AFFIRMATIVE ACTION IN INDIA: A MARKET DESIGN APPROACH WITH HISTORICAL AND LEGAL PERSPECTIVE

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ABSTRACT: Most of the controversies in India's reservation system stem primarily from Other Backward Classes (OBC) reservations. We analyze India's complex affirmative action policies through a market design lens, focusing on the fundamental tension between the two most important Supreme Court of India (SCI) judgments. Indra Sawhney vs. Union of India (1992) and Ashoka Kumar Thakur vs. Union of India (2008). We formalize the principles established in Indra Sawhney (1992) and Ashoka Kumar Thakur (2008) as mathematical axioms and reveal a fundamental incompatibility between these established principles and the directives. To address incompatibilities while maintaining the judicial norms, we propose a *family of assignment rules* that accommodate soft OBC reservations. Using the reserve policy as a tool to maintain the standard of excellence, we identify the *Forward Transfer* (FT) *assignment rule* as *the most meritorious* assignment rule within this family.

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1. INTRODUCTION

Affirmative action in India has been implemented through a reservation policy that impacts the lives of millions through admission to publicly funded higher education institutions and the allocation of government jobs. Reservation policies were initially put in place to ensure the representation of historically disadvantaged and discriminated Scheduled Castes (SC) and Scheduled Tribes (ST) in education and employment, thus promoting their social and economic advancement. Articles 15(4) and 16(4) of the Indian Constitution of 1950 provided reservations for SCs and STs in educational institutions and government jobs. In 1954, the Ministry of Education suggested that 20% of seats should be reserved for SCs and STs in educational institutions, with 15% for SCs and 5% for STs. The percentage of SC and ST reservation was increased to 15% and 7. 5%, respectively, in central government jobs in 1982.

Reservations for Other Backward Classes (OBC) were formally introduced in 1987 following the recommendations of the Mandal Commission that suggested 27% reservation for OBCs in government jobs. There was strong opposition from different segments of society.[1] Shortly after, OBC reservations were challenged in court. In 1992, the Supreme Court of India (SCI), in the landmark case Indra Sawhney vs. Union of India (henceforth Indra Sawhney (1992)), upheld the government's decision to provide 27% reservation to OBCs.[2] Reservations for OBCs still remain the most controversial in Indian society and have been challenged multiple times in the SCI.

Indra Sawhney (1992) refers to the reservations for SC, ST and OBC as **vertical** reservations. The residual (unreserved) positions are classified as **open category** and are accessible to all applicants, including those in vertical categories. Applicants who do not identify with SC, ST, or OBC are classified in the **general category**. Indra Sawhney (1992)

[1]While this issue encompasses deep-rooted social and political dimensions, we maintain a neutral stance, focusing solely on a theoretical analysis of reservations in India rather than offering personal opinions.
[2]The case can be accessed at https://indiankanoon.org/doc/1363234/. Last accessed on 04/02/2025. Sönmez and Yenmez (2022) analyze Indra Sawhney (1992) from the matching theoretical perspective and formulate principles laid out in Indra Sawhney as formal mathematical axioms.

considers reservations for SC, ST, and OBC as **hard reserves** in the sense that the positions left from the SC, ST, and OBC categories cannot be provided to others.

Indra Sawhney (1992) established three fundamental principles that must be respected in the reserve system.

(1) The **over-and-above** (OA) principle: Applicants with the highest scores must be admitted through the open category.

(2) The **within-category fairness** (WCF) principle: If an applicant with a lower score obtains a position, then an applicant from the same category with a higher score must also obtain a position.

(3) The **quota-filling subject to eligibility** (QFE) principle: In each category, all positions must be assigned as long as there are eligible applicants.

By admitting high-scoring applicants from vertical reserve categories in the open category, the OA principle ensures that positions earmarked for reserved categories can give an extra push to applicants from the reserved categories. The WCF principle is a fairness criterion that requires selecting candidates based on merit scores in each category. Finally, the QFE principle is a mild efficiency requirement.

When reserves are considered a **hard constraint** in each reserve category, the three principles above characterize the following simple assignment rule used in India for decades.

> *First, applicants are selected for open category positions one at a time following the merit score ranking up to the capacity of the open category. Then, applicants are selected for vertical reserve categories following the merit score ranking of applicants in each respective category up to their capacities.*

We refer to this assignment procedure as the "*Indra Sawhney (IS) assignment rule* ." In Theorem 0, we show that an assignment rule satisfies the OA, WCF, and QFE properties if and only if it is the Indra Sawhney assignment rule.[3]

[3] Sönmez and Yenmez (2022) also provides a similar characterization in plain English without formalization and proof. They gave a verbal description of the OA principle that is different from our definition. See Footnote 8 for further details.

The judicial challenges to reservations for OBCs continued well beyond the SCI's ruling in the Indra Sawhney case. A significant development occurred in 2006 when the constitutional validity of the Central Educational Institutions (Reservation in Admission) Act was contested in the historic case Ashoka Kumar Thakur vs. Union of India (2008) (henceforth Ashoka Kumar Thakur (2008)). This pivotal case addressed the Act's provision for 27% reservation for OBCs in central educational institutions. In its judgment, the SCI modified the eligibility criteria for OBC positions with the explicit aim of **maintaining academic excellence**. The Court specifically mandated that the minimum qualifying score (cut-off) for candidates in the OBC category must not fall more than 10 points below that established for the open category.

> *It is reasonable to balance reservation with other societal interests. To maintain standards of excellence, cutoff marks for OBCs should be set not more than 10 marks out of 100 below that of the general category.*[4]

In most of the resource allocation scenarios across India, a substantial number of OBC positions are still unfilled. Baswana, Chakrabarti, Chandran, Kanoria, and Patange (2019) identify the prevalence of vacant OBC positions as one of the most critical design challenges in technical university admissions. The SCI's mandate establishing a higher threshold for OBC positions potentially exacerbates the underutilization of these reserved seats. Recognizing the inefficiencies that could result from this elevated threshold, the Court advocated for the implementation of **flexible** (or "**soft**") reservations for the OBC category as a remedial measure. This approach would allow the reallocation of unfilled OBC positions to candidates from other categories. A significant excerpt from the Ashoka Kumar Thakur (2008) judgment articulates this position as follows.

[4]There is widespread confusion in India regarding the position types and applicant types. A cursory search on the Internet reveals that the term ""general seats" is used to describe the open category positions. The same confusion also arises in Supreme Court decisions. In the above quote, by "general category" in the last sentence, they mean the "open category." Aygün and Turhan (2022) show that there are significant unintended consequences if the remaining OBC positions are **only** provided to students in the general category. Consistent with our interpretation, vacant OBC positions have been assigned as an open category in admissions to technical universities. See Baswana, Chakrabarti, Chandran, Kanoria, and Patange (2019) for further details.

> *Only non-creamy layer OBCs can avail of reservations in college admissions, and once they graduate from college, they should no longer be eligible for post-graduate reservation. 27% is the upper limit for OBC reservations. The Government need not always provide the maximum limit. Reasonable cutoff marks should be set so that standards of excellence are not greatly affected. The unfilled seats should revert to the general category.*[5]

However, the Ashoka Kumar Thakur (2008) judgment failed to establish a procedural framework for simultaneously implementing both the mandated differential between the open category and OBC cutoff scores and the recommended soft reservation policy for OBC positions. In their recent reform of the engineering college admission process, Baswana, Chakrabarti, Chandran, Kanoria, and Patange (2019) identify the de-reservation of unfilled OBC positions as one of the most significant and complex design challenges to be addressed.

