Optimal Tariffs When Labor Income Taxes Are Distortionary

Narayana R. Kocherlakota*

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Abstract

It is a classic result in trade theory that it is socially optimal to set the tariff on a good equal to the inverse of the elasticity of its foreign supply. However, this result is based on the assumption that the government can use lump-sum taxes. The paper considers a simple open representative agent economy and characterizes *second-best* tariffs when the government's only non-tariff source of revenue is linear labor income taxation. If public spending needs are sufficiently large, and import demand is more (less) income-elastic than non-import demand, then the second-best tariff is lower (higher) than the standard optimal tariff.

*Simon Business School (University of Rochester) and NBER. I thank Yan Bai and especially Ivan Werning for helpful discussions.

1 Introduction

It is well-known from trade theory that a country can gain by using positive tariffs. The key to this proposition is the presumption that the country (taken as a whole) enjoys some amount of monopsony power in the market for its imports. In that instance, the reduction in demand induced by a positive tariff lowers the world price of the imported good, and that terms-of-trade effect increases the welfare of the country's residents. Consistent with this monopsonistic rationalization, the optimal tariff can be shown to be equal to the inverse of the foreign supply elasticity[1] (which I will term the *standard optimal tariff*).

This logic abstracts, though, from the distortions associated with other forms of taxation. This paper asks the following question: Suppose the revenue raised by the standard optimal tariff is insufficient to finance the country's public goods needs, and that the government's sole non-tariff source of revenue takes the form of distorting labor income taxes. How does this consideration affect the size of the socially optimal tariff?

To address this question, I use a simple representative agent open economy. As discussed above, the model has a terms-of-trade effect: the quantity of the country's imports affects their world price. The paper characterizes *second-best* tariffs when the government has a required amount of public spending, but is restricted to raising revenue using tariffs or linear labor income taxes. I find that the magnitude of the second-best tariff depends critically on the relative income elasticities of imports and non-imports. More specifically, assuming that government spending needs exceed what can be raised using the standard optimal tariff alone, I obtain the following three results.

- If import demand and non-import demand are equally income-elastic, the second-best tariff equals the standard optimal tariff.
- If import demand is less income-elastic than non-import demand, then the second-best tariff is larger than the standard optimal tariff.

[1] See, among many others, Donaldson (2018).

- If import demand is more income-elastic than non-import demand, then the second-best tariff is smaller than the standard optimal tariff.

The available evidence suggests that the last finding may be more relevant for the US than the other two.[2] I show through simulations that even when the income elasticity of import demand is only slightly larger than unity, the second-best tariff can be significantly smaller than the standard optimal tariff.[3]

To understand the intuition behind the results, it is best to start with the case in which agents' utility u over non-imports (c_1) and imports (c_2) can be written as:

$$u(c_1, c_2) = W(U(c_1, c_2))$$

where W is strictly increasing and concave and U is real-valued and homogeneous of degree one. This is economically equivalent to a situation in which the argument of W is a single (composite) commodity produced by a constant-returns-to-scale technology U with intermediate inputs (c_1, c_2). The Diamond-Mirrlees (1971) and Atkinson-Stiglitz (1972) theorems imply it is always suboptimal to distort the margin between these intermediate inputs, as it is a form of double taxation. The only change here is that, with the terms-of-trade effect, a non-distorting tariff is in fact positive and equal to the inverse of the elasticity of foreign supply.

Now suppose that agents' utility functions are such that imports are more income-elastic than non-imports. With these kinds of preferences, a distorting positive labor income tax, and its associated decline in overall consumption, leads agents to tilt their consumption bundle toward non-imports. The results in the paper show that it is socially optimal to mitigate this knock-on between-good distortion by subsidizing imports. With the terms-of-trade effect,

[2]Houthakker and Magee (1969) is a a classic reference in this regard. Fieler (2011) uses a structural estimation approach and finds that rich countries (like the US) import goods with higher income elasticities.

[3]Throughout, I assume that lump-sum transfers are available to the government. Without this assumption, it is optimal for a government with zero or low public spending needs to use tariff revenue to *subsidize* labor income. This labor subsidy increases overall consumption. Hence, the optimal tariff results in this case (without lump-sum transfers) are reversed from the rest of the paper: the second-best tariff is lower (higher) than the standard optimal tariff if imports are less (more) income-elastic than non-imports.

this subsidy translates into reducing tariffs below the inverse of foreign supply elasticity. The argument runs in reverse when imports are less income-elastic than non-imports.[4]

This paper combines core ideas in the optimal tariff literature (which, as Humphrey (1987) documents, dates back nearly two centuries) and the Ramsey taxation literature (stemming from the seminal paper of Atkinson and Stiglitz (1972)). Formally, I utilize the primal approach to optimal taxation that was originally applied in macroeconomics by Lucas and Stokey (1983). (See Chari and Kehoe (1999) for a thorough exposition.) Costinot, Donaldson, Vogel, and Werning (2015) also use the primal approach to study optimal tariffs, but they focus instead on the production allocation role of tariffs and abstract from distorting labor taxes.

As I do, Boadway, Maital, and Prachowny (1973) (BMP) consider the problem of optimal tariff design in a representative agent open economy with linear labor income taxes. Contrary to my results, they find that the second-best tariff is equal to the standard optimal tariff, regardless of the nature of the domestic demand for imports and non-imports. However, this conclusion hinges on two strong assumptions:

- the tariff-imposing country also produces the imported goods that are subject to the tariff
- the government can impose a distinct tax/subsidy on the consumption of that good (which is the same whether it is produced at home or abroad)

Under these assumptions, the optimal tariff is wholly determined by its effect on the domestic allocation of inputs between the production of imported goods and the production of other goods. The tax/subsidy is then used to address the potential consumption distortions stemming from income effects that are stressed in this paper. In contrast, I assume (as is the more typical modern approach) that consumers view imports as being at least somewhat

[4]I characterize optimal tariffs in terms of the sign of the difference in the first derivatives of aggregate import and aggregate non-import Engel curves. In a recent working paper, Costinot and Werning (2025) describe the positive effect of tariffs on deficits in terms of the difference in the second derivatives of these Engel curves.

imperfectly substitutable for domestically produced goods. Under this assumption, there is no domestic production of goods subject to tariffs, and the two instruments in BMP - a tariff on an imported good and a consumption tax/subsidy on that good - become equivalent public finance instruments. In this paper, I abstract from the consumption tax/subsidy as it is not typically used.

The baseline model is a deliberately stark two-good representative agent setup. I describe how the results can be extended to multi-import, multi-period models, and heterogeneous agent models in Section 5. The proofs of Propositions 2-4 and of Corollary 1 are in Appendix A. The technical details of the multi-period analysis are in Appendix B.

2 Model

Consider an open economy Home that produces only consumption good 1 (also called non-imports). It trades good 1 for consumption good 2 (also called imports) which is produced only in other countries. Home uses labor to produce non-imports with a linear technology: n units of labor generates n units of good 1, for any $n \geq 0$. Agents in Home are identical and have a utility function:

$$u(c_1, c_2) - v(n),$$

over the two kinds of consumption goods and over labor. Here, u and v are both strictly increasing, and the functions u and $-v$ are both strictly concave. For the main result, I will also require that u satisfies *decreasing marginal rates of substitution*, so that:

$$\frac{\partial \frac{u_2}{u_1}}{\partial c_2} < 0 \text{ and } \frac{\partial \frac{u_1}{u_2}}{\partial c_1} < 0.$$

where subscripts represent the relevant partial derivatives. This property is satisfied by any utility function with positive cross-partials and, more generally, by the utility functions

typically used in international economics or public finance.[5]

The world price of imports in terms[6] of good 1 depends on Home's consumption of good 2 through the strictly increasing function $P(c_2)$. This formulation captures a standard terms-of-trade effect: lower imports by Home reduces the demand for good 2 in the world market and hence its world price. Importantly, in making their import decisions, agents in Home abstract from their price impact. I assume that revenue $P(c_2)c_2$ is a convex function of quantity c_2.

Home is required to spend G units of good 1 on government purchases. The public finance question is how best to use tariffs and labor income taxes to finance those purchases. In addressing this question, I assume that the government's objective is to maximize the representative agent's utility function.

3 Optimal Tariffs with Lump-Sum Taxation

This section assumes that all taxes and transfers, other than tariffs, are lump-sum. For completeness (and as a proof-of concept for the model), it derives the well-known result that it is optimal to have a positive tariff equal to the inverse of the foreign supply elasticity, even though the government can raise revenue without distortions.

