YINSHAN SHANG

◇ (+1)510-495-7329 ◇ yinshans@princeton.edu

Education

Princeton University *Ph.D. in Economics* *2022–Now*
- Fields: Microeconomic Theory, Finance

University of Hong Kong *B.A in Economics and Mathematics, First Class Honor* *2016–2020*

Working Paper

Politics, Finance, and Environmental Policy *with Jonathan Payne*
We develop a general equilibrium asset pricing model to study how political and financial frictions shape environmental policy. In the model, heterogeneous investors bargain over a carbon tax, but political agreements cannot bind subsequent private capital allocation. This leads to an inefficient equilibrium with systematic under-investment in green technology. In an international setting, this inefficiency is amplified by a "financial leakage" channel, where unilateral climate policy can shift capital and pollution abroad, highlighting the limits of relying solely on carbon taxes.

Work-In-Progress

Robust Certification with Costly Manipulation
I study a principal's problem of designing a certification rule when agents can costly manipulate their signals. The principal is uncertain about the agents' manipulation technology (i.e., the cost of falsification). I characterize the optimal robust policy, which maximizes the principal's payoff in the worst-case scenario. When there are multiple desirable agent types, the optimal policy can be non-monotonic and may involve "productive falsification", where the principal intentionally induces some misrepresentation to better separate high- and low-quality agents. The model provides a theoretical framework for designing robust ESG ratings, green investment criteria, and other certification systems.

Strengthening Fraud Control To Improve Government Health Insurance *with Pascaline Dupas & Radhika Jain*
This project designs and evaluates monitoring mechanisms to address hospital fraud within India's public health insurance scheme. We use a large-scale RCT to test theoretically-motivated interventions designed to alter hospitals' strategic behavior by shifting their beliefs about fraud detection. These interventions include a broad, low-cost signal (warning letters) and a high-cost, targeted action (on-site audits), where targets are selected using a data-driven screening algorithm. We measure the impact of these different incentive structures on hospital claims, program costs, and patient welfare.

Experience

Pre-doctoral Research Specialist *2020–2022*
Julis-Rabinowitz Center for Public Policy & Finance, Princeton University
PIs: Atif Mian, Ernest Liu, Jonathan Payne

Teaching and Service

Teaching Assistant *Princeton University, 2024-now*
ECO202, Statistics and Data Analysis for Economics
ECO310, Microeconomic Theory: A Mathematical Approach

Environmental Economics Reading Group, Student Member *Princeton University, Spring 2023*

Graduate Student Theory Reading Group, Co-Organizer *Princeton University, 2023–now*

Selected Honors and Awards

Research Grant, Data-Driven Social Science Initiative (2024) *Princeton University*
University Fellowship in the Humanities and Social Sciences (2022–2027) *Princeton University*
Lilian Poon Memorial Scholarship (2018–2019) *University of Hong Kong*
HKSAR Reaching Out Award (2017) *University of Hong Kong*
Dean's Honors List (2016–2019) *University of Hong Kong*

Politics, Finance, and Environmental Policy *

Jonathan Payne and Yinshan Shang†

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Abstract

We study the interaction of political and financial frictions in shaping environmental policy. We build a general equilibrium asset pricing model where investors with heterogeneous preferences for carbon reduction ("green" or "brown") bargain over a carbon tax, which in turn influences portfolio decisions and real investment. We show that the politically-determined equilibrium is generically inefficient. Because political agreements cannot dictate subsequent private capital allocations, the carbon tax disproportionately imposes welfare costs on brown investors, leading to a low carbon tax and systematic under-investment in green technology relative to the Pareto frontier. In a two-country extension with integrated financial markets, this inefficiency is compounded by a financial leakage channel: unilateral climate policy can be counterproductive, as capital and pollution shift to the untaxed jurisdiction, potentially increasing aggregate global emissions. Our results highlight that a carbon tax alone is insufficient when capital is mobile and policy is endogenous, suggesting that complementary policies targeting international capital flows may be necessary to enable effective climate action.

Keywords: Carbon taxes, political economy, green finance.

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†Payne: Princeton, Department of Economics, Bendheim Center for Finance, jepayne@princeton.edu. Shang: Princeton, Department of Economics, yinshans@princeton.edu.

1 Introduction

Many researchers have argued that we can resolve global warming by taxing carbon emissions and letting the market respond to the resulting financial incentive to use "green" technologies. The reality is more complicated. Political frictions make environmental policies difficult to implement, financial frictions distort incentives to invest, and international capital markets allow investors to evade taxation policy. For all these reasons, economists need to study how politics and finance interact to determine environmental policy. Are carbon taxes a sufficiently rich policy tool to reduce carbon emissions? Do foreign carbon taxes and capital controls help or hinder domestic political bargaining? Does asset pricing reflect political frictions and risks? In this paper, we build a general equilibrium model with environmental, financial, and political frictions to address these questions.

We start by modeling a two-period closed economy where carbon tax policy is set through political bargaining. The economy is populated by two types of investors: "green" investors who care more about carbon emissions and "brown" investors who care less. The investors can purchase equity in two different sectors: a "green" sector with low pollution and returns, and a "brown" production sector with high pollution and returns. All markets are competitive, so investor portfolio decisions determine equilibrium production and carbon emissions in the economy. The key environmental friction is that no investors internalize the impact of their individual financial decisions on carbon emissions when they choose portfolios. Instead, before financial decisions are made, the different types of investors collectively engage in political bargaining to set a carbon tax on emissions. When bargaining, each investor group internalizes the impact of carbon taxes on both general equilibrium output and carbon emissions.

We first show in Section 4 that the outcome of the political bargaining problem is not generically efficient. This is due to two frictions. First, as is standard in public finance models, the carbon tax distorts the labor market, leading to inefficiently low production. Second, political bargaining is incomplete in the sense that the different groups can only agree upon a carbon tax rate; they cannot commit to future portfolio choices or future lump sum transfers between groups. This means that, unlike a social planner, policy makers cannot implement the efficient capital allocations and then use lump-sum transfers to compensate the brown group. Instead, they must incentivize green portfolio choices with carbon taxes. This focuses the welfare cost of climate action on brown investors and prevents the bargaining process from arriving

at a high carbon tax. The result is systematic under-investment in green capital compared to brown capital and the Pareto frontier. Indeed, even a "brown-friendly" planner that only cares about the welfare of brown investors would choose a higher green-sector allocation than the market outcome under political bargaining.

In Section 5, we introduce political risk into the economy by modeling the realized carbon tax as a random variable centered around the bargained agreement. We show that this mechanism generates a "political risk premium" in equilibrium, whereby investors demand higher expected returns as compensation for bearing this non-diversifiable political risk. Whether the political risk premium appears on green or brown equity depends on which type of investments are most exposed to political risk. In our baseline model, the agents bargain over carbon taxes so the risk is primarily born by brown firms and a "brown" premium emerges on brown equity. However, if the agents instead bargained over subsidies (or negative taxes) for non-polluting production, then the green firms primarily bear the risk and so a "green" premium emerges on green equity.

In Section 6, we expand the model to an international setting. We introduce a symmetric second country that also sets carbon emission taxes. This means that investors can now purchase equity from domestic or foreign green firms and domestic or foreign brown firms. The key features are that countries do not coordinate on carbon tax policy and that carbon taxes are collected and redistributed domestically, so investors potentially face different carbon taxes when investing domestically and abroad. We show that the international markets can amplify the frictions in the domestic political bargaining. If carbon taxes abroad are low, then investors can bargain for high domestic carbon taxes and then privately move their wealth abroad to earn higher returns. This potentially makes it easier to get domestic agreement on high carbon taxes but ultimately leads to higher global carbon emissions.

In Section 7, we consider a collection of extensions that complicate policy making. We show that incorporating workers into the bargaining problem creates additional barriers to carbon policy because the welfare costs of carbon tax policies are disproportionately borne by workers who are unable to move resources abroad. We then show that capital controls and investment mandates act as a helpful second-best policy in the presence of political bargaining frictions.

Literature review: Our paper brings together two strands of the literature: asset pricing with climate concerns and the political economy of climate policy.

There is a recent and growing finance literature on asset pricing with climate concerns (e.g., Heinkel et al. (2001); Engle et al. (2020); Bolton and Kacperczyk (2021); Flammer (2021); Giglio et al. (2021); Pástor et al. (2021); Pedersen et al. (2021); Goldstein et al. (2022); Pástor et al. (2022); Hsu et al. (2023); Acharya et al. (2025); Green and Roth (2025); Hambel and Van Der Ploeg (2025)). Recent work in finance has investigated how investor preferences and anticipated climate policies influence asset returns. While these studies emphasize market responses to exogenous climate risks or static policy measures, we introduce endogenous political bargaining over carbon taxes within a general equilibrium framework. Specifically, our model incorporates investors' heterogeneous preferences explicitly into policy formation rather than into a "warm glow" utility from holding green asset. This provides insights into how political frictions shape policy choices, subsequent asset allocations, and ultimately green or brown risk premia.

We also contribute to the political economy literature on climate policy, which examines domestic and international challenges in environmental policy cooperation (See Oates and Portney (2003); Nordhaus (2015); Battaglini and Harstad (2016, 2020); Cassidy Jr (2023)). We expand upon these works by highlighting the role of integrated financial markets as channels for leakage and international free-riding. By integrating a Nash bargaining framework with general equilibrium asset pricing, we reveal how domestic political frictions combined with international capital mobility exacerbate policy inefficiencies, underscoring the limitations of unilateral carbon taxes and the potential need for coordinated financial market regulations.

We outline the market equilibrium in a single-country general equilibrium model with given carbon tax policy in Section 2. We then describe the political process that endogenizes the carbon policy and the equilibrium outcomes in Section 3. In Section 4 we characterize the inefficiencies arising from the political process and financial decisions. In Section 5 we introduce political risks and demonstrate how such risks get priced into equity premia. In Section 6 we extend the model to a two-country model and highlight a new source of inefficiency due to international capital flows. We discuss additional extensions in Section 7 and conclude in Section 8.

2 Single-Country Market Equilibrium

This section describes the market equilibrium within a single-country, production economy for a given carbon tax policy. We first describe the market environment, then characterize general equilibrium portfolio allocations, production, and pollution, and finish with comparative statics of the equilibrium outcome with respect to carbon tax policies. In the next section, we model how different groups bargain over carbon tax policy to shape the market equilibrium.

2.1 Environment

The economy lasts for two periods, $t \in \{0, 1\}$. The economy is populated by a unit mass of investors and a collection of workers who provide labor for production. There is one consumption good and one carbon emission bad.

Preferences: All investors are endowed with wealth of y_{ij} goods at $t = 0$. There are two types of investors of equal size, $i \in I := \{g, b\}$, where g refers to "green" and b refers to "brown". Type i investors rank outcomes at $t = 1$ according to:

$$u_i(c_i; E) = -\exp(-\gamma c_i + \eta_i(E))$$

where c_i is their consumption at $t = 1$, $\eta_i(\cdot)$ is their disutility from carbon emissions, and E is aggregate carbon emissions. We assume $\eta_g(\cdot) > \eta_b(\cdot)$ so "green" investors are more concerned about carbon emissions than "brown" investors.

Production: There are two production sectors, green and brown ($k \in g, b$). Each sector k has a representative firm with a technology that inputs capital S_k and labor l_k at $t = 0$ to produce goods f_k and emissions h_k at $t = 1$ according to the functions:

$$f_k(S_k, l_k) = A_k S_k^{\alpha} l_k^{1-\alpha}, \quad h_k(S_k, l_k) = P_k S_k^{\alpha} l_k^{1-\alpha}$$

where productivity is stochastic with distribution $A_k \sim \mathcal{N}(\bar{A}_k, \sigma_k^2)$. Aggregate pollution $E = \sum_k h_k(S_k, l_k)$.

The Labor Market: Labor is supplied by a collection of workers who incur a constant marginal disutility w_k from supplying labor to sector k.[1] Workers do not receive

[1] In our baseline model, workers do not make any strategic decisions or participate in the political

endowments or own assets and consume their wage income for linear utility. As a result, labor supply is perfectly elastic; firms can hire any quantity of labor they demand at this prevailing wage.

Assets and Financial Markets: There are three financial assets in the economy: firm equity in each sector and a risk-free bond in zero net supply. All assets are traded competitively. We use goods as the numeraire. We denote the risk free rate in the bond market by r_f. We denote the price and return on equity in sector k by q_k and $r_k := \pi_k/q_k$, where π_k is the firm dividend. We guess and later verify that the return on firm equity in sector k follows the distribution:

$$r = (r_g, r_b)' \sim \mathcal{N}(\bar{r}, \Sigma_r)$$

for mean $\bar{r}$ and covariance Σ_r to be determined in equilibrium.

Environmental Policy: A government taxes domestic emissions at rate τ and redistributes tax revenue, τE, equally to every domestic household. We will later endogenize the tax policy τ by introducing a political economy process.

Timing: In period 0, environmental policy is decided, then households investors decide on their asset portfolios across the two sectors and the risk-free bond. In period 1, the firms produce goods and pollution with the received investments, and the investment dividend is paid to households; carbon emission tax is collected and redistributed, and households consume all assets.

Discussion of Environmental Externality: The fundamental environmental problem in this economy is that some households care about aggregate carbon emissions, E, but no households believe they personally impact aggregate emissions. This means that agents need to coordinate to reduce pollution. Ultimately, coordination is complicated by the heterogeneity across the population in how much they care about reducing emissions.

bargaining because they are always indifferent between working and not working which gives them zero value. In section 7.1, we discuss extensions where workers have positive value functions and so play a role in the political bargaining.

2.2 Agent Problems

Investor problem: Given an outcome of the political process, prices, and emission outcome, a type i investor chooses a portfolio $\theta_i = (\theta_{ig}, \theta_{ib})'$ to solve problem (2.1):

$$\begin{aligned} V(y_i; \tau, r, r_f, E) := \max_{\theta_i} \quad & \mathbb{E}\left[-\exp(-\gamma c_i + \eta_i(E))\right] \quad \text{s.t.} \\ & c_i = [r_f + (r - r_f \mathbf{1}_2)'\theta_i] y_i + \frac{1}{2}\tau E \end{aligned} \tag{2.1}$$

where $r = (r_g, r_b)'$ is the vector of returns, and $\mathbf{1}_2$ is the 2-dimensional vector of ones. The term $(r - r_f \mathbf{1}_2)$ is the vector of excess returns.

