that all other model parameters can be interpreted in the dollar units of mortgage payments.

The full set of model parameters $\Theta$ includes the parameters of $G_\theta$, as well as the vectors $\beta$ and $\lambda$. I discuss specific parametric assumptions in the estimation section.

### 2. Default decision

At the default stage, all borrower uncertainty is resolved and borrowers make a decision about whether to repay their mortgage balances or to default, given their modification outcomes. The borrower has already entered delinquency if he has reached this decision, so delinquency costs play no role in the choice; these costs are sunk. A delinquent borrower chooses to default whenever the disutility of payment exceeds the value of keeping the home:

- **Default if modified when:** $\tilde{p}_i > H_i(x_i'\beta, \xi, \varepsilon)$
- **Default if not modified when:** $p_i > H_i(x_i'\beta, \xi, \varepsilon)$

Whenever a borrower defaults, I assume that the modification realization has no impact on borrower payoffs: the servicer receives a foreclosure payout of $f_i$ and the borrower receives a payoff of $u_i^D$. Servicers bear a modification cost whenever they choose to modify the borrower's loan. Modification outcomes will impact all "no default" borrower payoffs. If a modification was realized and the borrower chose not to default then the servicer receives a payoff $\tilde{p}_i - c_j < p_i$ and the borrower receives a payoff of $u_i^L|M$, associated with a late payment, modification and no default. Finally if no modification was realized and the borrower chose not to default then the servicer receives a payout of $p_i$ and the borrower receives a payoff of $u_i^Q|NM$. It is the case that $u_i^Q|NM < u_i^Q|M$, meaning that a delinquent borrower at least weakly prefers to receive a modification.

### 3. Delinquency decision

At the delinquency stage, borrowers face uncertainty about whether or not they will realize a loan modification. A borrower $i$ chooses to go delinquent if the expected gain from delinquency exceeds the utility from repayment:

$$\begin{aligned} & m_i \cdot Max\{u_i^D, u_{i|M}{}^Q\} + (1-m_i)\cdot Max\{u_i^D, u_{i|NM}{}^Q\} \geq u_i^{NQ} \\ & = \underbrace{m_i \cdot Max\{p_i - H_i(\cdot), p_i - \tilde{p}_i\} + (1-m_i)\cdot Max\{p_i - H_i(\cdot), 0\}}_{\textit{Delinquency expected utility gain}} \geq \underbrace{Q_i(\cdot)}_{\textit{Delinquency cost}} \end{aligned} \tag{1}$$

The second line in equation 1 provides intuition about the borrower's delinquency decision. The left-hand side of the inequality is the expected utility gain from entering delinquency and the right-hand side is the borrower-specific delinquency cost. A borrower will go delinquent whenever the expected gain from delinquency exceeds the cost of delinquency. If a borrower chooses to default, he gets out of mortgage payment $p_i$ but suffers the loss of $H_i$ yielding the term $p_i - H_i$. Meanwhile if the borrower chooses to repay he either gets a utility gain of $p_i - \tilde{p}_i$ with a modification, or a utility gain of 0 when modification does not occur. The max operator appears in this equation because the borrower observes the modification outcome before making the final default decision.

## 4. Borrower types

I define borrower types based on their choice if they were hypothetically at the final default decision. Borrower types are continuous, but a borrower will fall into one of three distinct groups based on how modification impacts their default outcome. Neither the loan servicer nor the researcher can directly observe these types due to the unobservable components of $H_i$ and $Q_i$, but they can form probabilities for a given type of borrower given observable characteristics.

- **Low type - Always defaults**: For this type, $p_i - H_i > p_i - \tilde{p}_i$, so his delinquency decision simplifies to:

  $$p_i - H_i \geq Q_i$$

  The low-type defaults whether or not he receives a loan modification since he prefers to get out of payment disutility than to repay in all circumstances. This type will avoid delinquency (and thus default) only if the delinquency cost outweighs the gain of entering foreclosure. In practice, one would expect the magnitude of $p_i - H_i$ to greatly exceed $Q_i$. Offering modification to a low-type will not prevent a default, so a servicer would prefer not to grant a modification for this type of borrower. The always defaulting type can be interpreted as a borrower in significant financial distress or as a borrower willingly walking away from a home mortgage.

- **Medium type - Requires modification to avoid default**: For this type, $p_i - \tilde{p}_i > p_i - H_i > 0$, so his delinquency decision is given by:

  $$m_i \cdot (p_i - \tilde{p}_i) + (1 - m_i) \cdot (p_i - H_i) \geq Q_i$$

  The medium-type defaults only if he does not receive a loan modification. This is the type of borrower that the loan servicer finds optimal to grant loan modifications to in order to avoid foreclosure. This is the only borrower type that is "efficient" to modify because a modification helps prevent foreclosure.

- **High type - Never defaults**: For this type, $0 > p_i - H_i$ so his delinquency decision simplifies to:

  $$m_i \cdot (p_i - \tilde{p}_i) \geq Q_i$$

The high-type's home value never enters his delinquency decision because he will keep his home in all circumstances. This type's delinquency decision depends on whether or not the expected decrease in mortgage payments from modification outweighs the cost of delinquency. Though the high-type will not enter foreclosure, not all borrowers of this type can be interpreted as "strategic". As an example, individuals that face short-term liquidity constraints that force them into delinquency fall in this category even if they are not attempting to fake distress in order to obtain a modification. Estimates for the share of high-type borrowers entering delinquency therefore represent an upper bound on strategic behavior.

## *III.B* The servicer's problem

A loan servicer sets a modification policy, $m_i$, for all possible combinations of borrower observables, given a known distribution of unobservables. The servicer first forms the probabilities for borrower late payment and default for any given probability of modification. Using these probabilities, the servicer can calculate the expected cash flows from a given borrower $i$, conditional on $m_i$, borrower observable characteristics and model parameters.

I allow for multiple loan servicers indexed by $j \in [1,\ldots,J]$. To introduce heterogeneity across different loan servicers in their willingness to grant loan modifications, consistent with the findings in Agarwal et al. (2011), Agarwal et al. (2017) and Korgaonkar (2020), I incorporate a servicer-specific transaction cost of processing a modification, $c_j$.[10] This term reflects differences in servicing companies' ability to process modifications and screen borrowers, which can be driven by servicer specialization, staff availability, and training. Servicers have no interaction between one another and a loan $i$ is always affiliated with a specific servicer $j$.

Formally, the expected cash flow maximization problem for a servicer $j$ facing loan $i$ is given by:

$$\begin{aligned}
&\underset{m_i}{Max} \quad E[\pi_{ij}|X_i,\xi_i,p_i,m_i;\Theta] \\
&= \underset{m_i}{Max} \quad Pr(DLQ_i|X_i,\xi_i,p_i,m_i;\Theta) \times \Big[m_i(E[\pi_{ij}|X_i,\xi_i,p_i,DLQ_i,MOD_i;\Theta]-c_j) \\
&\qquad\qquad + (1-m_i)E[\pi_{ij}|X_i,\xi_i,p_i,DLQ_i,NO\ MOD_i;\Theta]\Big] \\
&\qquad\qquad + (1-Pr(DLQ_i|X_i,\xi_i,p_i,m_i;\Theta)) \quad \times \underbrace{p_i}_{E[\pi_i|NO\ DLQ_i]}
\end{aligned} \tag{2}$$

Where:

- $X_i = \{x_i, w_i\}$: The set of borrower observable characteristics that affect the value of keeping the home and the delinquency cost, respectively.
- $\xi_i$ : Borrower heterogeneity in home utility relative to default that is observable to the servicer but not

[10] Agarwal et al. (2011) finds that adding servicer fixed-effects to their baseline regressions for loan modification increases explanatory power by around 40%.

the econometrican.

- $p_i$: Discounted stream of outstanding mortgage payments. $\tilde{p}_i$ is a function of $p_i$ so does not appear in the conditioning of Equation 2. I also assume that the foreclosure payoff, $f_i$, is a function of $p_i$.
- $c_j$ : Servicer specific transaction cost of processing a modification.
- $DLQ_i, MOD_i$: Realizations of delinquency and a modification, respectively.
- $\Theta$ : Set of model parameters, which includes $\beta, \lambda$ and the parameters of the joint unobservable variable distribution, $G_\theta$.

Under this set-up I assume that the servicer knows the outcomes $\tilde{p}_i$ and $f_i$ when he is setting his modification probability $m_i$. I justify this assumption by having the loan servicer set the $m_i$ modification probability based on the mean expected losses from $\tilde{p}_i$ and $f_i$ rather than the exact value of these future outcomes for a given borrower. The interpretation is that the servicer commits to a policy based on expected losses, even if he doesn't know the exact losses for a given borrower when committing to a policy.

## *III.C* Equilibrium of the model

In equilibrium, each conditional modification policy, $m_i$ maximizes the associated servicer $j$'s expected cash flow $\forall i, j$ and each borrower, at each stage of the game, is maximizing his expected utility. Since borrower types are continuous and servicer policy is set ex-ante, a given borrower will not be indifferent in his delinquency and default decisions, except for knife-edge cases. The cost of modification term, $c_j$, also helps break indifference for the servicer in cases where modification has no effect on a borrower's outcome: the servicer will prefer not to modify if modification has no impact on borrower outcome probabilities.

Existence of equilibrium in the game of commitment is relatively straightforward. The servicer conditional modification probability $m_i$ lies on the set $[0,1]$ by definition of being a probability, so there must exist $m_i^* \in [0,1]$ such that $\pi_i$ is maximized. Once a modification policy has been set by the servicer, then borrowers make delinquency and potential default decisions conditional on this probability. The final default decision is static with full information. Borrowers know the payoffs associated with defaulting or not defaulting and servicer behavior no longer has an effect on borrower payoffs. Given a delinquency decision and a realized modification outcome, borrowers will never be indifferent between defaulting or not defaulting by the continuity of $H_i, Q_i$ and $p_i$. At the delinquency stage, a given borrower faces uncertainty over the modification outcome but will always have a unique utility maximizing option.

Uniqueness of equilibrium depends on uniqueness in the servicer's conditional expected cash flow maximization problem. Unlike the borrower decisions, uniqueness of the servicer's optimal $m_i$ does not immediately follow from continuity of borrower characteristics. For example, a servicer will be indifferent between different modification probabilities whenever the borrower's probability of entering delinquency remains near zero for all $m_i \in [0,1]$. This indifference arises from the fact that if a borrower never becomes

delinquent, then the servicer never risks incurring the cost $c_j$, associated with a modification. When delinquency does occur, the modification cost ensures that the servicer is never indifferent between modification and no modification if financial relief fails to shift default probabilities. To ensure uniqueness in the servicer's problem, and thus model equilibrium, I make the following assumption about unobservable servicer behavior:

- **Assumption E1 (Servicer tie-break rule):** Define the set of conditional modification rates $M_i$ as all $m_i \in [0,1]$ that maximize the servicer's expected cash flow problem for borrower $i$. Whenever the cardinality of the set exceeds 1, $|M_i| > 1$, then assume $m_i^* = Max(M_i)$.

Assumption $E1$ states that whenever multiple conditional modification probabilities yield identical expected cash flows for the servicer, it will set the highest rate of this set as the equilibrium policy. I make this assumption based the requirement that Fannie Mae-affiliated servicers were required to grant loan modifications whenever a modification had a positive expected net present value. This tie-break rule simply implies that the indifferent servicer sets a policy that benefits borrowers. Alternative tie-break rules yield similar parameter estimates.

## IV. DATA AND DESCRIPTIVE EVIDENCE

The primary data for my project comes from Fannie Mae's *Single-Family Loan Performance* dataset. This resource provides access to monthly loan performance information for around 45 million U.S. mortgages originated from the year 2000 up until present day. All mortgages in the sample are fully amortizing, full documentation, conventional fixed-rate loans. Focusing on conventional, full documentation loans helps avoid issues involved with misreporting and fraud that took place prior to the 2008 financial crisis (Kruger and Maturana, 2019). Fannie Mae is the single largest government-sponsored enterprise that purchases home mortgages and then securitizes them into residential mortgage-backed securities. The total value of Agency Mortgage-Backed Securities was around $4 trillion at the start of 2007; Fannie Mae loans accounted for half of this.[11]

Fannie Mae servicing data is an excellent sample for estimating my empirical model because it allows me to track individual mortgage performance from origination of a loan to its termination. For each month of a loan's existence, I can observe financial characteristics such as principal outstanding, monthly interest rate, term remaining and a borrower's payment delinquency status. When a loan receives a modification, I can observe any adjustments made to its characteristics. At the termination of the loan, I can observe whether it was paid off or foreclosed upon and the net proceeds from a foreclosure sale. The dataset also contains a rich set of borrower characteristics at loan origination, including borrower credit score, debt-to-income ratio, loan-to-value ratio, geographic location and intended property usage.

I merge regional home price indices and unemployment rates with my dataset to expand the set of observable characteristics. The house price index comes from the Federal Housing Finance Agency and the

[11] Urban Institute (2021), Housing Finance at a Glance, July 2021.

unemployment data comes from the Bureau of Labor Statistics. These two measures track changes in home values and regional unemployment over the life of a mortgage. The measure for home prices is particularly critical because it enables me to compute updated home values on a monthly basis.[12]

I make four primary restrictions when creating my estimation sample from the raw data. Firstly, I focus on loans originated in California to avoid challenges associated with cross-state differences in mortgage laws, regulations and borrower composition. Ghent and Kudlyak (2011) document important differences in state foreclosure laws and recourse policy that complicate a cross-state study of bank modification policy. National servicing organizations also behave differently given different rules across states. I chose California because it is the single largest state for Fannie Mae mortgages in my sample, and the most populous state in the United States.

Secondly, I divide the data based on the origination-year of the mortgage loans to allow model parameters to differ based on when a loan was made. Numerous papers have documented the decline in lending standards leading into the 2008 financial crisis (for example: Bajari et al. (2008) or Demyanyk and Van Hemert (2009)). In my set-up, changes in the composition of borrowers cannot simply be accounted for by controlling for origination timing in the observables: different distributions of borrowers respond differently to loan characteristics and draw their unobservables from entirely different distributions. If I were to pool all loans into a single sample, I would need to account for origination timing not only in observable borrower characteristics, but also in the unobservable distributions governing delinquency and default decisions. Separating the data by origination-year enables more flexible and straightforward estimation, while allowing me to quantify differences in the pool of borrowers based on when they received their loans.[13]

Thirdly, I restrict my sample to loans that were originated between the beginning of 2004 and the end of 2007, to avoid the period of declining mortgage interest rates between 2000 and 2004 and the effects of the 2008 financial crisis on loan payback and origination, and to focus on loans that were eligible for HAMP. The sharp decline in interest rates between 2000 and 2004 means that the majority of borrowers who bought a home in this period refinanced their fixed rate mortgages before the start of the 2008 financial crisis. Borrowers who did not refinance after such a substantial decline in interest rates are likely to be unobservably different from borrowers who did, possibly because they are financially worse off or otherwise ineligible for refinancing. I do not consider loans originated after 2007, because the ensuing economic downturn affected lending practices and borrower decisions to purchase homes. Furthermore, under HAMP, only loans that had been originated on or before January 1, 2009 were eligible. Finally, prior to 2008 there was little precedent for the net-present-value reducing loan modifications that were introduced to help prevent foreclosure.

Finally, I remove loans that were paid off prior to January 2010, which is one year after the formal roll-out of the HAMP. My three-stage model requires an assumption about which month's data should be

[12] A limitation with the Fannie Mae data is that a property's geographic region is restricted to a 3-digit ZIP code and an MSA code to protect a borrower's identity. In practice, this is similar to following county-level trends. As an example, there are 59 unique 3-digit ZIP codes in California and 58 unique counties. Figure XI in the Appendix compares the maps for California's 3-digit ZIP codes and its counties.

[13] Separate estimation samples also act as an additional robustness test for key parameters of interest.

TABLE II
SUMMARY STATISTICS FOR POOLED ESTIMATION SAMPLE

| | Full sample | Delinquent loans | Modified loans | Foreclosed loans |
|---|---|---|---|---|
| Observations | 161,974 | 37,378 | 16,846 | 20,083 |
| Share of loans entering delinquency | 23.0% | 100% | 100% | 100% |
| Share of loans receiving modification | 10.4% | 45.1% | 100% | 17.8% |
| Share of loans entering foreclosure | 12.4% | 53.73% | 21.2% | 100% |

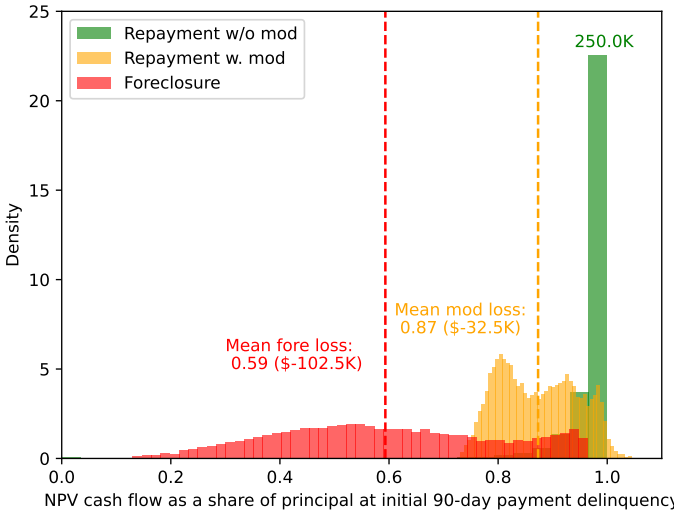
**At origination**

| | Full sample | | Delinquent loans | | Modified loans | | Foreclosed loans | |
|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. |
| Principal balance ($000s) | 243.54 | 85.48 | 262.28 | 83.05 | 276.93 | 79.74 | 255.99 | 82.2 |
| Home value ($000s) | 421.61 | 168.12 | 385.01 | 126.88 | 406.75 | 122.01 | 357.55 | 114.65 |
| Loan-to-value | 60.99 | 17.13 | 69.49 | 13.06 | 69.27 | 12.05 | 72.35 | 11.56 |
| Debt-to-income ratio | 0.39 | 0.12 | 0.42 | 0.11 | 0.43 | 0.1 | 0.42 | 0.11 |
| Credit Score | 721.22 | 56.7 | 693.58 | 54.7 | 686.62 | 53.86 | 698.2 | 54.21 |

**In January 2010**

| | Full sample | | Delinquent loans | | Modified loans | | Foreclosed loans | |
|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. |
| Principal balance ($000s) | 228.05 | 82.9 | 250.57 | 81.11 | 266.15 | 78.19 | 244.67 | 80.13 |
| Home value, ($000s) | 306.34 | 150.44 | 250.89 | 102.02 | 264.39 | 100.31 | 224.7 | 85.42 |
| Loan-to-value | 0.84 | 0.33 | 1.06 | 0.29 | 1.07 | 0.28 | 1.13 | 0.27 |
| Change in home value Jan'08 to Jan'10 | -27.0% | 8.0 | -29.0% | 8.0 | -29.0% | 8.0 | -30.0% | 7.0 |

FIGURE III
REALIZED LOAN PROCEEDS AS A SHARE OF OUTSTANDING PRINCIPAL




*Notes:* This figure depicts the ex-post realized losses that the servicer faces from modification and foreclosure compared to full repayment of mortgage principal at the time of initial 90-day delinquency. If full repayment is associated with a payback of 100% of expected future payments, then modification is on average associated with an 87% payback. The average realized payback from a foreclosure sale was only 59% of the original balance due. Mass exceeding 1.0 is driven by borrowers accruing and then paying late fees.

chosen to inform the borrower's decisions. Though my sample loans are *originated* between 2004 and 2007, loan information used before the onset of the 2008 financial crisis misses critical variation in home values, relative loan-to-value ratios and changes in national unemployment that help explain borrower decisions. I chose October 2009 to focus on loans that were exposed to the financial hardship of the financial crisis and about which decisions had to be made regarding mortgage delinquency and default, as home values declined.

My final sample looks at 161,974 loans originated in 2004, 2005, 2006 and 2007, tracking their monthly performance until the end of 2019. Table II provides summary statistics for the four years of data pooled together. In all samples, borrowers end up in all possible model outcomes, but the majority of them avoid delinquency altogether, where I define delinquency as 90 days behind on payments. Of the 23.0% of borrowers who become three-months late on their mortgage debt, a slight majority lost their home (53.73% of delinquent loans). Loans that entered delinquency and were awarded a loan modification tended to perform better than non-modified loans: only 21.2% of modified loans ended in foreclosure while 80.4% of delinquent, non-modified loans ended in foreclosure. Despite these trends, no single condition appears deterministic: loans that received modifications sometimes still "re-default" and ended in foreclosure while un-modified loans sometimes "self-cured" and started repaying without any debt forgiveness.

There are notable differences in loan outcomes across the four origination groups and pooled summary statistics conceal important trends that influence my empirical results. I report summary statistics by cohort in Table X in the Appendix. Mortgages originated closer to 2008 were more likely to enter delinquency and foreclosure than mortgages made earlier in the 2000s. For the full 2004 cohort, only 10.9% of mortgages ever reach 90-days delinquent on debt payments and 46.0% of these delinquent loans ultimately enter foreclosure. Delinquency rates climb to 20.7% for the 2005 cohort and then around 32.0% for both 2006 and 2007. Foreclosure rates climb in a similar way following the 2004 cohort: for delinquent loans these were 56.1%, 55.3% and 53.6% for 2005, 2006 and 2007 samples respectively. Difficulties with later repayment is partly explained by the fact that origination cohorts take on progressively more debt over time and have paid down less of it by the start of the financial crisis. Delinquent loans in the 2004 cohort hold a mean principal balance of $190k in January 2010, while the other three sample cohorts hold mean balances of $221k, $250k, and $260k in the same month respectively. Earlier cohorts also tended to have lower debt-to-income-ratios, lower loan-to-value ratios, and higher mean credit scores at the time of origination.

## *IV.A* Measures of $p_i$, $\tilde{p}_i$ and $f_i$

The choice of borrower-specific payment measures for unmodified and modified loans plays a central role in estimating my empirical model. The two values of $p_i$ and $\tilde{p}_i$ reflect the payment disutility in the borrowers' reduced-form utility functions. Due to a necessary scale-normalization in my discrete choice framework, all other parameters of the model can be interpreted relative to a change in the payment disutility.

Calculating the net present value of future payments for a fully amortizing, fixed rate mortgage is straightforward when information on a loan's interest rate, outstanding balance, and term remaining is available. I discount using an individual mortgage's own interest rate, consistent with approaches for mortgage loss calculations (see Hu and Cantor (2004) or Maturana (2017)). Using a mortgage's own interest rate means that the outstanding principal balance coincides with the net present value of future payments. Given this, I directly use a mortgage's outstanding principal balance on January 2010 for $p_i$.

I leverage the average realized losses from modification and foreclosure to set my $\tilde{p}_i$ and $f_i$ measures. Figure III provides information about the ex-post realized losses that the servicer faces from modification and foreclosure compared to full repayment. If full repayment is associated with a payback of 100% of expected future payments, then modification is on average associated with an 87% payback. The average realized payback from a foreclosure sale was only 59% of the original balance due, but the data suggests that there is a wide spread in how much of the outstanding debt the servicer can recoup from a foreclosed loan. My approach assumes that $\tilde{p}_i$ is deterministic based on $p_i$ and the realized mean of the modification distribution. Fannie Mae affiliated loan servicers followed prescribed rules for the size of a modification and were required to award one whenever borrowers satisfied a net-present-value criteria. These guidelines focused on specific debt-to-income shares for borrowers, limiting servicers' ability to adjust the size of modification. As shown in Figure III the distribution of repayment losses is relatively tight. Due to the greater heterogeneity in foreclosure loss, I develop a model of foreclosure loss expectations that feeds into the broader structural model.

### 1. Model of expected foreclosure losses

I incorporate loan-level heterogeneity in expected foreclosure losses to reflect the wide dispersion of realized foreclosure outcomes across mortgages. Assuming a common foreclosure loss across loans would bias parameter estimates, since variation in expected losses is a key driver of profit-maximizing modification behavior.

Whereas HAMP rules fixed the loss from modifications for fully repaying borrowers, foreclosure loss expectations depended on broader factors, including property values, loan location, and servicer-specific modeling assumptions. As part of the Making Home Affordable Base NPV Model Documentation, the estimated sale value from foreclosure after borrower default was often largely driven by the underlying property value as well as the relative value range of the property.[14]

[14]The HAMP methodology suggests that foreclosure sale losses can be larger for properties below $100,000. The specific

I approximate servicer foreclosure loss expectations by estimating a loan-level model of realized foreclosure losses from origination until July, 2010. The sample is truncated in 2010 to reflect the information available to servicers in the middle of the foreclosure crisis and to avoid overfitting to realized outcomes. Foreclosure loss expectations are computed loan by loan prior to estimation and are included as inputs to the broader empirical model. I provide further details on the loss estimation model in Appendix *IV.D*

## *IV.B* Servicer heterogeneity in loan modification and other outcomes

My sample focuses on the six largest initial loan servicers for Fannie Mae-purchased mortgages, as measured by the number of loans that they manage. In order of size, the top six servicers in California from 2004 to 2007 were: Bank of America, Citimortgage, GMAC, Wells Fargo, JP Morgan, and Suntrust. Mortgage servicing tends to be a relatively concentrated industry, with a long tail of small sub-servicing organizations.