In Theorem 1, we demonstrate that the concurrent implementation of the OBC cutoff score mandate and the recommended soft reservations in the OBC category fundamentally conflict with the principles established in the Indra Sawhney (1992) judgment. Our analysis reveals that no assignment rule can simultaneously satisfy all these requirements, highlighting an inherent incompatibility in the current legal framework.

The absence of explicit guidelines on the implementation of soft reservations for the OBC category has resulted in diverse inconsistent practices and considerable confusion throughout India. This ambiguity mainly concerns two critical aspects: first, whether the reverted positions from the unfilled OBC quota should be exclusively allocated to candidates in general categories or made available to all applicants, including those in reserved categories; and second, the appropriate sequencing to fill these reverted positions within the overall allocation process, specifically, whether they should be filled before or after processing the vertical reserve categories (Aygün and Turhan (2022)). For example, in a recently reformed admissions protocol for technical universities, the design team addresses

[5]The same confusion we pointed out in Footnote 4 applies here. The unfilled seats should revert to the "open category" is what is meant. See Aygün and Turhan (2022) for more on this point.

OBC de-reservation through a distinctive approach, re-executing the student-proposing deferred acceptance (DA) algorithm of Gale and Shapley (1962). This process systematically transfers unfilled OBC positions to the open category in a backward manner at the conclusion of each iteration by dynamically adjusting the distribution of positions across position categories. Baswana, Chakrabarti, Chandran, Kanoria, and Patange (2019) call this process *multi-run DA algorithm*.

The absence of explicit judicial directives on the implementation of soft reservations for OBCs presents an opportunity for innovative policy design. In this paper, we propose a family of assignment rules that simultaneously satisfies the OA principle, WCF, and QFE established in the Indra Sawhney (1992) judgment, while accommodating soft OBC reservations. The various assignment rules within this proposed family are differentiated primarily by the specific point in the precedence order at which unfilled positions from the OBC category are reallocated as open category positions. The Ashoka Kumar Thakur (2008) judgment placed considerable emphasis on maintaining the standard of excellence in the implementation of affirmative action programs. Modifying the set comparison concept of Gale (1968), we formulate a **merit-based comparison criterion** to compare assignment functions. Within our proposed family of assignment rules, we characterize a particular procedure, the **forward transfer (FT) assignment rule**, as **the most meritorious** among all assignment rules in this family. Thus, reinterpreting the policy goal of maintaining the standard of excellence, our analysis advocates the FT assignment rule to be implemented in India.

This paper is structured as follows. Section 2 discusses the related literature. Section 3 develops the theoretical framework that features a single institution in which reservations are formulated as hard constraints, following the judicial precedent established by Indra Sawhney (1992). Section 4 examines the policies and directives introduced in Ashoka Kumar Thakur (2008), which recommended the implementation of soft reservation for the OBC category. In Section 5, we design a family of assignment rules characterized by

established judicial principles. Section 6 concludes. All formal proofs are relegated to the appendix.

2. Literature Review

The closest paper to ours is Aygün and Turhan (2022), in which we examined the concurrent implementation of vertical reservations and OBC de-reservations, explicitly focusing on the admissions procedures for technical universities in India. Aygün and Turhan (2022) introduced the Backward Transfers (BT) assignment rule for institutions that ensures higher cutoff scores for the open category than those of the reserve categories. The BT assignment rule is in the family of choice rules developed in the present paper. The FT rule merit-dominates every other assignment rule in the family designed in this paper, including the BT assignment rule. The FT and BT assignment rules are the two extremes of this family of assignment rules. The BT rule guarantees a higher cutoff score for the open category, while the FT rule selects the best set of applicants.

Our paper complements Sönmez and Yenmez (2022) in which an axiomatic approach is taken to formulate policy goals as formal axioms and study the joint implementation of vertical and horizontal reservations. Unlike Sönmez and Yenmez (2022), our main focus is to propose effective ways to implement soft OBC reservations in conjugation with hard SC, ST and EWS reservations. Sönmez and Yenmez (2022) take OBC reservations as hard reserves and propose a choice rule to deal with overlapping horizontal reservations as their main innovation.

This paper contributes to the strand of market design literature on allocation problems in India under complex diversity constraints. Echenique and Yenmez (2015) is the first paper to present Indian affirmative action as an example of controlled school choice. Aygün and Turhan (2017) delves into the specific challenges faced in admissions to the Indian Institutes of Technology (IIT). Aygün and Turhan (2020) offer a comprehensive treatment of vertical reservations and de-reservations in the context of admissions to IITs by introducing a family of *dynamic reserves* choice rules. In a recent paper, Sönmez and Ünver (2024)

take an axiomatic approach to study the Supreme Court judgment in Janhit Abhiyan vs. Union of India (2022) and its consequences.

3. MODEL: INDRA SAWHNEY (1992)

Consider an institution s with $\overline{q}_s$ positions. Let $R = \{SC, ST, OBC, EWS\}$ be the set of reserve categories. We denote by q_s^r the number of positions reserved for category $r \in R$ such that $\sum_{r \in R} q_s^r < \overline{q}_s$. We let $q_s^o = q_s - \sum_{r \in R} q_s^r$ be the number of open (unreserved) positions. The vector $\mathbf{q_s} = \left(q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}, q_s^{EWS}\right)$ denotes the initial allocation of positions in categories.

Given a set of applicants A, the function $t : A \to \{g\} \bigcup R$ denotes the category membership of the applicants, where g denotes the "general category." $t(i) = g$ means that individual i does not belong to any vertical reserve category, or she did not report her category membership. In India, it is **optional** to report membership in the reserve categories. Members of the reserve categories who do not report their membership are considered general category applicants and are eligible only for open category positions. Members of the reserve category $r \in R$ are eligible for both the open category and reserved category r positions. We let $T = (t(i))_{i \in A}$ denote the profile of the applicants' reserve category membership.

Applicants' merit scores induce a priority ranking for institution s. We denote this linear order over $I \cup \{\emptyset\}$ by $\succ_s$.[6] We write $i \succ_s j$ to mean that the applicant i has a higher priority (i.e., a higher merit score) than the applicant j in institution s. $i \succ_s \emptyset$ means that the applicant i is acceptable for institution s. Similarly, we write $\emptyset \succ_s i$ to say that applicant i is unacceptable for institution s.

The priority ranking for applicants of type $r \in R$, denoted by $\succ_s^r$, is obtained from $\succ_s$ in a straightforward way as follows:

(i) for $i, j \in A$ such that $t(i) = r$, $t(j) \neq r$, $i \succ_s \emptyset$, and $j \succ_s \emptyset$, we have $i \succ_s^r \emptyset \succ_s^r j$,[7]

(ii) for any other $i, j \in A$, $i \succ_s^r j$ if and only if $i \succ_s j$.

[6] In India, when two or more applicants have the same score, ties are broken with a given objective criteria.

[7] $\emptyset \succ_t^s j$ means applicant j is unacceptable for category r at s.

The former property means that each category $r \in R$ is exclusive. That is, applicants who do not belong to category $r \in R$ are deemed unacceptable for positions in category r. The latter states that the ranking $\succ_s$ is preserved among the applicants in category r.