Firm problem: Each firm in sector k of country j takes its capital S_{kj}, the wage w_{kj}, and the tax τ_j as given, and chooses labor l_k to maximize expected payout to equity holders:

$$\max_{l_k} \quad \mathbb{E}[f_k(S_k, l_k) - w_k l_k - \tau h_k(S_k, l_k)] \tag{2.2}$$

2.3 Equilibrium Definition and Characterization

Given a policy τ, a competitive market equilibrium in a collection of prices, (r_f, r), portfolio choices, $(\theta_i)_{i \in I}$, capital allocation (S_g, S_b), and labor choices, $(l_k)_k$, such that (i) given prices, households solve problem (2.1) and firms solve problem (2.2), (ii) market clears in goods, labor, and financial markets:

$$\left(1 - \sum_k \bar{\theta}_k\right) \bar{y} = 0, \qquad S = \bar{\theta}\bar{y}, \qquad \sum_k S_k = \bar{y},$$
$$w_k = \frac{\partial f_k(S_k, l_k)}{\partial l_k} \ \forall k, \quad \sum_i c_i = \sum_k \left[f_k(S_k, l_k) - w_k l_k\right]$$

where $\bar{y} := \int_i y_i di$ is the average endowment of households, $\bar{\theta}_k = (\int_i \theta_{ik} y_i di)/(\int_i y_i di)$ is the market portfolio. The competitive equilibrium is characterized in Theorem 1.

Theorem 1. *Given tax rate τ, the competitive equilibrium satisfies* [2]

$$\bar{r}_k = \frac{(\bar{A}_k - \tau P_k)^2}{4w_k}, \qquad \mathrm{Var}(r_k) = \mathrm{Var}(A_k)\big(\tfrac{\bar{A}_k - \tau P_k}{2w_k}\big)^2 \quad \forall k$$
$$r_f = \frac{\mathbf{1}'\Sigma_r^{-1}\mathbf{r} - \gamma\,\bar{y}}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}}, \qquad \textit{where} \quad \Sigma_r = D\Sigma_A D, \quad D = diag((\frac{\bar{A}_k - \tau\, P_k}{2\, w_k})_k)$$

Individual portfolio choice θ_i and market portfolio $\bar{\theta}$ satisfy

$$\theta_i = \frac{1}{\gamma y_i}\Sigma_r^{-1}(\bar{r} - r_f\vec{1}), \qquad \bar{\theta} = \frac{1}{\gamma\bar{y}}\Sigma_r^{-1}(\bar{r} - r_f\vec{1}).$$

Labor and capital allocations satisfy:

$$l_k = \Big(\tfrac{\bar{A}_k - \tau_j P_k}{2w_k}\Big)^2 S_k \quad \textit{where} \quad S_k = \bar{y}\bar{\theta}_k.$$

See Appendix A for proof.

Remark 1. (Financial vs. Political Channel.) A central result of the market equilibrium characterization in Theorem 1 is that for a given tax policy τ, all investors, "green" and "brown", hold the exact same portfolio of risky assets. Their portfolio weights θ_i differ only in proportion to their wealth y_i, not their environmental preferences η_i. In other words, financial markets alone provide no channel for agents to express their heterogeneous preferences over the pollution externality. Because each investor is a price-taker and cannot individually influence aggregate emissions E, their investment decision reduces to a pure mean-variance optimization problem. The only avenue for investors to enact their environmental preferences is the political channel. This motivates our focus on the political economy of environmental policy.

We now characterize how the market equilibrium responds to changes in the carbon tax. We do this by differentiating the equilibrium quantities in Theorem 1 with respect to the tax rate τ under the assumption that the shocks are independent. Corollary 1 formalizes these comparative statics.

Corollary 1. *An increase in the carbon tax τ in the single-country economy changes the market equilibrium as follows:*

[2] Here we assume policy τ is not too large–specifically, $\bar{A}_k - \tau P_k > 0$ for both sectors. This is because the FOC for firm labor choice can only be true when this is the case, given a strictly positive w_{kj}. When carbon tax is so high such that $\bar{A}_{kj} - \tau_j P_{kj} \leq 0$, the FOC can never be satisfied and thus the firm will shut down.

1. ***Asset Returns****: The expected return on equity ($\bar{r}_g, \bar{r}_b$) decreases in both the green and brown sectors, with the decline more pronounced for the brown sector. The risk-free rate (r_f) also decreases.*

2. ***Capital Allocation****: Capital is reallocated from the brown sector to the green sector. The total stock of capital invested in the brown sector, S_b, decreases, while the stock of capital in the green sector, S_g, increases.*

3. ***Environmental Outcomes****: Aggregate pollution, E, unambiguously decreases.*

4. ***Household Welfare****: The effect on household welfare is ambiguous for both green and brown investors.*

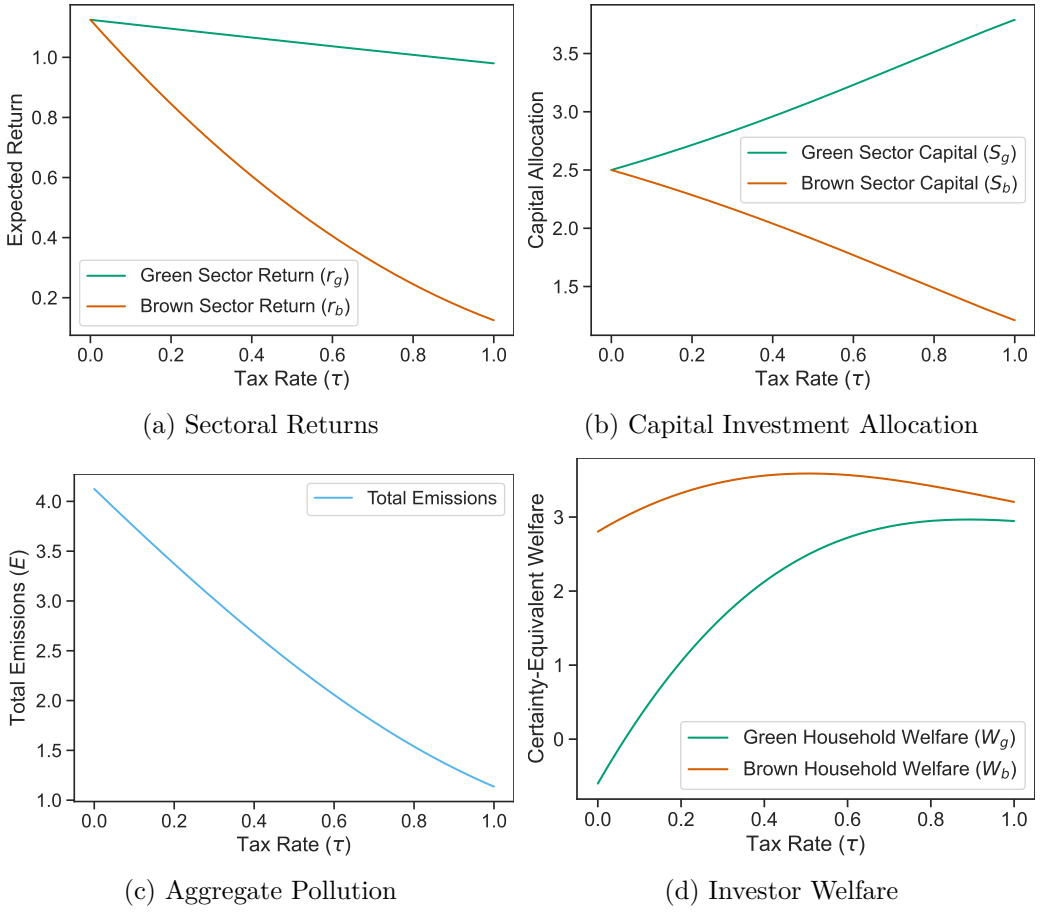
Proof. See Appendix A for the details. □

Discussion We illustrate the comparative statics from Proposition 1 with a numerical example in Figure 1. Both the Figure and the Proposition demonstrate that a carbon tax is an effective, albeit blunt, tool in a closed economy: increasing the carbon tax unambiguously reduces total emissions. This is due to two market mechanisms: it makes both sectors less productive and thus less pollution-intensive on the margin by cutting labor input per unit of capital; it also increases the relative return on green equity leading to an investment shift from "dirty" to "cleaner" sectors. However, the environmental benefit comes at the cost of lower consumption making the welfare effect ambiguous for both green and brown investors. Importantly, the welfare optimizing carbon tax is higher green households than brown households, which set the stage for the political bargaining problem we analyze in the next section.

3 Domestic Political Bargaining

The preceding analysis characterized the market equilibrium for a *given* carbon tax τ. Section 2 established that all investors hold identical risky portfolios, regardless of their environmental preferences. This finding motivates the shift in this section: if investors cannot express their preferences in the market, they must do so in the political process.

We thus endogenize the carbon tax τ by modeling its determination as the outcome of a political process. We model policy formation as a Nash bargaining game



(a) Sectoral Returns

(b) Capital Investment Allocation

(c) Aggregate Pollution

(d) Investor Welfare

Figure 1: Comparative Statics With Respect to Tax Policy

between the two groups of capital owners[3] in the economy: the "green" and "brown" investors, who differ in their preferences over environmental quality. This approach captures the trade-off between the collective benefit of environmental quality and the private costs of economic distortions, and allows us to study the equilibrium inefficiencies that arise when policy is not set by a benevolent planner but emerges from the conflicting interests of economic agents.

3.1 Setup: Nash Bargaining

We model the policy-setting process as a Nash bargaining game between the "green" and "brown" investor blocs. The key departure from the individual investor's problem is the objective function used in the negotiation. When bargaining, each group's representative internalizes the full general equilibrium impact of the policy on market outcomes.

The objective function of bloc $i \in \{g, b\}$ is its coordinated value function:

$$W_i(\tau) = W(y_i, \tau) := V(y_i; \tau, r(\tau), r_f(\tau), E(\tau))$$

where the individual value function V is as defined in equation (2.1). The main difference between the coordinated value function W and its individual counterpart V is that while individual value function takes market equilibrium outcomes r_f, r, and E as given, the bargaining bloc understands that its choice of τ directly shapes these equilibrium outcomes.

The disagreement point, or status quo outcome, for the negotiation is the "state of nature" outcome where no climate policy is enacted, i.e., $\tau = 0$. The disagreement utility for party i is therefore $W_i^0 \equiv W_i(0)$.

Assuming the green group has bargaining power $\beta \in [0, 1]$ and the brown group $1 - \beta$, the politically determined tax rate τ^* solves the Nash bargaining problem:

$$\begin{aligned} \max_{\tau \geq 0} \quad & [W_g(\tau) - W_g(0)]^{\beta} \times [W_b(\tau) - W_b(0)]^{1-\beta} \\ \text{subject to} \quad & W_i(\tau) \geq W_i(0) \quad \text{for } i \in g, b. \qquad \text{(Participation Constraint)} \end{aligned}$$

[3]This approach allows us to isolate the political-economic conflict that arises purely from the investor channel. We abstract from the political influence of labor by assuming a perfectly elastic supply at a fixed wage for each sector. This assumption renders laborers indifferent to the policy choices and focuses the analysis squarely on the preferences of capital owners.

3.2 Ideal Policies and Bargaining Outcome

To understand the political compromise, we first consider each group's ideal policy. The preferred tax rate for group i, denoted $\tau_i^* := \arg\max_{\tau \geq 0} W_i(\tau)$, is the one that maximizes its own welfare. This preference is shaped by three competing effects of the tax:

1. **Financial Portfolio Effect:** The tax distorts asset returns, changing both expected returns $\bar{r}$ and the variance-covariance matrix Σ_r. This alters households' financial wealth and is generally a source of welfare loss.

2. **Tax Rebate Effect:** Households receive a share of the collected tax revenue, $\lambda \tau E$. The value of this rebate is non-monotonic in τ, following a "Laffer curve" as the tax base E shrinks with higher taxes.

3. **Environmental Effect:** The tax reduces aggregate pollution E, which provides a utility gain through the term $\eta_i(E)$.

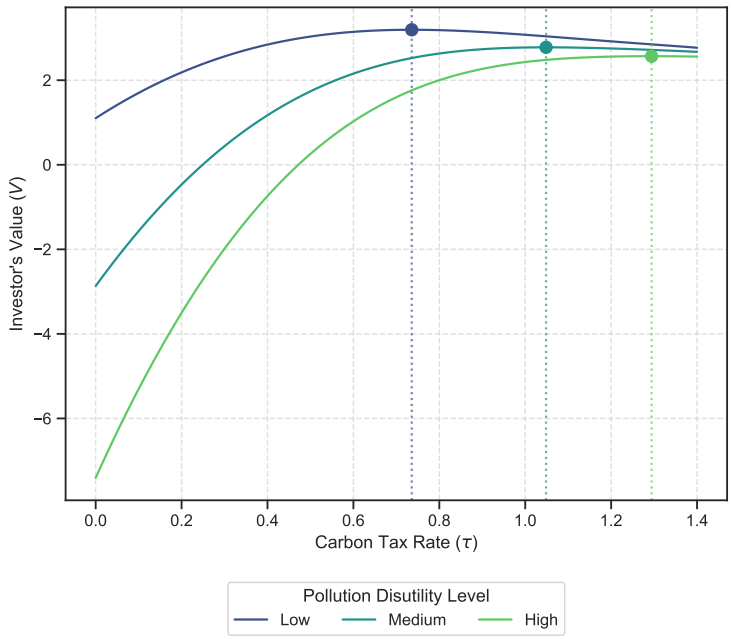
The fundamental political conflict arises because households weigh the third term differently. Since green households have a higher marginal disutility from pollution, they are willing to tolerate greater financial distortions to achieve a cleaner environment. Consequently, their ideal tax rate will be strictly higher than that of brown households.

Numerical Result 1 (Ordering of Preferred Policies): Assume welfare functions $W_i(\tau)$ are strictly quasi-concave in τ. Given that green investors have a strictly greater marginal disutility from pollution than brown investors (i.e., $\frac{\partial \eta_g(E)}{\partial E} > \frac{\partial \eta_b(E)}{\partial E} > 0$ for all aggregate emission levels $E > 0$), their ideal tax rates are strictly ordered:

$$\tau_g^* > \tau_b^* \geq 0$$

This ordering arises because, at the brown investor's ideal tax rate τ_b^*, the marginal financial cost of the tax is exactly balanced by the marginal environmental benefit for the brown investor. For the green investor, who values the environmental benefit more highly, the marginal benefit of reducing emissions further still outweighs the financial cost at this point. Therefore, the green investor's welfare is still increasing, and their optimal tax rate must be higher.

Figure 2: Investor's Value and Ideal Policies under Different Pollution Preferences



The bargained tax rate τ^* will therefore lie between these two ideal points, its exact position determined by the relative bargaining power and the curvatures of the welfare functions. This political process, while internally efficient for the bargainers, negotiates over a single, blunt instrument, thus generates inefficiency. We explore the consequence of such constraint on efficiency in the next section.