Let τ be the tariff imposed on imports (good 2). Then, an equilibrium in Home is defined by (c_1, c_2, n) that satisfy:

$$u_1(c_1, c_2)P(c_2)(1+\tau) = u_2(c_1, c_2) \tag{1}$$

$$u_1(c_1, c_2) = v'(n) \tag{2}$$

$$c_1 + P(c_2)c_2 + G = n \tag{3}$$

[5]However, unlike the assumption of diminishing marginal rates of substitution *along an indifference curve*, it is not satisfied by all concave functions. For example, consider the function $f(x, y) = -x^2 - y^2 - 1.5xy + 5x + 5y$. This function is strictly concave for all (x, y) and is strictly increasing if $(x + 3y/4) < 5/2$ and $(3x/4 + y) < 5/2$. But $f_2/f_1 = (5 - 2y - 1.5x)/(5 - 2x - 1.5y)$ is increasing in y at the point $(11/7, 1)$.

[6]Throughout, I treat good 1 as the numeraire.

The first restriction is the individual first-order condition that trades off purchases of goods 1 and 2. It captures the exogeneity of all prices from the point of view of individual agents, as it contains no derivative of the price function P. The second restriction is individual optimality in the labor market (given that there are no distorting labor taxes). The last restriction is the resource constraint. Agents produce good 1 using n units of labor (the right-hand side). Those n units of good 1 are used for consumption (c_1), to buy good 2 ($P(c_2)c_2$) and to resource government purchases (G). Note that a per-capita lump-sum transfer/tax M is determined residually so as to satisfy the government budget constraint:

$$M = \tau P(c_2)c_2 - G.$$

To solve for the optimal tariff, we use the primal approach. It is particularly simple in this setting with lump-sum taxes. Consider any (c_1, c_2, n) that satisfies the resource constraint (3) and labor optimality (2). Then, there exists a tariff

$$\tau(c_1, c_2) = \frac{u_2(c_1, c_2)}{u_1(c_1, c_2)P(c_2)} - 1$$

that satisfies (1) and so induces (c_1, c_2, n) as an equilibrium. Hence, to attack the optimal tariff problem, we can consider a government that solves:

$$\max_{c_1, c_2, n} u(c_1, c_2) - v(n)$$

$$s.t.\ u_1(c_1, c_2) = v'(n)$$

$$c_1 + P(c_2)c_2 + G = n$$

Note that, in solving its policy problem, the government incorporates the dependence of the world price of good 2 on imports c_2.

Intuitively, the first constraint (labor supply optimality) should be redundant (as there are no frictions in the labor market). Accordingly, consider the standard *first-best problem*

(with a larger set of (c_1, c_2, n) in the constraint set) as:

$$\max_{c_1,c_2,n} u(c_1, c_2) - v(n) \tag{4}$$
$$\text{s.t. } c_1 + P(c_2)c_2 + G = n$$

The solution $(c_1^{FB}, c_2^{FB}, n^{FB})$ to the first-best problem (4) depends on G, and satisfies the three equations:

$$\begin{aligned} u_1(c_1^{FB}, c_2^{FB}) &= v'(n^{FB}) \\ u_2(c_1^{FB}, c_2^{FB}) &= (P'(c_2^{FB})c_2^{FB} + P(c_2^{FB}))u_1(c_1^{FB}, c_2^{FB}) \\ c_1^{FB} + P(c_2^{FB})c_2^{FB} + G &= n^{FB}. \end{aligned}$$

As we intuited, this solution to the first-best problem satisfies the constraints of the original (unrelaxed) problem (since $u_1(c_1^{FB}, c_2^{FB}) = v'(n^{FB})$). The optimal allocation can be *implemented* as an equilibrium if the government sets the tariff equal to:

$$\tau^{FB} = \frac{P'(c_2^{FB})c_2^{FB}}{P(c_2^{FB})} \tag{5}$$

Equation (5) is the standard optimal tariff result. To see this more clearly, define $\hat{C}_2$ to be the inverse function of P. Then, we can rewrite (5) as:

$$\tau^{FB} = \frac{1}{\frac{dln(\hat{C}_2)}{dln(P)}},$$

Then, the first-best tariff τ_{FB} is equal to the inverse of the elasticity of foreign supply.

We turn in the next section to the more realistic setting in which labor income taxes are not lump-sum. Nonetheless, the first-best tariff will remain an important benchmark in our analysis. Accordingly, we define the function τ^{STD} to be the *standard optimal tariff*

associated with import level c_2.

$$\tau^{STD}(c_2) \equiv \frac{P'(c_2)c_2}{P(c_2)}.$$

If P has a constant elasticity (power) form, then τ^{STD} is independent of the import level c_2.

4 Optimal Tariffs With Linear Labor Income Taxes

This section contains the main results about optimal tariffs when the government is restricted to use a mixture of tariffs and linear labor income taxes. I assume that transfers (as opposed to taxes) can be lump-sum.

4.1 Equilibrium and Implementability Constraint

Suppose the government sets a tariff rate τ and a non-negative labor income tax rate Φ. Then, equilibrium allocations (c_1, c_2, n) satisfy the restrictions:

$$u_1(c_1, c_2)(1 - \Phi) = v'(n) \tag{6}$$

$$u_1(c_1, c_2)(1 + \tau)P(c_2) = u_2(c_1, c_2) \tag{7}$$

$$c_1 + P(c_2)c_2 + G = n \tag{8}$$

A tax Φ and tariff τ are budget-feasible if:

$$\Phi n + \tau P(c_2)c_2 \geq G \tag{9}$$

so that the government raises sufficient revenue to cover its funding needs. As mentioned above, the government hands out extra resources lump-sum if its budget constraint is slack.

Our aim is to use the primal approach, in which the government chooses directly among all budget-feasible equilibrium allocations. To that end, it is useful to substitute the budget

constraint (9) into the resource constraint (8) to obtain:

$$c_1 + P(c_2)c_2 + \Phi n + \tau P(c_2)c_2 \geq n$$

or:

$$c_1 + (1+\tau)P(c_2)c_2 \geq n(1-\Phi).$$

We can then eliminate the tax and tariff using (6) and (7) to arrive at an *implementability* constraint:

$$c_1 + \frac{u_2(c_1,c_2)c_2}{u_1(c_1,c_2)} \geq n\frac{v'(n)}{u_1(c_1,c_2)}$$

or equivalently:

$$u_1(c_1,c_2)c_1 + u_2(c_1,c_2)c_2 - nv'(n) \geq 0$$

Proposition 1 summarizes the above discussion and adds a straightforward converse characterization.

Proposition 1. *Suppose that (c_1, c_2, n) is an equilibrium for some (τ, Φ) such that the government budget constraint is satisfied given (τ, Φ, n, c_2). Then (c_1, c_2, n) satisfies the implementability and resource constraints:*

$$u_1(c_1,c_2)c_1 + u_2(c_1,c_2)c_2 - nv'(n) \geq 0$$
$$c_1 + c_2P(c_2) + G = n.$$

Conversely, suppose (c_1, c_2, n)satisfies the above (implementability and resource) constraints. Then, (c_1, c_2, n) is an equilibrium that satisfies the government's budget constraint if the tariff and tax are given by:

$$\tau = \frac{u_2(c_1,c_2)}{u_1(c_1,c_2)P(c_2)} - 1$$
$$\Phi = 1 - v'(n)/u_1(c_1,c_2)$$

In light of Proposition 1, we can write the *second-best policy problem* for the government as if it is choosing directly among the allocations that satisfy the implementability and resource constraints.

$$\begin{aligned} &\max_{c_1,c_2,n} u(c_1,c_2) - v(n) \\ &\text{s.t. } c_1 + P(c_2)c_2 + G = n \\ &u_1(c_1,c_2)c_1 + u_2(c_1,c_2)c_2 - nv'(n) \geq 0. \end{aligned} \tag{10}$$

Given a solution (c_1^*, c_2^*, n^*) to this policy problem, the corresponding tax and tariff are given by:

$$\begin{aligned} \Phi^* &= \frac{v'(n^*)}{u_1(c_1^*, c_2^*)} - 1 \\ \tau^* &= \frac{u_2(c_1^*, c_2^*)}{u_1(c_1^*, c_2^*)P(c_2^*)} - 1 \end{aligned}$$

4.2 Small G

In many (most) optimal tax problems, the implementability constraint ends up being binding, and so we can write it as an equality. However, the following proposition shows that, because of the terms-of-trade-effect, the implementability constraint is non-binding if G is low.