Given the set of applicants A and applicant $i \in A$, let $rank_A(i)$ be the rank of applicant i in the set A with respect to the merit ranking $\succ_s$. That is, $rank_A(i) = k$ if and only if

$$\mid \{j \in A \mid j \succ_s i\} \mid = k - 1.$$

Let $A^r = \{i \in A \mid i \succ_s^r \emptyset\}$ be the set of applicants in the set A who are eligible for category $r \in R$.

Conventionally, institutions' selection from a given set of applicants under diversity and affirmative action constraints is modeled via *choice rules*. Choice rules, which select a subset from any given set of applicants, are insufficient to formulate the problems we consider because they do not specify the position types under which applicants are chosen. In India, the type of positions to which applicants are assigned is made public. In what follows, we define an *assignment rule* that specifies, for each applicant, the type of position they are assigned to.

DEFINITION 1. *An **assignment rule** η is function, where, given a set of applicants A and a profile of category membership T,*

$$\eta(A, T) = \{(i, \tau) | i \in A \text{ and } \tau \in \{o\} \cup R\}$$

such that

(i) for any $i \in A$, we have at most one $(i, \tau) \in \eta(A, T)$, where

$$\begin{cases} \tau = o & \text{if } t(i) = g, \\ \tau \in \{t(i), o\} & \text{if } t(i) \in R, \end{cases}$$

(ii) $|\eta_s(A, T)| \leq \overline{q}_s$, and

(iii) $\mid \{(i, \tau) \in \eta(A, T) : \tau = r\} \mid \leq q_s^r$, for each $r \in R$.

Condition (i) is a feasibility requirement that says that applicants' eligibility for different position types must be respected. Condition (ii) is the standard capacity constraint. Condition (iii) states that capacities of reserve categories must be respected.

Each assignment rule η induces an associated choice rule C^{η}, which specifies a selected subset of any given set of applicants. That is, for any given $I \subseteq A$, $C^{\eta}(I) \subseteq I$. Formally, given a set of applicants $I \subseteq A$ and assignment function η_s, the **induced choice rule** C^{η} is defined as

$$C^{\eta}(I) = \{i \in I | \exists (i, r) \in \eta(I) \text{ for some } r \in R\}.$$

3.1. *Indra Sawhney (1992) Principles as Formal Axioms*

Indra Sawhney (1992) considers SC, ST, and OBC reservations as **hard** constraints and formulates three desiderata for an institution to make choices from a given set of applicants.

The first one requires that open category positions are allocated to the highest-scoring applicants up to the open category's capacity. In doing so, positions reserved for reserved categories can be more effectively assigned to individuals who would otherwise not qualify for these positions without the reservation policy.

DEFINITION 2. *An assignment rule η satisfies the **over-and-above principle** (OA) if each individual $i \in A$ such that $rank_A(i) \leq q_s^o$,*

$$(i, o) \in \eta(A, T).$$

The second is a fairness condition that requires that whenever an applicant with a lower priority obtains a position, then a higher-scoring applicant from the same category must assign a position, as well.

DEFINITION 3. *An assignment rule η satisfies **within-category fairness** (WCF) if, for any pair of individuals $i, j \in A$ such that $t_i = t_j = t$ and $i \succ_s j$,*

$$(j, \tau) \in \eta(A, T), \text{for some } \tau \in \{o, t\} \text{ implies } (i, \tau') \in \eta(A, T), \text{for some } \tau' \in \{o, t\}.$$

The last is a mild efficiency property.

DEFINITION 4. *An assignment rule η satisfies **quota-filling subject to eligibility** (QFE) if individual i with $t(i) = r$ is not assigned, then for all $r \in R$,*

$$|i \in A : (i, r) \in \eta(A, T)| = q^r.$$

3.2. *Assignment Rule Introduced in Indra Sawhney (1992)*

The three conditions described in the previous section characterize the following choice rule that has been used in India for decades when reserves in each vertical category are considered hard reserves.

Let $q_s = \left(q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}, q_s^{EWS}\right)$ be the initial distribution of positions in categories. Given a set of applicants A and the profile of category membership $T = (t(i))_{i \in A}$, the assignment of applicants to categories is calculated as follows:

Stage 1. Only open category positions are considered. For each applicant $i \in A$ such that $rank_A(i) \leq q_s^o$, add (i, o) to $\eta_s^o(A, T)$. Let $C_s^o(A, T)$ be the set of individuals chosen in this stage.

Stage 2. Among the remaining applicants $A' = A \setminus C_s^o(A, T)$, for each reserve category $r \in R$, applicants are chosen one at a time following $\succ_s^r$ up to capacity q_s^r. Let us call the set of applicants chosen for the reserved category r by $C_s^r\left(A', q_s^r\right)$. For each individual i chosen in category r, add (i, r) to $\eta_s^r(A', T)$.

Then $\eta_s^{IS}(A, T)$ is defined as

$$\eta_s^{IS}(A, T) = \eta_s^o(A, T) \cup \bigcup_{r \in \mathcal{R}} \eta_s^r\left(A', q_s^r\right).$$

THEOREM 0. *An assignment rule satisfies the OA principle, WCF, and QFE if and only if it is the Indra Sawhney assignment rule.*

Theorem 0 is the starting point of our axiomatic analysis.[8]

[8]An indirect (in plain English) characterization of the same choice rule is given in Section 2.2 of Sönmez and Yenmez (2022), in which the OA principle is defined differently as follows: *"Next, VR protected positions*

4. ASHOKA KUMAR THAKUR (2008)

The judgment in Ashoka Kumar Thakur (2008) brings about two crucial changes: (1) It changes the eligibility criteria for the OBC candidates, thereby modifies the QFE principle of Indra Sawhney (1992), and (2) it recommends OBC reservations to be implemented as soft reserves.

The judgment of Ashoka Kumar Thakur (2008) mandated a minimum cutoff score for the OBC category to **maintain the standard of excellence**. The SCI explicitly stated that there should be a reasonable difference in the OBC and open category cutoff scores.

> *It is reasonable to balance reservation with other societal interests. To maintain standards of excellence, cutoff marks for OBCs should be set not more than 10 marks out of 100 below that of the general category.*

In other words, the directive on cutoff scores in Ashoka Kumar Thakur (2008) modifies the eligibility criteria for candidates from OBC. We formulate this directive as a formal axiom below.

DEFINITION 5. *An assignment function η satisfies the **modified QFE** if for any set of applicants A and $T = (t(i))_{i \in A}$, the following holds.*

(1) If an individual i with $t(i) = r \in R \setminus \{OBC\}$ is not assigned, then the number of individuals who are matched to a category r position must be equal to q_s^r.

(2) If an individual i with $t(i) = OBC$ is not assigned and $\theta(i) \geq min\{\theta(j)|(j,o) \in \eta(A)\} - \kappa$, then the number of individuals who matched to an OBC position must be equal to q_s^{OBC}.

(3) If an individual i with $t(i) = OBC$ and $\theta(i) < min\{\theta(j)|(j,o) \in \eta(A)\} - \kappa$, then $(i, OBC) \notin \eta(A)$.

must be allocated on an "over-and-above" basis; that is, positions that can be received without invoking the VR protections do not count against VR-protected positions." Note that their definition does not require that the highest-scoring applicants be assigned to the open category.