4 (In)efficiency of the Decentralized Equilibrium

We now integrate the market equilibrium of Section 2 with the political bargaining model of Section 3 to analyze the overall outcome. While each of the asset pricing model and the Nash bargaining framework are efficient in isolation, their interaction in a decentralized setting is not. The resulting equilibrium is generally Pareto inefficient, a conclusion that challenges the folk wisdom that a well-designed carbon tax is a sufficient instrument for addressing climate externalities.

We establish this by first defining the Pareto frontier through a social planner's problem and then demonstrating that the decentralized equilibrium lies strictly inside this frontier. The inefficiency stems from two distinct but interacting frictions: (1) a conventional tax-induced distortion in the real economy, and (2) a more subtle

friction due to incomplete contracting in the political sphere, which prevents the economy from optimally allocating capital.

4.1 Planner's problem

To define Pareto efficiency, we consider the allocation chosen by a benevolent social planner who can directly control the economy's resources and distribute the resulting output. The planner is not constrained by decentralized markets or political bargaining. Their instruments are the direct allocation of capital to each sector, (S_b, S_g) and the allocation of final consumption to each household type, (c_b, c_g).

The planner's objective is to maximize a weighted sum of the households' utilities. Given the CARA-Normal framework, this is equivalent to maximizing a weighted sum of their certainty-equivalent wealth. Let the planner's weights for the green and brown groups be ω_g and ω_b respectively, with $\omega_g > 0, \omega_b > 0$, and $w_g + w_b = 1$.

The planner's problem is:

$$\max_{\{S_k, c_i(\cdot)\}_{k,i \in g,b}} \quad \omega_g W_g + \omega_b W_b \quad \text{subject to:}$$

$$(i) \quad S_g + S_b = \bar{y} \quad \text{(Capital Constraint)}$$

$$(ii) \quad \frac{1}{2}(c_g(A) + c_b(A)) \leq \sum_{k \in g,b} (f_k(S_k, l_k; A_k) - w_k l_k) \; \forall A \quad \text{(Goods Constraint)}$$

where $W_i = \mathbb{E}[-\exp(-\gamma c_i + \eta_i(E))]$ is the value of type i.

A key decision for the planner is the choice of labor l_k. Since the planner's objective is to maximize the total resources available, they are concerned with the physical productivity of labor, not with tax payments that are mere internal transfers. The planner thus chooses labor to maximize the expected net output $\mathbb{E}[f_k(S_k, l_k) - w_k l_k]$. The first-order condition yields the socially optimal labor allocation:

$$l_k^{\text{planner}}(S_k) = \left(\frac{\bar{A}_k}{2w_k}\right)^2 S_k \tag{4.1}$$

The set of solutions to this problem for all possible Pareto weights (ω_g, ω_b) traces out the economy's Pareto frontier.

Remark 2. (Planner's Allocation as a Complete Contract) Note that this same Pareto-optimal outcome could also be achieved without a central planner if the

green and brown investor blocs could collude and write a complete, binding contract. Such a contract would need to specify two things: (1) a direct, joint commitment to a specific capital allocation (S_g, S_b), and (2) a state-contingent rule for allocating the final consumption goods between the two groups. This arrangement would effectively bypass both the price-taking behavior in financial markets and the limitations of bargaining over a single tax rate. The inefficiency of the decentralized equilibrium, therefore, arises precisely because such complete contracts are unavailable. The political process is constrained to bargaining over only a tax rate and represents a form of incomplete contracting that prevents agents from reaching the Pareto frontier.

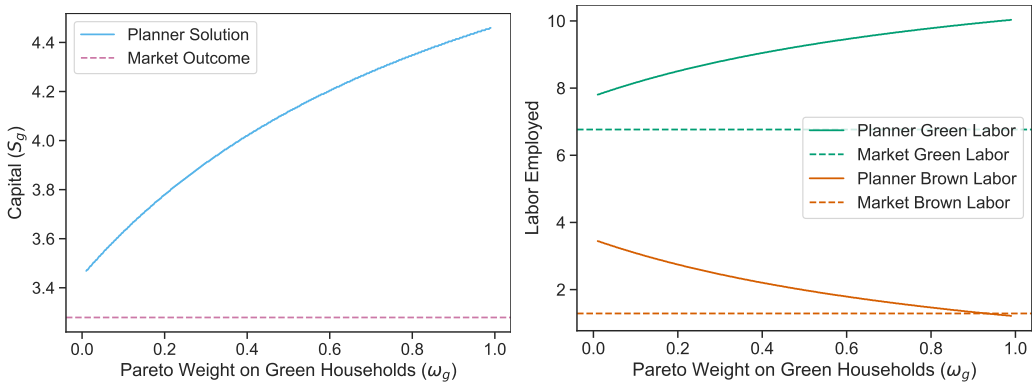
4.2 Solution and efficiency

Proposition 1 (Inefficiency of the Decentralized Equilibrium)**.** *The equilibrium resulting from the sequence of (1) Nash bargaining over the tax rate τ and (2) subsequent competitive market allocation of capital is generically Pareto inefficient.*

The formal proof of this proposition only requires demonstrating a single counterexample—an equilibrium that is Pareto-dominated. We provide such an example below. However, the logic behind this inefficiency is general. It stems from a mismatch between the complexity of the social problem and the simplicity of the tool used to solve it.

The social planner can navigate a two-dimensional choice space, independently trading off capital allocation (S_g vs. S_b) and distributional outcomes (via the Pareto weights (ω_g, ω_b). The decentralized political process, by contrast, collapses this problem into a single dimension. Bargaining over the tax rate τ creates a one-dimensional path through the space of possible outcomes; capital allocation, production, and consumption are all locked into a specific trajectory dictated by τ. It is therefore mathematically impossible for this one-dimensional path to coincide with the two-dimensional Pareto frontier, except by coincidence. Below, using a calibrated example (parameters in Appendix B), we illustrate the frictions to explain why and how the decentralized path diverges from efficiency.

Friction 1: Tax-Induced Production Distortion The first friction is a standard result from public finance. A carbon tax, while correctly pricing the pollution externality from a firm's perspective, simultaneously distorts the labor market. A



(a) Capital Investment in Green Sector

(b) Labor Employed

Figure 3: Capital and Labor With Respect to Pareto Weight

profit-maximizing firm facing a tax τ perceives its marginal revenue product of labor to be lower than its physical productivity. From Theorem 1, its labor demand is:

$$l_k^{\text{market}}(\tau, S_k) = \left(\frac{\bar{A}k - \tau Pk}{2w_k}\right)^2 S_k$$

Comparing this to the planner's optimal labor choice in (4.1), it is immediate that for any $\tau > 0$, $l_k^{\text{market}} < l_k^{\text{planner}}$. The tax drives a wedge between the private and social return to labor, leading to under-employment and rendering production inefficient for any given capital stock.

Figures 3a and 3b illustrate this distortion. Observe from Figure 3a that for any Pareto weight, the planner always invests more capital in the green sector than in the decentralized equilibrium. Figure 3b demonstrates that the decentralized market equilibrium employs strictly less labor in both the green and brown sectors than most planner solutions. This production inefficiency means the decentralized economy generates a smaller total consumption pie than the production frontier.

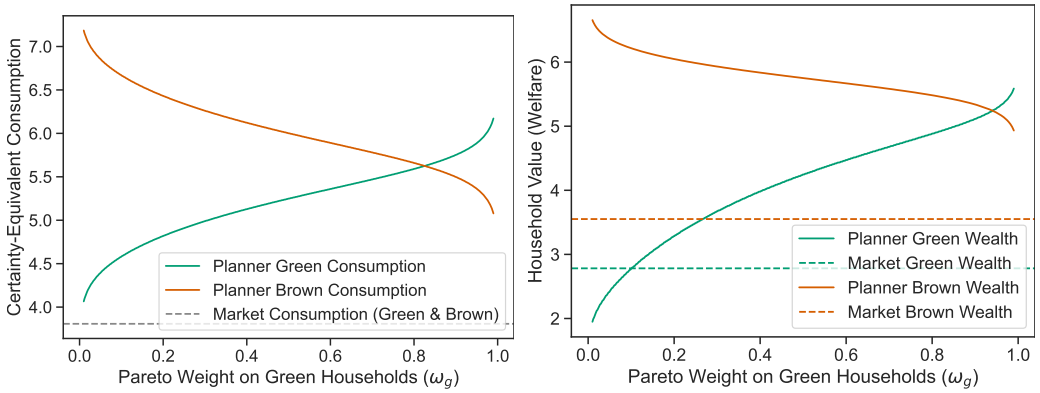
Friction 2: Incomplete Contracts and Blunt Policy Instruments A second friction emerges from the political process itself. Even if the production distortion could be eliminated, the equilibrium would remain inefficient because the political bargain is over an incomplete set of contracts. Households as political actors bargain only over the tax rate τ; they cannot write binding contracts on their future portfolio choices.

This turns the tax into a blunt instrument. It must simultaneously price the pollution externality and determine capital investment; But a single instrument cannot optimally perform both tasks. The optimal capital allocation from the planner's perspective directly balances risk-return trade-offs against the weighted social cost of emissions. In the decentralized equilibrium, capital is allocated by atomistic investors who, conditional on τ, care only about the financial risk-return profile of assets. The political bargain over τ only indirectly influence this allocation, leading to a suboptimal outcome.

This friction manifests as an under-investment in green capital relative to the efficient frontier. Figure 3a shows that for any Pareto weight, the planner allocates significantly more capital to the green sector than the decentralized political bargain. Even the most "brown-friendly" planner (with $\omega_g \to 0$) chooses a higher green-sector allocation. This is because the planner can directly implement the efficient capital stock and use lump-sum transfers to compensate the brown group. The political bargain, however, must "pay" for green investment with a higher tax, which imposes a direct distortionary cost on all firms and is thus heavily contested by the brown bloc, resulting in a compromise with less ambitious environmental investment.

Remark 3. (Incompleteness and the Lack of Transfers) One reason the political contract is incomplete is the absence of a lump-sum transfer mechanism between the investor blocs. In our model, the bargainers can only agree on a tax rate, τ; They cannot agree on a side payment from the green group to the brown group to compensate them for the financial costs of a higher tax. If such transfers were possible, the groups could agree to implement the planner's optimal capital allocation (and the tax required to support it), and then bargain over how to split the maximized surplus. This would decouple the allocative decision from the distributional one. Without transfers, the distributional conflict is fought entirely through the choice of the tax rate, a single, distortionary instrument, which inevitably leads to an inefficient compromise on the allocation of capital. In this sense, it is the absence of lump sum transfers that makes finance relevant to environmental bargaining.

Welfare Consequences These two frictions together generates welfare loss. The production distortion shrinks the total size of the economic pie for any given capital allocation (due to inefficient labor employment), while the incomplete contracting friction leads to inefficient allocation between green and brown production. Figure 4a shows the result: the realized value for both investor types is strictly dominated

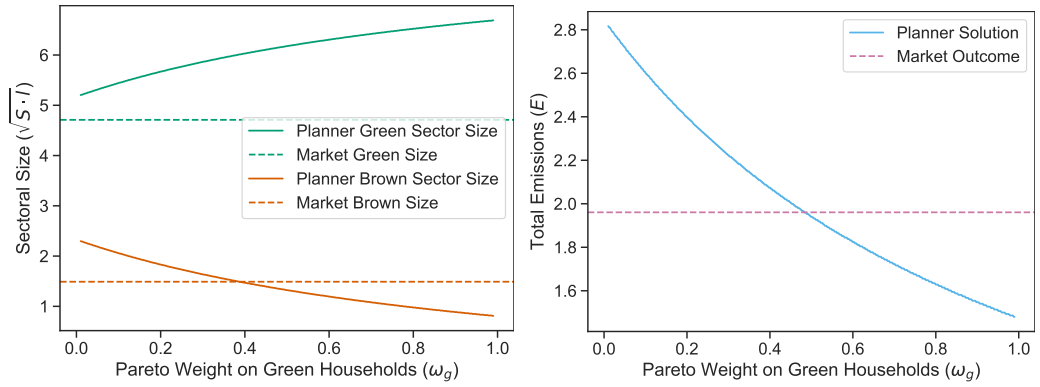


(a) Consumption By Household Type (b) Welfare of Green and Brown Investors

Figure 4: Consumption and Welfare With Respect to Pareto Weight

by the set of possible planner allocations. The decentralized equilibrium leaves "money on the table."

Figure 4b illustrates the welfare comparison. By plotting the welfare (including disutility from pollution) for all possible planner solutions, it traces out the Pareto frontier. The flat lines representing the decentralized equilibrium lies strictly below this frontier, confirming its inefficiency. The inability to write complete contracts over both policy and investment decisions constitutes a fundamental barrier to achieving an efficient environmental outcome.



(a) Sectoral Sizes (b) Emission Levels

Figure 5: Sectoral Sizes and Emission With Respect to Pareto Weight

Combined Effects on Production and Pollution The interplay between these two frictions—inefficiently low labor from the tax and inefficiently low green capital from incomplete contracts – together shapes the physical structure of the economy.

Given that the planner allocates more capital to the green sector (Friction 2) and employs labor more efficiently in it (Friction 1), the planner's green sector size, measured by $\sqrt{S_g l_g}$, is unambiguously larger than in the decentralized equilibrium across all Pareto weights. The more surprising result, shown in Figure 5a, is that a planner with a high weight on brown households may allocate less capital to the brown sector than the decentralized bargain, yet end up with a larger brown sector in terms of total output. This is a result of the production distortion: the planner, by using labor efficiently, can sustain a larger, more productive brown sector even with less capital.

This finding provides a clear mechanism for the results on aggregate pollution, shown in Figure 5b. Because a brown-leaning planner can support a larger brown sector, it is entirely possible for an efficient allocation to generate more pollution than the inefficient decentralized outcome. For the level of pollution it generates, the decentralized equilibrium delivers strictly less consumption and welfare than a planner could achieve. That is, the markets cannot simultaneously achieve efficient emission, consumption, and welfare.

5 Political Risk and Asset Prices

Our analysis so far has assumed that the political bargain yields a deterministic policy outcome, τ^*, which is known to all agents. In reality, policy processes are fraught with uncertainty. In this section, we extend the baseline model to introduce political risk into the economy. This allows us to study how uncertainty over future climate policy affects equilibrium asset prices and generates additional risk premia.

To allow this new aggregate political risk factor to potentially impact equilibrium returns in the economy, we make a second modification to the model: we introduce decreasing returns to scale in production. In the baseline constant-returns-to-scale (CRS) setting, the supply of returns on real investment is perfectly elastic at a level determined by technology. Consequently, any aggregate risk would be absorbed entirely through adjustments in the scale of investment (S_k), leaving expected returns unchanged. By introducing decreasing returns to scale (DRS), we create a downward-sloping demand for capital, ensuring that shifts in risk appetite or risk

exposure are reflected in equilibrium prices.

5.1 Revised Environment

We adapt the single-country model from Section 2 with two key changes.

