Higher Moment Price Risk in Rating Revenue Insurance

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Abstract

We develop and apply novel methods for estimating harvest price distributions in U.S. crop insurance premium rating. We compare the Risk Management Agency's (RMA) log-normal implied volatility approach with a risk-neutral density benchmark that incorporates skewness and kurtosis. We are the first to compute the "true" variance, skewness, and kurtosis of harvest price distributions using ex-post data, by applying Neuberger and Payne (2021)'s methodology to recover long-horizon moments from daily futures returns. Using corn and soybean market data, we assess predictive accuracy by comparing premium rates derived from alternative densities to those from the "true" distribution. Results show that the RMA's log-normal method performs at least as well as the higher-moment benchmark, suggesting limited pricing gains from added complexity. These findings have practical implications for improving crop insurance pricing and balancing model sophistication with operational efficiency.

Key words: Revenue protection insurance, price risk, risk-neutral price distribution, *ex-post* price distribution, insurance premium rates, Risk Management Agency.
JEL Codes: Q18.

1 Introduction

The Federal Crop Insurance Program (FCIP) in the United States provides agricultural producers with essential and diverse varieties of insurance products. In 2022, the total acreage under the FCIP was 494 million acres, and its total liability was about 33% of the total production value of the U.S. agricultural sector. The FCIP insurance policies

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can be classified into two broad categories: (1) individual-based and (2) area- and index-based policies. Individual-based policies provide insurance coverage for individual farms to protect against shortfalls in yield or revenue based on the realized yields or revenues of those farms. Area-based insurance products provide protection against such shortfalls based on the realized yields or revenues of a given area under consideration, say, at the county level. Index-based policies provide protection against shortfalls in yield or revenue utilizing an indirect measure of production, say, rainfall index or wind index. Notably, individual revenue-based policies are the most prominent insurance product available under the FCIP: they had a total liability of $128.58 billion or approximately 67% of the total liability under the FCIP program in 2022.[3]

Revenue protection (RP) insurance, a component of individual revenue-based insurance policies, has been at the center of U.S. agricultural policymaking since the 1990s. The RP program imposes a cost of $9 billion to U.S. taxpayers annually (Goodwin et al., 2018). The indemnity payout under the RP program is triggered by a combination of yield (i.e., yield risk) and crop prices (i.e., harvest price risk).[4] Therefore, the RMA makes assumptions about the price risk, yield risk (mainly computed from the historical loss-cost procedure), and a correlation between yield and prices using Gaussian copula to compute the actuarially fair RP insurance premiums (Goodwin et al., 2018). The RMA strictly adheres to Black and Scholes (1973)'s option pricing paradigm to compute a measure of the harvest price risk, i.e., a measure of the variability of the crop[5] futures prices from the planting season to the harvest season. The RMA assumes that the prices of a crop futures are log-normally distributed with a non-stochastic variance until the relevant option's expiration date, i.e., the harvest season. In particular, the RMA obtains the implied volatility (IV) measure as the average of the Black (1976)'s IVs of two close-to-the-money call and two close-to-the-money put options defined on the harvest-season futures for the crop (Goodwin et al., 2018).

However, a vast empirical literature has documented the existence of smile or smirk patterns in the Black and Scholes (1973)'s IV computed across different strikes for a given maturity, which implies a deviation from the assumption that the underlying asset's prices are log-normality distributed (Derman & Kani, 1994; François et al., 2022). McKenzie et al.

[3]Information available at the U.S. Department of Agriculture (USDA) Economic Research Service (ERS): https://www.ers.usda.gov/topics/farm-practices-management/risk-management/crop-insurance-at-a-glance/

[4]Price risk in this manuscript will refer to harvest price risk only. Therefore, we will use price risk and harvest price risk interchangeably.

[5]Since there are RP insurance products for many crops, here we use "crop futures" to refer to the commodity futures used to price an RP product.

(2022) characterize the IV functions for five agricultural commodities (including corn and soybean) and refute the claim of constant IVs for those commodities. However, Goodwin et al. (2018) delved deeper into this issue and, after comparing the IV estimates obtained from alternative option pricing models, they recommended using Black (1976)'s IV over other alternatives to measure the price risk for the crop insurance premium rating procedures. Goodwin et al. (2018) also emphasized that the trading volumes of the deep out-of-the-money (OTM) options causing the smile or smirk IV patterns are very low and do not represent the true market's expectation about the underlying corn, soybean, and wheat futures price variance. Moreover, Choe and Goodwin (2024) investigated the informational content embedded in the trading volume in thinly traded options markets, corn, soybean, wheat, and cotton. They found that prices manually settled by CME for options with zero trading volume are biased downward. Choe and Goodwin (2024) report that the RMA's IV factor can be significantly different than the IV factor one gets after dropping options with zero trading volume, i.e., RP insurance premium rates can be very different. Although RMA employs IVs from the close-to-the-money calls and puts, Choe and Goodwin (2024) find that 40% of such options can have zero trading volume.

It is critical to characterize the harvest price risk robustly, i.e., to estimate unbiased RP insurance premiums, which could have important welfare implications for crop producers, private insurance selling companies, and taxpayers in the U.S. The above-mentioned studies motivate us to investigate the issue of how to account for harvest price risk more effectively in RP insurance products.[6] Currently, the RMA characterizes the harvest price of a crop as log-normally distributed with a projected price (e.g., the average of the December corn or November soybean futures price as of the previous February) as its mean (adjusted by the variance in the logarithmic form) and Black (1976)'s IV defined on the respective corn or soybean futures as a variance parameter.[7] For context, Figure 1 illustrates the RMA's timeline to compute the projected price and the harvest price for corn and soybean. In a given year, the projected price is computed by averaging the prices for the relevant corn

[6] In line with Goodwin et al. (2018), we exclusively focus on the harvest price risk and do not investigate other assumptions embedded in obtaining actuarially fair RP premiums. Any discussion about insurance premiums in this study solely deals with the harvest price risk. Tangential to our work, Ramirez et al. (2011) exclusively focus on the yield risk to compute crop yield insurance premiums using alternative yield distributions. Ramirez et al. (2011) assume a constant harvest price and correlation between yield and harvest price to draw inferences about the accuracy of insurance premiums.

[7] The adjustment of the mean for a log-normal harvest price density function can be expressed as: $\mu = ln(F^*) - 0.5 \times VAR$; where F^* is the projected price, and μ and VAR are the mean and variance of a log-normal density function.

and soybean futures contracts during February. The harvest price is calculated in a similar fashion, but using the average prices for the same futures contracts over the following month of October. The RMA utilizes the December futures contract for corn and the November futures contract for soybean to compute the corresponding projected price and harvest price.

We relax the core assumption that the underlying corn and soybean futures prices are log-normally distributed by investigating whether risk-neutral skewness and kurtosis should be incorporated to characterize the harvest price risk. Using OTM option premiums, Bakshi et al. (2003) provide an intuitive approach to extract the risk-neutral variance, skewness, and kurtosis of the underlying asset's return distribution. We utilize these risk-neutral moments to empirically fit a risk-neutral density function of the harvest price by minimizing Kullback–Leibler (KL) divergence with respect to the RMA's (log-normal) harvest price density function obtained at the end of the projected price discovery period (i.e., February for corn and soybean).[8] To the best of the authors' knowledge, this is the first study that attempts to incorporate risk-neutral skewness and kurtosis to characterize the harvest price risk for the crop insurance rating procedure, which is an important contribution to the agricultural price risk literature (Boyd & Bellemare, 2020; Goodwin et al., 2018).

Another important research avenue that has remained largely unexplored in the crop insurance literature is how to estimate the *"true"* crop insurance premium using the time series of *ex-post* futures returns over the relevant insurance coverage period. The RMA provides the RP insurance premium rates shortly after the end of the projected price discovery period (i.e., February for corn and soybean) and computes the final indemnity payouts for the RP insurance products at the end of the harvest price discovery period (i.e., October for corn and soybean). Essentially, RP insurance products provide insurance coverage for the risks that occur between planting and harvest. Therefore, it is relevant to compare the (*ex-ante*) RP insurance premiums computed using risk-neutral moments shortly after the projected price discovery period with the *"ex-post"* insurance premiums based on the realized futures returns between projected and harvest price discovery periods.

To empirically fit the (*"true"*) realized harvest price density function by applying the KL-divergence optimization technique, we use the *ex-post* variance, skewness, and kurtosis measures computed from the realized futures prices between planting and harvest. The variance measure of returns grows linearly in time. Therefore, one can utilize the variance

[8]For robustness, we also empirically fit the harvest price density function by performing the entropy maximization with given risk-neutral variance, skewness, and kurtosis. Importantly, the main conclusions of this research remain unchanged.

of daily returns to compute the variance of the futures returns over the planting-harvest period. However, there is no direct one-to-one relationship between the skewness and kurtosis measures of short-horizon (e.g., daily) returns and long-horizon (e.g., planting-to-harvest) returns. Interestingly, a recent theoretical development in Neuberger and Payne (2021) allows us to compute skewness and kurtosis of long-horizon (planting-to-harvest) returns using short-horizon (daily) returns with the minimal assumptions that the price process is a strongly stationary martingale process and the relevant moments do exist. Neuberger and Payne (2021) apply their methodology to estimate the skewness and kurtosis of monthly and yearly U.S. stock index returns. Similarly, we utilize the setup of Neuberger and Payne (2021) to estimate the skewness and kurtosis of planting-to-harvest (futures) returns using daily futures returns.

To illustrate Neuberger and Payne (2021)'s approach, consider Figure 2. This graph depicts multiple plausible hypothetical paths for the futures prices covering the time span from t to $(t+\tau)$ for a specific year (e.g., planting 2021 to harvest 2021). If multiple realized price paths were available for that specific year, one could use their harvest prices to estimate the probability density function of the τ-period return depicted on the rightmost side of Figure 2. However, in practice, it is impossible to use this procedure to estimate the τ-period return density function because *ex-post* only one realized price path is available for that particular year (e.g., 2021), say, "P1" in Figure 2. Neuberger and Payne (2021) overcome this issue by deriving skewness and kurtosis measures of the τ-period return that can be estimated from a single realized path of the price process, i.e., the only realized return path that actually occurred. Subsequently, we fit the *"true"* realized empirical density function for harvest price for a crop year by means of that year's realized planting-to-harvest price path.

In sum, for each crop season, we compute (*ex-ante*) RP insurance premiums using option-implied variance, skewness, and kurtosis measures, and (*"true"*) ex-post RP insurance premiums using realized variance, skewness, and kurtosis measures. This enables us to compare those premiums and asses the efficacies of the RP insurance premium rating procedure, which makes a novel contribution to the crop insurance literature.

To summarize, this research attempts to address the following research questions: (1) How different are the RP insurance premium rates after augmenting the risk-neutral skewness and kurtosis in the rating process? (2) How different are the RMA RP insurance premium rates from the RP rates computed using the ("true") *ex-post* harvest price density? (3) Are the "true" RP insurance premium rates (i.e., computed from the *ex-post* harvest price density)

better approximated by the RP insurance premium rates based on densities with or without risk-neutral skewness and kurtosis?

The remainder of the study is organized as follows. Section 2 outlines the theories developed in Bakshi et al. (2003) and Neuberger and Payne (2021) for computing the risk-neutral and *ex-post* variance, skewness, and kurtosis measures. It also describes the empirical strategies used to fit continuous risk-neutral and ex-post harvest price densities, as well as the numerical procedures employed to compute RP insurance premium rates. Section 3 presents the data sources and estimation strategies. Section 4 reports and discusses the main empirical findings. Finally, Section 5 offers concluding remarks.

2 Price Risk Estimation Methodology

The RMA provides a detailed RP insurance rating mechanism that consists of a series of disjoint steps.[9] The RP insurance premium rates are calculated using a simulated revenue distribution, i.e., a function of yield and prices. The RMA simulates the yield distribution based on historical yield data and utilizes the planting-time futures prices along with close-to-the-money Black (1976)'s IV to measure the price risk embedded in the harvest price. In particular, the RMA assumes that the harvest price (F) follows a log-normal distribution, i.e., $ln(F) \sim N(\mu_p, \sigma_p^2)$, $\mu_p \equiv ln(F^*) - 0.5\sigma_p^2$, where F^* is the projected futures price and σ_p^2 is the close-to-the money Black (1976)'s IV measure. The projected futures price is calculated as the average of the futures prices during the price discovery period, as depicted in Figure 1. For example, in a given year, the RMA takes the average of the December corn futures prices in February and October months to compute the projected price and harvest price, respectively, for corn RP insurance products. Similarly, the RMA averages the November soybean futures prices in February and October months to compute soybean projected and harvest prices, respectively. Lastly, the RMA makes a specific set of assumptions about the dependence between yield and prices to estimate the RP premium rates.