Proposition 2. *Given $G \geq 0$, suppose $(c_1^{FB}, c_2^{FB}, n^{FB})$ solves the first-best policy problem (4):*

$$\begin{aligned} &\max_{(c_1,c_2,n)} u(c_1,c_2) - v(n) \\ &s.t. \ c_1 + P(c_2)c_2 + G = n. \end{aligned}$$

If the implied tariff revenue is sufficiently large to fund the required government purchases:

$$\tau^{STD}(c_2^{FB})P(c_2^{FB})c_2^{FB} \geq G$$

then $(c_1^{FB}, c_2^{FB}, n^{FB})$ *also solves the second-best policy problem. The second-best tariff is then the standard optimal tariff* $\tau^{STD}(c_2^{FB})$ *and the second-best labor tax is zero.*

The condition in Proposition 2 is satisfied if G is zero or (by continuity) sufficiently close to zero.

4.3 Second-Best

This subsection presents the main result. It treats the second-best policy problem under the condition that Home finds tariff revenue (when using the standard optimal tariff) to be insufficient to cover its government spending needs. In this case, the implementability condition binds.

The characterization depends on whether imports are more or less income-elastic than non-imports. These income elasticities are defined in the usual way. Specifically, let $(c_1^*(p, W), c_2^*(p, W))$ be the representative consumer's demand for non-imports and imports, given a (domestic) price p for imports and consumption expenditure W :

$$(c_1^*(p, W), c_2^*(p, W)) = \arg\max_{\hat{c}_1, \hat{c}_2} u(c_1, c_2)$$
$$\text{s.t. } c_1 + pc_2 = W$$

Then the income elasticities of non-imports and imports are defined as:

$$\frac{dc_1^*}{dW}\frac{W}{c_1^*} \text{ and } \frac{dc_2^*}{dW}\frac{W}{c_2^*}.$$

respectively. These are local measures that will (in general) depend on the price p and expenditure W. Given consumptions (c_1, c_2), I denote the income elasticity of imports by $\eta_{imp}(c_1, c_2)$ and the income elasticity of non-imports by $\eta_{dom}(c_1, c_2)$. Here, the price p and expenditure W needed to compute the elasticities depend on (c_1, c_2) through the consumer's

optimality conditions:

$$p = \frac{u_2(c_1, c_2)}{u_1(c_1, c_2)}$$
$$W = c_1 + \frac{u_2(c_1, c_2)}{u_1(c_1, c_2)} c_2$$

With this notation in hand, we are in a position to state the main result.

Proposition 3. *Suppose that the utility function u has decreasing marginal rates of substitution. Let $(c_1^{SB}, c_2^{SB}, n^{SB})$ solve the second-best policy problem (10) and suppose that the standard optimal tariff does not generate sufficient revenue to fund Home's government purchases (that is, $\tau^{STD}(c_2^{SB})P(c_2^{SB})c_2^{SB} < G$). Define the implied second-best tariff as:*

$$\tau^{SB} = \frac{u_2(c_1^{SB}, c_2^{SB})}{u_1(c_1^{SB}, c_2^{SB})P(c_2^{SB})} - 1$$

Then:

$$\eta_{imp}(c_1^{SB}, c_2^{SB}) = \eta_{dom}(c_1^{SB}, c_2^{SB}) \Rightarrow \tau^{SB} = \tau^{STD}(c_2^{SB})$$
$$\eta_{imp}(c_1^{SB}, c_2^{SB}) > \eta_{dom}(c_1^{SB}, c_2^{SB}) \Rightarrow \tau^{SB} < \tau^{STD}(c_2^{SB})$$
$$\eta_{imp}(c_1^{SB}, c_2^{SB}) < \eta_{dom}(c_1^{SB}, c_2^{SB}) \Rightarrow \tau^{SB} > \tau^{STD}(c_2^{SB})$$

The thrust of Proposition 3 is that whether the second-best tariff is larger or smaller than the standard optimal tariff depends on the local income elasticity of imports. However, the magnitude of the difference can potentially depend on all aspects of the model.

As discussed in the introduction, the intuition behind Proposition 3 is simple. Suppose that agents' utility functions are such that imports are more income-elastic than non-imports. With these kinds of preferences, a distorting positive labor income tax, and its associated decline in overall consumption, leads agents to tilt their consumption bundle toward non-imports. The crux of the analysis is that it is socially optimal to mitigate this knock-on

between-good distortion by subsidizing imports. With the terms-of-trade effect, this subsidy translates into reducing tariffs below the inverse of foreign supply elasticity. The argument runs in reverse when imports are less income-elastic than non-imports

The following corollary is a simple but important application of Proposition 3. It examines the case in which u is homothetic, which implies that the income elasticities of both import demand and non-import demand are globally equal to one.

Corollary 1. *Suppose u is homothetic and $(c_1^{SB}, c_2^{SB}, n^{SB})$ solves the second-best policy problem. Then the implied second-best policy problem is equal to the standard optimal tariff, given c_2^{SB} :*

$$\tau^{SB} = \tau^{STD}(c_2^{SB}).$$

The corollary underscores that the difference between the second-best and first-best tariffs is determined by relative income elasticities, not relative elasticities of substitution. For example, consider the utility function:

$$w(c_1, c_2) = c_1^{1/4} c_2^{1/4} + c_2^{1/2}.$$

With this utility function, the two goods (generically) differ in terms of their elasticities of substitution with labor. But the utility function is homothetic, and so Corollary 1 implies that there is no difference between the second-best tariff and the standard optimal tariff.

4.4 Numerics

Consider the following simple parameterization. Suppose $u(c_1, c_2) = 0.75ln(c_1)+0.25c_2^{(1-1/\psi)}/(1-1/\psi)$, where $\psi > 0$, and $v(n) = n^2/2$. Suppose too that $P(c_2) = c_2^{1/4}$, so that the elasticity of foreign supply is 4 and $\tau^{STD} = 0.25$. For this additively separable utility function, it is readily seen that the income elasticity of imports (good 2) is larger (smaller) than 1 if and only if ψ is. However, as long as the income elasticity differs from one, its exact magnitude depends not just on ψ, but on the full specification of the economic model.

The solution $(c_1^{SB}, c_2^{SB}, n^{SB})$ to the second-best policy problem is defined by the conditions:

$$\begin{aligned}
0.75/c_1^{SB} - \lambda &= 0 \\
0.25/(c_2^{SB})^{1/\psi} - 1.25\lambda(c_2^{SB})^{0.25} + 0.25\mu\psi(c_2^{SB})^{-1/\psi} &= 0 \\
n^{SB} - \lambda + 2\mu n^{SB} &= 0 \\
c_1^{SB} + (c_2^{SB})^{1.25} + G &= n^{SB} \\
0.75 + 0.25(c_2^{SB})^{1-1/\psi} &= (n^{SB})^2.
\end{aligned}$$

where λ is the multiplier on the resource constraint and μ is the multiplier on the implementability constraint. I solved these conditions numerically using Julia for various values of ψ and G. The implied optimal tariff and labor income tax are given by:

$$\begin{aligned}
\tau^{SB} &= c_1^{SB}(c_2^{SB})^{-0.25+1/\psi}/3 - 1 \\
\Phi &= 1 - 4c_1^{SB}n^{SB}/3
\end{aligned}$$

In Table 1, I fix $G = 0.2$ and vary ψ over the set $\{1/3, 0.5, 2/3, 1.000001, 3/2, 2, 3\}$. The table reports results for the second-best tariff, the import income elasticity, and the share of government purchases in the economy. The table demonstrates that the second-best tariff is decreasing in ψ, while the import income elasticity is increasing in ψ. As well, it shows that the second-best tariff can be materially higher or (especially) lower than the standard optimal tariff (0.25).

Table 1: Effect of ψ on Second-Best Tariffs

ψ	τ	η_{imp}	G/n
1/3	0.31	0.52	0.16
0.5	0.31	0.66	0.18
2/3	0.29	0.78	0.18
1	0.25	1	0.2
3/2	0.20	1.3	0.21
2	0.17	1.6	0.22
3	0.14	2	0.23

In Table 2, I fix $\psi = 2$ and vary the level of government purchases. Since $\psi > 1$, the income elasticity of import demand is larger than 1. As noted in the introduction, this "imports are luxuries" case is typically viewed to be the more empirically relevant case. However, the magnitude of the import income elasticity is not determined solely by the preference parameter ψ. The table shows that the second-best tariff can be much lower than the standard optimal tariff (still 0.25) and is in fact negative when the share of government purchases is sufficiently large.

