Portfolio Decisions with Higher-Order Dependence between Labor and Stock Markets

Wei Jiang* Shize Li† Jialu Shen‡

August 26, 2025

Abstract

Higher-order dependence measures extreme values jointly occurring in the labor and stock markets. Using Panel Study of Income Dynamics data, we document a higher-order dependence between labor and stock markets, and households adjust their portfolio decisions in response to this higher-order dependence risk. Higher-order dependence affects households' wealth risk profile via the skewness and kurtosis channels, forcing households to lower their risk exposures. Accounting for higher-order dependence reduces households' risky shares by 28% and increases participation thresholds by 22%. On the other hand, ignoring higher-order dependence generates certainty equivalent wealth losses of 3% and increases wealth inequality by 2.7%.

JEL: D15, D31, G11, J24.

Keywords: Higher-Order Dependence, Mean-Variance-Skewness-Kurtosis Model, Portfolio Choice, Stock Market Participation, Wealth Inequality, Wealth-Equivalent Loss.

*Hong Kong University of Science and Technology. E-mail: weijiang@ust.hk
†Hong Kong University of Science and Technology. E-mail: slidq@connect.ust.hk
‡Trulaske College of Business, University of Missouri. E-mail: jshen@missouri.edu

1 Introduction

With 58% of U.S. households owning stocks and consumer spending—primarily driven by labor income—accounting for 70% of GDP, both the labor and stock markets play a pivotal role in the U.S. economy. Are these markets interdependent, and how does this affect household portfolio choices? While prior studies focus on linear dependence between labor and stock markets and report mixed findings,[1] this paper shifts attention to higher-order dependence, documenting its existence and showing its substantial influence on household asset allocation.

High-order dependence can arise when economic shocks occur, simultaneously impacting both labor and stock markets. Both small and large economic shocks could occur and affect both labor and stock markets or just one of the two, leading to market links or comovements (e.g., Hall et al., 2000;Manuelli, 2000).[2] For instance, when a technology shock occurs, such as the advent of AI, it typically depresses the labor market due to AI replacing human labor, while simultaneously boosting the stock market as AI enhances firm revenues.[3] Conversely, a pandemic shock commonly depresses both labor and stock markets. These economic shocks usually cause labor and stock markets to exhibit higher-order dependence, which is not captured by linear dependence measures.

Indirect evidence for the potential existence of higher-order dependence is the existence of stand-alone higher-order risks in either labor income or stock returns. Recent literature, including studies by Guvenen et al. (2014) and Shen (2022), has extensively explored higher-order moments in the labor income process. Neuberger and Payne (2021) provide compelling evidence of higher-order moments in annual stock returns. These findings motivate our investigation into the possible existence of the higher-order dependence between labor and stock

[1] A line of literature suggests contemporaneous correlation between labor income shocks and stock returns is low or nonexist (Davis and Willen (2013) Campbell and Viceira (2002), Vissing-Jørgensen (2002), Cocco et al. (2005)), while another line of literature finds a significant correlation between labor income risk and stock returns (i.e., Bonaparte et al. (2014), Bagliano et al. (2024)).

[2] Such shocks include economic events (fiscal and monetary policy changes, financial crises), supply-demand shocks (technology shocks, geopolitical events), and natural disasters and epidemics (COVID-19, avian flu, tsunamis, earthquakes).

[3] Related, Manuelli (2000) highlights that technology shocks, specifically those related to 'information technology,' not only affect both labor and stock markets but also give rise to diverse phases and comovements.

markets. We aim to address three fundamental questions about higher-order dependence. First, does higher-order dependence exist in the data? Second, how does the higher-order dependence affect households' portfolio decisions? Lastly, what's the economic consequence of ignoring higher-order dependence?

Before addressing these questions, it is important to distinguish higher-order dependence from linear dependence. Higher-order dependence measures the risk that extreme values jointly occur in the labor and stock markets. Such a specification is independent of any linear dependence structure, such as correlation or cointegration, and thus is consistent with empirically observed low contemporaneous correlations and cointegration between market returns and labor income shocks.

Figure 1: Higher-order Dependence versus Linear Dependence

The scatterplots compare linear dependence, which is measured by linear correlation (Corr), with higher-order dependence, which is measured by between-squares correlation ($\text{Corr}^{\text{sq}}(Z_1, Z_2) := \text{Corr}((Z_1 - \mathbb{E}Z_1)^2, (Z_2 - \mathbb{E}Z_2)^2)$). In panel A, two random variables (Z_1, Z_2) follow the bivariate normal distribution with Corr=0.3, Corr^{sq}=0.0, and $\mathbb{E}[Z_1] = \mathbb{E}[Z_2] = 0$, $\mathbb{E}[Z_1^2] = \mathbb{E}[Z_2^2] = 0.04$. In panels B and C, (Z_1, Z_2) follows a bivariate mixture normal distribution with Corr=0.0, Corr^{sq}=0.3, and the stand-alone moments for Z_1, Z_2 follow $\mathbb{E}[Z_1] = \mathbb{E}[Z_2] = 0$, $\mathbb{E}[Z_1^2] = \mathbb{E}[Z_2^2] = 0.04$, $\mathbb{E}[Z_1^3] = \mathbb{E}[Z_2^3] = -0.5$, and $\mathbb{E}[Z_1^4]/\text{Var}^2[Z_1] = \mathbb{E}[Z_2^4]/\text{Var}^2[Z_2] = 3.5$.

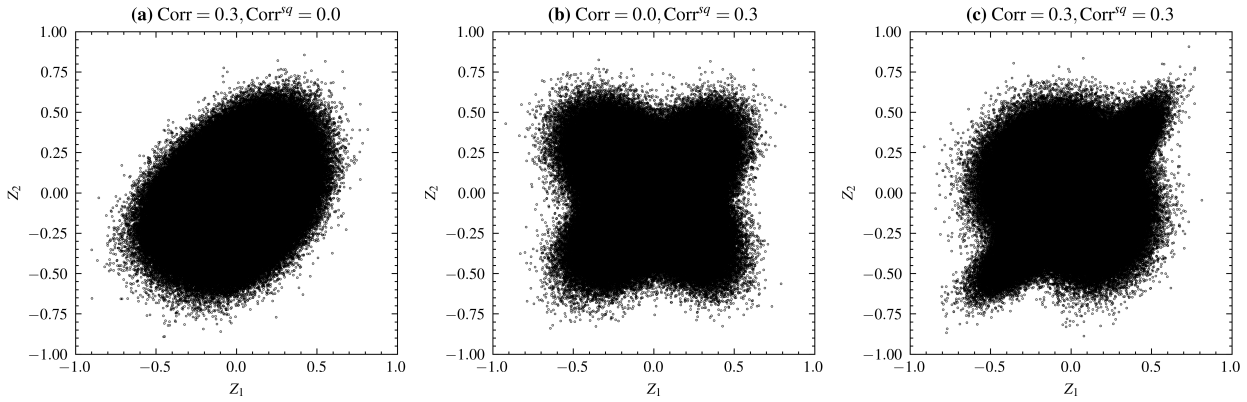


Figure 1 compares higher-order dependence with linear dependence using simulation data while controlling for the same mean and variance. In panel (a), when both variables satisfy normal distribution, a positive linear correlation implies a tendency to move in the same direction. Panels (b) and (c) offer a comparison, in which two random variables are subject to a mixture normal distribution such that higher-order moments and higher-order dependence exist. Both panels show that, when the between-squares correlation is positive, two types of comovements are possible: two variables may move in the same *and* opposite directions. Apparently, higher-order dependence can simultaneously capture both comovements

between the labor market and stock market, as shown in empirical studies.[4] Furthermore, higher-order dependence accounts for both downside and upside comovements, in contrast to downside tail risk, which only considers downside comovements. Lastly, panel (b) highlights the important point that the existence of higher-order dependence is unrelated to the presence of a linear correlation.

To investigate the existence of higher-order dependence in the data, we not only estimate the between-square correlation, but also five additional measures of higher-order dependence for labor and stock markets using the Panel Study of Income Dynamics (PSID) survey and the Center for Research in Security Prices (CRSP). These include two measures for the degree of coskewness and three for the degree of cokurtosis. Compared to these alternatives, the between-square correlation, a normalized form of cokurtosis, serves as a sufficient measure of higher-order dependence (see Equation 7). We find that all six measures of higher-order dependence differ significantly from the benchmark values derived under the assumption that labor and stock markets are independent at higher orders. This suggests the presence of nontrivial higher-order dependence and implies that the dynamics between labor and stock markets are nonlinear. Furthermore, we demonstrate that these findings are robust across different wealth levels and earnings growth rates.

Then a natural question arises: Do households make portfolio decisions in response to higher-order dependence? Using the PSID data, we first conduct probit participation regressions. Stock market participation drops as between-square correlation deviates further from zero.[5] Specifically, a one-standard-deviation shift in between-square correlation reduces stock market participation by 6.0%–6.96%. Meanwhile, using Tobit regressions, we find that households do adjust their portfolio holdings of risky assets when the between-squares correlation is present. In particular, the Tobit regression estimates indicate that a one-standard-deviation shift from zero in the between-squares correlation is associated with a 2.79% to 3.48% decrease in equity shares.

Next, to better understand the *mechanism* and *quantitative* effect of higher-order depen-

[4]The relationship between labor and stock markets extends beyond a simple linear correlation to exhibit a more intricate dependence structure. Several studies, including Michelacci and Quadrini (2009), Bronars and Famulari (2001), and Nickell and Wadhwani (1991), explore two types of comovements between shareholder value and wages.

[5]The results also hold with alternative higher-order dependence measures.

dence on portfolio decisions, we extend the one-period model of Harvey and Siddique (2000)[6] to a mean-variance-skewness-kurtosis framework for household portfolio choices. The higher-order dependence between labor and stock markets is modeled using a bivariate mixture normal distribution, capturing the heavy tails and low-frequency events typically observed in economic shocks. Introducing labor income that is higher-order dependent on stock returns significantly affects household's total portfolio risk, specifically the skewness and kurtosis. The skewness channel dominates for negative between-squares correlations, while the kurtosis channel becomes significant for positive ones, resulting in a nonlinear relationship between the between-squares correlation and portfolio allocations. This one-period model emphasizes how the risk stemming from higher-order dependence makes income to be more "stock-like" and suggests that a typical household should decrease their stock allocation to constrain the total portfolio risk.

The intuition behind households' response to higher-order dependence is as follows. Economic shocks can generate both positive and negative higher-order dependence between labor and stock markets. On the one hand, a higher positive higher-order dependence implies an increased likelihood of realized shocks driving comovements between labor and stock markets in both the same and opposite directions. This stronger same-direction comovement reduces the hedging effect of stocks against income risk, while opposite-direction comovement generates a hedging effect. In aggregate, households experience an increase in total portfolio risk, characterized by higher portfolio kurtosis, leading them to lower their holdings of risky assets. On the other hand, a more negative higher-order dependence indicates a lower likelihood of realized shocks driving comovements between the labor and stock markets. A reduced likelihood of comovements also diminishes the hedging effect and increases the skewness observed in households' total portfolios. Consequently, households are motivated to lower their holdings of risky assets optimally.

Moreover, given the availability of life-cycle profiles of households in the PSID dataset, we calibrate a life-cycle model to quantify the impact of higher-order dependence on portfolio decisions. We find that introducing the higher-order dependence into the model increases

[6]Harvey and Siddique (2000) use a mean-variance-skewness framework to understand the cross-sectional variation in asset returns.

the wealth threshold of household participation in the stock market by 22% and significantly lowers the shares of financial wealth invested in stocks conditional on stock market participation by 28%. As a result, higher-order dependence can simultaneously explain low stock market participation rates and moderate equity holdings for stock market participants even with a relative risk aversion coefficient of $\gamma = 4.3$ and a small participation cost.[7] Furthermore, we demonstrate that existing models, which assume linear dependence between labor and stock markets and incorporate higher-order risks within each market, produce nearly zero higher-order dependence.[8] This finding highlights the importance of explicitly modeling higher-order dependence to accurately capture its true impact.

Finally, what are the consequences for both households and society if the higher-order dependence in labor and stock markets is ignored? By measuring the wealth-equivalent loss, we reveal that disregarding higher-order dependence results in a wealth cost of 3.32% for households. Moreover, the equivalent wealth losses vary across households based on age, education, and wealth level, indicating that higher-order dependence may have a broad societal impact. To capture this effect, we then conduct a simulation study for a large number of households. It reveals that ignoring the higher-order dependence could lead to an increase of 2.73% in the Gini index for society overall and a 3.52% increase for college graduates.[9] Failing to consider the effects of higher-order dependence can lead to wealth losses and exacerbated wealth inequality, affecting both households and society at large. Therefore, understanding and accounting for this phenomenon are crucial for making informed decisions and implementing policies that promote more equitable outcomes.

Related Literature. This paper is part of the literature that investigates the comovement between labor and stock markets. Several papers examine the linear correlation between labor income and stock returns, but their findings are mixed. Davis and Willen (2013) suggest aggregate equity returns are uncorrelated with occupation-level income innovations. Campbell and Viceira (2002), Vissing-Jørgensen (2002), Cocco et al. (2005), and Massa and Simonov (2006) all find the contemporaneous correlation between labor income shocks and

[7]For comparison, Fagereng et al. (2017) estimate $\gamma = 11$; Shen (2022) generates $\gamma = 5.6$; and Catherine (2022) generates $\gamma = 6.1$ when including stand-alone higher-order moments in labor income or stock returns innovations.

[8]See Appendix E.5 for details.

[9]The magnitude of these changes is substantial as the U.S. Gini index grew $< 3\%$ from 1970 to 2017.

stock returns is low. However, Bonaparte et al. (2014) document a high correlation between labor income risk and stock returns, and Bagliano et al. (2024) show a significant correlation between aggregate shocks and earnings shocks, where aggregate shocks affect both earnings and stock returns. Moreover, Storesletten et al. (2007), Benzoni et al. (2007), and Lynch and Tan (2011) show stock returns tend to move together with labor income in the long run, but Huggett and Kaplan (2016) find that human capital and stock returns have a more negligible correlation than the one in Benzoni et al. (2007). Finally, Hall et al. (2000) and Manuelli (2000), among others, provide micro-foundations to understand how technology shocks cause linkages between labor and stock markets.

Our paper fills a crucial gap among these inconclusive findings. Higher-order dependence offers a novel explanation of the comovements between earnings risk and stock return innovations, distinct from the stand-alone risk inherent in the stock or labor market or the risk stemming from their linear dependence.[10] Our exhaustive estimations show that the higher-order dependence in labor and stock markets persistently exists across households with different education, labor growth rates, and wealth levels.

This paper also relates to the literature on how income hedging motives influence stock market participation and portfolio choice decisions.[11] Bonaparte et al. (2014) shows strong support for income hedging motives in stock market participation and asset allocation decisions using a Dutch data set. Guiso et al. (1996) find that Italian households with more background income risk hold more liquid assets and invest less in risky assets. Fagereng et al. (2018) finds substantial sensitivity of portfolio decisions to uninsured wage risk in a long panel data of firms and their workers. Shen (2022) shows that high-order income risk alone can generate moderate participation and risky asset holdings. Instead, we investigate the risk arising from the higher-order dependence in labor and stock markets and its impact on households' portfolio decisions. Our paper offers a new perspective on the income

[10]Technically, our higher-order dependence specification aligns with the general dependence function copula (e.g., Fan and Patton (2014)), which exists independently of the marginal distributions of two variables. We use higher-order dependence because it can be easily estimated using a single measure (under regularity conditions), while the use of copula is subject to difficulties in model selection and measurement.

[11]Households' hedging motives arise because of the nontradable nature of human capital, as moral hazard and adverse selection problems prevent most households from trading claims against their future labor income. This makes human capital a nontradable asset, the value of which depends on the yield of labor income.

hedging motive and a novel approach to quantify the hedging effect. This work suggests that higher-order dependence is able to predict both participation and risky asset holdings, consistent with the empirical data.

Finally, our paper builds on the literature on large economic shocks and rare disasters, pioneered by Rietz (1988), Barro (2006), and Barro (2009). While this body of work primarily focuses on the impact of negative economic shocks within a single market, our research broadens the scope by examining the effects of economic shocks across both labor and stock markets through the lens of higher-order dependence. Unlike existing approaches, our bivariate mixture normal distribution for higher-order dependence accounts for both negative and positive shocks, as well as the heavy tails and low frequencies of rare disasters. This provides a more comprehensive perspective on the interconnections between these shocks. By analyzing how higher-order dependence stemming from the co-occurrence of shocks across markets influences household asset allocation decisions, this paper offers fresh insights into the broader implications of economic shocks.

The rest of the paper is organized as follows. Section 2 documents the presence of a higher-order dependence in the data. Section 3 conducts the empirical analysis. Section 4 uses a one-period model that captures the essential ingredients for our analysis and highlights the main mechanisms. Section 5 uses a life-cycle model calibrated to data to show the substantial effects of higher-order dependence on households' optimal portfolio decisions. Section 6 reveals the economic implications of higher-order dependence. Section 7 concludes.

2 Does a Higher-Order Dependence Exist in the Data?

To examine the presence of a higher-order dependence in the data, we take a comprehensive approach. We employ a wide range of higher-order dependence measurements to thoroughly examine the relationship between labor income shocks and innovations in stock returns. We use the PSID data, which provides information on household labor income, and the CRSP data, which captures stock market performance. Then we present estimates obtained from various measures, including coskewness, cokurtosis and between-squares correlation, for different groups of households. Lastly, since the between-squares correlation is a standardized

measure of higher-order dependence and allows us to capture the entirety of higher-order dependence under certain regular conditions, we focus on this measure whenever discussing higher-order dependence and present more supporting evidence.

2.1 PSID and CRSP Data

The PSID is a comprehensive survey that encompasses a diverse range of information about the labor market and demographic variables, including age, education, household composition, and marital status. The survey was conducted annually from 1968 to 1997 and has been conducted biennially since 1997. However, income-related variables were bracketed in the 1968 and 1969 waves, making them unusable for our analysis. To optimize the use of data after 1997, we consider two time series: one with annual data ($\Delta t = 1$) from 1970 to 1997 and another with biennial data ($\Delta t = 2$) from 1970 to 2017. To construct the biennial series, we can directly use PSID data after 1997, while for data before 1997, we only retain observations from every other year, despite the annual collection.

The analysis was restricted to households with a male head of household.[12] We defined labor income as the total reported labor income plus unemployment compensation, workers' compensation, Social Security, Supplemental Social Security, other financial assistance, child support, and total transfers, all for both the head of household and, if present, his spouse. Then we deflate labor income using the 2016 Consumer Price Index from the U.S. Bureau of Labor Statistics.

First, we closely follow the sample selection strategy of Nakajima and Smirnyagin (2019).[13] We only include households in which the head has a minimum of 20 waves of positive labor income data. If a household head retires in the current survey year, we delete all information about this household. Then following Cocco et al. (2005), we decompose the logarithm of household i's labor income at age t (y_{it}) into two components: a deterministic function of age t and a vector of other individual characteristics ($f(t, \Omega_{it})$), and an error structure that

[12]The age profile may differ among households with a female head of household, necessitating a separate estimation. However, the subsample with a female head of household had insufficient observations, and we were unable to perform a reliable estimation.

[13]See Online Appendix A for details.

captures both transitory shocks and persistent shocks ($\delta_{i,t}$),

$$y_{it} = f(t, \Omega_{it}) + \delta_{it}. \tag{1}$$

To estimate δ_{it}, we first regress y_{it} on dummies for age, family, marital status, and household composition, from which we obtain $\hat{f}(t, \Omega_{it})$. Next, we define $\Delta y^*_{it} = y^*_{it+\Delta} - y^*_{it}$, where y^*_{it} is given by $y^*_{it} = y_{it} - \hat{f}(t, \Omega_{it})$. To quantitatively assess the excess return on our stylized risky asset, we use CRSP data on the New York Stock Exchange value-weighted stock return relative to the Treasury-bill rate, denoted as η_t. Let n_i be the number of periods in household i's time series data. The linear correlation and higher-order dependence between return innovations $\{\eta_t\}_{t=1}^{n_i}$ and persistent income shocks $\{\Delta y^*_{it}\}_{t=1}^{n_i}$ can be computed.

We categorize all households into two groups, college graduates and a no-college group, to account for the impact of education. This is based on the well-established observation that households exhibit distinct behavioral patterns across different education levels (see Attanasio (1995); Hubbard et al. (1995); Cocco et al. (2005)). Subsequently, we conduct separate estimations of higher-order dependence for each group to understand the effect of education on our results.

2.2 Measuring Higher-Order Dependence

To test for the existence of higher-order dependence, we use five measures of coskewness and cokurtosis, defined between two random variables Z_1 and Z_2 by

$$S(Z_1, Z_1, Z_2) \triangleq \frac{\mathbb{E}[(Z_1 - \mathbb{E}Z_1)^2(Z_2 - \mathbb{E}Z_2)]}{\sigma_1^2 \sigma_2}, \tag{2}$$

$$S(Z_1, Z_2, Z_2) \triangleq \frac{\mathbb{E}[(Z_1 - \mathbb{E}Z_1)(Z_2 - \mathbb{E}Z_2)^2]}{\sigma_1 \sigma_2^2}, \tag{3}$$

$$K(Z_1, Z_1, Z_1, Z_2) \triangleq \frac{\mathbb{E}[(Z_1 - \mathbb{E}Z_1)^3(Z_2 - \mathbb{E}Z_2)]}{\sigma_1^3 \sigma_2}, \tag{4}$$

$$K(Z_1, Z_1, Z_2, Z_2) \triangleq \frac{\mathbb{E}[(Z_1 - \mathbb{E}Z_1)^2(Z_2 - \mathbb{E}Z_2)^2]}{\sigma_1^2 \sigma_2^2}. \tag{5}$$

$$K(Z_1, Z_2, Z_2, Z_2) \triangleq \frac{\mathbb{E}[(Z_1 - \mathbb{E}Z_1)(Z_2 - \mathbb{E}Z_2)^3]}{\sigma_1 \sigma_2^3}, \tag{6}$$

where σ_i is the standard deviation of $Z_i\,, i = 1\,, 2$, as well as the between-squares correlation, which is defined as the Pearson correlation between the de-meaned squares of two random variables:

$$\text{Corr}^{\text{sq}}(Z_1, Z_2) := \text{Corr}\left((Z_1 - \mathbb{E}Z_1)^2\,, (Z_2 - \mathbb{E}Z_2)^2\right), \tag{7}$$

where $\text{Corr}(\cdot, \cdot)$ denotes the Pearson correlation coefficient function.[14] Two random variables with a high level of the between-squares correlation will tend to undergo both extremely positive and extremely negative deviations.[15]

Compared to coskewness and cokurtosis as defined in (2)-(6), between-squares correlation offers a more efficient estimation of higher-order dependence in two ways. First, between-squares correlation is a normalized measure that ranges within $[-1, 1]$, facilitating easier comparisons in analysis.[16] Second, as demonstrated in Theorem 1 (see Online Appendix Appendix B), under certain regularity conditions, between-squares correlation serves as a sufficient measure of higher-order dependence. Specifically, between-squares correlation, together with linear correlation and given marginal moments, fully characterizes the joint distribution of two random variables.[17]

[14]These measures capture the dependence of higher-order powers of two variables. However, they should not be interpreted as a relationship between the higher-order moments of the two variables. For instance, $S(Z_1, Z_1, Z_2)$ only measures the covariance between Z_2 and the squared value of Z_1, but it does not capture the relationship between Z_2 and the variance (volatility) of Z_1. Mathematical proofs are available upon request.

[15]Under a normality assumption, the between-squares correlation is completely determined by the linear correlation, that is, $\text{Corr}^{\text{sq}}(Z_1, Z_2) = \text{Corr}^2(Z_1, Z_2)$, when (Z_1, Z_2) is binormal. However, when data are subject to risks from higher-order moments (i.e., Shen (2022)), the between-squares correlation emerges and cannot be determined by the linear correlation. For instance, in this paper, we assume both labor income shocks and stock return innovations follow a mixture normal distribution, and thus both labor income and stock return are subject to higher-order moments and the between-squares correlation between them exists.

[16]The between-squares correlation is related to the cokurtosis defined in Equation (5) through the following equation: $\text{Corr}^{\text{sq}}(Z_1, Z_2) = K(Z_1, Z_1, Z_2, Z_2)/\sqrt{(\text{Kurt}[Z_1] - 1)(\text{Kurt}[Z_2] - 1)}$, which ensures its value remains within the range of $[-1, 1]$.

[17]For instance, all five coskewness and cokurtosis measures can be determined from the between-squares correlation, as shown in Online Appendix B.3.

Table 1: Coskewness, Cokurtosis, and Between-Squares Correlation

This table reports estimates and t-statistics (in parentheses) of coskewness, cokurtosis, and between-squared correlations between labor income growth shocks ($\Delta y^*_{it,\Delta}$) and stock return innovations (η_t), computed using PSID and CRSP data. Following Cocco et al. (2005), we estimate log labor income $y_{it} = f(t, \Omega_{it}) + \delta_{it}$, where $f(t, \Omega_{it})$ is a deterministic function of age t and household characteristics Ω_{it} (marital status, household composition). Detrended income y^*_{it} is obtained by subtracting the estimated component $\hat{f}(t, \Omega_{it})$. Labor income shocks $\Delta y^*_{it} \equiv y^*_{i,t+\Delta} - y^*_{it}$ are computed at $\Delta \in \{1, 2\}$-year frequencies. Stock return innovations η_t are derived from market returns minus the risk-free rate. Panel A presents full-sample results, while Panels B and C split households by college education status. Columns distinguish annual (1970–1997) versus biennial (1970–2017) sampling frequencies. The cokurtosis measure, $K(\Delta y^*_i, \Delta y^*_i, \eta, \eta)$, is tested against the null hypothesis $H_0 : K(\Delta y^*_i, \Delta y^*_i, \eta, \eta) = 1$, while all other measures are tested against zero.