Following the analysis of Goodwin et al. (2018), the present research solely focuses on the harvest price risk at the time of planting corn and soybean, which is an essential component for the RP insurance premium rating. Given the large number of studies providing evidence that the log-normal assumption for the underlying asset price doesn't hold, i.e., there exists smile, skew or smirk in Black (1976)'s IV across strikes for commodities (McKenzie et al.,

[9]Information on RMA's RP insurance rating mechanism is available at:
https://legacy.rma.usda.gov/pubs/2008/ratemethodology.pdf

2022), we investigate whether skewness and kurtosis should be incorporated to characterize the harvest price risk. For instance, the RMA computes the projected price for corn as the average of December corn futures prices in February and averages the Black (1976)'s IVs of two close-to-the-money calls and two close-to-the-money puts defined on the December corn futures. For corn and soybean, the RMA averages such IVs corresponding to the last week of February each year. Subsequently, the RMA establishes a log-normal harvest price distribution. The present study augments RMA's method of computing harvest price risk by incorporating risk-neutral skewness and kurtosis into the analysis to estimate a more flexible probability density function for harvest prices as compared to the log-normal distribution.

2.1 Risk-neutral Moments

Bakshi et al. (2003) provide a detailed methodology to compute the τ-period risk-neutral variance as of time t ($VAR^{BKM}_{t,t+\tau}$), skewness ($SKEW^{BKM}_{t,t+\tau}$), and kurtosis ($KURT^{BKM}_{t,t+\tau}$) measures from the option premiums defined on the relevant underlying asset, e.g., December corn futures or November soybean futures, which we adopt in this research.[10] Define the τ-period log return of the underlying futures prices as $ln\left[\frac{F(t+\tau)}{F(t)}\right]$ and let i designate the continuously compounded risk-free interest rate. Also, let $C(t,\tau;K)$ and $P(t,\tau;K)$ represent the corresponding premiums of OTM European puts and calls with strike price K. Then, risk-neutral variance, skewness, and kurtosis of τ-period log return can be expressed as follows:

$$VAR^{BKM}_{t,t+\tau} = e^{i\tau}V(t,t+\tau) - m(t,t+\tau)^2, \tag{1}$$

$$SKEW^{BKM}_{t,t+\tau} = \frac{e^{i\tau}W(t,t+\tau) - 3m(t,t+\tau)e^{i\tau}V(t,t+\tau) + 2m(t,t+\tau)^3}{[e^{i\tau}V(t,t+\tau) - m(t,t+\tau)^2]^{3/2}}, \tag{2}$$

$$\begin{aligned} KURT^{BKM}_{t,t+\tau} = {} & \frac{e^{i\tau}\,X(t,t+\tau) - 4\,m(t,t+\tau)\,e^{i\tau}\,W(t,t+\tau)}{[e^{i\tau}\,V(t,t+\tau) - m(t,t+\tau)^2]^2} \\ & + \frac{6\,e^{i\tau}\,m(t,t+\tau)^2V(t,t+\tau) - 3\,m(t,t+\tau)^4}{[e^{i\tau}\,V(t,t+\tau) - m(t,t+\tau)^2]^2} - 3, \end{aligned} \tag{3}$$

[10] Bakshi et al. (2003) derive the risk-neutral moments using OTM call and put premiums defined on the underlying stocks. Following Chabi-Yo et al. (2023) and Triantafyllou et al. (2015), we substitute the underlying stock prices with the futures prices in the risk-neutral moment equations to estimate the same and make no other substitutions.

where the relevant terms are defined as shown below:

$$m(t,t+\tau) = e^{i\tau} - 1 - \frac{e^{i\tau}}{2}V(t,t+\tau) - \frac{e^{i\tau}}{6}W(t,t+\tau) - \frac{e^{i\tau}}{24}X(t,t+\tau), \tag{4}$$

$$\begin{aligned} V(t,t+\tau) = &\int_{F(t)}^{\infty} \frac{2\left(1 - ln\left[\frac{K}{F(t)}\right]\right)}{K^2} C(t,\tau;K)dK \\ &+ \int_{0}^{F(t)} \frac{2\left(1 + ln\left[\frac{F(t)}{K}\right]\right)}{K^2} P(t,\tau;K)dK, \end{aligned} \tag{5}$$

$$\begin{aligned} W(t,t+\tau) = &\int_{F(t)}^{\infty} \frac{6ln\left[\frac{K}{F(t)}\right] - 3\left(ln\left[\frac{K}{F(t)}\right]\right)^2}{K^2} C(t,\tau;K)dK \\ &- \int_{0}^{F(t)} \frac{6ln\left[\frac{F(t)}{K}\right] + 3\left(ln\left[\frac{F(t)}{K}\right]\right)^2}{K^2} P(t,\tau;K)dK, \end{aligned} \tag{6}$$

$$\begin{aligned} X(t,t+\tau) = &\int_{F(t)}^{\infty} \frac{12\left(ln\left[\frac{K}{F(t)}\right]\right)^2 - 4\left(ln\left[\frac{K}{F(t)}\right]\right)^3}{K^2} C(t,\tau;K)dK \\ &+ \int_{0}^{F(t)} \frac{12\left(ln\left[\frac{F(t)}{K}\right]\right)^2 + 4\left(ln\left[\frac{F(t)}{K}\right]\right)^3}{K^2} P(t,\tau;K)dK. \end{aligned} \tag{7}$$

Intuitively, $m(t,t+\tau)$ denotes the (arbitrage-free) risk-neutral valuation of the (mean) τ-period log return of the underlying futures price, i.e., $ln\left[\frac{F(t+\tau)}{F(t)}\right]$. Similarly, $V(t,t+\tau)$, $W(t,t+\tau)$, and $X(t,t+\tau)$ are the (arbitrage-free) risk-neutral valuation of the (mean) squared contract with a payoff of $\left(ln\left[\frac{F(t+\tau)}{F(t)}\right]\right)^2$, cubic contract with a payoff of $\left(ln\left[\frac{F(t+\tau)}{F(t)}\right]\right)^3$, and quartic contract with a payoff of $\left(ln\left[\frac{F(t+\tau)}{F(t)}\right]\right)^4$, respectively. Bakshi and Madan (2000) provide a crucial theoretical result that the entire set of twice-continuously differential payoff functions with bounded expectations can be spanned algebraically by a continuum of OTM (European) call and put premiums. Building on this theoretical result, Bakshi et al. (2003) employ the payoffs of mean, squared, cubic, and quartic contracts to derive Equations (1) through (**??**).

2.2 Realized Moments

Typically, one would compute the skewness and kurtosis of τ-period returns from a sample of τ-period returns. However, for any particular crop season, only one realization of the τ-period

return is available; hence, recovering *ex-post* skewness and kurtosis for the planting-to-harvest returns is a challenging exercise (Amaya et al., 2015). Figure 2 depicts some of the infinite plausible price processes for the time period from t to $(t+\tau)$ in a specific crop year. However, we only observe one realization of this price process from the time period of t to $(t+\tau)$, and our objective is to recover *ex-post* skewness and kurtosis of the τ-period return using this single realized price process available at time $(t+\tau)$. To address this issue, Neuberger and Payne (2021) provide a methodology that enables one to compute (realized) skewness and kurtosis of long-horizon returns (e.g., planting-to-harvest) $R_t(\tau) \equiv ln\left[\frac{F(t+\tau)}{F(t)}\right]$, using short-horizon returns (e.g., daily) $r(t) \equiv ln\left[\frac{F(t+1)}{F(t)}\right]$. Neuberger and Payne (2021) develop this methodology based on a set of weak assumptions, namely, that the price process is a strongly stationary martingale and the relevant moments exist.

We utilize Neuberger and Payne (2021)'s methodology to retrieve the *ex-post* futures return density for the crop insurance coverage period. In particular, corn and soybean RP insurance products provide coverage for harvest price risk in each production year as of planting time. To reiterate, for corn and soybean, February is the projected price discovery period and October is the harvest price discovery period as per RMA's methodology, as shown in Figure 1. Essentially, corn and soybean RP insurance products provide revenue insurance coverage for risks that occur during an eight-month horizon (from March to October) every year. Therefore, we employ Neuberger and Payne (2021)'s methodology to compute variance ($VAR^{NP}_{t,t+\tau}$), skewness ($SKEW^{NP}_{t,t+\tau}$), and kurtosis ($KURT^{NP}_{t,t+\tau}$) of eight-month returns (i.e., $\tau = 244$ days) using daily futures returns data, as follows:

$$VAR^{NP}_{t,t+\tau} = var^L[R_t(\tau)] = var^L[r]\,\tau, \tag{8}$$

$$SKEW^{NP}_{t,t+\tau} = skew[R_t(\tau)] = \left(skew[r] + 3\,\frac{cov[y^{(1)}, x^{(2E)}(r)]}{var^L[r]^{3/2}}\right)\tau^{-1/2}, \tag{9}$$

$$KURT^{NP}_{t,t+\tau} = kurt[R_t(\tau)] = \left(kurt[r] + 4\,\frac{cov[y^{(1)}, x^{(3)}(r)]}{var^L[r]^2} + 6\,\frac{cov[y^{(2L)}, x^{(2L)}(r)]}{var^L[r]^2}\right)\tau^{-1}, \tag{10}$$

where variance, skewness, and kurtosis of daily returns distribution (along with other relevant terms) can be expressed as:

$$var^L[r] = E\left[x^{(2L)}(r)\right];\; x^{(2L)}(r) = 2\,(e^r - 1 - r), \tag{11}$$

$$var^{E}[r] = E\left[x^{(2E)}(r)\right]\,;\, x^{(2E)}(r) = 2\left(re^{r} - e^{r} + 1\right), \tag{12}$$

$$skew[r] = \left[\frac{E\left[x^{(3)}(r)\right]}{var^{L}[r]^{3/2}}\right]\,;\, x^{(3)}(r) = 6\left[(e^{r}+1)r - 2(e^{r}-1)\right], \tag{13}$$

$$kurt[r] = \left[\frac{E\left[x^{(4)}(r)\right]}{var^{L}[r]^{2}} - 3\right]\,;\, x^{(4)}(r) = 12\left[r^{2} + 2\times(e^{r}+2)r - 6\times(e^{r}-1)\right], \tag{14}$$

$$y_{t}^{(j)} = \left[\sum_{u=1}^{\tau-1}\frac{x^{(j)}[R_{t-1-u}(u)]}{\tau}\right]\;\forall\, j = 1, 2L, \tag{15}$$

$$x^{(1)}(r) = (e^{r} - 1). \tag{16}$$

The definitions of variance, skewness, and kurtosis of daily returns converge to their respective standard (Pearson's) definitions of moments due to the fact that $\lim_{r\to 0}\frac{x^{(n)}(r)}{r^{n}} = 1$ and $x^{(n)}(r)$ lies in the range of $[r^{n}, (e^{r}-1)^{n}]$. Intuitively, the variance of long-horizon return increases linearly with the time horizon (see Equation 8). The skewness of long-horizon return stems from the skewness of daily returns and the leverage effect, which is the correlation between variance ($x^{(2E)}(r)$) and lagged returns ($y^{(1)}$) (see Equation 9). Equation (9) implies that the contribution of skewness of daily returns to the skewness in the long-horizon returns diminishes as the horizon lengthens. According to Equation (10), the kurtosis of long-horizon return originates from three plausible sources, namely, kurtosis in daily returns, the correlation of daily skew ($x^{(3)}(r)$) with the lagged returns ($y^{(1)}$), and the GARCH effect consisting of the correlation between variance ($x^{(2L)}(r)$) and the past variance ($y^{(2L)}$). As the horizon lengthens, the kurtosis of daily returns generates a smaller ($\frac{1}{\tau}$) kurtosis in the long-horizon returns (see Equation 10).

The core assumption mentioned above that the price process is martingale entails that daily returns (r_t) should remain uncorrelated with the lagged rolling average of squared returns ($y^{(2L)}$) and the lagged rolling average of cubed returns ($y^{(3)}$). We empirically test these correlations to ensure the validity of Equations (9) and (10) for the data sample utilized in this research.

2.3 Approximation of Harvest Price Densities

We employ risk-neutral higher moments to approximate the risk-neutral probability density function for harvest price at the end of the projected price discovery period in a particular year, i.e., the *ex-ante* (risk-neutral) harvest price density (see Figure 1). Similarly, we utilize

ex-post higher moments to approximate the *ex-post* probability density function for harvest price in that year (see Figure 1). We adopt two basic statistical approaches to approximate probability density functions: (i) Kullback-Leibler (KL) divergence minimization (Joyce, 2011) and (ii) Entropy maximization (Guiasu & Shenitzer, 1985).

The KL divergence measures the difference between two probability density functions over the same variable z. It is a measure of extra information (or surprise) when $f(z)$ is used to approximate $g(z)$, where $g(z)$ is the baseline (theoretically) known probability density function and $f(z)$ is being used to approximate the $g(z)$. A KL divergence of zero means $f(z)$ perfectly matches $g(z)$, and a higher value indicates greater inefficiency or surprise when $f(z)$ is used instead of $g(z)$. For crop insurance purposes, the RMA assumes that the corn and soybean harvest prices are log-normally distributed with a non-stochastic variance (Goodwin et al., 2018). Therefore, we assume $g(z)$ to be the baseline probability density function of harvest price (F), i.e., $ln(F) \sim N(\mu_p, \sigma_p^2)$.