Table 2: Effect of G on Second-Best Tariffs

G	τ	η_{imp}	G/n
0	0.25	1.8	0
0.05	0.24	1.8	0.05
0.1	0.22	1.7	0.11
0.2	0.17	1.6	0.22
0.3	0.12	1.4	0.33
0.4	0.05	1.3	0.44
0.6	-0.17	1.2	0.67

5 Extensions

This section discusses extensions to a multi-import setting, to a multi-period model, and to a model with heterogeneity.

5.1 Multiple Import Goods

This subsection considers an extension of the baseline model with $K > 1$ imported goods. There is a terms-of-trade effect for each import: there exists a collection of increasing functions $\{P_k\}_{k=2}^{K+1}$ such that P_k describes the dependence of the world price of good k (in terms of good 1) on the import quantity c_k. The standard optimal tariff for import k associated with quantity c_k is defined as before:

$$\tau_k^{STD}(c_k) = \frac{P_k'(c_k)c_k}{P_k(c_k)}.$$

Agents' preferences over consumption goods and labor are defined as:

$$\sum_{k=1}^{K+1} f_k(c_k) - v(n)$$

where $f_k', -f_k'' > 0$ for all k. The (strong assumption of) additive separability between consumption goods is at least useful (and I suspect necessary) to prove results about import-specific tariffs. If Home's consumptions are given by the vector $\overrightarrow{c}$, denote the income elasticity of good $k \in \{2, ..., K+1\}$ by $\eta_k(\overrightarrow{c})$ and the income elasticity of the non-import good 1 by $\eta_{dom}(\overrightarrow{c})$.

It can be shown as in Section 3 that the set of budget-feasible equilibrium allocations

consists of $(\overrightarrow{c}, n)$ that satisfies a resource constraint and an implementability constraint:

$$c_1 + \sum_{k=2}^{K+1} P_k(c_k)c_k + G = n \tag{11}$$

$$\sum_{k=1}^{K+1} f_k'(c_k)c_k \geq v'(n)n. \tag{12}$$

where G is the required level of government purchases. The second-best policy problem in this setting is that the government maximizes

$$\sum_{k=1}^{K+1} f_k(c_k) - v(n)$$

subject to the two constraints (11)-(12). The first-best policy problem is that the government maximizes the same objective subject to only the resource constraint (11).

Intuitively, the second-best tariffs should be lower for imported goods with higher income elasticities and for imports with higher foreign supply elasticities. Proposition 4 uses the additive separability of the preferences over consumption goods to prove this result.

Proposition 4. *Suppose* $(\overrightarrow{c}^{SB}, n^{SB})$ *solves the second-best policy problem and that* $(\overrightarrow{c}^{FB}, n^{FB})$ *solves the first-best policy problem. Assume that the total revenue from the standard optimal tariffs is insufficient to fund the desired level of government purchases* G :

$$\sum_{n=2}^{K+1} \tau_k^{STD}(c_k^{FB})P(c_k^{FB})c_k^{FB} < G.$$

Then:

$$\eta_k(\overrightarrow{c}^{SB}) = \eta_{dom}(\overrightarrow{c}^{SB}) \Rightarrow \tau_k^{SB} = \tau_k^{STD}(c_k^{SB})$$

$$\eta_k(\overrightarrow{c}^{SB}) > \eta_{dom}(\overrightarrow{c}^{SB}) \Rightarrow \tau_k^{SB} < \tau_k^{STD}(c_k^{SB})$$

$$\eta_k(\overrightarrow{c}^{SB}) < \eta_{dom}(\overrightarrow{c}^{SB}) \Rightarrow \tau_k^{SB} > \tau_k^{STD}(c_k^{SB}).$$

5.2 Dynamic Model

The analysis in this paper is based on a static model, in which trade is necessarily balanced within the single period. Appendix B presents an extension to a multiperiod model in which Home may run a trade surplus or deficit at any date. The extended model assumes that:

- the residents of Home can borrow or lend from abroad at an exogenous world interest rate r
- the government of Home can borrow or lend from the residents of Home at an endogenously determined interest rate. The government of Home is not able to borrow and lend from abroad.[7]
- the government of Home can impose a common tax rate on Home residents' interest income from their holdings of international debt and Home government debt.

All intertemporal exchanges are real and denominated in terms of good 1 (non-imports).

Under these modeling assumptions, Appendix B shows how to derive an intertemporal implementability constraint similar to that in the baseline model. With that in hand, it is straightforward to prove a period-by-period analog of Proposition 3.

5.3 Ramsey versus Mirrlees

This paper uses the widely-used Ramsey approach to optimal taxation, in which labor income taxes are restricted to be linear, in a representative agent economy. In the alternative (and also widely-used) approach to optimal taxation based on Mirrlees (1971), agents are assumed to be heterogeneous and the government is allowed to use arbitrary nonlinear tax schedules. In this context, the results of Atkinson and Stiglitz (1976) imply that if the preferences between consumption and labor are additively separable (as we have assumed), it is always

[7]This modeling of intertemporal trade is somewhat unusual. To derive the implementability constraint, the government's discount rate must be the same as the *after-tax* interest rate facing Home residents. But in this world in which the supply of external financing is infinitely elastic, the government cannot levy a tax on foreign lenders.

suboptimal to distort the margin between consumption goods. This logic implies that the second-best tariff is equal to the standard optimal tariff, regardless of any difference in income elasticities between imports and non-imports.

What gives rise to the difference in results? Heterogeneity is not essential. Rather, the key is that the Ramsey approach imposes a functional form restriction on tax schedules which is not present in the Mirrleesisan analyses. Of course, neither Ramsey (linear tax schedules) nor Mirrlees (any tax schedule) seems realistic. Accordingly, there is a large body of work on optimal taxation that imposes a range of functional form restrictions that are weaker than linearity.[8] How is the optimal tariff affected by non-homotheticities in agents' preferences over imports and non-imports given these weaker restrictions on income tax schedules? This problem is a fruitful basis for future research.

6 Conclusion

It is well-known in trade theory that it is optimal for a country to impose a tariff on imported goods equal to the inverse of the elasticity of foreign supply (what I term the "standard optimal tariff"). But the logic for this result presumes that the government has access to lump-sum taxation. This paper instead considers a country that seeks to solve the *second-best* problem of financing public good expenditures in a socially optimal fashion using a mixture of tariffs and linear labor income taxes. It finds that, if public spending needs are sufficiently high, the *second-best* tariff is *lower* than the standard optimal tariff if imports are more income-elastic than non-imports. In a numerical exercise, I show that the difference can be substantial.

The paper focuses on a single country and abstracts from the important issue of retaliation. It would be useful to extend the analysis to allow for country interactions in policy-setting via a simultaneous move or possibly a Stackelberg formulation. The results in

[8]For example, Werning (2007) assumes that the government must use an affine tax schedule. Heathcote, Storesletten, and Violante (2017) assume that the government must use an after-tax schedule that takes a power form.

this paper suggest that the outcomes of such an analysis will depend in nontrivial ways on the different countries' demands for imports and non-imports.[9]

[9]Chari, Nicolini, and Teles (2023) consider the problem of optimal global co-ordination of tax and trade policy, given a range of supporting cross-country transfers. They show that any Pareto efficient outcome is consistent with zero tariffs.

Appendix A

This appendix includes the proofs of Proposition 2, Proposition 3, Proposition 4, and Corollary 1.

