The importance of being earnest (in search markets)*

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DISCUSSION DRAFT

Abstract

In some markets for highly illiquid assets most transactions are mediated through auctions. In others it is through brokers who hold no inventory. Why? We explore this with a novel framework of decentralized markets for large indivisible assets where the central friction is buyers' difficulty to credibly convey their private valuation. We show this friction can be costly to sellers and identify a novel role for brokers as certifiers of their own clients' earnest intentions. When sellers can choose between an auction and broker-mediation, the latter dominates whenever markets are not tight and/or when buyers' private valuations are sufficiently diverse.

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1 Introduction

Large indivisible assets, such as real estate, often trade in decentralized markets and without inventory-owning dealers (i.e., market makers). Such markets seem to predominately be organized in one of three ways: informal searching, matching, and bargaining; (non-inventory) broker networks; or individual auctions. Some assets, like art (or other collectibles) and real estate are sometimes transacted through auctions and sometimes through broker networks.[1] Other assets, like firms, are characterized by thin markets where acquisition transactions take place through informal mechanisms or via investment banks that effectively intermediate as brokers (Servaes and Zenner, 1996). What may account for which asset markets are organized in which way?

The goal of this paper is to develop a basic framework for the study of such markets and to understand the features that may predispose them towards one structure over another. Key considerations in our setup include heterogeneous private buyer values and whether or not counterparties are able to commit to initially agreed upon terms when meeting and, subsequently, attempting to transact. Looking ahead, this leads us to identify a novel role for intermediaries, like brokers, whose task is to increase the certainty of execution in bilateral bargaining by credibly conveying their client's sincere degree of interest (and therefore earnest intentions).[2] We show that overcoming such information frictions through "buyer representation" can produce better results for sellers than committed bid solicitations (essentially, auctions), unless markets are tight in which case the latter are preferred. Auctions, however, can yield such poor outcomes for sellers in thin markets that sellers would be better off bargaining with randomly selected buyers and at a costly information disadvantage.

Key to our search and matching framework is the presence of asymmetric information when counterparty selection can precede a settlement phase. We consider the arrival of multiple buyers who indicate interest in a seller's asset within the same period. Sellers must select a counterparty among the arriving buyers, after which a settlement process determines whether and how a transaction is consummated. We consider several variants

[1] Real estate auctions, for instance, are prevalent in countries such as New Zealand and Australia (see Mayer, 1998).

[2] Because brokers are fewer and involved in transactions at a much higher frequency than buyers and sellers, they have an incentive to maintain their reputation for honesty within their broker network (see, for instance Chemmanur and Fulghieri, 1994; Lizzeri, 1999).

for selection and settlement. At the outset, we assume that interested buyers cannot credibly commit to offers, and hence the seller must first *select* a buyer and then *bargain* with them directly.[3]

If private values cannot be credibly conveyed to a seller at the selection stage, sellers treat all prospective buyers symmetrically and must choose at random. As a result, high valuation buyers may end up being passed over. If, pre-selection, the seller learns buyer values (by some currently unspecified manner), then the seller *can* select the buyer with highest valuation, and will do so under appropriate conditions.[4]

In the bargaining stage, we follow the standard practice of assuming the Nash (1950) bargaining solution if the buyer's type is known to the seller. Since this solution is Pareto efficient, there are no breakdowns of negotiations for strategic or value-related reasons. However, what if the buyer's type remains unknown at the time of bargaining? In this case, due to strategic considerations and information rents, bargaining is sometimes inefficient: A selected buyer with valuation greater than the seller's reservation may fail to come to an agreement.[5]

To illustrate the features described above and motivate the quantitative significance of the information structure, we present a numerical example roughly calibrated to the US residential real estate market. For instance, equilibria with uninformed bargaining exhibit prices that are 15% to 40% lower than those with informed bargaining (depending on market tightness and turnover), while aggregate welfare differences can vary from 0.5% in thin markets with low turnover to 6% in tight markets with high turnover. In the example, as long as bargaining is informed, there is only a modest difference between equilibria with uninformed versus informed selection. In other words, information is much

[3]In real estate transactions, there are typically far more opportunities for buyers to withdraw or demand more favorable terms (for example, after claiming to discover something negative about the property during due diligence) than there are for the seller to do likewise (Daley, Geelen and Green, 2024). Hence, interpreted literally, buyers would overbid for the asset, but with both parties knowing that once a bid is accepted, the buyer will not honor the terms. This type of friction is prevalent in virtually all markets in which an inspection or discovery period is valuable to buyers.

[4]Interestingly, monotonic selection can fail to constitute equilibrium behavior. The reason is that the prioritization of higher buyer types in selection increases those buyers' continuation value, effectively strengthens their bargaining position, and makes them less attractive trading partners. If this effect is sufficiently strong, then sellers must sometimes select lower buyer types over higher ones to balance out this consideration.

[5]Ausubel, Cramton and Deneckere (2002) discusses the inefficiencies of search under asymmetric information. We employ a "random proposer" mechanism which converges to Nash bargaining as information asymmetry vanishes. Further motivation for this bargaining solution is provided by Peski (2022) and Myerson (1984).

more valuable at the bargaining stage than at the selection stage.

Having established the importance of information availability, we next study the strategic incentives around the provision of such information when certain commitment devices are available. First, we show that buyer representatives who can "certify" a buyer's type can facilitate a full-information equilibrium as long as the commitment to disclose through certification is costless to the buyer. This is akin to "prepaid" buyer intermediary services — a common brokerage contract provision in the US residential real estate market. That is, buyers will have an incentive to seek out certification by intermediaries that are trusted by the other side of the market. Buyers that fail to do so will be passed over. By contrast, when the decision to be certified is costly to the buyer, a full-information equilibrium cannot be generically attained. This is because low-value buyers will not find certification worth the cost.

We next analyze selection and settlement when buyers can commit to bids and sellers can commit to honoring a selected bid. This is equivalent to holding a first-price auction. Because bids are monotonic in type, the auction environment is informationally equivalent to one in which types are revealed (e.g., via a certifying broker).[6] However, price determination and therefore steady state values and division of surplus are different. One key finding is that competition among sellers will drive them to set their equilibrium reserve price equal to their private valuation. Another is that, under standard assumptions over the distribution of private benefits of ownership, auctions cannot be ex-post efficient for sellers when there is an alternative of free (or equivalent-cost) certification, say through an intermediary. This means that, to be effectively implemented, auctions require the means for ex-ante commitment by sellers.

Returning to our core motivation of understanding market structure, we allow sellers to choose between committed bid solicitation versus bargaining (with buyer access to certification). We then look for symmetric equilibria in which all sellers use the same protocol. We find for the US housing market calibrated parameters that certification is the unique such equilibrium. It is only when markets are significantly more tight or homogeneous that the preferred market structure shifts to auctions. The reason is that the efficiency of auctions in eliciting surplus for the seller depends on competition. In thinner markets, it is not rare for there to be only one bidder. Knowing this, and the

[6] We are also able to show that, unlike certification and auctions, buyer-chosen deposits (on their own) cannot lead to full revelation of buyer types.

fact that they can always visit an auction next period if they are outbid, buyers are unlikely to place bids that give up significant surplus. This effect is exacerbated by buyer heterogeneity where the reluctance by high-valuation buyers to forego surplus is more impactful on seller outcomes. For the same reason, auctions eventually become *ex-ante* dominant as market tightness (and competition among buyers) increases.

Speaking more to applications, the literature typically assumes that brokers, especially on the residential real estate buyer side, exist to provide their clients with information about the asset market (Han and Strange, 2015; Kurlat and Stroebel, 2015; Broxterman and Zhou, 2023). By contrast, our framework suggests a drastically different informational role for brokers, whereby they provide information to their client's counterparty about their own client's sincerity. Doing so maximizes the chance of transacting, by both improving the client's odds of being selected and reducing the probability of a breakdown in negotiations. In this way, brokers can play a role analogous to auctions in the sense that the latter incentivize the revelation of buyers' private values. The difference is that in thin markets, auctions fail to generate competitive outcomes for sellers. We show that in such markets brokers of the sort we envision deliver the greatest value. It is only in thin markets, where it would be too costly to implement a broker network, that informal searching, matching, and bargaining would be prevalent.

1.1 Related Literature

Our model features (*i*) search with a non-rival meeting technology (one buyer's arrival at a seller does not affect the probability that another buyer will arrive at the same seller); (*ii*) pricing via bilateral bargaining (Section 3) or competitive bidding (Section 4); and (*iii*) asymmetric information about heterogeneous valuations. This combination does not align with the typical assumptions made in either of the large literatures in directed-search-markets or over-the-counter (OTC) markets.

Regarding feature (*i*), our search model features the canonical non-rival "urn ball" meeting technology common in models of directed search (see Wright et al., 2021, for an extensive survey). However, in that literature, buyer search is "directed" by publicly announced seller choices—most commonly, the choice of posted prices. Under posted prices, if multiple buyers arrive at a seller, the typical assumption is that the asset is probabilistically rationed with each buyer having equal chance of acquiring the asset. By contrast,

in feature (*ii*) of our model, sellers may actively select among arriving buyers depending on what information they have at their disposal. Peters and Severinov (1997); Coles and Eeckhout (2003); Eeckhout and Kircher (2010); Albrecht, Gautier and Vroman (2016) consider a model in which sellers can publicly commit to more general mechanism such as auctions. Their main focus is on auctions versus posted prices, as opposed to auctions versus (possibly intermediated) negotiations, which are central to our comparison.[7]

In much of the directed search literature, buyer valuations are homogeneous ruling out asymmetric information as in our feature (*iii*). Moreover even with heterogeneous buyer valuations, if sellers are restricted to pre-search posted prices, privately known buyer values are irrelevant because the seller's pricing has already been determined. In this literature, Jiang et al. (2025) is perhaps the paper most related to our own. They consider a non-rival, directed search market for housing with heterogeneous buyers and seller posted prices. They find that the sellers will endogenously segment the market with different price offers, and higher value buyers will direct their search to homes with higher prices and greater probability of deal completion. Segmentation is impossible in our model when prices are determined by bilateral bargaining. Somewhat surprisingly, even when we allow sellers to hold auctions with reserve prices, they will all elect to have no reserve price and allow competitive bidding to determine prices.

A large literature explores OTC markets in Finance, which typically employs undirected, rival search (at most one buyer at a time can arrive at a seller) and prices determined by bilateral negotiations (see Weill, 2020, for an extensive survey). Whereas rival search contrasts with feature (*i*) of our model, we do consider bilateral bargaining. Starting with Duffie, Gârleanu and Pedersen (2005), buyer valuations are typically not their private information as in feature (*iii*) of our model, but exceptions include Zhu (2012); Cujean and Praz (2014); Zhang (2018); Glode and Opp (2019). The focus of Zhu (2012) is on repeated contact between the same seller-buyer pair and in Zhang (2018) on when parties are in long-term relationships, neither of which are relevant for our study. Cujean and Praz (2014) and Glode and Opp (2019) are more related to the spirit of our exercise by exploring the effects of varying manners of information asymmetry. As we discuss in Section 3, the introduction of private information means that the ubiquitous Nash bargaining solution can no longer be the assumed settlement mechanism. Each of

[7]Lovo and Spaenjers (2018) and Arefeva (2020) also study search markets with embedded auctions. Their motivations are very different from ours.

these paper assumes that one negotiating partner has the power to make a take-it-or-leave-it price offer, whereas we appeal to a bargaining solution that converges to Nash under symmetric information and is also micro-founded in recent work by Peski (2022).

A substantial literature studies search, matching, and pricing in the specific context of real estate (see Han and Strange, 2015, for an extensive survey). A notable feature in these markets is the prominence of brokers, prompting Han and Strange (2015) to title a subsection of their survey, "Why do real estate brokers exist?" They surmise that the literature's answer is to ameliorate incomplete information about asset and market characteristics and facilitate buyer and seller meetings (Stigler, 1961; Yinger, 1981; Wu and Colwell, 1986; Yavaş, 1992, 1994; Benjamin, Chinloy and Winkler, 2007). Given the rise of technological innovations in matching and information provision (e.g., Zillow), there is concern that the value created by broker networks has diminished while their high costs have not adjusted (see, for instance, Barwick and Wong, 2019). One of our main contributions is to establish that, by certifying their clients, brokers can potentially play a role that is both distinct from the prevailing view in this literature and less prone to technological innovations.

2 Model primitives

Figure 1 summarizes the timeline. Time is discrete and infinite. There is a unit mass of individuals with types uniformly distributed on $[0, 1]$. Likewise, the total inventory of ex-ante identical and indivisible assets is a mass denoted by I. One individual may own multiple assets but can only be "well-matched" with at most one asset they own (any other owned asset is said to be "mismatched"). An owner of type $x \in [0, 1]$ receives a private utility flow of $\delta(x)$ per period if well-matched with one of their assets. We assume that $\delta(0) \leq 0 < \delta(1)$, that δ is differentiable with $\delta'(x) > 0$ in $(0, 1)$, and that $\int_x^1 \delta(x')dx' < \infty$ for every $x \in [0, 1)$. We also assume that $(\delta(x) - \kappa)(1 - x)$ is single-peaked in $[0, 1]$ for every $\kappa \in \mathbb{R}$.[8] By contrast, a mismatched asset provides zero utility flow to its owner.[9]

[8]The private utility flow, $\delta(x)$, can be viewed as a quantile function and x as the associated percentile of private flow certainty equivalent. The conditions on $\delta(x)$ are implied by the regularity requirements on private value distributions prevalent in auction theory, and amount to a single crossing condition on the implied virtual valuation function. They are satisfied by a wide array of standard distributions, including uniform and normal distributions. Further generalizations of the conditions are possible.

[9]Setting mismatched utility flow to zero is a normalization. All of our results are robust to increasing well-matched and mismatched utility flows by the same constant.

Every individual is risk neutral and discounts next period's utility by a factor $\beta \in (0,1)$.

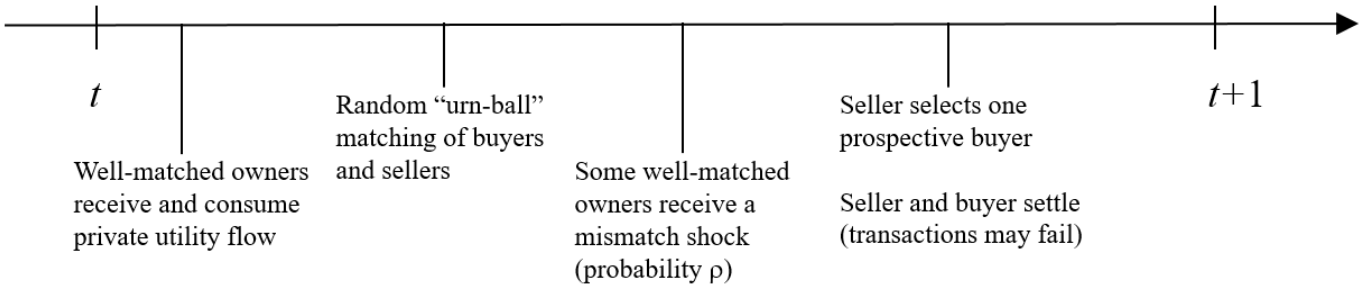


Figure 1: **Timeline of actions**
The seller's selection among arriving buyers is based on the the seller's information about buyer private values. The transaction settlement process is determined through either committed bidding or bargaining (transactions may fail).

Because there are no explicit dynamics to our model primitives, we restrict analysis to the steady state in which all quantities of interest are time-independent. Let $m(x)$ be the density of type x well-matched owners at the start of the period and $M = \int_0^1 m(x)dx$ be the mass of all well-matched owners (regardless of type). These well-matched owners do not search for new assets and enjoy their flow utility $\delta(x)$. At the end of the period they may, however, experience a mismatch shock turning them into potential searchers at the start of the next period. This occurs with probability ρ. If they do not experience the shock they remain well-matched owners at the start of the next period.

Correspondingly, $\big(1 - m(x)\big)dx$ is the infinitesimal mass of potential type-x searchers and $I - M$ is the mass of mismatched assets. Search is an "urn-ball" random assignment process where the probability that n buyers of type x "arrive at" (or "approach") a given seller is Poisson with intensity that equals the ratio of the mass of searchers of type x to the mass of assets for sale.[10] The arrival intensity of type-x searchers is, therefore, $\frac{1-m(x)}{I-M}dx$.[11]

Procedurally, upon being approached by a set of prospective buyers, a seller must

[10] See Petrongolo and Pissarides (2001) for a review of urn-ball processes, common in the labor market literature.

[11] Note that initially well-matched owners/assets have no interaction with searchers/available assets within a period. That is, newly mismatched owners do not join the search or make their assets available for sale in the period in which the mismatch shock is received. Starting with the following period, a newly mismatched asset ceases to provide utility flow of $\delta(x)$, where x is the type of its owner. At that point, the recently mismatched owner may participate in a new search and simultaneously put their mismatched asset up for sale. This separation simplifies the steady state equations.

select one with whom to potentially transact.[12] The selection is based on the information available to the seller about arriving buyer types. Following the selection phase, the seller and prospective buyer undertake a settlement process in an effort to consummate a transaction. Both the information available to sellers at the selection stage and the settlement process depend on available technology. For instance, if buyers credibly convey their types then sellers may choose the highest type buyer and settlement may consist of Nash bargaining. Alternatively, if buyers commit to bids and sellers commit to honoring a selected bid, then sellers may simply select the highest bidding buyer and settle based on the bid price. It is important to emphasize that the settlement process can include the possibility of endogenous transaction failure. We consider several alternative settings with the goal of characterizing market structure when buyers and sellers can choose among actions that influence both the selection and settlement phases.

Post-selection, if the settlement process results in a successful transaction, the buyer acquires the asset and immediately becomes well-matched with it: Starting with the following period, the new owner will receive a utility flow, $\delta(x)$, each period up to and including the period in which they experience a mismatch shock.

Before delving into a description of possible information and settlement structures, we derive a basic set of model results that always apply.

Steady state ownership and transaction probabilities

We are primarily interested in studying steady state equilibria in which the ownership density of the asset across owner types does not change. To establish the steady state condition, we must equate the outflow of well-matched owners of type x in a given period with the inflow of buyers of type x that successfully acquire the asset in that same period:

$$\rho m(x)dx = \frac{1-m(x)}{I-M}dx\, q(x)\,(I-M).$$

The term to the left of the equality is the (infinitesimal) mass of well-matched owners of type x that become mismatched during the period. The first term on the right is the

[12] If no buyers approach a given seller, they have no one with whom to transact and wait for the next period.

probability of arrival, at a given seller, of a type x buyer.[13] The second term, $q(x)$, denotes the probability that an arriving buyer of type x will ultimately succeed in acquiring the asset from the seller. The product of the two probabilities corresponds to the probability that a given seller will transact with a buyer of type x. This must then be applied to the mass of assets being sold, $I - M$. We emphasize that $q(x)$ is an endogenous quantity that depends on the model's information structure as well as the possibility that the settlement process (e.g., bargaining) fails. Solving for $q(x)$, we find that in steady state it is constrained to satisfy:

$$q(x) = \frac{\rho m(x)}{1 - m(x)}. \tag{1}$$

A similar steady state equation governs the per seller unconditional probability of a successful transaction, denoted q_s. Specifically, each period ρM assets transition from being well-matched to mismatched, and this must be equated with $q_s(I - M)$:

$$q_s = \frac{\rho M}{I - M}. \tag{2}$$

Values, Surpluses, and Prices

Because we assume sellers consume no flow utility from their mismatched asset(s), their reservation value depends wholly on the probability that they will transact next period and the price at which they will transact. A seller's reservation value is the asset's retention value into next period if the asset does not transact. This corresponds to the discounted probability-weighted sum of next period's reservation value and the expected value from successfully trading.

$$V_s = \beta\Big((1 - q_s)V_s + q_s E_{\mathbb{T}}\big[S_s(x)\big]\Big),$$

where V_s is the seller's reservation value, q_s (recall) is the probability that the seller will transact with some buyer next period, $S_s(x)$ is the seller's surplus from successfully transacting with a buyer of type x (i.e., the share of gains from trade earned by the seller). Because price could be random (even fixing the type x), $S_s(x)$ may be random,

[13] This follows from the fact that the arrival process of buyers of type x is Poisson with intensity equal to the number of searchers of type x divided by the number of sellers: $\big(1 - m(x)\big)dx/(I - M)$. Because dx is infinitesimal, the arrival probability of one type x buyer is, to first order in dx, equal to the arrival probability of one or more type x buyers.

and $E_{\mathbb{T}}[S_s(x) \mid x]$ is the expected seller's surplus from trading with a buyer of type x.