	Panel A: Full sample		*Panel B: No college*		*Panel C: College*	
	Annual	Biennial	Annual	Biennial	Annual	Biennial
$S(\Delta y^*_i, \Delta y^*_i, \eta)$	-0.053^{***}	-0.041^{***}	-0.040^{**}	-0.030^{*}	-0.070^{**}	-0.086^{**}
	(-3.215)	(-2.755)	(-2.209)	(-1.780)	(-2.370)	(-2.151)
$S(\Delta y^*_i, \eta, \eta)$	-0.063^{***}	-0.060^{***}	-0.062^{***}	-0.053^{***}	-0.063^{***}	-0.074^{**}
	(-5.430)	(-3.736)	(-4.813)	(-2.887)	(-2.641)	(-2.367)
$K(\Delta y^*_i, \Delta y^*_i, \Delta y^*_i, \eta)$	0.119^{**}	0.173^{***}	0.144^{**}	0.184^{***}	0.060	0.152^{*}
	(2.183)	(3.950)	(2.332)	(3.657)	(0.539)	(1.774)
$K(\Delta y^*_i, \eta, \eta, \eta)$	0.142^{***}	0.267^{***}	0.142^{***}	0.211^{***}	0.141^{**}	0.381^{***}
	(4.504)	(4.704)	(3.840)	(3.159)	(2.354)	(3.618)
$K(\Delta y^*_i, \Delta y^*_i, \eta, \eta)$	1.004	1.039^{*}	0.978	1.008	1.065^{*}	1.102^{**}
	(0.158)	(1.337)	(-0.855)	(0.258)	(1.360)	(1.698)
$\mathrm{Corr}^{\mathrm{sq}}(\Delta y^*_i, \eta)$	0.044^{***}	0.043^{***}	0.034^{**}	0.034^{***}	0.070^{***}	0.061^{***}
	(3.753)	(4.272)	(2.453)	(2.910)	(3.053)	(3.221)

$^{*}\, p < 0.1$; $^{**}\, p < 0.05$; $^{***}\, p < 0.01$

We use sample estimators to calculate the high-order dependence measures described above, based on time series data of household i's persistent income shocks ($\{\Delta y^*_{it}\}_{t=1}^{n_i}$) and market return innovations ($\{\eta_t\}_{t=1}^{n_i}$). Specifically, we estimate the between-squares correlation between labor income growth and stock returns for the i-th household as follows:

(8)

$$\widehat{\mathrm{Corr}}^{\mathrm{sq}}_{\mathrm{i}}(\Delta y^*_i, \eta) = \frac{1}{n_i} \sum_{t=1}^{n_i} \frac{\left[\left(\Delta y^*_{it} - \overline{\Delta y^*_{it}}\right)^2 - \overline{\left(\Delta y^*_{it} - \overline{\Delta y^*_{it}}\right)^2}\right] \left[\left(\eta_t - \overline{\eta_t}\right)^2 - \overline{\left(\eta_t - \overline{\eta_t}\right)^2}\right]}{\sqrt{\left[\left(\Delta y^*_{it} - \overline{\Delta y^*_{it}}\right)^2 - \overline{\left(\Delta y^*_{it} - \overline{\Delta y^*_{it}}\right)^2}\right]^2} \sqrt{\left[\left(\eta_t - \overline{\eta_t}\right)^2 - \overline{\left(\eta_t - \overline{\eta_t}\right)^2}\right]^2}}$$

where a variable with an overbar denotes the mean of the corresponding variable.[18]

Table 1 presents summary statistics comparing six measures of the higher-order dependence between labor income and stock returns for annual and biennial data. Panel A shows that all statistics in the full sample significantly deviate from the reference values without higher-order dependence. Moreover, we observe heterogeneity in the estimates of the higher-order dependence between households without college degrees and college graduates from panels B and C. This suggests that higher-order dependence varies depending on the household's educational background.

Figure 2 shows the coskewness, cokurtosis, and the between-squares correlation across various percentiles of the earnings growth distribution. The data reveal that all statistics deviate from the benchmark values (red dashed lines) if there is no higher-order dependence, which indicates the presence of nontrivial higher-order dependence.[19]

Our findings reveal that the connection between labor and stock markets goes beyond a simple linear correlation, exhibiting a complex dependence structure.[20] Nevertheless, a more crucial question remains unanswered: what drives this higher-order dependence? Multiple factors contribute. Macroeconomic policies, such as increased government spending, may negatively affect labor income by reducing workers' purchasing power. Surges in infrastructure investment can intensify competition for resources, raise prices, and compress industry profits. Similarly, fiscal and monetary policies that raise inflation or interest rates may dampen job opportunities and income levels. Furthermore, studies such as Michelacci and Quadrini (2009), Bronars and Famulari (2001), and Nickell and Wadhwani (1991) show that firm value can be either positively or negatively related to wages. Our paper does not address this aspect in detail, as we consider our contribution a first step in documenting the existence of higher-order dependence between these two markets.

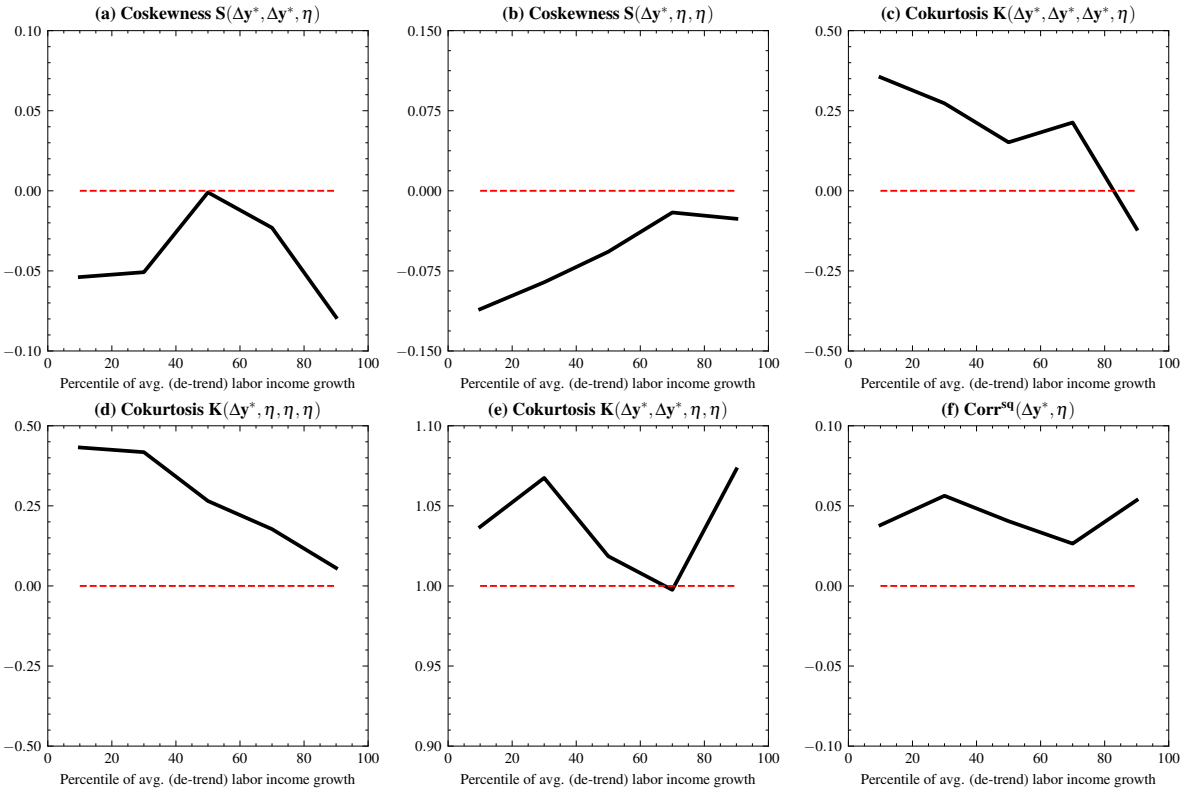
[18]For example, $\overline{\Delta y^*_{it,\Delta}} = \frac{1}{n_i}\sum_{t=1}^{n_i} \Delta y^*_{it,\Delta}$.

[19]Similarly, as shown in Figure G3 in Online Appendix G, when examining households grouped by wealth percentiles, we consistently observe that coskewness, cokurtosis and the between-squares correlation across different groups deviate from the benchmark values without higher-order dependence. The results also hold when we group households by wealth percentiles and earnings growth percentiles as shown in Figure G4 in Online Appendix G.

[20]Additional evidence for the existence of the higher-order dependence between labor and stock markets is provided in Online Appendix G.

Figure 2: Coskewness, Cokurtosis and Between-Squares Correlation between Detrended Labor Income Growth and Stock Return

The line graphs display the coskewness, cokurtosis, and the between-squares correlation between detrended labor income growth (the same as Table 1) and the stock return for the full sample. We use the biennial data from PSID and CRSP data, respectively. Households are grouped by percentiles of average labor income growth shock over all waves. The coskewness, cokurtosis, and between-squares correlation are calculated using the definitions in (2) to (7). Red dashed lines represent the benchmark values if there is zero higher-order dependence.



2.3 A Closer Look at Heterogeneity in Higher-Order Dependence

In this subsection, we take a closer look at higher-order dependence between labor and stock markets, focusing on its distribution and variations across different household groups. Since the between-squares correlation serves as an efficient estimation of higher-order dependence, the remainder of the paper focuses on the between-squares correlation between labor income and stock market returns. Furthermore, higher-order dependence and between-squares correlation are used interchangeably throughout the analysis.

Figure 3 is a histogram and kernel density of the estimations across the full sample, highlighting a high density of positive between-squares correlations. The long right tail indicates that many households earn income that is highly exposed to the stock market as

evidenced by the between-squares correlation. Though there are a few negative between-squares correlations, these negative estimates cluster around zero, suggesting that a positive between-squares correlation is a reasonable assumption for the model setup. Besides, the findings presented in Table 2 also support this assumption.

Figure 3: Distribution of higher-order dependence Measured by the Between-Squares Correlation

The panels display the histograms and kernel density of between-squares correlation estimates between labor income shock and excess stock return for the full sample. We use labor income and stock return data from PSID and CRSP, respectively. Panel A uses the annual data from 1970 to 1997, and panel B uses the biennial data covering the period from 1970 to 2017.

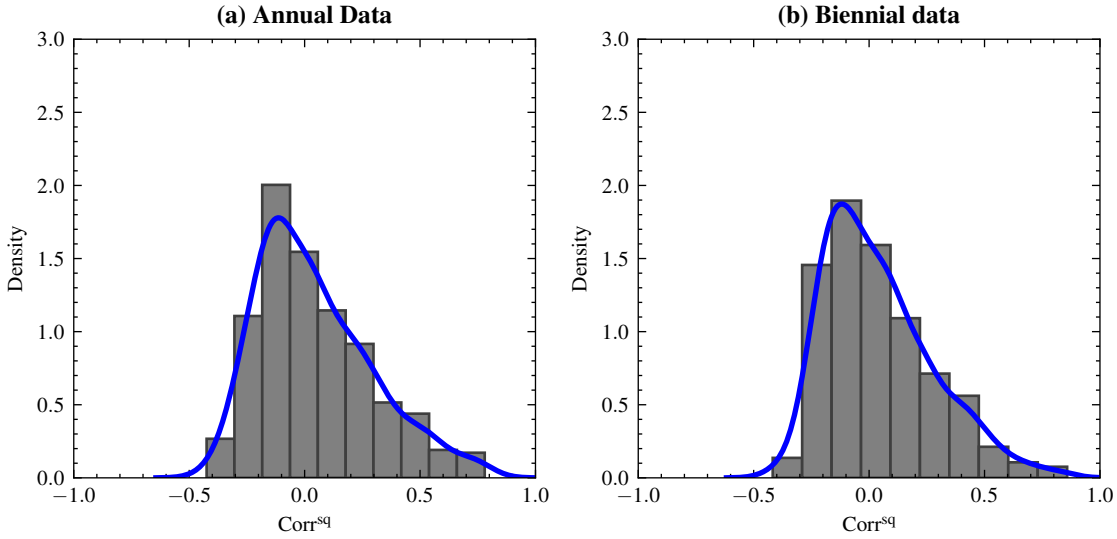


Table 2 presents summary statistics for the between-squares correlation, with results from different data frequencies supporting a significant and positive between-squares correlation. Panel A reveals that the full sample exhibits positive average values of 0.044 and 0.043 for the annual and biennial data, respectively. These values are significantly different from zero, with p-values of $< 0.2\%$. While medians are close to zero, substantial variability exists in the values of the between-squares correlation. Panel A also provides the interquartile range of the between-squares correlation for both data frequencies over the full sample. We find that the 75^{th} and 90^{th} percentiles are substantially positive at 0.2 and 0.4, respectively. Moreover, more than half of the estimations are positive, with about one-third featuring a substantially positive between-squares correlation (≥ 0.1), and over one-fifth exceeding 0.2. Therefore, we assume a positive between-squares correlation and use the sample mean as the calibration target for the model setup. Panels B and C in Table 2 show summary statistics for two

education groups, with results similar to those of the full sample. However, most statistics for the no-college group are lower than those for college graduates, with the average between-squares correlation for college graduates approximately twice that of the no-college group. Based on these findings, we perform separate calibrations for each group and investigate the disparities in the corresponding policy rules in Section 5.

Table 2: Summary Statistics of Higher-Order Dependence Measured by the Between-Squares Correlation

This table presents summary statistics from various samples using PSID and CRSP data. Panel A displays results for the entire sample, while panels B and C show results for households without and with college degrees, respectively. The column labeled "Annual" (or "Biennial") indicates whether the statistics are derived from annual (or biennial) data from the period 1970 to 1997 (or 2017). "Fraction > k" shows the percentage of samples in which the between-squares correlation exceeds k.

	Panel A: Full sample		*Panel B: No college*		*Panel C: College*	
Corr^{sq}	Annual	Biennial	Annual	Biennial	Annual	Biennial
Mean	0.044	0.043	0.034	0.034	0.070	0.061
St. dev	0.247	0.233	0.241	0.224	0.259	0.250
p-value	$< 1\%$	$< 1\%$	$< 2\%$	$< 1\%$	$< 1\%$	$< 1\%$
P10	-0.232	-0.208	-0.236	-0.214	-0.211	-0.186
P25	-0.142	-0.140	-0.152	-0.141	-0.115	-0.135
P50	0.006	0.001	0.007	0.000	0.005	0.004
P75	0.200	0.179	0.177	0.150	0.244	0.206
P90	0.400	0.378	0.357	0.366	0.433	0.406
Fraction > 0	51.26%	50.29%	51.63%	50.00%	50.39%	50.89%
Fraction > 0.1	34.71%	34.24%	34.64%	33.33%	34.88%	36.09%
Fraction > 0.2	24.83%	23.60%	22.55%	22.41%	30.23%	26.04%

3 Empirical Results

Section 2 documents the presence of the higher-order dependence between stock and labor income, and in this section we present our empirical evidence regarding how households make their portfolio decisions in response to higher-order dependence. We begin with graphical evidence. Then, we present results from market participation regressions. Lastly, we examine the portfolio allocation decisions of households.

3.1 Graphical Evidence

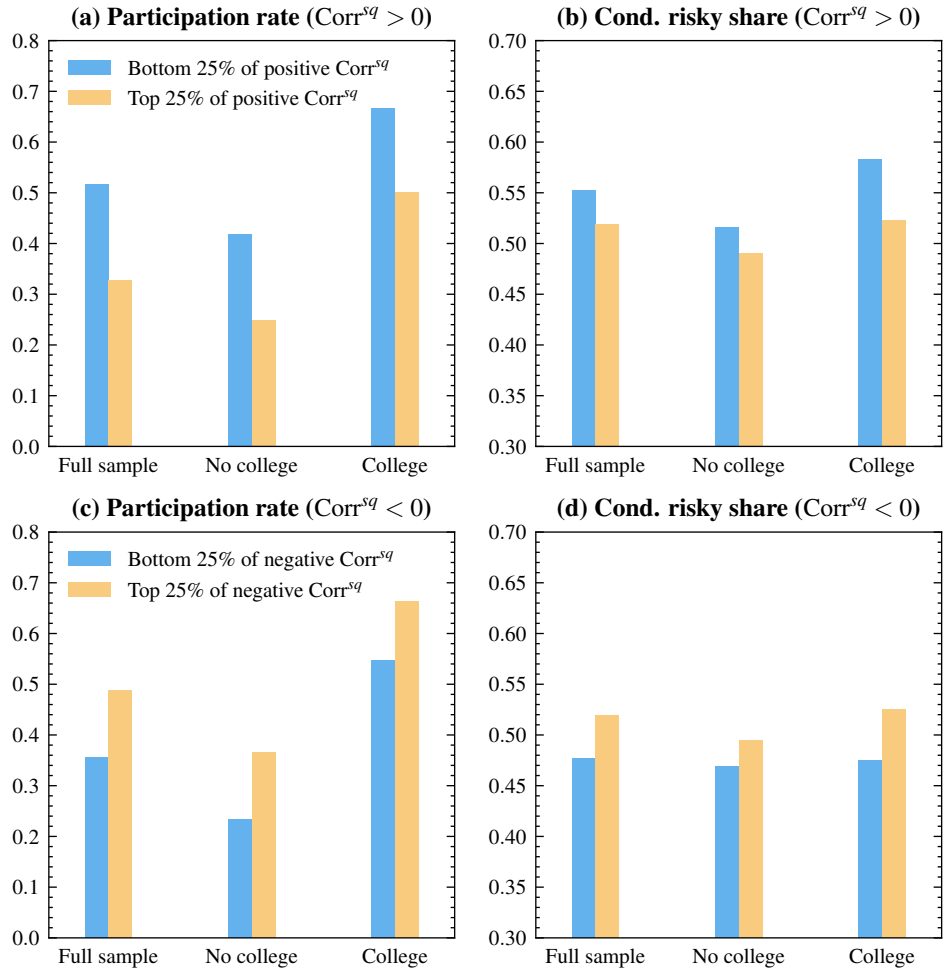
We provide graphical evidence of how higher-order dependence influences market participation and portfolio choice. Panels A and B in Figure 4 show participation rates and the conditional risky asset shares, conditional on the *positive* level of higher-order dependence. In panel A, we present the average participation rates for households with the smallest higher-order dependence (bottom quartile) and largest higher-order dependence (top quartile). We find that households with small higher-order dependence participate more in the market than those in the large higher-order dependence subsample. Meanwhile, conditional on participation, households with smaller higher-order dependence hold more risky asset shares.

Panels C and D in Figure 4 show participation rates and the conditional risky asset shares, conditional on the *negative* level of higher-order dependence. In contrast to *positive* higher-order dependence, when considering the average participation rates, households in the top quartile of *negative* higher-order dependence exhibit significantly higher participation rates compared to those in the bottom quartile of *negative* higher-order dependence. This suggests that more negative higher-order dependence is associated with a decreased likelihood of households participating in risky assets. Meanwhile, conditional on participation, our analysis reveals that households with a more negative higher-order dependence tend to hold a lower proportion of risky asset shares. This suggests that as negative higher-order dependence becomes less pronounced, households exhibit a greater inclination to allocate a larger share of their portfolio to risky assets.

Overall, the first look depicted in Figure 4 suggests that higher-order dependence plays a significant role in explaining the portfolio decisions of households. Moreover, it exhibits a nonlinear impact on both participation rates and conditional risky asset shares. These findings highlight the need for further exploration and analysis of higher-order dependence and households' portfolio decisions.

Figure 4: Participation Rate and Conditional Risky Share for Different Corr^{sq}

The panels present participation rates and the conditional risky share for different Corr^{sq} subsamples. The samples are by quartiles of Corr^{sq} between income growth and market returns. Panels use PSID and CRSP data.



3.2 Portfolio Decisions

We regress households' portfolio decisions on higher-order dependence measured by the between-squares correlation. While higher-order dependence is measured using labor income data dating back to 1968, the household portfolio data is limited to the period from 1997 to 2019, as the PSID has been reporting the value of stockholdings continuously since 1997.[21] This detailed information enables us to explore the empirical link between labor income and portfolio decisions. We define the variables following Brunnermeier and Nagel (2008). Financial wealth is calculated as the sum of equity in stocks and the value in safe

[21] Before 1997, only the 1984 and 1989 waves reported the data on stockholdings.

accounts, where the value in safe accounts is the money amount in checking and savings accounts, money market funds, certificates of deposit, government bonds, or Treasury bills. We calculate the ratio of financial wealth invested in stocks and assign a value of one if a household participates in the stock market and zero if not. To estimate the higher-order dependence in labor income and stock returns for each household, we use the biennial income level and the CRSP index. Moreover, we observe a nonlinear relationship between higher-order dependence and households' portfolio decisions from Section 3.1. Specifically, higher-order dependence, measured by between-squares correlation, has an almost opposite relationship with portfolio decisions at the tipping point of zero. Therefore, we distinguish between *positive* and *negative* higher-order dependence to better capture this nonlinear relationship.

We begin our empirical analysis by estimating a probit regression for stock market participation. Motivated by the nonlinear impact on portfolio decisions, where household portfolio choices differ depending on whether Corr^{sq} is positive or negative, as highlighted in the graphical evidence in Section 3.1, we propose the use of independent variables $\text{Corr}^{sq,+}$ and $\text{Corr}^{sq,-}$. These variables capture *positive* and *negative* higher-order dependence, respectively:

$$\text{Corr}_i^{sq,+} = \max\left(0, \widehat{\text{Corr}^{\text{sq}}}_i\right), \tag{9}$$

$$\text{Corr}_i^{sq,-} = \min\left(0, \widehat{\text{Corr}^{\text{sq}}}_i\right), \tag{10}$$

where $\widehat{\text{Corr}^{\text{sq}}}_i$ is given in (8).

We investigate whether a link exists between higher-order dependence and participation decisions and if there is any heterogeneity between college graduates and households without college degrees. Following the related literature, we control for household characteristics, such as the level of income and wealth, income risk, marriage status, age, and education. These household-level variables serve as a reasonable proxy for real and perceived market participation costs:

$$\mathbb{P}(\alpha_{it} > 0) = \Phi\Big(\beta_0 + \beta_1^{+}\text{Corr}_i^{sq,+} + \beta_1^{-}\text{Corr}_i^{sq,-} + \beta_2 Corr_i + \beta_3 \ln(y_{it}) + \beta_4 age_{it} + \beta_5 age_{it}^2$$

$$\beta_6 w_{it} + \beta_7 s_i + \beta_8 sk_i + \beta_9 k_i + marr_i + year_t + \xi_{it}\Big), \tag{11}$$

$$\alpha^*_{it} = \beta_0 + \beta_1^+ \text{Corr}_i^{sq,+} + \beta_1^- \text{Corr}_i^{sq,-} + \beta_2 Corr_i + \beta_3 \ln(y_{it}) + \beta_4 age_{it} + \beta_5 age_{it}^2$$

$$\beta_6 w_{it} + \beta_7 s_i + \beta_8 sk_i + \beta_9 k_i + marr_i + year_t + \xi_{it}, \tag{12}$$

where Φ is the cumulative distribution function of the standard normal distribution, $\alpha_{it}, y_{it}, w_{it}$ are the risky share, labor income, and financial wealth of household i at period t, s_i, sk_i, k_i, are the standard deviation, skewness, and kurtosis of labor income growth rate over the life cycle, $\text{Corr}_i, \text{Corr}_i^{sq,+}, \text{Corr}^{sq,-}$ are the correlation and between-squares correlations between labor income growth of household i and CRSP stock return over the life cycle. Additionally, we include marriage and year fixed effects. Finally, α^*_{it} is defined by

$$\alpha_{it} = \begin{cases} 0, & \alpha^*_{it} \leq 0 \\ \alpha^*_{it}, & 0 < \alpha^*_{it} < 1 \\ 1, & \alpha^*_{it} \geq 1. \end{cases} \tag{13}$$

Table 3 shows the marginal effects from probit market participation regressions. Our results indicate that the absolute value of higher-order dependence is significantly negatively correlated with households' participation decisions. In particular, we find that when the *positive* (*negative*) higher-order dependence increases, the propensity to participate in the stock market decreases (increases). This result still holds under different model specifications. Moreover, the coefficients of higher-order dependence remain statistically significant for both college graduates and the no-college group. Across all model specifications, we find that the relation between higher-order dependence and participation decisions is stronger for the no-college group than for college graduates. Specifically, for the coefficients in front of $\text{Corr}^{sq,+}$, we observe a value of -0.505 (model (4)) for the no-college group, while for college graduates, it is -0.411 (model (7)). As we include more control variables, the discrepancy between these two coefficients tends to grow. These differences indicate that households with college degrees exhibit greater resilience compared to those without college degrees.

Table 3: Probit Regressions of Stock Market Participation

This table reports marginal effects from probit regressions. The dependent variable is the dummy variable for stock market participation. Estimated coefficients and standard errors (in parentheses) are presented. We conduct the regressions using different sample populations and control for various households characteristics. Additionally, all regressions include year dummies, which are not reported. The estimations are based on PSID and CRSP data from 1997 to 2019.

	Full sample			No college			College		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$Corr^{sq,+}$	-0.492^{***}	-0.294^{***}	-0.237^{***}	-0.505^{***}	-0.339^{***}	-0.205^{**}	-0.411^{***}	-0.256^{**}	-0.241^{**}
	(0.060)	(0.063)	(0.071)	(0.079)	(0.083)	(0.093)	(0.098)	(0.102)	(0.112)
$Corr^{sq,-}$	0.640^{***}	0.356^{***}	0.263^{***}	0.771^{***}	0.497^{***}	0.302^{**}	0.473^{***}	0.281^{**}	0.295^{**}
	(0.075)	(0.078)	(0.089)	(0.100)	(0.104)	(0.117)	(0.122)	(0.125)	(0.141)
$Corr$		-0.028	-0.021		-0.055	-0.038		-0.047	-0.059
		(0.036)	(0.037)		(0.048)	(0.049)		(0.055)	(0.058)
$ln(y)$		0.933^{***}	0.669^{***}		0.860^{***}	0.550^{***}		0.684^{***}	0.529^{***}
		(0.034)	(0.041)		(0.047)	(0.056)		(0.052)	(0.067)
$Age/10$			-0.273^{*}			-0.001			-0.327
			(0.143)			(0.186)			(0.231)
$Age^2/100$			0.025			-0.001			0.034
			(0.017)			(0.022)			(0.027)
$Marriage$			-0.347^{***}			-0.152^{**}			-0.343^{***}
			(0.052)			(0.070)			(0.082)
$FWealth$			0.247^{***}			0.253^{***}			0.170^{***}
			(0.009)			(0.014)			(0.011)
$Std(d\delta)$			-0.001			-0.011			0.094
			(0.098)			(0.130)			(0.154)
$Skew(d\delta)$			-0.012			-0.019			0.010
			(0.023)			(0.030)			(0.035)
$Kurt(d\delta)$			-0.022			-0.031^{*}			-0.023
			(0.014)			(0.018)			(0.021)
N		7,954			4,794			3,160	

$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

Next, we will look for a link between higher-order dependence and risky asset shares. We run Tobit regressions for market participants. In this exercise, the dependent variable is the share of wealth in stocks, and the independent variables of interest are the measures of higher-order dependence ($\text{Corr}^{sq,+}$ and $\text{Corr}^{sq,-}$). In addition, we control for variables that may cause movement in risky asset holdings.

Table 4 reports regression results for the full sample, and separately for households without a college degree and those with a college degree. We focus on the estimated coefficients associated with higher-order dependence ($\text{Corr}^{sq,+}$ and $\text{Corr}^{sq,-}$). We find that the coefficients of $\text{Corr}^{sq,+}$ ($\text{Corr}^{sq,-}$) are consistently negative (positive) for all households, the no-college group, and college graduates, with a high statistical significance. For example, in

model specification (3), where we consider a broad set of control variables, the coefficient estimate of $\text{Corr}^{sq,+}$ (-0.095) remains significant. For college graduates, the coefficient estimate is -0.092, which is smaller than that (-0.100) for the no-college group, but both estimates are statistically significant.

Overall, we find that the regression analysis is consistent with the first look in Figure 4. Higher-order dependence has a nonlinear relationship with participation decisions and risky asset shares. For all sample groups, positive (negative) higher-order dependence is negatively (positively) correlated with participation in the stock market and risky asset holdings. This result is stronger for the no-college group compared to college graduates.