The modified QFE principle may require that some OBC positions be left unfilled to maintain the standard of excellence. Realizing this effect, to prevent waste in that case, the SCI recommended that OBC reservations be implemented as soft reserves.

> *To this end, the Government shall set up a committee to look into the question of setting the OBC cutoff no more than 10 marks below that of the general category. Under such a scheme, whenever the non-creamy layer OBCs fail to fill the 27% reservation, the remaining seats would revert to general category students.*

The judgment recommends a policy to provide unfilled OBC positions to others. We formulate this recommendation, the OBC de-reservation, as a formal axiom.

DEFINITION 6. *An assignment function η satisfies the **OBC de-reservation** if for any set of individuals A and $T = (t(i))_{i \in A}$,*

$$|\{(i,\tau) \in \eta(A,T) : \tau \in \{o, OBC\}\}| < q_s^o + q_s^{OBC} \implies |\eta(A,T)| = |A|.$$

The OBC de-reservation policy requires reverting the vacancies in OBC category to open category. When reverted open category positions are filled, that is, more than q_s^o positions are allocated as open category, the open category cutoff score becomes lower. In turn, some candidates for the OBC who were ineligible due to their low scores may become eligible with the lower open category cutoff score. In a sense, the two AKT directives contradict each other. Our next result formalizes this observation.

THEOREM 1. *No assignment function satisfies the OA principle, WCF, modified QFE, and OBC de-reservation.*

PROOF. Consider an institution with four positions. One position is reserved for SC, one for ST and one for OBC. The remaining position is an open category. Consider the set of individuals $A = \{i_1, i_2, i_3, i_4, i_5\}$ with the following scores and category membership:

Individual	*Category*	*Score*
i_1	g	100
i_2	g	90
i_3	OBC	89
i_4	SC	81
i_5	ST	80

The OA principle, WCF, and modified QFE imply that $(i_1, o), (i_4, SC), (i_5, ST) \in \eta_s(A, T)$. Given the category memberships of the individuals i_2 and i_3, one of the following must hold.

(1) $\eta_s(A, T) = \{(i_1, o), (i_3, OBC), (i_4, SC), (i_5, ST)\}$.

(2) $\eta_s(A, T) = \{(i_1, o), (i_2, o), (i_4, SC), (i_5, ST)\}$.

(3) $\eta_s(A, T) = \{(i_1, o), (i_3, o), (i_4, SC), (i_5, ST)\}$.

(4) $\eta_s(A), T = \{(i_1, o), (i_4, SC), (i_5, ST)\}$.

The assignment function in (1) violates the modified QFE because i_3 is assigned to an OBC position even though her score is 11 points lower than the cutoff score of the open category. The assignment function in (2) violates the modified QFE because the open category cutoff score is 90 (that is, the score of i_2) while i_3 was not chosen even though she is eligible for the OBC position. The assignment function in (3) violates WCF because i_2 has a higher score than i_3, assigned an open category position. Finally, the assignment function in (4) violates the OBC de-reservation policy because, from the open category and OBC combined, only i_1 is chosen. However, $|\eta_s(A, T)| = 3$ and $|A| = 5$. Thus, no assignment function can satisfy these four axioms concurrently.

■

5. Designing for the Standard of Excellence

In many applications in India, such as admissions to engineering schools, medical schools, and faculty recruitment in public universities, many OBC positions are left vacant each year.[9] Baswana, Chakrabarti, Chandran, Kanoria, and Patange (2019) documents a large number of vacancies in the OBC category in admissions to engineering colleges.

As we discussed earlier, mandating a minimum cutoff score for the OBC category by making it a function of the cutoff score of the open category is motivated by the policy goal of "maintaining the standard of excellence."[10] Given the impossibility result in Theorem 1, we reinterpret the principle of maintaining the standard of excellence as follows: Among all possible choice rules via different transfer policies, which choice rule always selects the most meritorious set of applicants?

Building on the impossibility result in Theorem 1, in this section, we introduce assignment functions by (i) weakening the modified QFE to the Indra Sawhney (1992)'s version of QFE principle and (ii) dropping the OBC de-reservation policy.

We consider assignment rules that satisfy the OA principle, WCF, QFE principles from Indra Sawhney (1992) together with the OBC de-reservation policy from Ashoka Kumar Thakur (2008). Throughout this section, we assume that any vacant position in the OBC category must be provided to others as an open category position, whenever possible.

An institution initially fills its positions in the following precedence order.

$$Open \rightarrow SC \rightarrow ST \rightarrow OBC \rightarrow EWS. \tag{1}$$

If there is no vacancy in the OBC category after applying the Indra Sawhney assignment rule according to the precedence order (1), then we are done. If there are vacancies

[9] For vacancies in the medical universities and the Indian Institute of Sciences (IISc), please see https://www.outlookindia.com/national/india-news-obc-reservation-55-reserved-posts-lying-vacant-in-govt-universities-news-390255.

[10] There is a similar cutoff score restriction in admissions to high schools in China (Hu and Zhang (2025)). Wuhan eliminated the 30-point cutoff score constraint for reserved seats in 2024. Hu and Zhang (2025) argues that there is a trend in China toward completely eliminating this constraint.

in the OBC category, they must be allocated as open category positions. Where in the precedence order these additional open category positions are filled has distributional consequences. We expand the precedence order (1) to locate new groups of open category positions T_1, T_2, T_3, and T_4, referred to as the **de-reservated open-categories** below.

$$Open \to T_1 \to SC \to T_2 \to ST \to T_3 \to OBC \to T_4 \to EWS \to T_5. \tag{2}$$

Each de-reserved open category T_i has the initial capacity of 0. When there are $\sum_{i=1}^{5} r_i$ many unfilled positions in the OBC category, the vector $(r_1, r_2, r_3, r_4, r_5)$ describes how these vacancies are distributed in the groups T_1, T_2, T_3, T_4, and T_5, where r_i is the number of positions allotted to the group T_i, for all $i = 1, ..., 5$.

Given the expanded precedence order (2), a set of individuals A, a category membership profile $T = (t(i))_{i \in A}$, and the capacities of the groups of positions $\mathbf{x} = (x_1, \ldots, x_{10}) \in \mathbb{Z}_+^{10}$, the assignment rule $\eta^{\mathbf{x}}(A, T)$ induced by the capacity vector $\mathbf{x}$ is calculated as follows:

Step 1. Consider all applicants. Select top-x_1 applicants according to merit scores. If there are fewer than x_1 applicants, select all of them. Add (i, o) to $\eta^{\mathbf{x}}(A, T)$ for any applicant $i \in A$ selected. Let A^1 be the set of remaining applicants.

Step $10 \geq k \geq 2$**.** Consider the relevant applicants[11] in A^{k-1}. The groups $\{T_i\}_{i=1}^{5}$ consider all applicants. Select top-x_k applicants according to merit scores. If there are fewer than x_k applicants, select all of them. For each applicant $i \in A^{k-1}$ selected in the groups $T_1, ..., T_5$, add (i, o) to $\eta^{\mathbf{x}}(A, T)$. Similarly, for each applicant i who is selected in category $r \in \{SC, ST, OBC, EWS\}$, add (i, r) to $\eta^{\mathbf{x}}(A, T)$. Let A^k be the set of remaining applicants.