Table III reports the share of delinquent loans that received a modification by servicer. The raw data reveal notable variation in modification rates across servicers. For example, Bank of America and Citimortgage— which serviced the two largest volumes of delinquent loans—exhibited a nearly 13 percentage point difference in the share of delinquent loans modified. Similarly, JP Morgan and Wells Fargo had comparable volumes of delinquent loans but differed in modification rates by approximately 10 percentage points. Despite these differences in delinquency volumes and modification propensities, the share of delinquent loans entering foreclosure remained within a relatively narrow range, between 24 and 29 percent across all servicers.

TABLE III
SHARE OF DELINQUENT LOANS RECEIVING A MODIFICATION OR ENDING IN FORECLOSURE, BY SERVICER IN POOLED ESTIMATION SAMPLE

| Servicer | N of Dlq. loans | Share Dlq. | Share $Mod\|Dlq.$ | Share $Fore\|Dlq.$ |
|---|---|---|---|---|
| Bank of America (BoA) | 18,601 | 25.8% | 39.7% | 27.3% |
| Citimortgage | 6,031 | 21.5% | 53.0% | 24.5% |
| GMAC | 4,540 | 19.1% | 48.0% | 29.2% |
| Wells Fargo | 3,315 | 26.2% | 54.9% | 23.9% |
| JP Morgan | 3,106 | 16.9% | 45.1% | 24.3% |
| Suntrust | 1,785 | 25.4% | 48.2% | 25.3% |

*Notes:* The table presents raw counts of delinquent loans and shares of loans receiving modifications by mortgage servicer in the pooled estimation sample. Before controlling for borrower characteristics and other potential factors affecting loan modification, there are significant differences in the rate of loan modification between servicers.

linear model outlined in their Making Home Affordable Base NPV Model Documentation is given by:

$$\text{Foreclosure sale value}_i = \beta_0 + \beta_1 \mathbb{1}\{PropValue_i <= \$50k\} + \beta_2 \mathbb{1}\{\$50k < PropValue <= \$100k\} + \beta_3 PropValue_i + \beta_4 PropValue_i \times \mathbb{1}\{\text{PropValue} <= \$50k\} + \beta_5 PropValue_i \times \mathbb{1}\{\$50k <= PropValue_i <= \$100k\} + \varepsilon_i$$

FIGURE IV
Probit model predicted probabilities, by servicer

FIGURE IV
Raw outcomes

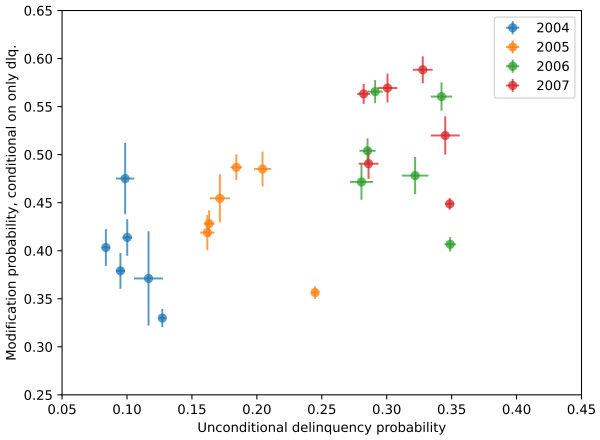
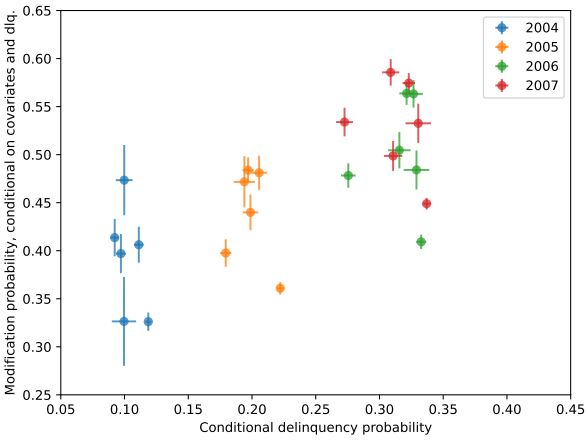



FIGURE IV
Conditional outcomes




*Notes:* This figure presents unconditional (left) and conditional (right) predicted probabilities of delinquency relative to predicted probabilities of modification across loan servicers and origination cohort samples. Points depict predicted mean probabilities for individual servicers and bars represent the standard errors on these estimates. For the model labeled 'raw', the only explanatory variable is a set of dummies for the servicer identities. The set of controls for the conditional model includes interest rate, home value, principal balance, loan-to-value ratio, debt-to-income ratio, credit score, imputed income, and dummies for the loan purpose and origination channel at the time of origination. In this same model, I also control for the percentage change in home value and local unemployment between January 2008 to January 2010. A central observation from this figure is that differences between servicer-specific delinquency rates can largely be explained by observable borrower characteristics, while observable characteristics cannot explain persistent differences in modification probabilities for delinquent loans across the same servicers.

Differences in servicer modification rates cannot be rationalized by borrower observable characteristics alone, and are likely to reflect differences in institutional willingness or ability to grant modifications. Figure IV shows predicted probabilities from simple probit models for borrower delinquency and modification conditional on delinquency; before and after controlling for observable characteristics. The set of controls for the conditional model includes interest rate, home value, principal balance, loan-to-value ratio, debt-to-income ratio, credit score, imputed income, and dummies for the loan purpose and origination channel at the time of origination. In this same model, I also control for the percentage change in home value and local unemployment between January 2008 to January 2010. Conditional on these observables, the probability of borrower 90 day delinquency is statistically similar across loan servicers within all estimation samples (though different across samples). I cannot reject the null hypothesis that approximately three to four servicers in each cohort exhibit equivalent delinquency rates after conditioning on borrower characteristics. However, despite this reduced-form evidence suggesting comparable borrower risk profiles, servicers exhibit substantial heterogeneity in their propensity to offer modifications to delinquent borrowers.

The gap in modification rates between the most and least generous servicers underscores institutional variation. These descriptive findings are consistent with Agarwal et al. (2011) and Agarwal et al. (2017). Agarwal et al. (2017) point out that modification rates "cannot be accounted for by differences in contract, borrower or regional characteristics of mortgages across servicers". In their paper, the authors

suggest that servicer-specific factors such as the quality and size of servicing staff played a role in overall modification rates. The cross-sectional heterogeneity in servicer modification rates provides the primary source of identifying variation in this paper.

# V. IDENTIFICATION AND ESTIMATION

## *V.A* Model identification and necessary assumptions

The primary challenge to identification in the model arises from the presence of the $\xi_i$ term, which drives heterogeneity in borrower home utility relative to default, and is assumed to be observed by borrowers and servicers, but not the econometrican. The presence of $\xi_i$ means that borrowers receiving loan modifications are unobservably different from those that do not receive modifications. Moreover, the separate identification of the two home utility terms, $\xi_i$ and $\varepsilon_i$, is non-trivial because they affect borrower default outcomes in an identical way. The only difference between the two unobservables is that a servicer is assumed to set its modification policy with knowledge of a particular borrower's $\xi_i$, but without knowing the private information governing home utility, given by $\varepsilon_i$. Overcoming this challenge relies on differences in servicer modification rates that are unrelated to borrower characteristics.

For cross-servicer variation to enable identification, I make two key assumptions.

**Identifying Assumption 1.** *No borrower sorting to servicers on unobservables.*

The no sorting assumption states that borrowers are not assigned to servicers on the basis of their *unobservable* characteristics. It implies that individual borrowers draw their home utility and delinquency cost heterogeneity from common distributions, irrespective of servicer affiliation.

The assumption does not rule out borrowers from sorting to servicers on the basis of *observables*. I allow certain servicers to cater to more quantifiably riskier borrowers, such as those with lower credit scores or higher interest rates. My sorting assumption leverages the fact that Fannie Mae's loan servicing data is rich at the time of loan origination, allowing me to control for almost all measures of riskiness known to the bank when the loan was first made. The interpretation of this assumption is that the distribution of idiosyncratic shocks governing borrower delinquency and default following the 2008 financial crisis was independent of the factors influencing original mortgage origination and servicer assignment.

The no-sorting assumption is required for the model to be identified. The central aspect is that multiple servicers face different modification costs, $c_j$, but a common joint distribution of unobservable characteristics, $G_\theta$. The borrower characteristics and servicers' identities will create cross-sectional differences in the share of delinquent loans receiving modification. After controlling for observable characteristics and servicer identity, the remaining differences in the share of loans receiving modifications for each servicer must be jointly rationalized by the common distribution of home utility heterogeneity known to the servicer ($\xi_i$) because servicers do not observe any other source of borrower heterogeneity. The no-sorting assumption can be relaxed by grouping servicers based on their borrower characteristics and assuming a common

distribution within these smaller sub-groups, but servicer groups must still contain multiple institutions and servicers within a group must face different costs of modification.

**Identifying Assumption 2.** *Servicer modification costs do not depend on borrower characteristics.*

The cost assumption says that the servicer's cost of modification, $c_j$, does not depend on borrower or loan characteristics. This implies that a borrower's financial health or his loan conditions do not complicate the loan modification process. Assumption 2 is less fundamental to identification than the no-sorting assumption, and can be relaxed easily by adding structure to the relationship between modification costs and borrower characteristics.

Using the assumptions above, I can identify the distributions of individual heterogeneity in home utility and delinquency costs introduced by $\xi_i, \varepsilon_i$ and $\eta_i$. I start by thinking about the servicers' modification decisions, which are driven by borrower characteristics ($x_i$ and $w_i$), costs of modification ($c_j$) and the $\xi_i$ draw. The no-sorting assumption and the fact that servicers offer different probabilities of modification due to differences in $c_j$ lead to exogenous variation in modification outcomes, which allows me to learn about the distribution of servicer observed borrower heterogeneity.

With $\sigma_\xi^2$ pinned down, identifying the remain variance terms becomes more straightforward. As stated above, only the borrower heterogeneity observed by the servicer, $\sigma_\xi^2$, can jointly rationalize observed shares of modifications across all loan servicers. The remaining differences in default outcomes for identically observable borrowers can only be explained by the borrower heterogeneity that is unknown to the servicer, $\sigma_\varepsilon^2$. The delinquency cost plays no role in the default outcomes so $\sigma_\eta^2$ cannot explain observed differences in borrower default outcomes. With the distributions of $\xi_i$ and $\varepsilon_i$ pinned down by modification and default outcomes respectively, I can look at the remaining variation in delinquency outcomes for observably identical borrowers to identify the delinquency cost heterogeneity parameter, $\sigma_\eta^2$.

## *V.B* Estimation details

This section outlines the central features of my estimation procedure. Appendix *V.B* provides additional details on the model estimation algorithm that are omitted here. I estimate my model using a simulated maximum likelihood ("SMLE") approach that jointly matches the probabilities of loan outcomes to those observed in the data. The potential outcomes for a given loan are: 1) No delinquency, 2) Delinquency, modification and foreclosure, 3) Delinquency, no modification and foreclosure, 4) Delinquency, no modification and no foreclosure, and 5) Delinquency, modification and no foreclosure.

I form the probabilities of the five potential outcomes by conditioning on observables and integrating out the unobservable variables. The probability of a given outcome $k$ for borrower $i$ is given by:

$$\Pr(y_i = k|\mathscr{X}_i;\Theta) = \int_{\varepsilon,\xi,\eta} \Pr(y_i = k|\mathscr{X}_i,\varepsilon_i,\xi_i,\eta;\Theta)dG(\varepsilon,\xi,\eta) \quad k \in \{1,\ldots,5\}$$

Where $\mathscr{X}_i$ includes all observables, $x_i, w_i, p_i$ and $\tilde{p}_i$. $\Theta$ is the full set of model parameters. $G$ is the

joint distribution of the unobservables governing home utility and delinquency cost. Taking the natural log of the combined probabilities yields the log-likelihood function:

$$\log \mathscr{L}\big(Y,X;\Theta\big) = \sum_{i=1}^{n}\sum_{k=1}^{5}\big[\mathbb{1}(y_i=k)\log\{\Pr(y_i=k|\mathscr{X}_i,\Theta)\}\big] \tag{3}$$

My SMLE estimation routine searches across the $\Theta$ space in order to maximize the log-likelihood function.

### 1. Parametric assumptions

To make estimation tractable, I make parametric assumptions on the distributions of the unobservable variables that govern home utility relative to default and delinquency cost.

**Estimation assumption 1.** *Linearity of home utility and delinquency costs.*

$$H_i(x_i'\beta,\xi_i,\varepsilon_i) = x_i'\beta+\xi_i+\varepsilon_i$$
$$Q_i(w_i'\lambda,\eta_i) = w_i'\lambda+\eta_i$$

**Estimation assumption 2.** *Parametric form of borrower unobservables.*

$$\xi_i \sim \text{Normal}(0,\sigma_\xi^2), \quad \begin{bmatrix}\varepsilon_i\\ \eta_i\end{bmatrix} \sim \text{Normal}\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}\sigma_\varepsilon^2 & \rho\sigma_\varepsilon\sigma_\eta\\ \rho\sigma_\varepsilon\sigma_\eta & \sigma_\eta^2\end{bmatrix}\right)$$

I intentionally allow all three unobservable components to take on both positive and negative values to reflect the fact that an individual will find a low relative value of repaying a loan in situations where they are financially distressed and the opportunity cost of repayment is high.

**Estimation assumption 3.** *Independence of default unobservables.* $\xi_i \perp \varepsilon_i$
I assume that the unobservable components of home value relative to default, $H_i$, are independent of one another. I prove that it is without loss of generality to assume that $\xi_i$ and $\varepsilon_i$ are uncorrelated in Appendix B.[15] The assumption of independence between $\xi_i$ and $\varepsilon_i$ is not as restrictive as an assumption of $\eta_i$ being uncorrelated with $\varepsilon_i$. I interpret $\varepsilon_i$ as the home utility residual that cannot be predicted by the servicer with knowledge about $\xi_i$. This interpretation implies that individual hetereogeneity in factors such as home-attachment, access to informal credit markets, and opportunity costs of debt payments are unrelated to information available to the mortgage servicer.

With Estimation Assumptions 1-3, I can express the distribution of the borrower's home utility and

[15] While uncorrelatedness does not imply independence in general, this will be the case under certain parametric assumptions such as a bivariate normal distribution.

delinquency cost respectively by:

$$H_i \sim \text{Normal}(x_i'\beta, \sigma_\xi^2 + \sigma_\varepsilon^2)$$

$$Q_i \sim \text{Normal}(w_i'\lambda, \sigma_\eta^2)$$

### 2. Conditional modification probabilities

I must solve for the conditional modification probabilities as part of the estimation. The probability that a borrower $i$, with observable characteristics $x_i$ and $w_i$, will receive a modification, $m_i$, is a crucial input into the likelihood but cannot be seen directly in the data because it is a function of unobservables. These probabilities establish model equilibria for a given set of observables and model parameters, but are computationally costly to solve for in every likelihood iteration. To overcome this computational challenge, I pre-solve the loan servicer's expected cashflow maximization problem on an eight-dimensional grid that accounts for borrower characteristics and unobservable parameters. The maximization routine loads in the grid and then draws modification probabilities from it for borrowers given a guess of model parameters. For points between grid values, I use linear interpolation.

Solving for a grid of conditional modification probabilities relies on evaluating the servicer's expected cashflow function with many different inputs. Specifically, the modification rate is a function of eight variables: $x_i'\beta + \xi_i, w_i'\lambda, p_i, f_i, \sigma_\varepsilon, \sigma_\eta, \rho\ c_j$. The first four terms are assumed observable characteristics of an individual borrower to the servicer, where the $x_i'\beta + \xi_i$ and $w_i'\lambda$ are scalar indices that can accommodate vectors of arbitrarily many variables and parameters. The fifth and sixth terms, and seventh terms are the standard deviations of borrower private information relating to home utility ($\varepsilon_i$), delinquency cost ($\eta_i$), as well as their correlation ($\rho$), while the $c_j$ is the servicer-specific cost of processing a modification. In solving its high-dimensional problem, the servicer must compute truncated distributions of the unobservable components of the borrower's indirect utility function. Details about the servicer's problem under my parametric assumptions are laid out in Appendix C.

The computation of cash flow maximizing $m_i$ is highly conducive to parallel computing since the optimum for a given set of inputs can be computed independently from the optimum for a different set of arguments. I use the Texas Advanced Computing Center's Stampede3 cluster to calculate the optimal modification policy grid. My final estimation routine employs a grid with 20 points of support of $p_i$, 17 points of support of $x_i'\beta + \xi_i$, 12 points of support of $w_i'\lambda$, 10 points in the support of $c_j$ and $\rho$, 7 points in the support of $\sigma_\eta$, and 5 points in the support of $\sigma_\varepsilon$ and $f_i$. Jointly this creates a grid with 71.4 million points. Forming this grid takes approximately 15 minutes with 40 computing nodes. Additional details on the formation of the loan modification grid are available in Appendix 1.

## VI. RESULTS

Table IV reports financial equivalents for my borrower home utility parameters and Table V reports financial equivalents for estimated servicer costs. Due to a standard scale-normalization in my discrete choice frame-

TABLE IV
ESTIMATED HOME UTILITY PARAMETERS, BY ORIGINATION COHORT

| | Q1-Q4 2004 Sample | | Q1-Q4 2005 Sample | | Q1-Q4 2006 Sample | | Q1-Q4 2007 Sample | |
|---|---|---|---|---|---|---|---|---|
| **Dollar-denominated variables** | | | | | | | | |
| | Ests | Std. Err. | Ests | Std. Err. | Ests | Std. Err. | Ests | Std. Err. |
| $\beta_{Home\ Value}$ | 0.52 | 0.0006 | 0.64 | 0.0002 | 0.77 | 0.0017 | 0.79 | 0.0022 |
| $\beta_{Income}$ | 0.24 | 0.0003 | 0.38 | 0.0001 | 0.44 | 0.0036 | 0.30 | 0.0103 |
| **Non-dollar-denominated variables** | | | | | | | | |
| | Ests | Std. Err. | Ests | Std. Err. | Ests | Std. Err. | Ests | Std. Err. |
| $\beta_0$ | \$-585.0 | 572.59 | \$131.0 | 740.98 | \$13.0 | 901.61 | \$0.0 | 791.12 |
| $\beta_{LTV}$ | \$-24,945.0 | 31.36 | \$-25,626.0 | 8.22 | \$-6,220.0 | 54.59 | \$-9,429.0 | 779.16 |
| $\beta_{DTI}$ | \$-76,815.0 | 96.38 | \$-58,217.0 | 16.60 | \$-52,310.0 | 448.34 | \$-147,732.0 | 1,858.18 |
| $\beta_{Credit\ Score}$ | \$395.0 | 0.42 | \$333.0 | 0.09 | \$225.0 | 0.55 | \$337.0 | 1.09 |
| $\beta_{\Delta Home\ Value}$ | \$15,480.0 | 19.46 | \$28,425.0 | 9.12 | \$19,243.0 | 168.94 | \$59,574.0 | 2,835.3 |
| $\beta_{\Delta UE}$ | \$-20,391.0 | 25.62 | \$-38,016.0 | 10.84 | \$-24,462.0 | 208.95 | \$-38,707.0 | 805.76 |
| $\sigma_\xi$ | \$128,477.0 | 132.70 | \$119,685.0 | 34.15 | \$126,796.0 | 313.04 | \$126,721.0 | 527.84 |
| $\sigma_\varepsilon$ | \$8,085.0 | 413.31 | \$7,941.0 | 222.77 | \$9,170.0 | 253.46 | \$13,125.0 | 344.27 |
| Loan purpose dummy | Yes | | Yes | | Yes | | Yes | |
| *N* | 43,546 | | 47,418 | | 31,105 | | 39,905 | |
| *Log LL* | 24,228 | | 43,710 | | 42,473 | | 56,965 | |

*Notes:* This table presents estimated model parameters that jointly explain the borrower home utility, $H_i$, for single-family Fannie Mae loans in the state of California. Parameters have been re-scaled so that they can be interpreted in terms of nominal dollar amounts. Estimation was conducted for each loan origination group separately, and there are no restrictions that would force parameters to be consistent across origination cohorts. Information asymmetry comes from the $\sigma_\xi$ and $\sigma_\varepsilon$ terms. These two parameters represent the standard deviations of unobservable borrower information that are either known to the servicer ($\xi_i$) or entirely private information to the borrower ($\varepsilon_i$).

work, I intentionally set the parameter on payment disutility, $p_i$ to -1. This offers the advantage that all other parameters of the model can then be interpreted relative to the payment disutility, measured in January 2010 principal balance dollars.

Coefficient interpretation depends on whether the variable is dollar-denominated or non-dollar-denominated. All dollar-denominated coefficients are reported as decimals for direct interpretation in dollar terms. As an example from the home utility parameters, the coefficient on home value suggests that a \$1 increase in home value has the same effect as a \$0.52-\$0.79 reduction in the outstanding mortgage principal in January 2010. Likewise, the income coefficient suggests that a \$1 increase in borrower income is equivalent to a \$0.24-\$0.44 reduction in mortgage principal. Non-dollar-denominated variables, such as loan-to-value and credit score, should also be interpreted in terms of their effect on mortgage principal. As an example of this type of variable, the credit score coefficient suggests that a 1-point increase in credit score is equivalent to a \$225-\$395 reduction in principal balance. Although the coefficient on debt-to-income ratio appears numerically large, note that a one-unit change corresponds to a 100-percentage-point shift. For the 2004 sample, a 10-percentage-point increase in debt-to-income ratio would be the equivalent to a \$7,681 increase in principal.

My results suggest that borrowers place significant value on avoiding foreclosure. Using the full datasets of borrowers, the expected value of home utility minus payment disutility, $u_i^{NQ} = H_i - p_i$, for a single, non-delinquent borrower monotonically declines from \$224,723 in 2004 to \$105,411 in 2007, suggesting rising probabilities of borrower default for loans originated closer to the start of the 2008 financial crisis. These amounts can be interpreted as the mean values of avoiding foreclosure and paying to remain in

TABLE V
ESTIMATED COSTS OF LOAN MODIFICATION, BY SERVICER AND ORIGINATION COHORT

| | **Q1-Q4 2004 Sample** | | **Q1-Q4 2005 Sample** | | **Q1-Q4 2006 Sample** | | **Q1-Q4 2007 Sample** | |
|---|---|---|---|---|---|---|---|---|
| | Ests | Std. Err. | Ests | Std. Err. | Ests | Std. Err. | Ests | Std. Err. |
| $c_{Bank\ of\ America}$ | \$1,787.0 | \$352.21 | \$1,191.0 | \$326.94 | \$1,138.0 | \$339.08 | \$2,505.0 | \$210.54 |
| $c_{Wells\ Fargo}$ | \$707.0 | \$536.60 | \$510.0 | \$276.20 | \$278.0 | \$284.35 | \$854.0 | \$227.03 |
| $c_{Citimortgage}$ | \$921.0 | \$353.57 | \$389.0 | \$204.43 | \$328.0 | \$240.35 | \$809.0 | \$137.10 |
| $c_{GMAC}$ | \$1,026.0 | \$353.04 | \$383.0 | \$193.92 | \$30.0 | \$30.26 | \$248.0 | \$226.90 |
| $c_{JP\ Morgan}$ | \$1,004.0 | \$345.16 | \$1,967.0 | \$878.94 | \$1,153.0 | \$792.20 | \$2,150.0 | \$623.55 |
| $c_{Suntrust}$ | \$971.0 | \$1,533.36 | \$1,721.0 | \$1,067.48 | \$918.0 | \$324.72 | \$3,609.0 | \$533.99 |
| *N* | 43,546 | | 47,418 | | 31,105 | | 39,905 | |
| *Log LL* | 24,228 | | 43,710 | | 42,473 | | 56,965 | |

*Notes:* This table presents estimated parameters for the costs of modification for single-family Fannie Mae loans in the state of California. Parameters have been re-scaled so that they can be interpreted in terms of nominal dollar amounts. Estimation has been conducted for each loan origination group separately and there are no restrictions that would force estimated costs of modification for the same servicer to be consistent across origination groups. In general, there are notable differences in the costs of modification between servicers and these differences tend to persist across origination groups. Higher cost servicers, such as servicers Bank of America and JP Morgan, tend to be high cost across all estimation groups. Low costs servicers, such as servicers Wells Fargo and Citimortgage tend to exhibit lower estimated costs to process loans. Differences in the estimated cost of modification *for the same servicer*, such as servicer JP Morgan, may reflect the servicer's differing treatment of mortgages based on origination timing.

a home for different borrower cohorts. There is significant value to borrowers from avoiding foreclosure: a foreclosed borrower in the U.S. cannot qualify for a conventional mortgage for seven years and will have a significantly reduced credit score that will restrict access to non-housing credit markets. Studies have also linked foreclosure to elevated levels of depression, divorce and suicide.[16] Applying the headline estimates from Bhutta et al. (2017) to my data suggests that a \$300k home would need to have a \$522k valued mortgage before the median borrower would choose to "walk away" from their mortgage obligation: this \$222k gap is consistent with the magnitude of my estimates. It is also worth commenting on the decline in the net home utility over time, a result that is driven by both differences in $\beta$ parameter estimates and increases in $p_i$ as Fannie Mae increased its conforming loan limits.[17] This finding suggests that financially less healthy borrowers were taking on larger mortgage balances than financially more healthy borrowers in the lead up to 2008, even in the prime mortgage market.