Table 4: Tobit Model of the Risky Share

This table reports the marginal effects from tobit regressions. The dependent variable is the risky share of households. Estimated coefficients and standard errors (in parentheses) are presented. We conduct the regressions using different sample populations and control for various households characteristics. Additionally, all regressions include marriage and year dummies, which are not reported. The estimations are based on PSID and CRSP data from 1997 to 2019.

	Full sample			No college			College		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\text{Corr}^{sq,+}$	−0.280***	−0.168***	−0.095***	−0.316***	−0.209***	−0.100*	−0.200***	−0.135***	−0.092**
	(0.034)	(0.032)	(0.034)	(0.05)	(0.049)	(0.051)	(0.042)	(0.042)	(0.044)
$\text{Corr}^{sq,-}$	0.404***	0.245***	0.153***	0.511***	0.33***	0.182***	0.293***	0.215***	0.152***
	(0.042)	(0.04)	(0.043)	(0.064)	(0.062)	(0.065)	(0.053)	(0.052)	(0.055)
Corr		−0.017	−0.011		−0.031	−0.027		−0.027	−0.021
		(0.018)	(0.018)		(0.029)	(0.027)		(0.023)	(0.023)
ln(y)		0.471***	0.461***		0.503***	0.330***		0.282***	0.282***
		(0.017)	(0.019)		(0.028)	(0.030)		(0.021)	(0.025)
Age/10			−0.428			0.836			0.435
			(0.686)			(1.021)			(0.893)
$Age^2/100$			1.162			−0.644			0.101
			(0.808)			(1.205)			(1.049)
Marriage			−0.191***			−0.089**			−0.147***
			(0.025)			(0.039)			(0.032)
FWealth			0.008***			0.075***			0.004***
			(0.001)			(0.005)			(0.001)
$Std(d\delta)$			0.079*			−0.002			0.145**
			(0.047)			(0.071)			(0.06)
$Skew(d\delta)$			−0.006			−0.019			0.012
			(0.011)			(0.016)			(0.013)
$Kurt(d\delta)$			−0.002			−0.004			−0.008
			(0.006)			(0.01)			(0.007)
N		7,954			4,794			3,160	

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

4 A Mean-Variance-Skewness-Kurtosis Model

To better understand the impact of higher-order dependence on portfolio allocation, as well as the underlying economic intuition behind its nonlinear relationship with portfolio choice, we extend the mean-variance-skewness framework from Harvey and Siddique (2000) and start with a mean-variance-skewness-kurtosis portfolio optimization problem.[22] Through this model, we show that higher-order dependence has a nonlinear effect on portfolio allocation as it affects households' portfolio allocation via two channels: the skewness channel and the kurtosis channel.

We consider a one-period economy with two assets, a stock with a random return R^S and a bond with a constant gross return R_f. For simplicity, we assume that the household is endowed with a fixed initial wealth (W_0), and that borrowing and short selling are not allowed.[23] She decides what proportion of her wealth (α) to invest in the stock and saves the rest. Meanwhile, she faces labor income risk, captured by $W_0 R^L$, where R^L is the income shock. So, her next-period wealth (W_1) is given by

$$W_1 = W_0 R^p = W_0(1 + \alpha R^S + (1-\alpha)R_f + R^L) = W_0(1 + R_f + \alpha R^E + R^L), \tag{14}$$

where $R^E = R^S - R_f$ is the excess return and $R^p = 1 + R_f + \alpha R^E + R^L$ is the portfolio return.[24]

The households then solves the following optimization problem, which accounts for the mean, variance, skewness, and kurtosis of portfolio returns (MVSK):[25]

$$\max_{\alpha} \mathbb{E}(R^p) - \lambda_1 \text{Var}(R^p) - \lambda_2 \text{Skew}(R^p) - \lambda_3 \text{Kurt}(R^p), \tag{15}$$

where $\lambda_1, \lambda_2, \lambda_3$ are risk weights contingent on the household's utility.[26]

[22] For simplicity, we do not consider participation decisions.

[23] For tractability, consumption is assumed to be exogenous here.

[24] This portfolio return has a constant unit of labor income return. This reflects the fact that labor income is nontradable and households can only adjust their holdings of risky assets to achieve a target portfolio return.

[25] According to Online Appendix C, the optimization problem (Equation (43)) for this household with generic utility $u(\cdot)$ can be approximated by the a mean-variance-skewness-kurtosis objective.

[26] In the special case of mean-variance utility, we set $\lambda_2 = 0$ and $\lambda_3 = 0$. However, in the case of more general utility functions, λ_2 and λ_3 take on nontrivial values, and their impact on the decision-making process

The main focus of our investigation is to examine the effect of higher-order dependence on households' portfolio allocation, as determined by Equation (15). The lemma 1 demonstrates that the higher-order dependence, measured by various measures of coskewness and cokurtosis, affects the skewness and kurtosis of the portfolio, while leaving the variance unchanged. However, it is important to note that relying solely on any of these measures fails to fully capture the effect of higher-order dependence on the household's portfolio risk profile.

Lemma 1. *Let $(R^E, R^L)^T$ be a random vector. A portfolio return $\Pi(\alpha, R^E, R^L)$ is defined as the linear combination $\alpha R^E + R^L$. Then the standard deviation, skewness, and kurtosis of this portfolio can be represented as:*

$$\begin{aligned}
\sigma_\Pi =& \sqrt{\alpha^2\sigma_E^2 + 2\alpha\sigma_E\sigma_L \mathrm{Corr}\left(R^E, R^L\right) + \sigma_L^2}, \\
\mathrm{Skew}_\Pi =& \frac{1}{\sigma_\Pi^3}\left[\alpha^3\sigma_E^3 S_E + \sigma_L^3 S_L + 3\alpha^2\sigma_E^2\sigma_L S(R^E, R^E, R^L) + 3\alpha\sigma_E\sigma_L^2 S(R^E, R^L, R^L)\right], \\
\mathrm{Kurt}_\Pi =& \frac{1}{\sigma_\Pi^4}\left[\alpha^4\sigma_E^4 K_E + \sigma_L^4 K_L + 4\alpha^3 K(R^E, R^E, R^E, R^L) + 6\alpha^2 K(R^E, R^E, R^L, R^L)\right. \\
& \left. + 4\alpha K(R^E, R^L, R^L, R^L)\right],
\end{aligned}$$

where σ_a, S_a, K_a, for $a \in \{E, L\}$ denote the standard deviation, skewness, and kurtosis of R^E and R^L, respectively.

Specifically, we model higher-order dependence between the labor and stock markets by assuming a bivariate mixture normal distribution (also see the definition in Appendix B) for the excess return, R^E, and labor income risk, R^L, where

$$(16)\quad R^E \sim \begin{cases} N(\mu_{E,1}, \sigma_{E,1}) & \text{with prob. } p_E, \\ N(\mu_{E,2}, \sigma_{E,2}) & \text{with prob. } 1 - p_E, \end{cases} \quad R^L \sim \begin{cases} N(\mu_{L,1}, \sigma_{L,1}) & \text{with prob. } p_L, \\ N(\mu_{L,2}, \sigma_{L,2}) & \text{with prob. } 1 - p_L, \end{cases}$$

and the pair $(N(\mu_{E,l}, \sigma_{E,l}), N(\mu_{L,m}, \sigma_{L,m}))$ is linearly correlated with a coefficient ρ_{lm} for $\forall l, m \in \{1, 2\}$. This joint distribution captures two distinct types of shocks affecting both labor and stock markets: one generating comovement and the other inducing divergence. The mixture normal specification for stock returns and labor income shocks effectively captures

becomes significant.

heavy tails and low-frequency events commonly observed in empirical data. Consequently, higher-order dependence naturally arises, independent of the mixture normality assumption.[27]

Moreover, to capture the entirety of higher-order dependence, we impose proper regularity conditions such that between-squares correlation turns to be a sufficient measure of higher-order dependence.[28] As demonstrated by the following theorem, given the shares of risky assets, the portfolio's risk profile, such as skewness and kurtosis, is solely determined by the higher-order dependence as measured by between-squares correlation.

Theorem 1. *Let* $(R^E, R^L)^T$ *be a random vector following a bivariate mixture normal distribution. Define the portfolio return* $\Pi(\alpha, R^E, R^L)$ *as the linear combination* $\alpha R^E + R^L$*. Under the regularity conditions stated in Theorem 3 in Appendix B, the standard deviation of this portfolio is independent of the between-squares correlation and is given by:*

$$\sigma_\Pi = \sqrt{\alpha^2\sigma_E^2 + 2\alpha\sigma_E\sigma_L \mathrm{Corr}\,(R^E, R^L) + \sigma_L^2}.$$

The skewness and kurtosis of this portfolio can be expressed as functions of $\mathrm{Corr}^{sq}(R^E, R^L)$*:*

$$\mathrm{Skew}_\Pi = \mathrm{sgn}(\rho_1 + \frac{a_1}{2a_2})k_s\sqrt{\mathrm{Corr}^{sq}(R^E, R^L) - \frac{4a_0a_2 - a_1^2}{4a_2}} + b_s,$$

$$\mathrm{Kurt}_\Pi = a_k\mathrm{Corr}^{sq}(R^E, R^L) + \mathrm{sgn}(\rho_1 + \frac{a_1}{2a_2})b_k\sqrt{\mathrm{Corr}^{sq}(R^E, R^L) - \frac{4a_0a_2 - a_1^2}{4a_2}} + c_k,$$

where $\mathrm{sgn}(\cdot)$ *denotes the sign function, and the constants* $\rho_1, a_0, a_1, a_2, k_s, b_s, a_k, c_k$ *are defined in Online Appendix B.*

Furthermore, it is important to note that portfolio variance depends solely on the linear correlation and marginal distributions, and thus fails to capture risks arising from higher-order dependence. Consequently, our results suggest that the use of utility functions that

[27] Alternative distributions, such as the Laplace distribution, can capture higher moments in the data-generating process while still allowing for the existence of higher-order dependence.

[28] Under the regularity conditions in Online Appendix B where the marginal distributions and linear correlation are fixed, any coskewness and kurtosis can be fully determined by the between-squares correlation. This is analogous to the concept of *copula* (i.e., Fan and Patton (2014)), which is independent of marginal information but captures only dependence.

incorporate higher-order moments provides a more effective framework for capturing and managing portfolio risks than the conventional Markowitz mean-variance approach.

Figure 5: Sensitivity of Portfolio Skewness and Kurtosis to Between-Squares Correlation

These panels illustrate how the higher-order moments of portfolio returns change as the between-squares correlation varies, while all other parameters are kept constant. Panel A shows the changes in skewness, and panel B shows the changes in kurtosis. The risky share of wealth (α) is set to 1, and the linear correlation is fixed at 0.1. The remaining moments are specified as follows: for labor income risk, $\mathbb{E}[R^L] = 0$, $\text{Var}(R^L) = 0.25^2$, $\text{Skew}(R^L) = -0.5$, and $\text{Kurt}(R^L) = 3.0$; for stock return innovations, $\mathbb{E}[R^S] = 0$, $\text{Var}(R^S) = 0.2^2$, $\text{Skew}(R^S) = -1$, and $\text{Kurt}(R^S) = 4.5$.

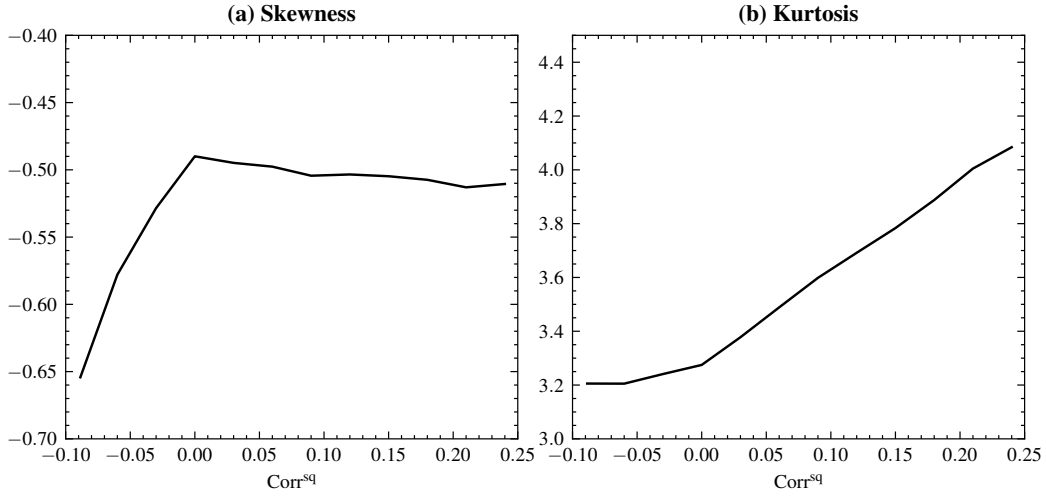


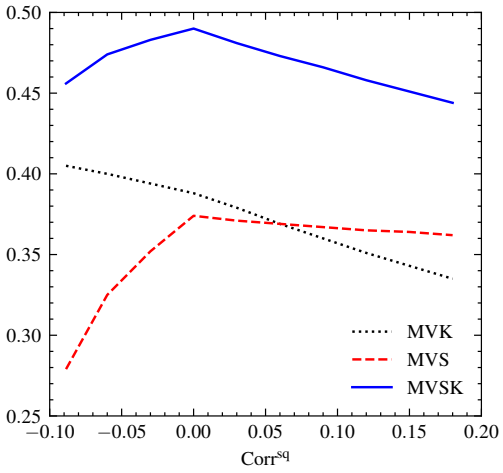
Figure 5 presents the skewness and kurtosis of portfolio returns as higher-order dependence varies, while linear dependence and all other parameters are held constant at $\alpha = 1$. The results indicate that changes in higher-order dependence have asymmetric effects on the portfolio's skewness and kurtosis. Specifically, a decrease in higher-order dependence from 0 to -0.10 leads to a substantial decline in skewness, from -0.50 to -0.65, while kurtosis remains unchanged. In contrast, an increase in higher-order dependence from 0 to 0.25 results in a significant increase in kurtosis, from 3.25 to 4.10, while skewness remains relatively stable, with only a slight decrease.

Overall, these findings indicate that the impact of higher-order dependence on portfolio risk is primarily driven by the skewness channel when higher-order dependence is negative, whereas the kurtosis channel becomes more prominent when it is positive. Therefore, households with nontradable labor income are expected to strategically reduce their risky asset share in order to manage portfolio risk and optimize their risk-adjusted objective when

higher-order dependence deviates from zero.

Figure 6: Optimal Risky Share

This figure compares the optimal risky asset share across varying levels of between-squares correlation, while holding all other parameters constant. The analysis is conducted under three distinct model specifications: MVSK, MVS, and MVK. The MVSK model incorporates the mean, variance, skewness, and kurtosis of portfolio returns; the MVS model includes only the mean, variance, and skewness; and the MVK model considers the mean, variance, and kurtosis. The parameters used in the simulation are: $\mathbb{E}[R^S] = 0.04$, $\text{Std}[R^S] = 0.2$, $\text{Skew}[R^S] = -0.5$, $\text{Kurt}[R^S] = 4.5$; $\mathbb{E}[R^L] = 0.01$, $\text{Std}[R^L] = 0.25$, $\text{Skew}[R^L] = -1$, $\text{Kurt}[R^L] = 3$; $\text{Corr}[R^S, R^L] = 0.1$; and initial wealth is set to $W_0 = 1$. The coefficients λ_1, λ_2, and λ_3 are calibrated using a CRRA utility function with a relative risk aversion of 3, based on the parameters above. The resulting values are $\lambda_1 = 0.9473$, $\lambda_2 = -0.0054$, and $\lambda_3 = 0.0011$.



As anticipated, the results shown in Figure 6 align with our previous findings. The figure illustrates the optimal risky asset share as the higher-order dependence varies. To further investigate the asymmetric impact of the higher-order dependence on skewness and kurtosis, we examine two additional model specifications. The first specification includes the mean, variance, and skewness (MVS), while the second considers the mean, variance, and kurtosis (MVK).

As higher-order dependence increases from -0.10 to 0, the optimal risky share rises, but it decreases when higher-order dependence increases from 0 to 0.20. These findings can be explained through the lens of the reduced models, MVK and MVS. When higher-order dependence is negative, the MVS model exhibits a more pronounced change compared to the MVK model, indicating that negative higher-order dependence has a greater impact on portfolio skewness than on kurtosis. Consequently, the MVSK model displays a pattern similar to that of MVS with a negative higher-order dependence, suggesting that the skewness channel is the dominant driver.

Conversely, when higher-order dependence becomes positive, its impact on portfolio skewness is minimal, as reflected by the relatively flat behavior of the MVS model. In contrast, the effect on portfolio kurtosis is substantial, as indicated by the MVK model under positive higher-order dependence. As a result, the MVSK model exhibits a pattern similar to that of MVK, suggesting that the effect of positive higher-order dependence on kurtosis dominates the determination of the optimal risk share, thus indicating a kurtosis channel.

5 A Life-Cycle Model

Since households make portfolio decisions over their lifetimes and the PSID tracks the same households and their descendants over time, it is important to study a life-cycle model with higher-order dependence and calibrate it to the data. To do so, we extend the framework of Shen (2022) by introducing bivariate mixture normally distributed labor and stock shocks. This extension allows us to quantitatively assess the impact of higher-order dependence on households' portfolio decisions. In each period, households earn labor income that nonlinearly depends on the stock return (measured by the between-squares correlation) and and decide on consumption, stock market participation, and conditional risky asset holdings. Borrowing against risk-free assets or future labor income, as well as short-selling of risky assets, is prohibited.

Households have Epstein-Zin (1989) preferences. For household i, let $X_{i,t}$ denote the cash-on-hand at the beginning of age t. The recursive utility function is given by

$$U_{i,t} = \left\{ (1-\beta) C_{i,t}^{1-1/\psi} + \beta \left(\mathbb{E}_t \left[p_{t+1} U_{i,t+1}^{1-\gamma} + b(1-p_{t+1}) X_{i,t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}, \tag{17}$$

where $C_{i,t}$ is consumption at the beginning of age t, β is the discount factor, b determines the strength of bequest motive, γ is the coefficient of relative risk aversion, and ψ is the elasticity of intertemporal substitution.

Labor Income Process and Financial Assets Return Given household i's labor income process during the working period y_{it}, as defined in (1), we further decompose the

labor income shock $\delta_{i,t}$ into a persistent shock, ν_{it}, and a transitory shock, ϵ_{it}:

$$\delta_{it} = \nu_{it} + \epsilon_{it}. \tag{18}$$

We assume that ϵ_{it} is normally distributed as $N(0, \sigma_\epsilon^2)$ and is independent of both ν_{it} and the stock return shock, η_{it}. The persistent shock ν_{it} is given by

$$\nu_{it} = \lambda \nu_{i(t-1)} + u_{it}, \tag{19}$$

where the noise term u_{it} is specified in (22).

Income during the retirement period is assumed to be exogenous and deterministic, with all households retiring in period K, corresponding to age 65. Income during retirement follows

$$y_{it} = log(\lambda) + f(K, \Omega_{iK}) + v_{iK}, \quad t > K, \tag{20}$$

where retirement income is modeled as a constant fraction (λ) of the permanent component of labor income in the last working period.

We assume financial market assets consist of two financial assets in which households can invest, one riskless and one risky asset. The riskless asset has a constant gross return R_f, and the return of the risky asset is given by

$$R_t^S = R_f + \mu + \eta_t, \tag{21}$$

where μ is the excess return of the risky asset and η_t is the shock to returns.

Dependence between Stock and Labor Markets We model the dependence between the labor and stock markets using a bivariate mixture normal distribution for the shocks to

the persistent component of labor income, $u_{i,t}$, and the shocks to asset returns, η_t:

(22)
$$u_{it} = \begin{cases} u_{it,1} \sim N(\mu_{u,1}, \sigma^2_{u,1}) & \text{with prob. } p_u, \\ u_{it,2} \sim N(\mu_{u,2}, \sigma^2_{u,2}) & \text{with prob. } 1 - p_u. \end{cases} \quad \eta_t \sim \begin{cases} \eta_{it,1} \sim N(\mu_{\eta,1}, \sigma_{\eta,1}) & \text{with prob. } p_\eta, \\ \eta_{it,2} \sim N(\mu_{\eta,2}, \sigma_{\eta,2}) & \text{with prob. } 1 - p_\eta, \end{cases}$$

Define the linear correlations between the components of u_{it} and η_{it} as follows:

$$\rho_{lm} = \text{Corr}\left(u_{it,l}, \eta_{it,m}\right), l = 1, 2, m = 1, 2. \tag{23}$$

As demonstrated in Appendix B, the dependence structure between labor and stock shocks is fully captured by the Pearson correlation and the between-squares correlation. Specifically, the linear and higher-order dependence between u_{it} and η_{it}, both determined by the four correlations ρ_{lm}, can be explicitly characterized by (31) and (32) in Appendix B, respectively.[29]

Optimization Problem In each period t, households manage their cash-on-hand X_t by deciding whether or not to participate in the market, determining their risky shares conditional on participation, and choosing consumption to maximize expected utility, subject to borrowing constraints. Let V_{it} denote household i's value at time t. We standardize the entire problem by using the persistent labor income, $P_{it} = e^{y_{it}}$, and derive normalized variables: normalized cash-on-hand ($x_{it} = \frac{X_{it}}{P_{it}}$), normalized consumption ($c_{it} = \frac{C_{it}}{P_{it}}$), and the normalized value function ($v_{it} = \frac{V_{it}}{P_{it}}$). Online Appendix D.1 provides further details.

5.1 Calibration

Labor Income, Stock Return, and Dependence We jointly estimate the parameters of labor income, stock return, and their dependence structure using the GMM. To estimate labor income shocks, we use PSID data from 1970 to 2017 and run a regression to estimate family-specific fixed effects $f(t, \Omega_{it})$ of labor income in Equation (1).[30] The residuals from the

[29] For more derivation details, see Appendix B.

[30] See Table A1 for variable definitions and Online Appendix A for the data selection criteria and additional details.

regression can be considered as realizations of the labor income shock, $\delta_{i,t}$. Next, we take the annual excess returns from the CRSP between 1970 and 2017, where the demeaned series are interpreted as realizations of stock return shocks, η_t. To capture the dependence structure between the two markets, we use the average between-squares correlation reported in Table 2, along with the average Pearson correlation estimated using the same methodology. For simplicity, we assume that $\rho_{11} = \rho_{22}$ and $\rho_{12} = \rho_{21}$. Consistent with Section 2, we control for education and divide households into two groups: those without a college degree and college graduates. For each group, we estimate the higher-order dependence and calibrate the corresponding parameters.

Preference and Bequest Motive We use SCF data from 2007 to 2019 and calibrate the preference parameters and fixed cost to match the average participation rate, risky asset share, and normalized wealth across different age groups. To capture behavioral differences between the two educational groups, we calibrate the parameters separately, except for relative risk aversion γ and the fixed cost F, which are assumed to be the same between groups.[31] To approximate the life-cycle profiles for an average household, we assume that households with college degrees make up 30% of the world's population, which is observed from the PSID.

Table 6 shows the calibration results for the preference parameters and the fixed cost. For comparison, we also calibrate the model without higher-order dependence and report the preference parameters in Table 6. This alternative model can be viewed as similar to the one presented in Shen (2022) and serves as a useful point of comparison. When higher-order dependence is included, the risk aversion estimate drops from 5.3 to 4.3. With this moderate risk aversion and low fixed cost of 0.005 (0.5% of the household's expected annual income), our model matches the data well. Therefore, including higher-order dependence fits the empirical data, especially the portfolio choice, with a lower but reasonable risk aversion estimate. In addition, the strength of the bequest motive (b) is 2.5, within the range of

[31] We consider a scenario in which risk aversion is allowed to vary between college graduates and households without college degrees in Online Appendix E.4. Figure E5 shows that the main results still hold qualitatively.

[31] Compared with the existing literature, the difference is even larger, for example, the models without the between-squares correlation have risk aversion estimates of $\gamma = 11$ in Fagereng et al. (2017), $\gamma = 5.6$ in Shen (2022), and $\gamma = 6.1$ in Catherine (2022).

Table 5: Baseline Calibration Parameters

This table presents the calibrated parameters for the life-cycle model. We use PSID and CRSP data to perform a two-step calibration. Panel A reports the parameters for the mixture normal shock, which governs the stock return process. Panel B provides the parameters for the persistent shock $\nu_{i,t}$ (modeled as mixture normally distributed) and the transitory shock $\epsilon_{i,t}$ (modeled as normally distributed), which define the labor income process for different groups. The two correlation parameters represent the correlations between the stock return shock components, η_t, and the persistent labor income shock, $\nu_{i,t}$. Panel C reports the dependence parameters. We assume that both groups share the same risk aversion and fixed cost.

Panel A: Stock return shock, η_t			
Mixture weight	p_η	0.171	
Mean of 1st component	$\mu_{\eta,1}$	-0.184	
Mean of 2nd component	$\mu_{\eta,2}$	0.038	
SD of 1st component	$\sigma_{\eta,1}$	0.394	
SD of 2nd component	$\sigma_{\eta,2}$	0.126	
Panel B: Labor income shocks $u_{i,t}$ and $\epsilon_{i,t}$			
		No college	College
Mixture weight of $u_{i,t}$	p_u	0.271	0.279
Mean of 1st component of $u_{i,t}$	$\mu_{u,1}$	-0.124	-0.156
Mean of 2nd component of $u_{i,t}$	$\mu_{u,2}$	0.046	0.060
SD of 1st component of $u_{i,t}$	$\sigma_{u,1}$	0.172	0.231
SD of 2nd component of $u_{i,t}$	$\sigma_{u,2}$	0.010	0.013
SD of transitory shock $\epsilon_{i,t}$	σ_ϵ	0.204	0.138
1st autocorrelation coefficient	λ	0.918	0.959
Replacement rate	k	0.903	0.945
Panel C: Dependence parameters			
		No college	College
Dependence parameter 1	ρ_{11}, ρ_{22}	0.832	0.769
Dependence parameter 2	ρ_{12}, ρ_{21}	-0.166	-0.222

Table 6: Preference Parameters

This table presents the preference parameters for both the baseline model and the model without higher-order dependence. For the latter, we adjust the two correlation parameters to remove any higher-order dependence. The calibration of the preference parameters follows the methodology outlined in Section 5.1.

		Baseline model		Model without Corr^{sq}	
		No college	College	No college	College
Relative risk aversion	γ	4.3	4.3	5.3	5.3
EIS	ϕ	0.28	0.95	0.30	0.90
Discount factor	β	0.902	0.985	0.92	0.986
Bequest motive	b	2.5	2.5	2.5	2.5
Fixed cost rate	F	0.005	0.005	0.005	0.005

existing empirical evidence and calibrations. Compared with college graduates, households without college degrees are more impatient, with a discount factor of 0.902. The no-college group accumulates less wealth and has a weaker incentive to pay the fixed cost, leading to a low participation rate. College graduates have an EIS of $\psi = 0.95$, and households without college degrees have a lower EIS of $\psi = 0.28$. These measures are consistent with those of Vissing-Jørgensen (2002), Brav et al. (2002), Malloy et al. (2009), and Gomes and Michaelides (2008).