Let $C^{\mathbf{x}}(A)$ be the induced choice function from $\eta^{\mathbf{x}}(A, T)$.

[11]The relevant applicants for the de-reserved open categories T_1, T_2, T_3, T_4 and T_5 are **all** applicants in the set A^{k-1}. The relevant applicants for the SC (similarly for the ST / OBC / EWS) categories are the applicants who belong to SC (similarly to ST/OBC/EWS) in the set A^{k-1}.

5.1. *A Family of Assignment Rules*

Let q_s^{OBC} be the capacity of the OBC category. We first define a transfer scheme, where κ denotes the number of vacant positions in OBC category. Note that $\kappa \in \{0, ..., q_s^{OBC}\}$.

DEFINITION 7. *A transfer scheme is a 5-dimensional vector function f such that*

(1) $f(\kappa) = (f_1(\kappa), f_2(\kappa), f_3(\kappa), f_4(\kappa), f_5(\kappa))$ *where* $\kappa = \sum_{j=1}^{5} f_j(\kappa)$, *for each* $\kappa \in \{0, \ldots, q_s^{OBC}\}$ *and* $f_i(\kappa) \geq 0$ *is the number of available positions in category* T_i.

(2) $f_i(x+1) - f_i(x) \in \{0, 1\}$, *for all* $i \in \{1, 2, 3, 4, 5\}$ *and* $\sum_{i=1}^{5}[f_i(x+1) - f_i(x)] = 1$, *for all* $x \in \{0, \ldots, q_s^{OBC} - 1\}$.

Let $\mathcal{F}$ be the set of all transfer schemes. The assignment rule induced by the transfer scheme $f \in \mathcal{F}$, η^f is constructed as follows.

Initially, we set the capacity of each group of transferred slots T_i to 0 and construct

$$\mathbf{x^1} = (q_s^o, 0, q_s^{SC}, 0, q_s^{ST}, 0, q_s^{OBC}, 0, q_s^{EWS}, 0).$$

Given a set of applicants A, a category membership profile $T = (t(i))_{i \in A}$, and the vector of initial position distribution $q_s = (q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}, q_s^{EWS})$, the assignment rule $\eta_s^f(A, T)$ selects the applicants in (possibly) multiple iterations as follows:

Iteration 1. Run $\eta_s^{\mathbf{x^1}}(A, T)$. Let κ^1 be the number of vacant positions in OBC. Update the number of positions by transferring κ^1 many positions to the de-reserved open categories via f. That is, $\mathbf{x^2} = (x_1^2, \ldots, x_{10}^2)$ is such that

$$x_{2j}^2 = f_j(\kappa^1), \text{ for all } j = 1, ..., 5,$$

$$x_7^2 = q_s^{OBC} - \kappa^1,$$

$$x_1^2 = x_1^1, x_3^2 = x_3^1, x_5^2 = x_5^1, \text{ and } x_9^2 = x_9^1.$$

Note that the capacities of the Open, SC, ST, and EWS categories remain unchanged.

Iteration n (n≥2). Run $\eta_s^{\mathbf{x^n}}(A,T)$. Let κ^n be the number of vacant OBC positions. Update the number of positions by transferring κ^n many positions in the transferred slots. That is, $\mathbf{x^{n+1}} = (x_1^{n+1},\ldots,x_{10}^{n+1})$ is such that

$$x_{2j}^{n+1} = x_{2j}^n + f_j(\kappa^n), \text{ for all } j = 1,...,5,$$

$$x_7^{n+1} = x_7^n - \kappa^n,$$

$$x_1^{n+1} = x_1^n, x_3^{n+1} = x_3^n, x_5^{n+1} = x_5^n, \text{ and } x_9^{n+1} = x_9^n.$$

The process ends when there are no vacant positions in the OBC category. Let N be the last iteration. Then,

$$\eta_s^f(A,T) = \eta_s^{\mathbf{x^N}}(A,T).$$

Let $C_s^f(A)$ be the induced choice rule from $\eta_s^f(A,T)$.

We denote by $\mathbb{A} \equiv \left(\eta^f\right)_{f\in\mathcal{F}}$ the set of assignment rules generated by different transfer schemes. Notable assignment rules in this family include the choice rule that moves all transferable vacant positions to T_4, which we call the assignment rule **forward transfer** (FT), and to T_1, which is the assignment rule **backward transfer** (BT) introduced in Aygün and Turhan (2022).

Forward Transfer (FT) Assignment Rule

The FT choice rule is such that $f(\kappa) = (0,0,0,0,\kappa)$ for all $\kappa \in \left\{0,\ldots,q_s^{OBC}\right\}$. We denote this assignment rule η_s^{FT}. Let C_s^{FT} be the choice rule induced by η_s^{FT}.

Backward Transfer (BT) Assignment Rule

The BT choice rule is such that $f(\kappa) = (\kappa,0,0,0,0)$ for all $\kappa \in \left\{0,\ldots,q_s^{OBC}\right\}$. We denote this assignment rule η_s^{BT}. Let C_s^{BT} be the choice rule induced by η_s^{BT}.

EXAMPLE 1. *Suppose that* $A = \{i_1,i_2,i_3,i_4,i_5,i_6,i_7,i_8\}$. *Applicants' test scores induce the following priority ranking:*

$$i_1 \succ i_2 \succ i_3 \succ i_4 \succ i_5 \succ i_6 \succ i_7 \succ i_8.$$

Applicants' category memberships are such that $t(i_1) = t(i_2) = t(i_3) = t(i_5) = g$, $t(i_4) = t(i_8) = SC$, $t(i_6) = OBC$, *and* $t(i_7) = ST$.

Consider an institution s *with seven positions. The initial distribution of seven positions over categories is* $(o, SC, ST, OBC, EWS) = (3, 1, 1, 2, 0)$.

We first compute $\eta_s^{FT}(A, T)$. i_1, i_2, *and* i_3 *are selected for open category positions.* i_4 *and* i_7 *are assigned to the reserved SC and ST positions, respectively.* i_6 *is chosen for a reserved OBC position. The other OBC position remains unfilled and is made an open category. Among the unassigned individuals,* i_5 *has the highest score and is chosen for the open category position. We have*

$$\eta_s^{FT}(A, T) = \{(i_1, o), (i_2, o), (i_3, o), (i_4, SC), (i_5, o), (i_6, OBC), (i_7, ST)\}.$$

We now compute $\eta_s^{BT}(A, T)$. *In the first iteration,* i_1, i_2, *and* i_3 *are selected in the open category,* i_4 *is selected in the SC category,* i_7 *is selected in the ST category, and the only OBC individual* i_6 *is chosen in the OBC category. One reserved OBC seat remains vacant. Therefore, it becomes an open category position by updating the capacity vector to* $q_s^2 = (4, 1, 1, 1, 0)$.

In the second iteration, i_1, i_2 , i_3, *and* i_4 *are selected in the open category,* i_8 *is selected in the SC category,* i_7 *is selected in the ST category and* i_6 *is chosen in the OBC category. We end the procedure because there is no vacancy in the OBC category. Thus, we have*

$$\eta_s^{BT}(A, T) = \{(i_1, o), (i_2, o), (i_3, o), (i_4, o), (i_6, OBC), (i_7, ST), (i_8, SC)\}.$$

Note that $\eta_s^{FT}(A, T) \setminus \eta_s^{BT}(A, T) = \{(i_5, o)\}$ *and* $\eta_s^{BT}(A, T) \setminus \eta_s^{FT}(A, T) = \{(i_8, SC)\}$.