Proof of Proposition 2

The first-best policy problem is a relaxed version of the second-best policy problem. Hence, a solution to the first-best policy policy problem necessarily solves the second-best policy problem if the solution satisfies the implementability constraint. Suppose:

$$\tau^{STD}(c_2^{FB})P(c_2^{FB})c_2^{FB} \geq G.$$

To check the implementability constraint in the second-best policy problem, we can use the following logic:

$$\begin{aligned}
&u_1(c_1^{FB}, c_2^{FB})c_1^{FB} + u_2(c_1^{FB}, c_2^{FB})c_2^{FB} - v'(n^{FB})n^{FB} \\
&= u_1(c_1^{FB}, c_2^{FB})(c_1^{FB} + \frac{u_2(c_1^{FB}, c_2^{FB})}{u_1(c_1^{FB}, c_2^{FB})}c_2^{FB} - \frac{v'(n^{FB})}{u_1(c_1^{FB}, c_2^{FB})}n^{FB}) \\
&= u_1(c_1^{FB}, c_2^{FB})(c_1^{FB} + P(c_2^{FB})(1 + \tau^{STD}(c_2^{FB}))c_2^{FB} - n^{FB}) \\
&= u_1(c_1^{FB}, c_2^{FB})(c_1^{FB} + P(c_2^{FB})c_2^{FB} + P(c_2^{FB})\tau^{STD}(c_2^{FB})c_2^{FB} - n^{FB}) \\
&\geq u_1(c_1^{FB}, c_2^{FB})(c_1^{FB} + P(c_2^{FB})c_2^{FB} + G - n^{FB}) \\
&= 0.
\end{aligned}$$

The implied optimal tariff is easily seen to be $\tau^{STD}(c_2^{FB})$ and the optimal labor income tax is zero.

Proof of Proposition 3

This subsection proves Proposition 3. It relies on the following lemma.

Lemma 1. *Suppose u exhibits decreasing marginal rates of substitution. Then:*

$$\eta_{imp}(c_1, c_2) > 1 > \eta_{dom}(c_1, c_2) \iff \frac{u_{11}c_1 + u_{12}c_2}{u_1} < \frac{u_{21}c_1 + u_{22}c_2}{u_2}$$
$$\eta_{imp}(c_1, c_2) < 1 < \eta_{dom}(c_1, c_2) \iff \frac{u_{21}c_1 + u_{22}c_2}{u_2} < \frac{u_{11}c_1 + u_{12}c_2}{u_1}$$
$$\eta_{imp}(c_1, c_2) = 1 = \eta_{dom}(c_1, c_2) \iff \frac{u_{21}c_1 + u_{22}c_2}{u_2} = \frac{u_{11}c_1 + u_{12}c_2}{u_1}.$$

Proof. The first order conditions to the consumer's optimization problem given wealth W and (domestic) import price p are:

$$u_1 = \varsigma$$
$$u_2 = p\varsigma$$
$$c_1 + pc_2 = W.$$

Differentiating, we obtain:

$$\frac{u_{11}dc_1 + u_{12}dc_2}{u_1} = \frac{d\varsigma}{u_1}$$
$$\frac{u_{21}dc_1 + u_{22}dc_2}{u_2} = \frac{d\varsigma}{u_1}$$
$$dc_1 + pdc_2 = dW.$$

We can convert to elasticities. Let $\eta_{dom} = \frac{dc_1}{c_1}\frac{W}{dW}$ and $\eta_{imp} = \frac{dc_2}{c_2}\frac{W}{dW}$. Then:

$$\frac{u_{11}c_1\eta_{dom} + u_{12}c_2\eta_{imp}}{u_1} = \frac{d\varsigma}{u_1}\frac{W}{dW}$$
$$\frac{u_{21}c_1\eta_{dom} + u_{22}c_2\eta_{imp}}{u_2} = \frac{d\varsigma}{u_1}\frac{W}{dW}$$
$$\frac{c_1}{W}\eta_{dom} + \frac{pc_2}{W}\eta_{imp} = 1.$$

It follows that:

$$\frac{u_{11}c_1\eta_{dom} + u_{12}c_2\eta_{imp}}{u_1} = \frac{u_{21}c_1\eta_{dom} + u_{22}c_2\eta_{imp}}{u_2} \tag{13}$$

$$\eta_{imp} < (=) > 1 \;\Rightarrow\; \eta_{dom} > (=) < 1 \tag{14}$$

The equation (13) implies that:

$$(\frac{u_{21}}{u_2} - \frac{u_{11}}{u_1})c_1\eta_{dom} = (\frac{u_{12}}{u_1} - \frac{u_{22}}{u_2})c_2\eta_{imp} \tag{15}$$

Since marginal rates of substitution are decreasing:

$$0 > \frac{\partial \frac{u_i}{u_j}}{\partial x_i} = \frac{u_{ii}u_j - u_{ji}u_i}{u_j^2}, i \neq j.$$

Hence:

$$u_{21}/u_2 - u_{11}/u_1 > 0$$

$$u_{12}/u_1 - u_{22}/u_2 > 0.. \tag{16}$$

Now suppose $\eta_{imp} > 1$. Then $\eta_{dom} < 1$, and (15) implies:

$$(\frac{u_{21}}{u_2} - \frac{u_{11}}{u_1})c_1 > (\frac{u_{12}}{u_1} - \frac{u_{22}}{u_2})c_2$$

The result follows: :

$$\frac{u_{11}c_1 + u_{12}c_2}{u_1} < \frac{u_{21}c_1 + u_{22}c_2}{u_2}.$$

The other cases ($\eta_{imp} < 1 < \eta_{dom}$ and $\eta_{imp} = 1 = \eta_{dom}$) can be established similarly. □

The remainder of the subsection is the proof of Proposition 3.

The solution to (10) satisfies the first order conditions:

$$u_1(c_1^{SB}, c_2^{SB}) - \lambda \tag{17}$$

$$= -\mu u_{11}(c_1^{SB}, c_2^{SB})c_1^{SB} - \mu u_{21}(c_1^{SB}, c_2^{SB})c_2^{SB} - \mu u_1(c_1^{SB}, c_2^{SB}) \tag{18}$$

$$u_2(c_1^{SB}, c_2^{SB}) - \lambda P'(c_2^{SB})c_2^{SB} - \lambda P(c_2^{SB}) \tag{19}$$

$$= -\mu u_{12}(c_1^{SB}, c_2^{SB})c_1^{SB} - \mu u_{22}(c_1^{SB}, c_2^{SB})c_2^{SB} - \mu u_2(c_1^{SB}, c_2^{SB}) \tag{20}$$

where λ is the multiplier on the resource constraint and μ is the multiplier on the implementability constraint. It is convenient to rewrite as:

$$1 - \frac{\lambda}{u_1(c_1^{SB}, c_2^{SB})} + \mu\left(\frac{u_{11}(c_1^{SB}, c_2^{SB})c_1^{SB} + u_{21}(c_1^{SB}, c_2^{SB})c_2^{SB}}{u_1(c_1^{SB}, c_2^{SB})} + 1\right) = 0 \tag{21}$$

$$1 - \frac{\lambda P'(c_2^{SB})c_2^{SB} + \lambda P(c_2^{SB})}{u_2(c_1^{SB}, c_2^{SB})} + \mu\left(\frac{u_{12}(c_1^{SB}, c_2^{SB})c_1^{SB} + u_{22}(c_1^{SB}, c_2^{SB})c_2^{SB}}{u_2(c_1^{SB}, c_2^{SB})} + 1\right) = 0 \tag{22}$$

If the multiplier μ equals zero, then applying the proof of Proposition 2 to these FOC shows that $\tau^{SB} = \tau^{STD}(c_2^{FB})$ and $\tau^{LAB} = 0$. However, the proposition assumes that this tariff and tax policy is not budget feasible. Hence, $\mu > 0$.

Suppose the import income elasticity $\eta_{imp}(c_1, c_2) > 1$. Then, Lemma 1 implies that:

$$\frac{u_{11}(c_1, c_2)c_1 + u_{12}(c_1, c_2)c_2}{u_1(c_1, c_2)} < \frac{u_{12}(c_1, c_2)c_1 + u_{22}(c_1, c_2)c_2}{u_2(c_1, c_2)}.$$

It follows from (21)-(22) that:

$$\frac{\lambda}{u_1(c_1^{SB}, c_2^{SB})} < \frac{\lambda(P'(c_2^{SB})c_2^{SB} + P(c_2^{SB}))}{u_2(c_1^{SB}, c_2^{SB})}$$

Multiplying by $\lambda^{-1}u_2(c_1^{SB}, c_2^{SB})/P(c_2^{SB})$, we get:

$$\frac{u_2(c_1^{SB}, c_2^{SB})}{u_1(c_1^{SB}, c_2^{SB})P(c_2^{SB})} < \frac{P'(c_2^{SB})c_2^{SB} + P(c_2^{SB})}{P(c_2^{SB})}$$

It follows that: :

$$\tau^{SB} < \tau^{STD}(c_2^{SB}).$$

The other cases (import income elasticity equal to or less than one) can be handled similarly.