Moreover, we consider a scenario in which we allow risk aversion to vary between college graduates and households without college degrees in Appendix E.4. Figure E5 shows that the main results still hold qualitatively. The existence of higher-order dependence increases labor income risk, and thus, labor income becomes more stock-like. Households need to balance their stockholdings to control for total risk. Introducing higher-order dependence provides us with a better understanding of labor income risk without a counterfactually high-risk aversion, which may complicate explanations of other economic behaviors.

The parameters of the dependence structure (college graduates: 0.769 and -0.222; households without college degrees: 0.832 and -0.166) in panel C imply linear dependence and higher-order dependence values of 0.046 and 0.069 for the college group, and 0.038 and 0.033 for households without college degrees. The calibration results for linear dependence, consistent with those of Cocco et al. (2005), are not significantly different from zero. In contrast, our estimations for higher-order dependence are statistically significant and large enough to

Table 7: Moments of Labor Income and Stock Return

This table reports the relative errors of the moments used to calibrate labor income and stock returns. Panels A and B present the mean, variance, and higher moments of the stock return shock and labor income shock. Panel C provides the errors in the dependence structure, including both linear dependence and higher-order dependence. The moments from the model are computed using the calibrated parameters under a mixture normal distribution, while the empirical moments are derived from PSID and CRSP data.

Panel A: Moments of stock return shock			
	Model	Data	
SD	0.216	0.216	
3rd moment	-1.412	-1.410	
4th moment	8.595	8.620	

Panel B: Moments of labor income shock					
		No college		College	
		Model	Data	Model	Data
SD:	$25-35$	0.378	0.408	0.386	0.405
	$35-45$	0.428	0.432	0.412	0.395
	$45-55$	0.449	0.443	0.416	0.392
	$55-65$	0.457	0.464	0.417	0.414
3rd moment:	$25-35$	-0.021	-0.021	-0.032	-0.026
	$35-45$	-0.027	-0.032	-0.034	-0.036
	$45-55$	-0.029	-0.032	-0.035	-0.040
	$55-65$	-0.029	-0.033	-0.035	-0.044
4th moment:	$25-35$	0.066	0.081	0.076	0.086
	$35-45$	0.107	0.106	0.097	0.094
	$45-55$	0.128	0.116	0.100	0.093
	$55-65$	0.138	0.130	0.101	0.105

Panel C: Dependence structure				
	No college		College	
	Model	Data	Model	Data
Linear dependence	0.038	0.038	0.046	0.046
Higher-order dependence	0.033	0.034	0.069	0.070

generate substantial differences in policy functions, as demonstrated in Section 5.2.

5.2 Quantitative Analysis of Higher-Order Dependence

Higher-order dependence, measured by the between-squares correlation, exposes households to a new source of risk and makes labor income more stock-like. To better understand its implications, we present the results of the baseline model with calibrated parameters and compare them to those of an alternative model that excludes higher-order dependence, using the same parameters. To ensure a fair comparison, we maintain the same linear dependence in both models.[32] Moreover, we show in Online Appendix E.2 that the effect of higher-order dependence persists even after varying the preference parameters.

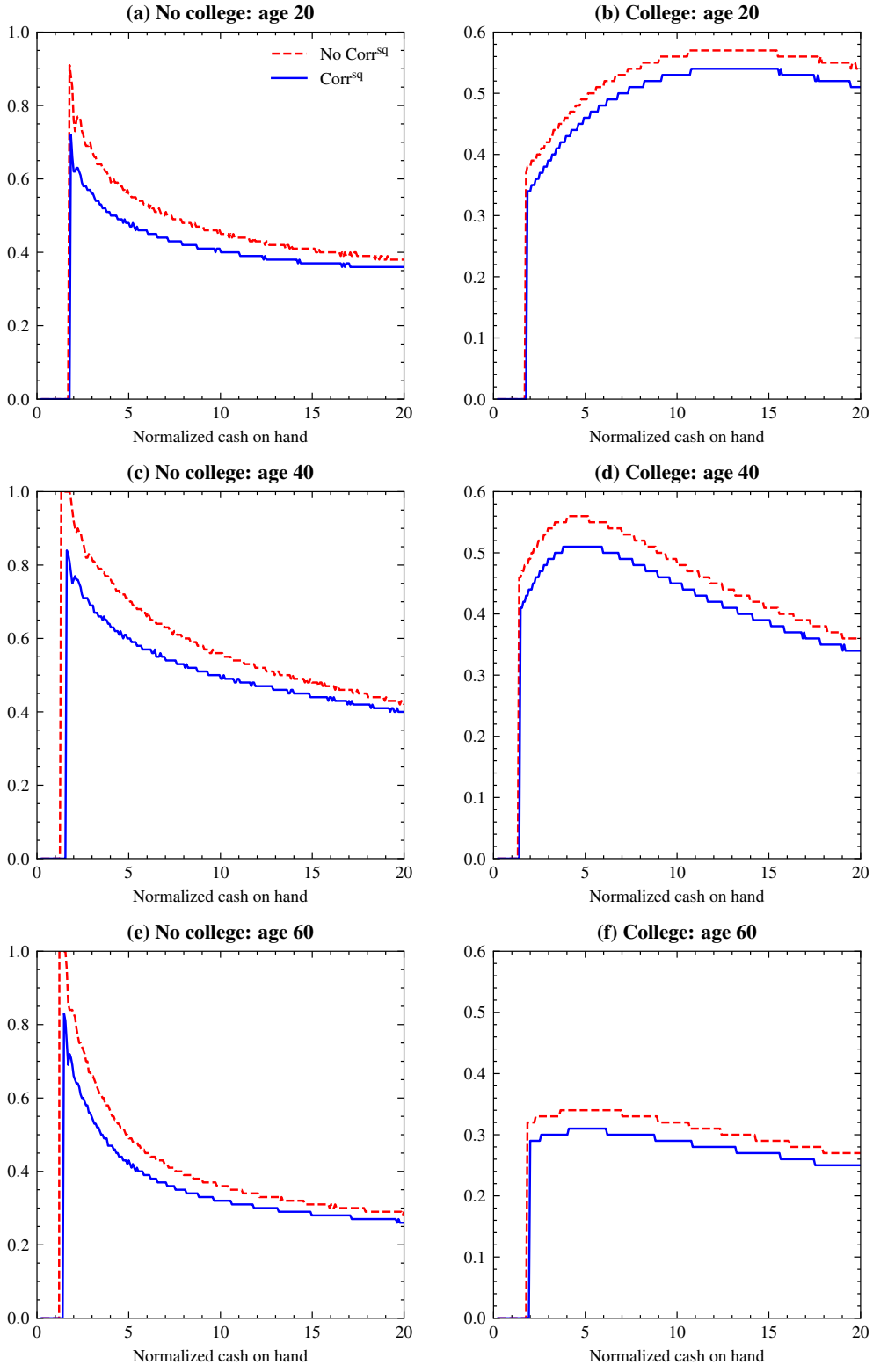
Figure 7 shows the policy functions for the share of wealth in stocks at different ages for both the no-college group and college graduates. Conditional on a household's participation in the stock market, introducing a small between-squares correlation significantly lowers the optimal risky share, with the largest drop of 14.6% at age 40 for the college group.

We also observe that the optimal risky share decreases with both financial wealth and age, consistent with the literature (e.g., Cocco et al. (2005)). This is mainly driven by the bond-like properties of labor income. The present value of labor income can be considered a substitute for riskless assets. Households assess the risk-free position provided by labor income and adjust their portfolio choices accordingly. The effect is weaker for wealthier households since labor income becomes trivial compared to wealth. Thus, households with less wealth hold more riskless assets through labor income and make more aggressive portfolio choices. However, labor income is not a perfect substitute for riskless assets, as it is subject to various risks from different shocks. As noted by Benzoni et al. (2007), labor income also exhibits stock-like characteristics. The higher-order dependence between stock returns and labor income further amplifies these stock-like features. Although the decreasing pattern remains, the model with higher-order dependence narrows the gap between the rich and the poor.

[32]This section primarily examines the policy functions. Online Appendix E analyzes the simulated life-cycle profiles and demonstrates that incorporating higher-order dependence reduces the participation rate and conditional risky asset holdings. Additionally, there is heterogeneity between college graduates and households without college degrees.

Figure 7: Optimal Risky Asset Shares Policy Function

The graphs display the policy functions for optimal risky asset shares across different ages and groups. The first column represents the no-college group, while the second corresponds to the college group. These policy functions are derived using calibrated parameters, with the adjusted persistent income shock $w_{i,t}$ set to 1.



To take a closer look, we calculate the differences in optimal portfolio rules between the baseline model and the model without higher-order dependence. Table 8 presents the results. We find that the difference is significantly larger for the no-college group, which tends to have less wealth. For example, among all age-20 households without college degrees, the largest decrease is 28.9%, whereas for the wealthiest age-20 households, this difference drops to 12.5%. This heterogeneity across different wealth levels is driven by the fact that households with less wealth place greater value on their labor income stream. The additional risk exposure from higher-order dependence in labor and stock markets causes these households to tilt their financial portfolios more aggressively toward safe assets.

Table 8: Change in Optimal Risky Asset Shares with Higher-Order Dependence
This table reports the differences in the optimal risky share α_t between the baseline model and the alternative model without higher-order dependence for college graduates and the no-college group. The data are derived from the policy functions discussed in Figure 7. The differences are calculated for each age and wealth ratio. For example, the second percentage (−12.7%) for age-20 college graduates is computed as the difference between the optimal share at a wealth ratio of 2.0 in the model with higher-order dependence (28.4%) and the model without higher-order dependence (32%). Additionally, we report the largest difference for each age.

Group	Age	Largest diff.	Wealth ratio			
			2.0	4.0	8.0	10.0
No college	20	−28.9%	−22.6%	−18.0%	−14.3%	−12.5%
	40	−26.7%	−21.1%	−20.6%	−13.2%	−14.3%
	60	−26.7%	−25.0%	−22.5%	−18.8%	−16.7%
College	20	−15.6%	−12.7%	−8.4%	−7.6%	−7.4%
	40	−21.1%	−12.8%	−10.4%	−10.3%	−10.1%
	60	−16.9%	−11.5%	−12.1%	−12.1%	−10.9%

Including a per-period participation cost allows us to analyze the wealth-participation threshold. Two factors may discourage households from participating in the stock market. First, the participation cost may be too high relative to the households' accumulated wealth, making it optimal for less wealthy households to abstain from paying for stock market participation. Second, labor income risk alone may already exceed households' risk tolerance. When households anticipate higher labor income risk, they may choose to avoid participating in the stock market to limit additional exposure to risk. Such a case is less likely for wealthier households since labor income makes up a smaller proportion of their wealth. Introducing

higher-order dependence between labor and stock markets makes labor income more stock-like, further influencing households' participation decisions through the second channel.

Figure 8: Participation Wealth Threshold

The graphs show the participation wealth threshold across various ages for the two groups. This threshold is calculated from the corresponding equity share policy function and represents the highest wealth ratio at which households choose not to participate in the stock market. The adjusted persistent income shock $w_{i,t}$ is set to 1.

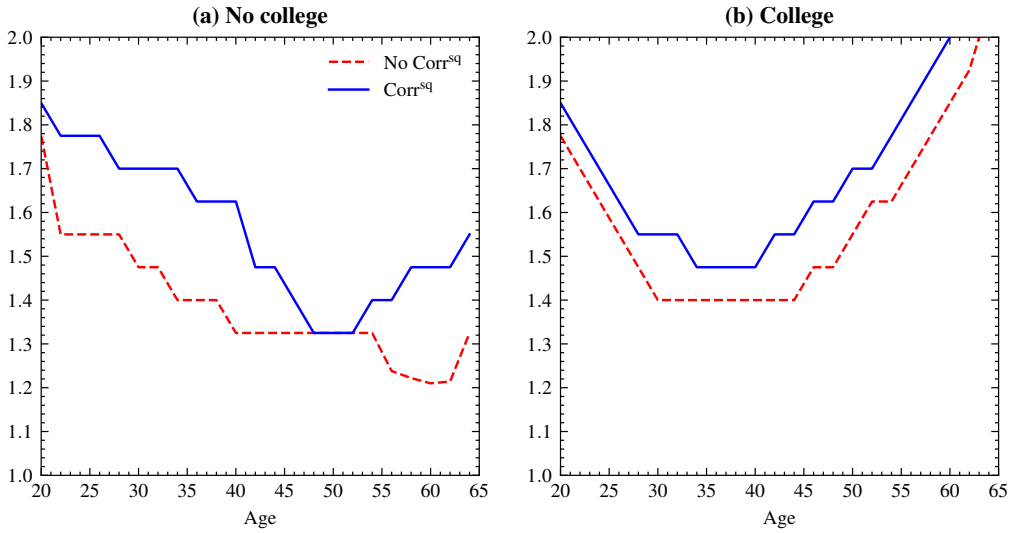


Figure 8 shows the wealth-participation threshold with respect to age. For both groups, incorporating higher-order dependence results in a higher threshold for participation. Generally, the wealth threshold increases by 0% to 22.64% for the no-college group and by 4.05% to 10.84% for college graduates. Additionally, the U-shaped pattern of the participation threshold aligns with the findings in Fagereng et al. (2017). At early ages, households have limited wealth, which reduces the benefit of participating in the stock market below the participation cost. As households age, they accumulate more wealth, leading to a higher optimal risky asset share, making stock market participation more appealing. Consequently, the wealth threshold decreases. However, as households approach retirement age, the decline in the present value of future labor income flows gradually dominates and leads to the rebalancing of the portfolio. In addition, we also observe heterogeneity between college graduates and the no-college group. Although the U-shaped pattern of the participation threshold diminishes for both groups when higher-order dependence is included, the effect is significantly stronger for households without college degrees. When higher-order dependence exists between labor and stock markets, households are less willing to participate in the stock

market due to the increased total risk exposure.

Further Discussion A potential argument suggests that including higher-order moments in the labor income process and stock return innovations, along with alternative linkages between labor and stock markets discussed in existing literature, would lead to significant higher-order dependence. However, our analysis in Online Appendix E.5 indicates that higher-order dependence is nearly negligible in this specific setup. Specifically, as shown in Online Appendix E.5, Table E2, panel D, using parameters from existing literature, higher-order dependence measured by the between-squares correlation is computed as 0.0056, which is effectively zero. Therefore, the effect of higher-order dependence on portfolio decisions is neglected. This highlights the importance of explicitly incorporating higher-order dependence into the model.

On the other hand, we show that without explicitly incorporating a higher-order dependence process, achieving a higher-order dependence of similar magnitude to that observed in our paper would require highly unrealistic parameters. For example, reducing the expected log stock returns during stock market crashes to -1.245 can produce a higher-order dependence of 0.071. However, this implies an average loss of 71.21% during crashes, which contradicts the data.

Overall, this discussion highlights the importance of explicitly incorporating higher-order dependence into the life-cycle model to capture its effects. It emphasizes that higher-order moments in both labor and stock markets alone are insufficient to account for the between-squares correlation. An explicit consideration of higher-order dependence is crucial for an accurate representation of the dynamics in the model.

6 Economic Implications

What happens to households and society when higher-order dependence is ignored? We investigate this question quantitatively by assessing wealth-equivalent loss at the household level and examining changes in wealth inequality at the societal level in the following analysis.

6.1 Wealth-Equivalent Loss

So far, we have shown that higher-order dependence significantly affects households' participation decisions and portfolio choices. However, it is also important to understand how higher-order dependence affects households' welfare. To answer this question, we consider the wealth-equivalent loss resulting from ignoring higher-order dependence between the labor and stock markets. Let v, v^0 represent the value functions of models with higher-order dependence and without higher-order dependence, respectively. We use the parameter values from Section 5.1. The wealth-equivalent loss, $\Delta x(t, w)$, at age t with the adjusted persistent income shock w, is defined as:

$$v(t, x, w) = v^0(t, x + \Delta x, w). \tag{24}$$

Table 9: Cost of Ignoring the Between-Squares Correlation
This table reports the relative wealth-equivalent loss, $\Delta x/x$ for two education groups at various ages and levels of wealth. The values in the table were obtained using the parameter values calibrated in Section 5.1 (Tables 5 and 7), with the adjusted persistent income shock set to $w = 1$.

Group	Age	Normalized wealth					
		1	2	4	6	8	10
No college	20	3.32%	2.86%	2.54%	2.05%	1.87%	1.74%
	40	3.14%	2.67%	1.97%	1.83%	1.58%	1.44%
	60	0.28%	0.18%	0.14%	0.15%	0.12%	0.11%
College	20	1.72%	1.59%	1.26%	1.01%	0.85%	0.72%
	40	1.26%	0.90%	0.53%	0.34%	0.23%	0.13%
	60	0.05%	0.09%	0.10%	0.09%	0.08%	0.07%

Table 9 reports the relative wealth-equivalent loss, $\Delta x/x$, across different ages and wealth, with an adjusted persistent income shock set to $w = 1$. The results are reported separately for households with college degrees and those without. Our findings suggest that ignoring the higher-order dependence between labor and stock markets can lead to substantial welfare losses for households. This effect is especially pronounced for households without college degrees. Specifically, the welfare losses for households without college degrees can be as high as 3.32% of their equivalent wealth. These losses are economically significant, given that the higher-order dependence, as measured by between-squares correlation, is estimated to be

0.033 for the no-college group. Moreover, we find that the wealth-equivalent loss decreases with age and wealth, consistent with the diminishing impact of higher-order dependence on optimal portfolio rules and the participation threshold.

Given the importance of higher-order dependence on household welfare, we next examine its potential impact on society as a whole, specifically in terms of wealth inequality.

6.2 Wealth Inequality

Recent studies have made significant progress in documenting the increasing trends in wealth inequality in the United States (see Piketty and Saez (2003); Saez and Zucman (2020)). The causes and consequences of these trends have become a prominent topic of debate in recent years. The ongoing disagreement about wealth inequality highlights the importance of understanding the role of higher-order dependence in shaping broader economic outcomes.

To investigate the potential impact of higher-order dependence on wealth inequality, we simulate 100,000 households over a period of 500 years. We then calculate the Gini index, a widely used measure of wealth inequality, and compare the Gini indexes obtained when higher-order dependence is recognized to those obtained when it is ignored. We acknowledge that dividing households into only two groups may not capture the full range of heterogeneity in their preferences and attitudes toward risk. Therefore, we explore various combinations of discount factors and risk aversion to better reflect the diversity of household behaviors in savings and investment decisions (see MaCurdy (1982); Gourinchas and Parker (2002)). Additionally, we account for preferences that may evolve over time due to external shocks, as demonstrated by Guiso et al. (2018).

Table 10 presents the difference in Gini indices between scenarios where higher-order dependence is ignored and those where it is recognized. Using the calibrated parameters from Section 5.1, the higher-order dependence for households with college degrees is 0.069, while for households without college degrees, it is 0.033. Panel A of Table 10 shows that when higher-order dependence is ignored, the Gini index increases inefficiently for the full sample of households. Specifically, across different combinations of discount rates and risk aversion parameters, the difference in Gini indexes increases by 0.61% to 2.73%, indicating

that ignoring higher-order dependence significantly affects wealth inequality.[33]

When households ignore the higher-order dependence between labor and stock markets, they overlook the interconnections between these two markets. This ignorance can result in a misallocation of resources, where households may disproportionately invest in the stock market. Moreover, misallocation can have a heterogeneous effect on households with different levels of wealth. Households with lower levels of wealth may increase their shares of risky assets more significantly than those with higher levels of wealth, even though the latter group may invest more in absolute dollar terms. The results from Section 6.1 demonstrate that households with lower levels of wealth incur higher losses due to higher-order dependence. Therefore, ignoring higher-order dependence exacerbates wealth inequality.

Meanwhile, Table 10, panel B, suggests that the difference between ignoring and recognizing higher-order dependence is even greater for households with college degrees compared to those without. Specifically, the difference in the Gini index can be as high as 2.32% for households with college degrees, while it is only 1.78% for households without college degrees. This finding suggests a greater concentration of wealth among highly educated households, potentially exacerbating overall inequality in society.

Moreover, given the right-skewed distribution of higher-order dependence observed in Section 2.3, investigating the effects of stronger higher-order dependence is essential for understanding its impact on wealth inequality. This motivates our examination of the differences in the Gini index, as shown in Table 11, particularly with respect to higher levels of higher-order dependence. To achieve this, we increase higher-order dependence by 50% for both education groups. Our findings show that as higher-order dependence increases, the difference in the Gini index between recognizing and ignoring the between-squares correlation for the full sample reaches a maximum of 3.44%, increasing from the initial 2.73%. Specifically, for households with college degrees, the difference is even larger, reaching 4.62%.

[33]Considering higher-order dependence yields a Gini index of 0.203, while ignoring it generates a higher Gini index of 0.210. Compared with empirical data, the models generate lower Gini coefficients because of two limitations in our baseline model. First, the model fails to capture the behavior of the wealthiest households, who accumulate wealth through channels other than labor income. Second, the model does not account for portfolio heterogeneity: wealthy households tend to have portfolios dominated by corporate and noncorporate equities, whereas middle-class households typically hold highly leveraged portfolios concentrated in residential real estate. Despite these limitations, the models still exhibit a valid trend, and incorporating these features could lead to even greater observed wealth inequality.

Table 10: Changes in Gini Index

This table examines the impact of higher-order dependence, measured by the between-squares correlation, on wealth inequality by comparing the Gini index obtained when this correlation is ignored to the Gini index obtained when it is recognized. Panel A presents the results for the full sample of households, considering various combinations of discount rates and risk aversion parameters. Panel B provides the results separately for households with and without college degrees, using the same combinations of discount rates and risk aversion parameters as in panel A. The notation (β^c, β^{nc}) in the first column indicates that β^c is used as the discount rate for the college group, while β^{nc} is used as the discount rate for the no-college group. This notation enables us to differentiate between the two education groups and analyze their respective impacts on wealth inequality.

Panel A: Full samples			
	Risk aversion		
Discount rate (β^c, β^{nc})	4.3	4.8	5.3
(92%, 92%)	1.49%	2.22%	2.73%
(94%, 92%)	1.67%	2.08%	2.21%
(96%, 92%)	1.44%	1.49%	1.45%
(98%, 92%)	0.93%	0.82%	0.61%
Panel B: Changes in Gini Index for each education group			
	Risk aversion		
Discount rate (β^c, β^{nc})	4.3	4.8	5.3
(92%, 92%): College	1.14%	2.32%	3.52%
(94%, 92%): College	1.44%	2.14%	2.47%
(96%, 92%): College	1.22%	1.40%	1.51%
(98%, 92%): College	1.62%	1.04%	0.88%
(92%, 92%): No college	1.28%	1.78%	2.18%

While these differences may not seem significant at first glance, it is worth noting that the Gini index increased by $< 3.0\%$ from 1970 to 2017, with the maximum change in the United States since 1969 being 6.76%.[34] When compared to these figures, ignoring higher-order dependence has a considerable impact on wealth inequality. Therefore, accounting for higher-order dependence between labor and stock markets is important.

Overall, recognizing the importance of higher-order dependence between labor and stock markets can help households make more informed investment and risk management decisions, leading to a more efficient allocation of resources and a more equitable distribution of wealth. Our findings emphasize a channel that has received little attention: higher-order dependence

[34]Figure F1 in Online Appendix F compares the Gini coefficient in each year to the benchmark year of 1969 and displays the relative changes over time.

between labor and stock markets induces significant shifts in wealth inequality.

Table 11: Changes in the Gini Index with Larger Higher-Order Dependence
This table examines the impact of higher-order dependence, measured by the between-squares correlation, on wealth inequality. We compare the Gini index obtained when this correlation is ignored to the index obtained when it is recognized, focusing on higher levels of the between-squares correlation. Panel A presents the results for the full sample of households, considering different combinations of discount rates and risk aversion parameters. Panel B provides the results separately for households with and without college degrees, using the same combinations of discount rates and risk aversion parameters as in Panel A. The notation (β^c, β^{nc}) indicates that β^c is used as the discount rate for the college group and β^{nc} for the no-college group. This notation allows us to distinguish between the two education groups and analyze their respective impacts on wealth inequality.

Panel A: Full sample			
	Risk aversion		
Discount rate (β^c, β^{nc})	4.3	4.8	5.3
(92%, 92%)	1.84%	2.76%	3.44%
(94%, 92%)	2.05%	2.66%	2.95%
(96%, 92%)	1.98%	2.05%	2.08%
(98%, 92%)	1.07%	0.91%	0.80%
Panel B: Changes in the Gini Index for each education group			
	Risk aversion		
Discount rate (β^c, β^{nc})	4.3	4.8	5.3
(92%, 92%): College	1.44%	3.15%	4.62%
(94%, 92%): College	1.98%	2.95%	3.61%
(96%, 92%): College	1.72%	2.07%	2.38%
(98%, 92%): College	0.99%	1.08%	1.14%
(92%, 92%): No college	1.41%	1.97%	2.36%

7 Conclusion

In this paper, we examine the optimal portfolio decisions of households in the presence of extreme values jointly occurring in the labor and stock markets. We document the existence of higher-order dependence in the data and show that it reduces households' willingness to participate in the market, leading to significant reductions in stock investments. Heterogeneity exists between college graduates and households without college degrees. Moreover, our paper shows that higher-order dependence is independent of the linear correlation, for which the literature provides mixed evidence. Thus, our study complements existing research.

Using a mean-variance-skewness-kurtosis framework, we further show that the impact of higher-order dependence portfolio decisions operates through two distinct mechanisms: the skewness channel and the kurtosis channel. The extent to which one channel dominates over the other is determined by the magnitude of higher-order dependence. Meanwhile, a life-cycle model that incorporates higher-order dependence provides an explanation for both the limited market participation puzzle and the moderate levels of risky asset holdings observed in the PSID data. More importantly, we find that ignoring higher-order dependence reduces households' welfare and increases wealth inequality.

Although our model accounts for the higher-order dependence between labor and stock markets, it does not endogenize this higher-order dependence or address asset pricing implications. Future work could study higher-order dependence within a general equilibrium framework, as demonstrated in Guvenen (2009) and Ai and Bhandari (2021) for macroeconomic models. Such an extension would offer a characterization of the risk premium while accounting for the complex and interdependent dynamics of labor and stock markets.

References

Ai, H. and Bhandari, A. (2021). Asset pricing with endogenously uninsurable tail risk. *Econometrica*, 89(3):1471–1505.

Attanasio, O. P. (1995). The intertemporal allocation of consumption: theory and evidence. In *Carnegie-Rochester conference series on public policy*, volume 42, pages 39–89. Elsevier.

Bagliano, F., Corvino, R., Carolina, F., and Nicodano, G. (2024). Hedging permanent income shocks. *Working paper*.

Barro, R. J. (2006). Rare disasters and asset markets in the twentieth century. *The Quarterly Journal of Economics*, 121(3):823–866.

Barro, R. J. (2009). Rare disasters, asset prices, and welfare costs. *American Economic Review*, 99(1):243–264.

Benzoni, L., Collin-Dufresne, P., and Goldstein, R. S. (2007). Portfolio choice over the life-cycle when the stock and labor markets are cointegrated. *The Journal of Finance*, 62(5):2123–2167.

Bonaparte, Y., Korniotis, G. M., and Kumar, A. (2014). Income hedging and portfolio decisions. *Journal of Financial Economics*, 113(2):300–324.