5.2. *Characterization of the FT Assignment Rule*

We first introduce the following criteria that compare two sets of individuals based on merit.[12]

DEFINITION 8. *The set of individuals* I merit dominates *the set of individuals* J *with* $\mid I \mid \geq \mid J \mid$ *at institution* s *if there exists an injection* $g : J \to I$ *such that*

[12]Our merit domination criterion is stronger than the Gale domination introduced in Gale (1968). Gale domination induces a reflexive binary relation, while ours is irreflexive.

(1) for all $j \in J$, $\theta(g(j), s) \geq \theta(j, s)$, *and,*

(2) there exists $j \in J$ *such that* $\theta(g(j), s) > \theta(j, s)$.

The merit-based comparison of two sets can be extended to compare choice rules.

DEFINITION 9. *A choice rule* C merit dominates *another choice rule* $C^{'}$ *if, for all sets of individuals* $I \subseteq A$, *either* $C(I) \supseteq C^{'}(I)$ *or* $C(I)$ *merit dominates* $C^{'}(I)$.

In Example 1, the set of individuals chosen induced by $\eta_s^{FT}(A, T)$ is a more meritorious set than the set of individuals chosen induced by $\eta_s^{BT}(A, T)$. We now generalize this observation and establish that the FT choice rule merit dominates other choice rules induced by the assignment rules in the family F.

THEOREM 2. *The FT choice rule merit dominates any choice rule induced by the assignment rules in* $\mathbb{A}$.

This result implies that if selecting the most meritorious set of applicants is the policy objective, then the FT choice rules are the right choice as a soft reserve policy.

6. CONCLUSION

In this paper, we have analyzed India's complex affirmative action policies through the lens of market design, focusing on the fundamental tension between two landmark Supreme Court of India judgments: Indra Sawhney (1992) and Ashoka Kumar Thakur (2008). Our theoretical analysis reveals important insights into the implementation challenges of India's reservation system.

First, we formalized the principles established in Indra Sawhney (1992) as axioms and characterized the "Indra Sawhney assignment rule" that uniquely satisfies these properties. We then demonstrated the incompatibility between these principles and the directives mandated in Ashoka Kumar Thakur (2008), particularly regarding OBC cutoff score restrictions and de-reservation policies. To address this incompatibility, we proposed a family of assignment rules that minimize deviations from established judicial

norms while allowing flexibility in the implementation of soft OBC reservations. We introduced the concept of merit domination to compare assignment rules and identified the Forward Transfer (FT) assignment rule as the most merit-maximizing option within our proposed family.

Our paper makes several key contributions to the literature on market design for diversity and affirmative action. By providing a formal analysis of India's complex reservation system, we bridge the gap between legal principles and practical mechanism design. The assignment rules we propose offer potential solutions to the ongoing challenges in implementing reservation policies in India's education and employment sectors.

7. APPENDIX

Proof of Theorem 1. By definition of η_s^{IS}, the first q_s^o top-ranked applicants are assigned open category positions. If there are fewer than q_s^o applicants, all of them are assigned open category positions. Thus, the over-and-above principle is satisfied. Because each category fills its positions following the merit score rankings, the within-category fairness is trivially satisfied. The only way for an individual i to be unassigned is that positions that i is eligible for are filled with higher merit score individuals. Thus, the capacity filling subject to eligibility is also satisfied.

Let η_s be an assignment rule that satisfies the three axioms. By the OA principle, we have $\eta_s^o(A,T) = \eta_s^{IS}(A,T)$. If $\mid A \mid \leq q^o$, then we are done. Let $\mid A \mid > q^o$. The QFE requires that if an individual i with $t(i) \in \mathcal{R}$ is rejected, then $\mid C^{t(i)} \mid = q_s^{t(i)}$. The QFE and WCF jointly imply that C^v chooses the top q_s^v individuals from the set $(A \setminus C^o(A)) \bigcap A^v$, for all $v \in \mathcal{R}$. Thus, $C(A) = C^{OA}(A)$.

Proof of Theorem 3.

Consider an institution $s \in \mathcal{S}$ and a set of individuals $A \subseteq \mathcal{I}$. Let $q_s = \left(q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}, q_s^{EWS}\right)$ be the vector of the initial distribution of capacities over categories. Let $\left((q_s^N)^o, q_s^{SC}, q_s^{ST}, (q_s^N)^{OBC}\right)$ and $\left(q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}, q_s^{EWS}, \lambda\right)$ be the final capacity vectors of categories under C_s^{BT} and

C_s^{FT}, respectively. Note that N denotes the final iteration of C_s^{BT} and κ denotes the number of unfilled OBC seats that are converted into positions under C_s^{FT}.

Let $A_t \subseteq A$ denote the set of individuals who belong to reserved category $t \in \mathcal{R}$. For each $t \in \mathcal{R}$, define

$$A_t^{'} = A_t \setminus C_s^o\left(A, q_s^o\right) \widetilde{A}_t = A_t \setminus C_s^o\left(A, (q_s^N)^o\right). \tag{3}$$

By definition of C_s^{BT}, we have

$$C_s^{BT}\left(A, q_s\right) = C_s^o\left(A, (q_s^N)^o\right)\left(\bigcup_{r\in\mathcal{R}} C_s^r\left(\widetilde{A}_r, (q_s^N)^r\right)\right). \tag{4}$$

Similarly, by definition of C_s^{FT}, we have

$$C_s^{FT}\left(A, q_s\right) = C_s^o\left(A, q_s^o\right)\left(\bigcup_{r\in\mathcal{R}} C_s^r(A_r^{'}, q_s^r)\right) \cup C_s^{OP}\left(Z, \tau_1\right), \tag{5}$$

where Z is the set of remaining individuals, i.e., $Z = A \setminus C_s^{OA}\left(A, q_s\right)$.

Because we have $(q_s^N)^o \geq q_s^o$ and each category chooses individuals following the merit scores up to their capacities, we have

$$C_s^o\left(A, q_s^o\right) \subseteq C_s^o\left(A, (q_s^N)^o\right).$$

For each $t \in \{SC, ST\}$, because $C_s^t\left(A_t^{'}, q_s^t\right)$ is the top q_s^t candidates, for each individual $i \in C_s^o\left(A_t^{'}, q_s^t\right)$, we have

$$i \in C_s^o\left(A, (q_s^N)^o\right) \cup C_s^t\left(\widetilde{A}_t, q_s^t\right),$$

which implies $C_s^t\left(A_t^{'}, q_s^t\right) \subseteq C_s^{BT}\left(A, q_s\right)$. That is, every SC and ST individual who are chosen from the reserved SC and ST categories under the FT choice rule are also chosen under the BT choice rule.

Let $m_t = \mid \left(C_s^{OP}\left(A, (q_s^N)^{OP}\right) \setminus C_s^{OP}\left(A, q_s^{OP}\right)\right) \cap A_t \mid$, for $t \in \{o, SC, ST\}$. By Lemma 1, we have either

(i) $m_o + m_{SC} + m_{ST} = \tau_1$, or

(ii) $m_o + m_{SC} + m_{ST} < \tau_1$ and $C_s^{BT}(A, q_s) = A$.