The estimated standard deviation parameters in the home utility, $\sigma_\xi$ (known to servicer) and $\sigma_\varepsilon$ (unknown to servicer), suggest significant unobservable heterogeneity across borrowers, much of which is known to loan servicers. The large and statistically significant combined dollar measures for the two unobservable parameters mean that unobservable factors typically drive borrower delinquency and default decisions. $\xi_i$ likely captures much of this variation because a borrower often informs his bank about adverse life events, even if it does not show up in my servicing data. The importance of unobservables supports recent mortgage literature that assigns the vast majority of mortgage defaults to adverse life events rather than negative equity concerns (Ganong and Noel, 2020b; Low, 2022). Though the scale of servicer-known

[16] Tsai (2015) conducts a meta-analysis on 42 papers that explore the linkages between mental health outcomes and foreclosure. Fowler et al. (2015) provide an original study on the link between foreclosure and suicide rates.

[17] Between 2004 and 2007 single family, Fannie Mae loan limits rose from \$333,700 to \$417,000.

heterogeneity is an order of magnitude larger than servicer-unknown heterogeneity, servicers still face economically meaningful uncertainty that impacts their decision making. Subsection *VI.C* below explores the effectiveness of loan modification allocations in more detail. I do not report the correlation coefficient between $\sigma_\varepsilon$ and $\sigma_\eta$, as its inclusion does not improve model fit. A formal likelihood ratio test rejects the specification in which $\sigma_\rho \neq 0$, indicating that the correlation parameter is not statistically supported in this setting.[18]

While loan, property, and borrower characteristics all play an important role in shifting the likelihood of default, my results suggest that delinquency costs play a smaller role for the typical borrower. At the point estimates for delinquency cost parameters, a borrower with the mean credit score behaves as if the costs of delinquency are \$3,046, \$6,220, \$3,233 and \$5,000 respectively across origination groups, with standard deviations of \$1,515, \$3,136, \$1,639 and \$2,557. These measures of delinquency cost are greatly outweighed by potential changes in payment disutility through modification and the cost of default, which tend to be upwards of tens of thousands of dollars. Low mean delinquency costs may reflect the fact that borrowers have a high opportunity cost of making payments on time and are not willing to forgo other spending to remain current on their payments. This sort of interpretation is consistent with thinking about temporary delinquency as a form of insurance (Bhutta et al., 2017).[19]

As expected from observable modification activity and past papers on loan renegotiation, there are statistically significant differences in the cost of processing loan modifications across servicers and this heterogeneity plays an important role in modification and foreclosure outcomes. The lowest estimated modification costs are associated with Wells Fargo and Citimortgage which behaves as if the transaction costs of modification were generally well below \$900. Estimates imply that Bank of America faces the largest costs of loan modification, with estimates that exceed \$1,100 in all four samples and go as high as \$2,500 in 2007. In my model, these high costs rationalize Bank of America's reluctance to grant loan modifications to its borrowers (Table III).

The modification cost parameters capture factors beyond the mortgage servicer's accounting transaction costs. First, the estimation set-up assumes all mortgage servicers operate with the same model for predicting foreclosure losses. Due to this restriction, modification costs may also capture differences between servicer housing market expectations. Second, the model does not allow servicers to differ in their ability to assess borrowers as a part of the loan modification process. Screening ability between banks would also be captured in the modification cost estimates. Third, large costs of modification could indicate that a servicer is understaffed or otherwise unable to service large quantities of loans.

FIGURE V
Within-sample model fit, outcome probabilities

FIGURE V
Actual outcomes

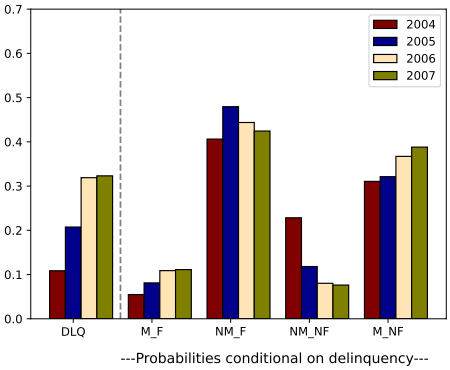
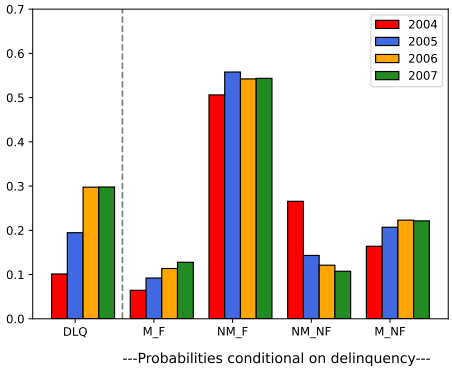



FIGURE V
Predicted outcomes




*Notes:* This figure presents actual (left) and model predicted (right) outcome probabilities across origination cohort samples. The *DLQ* bars are the share of loans that become at least three months late on their mortgage payments. The other four groups represent outcome probabilities *conditional on delinquency*. Each delinquent outcome is a combination of a modification, *M*, and foreclosure, *F*, outcome. An outcome with an *N* signifies either *Not modified* or *Not foreclosed*. The model implied probabilities are ordinally correct. The model fits delinquency probabilities across origination cohorts almost exactly and predicts the correct magnitude for non-modified foreclosures.

## *VI.A Model fit*

Figure V presents (a) the actual borrower outcome probabilities in the data, relative to (b) the in-sample model predicted borrower outcome probabilities. The model generally preserves both relative cardinality and ordinality of outcome probabilities: parameter estimates suggest that the majority of borrowers avoid delinquency all together and that delinquent borrowers are mostly likely to end in foreclosure, without having received a modification. The estimates slightly under-predict the probability of avoiding foreclosure given a modification and over-predict the probability of a foreclosure after a modification. Most obviously, the model suggests that servicers make more modification errors than is supported by the raw data.

There are various factors that affect the current model fit. Perhaps most importantly, the fact that over 75% of borrowers never enter delinquency means that estimation relies on a relatively small fraction of loans to fit the model to all outcome probabilities. The model performs well for the outcomes with the most observations: predicted probabilities of delinquency match the data almost exactly and predicted probabilities of foreclosure without modification also appear closely matched to observed outcomes. Discrepancies primarily arise with the two least likely outcomes: foreclosure after modification and no foreclosure without a modification.

Discrepancies between observed loan outcome probabilities and the model predicted outcome

[18]Because $\sigma_\rho$ term does not improve model fit, it is empirically advantageous to remove the parameter from the high-dimensional interpolation that takes place to solve for model equilibria.

[19]I separate delinquency and default more explicitly than much of the mortgage literature, which is often focused on a measure similar to my measure of delinquency (60 or 90 days behind on mortgage payments). It seems credible that borrowers are willing to go delinquent when facing a liquidity constraint but would still try to avoid the high costs of foreclosure.

TABLE VI
MEAN GAINS/LOSSES FROM MODIFICATION AS PERCENTAGE OF OUTSTANDING PRINCIPAL

| Result | **2004** | **2005** | **2006** | **2007** |
|---|---|---|---|---|
| **Borrowers** | | | | |
| Gain - Medium borrower type avoids foreclosure | \$7.90% | \$7.77% | \$7.72% | \$7.90% |
| **Servicers** | | | | |
| Loss - Low borrower receives modification | \$-0.64% | \$-0.43% | \$-0.41% | \$-0.87% |
| Gain - Medium borrower avoids foreclosure | \$24.39% | \$24.60% | \$24.62% | \$24.20% |
| Loss - High borrower modification | \$-15.71% | \$-15.48% | \$-15.47% | \$-16.00% |

*Notes:* This table presents mean gains and losses experienced by both borrowers and servicers through modification outcomes, as a percentage of outstanding principal. The model estimates suggest that high borrower types may receive more relative utility from modification than medium types, and this is primarily driven by the fact that medium borrower types have lower home values relative to default, as well as lower outstanding principal balances. Servicers tend to take small relative losses when modifications are incorrectly rewarded to low borrower types, but financial institutions take relatively large losses from incorrectly granting aid to a borrower that could have paid without assistance (high types). Benefits from awarding modifications to medium types that avoid foreclosure outweigh the losses of an incorrect modification to either a low or high borrower type. The loss to the servicer from modifying a low borrower type arises from the cost of processing the modification, $c_j$.

probabilities are also at least partially explained by two key restrictions: 1. setting modification as a fixed share of outstanding principal and 2. the use of a sparse grid in estimating model parameters. Firstly, as previously discussed, Figure III highlights the fact that financial institutions realized a wide range of potential losses through modification and foreclosure and the current model does not allow for this. This is not as extreme as it may seem at face value: servicers would not have known future loss realization when setting modification policy and generosity was largely prescribed by Federal guidelines. Secondly, the sparse grid used in estimation may introduce error due to the linear interpolation used to approximate the servicer's policy function. I mitigate this second potential challenge by solving the servicer's problem at tens of millions of potential states of borrower characteristics and model parameters.

## *VI.B Benefits of modification*

Model estimates suggest that there are meaningful benefits from appropriately targeted modification from both the private and social perspective. My results suggest that servicers have a strong incentive to grant modifications to borrowers that would enter foreclosure otherwise. This incentive is counteracted by meaningful, though relatively smaller, losses if modification is awarded to borrower types whose foreclosure statuses are unchanged by financial assistance. Borrowers also meaningfully gain if they are able to avoid foreclosure. Since modifications function as a transfer between the borrower and his servicer, social losses from incorrect allocation of modifications tend to be relatively lower than the private losses considered by a loan servicer.

**Private benefits and losses**

Using my estimates I can quantify the private benefits of foreclosure prevention. Table VI summarizes the mean gains and losses from modification as a percentage of the outstanding loan principal. The average gain in utility for a borrower who successfully avoids foreclosure will be given by: $E(H_i - \tilde{p}_i | H_i \leq p_i, H_i > \tilde{p}_i)$, which is the average home value relative to default minus payment disutility for the medium-type borrower. I find on average that this benefit is between 7.5% and 8.0% as a percentage of outstanding borrower principal, depending on the origination sample. In nominal dollar terms this is equivalent to about $23,000 in October, 2009 dollars.

Average servicer gains for the medium-type borrower can also be calculated as:
$E(\tilde{p} - f_i - c_j | H_i \leq p_i, H_i > \tilde{p}_i)$, which I find to be over 20% of the value of the outstanding principal due to large expected losses from foreclosure (Over $70,000 in nominal dollar terms for the servicer). These benefits must be weighed against the losses from incorrectly awarded debt relief. Creditors will on average lose $E(p_i - \tilde{p}_i | H_i > p_i)$ when they award relief to the high-type borrower. I estimate that servicers lose an average of 15.5% to 16.0% of the outstanding mortgage principal per borrower who could have avoided foreclosure without assistance, across origination samples. Again in nominal dollars, this translates to a loss upwards of $41,000.r

**Social benefits**

The social benefits of modification greatly outweigh the private benefits to financial institutions, a result driven by the fact that modification acts as a transfer between individual and servicer. Figure VI depicts both servicer and social losses from (a) a "modification error" in which a modification was awarded that did not affect ultimate foreclosure statues, and (b) a "no modification error" in which a modification would have shifted a borrower away from foreclosure but was not offered. Mean servicer losses from modification errors greatly outweigh the mean social losses from modification, because any reduction in servicer cashflow from modification directly improves the borrower's welfare. Social losses from modification error are driven exclusively by the avoidable $c_j$ costs. The model predicts that a servicer will perceive modification errors as a loss of about 5% of the value of the outstanding mortgage principal. For comparison, the social cost of a modification error is generally below 1%.

While servicers face greater losses from modification errors than a social planner, they also absorb smaller losses from "no modification errors" than the social loss. Figure VIIIb shows the average servicer and social loss as a share of outstanding mortgage principal. The servicer perceives the mean loss from a no modification error to be about 22% of the value of the outstanding mortgage balance while the total social loss is in the order of 33%. This gap is explained by the loss in welfare faced by the borrower that is forced into foreclosure. This loss of welfare is not internalized by the servicer. The gap between servicer private loss and social loss grows further when externalities of foreclosure on third-parties are included. Taken together with greater direct losses from modification errors, it is clear why a servicer will offer fewer loan modifications than a social planner who looks to maximize the combined welfare of the servicer and the

borrower.

FIGURE VI
Mean servicer error losses, private vs. social

FIGURE VI
Mean losses from *modification* error

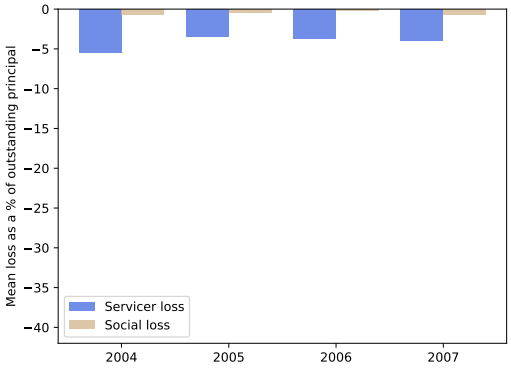
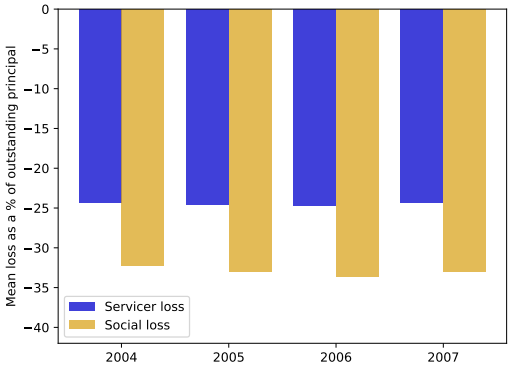



FIGURE VI
Mean losses from *no modification* error




*Notes:* This figure presents stimulated mean modification error losses (left) and no modification error losses (right) across origination cohorts in the samples based on model estimates. The main take-away is that the mean losses of "no modification" error, or failing to modify a borrower that could have avoided foreclosure, greatly outweigh the losses from a "modification error", modifying a borrower whose foreclosure status is unaffected by assistance. Though servicers face large private losses from a no modification error, the wider social loss is even greater, since borrower welfare declines meaningfully through foreclosure. Social losses from a modification error also tend to be smaller than the servicer's private losses from incorrectly awarding financial assistance. Taken together, this implies that a social planner maximizing social welfare would prefer to offer even more modifications than the servicer, because the relative losses from modification error are low and the losses from no modification error are relatively higher.

## *VI.C Accuracy of the servicers*

In addition to quantifying the losses arising from misallocated modifications, my model estimates allow me to simulate the accuracy of servicer decisions. "Accuracy" here simply reflects whether the servicer made the correct modification decision ex-post. Servicers granting modifications to a large number of borrowers who cannot avoid foreclosure are "inaccurate", while servicers that only grant modifications that succeed in preventing a foreclosure are "accurate".

Table VII reports the average model-predicted accuracy of servicers across my four origination year samples. Each cell of the table contains the percentage of delinquent loans based on whether modifications would have been successful in changing a foreclosure outcome (rows) and whether they actually received a modification (columns). Estimates show that servicers tend to make the correct modification decisions and prefer to grant too many modifications rather than too few. I find that servicers make the correct modification decision 78% of the time; the rest of the time they either grant a modification when it has no effect, or fail to grant a modification when it would have prevented foreclosure. The servicer preference for modification can be seen from the fact that they make many more "modification" errors (12.6% of delinquent loans) than "no modification" errors (9.6%). I report the predicted servicer modification accuracy for other estimation samples in Appendix *V.D*. The predicted servicer modification accuracy tends to be consistent across all four origination cohorts used for estimation.

TABLE VII
PREDICTED SERVICER MODIFICATION ACCURACY, CROSS-SAMPLE MEANS

| | | *Servicer action* | |
|---|---|---|---|
| | | No modification | Modification |
| *State of the world* | Modification has no effect | 60.8% *(Ex-Post Correct: No Modification)* | 12.6% *(Ex-Post Error: Modification)* |
| | Modification is effective | 9.6% *(Ex-Post Error: No modification)* | 17.0% *(Ex-Post Correct: Modification)* |

*Notes:* This table presents model-predicted servicer accuracy using the average estimated results across my four origination year samples. Each percentage in the table is the fraction of delinquent loans with the particular realized outcome. Around 78% of the time the servicer makes the correct modification decision: either not offering a modification when it has no effect or offering one when it will help avoid a foreclosure. In equilibrium, servicers prefer to make more modification errors than non-modification errors. Around 30% of delinquent loans receive a modification even when it makes no difference to their ex-post foreclosure outcome. The tendency to modify can be rationalized by the fact that servicers gain significantly when a modification succeeds in preventing a foreclosure. Model-predicted servicer modification effectiveness tends to be consistent across all four origination cohorts, and relative types of mistakes maintain ordinal rank. The incentive to modify grows with later origination cohorts that hold less home equity, leading to greater modification errors and fewer no modification errors (Appendix *V.D*).

## VII. Counterfactual analysis

The sizeable differences between private and social losses discussed in Section VI. suggest scope for policy intervention in foreclosure prevention. The gap between losses is driven by the fact that mortgage servicing institutions do not internalize borrower welfare losses from foreclosure, which can lead them to have an insufficient incentive to offer modifications from a social welfare perspective. The inclusion of negative foreclosure externalities on local housing markets (Campbell et al., 2011; Anenberg and Kung, 2014) would further widen the gap between private and social losses.

In this section of the paper I use my estimated model to test counterfactual policy solutions that could have been used to achieve greater levels of social welfare. I start by establishing two informative benchmarks: (a) the "First Best" case in which there is no information asymmetry and no inefficient foreclosure occurs, and (b) the "social planner" problem in which a hypothetical social planner sets modification policy to maximize total social welfare, taking as given information asymmetry and servicer modification costs.[20] I then predict equilibrium outcomes under counterfactual circumstances where: 1) servicers receive further subsidies that lower loan modification costs, and 2) servicers hold more information about borrower financial positions.

[20]The social planner faces an even more complex optimization problem than the servicer because it internalizes borrower welfare. The social planner must form beliefs about expected welfare for borrowers with private information. Estimation details for the social planner are provided in Appendix D.

## *VII.A Policy benchmarks*

TABLE VIII
MODIFICATIONS AND FORECLOSURES, RELATIVE TO FIRST BEST

| | % Modifications | % Foreclosures |
|---|---|---|
| First Best | 100.0 | 100.0 |
| Social Planner | 177.3 | 113.1 |
| *Baseline: Servicer* | 129.9 | 116.2 |

*Notes:* This table summarizes the model-predicted number of modifications and foreclosures using the average estimated results across my four origination year samples. The table depicts the baseline case and the social planner's problem as a percentage of the First Best benchmark. In the First Best case, mortgage servicers have full information about borrower financial health and only award modifications when they are effective in preventing foreclosure. Modification rates are significantly higher under both the baseline and social planner's problem since modifications cannot be perfectly targeted to those borrowers who need them to avoid foreclosure.

TABLE IX
BORROWER AND SERVICER SURPLUS, RELATIVE TO FIRST BEST

| | % Total surplus | % Borrower surplus | % Servicer surplus | % Medium-type surplus | % High-type surplus |
|---|---|---|---|---|---|
| First Best | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| Social planner | 99.47 | 100.37 | 98.79 | 97.65 | 100.39 |
| *Baseline: Servicer* | 99.46 | 100.00 | 99.05 | 66.48 | 100.09 |

*Notes:* This table summarizes the model-predicted borrower and servicer surplus using the average estimated results across my four origination year samples. The table depicts the baseline case and the social planner's problem as a percentage of the First Best benchmark. In the First Best case, mortgage servicers have full information about borrower financial health and only award modifications when they are effective in preventing foreclosure. Medium-type surplus tends to improve significantly with a decline in foreclosures. High-type borrowers benefit from higher rates of loan modification when servicers cannot perfectly screen. High-type borrowers make up the majority of unconditional borrower types in my sample and because they never enter foreclosure, drive the near First Best relative total surplus. Socially optimal increases in surplus are drive by the medium-type borrowers.

Table VIII and Table IX summarize the relative welfare of servicers and borrowers when comparing the predicted model outcomes with the servicer to the First Best and social planner solutions.

In the First Best case, servicers have complete information about borrower outcomes and are able to stop all preventable foreclosures. With full information, servicers offer no modifications to the low- or high-type borrowers for whom a modification will not change equilibrium outcomes. Based on my average model estimates across my four estimation cohorts, there would be 30.0% more modifications but also 16.2% more foreclosures under the base case, in which servicers maximize their expected cashflows, relative to the First Best case. Modifications decline in the First Best case because servicers face no uncertainty about which borrowers would benefit from such assistance: each modification offered successfully prevents a foreclosure. Medium-type borrowers are made significantly better off under this hypothetical scenario: their combined welfare improves by 50.4% relative to the base case in which a servicer is not fully informed about borrowers. Conversely, the high-type borrowers lose out because they never receive modifications when a servicer is fully informed; their combined welfare falls by around 0.9%. This decline is small because the vast majority of high-type borrowers never enter delinquency at all.

Under the social planner's problem, a planner attempts to maximize joint welfare of borrowers and servicers, but faces the same information asymmetry and modification costs as the servicers themselves.

Due to the social costs of foreclosures, the planner offers significantly more modifications than the servicers: there would be around 36% more modifications than the servicers and 77% more modifications than in the First Best case. The dramatically increased rates of modification modestly brings down foreclosures relative to the baseline case; there are still 13.1% more foreclosures with the social planner than in the First Best case. The sharp increase in modifications comes at the cost of modifications being misallocated to borrowers who do not need them. High type borrowers enjoy total welfare that is marginally higher because many of them receive modifications that decrease their debt burden.

These two policy benchmarks suggest a few important aspects of equilibrium outcomes. Firstly, given a fixed level of modification generosity, even a fully informed servicer can only prevent around 14.0% of foreclosures that occur in the model-implied equilibrium. This means that many foreclosures simply cannot be prevented without increased generosity of modification. Some borrowers could certainly have been under sufficient financial duress that no reasonable level of forgiveness would have allowed them to keep their homes. In a few other cases, borrowers may have also had low attachment to their properties and been interested in walking away from an underwater mortgage.[21] The benchmark results also highlight the tension between incorrectly awarding modifications and successfully preventing foreclosures. Total borrower welfare under all three outcomes appears similar because high-type borrowers benefit from loan modifications. A policy-maker wants to carefully consider the incorrect allocation of modifications to borrowers who do not need them because this hurts financial institutions without providing meaningful gains in social welfare.

## *VII.B Counterfactual policies*

FIGURE VII
Subsidies and information in foreclosure outcomes

FIGURE VII
Subsidies

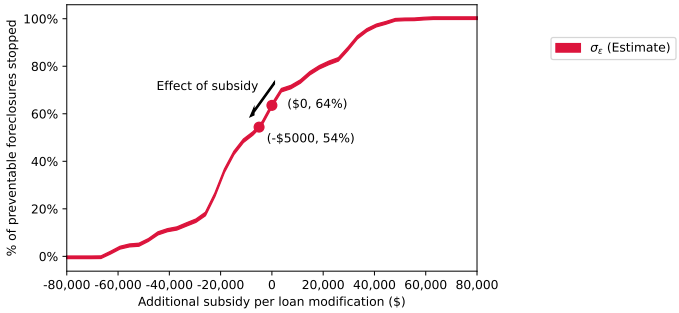
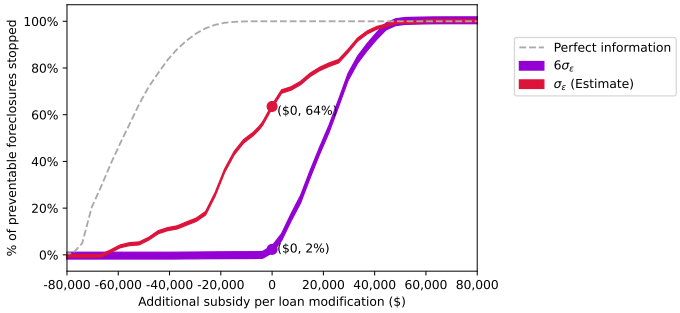



FIGURE VII
Information




*Notes:* This figure presents model-predicted effects of loan modification subsidies and lender information on mean foreclosures outcomes.