Brav, A., Constantinides, G. M., and Geczy, C. C. (2002). Asset pricing with heterogeneous consumers and limited participation: Empirical evidence. *Journal of Political Economy*, 110(4):793–824.

Bronars, S. G. and Famulari, M. (2001). Shareholder wealth and wages: Evidence for white-collar workers. *Journal of Political Economy*, 109(2):328–354.

Brunnermeier, M. K. and Nagel, S. (2008). Do wealth fluctuations generate time-varying risk aversion? micro-evidence on individuals. *American Economic Review*, 98(3):713–36.

Campbell, J. and Viceira, L. (2002). *Strategic asset allocation: Portfolio choice for long-term investors*. Oxford University Press.

Catherine, S. (2022). Countercyclical labor income risk and portfolio choices over the life cycle. *The Review of Financial Studies*, 35(9):4016–4054.

Cocco, J. F., Gomes, F. J., and Maenhout, P. J. (2005). Consumption and portfolio choice over the life cycle. *The Review of Financial Studies*, 18(2):491–533.

Davis, S. J. and Willen, P. S. (2013). Occupation-level income shocks and asset returns: Their covariance and implications for portfolio choice. *The Quarterly Journal of Finance*, 126(1):373–416.

Fagereng, A., Gottlieb, C., and Guiso, L. (2017). Asset market participation and portfolio choice over the life-cycle. *The Journal of Finance*, 72(2):705–750.

Fagereng, A., Guiso, L., and Pistaferri, L. (2018). Portfolio choices, firm shocks, and uninsurable wage risk. *Review of Economic Studies*, 85(1):396–436.

Fan, Y. and Patton, A. J. (2014). Copulas in econometrics. *Annual Review of Economics*, 6(1):179–200.

Gomes, F. and Michaelides, A. (2005). Optimal life-cycle asset allocation: Understanding the empirical evidence. *The Journal of Finance*, 60(2):869–904.

Gomes, F. and Michaelides, A. (2008). Asset pricing with limited risk sharing and heterogeneous agents. *The Review of Financial Studies*, 21(1):415–448.

Gourinchas, P.-O. and Parker, J. A. (2002). Consumption over the life cycle. econometrica. *Econometrica*, 70(1):41–89.

Guiso, L., Jappelli, T., and Terlizzese, D. (1996). Income risk, borrowing constraints, and portfolio choice. *The American Economic Review*, pages 158–172.

Guiso, L., Sapienza, P., and Zingales, L. (2018). Time varying risk aversion. *Journal of Financial Economics*, 128(3):403–421.

Guvenen, F. (2009). A parsimonious macroeconomic model for asset pricing. *Econometrica*, 77(6):1711–1750.

Guvenen, F., Ozkan, S., and Song, J. (2014). The nature of countercyclical income risk. *Journal of Political Economy*, 122(3):621–660.

Hall, R. E., Cummins, J. G., and Lamont, O. A. (2000). E-capital: The link between the stock market and the labor market in the 1990s. *Brookings Papers on Economic Activity*, 2000(2):73–118.

Harvey, C. R. and Siddique, A. (2000). Conditional skewness in asset pricing tests. *The Journal of Finance*, 55(3):1263–1295.

Hubbard, R. G., Skinner, J., and Zeldes, S. P. (1995). Precautionary saving and social insurance. *Journal of political Economy*, 103(2):360–399.

Huggett, M. and Kaplan, G. (2016). How large is the stock component of human capital? *Review of Economic Dynamics*, 22:21–51.

Lynch, A. W. and Tan, S. (2011). Labor income dynamics at business-cycle frequencies: Implications for portfolio choice. *Journal of Financial Economics*, 101(2):333–359.

MaCurdy, T. E. (1982). The use of time series processes to model the error structure of earnings in a longitudinal data analysis. *Journal of Econometrics*, 18(1):83–114.

Malloy, C. J., Moskowitz, T. J., and Vissing-Jørgensen, A. (2009). Long-run stockholder consumption risk and asset returns. *The Journal of Finance*, 64(6):2427–2479.

Manuelli, R. (2000). Technological change, the labor market and the stock market.

Massa, M. and Simonov, A. (2006). Hedging, familiarity and portfolio choice. *Review of Financial Studies*, 19(2):633–685.

Michelacci, C. and Quadrini, V. (2009). Financial markets and wages. *The Review of Economic Studies*, 76(2):795–827.

Nakajima, M. and Smirnyagin, V. (2019). Cyclical labor income risk. *Available at SSRN 3432213*.

Neuberger, A. and Payne, R. (2021). The skewness of the stock market over long horizons. *The Review of Financial Studies*, 34(3):1572–1616.

Nickell, S. and Wadhwani, S. (1991). Employment determination in british industry: investigations using micro-data. *The Review of Economic Studies*, 58(5):955–969.

Piketty, T. and Saez, E. (2003). Income inequality in the united states, 1913–1998. *The Quarterly journal of economics*, 118(1):1–41.

Rietz, T. A. (1988). The equity risk premium a solution. *Journal of monetary Economics*, 22(1):117–131.

Saez, E. and Zucman, G. (2020). The rise of income and wealth inequality in america: Evidence from distributional macroeconomic accounts. *Journal of Economic Perspectives*, 34(4):3–26.

Shen, J. (2022). Countercyclical risks and optimal life-cycle profile: Theory and evidence. *Management Science*.

Storesletten, K., Telmer, C. I., and Yaron, A. (2007). Asset pricing with idiosyncratic risk and overlapping generations. *Review of Economic Dynamics*, 10(4):519–548.

Vissing-Jørgensen, A. (2002). Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy*, 110(4):825–853.

Online Appendix

Portfolio Decisions with Higher-Order Dependence between Labor and Stock Markets

Appendix A Data

A.1 Panel Study of Income Dynamics

The Panel Study of Income Dynamics (PSID) is a longitudinal household survey spanning from 1968 to 2019, covering data from over 18,000 individuals residing in 5,000 families across the United States. The survey includes a broad range of topics, such as family demographics and financial situations, which are crucial for most life-cycle household models. PSID is particularly well-suited for our model, as it provides time-series data on labor income, which is necessary to estimate its dependence on stock returns. However, PSID has some well-documented challenges for researchers, and many studies have discussed various approaches to address these issues[1].

In this paper, we follow the sample selection criteria outlined in Nakajima and Smirnyagin (2019). First, we exclude data from years prior to 1970, as the 1968 and 1969 waves lack certain variables and are inconsistent with subsequent waves. Second, we drop the SEO and Latino samples, which were collected using unequal selection probabilities. Specifically, PSID includes approximately 2,000 low-income SEO samples and 2,000 Latino samples, both of which are excluded from our analysis. Third, we remove observations with missing or non-positive labor incomes for either the head or the spouse. Additionally, we trim the top 1% of labor incomes for both the head and the spouse, as PSID imposes upper brackets on many finance-related variables (such as labor income). This trimming aims to exclude bracketed or extreme values. Finally, households with income growth anomalies (annual log growth rate outside the range of 1/20 to 20) are also dropped from our analysis. We construct variables following the methodology of Brunnermeier and Nagel (2008), with variable definitions summarized in the following table.

[1]For example, see Cocco et al. (2005), Gomes and Michaelides (2005), and Nakajima and Smirnyagin (2019).

Table A1: Varible definitions

Variable	Definition
Labor Income Y_{it}	Includes the labor income of both reference person and spouse, labor part of farm income and business income, all transfer income of the family, and social securities.
Riskless Assets	Checking and savings accounts, money market funds, certificates of deposits, savings bonds, and treasury bills
	Bonds, bond funds, cash value in a life insurance, valuable collection for investment purposes, and rights in a trust or estate
Risky Assets	the combined value of shares of stock in publicly held corporations, mutual funds, and investment trusts
IRA Assets	Value of private annuities or Individual Retirement Accounts
Other Debts	Credit card debt, student loans, medical or legal bills, and loans from relatives
Home Equity	Value of the home minus remaining mortgage principal
Liquid Assets	Riskless Assets + Risky Assets
Financial Wealth	Liquid Assets - Other Debts + Home Equity

A.2 SCF Data

The Survey of Consumer Finances (SCF) is a triennial cross-sectional survey of U.S. families that collects detailed information on families' balance sheets, pensions, income, and demographic characteristics. It also includes data from related surveys of pension providers and earlier surveys conducted by the Federal Reserve Board. For this study, we use data from five survey waves, spanning 2007 to 2019, and construct variables based on the codebook and macro-variable definitions available on the Federal Reserve's website.

Following Gomes and Michaelides (2005), we construct variables for the SCF. Labor income is defined as the sum of wages and salaries (X5702), unemployment or workers' compensation (X5716), and Social Security or other pensions, annuities, or disability/retirement programs (X5722). Financial wealth is constructed in the same way as the variable FIN in the publicly available SCF dataset. This includes LIQ (all types of transaction accounts such as checking, savings, money market, and call accounts), CDS (certificates of deposit), total directly held mutual funds, stocks, bonds, total quasi-liquid financial assets (the sum of IRAs, thrift accounts, and future pensions), savings bonds, the cash value of whole life insurance, other managed assets (trusts, annuities, and managed investment accounts in

which the household has equity interest), and other financial assets (such as loans from the household to others, future proceeds, royalties, futures, nonpublic stock, and deferred compensation). Additionally, data on financial assets invested in risky assets is obtained from the variable EQUITY in the publicly available SCF dataset. This variable includes directly held stock, stock mutual funds, and amounts of stock in retirement accounts. We calculate the conditional risky share as EQUITY/FIN, conditional on EQUITY being positive.

Appendix B Bivariate Mixture Normal Distribution

This section begins by defining the mixture normal distribution and describing its first four moments. Then, we establish that the between-squares correlation can function as an adequate measure of higher-order dependence, provided certain regularity conditions are satisfied. It is important to note that the following definition provides a general mathematical representation of a bivariate mixture normal distribution, which is identical to the one given in (16).

Definition 1. *A random vector* $(Z_1, Z_2)^T$ *follows a bivariate mixture normal distribution with parameters* $p_1, \mu_{11}, \mu_{12}, \sigma_{11}, \sigma_{12}, p_2, \mu_{21}, \mu_{22}, \sigma_{21}, \sigma_{22}, \rho_{11}, \rho_{12}, \rho_{21}, \rho_{22}$ *if:*

$$Z_1 = I_1 Z_{11} + (1 - I_1) Z_{12}, \tag{25}$$

$$Z_2 = I_2 Z_{21} + (1 - I_2) Z_{22}, \tag{26}$$

where $I_i \sim \mathrm{B}(1, p_i)$*,* Z_{i1} *is independent of* Z_{i2} *for i = 1,2 and for* $i = 1, 2, j = 1, 2$*,* $\boldsymbol{Z}^{ij} = (Z_{1i}, Z_{2j})^T$ *is binomially distributed, subject to:*

$$\mathbb{E}(\boldsymbol{Z}^{ij}) = (\mu_{1i}, \mu_{2j})^T,$$
$$\mathrm{Cov}(\boldsymbol{Z}^{ij}) = \begin{bmatrix} \sigma_{1i}^2 & \rho_{ij}\sigma_{1i}\sigma_{2j} \\ \rho_{ij}\sigma_{1i}\sigma_{2j} & \sigma_{2j}^2 \end{bmatrix}.$$

Intuitively, $\{I_i\}_{i\in\{1,2\}}$ indicates the occurrence of potential economic shocks in the two markets, while $\{Z_{ij}\}_{i,j\in\{1,2\}}$ represents the corresponding sizes of the shocks. For instance, when $I_1 = 1$ and $I_2 = 1$, a shock simultaneously affects both markets: the first market experiences a movement driven by the random size Z_{11}, while the second market is influenced by

Z_{21}. Moreover, the bivariate mixture normal distribution effectively captures the heavy tails and low frequencies typically observed in large economic shocks. This is achieved through two key mechanisms. First, as shown in subsection B.1, the mixture normal distribution inherently exhibits heavy tails, characterized by nontrivial skewness and kurtosis. Second, the parameter p_i, associated with the shock indicator I_i, controls the likelihood of jumps. When p_i is small, the mixture normal distribution reflects a lower frequency of shocks, aligning with the observed behavior of rare but extreme economic events.

B.1 Central Moments and Correlations

Denote $\mu_i, \sigma_i, s_i, k_i$ as the mean, standard deviation, skewness, and kurtosis of Z_i, where $i = 1, 2$. For convenience, in the remainder of the online appendix, we use the shorthand notation $p_{11} = p_1, p_{12} = 1 - p_1, p_{21} = p_2, p_{22} = 1 - p_2$. The first four moments of Z_i are given by:

$$\mu_i = \mathbb{E}[Z_i] = p_{i1}\mu_{i1} + p_{i2}\mu_{i2}, \tag{27}$$

$$\sigma_i^2 = \mathrm{Var}[Z_i] = p_{i1}\sigma_{i1}^2 + p_{i2}\sigma_{i2}^2 + p_{i1}p_{i2}(\mu_{i1} - \mu_{i2})^2, \tag{28}$$

$$s_i = \mathrm{Skew}[Z_i] = \sigma_i^{-3} p_{i1}p_{i2}(\mu_{i1} - \mu_{i2}) \left(3(\sigma_{i1}^2 - \sigma_{i2}^2) + (1 - 2p_{i1})(\mu_{i1} - \mu_{i2})^2\right), \tag{29}$$

$$\begin{aligned} k_i = \mathrm{kurt}[Z_i] = \sigma_i^{-4} \big[&3p_{i2}\sigma_{i1}^4 + 3p_{i1}\sigma_{i2}^4 + p_{i1}p_{i2}(\mu_{i1} - \mu_{i2})^2 \big(6(p_{i1}\sigma_{i2}^2 + p_{i2}\sigma_{i1}^2) + \\ &(3p_{i1}^2 - 3p_{i1} + 1)(\mu_{i1} - \mu_{i2})^2\big)\big]. \end{aligned} \tag{30}$$

The Pearson correlation and the between-squares correlations between Z_1 and Z_2 are given by:

$$\mathrm{Corr}(Z_1, Z_2) = \frac{1}{\sigma_1\sigma_2} \sum_{i,j=1,2} p_{1i}p_{2j}\sigma_{1i}\sigma_{2j}\rho_{ij} \tag{31}$$

$$\begin{aligned} \mathrm{Corr}^{\mathrm{sq}}(Z_1, Z_2) = &\frac{1}{\sigma_1^2\sigma_2^2\sqrt{(k_1 - 1)(k_2 - 1)}} \Big(2 \sum_{i,j=1,2} p_{1i}p_{2j}\sigma_{1i}^2\sigma_{2j}^2\rho_{ij}^2 \\ &+ 4(\mu_{11} - \mu_{12})(\mu_{21} - \mu_{22})p_{11}p_{12}p_{21}p_{22} \sum_{i,j=1,2} (-1)^{i+j}\sigma_{1i}\sigma_{2j}\rho_{ij}\Big). \end{aligned} \tag{32}$$

The moments and correlations can be calculated straightforwardly using their definitions. However, we elaborate on the process of characterizing the between-squares correlation.

Denote $Z_{ij}^* = \dfrac{Z_{ij} - \mu_{ij}}{\sigma_{ij}}, i, j = 1, 2$, and construct Schmidt orthogonalization as follows:

$$
\begin{aligned}
Z_{11}^{**} =& Z_{11}^*, \\
Z_{12}^{**} =& Z_{12}^*, \\
Z_{21}^{**} =& Z_{21}^* - \rho_{11} Z_{11}^{**} - \rho_{21} Z_{12}^{**}, \\
Z_{22}^{**} =& Z^* 22 - \rho_{12} Z_{11}^{**} - \rho_{22} Z_{12}^{**} + \frac{\rho_{11}\rho_{12} + \rho_{21}\rho_{22}}{1 - \rho_{11}^2 - \rho_{21}^2} Z_{21}^{**}.
\end{aligned}
$$

Subsequently, we obtain

$$
\begin{aligned}
\mathrm{Corr}^{\mathrm{sq}}(Z_1, Z_2) =& \mathrm{Corr}\big((\sigma_{11} I_1 Z_{11}^* + \sigma_{12}(1 - I_1) Z_{12}^* + (I_1 - p_1)(\mu_{11} - \mu_{12}))^2, \\
& (\sigma_{21} I_2 Z_{21}^* + \sigma_{22}(1 - I_2) Z_{22}^* + (I_2 - p_2)(\mu_{21} - \mu_{22}))^2\big) \\
=& \frac{1}{\sigma_1^2 \sigma_2^2 \sqrt{(k_1 - 1)(k_2 - 1)}} \mathrm{Cov}\big((\sigma_{11} I_1 Z_{11}^{**} + \sigma_{12}(1 - I_1) Z_{12}^{**} + (I_1 - p_1)(\mu_{11} - \mu_{12}))^2, \\
& (\sigma_{21} I_2 (\rho_{11} Z_{11}^{**} + \rho_{21} Z_{12}^{**} + Z_{21}^{**}) + \sigma_{22}(1 - I_2)(\rho_{12} Z_{11}^{**} + \rho_{22} Z_{12}^{**} \\
& - \frac{\rho_{11}\rho_{12} + \rho_{21}\rho_{22}}{1 - \rho_{11}^2 - \rho_{21}^2} Z_{21}^{**} + Z_{22}^{**}) + (I_2 - p_2)(\mu_{21} - \mu_{22}))^2\big) \\
=& \frac{1}{\sigma_1^2 \sigma_2^2 \sqrt{(k_1 - 1)(k_2 - 1)}} \mathrm{Cov}\big((\sigma_{11} I_1 Z_{11}^{**} + \sigma_{12}(1 - I_1) Z_{12}^{**} + (I_1 - p_1)(\mu_{11} - \mu_{12}))^2, \\
& ((\sigma_{21} I_2 \rho_{11} + \sigma_{22}(1 - I_2)\rho_{12}) Z_{11}^{**} + (\sigma_{21} I_2 \rho_{21} + \sigma_{22}(1 - I_2)\rho_{22}) Z_{12}^{**} + \\
& (I_2 - p_2)(\mu_{21} - \mu_{22}))^2\big).
\end{aligned}
$$

By exploiting the mutual independence of y_{ij}, we derive Equation (32).

B.2 Sufficiency of the First Four Moments

In Appendix B.1, we provided the first four moments of a mixture normal distribution. In this section, we explore whether these moments can uniquely characterize the distribution. To do so, we outline specific regularity conditions that establish the uniqueness of a mixture normal distribution based solely on its first four moments.

Theorem 2. *Let Z_1 be a random variable that follows a mixture normal distribution as defined in* (25)*. Denote the mean, standard deviation, skewness, and kurtosis of Z_1 as $\mu, \sigma, s,$ and k, respectively. For convenience, we refer to the parameters of Z_1 as $p, \mu_1, \mu_2, \sigma_1, \sigma_2,$*

and $u_d(Z_1) = \mu_1 - \mu_2$. *If the following conditions hold:*

$$\frac{1}{2} + \frac{\sqrt{15}\sqrt{13 - 2\sqrt{31}}}{30} < p < 1, \tag{33}$$

$$-\frac{2p(1-p)(2p-1)}{\left(p^2 - p + \frac{1}{3}\right)}\left(\frac{1}{3}t^{3/2} - t^{1/2}\right) \leq s < 0, \tag{34}$$

$$0 < u_d \leq t\sigma^2, \tag{35}$$

where $t = \left(\sqrt{\frac{90k - 158}{31}} + 1.9\right)$, *then the first four moments* $((\mu, \sigma, s, k)^T)$ *are sufficient statistics for uniquely characterizing a mixture normal distribution. This implies that no other mixture normal distributed random variable* $\widetilde{Z}_1$ *exists with different parameters but the same first four moments as* Z_1.

Proof. From (28) and (29), we obtain

$$\sigma_1^2 = \frac{p^2 - 1}{3}u_d + \frac{\sigma^3 s}{3p}\frac{1}{\sqrt{u_d}} + \sigma^2, \tag{36}$$

$$\sigma_2^2 = \frac{p(1-2p)}{3}u_d - \frac{\sigma^3 s}{3(1-p)}\frac{1}{\sqrt{u_d}} + \sigma^2. \tag{37}$$

Then substitute (36) and (37) into (30), we get:

$$f(u_d) = 0, \tag{38}$$

where,

$$\begin{aligned} f(u_d) = &\left(-10(p^2 - p + \frac{1}{30})^2 + \frac{31}{90}\right)u_d^2 - 2\sigma^2(2p-1)^2 u_d + \frac{2s\sigma^3 \cdot (1-2p)}{3p(p-1)}u_d^{1/2} \\ &+ \frac{2s\sigma^5(2p-1)}{p(p-1)}u_d^{-1/2} + \frac{s^2\sigma^6\left(p^2 - p + \frac{1}{3}\right)}{p^2(p^2 - 2p + 1)}u_d^{-1} - \sigma^4(k-3). \end{aligned} \tag{39}$$

We can then demonstrate that, under the following three constraints, (38) has a unique solution:

1) $f(u_d)$ is convex. Examining its second derivative,

$$f''(u_d) = \frac{2\left(p^2 - p + \frac{1}{3}\right)}{p^2(p^2 - 2p + 1)}s^2\sigma^6 x_3^{-3} + \frac{3(2p-1)}{2p(p-1)}s\sigma^5 x_3^{-\frac{5}{2}} - \frac{(1-2p)}{6p(p-1)}s\sigma^3 x_3^{-\frac{3}{2}}$$

$$-20p^4+40p^3-\frac{64p^2}{3}+\frac{4p}{3}+\frac{2}{3}.$$

we find that the first three terms are positive since $p\in(1/2,1)$ and $s<0$. By applying the root formula of quartic equations, we can easily verify that

$$-10p^4+20p^3-\frac{32p^2}{3}+\frac{2p}{3}+\frac{1}{3}>0, p\in[1/2,1]$$

is equivalent to (33). Therefore, $f''(u_d)>0$ i.e. $f(u_d)$ is convex.

2) As the coefficient of u_d^{-1} in $f(u_d)$ is strictly positive, we conclude that $\lim_{u_d\to 0} f(u_d)=+\infty$.

3) $f(t\sigma^2)<0$. First, we rewrite the inequality as follows:

$$\begin{aligned}
&\left(-10(p^2-p+\frac{1}{30})^2+\frac{31}{90}\right)u_d^2-2\sigma^2\left(2p-1\right)^2u_d-\sigma^4(k-3)<0\\
\Leftrightarrow& u_d<\left(\frac{2(2p-1)^2}{-10(p^2-p+\frac{1}{30})^2+\frac{31}{90}}+\frac{(k-3)\sigma^2}{(-10(p^2-p+\frac{1}{30})^2+\frac{31}{90})}\frac{1}{u_d}\right)\sigma^2\\
\Leftarrow& u_d\le\left(3.8+\frac{90}{31}(k-3)\frac{\sigma^2}{u_d}\right)\sigma^2\\
\Leftarrow& u_d\le\left(\sqrt{\frac{90k-158}{31}}+1.9\right)\sigma^2,
\end{aligned}$$

where we have used the following inequality:[2]

$$\frac{(2p-1)^2}{-10(p^2-p+\frac{1}{30})^2+\frac{31}{90}}=\left(-\frac{5p^2}{2}+\frac{5p}{2}+\frac{11}{24}-\frac{1}{8\left(2p-1\right)^2}\right)^{-1}>1.9$$

which holds for $\frac{1}{2}+\frac{\sqrt{15}\sqrt{13-2\sqrt{31}}}{30}<p<1$. Second, we have

$$\begin{aligned}
&\left[\frac{2s\sigma^3\left(1-2p\right)}{3p\left(p-1\right)}u_d^{1/2}+\frac{2s\sigma^5\cdot\left(2p-1\right)}{p\left(p-1\right)}u_d^{-1/2}+\frac{s^2\sigma^6\left(p^2-p+\frac{1}{3}\right)}{p^2\left(p^2-2p+1\right)}u_d^{-1}\right]_{u_d=t\sigma^2}<0\\
\Leftrightarrow&\frac{2\left(1-2p\right)}{p\left(p-1\right)}\left(\frac{1}{3}t^{3/2}-t^{1/2}\right)+\frac{s\left(p^2-p+\frac{1}{3}\right)}{p^2\left(p^2-2p+1\right)}>0\\
\Leftrightarrow& s>-\frac{2p(1-p)\left(2p-1\right)}{\left(p^2-p+\frac{1}{3}\right)}\left(\frac{1}{3}t^{3/2}-t^{1/2}\right).
\end{aligned}$$

[2]This inequality can be verified using the first derivative. However, the exact lower bound is quite complicated; for simplicity, we have rounded it to two significant digits.

Given constraints 1, 2, and 3, the equation $f(u_d) = 0$ has one and only one root. Using Equations (27), (36), and (37), we conclude that the value of u_d uniquely determines the values of the other parameters. Therefore, $(\mu, \sigma, s, k)^T$ is a sufficient statistic. □

Theorem 2 establishes that the first four moments are sufficient statistics for characterizing a mixture normal distribution, provided that p satisfies both (33) and (34). The range of values for p satisfying (33) is approximately $(0.6763, 1)$, and conditions (34) and (35) further imply that this range is sufficiently wide to cover parameter values estimated from data. For example, if we consider $p = 0.7, \sigma = 0.2, k = 6$, the two constraints become: $-2.5457 \leq s < 0$, $0 < u_d \leq 0.2164$. It is worth mentioning that the calibrated parameters in Table 5 satisfy these three conditions. Therefore, we introduce the following assumption, which guarantees that the first four moments can serve as sufficient statistics for uniquely characterizing a mixture normal distribution:

- **Assumption 1:** The probability parameter and the first four moments of Z_1, Z_2 satisfy conditions (33), (34) and (35).

B.3 Sufficiency of Between-squares Correlation

Various measures of higher-order dependence have been employed in the literature to capture higher-order dependencies. For example, Harvey and Siddique (2000) uses coskewness to measure dependence between an individual asset and a market portfolio. In this section, we demonstrate that, subject to certain regularity conditions, the between-squares correlation is a sufficient measure to capture all higher-order dependencies.

First, let us examine the frequently used higher-order dependences in the 3rd and 4th degrees.

Assuming that Z_1 and Z_2 follow a mixture normal distribution, we can demonstrate that it is sufficient to use between-squares correlation to measure all of the coskewness and cokurtosis mentioned above, subject to a mild condition. As we have established the sufficiency of the first four moments of a mixture normal distribution in Appendix B.2, we will assume that all moments, except for Corr and Corr^{sq}, are fixed in this section.

Using the properties of the bivariate normal distribution,[3] Lemma 1 provides a method

[3] If $(Z_1, Z_2)^T$ follows a bivariate normal distribution with mean $(\mu_1, \mu_2)^T$ and covariance $\begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$, then: $S(Z_1, Z_1, Z_2) = 0, K(Z_1, Z_1, Z_1, Z_2) = K(Z_1, Z_2, Z_{21}, Z_2) = 3\rho, K(Z_1, Z_1, Z_2, Z_2) = 1 + 2\rho^2$.

for calculating coskewness and cokurtosis in bivariate mixture normal distributions.