An equivalent expression applies to the buyer's reservation value:

$$F(x) = \beta\Big(F(x) + q(x)E_{\mathbb{T}}\big[S_b(x) \mid x\big]\Big),$$

where $F(x)$ is the reservation value of a type x buyer, $q(x)$ (recall) is the probability that the buyer will successfully transact with some seller in next period's search, and $S_b(x)$ is the buyer's surplus from trading.

When a searching buyer acquires the asset from a seller they become well-matched with it. The valuation of a well-matched owner, which we denote as $G(x)$, is the discounted sum of next period's utility flow, $\delta(x)$, the probability-weighted value of continuing to be well-matched with the asset, and the probability-weighted value of becoming both a seller and a searching buyer in the event of a mismatch shock. Thus,

$$G(x) = \beta\Big(\delta(x) + (1-\rho)G(x) + \rho\big(V_s + F(x)\big)\Big).$$

The total available surplus from trade, between a buyer of type x and a seller, corresponds to the difference between the buyer's gain from acquiring the asset minus the seller's reservation value. The buyer's gain must be measured net of their outside option of continuing to search and is denoted $V(x) \equiv G(x) - F(x)$. Thus, the total trade surplus is

$$S_{\text{tot}}(x) = G(x) - F(x) - V_s = V(x) - V_s. \tag{3}$$

The following equations recast the reservation values and the value of being a well-matched owner in terms of the gains from trade:

$$V_s = \frac{\beta}{1-\beta} q_s E_{\mathbb{T}}\big[S_s(x)\big] \qquad \text{Seller's reservation value} \tag{4}$$

$$F(x) = \frac{\beta}{1-\beta} q(x) E_{\mathbb{T}}\big[S_b(x) \mid x\big] \qquad \text{Buyer's reservation value} \tag{5}$$

$$G(x) = \frac{\beta}{1-\beta}\Big(\delta(x) - \rho S_{\text{tot}}(x)\Big) \qquad \text{Well-matched owner's valuation} \tag{6}$$

If an asset transaction takes place, its price corresponds to the seller's reservation price

plus the seller's gains from trade. That is,

$$P(x) \equiv V_s + S_s(x). \tag{7}$$

Moreover,

$$S_{\text{tot}}(x) = S_b(x) + S_s(x) \tag{8}$$

$$S_b(x) = V(x) - P(x), \tag{9}$$

where the second equation results from using Eqns. (3) and (7) in Eqn. (8).

To solve for a model equilibrium, in addition to the parameters ρ, β, and the primitive function $\delta(x)$, one must specify the details of the information structure and available actions at the selection and price-setting stage.

Definition 1. *A **steady state** is specified by a pair $\{q(\cdot), P(\cdot)\}_{x\in[0,1]}$ from which $\{q_s, m(\cdot), M, V_s, F(\cdot), G(\cdot)\}_{x\in[0,1]}$ are derived to satisfy* (1)-(9). *A **steady state equilibrium** is a steady state such that*

i. (Voluntary sales and participation in search) For all x, $S_s(x)$ is non-negative, and $q(x) > 0$ if and only if $S_b(x) \geq 0$.

ii. (Equilibrium in selection and pricing) Given $\{V_s, F(\cdot), G(\cdot)\}_{x\in[0,1]}$, the pair $\{q(\cdot), P(\cdot)\}_{x\in[0,1]}$ are the result of equilibrium in the selection and settlement model.

3 Steady state equilibria without commitment

In this section we characterize the steady state equilibrium for each of three distinct types of information structures when commitment is not possible and settlement proceeds through *bargaining*. In the first model, we assume the seller has no information about arriving buyer types at the selection stage, but bargaining takes place with complete information. This is essentially the standard search paradigm in which matching is random and trade surplus is split via Nash bargaining. This partial-information and bargaining (PIB) model thus serves as a comparison with prevailing literature. In the second model, we assume that the seller has no information about arriving buyer types at both the selection and the bargaining stages. We refer to its setting as no-information

and bargaining (NIB). In the third model, we assume the seller knows the types of arriving buyers at both stages and refer to its setting as full-information and bargaining (FIB). Differences between the NIB and FIB models characterize the value of information in our framework, which we show can be profound. This value derives from two sources: informed (instead of random) selection, and symmetric-information (instead of asymmetric-information) bargaining. Comparisons to the PIB allow to explore this decomposition of the value of information (Sections 3.4 and 3.5).

We initially take these information structures as given. In Section 4 we investigate how commitment can take one from the NIB setting to one with full information. In particular, we show that allowing buyers to commit to credibly convey their type (e.g., through certification via an intermediary, like a broker) leads to an equilibrium that is equivalent to the FIB equilibrium. We then consider how allowing buyer commitment to bids and seller commitment to honoring such bids leads to a full-information auction-like equilibrium.

In the ensuing, we relegate proofs to the appendix where interested readers can also find details of explicit solutions to the various steady state equilbria that we consider.

3.1 Base Case: Partial-information and bargaining (PIB)

Search models often feature i) the random pairing of parties who then ii) bargain under full information.[14] In our framework this corresponds to the case in which (i) sellers have no ability to distinguish buyer types and buyers have no credible way to convey their types (or commit to bids) at the selection stage, but once a buyer is selected (ii) their type is revealed (through some unspecified mechanism).[15] Hence, even though it is informationally between the two extremes of no information at both stages and full information at both stages (studied below), this partial information and bargaining (PIB) environment is a natural initial base case to analyze. As is common, we assume bargaining in this model satisfies Nash (1950), meaning the parties will equally split the available gains from trade $V(x) - V_s$.

[14] In finance, see the large literature spurred by Duffie, Gârleanu and Pedersen (2005) and Lagos and Rocheteau (2009). In real estate markets, see Han and Strange (2015). In the broader economics literature, see Chade, Eeckhout and Smith (2017).

[15] An alternative interpretation of the standard framework is that buyer types are always observable, but in each period buyers arrive in random order and the seller (through some unspecified mechanism) is forced to only negotiate and transact with the first arriving buyer.

Four features of steady state equilibrium are straightforward. First, buyer valuation (relative to continuation) $V(x)$ is increasing in x because no type is advantaged in selection but higher types have higher flow value δ for the asset. Second, there exists a threshold type $\bar{x} = \min\{x : V(x) \geq V_s\} \geq 0$ such that a buyer participates in the search market if and only if their type is $x \geq \bar{x}$. Third, transactions never fail. That is, negotiations never break down due to strategic considerations. Fourth, the density of the ownership distribution, $m(\cdot)$ is constant above $\bar{x}$ (and zero below it)—any participating type is equally likely to be an owner.

Proposition 1. *Any steady state equilibrium of the PIB model is characterized by a participation threshold* $\bar{x} = \min\{x : V(x) \geq V_s\} \geq 0$ *and the* $\{q(\cdot), P(\cdot)\}_{x\in[0,1]}$ *functions:*

$$q(x) = \begin{cases} 0 & x < \bar{x} \\ \frac{\rho m}{1-m} & x \geq \bar{x} \end{cases} \qquad P(x) = \max\left\{\frac{1}{2}(V_s + V(x)), V_s\right\},$$

where

$$m(x) = m \equiv \frac{(1-e^{-\bar{\tau}})}{\rho\bar{\tau} + (1-e^{-\bar{\tau}})}, \quad \bar{\tau} = \frac{1-\bar{x}-M}{I-M} \quad \text{and} \quad M = (1-\bar{x})m,$$

and $\{q_s, m(\cdot), M, V_s, F(\cdot), G(\cdot)\}_{x\in[0,1]}$ *satisfy* (1)-(9) *given* $\{q(\cdot), P(\cdot)\}_{x\in[0,1]}$. *In addition, a steady state equilibrium of the PIB model exists.*

Remark 1. *The equilibrium may not be unique because* V_s *is not guaranteed to be decreasing in* $\bar{x}$. *A sufficient condition to ensure uniqueness is that* $\delta(x)(1-x)/\int_x^1 \delta(x')dx'$ *is increasing. The type utility flow distributions satisfying this condition include uniform, normal, and exponential.*

The subsequent subsections consider two departures from this base case: What if sellers knew less than the base case and were forced to bargain without knowledge of the buyers type? And then, what if sellers knew more than the base case and knew buyer types before having to make their selection?

3.2 No-information and bargaining (NIB)

Suppose now that, during the selection stage and the bargaining stage, sellers have no ability to distinguish buyer types and buyers have no credible way to convey their types

(or commit to bids). A seller therefore chooses randomly among arriving buyers. Once a buyer is selected, the parties bargain under asymmetric information. As in the PIB model and is common in the search and matching literature, rather than model out the particulars of a strategic bargaining game, we employ a simplified solution concept. We refer to it as the "random proposer" solution. We first describe the solution and then justify its use.

Understood literally, the solution gives each party a one-half chance to make a take-it-or-leave-it offer that maximizes their expected surplus (given their information set). These offers can be any mechanism—they are not restricted to simple price offers. While menus of price-quantity pairs are allowed, each party's optimal mechanism is a simple price offer. The buyer's offer is to pay V_s for the asset, leaving the seller with zero surplus. The seller's offer is to be paid the price, P^*, that maximizes $(P - V_s)\text{Prb}\big(V(x) \geq P\big)$ — i.e., the price that results in the highest expected seller surplus (Myerson, 1981). If x^* is the threshold buyer type with $V(x^*) = P^*$, then equilibrium transactions with selected buyers with type $x < x^*$ will be at a price of V_s but fail half the time, while transactions with types $x \geq x^*$ are always consummated and at an expected price of $\frac{1}{2}(V_s + P^*)$.

We are not asserting that buyers and sellers will actually toss a coin to see who makes a take-it-or-leave-it-offer. We only assert that, conditional on the buyer type, transactions will take place at rates and at expected prices that are equivalent to those generated by the random proposer solution. For example, these rates and expected prices would arise if the seller offers the buyer the choice between transacting at V_s with probability one half or transacting for certain at a price of $(P^* + V_s)/2$. Viewed in this way, the bargaining solution provides scope for the buyer to make a "low-ball" offer with a risk of transaction failure, or transact for certain at a higher price.

In addition to its simplicity, the main justifications for the random proposer solution come from Peski (2022) and Myerson (1984). When applied to our setting, Peski (2022) shows, remarkably, that the random proposer solution is the unique perfect Bayesian equilibrium of the discrete-time, no-deadline bargaining game in which both parties have equal probability of making the next offer and offers can be any mechanism (not merely simple prices). He also provides an alternative description of the resultant equilibrium mechanism as "the menu that is optimal for the [seller] among all menus, such that each type of the [buyer] receives at least her payoff under complete information."

In addition, Peski (2022) shows that when buyer values satisfy the standard monotonicity of virtual values property, the random proposer solution also satisfies the conditions for Myerson's (1984) *neutral solution*: an axiomatic solution concept for bargaining problems with asymmetric information proposed as a generalization of Nash (1950). Finally, it is important to note that the random proposer solution coincides with the Nash bargaining solution under full information. This nesting property is essential to comparing the no-information equilibrium with the full- and partial-information models because the latter models employ Nash bargaining solutions.

One important aspect of the random proposer solution is the possibility of transaction failure: If the seller selects a buyer of type $x < x^*$, then with probability one-half they will fail to reach a deal even though a deal would generate positive surplus. The next proposition characterizes the NIB steady state equilibrium and establishes its existence.

Proposition 2. *Any steady state equilibrium of the NIB model is characterized by a participation threshold $\bar{x} \geq 0$ and a pricing threshold $x^* \in [\bar{x}, 1)$, where*

- $\bar{x} = \min\{x : V(x) \geq V_s\}$
- $x^* = \arg\max_x (1-x)(V(x) - V_s)$.

Letting $P^* \equiv V(x^*)$, the resulting $\{q(\cdot), P(\cdot)\}_{x\in[0,1]}$ functions are

$$q(x) = \begin{cases} 0 & x < \bar{x} \\ \frac{\rho m_L}{1-m_L} & x \in [\bar{x}, x^*) \\ \frac{\rho m_H}{1-m_H} & x \geq x^* \end{cases} \qquad P(x) = \begin{cases} V_s & x < x^* \\ \frac{1}{2}(V_s + P^*) & x \geq x^*, \end{cases}$$

where

$$m(x) = \begin{cases} m_L \equiv \frac{(1-e^{-\bar{\tau}})}{2\rho\bar{\tau}+(1-e^{-\bar{\tau}})} & \bar{x} \leq x < x^* \\ m_H \equiv \frac{(1-e^{-\bar{\tau}})}{\rho\bar{\tau}+(1-e^{-\bar{\tau}})} & x^* \leq x \leq 1 \end{cases}$$

$$\bar{\tau} = \frac{1-\bar{x}-M}{I-M} \quad \text{and} \quad M = m_H(1-x^*) + m_L(x^*-\bar{x}),$$

and $\{q_s, m(\cdot), M, V_s, F(\cdot), G(\cdot)\}_{x\in[0,1]}$ satisfy (1)-(9) given $\{q(\cdot), P(\cdot)\}_{x\in[0,1]}$. In addition, a steady state equilibrium of the NIB model exists.

Remark 2. *Generically, there are multiple NIB steady state equilibria. They arise because the transaction rate at x^* is discontinuous and leads to a kink in the value of ownership at $x = x^*$. In turn, the price, P, that maximizes $(P - V_s)Prb\big(G(x) - F(x) \geq P\big)$ satisfies a first order inequality rather than an equation. Interested readers can consult the proofs for more details.*

As alluded to above, when bargaining under complete information, the random proposer and Nash bargaining solution coincide. It therefore merits mention that fixing a pair's $V(x)$ and V_s, the buyer always does weakly better bargaining with asymmetric information compared to complete information:[16] In expectation, the buyer gets half of the surplus from bargaining if $x \leq x^*$, but more than half of the surplus for $x > x^*$, which can be thought of as the buyer's *information rents*. Thus, it is sellers who bear the brunt of their ignorance. As we show in Section 4, this creates an incentive for individual sellers to adopt a mechanism, if one is available, that improves their information status — even at a cost. One issue is less clear: Given their benefit from informational rents in the bargaining stage, might individual buyers be willing to disclose their values in the selection stage to affect the likelihood that they will be selected? This too is addressed in Section 4.

3.3 Full-information and bargaining (FIB)

In the third and last information structure of this section, each seller knows the types of all their arriving buyers and use this information in the selection stage. As a result, the steady-state distributions of owners and of searchers will be non-uniform in type (unlike in the standard search paradigm, as captured by the PIB setting). Because the selected buyer's type is known, any ensuing bargaining is under full information.

How should a seller select among multiple buyers with different types? Because the two parties will evenly split the surplus of $V(x) - V_s$, they should choose the buyer with the highest $V(x)$, the endogenously determined buyer valuation of a transaction. It is natural to look for a *monotonic* equilibrium: one in which V is strictly increasing in the flow value from ownership (equivalently, in x). In a monotonic equilibrium, higher types are always selected over lower ones. The following proposition characterizes the steady

[16] Section 3.4 examines how these endogenous values change if the entire market's information structure changes.

state ownership distribution when selection is monotonic.

Lemma 1. *In any steady state with monotonic selection and a participation threshold $\bar{x} \in [0,1)$, the ownership density, $m(x)$, is non-zero only for $x \in [\bar{x},1]$, strictly increasing, and satisfies*

$$\ln\Big(\frac{\rho m(x)}{(1-m(x))}\Big) + \frac{1}{1-m(x)} - \frac{1}{\rho} = \frac{x-M+I-1}{I-M},$$

The total mass of owners, M, is given by

$$M = \frac{(1-e^{-\bar{\tau}})}{\rho\bar{\tau} + (1-e^{-\bar{\tau}})}(1-\bar{x}).$$

where $\bar{\tau} = \frac{1-\bar{x}-M}{I-M}$. In addition, a triplet $\{m(\cdot), M, \bar{\tau}\}_{x\in[\bar{x},1]}$ with $M < min\{I, 1-\bar{x}\}$ exists for any $\bar{x} \in [0,1)$ and $I > 0$.

In the PIB model, the steady state equilibrium density of the ownership distribution is flat above $\bar{x}$: Any type that participates in the market is equally likely to own an asset. Lemma 1 establishes that this is no longer true under monotonic selection in the FIB model. In a monotonic steady state, the density of ownership increases with type because high valuation buyers are more likely to be selected and then transact. Using this understanding of the steady state ownership dynamics under monotonic selection, we next characterize monotonic steady state equilibria.

Proposition 3. *Any monotonic steady state equilibrium of the FIB model is characterized by a participation threshold $\bar{x} = \min\{x : V(x) \geq V_s\} \geq 0$ and the $\{q(\cdot), P(\cdot)\}_{x\in[0,1]}$ functions:*

$$q(x) = \begin{cases} 0 & x < \bar{x} \\ \frac{\rho m(x)}{1-m(x)} & x \geq \bar{x} \end{cases} \qquad P(x) = \frac{1}{2}(V_s + V(x)),$$

$m(x)$ is from Lemma 1 and $\{q_s, m(\cdot), M, V_s, F(\cdot), G(\cdot)\}_{x\in[0,1]}$ satisfy (1)-(9) given $\{q(\cdot), P(\cdot)\}_{x\in[0,1]}$. In addition, a monotonic steady state equilibrium of the FIB model exists if and only if for every $x \in (\bar{x}, 1)$,

$$\delta'(x) \geq \frac{1-\beta}{\beta}\big(\ln q(x)\big)' F(x). \tag{10}$$

To understand the reasons for Condition (10) consider that, holding V_s fixed, the surplus from trade balances the difference between the buyer's annuity value of utility flow

from the asset against the buyer's search value (i.e., value of being a searcher). Suppose that the buyer's flow annuity does not increase very much with x, but the search value increases substantially. The former corresponds to small $\delta'(x)$. The latter can happen when the probability of being selected is sharply increasing (i.e., $q'(x)$ is large), which arises under monotonic selection if supply is very tight. In such a case, the increase in trade surplus coming from the flow annuity will be small and overwhelmed by the negative impact from the sharply increasing search value. In other words, the surplus will decrease with x and, under Nash bargaining, so will the price. Monotonic selection would not be optimal for the seller in this case and will not characterize an equilibrium.

What happens when (10) is violated? It remains true that in any steady state equilibrium of the FIB model, V must be non-decreasing, and higher types must be selected with higher probability than lower types. However, without (10), equilibrium can involve complicated mixing (i.e., an "ironing" process) on the part of the seller to offset the force described in the paragraph above. We leave a complete treatment of this case to future study.