Proof of Corollary 1

If u is homothetic, then:

$$\frac{d\frac{u_2(kc_1,kc_2)}{u_1(kc_1,kc_2)}}{dk}|_{k=1} = 0.$$

Hence:

$$\frac{(u_{21}c_1 + u_{22}c_2)}{u_2} = \frac{(u_{11}c_1 + u_{12}c_2)}{u_1}.$$

We can then apply the proof of Proposition 3 to establish the corollary.

Proof of Proposition 4

This subsection proves Proposition 4. We use the following simple lemma.

Lemma 2. *For any $\overrightarrow{c}$, there exists a positive constant $\gamma(\overrightarrow{c})$ such that $\eta_k(\overrightarrow{c})(-\frac{f_k''(c_k)c_k}{f_k'(c_k)}) = \gamma(\overrightarrow{c})$ for all $k \in \{1, 2, ..., K+1\}$.*

Proof. Consider the consumer choice problem:

$$\max_{c_1,c_2,...,c_{K+1}} \sum_{j=1}^{K+1} f_j(c_j)$$
$$\text{s.t. } c_1 + \sum_{j=2}^{K+1} p_j c_j = W$$

The first order conditions to this problem are:

$$f_j'(c_j) = p_j f_1'(c_1), j = 2, ..., K+1.$$

Differentiating with respect to $ln(W)$, we get:

$$f_j''(c_j)c_j\frac{d(ln(c_j))}{d(ln(W))} = p_j f''(c_1)c_1\frac{dln(c_1)}{dln(W)}$$

Since $p_j = f_j'(c_j)/f_1'(c_1)$, we can substitute:

$$\frac{f_j''(c_j)c_j}{f_j'(c_j)}\eta_j(\overrightarrow{c}) = \frac{f_1''(c_1)c_1}{f_1'(c_1)}\eta_1(\overrightarrow{c})$$

which proves the lemma. □

We now prove the following that there exists $\alpha > 0$ such that

$$\tau_k^{SB} = \frac{1+\tau_k^{STD}(c_k^{SB})}{\alpha[\frac{1}{\eta_{dom}(\overrightarrow{c}^{SB})} - \frac{1}{\eta_k(\overrightarrow{c}^{SB})}]+1} - 1, k = 2, ..., K+1. \tag{23}$$

This set of restrictions (23) on the second-best tariffs immediately implies the characterizations in Proposition 4.

The first order conditions to the second-best policy problem are:

$$f_1'(c_1^{SB}) - \lambda + \mu[f_1'(c_1^{SB}) + f_1''(c_1^{SB})c_1^{SB}] = 0$$
$$f_k'(c_k^{SB}) - \lambda[P_k'(c_k^{SB})c_k^{SB} + P_k(c_k^{SB})] + \mu[f_k'(c_k^{SB}) + f_k''(c_k^{SB})c_k^{SB}] = 0, k = 2, .., K.$$

It is easily shown, as in the proof of Proposition 3, that the insufficiency of tariff revenue to fund G implies $\mu > 0$. It follows from the first order conditions that:

$$1 + \mu[1 + \frac{f_1''(c_1^{SB})c_1^{SB}}{f_1'(c_1^{SB})}] = \frac{\lambda}{f_1'(c_1^{SB})}$$
$$1 + \mu[1 + \frac{f_k''(c_k^{SB})c_k^{SB}}{f_k'(c_k^{SB})}] = [P_k'(c_k^{SB})c_k^{SB} + P_k(c_k^{SB})]\frac{\lambda}{f_k'(c_k^{SB})}, k \geq 2.$$

Subtracting, we obtain:

$$\mu[\frac{f_k''(c_k^{SB})c_k^{SB}}{f_k'(c_k^{SB})} - \frac{f_1''(c_1^{SB})c_1^{SB}}{f_1'(c_1^{SB})}] = [P_k'(c_k^{SB})c_k^{SB} + P_k(c_k^{SB})]\frac{\lambda}{f_k'(c_k^{SB})} - \frac{\lambda}{f_1'(c_1^{SB})}, k \geq 2.$$

Re-arranging, we find that for $k \geq 2$:

$$[\frac{f_k''(c_k^{SB})c_k^{SB}}{f_k'(c_k^{SB})} - \frac{f_1''(c_1^{SB})c_1^{SB}}{f_1'(c_1^{SB})}] = \frac{\lambda}{\mu f_1'(c_1^{SB})}(\frac{P_k'(c_k^{SB})c_k^{SB} + P_k(c_k^{SB})f_1'(c_1^{SB})}{f_k'(c_k^{SB})} - 1)$$

By applying Lemma 2, we can conclude that for $k \geq 2$:

$$[\frac{1}{\eta_{dom}(\overrightarrow{c}^{SB})} - \frac{1}{\eta_k(\overrightarrow{c}^{SB})}] = \frac{\lambda}{\mu\gamma f_1'(c_1^{SB})}(\frac{P_k'(c_k^{SB})c_k^{SB} + P_k(c_k^{SB})f_1'(c_1^{SB})}{f_k'(c_k^{SB})} - 1).$$

Using the formula for the standard optimal tariff and the second-best tariff, we can show that:

$$\alpha[\frac{1}{\eta_{dom}(\overrightarrow{c}^{SB})} - \frac{1}{\eta_k(\overrightarrow{c}^{SB})}] = (\frac{1 + \tau_k^{STD}(c_k^{SB})}{1 + \tau_k^{SB}} - 1), k \geq 2.$$

where

$$\alpha = \frac{\mu\gamma f_1'(c_1^{SB})}{\lambda}.$$

The restrictions (23) follow by arranging terms so that τ_k^{SB} is on the left-hand side.

Appendix B

This appendix first derives an implementability constraint in a T period deterministic version of the model. (The deterministic model can easily be made stochastic if financial markets are assumed to be complete.) It is then straightforward to prove a generalization of Proposition 3 for this dynamic framework.

Consider an environment in which agents in Home live for T periods and have utility

function:

$$\sum_{t=1}^{T} \delta^{t-1}\{u(c_{1t}, c_{2t}) - v(n_t)\}, 0 < \delta < 1$$

over consumptions of good 1 and good 2, and over labor. Home agents can borrow and lend good 1 in a world market at a fixed interest rate r. The government needs to finance a sequence of purchases $\{G_t\}_{t=1}^{T}$ of good 1. It is able to use tariffs $(\tau_t)_{t=1}^{T}$, labor income taxes $(\Phi_t)_{t=1}^{T}$, and interest income taxes $(\alpha_t)_{t=2}^{T}$. Agents begin life with no assets, and so there is no interest income tax in period 1.

Home agents have linear technologies to turn labor into good 1 at each date, with labor productivity A_t at date t. Home agents take prices as given and so face budget constraints of the form:

$$\begin{aligned} c_{1t} + P_t c_{2t}(1+\tau_t) + B_{t+1} + b_{t+1} \\ \leq n_t A_t(1-\Phi_t) + B_t(1+r(1-\alpha_t)) + b_t(1+r(1-\alpha_t)) + M_t, T \geq t \geq 1 \\ B_{T+1}, b_{T+1} \geq 0 \end{aligned}$$

where $\{P_t\}_{t=1}^{T}$ represent the world prices of imports in terms of non-imports at each date. Here, B_t represents holdings of international debt (which pays off in terms of good 1) and b_t represents holdings of Home government debt that also pay off in terms of good 1. Both B_t and b_t are allowed to be negative prior to period $(T+1)$. In that case, the tax on interest income is actually a subsidy to borrowers. Finally, $\{M_t\}_{t=1}^{T}$ is a sequence of non-negative lump-sum transfers.

The world price P_t of good 2 is a fixed increasing function P of c_{2t} and there is an exogenous sequence $\{G_t\}_{t=1}^{T}$ of government purchases of good 1 that needs to be financed. It follows that the period-by-period flow resource constraints for Home is given by:

$$c_{1t} + P(c_{2t})c_{2t} + G_t + B_{t+1} = n_t A_t + B_t(1+r), T \geq t \geq 1$$

$$B_1 = 0, B_{T+1} = 0$$

These take account of the international inflows and outflows associated with asset purchases and payoffs. Note that if $A_t n_t < c_{1t} + G_t$ in period t, then Home is importing good 1 as well as good 2.