Lemma 2. *If* $(Z_1, Z_2)^T$ *follows a bivariate mixture normal distribution with parameters* $p_1, \mu_{11}, \mu_{12}, \sigma_{11}, \sigma_{12}, p_2, \mu_{21}, \mu_{22}, \sigma_{21}, \sigma_{22}, \rho_{11}, \rho_{12}, \rho_{21}, \rho_{22}$ *and* $\mu_i, \sigma_i, s_i, k_i, i = 1, 2$ *defined in Appendix B.1, then:*

$$\begin{aligned}
S(Z_1, Z_1, Z_2) =& \frac{2(\mu_{11} - \mu_{12})p_{11}p_{12}}{\sigma_1^2\sigma_2}\big[\sigma_{11}(p_{21}\sigma_{21}\rho_{11} + p_{22}\sigma_{22}\rho_{12}) - \sigma_{12}(p_{21}\sigma_{21}\rho_{21} + p_{22}\sigma_{22}\rho_{22})\big], \\
K(Z_1, Z_1, Z_1, Z_2) =& \frac{3}{\sigma_1^3\sigma_2}\big[p_{11}\sigma_{11}(\sigma_{11}^2 + p_{12}^2(\mu_{11} - \mu_{12})^2)(p_{21}\sigma_{21}\rho_{11} + p_{22}\sigma_{22}\rho_{12}) \\
&+ p_{21}\sigma_{12}(\sigma_{12}^2 + p_{11}^2(\mu_{11} - \mu_{12})^2)(p_{21}\sigma_{21}\rho_{21} + p_{22}\sigma_{22}\rho_{22})\big] \\
K(Z_1, Z_1, Z_2, Z_2) =& \sqrt{(k_1 - 1)(k_2 - 1)}\mathrm{Corr}^{\mathrm{sq}}(X_1, X_2) + 1.
\end{aligned}$$

Proof. Let Z_{11} and Z_{12} be the two normal components of Z_1, where $Z_1 = I_1 Z_{11} + (1 - I_1)Z_{12}$ and $I_1 \sim B(1, p_1)$. Similarly, let Z_{21} and Z_{22} be the components of Z_2. Using the properties of normal distributions, we have $S(Z_{1i}, Z_{1i}, Z_{2j}) = 0$. Therefore, we can rewrite the expression as follows:

$$\begin{aligned}
\mathbb{E}[(Z_1 - \mathbb{E}Z_1)^2(Z_2 - \mathbb{E}Z_2)] =& \sum_{i,j=1,2} p_{1i}p_{2j}\mathbb{E}[(Z_{1i} - \mathbb{E}Z_{1i}) + \mathbb{E}Z_{1i} - \mathbb{E}Z_1)^2((Z_{2j} - \mathbb{E}Z_{2j}) + \mathbb{E}Z_{2j} - \mathbb{E}Z_2)] \\
=& \sum_{i,j=1,2} p_{1i}p_{2j}\left[\sigma_{1i}^2\sigma_{2j}S(Z_{1i}, Z_{1i}, Z_{2j}) + 2(\mathbb{E}Z_{1i} - \mathbb{E}Z_1)\rho_{ij}\right], \\
=& 2\sum_{i,j=1,2} p_{1i}p_{2j}\rho_{ij}(\mathbb{E}Z_{1i} - \mathbb{E}Z_1).
\end{aligned}$$

where σ_{1i}^2 and σ_{2j}^2 are the variances of Z_{1i} and Z_{2j}, respectively, and ρ_{ij} is the correlation between Z_{1i} and Z_{2j}. Since $\mathbb{E}Z_{1i} = \mu_{1i}$ and $\mathbb{E}Z_1 = p_1\mu_{11} + (1 - p_1)\mu_{12}$, we can derive the equation for $S(Z_1, Z_1, Z_2)$. The calculation for cokurtosis is similar; thus, we will not provide the details here. □

It is evident that the cokurtosis $K(Z_1, Z_1, Z_2, Z_2)$ can be calculated using $\mathrm{Corr}^{\mathrm{sq}}(Z_1, Z_2)$. However, for the remaining cases, we require Lemma 2.

Lemma 3. *The bivariate mixture normal distribution with parameters* p_1, μ_{11}, μ_{12}, σ_{11}, σ_{12}, p_2, μ_{21}, μ_{22}, σ_{21}, σ_{22}, ρ_{11}, ρ_{12}, ρ_{21}, ρ_{22} *is represented by the random vector* $(Z_1, Z_2)^T$. *Here,* $\rho_{11} = \rho_{22} = \rho_1$, $\rho_{12} = \rho_{21} = \rho_2$, *and we define* $\iota_{ij} = p_{1i}p_{2j}\sigma_{1i}\sigma_{2j}$, *for* $i, j = 1, 2$. *Using these*

notations, we can express the following equations:

$$\rho_2 = k\rho_1 + b, \tag{40}$$

$$\mathrm{Corr}^{\mathrm{sq}}(\mathrm{Z}_1, \mathrm{Z}_2) = a_0 + a_1\rho_1 + a_2\rho_1^2, a_2 > 0, \tag{41}$$

where

$$\begin{aligned}
k =& -\frac{\iota_{11} + \iota_{22}}{\iota_{12} + \iota_{21}},\\
b =& \frac{\sigma_1\sigma_2 \mathrm{Corr}(Z_1, Z_2)}{\iota_{12} + \iota_{21}},\\
a_2 =& \frac{2}{\sigma_1^2\sigma_2^2\sqrt{(k_1-1)(k_2-1)}}\left(\frac{\iota_{11}^2}{p_{11}p_{21}} + \frac{k^2\iota_{12}^2}{p_{11}p_{22}} + \frac{k^2\iota_{21}^2}{p_{12}p_{21}} + \frac{\iota_{22}^2}{p_{12}p_{22}}\right),\\
a_1 =& \frac{4(\mu_{11}-\mu_{12})(\mu_{21}-\mu_{22})}{\sigma_1^2\sigma_2^2\sqrt{(k_1-1)(k_2-1)}}(p_{12}p_{22}\iota_{11} - kp_{12}p_{21}\iota_{12} - kp_{11}p_{22}\iota_{21} + p_{11}p_{21}\iota_{22})\\
&+ \frac{4kb}{\sigma_1^2\sigma_2^2\sqrt{(k_1-1)(k_2-1)}}\left(\frac{\iota_{12}^2}{p_{11}p_{22}} + \frac{\iota_{21}^2}{p_{12}p_{21}}\right)\\
a_0 =& \frac{2}{\sigma_1^2\sigma_2^2\sqrt{(k_1-1)(k_2-1)}}\left(\frac{\iota_{12}^2}{p_{11}p_{22}} + \frac{\iota_{21}^2}{p_{12}p_{21}}\right)b^2 - \frac{4(\mu_{11}-\mu_{12})(\mu_{21}-\mu_{22})}{\sigma_1^2\sigma_2^2\sqrt{(k_1-1)(k_2-1)}}(p_{12}p_{21}\iota_{12} + p_{11}p_{22}\iota_{21})\,b.
\end{aligned}$$

Proof. We can easily verify Lemma 2 using Equations (31) and (32). □

By solving (41), we obtain the following solutions:

$$\rho_1^+ = -\frac{a_1}{2a_2} + \left(\frac{1}{\sqrt{a_2}}\sqrt{\mathrm{Corr}^{\mathrm{sq}} - \frac{4a_0a_2 - a_1^2}{4a_2}}\right), \rho_1^- = -\frac{a_1}{2a_2} - \left(\frac{1}{\sqrt{a_2}}\sqrt{\mathrm{Corr}^{\mathrm{sq}} - \frac{4a_0a_2 - a_1^2}{4a_2}}\right) \tag{42}$$

Assuming the marginal distributions and linear correlation are given, Lemma 3 and (40) imply that $S(Z_1, Z_1, Z_2)$ and $K(Z_1, Z_1, Z_1, Z_2)$ are linearly dependent on ρ_1. Therefore, they can be determined uniquely by $\mathrm{Corr}^{\mathrm{sq}}(\mathrm{Z}_1, \mathrm{Z}_2)$, subject to the following assumption:

- **Assumption 2:** Suppose that $(Z_1, Z_2)^T$ follows a bivariate mixture normal distribution, where the parameters satisfy $\rho_{11} = \rho_{22} = \rho_1$, $\rho_{12} = \rho_{21} = \rho_2$, and only one of ρ_1^+ or ρ_1^- falls within the interval $[-1, 1]$. In this case, Equation (41) provides a solution for only one root within the interval $[-1, 1]$.

When $\mathrm{Corr}^{\mathrm{sq}}(\mathrm{Z}_1, \mathrm{Z}_2)$ is positive, the condition in Assumption 2 can be easily satisfied, which

is consistent with data estimation. The following theorem establishes that the between-squares correlation is a sufficient measure of higher-order dependence under mixture normal distributions.

Theorem 3. *Let $(Z_1, Z_2)^T$ follow a bivariate mixture normal distribution. Denote $\mu_i, \sigma_i, s_i, k_i$ as the mean, standard deviation, skewness and kurtosis of X_i, respectively. Assuming Assumptions 1 and 2 hold, the vector $(\mu_1, \sigma_1, s_1, k_1, \mu_2, \sigma_2, s_2, k_2, \mathrm{Corr}(Z_1, Z_2), \mathrm{Corr}^{\mathrm{sq}}(Z_1, Z_2))^T$ constitutes sufficient statistics that can uniquely determine the joint distribution of $(Z_1, Z_2)^T$.*

Proof. Based on Theorem 2 and assuming Assumption 1 holds, we can conclude that the vector $(\mu_i, \sigma_i, s_i, k_i)^T$ is sufficient for Z_i and can be used to determine the underlying parameters $(\mu_{i1}, \mu_{i2}, \sigma_{i1}, \sigma_{i2})^T$. Additionally, Assumption 2 implies that Equations (40) and (41) have only one solution. Therefore, the vector $(\mu_1, \sigma_1, s_1, k_1, \mu_2, \sigma_2, s_2, k_2, \mathrm{Corr}(Z_1, Z_2), \mathrm{Corr}^{\mathrm{sq}}(Z_1, Z_2))^T$ constitutes sufficient statistics to uniquely determine ρ_1, ρ_2. □

Theorem 3 establishes that, under Assumptions 1 and 2, the between-squares correlation is sufficient to measure the entirety of higher-order dependence between two random variables.

Appendix C A Mean-Variance-Skewness-Kurtosis Model

In this section, we first show that the household's one-period utility optimization problem can be simplified into a one-period model incorporating higher-order moments of portfolio returns. We then examine the household's portfolio risk profile, as measured by higher-order moments, within this simplified model.

C.1 A Mean-Variance-Skewness-Kurtosis Model as an Approximation to the Full model

As outlined in Section 4, W_0 represents the initial wealth, $W_1 = R^p W_0$ represents the wealth after one period, and $w_1 = \mathbb{E}[W_1]$ represents the expected wealth. Let $u(\cdot)$ denote the utility function for a representative household. The portfolio optimization problem, based on utility maximization, can be expressed as follows:

$$\max_{\alpha} \mathbb{E}[u(W_1)] \tag{43}$$

$$\text{s.t.} \quad W_1 = W_0 R^p = W_0(1 + R_f + \alpha R^E + R^L)\,.$$

where R_f represents the risk-free rate, R^E represents the excess return of the risky asset, R^L represents the income shock, and α represents the allocation to the risky asset.

We can approximate $u(W_1)$ using its Taylor series expansion around w_1. Specifically, we have:

$$u(W_1) = u(w_1) + \sum_{n=1}^{4} \frac{1}{n!} u^{(n)}(w_1)(W_1 - w_1)^n + O[(W_1 - w_1)^5]\,.$$

where $u^{(n)}(w_1)$ denotes the n-th derivative of $u(w_1)$, and $O[(W_1 - w_1)^5]$ represents the error term in the approximation.

Taking the expectation of both sides and retaining terms up to the fourth order, we obtain:

$$\begin{aligned}
\mathbb{E}[u(W_1)] \approx & \frac{u(w_1)}{w_1}\mathbb{E}[W_1] + \frac{1}{2}u^{(2)}(w_1)\mathrm{Var}(W_1) + \frac{1}{6}u^{(3)}(w_1)\mathrm{Var}^{1.5}(W_1)\mathrm{Skew}(W_1) \\
& + \frac{1}{24}u^{(4)}(w_1)\mathrm{Var}^2(W_1)\mathrm{Kurt}(W_1) \\
\approx & \frac{u(w_1)}{w_1} W_0 \bigg(\mathbb{E}(R^p) - \lambda_1 \mathrm{Var}(R^p) - \lambda_2 \mathrm{Skew}(R^p) - \lambda_3 \mathrm{Kurt}(R^p) \bigg)\,,
\end{aligned}$$

where $\mathrm{Var}(R^p)$, $\mathrm{Skew}(R^p)$, and $\mathrm{Kurt}(R^p)$ denote the portfolio's variance, skewness, and kurtosis, respectively, and the risk weights $\lambda_1, \lambda_2, \lambda_3$ satisfy

$$\lambda_1 = -\frac{1}{2}\frac{u^{(2)}(w_1)w_1}{u(w_1)}W_0, \lambda_2 = -\frac{1}{6}\frac{u^{(3)}(w_1)w_1}{u(w_1)}Var^{1.5}(R^p)W_0^3, \lambda_3 = -\frac{1}{24}\frac{u^{(4)}(w_1)w_1}{u(w_1)}Var^2(R^p)W_0^4\,.$$

The risk weights $\lambda_1, \lambda_2, \lambda_3$ can be estimated using the household's utility function, along with the conjectured wealth expectation and variance. For instance, if we denote the predicted wealth mean and portfolio return variance as $\widehat{w}_1$ and $\widehat{\mathrm{Var}}(R^p)$, respectively, we can approximate the objective function in (43) as follows:

$$\mathbb{E}[u(W_1)] \approx \frac{u(\widehat{w}_1)}{\widehat{w}_1} W_0 \bigg(\mathbb{E}(R^p) - \lambda_1 \mathrm{Var}(R^p) - \lambda_2 \mathrm{Skew}(R^p) - \lambda_3 \mathrm{Kurt}(R^p) \bigg)\,, \tag{44}$$

where the risk weights are approximated as follows

$$\lambda_1 \approx -\frac{1}{2}\frac{u^{(2)}(\widehat{w}_1)\widehat{w}_1}{u(\widehat{w}_1)}W_0, \lambda_2 \approx -\frac{1}{6}\frac{u^{(3)}(\widehat{w}_1)\widehat{w}_1}{u(\widehat{w}_1)}\widehat{\mathrm{Var}}(R^p)^{1.5}W_0^3, \lambda_3 \approx -\frac{1}{24}\frac{u^{(4)}(\widehat{w}_1)w_1}{u(\widehat{w}_1)}\widehat{\mathrm{Var}}(R^p)^2W_0^4\,.$$

Hence, we can estimate the one-period utility optimization problem using Equation (15), where the risk weights λ_1, λ_2, and λ_3 are related to the household's utility.

C.2 Higher Order Moments of the Portfolio Return

Using Lemma 2, we can calculate the skewness and kurtosis of the portfolio return in the first model, as shown in Lemma 1.

As detailed in Online Appendix B.2, assuming that $(R^E, R^L)^T$ follows a bivariate mixture normal distribution and satisfies Assumptions 1 and 2 with dependence parameters ρ_1 and ρ_2, we can express $S(R^E, R^E, R^L)$ and $K(R^E, R^E, R^E, R^L)$ as linear functions of ρ_1, given marginal distributions and linear correlation. Thus, we can represent them as follows:

$$\begin{aligned} S(R^E, R^E, R^L) =& a_{s,1}\rho_1 + b_{s,1}, \\ S(R^E, R^L, R^L) =& a_{s,2}\rho_1 + b_{s,2}, \\ K(R^E, R^E, R^E, R^L) =& a_{k,1}\rho_1 + b_{s,1}, \\ K(R^E, R^L, R^L, R^L) =& a_{k,2}\rho_1 + b_{s,2}. \end{aligned}$$

The coefficients a's and b's can be calculated using Lemmas 2 and 3.

Combining the above equations with Equation (42), we can rewrite the portfolio's skewness and kurtosis (as defined in Theorem 1) as follows:

$$\text{Skew}_\Pi = \text{sgn}(\rho_1 + \frac{a_1}{2a_2})k_s\sqrt{\text{Corr}^{sq}(R^E, R^L) - \frac{4a_0a_2 - a_1^2}{4a_2}} + b_s, \tag{45}$$

$$\text{Kurt}_\Pi = a_k\text{Corr}^{sq}(R^E, R^L) + \text{sgn}(\rho_1 + \frac{a_1}{2a_2})b_k\sqrt{\text{Corr}^{sq}(R^E, R^L) - \frac{4a_0a_2 - a_1^2}{4a_2}} + c_k, \tag{46}$$

where $\text{sgn}(\cdot)$ represents the sign function and

$$\begin{aligned} k_s &= \frac{3}{\sqrt{a_2}\sigma_\Pi^3}\left[\alpha^2\sigma_E^2\sigma_L a_{s,1} + \alpha\sigma_E\sigma_L^2 a_{s,2}\right], \\ b_s &= \frac{1}{\sigma_\Pi^3}\left[\alpha^3\sigma_E^3 S_E + \alpha^2\sigma_E^2\sigma_L b_{s,1} + 3\alpha\sigma_E\sigma_L^2 b_{s,2} + 3\sigma_L^3 S_L\right], \\ a_k &= \frac{6}{\sigma_\Pi^4}\alpha^2\sigma_E^2\sigma_L^2\sqrt{(K_E - 1)(K_L - 1)}, \\ b_k &= \frac{4}{\sqrt{a_2}\sigma_\Pi^4}\left[\alpha^3\sigma_E^3\sigma_L a_{k,1} + \alpha\sigma_E\sigma_L^3 a_{k,2}\right], \end{aligned}$$

$$c_k = \frac{1}{\sigma_\Pi^4}\left[\alpha^4\sigma_E^4 K_E + 4\alpha^3\sigma_E^3\sigma_L b_{k,1} + 4\alpha\sigma_E\sigma_L^3 b_{k,2} + \sigma_L^4 K_L\right].$$

We can use Equations (45) and (46) to solve the portfolio optimization problem (43) using the approximation form (44). This reveals that the nonlinear impact of the between-squares correlation is transmitted through the skewness and kurtosis channels.

Appendix D Life-cycle Model and Numerical Methods

D.1 The Life-cycle model

Cash-on-hand, Participation Costs, and Borrowing Constraints The participation decision is represented by a dummy variable I_P, which equals one if households decide to participate and zero otherwise. To participate in the stock market, households must pay a fixed cost, F, which represents the cost of acquiring information about the stock market and transaction fees. Therefore, cash-on-hand $X_{i,t}$ is

$$X_{i,t+1} = (X_{i,t} - C_{i,t})R^p_{i,t+1} - FI^p_{i,t}P_{i,t} + Y_{i,t+1}, \tag{47}$$

where $R^p_{i,t+1}$ is the portfolio return given by

$$R^p_{i,t+1} = \alpha_{i,t}R^S_{t+1} + (1-\alpha_{i,t})R_f. \tag{48}$$

We restrict borrowing against riskless assets or future labor income, as well as short sales of risky assets. Specifically, households face the following constraints:

$$0 \leq \alpha_{i,t} \leq 1, \tag{49}$$

$$0 < C_{i,t} \leq X_{i,t}. \tag{50}$$

Households' Optimization Problem In each period t, households decide whether to participate in the market and determine their consumption and risky shares $(I^p_{i,t}, C_{i,t}, \alpha_{i,t})$ based on cash-on-hand $X_{i,t}$ to maximize expected utility. The optimization problem can be

stated as follows:

$$V_{i,t} = \max_{\left\{I^p_{i,u}\right\}^T_{u=t},\{\alpha_{i,u}\}^T_{u=t},\{C_{i,u}\}^T_{u=t}} U_{i,t}, \tag{51}$$

where $U_{i,t}$ is defined in Equation (17) and is subject to the constraints given by Equations (1) to (50).

D.2 Numerical Solutions to the Household Optimization

Since analytical solutions do not exist for this problem, we use a numerical solution method. We standardize the entire problem with persistent labor income $P_{i,t}$, defined as $log(P_{it}) = y_{it}$, and introduce a new state, the adjusted persistent income shock $w_{i,t}$, defined as $\log(w_{i,t}) = (1-\lambda)\nu_{i,t}$.[4] Specifically, let $x_{i,t} = \frac{X_{i,t}}{P_{i,t}}$ and $c_{i,t} = \frac{C_{i,t}}{P_{i,t}}$ represent the normalized cash on hand and consumption, respectively. The normalized value function $v_{i,t} = \frac{V_{i,t}}{P_{i,t}}$ is

$$v_{i,t}(x_{i,t}, w_{i,t}) = \max_{\substack{\{\alpha_{i,u}\}^T_{u=t} \\ \{c_{i,u}\}^T_{u=t}}} \left\{ \begin{array}{l} (1-\beta)c_{i,t}^{1-\frac{1}{\psi}} \\ +\beta\left(\mathbb{E}_t\left(\frac{P_{i,t+1}}{P_{i,t}}\right)^{1-\gamma}\left[p_{t+1}v_{i,t+1}^{1-\gamma} + (1-p_{t+1})bx_{i,t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}} \end{array} \right\}^{\frac{1}{1-1/\psi}}, \tag{52}$$

subject to

$$x_{i,t+1} = \begin{cases} (x_{i,t} - c_{i,t})R^p_{i,t+1}\frac{P_{i,t}}{P_{i,t+1}} - FI^p_{i,t} + e^{\varepsilon_{i,t+1}} & \text{for } t \leq K\ , \\ (x_{i,t} - c_{i,t})R^p_{i,t+1} - FI^p_{i,t} + k & \text{for } t > K\ . \end{cases} \tag{53}$$

$$\ln w_{i,t+1} = \begin{cases} \lambda \ln w_{i,t} + (1-\lambda)u_{i,t+1} & \text{for } t \leq K\ , \\ 0. & \text{for } t > K\ . \end{cases} \tag{54}$$

$$\frac{P_{i,t}}{P_{i,t+1}} = \begin{cases} w_{i,t}e^{f(t,\Omega_{it}) - f(t+1,\Omega_{i,t+1}) - u_{i,t+1}} & \text{for } t \leq K\ , \\ 1 & \text{for } t > K\ . \end{cases} \tag{55}$$

The model can be solved numerically using the backward induction method. In this approach, the value function for each period depends on a continuous variable, the normalized

[4] We use $w_{i,t}$ as a state instead of $\nu_{i,t}$ for computational convenience.

cash-on-hand x_t, which must be discretized because we use a grid search to optimize the value function. The terminal condition in the final period is determined by the bequest motive, and the value function corresponds to the bequest function.

To solve the model, we iterate backward for each period t before T and for each point in the state space. We use a grid search to compute the value associated with each consumption and risky share grid and select the optimal grids that achieve the maximum value as the policy rules. This iterative procedure is repeated for each period t until we obtain the optimal policy rules for all periods.

To approximate the expected value in the backward induction process, we use the Gauss-Hermite method for a mixture normal distribution. Specifically, we evaluate the expectation $\mathbb{E}[h(\gamma_{t+1}, \eta_{t+1}, \epsilon_{t+1})]$ with respect to $(\gamma_{t+1}, \epsilon_{t+1}, \eta_{t+1})$, where $\gamma_{t+1} = I_1 Z_{11} + (1 - I_1) Z_{12}$, $\eta_{t+1} = I_2 Z_{21} + (1 - I_2) Z_{22}$, and $\epsilon_{t+1} \sim N(0, \sigma_\epsilon^2)$.

We apply the Gauss-Hermite method to the four normal distributions embedded in the mixture. Specifically, we have:

$$\mathbb{E}[h] = \sum_{i,j=1,2} p_{1i} p_{2j} \mathbb{E}[h(Z_{1i}, Z_{2j}, \epsilon_{t+1})],$$

where $p_{n1} = 1 - p_{n2} = \mathbb{P}[I_n = 1], n = 1, 2$. For each component of the summation, the vector $(Z_{1i}, Z_{2j}, \epsilon_{t+1})^T$ is normally distributed, and we use the Gauss-Hermite method to approximate the expected value.

D.3 Calibration

In this section, we present the explicit form of the moments that can be utilized in the GMM method discussed in Section 4.1.

Based on Equations (18) and (19), we obtain

$$\delta_{it} = \sum_{s=0}^{t} \lambda^{t-s} u_{is} + \epsilon_{it}.$$

We then proceed to compute the moments:

$$\text{Corr}(\eta_{t+1}, \delta_{it+1} - \delta_{it}) = \text{Cor}(\eta_{t+1}, u_{it+1}) \frac{\sigma_u}{\sqrt{\sigma_u^2 + 2\sigma_\epsilon^2}} = \frac{\sigma_u}{\sqrt{\sigma_u^2 + 2\sigma_\epsilon^2}} \text{Cor}(\eta_{t+1}, u_{t+1}),$$

$$
\begin{aligned}
\mathrm{Corr}^{\mathrm{bs}}(\eta_{t+1}, \delta_{it+1} - \delta_{it}) &= \mathrm{Cor}^{\mathrm{bs}}(\eta_{t+1}, u_{it+1}) \frac{\sqrt{\mathrm{Var}(u_{it+1}^2)}}{\sqrt{\mathrm{Var}((u_{it+1} + \epsilon_{it+1} - \epsilon_{it})^2)}} \\
&= \frac{\sqrt{\mathrm{kurt}_u - 1}\sigma_u^2}{\sqrt{(\mathrm{kurt}_u - 1)\sigma_u^4 + 8\sigma_\epsilon^4 + 8\sigma_u^2\sigma_\epsilon^2}} \mathrm{Cor}^{\mathrm{bs}}(\eta_{t+1}, u_{t+1}) \\
\mathrm{Var}[\delta_{i,t}] &= \frac{1 - \lambda^{2(t-s+1)}}{1 - \lambda^2}\sigma_u^2 + \sigma_\epsilon^2 \\
\mathrm{Skew}[\delta_{i,t}] &= \frac{\sigma_u^3}{(\mathrm{Var}[\delta_{i,t}])^{3/2}} \frac{1 - \lambda^{3(t-s+1)}}{1 - \lambda^3} \mathrm{Skew}_u, \\
\mathrm{Kurt}[\delta_{i,t}] &= \frac{1}{(\mathrm{Var}[\delta_{i,t}])^2} \left[3\sigma_\epsilon^4 + 6\frac{1 - \lambda^{2(t-s+1)}}{1 - \lambda^2}\sigma_\epsilon^2\sigma_u^2 + \right. \\
&\quad \left. 3\left(\left(\frac{1 - \lambda^{2(t-s+1)}}{1 - \lambda^2} \right)^2 - \frac{1 - \lambda^{4(t-s+1)}}{1 - \lambda^4} \right) \sigma_u^4 + \frac{1 - \lambda^{4(t-s+1)}}{1 - \lambda^4}\sigma_u^4 \mathrm{Kurt}_u \right],
\end{aligned}
$$

where $\mathrm{Cor}(\eta_{t+1}, u_{t+1}), \mathrm{Cor}^{\mathrm{bs}}(\eta_{t+1}, u_{t+1}), \sigma_u, \mathrm{Skew}_u$ and Kurt_u can be computed using the results presented in Appendix B.1.