To inform policy on foreclosure prevention, I use my model to explore the roles of modification subsidies and servicer information about borrowers on equilibrium outcomes. I consider the effects of subsidies and

[21] Studies have generally disproved underwater mortgages as a primary driver of borrower default, but there is still evidence that this occurred from time to time (Guiso et al., 2013).

information together to develop a more nuanced understanding of the borrower screening and debt-relief allocation. The explicit treatment of information as a part of equilibrium outcomes and optimal subsidy setting is one of the unique contributions of this paper.

Modification subsidies reduce the effective transaction costs that servicers face when rewarding modifications and can help reduce the losses associated with debt forgiveness. Though subsidies will reduce transaction costs to the servicer, they will not guarantee more modifications. Servicers will still consider their expected cash flows with and without modification and then choose a policy that maximizes their expected cash flows. In my model a subsidy, $s$, reduces the size of $c_j$ in the following way:

$$\hat{c}_j = c_j - s$$

A servicer's information about his borrower's financial position can also play a crucial role in modification decisions. Servicers who are fully-informed about borrowers' financial health prevent all inefficient foreclosure without subsidies because they have a financial incentive to do so. Policy only starts to play a role precisely when servicers face information asymmetry and have insufficient private incentive to grant modifications that would be socially beneficial.

To change the amount of information that loan servicers have about borrower unobservable financial information in the model, I shift the share of variance in $H_i$ that is known to the servicer when he makes loan modification decisions. Information about $H_i$ is central to the loan servicer's problem because a borrower's permanent default and foreclosure will drastically impact payouts; temporary delinquency has negligible effects on the servicer if a borrower ultimately resumes paying.[22] I vary the share of $\sigma_\xi^2 + \sigma_\varepsilon^2$ that is explained by $\sigma_\xi^2$. As the share explained by $\sigma_\xi^2$ rises, the servicer becomes more informed about borrower private information while the overall distribution of $\xi_i + \varepsilon_i$ is preserved.

The model predictions from shifting the per-modification subsidies and servicers' level of information reveal the importance of effective borrower screening for subsidy policy. Figure VII presents the average model predicted effects of loan modification subsidies and lender information on equilibrium modifications, delinquencies, and foreclosures for across my estimation samples. Information alone dramatically shifts equilibrium outcomes, which can be seen by comparing the model predictions in the right panel. As servicers become more informed about borrower financial health, they are better able to identify borrowers who require debt relief in order to avoid foreclosure. More informed servicers increase the number of modifications available and reduce foreclosures, even in the absence of government subsidies. An uninformed servicer will offer nearly zero loan modifications because it is unlikely that they will be allocated to the correct borrowers. As servicers observe a greater share of borrower heterogeneity, they dramatically increase the number of modifications offered. At the estimated level of borrower private information, servicers award modifications to around 35% of delinquent borrowers and this results in a substantial decline in foreclosures: they fall by about 20% relative to a servicer that observes close to 0% of borrower heterogeneity driven by

[22] Knowledge about $Q_i$ does allow the servicer to set a stricter modification policy for borderline borrowers for whom a reduction in the modification probability would push them out of delinquency entirely.

$\sigma_\xi^2 + \sigma_\varepsilon^2$, the variance of home utility relative to default.

Equilibrium delinquencies also shift with changes in servicer information. As servicers become more informed and offer a greater number of modifications, delinquencies rise as financially healthy borrowers attempt to earn loan modifications. This pattern reverses once servicers become more informed; the closer servicers get to perfect information, the less likely they are to mistakenly offer a modification to a borrower going delinquent strategically. Strategic borrowers respond to more informed servicers by choosing not to enter delinquency, in order to avoid the costs of late payment. Equilibrium levels of delinquency lie above the First Best level precisely because the perfectly-informed servicer will never make any modification errors, completely eliminating the benefits of strategic delinquency.

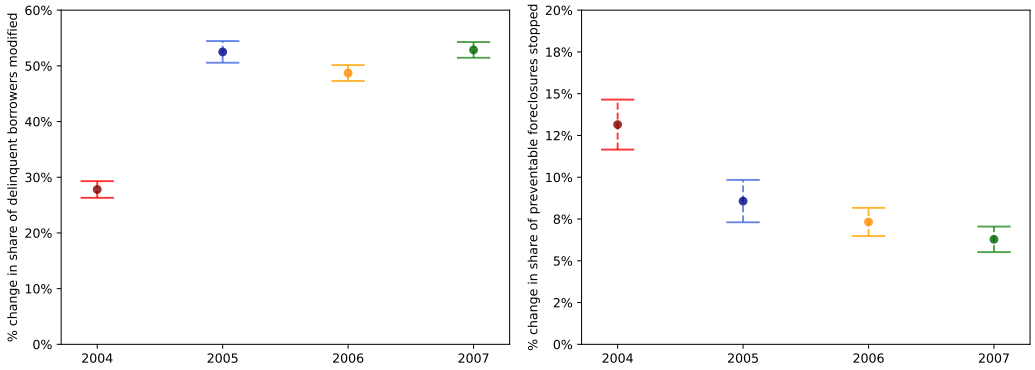
Given the effects that the servicer's information has on equilibrium outcomes, knowledge about borrowers also dictates the effectiveness of subsidy policy. In Figure VII, movement along the horizontal axis represents different levels of per-modification subsidies granted to loan servicers. As servicers become more informed about borrower financial health the outcome lines steepen, which reflects that servicers are more responsive to modification subsidies. An uninformed servicer ($\sigma_\varepsilon = 6 \times \hat{\sigma}_\varepsilon$) essentially cannot be incentivized to grant loan modifications because of the large losses associated with awarding them to borrowers who do not need aid. Subsidies per modification must be increased by over \$25,000 to acheive foreclosure prevention at the estimated level of information. The story differs for an informed servicer: at the estimated level of servicer informed-ness, subsidies can increase the number of modifications from around 20% of delinquent borrowers to over 70% of them, and most of this increase occurs with additional subsidies between \$0 and \$4,000 per modification. I find that the increase in modifications for the more informed servicer would lead to as much as a 2.4% decrease in the total number of foreclosures. Delinquency behavior also reflects how effective subsidies are in reaching the target population. If servicers are relatively uninformed, modification subsidies will tend to jointly drive up modification and delinquency, leading to a muted effect on foreclosures, as modifications are awarded to borrowers who do not need them or do not benefit from them. For highly informed servicers, modification subsidies have diminishing effects on borrower delinquency and a larger share of debt relief goes to preventing foreclosures.

Two main factors drive the result that foreclosure prevention requires a substantial shift in total modifications. Firstly, observed modifications took place under active Federal subsidy policies, meaning that any subsidies discussed above were *additional subsidies* on top of those that were granted to financial institutions in reality and incentive subsidies are likely to suffer from diminishing returns. Secondly, model estimates suggest that the majority of borrowers can be categorized as either Low Types or High Types, which means that modification is unable to shift outcomes for the a significant share of borrowers. The combination of imperfect information about borrower financial health and a small share of Medium Type borrowers makes it challenging for servicers to grant loans to precisely the borrowers that need them to avoid foreclosure.

FIGURE VIII
Estimated effect of modification subsidies on equilibrium outcomes

FIGURE VIII
Change in modifications

FIGURE VIII
Change in foreclosures




*Notes:* This figure presents model-predicted effects of subsidies on modification (left) and foreclosure outcomes (right). I find that subsidy policy tended to have a greater effect on the number of loan modifications made to borrowers in later cohorts but less of an effect on the number of foreclosure realizations. In later cohorts that tended to have higher delinquency rates, like 2006 and 2007, granting additional loan modifications to borrowers that re-default even after receiving assistance dampens the effect on foreclosure outcomes. As discussed in Section 6, private losses for the servicer of mistakenly granting a loan modification to a borrower that still defaults are relatively low.

## *VII.C Assessing subsidy policy and the costs of foreclosure prevention*

The empirical model enables a back-of-the-envelope calculation on the effectiveness of true foreclosure prevention policy. Loans in my sample would have generally been eligible for HAMP assistance since they were originated before the start of 2009. I consider the equilibrium response of modifications and foreclosures in a hypothetical, no-subsidy scenario by increasing the transaction costs by the amount servicers were entitled to under policy rules. The same assumption also allows me to quantify the total subsidy cost of foreclosure prevention to the government.

I simulate loan outcomes for each origination sample assuming that no subsidies are offered to mortgage servicers, and then compare them to the baseline case in which subsidies were available. I assume that the average loan subsidy awards $5,000 directly to a loan servicer for processing a modification (Hembre, 2018).

### 1. Effects on modifications and foreclosures

The counterfactual exercise suggests that modification subsidies did increase modifications and reduce foreclosures, but that the magnitude of effect varied across origination cohorts. Figure VIII reports the means and standard errors for the the percentage changes in the share of delinquent borrowers receiving a modification and the share of preventable foreclosures stopped. For loans originated in 2004, the mean result suggests that subsidies caused the share of delinquent borrowers receiving modifications to rise by 27.8% and that this in turn helped increase the share of preventable foreclosures stopped by about 13.2%. The share of delinquent loans that receive modification rises by even more in later cohorts, but the foreclosure

impact is less suggesting that modifications are less effective at preventing foreclosure for loans originated closer to 2008. For 2005, the counterfactual analysis suggests that there was a rise of 52.5% in the share of delinquent borrowers that received modification due to Federal subsidies but that there were was an only 8.6% rise in the share of preventable foreclosures stopped. For both 2006 and 2007, the change in the share of modifications rises by more than 45% and the share of preventable foreclosures stopped rises by less than 8%.

The differences between origination cohort outcomes are driven by both the underlying borrower financial health and mortgage servicer behavior. The mean value of keeping a home relative to defaulting progressively declines for loans originated later in the 2000's, at least partly expained by greater exposure to house price changes (Palmer, 2023). Estimated transaction costs of loan modification also differ within-servicer across estimation samples. For example, the results for Bank of America and Citimortgage suggest that two of the biggest loan servicers were more willing to offer loan modifications to the 2007 origination cohort than to any of the earlier ones. This might suggest that processing modifications was more labor-intensive or costly to complete for later cohort mortgages. It may also suggest differences in bank expectations about particular cohorts or knowledge that they held about borrower groups. High loan modification costs in the model may also reflect bank capacity constraints to process loan modifications driven by factors such as staff-shortages. A key takeaway is that subsidies alone may not be sufficient to influence servicer behavior if they are dealing with non-cost related impediments to processing modifications.

## 2. Welfare and optimal subsidies

A welfare calculation becomes necessary to assess modification subsidy policy as a means of foreclosure prevention. I am able to measure the joint welfare of mortgage borrowers and servicing institutions and weigh it against government subsidy spending and its associated social costs. The shadow cost of government spending plays an important role in assessing the effectiveness of a subsidy policy. I assume that the marginal cost of public funds for taxes on labor income ranges between $1.09 to $1.16 per dollar of tax revenue (Browning, 1976). In the context of my welfare analysis, the marginal cost of public funds will shift the marginal cost curve for subsidy spending upwards, reducing the optimal level of per-modification subsidy.

Residential mortgage foreclosures also create salient negative externalities that should also be factored into optimal policy design. Anenberg and Kung (2014) find that homes closest to a foreclosed property sell at a 1.6% lower price during the lender listing period, but that this effect only holds for properties within half a mile. Campbell et al. (2011) find similar local effects on prices and provide evidence that the negative effect can be as high as a 8.7% reduction in log-price for properties within 0.1-mile of the foreclosed property. Hembre (2018) assumes $16,000 as the social benefit to preventing a mortgage default that would end in foreclosure. Financial institutions will fail to internalize these local effects when designing borrower assistance programs, creating scope for welfare-improving policy intervention.

Figure IX presents my model implied optimal loan modification subsidies under different assumed levels for the social cost of government spending and third-party externality costs of foreclosure. The average of $5,000 per loan modification appears to exceed an optimal level for most cohorts, especially at higher assumed shadow costs of public funds. Nonetheless, optimal subsidies generally appear near $4,000, which provides justification for the level of support offered to mortgage institutions under HAMP. These results demonstrate the importance of accurate measures of foreclosure externalities when setting loan modification policy, and reiterate differences in the effectiveness of aid across loan origination cohorts. Appendix *V.C* provides further information on welfare calculations.

FIGURE IX
Optimal subsides with broader foreclosure externalities

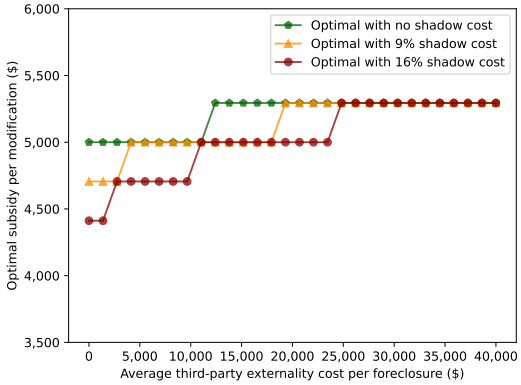



FIGURE IX
Q1-Q4 2004

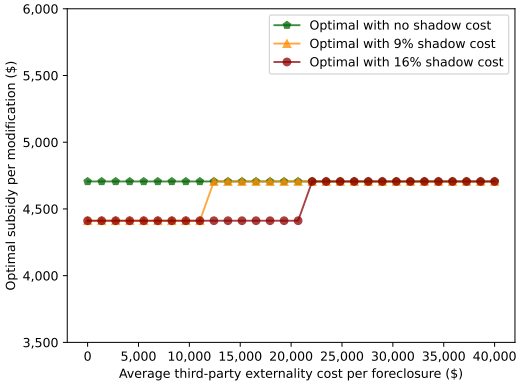



FIGURE IX
Q1-Q4 2005

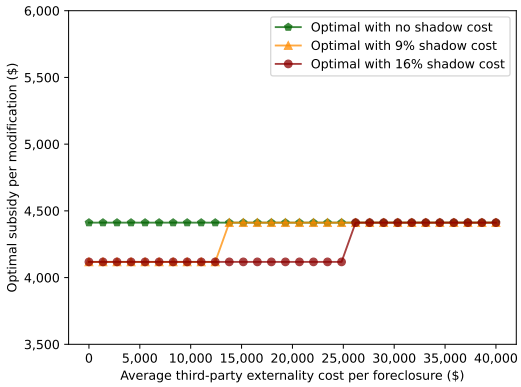



FIGURE IX
Q1-Q4 2006

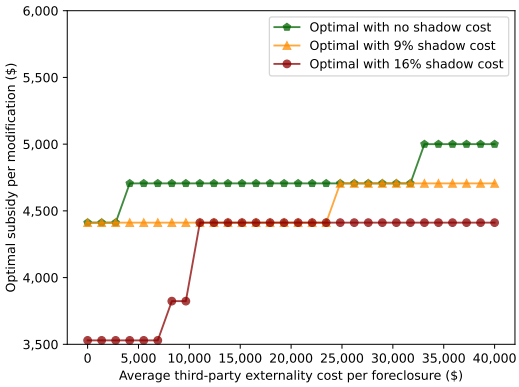



FIGURE IX
Q1-Q4 2007

*Notes:* This figure presents model-implied optimal subsidy policy based on a range of assumed shadow costs of government expenditure and foreclosure-induced cost externalities, by origination cohort. Each line reflects a different assumption for the level of government spending shadow costs with green, yellow, and red lines signifying no shadow costs, 9% shadow costs and 16% shadow costs respectively. For the 2004 sample, conservative assumptions on spending costs and externalities imply optimal subsidies exceed $4,500. For the later cohorts, lower shadow costs of government expenditure and higher externality assumptions are necessary to justify subsidies near $5,000 per modification. Appendix *V.C* provides further information on welfare calculations.

Collectively, the findings of this section support the use of loan modification subsidies for driving borrower assistance under the HAMP program, but highlight the importance of social costs of government spending and third-party externalities in policy-making. I tend to find that the benchmark of $5,000 of subsidy per loan modification may have exceeded socially optimal levels, but the "true" optimal level will differ based on the sample of loans and cost assumptions. Modification activity that fails to prevent foreclosure creates wasteful transaction costs as well as inefficiently diverts government funds that were costly to raise. For a fixed level of assistance, as in the HAMP program, deeply distressed borrowers cannot be brought out of foreclosure unless program generosity increases - yet increased generosity may also drive more strategic delinquency making aid targeting more difficult.

# VIII. CONCLUSION

In this paper I present a new empirical model to quantify the role of information asymmetry, transaction costs and relief sufficiency in foreclosure prevention efforts. The analysis offers a tractable approach for many other settings of debt renegotiation under imperfect information and threat of strategic agent behavior. I am able to reduce the computational burden involved with a multi-period, dynamic game with multiple players to tractably explore policy design in a principal-agent framework under commitment. I use this model to study how the distribution of the unobservable financial health of borrowers explains observed loan modification and foreclosure outcomes in my sample of Fannie Mae loans from their origination to the end of 2019. My results offer a comprehensive insight into how borrowers' financial wellbeing and the behavior of financial institutions influence debt relief policy.

I contribute several findings on the role of information in foreclosure prevention. The results highlight that borrower selection into delinquency and default are first order issues for policy design. The central implication is that financial institutions need to be highly informed about borrower default probabilities in order to offer loan modifications; greater information asymmetry between borrowers and their servicers leads to lower rates of modification and more foreclosures. I find that Fannie Mae loan servicers tended to be highly informed about borrower outcome probabilities in the wake of the 2008 financial crisis and that subsidies offered to banks under the program were more effective at preventing foreclosures in loans originated earlier in the 2000s, even though banks tended to be equally well informed about borrower financial health in all sample cohorts. In my welfare calculations, I find that average HAMP subsidies were generally above the social welfare maximizing level, but also that subsidy spending meaningfully improved welfare for vulnerable borrowers while having a limited effect on aggregate welfare.

My work also contributes valuable findings about mortgage borrower behavior. I find that borrower foreclosure costs are significant and larger than previously estimated in past research. My estimates suggest that borrowers are more responsive to changes in their debt burden than changes in the value of their home. I also find that there is meaningful variation in borrower financial health which is not explained by borrower or loan characteristics. The significant role of unobserved borrower heterogeneity in explaining default

supports recent mortgage literature that assigns the vast majority of mortgage defaults to adverse life events rather than negative equity alone (Ganong and Noel, 2020b; Low, 2022).

My findings open up several exciting new directions for academic inquiry. Debt relief programs typically face a trade-off between speedy delivery versus effective targeting. Faster delivery of aid often requires less screening of borrowers, which comes at the cost of poorly targeted relief that ends up being rewarded to borrowers who are less likely to need it. Better targeting of relief leads to better allocation of resources but also inherently requires more screening, leading to longer waits and potentially greater cost for organizations analyzing borrower credit-worthiness. Payroll Protection Program loans made out by the U.S. government in response to the COVID-19 pandemic tended to prioritize rapid disbursement, but have often been criticized for being poorly targeted across organizations (Bartik et al., 2020; Autor et al., 2022).[23] My study highlights and quantifies the importance of a lender's information about its borrowers. When policymakers rely on the voluntary participation of lenders to grant relief, then they must factor in the role of information for decision-making within these organizations. Less-informed organizations may not have a sufficient private incentive to grant loan modifications, leading to costlier and less effective incentive programs. Future projects can also look to better understand the source of servicer information and how best to improve the quantity of efficient modifications. There are many potential culprits for high costs of processing loan modifications which include servicer capacity constraints, poorly trained staff, and insufficient other resources.

*Acknowledgements.* I thank Dan Ackerberg, Eugenio Miravete, Sam Kruger, Victoria Marone, Jorge Balat, Robert Town and all other participants of the UT Austin Industrial Organization Writing Seminar for their guidance throughout this project. Additional thanks to Milena Almagro, James Brand, Dita Eckardt, Francisco Garrido, Carlos Garriga, Charles Hodgson, Mark Feldman, Anh Nguyen, Juan Sánchez and Shoshana Vasserman, as well as seminar participants at Toronto Rotman, Cornell SC Johnson, the Federal Reserve Bank of New York, the Federal Trade Commission, the Department of Justice, the Office of Financial Research, the 2022 Midwest Econometrics Group Conference, and the 2023 Stanford Institute for Theoretical Economics conference for feedback on my work. I acknowledge current and former UT PhDs Ivan Larsen, Cooper Howes, Justus Timmers, Richard Faltings, Ignacio Núñez and Lee Seltzer for their support during the darkest days of research amidst the COVID-19 pandemic. I need to give recognition to Xiaohan Wu, who worked tirelessly as my research assistant. I am grateful to Eric Schulman who helped me improve my coding skills over the course of my PhD program. I also thank the Finlandia Foundation for its generous financial support, which enabled me to successfully complete this study. Finally, I acknowledge the Texas Advanced Computing Center (TACC) at the University of Texas at Austin for providing high performance computing resources that have been essential to the results reported in this paper. All errors are my own.

[23] Popular media also gave significant coverage to poor loan targeting, see for example: New York Times (2020), *How Bad Was Virus Aid Fraud? One Banker Was 'Frustrated With Humanity'*, `https://www.nytimes.com/2020/12/09/business/ppp-fraud-paycheck-protection-program.html`.

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# A Online Appendix

## *I.A* Additional detail on loan modification eligibility

| TYPE OF HARDSHIP (CHECK ALL THAT APPLY) | REQUIRED HARDSHIP DOCUMENTATION |
|---|---|
| ☐ Unemployment | ▪ Not required |
| ☐ Reduction in income: a hardship that has caused a decrease in your income due to circumstances outside your control (e.g., elimination of overtime, reduction in regular working hours, a reduction in base pay) | ▪ Not required |
| ☐ Increase in housing-related expenses: a hardship that has caused an increase in your housing expenses due to circumstances outside your control (e.g., uninsured losses, increased property taxes, HOA special assessment) | ▪ Not required |
| ☐ Disaster (natural or man-made) impacting the property or borrower's place of employment | ▪ Not required |
| ☐ Long-term or permanent disability, or serious illness of a borrower/co-borrower or dependent family member | ▪ Written statement from the borrower, or other documentation verifying disability or illness<br>**Note**: Detailed medical information is not required, and information from a medical provider is not required |
| ☐ Divorce or legal separation | ▪ Final divorce decree or final separation agreement **OR**<br>▪ Recorded quitclaim deed |
| ☐ Separation of borrowers unrelated by marriage, civil union, or similar domestic partnership under applicable law | ▪ Recorded quitclaim deed **OR**<br>▪ Legally binding agreement evidencing that the non-occupying borrower or co-borrower has relinquished all rights to the property |
| ☐ Death of borrower or death of either the primary or secondary wage earner | ▪ Death certificate **OR**<br>▪ Obituary or newspaper article reporting the death |
| ☐ Distant employment transfer/relocation | ▪ **For active duty service members**: Permanent Change of Station (PCS) orders or letter showing transfer.<br>▪ **For employment transfers/new employment**: Copy of signed offer letter or notice from employer showing transfer to a new location or written explanation if employer documentation not applicable, **AND**<br>▪ Documentation that reflects the amount of any relocation assistance provided (not required for those with PCS orders) |
| ☐ Other – hardship that is not covered above: _____ | ▪ Written explanation describing the details of the hardship and any relevant documentation |

FIGURE X
Fannie Mae/Freddie Mac Form 710

## *I.B* 3-digit Zip codes and Counties

Fannie Mae's Single-Family loan performance dataset restricts geographic data to a 3-digit ZIP code and an MSA code for each individual loan. The figures above depict California's 3-digit ZIP code areas (left) and counties (right).