We empirically find a density function $f(z)$ which has the requisite risk-neutral or *ex-post* higher moments (i.e., mean, variance, skewness, and kurtosis) by minimizing the KL divergence between $f(z)$ and $g(z)$. The KL divergence minimization problem consists of

$$\min_{f(.)\in\Omega(f)} \int_{-\infty}^{\infty} g(z)\, ln\left[\frac{g(z)}{f(z)}\right] dz, \quad \text{s.t.} \quad \begin{aligned} &\int_{-\infty}^{\infty} e^z\, f(z)\, dz = F^*, \\ &\int_{-\infty}^{\infty} (z-\mu)^2\, f(z)\, dz = \sigma^2, \\ &\int_{-\infty}^{\infty} \left(\frac{z-\mu}{\sigma}\right)^3 f(z)\, dz = SKEW, \\ &\left[\int_{-\infty}^{\infty} \left(\frac{z-\mu}{\sigma}\right)^4 f(z)\, dz\right] - 3 = KURT, \\ &\int_{-\infty}^{\infty} f(z)\, dz = 1, \end{aligned} \tag{17}$$

where $\int_{-\infty}^{\infty} z f(z) dz = \mu$, F^* denotes the projected price, z represents the log harvest price (i.e., $ln(F)$), and $f(z)$ is the closest probability density function to $g(z)$ with the requisite skewness ($SKEW$) and kurtosis ($KURT$). Note that, by construction, $g(z)$ is a normal density function, i.e., it is characterized by mean and variance only, whereas the obtained $f(z)$ probability density function has the same mean and variance as $g(z)$ along with the requisite skewness and kurtosis.

For robustness, we also empirically find an empirical density function $f(z)$ by maximizing

the differential entropy. For a given continuous random variable Z with a probability density function $f(z)$, the differential entropy is defined as $\int_{-\infty}^{\infty} -f(z)\, ln\left[f(z)\right]\, dz$. In general, one can find many probability density functions with a set of given mean (μ) and variance (σ^2); however, a normal density function with the mean (μ) and variance (σ^2) has the maximum entropy (Kapur & Kesavan, 1992). Following this paradigm, we propose to find the requisite risk-neutral and *ex-post* probability density functions of harvest prices with a set of given moments (i.e., mean, variance, skewness, and kurtosis) using the following differential entropy maximization procedure:

$$\begin{aligned} \max_{f(.)\in\Omega(f)} \int_{-\infty}^{\infty} -f(z)\, ln\left[f(z)\right]\, dz, \quad \text{s.t.} \quad & \int_{-\infty}^{\infty} e^z\, f(z)\, dz = F^*, \\ & \int_{-\infty}^{\infty} (z-\mu)^2\, f(z)\, dz = \sigma^2, \\ & \int_{-\infty}^{\infty} \left(\frac{z-\mu}{\sigma}\right)^3 f(z)\, dz = SKEW, \\ & \left[\int_{-\infty}^{\infty} \left(\frac{z-\mu}{\sigma}\right)^4 f(z)\, dz\right] - 3 = KURT, \\ & \int_{-\infty}^{\infty} f(z)\, dz = 1. \end{aligned} \tag{18}$$

The computation of RMA RP insurance premium rates requires the probability density function of harvest price in levels. Thus, to obtain the RMA's probability density function of harvest prices in levels, i.e., to convert from a log-normal to a normal density function, we define the mean for the risk-neutral density function (i.e., μ^{BKM}) and for the *ex-post* density function (i.e., μ^{EP}) with the projected price F^* as follows:

$$\mu^{BKM} = ln(F^*) - 0.5 \times VAR^{BKM}, \tag{19}$$

$$\mu^{EP} = ln(F^*) - 0.5 \times VAR^{EP}. \tag{20}$$

The solutions of the KL divergence minimization (Equation 17) and entropy maximization (Equation 18) exercise provide the probability density functions for the log of harvest prices, i.e., $ln(F)$. Therefore, to obtain the probability density function for the harvest price in levels, we employ the Jacobian transformation. More specifically, assume S is a random

variable with known probability density function $f_S(s)$. Suppose we want to know the probability density function $h(F)$, where $F \equiv e^S$. Then, the Jacobian transformation to derive $h(F)$ from $f_S(s)$ is given by the following equation:

$$h(F) = f_S(s) \times \frac{1}{e^s}. \tag{21}$$

For the purpose of the present application, if we let random variable S be the log-futures and $f_S(s)$ is its probability density function estimated by KL divergence minimization or entropy maximization, then random variable F is the harvest price (in levels), and $h(F)$ is its probability density function. Subsequently, the harvest price density function can be utilized to compute the embedded price risk in the insurance premium rate.

2.4 Computation of Insurance Premium Rates

RMA's insurance policies can be classified into two major categories according to the price protection they provide, namely, policies with harvest price exclusion (HPE) and policies with harvest price option (HPO). HPE policies provide a simple type of price protection, as they pay for the amount by which the harvest price F falls short of a percentage ($\%Coverage$) of the projected price F^*. In the case of HPO policies, the price risk indemnity increases if the harvest price exceeds the projected price. Importantly, HPO indemnities depend on a more complex interaction between prices and yields. Hence, to maintain focus on the price risk in isolation of yield risk, in what follows we concentrate our attention on HPE policies, leaving the analysis of HPO policies for future work.

The price risk indemnity of an HPE policy consists of $\max(\%Coverage \times F^* - F, 0)$, where $Coverage \in \{55\%, 60\%, 65\%, 70\%, 75\%, 80\%, 85\%\}$ is the price coverage level. Therefore, the actuarially fair HPE insurance premium rate can be expressed as

$$HPEInsurancePremiumRate = \frac{100}{F^*} \int_0^{P^*} (P^* - F)\, h(F)\, dF, \tag{22}$$

where $h(F)$ is the probability density of the harvest price. Thus, to compute the price risk insurance premium rates under alternative harvest price density functions $h(F)$, we perform the numerical integration using the "RMA", "risk-neutral", and "true" density functions as $h(F)$. We compute the price risk insurance premium rate under three alternative assumptions regarding the probability density of harvest prices: (1) RMA assumption ("RMA"), (2) RMA augmented ("risk-neutral"), and (3) "true" *ex-post*.

3 Data

This research considers 2007–2022 as the time period of study. The RMA utilizes the December CME corn futures and November CME soybean futures to compute the projected price, harvest price, and IV factor to implement the RP insurance premium rating procedure. We source the daily December corn futures prices and November soybean futures prices along with the options premiums defined on those futures series from IVolatility.[11] We obtain the annualized interest rate for six-month and one-year risk-free loans from the Federal Reserve Economic Data (FRED), [12] A linear interpolation exercise gives us eight- and nine-month interest rates to compute risk-neutral moments for soybean and corn, respectively.

We only use the options with positive premiums, volume, and open interest on a given trading day to avoid the confounding effects (i.e., erroneous risk-neutral moments) that could originate from thinly traded strikes (Choe & Goodwin, 2024; Hollstein et al., 2019). The CME options are American, whereas the Bakshi et al. (2003)'s methodology is based on OTM European option premiums. Thus, we convert American futures options premiums to European futures options premiums using the method introduced by Chaudhury (1995). The American option premiums contain early exercise premiums attributable to the right to exercise before the option's expiration date. However, here we use only data on OTM commodity futures options for which the early exercise premiums would be negligible (Wong, 2023). Chaudhury (1995) provides an intuitive and tractable method to facilitate this conversion from American futures option prices to European futures option premiums, which is detailed in the Appendix.[13] Moreover, Chaudhury (1995)'s price conversion exercise provides the Black (1976)'s IV for each strike price available in the dataset. After converting to European premiums, we drop strikes violating standard arbitrage conditions (i.e., missing Black (1976)'s IV) or with Black (1976)'s IV greater than 300% to avoid extreme observations.[14]

The computation of risk-neutral moments using Equations (1) through (**??**) requires a continuum of OTM option premiums to apply the trapezoidal numerical integration proce-

[11] Information about the IVolatility can be found at https://www.ivolatility.com/

[12] Market Yield on U.S. Treasury Securities at 6-Month Constant Maturity is available at https://fred.stlouisfed.org/series/RIFLGFCM06NA. Market Yield on U.S. Treasury Securities at 1-Year Constant Maturity is available at https://fred.stlouisfed.org/series/RIFLGFCM06NA

[13] Barone-Adesi and Whaley (1987) propose another method utilized extensively in the literature. However, we follow Chaudhury (1995) to accomplish this price conversion exercise owing to its simple and tractable implementation. For instance, Barone-Adesi and Whaley (1987) require a 2×2 grid search over the critical underlying asset's value and the implied volatility, whereas Chaudhury (1995)'s procedure searches over critical implied volatility (σ) only.

[14] Less than 0.01% observations were dropped based on this criteria.

dure (Chang et al., 2013; Hollstein et al., 2019). However, on a given trading date, there are only a limited number of OTM option strikes with positive volume and open interest. For instance, Chabi-Yo et al. (2023) report such limited availability of OTM options for a wide variety of commodity futures options. Hence, we follow Trolle and Schwartz (2010) and Carr and Wu (2009), by implementing a linear interpolation and flat extrapolation of Black (1976)'s IV across strikes to get a continuum of OTM option prices. Specifically, we set a fine grid of 1,000 equidistant interpolated moneyness levels, i.e., the ratio of strike price to futures price ($\frac{K}{F}$), ranging from 1% to 300%. For the moneyness levels between the market available strikes, we perform a linear interpolation of Black (1976)'s IV on the moneyness levels. We assign the Black (1976)'s IV from the lowest (highest) available strike in the market data for all the strikes below (above) that strike level for the flat extrapolation. Figure 3 illustrates the scheme of linear interpolation of IVs between market-observed strikes and flat extrapolation of IVs for strikes outside the market-observed strike range. Subsequently, we compute the option premiums for all the interpolated strikes using Black (1976) futures option pricing model.

The RMA computes the IV factor by averaging Black (1976)'s IVs of two close-to-the-money calls and two close-to-the-money puts defined on the relevant harvest season futures contract on each trading date during the last week of the projected price discovery period, e.g., February for corn and soybean (see Figure 1). For simplicity, we compute RMA's style IV ($VAR^{RMA}_{t,t+244}$) by averaging Black (1976)'s IVs of two close-to-the-money calls and two close-to-the-money puts on the first trading date of March every year, i.e., the date on which risk-neutral moments ($VAR^{BKM}_{t,t+244}$, $SKEW^{BKM}_{t,t+244}$, $KURT^{BKM}_{t,t+244}$) are computed for the analysis in this paper. Tables 3 and 4 provide the dates on which risk-neutral moments and $VAR^{RMA}_{t,t+244}$ are computed to facilitate the approximation of (*ex-ante*) harvest price density functions for corn and soybean, respectively.

Lastly, for each year, we employ the constructed daily futures log return series to compute the moments of the *ex-post* return density for the first trading date of March using Neuberger and Payne (2021)'s methodology (i.e., using approximately historical 244 daily futures returns). Tables 3 and 4 provide the dates on which *ex-post* moments of the return density are computed to facilitate the approximation of *"true"* harvest price density functions during the insurance coverage period for corn and soybean, respectively.

4 Empirical Results

4.1 Robustness of Long-horizon Return Moments

Tables 1 and 2 provide results validating assumptions underlying the method advocated by Neuberger and Payne (2021) to estimate moments using *ex-post* returns data. The first two columns of Table 1 indicate that the daily futures log returns of corn (r) are uncorrelated with the lagged rolling averages of squared log returns ($y^{(2)}$) or cubed log returns ($y^{(3)}$), which are critical assumptions needed for the validity of the Neuberger and Payne (2021)'s estimates. In other words, there is no evidence that the predictability of daily log returns could cause biases in the estimates of long-horizon skewness and kurtosis for corn futures returns using Neuberger and Payne (2021)'s methodology. The leverage effect–a component of the long-horizon skewness–as measured by the correlation between variance ($x^{2E}(r)$) and lagged return ($y^{(1)}$) is negative and weakly significant. Neuberger and Payne (2021) also reported the leverage effect to be negative and significant for the U.S. stock market return. Similarly, the GARCH effect–a component of the long-horizon kurtosis–captured as the correlation of $x^{(2L)}(r)$ and $y^{(2)}$ is positive and significant. Lastly, the cube effect–a component of the long-horizon kurtosis–is insignificant. Neuberger and Payne (2021) also find similar GARCH and cube effects for the U.S. stock market return.

Analogous regressions for soybean yield similar results. Table 2 reports no statistically significant correlations between the daily futures log returns (r) and the lagged rolling averages of squared log returns ($y^{(2)}$) or cubed log returns ($y^{(3)}$). In essence, there are no statistically significant forecasting abilities of rolling averages of past squared or cubed values of log returns to predict daily log returns of soybean futures, i.e., long-horizon estimates of skewness and kurtosis of soybean futures log returns are unbiased. Similar to corn, the GARCH effect is positive and strongly significant, and the cube effect is statistically insignificant in the case of daily log returns of soybean. However, the leverage effect for soybean is insignificant. In sum, the findings about the daily log return predictability, leverage effect, GARCH effect, and the cube effect for soybean are largely consistent with findings reported in Neuberger and Payne (2021) using the daily U.S. market return.