Appendix E A Life-Cycle Model: Additional Quantitative Analysis

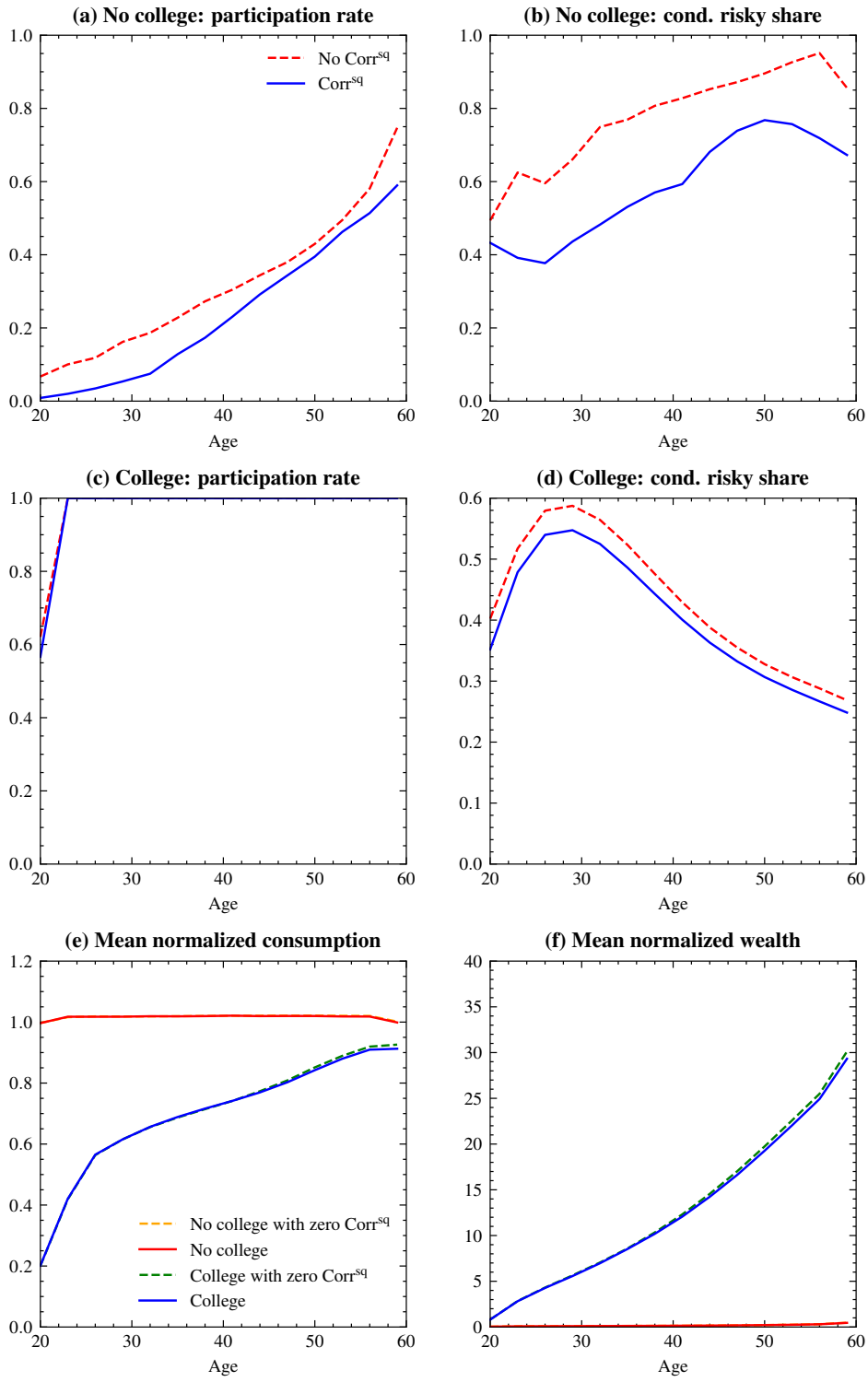
E.1 Simulated Life-cycle Profiles

We simulate the participation decisions, risky asset shares, and wealth accumulation of $100,000$ households over their life cycle. Figure E1 shows the average profiles for college graduates and the no-college group.

First, we find that the participation rate increases with age, regardless of whether the between-squares correlation is included (graphs (a) and (c)). Since households typically accumulate more wealth as they age, it becomes easier for older households to surpass the participation threshold. Second, average conditional risky asset shares are roughly hump-shaped with respect to age (graphs (b) and (d)). This finding aligns with Cocco et al. (2005); however, in Cocco et al. (2005), a large correlation (e.g., 0.2 or 0.4) is required to generate a significant hump-shaped pattern of conditional risky shares. In contrast, our model does not require a large correlation. Instead, a small between-squares correlation is sufficient to trigger this pattern. Since optimal portfolio rules decrease with wealth, relatively poor

Figure E1: Life-Cycle Profile of Portfolio Decisions

These six graphs plot the life-cycle profiles for the two groups and compare the results between the baseline model and the model without the between-squares correlation. The profiles are calculated from a simulation of 100,000 households.



investors tend to be very aggressive, often investing almost entirely in the stock market, while wealthier investors adopt more prudent strategies. At each period, some households will just cross the participation threshold and become aggressively poor investors.

In contrast, once households enter the market, they reduce their risky asset shares as they accumulate more wealth. During the early working years, when most households have very low wealth, the effect of new investors entering the market dominates, leading to an increase in average conditional risky asset shares. However, as more households participate in the market, the rebalancing effect from existing investors, who reduce their risky asset shares over time, begins to dominate, causing the average risky asset shares to decline. Consequently, the average optimal risky asset share follows a hump-shaped pattern, with its peak determined by the steepness of the participation rate. In our model, college graduates accumulate wealth much faster and are more likely to enter the stock market in their early twenties. As a result, conditional risky asset shares for college graduates peak much earlier than those for the non-college group.

Additionally, we find that the introduction of the between-squares correlation significantly reduces the share of wealth allocated to stocks, but it leads to only a small reduction in average wealth (graph F) and average consumption (graph E). The presence of a between-squares correlation between stock return innovations and labor income shocks implies that joint movements become less predictable, increasing labor income uncertainty and weakening its role as a riskless asset. This increased uncertainty lowers the average return on households' financial wealth, which, in turn, reduces their accumulated wealth over the life cycle.

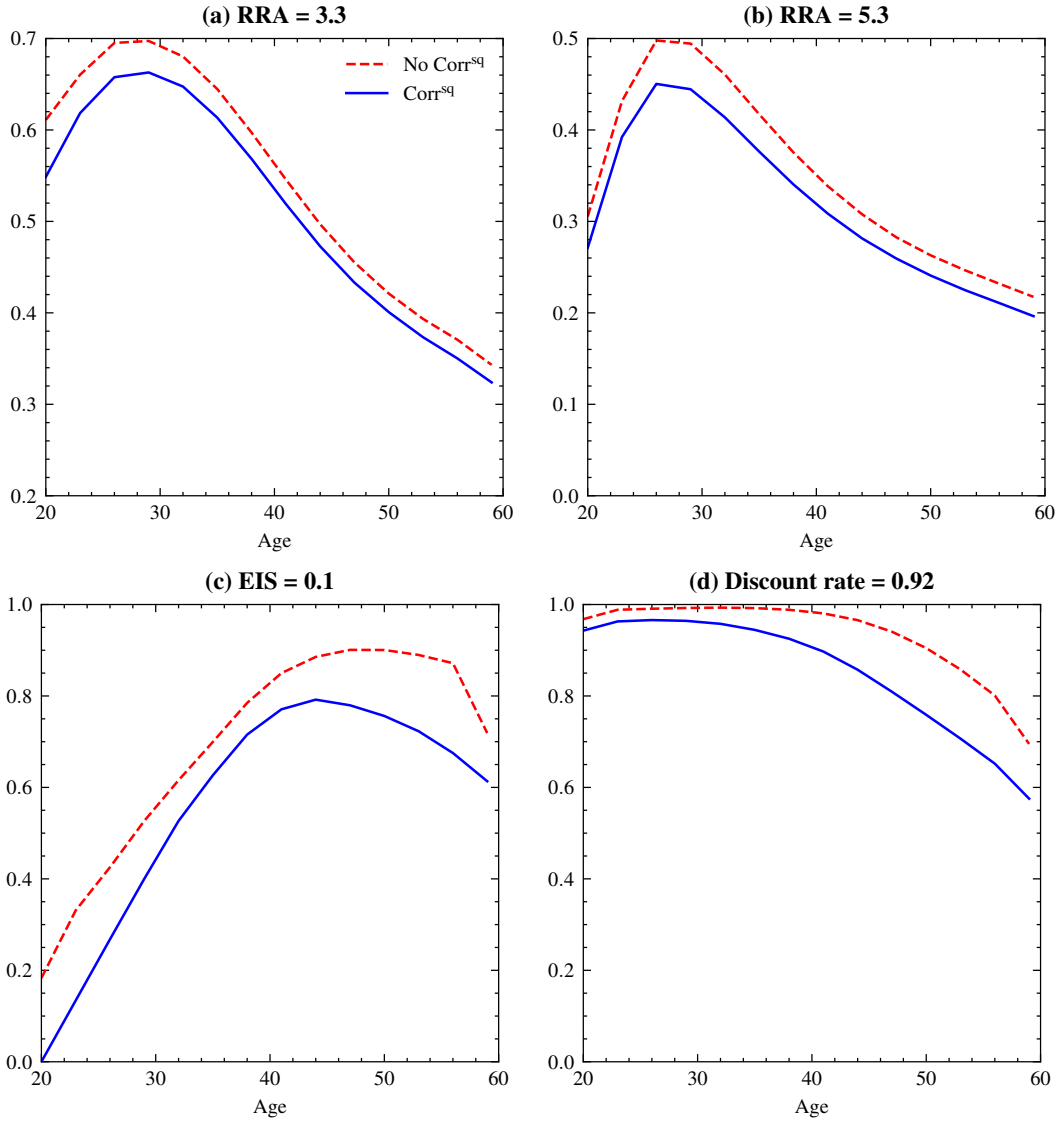
E.2 Sensitivity Analysis

To assess the robustness of the effect of the between-squares correlation, we conduct a sensitivity analysis with respect to relative risk aversion, EIS, and the discount rate. For simplicity, we present only the results for the college group at age 20. Results for the non-college group and other ages are qualitatively similar and can be provided upon request.

Figure E2 plots the life-cycle profile of risky asset shares under different parameters. We find that the between-squares correlation has a consistently robust effect on portfolio choices. While changes in parameters may significantly alter the shape of the risky asset shares curve, the impact of the between-squares correlation remains persistent and strong.

Figure E2: Sensitivity Analysis

The four graphs plot the mean conditional risky share across ages for different parameters. In each graph, we change only one parameter to test the robustness. The profiles are calculated from a simulation of 100,000 households.



E.3 Nonlinear Effect of the Between-squares Correlation

This section demonstrates that households facing a nonzero between-squares correlation between labor income and stock markets may delay their investment and opt for less risky shares. Our numerical simulations confirm that the nonlinear impact of between-squares correlation on household policy functions remains consistent in the full model, aligning with the findings of the first model presented in Section 4.

Figure E3 illustrates the policy functions of households at ages 20, 40, and 60 in our full model, with different levels of between-squares correlation, using the parameter values of the first model in Section 4. Figure E3 graph C displays the participation wealth threshold, which indicates that households are more likely to invest in the stock market when the between-squares correlation approaches a turning point near zero.

Additionally, graphs A and B show that households participating in the stock market are more willing to hold a larger proportion of risky assets when the between-squares correlation approaches the turning point near zero, regardless of their wealth levels. However, when the between-squares correlation deviates from the turning point, households tend to reduce their allocation to risky assets to decrease their portfolio risk.

These findings suggest that the between-squares correlation has a nonlinear effect on household portfolio decisions, consistent with the results of the first model. This effect operates through two channels: skewness and kurtosis. A negative between-squares correlation increases skewness, leading to delayed participation and reduced risky asset exposure. In contrast, a positive between-squares correlation increases kurtosis, resulting in similar decisions. It is worth noting that the effects of these two channels are asymmetric, with the kurtosis channel having a greater impact. In conclusion, the nonlinear relationship between labor income and stock returns, driven by the between-squares correlation, offers a new interpretation of the "stock-like" theory (i.e., Benzoni et al. (2007)), where households' labor income becomes more similar to stocks. This finding highlights the importance of accounting for the nonlinear dynamics of asset returns and their impact on portfolio decisions.

E.4 Comparison of Calibration Results

Figure E4 uses the preference parameters in Table 6 to compare the life-cycle profiles of participation rate, conditional risky asset share, and normalized wealth for an average household.

Figure E3: Changes in Portfolio Decisions

This figure shows the participation rate and conditional risky share derived from the policy functions, with the between-squares correlation set to -0.03, 0, and 0.06 for the college group. All other parameters remain consistent with those in Section 5.1. For this analysis, we assume the normalized persistent income shock is $w_{i,t} = 1$.

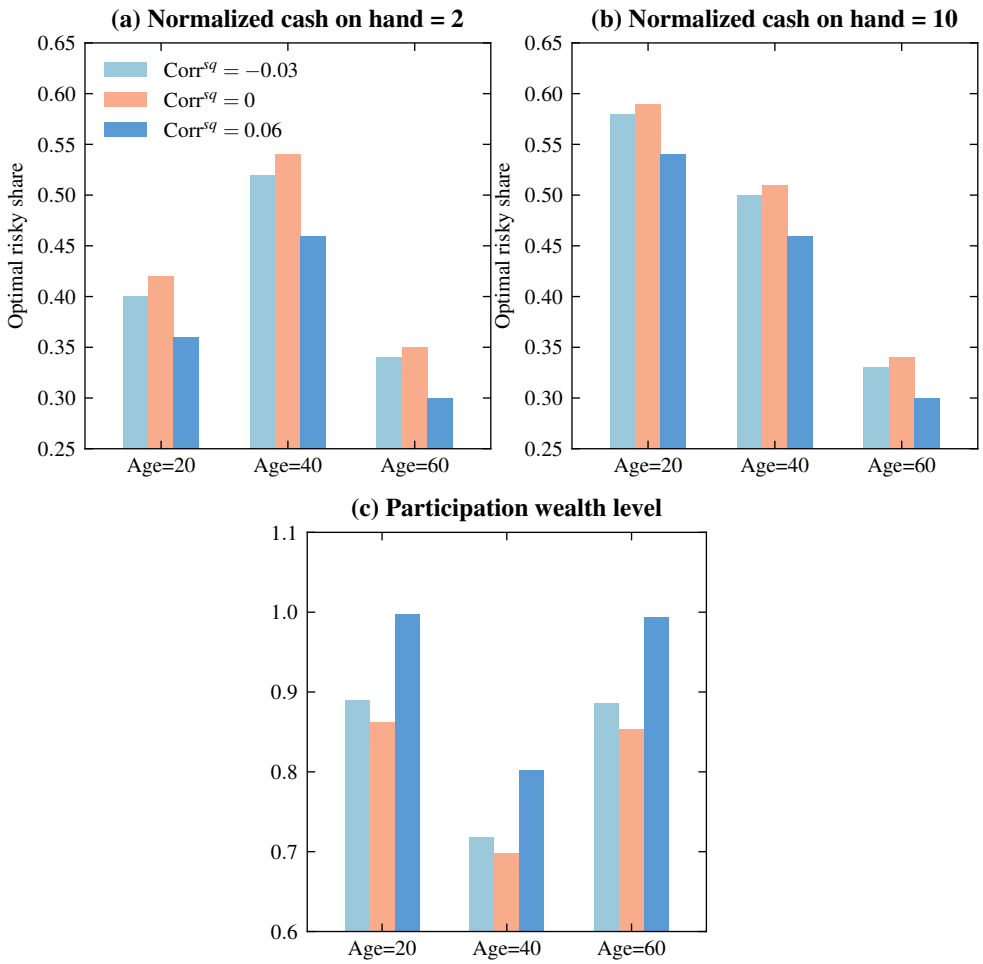
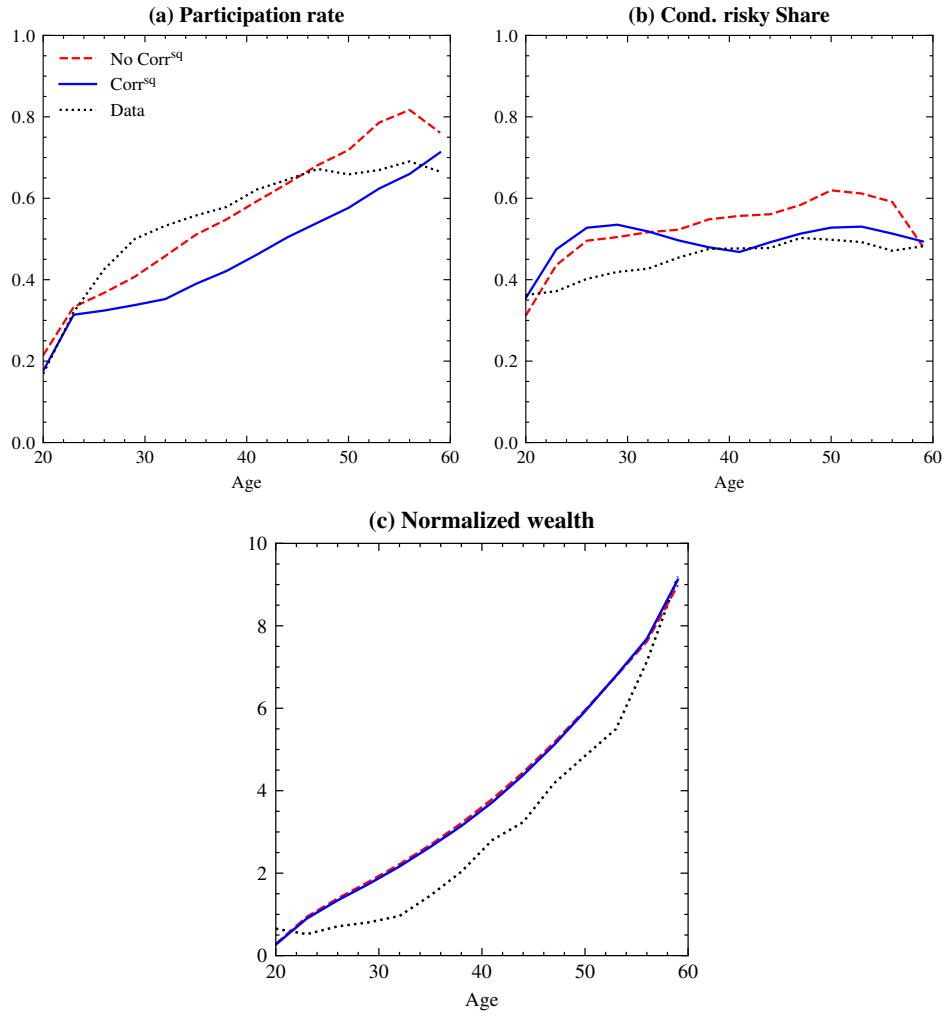


Figure E4: Calibration Results

The three graphs compare the baseline model, the model without between-squares correlation, and SCF data for an average household. Both models are calibrated to match the data moments and, as a result, utilize different preference parameters, as shown in Table 6. To align with the PSID, we assume that households with college degrees constitute 30% of the world's population. The parameters for the baseline model and the model without between-squares correlation are taken from Table 5 and Table 6. We use triennial SCF data from 2007 to 2019 to calculate the empirical moments. Moments are computed using non-overlapping age groups ranging from 20 to 65, with each age group containing samples spanning three successive ages. Graphs A, B, and C present the results for the participation rate, conditional risky share, and normalized wealth, respectively.



These profiles are derived from the baseline model and the model that excludes the between-squares correlation. Graph (a) compares the participation rates between the models and the data. Both models provide a more realistic representation of the participation rate. However, it is worth noting that participation rates are consistently higher across various age groups when the between-squares correlation is omitted. Graph (b) compares the share of wealth in stocks. The results indicate that both models successfully generate a low share of wealth in stocks, aligning with the data. However, it should be noted that without considering the between-squares correlation, the model tends to produce more imprudent portfolio choices compared to the baseline model. Graph (c) illustrates wealth accumulation profiles. Both models perform well in matching the observed patterns of wealth accumulation over the life cycle.

To explore the robustness of our findings, we relax our assumption and allow the risk aversion estimates to be different between the baseline model and the alternative model without between-squares correlation. Using the same calibration methodologies outlined in Section 5.1, we report the calibration results in Table E1 and Figure E5.

Table E1: Preference Parameters under Heterogeneous Risk Aversions
This table presents the preference parameters for both the baseline model and the model without between-squares correlation, assuming heterogeneous risk aversions. For the model without between-squares correlation, we adjust only the two correlation parameters to eliminate any between-squares correlation. The calibration of the preference parameters follows the methodology outlined in Section 5.1.

		Baseline model		Model without Corr^{sq}	
		No college	College	No college	College
Relative risk aversion	γ	4.3	4.8	4.8	5.3
EIS	ϕ	0.30	0.90	0.28	0.95
Discount factor	β	92.0%	98.5%	92.0%	98.0%
Bequest motive	b	2.5	2.5	2.5	2.5
Fixed cost rate	F	0.005	0.005	0.005	0.005

From this table, we find that incorporating between-squares correlation lowers the risk aversion estimates for both college graduates and households without college degrees, consistent with the calibration results in Table 6. Furthermore, after relaxing the assumption of identical risk aversion between college graduates and households without college degrees, the EIS of households without college degrees remains lower than that of stockholders. This aligns with findings by Gomes and Michaelides (2005) and Gomes and Michaelides (2008),

which show that households with low risk aversion and low EIS are less likely to participate in the stock market, whereas households with high risk aversion and high EIS are more inclined to participate. The calibration results in Table E1 appear consistent with these observations.

Figure E5: Calibration Results under Heterogeneous Risk Aversions

The three graphs show the comparison among the baseline model, the model without between-squares correlation, and SCF data for an average household under heterogeneous risk aversions. To align with the PSID, we assume that households with college degrees constitute 30% of the population. The parameters for the baseline model and the model without between-squares correlation have been taken from Table 5 and Table E1. We use triennial SCF data from 2007 to 2019 to compute empirical moments. Moments are calculated using nonoverlapping age groups ranging from 20 to 65, with each group containing samples spanning three successive ages. Graphs A, B, and C present results for the participation rate, conditional risky share, and normalized wealth, respectively.

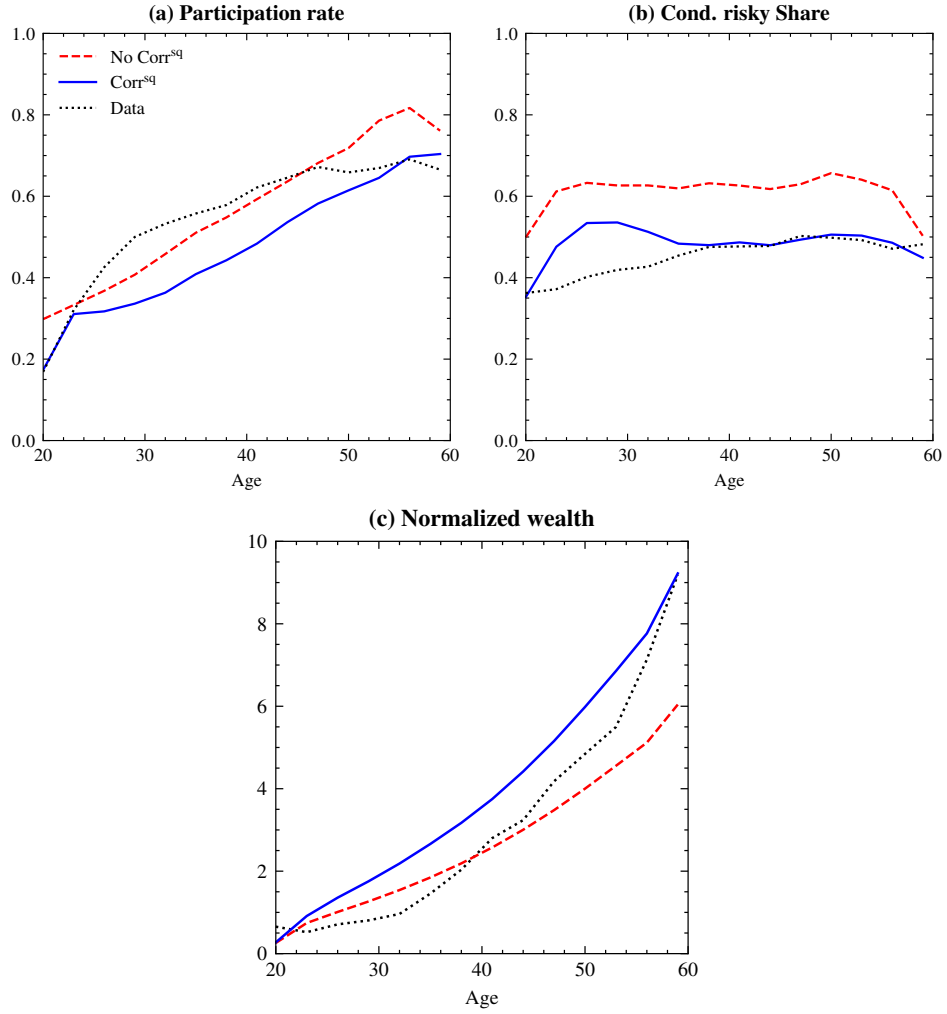


Figure E5 shows the life-cycle profiles for both the baseline model and the model without between-squares correlation. Overall, the baseline model generates the participation rate, conditional risky asset holdings, and wealth accumulation that closely align with the data.

In contrast, the model without between-squares correlation tends to overestimate conditional risky asset holdings relative to the data.

E.5 Simulated Higher-Order Dependence with Different Parameters from Existing Literature

In this appendix, we show that simulated higher-order dependence using the parameters from existing literature is almost zero. For example, in Table E2, panel D, the parameters from Catherine (2022),[5] result in higher-order dependence, as measured by between-squares correlation, of 0.0056, which is remarkably close to zero.[6] As a result, the effect of higher-order dependence on portfolio decisions is negligible. This raises the question: what parameter values would be necessary to achieve a higher-order dependence of similar magnitude to that observed in our paper, without explicitly modeling higher-order dependence?

Table E2 presents higher-order dependence under different parameter settings. In panel A, higher-order dependence is examined with varying values of $\overline{\mu_z^-}$. The results indicate that, without explicitly incorporating a higher-order dependence process, an average growth rate of -1.5 can only produce a higher-order dependence of 0.0233. In panel B, we show that reducing the expected log stock returns during years of stock market crashes to -1.245 can yield a higher-order dependence of 0.071. However, this implies an average loss of 71.21% during crashes, which is inconsistent with observed data.

In addition, panel C of Table E2 shows that increasing the frequency of important events in a worker's career to 0.846 leads to a higher-order dependence of 0.0715. However, imposing the constraint that p_z should not exceed 0.5, as specified by Catherine (2022), decreases the higher-order dependence to 0.0168, which is significantly smaller than the observed value of 0.0696. Panel D indicates that increasing the linear dependence between the growth rate of the wage index and stock returns to 0.72 produces a higher-order dependence of 0.0701. However, this adjustment also leads to a higher linear dependence of 0.234 between idiosyncratic income shocks and stock returns, which contradicts the observed evidence of

[5]More specifically, the frequency of important events in a worker's career (p_z) is 0.136; an expected log return in years of stock market crashes (μ_s^-) is -0.245; an average growth rate of labor shocks when important events occur ($\overline{\mu_z^-}$) is -0.086; and a correlation between the growth rate of the wage index and stock returns is 0.161.