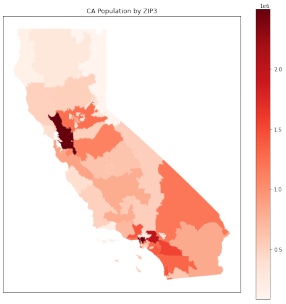



FIGURE XI
California 3-Digit Zip Codes

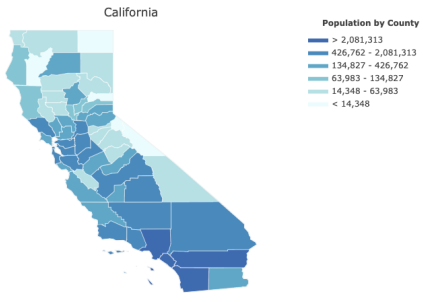



FIGURE XI
California Counties

FIGURE XI
California 3-Digit Zip codes and Counties Counties

## *I.C* Additional summary statistics

TABLE X
Cross cohort summaries

| | Q1-Q4 2004 Sample | | Q1-Q4 2005 Sample | | Q1-Q4 2006 Sample | | Q1-Q4 2007 Sample | |
|---|---|---|---|---|---|---|---|---|
| Observations | 43,546 | | 47,418 | | 31,105 | | 39,905 | |
| Share of loans entering delinquency | 10.85% | | 20.74% | | 31.90% | | 32.31% | |
| Share of loans receiving modification | 3.97% | | 8.35% | | 15.18% | | 16.13% | |
| Share of loans ending in foreclosure | 5.00% | | 11.63% | | 17.63% | | 17.31% | |
| **At origination** | | | | | | | | |
| | Q1-Q4 2004 Sample | | Q1-Q4 2005 Sample | | Q1-Q4 2006 Sample | | Q1-Q4 2007 Sample | |
| | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. |
| Principal balance, $000s | 211.27 | 72.77 | 238.53 | 76.06 | 262.87 | 91.67 | 269.63 | 91.22 |
| Home value, $000s | 378.85 | 164.81 | 426.96 | 164.34 | 449.49 | 166.66 | 440.19 | 168.42 |
| Loan-to-value | 59.86 | 17.39 | 59.21 | 16.65 | 61.00 | 16.69 | 64.33 | 17.25 |
| Debt-to-income ratio | 0.36 | 0.13 | 0.39 | 0.12 | 0.40 | 0.11 | 0.40 | 0.12 |
| Credit Score | 727.02 | 53.46 | 723.07 | 55.75 | 714.79 | 57.8 | 717.69 | 59.58 |
| **In January 2010** | | | | | | | | |
| | Q1-Q4 2004 Sample | | Q1-Q4 2005 Sample | | Q1-Q4 2006 Sample | | Q1-Q4 2007 Sample | |
| | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. |
| Principal balance, $000s | 190.31 | 66.99 | 221.17 | 71.47 | 249.88 | 87.84 | 260.41 | 88.81 |
| Home value, $000s | 348.74 | 176.65 | 300.69 | 142.12 | 280.86 | 130.54 | 286.65 | 132.81 |
| Loan-to-value | 0.62 | 0.23 | 0.82 | 0.29 | 0.98 | 0.33 | 1.00 | 0.32 |
| % change in HV Jan'08 to Jan'10 | -25.00 | 8.00 | -27.00 | 8.00 | -28.00 | 8.00 | -28.00 | 8.00 |

FIGURE XII
Select covariate distributions by loan servicer, Q1-Q4 2004

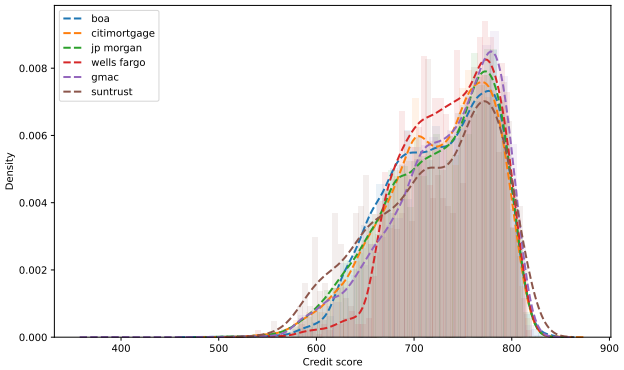



FIGURE XII
Credit score

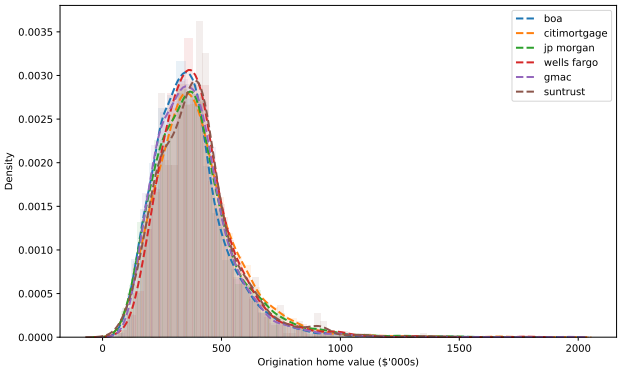



FIGURE XII
Home value at origination

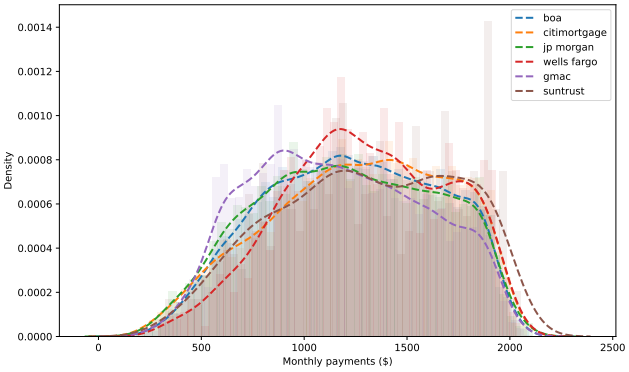



FIGURE XII
Monthly mortgage payments

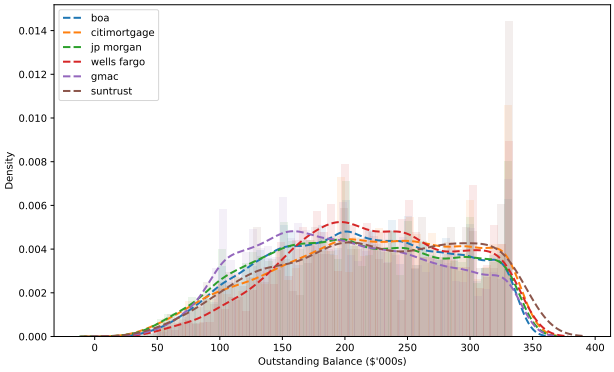



FIGURE XII
Principal balance at origination

*Notes:* The Figure shows kernel density plots of select borrower characteristics by servicer in the sample data. In general, Fannie Mae-affiliated servicers tend to manage observably similar types of borrowers.

FIGURE XIII
Select covariate distributions by loan servicer, Q1-Q4 2005

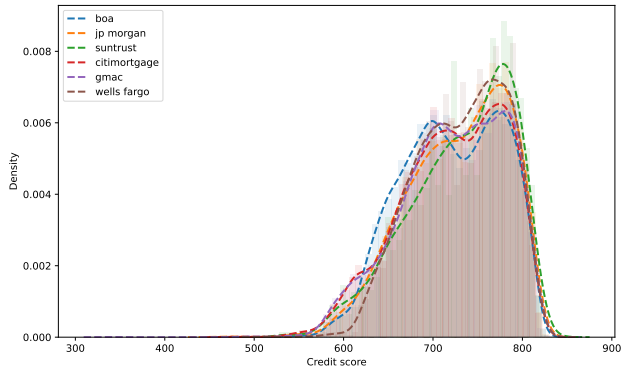



FIGURE XIII
Credit score

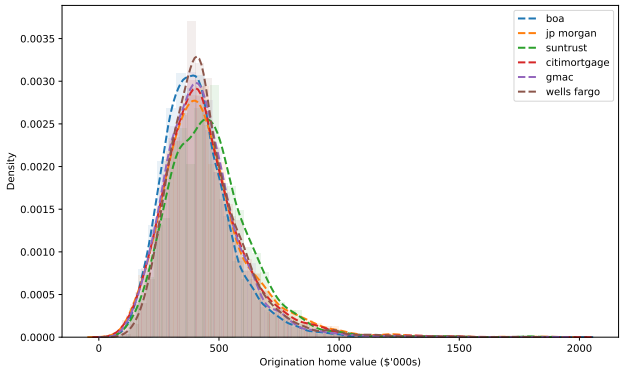



FIGURE XIII
Home value at origination

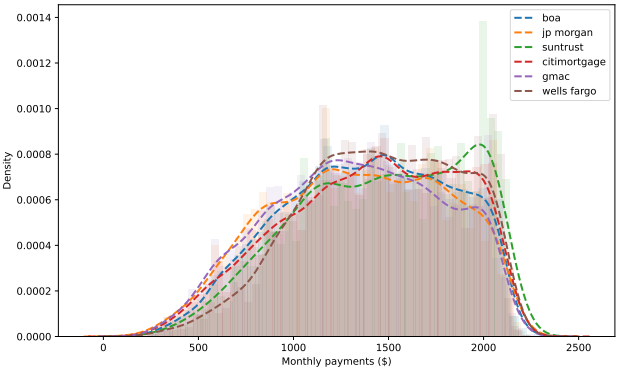



FIGURE XIII
Monthly mortgage payments

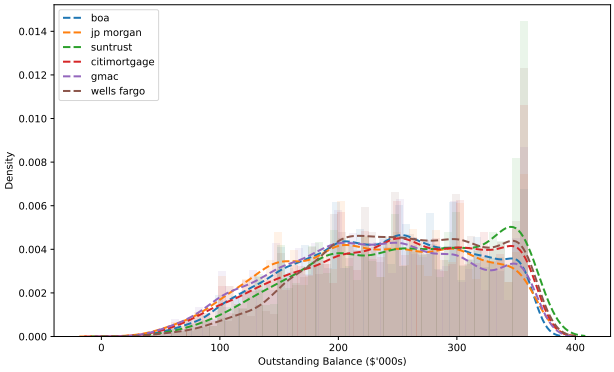



FIGURE XIII
Principal balance at origination

*Notes:* The Figure shows kernel density plots of select borrower characteristics by servicer in the sample data. In general, Fannie Mae-affiliated servicers tend to manage observably similar types of borrowers.

FIGURE XIV
Select covariate distributions by loan servicer, Q1-Q4 2006

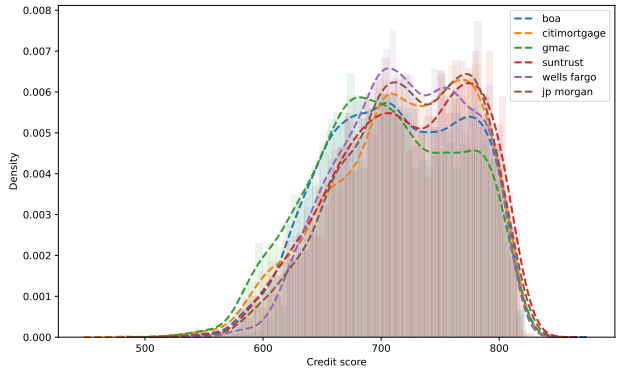



FIGURE XIV
Credit score

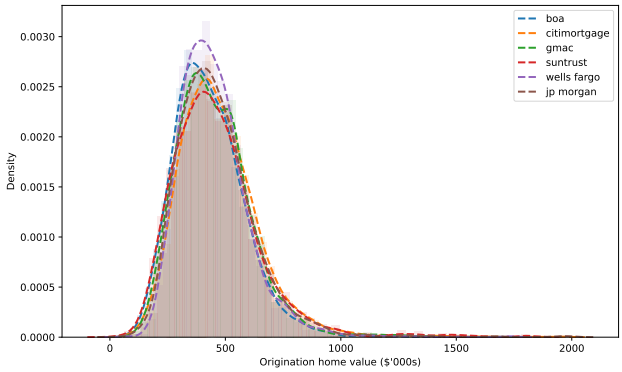



FIGURE XIV
Home value at origination

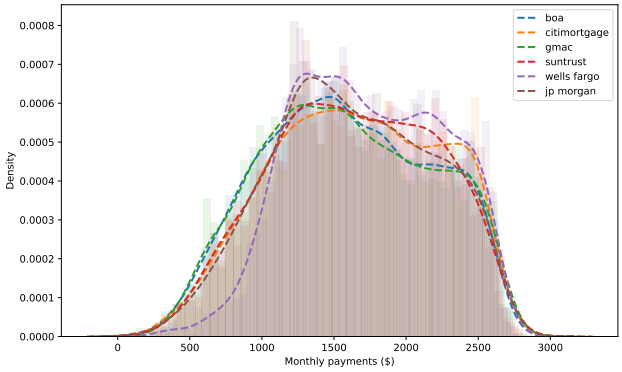



FIGURE XIV
Monthly mortgage payments

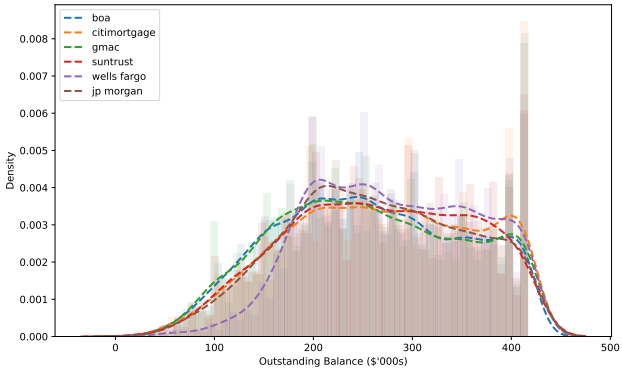



FIGURE XIV
Principal balance at origination

*Notes:* The Figure shows kernel density plots of select borrower characteristics by servicer in the sample data. In general, Fannie Mae-affiliated servicers tend to manage observably similar types of borrowers.

FIGURE XV
Select covariate distributions by loan servicer, Q1-Q4 2007

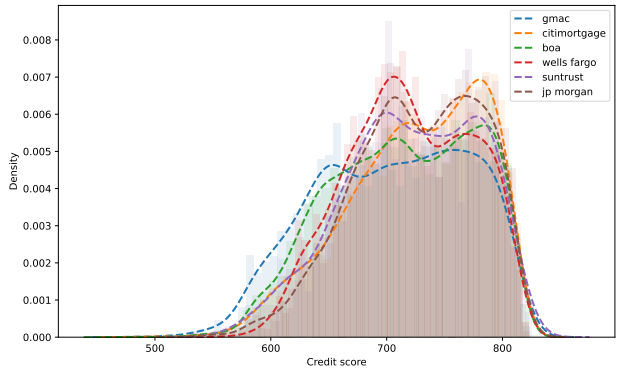



FIGURE XV
Credit score

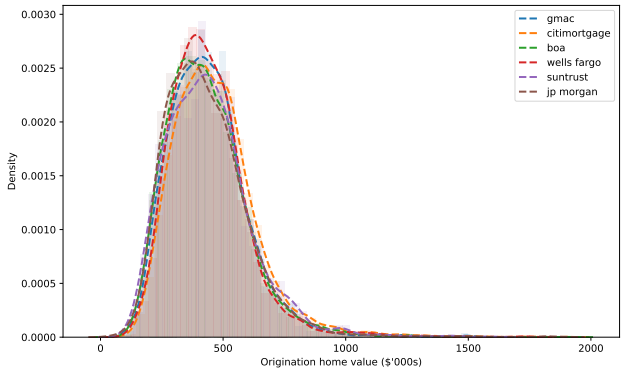



FIGURE XV
Home value at origination

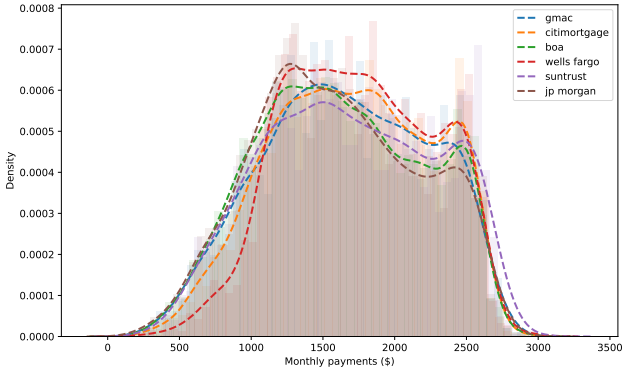



FIGURE XV
Monthly mortgage payments

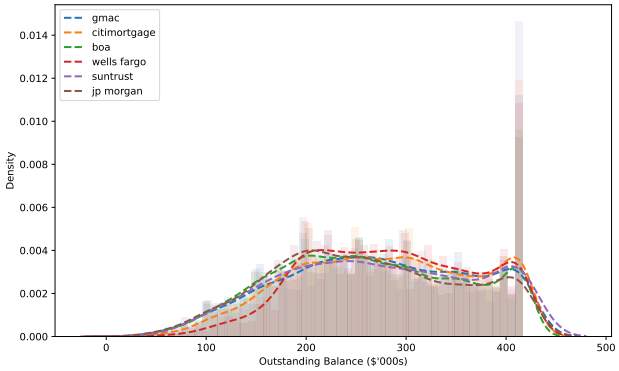



FIGURE XV
Principal balance at origination

*Notes:* The Figure shows kernel density plots of select borrower characteristics by servicer in the sample data. In general, Fannie Mae-affiliated servicers tend to manage observably similar types of borrowers.

## B Uncorrelatedness of $\xi$ and $\varepsilon$

**Proposition B.1.** *Because $\xi$ and $\varepsilon$ are assumed to be additively separable in the borrower's relative value from repaying their mortgage relative to a default, it is without loss of generality to assume that they are uncorrelated.*

*Proof.* Assume that $\xi$ and $\varepsilon$ are both correlated such that $\text{Cov}(\xi, \varepsilon) \neq 0$. We can always define new variables $\hat{\xi} := \xi + \mathbb{E}[\varepsilon|\xi]$ and $\hat{\varepsilon} := \varepsilon - \mathbb{E}[\varepsilon|\xi]$. Here we redefine $\hat{\xi}$ such that it contains all of the information that we could predict about $\varepsilon$ through their correlation. The $\hat{\varepsilon}$ is the remaining residual that cannot be predicted by knowledge about $\xi$. By construction, it is the case that:

$$\begin{aligned}\mathbb{E}[\hat{\varepsilon} \mid \xi] &= \mathbb{E}[\varepsilon - \mathbb{E}[\varepsilon \mid \xi] \mid \xi] \\ &= \mathbb{E}[\varepsilon \mid \xi] - \mathbb{E}[\mathbb{E}[\varepsilon \mid \xi]|\xi] \\ &= \mathbb{E}[\varepsilon \mid \xi] - \mathbb{E}[\varepsilon \mid \xi] \\ &= 0\end{aligned}$$

To complete the proof we want to show that $Cov(\hat{\xi}, \hat{\varepsilon}) = 0$. By definition:

$$\begin{aligned}\text{Cov}(\hat{\xi}, \hat{\varepsilon}) &= \text{Cov}(\xi + \mathbb{E}[\varepsilon \mid \xi], \varepsilon - \mathbb{E}[\varepsilon \mid \xi]) \\ &= \text{Cov}(\xi, \varepsilon) - \text{Cov}(\xi, \mathbb{E}[\varepsilon \mid \xi]) + \text{Cov}(\varepsilon, \mathbb{E}[\varepsilon \mid \xi]) - \text{Cov}(\mathbb{E}[\varepsilon \mid \xi], \mathbb{E}[\varepsilon \mid \xi])\end{aligned}$$

Each of these components can be re-written as (details below the proof):

$$\begin{aligned}&(1)\ \text{Cov}(\xi, \varepsilon) = \mathbb{E}[\xi \cdot \varepsilon] - \mathbb{E}[\xi]\mathbb{E}[\varepsilon] \\ &(2)\ \text{Cov}(\xi, \mathbb{E}[\varepsilon \mid \xi]) = \mathbb{E}[\xi \cdot \mathbb{E}[\varepsilon \mid \xi]] - \mathbb{E}[\xi]\mathbb{E}[\varepsilon] \\ &(3)\ \text{Cov}(\varepsilon, \mathbb{E}[\varepsilon \mid \xi]) = \mathbb{E}[\varepsilon \cdot \mathbb{E}[\varepsilon \mid \xi]] - \mathbb{E}[\varepsilon]^2 \\ &(4)\ \text{Cov}(\mathbb{E}[\varepsilon \mid \xi], \mathbb{E}[\varepsilon \mid \xi]) = Var(\mathbb{E}[\varepsilon \mid \xi]) = \mathbb{E}\left[\mathbb{E}[\varepsilon \mid \xi]^2\right] - \mathbb{E}[\varepsilon]^2\end{aligned}$$

$$\begin{aligned}\text{Cov}(\hat{\xi}, \hat{\varepsilon}) &= \mathbb{E}[\xi \cdot \varepsilon] - \mathbb{E}[\xi]\mathbb{E}[\varepsilon] - \mathbb{E}[\xi \cdot \mathbb{E}[\varepsilon \mid \xi]] + \mathbb{E}[\xi]\mathbb{E}[\varepsilon] + \mathbb{E}[\varepsilon \cdot \mathbb{E}[\varepsilon \mid \xi]] - \mathbb{E}[\varepsilon]^2 - \mathbb{E}\left[\mathbb{E}[\varepsilon \mid \xi]^2\right] + \mathbb{E}[\varepsilon]^2 \\ &= \mathbb{E}[\xi \cdot \varepsilon] - \mathbb{E}[\xi \cdot \mathbb{E}[\varepsilon \mid \xi]] + \mathbb{E}[\varepsilon \cdot \mathbb{E}[\varepsilon \mid \xi]] - \mathbb{E}\left[\mathbb{E}[\varepsilon \mid \xi]^2\right]\end{aligned}$$

We still need to simplify $Cov(\hat{\xi}, \hat{\varepsilon})$, note that by the law of iterated expectations:

$$\mathbb{E}[\xi \cdot \varepsilon] = \mathbb{E}[\mathbb{E}[\xi \cdot \varepsilon|\xi]] = \mathbb{E}[\xi \cdot \mathbb{E}[\varepsilon|\xi]]$$

Similarly, we can define $z := \varepsilon \cdot \mathbb{E}[\varepsilon \mid \xi]$ and leverage the law of iterated expectations in the fol-

lowing way:

$$\mathbb{E}[\varepsilon \cdot \mathbb{E}[\varepsilon \mid \xi] = \mathbb{E}[z] = \mathbb{E}[\mathbb{E}[z|\xi]] = \mathbb{E}[\mathbb{E}[\varepsilon \cdot \mathbb{E}[\varepsilon \mid \xi]|\xi]]]$$

Because we are again conditioning on $\xi$, the $\mathbb{E}[\varepsilon \mid \xi]$ object can be taken outside of the expectation resulting in:

$$\mathbb{E}[\mathbb{E}[\varepsilon \cdot \mathbb{E}[\varepsilon \mid \xi]|\xi]]] = \mathbb{E}[\mathbb{E}[\varepsilon \mid \xi]\mathbb{E}[\varepsilon \mid \xi]] = \mathbb{E}[\mathbb{E}[\varepsilon \mid \xi]^2]$$

Based on these two results, we have that:

$$\text{Cov}(\hat{\xi}, \hat{\varepsilon}) = \mathbb{E}[\xi \cdot \mathbb{E}[\varepsilon \mid \xi]] - \mathbb{E}[\xi \cdot \mathbb{E}[\varepsilon \mid \xi]] + \mathbb{E}\left[\mathbb{E}[\varepsilon \mid \xi]^2\right] - \mathbb{E}\left[\mathbb{E}[\varepsilon \mid \xi]^2\right] = 0$$

□

## Detail on covariance components

- $\text{Cov}(\xi, \varepsilon)$

$$\begin{aligned}
\text{Cov}(\xi, \varepsilon) &= \mathbb{E}[(\xi - \mathbb{E}(\xi))(\varepsilon - \mathbb{E}(\varepsilon))] \\
&= \mathbb{E}[\xi \cdot \varepsilon] - \mathbb{E}[\mathbb{E}[\xi]\varepsilon] - \mathbb{E}[\xi\mathbb{E}[\varepsilon]] + \mathbb{E}[\mathbb{E}[\xi]\mathbb{E}[\varepsilon]] \\
&= \mathbb{E}[\xi \cdot \varepsilon] - \mathbb{E}[\xi]\mathbb{E}[\varepsilon] - \mathbb{E}[\xi]\mathbb{E}[\varepsilon] + \mathbb{E}[\xi]\mathbb{E}[\varepsilon] \\
&= \mathbb{E}[\xi \cdot \varepsilon] - \mathbb{E}[\xi]\mathbb{E}[\varepsilon]
\end{aligned}$$

- $\text{Cov}(\xi, \mathbb{E}[\varepsilon \mid \xi])$

$$\begin{aligned}
\text{Cov}(\xi, \mathbb{E}[\varepsilon \mid \xi]) &= \mathbb{E}[(\xi - \mathbb{E}[\xi])(\mathbb{E}[\varepsilon \mid \xi] - \mathbb{E}[\mathbb{E}[\varepsilon \mid \xi]])] \\
&= \mathbb{E}[\xi \cdot \mathbb{E}[\varepsilon \mid \xi]] - \mathbb{E}[\xi]\mathbb{E}[\varepsilon] - \mathbb{E}[\xi]\mathbb{E}[\varepsilon] + \mathbb{E}[\xi]\mathbb{E}[\varepsilon] \\
&= \mathbb{E}[\xi \cdot \mathbb{E}[\varepsilon \mid \xi]] - \mathbb{E}[\xi]\mathbb{E}[\varepsilon]
\end{aligned}$$

- $\text{Cov}(\varepsilon, \mathbb{E}[\varepsilon \mid \xi])$

$$\begin{aligned}
\text{Cov}(\varepsilon, \mathbb{E}[\varepsilon \mid \xi]) &= \mathbb{E}[(\varepsilon - \mathbb{E}[\varepsilon])(\mathbb{E}[\varepsilon \mid \xi] - \mathbb{E}[\mathbb{E}[\varepsilon \mid \xi]])] \\
&= \mathbb{E}[\varepsilon \cdot \mathbb{E}[\varepsilon \mid \xi]] - \mathbb{E}[\mathbb{E}[\varepsilon]\mathbb{E}[\varepsilon \mid \xi]] - \mathbb{E}[\varepsilon \cdot \mathbb{E}[\varepsilon]] + \mathbb{E}[\varepsilon]^2 \\
&= \mathbb{E}[\varepsilon \cdot \mathbb{E}[\varepsilon \mid \xi]] - \mathbb{E}[\varepsilon]^2
\end{aligned}$$

- $\text{Cov}(\mathbb{E}[\varepsilon \mid \xi], \mathbb{E}[\varepsilon \mid \xi])$
  Same as variance, so by definition:

$$\text{Cov}(\mathbb{E}[\varepsilon \mid \xi], \mathbb{E}[\varepsilon \mid \xi]) = \text{Var}(\mathbb{E}[\varepsilon \mid \xi])) = \mathbb{E}[\mathbb{E}[\varepsilon \mid \xi]^2] - (\mathbb{E}[\mathbb{E}[\varepsilon \mid \xi]])^2 = \mathbb{E}[\mathbb{E}[\varepsilon \mid \xi]^2] - (\mathbb{E}[\varepsilon])^2$$

# C Truncated draws from the joint normal distribution

In this section, I derive expressions for the conditional probabilities of borrower delinquency and default outcomes under the parametric assumptions of the empirical model. These expressions rely on truncated distributions, as borrower types are defined by ex-post realizations that bound the unobservable error term once one conditions on outcomes and actions. I provide the full derivation of these conditional probabilities, which form the basis for likelihood evaluation in the main empirical analysis.