Monotonic equilibria of the FIB model do emerge in a wide variety of practical use cases. For instance, numerical calculations show that when utility flows are normally distributed with mean-to-standard deviation ratio of 2 or less, the condition for monotonic selection in Proposition 3 holds whenever $(I, \rho, 1-\beta) \geq (0.6, 0.01, 0.001)$.[17] As the mean-to-standard deviation ratio becomes large, the parameter region in which condition (10) fails likewise increases. For the remainder of the paper, we assume the model parameters are such that a monotonic equilibrium of the FIB model exists.

3.4 Numerical example and comparative analysis of PIB/NIB/FIB

To highlight key economic tensions in the three information settings, we provide an example roughly calibrated to the U.S. residential real estate market. Our main goal is to demonstrate that the NIB equilibrium tends to misallocate ownership and also leaves sellers worse off. This invites market structures that bridge the information gap between buyers and sellers—the topic of the remaining sections. Because the PIB model is consistent with standard search models, we use it as a benchmark for calibrating parameters

[17] If δ is linear, the condition for monotonic selection in Proposition 3 holds whenever $(I, \rho, 1-\beta) \geq (0.5, 0.01, 0.001)$.

to match stylized facts. We then compare differences in equilibrium quantities across the three models under these calibrated parameters.

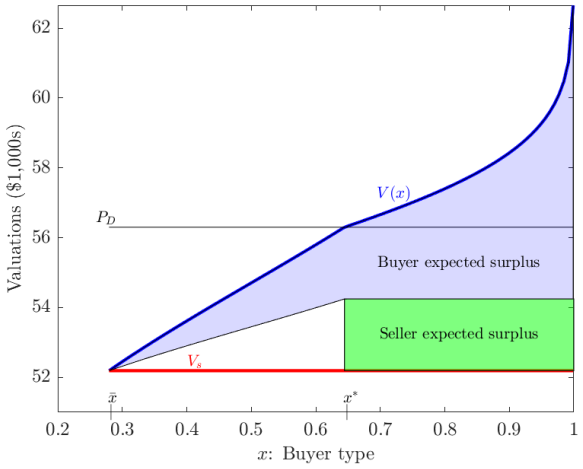
Each period represents a quarter, roughly coinciding with the shortest time between initial marketing of a property to the closing of a transaction. The mismatch shock parameter, ρ, is calibrated to observed turnover rates in owner-occupied homes. Specifically, we set $\rho = 1.718\%$ so that the median homeowner tenure is 10 years, consistent with figures in a recent Redfin study.[18] We set β to 0.9875, which, when annualized and for the holding horizon considered, is roughly consistent with the discount rates estimated in Figure 7 of Bracke, Pinchbeck and Wyatt (2018).

We assume that buyer utility flows are normally distributed with a mean and variance that reflect two aggregate assumptions when supply and participation are balanced (i.e., when $I = 1 - \bar{x}^{PIB}$): First, the average ownership rate is $\frac{2}{3}$, consistent with U.S. Census Bureau data.[19] Second, recalling that we normalize to zero the rental flow received by sellers, valuations and prices in the model must be interpreted as net of a rental equivalent. With that in mind, we calibrate the average model transaction price to \$60k, corresponding to capitalizing a perpetuity of \$3k per year using an annualized discount rate of 5%. The \$3k perpetuity crudely corresponds to the average annual premium home owners have paid relative to renters since 2000.[20] We stress that this calibration represents an oversimplification in which there are no private valuations in the rental market and where a mismatched owner that is unable to transact is always able to rent their house on a quarter-by-quarter basis.

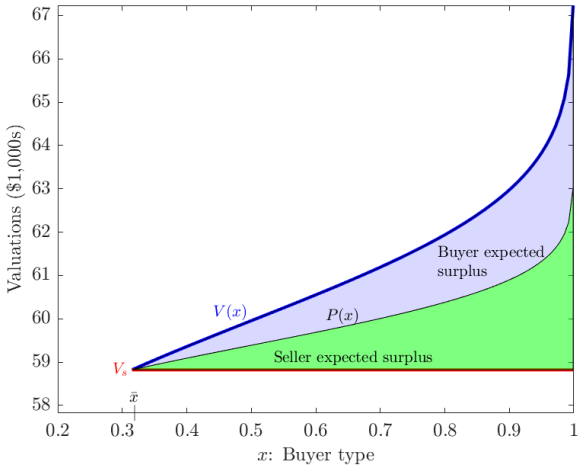
The resulting quantities are as follows. The stock of homes suitable for owner-occupiers is $I_{baseline} = 0.6848$. The mean value of $\delta(x)$ is \$1,137 per quarter, and the standard deviation is \$815. The mass of permanent renters, $\bar{x}^{PIB}$, is equal to 31.52% of the population and the mass of active searchers per quarter in the steady state is roughly 2% of the population.

Figure 2 plots the key valuation variables for the three information settings against the buyer's type, keeping all calibrated parameters constant across settings. Each plot documents the seller's valuation, V_s, and the buyer's valuation, $V(x)$. The region between these two graphs represents the surplus that is created through individual trade. For the

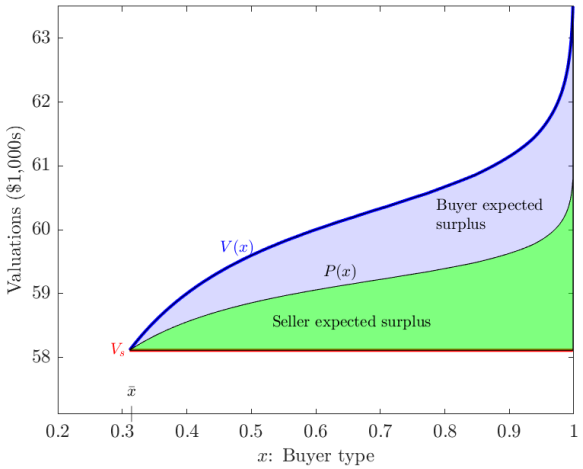
[18] See `https://www.redfin.com/news/homeowner-tenure-2023/`.
[19] See https://fred.stlouisfed.org/series/RHORUSQ156N.
[20] See http://bit.ly/41Tnj5a.



A. No information and bargaining (NIB) model



B. Partial information and bargaining (PIB) model ("standard search")



C. Full information and bargaining (FIB) model

Figure 2: **Comparison of models calibrated to US residential market**
Assuming buyer utility flows are normally distributed, we calibrate the model parameters to match stylized facts from the US residential market in the partial information steady state. The same parameters are then used for the full and no information models.

NIB setting, where there are multiple equilibria, we report the lowest aggregate welfare equilibrium results.[21]

In the NIB setting, bargaining fails half the time for $x < x^*$. Using the baseline calibration parameters, this roughly corresponds to a 25% reduction in transaction rates relative to a PIB or FIB economy with the same level of participation. When a transaction with a type $x < x^*$ succeeds the buyer pays V_s for the asset. This is why the seller receives no surplus from trade when bargaining with a buyer whose type is below x^*. For any type at or above x^*, the asset transacts at an average of price of $(P^* + V_s)/2$, which is why the seller's expected surplus is constant across types with $x \geq x^*$ in Figure 2. The figure makes clear that, in the NIB model, buyers get half the expected surplus from trade when $x \leq x^*$ and more when $x > x^*$. In other words, the bargaining outcome in the NIB is "buyer friendly" relative to Nash (1950), reflecting the buyer's information rents. In the baseline calibration this results in a lower NIB equilibrium seller value relative to the PIB settings. But it is not obvious whether such a result holds more generally. Precisely because transactions with low valuation buyers fail in the NIB setting and do not fail in the PIB setting, sellers in the NIB setting may transact proportionally more with high-valuation buyers than they do in the PIB setting. It is therefore conceivable that the average NIB transaction price could be higher and this may work in the NIB seller's favor. As the next result establishes, with standard utility flow distributions, sellers' higher share of surplus in PIB bargaining is more important than any potential "drag" from transacting with low valuation buyers.

Lemma 2. *Suppose that the distribution of utility flows is uniform, normal, or exponential. Then* $V_s^{PIB} \geq V_s^{NIB}$.[22]

Moving to comparing the PIB and FIB information settings in Figure 2, we recall that bargaining will always result in a transaction and the two parties split the surplus equally — this is represented by the green and blue regions, which are separated by the bargaining price, $P(x)$. In the PIB model, the surplus and the price are affine in $\delta(x)$.

[21] In the NIB setting with the baseline parameters, the lowest aggregate welfare is roughly 0.5% lower than the highest aggregate welfare equilbrium.

[22] A more general sufficient condition on δ is as follows. Define $\nu(v) = \min_{\{x \in [0,1] \mid \delta(x) \geq v\}} x$, and suppose that for any v such that $\nu(v) < 1$, it is true that $\delta'(y)(1-y) - \delta(y) + v = 0 \Rightarrow \delta(y) < \frac{1}{1-\nu(v)} \int_{\nu(v)}^{1} \delta(t)dt$. Note that this condition on δ is independent of other model parameters and is satisfied by the distributions in the lemma.

Both the surplus and price in the FIB model appear more concave, meaning that sellers in the FIB setting earn relatively more profit when transacting with low types but relatively less when transacting with high types. Moreover, $V_s^{PIB} > V_s^{FIB}$ suggesting that PIB sellers' information advantage in the selection stage, in equilibrium, somehow translates into a *lower* present value from future transaction profits.

To understand this, first consider that, because sellers are informed about buyer types in the selection stage, high-type buyers in the FIB setting are more likely to transact than in the PIB model. This means that, in the FIB setting, there are fewer high-type searchers *and* those that do search are in a relatively better bargaining position thanks to their high likelihood of transacting next period. Thus, when it comes to transacting with high-type buyers, sellers do not fare as well in the FIB than the PIB model. This explains the relatively suppressed FIB versus PIB surplus for high types in Figure 2.[23] On the other hand, sellers in the FIB setting are able to choose the highest arriving type, something that the sellers in the PIB setting are unable to do. Clearly, the two effects are in tension with one another and it is unclear whether sellers, on balance, are worse or better off in the FIB world. In the calibrated example, the decrease in surplus from trading with high types dominates the advantage of selection and this is why $V_s^{FIB} < V_s^{PIB}$. This, of course, is one of the most interesting features of the FIB model, in that sellers would individually always prefer to know buyers' types in the selection stage but, in aggregate, can be made worse off when all sellers have access to such information.

When supply is balanced in the FIB setting (i.e., $I = 1 - \bar{x}^{FIB}$), the following provides sufficient conditions for the effects discussed above.

Lemma 3. *Assume* $I = 1 - \bar{x}^{FIB}$ *and define* $A = 1 - \beta(1-\rho)$, $a = A - \frac{\rho}{2}$. *If* $\frac{1}{\rho e + 1} \geq \frac{A}{a} - \sqrt{\frac{A}{a}\left(\frac{A}{a} - 1\right)}$, *where* e *is Euler's Number, then* $V_s^{FIB} < V_s^{PIB}$.

We note that the parameter condition in Lemma 3 is satisfied for ρ and β in the calibration.

[23] The effect, also discussed following Proposition 3, rationalizes the condition in (10). The latter requires, for each buyer type, the marginal increase in flow utility to be higher than the seller surplus lost from increasing the buyer's probability of transacting next period.

3.5 Price and welfare across supply and turnover assumptions

Figure 3 provides further model comparisons across different assumptions of supply and tenure. We pick the longest and shortest median national homeownership tenure durations from the past 20 years according to the Redfin study: 13.4 years and 6.5 years, respectively. For each tenure assumption, we calculate a baseline balanced supply, $I_{baseline}$, and recalibrate a benchmark PIB model just as before. Then, keeping all other parameters constant, we vary the supply assumption, I, and the information setting. For each combination of tenure and supply, and each of the three information settings, we calculate the equilibrium seller valuation and a measure of aggregate welfare. The latter is defined as the aggregate perpetuity of utility flow from steady-state equilibrium ownership:

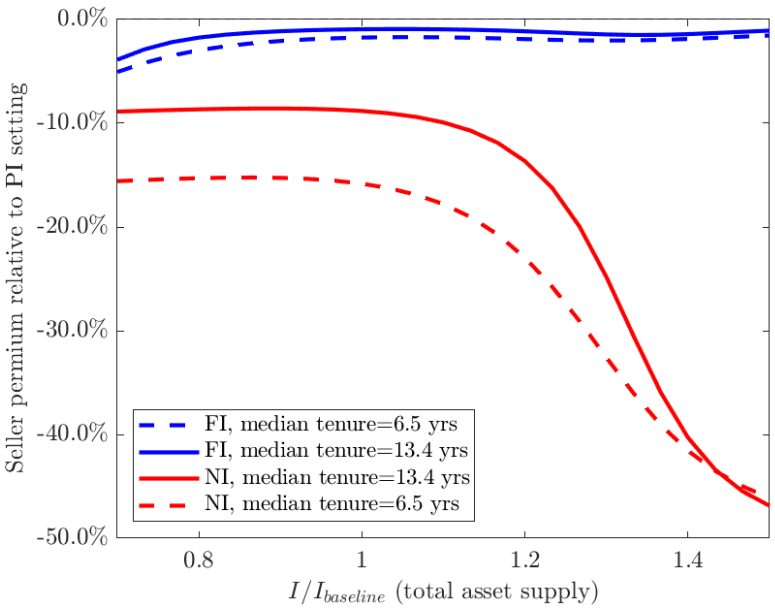
Definition 2. *The aggregate welfare in a steady-state equilibrium is*

$$W = \frac{\beta}{1-\beta}\int_0^1 m(x)\delta(x)dx.$$

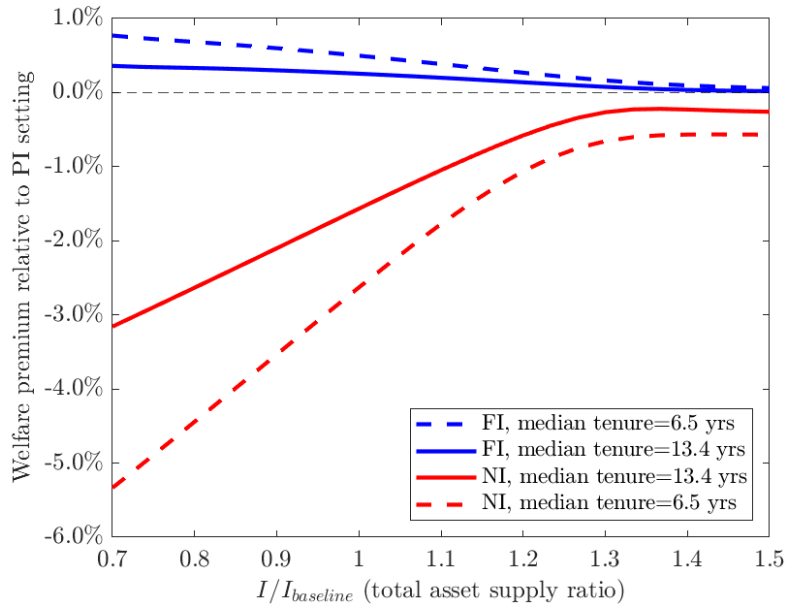
It can be shown that $W = (I-M)V_s + \int_0^1 \Big(m(x)G(x) + \big(1-m(x)\big)F(x)\Big)dx$, which is the sum of owners' valuations (both well-matched and mismatched) and searchers' valuations.

The seller valuation comparison demonstrates that, consistent with the discussion in the previous subsection, sellers' valuations are highest in the PIB setting. The difference between PIB and FIB settings diminishes with supply because the economic importance of selection diminishes when it is rare for sellers to be courted by more than one buyer. When supply is tight, however, high-valuation buyers wield increasingly more bargaining power and this overwhelms the benefits of informed selection. By contrast, V_s in the NIB setting ranges from being one half to 90% of V_s in the PIB setting. When there is an oversupply of mismatched assets, the market is flooded with ultra-low type buyers. This forces sellers to reduce P^* precipitously or risk a high probability of a zero profit transaction. As a result, V_s too drops markedly.

The welfare comparison figure depicts the ratio of aggregate welfare in the FIB (or NIB) setting to aggregate welfare in the PIB setting. In the PIB setting, random selection prevents the allocation of housing to buyers with the greatest utility flow. Because this improves in the FIB setting, the latter exhibits higher aggregate welfare. The effect, at least in our calibration, is small. It also declines with supply (I) and increases with



A. Seller valuations relative to the PIB (standard search) model



B. Aggregate welfare relative to the PIB (standard search) model

Figure 3:
Comparing equilibria across the information settings
Assuming buyer utility flows are normally distributed, we first calibrate the model parameters to match stylized facts from the US residential market in the partial information steady state. Two extreme scenarios of observed homeownership tenure are used. For each of the NIB, PIB, and FIB information settings the model is solved for different levels of supply (I) fixing all other parameters. Supply is normalized by $I_{baseline}$, corresponding to the supply in the PIB model that sets the total stock of housing fit for owner-occupiers equal to the mass of searchers.

turnover. As with the seller valuation figure, the supply dependence arises because the impact of selection diminishes as sellers face a declining chance of encountering more than one buyer. To understand the reason for welfare increase with turnover, first note that total ownership declines under high turnover. In the PIB setting, with random selection, the ownership density is constant across types. By contrast, in the FIB setting, the ownership density of high-valuation buyers remains relatively constant because they spend no more than one quarter unmatched. On the other hand, the density of FIB low-valuation buyers must decline substantially to account for the decline in total ownership as turnover increases. In other words, as turnover increases the gap in ownership density between high- and low-valuation buyers increases and this contributes to a more profound difference in aggregate welfare between the FIB and PIB settings.

The NIB setting additionally features the breakdown of bargaining (roughly a third of transactions in the example).[24] Intuitively, this would lead to lower ownership and thus lower aggregate welfare. One can see this most clearly in the case of oversupply as this is the limit in which selection does not matter but the bargaining setting does. Whereas the welfare differences between the PIB and FIB settings vanish with increasing supply, the NIB setting exhibits a persistent welfare gap. When supply limits participation, there is an offsetting force that potentially mitigates the NIB welfare gap. The lower seller valuation in the NIB setting invites greater participation by low valuation buyers who wouldn't otherwise enter and this additional participation can contribute somewhat to aggregate welfare. That said, the greater low type participation can also dilute ownership by high-valuation buyers (because of random selection), bringing down the latter's higher contribution to aggregate welfare. In the calibration, the marked decline in successful transactions overwhelms any positive welfare impact of higher participation by low valuation types. Overall, across the spectrum of supply scenarios considered in the figure, PIB and FIB aggregate welfare dominates NIB aggregate welfare.

One can generalize some of the welfare results observed in Figure 3. This is most easily done in the case where supply is balanced and when the endogenous entry threshold, $\bar{x}$, is the same across all three model — in other words, when one is able to control for the impact of endogenous participation and buyer arrival intensity in the search market.

[24]Here too, we select the lowest welfare equilibrium among the multiplicity of NIB equilibria, noting that the differences in welfare across the multiple equilibria tend to be small.

Lemma 4. *Fix $\beta, \rho, \delta(x), I$ and $\bar{x}$. Consider the three steady state solutions, not necessarily equilibria, corresponding to the specifications of $q(x)$ and $P(x)$ in each of the NIB, PIB, and FIB model. Then $W^{PIB} < W^{FIB}$. Furthermore if $I = 1 - \bar{x}$ then $W^{NIB} \leq W^{PIB}$.*

The Lemma compares welfare across three steady state solutions rather than steady state equilibria (see Definition 1). This is because, for some combinations of $\beta, \rho, \delta(x), I$, there may not exist equilibria with $\bar{x}$ equal across all three settings. Still, what we learn from Lemma 4 is that the welfare rankings demonstrated in Figure 3 can only be upset through endogenous entry or unbalanced supply.