Decreasing Returns to Scale: We introduce DRS by assuming that firms incur a quadratic cost related to their scale of investment, which can be interpreted as a span-of-control or capital adjustment cost. Specifically, the profit of a firm in sector k is now given by:

$$\pi_k = f_k(S_k, l_k) - w_k l_k - \tilde{\tau} h_k(S_k, l_k) - \frac{\phi_k}{2} S_k^2$$

where $\phi_k > 0$ is the cost parameter.

Political Risk: We model policy uncertainty by assuming the realized carbon tax, $\tilde{\tau}$, is a random variable. It is determined by the bargained outcome, τ^*, plus a normally distributed shock, ϵ_τ:

$$\tilde{\tau} = \tau^* + \epsilon_\tau, \quad \text{where} \quad \epsilon_\tau \sim \mathcal{N}(0, \sigma_\tau^2)$$

We assume the policy shock ϵ_τ is orthogonal to the productivity shocks A_k. This shock represents implementation uncertainty, last-minute political shifts, or electoral surprises that cause the final policy to deviate from the ex-ante negotiated compromise. This introduces a new, non-diversifiable source of aggregate risk into the economy.

5.2 Market Equilibrium with Political Risk

The introduction of DRS and political risk modifies the equilibrium characterization. The firm's problem for labor choice remains the same, as the quadratic cost is on capital and the firm optimizes labor based on the expected tax rate. However, the properties of asset returns and the resulting market clearing conditions change. The new equilibrium is characterized in Theorem 2.

Theorem 2 (Market Equilibrium with Political Risk)**.** *Given an expected tax rate $\tau^* = \mathbb{E}[\tilde{\tau}]$ and political risk variance σ_τ^2, the competitive equilibrium with DRS and political risk is characterized by a risk-free rate r_f and a market portfolio $\bar{\theta}$ that*

solve:

$$\bar{\theta} = \left(\gamma \bar{y} \Sigma_r + \frac{\bar{y}}{2}\Phi\right)^{-1} (\bar{R} - r_f \mathbf{1}) \tag{5.1}$$

$$r_f = \frac{\mathbf{1}' \left(\gamma \bar{y} \Sigma_r + \frac{\bar{y}}{2}\Phi\right)^{-1} \bar{R} - 1}{\mathbf{1}' \left(\gamma \bar{y} \Sigma_r + \frac{\bar{y}}{2}\Phi\right)^{-1} \mathbf{1}} \tag{5.2}$$

where:

- $\bar{R}$ *is the vector of expected returns net DRS costs, with elements* $\bar{R}_k = \frac{(\bar{A}_k - \tau^* P_k)^2}{4w_k}$.
- $\Phi = \text{diag}(\phi_k)$ *is the diagonal matrix of DRS cost parameters.*
- Σ_r *is the return variance-covariance matrix, with elements:*

$$\begin{aligned} \text{Var}(r_k) &= D_k^2 \sigma_{A_k}^2 + (P_k D_k)^2 \sigma_\tau^2 \\ \text{Cov}(r_g, r_b) &= (P_g D_g)(P_b D_b) \sigma_\tau^2 \end{aligned}$$

  where $D_k = \frac{\bar{A}_k - \tau^* P_k}{2w_k}$.

The equilibrium expected return on equity k *is* $\bar{r}_k = \bar{R}_k - \frac{\phi_k}{2} S_k$*, where the capital allocation is* $S_k = \bar{y}\bar{\theta}_k$.

Proof. See Appendix A. □

The key distinction from the baseline model (Theorem 1) is that equilibrium expected returns $\bar{r}$ are now a function of the capital allocations $S_k = \bar{y}\bar{\theta}_k$ for all k.

5.3 The Political Risk Premium

The presence of political risk creates a new priced factor. Since the carbon tax has a larger impact on the profits of the more pollution-intensive brown sector ($P_b > P_g$), investors demand a higher expected return to hold brown-sector equity as compensation for bearing this political risk. We formalize this as a "political risk premium".

Proposition 2 (Political Risk and Premia)**.** *Holding expected tax rate* τ^* *constant, an increase in political risk (an increase in* σ_τ^2*) has the following effects on the market equilibrium:*

1. ***Risk-Free Rate:*** *The equilibrium risk-free rate,* r_f*, decreases.*

2. ***Capital Reallocation:*** *Capital is reallocated away from the brown sector. The capital stock S_b decreases, while the ratio of green-to-brown capital, S_g/S_b, increases.*

3. ***The Brown Premium:*** *The expected return on brown-sector equity, $\bar{r}_b$, and its risk premium over the risk-free asset, $\bar{r}_b - r_f$, both increase.*

Proof. See Appendix A. □

Remark 4. (Policy Instruments and Risk Premia.) Under our framework, the risk premium—whether "brown" or "green"—depends on the policy instrument creating the uncertainty. The policy in our baseline model is a carbon tax that punishes polluters; Uncertainty in this regime makes brown assets risky, generating a brown premium, a channel consistent with the findings of Bolton and Kacperczyk (2021) and Hsu et al. (2023). If, instead, climate policy were implemented via a green subsidy (e.g., a per-unit subsidy on green output), policy uncertainty would be concentrated in the green sector. In that case, our model would then predict a *green premium*, as investors would demand compensation for bearing the risk of fluctuating green subsidies. The model thus provides a unified mechanism to rationalize these premia and to understand how the design of environmental policy shapes the pricing of climate-related risks.

6 Two-country Equilibrium

We now extend the model to a two-country framework to analyze the interplay between international capital flows and domestic climate policy. The core structure of the economy remains the same, but we introduce two countries, $j \in \{1, 2\}$, which interact through an integrated global financial market.

6.1 Model Modifications

The key changes involve adding a country dimension to households and firms, introducing country-specific policies, and allowing for correlation in productivity shocks across all sectors and countries.

6.1.1 Market setup

Households and Firms: The unit mass of households is now split equally between the two countries. All variables pertaining to households and firms are now indexed by country j in addition to their type. For instance, a type i household in country j has preferences given by $u_{ij}(c_{ij}) = \mathbb{E}\left[-\exp\left(-\gamma c_{ij} + \eta_{ij}(E)\right)\right]$ and endowment y_{ij}. Each country has a green and a brown sector, leading to four distinct sector-country firms, indexed by (k, j).

Production and Technology: Production and pollution functions for a firm in sector k and country j are:

$$f_{kj}(S_{kj}, l_{kj}) = A_{kj} S_{kj}^{\alpha} l_{kj}^{1-\alpha}, \quad h_{kj}(S_{kj}, l_{kj}) = P_{kj} S_{kj}^{\alpha} l_{kj}^{1-\alpha}$$

We allow productivity shocks to correlate across all four firms by assuming the vector of productivity $A = (A_{g1}, A_{b1}, A_{g2}, A_{b2}) \sim \mathcal{N}(\bar{A}, \Sigma_A)$, follows a multivariate normal distribution. Aggregate pollution is the sum over all sources, $E = \sum_{k,j} h_{kj}(S_{kj}, l_{kj})$; pollution produced in country j is $E_j = \sum_k h_{kj}(S_{kj}, l_{kj})$.

Financial Markets: Financial markets are fully integrated. Households in both countries can invest in four sectoral equities (one for each sector-country pair) and a single risk-free bond that is in zero net supply globally. Consequently, portfolio holdings θ_{ij} are now four-dimensional vectors.

Environmental Policy: Each country j sets its own carbon tax τ_j. Tax revenue is collected and redistributed domestically. Specifically, the government in country collects $\tau_j E_j$ and redistributes a fraction λ of this revenue equally to all households residing in country j.

6.1.2 Endogenous Policy: The Stackelberg Policy Game

We model the policy-setting process as a sequential game to capture the strategic interactions between sovereign nations linked through integrated financial markets.

The policy for each country j emerges from an internal Nash bargain between its green and brown households, as in the single-country case. However, the welfare of each domestic group, W_{ij}, now depends on the full policy vector (τ_j, τ_{-j}), as the

foreign policy affects global asset prices and total pollution.

To model the strategic interaction, we assume the countries move sequentially in a Stackelberg framework:

1. Leader's Move: Country 1 (the "Leader") chooses and commits to its carbon tax, τ_1
2. Follower's Move: Country 2 (the "Follower") observes τ_1 and then chooses its tax τ_2 as the outcome of its own domestic Nash bargaining process.

We characterize the follower's choice as a best-response function $\tau_2^*(\tau_1)$ derived from solving its internal political bargain for any given leader policy. The leader, in turn, anticipates this response and chooses τ_1 as its own domestic bargaining outcome.

6.2 Market Equilibrium in the Two-Country Model

For a given pair of tax policies (τ_1, τ_2), we solve for the global market equilibrium.

6.2.1 Agent Problems

The optimization problems for households and firms are direct extensions of the single-country case.

Household problem: A type i household in country j chooses a 4-dimensional portfolio θ_{ij} to solve:

$$\begin{aligned} \max_{\theta_{ij}} \quad & \mathbb{E}\left[-\exp(-\gamma c_{ij} + \eta_{ij}(E))\right] \\ \text{s.t.} \quad & c_{ij} = [r_f + (r - r_f \mathbf{1}_4) \cdot \theta_{ij}] y_{ij} + \lambda \tau_j E_j \end{aligned} \tag{6.1}$$

where $r = (r_{kj})_{(k,j)}$ is the vector of returns on the four equities; $\mathbf{1}_4$ is a 4-dimensional vector of ones.

Firm problem: A firm in sector k of country j chooses labor l_{kj} to maximize expected profit:

$$\max_{l_{kj}} \quad \mathbb{E}[f_{kj}(S_{kj}, l_{kj}) - w_{kj} l_{kj} - \tau_j h_{kj}(S_{kj}, l_{kj})] \tag{6.2}$$

6.2.2 Market Equilibrium Characterization

Given a policy $\tau = (\tau_1, \tau_2)$, a competitive market equilibrium in a collection of prices, (r_f, r), portfolio choices, $(\theta_i)_{i \in I}$, and labor choices, $(l_{kj})_{k,j}$, such that (i) given prices,

households solve problem (6.1) and firms solve problem (6.2), (ii) all markets clear. The market clearing conditions now aggregate across both countries. For example, the total capital invested in firm (k, j) is $S_{kj} = \bar{\theta}_{kj}\bar{y}$ where $\bar{y} := \int_{ij} y_{ij} dij$ is the average endowment of households globally, $\bar{\theta} := \frac{\int_{i,j} \theta_{ij} y_{ij} dij}{\int_{i,j} y_{i,j} dij}$ is market portfolio. The equilibrium is characterized in Theorem 3.

Theorem 3. *Given tax rates* (τ_1, τ_2),[4] *the competitive equilibrium satisfies*

$$\bar{r}_{kj} = \frac{(\bar{A}_{kj} - \tau_j P_{kj})^2}{4w_{kj}}, \qquad \mathrm{Var}(r_{kj}) = \mathrm{Var}(A_{kj})\left(\tfrac{\bar{A}_{kj} - \tau_j P_{kj}}{2w_{kj}}\right)^2 \quad \forall k, j$$

$$r_f = \frac{\mathbf{1}'\Sigma_r^{-1}\mathbf{r} - \gamma\,\bar{y}}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}}, \qquad \textit{where} \quad \Sigma_r = D\Sigma_A D, \quad D = diag\left(\left(\frac{\bar{A}_{kj} - \tau_j\, P_{kj}}{2\, w_{kj}}\right)_{kj}\right)$$

Individual portfolio choice θ_{ij} *and market portfolio* $\bar{\theta}$ *satisfy*

$$\theta_i = \frac{1}{\gamma y_{ij}}\Sigma_r^{-1}(\bar{r} - r_f\vec{1}), \qquad \bar{\theta} = \frac{1}{\gamma\bar{y}}\Sigma^{-1}(\bar{r} - r_f\vec{1}).$$

Labor employed satisfies

$$l_{kj} = \left(\tfrac{\bar{A}_{kj} - \tau_j P_{kj}}{2w_{kj}}\right)^2 S_{kj} \quad \textit{where} \quad S_{kj} = \bar{y}\bar{\theta}_{kj}.$$

Remark 5. (Internationalization and Financial Leakage) The two-country market equilibrium in Theorem 3 generalizes the single-country model in Theorem 1 and introduces a new mechanism: financial leakage.

The result on portfolio homogeneity holds across countries. All investors, regardless of nationality or environmental preference, hold the same relative portfolio of risky assets determined by the global risk-return profile $(\bar{r}, \Sigma_r)$. Non-financial factors, such as national tax rebates or pollution disutility, do not enter the portfolio optimization.

The key difference from the single-country model is international capital mobility. A unilateral policy change creates a return differential. If country 1 sets a higher tax than country 2 ($\tau_1 > \tau_2$), expected returns on its assets ($\bar{r}_{k1}$) fall relative to those in country 2 ($\bar{r}_{k2}$). The global market portfolio $\bar{\theta}$ rebalances, shifting capital from country 1 to country 2. This creates a financial leakage channel: a country's attempt

[4] As in the single-country case, this equilibrium characterization holds provided that $\bar{A}kj - \tau_j P_{kj} > 0$ for all firms (k, j). If a country's carbon tax τ_j is sufficiently high for a given sector k, that firm will shut down, employing no labor and attracting no capital.

to reduce domestic emissions is offset by capital flight and increased production and pollution abroad. This spillover complicates the domestic political bargaining process.

6.3 Comparative Statics

Proofs for results in this section are contained in Appendix A.