The government can borrow and lend from Home agents, but not from abroad. As a result, its flow budget constraint is given by:

$$\begin{aligned} &\tau_t P(c_{2t}) c_{2t} + n_t A_t \Phi_t + b_{t+1} + r\alpha_t b_t \\ &= G_t + b_t(1+r) + M_t, T \geq t \geq 1 \\ &b_1 = 0, b_{T+1} = 0 \end{aligned}$$

To summarize, given $(\tau_t, A_t, \Phi_t, M_t)_{t=1}^{\infty}$, an equilibrium $(c_{1t}, c_{2t}, n_t, b_t, B_t)_{t=1}^{T}$ in Home is defined by the first order conditions:

$$\begin{aligned} u_1(c_{1t}, c_{2t})(1-\Phi_t)A_t &= v'(n_t), T \geq t \geq 1 \\ u_1(c_{1t}, c_{2t})P(c_{2t})(1+\tau_t) &= u_2(c_{1t}, c_{2t}), T \geq t \geq 1 \\ \delta(1 + r(1-\alpha_{t+1}))u_1(c_{1,t+1}, c_{2,t+1}) &= u_1(c_{1t}, c_{2t}), (T-1) \geq t \geq 1 \end{aligned}$$

the resource constraints

$$\begin{aligned} c_{1t} + P(c_{2t})c_{2t} + G_t + B_{t+1} &= B_t(1+r) + n_t A_t, T \geq t \geq 1 \\ B_1, B_{T+1} &= 0 \end{aligned}$$

and the government's sequence of budget constraints:

$$\begin{aligned} c_{2t}P(c_{2t})\tau_t + n_t\Phi_t + \alpha_t r(b_t + B_t) &= b_{t+1} + G_t + M_t, T \geq t \geq 1 \\ b_1, b_{T+1} &= 0 \end{aligned}$$

We can then characterize the set of budget-feasible and resource-feasible allocations much

as we did in the static case.

Proposition 5. *An allocation* $(c_{1t}, c_{2t}, n_t, B_t)_{t=1}^T$ *is an equilibrium for some* $(\alpha_t, \tau_t, \Phi_t)_{t=1}^T$ *if and only if the allocation satisfies the intertemporal resource constraint:*

$$\sum_{t=1}^{T}(1+r)^{-t}[A_t n_t - c_{1t} + P(c_{2t})c_{2t} + G_t] = 0$$

and an implementability constraint:

$$\sum_{t=1}^{T}\delta^{t-1}(u_1(c_{1t}, c_{2t})c_{1t} + u_{2t}(c_{1t}, c_{2t})c_{2t} - n_t v'(n_t)) \geq 0.$$

Proof. Suppose $(c_{1t}, c_{2t}, n_t, B_t)_{t=1}^T$ is an equilibrium given $(\tau_t, \Phi_t, M_t)_{t=1}^T$ and $(\alpha_t)_{t=2}^T$. These satisfy the sequence of resource constraints:

$$c_{1t} + P(c_{2t})c_{2t} + G_t + B_{t+1} = B_t(1+r) + n_t A_t, T \geq t \geq 1$$
$$B_1, B_{T+1} = 0$$

These imply the intertemporal resource constraint in the usual way. The government's period t budget constraint implies:

$$P(c_{2t})c_{2t}\tau_t + n_t\Phi_t A_t + \alpha_t r(B_t + b_t) + b_{t+1} = G_t + b_t(1+r) + M_t$$

Add this together with the period t resource constraint, and we get:

$$c_{1t} + P(c_{2t})c_{2t}(1+\tau_t) - n_t(1-\Phi_t)A_t + (B_{t+1} + b_{t+1}) = (b_t + B_t)(1 + (1-\alpha_t)r) + M_t$$

Multiply by $\delta^{t-1}u_1(c_{1t}, c_{2t})$ and add up to obtain the implementability constraint:

$$\sum_{t=1}^{T}\delta^{t-1}(u_1(c_{1t}, c_{2t})c_{1t} + u_2(c_{1t}, c_{2t})c_{2t} - v'(n_t)n_t) \geq 0.$$

This adding up makes use of the non-negativity of transfers and of equilibrium first order conditions:

$$\begin{aligned}
\delta^{t-1}u_1(c_{1t}, c_{2t}) &= \delta^t u_1(c_{1,t+1}, c_{2,t+1})(1 + (1 - \alpha_{t+1})r) \\
u_1(c_{1t}, c_{2t})P(c_{2t})(1 + \tau_t) &= u_2(c_{1t}, c_{2t}) \\
u_1(c_{1t}, c_{2t})(1 - \Phi_t)A_t &= v'(n_t) \\
B_{T+1} = b_{T+1} &= 0
\end{aligned}$$

Now consider $(c_{1t}, c_{2t}, n_t)_{t=1}^T$ that satisfies the intertemporal resource constraint and the implementability constraints. Define $\{B_t\}_{t=2}^{T+1}$ by:

$$\begin{aligned}
B_{t+1} &= A_t n_t + B_t(1 + r) - c_{1t} - P(c_{2t})c_{2t} - G_t \\
B_1 &= 0.
\end{aligned}$$

The intertemporal resource constraint implies that $B_{T+1} = 0$.

Define:

$$\begin{aligned}
\tau_t &= \frac{u_2(c_{1t}, c_{2t})}{u_1(c_{1t}, c_{2t})P(c_t)} - 1 \\
\Phi_t &= 1 - \frac{v'(n_t)}{u_1(c_{1t}, c_{2t})} \\
\alpha_{t+1} &= 1 - \frac{\frac{u_1(c_{1t}, c_{2t})}{\delta u_1(c_{1,t+1}, c_{2,t+1})} - 1}{r}
\end{aligned}$$

to be taxes/tariff that satisfy the agents' first order conditions. Define a sequence of government bondholdings and a final-period government lump-sum transfer by:

$$\begin{aligned}
b_{t+1} &= G_t + b_t(1 + r) - P(c_{2t})c_{2t}\tau_t - \Phi_t A_t n_t - \alpha_t r(b_t + B_t), T - 1 \geq t \geq 1. \\
-M_T &= (G_T + b_T(1 + r) - P(c_{2T})c_{2T}\tau_T - \Phi_T A_T n_T - \alpha_T r(b_T + B_T))
\end{aligned}$$

Substitute for G_t using the resource constraint:

$$G_t = B_t(1+r) + n_t A_t - c_{1t} - P(c_{2t})c_{2t} - B_{t+1}, T \geq t \geq 1$$

and we get:

$$b_{t+1} = (B_t + b_t)(1 + r(1-\alpha_t)) - c_{1t} - P(c_{2t})c_{2t}(1+\tau_t) + n_t(1-\Phi_t)A_t - B_{t+1}, T-1 \geq t \geq 1$$
$$-M_T = (B_T + b_T)(1 + r(1-\alpha_T)) - c_{1T} - P(c_{2T})c_{2T}(1+\tau_T) + n_T(1-\Phi_T)A_T$$

since $B_{T+1} = 0$.

We need to verify that the lump-sum transfer in the final period is indeed non-negative. Multiply through by $\delta^{t-1}u_1(c_{1t}, c_{2t})$ and adding up. we obtain:

$$\delta^{t-1}u_1(c_{1t}, c_{2t})b_{t+1} = -\delta^{t-1}u_1(c_{1t}, c_{2t})B_{t+1} - \sum_{s=1}^{t} \delta^{s-1}(u_1(c_{1s}, c_{2s})c_{1s} + u_2(c_{1s}, c_{2s})c_{2s} - v'(n_s)n_s), T-1 \geq t \geq$$
$$-M_T = -\sum_{s=1}^{T} \delta^{s-1}(u_1(c_{1s}, c_{2s})c_{1s} + u_2(c_{1s}, c_{2s})c_{2s} - v'(n_s)n_s)$$

The implementability constraint then implies that the last period transfer is non-negative:

$$M_T \geq 0.$$

□

With this proposition in hand, we can define the second-best policy problem in this

dynamic setting as:

$$\max_{\{c_{1t},c_{2t},n_t\}_{t=1}^T} \sum_{t=1}^T \delta^{t-1}(u(c_{1t},c_{2t}) - v(n_t)\}$$

$$\text{s.t.} \sum_{t=1}^T (1+r)^{1-t}(c_{1t} + P(c_{2t})c_{2t} + G_t - A_t n_t) = 0$$

$$\sum_{t=1}^T \delta^{t-1}(u_1(c_{1t},c_{2t})c_{1t} + u_2(c_{1t},c_{2t})c_{2t} - v'(n_t)n_t) \geq 0.$$

The following proposition is a generalization of Proposition 3. The notation η_{imp} and η_{dom} refers, as in that Proposition, to within-period income elasticities of imports and non-imports respectively.