Recall that in our main calibration, we find that $V_s^{\text{FIB}} < V_s^{\text{PIB}}$: informed selection harms sellers in aggregate because it strengthens the bargaining position of high-valuation buyers. Lemma 3 provides sufficient conditions for this ranking. Whenever this is true, it follows that the participation threshold is lower in the FIB setting. Using an argument similar to one used in the Proof of Lemma 4, it must be that $W^{\text{PIB}} < W^{\text{FIB}}$ in all such cases. This means that whenever informed selection harms sellers (relative to the PIB setting), it encourages more buyers to participate in search and improves total welfare.

4 Steady state equilibria with commitment

An important aspect of the model is that each individual seller will always prefer to learn about the types of the arriving buyers. For sellers, this could induce a tragedy of the commons where the production of information through the actions of one seller benefits that seller individually without impacting the demand side of the market, but when every seller acquires such information, high-valuation searchers wield more bargaining power relative to a world where their types are not known (e.g., see the calibrated example in the last section).

Proposition 4. *A seller in an NIB setting would always strictly prefer to know the arriving buyers' types just before the selection stage and/or during the bargaining stage, and a seller in a PIB setting would always strictly prefer to know the arriving buyer types just before the selection stage.*

In both the NIB and the PIB settings, just before the selection stage, the seller knows that the probability that more than one buyer may arrive is strictly positive. Moreover,

the expected transaction price in both settings strictly increases with type. Thus, the ability to pick the highest type buyer among arrivals makes the seller strictly better off in expectation.

An immediate implication is that whenever a technology for information revelation is available to sellers, they will always be willing to pay some non-zero cost to adopt it. In this section we explore two such devices. The first is certification via an intermediary. We show that when the services of a "perfect" information intermediary are available and their use is costless to buyers, buyers will commit to report their type through intermediation; an intermediation equilibrium equivalent to the FIB always exists whenever the latter exists. This analysis, provides a new foundation for brokers in an illiquid asset market as information intermediaries. Specifically, the role of the broker is not to provide their client with information about the asset but to provide information to their client's counterparty about their client's willingness and ability to pay. In doing so, a buyer broker increases their client's probability of transacting by both improving their chance of being selected and, subsequently, minimizing the likelihood of transaction failure. This role can exist even if there is no asymmetric information about the asset itself (as is indeed the case in our model). Moreover, by Proposition 4 sellers will always be willing to pre-pay some positive amount for intermediation that would be costless to buyers, an arrangement that is frequently seen in practice.

The second device we consider allows buyers to make committed bids and, in turn, allows sellers to commit to honor a selected bid. We show that even if buyers can commit to bids, which ultimately reveal their types, a seller who solicits (and honors) bids may be worse off than they would be from bargaining *even without knowing the buyer type*.

4.1 When buyers can commit to certification: Intermediation

To begin, suppose that a buyer can commit to engaging the services of a zero-cost "broker" who works closely with the buyer and (because they are part of a reputational network) will faithfully and credibly convey the buyer's type to the seller.[25] Essentially, the broker acts to vouch for or certify their client's type and, by doing so, maximizes their chance of transacting in the competitive environment. Will the buyer have the incentive to utilize

[25]While we do not explicitly model the details, we note that certification networks have been explored in the literature in other contexts (see, for instance Chemmanur and Fulghieri, 1994; Lizzeri, 1999).

such intermediation?

We say that an equilibrium is *fully revealing* if the set of participating buyer types that do not reveal their private information through certification is of measure zero.

Proposition 5. *Assume that,*

i. At zero cost, a buyer can have their type credibly certified prior to either the selection or the bargaining stage.

ii. The bargaining solution concept remains random proposer (and, given that the technology exits, a proposal may specify if the buyer must have their type certified and a price contingent on the result).

Then any steady state equilibrium is fully revealing.

That a proposal can include a specification regarding certification is conceptually consistent with the Peski's 2022) framework in which stakeholders bargain over *mechanisms* for splitting surplus under one-sided asymmetric information. The existence of the certification technology then plays a key part at both the selection and bargaining stages. Using backward induction: at the bargaining stage, regardless of the seller's belief, each party will get their Nash-split of the true type-dependent surplus. This is because the buyer's offer is (as always) V_s, and the seller's offer is for the buyer to get certified and pay $V(x)$. So, in expectation, each receives half the available surplus, as they would under complete-information Nash bargaining.

Moving back to the selection stage, because the transaction surplus will be the same whether they reveal or not, buyers need only consider how revelation affects selection. Relative to the standard disclosure games with exogenously specified payoffs, there are subtleties due to our value functions being endogenous. Nevertheless, we still establish the familiar "unraveling from the top" property (see, for example, the review article by Verrecchia, 2001), as high types have the most to gain at the selection stage by revealing their type.[26]

[26]Glode, Opp and Zhang (2018) study the incentives for voluntary disclosure from the buyer in a model without selection pressure (the seller and buyer are already paired) and in which the seller will subsequently make a take-it-or-leave-it price offer. They show that the buyer partially discloses her private information to avoid the inefficiencies of the seller-optimal screening via price.

Corollary to Proposition 5. *If* (10) *is satisfied, then the monotonic steady state equilibrium from the FIB model coincides with the steady state equilibrium of the intermediation model.*

The "broker" device in Proposition 5 need not be costless for the results to go through. What is required is that the buyer's choice to use the device does not reduce the total surplus to be split between the buyer and seller.[27] Indeed, in the residential real estate market of the United States, at least until recently, the buyer's broker compensation has typically been specified in the brokerage contract between the seller and their representative. These contracts typically stipulate a fixed percentage commission that is split between the seller's and buyer's brokers (if the buyer is represented), or paid entirely to the seller's broker if the buyer has no representation. As long as such a contract is prevalent, buyers have little to no incentive to avoid the help of a broker and it is under such conditions that Proposition 5 holds.

If, on the other hand, the decision to credibly disclose their type is costly to the buyer, full unraveling will fail.

Lemma 5. *Assume that,*

i. Subject to a non-zero reduction in total trade surplus (if selected), a buyer can credibly report their type at both the selection and the bargaining stage.

ii. The solution concept for bargaining with a buyer of unknown (known) type is random proposer (Nash) and the seller's take-it-or-leave-it offer may include requiring the buyer to credibly report their type.

Then a fully revealing steady state equilibrium does not exist.

Under costly disclosure, sellers each face a dilemma. They must either accept a certain degree of transaction inefficiency (Lemma 5) or agree to a reduction of an ex-ante trade surplus (by pre-paying for buyer broker services whether or not a broker is used). The dominant effect depends on the model parameters, including the cost of certification. Some model parameters will, while other will not, favor an equilibrium in which sellers agree ex-ante to pay for buyers' brokers.

[27] In particular, the results hold as long as a seller cannot (or has no incentive to) dispense with prepaying for intermediation and that the total transaction surplus, net of disclosure costs, is non-decreasing in x.

4.2 When buyers can commit to bids: Auctions

Until now, we have assumed that arriving buyers cannot make credible, committed bids prior to the seller's counterparty selection. In this subsection, we endow buyers with this ability. To begin, assume that upon arrival, every buyer submits a bid to the seller, the seller selects a buyer who submitted the highest bid (ties broken randomly), and the transaction price equals the selected buyer's bid.[28] In particular, buyers do not know the number, types, or bids of the other arriving buyers when they submit their bids.

We are interested in equilibria with a committed bid-price schedule, $P^A(x)$, such that

i. $P^A(x)$ is strictly monotonic in x above the participation threshold.

ii. $P^A(x)$ is optimal and incentive compatible (i.e., a buyer of type x has no incentive to make a bid that deviates from $P^A(x)$).

Note that under such a price schedule, a steady state equilibrium is also one that fully reveals the types and is, therefore, informationally equivalent to the FIB equilibrium.

Remark 3. *There is no requirement at this point that transacting at the highest bid is better for the seller than bargaining. Determining whether the seller might prefer to do something else, if other options are available, is something we consider after characterizing the steady state committed-bid equilibrium.*

We use the term *auction* informally to refer to this solicitation of committed bids, coupled with the understanding that sellers will honor the bid price of the highest bidder. We refer to $P^A(x)$ as an auction bid schedule. Define V_s^A as the seller's valuation in a steady state equilibrium with price schedule $P^A(x)$ and the FIB (i.e., full information) transaction probabilities $q(x)$. Correspondingly, define $\bar{x}^A$ to be the equilibrium participation type threshold.

Proposition 6. *For P^A to be a committed bid schedule that is strictly monotonic and incentive compatible for the buyer, it must satisfy the ODE (ordinary differential equation),*

$$P'(x) = \frac{1 - m(x)}{I - M} S_b(x) \quad \textit{such that} \quad P(\bar{x}) \geq V_s^A. \tag{11}$$

[28] For example, the buyers are made to commit the necessary funds to an escrow account.

To help interpret the differential equation for the auction price, we note that $\frac{1-m(x)}{I-M} = \frac{q'(x)}{q(x)}$ so that (11) can be rewritten as $q(x)P'(x) = q'(x)S_b(x)$. A type-x buyer will be willing to pay a higher price for a greater chance at being selected, right up to the optimal bid where the marginal benefits from receiving their surplus from trade is offset by the higher price to which they must commit. Because the probability, $q(x)$ is monotonic in x, the price is also monotonic as long as the buyer's surplus is weakly positive (which is a precondition for trade).[29]

The threshold bid price $P(\bar{x})$ can be interpreted as a reserve price adopted by all sellers, as would be consistent in a symmetric equilibrium, and must satisfy the condition in Proposition 6. To be an equilibrium, the reserve price cannot be less than the seller's valuation, V_s^A. The next proposition establishes that an infinitesimal seller, if given the choice and if the prevailing reserve price is higher than V_s^A, will strictly prefer to reduce their own reserve price to attract more buyers. As a result, $P(\bar{x}) = V_s^A$ characterizes the unique steady state equilibrium that is symmetric across sellers and that satisfies Proposition 6 with an optimally set seller reserve price.

Proposition 7. *Assume that buyers can make committed bids, and that each seller selects a reserve price and must honor the highest submitted bid that exceeds the reserve price. Then there exists a unique symmetric steady state equilibrium. The equilibrium is characterized by $P^A(x)$, satisfying* (11)*, with reserve price/initial condition $P^A(\bar{x}) = V_s^A$, and $q(x)$ as specified in Proposition 3.*

Intuitively, this result is driven by competition between sellers and competition between buyers. The former means that the actions of a single seller cannot influence the rest of the market. Relying on the latter, the proof demonstrates that, when a single seller deviates from the prevailing reserve price, their corresponding flow of participating buyers of a given type adjusts so that the arrival rate and bid price equate to those in prevailing auctions. As a result, increasing or decreasing a price-taking seller's reserve price amount to shifting the participation threshold without impacting the price or arrival rate above that threshold. In turn, monotonic selection means that increasing participation (by reducing the reserve price) makes the deviating seller strictly better off.

[29]The proof to Proposition 6 establishes that $P(x)$ is indeed strictly monotonic in the interval of participating searchers. Below the participation threshold, $P(x)$ is set equal to $P(\bar{x})$.

Seller incentives when buyers can make committed bids

In Section 3, we assumed that buyers cannot credibly commit to bids and hence do not compete on price — they would always simply renege on any promised high bid and resort to bilateral bargaining. Proposition 7 is derived under the assumption that buyers have the ability to commit to bids that are incentive compatible prior to selection. But what about sellers? The proposition assumes that they have no choice but to honor an accepted (highest) bid. If it is possible for a seller to renegotiate with the winning bidder then, absent the ability pre-commit she may wish to do so. To maintain consistency with previous assumptions, we assume that any renegotiation would result in the Nash bargaining outcome.[30]

The auction price cannot be ex-post optimal for the seller relative to Nash bargaining. This is because endogenous entry at $\bar{x}^A > 0$ requires there to be zero trade surplus at $\bar{x}^A$, which means that the auction price is locally flat at $\bar{x}^A$. The slope of the transaction price resulting from Nash bargaining, however, is strictly positive at $\bar{x}^A$.[31] As a result, there will always be some buyer types with whom the seller would prefer Nash bargaining over a winning bid of $P^A(x)$. This is formalized in the following:

Lemma 6. *In the steady state auction equilibrium of Proposition 7, from the perspective of a price-taking seller, the Nash bargaining transaction price will dominate $P^A(x)$ over some non-empty interval of buyer types in $[\bar{x}^A, 1]$.*

Summarizing, from a seller's perspective, accepting the highest committed competitive bid may not be ex-post optimal relative to Nash bargaining, and will certainly fail to be so with low valuation buyers under standard utility flow distributions (e.g., normal). This does not mean that committed bidding is bad for the seller. It does mean that additional assumptions are required to ensure that the seller's behavior is compatible with honoring committed bids. For instance, if honoring bids is ex-ante optimal for the seller, they may be willing to pre-commit to accepting a selected buyer's bid via an enforceable contract. When will this be the case? The ODE in Equation (11) suggests that the bid price

[30]Because the equilibrium bid schedule P^A is strictly increasing, the seller learns the buyer valuations. If she then reneges on the auction and attempts to negotiate with the highest-value buyer, this is a bargaining problem analogous to in the FIB environment. We therefore assume adherence to the same bargaining solution concept. Notice however, because this represents the deviation of a single seller, the endogenous auction-equilibrium valuations of the seller and buyer that form their respective disagreement values will differ from their equilibrium values in the FIB environment (Section 3.3).

[31]This can be gleaned from the explicit solutions in the Proof to Proposition 3.

derivative is higher in tight markets (i.e., when $I - M$ is small). For sufficiently low supply, I, and for enough high-valuation types, the transaction price from competitive bidding may exceed the Nash split price and the seller will be willing to commit, ex-ante, to the bid price. On the other hand, in thin markets the bid price is slower to increase with type and can be so flat so as to be ex-post dominated by the Nash split price.[32] In such markets, sellers could not be enticed to to commit, ex-ante, to the bid price.

Figure 4 illustrates this situation in the context of the housing market calibration explored earlier. Using the benchmark calibration parameters, the top plot compares the equilibrium transaction prices, for a steady state equilibrium with auction prices that solve the ODE in (11), to the Nash split price corresponding to sellers' valuation plus half the auction surplus. For all participating types, committed bids honored by the seller result in a lower price than would a Nash split of the surplus. Thus any individual seller will prefer certification over an auction, if the former were available. This also suggests that an auction-like steady state equilibrium may not arise as an exclusive solution to address information asymmetry in search markets with parameters similar to those in our example.

The reason for the lower auction price is the relative lack of competition in the baseline parameterization (where the stock of housing roughly equals the mass of participants). Specifically, there is a 22% chance that an arriving buyer is the only bidder in an auction, creating a disincentive to submitting high bids. It should therefore not come as a surprise that seller valuations are lower in an auction equilibrium than in a Nash bargaining equilibrium ($V_s^A = 56.8$ versus $V_s^{FIB} = 58.1$) for the benchmark parameters. This, despite the fact that the two settings are informationally equivalent.

The preceding discussion suggests that a lower asset supply (i.e., greater market tightness) will improve the auction's ability to secure high prices for the seller (much as they do in a one-period auction where more than one bidder is required to show up). This is confirmed by the bottom plot of Figure 4 where we consider the pre-selection expected seller surplus generated by an auction steady state equilibrium solution, as well as that of the out-of-equilibrium alternatives of no-information and full-information selection and

[32]Intuitively, thin markets are those in which, it is unlikely that more than one bidder arrives. In such a situation, all arriving buyers will bid close to the seller's reservation price because they do not anticipate competition and believe that the seller will honor the winning (and only) bid.

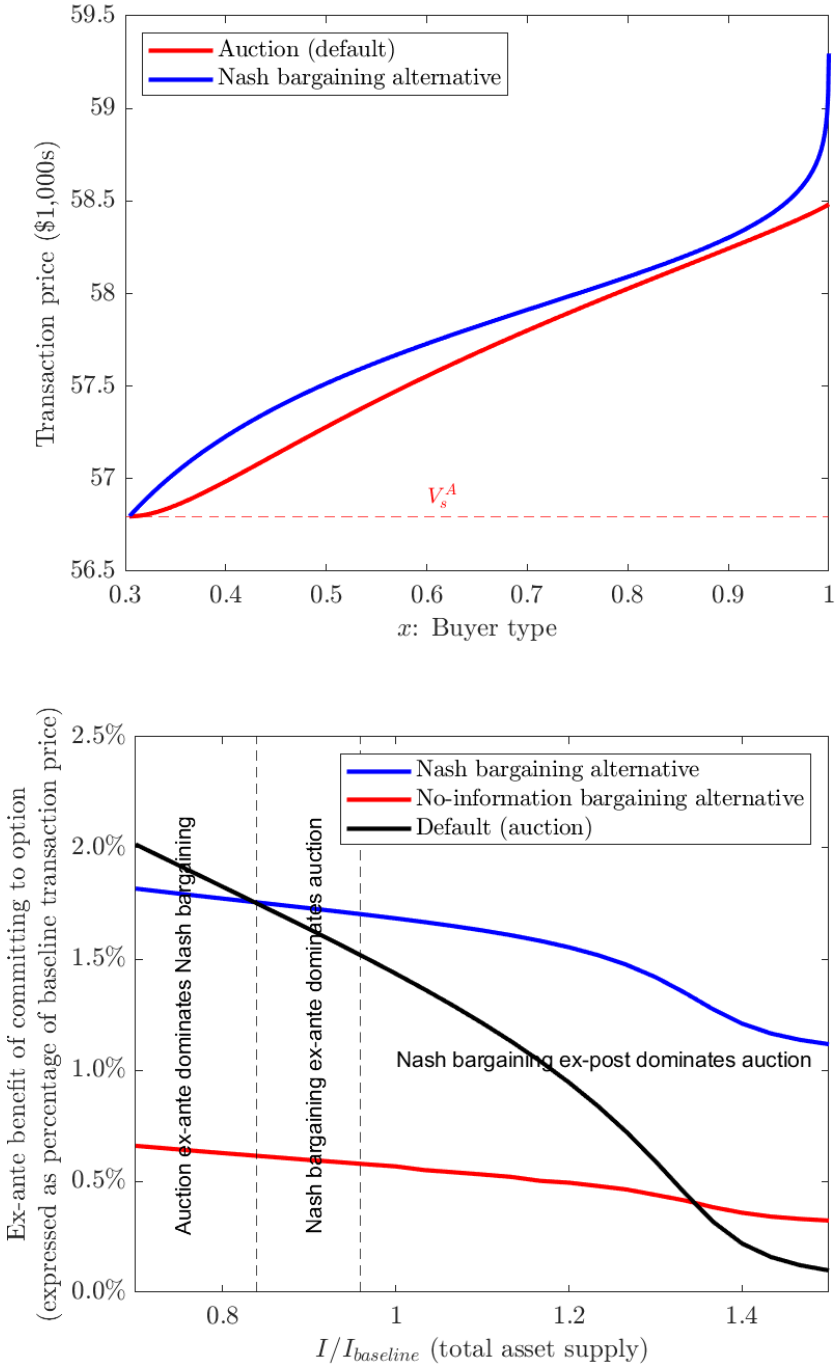


Figure 4: **Solution comparisons in a committed bidding setting**
The top plot depicts, for the benchmark housing market calibration, the Nash split price and $P^A(x)$ from (11) in a steady state equlibrium settings where buyers can commit to bids and sellers must honor the selected bid (i.e., an auction). The bottom plot depicts the benefits, to an infinitesimal seller, of deviating from the auction equilibrium. Two deviations are considered from the default (auction) choice: Nash bargaining and no-information bargaining. The parameters used are from the calibration to the real estate market explored earlier. Supply is normalized by $I_{baseline}$, corresponding to the supply in the PIB model that sets the total stock of housing fit for owner-occupiers equal to the mass of searchers.

bargaining. In considering the alternatives, it is assumed that search is not directed.[33] In the figure, and slightly below the balanced supply point, Nash bargaining (and therefore intermediation) no longer ex-post dominates the auction. When supply declines about 15% relative to benchmark, the auction becomes ex-ante preferred which means that the seller would be willing to commit to it before buyers arrive. Commitment is always necessary, however, because the auction can never be ex-post optimal (as guaranteed by Lemma 6).