[6]To maintain consistency with the notations used in Catherine (2022), we adopt the same terminology. Since $\overline{\mu_z^-}$ is just a constant term in Equation (10) in Catherine (2022), for illustration, we refer to it as the average growth rate of labor shocks when important events happen in a worker's career.

Table E2: Simulated Higher-Order Dependence with Different Parameters from Existing Literature

This table reports simulated between-squared correlation using parameters from existing literature. For each experiment, one or two parameters are adjusted, while the remaining parameters are based on the estimations from Catherine (2022): the frequency of important events in a worker's career (p_z) is 0.136; the expected log return in years of stock market crashes (μ_s^-) is -0.245; the average growth rate of labor shocks when important events occur ($\overline{\mu_z^-}$) is -0.086; and the linear dependence between the growth rate of the wage index and stock returns is 0.161.

Panel A: $Corr^{sq}$ with different average growth rate of income shocks in crashes ($\overline{\mu_z^-}$)

	Average growth rate of income shocks in crashes ($\overline{\mu_z^-}$)						
	-0.086	-0.1	-0.2	-0.3	-0.4	-0.5	-1.5
$Corr^{sq}$	0.0056	0.0059	0.0087	0.0141	0.0154	0.0161	0.0233

Panel B: $Corr^{sq}$ with different expected log stock return in crashes (μ_s^-)

	Expected stock return in crashes (μ_s^-)						
	-0.245	-0.3	-0.4	-0.5	-0.6	-0.7	-1.245
$Corr^{sq}$	0.0056	0.0084	0.0103	0.0154	0.0219	0.0281	0.0710

Panel C: $Corr^{sq}$ with different frequency of important events (p_z)

	Frequency of important events (p_z)						
	0.136	0.2	0.3	0.4	0.5	0.6	0.846
$Corr^{sq}$	0.0056	0.0069	0.0090	0.0122	0.0164	0.0223	0.0715

Panel D: $Corr^{sq}$ with different linear dependence between the growth rate of the wage index and stock returns (λ_{ls})

	Linear dependence between the growth rate of the wage index and stock returns (λ_{ls})						
	0.161	0.2	0.3	0.4	0.5	0.6	0.72
$Corr^{sq}$	0.0056	0.0069	0.0084	0.0168	0.0395	0.0531	0.0701

Panel E: Robust check of Corr^{sq} *in Catherine (2022)*

		Average growth rate of income shocks in crashes ($\overline{\mu_z^-}$)					
		-0.3	-0.4	-0.5	-0.6	-0.7	-1.245
Expected log stock return in crashes (μ_s^-)	-0.245	0.0056	0.0059	0.0087	0.0141	0.0154	0.0161
	-0.3	0.0084	0.0093	0.0109	0.0163	0.0169	0.0186
	-0.4	0.0103	0.0117	0.0168	0.0210	0.0251	0.0308
	-0.5	0.0154	0.0180	0.0220	0.0278	0.0327	0.0345
	-0.6	0.0219	0.0226	0.0272	0.0363	0.0403	0.0440
	-0.7	0.0281	0.0273	0.0362	0.0425	0.0499	0.0520

no significant linear dependence. Panel E explores different combinations of expected stock returns during crashes and the average growth rate of income shocks, while holding other parameters constant. Even with these variations, the maximum higher-order dependence achieved is 0.052, which remains below the observed value of 0.0692.

In summary, we conclude it is important to explicitly integrating higher-order dependence into the life-cycle model in order to accurately capture its impact. Relying solely on higher-order moments in labor and stock markets is inadequate for explaining the correlation between them. A deliberate inclusion of higher-order dependence is essential for a precise portrayal of the dynamics between labor and stock markets.

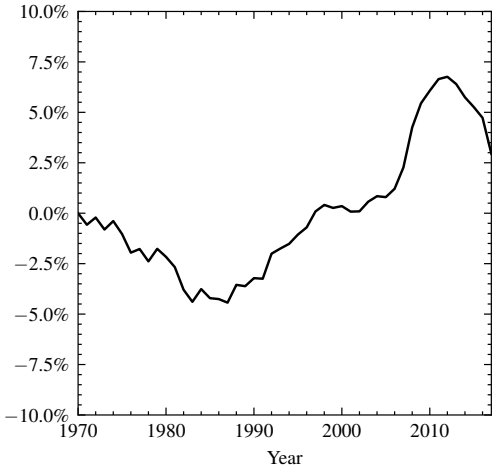
Appendix F Gini Coefficient Evolution in the USA

Figure F1 displays the evolution of the Gini coefficient in the United States from 1970 to 2017, based on data sourced from The World Inequality Database.

To analyze the evolution of income inequality over time, we have chosen 1970 as the benchmark year and calculated the relative changes in the Gini coefficient over the 48-year period. Our findings indicate that, except for the years surrounding the economic crises in 1987 and 2007, the variations in the Gini coefficient compared to 1969 remained within the range of $[-5\%, 5\%]$.

Figure F1: Gini Coefficient Evolution in the US

This figure shows the evolution of the Gini coefficient in the United States from 1970 to 2017, based on data from The World Inequality Database. Using 1962 as the benchmark year, relative changes in the Gini coefficient have been calculated to analyze trends over time.



Appendix G Additional Evidence of Higher-Order Dependence

In this appendix, we present additional evidence on the higher-order dependence between labor and stock markets. Figures G1 and G2 display the distributions of coskewness and cokurtosis, respectively. Both distributions exhibit positive means for annual and biennial frequencies. These findings indicate the presence of nonlinearity in the data, highlighting the importance of including higher-order dependence to capture the complexity of the underlying distributions.

Figure G3 shows the coskewness and cokurtosis when households are ranked by wealth. Across the five statistics, nearly all wealth groups exhibit significantly positive values, with particularly strong results for the two cokurtosis statistics. These findings demonstrate that higher-order dependencies, as captured by coskewness and cokurtosis, are present across different wealth groups.

Figure G4 presents the coskewness and cokurtosis values when households are grouped based on both earnings growth and wealth, while controlling for household characteristics. The results indicate consistency with the findings shown in Figures 2. Across all subgroups, the coskewness and cokurtosis statistics deviate from zero. Moreover, for the majority of subgroups, the five examined statistics show positive values. The similarity of these results to the previous analyses further supports the presence of nonlinear relationships.

Table G1 reports the marginal effects from probit and tobit regressions. These models, specified in Equations 11 and 12, incorporate different higher-order dependence measures. Following the methodology in Tables 3 and 4, we treat the positive and negative values of the higher-order dependence measures as separate independent variables.[7] We find that, for each model and group, at least one of the coskewness and cokurtosis measures significantly affects participation decisions. This finding provides empirical support for the impact of higher-order dependence on households' portfolio decisions. However, none of the coskewness or cokurtosis measures consistently exhibit significance across all models and groups. This indicates that between-squares correlation may serve as a more robust measure to capture higher-order dependence and its effect on households' portfolio decisions.

[7]The symmetric cokurtosis $K(\Delta y^*_{it,\Delta t}, \Delta y^*_{it,\Delta t}, \eta_t, \eta_t)$ is always positive; hence, we use 1, corresponding to the case of independent $y^*_{it,\Delta t}$ and η_t, to distinguish between positive and negative variables.

Figure G1: Coskewness Distribution

This graph presents the histogram and kernel density estimates of two types of coskewness between labor income shocks and excess stock returns for the full sample. We use labor income and stock return data from the PSID and CRSP, respectively. Panel (a) uses annual data from 1970 to 1997, while panel (b) uses biennial data covering the period from 1970 to 2017.

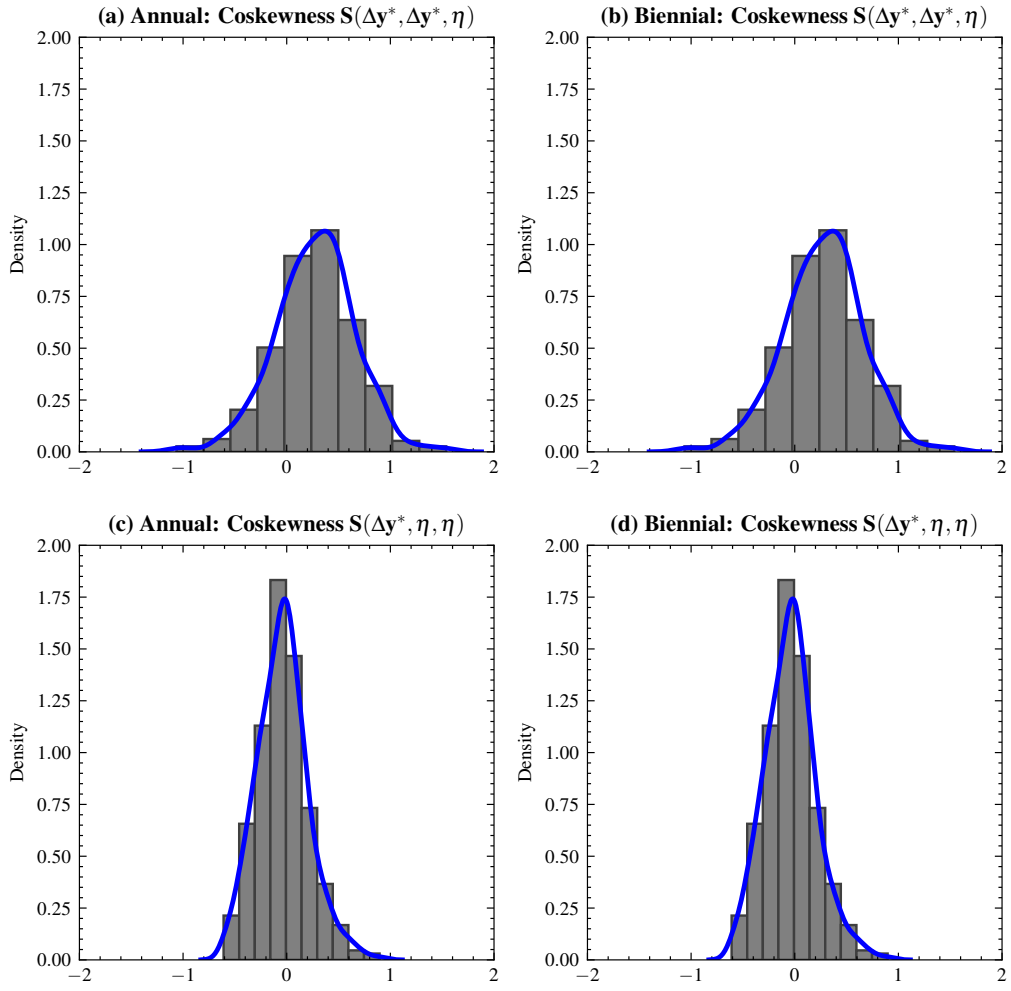


Figure G2: Cokurtosis Distribution

This graph shows the histogram and kernel density estimates of three types of cokurtosis between labor income shocks and excess stock returns for the full sample. We use labor income and stock return data from the PSID and CRSP, respectively. Panel (a) uses annual data from 1970 to 1997, while panel (b) uses biennial data covering the period from 1970 to 2017.

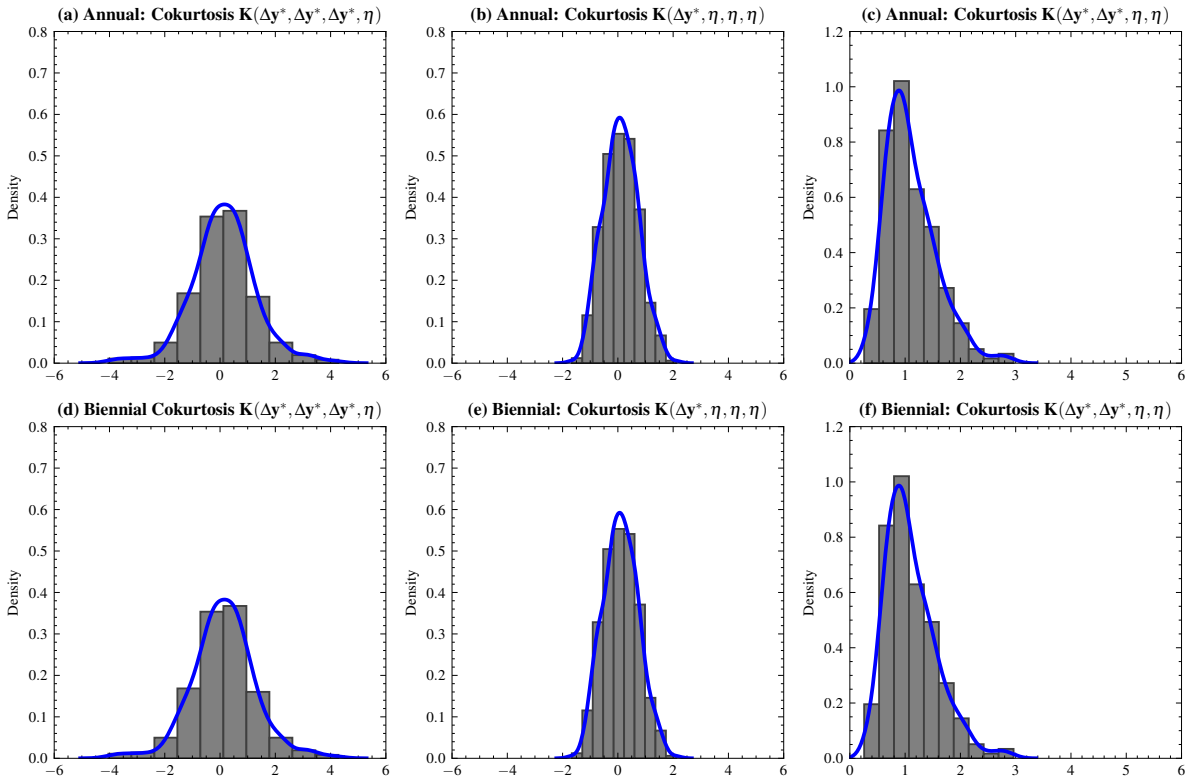


Figure G3: Coskewness and Cokurtosis by Wealth

These graphs display the coskewness and cokurtosis between the labor income growth rate and stock return for the full sample. We use biennial labor income data and stock return from the PSID and CRSP, respectively. Households are grouped by percentiles of average financial wealth across all waves. The coskewness and cokurtosis values are calculated based on the definitions provided in the Appendix B.3.

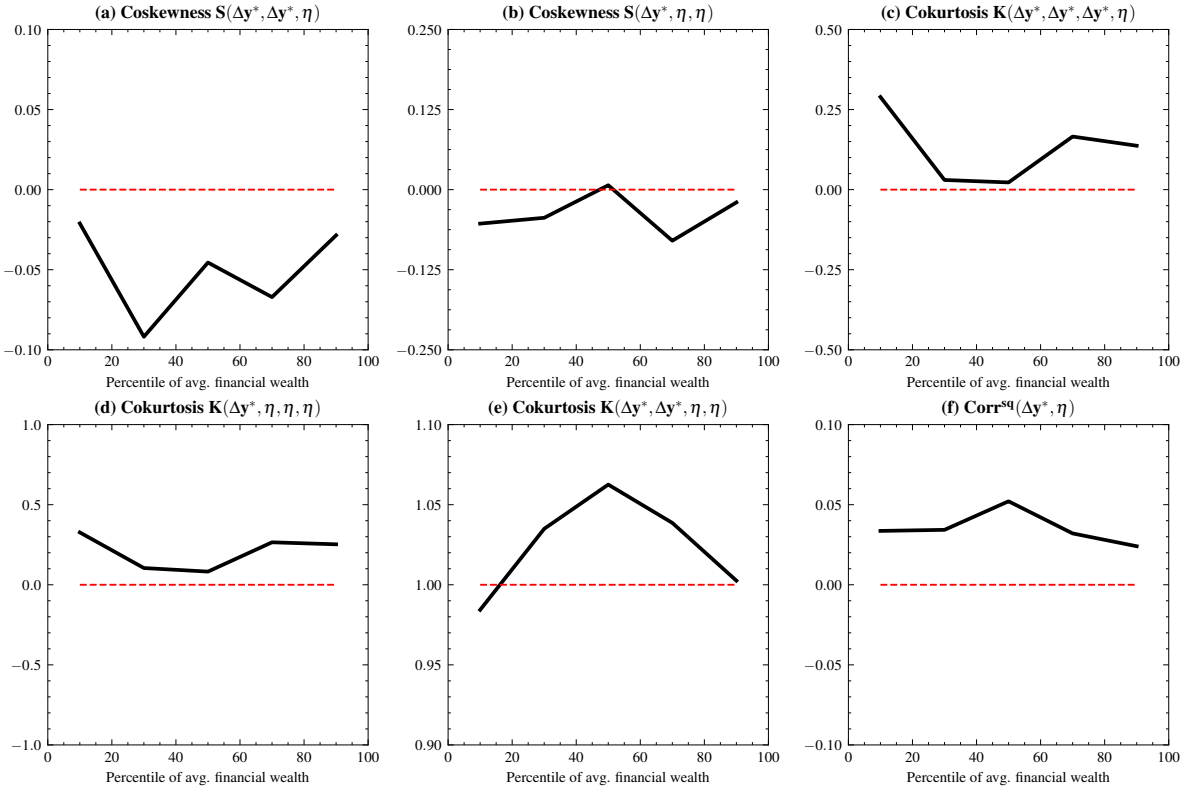


Figure G4: Coskewness and Cokurtosis by Wealth and Labor Income

These graphs display the coskewness and cokurtosis between de-trend labor income growth and stock return for the full sample. We use biennial labor income data and stock return from the PSID and CRSP, respectively. Households are grouped by percentiles of average financial wealth and average labor income growth rates across all waves. The coskewness and cokurtosis values are computed using the definitions provided in Equations (2) to (6).

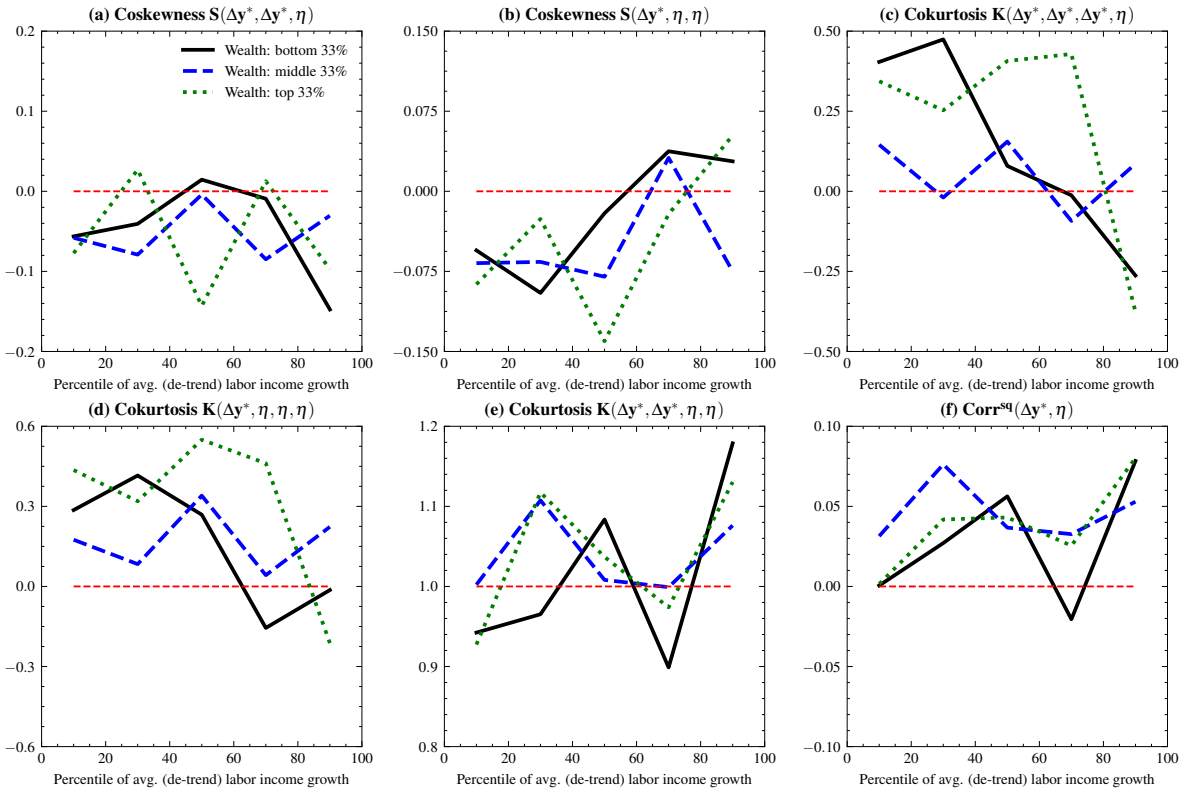


Table G1: Regression Estimates with Coskewness/Cokurtosis

This table reports marginal effects from probit and tobit regressions. In Panel A, the dependent variable is a dummy variable indicating whether the household participates in the stock market. In Panel B, the dependent variable is the risky share of households. The estimated coefficients and standard errors (in parentheses) are presented. We conduct the regressions using different sample populations, controlling for various household characteristics, as specified in Table 3. For simplicity, we report only the coefficients of Coskewness and Cokurtosis. The estimations are based on PSID and CRSP data from 1997 to 2019.

Panel A: Probit Participation Regression Estimates

	Full sample			No college			College		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$S^+(\Delta y_i^*, \Delta y_i^*, \eta)$	0.071*	0.017	−0.014	0.048	−0.027	−0.084	0.150**	0.124*	0.112*
$S^-(\Delta y_i^*, \Delta y_i^*, \eta)$	0.317*	0.343*	0.316*	0.304	0.360	0.405*	0.347	0.349	0.149*
$S^+(\Delta y_i^*, \eta, \eta)$	−0.157**	−0.177**	−0.148**	−0.142**	−0.134**	−0.118*	−0.204**	−0.237**	−0.204**
$S^-(\Delta y_i^*, \eta, \eta)$	0.031	−0.097**	−0.088*	0.021	−0.049	−0.061	0.018	−0.124*	−0.133*
$K^+(\Delta y_i^*, \Delta y_i^*, \Delta y_i^*, \eta)$	0.076**	0.020	0.028	0.087**	0.038	0.020	0.036	0.035	0.072
$K^-(\Delta y_i^*, \Delta y_i^*, \Delta y_i^*, \eta)$	−0.052	0.040	0.089*	−0.081*	0.009	0.061	0.031	0.131**	0.159**
$K^+(\Delta y_i^*, \eta, \eta, \eta)$	−0.029**	−0.035**	−0.033**	−0.033**	−0.032*	−0.029	−0.031*	−0.037*	−0.035*
$K^-(\Delta y_i^*, \eta, \eta, \eta)$	0.015*	−0.009	−0.008	0.013	−0.001	−0.003	0.011	−0.011	−0.013
$(K(\Delta y_i^*, \Delta y_i^*, \eta, \eta) - 1)^+$	−0.007	−0.007	−0.008	−0.006	−0.028	−0.034*	0.020	0.020	0.026
$(K(\Delta y_i^*, \Delta y_i^*, \eta, \eta) - 1)^-$	−0.273**	−0.170**	−0.085	−0.366**	−0.262**	−0.146*	0.218**	−0.153	−0.077

Panel B: Tobit Participation Regression Estimates

	Full sample			No college			College		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$S^+(\Delta y_i^*, \Delta y_i^*, \eta)$	0.043**	0.011	0.005	0.038	−0.010	−0.039	0.059**	0.044*	0.039*
$S^-(\Delta y_i^*, \Delta y_i^*, \eta)$	0.197**	0.191**	0.139*	0.163	0.179	0.162	0.230**	0.218**	0.162
$S^+(\Delta y_i^*, \eta, \eta)$	−0.086**	−0.086**	−0.064**	−0.087**	−0.076*	−0.057	−0.087**	−0.091**	−0.078**
$S^-(\Delta y_i^*, \eta, \eta)$	0.021	−0.041*	−0.066**	0.019	−0.021	−0.040	0.010	−0.042	−0.060*
$K^+(\Delta y_i^*, \Delta y_i^*, \Delta y_i^*, \eta)$	0.049**	0.019	0.024	0.060**	0.030	0.014	0.025	0.024	0.025
$K^-(\Delta y_i^*, \Delta y_i^*, \Delta y_i^*, \eta)$	−0.037*	0.009	0.039*	−0.050*	0.008	0.052*	−0.006	0.026	0.045*
$K^+(\Delta y_i^*, \eta, \eta, \eta)$	−0.016**	−0.019**	−0.016**	−0.020**	−0.017	−0.014	−0.016**	−0.018**	−0.017*
$K^-(\Delta y_i^*, \eta, \eta, \eta)$	0.009*	−0.003	−0.007	0.010	0.002	−0.002	0.005	−0.004	−0.007
$(K(\Delta y_i^*, \Delta y_i^*, \eta, \eta) - 1)^+$	−0.003	−0.003	−0.003	−0.011	−0.013	−0.015	0.003	0.003	0.005
$(K(\Delta y_i^*, \Delta y_i^*, \eta, \eta) - 1)^-$	−0.154**	−0.090**	−0.059*	−0.222**	−0.150**	−0.092*	−0.114**	−0.078**	−0.041
N	7,954			4,794			3,160		

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Finally, instead of measuring higher-order dependence between each household's labor income and market returns individually, we aggregate labor income across households and estimate the between-squares correlation between aggregate labor income and market returns. This approach uses annual time series data with over 30 observations for both labor income and market returns to compute the statistics. Table G2 presents the results, revealing a significant level of between-squares correlation between the labor and stock markets.

Table G2: Between-squares Correlation of Aggregate Income Shock and Stock Return

This table reports estimates of between-squared correlations between aggregate labor income growth shocks ($\Delta^a y_t^*$) and stock return innovations (η_t), computed using PSID annual data (1970–1997) and CRSP data. Following Cocco et al. (2005), we estimate log labor income $y_{it} = f(t, \Omega_{it}) + \delta_{it}$, where $f(t, \Omega_{it})$ is a deterministic function of age t and household characteristics Ω_{it} (marital status, household composition). Detrended income y_{it}^* is obtained by subtracting the estimated component $\hat{f}(t, \Omega_{it})$. For labor income shocks $\Delta y_{it}^* \equiv y_{i,t+\Delta}^* - y_{it}^*$, the aggregate labor income shocks are defined as $\Delta^a y_t^* = \sum_i w_i \Delta y_{it}^*$, where w_i is weight. Stock return innovations η_t are derived from market returns minus the risk-free rate. Panel A presents full-sample results, while Panels B and C split households by college education status.

	Panel A: Full sample	*Panel B: No college*	*Panel C: College*
$\text{Corr}^{\text{sq}}(\Delta^a y^*, \eta)$ (equally weighted)	0.179	0.134	0.141
$\text{Corr}^{\text{sq}}(\Delta^a y^*, \eta)$ (income weighted)	0.165	0.114	0.147