4.2 Risk-neutral and *Ex-post* Moments for Corn

Table 3 provides the variance, skewness, and kurtosis of risk-neutral and *ex-post* density functions of the natural logarithm of harvest price (i.e., $ln(F)$) for corn across all the cropping

seasons in the dataset, with the corresponding trading dates on which these moments are computed. $VAR^{RMA}_{t,t+244}$ is computed using RMA's methodology, as the average Black (1976)' IVs of two close-to-the-money calls and two close-to-the-money puts. Figure 4 graphically illustrates the risk-neutral and *ex-post* moments for corn futures returns from planting to harvest over the period under study.

Like Goodwin et al. (2018), we find that the $VAR^{RMA}_{t,t+244}$ and $VAR^{BKM}_{t,t+244}$ for corn are very close to each other, i.e., log-normal density functions for harvest price fitted using these variances will be very similar. The correlation coefficient between $VAR^{RMA}_{t,t+244}$ and $VAR^{BKM}_{t,t+244}$ is 0.99. The correlation between $VAR^{BKM}_{t,t+244}$ and $VAR^{NP}_{t,t+244}$ is 0.80. During the financial crisis of 2008, $VAR^{NP}_{t,t+244} > VAR^{BKM}_{t,t+244}$, as the market participants could not price the market crisis. However, this relationship reversed in 2009, i.e., $VAR^{BKM}_{t,t+244} > VAR^{NP}_{t,t+244}$, suggesting a more risk-averse attitude of the market participants. Notably, risk-neutral and *ex-post* variances for corn remain very close to each other during the 2012 drought season and the COVID-19 crisis in 2020. In 2021, in the aftermath of the COVID-19 crisis, the *ex-post* variance is higher than the risk-neutral variance for corn futures returns. In essence, a higher risk-neutral variance could plausibly imply growing uncertainty in the corn futures market.

Figure 4 indicates some differences between $SKEW^{BKM}_{t,t+244}$ and $SKEW^{NP}_{t,t+244}$, and frequent divergences between $KURT^{BKM}_{t,t+244}$ and $KURT^{NP}_{t,t+244}$ during the sample period. The correlation between $SKEW^{BKM}_{t,t+244}$ and $SKEW^{NP}_{t,t+244}$ is 0.47. In contrast, the correlation between $KURT^{BKM}_{t,t+244}$ and $KURT^{NP}_{t,t+244}$ is -0.08. $KURT^{BKM}_{t,t+244}$ values are always positive, whereas $KURT^{NP}_{t,t+244}$ take positive and negative values. Also, for almost all years, $SKEW^{BKM}_{t,t+244} > SKEW^{NP}_{t,t+244}$ and $KURT^{BKM}_{t,t+244} > KURT^{NP}_{t,t+244}$.

In addition, one could gauge the plausibility of price risk-based corn insurance indemnity payouts using the projected price–harvest price pair in a given year. For instance, during the drought season of 2012, the lower corn supply pushed the harvest price (\$7.501 per bushel) higher than the projected price (\$5.678 per bushel) (see Table 3). In 2013, one could observe the repercussions of higher corn futures prices in 2012, when the excess corn supply pushed the harvest price (\$4.392 per bushel) lower than the projected price (\$5.653 per bushel), i.e., a possibility of an indemnity payout (see Table 3).

4.3 Risk-neutral and *Ex-post* Moments for Soybean

The variance, skewness, and kurtosis of risk-neutral and *ex-post* density functions of the natural logarithm of soybean harvest prices (i.e., $ln(F)$) are reported in Table 4. Figure 5 depicts the risk-neutral and realized moments for soybean over the period analyzed. In line with Goodwin et al. (2018), $VAR^{RMA}_{t,t+244}$ and $VAR^{BKM}_{t,t+244}$ are very similar, with a correlation coefficient of 0.99. Therefore, log-normal harvest price density functions fitted using these two alternative variances would be very close to each other. The correlation coefficient between $VAR^{BKM}_{t,t+244}$ and $VAR^{NP}_{t,t+244}$ is 0.87, i.e., risk-neutral variance is indeed a strong predictor for *ex-post* variance in the soybean futures market. Similar to corn, the risk-neutral variance is higher than the *ex-post* variance in 2009, a representation of risk aversion in the market, which could be plausibly attributed to the financial crisis of 2008. The risk-neutral and *ex-post* variances remain very close to each other during and in the aftermath of the COVID-19 crisis, i.e., 2020, 2021, and 2022.

In contrast with corn, the correlation coefficient between $SKEW^{BKM}_{t,t+244}$ and $SKEW^{NP}_{t,t+244}$ is low at 0.06, whereas $KURT^{BKM}_{t,t+244}$ and $KURT^{NP}_{t,t+244}$ are moderately correlated with each other with a correlation coefficient of 0.60. The risk-neutral skewness is almost always positive, whereas the *ex-post* skewness contains both positive and negative values. The risk-neutral kurtosis is always positive, whereas the *ex-post* kurtosis is negative in some years. Figure 5 indicates frequent divergence between risk-neutral skewness and *ex-post* skewness, whereas the risk-neutral kurtosis and *ex-post* kurtosis tend to closely follow each other with some gaps. Notably, $KURT^{BKM}_{t,t+244}$ is always larger than $KURT^{NP}_{t,t+244}$.

Lastly, the projected price–harvest price pair indicates the plausibility of indemnity payouts. For instance, in 2008, the projected price and harvest price for soybean were reported as \$13.364 per bushel and \$9.220 per bushel, respectively, i.e., a plausibility of indemnity payouts (see Table 4). The relatively high projected price in 2008 corresponded to a price rally for soybeans in the last few months of 2007 and 2008. In 2007, the USDA recorded the lowest planted and harvested area for soybean since 1995, which caused subsequent spikes in soybean futures prices.[15]

[15] More information is available at https://downloads.usda.library.cornell.edu/usda-esmis/files/j098zb09z/1n79h637s/sj139444v/Acre-06-29-2007.pdf

4.4 Risk-neutral and *Ex-post* Harvest Price Densities

The above-mentioned correlation coefficients between risk-neutral moments and their corresponding *ex-post* counterparts motivate us to relax the core RMA's assumption of log-normal harvest prices and incorporate higher moments to characterize the price risk in the corn and soybean harvest prices. For instance, the 2016 production season for soybean is found to be characterized by high variance, skewness, and kurtosis in both risk-neutral and *ex-post* distributions. We employ the first four risk-neutral and *ex-post* moments to recover alternative harvest price densities described as follows:

1. **'RMA_IV'**: a log-normal density function with $VAR^{RMA}_{t,t+244}$ as the variance measure. This is the baseline harvest price density assumed by the RMA.

2. **'RMA_RN_IV'**: a log-normal density function with $VAR^{BKM}_{t,t+244}$ as the variance measure.

3. **'RN_KL'**: risk-neutral density based on $VAR^{BKM}_{t,t+244}$, $SKEW^{BKM}_{t,t+244}$, and $KURT^{BKM}_{t,t+244}$; obtained via minimizing KL divergence with respect to a log-normal density function with $VAR^{BKM}_{t,t+244}$ as the variance measure.

4. **'RN_Entropy'**: risk-neutral density based on $VAR^{BKM}_{t,t+244}$, $SKEW^{BKM}_{t,t+244}$, and $KURT^{BKM}_{t,t+244}$; obtained via entropy maximization.

5. **'EP_KL'**: *ex-post* density based on $VAR^{NP}_{t,t+244}$, $SKEW^{NP}_{t,t+244}$, and $KURT^{NP}_{t,t+244}$; obtained via minimizing KL divergence with respect to a log-normal density function with $VAR^{NP}_{t,t+244}$ as the variance measure.

6. **'EP_Entropy'**: *ex-post* density based on $VAR^{NP}_{t,t+244}$, $SKEW^{NP}_{t,t+244}$, and $KURT^{NP}_{t,t+244}$; obtained via entropy maximization.

As mentioned in Section 2, we recover the alternative density functions for the natural logarithm of harvest price (i.e., $ln(F)$) and then apply the Jacobian transformation to obtain the harvest price densities in levels. For illustration, Figure 6 (Panel a) shows the harvest price densities for corn in 2019 obtained under 'RMA_IV', 'RN_KL', and 'EP_KL'. One can observe that the 'RN_KL' is slightly positively skewed ($SKEW^{BKM}_{t,t+244} = 0.796$) and has a higher peak ($KURT^{BKM}_{t,t+244} = 1.571$) as compared to the 'RMA_IV' distribution. 'EP_KL' has a higher variance and thicker tails as compared to 'RMA_IV' and 'RN_KL'. Figure 6 (Panel b) illustrates the alternative harvest price densities for corn in 2022. We find that

'EP_KL' is negatively skewed as compared to 'RMA_IV' and 'RN_KL' densities. Similarly, Figure 7 illustrates the alternative soybean harvest price densities for 2019 (Panel a) and 2022 (Panel b). Note, 'RMA_IV', 'RN_KL', and 'EP_KL' densities for soybean are very close to each other in 2019 compared to those in 2022, due to wide differences in the risk-neutral and their counterpart *ex-post* moments in 2022. In sum, Figures 6 and 7 intuitively provide graphical illustrations of how alternative harvest price probability density functions could yield different price risk insurance premium rates. For a given set of moments, harvest price densities obtained under KL divergence minimization and entropy maximization are extremely very close to each other; therefore, the latter are ommitted to save space.

4.5 Price Risk Insurance Premium Rates

Tables 5 and 6 report the insurance premium rates for corn and soybean, respectively, for the period 2007–2022 at 80% and 85% price coverage levels. Figures 8 and 9 provide the corresponding time-series graphs of the insurance premium rates. For brevity, we focus only on the 80% and 85% coverage levels to draw the main conclusions of our study, as these coverage levels are the most popular crop insurance choices (Schnitkey, 2019). Also, insurance premium rates are too small at lower coverage levels to draw meaningful conclusions from the comparison of insurance premium rates obtained from alternative harvest price densities.

We compute the insurance premium rates using the above-mentioned six alternative harvest price densities. However, we focus on the insurance premium rates obtained from the following density functions: 'RMA_IV', 'RN_KL', and 'EP_KL'. Assuming price risk only, the RMA would provide its RP insurance premium rates to the market participants utilizing 'RMA_IV'. Alternatively, we propose 'RN_KL' as another benchmark density that utilizes risk-neutral variance, skewness, and kurtosis to characterize the price risk. Lastly, 'EP_KL' is the *ex-post* (realized) harvest price density, and insurance premium rates based on it provide a standard benchmark to compare premium rates derived from alternative harvest price densities based on the risk-neutral moments. We arrive at similar conclusions from the other three remaining densities, namely: 'RMA_RN_IV' (using $VAR^{BKM}_{t,t+244}$), 'RN_Entropy' (using $VAR^{BKM}_{t,t+244}$, $SKEW^{BKM}_{t,t+244}$, and $KURT^{BKM}_{t,t+244}$), and 'EP_Entropy' (using $VAR^{NP}_{t,t+244}$, $SKEW^{NP}_{t,t+244}$, and $KURT^{NP}_{t,t+244}$). The findings from these three density functions serve as the robustness check for our main conclusions.

In an ideal world, one would like to know the insurance premium rates based on 'EP_KL'

during the planting season of corn and soybean, i.e., February of each year. Corn and soybean futures prices are stochastic in nature. Therefore, one can only compute this ("true") *ex-post* harvest price density using realized futures prices. To facilitate comparisons, we compute the following percentage differences between insurance premium rates at a given coverage level:

$$d(IV, RN) = \left(\frac{RMA_IV - RN_KL}{RN_KL} \right) \times 100, \tag{23}$$

$$d(IV, EP) = \left(\frac{RMA_IV - EP_KL}{EP_KL} \right) \times 100, \tag{24}$$

$$d(RN, EP) = \left(\frac{RN_KL - EP_KL}{EP_KL} \right) \times 100. \tag{25}$$

To interpret, at a given coverage level, say 80%, $d(IV, RN)$ ($d(IV, EP)$, $d(RN, EP)$) in Equations (23) (24, 25) entails the percentage difference between the premium rates for 80% coverage computed from the 'RMA_IV' and 'RN_KL' ('RMA_IV' and 'EP_KL', 'RN_KL' and 'EP_KL') densities. In sum, the percentage differences, $d(IV, RN)$, $d(IV, EP)$, and $d(RN, EP)$, directly address the main research objectives of this study.

In essence, a comparison of absolute values of $d(IV, EP)$ and $d(RN, EP)$ in Tables 5 and 6 indicate that the insurance premium rates computed from 'RMA_IV' are closer to the ("true") insurance premium rates based on 'EP_KL' for both corn and soybean, respectively. In other words, for both corn and soybean, the insurance premium rates based on 'RMA_IV' track the insurance premium rates estimated from 'EP_KL' better as compared to the insurance premium rates computed from our benchmark harvest price density, i.e., 'RN_KL'.