The graph also demonstrates that the auction is typically preferred to the no-information selection and bargaining alternative. However, at some point as supply increases, lack of competition among auction bidders becomes more costly to the seller than the inefficiency of bargaining under incomplete information, and dispensing with the auction in favor of no-information bargaining is preferable. In other words, sellers do poorly with auctions when markets are thin.

4.2.1 Deposits

It is sensible to ask whether buyers are able to signal their types without an external device, such as intermediation, and without auctioning the asset, which we've shown may not serve the seller well. One device that naturally comes to mind is a weakly positive deposit, which in practice is often called "earnest money". Can a deposit signal earnestness in our model? To do so, a deposit would have to be positive, monotonic in types, and incentive compatible from the perspective of buyers. Here we show that such deposits cannot be fully revealing. The intuition is as follows: A process that differentiates buyers through their willingness to commit deposits is essentially an auction for the right to be selected. Because under full information revelation transacting under Nash bargaining is certain, such an auction is intuitively equivalent to directly auctioning the asset with the auction price being the sum of the deposit and usual Nash bargaining price. Lemma 6 and the example at the top of Figure 4, however, demonstrate that the auction price is less than the usual Nash bargaining price for some buyer type. The implication is that, to be fully revealing and optimal, the deposit must be negative for some participating buyer type.

[33] In other words, sellers do not "advertise" the process by which the asset is sold, but the process is determined when the first buyer arrives. We discuss this assumption more fully in the next section.

Suppose that buyers, upon arrival, place a deposit of their choosing in an escrow account observable to the seller. The seller chooses among the buyers and then bargains (Nash, if types are known and random proposer otherwise). The deposit is refunded to unselected buyers. If bargaining with the selected buyer results in a transaction, the deposit is applied against the bargaining price (i.e., it is refunded to the buyer). Otherwise, the seller keeps the deposit. We are interested in searching for a solution for the incentive compatible deposit schedule that is strictly monotonic and weakly positive. The former condition is required for full information revelation. The latter condition reflects the assumption that sellers cannot or will not commit to compensating buyers in the event of a failed transaction.

We start by assuming an equilibrium deposit schedule, $D(x)$, that is fully revealing and characterize the corresponding bargaining solution. In bargaining with a type x buyer, the seller's outside option, should the transaction fail, is $V_s + D(x)$. The buyer's outside option remains their search value, $F(x)$, because their deposit is sunk. If a transaction is consummated at a contract price of $P(x)$, then the buyer only pays $P(x) - D(x)$ but the seller receives $P(x)$ (because the deposit is used to pay for the shortfall). Thus the seller's surplus from trade is $S_s(x) = P(x) - \big(V_s + D(x)\big)$ and the buyer's is $S_b(x) = V(x) - \big(P(x) - D(x)\big)$. The total surplus from trade is $S_{\text{tot}}(x) = V(x) - V_s$, as before. Nash bargaining then yields a price equal to the seller's outside option plus half of the total surplus:

$$P(x) = V_s + D(x) + S_{\text{tot}}(x)/2 = D(x) + P^N(x),$$

where $P^N(x) = \frac{1}{2}\big(V(x) + V_s\big)$ is the usual Nash bargaining solution ignoring the deposit. In other words the deposit amounts to a transfer from the buyer to the seller above and beyond the usual Nash bargaining solution.

An arriving buyer of type x, taking the fully revealing deposit and bargaining price schedule as given, will place a deposit as if they were type y to maximize their expected surplus from being selected and then transacting at $P(y)$. Note that this assumes the buyer bargains as if they are type y, which is consistent with the seller's beliefs in a fully

revealing equilibrium.[34] The buyer's expected payoffs from this strategy is

$$U(y,x) = -D(y) + q(y)\big(G(x) - P(y) + D(y)\big) + (1 - q(x))\big(F(x) + D(y)\big).$$

Because transactions do not fail in a fully revealing equilibrium, deposits are always refunded. In turn, this means that $U(y,x) = F(x) + q(y)\big(V(x) - P(y)\big)$. The deposit only enters into the arriving buyer's utility through the bargaining price, $P(y) = D(y) + P^N(y)$.

Arriving buyers optimize over y. Solving for the first order condition and setting y to x, which is the equilibrium condition, results in an ODE for $P(x)$ that is identical to equation (11) for the auction price. Because buyer surplus is zero for the threshold entrant type, $\bar{x}$, it must be that $D(\bar{x}) = 0$ and $P'(\bar{x}) = 0$. Moreover, because $\frac{d}{dx}P^N(\bar{x}) > 0$, the identity $P(x) = D(x) + P^N(x)$ implies that $D'(\bar{x}) < 0$.[35] Thus, for a fully revealing deposit equilibrium to exist, there must be some type above the threshold entrant whose deposit is negative. Said differently, there is no weakly positive deposit schedule that results in a fully revealing equilibrium. We also note that, at least for some parameter values including our baseline calibration, the optimal deposit schedule is non-monotonic which precludes full type revelation.

In practice, non-refundable deposits are prevalent. Based on the preceeding analysis, their role may not be to stratify all serious buyers. Instead, they may exist to weed out "irrational" or "noise" buyers whose private valuation may be too low to transact. Likewise, a non-refundable deposit can compensate the seller if the buyer changes their mind after due diligence (i.e., because of imperfect information about the asset). In both cases, the deposit compensates the seller for carrying the asset an extra period (i.e., time value of money). In our property market calibration, this would amount to 1.25% of the value of the property, which is roughly consistent with typical deposits in that market.

4.3 Symmetric pure equilibria: auctions or intermediation?

We now combine key elements of the certification and auction approaches into our search framework. Our goal is to explore circumstances under which distinct market solutions may arise in response to the problem of searching and matching under incomplete infor-

[34] Alternatively, one can view this as assuming that a selected buyer that deposits $D(y)$ must either pay the price that would result from Nash bargaining with type y or walk away from the transaction.

[35] The proof to Lemma 6 establishes that $\frac{d}{dx}P^N(x) > 0$ for all entrants in an auction equilibrium.

mation when counterparty selection precedes a settlement phase. Specifically, we assume that, upon arrival, buyers are able to make committed bids or credibly report their types (e.g., by engaging a broker) at the seller's behest. In turn, sellers are able to commit, just before the selection stage and buyer arrival, to either bargain with a selected buyer or to honor any selected buyer's committed bid.[36] We also assume that there is no direct cost to buyer and seller choices. Note that under our zero-transaction-costs assumption there is no need to consider the option of no-information bargaining because it will always be dominated by certification (Propositions 4 and 5). We later return to briefly discuss the impact of transaction costs.

The analysis in the previous section suggests that market tightness is a key determinant of the benefits of certification over auctions, and vice versa. To explore this, we return to the housing example and look for pure symmetric steady state equilibria — those in which, just prior to the arrival of buyers, either all sellers find it ex-ante optimal to opt for credible bid solicitation (i.e., an auction) or all sellers instead opt for bargaining. In other words, we look for stable pure market structures as a function of model parameters.

Given a fixed set of parameters, we test for stability in a pure market structure by first solving for the each of equilibria in Propositions 5 and 7. Next, for each of these pure equilibrium market structures, we test whether a single seller would prefer to switch strategies. For instance, in the pure certification equilibrium, we test whether a seller will optimally deviate, prior to the selection stage, by committing to honor any selected committed bid from arriving buyers. A pure certification equilibrium is one in which individual sellers have no incentive to deviate. A similar exercise is undertaken to ascertain whether, for the same fixed parameter set, a pure auction equilibrium exists. In principle, it is possible that a choice of parameters is characterized by multiple, unique, or even no pure equilibria.

The plots in Figure 5 depict the results from an exercise in which we explore parameter sets in the housing example that are characterized by stable and pure market structures. Both plots exhibit three distinct regions. In the first region, bounded by the black solid curve, the unique pure equilibrium is certification (i.e., "Intermediation", presumably

[36]Note that the sellers' choices of auction or bargaining do not direct buyer searches. Beyond the technical complications involved, sellers in practice rarely advertise their commitment to a specific transaction framework in order to compete for buyer flow (e.g., "we'll be entertaining sincere bids but no bargaining will follow"). We note that this is distinct from the practice of posting minimum acceptable bids ahead of bid solicitation. Footnote 37 discusses how this affects results.

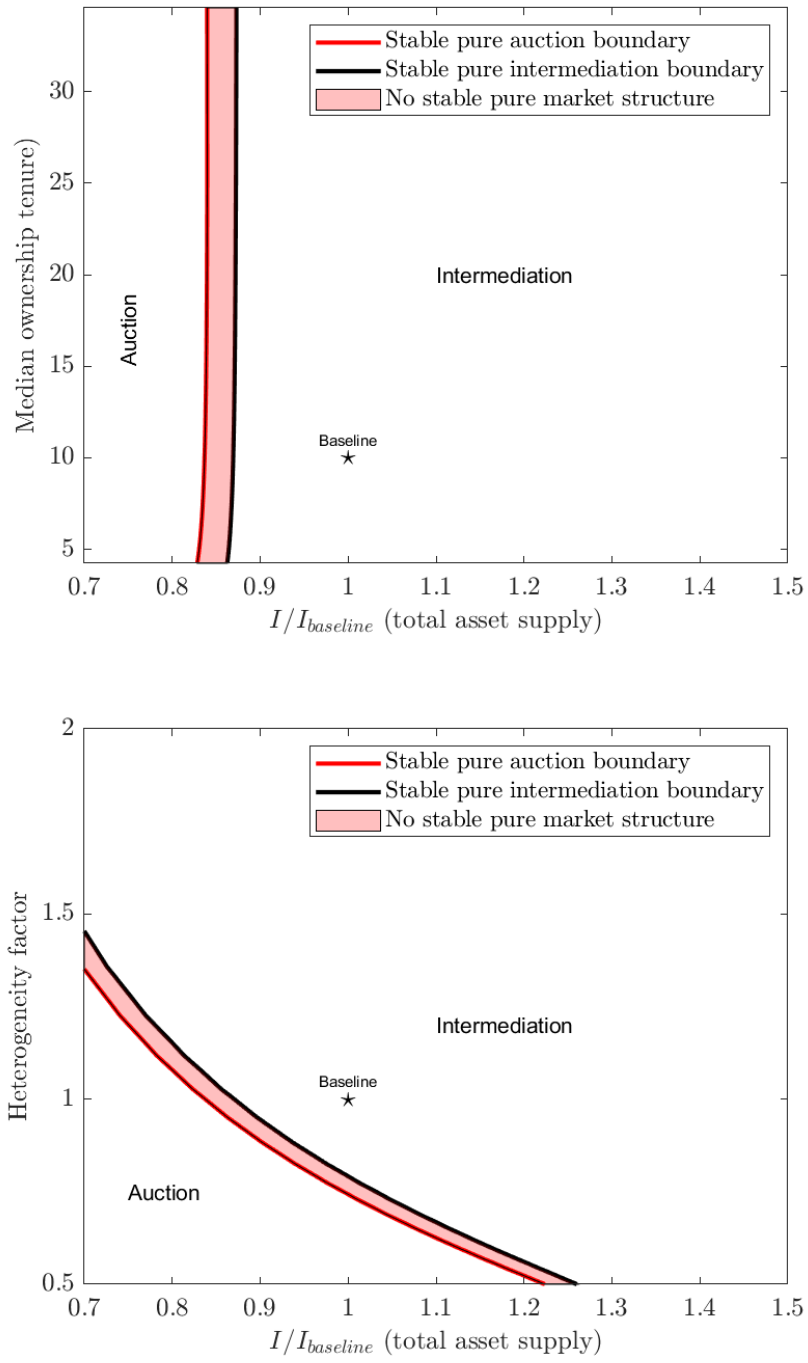


Figure 5: **Pure market structures**
Each plot depicts the prevailing search market structure as a function of key model parameters in the housing example of Section 3.4. Buyers can credibly commit to bids or to certification, whereas sellers can commit to either bid solicitation or bargaining just before buyers arrive. The baseline parameters are denoted by a "star" in the plots. The Intermediation (Auction) region corresponds to parameter combinations in which certification (bid solicitation) is the unique pure steady state equilibrium. Supply, on the horizontal axis, is normalized by the baseline supply $I_{baseline}$. Median ownership tenure is linked to ρ in the model. The heterogeneity factor is a multiplier applied to the baseline standard deviation of buyers' utility flow distribution.

through a broker). This region includes the baseline parameter set (denoted with a star symbol). A second region, bounded by the solid red line, admits only a pure auction equilibrium. The third region, shaded between the black and red curves, is one in which no pure equilibrium exists. Although we do not explore this here, it seems reasonable to expect that a mixed equilibrium can exist in this third region (i.e., some sellers selecting certification and others selecting auctions). Absent is a region in which both types of pure equilibria exist.[37]

The top plot in Figure 5 suggests that the equilibrium structure in the example does not critically depend on the median asset holding period (i.e., on ρ). As expected, though, the structure is sensitive to supply as this impacts market tightness and thus sellers ability to extract value from buyers in an auction where the number of participants is not known.

In the bottom plot, we vary both supply (relative to the baseline) and heterogeneity of buyer utility flow from ownership. Recall that, in the example, buyers' utility flow is assumed to be normally distributed across types. The 'Heterogeneity factor' is a multiplier applied to the standard deviation of this distribution. When buyers are very similar (low heterogeneity), there is little uncertainty across types and certification provides little informational benefits. Utility flow will be 'flatter' with respect to type, which is a property that will be inherited by bargaining prices. In particular, the fact that auction prices are flat near the participation threshold will no longer provide bargaining with as much of an advantage. Finally, because bilateral Nash bargaining limits sets the proportion of surplus that a seller can extract to one half, the limit of identical buyers leads to no informational benefits from certification while at the same time limiting the benefits from direct price competition among buyers. Thus, unless markets are very thin, when buyers are relatively homogeneous the competitive element of auctions will be more appealing to the seller than the pure information provision role of certification. Correspondingly, when there is a great deal of buyer heterogeneity, the informational role played by certification is critical while also allowing sellers to extract more from low valuation buyers than would an auction.

[37] Consider the alternative specification from footnote 36 in which arrival flows are influenced by sellers' choices to either bargaining or solicit bids. Although tedious, one can show that the same unique pure equilibria reappear when markets are sufficiently tight or sufficiently thin. However, under the baseline parametric assumptions, there exists an intermediate region in which both pure equilibria exist.

5 Further Discussion

Our idealized brokers, upon being hired by a buyer, receive a perfect signal of the buyer's type and convey this to the seller (or the seller's representative). In practice, any signal a buyer's broker may receive about their client's type is unlikely to be perfect. We note, however, that auctions too are neither costless nor perfectly aligned with the informational assumptions we model. Our goal here is to outline the performance of "ideal" devices. As long as brokers and buyers both wish to minimize the likelihood of transaction failure, and brokers' credibility is motivated by their membership in a reputational network, it seems plausible that they will accurately vouch for their clients.

5.1 The impact of transaction costs

Thus far, we have treated certification and committed bid solicitations as costless alternatives. In practice, effective certification through a trusted intermediary (e.g., a broker) requires setup and maintenance costs for the reputational network, as well as direct compensation for the intermediaries who perform the service. Likewise, committed bid solicitation involves costs including those needed to set up and maintain the necessary contracting and enforcement mechanisms. This may be complicated by buyers' need for raising outside capital, as a committed bid would entail the additional commitment of stakeholders who will not have direct ownership stake in the asset. In short, one anticipates that costs would be associated with both transaction processes.

To the extent that the costs to certification and committed bid solicitation are similar, one might expect that the analysis depicted in Figure 5 might remain largely intact. What may change, however, is the relative value to no-information bargaining. For instance, in a market in which transactions are neither frequent nor sufficiently lucrative, auctions would yield poor expected outcomes (see the bottom plot in Figure 4). In addition, there may not be a rationale for the development of a widely used broker (i.e., certification) reputational network. In other words, certification could exist but only to serve the highest valuation buyers. One might expect that lower valuation searchers would largely be confined to informal search (e.g., classifieds) and that the associated bargaining would be prone to failure. The Corollary to Proposition 5 hints at this by establishing that whenever participation is not full and buyers have to pay for brokers, a pure certification

equilibrium is not possible.

Summarizing, although we do not explicitly model transactions costs, our work does provide clear intuition for how they may be incorporated into the framework as well as the impact they would have on market structure.

5.2 Future directions

There are several compelling directions to further explore. The first concerns the nature of the search market when supply can change in response to production or depreciation of the mass of available assets. Yet another involves incorporating heterogeneous seller private values (e.g., heterogeneous rental rates). This would, presumably, increase the value of intermediation as each side would benefit from the employment of an intermediary by the other. It may also, however, lead to non-trivial heterogeneity in the use of market solution approaches. I.e., some seller types may prefer auctions while others may prefer bid solicitation.

While we have referred to certification as a mechanism that requires intermediation, it would be illuminating to more formally model a reputational broker network where the intermediaries receive an imperfect signal of their client's type and are compensated for their services in a competitive environment.

Finally, with some modifications, our framework could be applied to the labor search markets, where similar informational frictions exist (selection and bargaining) as do broker networks. In this context, "owners" would be workers and "buyers" would be firms. One key difference would be the potential inability of firms to sell labor contracts, impacting the formulation of buyers' continuation value. Because of the inalienability of human capital, another key difference would be inability of workers to commit to accepting a solicited bid for a labor contract.

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Appendices

A Proofs

* *

Preliminary Draft

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Proof of Proposition 1 After setting $x^* = \bar{x}$ in the NIB model, the expressions for m and M, and their continuity in the model parameters (and $\bar{x}$) are a special case of the more general derivation in Proposition 2. The reader is therefore referred to the proof of the latter. The expressions for q_s and $q(x) = q$ follow from Equation (1) and (2), and the expressions for m, M and $\bar{\tau}$.