Corollary 2. *Consider an increase in domestic tax rate τ_j holding foreign tax rate τ_{-j} constant. This will shift the market equilibrium in the following way:*

1. ***Domestic Firm Returns and Investment:*** *The expected returns on equity for firms in the taxing country j ($\bar{r}_{gj}$ and $\bar{r}_{bj}$) decrease. Consequently, total capital invested in country j, $S_j = S_{gj} + S_{bj}$, decreases, leading to a reduction in labor employed and production. See Figures 6a and 6b.*

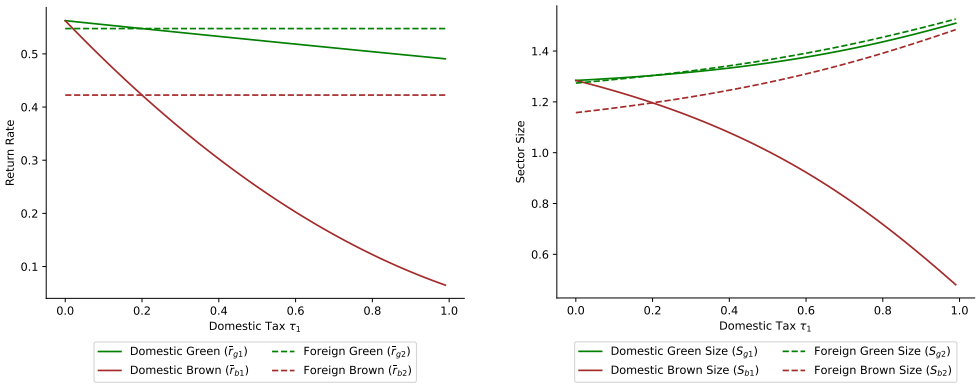
2. ***International Financial Flow:*** *As domestic assets become less attractive, capital flows from the taxing country j to the non-taxing country j'. The total capital invested in country j', $S_{j'} = S_{gj'} + S_{bj'}$, increases.*

3. ***Environmental Outcomes:*** *Pollution produced in the taxing country j, E_j, decreases. However, pollution produced in the non-taxing country j', $E_{j'}$, increases due to the inflow of capital. The net effect on aggregate global pollution, $E = E_j + E_{j'}$, is **ambiguous** due to this leakage and depends on the relative pollution intensities (P_{kj}) and productivity levels (A_{kj}) across countries. See Figures 6c and 6d.*

4. ***Household Welfare:*** *The effect on household welfare is **ambiguous** for all agents due to competing effects. Households in country j may see lower portfolio returns but benefit from potential reductions in global pollution and receive tax rebates. Households in country j' may see higher portfolio returns but suffer from any increase in global pollution without compensation.*

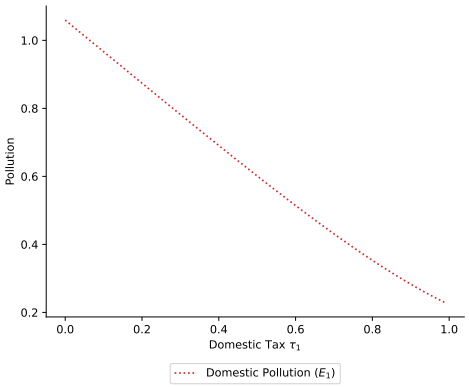
Corollary 2 introduces the key friction in the international setting: financial leakage. A unilateral tax makes domestic investment less attractive, causing capital to flow to the untaxed country. This spillover stimulates foreign production and pollution, which can undermine the policy's goals.

Beyond this direct leakage, integrated markets create a political economy friction that compounds the inefficiency.

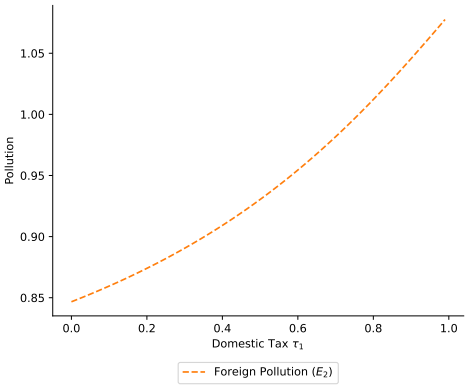


(a) Sectoral Returns

(b) Sectoral Sizes



(c) Domestic Pollution



(d) Foreign Pollution

Figure 6: Comparative Statics With Respect to Domestic Tax Policy

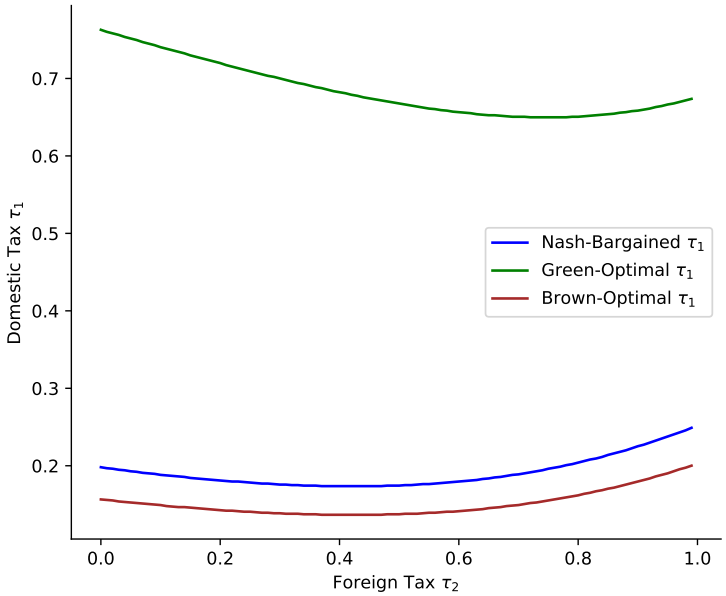


Figure 7: Ideal and Nash Bargained Policies in Follower Country

Remark 6. (International Political Friction from Sequential Commitment) Sequential policy-setting creates a strategic interdependence that changes the domestic bargain. Leader country bargainers must anticipate the follower's reaction, $\tau_2^*(\tau_1)$, so their welfare function becomes $W_{i1}(\tau_1, \tau_2^*(\tau_1))$, forcing them to internalize the strategic response of the other nation.

This introduces an additional political friction.the leader's climate policy can be actively undermined by the follower's reaction. As illustrated in Figure 7, the follower may respond to a higher tax in the leader country by lowering its own—a form of strategic substitution. Anticipating this policy neutralization, the leader's green bloc has a weaker bargaining position; the global environmental benefit of a high domestic tax is diminished, pushing the political equilibrium towards an inefficiently low tax.

Capital flight also alters the brown investors' incentives. Their opposition to a domestic tax is lower if they can evade its cost by investing abroad. This may ease domestic agreement on a tax, but at the cost of higher global pollution. The open capital channel thus causes leakage and distorts domestic politics, undermining unilateral action.

To isolate the interaction between policy and financial market, we now impose

a set of symmetry assumptions to explore more comparative static results.

Assumption 1. Symmetry in production sectors.

- **Productivity:** All sector productivities are independently and identically distributed, i.e., $A_{kj} \sim \mathcal{N}(\bar{A}, \sigma_A^2)$ for all k, j, which implies $\Sigma_A = \sigma_A^2 I$.
- **Pollution Technology:** The two green sectors have the same pollution intensity, $P_{g1} = P_{g2} = P_g$. The two brown sectors also have the same intensity, $P_{b1} = P_{b2} = P_b$. We maintain that green sectors are cleaner, so $P_g < P_b$.
- **Other Parameters:** We also assume symmetric wages, $w_{kj} = w$, and endowments, $y_{ij} = \bar{y}$.

These assumptions allow us to resolve the ambiguities regarding the change in the risk-free rate, as stated in the following corollary.

Corollary 3. *Under the symmetry assumptions, a unilateral increase in the carbon tax τ_j reduce the global risk-free rate, r_f.*

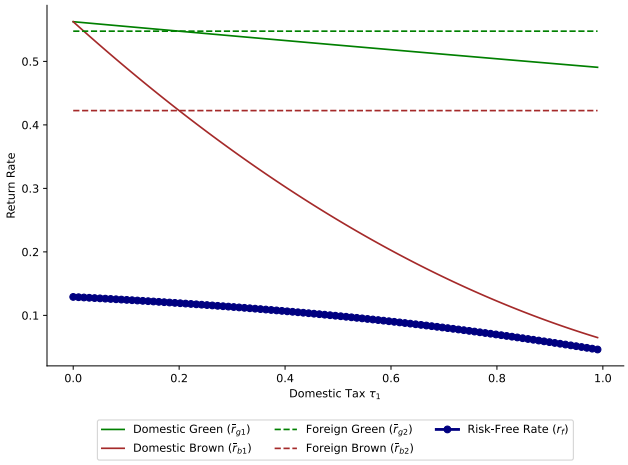


Figure 8: Sectoral Returns and Risk-Free Rate vs. Domestic Tax

We now characterize a condition under which a unilateral increase in domestic tax reduces global pollution:

Corollary 4. *Under the symmetry assumptions, a unilateral increase in the carbon tax τ_j reduces aggregate global pollution ($dE/d\tau_j < 0$) if and only if the pollution*

abatement from the domestic intensity effect outweighs the net pollution change from capital re-allocation. Formally, this condition is:

$$\underbrace{\sum_{k\in\{g,b\}} S_{kj}\frac{P_k^2}{2w}}_{\textit{Intensity Effect}} > \underbrace{\sum_{k\in\{g,b\}}\left(\frac{dS_{kj'}}{d\tau_j}\pi_{kj'} + \frac{dS_{kj}}{d\tau_j}\pi_{kj}\right)}_{\textit{Net Re-allocation Effect}}$$

where $\pi_{kj} = P_k\left(\frac{\bar{A}-\tau_j P_k}{2w}\right)$ *is the pollution generated per dollar of capital in sector* k *of country* j*.*

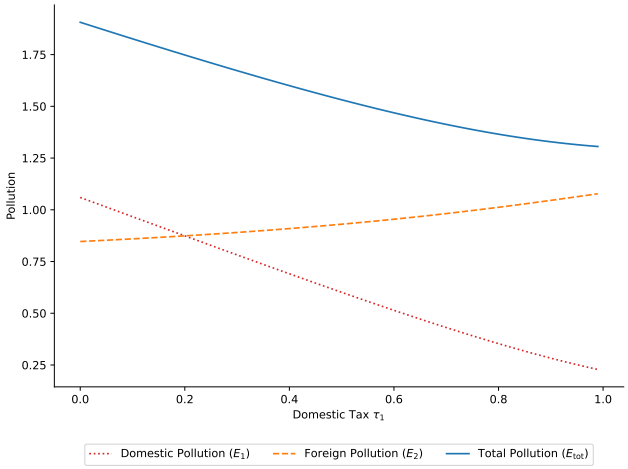


Figure 9: Global and National Pollution Levels

Finally, we characterize the welfare effects of the tax policy, which are heterogeneous across investor types and can be non-monotonic. To prepare for this, we introduce the following two assumptions:

Assumption 2. (Symmetric endowment). All households share the same level of endowment $\bar{y}$.

Assumption 3. (No benefit from taxing foreign investors). The tax friction $\lambda = \frac{1}{2}$. This means that while a country could indirectly tax foreign investors by taxing domestic firms, half of the tax revenue is lost and cannot be redistributed. Since all households hold the same portfolio and endowment is symmetric, effectively the tax income from foreign investors are lost.

Corollary 5. *Under assumptions A1-A3, the following comparative statics of welfare with respect to a unilateral increase in country* j*'s tax rate* τ_j *holds:*

1. ***Optimal Tax for Domestic Investors:*** *Assuming household utility is a concave function of the tax rate τ_j, there exists a unique optimal tax rate τ_i^* for each domestic investor type i. The most preferred tax rate for domestic green investors is strictly higher than that for domestic brown investors:*

$$\tau_g^* > \tau_b^* \geq 0$$

2. ***Zero-Tax Preference:*** *A domestic brown investor may prefer a zero tax ($\tau_b^* = 0$). Specifically, this holds when the marginal private welfare effect of introducing the tax is negative and outweighs their marginal environmental benefit at $\tau_j = 0$.*

3. ***Divergence among Foreign Investors:*** *The welfare of a foreign green investor ($W_{g,j'}$) can increase while the welfare of a foreign brown investor ($W_{b,j'}$) decreases. This occurs if the marginal certainty-equivalent cost of the tax on financial portfolios, $MFC(\tau_j)$, falls between the marginal environmental benefits for the two types.*

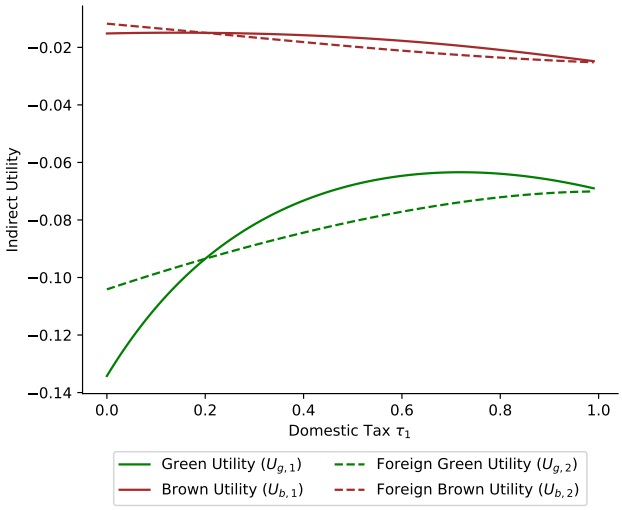


Figure 10: Utility Levels vs. Domestic Tax Policy

7 Extensions

Our framework provides a foundation for exploring several critical dimensions of the interplay between climate policy, finance, and politics. We briefly outline three promising extensions below.

7.1 The Role of Labor and Distributional Conflict

Our main analysis abstracts from the welfare of labor by assuming a perfectly elastic supply at a fixed wage, rendering laborers indifferent to the policy outcome. This simplification focuses the core conflict on capital owners with heterogeneous environmental preferences.

A straightforward and important extension is to consider the distributional consequences for labor. One could, for instance, assume the exogenous wage w_k represents a binding minimum wage that is above labor's true reservation wage (e.g., unemployment benefits). In this case, aggregate employment $L = \sum_{j,k} l_{kj}$ becomes a direct, albeit imperfect, proxy for labor's welfare. Since our model predicts that labor demand l_k is decreasing in the tax rate τ, any positive carbon tax would create an explicit conflict between environmental goals and labor interests. The policy bargain would then involve a trade-off not only between green and brown investors but also between capital owners and workers.

A richer model would endogenize wage determination and include labor as a third strategic actor in the political bargain. The resulting policy would depend on the coalition formation between labor and the two capital-owning groups. For instance, labor in the brown sector might align with brown investors to resist high taxes to preserve jobs; if not properly compensated, laborers with green preference may side with brown investors due to high marginal cost of losing jobs. Alternatively, labor could be induced to support green policy if a sufficient portion of the carbon tax revenue is used to fund programs like targeted wage subsidies or job retraining. Analyzing these dynamics would provide deeper insights into the political coalitions that can either enable or block effective climate action.

7.2 Capital Controls and Financial Leakage

Our two-country model highlights a central challenge of unilateral climate policy: financial leakage. When capital markets are integrated, imposing a domestic carbon tax can drive investment abroad to jurisdictions with laxer environmental standards.

This capital flight not only undermines the policy's effectiveness—potentially increasing global emissions—but also weakens domestic political support for climate action, as the economic costs are borne locally while the environmental benefits are globally dispersed and uncertain.

This raises the question: if perfect international policy coordination is politically unattainable, could other tools help? This extension explores the role of capital controls as a second-best policy to mitigate financial leakage. We can think of capital controls as any friction—such as a tax on foreign investment returns or other regulatory barriers—that makes cross-border investment less attractive. In Appendix C.1, we include an attempt to extend our model to allow for cross-border investment restrictions, and derive some preliminary results.