4.5.1 Insurance Premium Rates for Corn

Table 5 shows that the premium rates based on 'RMA_IV' and 'RN_KL' densities are very close to each other for moderate values of risk-neutral skewness and kurtosis, and $d(IV, RN)$ take small values at coverage levels of 80% and 85%. However, at the 80% coverage level, $d(IV, RN)$ takes large positive values for 2016, 2018, 2019, 2020, and 2021, i.e., premiums based on 'RMA_IV' density are larger than the ones based on 'RN_KL' density. For instance, higher risk-neutral skewness ($SKEW^{BKM}_{t,t+244} = 0.809$) and kurtosis ($KURT^{BKM}_{t,t+244} = 1.919$) of corn futures returns in 2016 compel premium rates based on 'RMA_IV' density to be

22% higher than the ones based on 'RN_KL' density. In 2019, at the variance level of $VAR^{RMA}_{t,t+244} = VAR^{BKM}_{t,t+244} = 0.023$, higher risk-neutral skewness ($SKEW^{BKM}_{t,t+244} = 0.797$) and kurtosis ($KURT^{BKM}_{t,t+244} = 1.571$) make the premium rates based on 'RMA_IV' density 59% higher than the one based on 'RN_KL' density (see Figure 8). In 2021, the higher variance of 'RMA_IV' density coupled with moderate values of risk-neutral skewness and kurtosis yields $d(IV, RN) = 25\%$ at 80% coverage level. We find that the premium rates based on 'RMA_IV' density are larger compared to their counterpart premiums based on the risk-neutral density, i.e., 'RN_KL', either due to the higher variance level (i.e. $VAR^{RMA}_{t,t+244} > VAR^{BKM}_{t,t+244}$) or a combination of higher risk-neutral skewness and kurtosis values. Also, the magnitude of positive $d(IV, RN)$ values tends to become smaller at a higher coverage level. For example, in 2019, $d(IV, RN)$ takes the value of 59% at the 80% coverage level, compared to 21% at the 85% coverage level.

Corn insurance premium rates based on 'RMA_IV' and 'RN_KL' densities are larger than the premiums based on (true) *ex-post* harvest price density (i.e., $d(IV, EP) > 0$ and $d(RN, EP) > 0$) when the risk-neutral variances ($VAR^{BKM}_{t,t+244}$ or $VAR^{RMA}_{t,t+244}$) are larger than the corresponding realized variance ($VAR^{NP}_{t,t+244}$) at both coverage levels of 80% and 85%. The opposite holds true (i.e., $d(IV, EP) < 0$ and $d(RN, EP) < 0$) in years characterized by higher *ex-post* variances, i.e., $VAR^{NP}_{t,t+244} > VAR^{BKM}_{t,t+244}$ (see Figures 4 and 8). Notably, in 2008, risk-neutral and realized variances are approximately comparable; however, a larger realized skewness ($SKEW^{NP}_{t,t+244} = -1.095$) yields $d(IV, EP) = 30\%$ and $d(RN, EP) = 31\%$ at 85% coverage level. In 2009, a higher risk-neutral variance compared to the realized variance leads to $d(IV, EP)$ and $d(RN, EP)$ greater than 16% at 85% coverage level. Similarly, the premium rates based on 'RMA_IV' and 'RN_KL' densities are smaller compared to the ones based on 'EP_KL' density in 2012 and 2013 due to higher realized variances of corn futures returns. In 2019, $VAR^{BKM}_{t,t+244} \approxeq VAR^{RMA}_{t,t+244} = 0.023$ are considerably smaller than $VAR^{NP}_{t,t+244} = 0.038$; therefore, *ex-post* premium rates are much higher than the ones based on 'RMA_IV' and 'RN_KL' densities.

Lastly, we compare the absolute values of $d(IV, EP)$ and $d(RN, EP)$ to assess the relative performance of the premium rates based on the 'RMA_IV' and 'RN_KL' densities to the premium rates based on the 'EP_KL' density. At the price coverage level of 80%, we find that the absolute value of $d(IV, EP)$ is smaller than the absolute value of $d(RN, EP)$ for 9 cropping seasons out of 16 cropping seasons in the sample. For the 85% price coverage level, $|d(IV, EP)| < |d(RN, EP)|$ for 10 cropping seasons out of 16 cropping seasons in the sample. For instance, in 2019, at 85% price coverage level, we find $d(IV, EP) = -34.54\%$ and

$d(RN, EP) = -46.09\%$ which could be attributed to high realized variance ($VAR^{NP}_{t,t+244} = 0.038$) for the 'EP_KL' density and high risk-neutral skewness ($SKEW^{BKM}_{t,t+244} = 0.797$) and kurtosis ($KURT^{BKM}_{t,t+244} = 1.571$) for the 'RN_KL' density. Therefore, based on the magnitudes of $d(IV, EP)$ and $d(RN, EP)$, it can be argued that the log-normal harvest price density ('RMA_IV') yields insurance premium rates for corn that are at least as close to the "true" rates as those based on our benchmark harvest price density ('RN_KL'), which incorporates risk-neutral skewness and kurtosis.

4.5.2 Insurance Premium Rates for Soybean

We document the soybean insurance premium rates for 2007–2022 at 80% and 85% coverage levels in Table 6, along with a graphical depiction in Figure 9. We find that premium rates based on 'RMA_IV' density are consistently lower than the premium rates based on the risk-neutral density ('RN_KL'), i.e., $d(IV, RN) < 0$ at 80% and 85% coverage levels. However, in 2018, we have a notable exception with $d(IV, RN) = 127\%$ at 80% coverage level and diminishes to 34% at the 85% coverage level, primarily being driven by $VAR^{RMA}_{t,t+244} > VAR^{BKM}_{t,t+244}$. Similar to the corn insurance premium rate, the RMA's methodology yields lower insurance premium rates as compared to our benchmark harvest price density (i.e., 'RN_KL') for moderate risk-neutral skewness and kurtosis values.

The insurance premiums based on the RMA's methodology are higher as compared to those based on the (true) *ex-post* density ('EP_KL'), i.e., $d(IV, EP) > 0$. However, there are several observations with $d(IV, EP) < 0$ (for seven out of sixteen crop years), i.e., premiums based on the RMA's methodology are smaller as compared to the premiums based on the 'EP_KL' density. As noted for corn, observations with $d(IV, EP) < 0$ are primarily driven by higher realized variance than their risk-neutral counterparts, i.e., i.e., $VAR^{NP}_{t,t+244} > VAR^{BKM}_{t,t+244}$. For illustration, *ex-post* moments for soybean in 2016 are as follows: $VAR^{NP}_{t,t+244} = 0.036$, $SKEW^{NP}_{t,t+244} = 0.571$, and $KURT^{NP}_{t,t+244} = 1.299$, whereas $VAR^{RMA}_{t,t+244} = 0.016$; the resulting premium rates based on the 'RMA_IV' density are almost half of the premiums based on the 'EP_KL' density.

We compare insurance premium rates based on our benchmark density ('RN_KL') with those based on the 'EP_KL' density, i.e., $d(RN, EP)$. We find that the signs of $d(IV, EP)$ and $d(RN, EP)$ remain the same for a particular year and at both coverage levels. In other words, premiums based on 'RN_KL' density are smaller than those based on 'EP_KL' density, primarily due to higher realized variances. For illustration, in 2018, we find that the *ex-post* variance is higher than the risk-neutral variance, the risk-neutral density is positively

skewed with an elongated peak, and the *ex-post* density is negatively skewed; as a result, the premiums based on the risk-neutral density are smaller as compared to those based on the *ex-post* density. Lastly, $|d(IV, EP)| < |d(RN, EP)|$ for nine out of sixteen crop years. Therefore, in the case of soybean it can also be argued that insurance premium rates based on the 'RMA_IV' density are as accurate as the premium rates based on the 'EP_KL' density as compared to the ones based on our benchmark risk-neutral density ('RN_KL').

5 Concluding Remarks

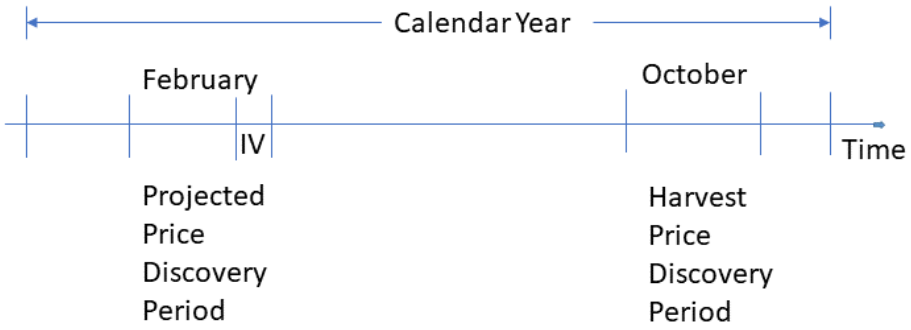
This study develops and applies new empirical methods to refine the price risk component of U.S. crop insurance premium rating by incorporating risk-neutral skewness and kurtosis into the harvest price density. We relax the Risk Management Agency's (RMA) core assumption that corn and soybean harvest prices are log-normally distributed, using the Bakshi et al. (2003) framework to estimate risk-neutral variance, skewness, and kurtosis from planting to harvest for the crop years 2007 through 2022. These higher moments are incorporated into alternative harvest price densities via Kullback–Leibler divergence minimization and entropy maximization.

A key novelty is that we are the first to estimate the "true" variance, skewness, and kurtosis of the harvest price distribution from ex-post data, applying Neuberger and Payne (2021)'s method to recover long-horizon moments from daily futures returns. This allows us to directly compare the premium rates computed from RMA's log-normal method, a higher-moment risk-neutral benchmark, and the ex-post "true" distribution.

For both corn and soybean, we find that the RMA's log-normal premiums are generally as close—or closer—to "true" premiums as those from the more complex benchmark. Differences in performance are largely driven by whether risk-neutral or ex-post variance is higher in a given year. These results indicate that, for the crops and periods studied, the simpler RMA methodology is robust and efficient, with no systematic disadvantage relative to higher-moment models.

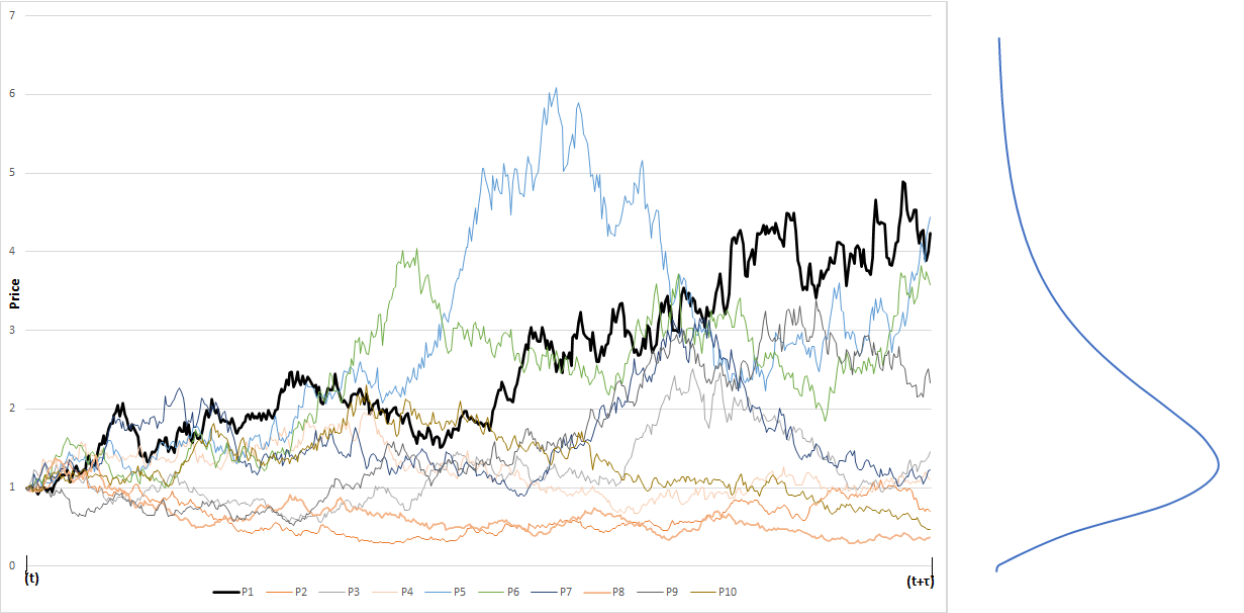
The findings support continued use of the current RMA approach, while providing a framework for evaluating alternative pricing methods. They also highlight the potential value of integrating both price and yield risk refinements, as in Ramirez et al. (2011), to improve accuracy further. Broader application to other crops, market conditions, and time periods would help assess the generality of these conclusions and guide policy toward more precise and cost-effective crop insurance pricing.

Figure 1: Timeline of the Price Discovery Period for RP Insurance.