In the case of (ii), we have $\mid A \mid < q_s^o + q_s^{OBC}$. This implies $C_s^{FT}(A, q_s) = A$. For the case (i), we have

$$\mid C_s^o\left(A, (q_s^N)^o\right) \mid + \mid C_s^{OBC}\left(\widetilde{A}_{OBC}, (q_s^N)^{OBC}\right) \mid = q_s^o + q_s^{OBC} \qquad (3)$$

which implies

$$\mid C_s^o(A, q_s^o) \mid + \mid C_s^{OBC}\left(A'_{OBC}, (q_s^N)^{OBC}\right) \mid = q_s^o + q_s^{OBC} - \tau_1 \qquad (4)$$

We also have the following inequalities

$$0 < \mid C_s^{SC}\left(A'_{SC}, q_s^{SC}\right) \mid - \mid C_s^{SC}\left(\widetilde{A}_{SC}, q_s^{SC}\right) \mid < m_{SC} \qquad (5)$$

$$0 < \mid C_s^{ST}\left(A'_{ST}, q_s^{ST}\right) \mid - \mid C_s^{ST}\left(\widetilde{A}_{ST}, q_s^{ST}\right) \mid < m_{ST} \qquad (6)$$

Given the equalities (1) and (2), summing over equalities (3) and (4) and inequalities (5) and (6), we have the following inequality:

$$-\tau_1 < \mid C_s^{FT}(A, q_s) \mid - \mid C_s^o(R, \tau_1) \mid + \mid C_s^{BT}(A, q_s) \mid < m_{SC} + m_{ST} - \tau_1 \qquad (7)$$

There are two cases to consider given inequality (7):

Case 1. $\mid C_s^o(R, \tau_1) \mid = \mid R \mid$**.**

In this case, we have $C_s^{FT}(A, q_s) = A \supseteq C_s^{BT}(A, q_s)$. This implies $\mid C_s^{FT}(A, q_s) \mid \geq \mid C_s^{BT}(A, q_s) \mid$.

Case 2. $\mid C_s^o(R, \tau_1) \mid = \tau_1$**.**

In this case, from inequality (7), we have

$$0 < \mid C_s^{FT}(A, q_s) \mid - \mid C_s^{BT}(A, q_s) \mid < m_{SC} + m_{ST},$$

which also implies $\mid C_s^{FT}(A, q_s) \mid \geq \mid C_s^{BT}(A, q_s) \mid$.

Therefore, for any given set $A \subseteq \mathcal{I}$, $\mid C_s^{FT}(A, q_s) \mid \geq \mid C_s^{BT}(A, q_s) \mid$.

We now show that $C_s^{FT}(A, q_s)$ merit-based dominates $C_s^{BT}(A, q_s)$. First, note that we can write the set of selected individuals under the FT choice rule $C_s^{FT}(A, q_s)$ as

$$C_s^{OA}(A, q_s) \cup C_s^{OP}\left(R, \tau^1\right),$$

where $R = A \setminus C_s^{OA}(A, q_s)$ and τ^1 is the number of unfilled OBC positions that are made positions. We have already shown that if $i \in C_s^{OA}(A, q_s)$, then $i \in C_s^{BT}(A, q_s)$. That is, any individual who are chosen either in Stage 1 or Stage 2 of the FT choice rule is chosen by the BT choice rule. We will now construct an injection $g : C_s^{BT}(A, q_s) \to C_s^{FT}(A, q_s)$. For all individuals in $C_s^{OA}(A, q_s)$, which is a subset of both $C_s^{FT}(A, q_s)$ and $C_s^{BT}(A, q_s)$, we set $g(i) = i$.

Because we have $\mid C_s^{FT}(A, q_s) \mid \geq \mid C_s^{BT}(A, q_s) \mid$, we also have

$$\mid C_s^{FT}(A, q_s) \setminus C_s^{OA}(A, q_s) \mid \geq \mid C_s^{BT}(A, q_s) \setminus C_s^{OA}(A, q_s) \mid .$$

We call $C_s^{FT}(A, q_s) \setminus C_s^{OA}(A, q_s) \mid$ as $C_s^o\left(R, \tau^1\right)$, which is a q-responsive choice function. That is, among the remaining individuals either all of them or the top τ^1 of them will be chosen following the merit scores.

Given the set of individuals $C_s^{BT}(A, q_s) \setminus C_s^{OA}(A, q_s)$ and $C_s^{FT}(A, q_s) \setminus C_s^{OA}(A, q_s)$, we map the top-scoring individual in $C_s^{BT}(A, q_s) \setminus C_s^{OA}(A, q_s)$, call her j_1, to the top-scoring individual in $C_s^{FT}(A, q_s) \setminus C_s^{OA}(A, q_s)$, call her $\widetilde{j_1}$. That is, $g(j_1) = \widetilde{j_1}$. We map the second top-scoring individual in $C_s^{BT}(A, q_s) \setminus C_s^{OA}(A, q_s)$, call her j_2, to the second top-scoring individual in $C_s^{FT}(A, q_s) \setminus C_s^{OA}(A, q_s)$, call her $\widetilde{j_2}$. That is, $g(j_2) = \widetilde{j_2}$. We process in the same fashion until we exhaust all individuals in the set $C_s^{BT}(A, q_s) \setminus C_s^{OA}(A, q_s)$, in K steps. If $\{j_1, ..., j_K\} = \left\{\widetilde{j_1}, ..., \widetilde{j_K}\right\}$, then we will have

$$C_s^{FT}(A, q_s) \setminus C_s^{OA}(A, q_s) \supseteq C_s^{BT}(A, q_s) \setminus C_s^{OA}(A, q_s),$$

which implies

$$C_s^{FT}(A, q_s) \supseteq C_s^{BT}(A, q_s).$$

In this case, we are done. Otherwise, i.e., $\{j_1, ..., j_K\} \neq \left\{\widetilde{j}_1, ..., \widetilde{j}_K\right\}$, then there must exist an individual in $C_s^{FT}(A, q_s) \setminus C_s^{OA}(A, q_s)$, call her $g(j')$, who has a strictly higher score than the individual j' in $C_s^{BT}(A, q_s) \setminus C_s^{OA}(A, q_s)$ because $C_s^o\left(R, \tau^1\right)$, which is a q-responsive choice function. Therefore, in this case, $C_s^{FT}(A, q_s)$ is a better set of individuals than $C_s^{BT}(A, q_s)$ based on merit.

We first prove two lemmas.

LEMMA 1. *Given a set of individuals $A \subseteq \mathcal{I}$, $i \notin C_s^{\lambda}(A)$ implies $i \notin C_s^{OA}(A)$ for any $\lambda \in \mathcal{F}$.*

PROOF. If $L = 0$ then we have $C_s^{\lambda}(A) = C_s^{OA}(A)$. In this case, the result straightforwardly follows. Now, suppose $L \neq 0$. Let $t(i) = GC$. Then, $i \notin C^{\lambda}(A)$ implies that individual i is not among the top $q_s^o + \lambda(L)$ scoring applicants. Therefore, $i \notin C^{OA}(A)$. Now suppose $t(i) = OBC$. The result is trivially satisfied because when $L \neq 0$, all OBC individuals are chosen by C_s^{λ} and C_s^{OA}. Finally, suppose $t(i) = SC$ or $t(i) = ST$. Since C_s^{λ} may provide additional provisions to SC and ST candidates by assigning high-scoring SC/ST applicants to open categories before processing the reserved category positions, $i \notin C^{\lambda}(A)$ straightforwardly implies $i \notin C^{OA}(A)$. ■

Lemma 1 states that $C_s^{OA}(A) \subseteq C_s^{\lambda}(A)$ for any set of individuals $A \subseteq \mathcal{I}$.