## Low type

### Type probability

For a borrower to be a low-type. It must be the case that he would prefer to default on his outstanding debt even if he received a loan modification:

$$\begin{aligned} p_i - H_i &> p_i - \tilde{p}_i \\ \tilde{p}_i - x_i'\beta - \xi_i - \varepsilon_i &> 0 \\ \Rightarrow \tilde{p}_i - x_i'\beta - \xi_i &> \varepsilon_i \end{aligned}$$

Given this, the probability of being the low-type depends only on the distribution of $\varepsilon$, $F$, and can be written as:

$$Pr(Low\ type|\tilde{p}_i, x_i, \xi_i; \theta) = F(\tilde{p}_i - x_i'\beta - \xi_i) \tag{4}$$

### Delinquency probability

Conditional on being a low-type, the probability of delinquency becomes more complicated due to the fact that we are truncating the distribution of $\varepsilon$ and there is a relationship between $\varepsilon$ and $\eta$ through the correlation term $\rho$.

The probability of delinquency given a low type is given:

$$\begin{aligned} &Pr(Delinquency|Low\ type, p_i, x_i, \xi_i; \theta) \\ &= Pr(p_i - x_i'\beta - \xi_i - \varepsilon_i^{Low} \geq x_i'\lambda + \eta_i^{Low}) \\ &= Pr(p_i - x_i'(\beta + \lambda) - \xi_i \geq \varepsilon_i^{Low} + \eta_i^{Low}) \end{aligned}$$

This now requires us to use the joint distribution $\varepsilon_i^{Low} + \eta_i^{Low} \sim \mathscr{G}^{Low}$.

$$Pr(Delinquency|Low\ type, p_i, \tilde{p}_i, x_i, \xi_i; \theta) = \mathscr{G}^{Low}(p_i - x_i'(\beta + \lambda) - \xi_i) \tag{5}$$

Since I assume the unobservables to be normally distributed then we can solve for the mean and standard deviation of this distribution,

defining $\underline{\mathscr{C}} := \tilde{p}_i - x_i'\beta - \xi_i$:

$$
\begin{aligned}
E[\varepsilon_i + \eta_i | \underline{\mathscr{C}} > \varepsilon] &= E[\varepsilon_i | \underline{\mathscr{C}} > \varepsilon] + E[\eta_i | \underline{\mathscr{C}} > \varepsilon] \\
&= \mu_\varepsilon - \sigma_\varepsilon \frac{\phi(\alpha)}{\Phi(\alpha)} + \mu_\eta - \rho\sigma_\eta \frac{\phi(\alpha)}{\Phi(\alpha)} \\
&\textit{With zero means} \\
&= -(\sigma_\varepsilon + \rho\sigma_\eta)\frac{\phi(\alpha)}{\Phi(\alpha)} \\
\textit{With } \alpha = \frac{\underline{\mathscr{C}} - \mu_\varepsilon}{\sigma_\varepsilon} &\textit{ and in our context } \alpha = \frac{\underline{\mathscr{C}}}{\sigma_\varepsilon}
\end{aligned}
$$

$$Var(\varepsilon_i + \eta_i | \underline{\mathscr{C}} > \varepsilon) = Var(\varepsilon_i | \underline{\mathscr{C}} > \varepsilon) + Var(\eta_i | \underline{\mathscr{C}} > \varepsilon) + 2Cov(\varepsilon_i, \eta_i | \underline{\mathscr{C}} > \varepsilon)$$

---

$$Var(\varepsilon_i | \underline{\mathscr{C}} > \varepsilon) = \sigma_\varepsilon^2 \left[ 1 - \alpha \cdot \frac{\phi(\alpha)}{\Phi(\alpha)} - \left( \frac{\phi(\alpha)}{\Phi(\alpha)} \right)^2 \right]$$

$$Var(\eta_i | \underline{\mathscr{C}} > \varepsilon) = \sigma_\eta^2 \left[ 1 - \rho^2 \left\{ \alpha \frac{\phi(\alpha)}{\Phi(\alpha)} + \left( \frac{\phi(\alpha)}{\Phi(\alpha)} \right)^2 \right\} \right]$$

$$
\begin{aligned}
Cov(\varepsilon_i, \eta_i | \underline{\mathscr{C}} > \varepsilon) &= E[\varepsilon_i \cdot \eta_i | \underline{\mathscr{C}} > \varepsilon_i] - E[\varepsilon_i | \underline{\mathscr{C}} > \varepsilon_i] \cdot E[\eta_i | \underline{\mathscr{C}} > \varepsilon_i] \\
&\textit{Leverage law of total expectation} \\
&= E[E[\varepsilon_i \cdot \eta_i | \varepsilon_i] | C > \varepsilon_i] - E[\varepsilon_i | \underline{\mathscr{C}} > \varepsilon_i] \cdot E[\eta_i | \underline{\mathscr{C}} > \varepsilon_i] \\
&= E[\varepsilon_i E[\eta_i | \varepsilon_i] | C > \varepsilon_i] - E[\varepsilon_i | \underline{\mathscr{C}} > \varepsilon_i] \cdot E[\eta_i | \underline{\mathscr{C}} > \varepsilon_i] \\
&= E[\rho \frac{\sigma_\eta}{\sigma_\varepsilon} \varepsilon_i^2 | C > \varepsilon_i] - E[\varepsilon_i | \underline{\mathscr{C}} > \varepsilon_i] \cdot E[\eta_i | \underline{\mathscr{C}} > \varepsilon_i] \\
&= \rho \frac{\sigma_\eta}{\sigma_\varepsilon} E[\varepsilon_i^2 | \underline{\mathscr{C}} > \varepsilon_i] - E[\varepsilon_i | \underline{\mathscr{C}} > \varepsilon_i] \cdot E[\eta_i | \underline{\mathscr{C}} > \varepsilon_i] \\
&= \rho \frac{\sigma_\eta}{\sigma_\varepsilon} \left[ \underbrace{\sigma_\varepsilon^2 \left\{ 1 - \alpha \frac{\phi(\alpha)}{\Phi(\alpha)} - \left( \frac{\phi(\alpha)}{\Phi(\alpha)} \right)^2 \right\}}_{Var[\varepsilon_i | \underline{\mathscr{C}} > \varepsilon]} + \underbrace{\sigma_\varepsilon^2 \left( \frac{\phi(\alpha)}{\Phi(\alpha)} \right)^2}_{(E[\varepsilon | \underline{\mathscr{C}} > \varepsilon])^2} \right] - \rho\sigma_\varepsilon\sigma_\eta \left( \frac{\phi(\alpha)}{\Phi(\alpha)} \right)^2 \\
&= \rho\sigma_\varepsilon\sigma_\eta \left[ 1 - \alpha \frac{\phi(\alpha)}{\Phi(\alpha)} \right] - \rho\sigma_\varepsilon\sigma_\eta \left( \frac{\phi(\alpha)}{\Phi(\alpha)} \right)^2 \\
&= \rho\sigma_\varepsilon\sigma_\eta \left[ 1 - \alpha \frac{\phi(\alpha)}{\Phi(\alpha)} - \left( \frac{\phi(\alpha)}{\Phi(\alpha)} \right)^2 \right]
\end{aligned}
$$

---

Putting all of these pieces together, we can write down:

$$
\begin{aligned}
&Var(\varepsilon_i + \eta_i | \underline{\mathscr{C}} > \varepsilon) \\
&= (\sigma_\varepsilon^2 + 2\rho\sigma_\varepsilon\sigma_\eta)\left[1 - \alpha\frac{\phi(\alpha)}{\Phi(\alpha)} - \left(\frac{\phi(\alpha)}{\Phi(\alpha)}\right)^2\right] \\
&\quad + \sigma_\eta^2\left[1 - \rho^2\left\{\alpha\frac{\phi(\alpha)}{\Phi(\alpha)} + \left(\frac{\phi(\alpha)}{\Phi(\alpha)}\right)^2\right\}\right] \\
&With\ \alpha = \frac{\underline{\mathscr{C}} - \mu_\varepsilon}{\sigma_\varepsilon}\ and\ in\ our\ context\ \alpha = \frac{\underline{\mathscr{C}}}{\sigma_\varepsilon}
\end{aligned}
\tag{6}
$$

Putting these together, we can evaluate the probability of low type delinquency with the following equation:

$$
\begin{aligned}
\mathscr{G}^{Low} \sim \mathscr{N}\Bigg( &-(\sigma_\varepsilon + \rho\sigma_\eta)\frac{\phi(\alpha)}{\Phi(\alpha)}, \\
&(\sigma_\varepsilon^2 + 2\rho\sigma_\varepsilon\sigma_\eta)\left[1 - \alpha\frac{\phi(\alpha)}{\Phi(\alpha)} - \left(\frac{\phi(\alpha)}{\Phi(\alpha)}\right)^2\right] \\
&+ \sigma_\eta^2\left[1 - \rho^2\left\{\alpha\frac{\phi(\alpha)}{\Phi(\alpha)} + \left(\frac{\phi(\alpha)}{\Phi(\alpha)}\right)^2\right\}\right]\Bigg)
\end{aligned}
$$

## Medium type

### Type probability

This is an intermediate type that has: $p_i - \tilde{p}_i > p_i - H_i > 0$. This inequality implies two things:

1. **No default with modification** $\varepsilon_i > \tilde{p}_i - x_i'\beta - \xi_i$
2. **Default without modification** $p_i - x_i'\beta - \xi_i > \varepsilon_i$

The probability of begin the middle type again depends on the distribution of $\varepsilon$, F and can be written as:

$$
Pr(Medium\ type | p_i, \tilde{p}_i, x_i, \xi_i; \theta) = F(p_i - x_i'\beta - \xi_i) - F(\tilde{p}_i - x_i'\beta - \xi_i) \tag{7}
$$

### Delinquency probability

The medium type borrower enters delinquency whenever:

$$
\begin{aligned}
&m_i \cdot (p_i - \tilde{p}_i) + (1 - m_i) \cdot (p_i - x_i'\beta - \xi_i - \varepsilon_i) \geq x_i'\lambda + \eta_i \\
&\Rightarrow m_i \cdot (p_i - \tilde{p}_i) + (1 - m_i) \cdot (p_i - x_i'\beta - \xi_i) - x_i'\lambda \geq (1 - m_i) \cdot \varepsilon_i + \eta_i \\
&Pr(Delinquency | Medium\ type, m_i, p_i, \tilde{p}_i, x_i, \xi_i; \theta) = \\
&\qquad Pr(m_i \cdot (p_i - \tilde{p}_i) + (1 - m_i) \cdot (p_i - x_i'\beta - \xi_i) - x_i'\lambda \geq (1 - m_i) \cdot \varepsilon_i + \eta_i)
\end{aligned}
$$

This now requires us to use the joint distribution $(1-m_i)\cdot\varepsilon_i^{Med}+\eta_i^{Med}\sim\mathscr{G}^{Med}$.

Now working with the truncation above and below:

$$p_i - x_i'\beta - \xi_i > \varepsilon_i > \tilde{p}_i - x_i'\beta - \xi_i$$

Define $\bar{\mathscr{C}} := p_i - x_i'\beta - \xi_i$ and $\underline{\mathscr{C}} := \tilde{p}_i - x_i'\beta - \xi_i$
Re-writing the truncation as $\bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}$. Also define: $\bar{\alpha}^{Med} = \frac{\bar{\mathscr{C}}}{\sigma_\varepsilon}$ $\underline{\alpha}^{Med} = \frac{\underline{\mathscr{C}}}{\sigma_\varepsilon}$.

$$E[(1-m_i)\varepsilon_i+\eta_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}] = (1-m_i)\cdot E[\varepsilon_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]+E[\eta_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]$$

*Applying law of total expectation for the second expectation:*

$$= (1-m_i)\cdot\left(-\sigma_\varepsilon\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)-\rho\sigma_\eta\left(\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)$$

$$= -\left((1-m_i)\,\sigma_\varepsilon+\rho\,\sigma_\eta\right)\cdot\left(\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)$$

*OR ALTERNATIVELY FLIPPING NUMERATOR WITH PDFS:*

$$= \left((1-m_i)\,\sigma_\varepsilon+\rho\,\sigma_\eta\right)\cdot\left(\frac{\phi(\underline{\alpha}^{Med})-\phi(\bar{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)$$

$$Var[(1-m_i)\varepsilon_i+\eta_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]$$
$$=(1-m_i)^2 Var[\varepsilon_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]+Var[\eta_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]+2(1-m_i)Cov[\varepsilon_i,\eta_i||\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]$$

---

$$Var[\varepsilon_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]=\sigma_\varepsilon^2\left[1-\frac{\bar{\alpha}^{Med}\phi(\bar{\alpha}^{Med})-\underline{\alpha}^{Med}\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}-\left(\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)^2\right]$$

*This equation is taken from established results on truncated distributions. John Burkardt's 2023 notes have this equation.*

$$Var[\eta_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]$$

*By the law of total variance...*

$$=\underbrace{E[Var(\eta_i|\varepsilon_i)|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]}_{\textit{Unaffected by the truncation}}+Var[E[\eta_i|\varepsilon_i]|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]$$
$$=\sigma_\eta^2(1-\rho^2)+Var[\rho\frac{\sigma_\eta}{\sigma_\varepsilon}(\varepsilon_i)|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]$$
$$=\sigma_\eta^2(1-\rho^2)+(\rho\frac{\sigma_\eta}{\sigma_\varepsilon})^2 Var[\varepsilon_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]$$
$$=\sigma_\eta^2(1-\rho^2)+\sigma_\eta^2\rho^2\underbrace{\left[1-\frac{\bar{\alpha}^{Med}\phi(\bar{\alpha}^{Med})-\underline{\alpha}^{Med}\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}-\left(\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)^2\right]}_{\frac{Var[\varepsilon_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}]}{\sigma_\varepsilon^2}}$$

$$
\begin{aligned}
&Cov[\varepsilon_i, \eta_i || \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] \\
&\quad = E[\varepsilon_i \cdot \eta_i | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] - \underbrace{E[\varepsilon_i | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] \cdot E[\eta_i | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}]}_{\textit{Established above!}}
\end{aligned}
$$

---

*Focusing on the* $E[\varepsilon_i \cdot \eta_i | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}]$ *term...*

*Leveraging law of total expectation*

$$
\begin{aligned}
&= E[\varepsilon E[\eta|\varepsilon]] | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] = E[\rho \sigma_\eta \frac{\varepsilon_i^2}{\sigma_\varepsilon} | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] \\
&= \frac{\rho \sigma_\eta}{\sigma_\varepsilon} E[\varepsilon_i^2 | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] = \frac{\rho \sigma_\eta}{\sigma_\varepsilon} \left[ Var[\varepsilon_i | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] + (E[\varepsilon_i | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}])^2 \right] \\
&= \frac{\rho \sigma_\eta}{\sigma_\varepsilon} \left( \sigma_\varepsilon^2 \left[ 1 - \frac{\bar{\alpha}^{Med} \phi(\bar{\alpha}^{Med}) - \underline{\alpha}^{Med} \phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})} - \left( \frac{\phi(\bar{\alpha}^{Med}) - \phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})} \right)^2 \right] \right. \\
&\qquad \left. + \sigma_\varepsilon^2 \left( \frac{\phi(\bar{\alpha}^{Med}) - \phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})} \right)^2 \right) \\
&= \rho \sigma_\varepsilon \sigma_\eta \left( 1 - \frac{\bar{\alpha}^{Med} \phi(\bar{\alpha}^{Med}) - \underline{\alpha}^{Med} \phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})} \right)
\end{aligned}
$$

---

Bringing this together, then we have:

$$
\begin{aligned}
&Cov[\varepsilon_i, \eta_i || \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] \\
&\quad = E[\varepsilon_i \cdot \eta_i | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] - E[\varepsilon_i | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] \cdot E[\eta_i | \bar{\mathscr{C}} > \varepsilon_i > \underline{\mathscr{C}}] \\
&\quad = \rho \sigma_\varepsilon \sigma_\eta \left( 1 - \frac{\bar{\alpha}^{Med} \phi(\bar{\alpha}^{Med}) - \underline{\alpha}^{Med} \phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})} \right) - \rho \sigma_\varepsilon \sigma_\eta \left( \frac{\phi(\bar{\alpha}^{Med}) - \phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})} \right)^2 \\
&\quad = \rho \sigma_\varepsilon \sigma_\eta \left( 1 - \frac{\bar{\alpha}^{Med} \phi(\bar{\alpha}^{Med}) - \underline{\alpha}^{Med} \phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})} - \left( \frac{\phi(\bar{\alpha}^{Med}) - \phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})} \right)^2 \right)
\end{aligned}
$$

$$
\begin{aligned}
&Var[(1-m_i)\varepsilon_i+\eta_i|\bar{\mathscr{C}}>\varepsilon_i>\underline{\mathscr{C}}] \\
&\quad=(1-m_i)^2\sigma_\varepsilon^2\left[1-\frac{\bar{\alpha}^{Med}\phi(\bar{\alpha}^{Med})-\underline{\alpha}^{Med}\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}-\left(\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)^2\right] \\
&\qquad+\sigma_\eta^2(1-\rho^2)+\sigma_\eta^2\rho^2\left[1-\frac{\bar{\alpha}^{Med}\phi(\bar{\alpha}^{Med})-\underline{\alpha}^{Med}\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}-\left(\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)^2\right] \\
&\qquad+2(1-m_i)\rho\sigma_\varepsilon\sigma_\eta\left[1-\frac{\bar{\alpha}^{Med}\phi(\bar{\alpha}^{Med})-\underline{\alpha}^{Med}\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}-\left(\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)^2\right] \\
&\quad=\sigma_\eta^2(1-\rho^2)+\left\{(1-m_i)^2\sigma_\varepsilon^2+\sigma_\eta^2\rho^2+2(1-m_i)\sigma_\varepsilon\sigma_\eta\right\}\left[1-\frac{\bar{\alpha}^{Med}\phi(\bar{\alpha}^{Med})-\underline{\alpha}^{Med}\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right. \\
&\qquad\qquad\left.-\left(\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)^2\right]
\end{aligned}
$$

Full distribution is given by:

$$
\begin{aligned}
\mathscr{G}^{Med}\sim\mathscr{N}\Bigg(&-((1-m_i)\,\sigma_\varepsilon+\rho\,\sigma_\eta)\cdot\left(\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right), \\
&\sigma_\eta^2(1-\rho^2)+\left\{(1-m_i)^2\sigma_\varepsilon^2+\sigma_\eta^2\rho^2+2(1-m_i)\sigma_\varepsilon\sigma_\eta\right\} \\
&\qquad\times\left[1-\frac{\bar{\alpha}^{Med}\phi(\bar{\alpha}^{Med})-\underline{\alpha}^{Med}\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}-\left(\frac{\phi(\bar{\alpha}^{Med})-\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med})-\Phi(\underline{\alpha}^{Med})}\right)^2\right]\Bigg)
\end{aligned}
$$

## High type

### Type probability

For a borrower to be the high-type, it must be the case that he would prefer to pay his outstanding debt even if he does not receive a loan modification:

$$\begin{aligned} 0 &> p_i - H_i \\ 0 &> p_i - x_i'\beta - \xi_i - \varepsilon_i \\ \Rightarrow \varepsilon_i &> p_i - x_i'\beta - \xi_i \end{aligned}$$

The probability of being the high-type depends only on the distribution of $\varepsilon$, $F$, and can be written as:

$$Pr(High\ type|p_i, x_i, \xi_i; \theta) = 1 - F(p_i - x_i'\beta - \xi_i) \tag{8}$$

### Delinquency probability

The conditional probability of delinquency for the high-type does not explicitly feature the $\varepsilon_i$, though it still affects the $\eta_i$ through their joint distribution. The probability of delinquency given a high-type is given:

$$\begin{aligned} &Pr(Delinquency|High\ type, p_i, x_i, \xi_i; \theta) \\ &= Pr(m_i \cdot (p_i - \tilde{p}_i) \geq x_i'\lambda + \eta_i^{High}) \\ &= Pr(m_i \cdot (p_i - \tilde{p}_i) - x_i'\lambda \geq \eta_i^{High}) \end{aligned}$$

If we define the truncated distribution of $\eta_i$ as $\mathscr{G}^{High}$ then this probability can be evaluated as: $\mathscr{G}^{High}(m_i \cdot (p_i - \tilde{p}_i) - x_i'\lambda)$. We can again solve for the mean and standard deviation of this distribution, which is more straightforward than the previous calculations and directly follows the moments of the incidentally truncated bivariate normal distribution (Green Theorem 22.5).

Define $\bar{C}(X_i) := p_i - x_i'\beta - \xi_i$ and $\alpha^H := \frac{\bar{C}(X_i)}{\sigma_\varepsilon}$.

$$\begin{aligned} E[\eta_i|\varepsilon_i > \bar{C}(X_i)] &= \rho\sigma_\eta \frac{\phi(\alpha^H)}{1 - \Phi(\alpha^H)} \\ Var[\eta_i|\varepsilon_i > \bar{C}(X_i)] &= \sigma_\eta^2 \left[1 - \rho^2 \left\{ \left( \frac{\phi(\alpha^H)}{1 - \Phi(\alpha^H)} \right)^2 - \frac{\alpha^H \phi(\alpha^H)}{1 - \Phi(\alpha^H)} \right\} \right] \end{aligned}$$

Putting these together, we can evaluate the probability of high type delinquency with the following equation:

$$\mathscr{G}^{High} \sim \mathscr{N}\left( \rho\sigma_\eta \frac{\phi(\alpha^H)}{1 - \Phi(\alpha^H)}, \sigma_\eta^2 \left[1 - \rho^2 \left\{ \left( \frac{\phi(\alpha^H)}{1 - \Phi(\alpha^H)} \right)^2 - \frac{\alpha^H \phi(\alpha^H)}{1 - \Phi(\alpha^H)} \right\} \right] \right)$$

No simulation is required for the servicer.

## Type-delinquency distribution summary

For all types, $F \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.

### Low type

$$Pr(Low|\tilde{p}_i, x_i, \xi_i; \theta) = F(\tilde{p}_i - x_i'\beta - \xi_i) \tag{9}$$

$$Pr(Dlq|Low, \cdot; \theta) = \mathscr{G}^{Low}(p_i - x_i'(\beta + \lambda) - \xi_i) \tag{10}$$

$$\mathscr{G}^{Low} \sim \mathcal{N}\Bigg( -(\sigma_\varepsilon + \rho\sigma_\eta)\frac{\phi(\alpha^L)}{\Phi(\alpha^L)}, \\ (\sigma_\varepsilon^2 + 2\rho\sigma_\varepsilon\sigma_\eta)\left[1 - \alpha^L\frac{\phi(\alpha^L)}{\Phi(\alpha^L)} - \left(\frac{\phi(\alpha^L)}{\Phi(\alpha^L)}\right)^2\right] + \sigma_\eta^2\left[1 - \rho^2\left\{\alpha^L\frac{\phi(\alpha^L)}{\Phi(\alpha^L)} + \left(\frac{\phi(\alpha^L)}{\Phi(\alpha^L)}\right)^2\right\}\right]\Bigg)$$

$\alpha^L = \frac{\mathscr{C}}{\sigma_\varepsilon}$. $\mathscr{C}$ is the upper bound for $\varepsilon_i$.