By trapping capital within national borders, such controls can restore the potency of domestic environmental policy. If investors cannot easily move their funds to a brown-friendly foreign country, a domestic carbon tax becomes a more powerful lever for shifting capital from the domestic brown sector to the domestic green sector. This reduces the leakage channel, ensuring that a higher domestic tax leads to a more meaningful reduction in global emissions.

However, this approach introduces a trade-off. While capital controls can enhance the effectiveness of domestic climate policy, they do so by sacrificing the benefits of financial integration, such as efficient global capital allocation and risk-sharing. The analysis suggests that when international cooperation fails, financial regulations targeting capital flows may become a necessary, albeit costly, component of a country's climate toolkit.

8 Conclusion

This paper investigates the effectiveness of carbon taxation when environmental policy is determined through political bargaining in an economy with integrated financial markets. We develop a general equilibrium model where heterogeneous "green" and "brown" investors negotiate a carbon tax, which in turn shapes production, pollution, and asset prices. Our analysis reveals that the interaction between political compromise and financial market responses leads to fundamentally inefficient outcomes, challenging the notion that a simple carbon tax is a sufficient tool for addressing climate externalities.

In a single-country setting, we illustrates two key channels of inefficiency. First,

the carbon tax distorts labor allocation, leading to production inefficiency relative to a social planner's benchmark. Second, and more subtly, the political process suffers from an incomplete contracting problem: agents bargain over a single, blunt policy instrument (the tax rate τ) but cannot commit to subsequent investment allocations. This prevents the economy from achieving an optimal balance between environmental quality and risk-sharing, resulting in systematic under-investment in green technology compared to the Pareto frontier.

Furthermore, we show how uncertainty inherent in the political process translates into asset prices. When the realized carbon tax is uncertain, policy risk becomes a priced factor. The sector that is more exposed to this political risk will command a "political risk premium" in equilibrium to compensate investors, generating higher expected returns. This model thus provides a unified mechanism to rationalize climate premia and understand how environmental policy shapes the pricing of climate-related risks.

Extending the model to a two-country framework with integrated capital markets highlights a further source of inefficiency: financial leakage. A unilateral carbon tax in one country incentivizes capital to flow to the untaxed jurisdiction, partially or even fully offsetting the intended reduction in global emissions. This international spillover complicates the domestic political bargain, as the benefits of local policy are diluted while the costs of capital flight are borne locally.

Our findings suggest that effective climate policy may require a multi-instrument approach. The very financial channels that create leakage could also be part of the solution. Our preliminary analysis of capital controls indicates that segmenting financial markets can restore some degree of policy autonomy. Future research should further explore how other financial regulations or innovations might interact with the political process to mitigate these frictions and steer the economy toward more efficient environmental outcomes.

References

Acharya, V. V., Giglio, S., Pastore, S., Stroebel, J., Tan, Z., and Yong, T. (2025). Climate transition risks and the energy sector. Technical report, CESifo.

Battaglini, M. and Harstad, B. (2016). Participation and duration of environmental agreements. *Journal of Political Economy*, 124(1):160–204.

Battaglini, M. and Harstad, B. (2020). The political economy of weak treaties. *Journal of Political Economy*, 128(2):544–590.

Bolton, P. and Kacperczyk, M. (2021). Do investors care about carbon risk? *Journal of financial economics*, 142(2):517–549.

Cassidy Jr, W. M. (2023). Elections have consequences: The impact of political agency on climate policy and asset prices. Technical report.

Engle, R. F., Giglio, S., Kelly, B., Lee, H., and Stroebel, J. (2020). Hedging climate change news. *The Review of Financial Studies*, 33(3):1184–1216.

Flammer, C. (2021). Corporate green bonds. *Journal of financial economics*, 142(2):499–516.

Giglio, S., Kelly, B., and Stroebel, J. (2021). Climate finance. *Annual review of financial economics*, 13(1):15–36.

Goldstein, I., Kopytov, A., Shen, L., and Xiang, H. (2022). On esg investing: Heterogeneous preferences, information, and asset prices. Technical report, National Bureau of Economic Research.

Green, D. and Roth, B. N. (2025). The allocation of socially responsible capital. *The Journal of Finance*, 80(2):755–781.

Hambel, C. and Van Der Ploeg, F. (2025). Policy transition risk, carbon premiums, and asset prices. *Journal of Monetary Economics*, page 103780.

Heinkel, R., Kraus, A., and Zechner, J. (2001). The effect of green investment on corporate behavior. *Journal of financial and quantitative analysis*, 36(4):431–449.

Hsu, P.-H., Li, K., and Tsou, C.-Y. (2023). The pollution premium. *The Journal of Finance*, 78(3):1343–1392.

Nordhaus, W. (2015). Climate clubs: Overcoming free-riding in international climate policy. *American Economic Review*, 105(4):1339–1370.

Oates, W. E. and Portney, P. R. (2003). The political economy of environmental policy. In *Handbook of environmental economics*, volume 1, pages 325–354. Elsevier.

Pástor, L., Stambaugh, R. F., and Taylor, L. A. (2021). Sustainable investing in equilibrium. *Journal of financial economics*, 142(2):550–571.

Pástor, L., Stambaugh, R. F., and Taylor, L. A. (2022). Dissecting green returns. *Journal of financial economics*, 146(2):403–424.

Pedersen, L. H., Fitzgibbons, S., and Pomorski, L. (2021). Responsible investing: The esg-efficient frontier. *Journal of financial economics*, 142(2):572–597.

A Proofs

Proof for Theorem 1

Proof of Theorem 1. The proof proceeds in four steps. We first solve the firm's problem to determine optimal labor demand. Second, we characterize the statistical properties of asset returns. Third, we solve the investor's portfolio choice problem. Finally, we impose market clearing conditions to solve for the equilibrium risk-free rate and capital allocation.

To derive the expressions in the theorem, we assume the Cobb-Douglas production function has an exponent $\alpha = 1/2$.

Step 1: Firm's Problem and Optimal Labor

A representative firm in sector k chooses its labor input l_k to maximize expected profits, taking its capital stock S_k and the carbon tax τ as given. The firm's problem is:

$$\max_{l_k \geq 0} \quad \mathbb{E}\left[A_k S_k^{1/2} l_k^{1/2} - w_k l_k - \tau P_k S_k^{1/2} l_k^{1/2}\right]$$

Since the firm chooses labor before the productivity shock A_k is realized, it optimizes based on the expected productivity, $\bar{A}_k$:

$$\max_{l_k \geq 0} \quad (\bar{A}_k - \tau P_k) S_k^{1/2} l_k^{1/2} - w_k l_k$$

For an interior solution with positive production, we require the expected post-tax productivity to be positive, i.e., $\bar{A}_k - \tau P_k > 0$. The first-order condition with respect to l_k is:

$$\frac{1}{2}(\bar{A}_k - \tau P_k) S_k^{1/2} l_k^{-1/2} - w_k = 0$$

Solving for the optimal labor demand l_k yields:

$$l_k = \left(\frac{\bar{A}_k - \tau P_k}{2 w_k}\right)^2 S_k \tag{A.1}$$

This matches the expression for the labor allocation in the theorem.

Step 2: Asset Returns

The dividend (profit) π_k paid to shareholders is the firm's realized revenue net of labor costs and taxes: $\pi_k = (A_k - \tau P_k) S_k^{1/2} l_k^{1/2} - w_k l_k$. The equity return is

$r_k = \pi_k / S_k$, as the initial equity value is the invested capital S_k. Substituting the optimal l_k from (A.1):

$$\begin{aligned} r_k &= \frac{(A_k - \tau P_k) S_k^{1/2} \left(\frac{\bar{A}_k - \tau P_k}{2w_k} S_k^{1/2} \right) - w_k \left(\frac{\bar{A}_k - \tau P_k}{2w_k} \right)^2 S_k}{S_k} \\ &= (A_k - \tau P_k) \left(\frac{\bar{A}_k - \tau P_k}{2w_k} \right) - w_k \left(\frac{\bar{A}_k - \tau P_k}{2w_k} \right)^2 \end{aligned}$$

The expected return $\bar{r}_k = \mathbb{E}[r_k]$ is:

$$\begin{aligned} \bar{r}_k &= (\bar{A}_k - \tau P_k) \left(\frac{\bar{A}_k - \tau P_k}{2w_k} \right) - \frac{(\bar{A}_k - \tau P_k)^2}{4w_k} \\ &= \frac{2(\bar{A}_k - \tau P_k)^2 - (\bar{A}_k - \tau P_k)^2}{4w_k} = \frac{(\bar{A}_k - \tau P_k)^2}{4w_k} \end{aligned}$$

The variance of the return, $\mathrm{Var}(r_k) = \sigma_{r_k}^2$, depends only on the variance of the productivity shock A_k:

$$\mathrm{Var}(r_k) = \mathrm{Var}\left(A_k \left(\frac{\bar{A}_k - \tau P_k}{2w_k} \right) \right) = \left(\frac{\bar{A}_k - \tau P_k}{2w_k} \right)^2 \mathrm{Var}(A_k)$$

Let D be a diagonal matrix with diagonal elements $D_{kk} = \frac{\bar{A}_k - \tau P_k}{2w_k}$. The vector of returns can be written as $r = DA + \text{const}$, where $A = (A_g, A_b)'$. The covariance matrix of returns is therefore $\Sigma_r = D\Sigma_A D' = D\Sigma_A D$, since D is diagonal.

Step 3: Investor's Problem

Investor i chooses a portfolio θ_i to maximize their CARA utility. Since aggregate emissions E are determined by aggregate capital and are taken as given by the individual investor, the term $\eta_i(E)$ is a constant in the optimization. The problem (2.1) simplifies to maximizing the standard CARA objective over terminal wealth. With normally distributed returns, this is equivalent to the mean-variance objective:

$$\max_{\theta_i} \quad \mathbb{E}[c_i] - \frac{\gamma}{2} \mathrm{Var}(c_i)$$

where $\mathbb{E}[c_i] = [r_f + (\bar{r} - r_f \mathbf{1})' \theta_i] y_i + \frac{1}{2} \tau E$ and $\mathrm{Var}(c_i) = y_i^2 \theta_i' \Sigma_r \theta_i$. The first-order condition with respect to the portfolio vector θ_i is:

$$\frac{\partial}{\partial \theta_i} \left([r_f + (\bar{r} - r_f \mathbf{1})' \theta_i] y_i - \frac{\gamma}{2} y_i^2 \theta_i' \Sigma_r \theta_i \right) = y_i (\bar{r} - r_f \mathbf{1}) - \gamma y_i^2 \Sigma_r \theta_i = 0$$

Solving for θ_i gives the individual's optimal portfolio demand:

$$\theta_i = \frac{1}{\gamma y_i}\Sigma_r^{-1}(\bar{r} - r_f\mathbf{1})$$

Step 4: Market Clearing

We impose two market clearing conditions: for the risky assets (equity) and the risk-free asset (bonds). The market portfolio $\bar{\theta}$ is the wealth-weighted average of individual portfolios:

$$\bar{\theta} = \int_i \frac{y_i}{\bar{y}}\theta_i di = \int_i \frac{y_i}{\bar{y}}\left(\frac{1}{\gamma y_i}\Sigma_r^{-1}(\bar{r} - r_f\mathbf{1})\right) di = \frac{1}{\gamma\bar{y}}\Sigma_r^{-1}(\bar{r} - r_f\mathbf{1})$$

The market for the risk-free bond, which is in zero net supply, must clear. This implies that the aggregate holdings of the risky assets must sum to the total wealth, so $\mathbf{1}'\bar{\theta} = 1$. We use this to solve for the risk-free rate r_f:

$$\begin{aligned}
\mathbf{1}'\left(\frac{1}{\gamma\bar{y}}\Sigma_r^{-1}(\bar{r} - r_f\mathbf{1})\right) &= 1 \\
\mathbf{1}'\Sigma_r^{-1}\bar{r} - r_f(\mathbf{1}'\Sigma_r^{-1}\mathbf{1}) &= \gamma\bar{y} \\
r_f &= \frac{\mathbf{1}'\Sigma_r^{-1}\bar{r} - \gamma\bar{y}}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}}
\end{aligned}$$

Finally, the capital market for each sector must clear. The total capital invested in sector k, S_k, is equal to the total wealth allocated to that sector's equity, which is given by the market portfolio:

$$S_k = \bar{y}\bar{\theta}_k$$

This completes the characterization of the competitive equilibrium. □

Proof for Corollary 1

Proof. 1. **Asset Returns**:

- **Equity Returns**: We differentiate the expected return $\bar{r}_k = \frac{(\bar{A}_k - \tau P_k)^2}{4w_k}$ with respect to τ:

$$\frac{\partial\bar{r}_k}{\partial\tau} = \frac{2(\bar{A}_k - \tau P_k)(-P_k)}{4w_k} = -\frac{P_k(\bar{A}_k - \tau P_k)}{2w_k} < 0$$

The derivative is negative as firms are operational ($\bar{A}_k - \tau P_k > 0$). Since $P_b > P_g$, the magnitude of the decrease is larger for the brown sector.

- **Risk-Free Rate**: With independent shocks, Σ_r is diagonal. The risk-free rate is $r_f = \frac{C_1 - \gamma\bar{y}}{C_2(\tau)}$, where $C_1 = \mathbf{1}'\Sigma_r^{-1}\bar{r} = \sum_k \frac{w_k}{\sigma_k^2}$ is a constant independent of τ, and $C_2(\tau) = \mathbf{1}'\Sigma_r^{-1}\mathbf{1} = \sum_k \frac{1}{\sigma_{rk}^2} = \sum_k \frac{4w_k^2}{\sigma_k^2(\bar{A}_k - \tau P_k)^2}$. Differentiating the denominator $C_2(\tau)$ with respect to τ:

$$\frac{\partial C_2(\tau)}{\partial \tau} = \sum_k \frac{-2 \cdot 4w_k^2}{\sigma_k^2(\bar{A}_k - \tau P_k)^3}(-P_k) > 0$$

Since the numerator of r_f is constant and its denominator is strictly increasing in τ, the risk-free rate r_f must decrease.

2. **Capital Allocation**: The market portfolio weight for sector k is $\bar{\theta}_k = \frac{1}{\gamma\bar{y}} \frac{\bar{r}_k - r_f}{\sigma_{rk}^2}$. Since a higher tax makes the brown sector relatively less attractive than the green sector (i.e., its risk-adjusted return falls more), investors rebalance their portfolios away from the brown sector and towards the green sector. This leads to a decrease in the market portfolio share of the brown sector and an increase in that of the green sector:

$$\frac{\partial \bar{\theta}_b}{\partial \tau} < 0 \quad \text{and} \quad \frac{\partial \bar{\theta}_g}{\partial \tau} > 0$$

As capital allocation is given by $S_k = \bar{y}\bar{\theta}_k$, it follows directly that S_b decreases and S_g increases.