Proposition 6. *Suppose u has decreasing marginal rates of substitution. Suppose too that $(c_{1t}^{SB}, c_{2t}^{SB}, n_t^{SB})_{t=1}^T$ solves the above second-best policy problem and that $(c_{1t}^{FB}, c_{2t}^{FB}, n_t^{FB})_{t=1}^T$ solves the analog first-best policy problem without the implementability constraint. Assume that the standard optimal tariff revenue does not raise sufficient revenue to fund the stream of government purchases:*

$$\sum_{t=1}^T (1+r)^{1-t}(\tau_{STD}(c_{2t}^{FB})P(c_{2t}^{FB})c_{2t}^{FB} - G_t) < 0$$

Then:

$$\eta_{imp}(c_{1t}^{SB}, c_{2t}^{SB}) = \eta_{dom}(c_{1t}^{SB}, c_{2t}^{SB}) \Rightarrow \tau_t^{SB} = \tau_{STD}(c_{2t}^{SB})$$

$$\eta_{imp}(c_{1t}^{SB}, c_{2t}^{SB}) > \eta_{dom}(c_{1t}^{SB}, c_{2t}^{SB}) \Rightarrow \tau_t^{SB} < \tau_{STD}(c_{2t}^{SB})$$

$$\eta_{imp}(c_{1t}^{SB}, c_{2t}^{SB}) < \eta_{dom}(c_1^{SB}, c_2^{SB}) \Rightarrow \tau^{SB} > \tau_{STD}(c_2^{SB}).$$

Proof. Let λ be a multiplier on the resource constraint and μ be a multiplier on the implementability constraint. Then at any date t, $(c_{1t}^{SB}, c_{2t}^{SB}, n_t^{SB})$ satisfies the same first order

necessary conditions that arose in the proof of Proposition 3:

$$u_1(c_{1t}^{SB}, c_{2t}^{SB}) - \lambda \tag{24}$$

$$= -\mu u_{11}(c_{1t}^{SB}, c_{2t}^{SB})c_{1t}^{SB} - \mu u_{21}(c_{1t}^{SB}, c_{2t}^{SB})c_{2t}^{SB} - \mu u_1(c_{1t}^{SB}, c_{2t}^{SB}) \tag{25}$$

$$u_2(c_{1t}^{SB}, c_{2t}^{SB}) - \lambda P'(c_2^{SB})c_2^{SB} - \lambda P(c_2^{SB}) \tag{26}$$

$$= -\mu u_{12}(c_{1t}^{SB}, c_{2t}^{SB})c_{1t}^{SB} - \mu u_{22}(c_{1t}^{SB}, c_{2t}^{SB})c_{2t}^{SB} - \mu u_2(c_{1t}^{SB}, c_{2t}^{SB}) \tag{27}$$

As long as $\mu > 0$, we can follow the proof of Proposition 3 to use these first order conditions to prove the current Proposition.

All that is left to prove is that $\mu > 0$. Suppose $\mu = 0$, and $(c_{1t}^{SB}, c_{2t}^{SB}, n_t^{SB}) = (c_{1t}^{FB}, c_{2t}^{FB}, n_t^{FB})$ for all $t = 1, ..., T$. The first-best allocation satisfies the optimality conditions:

$$\begin{aligned}
u_1(c_{1t}^{FB}, c_{2t}^{FB})A_t &= v'(n_t^{FB}), t = 1, ..., T \\
\frac{u_1(c_{1t}^{FB}, c_{2t}^{FB})}{u_2(c_{1t}^{FB}, c_{2t}^{FB})} &= P(c_2^{FB})(1 + \tau_{STD}(c_2^{FB})), t = 1, ..., T \\
u_1(c_{11}^{FB}, c_{21}^{FB}) &= \delta^{t-1}(1+r)^{1-t}u_1(c_{1t}^{FB}, c_{2t}^{FB}), t = 2, .., T
\end{aligned}$$

Then, we can plug these into the implementability constraint:

$$\begin{aligned}
&\sum_{t=1}^{T} \delta^{t-1}(u_1(c_{1t}^{FB}, c_{2t}^{FB})c_{1t}^{FB} + u_2(c_{1t}^{FB}, c_{2t}^{FB})c_{2t}^{FB} - v'(n_t^{FB})n_t^{FB}) \\
&= \sum_{t=1}^{T} \delta^{t-1}u_1(c_{1t}^{FB}, c_{2t}^{FB})(c_{1t}^{FB} + \frac{u_2(c_{1t}^{FB}, c_{2t}^{FB})}{u_1(c_{1t}^{FB}, c_{2t}^{FB})}c_{2t}^{FB} - \frac{v'(n_t^{FB})}{u_1(c_{1t}^{FB}, c_{2t}^{FB})}n_t^{FB}) \\
&= u_1(c_{11}^{FB}, c_{21}^{FB}) \sum_{t=1}^{T}(1+r)^{1-t}(c_{1t}^{FB} + P(c_{2t}^{FB})(1 + \tau_{STD}(c_2^{FB}))c_{2t}^{FB} - A_t n_t^{FB}) \\
&= u_1(c_{11}^{FB}, c_{21}^{FB}) \sum_{t=1}^{T}(1+r)^{1-t}(c_{1t}^{FB} + P(c_{2t}^{FB})(1 + \tau_{STD}(c_2^{FB}))c_{2t}^{FB} - A_t n_t^{FB}) \\
&= u_1(c_{11}^{FB}, c_{21}^{FB}) \sum_{t=1}^{T}(1+r)^{1-t}(\tau_{STD}(c_2^{FB}))P(c_2^{FB})c_{2t}^{FB} - G_t) \\
&< 0.
\end{aligned}$$

The assumption that $\mu = 0$ leads to a contradiction and so $\mu > 0$. □

References

[1] Atkinson, A. A, and Stiglitz, J. E., 1972, The structure of indirect taxation and economic efficiency, *Journal of Public Economics* 1, 97-119,

[2] Atkinson, A. A., and Stiglitz, J. E., 1976, The design of tax structure: Direct versus indirect taxation, *Journal of Public Economics* 6, 55-75.

[3] Boadway, R., Maital, S., and Prachowny, M., 1973, Optimal tariffs, optimal taxes, and optimal public goods, *Journal of Public Economics* 2, 391-403.

[4] Chari, V. V., and Kehoe, P., 1999, Optimal fiscal and monetary policy, *Handbook of Macroeconomics, Volume 1C,* 1671-1745.

[5] Chari, V. V., Nicolini, J.-P., and Teles, P., 2023, Optimal cooperative taxation in the global economy, *Journal of Political Economy* 131, 95-130.

[6] Costinot, A., Donaldson, D., Vogel, J., and Werning, I., 2015, Comparative advantage and optimal trade policy, *Quarterly Journal of Economics* 130, 659-702.

[7] Costinot, A., and Werning, I., 2025, How tariffs affect trade deficits, NBER WP 33709.

[8] Diamond, P., and Mirrlees, J. A., 1971, Optimal taxation and public production I: Production efficiency, *American Economic Review* 61, 8–27.

[9] Donaldson, D., 2018, Lecture notes for international economics (Lecture 20), https://dave-donaldson.com/2018-fall-international-economics-mit-14-581/.

[10] Fieler, A. C., 2011, Nonhomotheticity and bilateral trade: Evidence and a quantitative explanation, *Econometrica* 79, 1069-1101.

[11] Heathcote, J. A., Storesletten, K., and Violante, G., 2017, Optimal tax progressivity: An analytical framework, *Quarterly Journal of Economics* 132, 1693-1754.

[12] Houthakker, H., and Magee, S., 1969, Income and price elasticities in world trade, *Review of Economics and Statistics* 2, 111-25.

[13] Humphrey, T., 1987, Classical and neoclassical roots of the theory of optimum tariffs, *Federal Reserve Bank of Richmond Economic Review Jul/Aug.*

[14] Lucas, R. E, Jr., and Stokey, N., 1983, Optimal fiscal and monetary policy in an economy without capital, *Journal of Monetary Economics* 12, 55-93.

[15] Mirrlees, J. A., 1971, Explorations in the theory of optimum income taxation, *The Review of Economic Studies* 38, 175-208.

[16] Werning, I., 2007, Optimal fiscal policy with redistribution, *Quarterly Journal of Economics* 122, 925-967.