### Medium type

$$Pr(Medium|p_i, \tilde{p}_i, x_i, \xi_i; \theta) = F(p_i - x_i'\beta - \xi_i) - F(\tilde{p}_i - x_i'\beta - \xi_i) \tag{11}$$

$$Pr(Dlq|Medium, \cdot; \theta) = \mathscr{G}^{Med}(m_i \cdot (p_i - \tilde{p}_i) + (1 - m_i) \cdot (p_i - x_i'\beta - \xi_i) - x_i'\lambda) \tag{12}$$

$$\mathscr{G}^{Med} \sim \mathcal{N}\Bigg( -((1 - m_i)\,\sigma_\varepsilon + \rho\sigma_\eta) \cdot \left(\frac{\phi(\bar{\alpha}^{Med}) - \phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})}\right), \\ \sigma_\eta^2(1 - \rho^2) + \left\{(1 - m_i)^2\sigma_\varepsilon^2 + \sigma_\eta^2\rho^2 + 2(1 - m_i)\sigma_\varepsilon\sigma_\eta\right\} \\ \times\left[1 - \frac{\bar{\alpha}^{Med}\phi(\bar{\alpha}^{Med}) - \underline{\alpha}^{Med}\phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})} - \left(\frac{\phi(\bar{\alpha}^{Med}) - \phi(\underline{\alpha}^{Med})}{\Phi(\bar{\alpha}^{Med}) - \Phi(\underline{\alpha}^{Med})}\right)^2\right]\Bigg)$$

With $\bar{\alpha}^{Med} = \frac{\bar{\mathscr{C}}}{\sigma_\varepsilon}$ $\underline{\alpha}^{Med} = \frac{\underline{\mathscr{C}}}{\sigma_\varepsilon}$. $\bar{\mathscr{C}}$ is the upper bound for $\varepsilon_i$ and $\underline{\mathscr{C}}$ is the lower bound for $\varepsilon_i$.

### High type

$$Pr(High|p_i, x_i, \xi_i; \theta) = 1 - F(p_i - x_i'\beta - \xi_i) \tag{13}$$

$$Pr(Dlq|High, \cdot; \theta) = \mathscr{G}^{High}(m_i \cdot (p_i - \tilde{p}_i) - x_i'\lambda) \tag{14}$$

$$\mathscr{G}^{High} \sim \mathcal{N}\left(\rho\sigma_\eta\frac{\phi(\alpha^H)}{1 - \Phi(\alpha^H)}, \sigma_\eta^2\left[1 - \rho^2\left\{\left(\frac{\phi(\alpha^H)}{1 - \Phi(\alpha^H)}\right)^2 - \frac{\alpha^H\phi(\alpha^H)}{1 - \Phi(\alpha^H)}\right\}\right]\right)$$

Where $\bar{C}(X_i) := p_i - x_i'\beta - \xi_i$ and $\alpha^H := \frac{\bar{C}(X_i)}{\sigma_\varepsilon}$. $\bar{C}$ is the lower bound for $\varepsilon_i$.

# D Planner type expectations

The planner considers more payoff than the servicers when making their modification decisions. This introduces complexity in the expected unobservable draws. This section formalizes the math behind this.

## *IV.A* Low type

We need to form the expectation of home-value for low-types in the case that they avoid delinquency. Defining $\underline{\mathscr{C}} := \tilde{p}_i - x_i'\beta - \xi_i$ and
$\mathscr{D}^{Low}(X,\Theta) := p_i - x_i'(\beta+\lambda) - \xi_i$ this will be given by solving:

$$E[\varepsilon_i | \varepsilon_i < \underline{\mathscr{C}}, \varepsilon_i^{Low} + \eta_i^{Low} > \mathscr{D}^{Low}]$$

Noting again that in this equation, the low-type borrower avoids delinquency.

$$(\varepsilon,\eta) \sim \mathscr{N}_2(\mu,\Sigma), \text{ with } \mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \Sigma_{\varepsilon,\eta} = \begin{bmatrix} \sigma_\varepsilon^2 & \rho\sigma_\varepsilon\sigma_\eta \\ \rho\sigma_\varepsilon\sigma_\eta & \sigma_\eta^2 \end{bmatrix}$$

In a change of variables, define: $U = \varepsilon$ and $V = \varepsilon + \eta$ then the condition we are looking for becomes:

$$E[U | U < \underline{\mathscr{C}}, V > \mathscr{D}^{Low}]$$

Properties of $U$ and $V$ are straightforward to derive and are given by:

- E[U] = E[V] = 0
- $Var(U) = \sigma_\varepsilon^2$
- $Var(V) = \sigma_\varepsilon^2 + 2\rho\sigma_\varepsilon\sigma_\eta + \sigma_\eta^2$
- $Cov(U,V) = \sigma_\varepsilon^2 + \rho\sigma_\varepsilon\sigma_\eta$

$$\Sigma_{U,V} = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_\varepsilon^2 + \rho\sigma_\varepsilon\sigma_\eta \\ \sigma_\varepsilon^2 + \rho\sigma_\varepsilon\sigma_\eta & \sigma_\varepsilon^2 + 2\rho\sigma_\varepsilon\sigma_\eta + \sigma_\eta^2 \end{bmatrix}$$

Standardizing variables for the evaluation of probabilities:

$$Z_1 = \frac{U}{\sigma_\varepsilon}, Z_2 = \frac{V}{\sigma_V}$$

With $\sigma_V = \sqrt{\sigma_\varepsilon^2 + 2\rho\sigma_\varepsilon\sigma_\eta + \sigma_\eta^2}$

Also define : $a = \frac{\underline{\mathscr{C}}}{\sigma_\varepsilon}, b = \frac{\mathscr{D}^{Low}}{\sigma_V}, \rho^* = \frac{Cov(U,V)}{\sigma_\varepsilon \sigma_V} = \frac{\sigma_\varepsilon + \rho\varepsilon_\eta}{\sigma_V} = \frac{\sigma_\varepsilon + \rho\varepsilon_\eta}{\sqrt{\sigma_\varepsilon^2 + 2\rho\sigma_\varepsilon\sigma_\eta + \sigma_\eta^2}}$

$$(Z_1, Z_2) \sim \mathscr{N}_2(0, \Sigma_{Z1,Z2}), \text{ with } \Sigma_{\varepsilon,\eta} = \begin{bmatrix} 1 & \rho^* \\ \rho^* & 1 \end{bmatrix}$$

Under this standardization:

$$E[\varepsilon_i | \varepsilon_i < \underline{\mathscr{C}}, \varepsilon_i^{Low} + \eta_i^{Low} > \mathscr{D}^{Low}] = \sigma_\varepsilon \cdot \mathbb{E}[Z_1 \mid Z_1 < a, Z_2 > b]$$

We can now solve for our expectation under the standardized case:

$$\mathbb{E}[Z_1 \mid Z_1 < a, Z_2 > b] = \frac{1}{P} \int_{-\infty}^{a} \underbrace{z \cdot \phi(z)}_{Expected\ val.} \cdot \underbrace{\left[ 1 - \Phi\left( \frac{b - \rho^* z}{\sqrt{1 - (\rho^*)^2}} \right) \right]}_{\mathbb{P}(Z_2 > b | Z_1 = z)} dz$$

where the normalizing constant is the joint probability:

$$P = \mathbb{P}(Z_1 < a, Z_2 > b) = \int_{-\infty}^{a} \phi(z) \cdot \left[ 1 - \Phi\left( \frac{b - \rho^* z}{\sqrt{1 - (\rho^*)^2}} \right) \right] dz$$

## 1. Low type delinquency $\eta$

We aim to compute the conditional expectation:

$$\mathbb{E}\left[\eta_i \mid \varepsilon_i < \bar{\mathscr{C}},\ \varepsilon_i + \eta_i < \mathscr{D}^{Low}\right]$$

This region represents a wedge in $(\varepsilon_i, \eta_i)$ space bounded above by both $\bar{\mathscr{C}}$ on $\varepsilon_i$ and $\mathscr{D}^{Low}$ on the sum $\varepsilon_i + \eta_i$.

**Step 1: Law of Iterated Expectations**

$$\mathbb{E}[\eta_i \mid \cdot] = \frac{1}{P} \int_{-\infty}^{\bar{\mathscr{C}}} f_\varepsilon(x) \cdot \mathbb{E}[\eta_i \mid \varepsilon_i = x,\ \eta_i < \mathscr{D}^{Low} - x] \cdot \mathbb{P}(\eta_i < \mathscr{D}^{Low} - x \mid \varepsilon_i = x) dx$$

where:

$$f_\varepsilon(x) = \frac{1}{\sigma_\varepsilon} \phi\left( \frac{x - \mu_\varepsilon}{\sigma_\varepsilon} \right) \quad \text{and} \quad P = \mathbb{P}\left(\varepsilon_i < \bar{\mathscr{C}},\ \varepsilon_i + \eta_i < \mathscr{D}^{Low}\right)$$

**Step 2: Conditional Distribution of** $\eta_i \mid \varepsilon_i = x$

$$\eta_i \mid \varepsilon_i = x \sim \mathcal{N}\left(\mu_{\eta|x},\ \sigma^2_{\eta|\varepsilon}\right)$$

with:

$$\mu_{\eta|x} = \mu_\eta + \rho\frac{\sigma_\eta}{\sigma_\varepsilon}(x-\mu_\varepsilon), \quad \sigma^2_{\eta|\varepsilon} = \sigma^2_\eta(1-\rho^2)$$

Define the truncation point:

$$b(x) = \mathscr{D}^{Low} - x$$

and the standardized upper limit:

$$z(x) = \frac{b(x)-\mu_{\eta|x}}{\sigma_{\eta|\varepsilon}} = \frac{\mathscr{D}^{Low} - x - \mu_{\eta|x}}{\sigma_{\eta|\varepsilon}}$$

Then the expectation of the truncated normal is:

$$\mathbb{E}[\eta_i \mid \varepsilon_i = x,\ \eta_i < b(x)] = \mu_{\eta|x} - \sigma_{\eta|\varepsilon}\cdot\frac{\phi(z(x))}{\Phi(z(x))}$$

**Step 3: Plug into Full Expectation**

$$\boxed{\mathbb{E}[\eta_i \mid \varepsilon_i < \bar{\mathscr{C}},\ \varepsilon_i + \eta_i < \mathscr{D}^{Low}] = \frac{\int_{-\infty}^{\bar{\mathscr{C}}} f_\varepsilon(x)\left[\mu_{\eta|x} - \sigma_{\eta|\varepsilon}\cdot\frac{\phi(z(x))}{\Phi(z(x))}\right]dx}{\int_{-\infty}^{\bar{\mathscr{C}}} f_\varepsilon(x)\cdot\Phi(z(x))\,dx}}$$

Where:

- $\mu_{\eta|x} = \mu_\eta + \rho\frac{\sigma_\eta}{\sigma_\varepsilon}(x-\mu_\varepsilon)$
- $\sigma_{\eta|\varepsilon} = \sigma_\eta\sqrt{1-\rho^2}$
- $z(x) = \dfrac{\mathscr{D}^{Low} - x - \mu_{\eta|x}}{\sigma_{\eta|\varepsilon}}$
- $\phi(\cdot)$: standard normal PDF
- $\Phi(\cdot)$: standard normal CDF

## *IV.B* Medium type

1. *Delinquent medium-type*
   Defining:

   - $\underline{\mathscr{C}} := \tilde{p}_i - x_i'\beta - \xi_i$
   - $\bar{\mathscr{C}} := p_i - x_i'\beta - \xi_i$
   - $\mathscr{D}^{Med}(m_i; X, \Theta) := m_i(p_i - \tilde{p}_i) + (1-m_i)(p_i - x_i'\beta - \xi_i) - x_i'\lambda$
   - $\alpha(m_i) := (1-m_i) + \rho\frac{\sigma_\eta}{\sigma_\varepsilon}$

   To reduce notation, also define:

$$M(\varepsilon_i, m_i) := Pr((1-m_i)\cdot\varepsilon_i + \eta_i < \bar{\mathscr{D}}(m_i)|\varepsilon_i, m_i) = \Phi\left(\frac{\mathscr{D}^{Med}(m_i) - \alpha(m_i)z}{\sigma_\eta\sqrt{1-\rho^2}}\right)$$

   Now we can move to the expected values.

$$\mathbb{E}[\varepsilon_i \mid \underline{C} < \varepsilon_i < \bar{\mathscr{C}}, (1-m_i)\cdot\varepsilon_i + \eta_i < \mathscr{D}^{Med}(m_i), m_i] = \frac{1}{\underline{\mathscr{M}}}\int_{\underline{\mathscr{C}}}^{\bar{\mathscr{C}}} z\cdot f_\varepsilon(z)\cdot M(z, m_i)dz$$

   Where the denominator term $\underline{\mathscr{M}}$ is:

$$\underline{\mathscr{M}} = \int_{\underline{\mathscr{C}}}^{\bar{\mathscr{C}}} f_\varepsilon(z)\cdot M(z, m_i)dz$$

2. *Non-delinquent medium-type*

$$\mathbb{E}[\varepsilon_i \mid \underline{C} < \varepsilon_i < \bar{\mathscr{C}}, (1-m_i)\cdot\varepsilon_i + \eta_i > \mathscr{D}^{Med}(m_i), m_i] = \frac{1}{\bar{\mathscr{M}}}\int_{\underline{\mathscr{C}}}^{\bar{\mathscr{C}}} z\cdot f_\varepsilon(z)\cdot(1-M(z, m_i))dz$$

   Where the denominator term $\bar{\mathscr{M}}$ is:

$$\bar{\mathscr{M}} = \int_{\underline{\mathscr{C}}}^{\bar{\mathscr{C}}} f_\varepsilon(z)\cdot(1-M(z, m_i))dz$$

### 1. Medium-type delinquency $\eta$

The planner's problem also requires the medium-type's expected delinquency cost, given by:

$$\mathbb{E}[\eta_i \mid \underline{C} < \varepsilon_i < \bar{\mathscr{C}}, (1-m_i)\cdot\varepsilon_i + \eta_i < \mathscr{D}^{Med}(m_i), m_i]$$

Using the law of iterated expectations, this is:

$$\mathbb{E}[\eta_i \mid \cdot] = \frac{1}{P}\int_{\underline{\mathscr{C}}}^{\bar{\mathscr{C}}} f_\varepsilon(x)\cdot \mathbb{E}[\eta_i \mid \varepsilon_i = x,\ \eta_i < \mathscr{D}^{Med}(m_i) - (1-m_i)x] \\ \times \mathbb{P}\left(\eta_i < \mathscr{D}^{Med}(m_i) - (1-m_i)x \mid \varepsilon_i = x\right) dx$$

where:

$$f_\varepsilon(x) = \frac{1}{\sigma_\varepsilon}\phi\left(\frac{x-\mu_\varepsilon}{\sigma_\varepsilon}\right) \quad \text{and} \quad P = \mathbb{P}\left(\underline{\mathscr{C}} < \varepsilon_i < \bar{\mathscr{C}},\ (1-m_i)\varepsilon_i + \eta_i < \mathscr{D}^{Med}(m_i)\right)$$

The conditional distribution of $\eta_i \mid \varepsilon_i = x$ is:

$$\eta_i \mid \varepsilon_i = x \sim \mathscr{N}\left(\mu_{\eta|x},\ \sigma^2_{\eta|\varepsilon}\right)$$

where:

$$\mu_{\eta|x} = \mu_\eta + \rho\frac{\sigma_\eta}{\sigma_\varepsilon}(x-\mu_\varepsilon), \quad \sigma^2_{\eta|\varepsilon} = \sigma^2_\eta(1-\rho^2)$$

Define the truncation threshold:

$$b(x) = \mathscr{D}^{Med}(m_i) - (1-m_i)x$$

and standardized truncation point:

$$z(x) = \frac{b(x) - \mu_{\eta|x}}{\sigma_{\eta|\varepsilon}}$$

Then the conditional expectation under truncation is:

$$\mathbb{E}[\eta_i \mid \varepsilon_i = x,\ \eta_i < b(x)] = \mu_{\eta|x} - \sigma_{\eta|\varepsilon}\cdot\frac{\phi(z(x))}{\Phi(z(x))}$$

Putting it together, the final expression is:

$$\boxed{\mathbb{E}[\eta_i \mid \underline{\mathscr{C}} < \varepsilon_i < \bar{\mathscr{C}},\ (1-m_i)\varepsilon_i + \eta_i < \mathscr{D}^{Med}(m_i)] = \frac{\int_{\underline{\mathscr{C}}}^{\bar{\mathscr{C}}} f_\varepsilon(x)\left[\mu_{\eta|x} - \sigma_{\eta|\varepsilon}\cdot\frac{\phi(z(x))}{\Phi(z(x))}\right]dx}{\int_{\underline{\mathscr{C}}}^{\bar{\mathscr{C}}} f_\varepsilon(x)\cdot\Phi(z(x))\,dx}}$$

## *IV.C* High type

### 1. High type $\varepsilon$

For the high-type, the planner needs to form expectations of home-value for high-types in the case that they enter delinquency and in the case that they avoid delinquency. Defining $\bar{\mathscr{C}} := p_i - x_i'\beta - \xi_i$ and $\mathscr{D}^{High}(m_i;X,\Theta) := m_i(p_i - \tilde{p}_i) - x_i'\lambda$.

1. *Delinquent high-type*

$$E[\varepsilon_i|\varepsilon_i > \bar{\mathscr{C}}, \eta_i^{High} < \mathscr{D}^{High}(m_i)]$$

First define our conditional probability of observing a low enough $\eta_i$ draw for observing a high-type delinquency:

$$H(\varepsilon_i, m_i) := Pr(\eta_i < \mathscr{D}^{High}(m_i)|\varepsilon_i, m_i) = \Phi\left(\frac{\mathscr{D}^{High}(m_i) - \mu_\eta - \rho\frac{\sigma_\eta}{\sigma_\varepsilon}(\varepsilon_i - \mu_\varepsilon)}{\sigma_\eta\sqrt{1-\rho^2}}\right)$$

We can then find our desired expected value through the following expression:

$$E[\varepsilon_i|\varepsilon_i > \bar{\mathscr{C}}, \eta_i^{High} < \mathscr{D}^{High}(m_i)] = \mu_\varepsilon + \frac{1}{\underline{\mathscr{H}}}\int_{\bar{C}}^{\infty}(z-\mu_\varepsilon)\, f_\varepsilon(z)\cdot H(z)dz$$

Where the denominator $\underline{\mathscr{H}}$ is:

$$\underline{\mathscr{H}} = \int_{\bar{C}}^{\infty} f_\varepsilon(z)\cdot H(z)dz$$

Noting also that $f_\varepsilon(z) = \frac{1}{\sigma_\varepsilon}\phi\left(\frac{z-\mu_\varepsilon}{\sigma_\varepsilon}\right)$ where $\phi$ is the standard-normal probability density function.

2. *Non-delinquent high-type*
Using similar notation from above:

$$E[\varepsilon_i|\varepsilon_i > \bar{\mathscr{C}}, \eta_i^{High} > \mathscr{D}^{High}(m_i)] = \mu_\varepsilon + \frac{1}{\bar{\mathscr{H}}}\int_{\bar{C}}^{\infty}(z-\mu_\varepsilon)\, f_\varepsilon(z)\cdot(1-H(z))\, dz$$

Where the denominator $\bar{\mathscr{H}}$ is:

$$\bar{\mathscr{H}} = \int_{\bar{C}}^{\infty} f_\varepsilon(z)\cdot(1-H(z))\, dz$$

### 2. High type delinquency $\eta$

We are also interested in $E[\eta_i|\varepsilon_i > \bar{\mathscr{C}}, \eta_i^{High} < \mathscr{D}^{High}(m_i)]$ when delinquency does occur for a high type.

We'll start with the fact that the truncated distribution will be given by:

$$\eta\,\Big|\,\varepsilon = \eta_i \sim \mathscr{N}\left(\mu_\eta + \rho\frac{\sigma_\eta}{\sigma_\varepsilon}(\varepsilon - \mu_\varepsilon), \sigma_\eta^2(1-\rho^2)\right) \tag{15}$$

Define the parameters:

$$\mu_{\eta|\varepsilon} = \mu_\eta + \rho \frac{\sigma_\eta}{\sigma_\varepsilon}(\varepsilon_i - \mu_\varepsilon)$$
$$\sigma_{\eta|\varepsilon} = \sigma_\eta \sqrt{1-\rho^2}$$

We can write out our desired expectation:

$$E[\eta_i|\varepsilon_i > \bar{\mathscr{C}}, \eta_i^{High} < \mathscr{D}^{High}(m_i)] = \frac{1}{P}\int_{\bar{\mathscr{C}}}^{\infty} f_\varepsilon(z) \cdot E[\eta_i|\eta < \mathscr{D}^{High}(m_i), \varepsilon = \varepsilon_i]dz$$

With (very standard, using inverse mills ratio):

$$E[\eta_i|\eta < \mathscr{D}^{High}(m_i), \varepsilon = \varepsilon_i] = \mu_{\eta|\varepsilon} - \sigma_{\eta|\varepsilon}\frac{\phi\left(\frac{\mathscr{D}^{High}(m_i)-\mu_{\eta|\varepsilon}}{\sigma_{\eta|\varepsilon}}\right)}{\Phi\left(\frac{\mathscr{D}^{High}(m_i)-\mu_{\eta|\varepsilon}}{\sigma_{\eta|\varepsilon}}\right)}$$

Once again, the $P$ term is the normalizing constant for the conditional expectation:

$$P = P\left(\varepsilon > \bar{\mathscr{C}}, \eta < \mathscr{D}^{High}(m_i)\right)$$

$$P = \int_{\bar{\mathscr{C}}}^{\infty} f_\varepsilon(z) P(\eta < \mathscr{D}^{High}(m_i)|\varepsilon = \varepsilon_i)dz$$

Or more explicitly:

$$P = \int_{\bar{\mathscr{C}}}^{\infty} f_\varepsilon(z)\Phi\left(\frac{\mathscr{D}^{High}(m_i) - \mu_\eta - \rho\frac{\sigma_\eta}{\sigma_\varepsilon}(z-\mu_\varepsilon)}{\sigma_\eta\sqrt{1-\rho^2}}\right)dz$$

## *IV.D* Model of foreclosure loss expectations

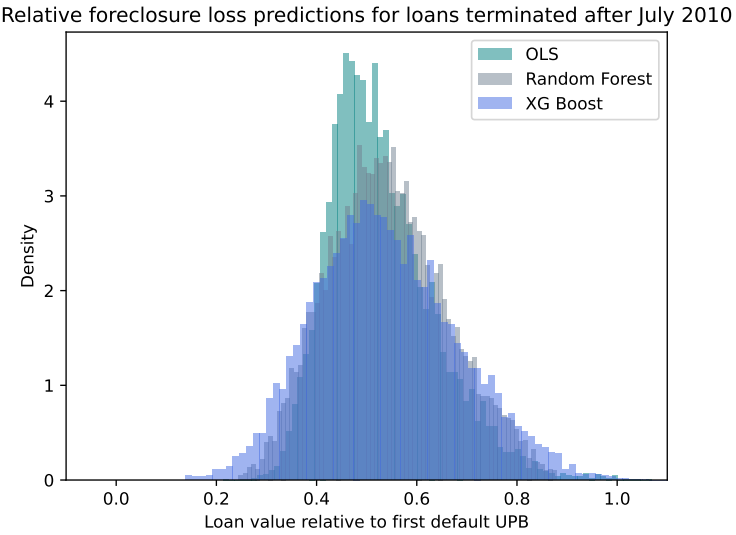



FIGURE XVI
Foreclosure loss model

# E Model of foreclosure loss expectations

This section describes the reduced-form model of foreclosure loss expectations that is used as an input into the broader structural framework of loan modification and foreclosure prevention policy. The purpose of this model is to generate predictions of foreclosure losses consistent with the types of decision rules loan servicers relied upon during the financial crisis and its aftermath. These predicted losses serve as a crucial component in the larger model, where foreclosure decisions interact with modification probabilities and borrower repayment outcomes.

The model is estimated using a sample of California loans purchased by Fannie Mae in the years 2004 through 2007. To ensure that the loss model is fit on a sufficiently large pool of foreclosed loans, I estimate it using outcomes for loans that entered foreclosure prior to July 2010. This timing strikes a balance between capturing a representative cross-section of crisis-era foreclosures and avoiding overfitting to realized outcomes observed well after the crisis had unfolded. In practice, the truncation ensures that the estimated model reflects information available to servicers in real time, rather than patterns that emerged as the housing market stabilized.

I estimate the foreclosure loss model using three approaches: ordinary least squares (OLS), a random forest, and an extreme gradient boosting (XGBoost) algorithm. Across these specifications, the models produce broadly similar predictions of foreclosure losses. The OLS results exhibit somewhat lower dispersion relative to the machine learning methods, which tend to produce a wider range of predicted values. Figure XVI illustrates this contrast across models. The similarity of results across approaches provides confidence that the reduced-form model captures the central patterns in foreclosure loss realizations without

being overly sensitive to the chosen estimation method.