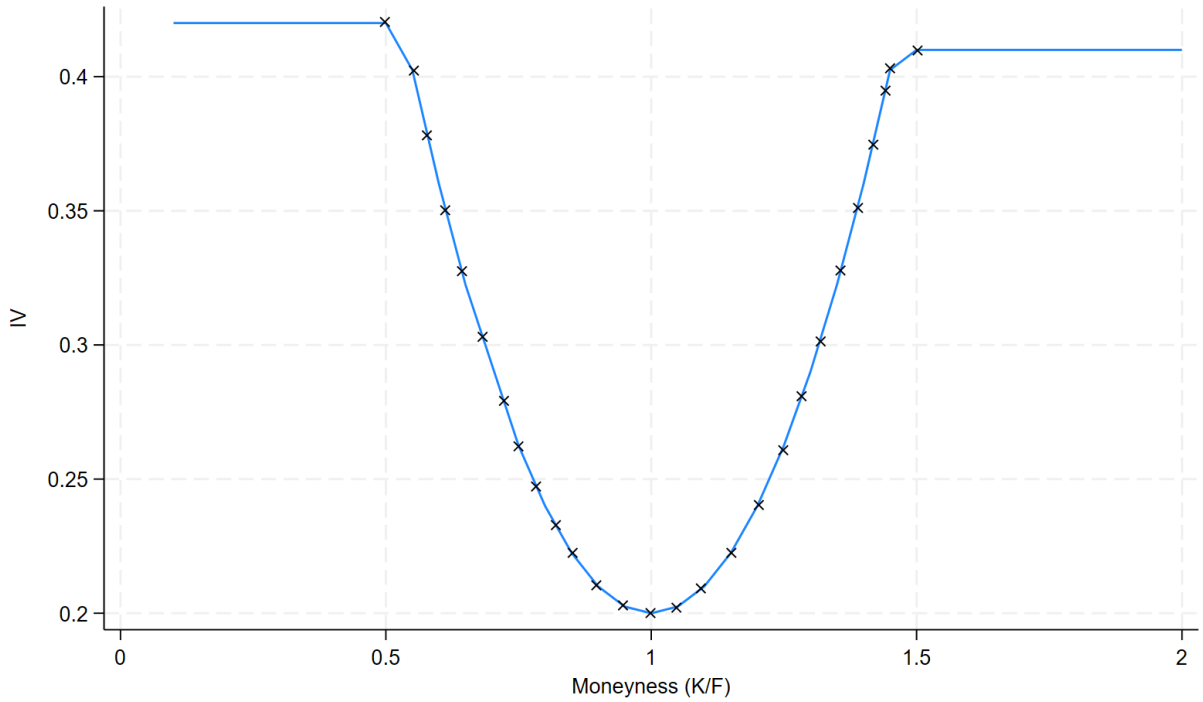
Notes: In a given calendar year, the average of the futures prices during the months of February and October are utilized to compute the projected price and harvest price, respectively, for corn and soybean. The RMA takes the December futures contract for corn and the November futures contract for soybean to compute the corresponding projected price and harvest price. In addition, during the last week of February in a given calendar year, the RMA averages the Black (1976)'s implied volatility (IV) of the December futures contract for corn and the November futures contract for soybean to compute the corresponding volatility factor to characterize the harvest price risk.

Figure 2: Plausible Hypothetical Realized Price Paths For a Specific Year.



Notes: "Pi"s depict plausible hypothetical price realizations from time-period t to $(t+\tau)$. An illustrative probability density function of the prices is indicated on the right-hand side of the graph. "P1" shows the (only) price path that actually occurred.

Figure 3: Interpolation of Implied Volatility As a Function of Moneyness.



Notes: "IV" denotes Black (1976)'s implied volatility. The Moneyness level is the ratio of the strike price (K) to the futures price (F). Symbol "$\times$" denotes the option premiums available in the market data.

Figure 4: Risk-neutral and *Ex-post* Moments for Corn.



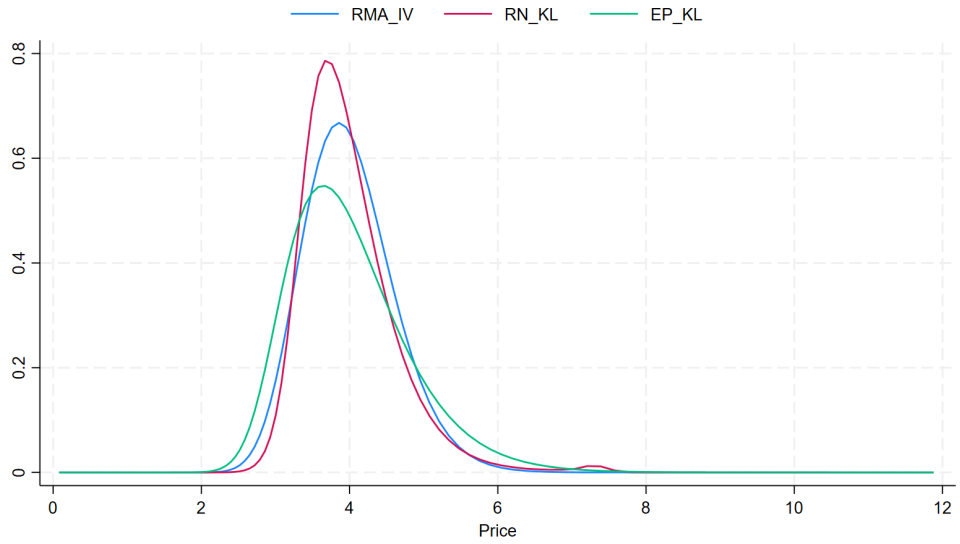
Notes: Risk-neutral and *ex-post* moments for corn from projected price to harvest price discovery period. VAR^{BKM}, $SKEW^{BKM}$, and $KURT^{BKM}$ denote risk-neutral variance ($VAR^{BKM}_{t,t+244}$), skewness ($SKEW^{BKM}_{t,t+244}$), and kurtosis ($KURT^{BKM}_{t,t+244}$) computed using Bakshi et al. (2003)'s methodology, respectively. VAR^{RMA} is the risk-neutral variance computed using RMA's method ($VAR^{RMA}_{t,t+244}$). VAR^{NP}, $SKEW^{NP}$, and $KURT^{NP}$ are realized (physical) variance ($VAR^{NP}_{t,t+244}$), skewness ($SKEW^{NP}_{t,t+244}$), and kurtosis ($VAR^{NP}_{t,t+244}$) computed using Neuberger and Payne (2021)'s methodology, respectively.

Figure 5: Risk-neutral and *Ex-post* Moments for Soybean.

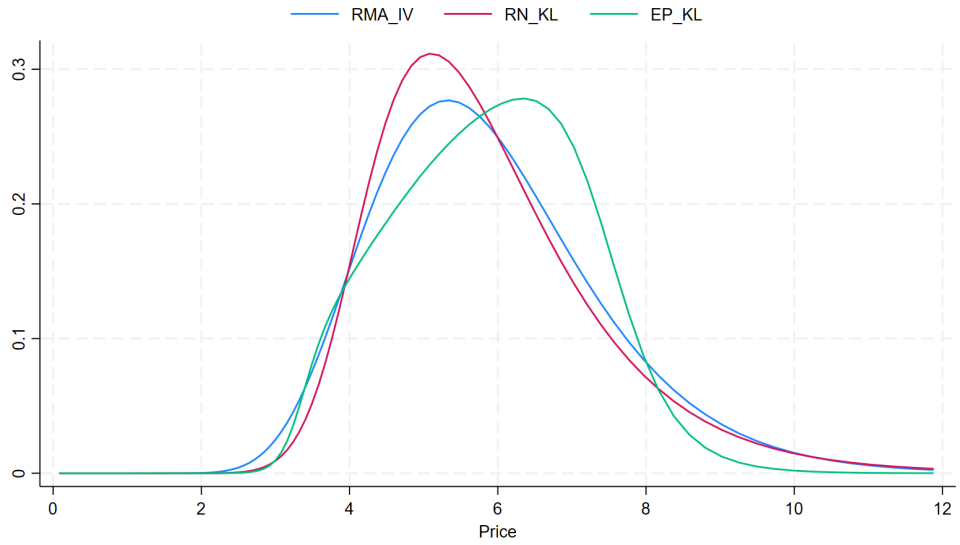


Notes: Risk-neutral and *ex-post* moments for soybean from projected price to harvest price discovery period. VAR^{BKM}, $SKEW^{BKM}$, and $KURT^{BKM}$ denote risk-neutral variance ($VAR^{BKM}_{t,t+244}$), skewness ($SKEW^{BKM}_{t,t+244}$), and kurtosis ($KURT^{BKM}_{t,t+244}$) computed using Bakshi et al. (2003)'s methodology, respectively. VAR^{RMA} is the risk-neutral variance computed using RMA's method ($VAR^{RMA}_{t,t+244}$). VAR^{NP}, $SKEW^{NP}$, and $KURT^{NP}$ are realized (physical) variance ($VAR^{NP}_{t,t+244}$), skewness ($SKEW^{NP}_{t,t+244}$), and kurtosis ($KURT^{NP}_{t,t+244}$) computed using Neuberger and Payne (2021)'s methodology, respectively.

Figure 6: Alternative Harvest Price Distributions for Corn.



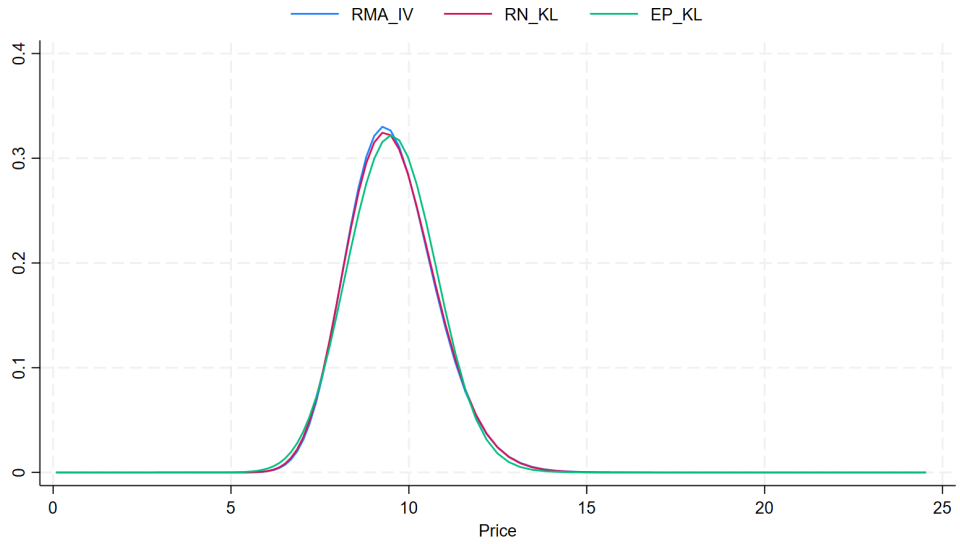
((a)) 2019



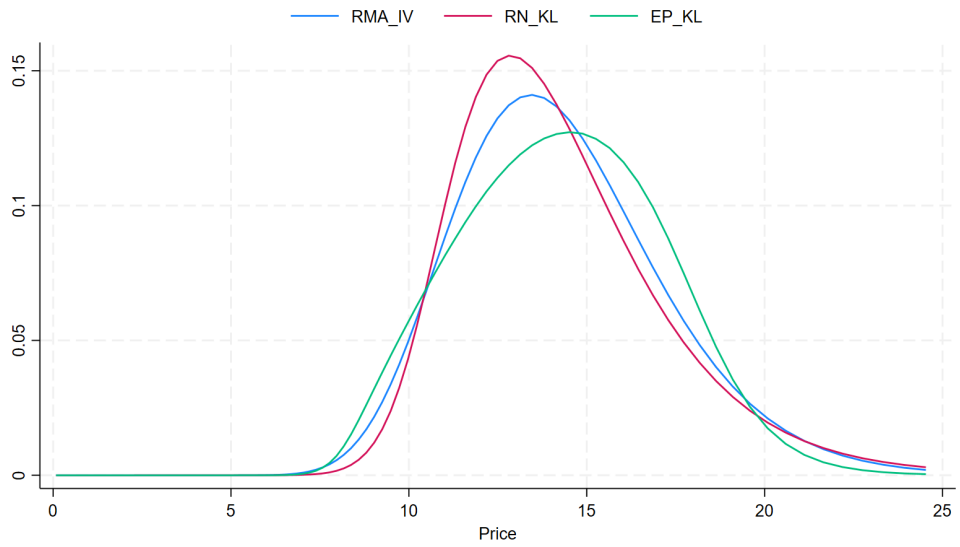
((b)) 2022

Notes: 'Price': denotes the harvest price; 'RMA_IV: log-normal probability distribution with a variance obtained using RMA's methodology; 'RN_KL': risk-neutral probability distribution estimated using $VAR^{BKM}_{t,t+244}$, $SKEW^{BKM}_{t,t+244}$, and $KURT^{BKM}_{t,t+244}$, via minimizing Kullback–Leibler divergence; 'EP_KL': *Ex-post* probability distribution estimated using $VAR^{NP}_{t,t+244}$, $SKEW^{NP}_{t,t+244}$, and $KURT^{NP}_{t,t+244}$, via minimizing Kullback–Leibler divergence.

Figure 7: Alternative Harvest Price Distributions for Soybean.