3. **Environmental Outcomes**: Total pollution is $E = \sum_k E_k = \sum_k S_k P_k \left(\frac{\bar{A}_k - \tau P_k}{2w_k}\right)$. We differentiate with respect to τ using the product rule:

$$\frac{dE}{d\tau} = \sum_{k \in \{g,b\}} \left(\frac{dS_k}{d\tau} \cdot P_k \frac{\bar{A}_k - \tau P_k}{2w_k} + S_k \cdot \frac{-P_k^2}{2w_k} \right)$$

This can be decomposed into two effects:

$$\frac{dE}{d\tau} = \underbrace{\left(\frac{dS_g}{d\tau}\pi_g - \left|\frac{dS_b}{d\tau}\right| \pi_b \right)}_{\text{Re-allocation Effect}} - \underbrace{\left(S_g \frac{P_g^2}{2w_g} + S_b \frac{P_b^2}{2w_b} \right)}_{\text{Intensity Effect}}$$

where $\pi_k = P_k(\bar{A}_k - \tau P_k)/(2w_k)$ is the pollution per unit of capital. The

intensity effect is unambiguously negative. The **re-allocation effect** is also negative because capital shifts from a high-pollution-intensity sector (π_b) to a low-intensity one (π_g). Since both effects are negative, total pollution $\frac{dE}{d\tau}$ is unambiguously negative.

4. **Household Welfare**: Welfare depends on consumption and the disutility from pollution. An investor's consumption is derived from their portfolio returns and the tax rebate: $c_i = [r_f + (r - r_f\mathbf{1})'\theta_i]y_i + \frac{1}{2}\tau E$. A higher tax τ leads to: (i) lower financial returns (as shown in part 1), which reduces consumption; (ii) a reduction in pollution E, which increases utility; and (iii) a change in the tax rebate $\frac{1}{2}\tau E$, the sign of which is uncertain. Due to these conflicting effects, the net impact on welfare is ambiguous and depends on the specific parameters and an investor's green preference η_i.

□

Proof for Theorem 2

Proof. The proof proceeds in four steps, same as the structure of the proof for Theorem 1.

Step 1: Firm's Problem

A firm in sector k with capital S_k chooses labor l_k to maximize expected profit, taking the distribution of the tax $\tilde{\tau}$ as given.

$$\max_{l_k \geq 0} \quad \mathbb{E}\left[A_k S_k^{1/2} l_k^{1/2} - w_k l_k - \tilde{\tau} P_k S_k^{1/2} l_k^{1/2} - \frac{\phi_k}{2} S_k^2\right]$$

The firm optimizes based on expected values. Let $\mathbb{E}[\tilde{\tau}] = \tau^*$.

$$\max_{l_k \geq 0} \quad (\bar{A}_k - \tau^* P_k) S_k^{1/2} l_k^{1/2} - w_k l_k - \frac{\phi_k}{2} S_k^2$$

The first-order condition with respect to l_k is identical to the baseline model:

$$\frac{1}{2}(\bar{A}_k - \tau^* P_k) S_k^{1/2} l_k^{-1/2} - w_k = 0 \quad \Longrightarrow \quad l_k^* = \left(\frac{\bar{A}_k - \tau^* P_k}{2w_k}\right)^2 S_k$$

Step 2: Asset Returns

The realized return on equity in sector k is $r_k = \pi_k / S_k$. Substituting the optimal

labor choice l_k^* into the profit function:

$$\begin{aligned} r_k &= \frac{1}{S_k}\left[(A_k - \tilde{\tau}P_k)S_k^{1/2} l_k^{*1/2} - w_k l_k^* - \frac{\phi_k}{2}S_k^2\right] \\ &= (A_k - \tilde{\tau}P_k)\left(\frac{\bar{A}_k - \tau^* P_k}{2w_k}\right) - w_k\left(\frac{\bar{A}_k - \tau^* P_k}{2w_k}\right)^2 - \frac{\phi_k}{2}S_k \end{aligned}$$

Let $D_k := \frac{\bar{A}_k - \tau^* P_k}{2w_k}$. The return can be written as $r_k = (A_k - \tilde{\tau}P_k)D_k - w_k D_k^2 - \frac{\phi_k}{2}S_k$. This is linear in the random variables A_k and $\tilde{\tau} = \tau^* + \epsilon_\tau$.

The expected return $\bar{r}_k = \mathbb{E}[r_k]$ is:

$$\begin{aligned} \bar{r}_k &= (\bar{A}_k - \tau^* P_k)D_k - w_k D_k^2 - \frac{\phi_k}{2}S_k \\ &= \frac{(\bar{A}_k - \tau^* P_k)^2}{2w_k} - \frac{(\bar{A}_k - \tau^* P_k)^2}{4w_k} - \frac{\phi_k}{2}S_k \\ &= \frac{(\bar{A}_k - \tau^* P_k)^2}{4w_k} - \frac{\phi_k}{2}S_k = \bar{R}_k - \frac{\phi_k}{2}S_k \end{aligned}$$

The variance of returns is derived from $r_k - \bar{r}_k = (A_k - \bar{A}_k)D_k - (\tilde{\tau} - \tau^*)P_k D_k = (A_k - \bar{A}_k)D_k - \epsilon_\tau P_k D_k$. Since A_k and ϵ_τ are independent:

$$\mathrm{Var}(r_k) = \mathbb{E}[(r_k - \bar{r}_k)^2] = D_k^2\sigma_{A_k}^2 + (P_k D_k)^2\sigma_\tau^2$$

The covariance between the returns of the green and brown sectors is:

$$\mathrm{Cov}(r_g, r_b) = \mathbb{E}[(r_g - \bar{r}_g)(r_b - \bar{r}_b)] = \mathbb{E}[(-\epsilon_\tau P_g D_g)(-\epsilon_\tau P_b D_b)] = (P_g D_g)(P_b D_b)\sigma_\tau^2$$

This defines the covariance matrix Σ_r.

Step 3: Investor's Problem

The investor's problem is standard CARA-Normal optimization, yielding portfolio demand:

$$\theta_i = \frac{1}{\gamma y_i}\Sigma_r^{-1}(\bar{r} - r_f\mathbf{1})$$

Step 4: Market Clearing

The market portfolio $\bar{\theta}$ is the wealth-weighted average of individual portfolios:

$$\bar{\theta} = \frac{1}{\gamma\bar{y}}\Sigma_r^{-1}(\bar{r} - r_f\mathbf{1})$$

Crucially, the vector of expected returns $\bar{r}$ depends on the capital allocation, $S_k = \bar{y}\bar{\theta}_k$. Let $\Phi = \text{diag}(\phi_k)$. In vector form, $\bar{r} = \bar{R} - \frac{\bar{y}}{2}\Phi\bar{\theta}$. Substituting this into the market portfolio equation:

$$\bar{\theta} = \frac{1}{\gamma\bar{y}}\Sigma_r^{-1}\left(\bar{R} - \frac{\bar{y}}{2}\Phi\bar{\theta} - r_f\mathbf{1}\right)$$
$$\gamma\bar{y}\bar{\theta} = \Sigma_r^{-1}\bar{R} - \frac{\bar{y}}{2}\Sigma_r^{-1}\Phi\bar{\theta} - r_f\Sigma_r^{-1}\mathbf{1}$$

Multiplying by Σ_r and rearranging to solve for $\bar{\theta}$:

$$\gamma\bar{y}\Sigma_r\bar{\theta} + \frac{\bar{y}}{2}\Phi\bar{\theta} = \bar{R} - r_f\mathbf{1}$$
$$\left(\gamma\bar{y}\Sigma_r + \frac{\bar{y}}{2}\Phi\right)\bar{\theta} = \bar{R} - r_f\mathbf{1}$$
$$\bar{\theta} = \left(\gamma\bar{y}\Sigma_r + \frac{\bar{y}}{2}\Phi\right)^{-1}(\bar{R} - r_f\mathbf{1})$$

This gives equation (5.1). To find the risk-free rate r_f, we impose the bond market clearing condition, $\mathbf{1}'\bar{\theta} = 1$:

$$1 = \mathbf{1}'\left(\gamma\bar{y}\Sigma_r + \frac{\bar{y}}{2}\Phi\right)^{-1}(\bar{R} - r_f\mathbf{1})$$
$$1 = \mathbf{1}'\left(\gamma\bar{y}\Sigma_r + \frac{\bar{y}}{2}\Phi\right)^{-1}\bar{R} - r_f\left[\mathbf{1}'\left(\gamma\bar{y}\Sigma_r + \frac{\bar{y}}{2}\Phi\right)^{-1}\mathbf{1}\right]$$

Solving for r_f yields equation (5.2). This completes the characterization. □

Proof Outline for Theorem 2. The results are derived by differentiating the equilibrium conditions in Theorem 1 with respect to τ_j.

1. **Direct Effect on Returns:** We first find the direct effect of the tax on the expected returns of firms in country j. Differentiating $\bar{r}_{kj}$ with respect to τ_j:

$$\frac{\partial\bar{r}_{kj}}{\partial\tau_j} = \frac{\partial}{\partial\tau_j}\left(\frac{(\bar{A}_{kj} - \tau_j P_{kj})^2}{4w_{kj}}\right) = \frac{2(\bar{A}_{kj} - \tau_j P_{kj})(-P_{kj})}{4w_{kj}} < 0$$

   The derivative is negative given the assumption that firms are operating ($\bar{A}_{kj} - \tau_j P_{kj} > 0$). The direct effect on returns in country j' is zero.

2. **Capital Allocation and Leakage:** The market portfolio is $\bar{\theta} = \frac{1}{\gamma\bar{y}}\Sigma_r^{-1}(\bar{r} - r_f\vec{1})$. A change in τ_j lowers elements of $\bar{r}$ and changes the variance-covariance

matrix Σ_r. In the new equilibrium, investors rebalance away from the less attractive assets in country j and towards assets in country j'. This implies a change in the market portfolio shares such that:

$$\frac{\partial \bar{\theta}_{kj}}{\partial \tau_j} < 0 \quad \text{and} \quad \frac{\partial \bar{\theta}_{kj'}}{\partial \tau_j} > 0.$$

Since capital allocation is $S_{kj} = \bar{y}\bar{\theta}_{kj}$, capital flows out of country j and into country j', proving financial leakage.

3. **Pollution and Environmental Leakage:** Pollution produced in country j is $E_j = \sum_k h_{kj}(S_{kj}, l_{kj})$. Since both capital S_{kj} and the labor-to-capital ratio, $(l/S)_{kj} = (\frac{\bar{A}_{kj} - \tau_j P_{kj}}{2w_{kj}})^2$, decrease for firms in country j, pollution E_j must fall. Conversely, since capital $S_{kj'}$ increases in country j' while its policy parameters are fixed, pollution $E_{j'}$ must rise. The change in total pollution is:

$$\frac{\partial E}{\partial \tau_j} = \underbrace{\frac{\partial E_j}{\partial \tau_j}}_{<0} + \underbrace{\frac{\partial E_{j'}}{\partial \tau_j}}_{>0}.$$

The sign of the net effect is therefore ambiguous.

4. **Household Welfare:** Household welfare depends on consumption c_{ij} and disutility from pollution $\eta_{ij}(E)$. For a household in the taxing country j, consumption is $c_{ij} = (\text{portfolio income}) + \tau_j E_j$. The effect of the tax increase is ambiguous because: (i) portfolio income is likely to fall; (ii) the change in the tax rebate, $\frac{\partial(\tau_j E_j)}{\partial \tau_j} = E_j + \tau_j \frac{\partial E_j}{\partial \tau_j}$, is of uncertain sign (a Laffer curve effect); and (iii) the change in utility from pollution depends on the ambiguous sign of $\frac{\partial E}{\partial \tau_j}$. Similar ambiguities apply to households in country j'.

Proof Sketch for Proposition 2.

Proof. The proof proceeds by analyzing the derivatives of the equilibrium quantities with respect to σ_τ^2. Let $M(\sigma_\tau^2) = \gamma \bar{y} \Sigma_r + \frac{\bar{y}}{2}\Phi$.

1. **Effect on the Risk-Free Rate:** From equation (5.2), the risk-free rate is $r_f = (\mathbf{1}' M^{-1} \bar{R} - 1)/(\mathbf{1}' M^{-1} \mathbf{1})$. To find the sign of $dr_f/d\sigma_\tau^2$, we first analyze how M^{-1} changes. Using the matrix derivative formula, $\frac{dM^{-1}}{d\sigma_\tau^2} =$

$-M^{-1}\frac{dM}{d\sigma_\tau^2}M^{-1}$. Since $\frac{dM}{d\sigma_\tau^2} = \gamma\bar{y}\Sigma_P = \gamma\bar{y}v_P v_P'$, which is a positive semi-definite matrix, the derivative $\frac{dM^{-1}}{d\sigma_\tau^2}$ is negative semi-definite. This means that for any vector x, $x'\frac{dM^{-1}}{d\sigma_\tau^2}x \leq 0$.

Intuitively, an increase in risk makes risky assets less desirable. To clear the bond market (which is in zero net supply), the return on the safe asset must fall to encourage investors to hold the existing stock of risky assets. Formally, $(-1,1)'v_P v_P' M^{-1}1 \geq 0$ implies $\frac{dr_f}{d\sigma_\tau^2} < 0$.[5]

2. **Effect on Capital Allocation:** The market portfolio is given by $\bar{\theta} = M^{-1}(\bar{R} - r_f\mathbf{1})$. We analyze its total derivative:

$$\frac{d\bar{\theta}}{d\sigma_\tau^2} = \frac{\partial M^{-1}}{\partial\sigma_\tau^2}(\bar{R} - r_f\mathbf{1}) - M^{-1}\mathbf{1}\frac{dr_f}{d\sigma_\tau^2}$$

The first term is a substitution effect, and the second is a wealth effect. The substitution effect is $\frac{\partial M^{-1}}{\partial\sigma_\tau^2}(\bar{R} - r_f\mathbf{1}) = -M^{-1}(\gamma\bar{y}v_P v_P')M^{-1}(\bar{R} - r_f\mathbf{1})$. Since $\bar{R} - r_f\mathbf{1}$ is the vector of risk premia (which is positive), this term is negative. It captures investors shifting away from assets whose risk has increased. The term $v_P = (P_g D_g, P_b D_b)'$ has a larger second element since $P_b > P_g$ and we assumed symmetry between two sectors in production technology. The political risk shock therefore loads more heavily on the brown sector. Consequently, the substitution effect is stronger for the brown sector, pushing its allocation down more significantly than the green sector's. The wealth effect from the falling risk-free rate ($-\frac{dr_f}{d\sigma_\tau^2} > 0$) pushes up demand for both risky assets. However, the stronger negative substitution effect on the brown sector dominates, leading to a net decrease in its allocation: $\frac{d\bar{\theta}_b}{d\sigma_\tau^2} < 0$, which implies $\frac{dS_b}{d\sigma_\tau^2} < 0$. The reallocation towards the relatively safer green asset causes the ratio S_g/S_b to increase.