From Equations (5) and (6), we can write a buyer's value of ownership net of the opportunity cost of continuing to search as

$$V(x) = \frac{\beta}{1-\beta}\Big(\delta(x) - \rho S_{\text{tot}}(x) - q(x)E_{\mathbb{T}}\big[S_b(x) \mid x\big]\Big).$$

Using Equations (7)-(9) this can be manipulated to yield

$$V(x) = \frac{\beta\Big(\delta(x) + \rho V_s + q(x)E_{\mathbb{T}}\big[P(x) \mid x\big]\Big)}{1 - \beta\big(1 - \rho - q(x)\big)}, \tag{12}$$

where $E_{\mathbb{T}}\big[P(x) \mid x\big]$ is the buyer's expectations of the transaction price at which they will acquire the asset next period if they continue to search, are successful in reaching the bargaining stage, *and* successfully transact. $E_{\mathbb{T}}\big[P(x) \mid x\big]$ can instead be written as $P(x)$ because, under Nash bargaining, the price is fixed (given x) at V_s plus half of the total surplus: $P(x) = \frac{1}{2}\big(V_s + V(x)\big)$. After some manipulation one arrives at

$$P(x) = V_s + \frac{1}{2}\frac{\beta\delta(x) - (1-\beta)V_s}{1 - \beta(1 - \rho - q/2)}.$$

Recalling that $V_s = \frac{\beta}{1-\beta} q_s E_{\mathbb{T}}\big[P(x) - V_s\big]$, plugging in the expression for the price, and solving for V_s, leads to

$$V_s = \frac{\beta \frac{q_s}{2}}{1 - \beta(1 - \rho - \frac{q+q_s}{2})} \frac{\beta}{1-\beta} E_{\mathbb{T}}\big[\delta(x)\big]$$

Because every participating type has the same chance of getting to the bargaining stage (and therefore transacting), the distribution of transactions is uniform with respect to types and $E_{\mathbb{T}}\big[\delta(x)\big] = \frac{1}{1-\bar{x}} \int_{\bar{x}}^{1} \delta(x) dx$. This leads to

$$V_s = \frac{\beta \frac{q_s}{2}}{1 - \beta(1 - \rho - \frac{q+q_s}{2})} \frac{\beta}{1-\beta} \frac{1}{1-\bar{x}} \int_{\bar{x}}^{1} \delta(x) dx.$$

To derive the participation threshold type, consider that a buyer of type x will only participate if the total surplus is non-negative, $V(x) - V_s \geq 0$. Using the earlier derived expression for $V(x)$ yields, after some algebraic manipulation, the participation condition $V_s \leq \frac{\beta}{1-\beta}\delta(x)$. The participation threshold condition can therefore be written as

$$\bar{x}^* \equiv \min_{\{x \in [0,1] \,|\, \frac{\beta}{1-\beta}\delta(x) \geq V_s\}} x$$

Because $\frac{\beta}{1-\beta}\delta(1) > V_s$ and $\delta(x)$ is increasing, $\bar{x}^*$ exists (and is unique). Moreover, because V_s is a continuous function of $\bar{x}$, Berge's Maximum Theorem implies that $\bar{x}^*$ is a continuous function of $\bar{x}$ in $[0, 1]$. The Brouwer Fixed Point Theorem can be applied to $\bar{x}^*(\bar{x}) = \bar{x}$, establishing the existence of an equilibrium threshold and, with it, the existence of a steady state equilibrium. □

Proof of Proposition 2 We begin by conjecturing an equilibrium in which only types $x \geq \bar{x}$ participate and such that the seller's offer price, P^* equals the buyer surplus, $S_b(\cdot)$, for some type $x^* < 1$ with $\bar{x} \leq x^*$.

Under the random selection assumption of the no-information equilibrium, an arriving buyer has a $\frac{1}{n}$ chance of being selected, where $n - 1$ is the number of other arriving buyers with the same seller. The arrival intensity, irrespective of type, is $\int_{\bar{x}}^{1} \frac{1-m(x)}{I-M} dx = \frac{1-\bar{x}-M}{I-M} \equiv \bar{\tau}$. Thus, the probability that k other buyers arrive is $e^{-\bar{\tau}} \frac{\bar{\tau}^k}{k!}$. Weighting $\frac{1}{n}$ by

the probability of $k = n - 1$ other arrivals, and summing over $k \geq 0$ yields a selection probability of $\frac{1-e^{-\bar{\tau}}}{\bar{\tau}}$. If a type $x \geq x^*$ is selected they will transact with probability one, so from Equation (1),

$$\frac{\rho m(x)}{1 - m(x)} = \frac{1 - e^{-\bar{\tau}}}{\bar{\tau}}.$$

Because the right side is constant for all types such that $x \geq x^*$, we simply write $m(x) = m_H$. Solving for m_H yields,

$$m_H = \frac{(1 - e^{-\bar{\tau}})}{\rho\bar{\tau} + (1 - e^{-\bar{\tau}})}.$$

To calculate m_L we note that, by conjecture, the bargaining stage fails to deliver a transaction half of the time (because the seller's take-it-or-leave-it offer would otherwise result in negative buyer surplus). Thus $\frac{\rho m_L}{1-m_L} = \frac{1-e^{-\bar{\tau}}}{2\bar{\tau}}$, or

$$m_L = \frac{(1 - e^{-\bar{\tau}})}{2\rho\bar{\tau} + (1 - e^{-\bar{\tau}})}.$$

By definition, $M = \int_{\bar{x}}^{1} m(x)dx \Rightarrow M = (1-x^*)m_H+(x^*-\bar{x})m_L$. The right side depends on M through $\bar{\tau}$. To ensure the arrival intensity is positive and that ownership doesn't exceed the stock of assets, we look for a solution for M in $[0, \min\{I, 1-\bar{x}\}]$. Fixing $\bar{x}$ and x^*, note that $\bar{\tau}$ is increasing in M if $1 - \bar{x} > I$ and decreasing if $1 - \bar{x} < I$ (if $I = 1 - \bar{x}$ then $\bar{\tau} = 1$ and M is uniquely pinned down). In the former case, note first that both m_H and m_L are decreasing in $\bar{\tau}$.[1] So if $\bar{\tau}$ is increasing in M, the right side of $M = (1-x^*)m_H+(x^*-\bar{x})m_L$ is decreasing in M. When $M = 0$ the right side is strictly positive, and when M approaches I from below the right side approaches zero. Thus, there is a unique solution for M in $(0, I)$ whenever $I < 1 - \bar{x}$. Next, assume that $I > 1 - \bar{x}$. The expression for M can be algebraically manipulated to yield $\rho M = (1 - e^{-\bar{\tau}})(I - M) - \rho m_L(x^* - \bar{x})$. Because $\bar{\tau}$ is decreasing in M, the right side of this equation is also decreasing in M. When M is zero the right side is positive. When $M = 1 - \bar{x}$ the right side is zero. So here too, there exists a unique solution to M in $(0, 1 - \bar{x})$. Finally, note that, because the implicit equations for M are continuous (and differentiable) in x^* and $\bar{x}$, M too must be continuous in x^* and $\bar{x}$.

[1] To see this, divide the numerator and denominator of m_H by $\bar{\tau}$ and note that $\frac{1-e^{-\bar{\tau}}}{\bar{\tau}}$ is decreasing in $\bar{\tau}$. The case of m_L is treated similarly.

Given the random proposer bargaining solution,

$$E_{\mathbb{T}}\big[P(x)\,|\,x\big] = \begin{cases} V_s & \bar{x} \le x < x^* \\ \frac{1}{2}(V_s + P^*) & x \ge x^* \end{cases}$$

and applying Equation (1) to note that

$$q(x) = \begin{cases} \frac{q}{2} \equiv \frac{1-e^{-\bar{\tau}}}{2\bar{\tau}} & \bar{x} \le x < x^* \\ q & x \ge x^* \end{cases}$$

one can use Equation (12), which holds across models, to write:

$$V(x) = \begin{cases} \frac{\beta\left(\delta(x)+(\rho+\frac{q}{2})V_s\right)}{1-\beta\left(1-\rho-\frac{q}{2}\right)} & \bar{x} \le x < x^* \\ \frac{\beta\left(\delta(x)+(\rho+\frac{q}{2})V_s+\frac{q}{2}P^*\right)}{1-\beta\left(1-\rho-q\right)} & x \ge x^* \end{cases}$$

The conjecture specifies that a buyer of type x^* is indifferent between accepting and rejecting the take-it-or-leave-it offer of paying P^* for the asset. This requires setting $V(x^*) = P^*$, or

$$P^* = \beta\frac{\delta(x^*) + (\rho + \frac{q}{2})V_s}{1 - \beta(1 - \rho - \frac{q}{2})}.$$

It is easy to check that this also implies $V(x)$ is continuous (but generally kinked) at x^*.

Under the conjecture, if a transaction takes place, the seller receives a surplus from trade of $P^* - V_s$ with probability one half only if the buyer happens to be of type $x \ge x^*$. Define $\tau_H = (1-x^*)\frac{1-m_H}{I-M}$ and $\tau_L = (x^*-\bar{x})\frac{1-m_L}{I-M}$ to be the intensities of "High" and "Low" buyer type arrivals. The probability that a "High" type is *transacted* among arriving types is simply $\frac{\tau_H}{\tau_H+\frac{1}{2}\tau_L}$. Noting that $q_s = (1 - e^{-\bar{\tau}})\frac{\tau_H+\frac{1}{2}\tau_L}{\bar{\tau}}$, and using Equation (9), one can write Equation (4) for the seller's valuation as

$$V_s = \frac{\beta}{1-\beta}(1 - e^{-\bar{\tau}})\frac{(1 - x^*)(1 - m_H)}{1 - \bar{x} - M}\frac{1}{2}(P^* - V_s) = \frac{\beta}{1-\beta}\frac{Q_s}{2}(P^* - V_s),$$

where $Q_s \equiv (1 - e^{-\bar{\tau}})\frac{(1-x^*)(1-m_H)}{1-\bar{x}-M}$ and the factor of $\frac{1}{2}$ arises because the transaction price is P^* only half the time when transacting with a High type. Using the expression earlier

derived for P^* and solving for V_s yields

$$V_s = \beta \frac{Q_s}{2} \frac{\frac{\beta}{1-\beta}\delta(x^*)}{1-\beta(1-\rho-\frac{q+Q_s}{2})}. \tag{13}$$

We note that, because M is a continuous function of $\bar{x}$ and x^*, so is V_s.

Just as in the PIB case, the participation condition is $V_s \leq \frac{\beta}{1-\beta}\delta(x)$. The participation threshold can therefore be written as

$$\bar{x}^* \equiv \min_{\{x \in [0,x^*] \,|\, \frac{\beta}{1-\beta}\delta(x) \geq V_s\}} x$$

Because $\frac{\beta}{1-\beta}\delta(x^*) > V_s$ and $\delta(x)$ is increasing, $\bar{x}^*$ exists (and is unique). Moreover, because V_s is a continuous function of $\bar{x}$ and x^*, Berge's Maximum Theorem implies that $\bar{x}^*$ is a continuous function of $\bar{x}$ and x^* in the simplex $\Delta = \{(\bar{x}, x^*) \,|\, x^* \in [0,1], 0 \leq \bar{x} \leq x^*\}$.

To pin down the High type threshold, x^*, we recall that the conjectured offer price, P^*, can only be rationalized in equilibrium if it is the solution to the seller's optimization: $P^* = \underset{P \geq V_s}{\text{argmax}}(P - V_s)\text{Prb}\big(V(x) \geq P\big)$. Note that $P - V_s$ in this optimization is equal to the total surplus generated by trading with the lowest buyer type that will accept the seller's offer. So the optimization can instead be recast as:

$$x^{**} = \underset{x \in [0,1]}{\text{argmax}}\big(V(x) - V_s\big)(1-x).$$

Substituting the expressions for $V(x)$, V_s, and P^*, up to a factor, this becomes

$$x^{**} = \underset{x \in [0,1]}{\text{argmax}}\Big(\delta(x) - \frac{1-\beta}{\beta}V_s - \chi_{x \geq x^*}\frac{\beta q}{2}\frac{\delta(x)-\delta(x^*)}{1-\beta(1-\rho-q)}\Big)(1-x) \equiv \underset{x \in [0,1]}{\text{argmax}}\, f(x; \bar{x}, x^*),$$

where $\chi_{x \geq x^*}$ is an indicator function (zero when $x < x^*$ and one when $x \geq x^*$).

To characterize x^{**}, begin by recalling that, by assumption, $\phi(x, \kappa) = (\delta(x) - \kappa)(1-x)$ is continuous and single-peaked in $[0,1]$ for every $\kappa \in \mathbb{R}$. We use this to show that the correspondence, $x^{**} = \underset{x \in [0,1]}{\text{argmax}}\, f(x; \bar{x}, x^*)$, is non-empty, convex-valued and continuous in Δ. Existence and continuity of x^{**} follows from the Maximum Theorem. To prove that x^{**} is convex-valued, first note that $f(x; \bar{x}, x^*)$ takes the form $\phi(x, \kappa_L)\chi_{x \leq x^*} + \xi\phi(x, \kappa_H)\chi_{x \geq x^*}$

with $\phi(x,\kappa_L) = \xi\phi(x,\kappa_H)$.[2] Moreover, $\xi\phi(x,\kappa_H)$ weakly dominates $\phi(x,\kappa_L)$ in $[0,x^*]$ and vice versa in $[x^*,1]$. Because each of $\phi(x,\kappa_L)$ and $\phi(x,\kappa_H)$ are single-peaked, whenever $\phi(x,\kappa_L)$ is maximized in $[0,x^*]$, $f(x;\bar{x},x^*)$ must be maximized at a single value in $[0,x^*]$ and is also single-peaked in $[0,1]$; correspondingly, whenever $\phi(x,\kappa_H)$ is maximized in $[x^*,1]$, $f(x;\bar{x},x^*)$ is maximized at a single value in $[x^*,1]$ and is also single-peaked in $[0,1]$. Finally, in case $\phi(x,\kappa_L)$ is maximized in $[x^*,1]$ *and* $\phi(x,\kappa_H)$ is maximized in $[0,x^*]$, $f(x)$ must be single peaked (and maximized) at x^*. This demonstrates that $\underset{x\in[0,1]}{\operatorname{argmax}} f(x;\bar{x},x^*)$ is continuous and single-valued.

The Brouwer Fixed Point Theorem can be applied to $F:\Delta\mapsto\Delta$ defined by $F(x^*,\bar{x}) = (x^{**},\bar{x}^*)$. Specifically, there exists some $(x^*,\bar{x})\in\Delta$ such that $(x^{**},\bar{x}^*) = (x^*,\bar{x})$. This establishes the existence of a steady state equilibrium.

To see that the equilibrium x^* is not unique, consider that, because of the kink at x^*, requiring that $x^* = \underset{x\in[0,1]}{\operatorname{argmax}} f(x;\bar{x},x^*)$ leads to a first order inequality: The derivative at x^* must be non-negative from the left and non-positive from the right. This leads to

$$\delta(x^*) - \frac{1-\beta}{\beta}V_s \leq (1-x^*)\delta'(x^*) \leq \Big(\delta(x^*) - \frac{1-\beta}{\beta}V_s\Big)\frac{1-\beta(1-\rho-q)}{1-\beta(1-\rho-q/2)}.$$

Note that the bound on the left side is strictly smaller than the bound on the right, so there is always a range of solutions for x^*. □

Proof of Lemma 1 Under monotonic selection, the probability that an arriving buyer of type x will be selected is the probability that no buyer of type higher than x arrives. That is, $q(x) = \exp\big(-\tau(x)\big)$, where $\tau(x) = \int_x^1 \frac{1-m(x)}{I-M}dx$. The cumulative intensity, $\tau(x)$ is simply the integral over all arrival intensities of types higher than x. Substituting for $q(x)$ in Equation (1), taking the natural logarithm of both sides, and differentiating with respect to x, yields the following ordinary differential equation:

$$m'(x)\Big(\frac{1}{m(x)} + \frac{1}{1-m(x)}\Big) = \frac{1-m(x)}{I-M}.$$

Note that for $m(x)\in(0,1)$, $m'(x)$ is strictly positive. It can be easily verified that the

[2]Set $\kappa_L = \frac{1-\beta}{\beta}V_s$, $\kappa_H = \frac{1}{\xi}\Big(\kappa_L - \frac{\frac{\beta q}{2}\delta(x^*)}{1-\beta(1-\rho-q)}\Big)$, and $\xi = \frac{1-\beta(1-\rho-\frac{q}{2})}{1-\beta(1-\rho-q)}$. For notational convenience we suppress the dependence of κ_L,κ_H and ξ on $\bar{x}$ and x^*.

following implicit equation for $m(x)$ solves the ODE:

$$\ln\Big(\frac{m(x)}{1-m(x)}\Big)+\frac{1}{1-m(x)}=\frac{x}{I-M}+A.$$

When $x=1$, $q(x)=$, and Equation (1) implies $m(1)=\frac{1}{\rho+1}$. Solving for A leads to

$$A=\ln\big(\frac{1}{\rho}\big)+\frac{\rho+1}{\rho}-\frac{1}{I-M},$$

and the desired implicit equation for $m(x)$ in the statement of the Proposition.

If the participation thresholds are the same in the partial-information and the full-information settings, so must be the solution to the total mass equation. This is because, in both settings, the probability of some transaction taking place, for a given seller, only depends on β, ρ, I and $\bar{x}$. Thus, the total ownership, M must be given by the density, m, in Proposition 1 multiplied by $1-\bar{x}$. □

Proof of Proposition 3 Here too the expression for $V(x)$ is the same as in Equation (12), with $E_{\mathbb{T}}\big[P(x)\mid x\big]$ set to $P(x)$ because the price is fixed, given x. Under Nash bargaining, $S_b(x)=S_s(x)=\frac{1}{2}S_{\text{tot}}(x)$. Using the latter with the expression for $V(x)$ and then solving for $P(x)$ yields

$$P(x)=V_s+\frac{1}{2}\frac{\beta\delta(x)-(1-\beta)V_s}{1-\beta(1-\rho-q(x)/2)}. \tag{14}$$

From Equations (4) and (7), $V_s=\frac{\beta}{1-\beta}q_sE_{\mathbb{T}}[P(x)-V_s]$. The expression $q_sE_{\mathbb{T}}[P(x)-V_s]$ is the integral of $P(x)-V_s$ over $[0,1]$ weighted by $q_b(x)$, the probability density that a buyer of type x is selected by a given seller for bargaining.[3] To calculate $q_b(x)$ we note that the probability density for a type x arrival is $\frac{1-m(x)}{I-M}$, while the probability that this arriving type will ultimately transact with the seller is, by definition, $q(x)$. Using Equation (1), it follows that $q_b(x)=\frac{1-m(x)}{I-M}q(x)=\frac{\rho m(x)}{I-M}$. Thus

$$V_s=\frac{\beta}{1-\beta}\int_0^1\frac{\rho m(x)}{I-M}\big(P(x)-V_s\big)dx.$$

[3]Recall that q_s is the probability that the seller will bargain with some buyer, while $E_{\mathbb{T}}[\cdot]$ is an expectation conditional on bargaining with someone.

If the buyer participation threshold is $\bar{x}$, then $m(x) = 0$ for $x < \bar{x}$, so the lower integral limit can be set to $\bar{x}$. Plugging in the earlier expression for $P(x)$, after some algebraic manipulation, leads to:

$$V_s = \frac{\beta}{1-\beta} \frac{\frac{\rho\beta}{(I-M)} \int_{\bar{x}}^{1} m(x) \left[\frac{\delta(x)}{1-\beta(1-\rho-q(x)/2)} \right] dx}{2 + \frac{\rho\beta}{(I-M)} \int_{\bar{x}}^{1} m(x) \left[\frac{1}{1-\beta(1-\rho-q(x)/2)} \right] dx}$$

As can be seen from the expression for the seller surplus, $P(x) - V_s$, which must be non-negative, participating types must satisfy $V_s \leq \frac{\beta}{1-\beta}\delta(x)$. This is the same requirement appearing in the no-information and partial-information settings. As in the latter, the Brouwer Fixed Point Theorem ensures the existence of a threshold, $\bar{x}$, coinciding with the lowest type in $[0, 1]$ satisfying $V_s \leq \frac{\beta}{1-\beta}\delta(x)$.