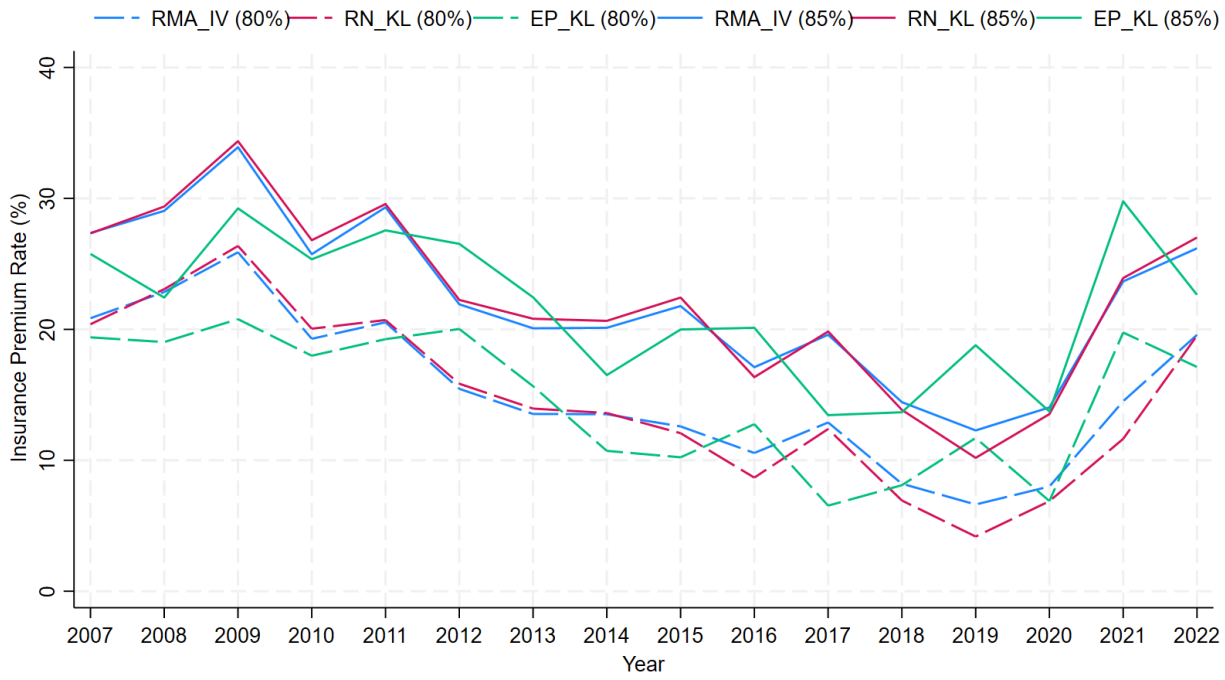
((a)) 2019



((b)) 2022

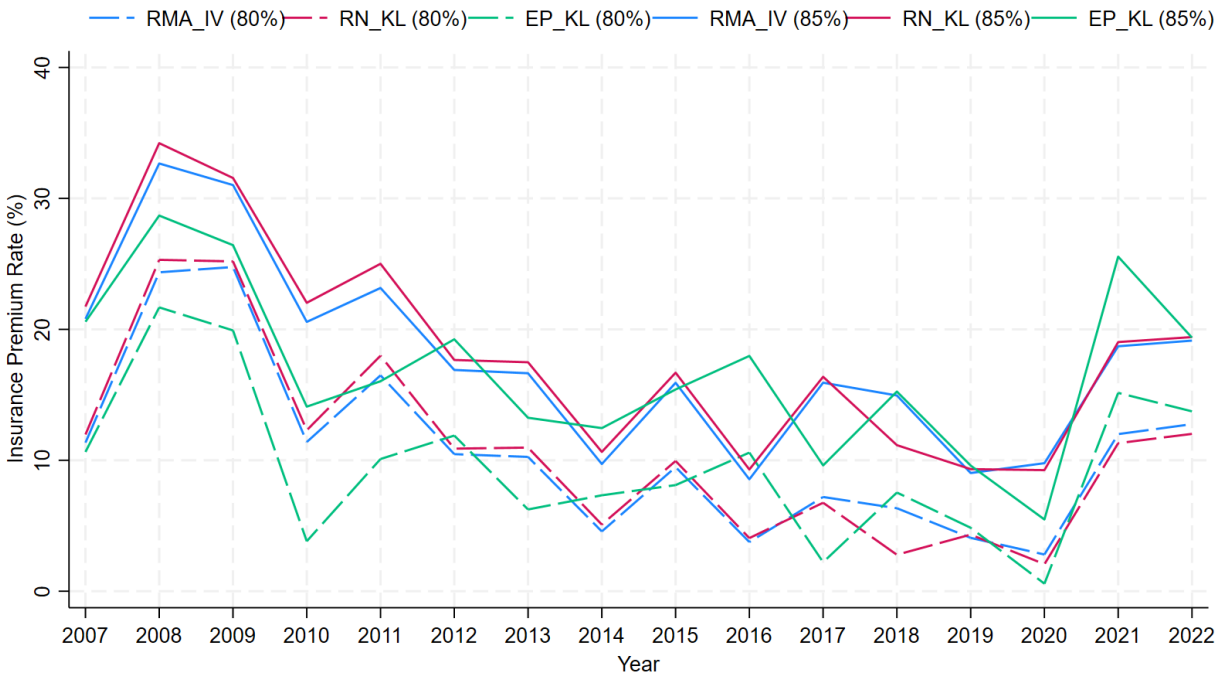
Notes: 'Price': denotes the harvest price; 'RMA_IV: log-normal probability distribution with a variance obtained using RMA's methodology; 'RN_KL': risk-neutral probability distribution estimated using $VAR^{BKM}_{t,t+244}$, $SKEW^{BKM}_{t,t+244}$, and $KURT^{BKM}_{t,t+244}$, via minimizing Kullback–Leibler divergence; 'EP_KL': *Ex-post* probability distribution estimated using $VAR^{NP}_{t,t+244}$, $SKEW^{NP}_{t,t+244}$, and $KURT^{NP}_{t,t+244}$, via minimizing Kullback–Leibler divergence.

Figure 8: Price Risk Insurance Premium Rates for Corn (%).



Notes: Price risk insurance premium rates in percentage (%) term for corn at 80% and 85% price coverage levels from 2007–2022. 'RMA_IV': a log-normal probability distribution with RMA's implied variance measure; 'RN_KL': Risk-neutral distribution obtained via minimizing Kullback–Leibler divergence; 'EP_KL': *Ex-post* distribution obtained via minimizing Kullback–Leibler divergence.

Figure 9: Price Risk Insurance Premium Rates for Soybean (%).



Notes: Price risk insurance premium rates in percentage (%) term for corn at 80% and 85% price coverage levels from 2007–2022. 'RMA_IV': a log-normal probability distribution with RMA's implied variance measure; 'RN_KL': Risk-neutral distribution obtained via minimizing Kullback–Leibler divergence; 'EP_KL': *Ex-post* distribution obtained via minimizing Kullback–Leibler divergence.

Table 1: Regression Estimates of Covariance terms in Eight-month Skewness and Kurtosis for Corn Futures Returns.

	Dependent variable				
	(1)	(2)	(3)	(4)	(5)
	r	r	$x^{(2E)}(r)$	$x^{(2L)}(r)$	$x^{(3)}(r)$
$y^{(1)}$			-0.0002*		-1.13×10^{-06}
			(0.0001)		(7.97×10^{-06})
$y^{(2)}$	-0.0069			0.0049***	
	(0.0147)			(0.0006)	
$y^{(3)}$		-0.0121			
		(0.0347)			
Constant	6.22×10^{-05}	-7.45×10^{-05}	0.0003***	0.0002***	-1.67×10^{-07}
	(0.0003)	(0.0003)	(9.72×10^{-06})	(1.22×10^{-05})	(6.12×10^{-07})
No. of Obs.	4,053	4,053	4,053	4,053	4,053
R^2	0.000	0.000	0.002	0.052	0.000

Notes: Robust standard errors within parentheses. *** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table 2: Regression Estimates of Covariance terms in Eight-month Skewness and Kurtosis for Soybean Futures Returns.

	Dependent variable				
	(1)	(2)	(3)	(4)	(5)
	r	r	$x^{(2E)}(r)$	$x^{(2L)}(r)$	$x^{(3)}(r)$
$y^{(1)}$			-0.0001		-2.71×10^{-06}
			(0.0001)		(6.53×10^{-06})
$y^{(2)}$	-0.0032			0.0047***	
	(0.0157)			(0.0006)	
$y^{(3)}$		-0.0119			
		(0.0397)			
Constant	6.86×10^{-05}	2.43×10^{-05}	0.0002***	0.0001***	-5.73×10^{-07}
	(0.0002)	(0.0002)	(6.88×10^{-06})	(7.50×10^{-06})	(3.92×10^{-07})
No. of Obs.	4,366	4,366	4,366	4,366	4,366
R^2	0.000	0.000	0.001	0.059	0.000

Notes: Robust standard errors within parentheses. *** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table 3: Risk-neutral and Ex-post Moments for Corn Futures Returns.

$Date^{Planting}$	$VAR^{RMA}_{t,t+244}$	$VAR^{BKM}_{t,t+244}$	$SKEW^{BKM}_{t,t+244}$	$KURT^{BKM}_{t,t+244}$	Projected Price	$Date^{Harvest}$	$VAR^{NP}_{t,t+244}$	$SKEW^{NP}_{t,t+244}$	$KURT^{NP}_{t,t+244}$	Harvest Price
3/1/2007	0.077	0.070	0.221	0.276	4.061	11/1/2007	0.065	-0.092	-0.263	3.579
3/3/2008	0.098	0.097	0.077	0.120	5.396	11/3/2008	0.120	-1.095	0.307	4.126
3/2/2009	0.136	0.142	-0.002	0.699	4.037	11/2/2009	0.089	-0.090	0.016	3.720
3/1/2010	0.071	0.073	0.265	0.867	3.985	11/1/2010	0.061	0.420	0.797	5.455
3/1/2011	0.086	0.086	0.036	0.561	6.012	11/1/2011	0.076	-0.184	-0.187	6.321
3/1/2012	0.053	0.055	-0.070	0.814	5.678	11/1/2012	0.060	-0.011	-0.783	7.501
3/1/2013	0.044	0.044	0.116	0.504	5.653	11/1/2013	0.050	0.068	-0.138	4.392
3/3/2014	0.043	0.043	0.242	0.549	4.617	11/3/2014	0.032	-0.353	0.425	3.494
3/2/2015	0.045	0.045	0.519	1.291	4.148	11/2/2015	0.037	0.147	0.004	3.831
3/1/2016	0.032	0.031	0.809	1.919	3.859	11/1/2016	0.038	0.248	0.124	3.495
3/1/2017	0.039	0.039	0.543	1.584	3.957	11/1/2017	0.023	0.264	-0.043	3.494
3/1/2018	0.026	0.025	0.560	1.270	3.959	11/1/2018	0.023	-0.330	-0.005	3.682
3/1/2019	0.023	0.023	0.797	1.571	3.996	11/1/2019	0.038	0.275	0.168	3.898
3/2/2020	0.026	0.027	0.631	1.829	3.883	11/2/2020	0.025	0.214	-0.196	3.989
3/1/2021	0.054	0.047	0.735	0.728	4.584	11/1/2021	0.078	0.504	0.808	5.366
3/1/2022	0.068	0.065	0.509	0.962	5.899	11/1/2022	0.052	-0.433	-0.290	6.856

Notes: $Date^{Planting}$: risk-neutral and *ex-post* moments computation dates; $VAR^{RMA}_{t,t+244}$: variance estimated using RMA's methodology; $VAR^{BKM}_{t,t+244}$, $SKEW^{BKM}_{t,t+244}$, and $KURT^{BKM}_{t,t+244}$ denote the risk-neutral variance, skewness, and kurtosis, respectively, estimated using methodology in Bakshi et al. (2003); 'Projected price': projected harvest price, i.e., average of December corn futures price ($/Bu) during February of the corresponding year; $Date^{Harvest}$: harvest price computation date; $VAR^{NP}_{t,t+244}$, $SKEW^{NP}_{t,t+244}$, and $KURT^{NP}_{t,t+244}$ denote variance, skewness, and kurtosis of the *ex-post* corn futures returns distribution estimated using Neuberger and Payne (2021)'s methodology. 'Harvest price': average of December corn futures price ($/Bu) during October of the corresponding year.

Table 4: Risk-neutral and Ex-post Moments for Soybean Futures Returns.