LEMMA 2. *Either $\mid C_s^{FT}(A) \mid = \mid A \mid$ or $\mid C_s^{FT}(A) \setminus C_s^{OA}(A) \mid = L$.*

PROOF. By definition of a choice rule $C_s^{FT}(A) \subseteq A$. That is, either $C_s^{FT}(A) = A$ or $C_s^{FT}(A) \subset A$. In the former case, we have $\mid C_s^{FT}(A) \mid = \mid A \mid$. In the latter case, some individuals are not chosen by C_s^{FT}. By Lemma 1, we know $C_s^{OA}(A) \subseteq C_s^{FT}(A)$. Note that, by definition of C_s^{FT}, C_s^{OA} is run first. There are L unfilled OBC positions that are filled by the remaining applicants $A \setminus C_s^{OA}(A)$. All L positions are filled because $C_s^{FT}(A) \subset A$. Therefore, in this case, $\mid C_s^{FT}(A) \setminus C_s^{OA}(A) \mid = L$. ■

Lemma 2 shows that among the choice rules in $\mathcal{C}$, no rule chooses more individuals than C^{FT} from any given set of individuals $A \subseteq \mathcal{I}$.

Toward a contradiction, let $\widetilde{C}_s \in \mathcal{C}$ be a choice rule that merit dominates C_s^{FT}. By Lemma 1, $C_s^{OA}(A) \subseteq \widetilde{C}_s(A)$. Hence, both $\widetilde{C}_s$ and C_s^{FT} choose all individuals who are in $C_s^{OA}(A)$. Since C_s^{FT} chooses the top-L scoring individuals from $A \setminus C_s^{OA}(A)$, $C_s^{FT}(A)$ merit dominates $\widetilde{C}_s(A)$.

Proof of Theorem 4.. We first show that C_s^{AKT} satisfies the substitutes condition. Consider $i, j \in I$ and $A \subset I \setminus \{i, j\}$ such that $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$. We need to show that $i \notin C_s^{AKT}(A \cup \{i, j\}, q_s)$.

Since $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$, then $\nexists t \in \mathcal{C}$ such that $(i, t) \in \eta_s^{AKT}(A \cup \{i\})$. Let $\mathbf{t}(i) = GC$. Then, it must be the case that $rank_{A\cup\{i\}}(i) > q_s^o$. When an individual j joins, we still have $rank_{A\cup\{i,j\}}(i) > q_s^o$. Thus, we have $i \notin C_s^{AKT}(A \cup \{i, j\}, q_s)$.

Now, let $\mathbf{t}(i) \in \{SC, ST\}$. Without loss of generality, suppose that $\mathbf{t}(i) = SC$. $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$ implies that (1) $rank_{A\cup\{i\}}(i) > q_s^o$ so that i could not obtain an open category position and (2) $rank_{A'_{SC}\cup\{i\}}(i) > q_s^{SC}$, where $A'_{SC} \cup \{i\}$ is the set of remaining unassigned SC applicants after the open category positions were filled. When j is added to the set $A \cup \{i\}$, both (1) and (2) still hold. Thus, $i \notin C_s^{AKT}(A \cup \{i, j\}, q_s)$.

Now, consider $\mathbf{t}(i) = OBC$. $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$ implies that $rank_{A\cup\{i\}}(i) > q_s^o$ and either (1) $rank_{A'_{OBC}\cup\{i\}}(i) > q_s^{OBC}$, where $A'_{OBC} \cup \{i\}$ is the set of remaining unassigned OBC applicants after the open category positions were filled, or (2) $\theta(i) < min\{\theta(j) | (j, o) \in \eta_s(A)\} - 10$. Including j to $A \cup \{i\}$ does not affect either condition. Therefore, $i \notin C_s^{AKT}(A \cup \{i, j\}, q_s)$.

Next, we show that C_s^{AKT} satisfies consistency. Suppose $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$. We must have $rank_{A\cup\{i\}}(i) > q_s^o$. By the definition of C_s^{AKT}, the same set of individuals is assigned to open category positions when i is removed from the set of alternatives.

If $\mathbf{t}(i) \in \{SC, ST\}$, then the positions in category $\mathbf{t}(i)$ must be filled with individuals with higher scores. Thus, removing i from the set of alternatives does not affect who is chosen in the SC and ST categories.

Finally, if $\mathbf{t}(i) = OBC$, then $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$ implies that either $\theta(i) < min\{\theta(j)|(j,o) \in \eta_s(A)\} - 10$ or $rank_{A'_{OBC} \cup \{i\}}(i) > q_s^{OBC}$, where $A'_{OBC} \cup \{i\}$ is the set of remaining unassigned OBC applicants after the open category positions were filled. In both cases, removing i from the set of individuals available does not change the set of chosen individuals for OBC positions.

Thus, we conclude

$$C_s^{AKT}(A \cup \{i\}, q_s) = C_s^{AKT}(A, q_s).$$

Aizerman and Malishevski (1981) showed the equivalence of path-independence and the conjugation of substitutes condition and consistency. This concludes our proof.

REFERENCES

AIZERMAN, M., AND A. MALISHEVSKI (1981): "General theory of best variants choice: Some aspects," *IEEE Transactions on Automatic Control*, 26(5), 1030–1040.

AYGÜN, O., AND B. TURHAN (2017): "Large-scale affirmative action in school choice: Admissions to IITs in India," *American Economic Review*, 107(5), 210–213.

——— (2020): "Dynamic reserves in matching markets," *Journal of Economic Theory*, 188, 105069.

——— (2022): "How to de-reserve reserves: Admissions to technical colleges in India," *Management Science*, 69(10), 5695–6415.

BASWANA, S., P. P. CHAKRABARTI, S. CHANDRAN, Y. KANORIA, AND U. PATANGE (2019): "Centralized admissions for engineering colleges in India," *INFORMS Journal on Applied Analytics*, 45(5), 338–354.

ECHENIQUE, F., AND M. B. YENMEZ (2015): "How to control controlled school choice," *American Economic Review*, 105(8), 2679–94.

GALE, D. (1968): "Optimal assignments in an ordered set: an application of matroid theory," *Journal of Combinatorial Theory*, 4(2), 176–180.

GALE, D., AND L. S. SHAPLEY (1962): "College admissions and the stability of marriage," *The American Mathematical Monthly*, 69(1), 9–15.

HU, X., AND J. ZHANG (2025): "Verifiable affirmative action in Chinese high school admissions," *arXiv preprint arXiv:2504.04689*.

SÖNMEZ, T., AND M. B. YENMEZ (2022): "Affirmative action in India via vertical, horizontal, and overlapping reservations," *Econometrica*, 90(3), 1143–1176.

SÖNMEZ, T., AND M. U. ÜNVER (2024): "Informed neutrality in minimalist market design: A case study on a constitutional crisis in India," *ArXiv Working Paper*.