For selection to be monotonic, so must be the seller surplus. This is true iff $P(x)$ is monotonic in x. It is straight forward to show that the derivative of $P(x)$ with respect to x is non-decreasing iff $\delta'(x) \geq q'(x)\big(P(x) - V_s\big)$. From the proof of Lemma 1,

$$q'(x) = \exp\big(-\tau(x)\big)\frac{1-m(x)}{I-M} = q(x)\frac{1-m(x)}{I-M} = \frac{\rho m(x)}{I-M}.$$

Where the last equality follows from Equation (1). The derivative condition in the statement of the Proposition follows from substituting the rightmost expression for $q'(x)$ in the derivative condition for $\delta'(x)$. □

Proof of Lemma 2 Fixing model parameters and $\delta(x)$, consistent with the hypothesis, let V_s^{NIB} be the NIB equilibrium solution in Equation (13) with participation threshold of $\bar{x}^{NIB}$ and transaction failure boundary x^*. Let $M(\bar{x}^{NIB}, x^*)$ correspond to the mass of steady state equilibrium ownership and define $\bar{\tau}(\bar{x}^{NIB}, x^*) = \frac{1-\bar{x}^{NIB}-M(\bar{x}^{NIB},x^*)}{I-M(\bar{x}^{NIB},x^*)}$. Define $\hat{q}_s(\bar{x}^{NIB}, x^*) = \big(1 - \exp(-\bar{\tau}(\bar{x}^{NIB}, x^*)\big)$ and note that Q_s, defined above Equation (13), can be written as

$$Q_s(\bar{x}^{NIB}, x^*) = \hat{q}_s(\bar{x}^{NIB}, x^*)\frac{(1-x^*)(1-m_H)}{(1-x^*)(1-m_H) + (x^* - \bar{x}^{NIB})(1-m_L)}.$$

clarifying that $\hat{q}_s^{NIB}(\bar{x}^{NIB}, x^*) \geq Q_s(\bar{x}^{NIB}, x^*)$. Next, define

$$\theta(\bar{x}^{NIB}, x^*) = \frac{\frac{\beta Q_s(\bar{x}^{NIB}, x^*)}{2}}{1 - \beta(1 - \rho - \frac{q(\bar{x}^{NIB}, x^*) + Q(\bar{x}^{NIB}, x^*)}{2})},$$

where $q(\bar{x}^{NIB}, x^*) = \hat{q}_s(\bar{x}^{NIB}, x^*)/\bar{\tau}(\bar{x}^{NIB}, x^*)$ corresponds to q as derived in the proof to Proposition 2. Note that, from the proof to Proposition 2, one can write $V_s^{NIB} = \theta(\bar{x}^{NIB}, x^*)\frac{\beta}{1-\beta}\delta(x^*)$.

We now consider the effect on $\theta(\bar{x}^{NIB}, x^*)$ of shifting x^* to $\bar{x}^{NIB}$. This is akin to calculating the factor analogous to θ in the PIB setting under the (out of equilibrium) assumption that the participation threshold remains at $\bar{x}^{NIB}$. After a bit of algebra, one can derive

$$\theta(\bar{x}^{NIB}, \bar{x}^{NIB}) \geq \theta(\bar{x}^{NIB}, x^*) \;\Leftrightarrow\; \frac{\bar{\tau}(\bar{x}^{NIB}, \bar{x}^{NIB})}{\bar{\tau}(\bar{x}^{NIB}, x^*)} \geq \frac{(1 - x^*)(1 - m_H)}{(1 - x^*)(1 - m_H) + (x^* - \bar{x}^{NIB})(1 - m_L)}.$$

Using the definitions of $\bar{\tau}$ and M^{NIB}, the condition on the right can be further manipulated to become

$$(1-\bar{x}^{NIB})\big(I-M(\bar{x}^{NIB}, x^*)\big)\big(1-m_H(\bar{x}^{NIB}, \bar{x}^{NIB})\big) \geq (1-x^*)\big(I-M(\bar{x}^{NIB}, \bar{x}^{NIB})\big)\big(1-m_H(\bar{x}^{NIB}, x^*)\big).$$

Note that $M(\bar{x}^{NIB}, \bar{x}^{NIB}) \geq M(\bar{x}^{NIB}, x^*)$ because reducing x^* while keeping the participation threshold constant means reducing the rate of transaction failures.[4] Moreover, when $I \geq 1 - \bar{x}^{NIB}$, it is straight forward to show that $\frac{dm_H}{dx^*} > 0$, so that the inequality holds and $\theta(\bar{x}^{NIB}, \bar{x}^{NIB}) \geq \theta(\bar{x}^{NIB}, x^*)$.

In the case $I < 1 - \bar{x}^{NIB}$ the inequality can be established by demonstrating that $\frac{d}{dx^*}\Big((1 - x^*)\big(1 - m_H(\bar{x}^{NIB}, x^*)\big)\Big) < 0$. Though tedious, one can calculate that

$$\frac{d}{dx^*}\Big((1-x^*)(1-m_H)\Big) = -(1-m_H)\Big(1-\frac{(1 - x^*)(1 - m_H)(m_H - m_L)}{\kappa + (1 - x^*)(1 - m_H)^2 + \frac{1}{2}(x^* - \bar{x}^{NIB})(1 - m_L)^2}\Big),$$

where $\kappa > 0$. The result, $\frac{d}{dx^*}\Big((1-x^*)(1-m_H)\Big) < 0$, follows from the fact that $1 - m_H > m_H - m_L$, which itself follows from the definitions of m_H and m_L in Proposition 2. This establishes that $\theta(\bar{x}^{NIB}, \bar{x}^{NIB}) \geq \theta(\bar{x}^{NIB}, x^*)$ and, thus, $V_s^{NIB} \leq \theta(\bar{x}^{NIB}, \bar{x}^{NIB})\frac{\beta}{1-\beta}\delta(x^*)$.

[4] One can more formally show that $M(\bar{x}^{NIB}, x^*)$ decreases in x^* by an argument similar to that used to establish the existence and uniqueness of $M(\bar{x}^{NIB}, x^*)$ in Proposition 2.

Now, consider that x^* must be smaller than or equal to y^{NIB}, the unique solution to $\delta'(y)(1-y) - \delta(y) + V_s^{NIB} = 0$. This is because the the regularity condition on $\delta(x)$ ensures that $y(\bar{x}^{NIB})$ is the largest $y \in [0,1]$ that satisfies $\delta(y) - V_s^{NIB} \leq \delta'(y)(1-y)$, which must also be satisfied by x^* according to Proposition 2. Thus, by hypothesis,

$$\delta(x^*) \leq \delta(y^{NIB}) \leq \frac{1}{1-\bar{x}^{NIB}} \int_{\bar{x}^{NIB}}^{1} \delta(x)dx.$$

Summarizing, we've established that $V_s^{NIB} \leq \theta(\bar{x}^{NIB}, \bar{x}^{NIB}) \frac{\beta}{1-\beta} \delta(x^*)$ and that $\delta(x^*) \leq \frac{1}{1-\bar{x}^{NIB}} \int_{\bar{x}^{NIB}}^{1} \delta(x)dx$. Putting both together yields

$$V_s^{NIB} \leq \theta(\bar{x}^{NIB}, \bar{x}^{NIB}) \frac{\beta}{1-\beta} \frac{1}{1-\bar{x}^{NIB}} \int_{\bar{x}^{NIB}}^{1} \delta(x)dx.$$

A similar argument to that given in the proof of Lemma 3 establishes that $\bar{x}^{NIB} \leq \bar{x}^{PIB}$, and that this implies $V_s^{NIB} \leq V_s^{PIB}$.

□

Proof of Lemma 3 Each of V_s^{FIB} and V_s^{PIB} can be written as

$$V_s^{\mathcal{I}} = \frac{\beta}{1-\beta} \frac{\int_{\bar{x}^{\mathcal{I}}}^{1} \gamma \circ m^{\mathcal{I}}(x)\delta(x)dx \;/\; \int_{\bar{x}^{\mathcal{I}}}^{1} \gamma \circ m^{\mathcal{I}}(x)dx}{1 + 2(I - M^{\mathcal{I}}) \;/\; \left(\rho\beta \int_{\bar{x}^{\mathcal{I}}}^{1} \gamma \circ m^{\mathcal{I}}(x)dx\right)},$$

where $\mathcal{I}$ corresponds to the information setting (PIB or FIB),

$$\gamma(m) = \frac{m\big(1-m\big)}{A - am},$$

which is derived by replacing $q(x)$ with $\frac{\rho m(x)}{1-m(x)}$, per Equation (1), and where $A = 1 - \beta(1-\rho)$ while $a = 1 - \beta(1-\rho/2)$. By Proposition 1, $m^{PIB}(x)$ is constant, while by Proposition **??**, $m^{FIB}(x)$ is increasing in x. Differentiating $\gamma \circ m^{FIB}(x)$ with respect to x yields,

$$\frac{d}{dx}\gamma \circ m^{FIB}(x) = \frac{a}{\big(A - am^{FIB}(x)\big)^2} \Big(\frac{A}{a} - 2m^{FIB}(x)\frac{A}{a} + m^{FIB}(x)^2\Big) \frac{d}{dx} m^{FIB}(x).$$

The quadratic form in the derivative is negative when m^{FIB} is between $\frac{A}{a} - \sqrt{\frac{A}{a}\big(\frac{A}{a} - 1\big)}$ and $\frac{A}{a} + \sqrt{\frac{A}{a}\big(\frac{A}{a} - 1\big)}$, and is minimized at $\frac{A}{a}$ (which is greater than one).

By hypothesis, $I = 1 - \bar{x}^{FIB}$. Thus $\bar{\tau}^{FIB} = 1$. Note that, from Equation (1) and

the fact that $q^{FIB}(\bar{x}) = \exp(-\bar{\tau}) = e^{-1}$ as derived in Proposition 3, that $m^{FIB}(\bar{x}^{FIB}) = \frac{1}{\rho e^1+1}$. The hypothesis therefore ensures that $\frac{d}{dx}\gamma \circ m^{FIB}(x) < 0$ over the interior range of $m^{FIB}(x)$. This establishes that, under the hypothesis, $\gamma \circ m^{\mathcal{I}}(x)$ is decreasing in the integral expression, $\int_{\bar{x}^{\mathcal{I}}}^{1} \gamma \circ m^{\mathcal{I}}(x)\delta(x)dx$. Because $\delta(x)$ is increasing, this also means that

$$V_s^{FIB} < \frac{\beta}{1-\beta} \frac{\frac{1}{1-\bar{x}^{FIB}} \int_{\bar{x}^{FIB}}^{1} \delta(x)dx}{1 + 2(I-M) \;/\; \left(\rho\beta \int_{\bar{x}^{FIB}}^{1} \gamma \circ m^{FIB}(x)dx\right)}.$$

Next, we note that $\gamma(m) = \frac{m\left(1-m\right)}{A-am}$ is concave in the range of $m^{FIB}(x)$ considered under the hypothesis. By Jensen's Inequality,

$$\int_{\bar{x}^{FIB}}^{1} \gamma \circ m^{FIB}(x)dx \leq \frac{\bar{m}^{FIB}\left(1-\bar{m}^{FIB}\right)}{A - a\bar{m}^{FIB}}(1-\bar{x}^{FIB}),$$

where $\bar{m}^{FIB} = \frac{1}{1-\bar{x}^{FIB}} \int_{\bar{x}^{FIB}}^{1} m^{FIB}(x)dx$. This allows us to replace the bound for V_s^{FIB} with,

$$V_s^{FIB} < \frac{\beta}{1-\beta} \frac{\frac{1}{1-\bar{x}^{FIB}} \int_{\bar{x}^{FIB}}^{1} \delta(x)dx}{1 + 2(I-M) \;/\; \left(\rho\beta \frac{\bar{m}^{FIB}\left(1-\bar{m}^{FIB}\right)}{A-a\bar{m}^{FIB}}(1-\bar{x}^{FIB})\right)}.$$

This last expression can be manipulated, after some tedious algebra, to

$$V_s^{FIB} < \frac{\beta}{1-\beta} \frac{\frac{\beta q_s(\bar{x}^{FIB})}{2}}{1-\beta\left(1-\rho-\frac{q_s(\bar{x}^{FIB})+\bar{q}(\bar{x}^{FIB})}{2}\right)} \frac{1}{1-\bar{x}^{FIB}} \int_{\bar{x}^{FIB}}^{1} \delta(x)dx,$$

where $q_s(\bar{x}^{FIB}) = \frac{\rho M(\bar{x}^{FIB})}{I-M(\bar{x}^{FIB})}$, $\bar{q}(\bar{x}^{FIB}) = \frac{\rho\bar{m}(\bar{x}^{FIB})}{1-\bar{m}(\bar{x}^{FIB})}$, and $\bar{m}(\bar{x}^{FIB}) = \frac{M(\bar{x}^{FIB})}{1-\bar{x}^{FIB}}$. The right side of the inequality coincides with the expression for V_s^{PIB} if it were the case that $\bar{x}^{PIB} = \bar{x}^{FIB}$.

It cannot be that $\bar{x}^{PIB} = \bar{x}^{FIB}$, however. As established in the proofs to Propositions 1 and 3, equality of $\bar{x}^{PIB}$ and $\bar{x}^{FIB}$ implies that $V_s^{PIB} = \frac{\beta}{1-\beta}\delta(\bar{x}^{PIB}) = \frac{\beta}{1-\beta}\delta(\bar{x}^{FIB}) = V_s^{FIB}$, which contradicts $V_s^{FIB} < V_s^{PIB}$ when $\bar{x}^{PIB} = \bar{x}^{FIB}$. We now use these insights to show that $\bar{x}^{PIB} > \bar{x}^{FIB}$, which in turn implies that $V_s^{PIB} > V_s^{FIB}$. To see that, first consider the following:

Claim: $\theta(\bar{x}) \equiv \frac{\frac{\beta q_s(\bar{x})}{2}}{1-\beta\left(1-\rho-\frac{q_s(\bar{x})+\bar{q}(\bar{x})}{2}\right)}$ is decreasing in $\bar{x}$.

Proof of Claim: We first establish that $\frac{dM}{d\bar{x}} < 0$. Suppose that $I \geq 1 - \bar{x}$. Differentiating $\frac{\rho M}{I-M} = 1 - e^{-\bar{\tau}}$ yields, after some algebra, $\frac{dM}{d\bar{x}} = -\frac{I-M}{(\rho e^{\bar{\tau}}+1)I+\bar{x}-1}$, which is negative. Now suppose that $I < 1 - \bar{x}$, differentiating $M = (1 - \bar{x})m$, where $m = \frac{(1-e^{-\bar{\tau}})/\bar{\tau}}{\rho+(1-e^{-\bar{\tau}})/\bar{\tau}}$ yields $\frac{dM}{d\bar{x}} = -\big(m + \frac{dm}{d\bar{\tau}}\frac{1-\bar{x}}{I-M}\big) \div \Big(1 - (1 - \bar{x})\frac{dm}{d\bar{\tau}}\frac{d\bar{\tau}}{dM}\Big)$. After some algebra, substituting $\frac{\rho M}{I-M} = 1 - e^{-\bar{\tau}}$ and $\bar{\tau} = \frac{1-\bar{x}-M}{I-M}$, one can write $\frac{dM}{d\bar{x}} = -\frac{1-m}{\rho}e^{-\bar{\tau}} \div \Big(1 - (1 - \bar{x})\frac{dm}{d\bar{\tau}}\frac{d\bar{\tau}}{dM}\Big)$, which is negative because $\frac{dm}{d\bar{\tau}} < 0$ and $\frac{d\bar{\tau}}{dM}$ is positive whenever $1 - \bar{x} > I$. Now note that $q_s = \frac{\rho M}{I-M}$ is increasing in M, so it must be decreasing in $\bar{x}$. Moreover, because $q_s = 1-e^{-\bar{\tau}}$ is increasing in $\bar{\tau}$, $\bar{\tau}$ must also be decreasing in $\bar{x}$. Finally, $\bar{q} = \frac{1-e^{-\bar{\tau}}}{\bar{\tau}}$ is decreasing in $\bar{\tau}$, so it must be increasing in $\bar{x}$. Because q_s decreases while $\bar{q}$ increases with $\bar{x}$, θ decreases with $\bar{x}$ thus proving the claim.

Specializing to $\delta(x) = x$, in which case $\frac{1}{1-\bar{x}^{FIB}}\int_{\bar{x}^{FIB}}^{1}\delta(x)dx = \frac{1+\bar{x}^{PIB}}{2}$, $\bar{x}^{PIB}$ is the solution to $\bar{x}^{PIB} = \theta(\bar{x})\frac{1+\bar{x}^{PIB}}{2}$, or alternatively, $\frac{\bar{x}^{PIB}}{1+\bar{x}^{PIB}} = \frac{1}{2}\theta(\bar{x}^{PIB})$. Because the left side of this equation is increasing while the left is decreasing, the solution is unique.[5] We earlier demonstrated that

$$\frac{\beta}{1-\beta}\bar{x}^{FIB} = \frac{\beta}{1-\beta}\delta(\bar{x}^{FIB}) = V_s^{FIB} < \frac{\beta}{1-\beta}\theta(\bar{x}^{FIB})\frac{1+\bar{x}^{FIB}}{2}.$$

In other words, $\frac{\bar{x}^{FIB}}{1+\bar{x}^{FIB}} < \frac{1}{2}\theta(\bar{x}^{FIB})$, which means that $\bar{x}^{FIB} < \bar{x}^{PIB}$ and, therefore, $V_s^{FIB} < V_s^{PIB}$ in the case $\delta(x) = x$.

Now suppose that one can find some $\delta(x)$ such that $V_s^{FIB} > V_s^{PIB}$. Define $\hat{\delta}(x) = \alpha\delta(x) + (1-\alpha)x$ for $\alpha \in [0, 1]$. Continuity of the seller valuations in α implies there exists some α for which $V_s^{FIB} = V_s^{PIB}$, which we have proven is not possible. This establishes that $V_s^{FIB} < V_s^{PIB}$ more generally.

Because $V_s^{FIB} < V_s^{PIB}$, the participation threshold in the FIB setting is lower than in the PIB setting. In the earlier claim it is proven that M decreases with $\bar{x}$, implying a higher total ownership in the FIB than in the PIB setting (i.e., $M^{FIB} > M^{PIB}$). The implicit equation for $m(x)$ in Lemma 1 depends on the threshold only through M. Moreover, as M increases so does $m(x)$ (for every x). This means that the average of $m(x)$ between $\bar{x}^{FIB}$ and one is larger than the constant ownership density in the PIB setting, m^{PIB}. From this it follows that $W^{FIB} > W^{PIB}$. □

[5] $\theta(\bar{x}^{PIB}) \in (0, \frac{1}{2})$ so a solution exists.

Proof of Lemma 4 Let $\bar{x}$ be the participation threshold, assumed to be equal across the three model settings. To derive the welfare ranking, first note from Propositions 2 and 1 that when supply is balanced $\bar{\tau} = 1$, so that $m^{\text{NIB}}(x) \leq m^{\text{PIB}}(x)$ for all $x \in [\bar{x}, 1]$. Thus $W^{\text{NIB}} \leq W^{\text{PIB}}$. Next, we note that $m^{\text{FIB}}(x)$ is strictly increasing and its average over $[\bar{x}, 1]$ coincides with $m^{\text{PIB}}(x)$. Thus $m^{\text{FIB}}(x)$ stochastically dominates $m^{\text{PIB}}(x)$, implying that $W^{\text{PIB}} < W^{\text{FIB}}$ (because $\delta(x)$ is increasing). □

Proof of Proposition 4 Because sellers in both the NIB and PIB settings are price takers, if a seller in either setting is presented with the opportunity to know arriving buyer types, any action they take based on that information will not distort the equilibrium.