The loss model is ultimately used to provide foreclosure loss predictions at the loan level in advance of estimation of the structural model. By fixing these loss expectations outside of the equilibrium modification problem, I reduce the computational complexity of estimation while maintaining consistency with the servicer decision environment during the crisis. Joint identification of loss beliefs and unobserved information distribution would not be possible.

### 1. Alternative approach based on NPV-documentation

Based on the *Home Affordable Modification Program Base Net Present Value (NPV) Model v7.0 Model Documentation*.[24]

On p30, the NPV model documentation lays out the specification for estimating the REO sale value of the property for any specific state. The question is as follows:

$$\begin{aligned}\text{REO}_{\text{s}} =& \beta_{0\ \text{s}} + \beta_{1\ \text{s}} * I_{\text{PropValue} \leq \$50\text{k}} + \beta_{2\ \text{s}} * I_{\$50\text{k} < \text{PropValue} \leq \$100\text{k}} + \beta_{3\ \text{s}} * \text{PropValue} \\ &+ \beta_{4\ \text{s}} * \text{PropValue} * I_{\text{VPropValue} \leq \$50\text{k}} + \beta_{5\ \text{s}} * \text{PropValue}\, I_{\$50\text{k} < \text{PropValue} \leq \$100k}\end{aligned} \tag{16}$$

Where:

- $REO_s$ is the estimated REO sale value in a given state
- $I(\cdot)$ indicators are set to 1 if the property value is less than or equal to 50k or greater than 50k and less than or equal to 100k.
- Prop value is the property valuation as is at the disposition date

I will adjust this approach to make loss projections using the Fannie Mae data as follows:

$$\begin{aligned}\text{NetSalesGain}_{\text{s}} =& \beta_{0\ \text{s}} + \beta_{1\ \text{s}} * I_{\text{DLQ PropValue} \leq \$50\text{k}} + \beta_{2\ \text{s}} * I_{\$50\text{k} < \text{DLQ PropValue} \leq \$100\text{k}} + \beta_{3\ \text{s}} * \text{DLQ PropValue} \\ &+ \beta_{4\ \text{s}} * \text{DLQ PropValue} * I_{\text{VPropValue} \leq \$50\text{k}} + \beta_{5\ \text{s}} * \text{DLQ PropValue} I_{\$50\text{k} < \text{DLQ PropValue} \leq \$100k}\end{aligned} \tag{17}$$

After computing a fitted Net Sales Gain I then divide the outstanding principal balance at the time the borrower first enters 90-day delinquency. So that the final loss figure is given by:

$$\text{Predicted loss on principal} = \frac{\widehat{\text{NetSalesGain}}}{\text{DLQ UPB}} \tag{18}$$

[24] Documentation updated on October 26, 2015.

## *V.A* Loan modification equilibrium grid

### 1. Construction of the equilibrium grid

This section provides additional detail on the construction of the equilibrium loan modification grid discussed in sub-section 2. The grid solves the servicer's cashflow maximization problem, and in doing so, also solves for equilibrium in the loan modification game for a large number of potential borrower characteristics, repayment expectations, unobservable variable distributions, and servicer modification costs. Solving for modification probabilities set by servicers, $m_i \in [0,1]$, is central to estimating the model because these probabilities are not directly observable in the data. The probabilities rely on the structural model because they will depend on unobservables and because modification outcomes are censored for any borrower that avoids delinquency.

The simulated maximum likelihood approach used in estimation can become numerically demanding to implement because one must solve for model equilibrium for every observation in the dataset, for every guess of parameters. Even if borrowers are grouped into discrete types based on their observable characteristics, one must still compute potentially many equilibrium probabilities across types. Within the maximum likelihood routine, one must then also re-solve the model for every borrower (or borrower type) for every potential guess of model parameters. To mitigate the computational burden involved with solving for model equilibria, I pre-solve them using a eight-dimensional grid that can be loaded into the maximum likelihood routine. The eight-dimensions are chosen to reduce the complexity of the servicer problem, which in practice can be very high dimensional. Without some form of simplification, the grid quickly suffers from a Curse of Dimensionality.

Dimensionality reduction takes place on borrower home utility, delinquency costs and expected repayments. The linearity assumption in the borrower's indirect utility function allows for a wide set of borrower observable characteristics to be reduced to the scalar index $x_i'\beta$. A benefit of this assumption is that additional observable characteristics and parameters can be added without affecting the dimensionality of the servicer's maximization problem. The $\xi_i$ term is assumed to be known to the servicer and additively separable in the indirect utility so servicers set policy based on one dimension given by $x_i'\beta + \xi_i$. A similar linearity assumption in the borrower's delinquency cost also means that many observable characteristics collapse into the scalar $w_i'\lambda$ in the $\mu$ parameter of the delinquency cost. On repayment expectations, my model assumes that $\tilde{p}_i$ is a rescaling of $p_i$, $\tilde{p}_i = \rho_m \cdot p_i$. For foreclosure losses, I predict a model of foreclosure loss outside of the empirical model and assume that loss-model fitted losses are known to the servicer. Given the assumptions above, the eight dimensions of the grid are:

1. $x_i'\beta + \xi_i$ - Mean home utility parameter index
2. $\sigma_\varepsilon$ - Unobserved standard deviation of home utility
3. $w_i'\lambda$ - Mean of delinquency cost index
4. $\sigma_\eta$ - Unobserved standard deviation of delinquency cost

5. $p_i$ - Net present value of future stream of payments

6. $c_j$ - Lender cost of modification

7. $\rho$ - Correlation between unobserved home utility ($\varepsilon$) and delinquency costs ($\eta$)

8. $f_i$ - Expected loss from foreclosure

The grid is constructed by maximizing the servicer's objective function on several points of support in each dimension. My final estimation routine employs a grid with 20 points of support of $p_i$, 17 points of support of $x_i'\beta + \xi_i$, 12 points of support of $w_i'\lambda$, 10 points in the support of $c_j$ and $\rho$, 7 points in the support of $\sigma_\eta$, and 5 points in the support of $\sigma_\varepsilon$ and $f_i$. Jointly these supports form a grid with $20 \times 17 \times 12 \times 10^2 \times 7 \times 5^2 = 71,400,000$ points.[25] For any set of inputs, I use Halton draws to integrate out the servicer-known private information $\xi_i$ within the servicer's cashflow maximization problem, while private information about the home utility ($\varepsilon$) and delinquency cost ($\eta_i$) is integrated out analytically (see Appendix C). The grid placement and dimensions were carefully chosen through extensive exploration and experimentation. $x_i'\beta + \xi$, $p_i$ and $c_j$ dimensions have the greatest number of points in the support because these values will vary across borrowers for any guess of model parameters. The unobserved distribution parameters, namely $\sigma_\varepsilon$, $\sigma_\eta$, and $\rho$, will apply to all borrowers in the same way because the servicer only observes the distribution parameters rather than the specific draws from that distribution. In theory, it would be possible to select only one point for these two dimensions if the true values were known. However, counterfactual analysis in which $\sigma_\varepsilon$ is shifted will require more points to evaluate the new equilibrium. The $w_i'\lambda$ dimension does vary with borrower characteristics but plays a less critical role for model fit than the dimensions associated with the borrower home utility, $H_i$.

Parallel computing dramatically speeds up solving for model equilibrium at the over 70 million points on the grid. This parallel computing task is "embarrassingly parallel" in the sense that the solution for a given set of inputs can be computed completely independently from some other set of inputs. By leveraging the Texas Advanced Computing Center's Stampede3 supercomputer, I am able to use 40 compute nodes simultaneously to achieve an almost $40\times$ improvement in computation speed of the full grid relative to using a single node.[26] The number of hardware threads (workers) available per node also plays a crucial role in speeding up this computation. By using Python's *multiprocessing* library I am able to take advantage of either 112 hardware threads on Stampede3's SPR Compute Nodes, or 80 hardware threads on the ICX Compute Nodes. Using these resources, I am able to form the grid in a minimum run time of 15 minutes.[27]

[25] In practice, the appropriate support for parameter estimates must be identified through iteration estimation of the model. If the truth for $\sigma_\varepsilon, \sigma_\eta$ and $\rho$ were known, the econometrician would only require a single point of support in each dimension. Likewise, if each servicer's cost of modification, $c_j$, were known, one would only need a point of support for each institution.

[26] Interested readers can learn more about Stampede3 here: `https://portal.tacc.utexas.edu/user-guides/stampede3`

[27] It took about 65 hours to form an early iteration of this grid using a single node with eight threads.

## *V.B* Further estimation detail

This section provides further detail on the simulated maximum likelihood approach discussed in section V. of the paper. The full estimation routine incorporates the servicer's cashflow maximization problem (previous Appendix section), simulates borrower unobservable draws and searches across potential parameter values to maximize the log-likelihood function. The log-likelihood function is also conditional on observed outcomes in the data. The estimation was done using Python. The complete process can be summarized in the following four steps:

1. **Load in data and the pre-solved servicer's equilibrium grid:** After data cleaning and forming the equilibrium grid, both must be loaded into the maximization routine. Since the grid has been computed in parts to facilitate parallel computing, it must be rejoined into a single Python object.

2. **Take $S$ simulation draws for $\xi_i$** I take $S$ Halton draws for $\xi_i$. For any given set of model parameters, I then scale these draws up or down. To illustrate this: with $\xi_i \sim N(\mu_\xi, \sigma_\xi^2)$ the simulation draws $s_\xi \sim N(0,1)$ are re-scaled as $\hat{s}_\xi = \mu_\xi + s_\xi \sigma_\xi$ (Train, 2009).[28]

3. **Calibrate the lender's loss on the $\tilde{p}_i$ and $f_i$:** To simplify the computational burden, my model assumes that $\tilde{p}_i$ is a functions of $p_i$, explicitly that $\tilde{p}_i = \rho_m \cdot p_i$. For $f_i$, I pre-estimate a model of foreclosure losses and use the fitted values, $\hat{f}_i$ to estimate the equilibrium model of loan modification. My estimation routine takes the appropriate re-scale factors $\rho_m$ and $\hat{f}_i \; \forall \; i \in \{1, \ldots, N\}$ as arguments. I test my model with alternative inputs as a robustness test.

4. **Maximize the likelihood function using global search algorithm followed by a local search:**

   My log-likelihood function is not globally concave and features flat sections that create challenges for computational maximization routines. To increase the probability of finding the parameters that reach the global maximum of my log-likelihood function, I conduct a global search algorithm that emphasizes shifting away from potential local extrema. At the completion of the global parameter search I allow for a more extensive non-linear local maximization of the function based on the maximizing value found in the global search.

   (a) *Global search:* I set the global maximizing routine to take 500 random "steps" or iterations in the initial parameter guesses before conducting 200 iterations of a local Nelder-Mead, non-linear parameter search. The Nelder-Mead iterations are intentionally constrained in this step to limit estimation run-time. This procedure is conducted using Python's *basinhopping* function within the *scipy.optimize* library.[29] Basin-Hopping is a more general version of Simulated Annealing.

[28] Bhat (2001) shows that Halton draws can greatly improve simulation precision in mixed logit setting over conventional sampling.

[29] See: `https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.basinhopping.html`

(b) *Local search:* The local search takes the results of the global search as the initial guess for parameters and then again conducts a non-linear parameter search using the Nelder-Mead simplex method. This search is allowed to continue until convergence. This procedure is conducted using Python's *minimize* function within the *scipy.optimize* library.[30]

## *V.C* Additional detail on welfare effects of policy

Figure XVII presents the welfare per loan for all borrowers at different levels of subsidy-per-modification offers and with different assumed social costs of government spending but no assumed negative externalities from foreclosure. The results suggest that some subsidy spending was worthwhile from a social welfare perspective, but that subsidy spending had a relatively small effect on overall social welfare. The assumed $5,000 per-modification level exceeds the socially optimal level for all origination cohorts, but overall welfare per loan does not appear sensitive to policy until around $4,000 per modification. At this point average welfare begins to decline due to a rise in unnecessary transaction costs associated with processing modifications that fail to prevent foreclosure. I find that the optimal level of subsidy spending tends to be higher in earlier origination cohorts: the optimal level may have been near $5,000 in both 2004 and 2005, but lower subsidy offers would have been socially optimal in 2006 and 2007. These results reflect the fact that borrower distributions and servicer behavior across origination years influenced the effectiveness of Federal policy.

[30] See: `https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.minimize.html`

FIGURE XVII
Optimal subsides and welfare without foreclosure externalities

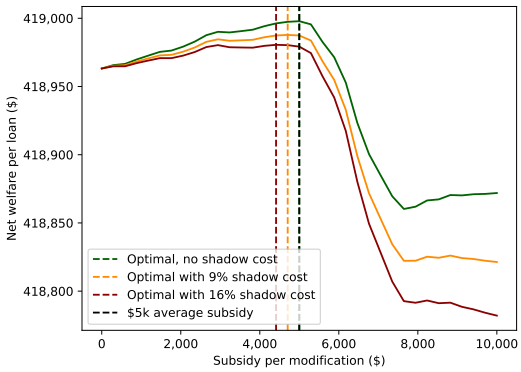



FIGURE XVII
Q1-Q4 2004

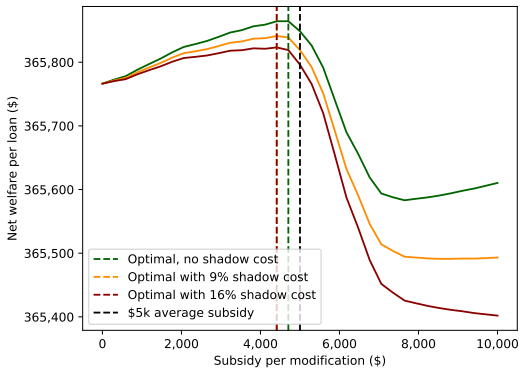



FIGURE XVII
Q1-Q4 2005

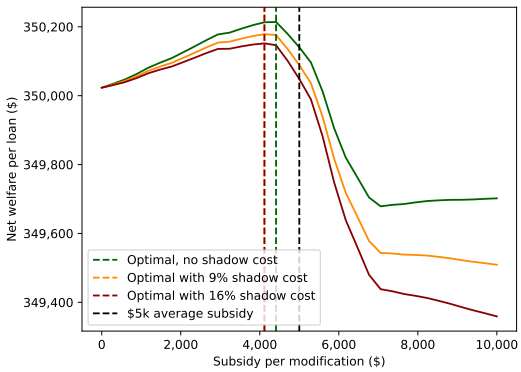



FIGURE XVII
Q1-Q4 2006

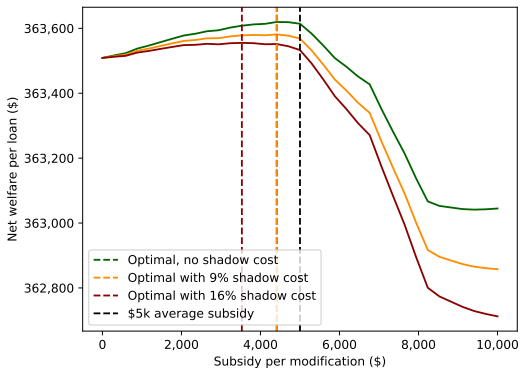



FIGURE XVII
Q1-Q4 2007

*Notes:* This figure presents average overall net welfare per loan, by origination cohort, under different levels of loan modification subsidy. The solid lines reflect how average net welfare changes with the subsidy level and the dotted lines mark the optimal subsidy level under different assumptions for the shadow cost of government spending. The optimal subsidy level is weakly decreasing in the level of shadow costs of public funds. There are notable differences across origination samples, suggesting that the underlying borrower populations played a role in the effectiveness of loan modification policy. Though in practice subsidy levels were not conditioned on origination cohort, the figures suggest that subsidies were more effective for mortgages originated further from the financial crisis. The change in overall net welfare appears small because the vast majority of borrowers in my sample never enter mortgage delinquency (90-days delinquent).The welfare effects of increased loan modification subsidies were significantly larger for liquidity constrained borrowers at risk of entering foreclosure.

The shadow cost of government spending plays an important role in assessing the effectiveness of a subsidy policy. I assume that the marginal cost of public funds for taxes on labor income ranges between \$1.09 to \$1.16 per dollar of tax revenue (Browning, 1976). In the context of my welfare analysis, the marginal cost of public funds will shift the marginal cost curve for subsidy spending upwards, reducing the optimal level of per-modification subsidy. For the 2006 and 2007 origination cohorts, where marginal welfare benefits and marginal costs of subsidies are of a similar magnitude, accounting for the marginal cost of public funds may push the optimal subsidy towards zero because the cost of public spending may ultimately exceed the welfare benefit for market participants.

Though the subsidy effects on overall net welfare appear small, the aggregate figures are misleading because certain borrowers stood to significantly benefit from loan modification subsidies. The effect of loan subsidies on overall welfare tend to be muted because over 80% of borrowers in my sample never enter 90 days of payment delinquency, which means that they were generally unaffected by loan modification policy. Appendix *V.E* presents the average welfare for medium-type borrowers, that have the possibility of avoiding foreclosure if, and only if, they are allocated a loan modification. For this borrower group, larger loan subsides could have increased average welfare by upwards of 30%. Large potential effects for a vulnerable sub-group of borrowers and relatively small effects on overall welfare imply that loan modification policy was generally worthwhile for my sample of Fannie Mae single-family mortgages. Applying a larger weight to the well-being of these borrowers would provide further support for the use of loan modification subsidies.

## *V.D* Servicer simulated accuracy across origination samples

FIGURE XVIII
Servicer modification accuracy, by origination cohort

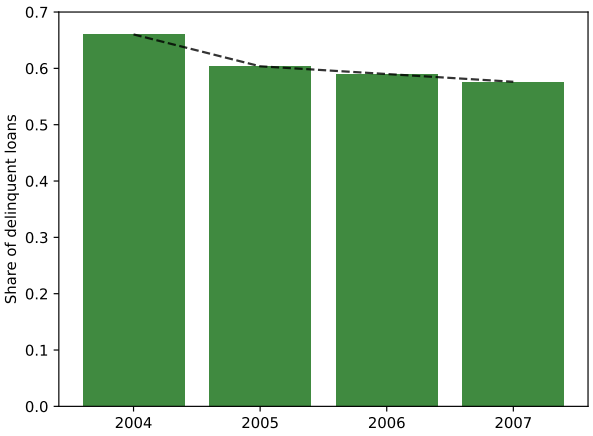



FIGURE XVIII
Servicer correct: No Modification

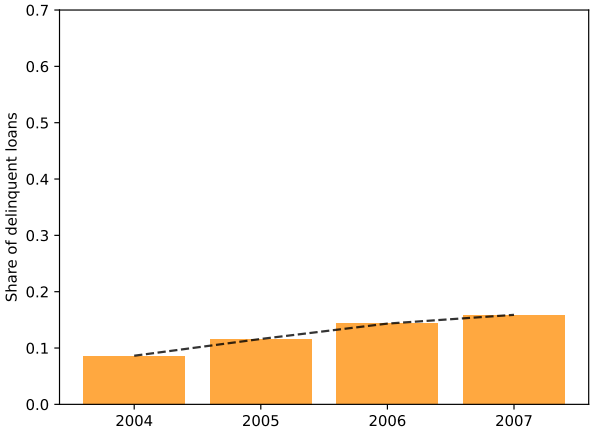



FIGURE XVIII
Servicer Mistake: Modification

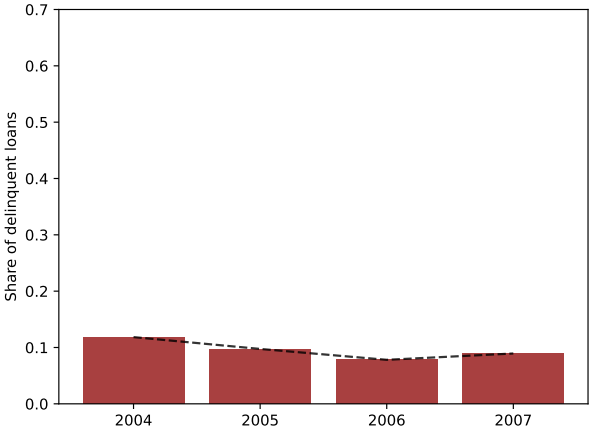



FIGURE XVIII
Servicer mistake: No Modification

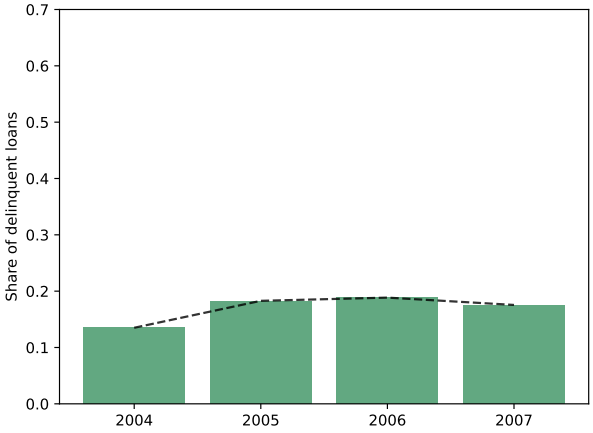



FIGURE XVIII
Servicer correct: Modification

## *V.E* Medium borrower average welfare

FIGURE XIX
Average welfare of medium borrower types and modification subsidies

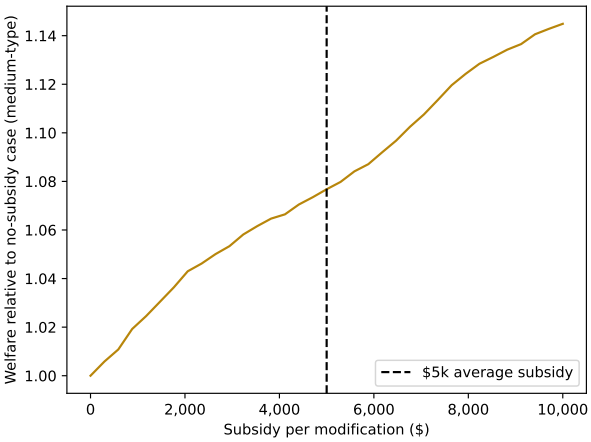



FIGURE XIX
Q1-Q4 2004

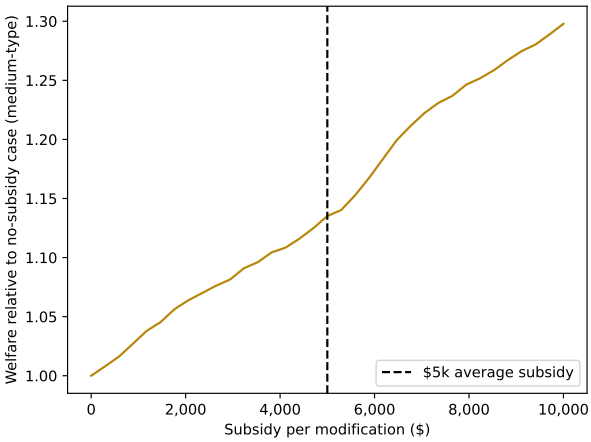



FIGURE XIX
Q1-Q4 2005

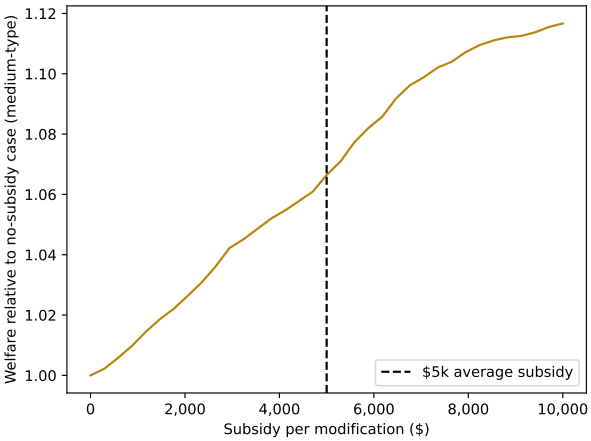



FIGURE XIX
Q1-Q4 2006

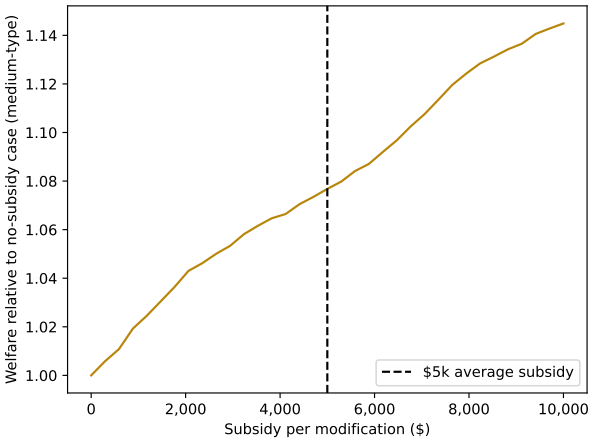



FIGURE XIX
Q1-Q4 2007