$Date^{Planting}$	$VAR^{RMA}_{t,t+244}$	$VAR^{BKM}_{t,t+244}$	$SKEW^{BKM}_{t,t+244}$	$KURT^{BKM}_{t,t+244}$	Projected Price	$Date^{Harvest}$	$VAR^{NP}_{t,t+244}$	$SKEW^{NP}_{t,t+244}$	$KURT^{NP}_{t,t+244}$	Harvest Price
3/1/2007	0.038	0.041	0.172	1.066	8.087	11/1/2007	0.035	0.063	0.047	9.754
3/3/2008	0.117	0.115	0.369	0.693	13.364	11/3/2008	0.121	-0.659	0.506	9.220
3/2/2009	0.106	0.108	0.092	0.483	8.795	11/2/2009	0.069	-0.054	-0.006	9.658
3/1/2010	0.041	0.045	0.352	1.642	9.229	11/1/2010	0.025	0.551	0.129	11.626
3/1/2011	0.054	0.060	0.348	1.424	13.493	11/1/2011	0.029	-0.283	0.264	12.139
3/1/2012	0.033	0.034	0.215	0.994	12.548	11/1/2012	0.038	0.411	0.706	15.391
3/1/2013	0.032	0.034	0.005	0.875	12.871	11/1/2013	0.024	0.314	-0.111	12.868
3/3/2014	0.018	0.020	0.272	2.042	11.363	11/3/2014	0.024	-0.565	1.034	9.646
3/2/2015	0.029	0.030	0.090	1.219	9.7307	11/2/2015	0.027	0.300	0.006	8.911
3/1/2016	0.016	0.018	0.480	4.322	8.850	11/1/2016	0.036	0.571	1.299	9.754
3/1/2017	0.027	0.029	0.642	2.447	10.185	11/1/2017	0.016	0.213	-0.021	9.751
3/1/2018	0.024	0.020	0.636	1.175	10.162	11/1/2018	0.026	-0.413	0.252	8.956
3/1/2019	0.017	0.017	-0.073	0.682	9.537	11/1/2019	0.017	-0.311	0.335	9.250
3/2/2020	0.015	0.016	0.508	1.425	9.173	11/2/2020	0.014	0.848	0.706	10.548
3/1/2021	0.035	0.036	0.525	0.942	11.867	11/1/2021	0.041	0.192	-0.016	12.302
3/1/2022	0.042	0.042	0.497	1.039	14.326	11/1/2022	0.042	-0.346	-0.270	13.813

Notes: $Date^{Planting}$: risk-neutral and *ex-post* moments computation dates; $VAR^{RMA}_{t,t+244}$: variance estimated using RMA's methodology; $VAR^{BKM}_{t,t+244}$, $SKEW^{BKM}_{t,t+244}$, and $KURT^{BKM}_{t,t+244}$ denote the risk-neutral variance, skewness, and kurtosis, respectively, estimated using methodology in Bakshi et al. (2003); 'Projected price': projected harvest price, i.e., average of November soybean futures price ($/Bu) during February of the corresponding year; $Date^{Harvest}$: harvest price computation dates; $VAR^{NP}_{t,t+244}$, $SKEW^{NP}_{t,t+244}$, and $KURT^{NP}_{t,t+244}$ denote variance, skewness, and kurtosis of the *ex-post* corn futures returns distribution estimated using Neuberger and Payne (2021)'s methodology. 'Harvest price': average of November soybean futures price ($/Bu) during October of the corresponding year.

Table 5: Corn Insurance Premium Rates (%).

Year	Coverage	RMA_IV	RMA_RN_IV	RN_KL	RN_Entropy	EP_KL	EP_Entropy	d(IV, RN)	d(IV, EP)	d(RN, EP)
2007	0.80	20.85	19.77	20.39	20.27	19.39	19.73	2.29	7.52	5.12
2008	0.80	22.85	22.66	23.07	22.99	19.03	18.80	-0.94	20.08	21.23
2009	0.80	25.89	26.39	26.37	26.26	20.78	20.74	-1.84	24.57	26.90
2010	0.80	19.28	19.67	20.05	20.02	17.99	17.83	-3.81	7.21	11.46
2011	0.80	20.54	20.65	20.71	20.62	19.25	19.23	-0.84	6.68	7.58
2012	0.80	15.46	15.95	15.85	15.79	20.03	19.55	-2.47	-22.84	-20.89
2013	0.80	13.54	13.75	13.95	13.71	15.66	15.97	-2.91	-13.52	-10.93
2014	0.80	13.52	13.62	13.61	13.58	10.72	10.74	-0.65	26.13	26.96
2015	0.80	12.59	12.60	12.07	12.14	10.23	10.26	4.28	23.04	18.00
2016	0.80	10.56	10.44	8.68	8.93	12.76	13.03	21.70	-17.23	-31.99
2017	0.80	12.89	12.93	12.39	12.46	6.54	6.70	4.07	97.27	89.55
2018	0.80	8.21	8.19	6.93	7.01	8.10	8.31	18.34	1.35	-14.36
2019	0.80	6.63	6.45	4.18	4.33	11.71	11.94	58.68	-43.41	-64.33
2020	0.80	8.00	8.18	6.88	6.76	6.89	7.05	16.24	16.05	-0.16
2021	0.80	14.51	12.98	11.64	12.16	19.76	19.74	24.67	-26.58	-41.11
2022	0.80	19.58	18.94	19.52	19.53	17.13	16.90	0.31	14.29	13.94
2007	0.85	27.36	26.34	27.33	27.20	25.76	26.09	0.10	6.21	6.11
2008	0.85	29.05	28.87	29.38	29.31	22.43	22.13	-1.13	29.52	30.99
2009	0.85	33.91	34.34	34.37	34.32	29.24	29.22	-1.35	15.97	17.56
2010	0.85	25.74	26.11	26.81	26.81	25.35	25.03	-3.99	1.52	5.74
2011	0.85	29.32	29.42	29.57	29.53	27.56	27.59	-0.84	6.38	7.28
2012	0.85	21.91	22.40	22.25	22.19	26.53	26.07	-1.56	-17.42	-16.12
2013	0.85	20.08	20.29	20.81	20.45	22.45	22.81	-3.51	-10.55	-7.29

2014	0.85	20.12	20.22	20.65	20.61	16.51	16.53	-2.57	21.92	25.13
2015	0.85	21.78	21.79	22.43	22.72	19.99	19.93	-2.89	8.96	12.21
2016	0.85	17.11	16.98	16.35	16.65	20.12	20.51	4.68	-14.95	-18.75
2017	0.85	19.59	19.63	19.84	20.13	13.45	13.54	-1.27	45.65	47.52
2018	0.85	14.44	14.42	13.85	13.98	13.67	14.03	4.24	5.65	1.36
2019	0.85	12.28	12.05	10.19	10.55	18.79	19.14	20.48	-34.66	-45.76
2020	0.85	14.04	14.26	13.53	13.66	13.73	13.80	3.81	2.25	-1.51
2021	0.85	23.66	22.17	23.92	23.96	29.78	29.69	-1.06	-20.54	-19.69
2022	0.85	26.19	25.59	27.01	26.99	22.64	22.51	-3.05	15.69	19.32

Notes: Insurance premium rates are in percentage (%) terms; Coverage: 'x' indicates x% of the projected price is insured. 'RMA_IV': a log-normal probability distribution with RMA's implied variance measure; 'RMA_RN_IV': a log-normal probability distribution with $VAR^{BKM}_{t,t+244}$ as variance measure; 'RN_KL': Risk-neutral distribution obtained via minimizing Kullback–Leibler divergence with respect to 'RMA_RN_IV'; 'RN_Entropy': Risk-neutral distribution obtained via entropy maximization; 'EP_KL': *Ex-post* distribution obtained via minimizing Kullback–Leibler divergence; 'EP_Entropy': *Ex-post* distribution obtained via entropy maximization; $d(IV, RN) = \left(\frac{RMA_IV - RN_KL}{RN_KL}\right) \times 100$; $d(IV, EP) = \left(\frac{RMA_IV - EP_KL}{EP_KL}\right) \times 100$; $d(RN, EP) = \left(\frac{RN_KL - EP_KL}{EP_KL}\right) \times 100$.

Table 6: Soybean Insurance Premiums Rates (%).

Year	Coverage	RMA_IV	RMA_RN_IV	RN_KL	RN_Entropy	EP_KL	EP_Entropy	d(IV, RN)	d(IV, EP)	d(RN, EP)
2007	0.80	11.35	12.09	11.97	11.91	10.64	10.66	-5.17	6.67	12.49
2008	0.80	24.36	24.16	25.31	25.12	21.68	21.74	-3.72	12.38	16.72
2009	0.80	24.76	24.89	25.19	25.38	19.92	20.26	-1.73	24.26	26.45
2010	0.80	11.40	12.57	12.30	12.31	3.82	4.14	-7.37	198.07	221.79
2011	0.80	16.49	17.87	17.99	18.05	10.10	10.08	-8.35	63.33	78.22
2012	0.80	10.47	11.10	10.89	10.82	11.89	11.93	-3.86	-11.95	-8.41
2013	0.80	10.26	11.04	10.97	11.02	6.25	6.41	-6.49	64.27	75.68
2014	0.80	4.57	5.50	5.09	4.99	7.33	7.50	-10.17	-37.64	-30.58
2015	0.80	9.48	10.10	9.95	9.91	8.11	8.29	-4.73	16.83	22.62
2016	0.80	3.76	4.82	4.06	3.93	10.59	10.71	-7.30	-64.48	-61.69
2017	0.80	7.20	7.91	6.76	6.71	2.22	2.28	6.40	223.49	204.03
2018	0.80	6.34	4.47	2.79	2.83	7.54	7.55	126.89	-15.89	-62.93
2019	0.80	4.08	4.26	4.33	4.35	4.85	5.05	-5.72	-15.85	-10.75
2020	0.80	2.81	3.05	2.07	1.96	0.58	0.34	35.96	383.96	255.97
2021	0.80	12.00	12.02	11.31	11.43	15.15	13.92	6.10	-20.79	-25.34
2022	0.80	12.76	12.60	12.02	12.12	13.74	13.98	6.18	-7.11	-12.52
2007	0.85	20.80	21.56	21.74	21.77	20.58	20.58	-4.34	1.05	5.63
2008	0.85	32.67	32.49	34.22	34.09	28.69	28.83	-4.52	13.89	19.28
2009	0.85	31.02	31.15	31.56	31.78	26.43	26.75	-1.70	17.37	19.40
2010	0.85	20.57	21.77	22.03	22.15	14.10	14.49	-6.64	45.87	56.24
2011	0.85	23.16	24.50	25.01	25.28	16.04	16.04	-7.38	44.43	55.94
2012	0.85	16.90	17.58	17.66	17.64	19.24	19.22	-4.28	-12.16	-8.23
2013	0.85	16.65	17.50	17.49	17.60	13.25	13.58	-4.79	25.65	31.96

2014	0.85	9.72	10.97	10.64	10.63	12.46	12.52	-8.73	-22.02	-14.56
2015	0.85	15.92	16.62	16.69	16.66	15.41	15.50	-4.61	3.31	8.30
2016	0.85	8.55	10.06	9.30	9.31	17.97	18.09	-8.11	-52.42	-48.23
2017	0.85	15.93	16.78	16.38	16.63	9.61	9.71	-2.77	65.81	70.53
2018	0.85	14.95	12.38	11.16	11.39	15.25	15.26	33.87	-1.99	-26.79
2019	0.85	9.03	9.27	9.33	9.32	9.60	9.95	-3.21	-5.93	-2.81
2020	0.85	9.78	10.21	9.25	9.28	5.48	6.13	5.72	78.43	68.79
2021	0.85	18.71	18.73	19.03	19.07	25.55	21.20	-1.70	-26.78	-25.51
2022	0.85	19.14	18.97	19.41	19.46	19.37	19.68	-1.38	-1.15	0.23

Notes: Insurance premium rates are in percentage (%) terms; Coverage: 'x' indicates x% of the projected price is insured. 'RMA_IV': a log-normal probability distribution with RMA's implied variance measure; 'RMA_RN_IV': a log-normal probability distribution with $VAR^{BKM}_{t,t+244}$ as variance measure; 'RN_KL': Risk-neutral distribution obtained via minimizing Kullback–Leibler divergence with respect to 'RMA_RN_IV'; 'RN_Entropy': Risk-neutral distribution obtained via entropy maximization; 'EP_KL': *Ex-post* distribution obtained via minimizing Kullback–Leibler divergence; 'EP_Entropy': *Ex-post* distribution obtained via entropy maximization; $d(IV, RN) = \left(\frac{RMA_IV - RN_KL}{RN_KL}\right) \times 100$; $d(IV, EP) = \left(\frac{RMA_IV - EP_KL}{EP_KL}\right) \times 100$; $d(RN, EP) = \left(\frac{RN_KL - EP_KL}{EP_KL}\right) \times 100$.

Appendix: Converting American Futures Options Prices to European Futures Options Prices

Chaudhury (1995) provides the following formulas that allow one to retrieve the premiums for European calls (C) and puts (P) and implied volatilities from the premiums of the corresponding American calls (c) and puts (p):

$$c = \max\{F - K, C\ \exp[0.5\,\varphi(d_2)\,i\,\tau]\}, \tag{26}$$

$$p = \max\{K - F, P\ \exp[0.5\,\varphi(-d_2)\,i\,\tau]\}, \tag{27}$$

where $\varphi(.)$ is the cumulative standard normal distribution function. The premiums for European calls (C) and puts (P) are given by Black (1976)'s formulas (28) and (29),

$$C = \exp(-i\,\tau)\ [F\,\varphi(d_1) - K\,\varphi(d_2)] \tag{28}$$

$$P = \exp(-i\,\tau)\ [K\,\varphi(-d_2) - F\,\varphi(-d_1)] \tag{29}$$

$$d_1 = \frac{\left[\ln\left(\frac{F}{K}\right) + 0.5\,\sigma^2\,\tau\right]}{\sigma\sqrt{\tau}} \tag{30}$$

$$d_2 = d_1 - \sigma\sqrt{\tau} \tag{31}$$

By substituting Equation (28) into Equation (26), we obtain an equation for the American call price where the single unknown is σ, and perform an iterative search to solve for σ using the interval bisection method algorithm. We then plug the resulting value of σ into Equation (28) to obtain the European call price. We use a similar procedure to obtain implied volatilities (σ) from American futures put premiums by substituting Equation (29) into Equation (27) and subsequently obtain European futures put prices using Equation (29).

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