Consider first that, in the bargaining stage in the NIB setting, the seller receives at most half of the total surplus and this happens only if the buyer type is x^*. Because knowing the buyer type reduces the bargaining solution to (full information) Nash, in which the total surplus is equally split, knowledge of the seller type just before the bargaining stage makes the seller strictly better off ex-ante.

It is easy to check that $V(x)$ is strictly increasing in x in both the NIB and the PIB settings. Under full-information bargaining the seller and buyer equally split the surplus, so the seller can do no better than to choose the buyer with the highest x at the selection stage. Because there is a non-zero probability that more than one buyer arrives, informed selection makes the seller strictly better off in expectations (in either the NIB or PIB setting). □

Proof of Proposition 5 Consider the bargaining stage of a steady state equilibrium. Under the modified random proposal solution, a proposing buyer receives all the surplus with probability half. If the seller is the proposer, the equilibrium solution is for the buyer to acquire certification and grant all their surplus to the seller.[6] Both buyer and seller, in expectation, receive half the surplus, which is equivalent to the Nash split.

In the selection stage, buyers know that they will get their full-information Nash split.

[6]The buyer is indifferent between walking away from bargaining and accepting the seller's offer. In equilibrium, the seller would only make the offer if the buyer accepts with probability one (any other buyer response is not an equilibrium as the seller would adjust their offer — say, by offering an arbitrarily small payoff).

Thus buyers in the selection stage only care about maximizing the probability of being chosen. And, among disclosing buyers, the seller prefers the highest $V(x)$ value (sellers can do no better within this subgroup).

Let D and N be the sets of disclosing and non-disclosing types at the selection stage, respectively. Let $q(N)$ be the probability of selection for any $x \in N$ (which must be the same across non-disclosers). Then for all $x \in D$, it must be that $q(x)$ is greater than or equal to $q(N)$, otherwise the anomalous member of D would prefer not to disclose. Whereas each type in N has the same probability of being selected, $V(x)$ increases in x within N. The reason is that the equilibrium price will correspond to the Nash split and, whenever that is the case, will satisfy Equation (14). Fixing $q(x) = q(N)$ for non-disclosers implies that $P(x)$ is monotonic in x *within* N. Correspondingly, because $V(x) = P(x) - V_s$ when the price evenly splits the surplus, it must also be that $V(x)$ is monotonic in x within N.

Now consider the case where N has non-zero measure. Let $\bar{V}(N)$ be the average $V(x)$ in N. Because $V(x)$ is strictly monotonic within N, any non-measure-zero set N has some x_h such that $V(x_h) > \bar{V}(N)$. We now show that x_h prefers to disclose, which presents a contradiction. Because sellers will select the arriving buyer with their highest estimate of V, disclosure for x_h improves the chance of being selected relative to other non-disclosers and does not reduce the chance of being selected over disclosers. Hence, x_h will disclose because disclosure strictly increases the probability of being selected while leaving payoffs unchanged.

The contradiction implies that N is a measure zero set, meaning that the equilibrium must be fully revealing. □

Proof of Lemma 5 Under our assumption that $\delta(0) < 0$, it must be the case that $\bar{x} > 0$ and there is no trade surplus with the lowest participating type. So either type $\bar{x}$ would not undertake costly type revelation or there is a type $x < \bar{x}$ that can enter, be selected with non-zero probability, and benefit from bargaining as long as they do not disclose their type (meaning that $\bar{x}$ is not the threshold type—a contradiction).

□

Proof of Proposition 6 Suppose that, when an arriving buyer optimally bids for

the asset, they assume a bidding schedule on the part of other prospective bidders that is strictly monotonic in type. Denote this schedule as $P(x)$. Correspondingly, by selecting the highest bid, the seller is also selecting the highest bidding type (i.e., buyer selection is monotonic).

A searching buyer of type y that participates in the auction selects a bid, $\hat{P}(y)$ that maximizes their utility:

$$U(y,\hat{P}) = q\big(x(\hat{P})\big)\Big(G(y) - \hat{P}\Big) + \Big(1 - q\big(x(\hat{P})\big)\Big)F(y),$$

where $F(y)$ is the opportunity cost of the bidder, $G(y)$ is the bidder's gross benefits to ownership, $x(\hat{P})$ is the type associated with the monotonic bidding schedule when the bid is $\hat{P}$, and $q\big(x\big)$ is the probability that a bid by an arriving buyer of type x is selected and transacted, assuming monotonic selection. In other words $q(x) = \exp\big(-\tau(x)\big)$, as derived in the proof to Lemma 1.

The first order condition with respect to $\hat{P}$ implies

$$q'\big(x(\hat{P})\big)x'(\hat{P})\Big(V(y) - \hat{P}\Big) = q\big(x(\hat{P})\big).$$

In a monotonic bidding equilibrium, a type y buyer bids like a type x only if they are type x, and the bidding price is $\hat{P} = P(x)$. Moreover, if $P(x)$ is monotonic in x, then it is invertible and $x'(\hat{P}) = \frac{1}{P'\left(x(\hat{P})\right)}$. Thus, an equilibrium would be characterized by a bidding price schedule that satisfies the following ordinary differential equation (ODE):

$$\frac{q'(x)}{q(x)}\Big(V(x) - P(x)\Big) = P'\big(x\big).$$

The term in parentheses on the left is the buyer's surplus, $S_b(x)$, from an equilibrium transaction. The differential equation in the statement of the Proposition follows from noting that $\frac{q'(x)}{q(x)} = \frac{1-m(x)}{I-M}$. Note, further, that $1 - m(x) > 0$ implies that $q'(x) > 0$.

The required second order condition is:

$$\frac{q''(x)}{\big(P'(x)\big)^2}S_b(x) - \frac{q'(x)P''(x)}{\big(P'(x)\big)^3}S_b(x) - 2\frac{q'(x)}{P'(x)} < 0$$

which, using the first order condition, can be further manipulated to yield

$$-\frac{q(x)}{P'(x)}\frac{d}{dx}\ln\Big(S_b(x)q(x)\Big) < 0.$$

Because $q'(x) > 0$, monotonicity of $P(x)$ and the second order condition are simultaneously satisfied if and only if $S_b(x) > 0$ and $\frac{d}{dx}\ln\Big(S_b(x)q(x)\Big) > 0$.

We first establish that $S_b(x)$ is strictly positive on $(\bar{x}, 1]$ where $\bar{x}$ is the lowest participating type. To do that, note that, the expression for $V(x)$ is the same as in Equation (12), with $P(x) = E_{\mathbb{T}}\big[P(x) \mid x\big]$ so that the buyer's surplus can be written as

$$S_b(x) = \frac{\beta\delta(x) + \beta\rho V_s - (1-\beta(1-\rho))P(x)}{1-\beta(1-\rho-q(x))},$$

After some algebra, and using the ODE for $P(x)$, one can calculate that

$$S_b'(x) = \beta\frac{\delta'(x)}{1-\beta\big(1-\rho-q(x)\big)} - \frac{q'(x)}{q(x)}S_b(x).$$

Noting that with endogenous entry, $S_b(\bar{x}) = 0$, this leads to

$$S_b(x) = \int_{\bar{x}}^{x} \exp\Big(-\int_{x'}^{x}\alpha(x'')dx''\Big)\,\Gamma(x')dx',$$

for strictly positive functions, $\alpha(x)$ and $\Gamma(x)$. Thus $S_b(x)$ must be strictly positive in $(\bar{x}, 1]$, meaning that $P(x)$ is strictly monotonic in $[\bar{x}, 1]$.

Finally, by using the explicit expression derived above for $S_b'(x)$, one can write

$$\frac{d}{dx}\ln\Big(S_b(x)q(x)\Big) = \beta\frac{\delta'(x)}{1-\beta\big(1-\rho-q(x)\big)}\frac{1}{S_b(x)} > 0.$$

This establishes the second order condition. □

Proof of Proposition 7 We first establish that a fully-revealing auction equilibrium with $V_s = P(\bar{x})$ exists. To do this, we simply need to prove that there exists a threshold type in $[0, 1)$ such that all of the equilibrium conditions hold. We first prove that $V_s \leq P(x) \leq \frac{\beta}{1-\beta}\delta(x)$. To begin, note that $P(x)$ is the solution to (11) with some initial condition at the participation threshold, $\bar{x}$. According to Definition 1, the seller will

only agree to sell if $P(\bar{x}) \geq V_s$. If the initial condition is set to satisfy this equilibrium condition, monotonicity of $P(x)$ immediately delivers $V_s \leq P(x)$. Next, we explicitly write out $S_b(x)$ in (11):

$$P'(x) = \frac{1-m(x)}{I-M} \, \frac{(1-\beta)\Big(\frac{\beta}{1-\beta}\delta(x) - P(x)\Big) + \beta\rho\Big(V_s - P(x)\Big)}{1-\beta(1-q(x)-\rho)}.$$

We've already established that the right side of the equation is positive. Thus, because $V_s \leq P(x)$, it must be that $\frac{\beta}{1-\beta}\delta(x) \geq P(x)$.

Next, let $\hat{M}$ to be the mass of ownership under full participation and note that this must be greater than the mass of ownership when the participation threshold is greater than zero (i.e., under partial participation). Define

$$\bar{v} = \Big(\frac{\beta}{1-\beta}\Big)^2 \frac{\rho}{I-\hat{M}} \int_0^1 \delta(x)dx,$$

which, in the ensuing, will furnish an upper bound to V_s. Consider now the function $F : \mathbf{R} \mapsto \mathbf{R}$ defined by

$$F(v) = \frac{\beta}{1-\beta} \frac{\rho}{I - M\big(\bar{x}(v)\big)} \int_{\bar{x}(v)}^1 m(x, \bar{x}(v))\big(P(x;v) - v\big)dx,$$

where $\bar{x}(v) = \min_{\{x\in[0,1] \,|\, \beta\delta(x)\geq(1-\beta)v\}} x$, while $m(x, \bar{x}(v))$ and $M\big(\bar{x}(v)\big)$ are the ownership density and mass, respectively, when the participation threshold is $\bar{x}(v)$, and $P(x;v)$ is the solution to (11) with initial condition

$$P\big(\bar{x}(v)\big) = v + \frac{1-\beta}{2} \frac{\frac{\beta}{1-\beta}\delta\big(\bar{x}(v)\big) - v}{1-\beta\Big(1-\rho-q\big(\bar{x}(v)\big)/2\Big)}.$$

It should be clear that $F(v)$ is continuous.

Any fixed point of the function $F(v)$ is an auction equilibrium solution. To see that, first note that, based the argument in the beginning of the proof to Proposition 3, one can equivalently write $F(v) = \frac{\beta}{1-\beta} q_s(\bar{x}(v)\big) E_{\mathbb{T}}\big[P(x;v) - v\big]$, where $q_s(\bar{x}(v)\big)$ is the probability that a seller transacts and $E_{\mathbb{T}}\big[\,\cdot\,\big]$ is an expectation conditional on transacting, both calculated assuming that the participation threshold is $\bar{x}(v)$. So $F(V_s^A) = V_s^A$ would be a solution to (4) with participating threshold satisfying $\bar{x}^A = \min_{\{x\in[0,1] \,|\, \beta\delta(x)\geq(1-\beta)V_s^A\}} x$, and

with initial condition satisfying $S_b(\bar{x}^A) = \frac{1}{2}S_{\text{tot}}(\bar{x}^A)$.[7]

It remains to demonstrate that F has a fixed point. To establish that, first note that, by definition, $P\big(\bar{x}(v)\big) - v \geq 0$, with equality holding whenever $v \geq \frac{\beta}{1-\beta}\delta(0)$. Thus the integrand in the definition of $F(v)$ is always positive. Next, note that, by definition, $\bar{x}(v) \geq 0$ and that $\frac{m(x)}{I-M\left(\bar{x}(v)\right)} \leq \frac{1}{I-M(0)}$. Combining these facts implies

$$F(v) \leq \frac{\beta}{1-\beta}\frac{\rho}{I-M(0)}\int_{\bar{x}(v)}^{1}\big(P(x;v)-v\big)dx \leq \bar{v},$$

where the last inequality derives from the fact that $P(x) \leq \frac{\beta}{1-\beta}\delta(x)$. Defining $\hat{F}$ as the restriction of F to the compact interval $[0,\bar{v}]$, it should be clear that $\hat{F}$ maps $[0,\bar{v}]$ into itself. By the Brouwer Fixed Point Theorem, continuity of $\hat{F}$ implies that it has a fixed point. This establishes the existence of an auction equilibrium solution with initial condition $S_b(\bar{x}^A) = \frac{1}{2}S_{\text{tot}}(\bar{x}^A)$ (which amounts to $P(\bar{x}^A) = V_s^A$ whenever $\delta(0) \leq 0$).

We now prove that the equilibrium with $P(\bar{x}^A) = V_s^A$ is the unique symmetric equilibrium in which every seller optimally sets their auction reserve price. To that end, let $\hat{P}(x)$ correspond to any steady equilibrium with monotonic selection satisfying Proposition 6 with participation threshold $\hat{x}$ such that $\hat{P}(\bar{x}^A) > V_s^A$. Consider next an infinitesimal seller who, prior to the arrival of buyers, deviates from the symmetric equilibrium reserve price of $\hat{P}(\bar{x}^A)$. Denote the deviating reserve price as V_r. Searching buyers that participate in the prevailing auction and with private valuation weakly greater than V_r will be directed to the deviating seller's auction until they are indifferent between the two auctions. Because the deviating seller is infinitesimal, there is neither price nor buyer flow impact on prevailing auctions. The subsequent equalization of buyer expected utility across the two auctions can therefore be written as:

$$q(x)\Big(\hat{V}(x) - \hat{P}(x)\Big) = q_d(x)\Big(\hat{V}(x) - P_d(x)\Big) \quad \forall x \in [\max\{\hat{x}, V_r\}, 1],$$

where $q_d(x)$ is the probability that no buyer of type higher than x will arrive at the deviating auction and $P_d(x)$ is the deviating auction bidding schedule. Note that the latter must satisfy Proposition 6 to ensure buyer incentive compatibility and monotonic

[7] This corresponds to setting the auction price equal to the Nash split price at the threshold, $\bar{x}$. This is generically true when $P(\bar{x}) = V_s^A$.

seller selection. Taking the derivative of the expected utilities of both sides of the above equation and applying Proposition 6 leads to

$$q(x)\hat{V}'(x) = q_d(x)\hat{V}'(x) \quad \forall x \in [\max\{\hat{x}, V_r\}, 1].$$

We claim that the buyer gain value, $\hat{V}(x)$, is increasing in type. To see that, note that its derivative can be written as

$$\hat{V}'(x) = \frac{\beta}{1-\beta(1-q(x)-\rho)}\Big(\delta'(x) + \hat{P}'(x)\hat{q}(x) + \hat{P}(x)\hat{q}'(x) - \beta\hat{V}(x)\hat{q}'(x)\Big).$$

Because $S_b(x) = \hat{V}(x) - \hat{P}(x)$, after applying Eqn. 11, the term in parentheses becomes $\delta'(x) + (1-\beta)\big(\hat{P}(x)\hat{q}(x)\big)'$. The latter is strictly positive, establishing that $\hat{V}'(x) > 0$. It must therefore be that $q(x) = q_d(x)$ for every $x \in [\max\{\hat{x}, V_r\}, 1]$. In turn, utility equalization for buyers participating in both auctions implies $\hat{P}(x) = P_d(x)$ for every $x \in [\max\{\hat{x}, V_r\}, 1]$.

Next, we show that $q(x) = q_d(x)$ uniquely pins down the buyer arrival rate at the deviating auction. Let $p_d(x)dx$ be the probability that a buyer of type x arrives at the deviating auction (note that $p_d(x)$ is a density). Assuming that type arrivals are independent, the probability that no buyer of type higher than x arrives at the deviating auction is $q_d(x) = \exp(-\int_x^1 p_d(x')dx')$.[8] Because $q(x) = q_d(x)$ holds for every type participating in both auction, it must also be that $p_d(x) = \frac{1-m(x)}{I-M}$, where the latter is the type x probability density of arrival in a prevailing auction. This also means that the deviating seller's conditional probability of transacting with a participating type x is the same as that of any prevailing auction seller. In other words, a seller's expected benefit from buyers that participate in both auctions is the same across the two auction types.

The preceding proves that, from the deviating seller's perspective, the impact of deviating from the prevailing reserve price comes solely from those buyers who do not participate in both auctions. Raising the reserve price will eliminate some participants, resulting in lower expected gains from sale. But lowering the reserve price will incentivize searchers with $x < \bar{x}$ to participate in the deviating auction.[9] Thus, if $\hat{P}(\bar{x}^A) > V_s^A$,

[8] This follows from the fact that arrivals are independent and $q_d(x) = \lim_{dx'\to 0}\prod_{x'>x}[1-p_d(x')dx'] \to \exp(-\int_x^1 p_d(x')dx')$.

[9] Note that monotonic selection means that buyers with $x < \bar{x}$ are indifferent to the participation of buyers with $x \geq \bar{x}$.

then reducing the reserve price is optimal for any given seller. As a result, the unique symmetric (across sellers) equilibrium in which the reserve price is optimal for every seller is the one in which $P^A(\bar{x}^A) = V_s^A$. □

Proof of Lemma 6 Assume an auction equilibrium with $V_s^A \geq \frac{\beta}{1-\beta}\delta(0)$. The total surplus can be written

$$S_{\text{tot}}(x) = \frac{\big(\beta\delta(x) - (1-\beta)V_s^A\big) + \beta q(x)\big(P(x) - V_s^A\big)}{1 - \beta\big(1 - \rho - q(x)\big)},$$

and the buyer's surplus can be written as

$$S_b(x) = \frac{\big(\beta\delta(x) - (1-\beta)P(x)\big) + \beta\rho\big(V_s^A - P(x)\big)}{1 - \beta\big(1 - \rho - q(x)\big)}.$$

If $S_{\text{tot}}(0) > 0$ then it must be that $P(0) > V_s$, but this must mean that either $S_b < 0$ or that $\frac{\beta}{1-\beta}\delta(0) > P(0)$, which in turn means $V_s^A > P(0)$. In the latter case, entry at $\bar{x}^A = 0$ implies negative seller surplus for some participating buyer which would be inferior to Nash bargaining. In the former case ($S_b(0) < 0$), entry is endogenous with $\bar{x}^A > 0$ so that $S_b(\bar{x}^A) = 0$ and, from Equation (11) $P'(\bar{x}^A) = 0$. If the seller were to "force" Nash bargaining for some type, $x > \bar{x}^A$, the resulting price, would be $P^N(x) = V_s^A + \frac{1}{2}S_{\text{tot}}(x)$. Differentiating and using Equation (11) yields:

$$\frac{dP^N(x)}{dx} = \frac{1}{2}\frac{\beta\delta'(x)}{1 - \beta(1 - \rho - q(x))}.$$

Because $\delta'(x) > 0$, $\frac{dP^N(x)}{dx} > 0$, and $P(\bar{x}^A) = P^N(\bar{x}^A)$. Thus, for x sufficiently close to $\bar{x}^A$, it must be that $P^N(x) > P(x)$. □