3. **Effect on Expected Returns (The Political Risk Premium):** The equilibrium expected return is $\bar{r}_k = \bar{R}_k - \frac{\phi_k}{2}S_k = \bar{R}_k - \frac{\phi_k\bar{y}}{2}\bar{\theta}_k$. Its derivative is:

$$\frac{d\bar{r}_k}{d\sigma_\tau^2} = -\frac{\phi_k\bar{y}}{2}\frac{d\bar{\theta}_k}{d\sigma_\tau^2}$$

Since we established that $\frac{d\bar{\theta}_b}{d\sigma_\tau^2} < 0$, it follows directly that $\frac{d\bar{r}_b}{d\sigma_\tau^2} > 0$. The

[5]This is a sufficient but not necessary condition for risk-free rate to be decreasing. We are working on relaxing the assumption to more general parameter ranges.

expected return on brown equity must rise to compensate investors for the increased political risk they are required to bear. The risk premium for the brown sector is $RP_b = \bar{r}_b - r_f$. Its derivative is:

$$\frac{d(RP_b)}{d\sigma_\tau^2} = \frac{d\bar{r}_b}{d\sigma_\tau^2} - \frac{dr_f}{d\sigma_\tau^2}$$

Since $\frac{d\bar{r}_b}{d\sigma_\tau^2} > 0$ and $\frac{dr_f}{d\sigma_\tau^2} < 0$, the change in the brown sector's risk premium is unambiguously positive. The greater the political risk, the larger the compensation required to hold brown assets.

□

Proof for Corollary 3. With symmetric i.i.d. shocks, the variance-covariance matrix Σ_r becomes diagonal. The expression for the risk-free rate in Theorem 1 simplifies significantly. The term $\mathbf{1}'\Sigma_r^{-1}\bar{\mathbf{r}}$ becomes a constant independent of taxes, and the denominator $\mathbf{1}'\Sigma_r^{-1}\mathbf{1}$ becomes proportional to $\sum_{k,j} \frac{1}{(\bar{A}-\tau_j P_k)^2}$.

$$r_f = \frac{4 - \gamma\bar{y}\sigma_A^2}{4w^2\left(\sum_k \frac{1}{(\bar{A}-\tau_j P_k)^2} + \sum_k \frac{1}{(\bar{A}-\tau_{j'} P_k)^2}\right)}$$

When country j increases its tax τ_j, the term $(\bar{A} - \tau_j P_k)^2$ decreases for its firms. This increases the corresponding terms in the summation in the denominator. As the denominator of the expression for r_f increases, the risk-free rate r_f must fall. □

Proof for Corollary 4. The total change in global pollution is the sum of changes in country j (the taxing country) and country j' (the non-taxing country), $dE/d\tau_j = dE_j/d\tau_j + dE_{j'}/d\tau_j$. We find each component by differentiating $E_m = \sum_k S_{km}\pi_{km}$ for $m \in \{j, j'\}$.

For the taxing country j, applying the product rule yields:

$$\frac{dE_j}{d\tau_j} = \sum_k \left(\frac{dS_{kj}}{d\tau_j}\pi_{kj} + S_{kj}\frac{d\pi_{kj}}{d\tau_j}\right)$$

For the non-taxing country j', whose policy is unchanged, the pollution intensity

$\pi_{kj'}$ is constant with respect to τ_j, so $\frac{d\pi_{kj'}}{d\tau_j} = 0$. This gives:

$$\frac{dE_{j'}}{d\tau_j} = \sum_k \frac{dS_{kj'}}{d\tau_j}\pi_{kj'}$$

The intensity of production in country j does change with the tax: $\frac{d\pi_{kj}}{d\tau_j} = -\frac{P_k^2}{2w}$. The condition for global pollution to fall, $dE/d\tau_j < 0$, is therefore:

$$\left(\sum_k \frac{dS_{kj}}{d\tau_j}\pi_{kj} - \sum_k S_{kj}\frac{P_k^2}{2w}\right) + \left(\sum_k \frac{dS_{kj'}}{d\tau_j}\pi_{kj'}\right) < 0$$

Rearranging this inequality to place the unambiguously positive intensity effect on the left-hand side yields the condition in the corollary. The left-hand side captures the reduction in pollution from making all inframarginal domestic production cleaner. The right-hand side captures the net pollution change from capital flowing out of country j and into country j'. Global pollution falls only when the former dominates the latter. □

Proof idea for Corollary 5. A household's utility is maximized by maximizing their certainty-equivalent wealth $W_{im}(\tau_j) = \mathbb{E}[c_{im}] - \frac{\gamma}{2}\text{Var}(c_{im}) - \frac{1}{\gamma}\eta_i(E)$, where $m \in \{j, j'\}$ is the household's country of residence. We analyze its derivative with respect to τ_j.

Let $c_{fin} = (r_f + (r - r_f\vec{1}) \cdot \bar{\theta})\bar{y}$ be the consumption derived from financial portfolio holdings, which is identical for all agents under the symmetric endowment assumption.

Define *Marginal Financial Cost* be $MFC(\tau_j) := -\frac{d}{d\tau_j}\left(\mathbb{E}[c_{fin}] - \frac{\gamma}{2}\text{Var}(c_{fin})\right)$. We assume $MFC > 0$ as the tax creates financial market distortions.

Let the *Marginal Private Effect* of the tax be the sum of its impact on financial wealth and the tax rebate: $MPE(\tau_j) := -MFC(\tau_j) + \frac{d}{d\tau_j}\left(\mathbb{E}[\lambda\tau_j E_j] - \frac{\gamma}{2}\text{Var}(\lambda\tau_j E_j)\right)$. This term applies to domestic investors whose tax rebate is also affected by the tax policy change, and is identical for green and brown domestic investors.

Let *Marginal Environmental Benefit* for type i be $MEB_i(\tau_j) := -\frac{1}{\gamma}\eta_i'(E)\frac{dE}{d\tau_j}$. Since $\frac{dE}{d\tau_j} < 0$ and $\eta_g > \eta_b > 0$, we have $MEB_g(\tau_j) > MEB_b(\tau_j) > 0$.

1. **Domestic Investors:** For a household in country j, the total welfare change is $\frac{dW_{ij}}{d\tau_j} = MPE(\tau_j) + MEB_i(\tau_j)$.

Assuming convexity, the optimal tax τ_i^* is found by the first-order condition $\frac{dW_{ij}}{d\tau_j} = 0$, which implies:

$$-MPE(\tau_i^*) = MEB_i(\tau_i^*)$$

Since MPE is the same for both types of households, but $MEB_g > MEB_b$, the solution must occur at a higher tax rate for the green investor, thus $\tau_g^* > \tau_b^*$.

2. **Zero-Tax Preference:** The optimal tax for a domestic brown investor, τ_b^*, is zero if their utility is decreasing in the tax rate at the origin. This occurs when $\frac{dW_{bj}}{d\tau_j}|_{\tau_j=0} < 0$, which, written in terms of our defined components, is the condition $MPE(0) + MEB_b(0) < 0$.

3. **Foreign Investors:** For a household in country j', consumption is $c_{ij'} = c_{fin}$. The change in their welfare is $\frac{dW_{ij'}}{d\tau_j} = -MFC(\tau_j) + MEB_i(\tau_j)$. The welfare of the green investor rises while the brown investor's falls if $MFC(\tau_j) + MEB_g(\tau_j) > 0$ and $MFC(\tau_j) + MEB_b(\tau_j) < 0$. Rearranging gives the condition $MEB_g(\tau_j) > MFC(\tau_j) > MEB_b(\tau_j)$.

B Simulation Parameters

Table 1: Simulation Parameters

Parameter	**Symbol**	**Value**
A. Household Preferences		
Risk Aversion	γ	3.0
Green Disutility from Pollution	α_g	0.8
Brown Disutility from Pollution	α_b	0.2
B. Production Technology		
Mean Productivity (Green, Brown)	$\bar{A}_k$	(1.5, 1.5)
Productivity Volatility (Green, Brown)	σ_k	(0.2, 0.2)
Pollution Intensity (Green, Brown)	P_k	(0.1, 1.0)
Sectoral Wage (Green, Brown)	w_k	(0.5, 0.5)
C. Economy-wide Parameters		
Aggregate Endowment	$\bar{y}$	5.0

C Technical Appendix

C.1 Model with Capital Controls

Our two-country model highlights how financial integration can undermine unilateral climate policy through a financial leakage channel. Here we introduce an extension that models capital controls via a cross-border investment friction δ.

C.1.1 Setup: Introduce Cross-Border Investment Friction

We depart from the baseline model by introducing a friction, $\delta \in [0, 1]$, on returns from foreign assets. An investor residing in country j who invests in a firm domiciled in country $-j$ receives only a fraction $(1-\delta)$ of the realized return. A value of $\delta = 0$ corresponds to our baseline model with perfect financial integration, while $\delta = 1$ represents complete financial autarky. This friction can be interpreted as a capital control tax, transaction costs, or other barriers that create a wedge between domestic and foreign investment returns. We assume the friction results in deadweight loss to abstract away from redistributional effects and isolate the effect of the friction itself.

The immediate consequence of this friction is the segmentation of capital markets. Investors in different countries now face distinct vectors of effective asset returns. For an investor in country j, the vector of expected returns from the four global equities is no longer $\bar{\mathbf{r}}$, but a country-specific vector $\bar{\mathbf{r}}_j$.

This segmentation implies we need two risk-free bonds, one for each country, in zero net supply within that country. Their respective risk-free rates, r_{f1} and r_{f2}, are determined by clearing the bond market within each country's borders.

The household problem is modified accordingly. For a household i in country j with endowment y_{ij}, the optimal portfolio holding of risky assets, θ_{ij}, is now:

$$\theta_{ij} = \frac{1}{\gamma\, y_{ij}} \Sigma_r^{-1} \left(\bar{\mathbf{r}}_j - r_{fj}\, \mathbf{1} \right)$$

where $\bar{\mathbf{r}}_j$ is the vector of expected returns faced by the investor in country j, adjusted for the cross-border friction. For instance, for an investor in country 1, $\bar{\mathbf{r}}_1 = (\bar{r}_{g1}, \bar{r}_{b1}, (1-\delta)\bar{r}_{g2}, (1-\delta)\bar{r}_{b2})'$.

With segmented markets, each risk-free rate is set to ensure that the aggregate demand for the local risk-free bond by that country's own investors is zero. This yields a country-specific risk-free rate r_{fj} that depends on the average endowment

of its own citizens, $\bar{y}_j := \int_i y_{ij} di$:

$$r_{fj} = \frac{\mathbf{1}'\Sigma_r^{-1}\bar{\mathbf{r}}_j - \gamma\,\bar{y}_j}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}}$$

This structure introduces a form of "home bias" into portfolios and ensures that shocks and policy changes in one country are no longer transmitted perfectly through a single global asset price, r_f.

C.1.2 Market Equilibrium Comparative Statics

A key observation, which simplifies the subsequent analysis, is that under a set of plausible symmetry conditions, the segmentation of capital markets does not lead to different prices for risk-free capital.

Lemma 1 (Equivalence of Risk-Free Rates)**.** *Under the capital control friction $\delta > 0$, if (i) productivity shocks are uncorrelated across sectors (Σ_A is diagonal), (ii) sectoral parameters for wages and productivity variance are symmetric across countries (i.e., for each sector $k \in \{g, b\}$, $w_{k1} = w_{k2}$ and $\sigma^2_{k1} = \sigma^2_{k2}$), and (iii) average endowments are symmetric ($\bar{y}_1 = \bar{y}_2$), then the country-specific risk-free rates are identical,*

$$r_{f1} = r_{f2}$$

This result holds for any combination of tax policies (τ_1, τ_2) and any level of capital control $\delta \in (0, 1]$.

Proof. The risk-free rates are equal if and only if the numerators of their respective expressions are equal, given the common denominator. The condition $r_{f1} = r_{f2}$ is thus equivalent to:

$$\mathbf{1}'\Sigma_r^{-1}\bar{\mathbf{r}}_1 - \gamma\,\bar{y}_1 = \mathbf{1}'\Sigma_r^{-1}\bar{\mathbf{r}}_2 - \gamma\,\bar{y}_2$$

Using the assumption of symmetric endowments ($\bar{y}_1 = \bar{y}_2$), this simplifies to:

$$\mathbf{1}'\Sigma_r^{-1}(\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2) = 0 \tag{C.1}$$

The difference in the effective expected return vectors is $\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2 = \delta \cdot (\bar{r}_{g1}, \bar{r}_{b1}, -\bar{r}_{g2}, -\bar{r}_{b2})'$. Substituting this into (C.1), and noting $\delta \neq 0$, the condition becomes:

$$\mathbf{1}'\Sigma_r^{-1}(\bar{r}_{g1}, \bar{r}_{b1}, -\bar{r}_{g2}, -\bar{r}_{b2})' = 0 \tag{C.2}$$

With uncorrelated productivity shocks (Σ_A is diagonal), the return covariance matrix $\Sigma_r = D\Sigma_A D$ is also diagonal, with entries $\text{Var}(r_{kj}) = \sigma_{kj}^2 d_{kj}^2$, where $d_{kj} = (\bar{A}_{kj} - \tau_j P_{kj})/(2w_{kj})$. The expression (C.2) expands to:

$$\frac{\bar{r}_{g1}}{\text{Var}(r_{g1})} + \frac{\bar{r}_{b1}}{\text{Var}(r_{b1})} - \frac{\bar{r}_{g2}}{\text{Var}(r_{g2})} - \frac{\bar{r}_{b2}}{\text{Var}(r_{b2})} = 0$$

A key simplification arises from the model's structure. For any sector k, j:

$$\frac{\bar{r}_{kj}}{\text{Var}(r_{kj})} = \frac{w_{kj} d_{kj}^2}{\sigma_{kj}^2 d_{kj}^2} = \frac{w_{kj}}{\sigma_{kj}^2}$$

This ratio, which can be interpreted as a policy-invariant measure of a sector's risk-return potential, is independent of the tax rate τ_j. The condition for equal risk-free rates thus reduces to a condition solely on these primitive parameters:

$$\left(\frac{w_{g1}}{\sigma_{g1}^2} + \frac{w_{b1}}{\sigma_{b1}^2}\right) - \left(\frac{w_{g2}}{\sigma_{g2}^2} + \frac{w_{b2}}{\sigma_{b2}^2}\right) = 0$$

This equation is satisfied if the sectoral parameters are symmetric across countries, i.e., $w_{k1} = w_{k2}$ and $\sigma_{k1}^2 = \sigma_{k2}^2$ for $k \in \{g, b\}$. As these conditions hold by assumption, the equality $r_{f1} = r_{f2}$ is established for any tax rates. □